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Isoparametric elements

Summary:

This document presents the bases of the isoparametric elements introduced into *Code_Aster* for modeling of the continuous mediums 2D and 3D. One first of all recalls the passage of a strong formulation to one

variational formulation, then one details the discretization by finite elements: use of an element of reference, calculation of the functions of form and evaluation of the elementary terms. One also briefly describes

the principle of the assembly of these terms and the imposition of the boundary conditions, and they are evoked

methods of matric resolution used. Finally the principal stages of a calculation by elements are exposed stop such as it is conceived and established in *Code_Aster*.

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1

Principle of the finite element method

The finite element method is employed in many scientific fields to solve partial derivative equations. It makes it possible to build a simple approximation of unknown factors to transform these continuous equations into a system of equations of finished size,

that one can write schematically in the form $\mathbf{A}\mathbf{U} = \mathbf{L}$, where \mathbf{U} is the vector of the unknown factors, \mathbf{A} a matrix and \mathbf{L} a vector.

Initially, one transforms the partial derivative equations (or strong formulation of problem) in a variational formulation (or weak formulation). The approximate solution is sought like linear combination of functions given. These functions must be simple but enough general to be able “well” to approach the solution. They must in particular make it possible to generate one

space of finished size which is as close as one wants space of functions in which find the solution. From this old idea (method of the balanced residues), various ways to choose these functions give place to various numerical methods (collocation, methods spectral, finite elements).

The originality of the finite element method is to take as functions of approximation of polynomials which are null on almost all the field, and thus take part in calculation only with particular point neighborhood. Thus, matrix \mathbf{A} is very hollow, containing only the terms of interaction between “close points”, which reduces the computing time and the place memory necessary to

storage. Moreover, matrix \mathbf{A} and the vector \mathbf{L} can be built by assembly of matrices and elementary vectors, calculated locally.

2

Obtaining a variational formulation

One can obtain the variational formulation of a problem starting from the equations with the derivative partial, by multiplying those by functions tests and while integrating by parts. In mechanics of solids, the weak formulation then obtained is identical to that given by the Principle of Work Virtual or in certain cases the minimization of the total potential energy of the structure. Let us note however that for certain problems, the equations of the model are easier to establish in tally variational (case of the plates and hulls for example).

2.1 Formulation

strong

Let us take an example resulting from the mechanics of the solids. Local equations of static balance of one

structure, subjected to forces of volume \mathbf{F} , the displacements imposed \mathbf{u}_D on part of its border D and of the forces imposed \mathbf{G} on a part NR of its border, are written:

div

$$+\mathbf{F} = 0$$

in

$$\mathbf{U} = \mathbf{u}_D \text{ on } D$$

.n =

\mathbf{G} on NR

where is the tensor of the constraints and \mathbf{N} the outgoing normal on the border. The relations which bind the tensor of the constraints to displacements \mathbf{U} .S. 'call relations of behavior.

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The boundary conditions applying to the primal unknown factors (displacements here) are called boundary conditions of DIRICHLET, or “essential”. Boundary conditions relating to the forces (or flows in thermics) are called boundary conditions of NEUMANN, or “natural”.

2.2 Formulation**weak****2.2.1 Functions****tests**

That is to say a space V of functions called functions tests, “sufficiently” regular and cancelling itself on D . In mechanics kinematically, this space is called the space of virtual displacements acceptable. By multiplying the local equations by a function test \mathbf{v} pertaining to space V and in integrating on the field, one obtains a variational form of the problem, rigorously equivalent to the form preceding, known as so operational:

$$V D = \left\{ \begin{array}{l} D \\ \mathbf{v} \text{ “regular”, } \mathbf{v} = \mathbf{U} \text{ on } D \\ \end{array} \right\}$$

$$V = \left\{ \mathbf{v} \text{ “regular”, } \mathbf{v} = 0 \text{ S} \right.$$

$$\text{ur } D$$

$$\left. \right\}$$

$$\mathbf{v}$$

$$V: (\text{div} + \mathbf{F}) \mathbf{v} D = 0$$

 D To find $\mathbf{U} V$

such as

:

éq 2.2.1-1

$\mathbf{n} = \mathbf{G}$ on

NR

In what follows, it will be supposed to simplify that the conditions of DIRICHLET are homogeneous, i.e. $\mathbf{u}\mathbf{D} = 0$; thus, spaces $V D$ and V are confused. Treatment of the conditions to the limits of nonhomogeneous DIRICHLET is exposed in the document [R3.03.01].

Note:

One will not discuss in this document functional spaces to which must belong the functions tests (cf [bib1]).

2.2.2 Formulate of GREEN

The analogue of integration by parts for an unspecified field of border is called formulate of GREEN and states itself as follows, in its simplest form:

$id =$
 $in D,$

where I indicates the derivative compared to direction I , and \mathbf{N} the outgoing normal with the field.

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When $= \mathbf{Q}$, where \mathbf{Q} is a vector and a scalar, it is expressed in the following way:

$I iq D = iq in D - I Q$

$D,$ I

maybe, by using vectorial notations (it. indicate the scalar product):

$\text{div}(\mathbf{Q}) D + \text{will gra}$

$D \cdot \mathbf{Q}$

$D =$

(\mathbf{q}, \mathbf{n})

 D

2.2.3 Formulation

variational

By applying the formula of GREEN to the integral [éq 2.2.1-1] and by taking account of the condition with

limits of NEUMANN

$ij n J = I$

G on NR and of the condition $\mathbf{v} = 0$ on D , one obtains the form known as variational of the problem:

To find $\mathbf{U} V D$ such as:

\mathbf{v}

V

$v D$

$ij J I =$

$F v D$

$I I +$

$G v D$

$I I$

NR

As the tensor of the constraints is symmetrical, the equation can be also written:

$ij (\mathbf{U}) ij (\mathbf{v}) D =$

$fi iv D +$

*I**G iv D**NR*

where

1

$$ij(\mathbf{v}) = (I v_j + j i v)$$

2

is the tensor of the linearized deformations. One thus finds exactly the expression given by Principle of Virtual Work in small displacements. The relation between the tensor of the constraints of CAUCHY and displacements \mathbf{U} will be given by a relation of behavior, and are independent of the writing of the variational formulation (in the elastic case, one has for example $ij(\mathbf{U}) = ijklkl(\mathbf{U})$).

One of the advantages of the variational formulation is that it integrates all the boundary conditions: the conditions of DIRICHLET are taken into account in the definition of space V of the functions tests, while the conditions of NEUMANN appear naturally after integration by parts.

This integration by parts also makes it possible to lower the order of derivation on the unknown factors.

Of

more, in a certain number of cases, it symmetrizes the problem out of \mathbf{U} and \mathbf{v} .

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Two things remain to be made lorque one wrote the variational formulation (or the Principle of Work Virtual): to take into account the relation of behavior and to set up the algorithmic one resolution. For this last point, let us give some examples: writing of an algorithm of resolution

of nonlinear system (method of NEWTON for example) for the nonlinear problems, writing of a diagram of integration in time for the problems of evolution in dynamics (method of NEWMARK for example)... Consequently, the majority of the variational problems are brought back to find

U, V, D such as:

$\mathbf{v} \in V$, has $(\mathbf{U}, \mathbf{v}) = L(\mathbf{v})$,

where $has(\cdot, \cdot)$ is a bilinear, symmetrical form or not, and $L(\cdot)$ a linear form. If

bilinear form is symmetrical and positive, the problem arising is equivalent to a problem of minimization of a functional calculus, which in static mechanics of the solids is total potential energy structure.

2.3

Method of resolution

In this document, one presents only the method finite elements in displacements, where unknown factors are, as its name indicates it, the variables known as primal (displacements in mechanics),

in opposition to the methods in constraints, or the mixed methods. Space V is represented by a discrete space V_h . For the methods finite elements in conformity to which us

let us restrict here, space V_h is included in V : the approximate solution is thus "more rigid" than exact solution (it over-estimates energy).

It is pointed out that one places oneself here if the boundary conditions of DIRICHLET are homogeneous. In addition, one confines oneself with the finite elements of LAGRANGE for which variables are the values of the unknown fields.

For the finite element method of GALERKIN described in this document, the unknown factors and their functions tests are represented in the same way, by defining a base of functions $\{w_i(\mathbf{X})\}$

) of

space V_h .

One calls nodes the points of the field where the unknown factors are calculated, and variable nodal or degrees of freedom scalar unknown factors to the nodes (components of displacement for example).

a many basic functions necessary are equal to the number of nodal variables: for a problem three-dimensional where the unknown factors are the three components of the vector displacement, the dimension of

the base is three times the number of nodes.

One will use indices I, J, \dots to indicate the numbers of the nodes (NR on the whole), and the indices \dots to indicate the numbers of the unknown factors (M unknown per node). Thus, the vector displacement discretized V_h is written: $\mathbf{u}_h(\mathbf{X})$

H

) = $U(\mathbf{X}) \mathbf{E}$

, where the vectors \mathbf{E} are the vectors of

base Cartesian. **Thereafter, one will omit the index H of the writings.**

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The basic function associated node I for the unknown factor number will be noted $wI(\mathbf{X})$. In this base $\{wI(\mathbf{X})\}$

) I , the unknown field is written:

1

 M NR

$$U(\mathbf{X}) = uI W$$

 $I(\mathbf{X})$. $I=1$

where uI is the nodal variables.

Thus, the problem amounts finding $U V D$ such as:

 $NR M$

$\mathbf{v} V$, has $(W \mathbf{E}, \mathbf{v}) uI = L(\mathbf{v}$

 I

)

.

 $I=1=1$

Each choice of \mathbf{v} makes it possible to obtain an equation. In the finite element method, one takes like function test \mathbf{v} successively each function W

 D $I(\mathbf{v} V$ $= V)$. Thanks to

linearity of $A(\cdot, \cdot)$ and $L(\cdot)$, one can write the discrete problem like:

 $NR M$

J , has $(W \mathbf{E}, W \mathbf{E}) uI$

= $L (W$
 I
 J
 $J \mathbf{E}),$

$I=1 =1$

from where the matrix system to solve: $\mathbf{WITH} = \mathbf{L},$

with:

With

= *has* $(W \mathbf{E}, W \mathbf{E}), L$

1

1

= $L (W$

$\mathbf{E}),$ and \mathbf{U}

=

$\{U... U \dots uI$
 $\dots UI... UI \dots U NR... U NR T$

$J I$

I

J

J

J

1

M

1

M

1

$M\},$

where NR is the number of nodes and M the number of scalar unknown factors per node (3 for displacements in 3D).

In fact, one “condenses” the indices two by two: each new index I contains at the same time information on the number of node I and the local number of the unknown factor (the index condensed I is called its number of degree of freedom). A A_{eij} term of the matrix thus contains information on the interaction between the degrees of freedom I and J (for example, I represents displacement according to y with node

12 and J it

displacement according to Z with the node 23).

In many cases, the basic functions used for the various unknown factors in a given node are the same ones: W

= W

I

I . One calls then the common basic function the function associated with node I , and it is noted wI . Subsequently, it will be supposed to simplify the writings that there is not that only one scalar basic function associated each node.

N.B.:

Let us note that certain authors, of Anglo-Saxon culture for the majority, describe for reasons histories finite element method like a method of RITZ per pieces.

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3 Discretization

The discretization consists in choosing a base of space $V H$ and calculating the terms numerically matrix \mathbf{A} and vector \mathbf{L} . For that, one expresses the bilinear form *has* (\cdot, \cdot) and the linear form $L(\cdot)$ like a sum on elements, defined by basic field division. If one begins again the mechanical problem presented in the paragraph [§1.2], that gives:

has

$$(W, W) = (W) (W)$$

I

J

$kl I kl J$)

elements

E

*E**L* $(W) =$ $F W +$ $G W$ *I**IIII**elements**E**E**E*

The A_{ij} terms, which represent the interaction between two degrees of freedom I and J are built in “assembling” the contributions coming from each element which contains the nodes correspondents; one proceeds in the same way to build the vector \mathbf{L} . These contributions, called elementary terms, are calculated at the time of a loop on the elements and only depend only variables of the element E .

3.1

Cutting in finite elements

The structure is cut out of “pieces” called elements. The data of the co-ordinates of the nodes elements and connectivities (list of the tops of each element) constitutes a grid.

cutting must respect a certain number of rules: in particular, there should not be nor covering nor hole.

Let us recall that one calls nodes the points where the unknown factors are calculated. The nodes can be tops of the grid or not (mediums on the sides for example). The number of scalar unknown factors (or nodal variables) in an element the number of degrees of freedom of the element is called.

3.2

Choice of the functions tests

The functions tests (or functions of the base $\{w_i(\mathbf{X})$

$\})$ must be dense in space V of

unknown functions, being continuous from one element to another, to allow to calculate the terms simply elementary A_{ij} and L_i , and to generate a matrix \mathbf{A} hollow and conditioned well. Three first conditions are met in particular by the choice of polynomial functions. Moreover, to have one stamp \mathbf{A} hollow, one will make so that the supports of two basic functions associated two “distant” nodes are disjointed: thus, the corresponding terms of the matrix will be null.

It is pointed out that one places oneself to simplify the writings if only one function is used basic by node I for all the unknown factors. In this case: $W = W = W$

I

*I**I*, where *I* is itcondensed number of degree of freedom for the unknown factor of node *I*.*Handbook of Reference**R3.01 booklet: General references**HI-75/95/023/A***Code_Aster** ®*Version*

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The choice of the basic functions is then the following: one associates each node *I* a basic function w_I which is a polynomial per pieces being cancelled on all the elements not containing node *I*

[Figure 3.2-a]. So has $(W, W$

I

$J) = 0$ if the ddl of numbers *I* and *J* are carried by nodes *I* and *J*

who do not belong to the same element. One forces moreover this polynomial to be worth 1 with node *I*, and 0 in all the other nodes. In other words, $W(X, J$

*J**I*

$) = I$. Thus, nodal values of

unknown factors will be the values taken with the nodes by the exact solution: $U(X, J$

u, J

(

)=

.

Appear 3.2-a: Basic function associated a node

In the continuation, one will call function of form associated with node *I* the trace (or restriction) on the element

considered basic function w_I , and it will be noted N_i .

3.3

Representation of the geometry

The calculation of the functions of form for an unspecified element can be rather complicated. In the case

triangles, one can for example use the concept of barycentric co-ordinates of a point by report/ratio at the three tops. However, in the case of the quadrangles, such a concept is less current and calculations can be delicate to carry out analytically. This is why one prefers often to bring back itself to an element known as of reference, simple form, and from which one can generate all elements of the same family by a geometrical transformation. Functions of form are then calculated on this generic element noted R , and the transport of the sizes on the element real E is carried out thanks to the knowledge of the geometrical transformation. Let us note however that the thermohydraulic code N3S uses for reasons of performance of the analytical formulas explicit and not concept of element of reference.

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3.3.1 Element of reference

Let us note X the punctual coordinates \mathbf{X} in the absolute reference mark. Points of the element of reference

will be described in terms of co-ordinates known as parametric. The figure [Figure 3.3.1-a] gives for a triangular element in 2D the element of reference and the real element. The transformation must be bijective and to transform the tops and sides of the element of reference into tops and sides of the real element.

2

 x_2

1

element of reference

real element

0

1

1

 x_1 **Appear 3.3.1-a: Geometrical transformation****3.3.2 Functions of geometrical interpolation**

The geometry of the element will be approximate by the means of functions known as of geometrical interpolation:

thus for example, the curved lines of the real element can be represented by segments on the element of reference.

These noted functions $NR()$ are defined on the element of reference; they make it possible to know them co-ordinates X of an unspecified point of the real element starting from its co-ordinates of sound antecedent in the element of reference and co-ordinates X_I of the nodes (of local number I) of the real element:

 N $X = NR() x_I$ I

,

 $I = 1$

where N is the number of nodes of the element, and I the number of each node locally to the element.

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3.3.3 Stamp jacobienne transformation

The jacobienne of the transformation is the matrix of the derivative partial of the real co-ordinates X compared to the co-ordinates in the element of reference:

X

J

=

.

By taking account of the definition of co-ordinates X according to co-ordinates xI of the nodes, one obtains an equivalent expression of the matrix jacobienne:

$N \quad NR$

J

I

=

xI

,

$I = 1$

NR

NR T

where

I

are the terms of the tensor

, the number of lines is the number of directions

space, and numbers it columns the number of nodes of the element.

NR T

Let us note that the tensor

depends only on the definition of the element of reference and not on

that of the real element.

The determinant of the matrix jacobienne, useful in calculations which will follow, is called the jacobien the geometrical transformation. It is nonnull when the transformation which makes pass from the element of

reference to the real element is bijective, and positive when respects the orientation of space.

3.4

Representation of the unknown factors

There are two equivalent ways to represent the unknown factors (component displacement in the mechanical example) in an element: by the coefficients of their polynomial approximation, or by their nodal values. These two possibilities correspond to the two manners complementary to to define an element: by the data of a base of students'rag processions, or by the data of the functions of form

associated the nodes. In addition, let us note that an element is known as isoparametric when its functions of form are identical to its functions of geometrical interpolation. In *Code_Aster*, all them finite elements of continuous medium (2D and 3D) are isoparametric.

3.4.1 Base

polynomial

The way simplest to define an element is to choose a polynomial base made up of one certain number of independent students'rag processions. For a given unknown factor, the number of students'rag processions

used must be equal to the number of nodal variables, i.e. with the number of nodes used for to represent the unknown factor. In the case of a triangular finite element where one wishes to have them linear displacements and constant pressure in each element, polynomial bases used

are respectively $\{1, X,$

$1 x2\}$ and $\{\}$

1. Consequently, one can choose to calculate displacements with the 3 nodes tops and the pressure with the central node.

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One generally defines the polynomial base on the element of reference; it contains students'rag processions of

the form

$1 \ 2 \ 3$, where, and are positive or null whole exhibitors. The degree of such students'rag procession is the entirety $+$ $+$. The base is known as complete degree N when all the students'rag processions of degree N are present. In certain cases, one employs incomplete bases. For example, for QI quadrangle in 2D, displacements are linear compared to each direction: the base used is $\{1,$

$1 \ 2 \ 1$

2

$\}$. The components $u1$ and $u2$ of displacement are thus written:

U

$(,) = has + has + has + has$

$1 \ 1 \ 2$

1

$2 \ 1$

$3 \ 2$

$4 \ 1$

2

$U(,) = B + b + b + b$

$2 \ 1 \ 2$

1

$2 \ 1$

$3 \ 2$

$4 \ 1$

2

One notes $pi()$ the ième students'rag procession of the base (which includes/understands m of them). Components of the vector

displacement (U) in the element are then given by the formula:

$$U = \sum_{i=1}^m U_i P_i$$

$$I = \sum_{i=1}^m I_i$$

One will note the matrix giving the values taken by the students'rag processions of the polynomial base on nodes of the element of reference:

$$H = I P$$

where I is the sequence number of the students'rag procession in the base, I the number of the node locally to the element and I co-ordinates of node I in the element of reference. This matrix is square, its dimension is the square of the number of nodes of the element.

3.4.2 Functions of form

An equivalent way to define a finite element is to give, for each unknown factor, the expression of functions of form of the element. For a given scalar unknown factor (component of displacement according to y for example), there is as much as nodes where the unknown factor must be calculated. In much

case, one uses the same functions of form for all the components of an unknown vector, but it is not obligatory. In what follows, one will suppose however to simplify the writings that it is the case.

The functions of form can be defined on the element real E : they then are noted $NR E (\mathbf{X})$, they depend on the geometry of the real element, and are thus different from one element to another. It is more

simple to express them on the element of reference, which gives the functions $NR ()$ independent of geometry of the real element. Let us recall that these functions are polynomial on the element, and that function of form associated with a node given there takes value 1, whereas it cancels in all them other nodes of the element.

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The unknown factors are expressed then like linear combination of the functions of form, the coefficients u_I of the combination being called nodal variables:

 N

$$U() = NR() u_I$$

 I

·

 $I=1$ N

$$\text{or } U(\mathbf{X}) = NR$$

-1

$$((\mathbf{X}))u_I$$

 I $I=1$

3.4.3 Element

isoparametric

Two types of interpolation thus intervene in the construction of a finite element: the interpolation geometrical (using the functions $NR()$) and the interpolation of the unknown factors (using the functions $NR()$). An element is known as isoparametric when it is based on identical interpolations for its geometry and its unknown factors: $NR() = NR()$.

3.4.4 Correspondence between polynomial base and functions of form

There are the relations:

$$U() = \text{have } P$$

 I

$$I() \text{ and } U() = NR() U$$

 I

·

 I I m

Moreover, it is clear that one a: $u_I = \text{have } P I$

$$() = \text{have}$$

I

II. One deduces from it the following relation between

$i=1$

base polynomial and the functions of form: $II N_i () = I$

$P ()$.

Example: P1 triangle in 2D

One will note $= (1,2)$ the parametric co-ordinates in the element of reference.

$P () = 1, P () =, P$

1

2

1

$3 () = 2$

,

1 0

0

1 0

0

$= 1 1$

0

-1

,

$= -1 1$

0,

1 0 1

-1 0 1

from where functions of form:

$NI () = 1 - 1 - 2$

$N2 () = 1$

NR

$3 () = 2$

It is checked well that NR

J

J

$I() = I.$

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3.5

Calculation of the elementary terms

The elementary terms to calculate are form:

$U(\mathbf{X})$ $2u(\mathbf{X})$

$F(U(\mathbf{X}),$

,

...) $d\mathbf{x}$

2

.

E

\mathbf{X}

\mathbf{X}

Three types of operations are to be carried out: the transformation of derived compared to \mathbf{X} into derivative

compared to, the passage of an integration on the real element with an integration on the element of reference, and the numerical realization of this integration which is generally made by a formula of squaring.

3.5.1 Transformation of the derivative

The transformation of derived is carried out thanks to the matrix jacobienne \mathbf{J} , according to the rule of derivation in chain:

U

T

U 1 **NR**

-

nod

=

=

J**U** XX where **unod**

is the vector of the nodal values of the component of displacement.

The derivative of a higher nature are also obtained by using this rule, even if that gives place with expressions more complex than we will not clarify here.

3.5.2 Change of field of integration

The passage to integration on the element of reference is carried out by multiplying the intégrande by determinant of the matrix jacobienne, called jacobien:

 $\mathbf{U}(\mathbf{X}) \mathbf{u}(\mathbf{X})$ $\mathbf{U}(\mathbf{X}) \mathbf{u}(\mathbf{X})$ $F(\mathbf{U}(\mathbf{X}),$

,

...,) $d\mathbf{x} =$ $F(\mathbf{U}(\mathbf{X}),$

,

...,) $\det(\mathbf{J}(\mathbf{X})) d\mathbf{X}$

2

.

2

 E \mathbf{X} \mathbf{X} R

3.5.3 Numerical integration: points of GAUSS

In certain particular cases, one can calculate the integrals analytically. For example, for one triangle in 2D, Jacobien is constant on the triangle, and the intégrandes are brought back to students'rag processions

that one can integrate exactly:

1 1

-

!!

1 2 $d1d2 =$

.

0 0

(+ + 2)!

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However, these particular cases are rare, and one prefers to evaluate the integrals numerically in calling upon formulas of squaring. Those give an approximation of the integral under form of a balanced sum of the values of the intégrende in a certain number of points of the element called points of integration:

R

$G ($

$) D G$

G

$(G).$

R

$g=1$

The scalars G are called the weights of integration, and the co-ordinates G are the co-ordinates R points of integration in the element of reference.

In the methods of integration of GAUSS, the points and weights of integration are given of manner to integrate exactly polynomials of a nature given. It is this type of method which one uses

in *Code_Aster* then, the points of integration are called points of GAUSS.

Note:

The number of points of selected GAUSS makes it possible to integrate exactly in the element of reference. In fact, because of the possible non-linearity of the geometrical transformation or space dependence of the coefficients, integration is not exact in the real element.

However, it is shown that the made error is of an order lower than the error of discretization induced by the finite element method.

To illustrate the use of the points of GAUSS, let us take as example the case 3D, where one supposes that one uses r_1 points in direction 1, r_2 in the direction in the direction

2 and r_3

3, is one

total of $R = R_1 R_2 R_3$

$r_1 r_2 r_3$ points of GAUSS. It is shown whereas the expression:

R

R

R

1

2

3

G (

) $D_{G I J K}$

I

$J K$

(,

1,

2)

3

R

$i=1 j=1 K=1$

allows to integrate exactly students'rag processions of the type (1) (2) (3), with

$2 R - 1, 2 R - 1,$

1

2

and $2 r_3 - 1$.

3.6 Example

One proposes to detail the calculation of the variational formulation, of the functions of form, of elementary matrix of thermal rigidity and the elementary vector loading in the case of

the equation of heat (Laplacian) in 2D, for elements of the type $Q1$ quadrangle.

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3.6.1 Formulation

variational

If one calls K the coefficient of conduction, and T the temperature, the local equations of balance are :

$$- \operatorname{div} (K \operatorname{grad} (T)) = F \text{ in}$$

$$T = 0 \text{ out of } 0$$

$$- K \operatorname{grad} (T) \cdot \mathbf{n} =$$

known R 1

where $- K \operatorname{grad} (T)$ is the heat flow and \mathbf{N} the outgoing normal with the field. One is imposed T_0 temperature = 0 on edge 0 of the field, and a heat flow on edge 1.

That is to say the virtual variable associated the temperature. By multiplying the equilibrium equation by, in

integrating by parts and by holding account of the boundary conditions, the formulation is obtained variational:

$$K \operatorname{grad} (T) \operatorname{grad} () D =$$

$$F D -$$

D

1

The elementary terms that one will have to calculate will be thus:

· the matrix of thermal rigidity elementary: $Ae =$

K

$NR) \text{ grad } (NR) D$

ij

$\text{grad } (I$

J

E

· the elementary vector of surface loading: =

$F NR D$

S

I

I

E

· the elementary vector of linear loading: =

$NR D$

L

I

I

$1e$

In fact, the term corresponding to the linear loading Lel is calculated in *Code_Aster* on one

I

element of particular edge and not on the edge of the element E . The functions of form are thus used

element of edge (which is the traces on the edge of the functions of form of the surface element). **It**

is thus necessary always to use 2 elements when one wishes to impose a loading or one

boundary condition: an element of “volume” (for E) and an element of edge (for E).

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3.6.2 Functions of form

One will use QI quadrangles, where the unknown factors are represented on the polynomial basis $\{1, 1, 2, 1, 2\}$. The element of reference is the square represented on the figure [Figure 3.6.2-a]:

2
1
-1
1
1
-1

Appear 3.6.2-a: Square of reference

One thus has:

1 -1 -1

1
1
1
1

1
1 1 -1 -

1
1 - 1
1
1 -
1
=
,
-1

,
1
1
1

=
1
4 -1 -1
1

1

1 -1

1 -

1

1 -1

1 -

1

and by using relation $II Ni () = I$

$P ()$, one obtains the expressions of the four functions of form associated the tops:

1

NR

1 (1

, 2

) = 1

(- 1

) 1

(- 2

)

4

1

$$N2 (1, 2) = 1 (+ 1) 1 (- 2)$$

4

1

$$N3 (1, 2) = 1 (+ 1) 1 (+ 2)$$

4

1

$N4 (1, 2$

) = 1

(- 1

) 1

(+ 2

)

4

The matrix of derived from the functions of form in the element of reference is:

NT NI

NR

,
2,

,

,

1

1

1

1

1

1

N3

1

N4

1

1

- + 2

- 2

+ 2 - - 2

=

=

,

NI

NR

,

2,

,

,

4

1

1

1

1

2

N3

2

N4

2

2

- + 1 - - 1

+ 1

- 1

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from where the matrix jacobienne **J** which makes it possible to pass from the element of reference to a real element of which

4 NR

the tops have the co-ordinates (X, y

I

I

I

I), obtained thanks to the relation *J*

=

X

:

I=1

1 1

(- 2

) (2
 $X - 1$
 $X) + 1$
 $(+ 2$
 $) (x^3 - X) 1$
 4
 $(- 2$
 $) (y^2 - 1$
 $y) + 1$
 $(+ 2$
 $) (3$
 $y - y^4)$
J =

$4 1$
 $(- 1$
 $) (4$
 $X - 1$
 $X) + 1$
 $(+ 1$
 $) (x^3 - X)$
 1
 2
 $(- 1$
 $) (y^4 - 1$
 $y) + 1$
 $(+ 1$
 $) (3$
 $y - 2$
 $y)$

This matrix of order 2 could be calculated at the points of GAUSS when one needs some and easily reversed.

3.6.3 Calculation of the elementary terms

The elementary matrix $Ae =$

K
 $NR) \text{ grad } (NR) D$
 ij
 $\text{grad } (\mathbf{X} I$
 \mathbf{X}
 J

4 X include/understand $4 = 16$ terms, but

E

as it is symmetrical, only 10 are to be calculated. It is necessary to carry out three operations for

to evaluate each term of the elementary matrix:

- sum balanced on the points of GAUSS,
- transformation of the derived ones: $\text{grad}(NR) = -1 \text{ grad}(NR)$

X

J

I

I,

- integration on the element of reference while multiplying by the jacobien (determinant of **J**).

One notes grad_x here the gradient whose components are the derivative of the functions compared to co-ordinates **X**, and grad the gradient whose components are the derivative of the functions by

NT

report/ratio with the co-ordinates (they are the columns of the matrix

).

One deduces the final expression from it from the elementary term A_{eij} :

NPG

$Ae = K$

-

J 1

NR

-

J 1

$\text{grad} ($

$($

$)$

$\text{grad}(NR$

ij

G

I

G

$J(G) \det(\mathbf{J}(G))$

,

$g=1$

where *NPG* indicates the number of points of GAUSS. One of the families of possible points of GAUSS (because it integrates the *QI* elements exactly) for the square of reference $[-1,]$

$1 \times [-1,]$

1 is that where

1

1

the points of GAUSS have as co-ordinates (\pm

,±

) and where the weights of integration are worth 1.

3

3

The components of the elementary vector corresponding to the surface loading T_{em} are calculated

 I

in a way even simpler:

 NPG

$$= F() NR$$
 S G G

$$I(G) \det(\mathbf{J}(G)),$$
 I $g=1$

where the surface loading F is interpolated at the points of GAUSS of parametric co-ordinates G .

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4 System

matric

For each element E , one knew to calculate the terms known as elementary: elementary matrix \mathbf{Ae} and elementary vector \mathbf{It} . The matrix \mathbf{A} and the vector \mathbf{L} are obtained by a procedure that one calls assembly of the elementary terms, described below. One exposes then the principle of the imposition of the boundary conditions, then one gives a list of methods usable to solve it matric system obtained. These the last two points are evoked very briefly because they are treated in other booklets of the reference material [in particular R6].

4.1

Assembly of the matrices and vectors elementary

The assembly consists in deferring the Ae terms

E

ij and Li of each elementary matrix \mathbf{Ae} and of

each elementary vector \mathbf{It} in corresponding boxes AIJ and LI of matrix \mathbf{A} and of

vector \mathbf{L} . the correspondence enters the local numbers I and J of the degrees of freedom, and their numbers

total I and J are given by the table of connectivities belonging to the grid.

Indeed, the table of connectivities gives, for each element, the absolute numbers of its nodes

(tops or not). The order in which the nodes of the element are described gives their numbers

buildings in the element of reference (K ième node described will have the number K locally). In

addition,

one knows for each node the order of the degrees of freedom: for example, displacement according to X ,

then displacement according to y , then pressure. That makes it possible to number the degrees of freedom

locally in each element. As for the numbers of the degrees of freedom of the total system, they are

obtained after renumerotation of the unknown factors [R2.02.03]. One thus knows, for a given element,

to associate numbers I and J local degrees of freedom numbers I and J of the degrees of freedom

total.

To carry out the assembly, one carries out a loop on the elements. For each element, one

determine the nodes which it comprises and thus the total numbers of the degrees of freedom considered,

and

one adds at the end A

E

IJ the Aij term corresponding to him.

4.2

Imposition of the boundary conditions kinematics

The treatment of the boundary conditions kinematics of the type $\mathbf{U} = \mathbf{uD}$ is described in detail in

booklet [R3.03.01]. They are imposed by a method of duality, by introducing a vector of

multipliers (or parameters) of LAGRANGE, which leads to the mixed matrix system:

$$\mathbf{WITH} + \mathbf{BT} = \mathbf{L}$$

$$\mathbf{DRUNK}$$

$$= \mathbf{UD}$$

4.3

Resolution of the matrix system

The preceding linear system can be solved by a certain number of numerical methods.

methods used in *Code_Aster* are factorization LDLT per blocks [R6.02.01], the method

multifrontale [R6.02.02], and the packaged combined gradient [R6.01.01].

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4.4

Estimate of error and improvement of the precision of calculations

After having carried out a calculation by finite elements, it is possible to make an allowing postprocessing

to consider the error made: to see on this subject the documents [R4.10.01] and [R4.10.02].

To improve the precision of the results, two tactics are possible:

- to refine the grid
- to use an approximation of a higher nature
- is by increasing the number of nodes of interpolation (family of the elements of LAGRANGE);
- is by increasing the number of nodal variables, by adding for example them derived from the unknown factors (family of the elements of HERMIT); this method is not used in *Code_Aster*.

5 Organization of a calculation by finite elements in

Code_Aster

One very briefly describes how and in which place the aspects evoked in this document are established in *Code_Aster*.

5.1

Concept of finite element in *Code_Aster*

A type of finite element is defined by:

- a type of mesh
- a list of nodes
- of the functions of form
- of the options of calculation

An element in the grid is defined by a type of mesh, a geometry (coordinated nodes) and a topology (ordered list of the nodes). It is the type of modeling chosen in the file of order which makes it possible to assign to each mesh grid a type of finite element. The order AFFE_MODELE [U4.22.01] assigns to each mesh a type of finite element corresponding to modeling specified for this mesh. When same modeling is retained for all it grid, the use of AFFE_MODELE is simple thanks to the use of the key word ALL: "YES".

Important remark:

In the contrary case, one should not forget to assign finite elements to the meshes of edge of which one has need to impose the boundary conditions and loadings, and which one will have taken care of to create during the manufacture of the grid.

Operator AFFE_CHAR_MECA [U4.25.01], which affects boundary conditions and loadings, goes also to create finite elements, for example the finite elements which will carry the degrees of freedom LAGRANGE used in the dualisation of the boundary conditions [R3.03.01].

Operator AFFE_CARA_ELEM [U4.24.01] allows to define additional characteristics for certain types of elements: for example, the thickness of the hulls, orientation of the beams, matrices of mass and rigidity of the discrete elements.

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An option of calculation indicates the elementary type of calculation that the element is able to calculate.

By

example RIGI_MECA relates to the calculation of the elementary matrix of mechanical rigidity:

Ae

E

E

$= ijkl ij (NR(\mathbf{X}))kl (NR(\mathbf{X}))d\mathbf{x}$,

R

The “data” of this option are the geometry (R) and the material (E), supplemented by temperature if the material depends on it.

Option CHAR_MECA refers to the calculation of the elementary vector for a mechanical loading imposed on the border:

$=$

$gN E$

(X) dx.

R

Let us recall that to apply the loadings of border, one uses finite elements of edge private individuals, and not borders of the finite elements of volume (3D) or surface (2D).

Note:

A developer can sometimes have the choice between creating a new finite element or adding one option of calculation to an existing element; the choice between these two solutions holds in general count criteria of data-processing facility (e.g. under-integrated elements).

5.2**Initializations of the elements**

The use of elements of reference makes it possible to once carry out a certain number of calculations for all at the beginning of the execution. These calculations are carried out in routines INI.... called routines of initialization of the elements. One defines, for each type of element of reference:

- the number of nodes and their co-ordinates;
- the number of families of points of GAUSS;
- the number of points of GAUSS;
- weights of integration G ;
- values of the functions of form at the points of GAUSS $Nor (G)$;

$Nor (G)$

- values of derived from the functions of form at the points of GAUSS

.

For a given element, one inevitably does not integrate all the elementary terms with the same one a number of points of GAUSS: for example, one in general uses more points of GAUSS for stamp of mass than for the matrix of rigidity, because the products of functions of form are degree higher than the products of their derivative. Another example is the under-integration used in certain cases. One calls family of points of GAUSS each whole of points of GAUSS likely to be used.

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5.3

Calculation of the elementary terms

During the calculation of the elementary terms (in the routines YOU...), one carries out for each point of GAUSS following operations:

- calculation of derived from the functions from form, \mathbf{X} on the real element starting from the coordinates nodes of the element and derivative of the functions of form \mathbf{NR} , on the element of reference;
- calculation of the matrix jacobienne;
- recovery of the weight of integration multiplied by Jacobien at the point of GAUSS considered;
- evaluation of the intégrande (according to the calculated option).

The elementary term is calculated by nap on the points of Gauss while balancing by the weights of integration.

5.4 Resolution

total

The total resolution takes place in routines COp... high level corresponding to the orders user (MECA_STATIQUE [U4.31.01], STAT_NON_LINE [U4.32.01], THER_LINEAIRE [U4.33.02], etc).

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***Functions of form and points of integration
finite elements***

Summary:

One describes the geometry and the topology of the elements established in Code_Aster, the expression of the functions of form and the various families of points of integration and the associated weights are detailed.

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1 Introduction

In Code_Aster, one calls “finite element”, a triplet (phenomenon, modeling, type of mesh). There is three principal phenomena: MECHANICS, THERMICS and ACCOUSTICS.

There are many modelings; for example, for the MECHANICAL phenomenon: 3D, C_PLAN, D_PLAN, AXIS, DKT, POU_D_E,...

For a given modeling (for example 3D) of a phenomenon (for example MECHANICAL), there exists in general several finite elements: an element by type of mesh supported: HEXA8, HEXA20, PENTA6,...

With final, there thus exists of very many finite elements (more than 500 in July 2004).

On the other hand, the types of mesh are them numbers some reduced: POI1, SEG2, SEG3, SEG4,

TRIA3,
TRIA6, TRIA7, QUAD4, QUAD8, HEXA8, HEXA20, ..., TETRA4, TETRA10.

In general, each finite element, to carry out its elementary calculations, uses the concepts of function of interpolation (or function of form) and of diagram of integration. In general also, these functions of form and these diagrams of integration are defined on an element known as “of reference” whose geometry is defined in a frame of reference often called: (,) the passage of the element of reference to the real element is done thanks to a geometrical transformation which uses the same ones functions of interpolation. The element is then known as “isoparametric”. These concepts are very well explained in [bib1].

The high number of finite elements of the code combined with the restricted number of the types of mesh, conduit with the fact that there are several finite elements being based on the same type of mesh; for example it quadrilateral with 8 nodes called QUAD8 supports more than 60 different finite elements.

In theory, each finite element can choose its functions of interpolation and its diagrams of integration as it hears it. But in practice, almost all finite elements being based on the same type of mesh, use the same element of reference, the same functions of form and the same ones diagrams of integration. The goal of this document is to describe these various elements of reference

For each element of reference (called in the continuation of document ELREFE), one will indicate:

- the mesh support, the number of the nodes, their local classification and their co-ordinates,*
- algebraical expressions of the functions of form and their derivative first (and sometimes seconds)*
- families of points of integration which one will name. For each family, one will give it a number of points, their co-ordinates and their “weights” of integration. The sum of these weights, must give the “volume” of the element of reference. For example, the “volume” of the quadrangle of reference ($-1 \leq \xi \leq +1, 1 \leq \eta \leq +1$) is worth: 4.*

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2

Linear elements: SE2, SE3 and SE4

SE2: segment with 2 nodes

a number of nodes

: 2

a number of nodes tops

: 2

SE3: segment with 3 nodes

a number of nodes

: 3

a number of nodes tops

: 2

X

N1 -1.0

N2 1.0

N3 0.0

N1

X

N3

N2

SE4: segment with 4 nodes

a number of nodes

: 4

a number of nodes tops

: 2

X

- $N1 -1.0$
- $N2 1.0$
- $N3 -1. /3.$
- $N4 +1. /3.$

- $N1$
- X
- $N3$
- $N4$
- $N2$

functions of form of the segment with 2 nodes:

$$W(X) = 0.5$$

\cdot

$(- X)$

$$W(X) = 0.5$$

\cdot

$(+ X$

1

2

)

functions of form of the segment with 3 nodes:

$$W(X) = - 5$$

\cdot

0

1

$(- X) X$

$$W(X) =$$

5

\cdot

0

1

$(+ X) X$

$$W(X) =$$

1

$(+ X 1$

) $(- X)$

1

2

3

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functions of form of the segment with 4 nodes:

1

$$W(X) = 16(1 - X)$$

1

$$X + (X - 1)$$

3

9

3

$$w_2(X) = -16(1 + X)$$

1

$$-X$$

$$X +$$

9

3

3

3

$$W(X) = 16(X -)$$

$$1(X +)$$

1

$$1 X -$$

27

3

$$w4(X) = -16(X -)$$

$$1(X +)$$

1

$$1 X +$$

27

3

Nb of pts

Not

X

Weight

of intégr.

1 1

0.0

2.0

2 1

0.577350269189626

1.0

2

-0.577350269189626

1.0

3 1

-0.774596669241

0.55555...

2

0

0.88888...

3

0.774596669241

0.55555...

4 1

0.339981043584856

0.652145154862546

2

-0.339981043584856

0.652145154862546

3

0.861136311594053

0.347854845137454

4

-0.861136311594053

0.347854845137454

3

Surface elements

3.1 Triangles

:
ELREFE TR3, TR6, TR7

N3

N6

N5

N7

N1

N4

N2

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Co-ordinates of the nodes:

N1

0.0

0.0

N2

1.0

0.0

N3

0.0

1.0

N4

0.5

0.0

N5

0.5

0.5

N6

0.0

0.5

N7

1/3

1/3

Family Not

Weight

FPG1 1 1/3 1/3 1/2

FPG3 1 1/6 1/6 1/6

2 2/3

1/6 1/6

3 1/6

2/3 1/6

FPG4 1 1/5 1/5

*25/(24*4)*

2 3/5

1/5

*25/(24*4)*

3 1/5

3/5

*25/(24*4)*

4 1/3

1/3

*-27/(24*4)*

FPG6 1

B

B

P2

2

1 2 B

B

P2

3

B

1 2 B

P2

4

has

1 2 A

P1

5 A *has P1*

6

1 2 A

has

P1

COT3 1 1/2 1/2 1/6

2 0 1/2 1/6

3 1/2 0 1/6

With

P1 = 0.11169079483905,

P2 = 0.0549758718227661,

To = 0.445948490915965,

B = 0.091576213509771

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Family Not

Weight

FPG7 1 1/3 1/3 9/80

2 A A P1

3

1-2A A P1

4 A 1-2A P1

5 B B P2

6

1-2B B P2

7 B 1-2B P2

With

$To = 0.470142064105115$

$B = 0.101286507323456$

$P1 = 0.066197076394253$

$$P2 = 0.062969590272413$$

Family Not

Weight

FPG12 1

With

With

P1

2

1-2A A P1

3 A 1-2A P1

4 B B P2

5

1-2B B P2

6 B 1-2B P2

7 C D P3

8 D C P3

9

1-C-D

C P3

10

1-C-D D P3

11 C 1-C-D P3

12 D 1-C-D P3

With

$$To = 0.063089014491502$$

$$B = 0.249286745170910$$

$$C = 0.310352451033785$$

$D = 0.053145049844816$

$P1 = 0.025422453185103$

$P2 = 0.058393137863189$

$P3 = 0.041425537809187$

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TR3: *triangle with 3 nodes*

a number of nodes

: 3

a number of nodes tops

: 3

functions of form and derived first of the triangle with 3 nodes:

{NR}

{NR/

}

{NR/

}
1- -
-1
-1

1
0

0
1

TR6: triangle with 6 nodes

a number of nodes

: 6

a number of nodes tops

: 3

functions of form, derived first of the triangle with 6 nodes:

{NR}
{NR/
}
{NR/
}
- 1
(- - 1
) (- 1
(
2 - -))
1 - 1
(
4 - -)
1 - 1
(
4 - -)
- 1
(- 2)
-1+
4
0
- 1
(- 2)

0
 $-1+$
 4
 $4\ 1$
 $(\ -\ -)$
 1
 $($
 $4\ -\ 2\ -)$
 $-$
 4

 4

 4

 4
 $4\ 1$
 $(\ -\ -)$
 $-$
 4
 1
 $($
 $4\ -\ -\ 2)$

derived seconds from the triangle with 6 nodes:

$\{2$
 2
 $NR/$
 $\}$
 $\{2N/$
 $\}$
 $\{2$
 2
 $NR/$
 $\}$

$4\ 4\ 4$
 $4\ 0\ 0$
 $0\ 0\ 4$
 $-8\ -4\ 0$

0 4 0

0 -4 -8

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TR7: *triangle with 7 nodes*

a number of nodes

: 7

a number of nodes tops

: 3

functions of form of the triangle with 7 nodes:

{NR}

1- (

3 +) + (

2 2

2

+) + 7 - 3(+)

(-1+ 2 + 3 - 3(+))

(1

- + 2 + 3 - 3 (+))

4 1

(- - 4 + 3(+))

4 (2

- + (
 3 +))
 4 1
 (- 4 - + 3 (+))
 27 1
 (- -)

derived first from the triangle with 7 nodes:

{NR/
 }
 {NR/
 }
 2
 - 3 + 4 + 7 - 6 - 3
 2
 - 3 + 7 + 4 - 6 - 3
 2
 -1 + 4 + 3 - 6 - 3
 3 1
 (- - 2)
 3 1
 (- 2 -)
 2
 -1 + 3 + 4 - 6 - 3
 1
 (
 4 - 2 - 4 + 6 + 3 2
)
 4 (4
 - + 3 + 6)
 4 (2
 - + 6 + 3)
 4 (2
 - + 3 + 6)
 4 (4
 - + 6 + 3)
 (
 4 -1 - 4 - 2 + 6 + 3 2
)
 27 1
 (- 2 -)
 27 1

(- - 2)

derived seconds from the triangle with 7 nodes:

{2
 2
 NR/
 }
 {2N/
 }
 {2
 2
 NR/
 }

4 -
 6
 7 -
 6 -
 6
 4 -
 6
 4 -
 6
 3 -
 6 -
 6
 -
 6
 -
 6
 3 -
 6 -
 6
 4 -
 6
 (
 4 - 2 + 6)
 (
 4 4
 - + 6 + 6)

24

24

(
4 2
- + 6 + 6)

24

24

(
4 4
- + 6 + 6)

(
4 -2 + 6)

-
54

27 1
(- 2 - 2)

-
54

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3.2 Quadrangles

**:
ELREFE QU4, QU8, QU9**

N4

N7

N3

N8

N6

N9

N1

N5

N2

Co-ordinates of the nodes:

N1 -1.0 -1.0

N2 1.0 -1.0

N3 1.0 1.0

N4 -1.0 1.0

N5 0.0 -1.0

N6 1.0 0.0

N7 0.0 1.0

N8 -1.0 0.0

N9 0.0 0.0

Family Not

Weight

FPG1 1

0

0

4

FPG4 1

- has

- has 1.0

2

has

- *has*

1.0

3

has

has

1.0

4 - *has*

has

1.0

= 1 *has* 3

FPG9 1

- *has*

- *has* 25/81

2

has

- *has*

25/81

3

has

has

25/81

4 - *has*

has

25/81

5.0.0 - *has*

40/81

6

has

0.0

40/81

7 0.0

has

40/81

8 - *has* 0.0

40/81
9 0.0 0.0
64/81

$a = 0.774596669241483$

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QU4: *quadrangle with 4 nodes*

a number of nodes

: 4

a number of nodes tops

: 4

functions of form, derived first and seconds of the quadrangle with 4 nodes:

{NR}

{NR/

}

{NR/

}

1

(- 1

$(-)/4$
 -1
 $(-)/4$
 -1
 $(-)/4$
 1
 $(+1$
 $)(-)/4$
 1
 $(-)/4$
 -1
 $(+)/4$
 1
 $(+1$
 $)(+)/4$
 1
 $(+)/4$
 1
 $(+)/4$
 1
 $(-1$
 $)(+)/4$
 -1
 $(+)/4$
 1
 $(-)/4$

$\{2$
 2
 $NR/$
 $\}$
 $\{2N/$
 $\}$
 $\{2$
 2
 $NR/$
 $\}$

$0 1/4$

0

$$0 -1/4 0$$

$$0 1/4 0$$

$$0 -1/4 0$$

QU8: *quadrangle with 8 nodes*

a number of nodes

: 8

a number of nodes tops

: 4

functions of form and derived first of the quadrangle with 8 nodes:

{NR}

{NR/

}

{NR/

}

1

(- 1

)(-) (1

- - -) / 4

1

(-)(2 +) / 4

1

(-)(+ 2) / 4

1

(+ 1

)(-)(-1+ -) / 4

1

(-)(2 -) / 4

- 1

(+)(- 2) / 4

1

(+ 1

)(+) (1

- + +) / 4

1

(+)(2 +) / 4

1

(+)(+ 2) / 4

1

(- 1
)(+)(-1- +) / 4
 - 1
 (+) (2
 - +) / 4
 1
 (-)(
 - + 2) / 4
 1
 (-) 2 1
 (-) / 2
 - 1
 (-)
 - 1
 (
 2
 -) / 2
 1
 (+ 1
)(-) 2
 / 2
 1
 (
 2
 -) / 2
 - 1
 (+)
 1
 (-) 2 1
 (+) / 2
 - 1
 (+)
 1
 (
 2
 -) / 2
 1
 (- 1
)(-) 2
 / 2
 - 1
 (
 2

-) / 2

- 1

(-)

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derived seconds from the quadrangle with 8 nodes:

{2

2

NR/

}

{2N/

}

{2

2

NR/

}

1

(-) / 2

1

(- 2 - 2) / 4

1

$(-)/2$
 1
 $(-)/2$
 -1
 $(+2-2)/4$
 1
 $(+)/2$
 1
 $(+)/2$
 1
 $(+2+2)/4$
 1
 $(+)/2$
 1
 $(+)/2$
 -1
 $(-2+2)/4$
 1
 $(-)/2$
 $-1+$

 0
 0
 $-$
 $-1-$
 $-1-$
 $-$
 0
 0

 $-1+$

QU9: quadrangle with 9 nodes

a number of nodes

: 9

a number of nodes tops

: 4

functions of form and derived first of the quadrangle with 9 nodes:

$\{NR\}$
 $\{NR/$
 $\}$

{NR/
}
(-)(
1 -)
1 / 4
(2 -)
1 (-)
1 / 4
(-)(
1 2 -)
1 / 4
(+)(
1 -)
1 / 4
(2 +)
1 (-)
1 / 4
(+)(
1 2 -)
1 / 4
(+)(
1 +)
1 / 4
(2 +)
1 (+)
1 / 4
(+)(
1 2 +)
1 / 4
(-)(
1 +)
1 / 4
(2 -)
1 (+)
1 / 4
(-)(
1 2 +)
1 / 4
1
(
2
-)(-)
1 / 2

- (-)
 1
 1
 (
 2
 -)(2 -)
 1 / 2
 (+ 1
)(
 1
 2
 -) / 2
 (2 + 1
)(
 1
 2
 -) / 2
 - (+)
 1
 1
 (
 2
 -)(+)
 1 / 2
 - (+)
 1
 1
 (
 2
 -)(2 +)
 1 / 2
 (- 1
)(
 1
 2
 -) / 2
 (2 - 1
)(
 1
 2
 -) / 2
 - (-)
 1

1
 $($
 2
 $- 1$
 $)($
 2
 $-)$
 $- 2 1$
 $($
 2
 $-)$
 $- 2 1$
 $($
 2
 $-)$

derived seconds from the quadrangle with 9 nodes:

$\{2$
 2
 $NR/$
 $\}$
 $\{2N/$
 $\}$
 $\{2$
 2
 $NR/$
 $\}$

$(-)$
 $1 / 2$
 $(-1/)($
 $2 -1/)$
 $2 / 4$
 $(-)$
 $1 / 2$
 $(-)$
 $1 / 2$
 $(+1/)($
 $2 -1/)$
 $2 / 4$

(+)
1 / 2
(+)
1 / 2
(+ 1 /)(
2 + 1 /)
2 / 4
(+)
1 / 2
(+)
1 / 2
(- 1 /)(
2 + 1 /)
2 / 4
(-)
1 / 2
-(-)
1
-(2 -)
1
2
1 -
2
1 -
-(2 +)
1
-(+)
1
-(+)
1
-(2 +)
1
2
1 -
2
1 -
-(2 -)
1
-(-)
1
- 1
(
2

2

-)

4

- 1

(

2

2

-)

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4

Voluminal elements

4.1 Tetrahedrons

:

ELREFE TE4, T10

Z

N2

N5

N6

N9

N3

N7
N1
N10
y
N8
N4
X

X
y
Z
N1 0. 1. 0.
N2 0. 0. 1.
N3 0. 0. 0.
N4 1. 0. 0.
N5 0. 0.5 0.5
N6 0. 0. 0.5
N7 0. 0.5 0.
N8 0.5.0.5 0.
N9 0.5 0. 0.5
N10 0.5 0. 0.

Functions of form:

Formulate with 4 nodes

$$w1 (X, y, Z) = y$$

$$w2 (X, y, Z) = Z$$

$$W3 (X, y, Z) = 1 - X - y - Z$$

W
$$4 (X, y, Z) = X$$

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Formulate with 10 nodes

W
= y
W
= 4 Z
6
(1 - X - y - Z)
1
(2y -) 1
W
= Z (2z -)
W
= 4 y
7
(1 - X - y - Z)
2
1
W
W
= 4 X y
3
= (1 - X - y - Z) (1 - 2x - 2y - 2z)
8
W
= X (2x -)
W
= 4 X Z
4

1
 9
 W
 $= 4 y Z$
 W
 $= 4 X$
 10
 $(1 - X - y - Z)$
 5

Formulate numerical integration:

Formulate at 4 points, of order 2 in X, y, Z: (FPG4)

Not

X

y

Z

Weight

1

has

has

has

1/24

2

has

has

B

1/24

3

has

B

has

1/24

4

B

has

has

1/24

5 - 5

5 + 3 5

with: has =

B =

20
20

Formulate at 5 points, of order 3 in X, y, Z: (FPG5)

Not

X

y

Z

Weight

1

has

has

has

- 2/15

2

B

B

B

3/40

3

B

B

C

3/40

4

B

C

B

3/40

5

C

B

B

3/40

1

with: has

= 0 2

. 5

B =

C = 0 5

.

6

Formulate at 15 points, of order 5 in X, y, Z: (FPG15)

Not

X

y

Z

Weight

1

has

has

has

8/405

2

B

1

b1

b1

3

B

2,665.-14 15

1

b1

c1

4

b1

c1

b1

226 800

5

c1

b1

b1

6

B

2

b2

b2

7

B

2.665 + 14 15

2

b2

c2

8

b2

c2

b2

226 800

9

c2

b2

b2

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10

D

D

E

11

D

E

D

12

E

D

D

5

13

D

E

E

14

567

E

D

E

15

E

E

D

with:

has = 0 2

. 5

7 + 15

13 - 3 15

5 - 15

b1 =

C

=

D =

34

1

34

20

7 - 15

13 + 3 15

5 + 15

b2 =

C

=

E =

34
2
34
20

4.2 Pentahedrons

:
ELREFE PE6, P15

Z
N2
N11
N8
N7
N3
N1
N9
N5
N12
N10
y
N14
N13
N6
N15
N4
X

X
y
Z
N1 -1. 1. 0.
N2 -1. 0. 1.
N3 -1. 0. 0.
N4 1.
1. 0.
N5 1.
0. 1.
N6 1.

0. 0.
N7 -1. 0.5.0.5.
N8 -1. 0. 0.5.
N9 -1. 0.5 0.
N10 0.
1. 0.
N11 0.
0. 1.
N12 0.
0. 0.
N13 1.
0.5 0.5
N14 1.
0. 0.5
N15 1.
0.5 0.

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Functions of form:
Formulate with 6 nodes

1
1
 $W = y$

$$\begin{aligned}
 &W \\
 &= \\
 &y(X +) \\
 &1 \\
 &(1 - X) \\
 &1 \\
 &2 \\
 &4 \\
 &2 \\
 &1 \\
 &1 \\
 &W = Z \\
 &W \\
 &= \\
 &Z(X +) \\
 &2 \\
 &(1 - X)
 \end{aligned}$$

$$\begin{aligned}
 &1 \\
 &2 \\
 &5 \\
 &2 \\
 &1 \\
 &1 \\
 &W = \\
 &w6 = \\
 &(1 - y - Z)(X +) \\
 &3 \\
 &(1 - y - Z)(1 - X) \\
 &1 \\
 &2 \\
 &2
 \end{aligned}$$

Formulate with 15 nodes

$$\begin{aligned}
 &W \\
 &= y(1 - X) \\
 &W \\
 &= 2y \\
 &9 \\
 &(1 - y - Z)(1 - X) \\
 &1
 \end{aligned}$$

$$(2y - 2 - X)/2$$

W

$$= Z(1 - X)(2z - 2 - X)$$

2

2

/2

W

$$= y$$

10

$$(1 - X)$$

W

$$= (X -)$$

3

$$1(1 - y - Z)(X + 2y + 2z)/2$$

W

$$= Z$$

2

11

$$(1 - X)$$

W

$$= y(1 + X)$$

4

$$(2y - 2 + X)/2$$

2

$$w12 = (1 - y - Z)(1 - X)$$

W

$$= Z(1 + X)(2z - 2 + X)$$

5

/2

W

$$= 2yz(1 + X)$$

13

W

$$= (-X -)$$

6

$$1(1 - y - Z)(-X + 2y + 2z)/2$$

W

$$= 2z$$

14

$$(1 - y - Z)(1 + X)$$

W

$$= 2yz(1 - X)$$

$$\begin{aligned}
&7 \\
&W \\
&= 2y \\
&15 \\
&(1 - y - Z) (1 + X) \\
&W \\
&= 2z \\
&8 \\
&(1 - y - Z) (1 - X)
\end{aligned}$$

Formulas of numerical integration at 6 points (order 3 in X, order 2 in y and Z) (FPG6)

Not

X

y

Z

Weight

- 1
- 3 3
- 0.5 0.5 1/6
- 2
- 3 3
- 0. 0.5 1/6
- 3
- 3 3
- 0.5 0.
- 1/6
- 4
- 3 3
- 0.5 0.5 1/6
- 5
- 3 3
- 0. 0.5 1/6
- 6
- 3 3
- 0.5 0.
- 1/6

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Formulate numerical integration at 8 points: (FPG8)

2 points of Gauss in X (order 3).

4 points of Hammer in y and Z (order 3).

Not

X

y

Z

Weight

1 - has

1/3

1/3 -27/96

2 - has

0.6

0.2 25/96

3 - has

0.2

0.6 25/96

4 - has

0.2

0.2 25/96

5 +a

1/3

1/3 -27/96

6 +a

0.6
0.2 25/96
7 +a
0.2
0.6 25/96
8 +a
0.2
0.2 25/96

With $A = 0.577350269189626$

Formulate numerical integration at 21 points: (FPG21)

3 points of Gauss in X (order 5).
7 points of Hammer in y and Z (order 5 in y and Z).

Not

X

y

Z

Weight

1

-

1/3 1/3

C

9

1 × 80

2

-

has

has

155+ 15

3

-

1-2a

has

×

4

-

has

1-2a

c1 2 400

5

-

B

B

155 - 15

6

-

1-2b

B

C ×

7

1

-

B

1-2b

2 400

8 0.

1/3

1/3 C 9

2 × 80

9

0.

has

has

155+ 15

10

0.

1-2a

has

C ×

11

0.

has

1-2a

2

2 400

12

0.

B

B

155 - 15

13

0.

1-2b

B

C ×

14

0.

2

B

1-2b

2 400

15

1/3 1/3

C

9

1 × 80

16

has

has

155+ 15

17

1-2a

has

C ×

18

has

1-2a

1

2 400

19

B

B

155 - 15

20

1-2b

B

C ×

21

1

B

1-2b

2 400

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with:

3

5

8

=

c1 =

c2 =

5

9

9

$6 + 15$

$6 - 15$

$has =$

$B =$

21

21

4.3 Hexahedrons

**:
ELREFE HE8, H20, H27**

Z

$N5$

$N8$

$N26$

$N6$

$N25$

$N7$

$N27$

y

$N24$

$N1$

$N22$

$N4$

$N23$

$N21$

$N14$

X

$N2$

$N3$

X

y

Z

$N1 -1.$

$-1.$

$-1.$

$N2 1.$

$-1.$

-1.
N3 1.
1.
-1.
N4 -1.
1.
-1.
N5 -1.
-1.
1.
N6 1.
-1.
1.
N7 1.
1.
1.
N8
-1.
1.
1.
N9 0.
-1.
-1.
N10 1.
0.
-1.
N11 0.
1.
-1.
N12 -1.
0.
-1.
N13 -1.
-1.
0.
N14 1.
-1.
0.
N15 1.
1.
0.
N16
-1.

1.
0.
N17 0.
-1.
1.
N18 1.
0.
1.
N19 0.
1.
1.
N20
-1.
0.
1.
N21 0.
0.
-1.
N22 0.
-1.
0.
N23 1.
0.
0.
N24 0.
1.
0.
N25
-1.
0.
0.
N26 0.
0.
1.
N27 0.
0.
0.

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Functions of form:

Formulate with 8 nodes

1

1

W

=

$(1 - X)$

W

=

$(1 - X)$

5

$(1 - y) (1 + Z)$

1

$(1 - y) (1 - Z)$

8

8

1

1

W

=

$(1 + X)$

W

=

$(1 + X)$

6

$(1 - y) (1 + Z)$

2

$$(1 - y) (1 - Z)$$

8

8

1

1

W

=

$$(1 + X)$$

W

=

$$(1 + X)$$

7

$$(1 + y) (1 + Z)$$

3

$$(1 + y) (1 - Z)$$

8

8

1

1

W

=

$$(1 - X)$$

W

=

$$(1 - X)$$

8

$$(1 + y) (1 + Z)$$

4

$$(1 + y) (1 - Z)$$

8

8

Formulate with 20 nodes

1

1

W

=

$$(1 - X)$$

W

2

11

=

$$(1 - X) (1 + y) (1 - z)$$

1

$$(1 - y) (1 - Z) (- 2 - X - y - Z)$$

8

4

1

1

W

=

$$(1 + X)$$

W

2

12

=

$$(1 - y) (1 - X) (1 - z)$$

2

$$(1 - y) (1 - Z) (- 2 + X - y - Z)$$

8

4

1

1

W

=

$$(1 + X)$$

W

2

13

=

$$(1 - z) (1 - X) (1 - y)$$

3

$$(1 + y) (1 - Z) (- 2 + X + y - Z)$$

8

4

1

1

W

=

$$(1 - X)$$

W

2

14

=

$$(1-z)(1+X)(1-y)$$

4

$$(1+y)(1-Z)(-2-X+y-Z)$$

8

4

1

1

W

=

$$(1-X)$$

W

2

15

=

$$(1-z)(1+X)(1+y)$$

5

$$(1-y)(1+Z)(-2-X-y+Z)$$

8

4

1

1

W

=

$$(1+X)$$

W

2

16

=

$$(1-z)(1-X)(1+y)$$

6

$$(1-y)(1+Z)(-2+X-y+Z)$$

8

4

1

1

W

=

$$(1+X)$$

(

Z)

W

2

17

=

$$(1 - X) (1 - y) (1+z)$$

7

$$(1+ y) (1+ Z) -2+ X + y +$$

8

4

1

1

W

=

$$(1 - X)$$

W

2

18

=

$$(1 - y) (1+ X) (1+z)$$

8

$$(1+ y) (1+ Z) (- 2 - X + y + Z)$$

8

4

1

1

W

2

W

2

19

=

$$(1 - X) (1+ y) (1+z)$$

9

=

$$(1 - X) (1 - y) (1-z)$$

4

4

1

1

W

2

W

2

20

=

$(1 - y) (1 - X) (1+z)$

10

=

$(1 - y) (1+ X) (1-z)$

4

4

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Formulate with 27 nodes

1

1

W

=

$X (X -)$

1 y

W

=

$X (X +)$

1 y

2

15

$$(y +) I (1 - Z)$$

1

$$(y -) I Z (Z -) I$$

8

4

1

1

W

=

$$X (X +)$$

1 y

W

=

$$X (X -)$$

1 y

2

16

$$(y +) I (1 - Z)$$

2

$$(y -) I Z (Z -) I$$

8

4

1

1

W

=

$$X (X +)$$

1 y

W

2

17

=

$$(1 - X) y (y) I Z (z+)$$

3

$$(y +) I Z (Z -) I$$

1

8

4

1

1

W

=

$$X (X -)$$

1 y

W

=

X (X +)

2

18

1 (1 - y) Z (Z +)

4

(y +) 1 Z (Z -) 1

1

8

4

1

1

W

=

X (X +)

1 y

W

2

19

=

(1 - X) y (y +) 1 Z (z +)

5

(y -) 1 Z (Z +) 1

1

8

4

1

1

W

=

X (X +)

1 y

W

=

X (X -)

2

20

1 (1 - y) Z (Z +)

6

(y -) 1 Z (Z +) 1

1

8

4

1

1

W

=

X (X +)

1 y

W

2

2

21

=

(1 - X) (1 - y) Z (Z)

7

(y +) 1 Z (Z +) 1

1

8

2

1

1

W

=

X (X -)

1 y

W

2

2

22

=

(1 - X) y (y) 1 (1-z)

8

(y +) 1 Z (Z +) 1

8

2

1

1

W

2

W

=

X (X +)

2

$$\begin{aligned}
& 2 \\
& 23 \\
& 1 (1 - y) (1 - Z) \\
& 9 \\
& =
\end{aligned}$$

$$\begin{aligned}
& (1 - X) y (y) 1 Z (Z) 1 \\
& 4 \\
& 2 \\
& 1 \\
& 1 \\
& W \\
& =
\end{aligned}$$

$$\begin{aligned}
& X (X +) \\
& 2 \\
& W \\
& 2 \\
& 2 \\
& 24 \\
& =
\end{aligned}$$

$$\begin{aligned}
& (1 - X) y (y+) 1 (1-z) \\
& 10 \\
& 1 (1 - y) Z (Z -) \\
& 1 \\
& 4 \\
& 2 \\
& 1 \\
& 1 \\
& W \\
& 2 \\
& W \\
& =
\end{aligned}$$

$$\begin{aligned}
& X (X -) \\
& 2 \\
& 2 \\
& 25 \\
& 1 (1 - y) (1 - Z) \\
& 11 \\
& =
\end{aligned}$$

$$\begin{aligned}
& (1 - X) y (y+) 1 Z (Z) 1 \\
& 4 \\
& 2
\end{aligned}$$

$$\begin{aligned}
& 1 \\
& 1 \\
& W \\
& = \\
& X(X -) \\
& 2 \\
& W \\
& 2 \\
& 2 \\
& 26 \\
& = \\
& (1 - X)(1 - y)Z(z+) \\
& 12 \\
& 1(1 - y)Z(Z -) \\
& 1 \\
& 1 \\
& 4 \\
& 2 \\
& 1 \\
& W \\
& 2 \\
& 2 \\
& 2 \\
& 27 \\
& = (1 - X)(1 - y)(1 - Z) \\
& W \\
& = \\
& X(X -) \\
& 1y \\
& 2 \\
& 13 \\
& (y -)1(1 - Z) \\
& 4 \\
& 1 \\
& W \\
& = \\
& X(X +) \\
& 1y \\
& 2 \\
& 14 \\
& (y -)1(1 - Z) \\
& 4
\end{aligned}$$

Formulate squaring of Gauss at 2 points in each direction (order 3) (FPG8)

Not

X

y

Z

Weight

1

- 3 3

- 3 3

- 3 3

1.

2

- 3 3

- 3 3

3 3

1.

3

- 3 3

3 3

- 3 3

1.

4

- 3 3

3 3

+ 3 3

1.

5

3 3

- 3 3

- 3 3

1.

6

3 3

- 3 3

3 3

1.

7

3 3

3 3

- 3 3

1.

8

3 3
3 3
3 3

1.
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Formulate squaring of Gauss at 3 points in each direction (order 5): (FPG27)

Not
X
y
Z

Weight
1

-
-
-
c31
2
-
-
0.

c2 C
1
2

3

-

-

c31

4

-

0.

-

c2 C

1

2

5

-

0. 0.

C c2

1 2

6

-

0.

c2 C

1

2

7

-

-

c31

8

-

0.

c2 C

1

2

9

-

c3

1

10 0.

-

-

c2 C

1

2

11 0.

-

0.

C c2

1 2

12 0.

-

c2 C

1

2

13 0.

0.

-

C c2

1 2

14 0.

0. 0.

c32

15 0.

0.

C c2

1 2

16 0.

-

c2 C

1

2

17 0.

0.

C c2

1 2

18

0.

c2 C

1

2

19

-

-

c31

20

-

0.

c2 C

1

2

21

-

c31

22

0.

-

c2 C

1

2

23

0. 0.

C c2

1 2

24

0.

c2 C

1

2

25

-

c31
26

0.
c2 C
1
2
27

c31

with:

3
5
8
=

c1 =

c2 =

5
9
9

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4.4 Pyramids

:

ELREFE PY5, P13

Z

N5

N12

N3

N13

N8

N7

N11

N10

N4

N2

y

N9

N6

N1

X

The square base is consisted the quadrangle N1 N2 N3 N4 and N5 is the top of the pyramid.

X

y

Z

N1 1. 0.

0.

N2 0. 1.

0.

N3 1. 0. 0.

N4 0.

-1.

0.

N5 0. 0.

1.

N6 0.5

0.5

0.

N7 0.5 0.5

0.

N8 0.5

-0.5

0.

N9 0.5

-0.5

0.

N10 0.5

0.

0.5

N11 0. 0.5

0.5

N12 0.5 0. 0.5

N13 0.

-0.5

0.5

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Functions of form:

Formulate with 5 nodes

$$(-X + y + Z -) I (-X - y + Z -) I$$

$$w1 =$$

$$4 (1 - Z)$$

$$(-X - y + Z -) I (X - y + Z -) I$$

$$w2 =$$

$$4 (1 - Z)$$

$$(X + y + Z -) I (X - y + Z -) I$$

$$W3 =$$

$$4 (1 - Z)$$

$$(X + y + Z -) I (-X + y + Z -) I$$

$$w4 =$$

$$4 (1 - Z)$$

$$W = 1 - Z$$

$$5$$

Formulate with 13 nodes

$$(-X + y + Z -) I (-X - y + Z -) I (X - 0.) 5$$

$$w1 =$$

$$2 (1 - Z)$$

$$(-X - y + Z -) I (X - y + Z -) I (y - 0.) 5$$

$$w2 =$$

$$2 (1 - Z)$$

$$(X - y + Z -) I (X + y + Z -) I (-X - 0.) 5$$

$$W3 =$$

$$2 (1 - Z)$$

$$(X + y + Z -) I (-X + y + Z -) I (-y - 0.) 5$$

$w4 =$
 $2 (1 - Z)$
 $W = 2 Z (Z - 0.)$
5
5
 $(- X + y + Z -) I (- X - y + Z -) I (X - y + Z -) I$
 $w6 = -$
 $2 (1 - Z)$
 $(- X - y + Z -) I (X - y + Z -) I (X + y + Z) I$
 $w7 = -$
 $2 (1 - Z)$
 $(X - y + Z -) I (X + y + Z -) I (- X + y + Z -) I$
 $W = -$
8
 $2 (1 - Z)$
 $(X + y + Z -) I (- X + y + Z -) I (- X - y + Z -) I$
 $W = -$
9
 $2 (1 - Z)$

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$Z (- X + y + Z -)$

$I (- X - y + Z -)$

I

$w10 =$

$$\begin{aligned}
 &1 - Z \\
 &Z (-X - y + Z -) \\
 &1 (X - y + Z -) \\
 &1 \\
 &w11 = \\
 &1 - Z \\
 &Z (X - y + Z -) \\
 &1 (X + y + Z -) \\
 &1 \\
 &w12 = \\
 &1 - Z \\
 &Z (X + y + Z -) \\
 &1 (-X + y + Z -) \\
 &1 \\
 &w13 = \\
 &1 - Z
 \end{aligned}$$

Formulate numerical integration at 5 points (FPG5):

Not X y Z Weight

- 1 0.5
- 0.
- h1 2/15
- 2 0.
- 0.5
- h1 2/15
- 3 -0.5
- 0.
- h1 2/15
- 4 0.
- 0.5
- h1 2/15
- 5 0.
- 0.
- H2 2/15

with:

$$\begin{aligned}
 h1 &= 0.1531754163448146 \\
 H2 &= 0.6372983346207416
 \end{aligned}$$

Formulate numerical integration at 6 points (FPG6):

Not X y Z Weight

1 A 0.

h1

p1

2 0.

has

h1

p1

3 -

0. h1 has

p1

4 0.

-

has

h1

p1

5 0.

0.

H2

p2

6 0.

0.

h3

p3

with:

$$p1 = 0.1024890634400000$$

$$p2 = 0.1100000000000000$$

$$p3 = 0.1467104129066667$$

$$= 0.5702963741068025 \text{ have}$$

$$h1 = 0.1666666666666666$$

$$H2 = 0.08063183038464675$$

$$h3 = 0.6098484849057127$$

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Formulate numerical integration at 27 points (FPG27):

Not X y Z Weight

1 0.

0.

1/2 a1

2

B

B

1 (

1

1 - Z)

1/2

b6

1

(- Z)

2

2

3

B

B

- 1 1
(- Z)
1 1
(- Z)
1/2
b6
2
2
4
B
B

- 1 1
(- Z)
- 1 1
(- Z)
1/2
b6
2
2
5
B
B

1 (
1
1 - Z)
1/2
b6

-
1
(- Z)
2
2
6

0.
0.
1 - b1
B

6
2
7

0.
0.
 $1 + b1$
B

6
2
8
C (
Z)
C
1 1 -

0.
 $(1-c)/2$
8
1

9 0.
C (
Z)
C
1 1 -

$(1-c)/2$
8
1

10
- C (- Z)
C
1 1

0.
 $(1-c)/2$
8
1

11 0.
- C (

)
C
1 1 - Z
(1-c) /2
8
1

12
C (
Z)
C
1 1 -

0.
(1+c)/2
8
1

13 0.
C (
Z)
C
1 1 -

(1+c)/2
8
1

14
- C (
)
C
1 1 - Z

0.
(1+c)/2
8
1

15 0.
- C (
)
C
1 1 - Z
(1+c)/2

8

1

16

d1

D

(

)

1

(- *Z*)

$1 - D/2$

1 1

(- *Z*)

1

d12

2

2

17

D

D

- 1

(

)

1

(- *Z*)

$1 - D/2$

1 1

(- *Z*)

1

d12

2

2

18

D

D

- 1

(

)

1

(- *Z*)

1 - D/2

- 1 1

(- Z)

1

d12

2

2

19

d1

D

(

)

1

(- Z)

1 - D/2

- 1 1

(- Z)

1

d12

2

2

20

D (

)

1 1 - Z

0. 1/2 d12

21 0.

D (

)

1 1 - Z

1/2 d12

22

- D (- Z)

1 1

0. 1/2 d12

23 0.

- D (

)

1 1 - Z

$1/2 d12$

24

$d1$

D

D

1

$(- Z)$

1 1

$(- Z)$

$(1+d) /2$

12

1

2

2

25

D

D

- 1

(

)

1

$(- Z)$

$1 + D/2$

1 1

$(- Z)$

1

$d12$

2

2

26

D

D

- 1

(

)

1

$(- Z)$

$1 + D/2$

- 1 1

$(- Z)$

1

d12

2

2

27

d1

D

(

)

1

(- Z)

$1 + D/2$

- 1 1

(- Z)

1

d12

2

2

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15/09/05

Author (S):

J. PELLET, X. DESROCHES *Key*

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with:

$a1 = 0.788073483$

$b6 = 0.499369002$

$b1 = 0.848418011$

$c8 = 0.478508449$

$c1 = 0.652816472$

$d12 = 0.032303742$

$d1 = 1.106412899$

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Editor: MALOINE S.A. Year 984

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R3.03 booklet: Boundary conditions and loadings

Document: R3.03.01

Dualisation of the boundary conditions

Summary:

One explains the principle of the **multipliers of Lagrange to solve the linear systems under constraints closely connected** resulting from the imposition of the boundary conditions of the kinematic type. The matrix of rigidity

obtained not being more definite positive, certain algorithms of resolution thus become unusable. One thus seek a technique to be able to continue to use the algorithm of factorization **LDLT without permutation and without elimination**. The technique suggested is that of the **“Lagrange doubles”** (used in

Code Castem 2000). It is shown that this technique is effective. One gives some indications on conditioning of the matrices obtained by this technique.

The problem of the research of the **clean modes** of the **constrained systems** is then examined. One shows

that a possible solution is to add the boundary conditions dualized to the matrix of “rigidity” and of not to touch with the matrix of “mass”.

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1 Introduction

We are interested in this document in the dualisation of the boundary conditions (known as kinematics). Two problems distinct from linear algebra are examined:

- the resolution of the linear systems: paragraphs 2, 3, 4, 5, 6,
- the research of the clean modes: paragraph 7.

2 Dualisation of the boundary conditions kinematics, principle of the multipliers of Lagrange

In *Code_Aster* (as in the other codes of finite elements), one is brought to solve many linear systems.

Often such a system can be regarded as the algebraical expression of a problem of minimization of a positive quadratic functional calculus $J(\mathbf{U})$ where \mathbf{U} belongs to \mathbb{R}^N or \mathbb{N} is the number nodal unknown factors while being constrained by a certain number of relations closely connected $\mathbf{C}(\mathbf{U})$

$\mathbf{I} - d\mathbf{i} = 0$

(boundary conditions of the Dirichlet type).

It is with this problem of minimization under constraints closely connected that one is interested here. In the continuation of

document, one will take as example the case (and the vocabulary) of linear static mechanics. One will speak about matrix of rigidity, vector displacement,... but the technique suggested remains valid for the problems of thermal evolution, into linear or non-linear.

That is to say the discretized problem:

Pb1: min

$J(\mathbf{U})$

$\mathbf{U} \in \mathbf{V}$

\mathbb{R}^N

where:

.

$J(\mathbf{U})$ is a quadratic form (total potential Energy)

1

$J(\mathbf{U}) =$

$(\mathbf{A}, \mathbf{U}) - (\mathbf{B}, \mathbf{U})$

2

\mathbf{A} is a positive symmetrical matrix $(\mathbf{A}, \mathbf{U}) \geq 0$ $\forall \mathbf{U} \in \mathbf{V}$

) but not inevitably definite

($\mathbf{A} = \mathbf{0}$ is possible for $\mathbf{U} \neq \mathbf{0}$)

.

\mathbf{V} is the space of displacements kinematically acceptable

(it is under space closely connected of \mathbb{R}^N).

This discrete problem is solved numerically while expressing that the “derivative” of $J(\mathbf{U})$ in \mathbf{V} is null. One is then brought back to solve a linear system of equations.

The problem is “to derive” $J(\mathbf{U})$ in $\mathbf{V} \subset \mathbb{R}^N$.

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In general, for practical reasons, the expression of $J(\mathbf{U})$ is calculated in the base of all them nodal displacements (without taking account of the constraints): \mathbf{U} belongs then to \mathbb{R}^N where N is the number

total of nodal unknown factors.

If \mathbf{V} is under vector space of \mathbb{R}^N generated by $(N - p)$ of basic nodal displacements, derivation of $J(\mathbf{U})$ in \mathbf{V} is done very simply: it is enough “to forget” in the matrices \mathbf{A} and \mathbf{B} them lines and columns correspondents with the removed ddl ($u_i =$)

0 .

If the constrained ddl are not put at zero but not assigned to a given value: $U = D$

I

I , it is necessary to modify

\mathbf{B} .

Finally if the constraints “mix” the ddl between them (linear relations between unknown factors) it \mathbf{A} should be modified

and \mathbf{B} .

The principle of the multipliers of Lagrange makes it possible to solve the problem without touching with matrices \mathbf{A} and \mathbf{B} . The price to be paid is an increase in the number of unknown factors in the system with

to solve.

Instead of solving the problem in space \mathbf{V} of dimension $N - p$, one solves it in \mathbb{R}^{N+p} space, noted additional unknown factors I being called multiplying of Lagrange.

Principle and justification

Let us take again the preceding problem by clarifying space \mathbf{V}

Problem 1:

$\min J(\mathbf{U})$

$\mathbf{U} \in \mathbf{V}$

$$J(\mathbf{U}) = (\mathbf{With}, \mathbf{U}) - (\mathbf{B}, \mathbf{U})$$

$$\mathbf{V} = N$$

$$\{ \mathbf{uR} C(\mathbf{U}) = d, I =, 1 \}$$

\mathbf{C}_i are linear forms on \mathbb{R}^N , the d_i are constant data. One supposes more than the p forms \mathbf{C}_i are independent between them: the dimension of the space generated by \mathbf{C}_i are p .

One can show (cf [Appendix 1]) that this problem is equivalent to the following:

Problem 2:

to find \mathbf{U}, \mathbf{V} where

$$\mathbf{V} = \{ \mathbf{U} \in \mathbb{R}^N \mid \mathbf{C}_i(\mathbf{U}) = I \}$$

such $(\mathbf{With} - \mathbf{b})(\mathbf{v})$ that where

$$= 0 \mathbf{v} \in \mathbf{V}$$

$$\mathbf{0} \in \mathbf{V} \subset \mathbb{R}^N$$

{ \mathbf{v} such \mathbf{R} that $\mathbf{C}_i(\mathbf{v}) = 0$ } $I =$

$$\{ \mathbf{v} \in \mathbb{R}^N \mid \mathbf{C}_i(\mathbf{v}) = 0 \}$$

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Let us rewrite problem 2 differently:

Problem 3:to find \mathbf{U} \mathbf{R} \mathbf{N} such as **$\mathbf{I} \mathbf{C} \mathbf{U} - \mathbf{D}$** **$\mathbf{I}$** **$\mathbf{I}$** **$= 0$** **éq 2-1** **$\mathbf{v} \mathbf{V}, \mathbf{C} \mathbf{v}$** **$\mathbf{0}$** **$\mathbf{I}$** **$= 0$** **éq 2-2** **$\mathbf{v} \mathbf{V}$** **(With - b) \mathbf{v}** **$\mathbf{0}$** , **$= 0$** **éq 2-3**The equations [éq 2-2] and [éq 2-3] show (by identifying $\mathbf{R}\mathbf{N}$ and its dual) that:**(I) \mathbf{C}_i** is orthogonal in **\mathbf{V}_0** **(With - b)**is orthogonal in **\mathbf{V}_0** **\mathbf{V}** **$\{\mathbf{C}_i \mid i=1, p\}$** **$\mathbf{0}$** are under vector space of $\mathbf{R}\mathbf{N}$ orthogonal with**(**

) (\mathbf{V}_0 is of dimension $(N - p)$

because the p conditions \mathbf{C}_i are supposed to be independent).

Since the decomposition of \mathbb{R}^N all in all direct of 2 pennies orthogonal spaces is single, one in deduced that ($\mathbf{With} - \mathbf{b}$) belongs to the vector space generated by \mathbf{C}_i .

There is thus a family of scalars I called multipliers of Lagrange such as:

$$(\mathbf{With} - \mathbf{B}) + \mathbf{C}$$

I

\mathbf{I}

$$= 0$$

I

This equality is true in \mathbb{R}^N .

Problem 3 becomes then:

Problem 4:

To find $\mathbf{U} \in \mathbb{R}^N$, $I \in \mathbb{R}$, ($I = 1 \dots p$)

(

$$I =,$$

$$1 \dots p) \mathbf{C} \mathbf{U} - \mathbf{D}$$

\mathbf{I}

\mathbf{I}

$$= 0$$

(

With - B

$$) + \mathbf{C}$$

$$I \mathbf{I} = 0$$

I

The reciprocal one ($\text{Pb4} \Rightarrow \text{Pb3}$) is obvious: if there is I such as:

$$(\mathbf{With} - \mathbf{b}) + \mathbf{C}$$

I

$$= 0, \text{ then } \mathbf{v}$$

$$\mathbf{V} (\mathbf{With} - \mathbf{b}) \mathbf{v} = - \mathbf{C} \cdot \mathbf{v}$$

\mathbf{I}

$$I I$$

$$= 0$$

0

I

I
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Problem 4 is the sought problem. It will be said that it is the problem with conditions dualized kinematics. Matriciellement one can write it:

KX = F

With CT U

B

=

C

0

éq 2-4

D

(K) (X) (F)

where:

U N

p

X n+

R

R

R p

;

;

WITH A; **C** N, N **With**; **K** p, N **An** $+ p, n + p$

One realizes that this system can be obtained while seeking to make extreme the functional calculus:

$$L(\mathbf{U}, \mathbf{C}) = (\mathbf{With}, \mathbf{U}) - (\mathbf{B}, \mathbf{U}) + (\mathbf{C}\mathbf{u} -) \mathbf{D}$$

éq 2-5

2

This functional calculus is called Lagrangian initial problem. Principal interest of this method is to free itself from the constraints: \mathbf{U} and are sought in \mathbb{R}^N and \mathbb{R}^P (\mathbf{X} in \mathbb{R}^{n+p}).

Coefficients I are called coefficients of Lagrange of the problem (one will say sometimes them "Lagrange").

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Disadvantages of this dualisation

One sees according to the expression of Lagrangian that the matrix \mathbf{K} is not positive any more (what was the case of

With). Indeed:

1

$$\mathbf{U} \mathbf{C}\mathbf{u} \mathbf{0} L(\mathbf{U}, \mathbf{C})! =$$

(With, U) + Cu

0

0

0

0

0

0

0

0

$0 < 0$

2

The loss of the positivity of the matrix **K** involves that the resolution of the system **KX = F** cannot any more

to be done in general by the traditional algorithms of gradient or, by the factorization of Cholesky.

The algorithm of factorization LDLT without permutation of the lines and columns is not guaranteed any more either:

it is the latter algorithm which one wants to be able to continue to use.

Let us illustrate the problem on the following example:

Example 1:

a spring of stiffness *K* connects 2 nodes N1 and N2

X

N1

K

N2

2 unknown factors: *u*₁, *u*₂; 2 modal forces: *f*₁, *f*₂ (*N* = 2)

K

- *K*

With =

- *K*

K

1 constraint: *U* + *U*

1

2

= (*p* = 1)

The dualized problem is written: **KX = F**

with:

K

- *K*

U

F

1

1
K =
- *K*
K
X =
U F

2
= *f*²

0

Recall of the condition necessary and sufficient so that algorithm LDLT-SP (without permutation) function:

Let us note **K_I** under matrix of **K** formed of *I* first lines and columns of **K**.

(If **K** is of order *N*, **K_N** = **K**, **K₁** = [*k*₁₁]).

There will be no null pivot in algorithm LDLT-SP if and only if (**K_i** is invertible for any *I* ranging between 1 and *N*).

This condition will be noted: cond1

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The matrix **K** above is written with like classification of the unknown factors, the order of the components

X:

.

$\mathbf{X} = (U, U$

1

2,)

K

- K

$\mathbf{K} =$

-

K

K

0

K

- K

$\mathbf{K2} =$

do not check the condition cond1.

- K

K

On example 1 let us test new classifications:

.

$\mathbf{X} = (, U, U$

1

2)

0

$\mathbf{K} =$

K

-

K

- $K K$

$\mathbf{K1}$ does not check the condition cond1.

.

$\mathbf{X} = (U, U$

1

2)

 K - K det $\mathbf{K1} = K$ $\mathbf{K} =$

0

det \mathbf{K}

2

2

=

2

- $K K$ det $\mathbf{K3} = - K (+)$

-

 K is supposed > 0 (stiffness of the spring)

-

 $\mathbf{K3}$ is invertible only if $\neq 0$. The case $= 0$ corresponds indeed to one"bad" physical blocking: the condition: $U - U$

1

2 = cte does not block the "movements

rigid body " for \mathbf{A} (without energy).

So that the total problem has a single solution, it is necessary indeed that the conditions

 $\mathbf{C U} = \mathbf{D}$ \mathbf{I} \mathbf{I} generate a space of acceptable displacements which does not contain any movements of rigid body of \mathbf{A} .

With the notations of [§1] one will write:

ker $\mathbf{A V0} = \{ \}$

0

One will suppose in the continuation of the document that this condition is checked. I.e.

constraints \mathbf{Ci} block at least the movements of rigid body of \mathbf{A} (they

can be more numerous of course). When this condition is checked and that them

conditions \mathbf{Ci} are independent between them, one will say that the problem is physically posed well.*Handbook of Reference**R3.03 booklet: Boundary conditions and loadings**HI-75/01/001/A*

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K2 is invertible only if 0

It is thus seen that classification (U, U

1

2) checks the condition cond1 if 0.

If blocking $U + U$

1

2 = is reduced to:

$u1 =$

$(,) = (1,)$

0 it goes

$u2 =$

$(,) = (0,)$

1 that does not go

The symmetry of the problem shows that to be able to deal with the problem $(,) = (,$

0)

1 it is necessary to number

$X = (U, U$

2

1) .

From this very simple example, one can draw some general conclusions (all negative):

· if one numbers all I after the ui , if **A** is singular, the condition cond1 will not be checked for **K**

With

I =

· is a condition $C U = D$ and the multiplier of associated Lagrange. The equation $C U = D$

in general does not utilize all the unknown factors **ui**: the equation constrained some ddl. If

is numbered before the ddl that it constrained the condition cond1 will not be checked. Indeed

let us look at under matrix **K J** where J is the number of the equation giving.

K**0**

-1

K**J****J****= 0 0**

· Is a physical structure which, by bad luck, “is retained by its physical last ddl” i.e. such as if this ddl is not blocked, the matrix is singular, and such as if one it block the matrix is invertible. The use of a ddl of Lagrange for this blocking is impossible. Indeed, if one numbers before the physical last ddl, there will be a null pivot with level of, and if one numbers it after (thus in any last ddl), matrix **kN-1** will not be not invertible since blocking is not taken yet into account. One will see with [§4] that technique of the “double lagrange” makes it possible to solve this problem. To finish this paragraph we can make the following remark: If **K** is invertible, one knows that there is a classification of the unknown factors making it possible to factorize **K** by LDLT. This

classification is for example that resulting from algorithm LDLT with permutation (pivot maximum for example). But this renumerotation relates to only the lines of the matrix; it y thus has loss of the symmetry of **K**. It is enough to consider the following example:

Example 2:**With** = []

0

0

1

U**K** =**X****C** = []

=

1

1

0

K is invertible, but there is not any common permutation of the lines and columns of **K** allowing a resolution by LDLT.

All these remarks show that the dualisation proposed in this paragraph does not allow

not to use LDLT_SP.

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Principle of the “Lagrange doubles”

The method suggested here is that implemented in code CASTEM 2000 (communication personal of Th. CHARAS and P. VERPEAUX). An intuitive presentation could be of it following:

It is seen that the dualized problem [éq 2-4] has null terms on the diagonal: those corresponding to ddl of Lagrange. This property is also noticed on the Lagrangian one [éq 2-5]: there are no terms quadratic in.

This nullity of the diagonal terms prevents certain permutations of lines and columns: one cannot to place Lagrange before the physical ddl that it constrained.

The idea is then to break up each coefficient of Lagrange into 2 equal parts 1 and 2.

The equation $\mathbf{C} \mathbf{U} = \mathbf{D}$ is then replaced by:

$$\mathbf{C} \mathbf{U} - (1 - 2) = \mathbf{D}$$

$$\mathbf{C} \mathbf{U} + (1 - 2) = \mathbf{D}$$

where is a nonnull constant.

Let us show the equivalence of the old problem and the new one:

Problem 1: “simple Lagrange”

\mathbf{U}

RN

With + CT

= B

to find:

such as (S):

p

$$\mathbf{Cu} = \mathbf{D}$$

R

$$1 = 2$$

$$\mathbf{With} + \mathbf{CT1} + \mathbf{CT2} = \mathbf{B}$$
$$= 1 + 2$$

(S)

$$\mathbf{Cu} - 1 + 2 = \mathbf{D}$$
$$\mathbf{With} + \mathbf{CT} = \mathbf{B}$$

$$\mathbf{Cu} + 1 - 2 = \mathbf{D}$$
$$\mathbf{Cu} =$$

D

is a constant 0

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From where the problem are equivalent to the precedent:

Problem 2: “double Lagrange”

With + CT1 CT2 = B

U

N

R

to find:

such as (): **Cu**

- 1 + 2 = **D**

1 2 *p*

p

,

R

× **R**

Cu

+ 1 - 2 = **D**

The new problem can be written:

K' X' = F'

with:

X'

= (**U**, 1 2)

F' =

(B, D,) D
WITH CT
CT

K' =
C

- L L
C L - L

The problem corresponds to make extreme the functional calculus:

1
 $(\mathbf{U}, 1, 2) =$
 $(\mathbf{With}, \mathbf{U}) - (\mathbf{B}, \mathbf{U}) + (1, \mathbf{Cu} -) \mathbf{D}$
 2
 2

+ ($\mathbf{Cu} -$)
 $\mathbf{D} - (1 - 2, 1 - 2)$
 2

One can show (cf Appendix 2) that if one observes a certain rule of classification of the unknown factors, and

by choosing the constant > 0 , the \mathbf{K}' matrix check the condition cond1.

This rule is as follows:

That is to say a relation of blocking $\mathbf{Cu} - \mathbf{D} = 0$, it corresponds to him 2 multipliers of Lagrange 1 and 2. This relation utilizes a certain number of ddl physical.

Regulate R0:

For each relation of blocking, it is necessary to place 1 before the constrained first ddl physical and 2 after the constrained last ddl physical.

To decrease the occupation memory of the matrix \mathbf{K} , it is necessary to seek to minimize the bandwidth. It is what one does in *Code_Aster* in "framing" the relations "with nearest": 1 is placed just before the constrained first ddl, 2 is placed just after the constrained last ddl.

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Illustration:

That is to say a problem with 4 ddls physical: U, U, U, U
1 2 3 4 .

This system is subjected to 2 conditions:

$U + has U = B has$

11 1

13 3

1

$U + has U = B has$

22 2

24 4

2

let us call 1

2

1

2

1, 1 2 ddls of Lagrange associated with the 1st condition and 2, 2 those associated
2nd condition.

By supposing that the physical ddls were numbered in the order: U, U, U, U

1 2 3 4, classification

total of the ddls retained by *Aster* is then:

1

1

2

2

1, 1

$U, 2, 2$

$U, 3$

$U, 1, 4$

$U, 2$

1

2

1 and 1 frames “with more close” the constrained ddls (U and U

1

3)

1
2
2
4)

2 and 1 frame “with more close” the constrained ddls (U and U)

The technique of the “Lagrange doubles” associated with the R0 rule thus makes it possible to solve any system

linear posed physically well with the algorithm of LDLT without permutation. The demonstration suppose nevertheless that **matrix A is symmetrical and positive** (not inevitably definite).

Note:

*Assumptions: **With** symmetrical and **A** positive are necessary to use LDLT (or LU) without permutation as the two following counterexamples show it:*

0

1

A1 = 1 0 is symmetrical but nonpositive,

0 1

A2 =
is positive but nonsymmetrical.

- 1 1

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**5 Advantage
additional**

One will show in this paragraph that the technique of the “Lagrange doubles” can allow

economically to solve a series of problems which would differ only by their conditions with limits kinematics (for example a variable zone of contact).

Note:

This possibility is not currently used in the code.

That is to say a system with constraints $\mathbf{KX} = \mathbf{F}$:

Let us write this system while emphasizing a particular constraint (this calculation remains obviously valid when there are several constraints) $\mathbf{Cu} - \mathbf{D} = 0$. To simplify the writing, $\alpha = 1$ is chosen.

That is to say

1 the first ddl of Lagrange associated with the constraint

2 the second ddl of Lagrange associated with the constraint

$$\mathbf{U} = \mathbf{X} - \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

,

}

~

\mathbf{K} = matrix \mathbf{K} projected on \mathbf{U} ; \mathbf{B} = vector \mathbf{F} projected on \mathbf{U}

The system is written with these variables:

~

\mathbf{K}

$\mathbf{C}^T \mathbf{C}^T \mathbf{U}$

\mathbf{B}

1

\mathbf{C}

-1

1

=

\mathbf{D}

$\mathbf{C} \begin{Bmatrix} 1 \\ -1 \\ 2 \end{Bmatrix}$

\mathbf{D}

Let us change the coefficient (2 2

),): - 1 3 and let us write the new system:

~

K**U (1 2) CT**

+

+

=

B $(1 + 2) = 0$ **éq 5 - 1****(S) C****U - 1 + 2 = D****C U - 1 + 2 = D****éq 5 - 2**

1

2

~

C U + +

3

= D**K U = B****éq 5 - 3**

This last system is uncoupled: one can solve [éq 5-3] to obtain **U** then to calculate 1 and 2.

It is noticed that the resolution of [éq 5-3] corresponds to the initial problem without the constraint $\mathbf{C}\mathbf{u} - \mathbf{D} = 0$. The values of 1 and 2 then do not have any more the same physical significance. In others

~

terms, the system (S) with the same solution out of **U** as the subsystem $\mathbf{K}\mathbf{U} = \mathbf{B}$; 2 unknown factors additional 1 and 2 does not disturb the solution out of **U**. The total system can appear of size higher (+2) with what is necessary, but by means of computer, that can be very convenient.

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Indeed, let us imagine now that we know in advance that certain relations kinematics are likely to be slackened. Let us number the 2 associated ones with these relations at the end of the system. Us

let us can then triangulate partially once and for all the system while stopping before these ddl.

left the triangulated matrix is most important in volume: all physical ddl and all the 1.

When a concrete problem arises, i.e. when one knows the list of the linear relations active, it is enough to update the last lines of the matrix (- if the relation is active, + 3 if it is not it). One can then finish the triangulation and solve the problem economically.

6

Notice on the conditioning of the system

When one looks at the form of the matrix which one finally will factorize \mathbf{K}' (cf [§3]), one sees that its various submatrices \mathbf{A} , \mathbf{C} , \mathbf{I} can be of order of magnitude very different. One knows that in general this situation is not favorable numerically (limited precision of the computers). It should be noticed that the equations of connection $\mathbf{C}\mathbf{u} - \mathbf{D} = 0$ can be multiplied by a constant arbitrary () without changing the problem. Moreover, we saw that matrices \mathbf{I} were also arbitrary (>)

0. We thus have two parameters allowing “to regulate” it conditioning of the matrix.

We will not make a general demonstration but we are satisfied to examine the case more commonplace which is: a spring, a ddl, a connection.

The \mathbf{K}' matrix is written if K is the rigidity of the spring:

K

$\mathbf{K}' =$

- +

-

The conditioning of this matrix is related to the dispersion of its eigenvalues μ_i :

Let us calculate the polynomial characteristic of \mathbf{K}' :

$$P(\mu) = (\mu + 2) (-\mu^2 + k\mu + 22)$$

$$\mu_1 = -$$

2

< 0

+ K +

2

$K + 82$

$\mu_2 =$

> 0

2

+ $K -$

2

$K + 82$

$\mu_3 =$

< 0

2

K is the eigenvalue of the nonconstrained system. This eigenvalue is the required order of magnitude for μ_1, μ_2, μ_3 .

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It is noticed that $\mu_1 < 0, \mu_3 < 0$ and $\mu_2, > 0$ i.e. the 2 eigenvalues added by coefficients of Lagrange is < 0 (it is besides because of that that LDLT_SP is not guaranteed without precautions).

One seeks to obtain eigenvalues of the same order of magnitude:

$\mu_1 \mu_2 \mu_3$

μ

2

2

2

2 μ_3

μ_1

2

éq 6-1

If $\ll K$, then $\mu_3 \approx 0$, $\mu_2 \approx K$: it is not the sought result.

If $\gg K$, then $\mu_1 \approx 2\mu_2 \approx \mu_3$

2

3

1

The three eigenvalues are then in absolute value about which is an arbitrary constant very large in front of K . This solution is not that which one will retain because the value K is in the case general (with great number of ddl) of an order of magnitude comparable with the other eigenvalues of system.

One will choose rather:

=

$$K \mu_1 - 2k$$

$$\mu_2 \approx 2k$$

$$\mu_3 \approx K$$

Practically in *Code_Aster*, one chooses a value of single for all the system. This value is the average of the extreme values of the diagonal terms associated the physical ddl: $(\min(a) + \max(a)) / 2$

II

II)/2. Moreover, one takes =.

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7

Clean modes and parameters of Lagrange

7.1 Introduction

This paragraph wants to answer the two following questions:

Q1:

Which is the system of values (and vectors) clean **tiny room** to be solved when one

mechanical model is subjected to constraints **linear** kinematics

homogeneous?

Q2:

Which is the model dualized (with parameters of Lagrange) equivalent or precedent?

7.2

Mechanical problem to solve

One supposes a mechanical system already discretized by finite elements.

The nodal unknown factors are noted $\mathbf{U} = \{U\}$ ($I = N$

I

1,).

Nodal displacements are not all independent: there is p ($< N$) linear relations

homogeneous between these displacements: $\mathbf{B} ()$

$0 (J 1, p$

$J \mathbf{U} =$

$=$

$).$

These linear relations are independent between them, i.e. the row of the matrix \mathbf{B}

containing the coefficients of the p linear relations is p .

That is to say \mathbf{K} the matrix of rigidity of the mechanical system without constraints.

That is to say \mathbf{M} the matrix of mass of the mechanical system without constraints.

Which is the system with the eigenvalues to solve to find the modes clean of the structure constraint?

7.3 System

reduced

Let us notice that if one writes the linear relations kinematics in the form:

$\mathbf{B} \mathbf{U} = 0$

éq 7.3-1

where:

\mathbf{B} is a matrix $p \times N$

\mathbf{U} is the vector of nodal unknown factors \mathbb{R}^N

then:

$\mathbf{B} \mathbf{U}'' = 0$

éq 7.3-2

and the relation is also valid for speeds.

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Moreover, if **B** is of row p , then there is a **square** submatrix of **B** of row p . Notons **B1** this submatrix.

Then let us make a partition of the unknown factors of **U** in **U1** and **U2** such as:

U1

B U = 0

[

B

B]

0

1

2

U

=

2

U

p

1

R

U

Np

2

-

R

B1

= matrix $p \times p$

B2 = matrix $p \times (N - p)$

The linear relations can then be written:

B U + B U

1

1

2

2

= 0

what makes it possible to express the **U1** unknown factors according to **U2** since **B1** is invertible.

U

= - **B** 1

- **B** U

1

1

2

2

éq 7.3-3

Stamp reduced rigidity:

1

The elastic deformation energy of the not forced discretized structure is W

T

def =

THE U.K.U.

2

If one partitionne the matrix **K** in the same way that one partitionné **U**, one obtains:

K

K

1

12

$K = T$

K

K

12

2

then:

2 W

T

T

T

T

T

def

= **U1 K1 U1 + U2 K2 U2 + U2 K12 U1 + U1 K12 U2**

Let us introduce the linear constraints then [éq 7.3-3]:

2

T

T

- T

1

W

-

T

def
= **U2 B2 B1 K1 B1 B2 U2 + U2 K2 U2**

T
1
-
T
T
- *T T*
- **U2 K12 B1 B2 U2 - U2 B2 B1 K12 U2**

T
=
~
U2 K2 U2

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with:

~

K
= **K + BT BT K B 1**

- **B - K B 1**

- **B - BT B-TKT**

2

2

2

1

1

1

2

12
1
2
2
1
12

éq 7.3-4

It is thus seen that one expressed the deformation energy of the reduced structure like a form bilinear of **U2**. The nodal unknown factors of **U1** **were eliminated**. The nodal unknown factors **U2** **are forced**.

Stamp of reduced mass:

The same partition for the matrix of mass **Mr**. We adopt can write the relation [éq 7.3-2]:

$$\mathbf{B} \mathbf{U}'' + \mathbf{B} \mathbf{U}$$

1
1
2 "2
= 0

Same calculation as previously then leads us to:

2 *W*
T
cin
=
" ~

$$\mathbf{U2} \mathbf{m2} \mathbf{U2}$$

with:

$$\mathbf{M} = \mathbf{M} + \mathbf{B}^T \mathbf{B}^T \mathbf{M} \mathbf{R} \mathbf{B} \mathbf{1} - \mathbf{B} - \mathbf{M} \mathbf{r} \mathbf{B} - \mathbf{1} \mathbf{B} - \mathbf{B}^T \mathbf{B} - \mathbf{T} \mathbf{M} \mathbf{T}$$

2
2
2
1
1
1
2
12
1
2
2
1
12

éq 7.3-5

Conclusion:

The system to solve to find the modes (and the frequencies) clean of the structure forced is:

$$\begin{matrix}
 2 \\
 \sim \\
 2 \sim \\
 \text{To find them } (N - p) (\\
 N p \\
 \mathbf{X}_i, I) \mathbf{R} \\
 \times \mathbf{R} \text{ such as: } (\mathbf{K}_2 - I \mathbf{m}_2) I \\
 \mathbf{X} \\
 = 0 \\
 \sim \\
 \sim
 \end{matrix}$$

with \mathbf{K}_2 and \mathbf{m}_2 defined by [éq 7.3-4] and [éq 7.3-5].

Application to the blocked ddl:

In this case:

$$\begin{matrix}
 \mathbf{B} \\
 \\
 = \mathbf{I} \\
 1 \\
 \mathbf{B} \\
 2 = 0 \\
 \sim \\
 \sim
 \end{matrix}$$

from where: $\mathbf{K} = \mathbf{K}$

$$\begin{matrix}
 2 \\
 2 \text{ and } \mathbf{M} = \mathbf{M} \\
 2 \\
 2
 \end{matrix}$$

i.e. it is enough “to forget” in \mathbf{K} and \mathbf{M} the lines and columns corresponding to the blocked ddl.

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7.4 System

Dualisé

We saw with [§5] that the taking into account of the coefficients of Lagrange (double) in a matrix **A** led to the **A'** matrix:

$$\mathbf{A} \mathbf{T} \mathbf{B} \mathbf{T} \mathbf{B} \mathbf{T}$$

$$\mathbf{A}' =$$

$$\mathbf{B}$$

$$- \mathbf{I} + \mathbf{I}$$

$$\mathbf{B}$$

$$\mathbf{I}$$

$$- \mathbf{I}$$

where: $\cdot \mathbf{B}$ is the matrix of the conditions kinematics: $\mathbf{B} \mathbf{U} = 0$,

.

+

R arbitrary

0 ,

.

R arbitrary 0,

Let us apply the dualisation of the C.L to the matrices **K** and **M**, by partitionnant the ddl in **X**, **X**

1

2 like

with [§7.3]. We obtain the problem with the eigenvalues according to:

T

T

T

T

K

K

1

12

K
B1
K
B
M
M
1
1
12
m
B1
m
B1

T
T

T
T
(K
K

B
B
M
M

B
B
S)
12
2
K
2
K
2
2
2
12
2
m
2
m

2

-

$$\mathbf{X} = 0$$

K

$$\mathbf{B1} K$$

$$\mathbf{B2} - K$$

$$\mathbf{I} + K$$

\mathbf{I}

m

$$\mathbf{B1} m$$

$$\mathbf{B2} - m \mathbf{I} m \mathbf{I}$$

K

$$\mathbf{B1} K$$

$$\mathbf{B2}$$

K

$$\mathbf{I} - K$$

\mathbf{I}

m

$$\mathbf{B1} m$$

$$\mathbf{B2}$$

m

$$\mathbf{I} - m$$

\mathbf{I}

for an own pulsation and a clean vector X of this system, one can write:

$$\mathbf{K} \mathbf{X} + \mathbf{K} \mathbf{X} + \mathbf{B} \mathbf{T}$$

1

1

12

2

1 (1

+ 2

) = 0

éq 7.4-1

K

X + K X + BT

12

1

2

2

2 (1

+ 2

) = 0

éq 7.4-2

(B X + B X

1

1

2

2 -

) (1 -2) = 0

éq 7.4-3

(B X + B X

1

1

2

2 +

) (1 -2) = 0

éq 7.4-4

with: **K = K - 2 M**

2

2

I

I

I; = *K*

- *m*

;

= *K* -

m

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N

= a number of ddls physical

The system (*S*) is of order ($N + 2 p$) if

·

p = a number of relations kinematicsThe characteristic polynomial in λ is **a priori** degree $N + 2 p$. Its term of higher degree is worth:*N*(- *m* $2 p$) (+ *m*)if *semi* is the *i*ème diagonal term of **Mr.** $i=1$ · one thus sees **that if $m \neq 0$** , the term of higher degree is $\neq 0$ (because the *semi ones* are > 0) and thusthe dualized system (*S*) has more eigenvalues than the reduced system: ($N - p$). Both

systems are thus not equivalent. It is what one notes on the example of [§7.5],

· let us choose

N = N = 0:

[éq 7.4-3] and [éq 7.4-4]

 $1 = 2$

1

-

1

X= - **B1 B2 X2**

[éq 7.4-1]

-1

- *T*

1
-
1
= 2 =
B1 (- K1 B1 B2 + K12) X

2
2
[éq 7.4-2]

-
-1
T
T
1
2
2 +
2
2 -
-
2
1
(-
-
K
B
B X
K X
B B
K B
B

1
1
2 + **K 12) X**
12
1
2
= 0
(~ - 2 ~

K
M2) X
2
2
= 0
with: ~K

= - **KT B 1**

- **B + K + BT BT K B-1 B - BT BT K**

2

12

1

2

2

2

1

1

1

2

2

1

12

~

M

= - **MT B 1**

- **B + M + BT BT MR. B 1**

- **B - BT BT M**

2

12

1

2

2

2

1

1

1

2

2

1

12

~

~

It is noted that the definitions of **K 2** and **m2** are identical to those of the equations [éq7.3-4] and [éq 7.3-5].

It is thus seen that any clean vector **X** of the dualized system is also clean vector of reduced system (with the same own pulsation) if one projects it on *U2* space.

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Reciprocally, any clean vector **X2** of the reduced problem can be prolonged in a clean vector system dualized **XT**

[**XT**, **XT** T T

= 1 2 ,1 ,2].

with:

$$\mathbf{X} = -\mathbf{B}^{-1} - \mathbf{B} \mathbf{X}$$

1

1

2

2

1

= -

BT

1

-

1

1 (- **K B**

B + K

1

1

2

12) **X2**

2

1

= -

BT

1

-

2

1 (- **K B****B + K**

1

1

2

12) **X2**

2

The two systems are thus equivalent, they have the same clean modes and the same values clean.

The dualized system, although of size higher than the reduced system, does not have more values clean than the reduced system (the dimension of clean space is the same one).

Conclusion:

The dualized system is equivalent to the reduced system as soon as one chooses $\mathbf{m} = \mathbf{m} = \mathbf{0}$, i.e. if one takes the matrix of dualized rigidity but which one does not modify the matrix of mass. It is what is made in Aster.

7.5 Example

That is to say the system:

K

m

m

u1

u2

K- **K**

m 0

K =**M**

=

- **K****K**

0

m

the constraint **U** is added to him

U

1 +

2 = 0

()

0; That is to say =.

The reduced system is then:

.

K = K; K = K; K = - K; M = m; M = m; M

1

2

12

1

2

12 = 0

.

B =; B

1

2 =

~

= K (1+) 2

~

K

; M

2

2

2

=

(m1+)

2

K 1

2

(+)

=

; X = 1

m 1+ 2

2

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It is noted that the eigenvalue 2 depends on the report/ratio =.

K

If = 0

2

,

= m

2

If = 1

2

K

= m

If

2

K

m

Let us choose (= =)

1 to simplify and let us write the dualized system:

K

- K

m

0

X

K

K

m

m

1

- K
 K

0
 m

X
 K
 K
 m
 m

2

-
= 0
 K

K

- K

K

m

m

- m

m

1

K
 K
 K
 $- K$
 m
 m
 m
 $- m^2$

the eigenvalues of this system are:

K
 2
 $K K 2$
 $=$
 K
,

m
 m

It is noted that one finds the real eigenvalue (the 4°), but that one finds 3 eigenvalues parasites due to nonthe nullity of m and Mr .

If one chooses $m = m = 0$, calculation shows that the characteristic polynomial is degree 1 and that its only solution is:

K
 2
 2
 $=$
 m

X =

{-1, + }

1
who is the sought solution.

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7.6 Conclusions

· If

K and **M** are the matrices of rigidity and mass of a not-constrained system.

· If the linear constraints can be written in the form:

[

U

B

B] 1

0

B

1

2

invertible

U =

with 1

2

· Alors the clean modes of the forced structure are those of the reduced system:

(~

2 ~

K - M2) X

2

2

= 0

with:

~

K

= - **KT B 1**

- **B + K + BT BT K B 1**

- **B - BT BT K**

2

12

1

2

2

2

1

1

1

2

2

1

12

~

M

= - **MT B 1**

- **B + M + BT BT MR. B 1**

- **B - BT BT M**

2

12

1

2

2

2

1

1

1

2

2

1

12

· The dualized system (double Lagrange) which is written:

(~~

~

~

-2

)~~

K

MR. X = 0

with:

~

~

XT

= [**X X**

1

12

1

2

]

K

K

BT BT

1

12

1

1

~

~

KT

K

BT BT

K = 12

2

2

2

B

B

1

2

- **I**

I

B

B

I

1
2
- **I**
M
M
1
12
0

0
T

~
~
M
M
0 0
M = 12
2

0
0
0

0

0
0
0

0

has the same solutions then as the reduced system.
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Appendix 1

That is to say problem 1

min $J(\mathbf{U})$

$\mathbf{U} \in \mathbf{V}$

1

$J(\mathbf{U}) =$

$(\mathbf{W}, \mathbf{U}) - (\mathbf{B}, \mathbf{U})$

2

\mathbf{V} under space refines $\mathbf{RN} = \{\mathbf{U} \in \mathbf{RN} \text{ such}$

\mathbf{C}

that

$\mathbf{U} \in \mathbf{D}$

\mathbf{I}

$\mathbf{I} = 0, I = 1$

$\}, \}$

p

\mathbf{W} and \mathbf{B} are defined on \mathbf{RN}

\mathbf{W} positive symmetrical matrix of order N .

Problem 2

To find $\mathbf{U} \in \mathbf{V}$ such as: $(\mathbf{W}, \mathbf{v}) - (\mathbf{B}, \mathbf{v}) = 0 \quad \forall \mathbf{v} \in \mathbf{V}$

\mathbf{V}

0

0

0

0)

$\mathbf{V} = \{\mathbf{U} \in \mathbf{RN} \text{ such as } \mathbf{C} \in \mathbf{U} - \mathbf{D}$

\mathbf{I}

$\mathbf{I} = 0, I = 1, \}$

p

$\mathbf{V}_0 = \{\mathbf{v} \in \mathbf{RN} \text{ such}$

\mathbf{C}

that

$\mathbf{U} \in$

0

\mathbf{I}

$= 0, I = 1, \}$

p

One will show that the two preceding problems are equivalent.

Let us notice first of all that problem 2 is equivalent to problem 2'.

Problem 2'

To find \mathbf{U}, \mathbf{V} such as: $(\mathbf{With}, \mathbf{v} - \mathbf{U}) - (\mathbf{B}, \mathbf{v} - \mathbf{U}) = 0 \mathbf{v} \mathbf{V}$

$\mathbf{V} = \{\mathbf{U} \text{ RN such as } \mathbf{C} \mathbf{U} - \mathbf{D}$

 \mathbf{I}

$\mathbf{I} = 0, I = 1, \}$

 p

There is indeed bijection between the whole of the $\{\mathbf{v} - \mathbf{U}, \mathbf{U} \mathbf{V}, \mathbf{v}\}$

\mathbf{V} and the $\mathbf{V0}$ unit.

Let us show that problem 2' is equivalent to problem 1:

That is to say \mathbf{U} solution of 2'

Then, $\mathbf{v} \mathbf{V}$

1

1

$J(\mathbf{v}) - J(\mathbf{U}) =$

$(\mathbf{AV}, \mathbf{v}) - (\mathbf{B}, \mathbf{v}) - (\mathbf{With}, \mathbf{U}) + (\mathbf{B}, \mathbf{U})$

2

2

1

1

=

$(\mathbf{AV}, \mathbf{v}) - (\mathbf{With}, \mathbf{v} - \mathbf{U}) - (\mathbf{With}, \mathbf{U})$

2

2

1

1

=

(

S

$\mathbf{AV}, \mathbf{v}) - 2 (\mathbf{With}, \mathbf{v}) + (\mathbf{With}, \mathbf{U}) =$

$(\mathbf{A} (\mathbf{UV}, \mathbf{UV}) 0$

2

2

 \mathbf{v} \mathbf{U} \mathbf{v} \mathbf{U} \mathbf{v} \mathbf{U} \mathbf{v} \mathbf{U} \mathbf{v} \mathbf{U} \mathbf{v} \mathbf{U}

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Let us calculate first of all the derivative of $J(\mathbf{U})$: $\mathbf{U} \mathbf{V}, \mathbf{v} \mathbf{V}$

,

0

0

 $J(\mathbf{U} + \mathbf{v}$ 0) - $J(\mathbf{U})$ $I(\mathbf{U}) \mathbf{v} \mathbf{0} = \lim$

0

=

 $\lim (\mathbf{With}, \mathbf{v} \mathbf{0}) + (\mathbf{AV}, \mathbf{v}$

0

0) - $(\mathbf{B}, \mathbf{v} \mathbf{0})$ $(\mathbf{With the b}) \mathbf{v}$

0

0

2

=

-

That is to say \mathbf{U} the solution of Pb1 $\mathbf{v} \mathbf{V}$, let us pose $\mathbf{v} = \mathbf{v} - \mathbf{U}$; $\mathbf{v} \mathbf{V}$

0

0

0.

 $J(\mathbf{U} + \mathbf{v} \mathbf{0}) - J(\mathbf{U})$ 0 $(I(\mathbf{U}) \mathbf{v} \mathbf{0} 0, \mathbf{v} \mathbf{0} \mathbf{V} \mathbf{0})$ It is seen that $I(\mathbf{U})$ who am a linear form on $\mathbf{V} \mathbf{0}$ must be systematically positive. This is not

possible that if this form is identically null.

One concludes from it that:

$I(\mathbf{U} \mathbf{v} - \mathbf{U}$

=

With - $\mathbf{B} \mathbf{v} - \mathbf{U}$

= 0

) (

)

(

)(

)

,

v

V

S

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Appendix 2

Definitions, notations

A is the matrix of unconstrained rigidity ($N \times N$) (symmetrical and positive)

C is the matrix of blocking: $\mathbf{C} \mathbf{u} - \mathbf{D} = 0$ (**C** stamps $N \times p$ ($p <$

N)

U is the vector of physical ddl R^N

1 is the vector of the first ddl of Lagrange R^p

2 is the vector of the second ddl of Lagrange R^p

X = (**U**, **1**, **2**) $R^N \times R^p \times R^p$

One notes:

.

U the whole of the physical ddl,

.

1 the whole of the first ddl of Lagrange,

.

2 the whole of the second ddl of Lagrange.

+

R

K stamps of a symmetrical nature $2p + N$

WITH CT

CT

K =

C

- **L L**

C L - L

The matrix **K** written above corresponds to a certain classification of the unknown factors:

X = (**U**, 1, 2)

The genuine matrix **K** which one seeks to show that it is factorisable by LDLT without permutation is not written with this classification. The only rule of classification taken into account is following:

Regulate R0:

Both ddl of Lagrange associated with an equation with connection **C U - D**

I

I = 0 frame the ddl

physiques constrained by this equation.

Thereafter, to simplify the writing, = 1 will be taken.

One seeks to show that very under **Ki** matrix of **K** is invertible.

That is to say under **Ki** matrix given. It corresponds to a division of the ddl: those of row **I**, those of row $> I$.

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We will note:

~

U the subset of U corresponding to the ddl of row I.

~

~

U the subset of U corresponding to the ddl of row > I.

L

1

2

1

2

1 is the whole of the couples (1, 1)

ruffle

such as

(1)<

(

ng

ruffle

1)

ng

I

L1 = {1

2

1}; L2

1

1 = {1 }

L

1

2

1

2

3 is the whole of the couples (3, 3)

ruffle

such as I <

(3)<

(3

ng

ruffle

)

ng

$L1 = \{1$

2

$\}; L2$

3

$\} = \{3\}$

L

1

2

1

2

2 is the whole of the couples (2, 2)

ruffle

such as

(2)

ng

$I < \text{ruffle } (2)$

ng

$L1 = \{1$

2

$\}; L2$

2

$\} = \{2\}$

J

There are $L =$

L_i

$\# .$

$i=1,3$

$j=1,2$

The matrix **C** can cut out in 3 parts corresponding to cutting (L1, L2, L3)

C1

C2

C3

~ ~ ~

Each matrix **C**

(**U**, **U**)

I can cut out in 2 parts corresponding to cutting

~

C

~

~

I

Ci

~ ~ ~

Matrix A can cut out in 4 parts corresponding to cutting (**U**, **U**)

~A

With

With =

~

~

T

With

With

Using these notations, the problem to be solved is to show that the **Ki** matrix is invertible.

-

~

I

I

0

C

1

~

I

- **I**

0

C

K

1

I

=

~

0

0

- **IC**

2

~

~

~

~
CT CT CT

With

1
1
2

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1

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1

This matrix corresponds to the vector $\mathbf{Xi} = 1$

2

~

U

It should be shown that: $\mathbf{K} \cdot \mathbf{X} = 0$

X

I

I

$I = 0$

The problem is equivalent to:

Problem 1:

- 1

2

~ ~

$$1 + 1 + \\ 1 \\ \mathbf{C U} = 0$$

$$\sim \\ \sim \\ \mathbf{U} = 0$$

$$1 \\ 2 \\ \sim \\ 1 - 1 + \\ 1 \\ \mathbf{C U} = 0 \\ ($$

$$S) = \\ \sim$$

$$1 \\ 2 \\ 0 \\ 1 \\ 1 = 1 =$$

$$- \\ \sim \\ 2 + \mathbf{C^2 U} =$$

$$0$$

$$1 \\ 0 \\ \sim \\ 1 \\ 2 \\ \sim \\ 1 \\ \sim \\ 2 = \\ T$$

$$1 \\ \mathbf{C (1 + 1)}$$

$$T$$

$$+ C^2_2 + A \cdot u = 0$$

Case general:

~
It is supposed that U; L1; L2

(S)

$$1$$

$$2$$

$$1$$

$$= 1$$

éq An2-1

$$\sim \sim$$

$$C U$$

$$1$$

$$= 0$$

éq An2-2

$$1_2 = \sim \sim$$

$$2$$

éq An2-3

$$C U$$

$$\sim T$$

$$\sim T \sim$$

$$\sim$$

$$2$$

$$1$$

$$+ ($$

$$\sim$$

$$C$$

$$2$$

$$C C^2 + A) U = 0$$

éq An2-4

From [éq An2-4], one deduces:

$$T T$$

$$T \sim T \sim$$

$$\sim$$

$$2 \sim \sim$$

$$1$$

$$\sim$$

$$+$$

$$($$

$$\sim$$

u.a.

U

2

C

2

$$\mathbf{C} + \mathbf{A}) \mathbf{U} = 0$$

1 1

From [éq An2-2], one obtains:

$$\sim T \sim T$$

$$T \sim T \sim$$

~

u.a.

$$= 0 \sim u$$

1

$$(\mathbf{C} \mathbf{C} + \mathbf{A}$$

2

2

$$) \sim u = 0$$

$$\sim T \sim T \sim \sim$$

$$\sim T \sim \sim$$

u.a. C U + U With

2

2

$$= 0$$

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~

$$\sim T \sim$$

However **A** is symmetrical positive (under matrix of a positive matrix) and **C C**

2

2 is also a matrix

symmetrical positive, therefore this sum can be null only if the two terms are null.

$T T \sim$

$\sim \sim$

u.a.

\sim

C U

2

2

= 0

$\sim T \sim \sim$

U With =

0

$\sim \sim$

C U

1

2

= 0

2

= 0

éq An2-5

\sim

A is a positive matrix, one wants to show that:

$\sim T \sim \sim$

\sim

U To = 0 U = 0

éq An2-6

It remains us to be shown that: \sim

U = 0

1

and

= 0

1

.

\sim

U = 0

Let us prolong \sim

U on RN by $\sim \sim = 0 = (\sim, \sim \sim$

U

U

U U):

~

C With

U

~T

=

$$\mathbf{A} \mathbf{u} \sim = 0$$

U ker A

~

C U

C u~

1

0

1

~

~

$$\mathbf{C} \mathbf{u} = \mathbf{C} \mathbf{U}$$

2

$$= \mathbf{C} \mathbf{U}$$

2

=

0

C U

3

$$\mathbf{C} \sim \mathbf{u} \sim$$

3

0

~

Indeed $\mathbf{C} \mathbf{3} = 0$ but if not, there would not yet exist ddl of $\sim u$ constrained by equations taken into account (of row $> I$) what is contrary in R_0 .

The prolongation \mathbf{U} of \sim

\mathbf{U} is thus in the cores of \mathbf{A} and \mathbf{C} . One will show that it is then null.

Let us take again the problem with "simple Lagrange".

(

$$\mathbf{With} + \mathbf{C} \mathbf{T} = \mathbf{B}$$

$$\mathbf{S}2) = \mathbf{C} \mathbf{u} =$$

D

If u_0 is such as $Au_0 = 0$ and $Cu_0 = 0$.

If u_1 is solution of S2, it is seen that then $U + U_1$

u_0 is also solution. What is impossible but we suppose our problem posed physically well.

One concludes from it that $U = 0$

$U = 0$.

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$11 = 0$

[*éq An2-4*] gives:

\sim

CT 1

1 1

= 0

éq An2-7

\sim

\sim

\sim

In the same way that the rule imposing $R_0 C_3 = 0$, one sees that $C_1 = 0$.

[*éq An2-6*] gives:

\sim CT 1

0

CT 1

1
1
1 1
= ~
=
0
=

éq An2-8
CT 1

0
1
1

Let us reason by the absurdity: if 1

T1

1 0 is such as **C1** 1

= 0, it is that there is a combination

linear of the lines of **C1** which is null, which is contradictory with the fact that the lines of **C1** from/to each other are independent (physical problem good posed).

Thus $11 = 0$.

Particular case:

When one (or more) of the ~ sets

U, **L1**, **L2** are empty, the system (S) is simplified. One can

to check that the reasoning which one made in the case general, makes it possible to show similar results.

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Summary

One presents in this documentation a manner of modelling indeformable parts of structure, in small displacements and rotations, thanks to key word LIAISON_SOLIDE of AFFE_CHAR_MECA.

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1 Introduction

Key word LIAISON_SOLIDE of orders AFFE_CHAR_MECA and (AFFE_CHAR_MECA_F) allows to model an indeformable part of a structure.

The principle selected is to write linear relations between the ddls “solid” part; these relations expressing the fact that the distances between the nodes are invariable.

Important remark:

The relations expressing the indeformability of a solid are in general not linear.

linearization of the problem supposes that the problem can be solved in theory of “small displacements”. To be convinced some, let us take the example of a segment AB in 2D:

B

With

The indeformability of AB is written:

([

2

2

2

2

$X + dx$

With

$With) - (X + dx$

B

$B)] + ($

$[y + dy$

With

$With) - (y + Dy$

B

$B)] = (X - X$

With

$B) + (y - y$

With

$B)$

(

2

2

$dx - dx$

B

$With) + ($

$2 X - X$

B

$With) (dx - dx$

B

$With) + (Dy - Dy$

B

$With) + ($

$2 y - y$

B

$With) (Dy - Dy$

B

$To) = 0$

X, y, X, y

With

With

B

B co-ordinates of A and B

by noting dx , Dy , dx , Dy

With

With

B

B displacements of A and B

it is seen that the expression is quadratic in dx , dx , Dy and Dy

With

B

With

B. To be able to linearize it,

it is necessary to eliminate the quadratic terms and for this reason, one is obliged to suppose that them elements dx , dx , Dy and Dy

With

B

With

B are small compared to the length of AB.

This remark wants to say that one cannot use this key word when the structure becomes deformed (or turn) too much. In such situations, “to rigidify” a solid part, one is obliged to use one “hard” material (compared to the remainder of the structure).

2

Principle of the use of the key word

Key word LIAISON_SOLIDE is a key word factor répétable at will. With each occurrence of the word key, the user defines a “piece of model” which it wishes to rigidify.

This “piece of model” defined by key words GROUP_MA, GROUP_NO, MESH and NODE, one deduced the **list from the nodes** to be rigidified.

Once this drawn up list, one writes the linear relations necessary to express that the “piece rigid ” has nothing any more but the degrees of freedom of a solid (3 in 2D or 6 in 3D).

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3

Which are the treated cases of figure?

According to ddls carried by the nodes of the list of the nodes to rigidify, one places oneself in one of the four

following cases of figure. If one does not find oneself in one of these cases of figure, the code stops in fatal error

The cases 2DA and 2DB correspond to plane” or axisymmetric problems “.

The cases 3DA and 3DB correspond to 3D problems.

Case 2DA:

All the nodes of the list of the nodes to be rigidified carry the ddls DX, DY (and possibly DRZ) but they do not carry DRX, DRY and DZ and there is at least a node of the list of nodes to be rigidified which carries DRZ.

Case 2DB:

All the nodes of the list of the nodes to be rigidified carry DX, DY but they do not carry DRX, DRY and DZ.

Case 3DA:

All the nodes of the list of the nodes to be rigidified carry DX, DY, DZ (and possibly DRX, DRY, DRZ) and there exist a node of the list of the nodes to be rigidified which carries DRX, DRY, DRZ.

Case 3DB:

All the nodes of the list of the nodes to be rigidified carry DX, DY, DZ and there is not node list of the nodes to be rigidified carrying at the same time DRX, DRY, DRZ.

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Treatment of the cases 2DA and 3DA

In these 2 cases of figure, one could find a node of the list of the nodes to be rigidified which carried **all** degrees of freedom of the solid. That is to say this node.

in 2D: DX, DY, DRZ

in 3D: $DX, DY, DZ, DRX, DRY, DRZ$

That is to say a node M of the list of the nodes to be rigidified unspecified.

In theory of small displacements, the movement of a solid body is expressed by:

\mathbf{U} is of displacement of A

\mathbf{U}

$= \mathbf{U} + \mathbf{AM}$

With

M

With

where the vector rotation of the solid

4.1 Case

2DA

The linear relations are written:

X

$MR. A: DX (M) - DX ()$

With $+ y DRZ ()$

$To = 0$

with $\mathbf{AM} =$

y

$DY (M)$

$- DY ()$

With $- X DRZ ()$

$To = 0$

+ if M

por DRZ

you

$: DRZ (M) - DRZ ()$

$To = 0$

4.2 Case

3DA

$DX (M) - DX ()$

$WITH - DRY ()$

$WITH .Z + DRZ ()$

A. $y = 0$

$MR. A: DY (M) - DY () A DRZ () A.x + DRX (A) .z = 0$

$DZ (M)$

- $DZ ()$
 WITH - $DRX ()$
 A. $y + DRY ()$
 With $.x = 0$
 $DRX (M) - DRX ()$
 $To = 0$

+ if M
 por DRX
 you, $DRY, DRZ: DRY (M) - DRY ()$
 $To = 0$
 $DRZ (M) - DRZ ()$
 With =

0
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5

Treatment of the case 2DB

5.1 Case

general

With and B in the list of the nodes to be rigidified/ $D (A, B) 0$

· determination

of

:

nx

That is to say $\mathbf{N} = \mathbf{AB} \mathbf{K} =$

(\mathbf{K} unit vector according to O

N

Z).

y
(U

- U

B

With). $AB = 0$

U - U - K AB

B

With

(U - U

B

With) .n - (K AB) .n = 0

· since

AB 0, one can determine:

1

= (

DX B .N

DX A .N

DY B .N

DY B .N

K AB) (

() X -

() X + () y - () y)

.n

,

N

1

That is to say =

X

(

;

N

K AB)

=

=

.n

,

ny

· equations to be written:

-

(U - U) .AB

B

With

= 0

(1 equation for 2 points A and B)

X

-

M (A, B):

AM =

y

DX (M) - DX ()

With + y

,

,

,

,

(DX (B).n - DX

X

(a).n + DY

X

(B).n - DY

y

()

With ny) = 0

DY (M) - DY () A X

,

,

,

,

(DX (B).n - DX

X

()

WITH .N + DY

X

(B).n - DY

y

()

With ny) = 0

5.2 Case

private individuals

· list of the nodes to be rigidified = { }

With there is nothing to write *A* tear,

· list of the nodes to be rigidified = { *Have* } where all *Have* them have the same co-ordinates.

That is to say *As* the first node of the list of the nodes to be rigidified

DX

$(Have) - DX (A0) = 0$

WITH A

I

O it is necessary to write *DY*

$(Have) - DY (A0) = 0$

Note:

is unspecified

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Treatment of the case 3DB

6.1 Case

general

With, *B*, *C* in the list of the nodes to be rigidified such as *ABC* is a triangle of nonnull surface

6.1.1 Treatment of the points A, B, C and determination of the vector rotation

B

B = AB; C = AC; m = AM

B

That is to say **N = B C**

b' = bn; c' = C N

With

C

C

$$M, \mathbf{UM} - \mathbf{UA} = \mathbf{M}$$

éq 6.1-1

U

$$- \mathbf{U} = \mathbf{B}$$

.

B

With

for the points B and C: $\mathbf{U} - \mathbf{U} = \mathbf{C}$

C

With

$$\mathbf{U} - \mathbf{U} \cdot \mathbf{N} = .$$

B

B

With

,

éq 6.1-2

$$\mathbf{U} - \mathbf{U} \cdot \mathbf{C} = .$$

N

B

With

éq 6.1-3

$$\mathbf{U} - \mathbf{U} \cdot \mathbf{B}$$

B

With

$$= 0$$

éq 6.1-4

$$\mathbf{U} - \mathbf{U} \cdot \mathbf{N} = .$$

C

C

With

,

éq 6.1-5

$$\mathbf{U} - \mathbf{U} \cdot \mathbf{B} = -.$$

N

C

With

éq 6.1-6

$U - U .C$

C

With

$= 0$

éq 6.1-7

$U - U .C + U - U .B$

B

With

C

With

$= 0$

éq 6.1-8

Of the 6 equations concerning the points B and C,

-

3 are to be written: [éq 6.1-4], [éq 6.1-7] and [éq 6.1-8] (they do not utilize)

-

3 are used to determine:

$.B' = (UB - UA) .N$

$.c' = (CPU - UA) .n$ éq

6.1-9

$.n' =$

$(UB - UA) .C$

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6.1.2 Relations concerning a point M (A, B, C)

12

That is to say **U**

the vector

$ABCM$

: $(U A V$

, $A, WA, U B V$

, B, \dots, C

$W, U MR. V$

, $M, WM)$

R

the vector

3

(

$X, y, Z) R$

The equation [éq 6.1-9] can be written: $Mr. = MR. U$

1

2

$ABCM$

$b' b' b'$

X

y

Z

with $M = it$

it

it

X

y

$Z M$ is invertible bus ABC is of nonnull surface

1

1

N

N

N

X
y
Z
- nx - ny - N N N N 0 0 0 0 0 0

Z
X
y
Z

and
M2 = - nx - ny - N

0
0
0
N
N
N
0 0 0

Z
X
y
Z

- cx - cy - C C C C 0 0 0 0 0 0

Z
X
y
Z

= -
MR. M

1
U
1
2 .

ABCM
éq 6.1-10

The equation [éq 6.1-1] **U**

- **U - M**
M

With
= 0 can be written:

M. U

M

4

ABCM +

3. = 0

éq 6.1-11

0

- *M*

M

Z

y

with

M = M

0

Z

- *M*

where

M

3

X

= (*M, M, M*

X

y

Z)

- *M*

M

0

y

X

- 1

1

and

M4 =

- 1

0

0

1

- 1

0

1

M. U

+ **Mr. Mr. M**

1

. **U**

= 0 **M. U**

4

ABCM

3

1

2

ABCM

5

= 0

ABCM

M (A, B, C), the 3 equations should be written corresponding to the 3 lines of the matrix

M = M + M - Mr. M

1

5

4

3

1

2

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6.1.3 Summary of the equation to be written

· calculation

of

B, C, N, b', it

· calculation

of

M-1M

1

2

U - U B

B

A. = 0

· for the points B and C: **U - U .C**

C

With

= 0

U - U .c + U - U B

B

With

C

A. = 0

·

M (A, B, C):

- calculation

of

M = M + MR. M 1

- **M**

5

4

3

1

2

-

writing of the 3 equations corresponding to **M**

6.2 Case

private individuals

· list of the nodes to be rigidified = { }

With there is nothing to write **A** tear,

· list of the nodes to be rigidified = { *Have* } where all *Have* them have the same co-ordinates.

That is to say *As the* first point of the list of the nodes to be rigidified

$$DX (Have) - DX (A0) = 0$$

$$WITH A DY (Have) - DY (A0)$$

$$I$$

$$O$$

$$= 0$$

$$DZ$$

$$(Have) - DZ (A0) = 0$$

is unspecified, which does not pose a problem.

· list of the nodes to be rigidified = {*Have*} where all *Have* them are aligned (right).

The solid {*Have*} does not have more whereas 5 movements of possible rigid bodies.

It misses rotation around.

That is to say:

- two

points

With and *B/*

AB 0

-

B = AB

-

n1 a vector not no orthogonal with **B** (thus with)

-

N = bn

2

1

B

B

n1

With

N2

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· not b: $\mathbf{U} - \mathbf{U} = \mathbf{B}$

B

With

$(\mathbf{U} - \mathbf{U}) \cdot \mathbf{B}$

B

With

= 0

éq 6.2-1

$(\mathbf{U} - \mathbf{U}) \cdot \mathbf{n1} = (\mathbf{n1}) \cdot \mathbf{b}$

B

With

éq 6.2-2

$(\mathbf{U} - \mathbf{U}) \cdot \mathbf{n2} = (\mathbf{N2}) \cdot \mathbf{b}$

B

With

éq 6.2-3

-

the equation [éq 6.2-1] is to be written

-

the equations [éq 6.2-2] and [éq 6.2-3] are used to calculate

The component of on \mathbf{B} is unspecified, one does not hold account of it:

= $1n1 + 2n2$

$(\mathbf{n1}) = 2n1 \mathbf{N}$

2

that is to say $K = \mathbf{n1n2} \cdot \mathbf{b}$

$K 0$

(

$K 1 = - \mathbf{U} \mathbf{B} - \mathbf{U} \cdot \mathbf{n}$

With

2

\mathbf{N}

2)

(

)

= 1n2 N
1

$K_2 = - (U B - U A) \cdot n_1$
 $K =$
 $K_1 n_1 +$
 $K_2 n_2$
has

That is to say N

(
 $B,$
 B
 $W, U MR. V$
 $, M, WM)$
 $1 = B$
; N

2 =
;

U

$ABM = U$
 V

,
 $W, U V$

With
With
With

$B,$
R9

C

$K = M_1 U ABM$ with: $M_1 = [M,$
 $2 - M,$
2

] 0

0

has

- *B*

has

- *C*

M2 =

B - has

0

B

- *C*

C

- *has*

C - B

0

M (A, B)

U M - U A - m

= 0

m = AM = (M, M, M

X

y

Z)

MR. U

4

ABM + m3 = 0

0

- *M*

M

Z

y

- 1 0

0

1

.
with $\mathbf{m3} = Mz$

0
- MX
 $\mathbf{M4} = 0 -$

1
0
0
1

-

M
 M
 y
 X
0

0
0
- 1

1
1
 $\mathbf{MR. U}$
5

$ABM = 0$
with

$\mathbf{M5} = \mathbf{M4} + \mathbf{M. M}$

K
3
1

for each point M , it is necessary to write the 3 equations corresponding to the 3 lines of the matrix $\mathbf{M5}$.

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· summarized equations to be written:

- calculation

of

B, N, N,

1

2 **K**

- calculation

of

M1

-

for point b: **U - U B**

B

A. = 0

-

M (A, B)

1

- calculation

of

M = M + M. M

5

4

3

1

K

-

writing of the 3 equations corresponding to **M5**

7

How to detect the particular cases?

In the paragraphs [§6] and [§7], we saw that it could arrive of the particular cases when certain nodes geometrically were confused or aligned on the same line.

The numerical criteria selected to detect these particular cases are:

· 2 points A and B are confused if:

AB

-

10 6. *DMIN*

· 3 points A, B, C are aligned if:

(AB AC

) 1/2

-6

10 .*DMIN*

where: *DMIN* notes the length of smallest stops meshes of the grid.

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Document: R3.03.03

Connection 3D - Beam

Summary:

This document explains the principle retained in *Aster* to connect a modeling continuous medium 3D and a modeling beam.

This connection results in 6 linear relations connecting displacements of the whole of nodes “3D” (3 degrees of freedom per node) dependent with the node of beam with the 6 degrees of freedom of this node.

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1**Presentation of the document**

This document is shown of a note of S. ANDRIEUX [bib1] in the course of publication. We added there some precise details concerning the establishment of modeling “connection 3D/Poutre”. Order AFFE_CHAR_MECA [U4.25.01] key words LIAISON_ELEM and OPTION: “3D_POU”.

2**The connection 3D-beam****2.1****Objectives and excluded solutions**

When one wishes to finely analyze part of a slim structure complexes [Figure 2.1-a], one can, to minimize the size of the grid to be handled, to want to represent the structure by a beam “far” from the part being analyzed. The goal of schematization by a beam is to bring conditions with the realistic limits at the edges of the part modelled and with a grid in continuous medium 3D. The connection

3D-beam must thus meet the following requirements:

P1

To be able to transmit the efforts of beam (torque) to the grid 3D

P2

Not to generate “parasitic” constraints (even of stress concentration), because it would then be necessary to place the connection far from the zone to be sufficiently analyzed so that these disturbances are attenuated in the zone of study.

P3

Not to support the conditions kinematics or the static conditions of connection one by report/ratio with the other. It must be equivalent to bring back a torque of effort or displacement to limits of the field 3D.

P4

To admit unspecified behaviors on both sides of the connection (elasticity, plasticity...) and to also allow a dynamic analysis.

modeling 3D

modeling beam

modeling beam

fissure

Appear 2.1-a

If these objectives are achieved one will be able to also use the rules of connection to deal with the problem

embedding of a beam in a solid mass 3D. However the distribution of the constraints in solid mass around embedding will remain rather coarse and will have to be used with precaution. It is preferable to net the connection in 3D then to prolong the starter of the grid 3D of the section of beam by one of the elements of beam with connection 3D/Poutre [Figure 2.1-b].

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modeling beam

modeling 3D

Appear 2.1-b

Within sight of objectives 1 to 4, one can eliminate two current techniques of connection right now:

1) the first which brings back all the connection to the treatment of conditions of connections between the points

in opposite with the intersection of the neutral axis of the beam and the solid 3D. Except the difficulty of to correctly define the "specific" rotation of the material point pertaining to the solid 3D, one concentrate the efforts (concentrated reaction, couple) in this point and one breaks symmetry

static kinematics/by privileging a particular kinematics.

2) the second solution which imposes completely a displacement of beam (NAVIER-BERNOULLI) at the points of the solid mass 3D being with the intersection of the solid 3D and the section of the beam. In elasticity, one knows that the assumption of indeformability of the sections in their plan is only one approximation. Correct from the energy point of view for the beam, it leads to stress concentrations in the vicinity of the limits of the section of junction for the solid 3D.

Note:

It goes without saying that all that is presented here is valid only on the assumption of small disturbances (small displacements).

2.2 Orientation

We will leave the machine elements of the connection:

- the field of definite vector forced $\cdot n$ on the trace of the section S of the beam on solid mass 3D, N being the normal in the plan of S ,
- and the field of definite $\mathbf{u}3D$ displacement on this same field, for the three-dimensional solid, like:

· it

torque

\mathbf{T} of the efforts of beam in the geometrical centre of inertia G of S ,

- and the torque \mathbf{D} of displacements of beam in this same point, for the beam.

These mechanical magnitudes are connected by:

- conditions of kinematic continuity,
- equilibrium conditions of the connection.

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The first conditions are the conditions of connections to impose in an approach “displacement”, the seconds result from the weak formulation of balance via the virtual work of the actions from

contact between beam and solid mass (which is not other than the expression of the “principle” of the action and the reaction

writing for the interface S). On surface S , one has indeed for any virtual displacement $(v, T,)$ licit:

$$\dots$$

$$N v dS =$$

.

+

F T Mr.

éq 2.2-1

S

where:

· T and is respectively the translation and the infinitesimal rotation of the beam: $D = (T,)$

· F

M

and

are respectively the resultant and the moment in the beam at the point of connection:

$$\mathbf{T} = (F, M)$$

The first member of this equality will provide the scalar product thanks to which one will define “component beam” of a field of displacement 3D defined on S . By using this scalar product, one the symmetry of the approach between conditions kinematics and statics of connection (P3) will ensure like

the possibility of treating unspecified behaviors on both sides of the connection (P4) since none aspect of behavior does not appear in the equality of balance used.

The step:

One will break up the field of displacement 3D into a part “beam” and a part “complementary”. This will lead us to rather naturally define the conditions of connection kinematics between beam and solid 3D like the equality of the displacement (torque) of beam and of beam part of the field of displacement 3D [(§ 2.3)]. Once this made, the equality [éq 2.2-1] us will allow to interpret in static term the conditions of connection and to thus reach the conditions of static connection [(§2.4)].

2.3

Decomposition of displacement 3D on the interface

The junction between the three-dimensional solid B and the beam of section S is supposed to be plane and of

normal N parallel with the tangent with the beam at the point of contact G , geometrical centre of inertia section S [Figure 2.3-a].

$N =$

G

G

N

S

(A) Normal with the solid = tangent with the beam

(b) Normal with the solid \circ tangent with the beam

Appear 2.3-a

One thus excludes the case (b) where the beam “does not leave” by perpendicular to surface the solid. It is necessary

to understand well that this restriction is necessary to the coherence of the connection such as it is considered here

since the theory of the beams knows only cuts normal with average fibre: the condition of balance [éq 2.2-1] no direction has if S is not the cross-section of the beam. If this condition is violated, one will be able to modify the grid to carry out it as the diagram indicates it below.

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$N =$

S

G

Appear 2.3-b

One notes:

x_2

x_1

G

S

.

(G, E, E

1

2) a principal reference mark of geometrical inertia of S having for origin the center of inertia G , and (X, X

1

2) associated co-ordinates,

.

N or \mathbf{e}_3 the normal in the plan S , outgoing with the solid mass $3D$,

.

$3 = ($

$\mathbf{E}, \mathbf{E}, \mathbf{e}_3$) the alternate shape of the mixed product of the basic vectors,

finally \mathbf{I} the geometrical tensor of inertia of S (diagonal in the reference mark $(\mathbf{E}, \mathbf{E}$

$\mathbf{1}$

$\mathbf{2}$)) and $A = S$ the surface

section S .

Let us recall that the tensor of inertia \mathbf{I} can be defined in an equivalent way by an application linear (mixed representative):

$$(\mathbf{I}\mathbf{U}) = \mathbf{G}\mathbf{M}(X) (\mathbf{U} \mathbf{G}\mathbf{M}(X)) dx$$

S

or a symmetrical bilinear application (covariant representative):

$$(\mathbf{I}\mathbf{U}, \mathbf{V}) = (\mathbf{U} \mathbf{G}\mathbf{M}(X)). (\mathbf{V} \mathbf{G}\mathbf{M}(X)) dx$$

S

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These two expressions will be useful, in the reference mark $(G, \mathbf{E}, \mathbf{E}, \mathbf{E}$

$\mathbf{1}$

$\mathbf{2}$

$\mathbf{3}$) the matrix representative of \mathbf{I} is:

I

0

0

1

[

$$\mathbf{I} = 0 \quad I_2$$

$$0 \quad I_1 + I_2$$

with I geometrical moment of inertia of S compared to the axis (G, \mathbf{E}) . By convention indices Greeks take values 1 or 2.

Useful space for the fields of displacements and vectors forced definite on S be V

\mathbf{T}

= $L^2(S)$ 3. One introduces space \mathbf{T} of the fields associated with a torque (defined by two vectors):

$$\mathbf{T} = \{ \mathbf{v} \in V / (\mathbf{T}, \cdot) \text{ such}$$

\mathbf{v}

$$\text{that } (M) = \mathbf{T} + \mathbf{G} \}$$

\mathbf{M}

éq 2.3-1

For the fields of displacement of S , \mathbf{T} is the translation of the section (or the point G), infinitesimal rotation and fields \mathbf{v} are displacements preserving the section S plane and not deformation (Assumptions of NAVIER-BERNOULLI).

For the fields of vectors forced, S \mathbf{T} is the resultant F of the actions applied to S , and $(\mathbf{I}) M$ in G is the resulting moment. Fields \mathbf{v} correspond then to distributions of constraints closely connected in the section. Indeed, one a:

$$F(\cdot) \cdot \mathbf{n} \, dS =$$

$$\mathbf{T} \, dS = S \, \mathbf{T}$$

S

S

$$M(\cdot) \mathbf{GM}(X) \cdot \mathbf{n} \, dS = \mathbf{GM}(X) (\mathbf{GM}) \, dS = ($$

$\mathbf{I})$

S

S

The fact here was used that G is geometrical centre of inertia thus:

$$X \, dS$$

S

=

0. The subspace

vectorial \mathbf{T} being of finished size (equal to 6) has additional orthogonal for the product scalar defined on V :

$$\mathbf{T} = \{ \mathbf{v} \in V / \mathbf{v} \cdot \mathbf{w} \, dS = 0 \quad \mathbf{W}$$

éq 2.3-2

S
 $\} \mathbf{T}$
 Maybe, in a more explicit way:
 $\mathbf{T} = \{ \mathbf{v} \cdot \mathbf{V} / \mathbf{v} \cdot \mathbf{d}\mathbf{S} = 0 \text{ and } \mathbf{GM} \cdot \mathbf{v} \cdot \mathbf{d}\mathbf{S} =$

0
éq 2.3-3

S
 S
 $\}$

Any field of V all in all breaks up in a single way of an element of \mathbf{T} and an element of \mathbf{T} .

U up custom up T
custom

=
 +

\mathbf{T}

,
éq 2.3-4

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One has moreover the following property:

For any couple of field 3D (\mathbf{U}, \mathbf{v}) definite on S ,

$\mathbf{U} = \mathbf{up} + \mathbf{custom}$

$\mathbf{v} \cdot \mathbf{W} \, dS = \mathbf{vp} \cdot \mathbf{wp} \, dS + \mathbf{vs} \cdot \mathbf{ws}$

dS **éq 2.3-5** S S S

$$\mathbf{v} = \mathbf{v}_p + \mathbf{v}_s$$

The following definition is thus natural:

Definition:

One calls component of displacement of beam of a field \mathbf{U} defined on the section component \mathbf{u}_p

 \mathbf{U}

of on the subspace.

The calculation of the beam part of a field 3D $\mathbf{U.S.}$ 'operates by using the property of projection orthogonal since T and T are orthogonal by definition.

If one notes $\mathbf{u}_p = T + \mathbf{GM}$

 \mathbf{U} \mathbf{U}

, then:

(

Argmin

2

You, \mathbf{U}) = ($\mathbf{U} - T -$ T), (\mathbf{GM})**éq 2.3-6** S

One will note in the passing the interpretation of the component beam of \mathbf{U} : it is the field of displacement of beam nearest to \mathbf{U} within the meaning of least squares. The calculation of the led minimum immediately with the characterization:

1

 T

-1

 $\mathbf{U} =$ $\mathbf{U} dS,$ $= \mathbf{I}$ $\mathbf{GM} \mathbf{U}$ dS **éq 2.3-7** S

U
 S

S

The kinematic condition of connection sought is thus the following linear constraint between the field 3D on S and elements of the torque of displacement of the beam in G :

$$S \mathbf{T} - U dS,$$

$$(\mathbf{I}) - \mathbf{GM} \mathbf{u} dS =$$

0

éq 2.3-8

S

S

2.4

Expression of the static condition of connection

While returning to the weak formulation of the balance of the interface [éq 2.2-1], one can deduce them conditions necessary and sufficient of static connection. Indeed, one a:

..

$$N \mathbf{v} dS = \mathbf{R.T} + \mathbf{M}r.$$

$\mathbf{v} V$

éq 2.4-1

S

\mathbf{v}

\mathbf{v}

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Thanks to the expressions [éq 2.3-7] and the decomposition of space V , and the property [éq 2.3-5], one has

immediately three equations:

$$F = \int_{.ndS} S$$

$$S$$

$$M = \mathbf{GM}(X)$$

$$\int_{.ndS}$$

éq 2.4-2

$$S$$

$$(\int_{.n}) \mathbf{S} = 0$$

or in an equivalent way

$$\dots$$

$$N \mathbf{v} dS = 0 \mathbf{v}$$

$$T$$

$$S$$

The conditions of static connection are thus:

- transmission of the torque of the efforts of beam, (satisfied the P1 property),
- nullity of the complementary part (“not beam”) of the field of vector forced 3D on section of the solid 3D (satisfied the P2 property).

One will also notice static and kinematic symmetry (P3 property) since conditions of connection are also interpreted like:

- equality within the meaning of least squares between displacement 3D and the displacement of the beam,
- equality within the meaning of least squares between the field of vector forced and the elements of reduction of the torque of the efforts of beam.

3

Establishment of the method of connection

For each connection, the user must define:

S:

the trace of the section of the beam on the solid mass 3D: it does it by key words MAILLE_1 and/or GROUP_MA_1; i.e. it gives the list of the meshes (lma) surface (affected elements "edge" of modeling 3D) which represent this section geometrically.

P:

a node (key word NOEUD_1 or GROUP_NO_1) carrying the 6 ddl traditional of beam: DX, DY, DZ, DRX, DRY, DRZ

Note:

- the node P can be a node of element of beam or discrete element,
- the list of the meshes lma must represent the section of the beam *exactly*. It is one important constraint for the grid.

For each node, the program calculates the coefficients of the 6 linear relations [éq 2.3-8] which connect:

- 6 ddl of the node P,
- with the ddl of **all the** nodes of lma.

These linear relations will be dualisées, like all the linear relations resulting for example from key word LIAISON_DDL of AFFE_CHAR_MECA.

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The calculation of the coefficients of the linear relations is done in several stages:

- calculation of elementary quantities on the elements of lma: (OPTION: CARA_SECT_POUT3)
- surface

= 1

;

X;

y;

x2

; !

elt

elt

elt

elt

· summation of these quantities on (*S*) from where the calculation of:

-

WITH = S

- position

of

G

- tensor

of inertia

· knowing *G*, elementary calculation on the elements of lma of:

(OPTION: CARA_SECT_POUT4)

GM {*X*, *y*,}

Z

Ni;

xNi;

yNi;

zNi

=

where:

elt

elt

elt

elt

Nor = functions of form of the element

· “assembly” of the terms calculated above to obtain of each node of lma, them coefficients of the terms of the linear relations.

4

Which uses can one make this modeling?

In addition to the two aimed uses to [§2] [Figure 2.1-a] and [Figure 2.1-b], this connection can also be used for:

· to apply a torque of efforts to a known surface of a modeling 3D:

For that, the user defines the surface of load application (lma), it “connects it” with one

node (P) of discrete element (DIS_TR_N) **without rigidity** then it applies the torque wanted to it node (FORCE_NODALE).

In this way, the torque is applied in “softness”, without generating secondary stresses on surface.

· “to retain” a structure without too the encaster:

For example, if there is with a grid in 3D a pipe and that one wants to prevent his movements of solid body

S

()

P

one connects (S) to P then one blocks the 6 ddls P.

The structure is then retained, without (S) is embedded. In particular, the section (S) can ovalize itself.

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Efforts external of pressure into large displacements

Summary:

A loading of pressure in great displacements is a following loading. By employing elements of skin, one is brought to calculate, on the one hand, a second member to which calculation is close to that into small

displacements, and in addition, an additional term of rigidity, which is in general not symmetrical. One chooses nevertheless to symmetrize it, discounting an appreciable saving of time even if some iterations additional can be necessary to converge.

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1 Introduction

The taking into account of loadings of the type pressure (key word PRES_REP in the order AFFE_CHAR_MECA [U4.25.01]) poses a certain number of difficulties in the absence of the assumption of

small displacements. Indeed, unlike the dead loads evoked with [R5.03.20], the pressure depends on displacements since it is about an effort whose direction is normal with the field; one speaks then of following forces, activated by key word TYPE_CHARGE: "SUIV" in the order STAT_NON_LINE [U4.32.01]. Nevertheless, the choice of the current configuration like configuration of reference (Lagrangian updated) led to simple expressions - with the help of some concepts of differential geometry - work of the efforts of pressure and its variation first compared to displacement, the latter being a nonsymmetrical bilinear form.

2

Virtual work of the efforts external of pressure

R

$\mathbf{X} = (\mathbf{X})$

= \mathbf{F}

R

N

p

NR

p

P (\mathbf{X})

p (\mathbf{X})

Appear 2-a: Configuration of reference and current configuration

In the current configuration, the virtual work of the efforts external of pressure is written simply [Figure 2-a]:

W (). =

- p

p \mathbf{U}

v

N v

ds

éq 2-1

p (\mathbf{U})

Moreover, one supposes henceforth that the value of the pressure does not depend explicitly on displacement but only of the material point of application:

$$p(\mathbf{X}) = P(\mathbf{X})$$

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In this case, one can then express the virtual work of the efforts of pressure in the configuration of reference:

R

$W(\cdot) =$

$- \int_R P \det(\cdot)$

$-1. \int_R$

$p \mathbf{U}$

\mathbf{v}

$\mathbf{F} \mathbf{F}$

\mathbf{N}

\mathbf{v}

T

$((\mathbf{X})) ds$

éq 2-2

R

p

On the practical level, one will use the formula [éq 2-1] to calculate the work of the efforts of pressure. However, the formula [éq 2-2] is adapted best to a derivation compared to the displacement, of which one will see the need in the following paragraph.

3 Variation of the virtual work of the efforts external of pressure

In the optics of a resolution of the problem of balance of the structure by a method of Newton, one is brought to express the variation of the virtual work of the efforts external of pressure compared to displacement, in a way similar to what was made for the virtual work of the interior efforts with [R5.03.20]. The field of integration being fixed in the expression [éq 2-2], derivation under the sign nap is licit, (cf [bib2]):

$W_p(\mathbf{U})$

$\int_V \mathbf{U} \cdot \mathbf{v} = -P$

$[\det(\mathbf{F}) TFI]$
 \mathbf{R}

R

\mathbf{U}

\mathbf{N}

\mathbf{v}

ds

\mathbf{U}

R

\cdot

\cdot

\mathbf{U}

p

We decide to choose like configuration of reference the current configuration, for which $\mathbf{F} = \mathbf{Id}$. This choice led to a simple expression of derived from the term between hooks:

$[\det(\mathbf{F}) \mathbf{F} \cdot \mathbf{T}]. \mathbf{U} = \text{div}(\mathbf{U}) - T$

\mathbf{Id}

\mathbf{U}

\mathbf{U}

Finally, the variation of the virtual work of the efforts external of pressure is written in the configuration current:

$W_p(\mathbf{U}). \mathbf{U} \cdot \mathbf{v} = -p$

[div (U
) Id-T U
]

. N v

ds

éq 3-1

U

p (U)

In the expression [éq 3-1] remains a difficulty. Indeed, one expects to obtain a size primarily surface whereas the intégrande reveals terms of normal derivation with surface. In other words, it is necessary to know the expression of virtual displacements not only on surface field but also inside this one (in a vicinity of surface to be able to express the derivative normals). This disadvantage is not pain-killer since in *Code_Aster*, for to calculate the elementary terms due to the surface efforts, one employs elements of skin for which a normal variation does not have a direction.

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Adoption of a curvilinear parameter setting of surface

To cure the problem mentioned previously, it is necessary to seek to express the relation [éq 3-1] with assistance of surface sizes only. For that, one has recourse to elements of geometry differential, [bib1], which one adopts the notations in particular (, one adopts the convention of summation

repeated indices where the Greek indices take values 1 and 2 while the Latin indices take values 1 to 3).

3

1

N
2
P
S
M

Appear 4-a: Curvilinear parameter setting of the vicinity of subjected surface with the pressure

That is to say (1 2

,
) an acceptable parameter setting of surface. To describe volume made up of one vicinity of this surface, one associates a third variable to him, 3, which measures the progression according to unit normal **N** in (1 2

,
) . One has thus [4-a]:

$$\mathbf{OM}(1\ 2\ 3) = (\mathbf{OS}\ 1\ 2) + 3 (\mathbf{N}\ 1\ 2)$$

,
,
,
,
)

With this choice of parameter setting, the natural base covariante (**G**, **G**, **G**

1
2
3) and the metric tensor **G** are:

$$\mathbf{G} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}$$

$$\mathbf{G} = \mathbf{G} = \mathbf{G} = \mathbf{N}$$

$$g_{ij} = \mathbf{G} \cdot \mathbf{g}$$

1

I

$$J = G$$

G

0

1

2

2

3

3

21

22

0

0

1

In this curvilinear parameter setting, the intégrende [éq 3-1] has as an expression:

- *p G nor [the U.K. v J*

J

K

K

- *U*

v

ij

K

]

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This term is simplified considerably. Indeed, one can already note that when $J = K$, the term enters hook is null. Moreover, in the adopted curvilinear system, the components contravariantes of \mathbf{N} are: $n1 =, N2 =, n3$

0

0

= 1. Lastly, by taking account of the particular form of \mathbf{G} , the variation of work is written simply:

W

p (

3

3

 $\mathbf{U} \cdot \mathbf{U}$. \mathbf{v}

=

- p

 $U v - U$

v

ds

éq 4-1 \mathbf{U}

p (

 \mathbf{U})

On this expression, one notes that only intervene of the surface differential operators (derivation covariante compared to 1 and 2 only), which is well the required goal. In introducing the base contravariante ($\mathbf{G}, \mathbf{G}, \mathbf{G} = \mathbf{N}$

1

2

3

), also called bases dual and which is expressed to leave

ij

base covariante by $\mathbf{g}_i = [\mathbf{g}^{-1}]^j_i \mathbf{G}_j$, one can be freed from the curvilinear components:

$\mathbf{W}_p ($

\mathbf{U}

\mathbf{U}

$\mathbf{U})$

$\cdot \mathbf{U}$

$\cdot \mathbf{v}$

p

$=$

$-$

$\cdot \mathbf{G}$

$(\mathbf{v} \cdot \mathbf{N}) \cdot \mathbf{N} (\mathbf{v} \cdot \mathbf{G}) ds$

éq 4-2

\mathbf{U}

$-$

$p (\mathbf{U})$

It is henceforth the expression [éq 4-2] which will be used to calculate the variation of the virtual work of efforts of pressure.

5

Introduction into *Code_Aster*

In *Code_Aster*, finite elements of skin (surface elements plunged in a space three-dimensional) are employed to discretize real and virtual displacements intervening in surface expressions such as [éq 2-1] and [éq 4-2]. These last make it possible to express respectively the vector second member and the matrix of rigidity due to the pressure, of which employment by

the algorithm of STAT_NON_LINE is specified in [R5.03.01] and which call some note:

- The calculation of the virtual work of the efforts of pressure [éq 2-1] is in fact identical to that carried out

in small displacements, with the help of a preliminary reactualization of the geometry. Let us recall that it is carried out with each iteration.

- The calculus of the variation of the virtual work of the efforts of pressure [éq 4-2], carried out with each construction of the matrix of rigidity, proves a little more delicate insofar as it requires the knowledge of metric of the element of skin of each one of its points of Gauss. If one calls N_n the functions of form and \mathbf{x}_n the position of the nodes of the element, then the metric one is calculated as follows:

$$\begin{aligned}
 &NR \\
 &\mathbf{G} \\
 &N \\
 &N \\
 &1 \mathbf{G} \\
 &\mathbf{G} = \\
 &\mathbf{X} \\
 &\mathbf{N} = \\
 &2 \\
 &[\mathbf{G}] = \mathbf{G} \mathbf{G} \mathbf{G} \\
 &\mu \\
 &= -1 \\
 &.
 \end{aligned}$$

[G] G

N
 $\mathbf{g}_1 \mathbf{G}$
 μ
 2

Moreover, this variation behaves like a term complementary to the matrix of tangent rigidity; in general, it is not **symmetrical** (except particular case of a structure subjected to an internal or external pressure constant, cf [§6]). It is then desirable to spread out the strategy of resolution. Initially, only the part is considered symmetrical of this complementary term: the problem remains symmetrical, even if it requires (perhaps) some additional iterations. It is the choice carried out in *Code_Aster*.

In the event of problems of convergence, one could consider this complementary term in its integrality while being ready to pay the price of a nonsymmetrical resolution.

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6 particular Cases of a structure subjected to one internal or external pressure constant

$v = 0$

N

p

p

N

p

p

Appear 6-a: Structure under internal or external pressure constant

In the particular case of a constant pressure in a cavity [Figure 6-a], one shows that the efforts from pressure derive from a potential which is not other than the product of the pressure by the volume of

cavity. This result extends to the case from a structure plunged in a fluid with constant pressure.

R

$$P = p$$

$$D = p \det(\mathbf{F})$$

D

p

Pr

Again, one chooses like configuration of reference the current configuration. Variation of P

conduit then well with the virtual work of the efforts external of pressure:

$$\text{Statement} = p \operatorname{div} (\mathbf{v})$$

$$) D = - p$$

$$dS = W$$

P

v

N

v

U

p

p

In this particular case, the variation of virtual work is also the second variation of the potential P, i.e. a **symmetrical** bilinear form:

W

2

p (

P

U)

. U

. v

=

(U)

. U

. v

U

u2

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Connection hull-beam

Summary:

Here the connection hull-beam is described, which makes it possible to connect two parts of grid, one made up

elements of beams (or a discrete element), and the other with a grid one in elements of hulls (to represent phenomena except kinematics of beam). This development thus functions under assumptions translating that it is the same kinematics of beam which is transmitted between the two grids, on both sides of connection. It results in 6 linear relations connecting displacements of the whole of the nodes of the edge of

hull with the 6 degrees of freedom of the node end of the beam.

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1 Assumptions and applications

1.1 Assumptions and limitations

Here the connection hull-beam is described, which is used to connect two grids, one comprising of the elements

hulls (or plates), the other comprising of the elements of beams. This functionality allows to model a slim structure in two parts: a part with a grid with traditional elements of beams, representing a kinematics and a behavior of beams, and the other part with a grid in elements of hulls, to reveal other phenomena (ovalization, swelling, plasticity located).

The following assumptions however are made:

- 1) the transverse sectional surface of the end of the grid of hulls is identical to right sectional surface of the element of beam which corresponds to him,
- the 2) centres of gravity are identical,
- the 3) sections are plane and coplanar,
- 4) the normal with the section of hulls is confused with the axis of the beam.

Limitations:

- 1) one does not hold account in the connection of the ovalization of the cross-sections,
- 2) account of warping is not taken.

1.2 Applications concerned:

1.2.1 Modeling of pipings

One of the major applications relates to pipings. The bent parts or prickings are then with a grid in hulls, which makes it possible to reveal an ovalization, a behavior elastoplastic room or a swelling in the event of internal pressure. This connection does not transmit the ovalization of the pipes since this one is not modelled in the elements of beams. For it to make, it is necessary to use the connection hull-pipe or to net a sufficient length of right piping in elements of hulls so that ovalization on the level of the connection is negligible.

Circular piping of section (or rectangular...) with a grid in hull then in beam.

1.2.2 Connection plates beam

Connection plate-beam (mean rectangular section).

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1.2.3 Beam with symmetrical profile

Beam with symmetrical profile with a grid partly in hulls.

1.2.4 Application of a loading or boundary conditions of the type “beam”

At the end of a slim structure with a grid in hulls, it is often useful to impose is one loading of the type “beam” i.e. a torque of efforts, is boundary conditions (embedding) compatible with the kinematics of beam. One can then connect the section transverse of end of the grid hulls to a discrete element to which one will apply this torque or this embedding.

1.2.5 Application not considered:

This functionality does not make it possible to model the “prickings transverse or orthogonal” of one beam on a plate or a hull:

2

Application of the method of the connection 3D-beam. Equations of connection

The step is identical to that of the connection beam-3D [R3.03.03]: the connection results in 6 linear relations connecting displacements of the whole of the nodes hull of the section of connection (6 degrees of freedom per node, compared to 3 ddl by node in 3D) with the 6 degrees of freedom of the node of beam. The section of connection of hull is made up of elements of edge hulls (segments). On the section crosses connection, one breaks up the field of displacement “hull” in a part “beam” and a “complementary” part. This brings us to to define the conditions of kinematic connection between beam and hull like the equality of displacement

(torque distributor or kinematic torque) of beam and the beam part of the field of displacement hull

As in [R3.03.03], one introduces space \mathbf{T} of the fields associated with a kinematic torque (definite by two vectors):

$$\mathbf{T} = \{ \mathbf{v} \vee (T,)$$

$$\mathbf{v} \text{ such as } (M) = \mathbf{T} + \mathbf{G} \}$$

M

éq 2-1

Here, \mathbf{G} represents the centre of gravity of the section of connection (having to be identical to that of the beam). For the fields of displacement of \mathbf{T} , \mathbf{T} is the translation of the section (or of the point \mathbf{G}), infinitesimal rotation and fields \mathbf{v} are displacements of the space of displacements there acceptable V preserving the section S plane and not deformed (One uses still the Assumptions of NAVIER-BERNOULLI).

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The vectorial subspace **T** being of finished size (equalizes to 6) has additional orthogonal for the definite scalar product on V :

$$\mathbf{T} = \mathbf{v} \int_V (M) \cdot \mathbf{w} (M)$$

$$\left\{ \begin{array}{l} dS = 0 \\ \mathbf{W} \end{array} \right.$$

éq 2-2 S } **T**

Maybe, in a more explicit way:

$$\mathbf{T} = \left\{ \mathbf{v} \int_V (M) dS = 0 \text{ and } \mathbf{GM} \int \mathbf{v} dS = \right.$$

0

éq 2-3 S S

}

Any field of V all in all breaks up in a single way of an element of **T** and an element of**T**.**U up custom up****T****custom**

=

+

T

éq 2-4

One has moreover the following property:
 For any couple of field hull (\mathbf{W} , \mathbf{v}) definite on S ,
 $\mathbf{W} = \mathbf{w}_p + \mathbf{w}_s \quad \mathbf{v} = \mathbf{v}_p + \mathbf{v}_s$
 $\mathbf{W} \cdot \mathbf{v} = \mathbf{w}_p \cdot \mathbf{v}_p + \mathbf{w}_s \cdot \mathbf{v}_s$

éq 2-5

$\mathbf{v} = \mathbf{v}_p + \mathbf{v}_s$

Definition:

One calls component of displacement of beam of a field of hull \mathbf{U} defined on the section component \mathbf{u}_p of \mathbf{U} on the subspace \mathbf{T} .

The characterization immediately is obtained:

$\mathbf{U} = \mathbf{u}_p + \mathbf{u}_s$

éq 2-7

$\mathbf{U} = \mathbf{I} \cdot \mathbf{G}$

S

where S represents the surface of the section S and \mathbf{I} the geometrical tensor of inertia of surface S , expressed in \mathbf{G} .

In other words, one can as say as the calculation of the beam part of a field hull \mathbf{U} . 'operates in using the property of orthogonal projection since \mathbf{T} and \mathbf{T}^\perp are orthogonal by definition.

If one notes $\mathbf{u}_p = \mathbf{T} \cdot \mathbf{G}$

\mathbf{U}

, then:

(
 2
 $\mathbf{T},) = \text{Argmin}$
éq 2-6
 $S (- \mathbf{T}$
 \mathbf{U}
 \mathbf{U}
 \mathbf{U}
 $- \mathbf{GM})$
 $(\mathbf{T},)$

The component beam of \mathbf{U} can thus be interpreted like the field of displacement of beam it nearer to \mathbf{U} within the meaning of least squares.

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The kinematic condition of connection sought between the field hull on S and the elements of the torque of displacement of the beam in G is given by:

$ST -$

$dS = 0$

$() - \mathbf{GM} D$

$\mathbf{U.S.} =$

\mathbf{U}

\mathbf{I}

0

éq 2-8

S

S

The equation [éq 2-8] shows that the situation is identical to the case 3D-beam. Linear relations

will have the same form. The only difference comes from the integrals on S (which represents a curve here

corresponding to the section of the hull, modelled by elements of edge of hull). Moreover, the field of displacement of hull utilizes DDL of rotation.

To translate the equation [éq 2-8] into linear relations, the two integrals should be calculated:

· average displacement: $\mathbf{u}dS$

S

· average rotation: $\mathbf{GM} U$

dS

S

3

Integrals to be calculated. Kinematics of hull.

For each node, the program calculates the coefficients of the 6 linear relations [éq 2-8] which connect:

· 6 ddl of the node of beam P (geometrically confused with the centre of gravity G of

transverse section of the grid hulls)

· with the ddl of **all the** nodes of the list of the meshes of the edge of hull.

These linear relations are dualisées, like all the linear relations resulting, for example, of the word key LIAISON_DDL of AFFE_CHAR_MECA. They are built as for the connection 3D-beam with to leave the assembly of elementary terms.

x^3

y^3

N

M

y^3

H

Q

$Q y^2$

T

E

1=e1

3rd

Edge of the transverse section (of hull) of

2

connection

G

e1

$x1 S = L \times I$

H

L : line of the points Q on the average layer

$H H$

$I = -$, interval describing the épaisseur .r

2 2
Kinematics of hull or linear plate in the thickness:

$$\begin{pmatrix} \mathbf{U} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \mathbf{U} \\ \mathbf{Q} \end{pmatrix} + \left(\frac{\partial \mathbf{U}}{\partial y_3} \right) y_3$$

\mathbf{U} is the vector displacement of average surface in Q ,

\mathbf{N} is the normal vector on the average surface of the hull in Q ,

\mathbf{H} is the vector rotation in Q of the normal according to directions' T1 and t2 of the tangent plan HH

y_3 is the co-ordinate in the thickness ($y_3 -$,
).

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3.1

Calculation of average displacement on the section S

It is a question of calculating the integral $\int_S \mathbf{u} dS$

, where \mathbf{U} is the displacement of hull (comprising 6 ddl by node),
 S

S is the edge of hull of the transverse section of connection.

Average displacement on the section S is written:

$$\int_{-h/2}^{h/2} U M) ds = H$$

$$\int_{-h/2}^{h/2} (uQ) ds + ((Q) N) y Dy ds$$

S
 L
 L

H
 3
 3
 $/2$

-
 that is to say

$$\int_{-h/2}^{h/2} U M) ds = H$$

$$\int_{-h/2}^{h/2} (uQ) ds$$

S
 L

One neglects in this expression the variations of metric in the thickness of the hull.

3.2
Calculation of the average rotation of the section S

$$\int_{-h/2}^{h/2} GM (U M) ds =$$

$$\int_{-h/2}^{h/2} (GQ + y^3 (nQ) ((uQ) + (Q) (nQ) .y^3) dsdy$$

S
 $L - H$

3
 $/2$
 H

$=$
 $/2$
 $H GQ U$

$$(Q) ds + GQ$$

$$((Q) (nQ) ds y Dy$$

L

L

$- H$

3

3

$/2$

H

$+ ($

$h/2$

$N Q) ($

$U Q)$

$y Dy ds$

N

2

2

3

3

$.$

$/2$

$-$

$+ (Q) ((Q) (nQ)$

$y Dy ds$

L

H

H

L

3

3

$- 2$

h^3

that is to say $GM ($

$U M) ds = H GQ$

$(uQ) ds +$

\cdot
 $12 ($
 $N Q) ((Q)$

$(nQ) ds$
 S
 L
 L

3.3

Calculation of the tensor of inertia

The tensor of inertia is defined by [R3.03.03]:

$$I () = \int_S GM (GM) ds$$

S
while posing: $GM = GQ + N (Q)$. y.

3
h3

One obtains: $I () = H GQ$

$$(GQ) ds + 12 (nQ) (N (Q) ds$$

L
 L

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3.4

Establishment of the method

The calculation of the coefficients of the linear relations is done in two times:

· calculation of elementary quantities on the elements of the list of the meshes of edges of hulls (mesh of type SEG2 or SEG3):

-
the 9 terms are calculated:

-
- ds ; xds ;
- yds ;
- $x2ds$;
- $y2ds$;
- $z2ds$;
- $xyds$;
- $xzds$;
- $yzds$

- elt
- elt
- elt
- elt
- elt
- elt
- elt
- elt
- elt
- elt
- elt
- $h3$

as well as terms resulting from $I ()$:
 $N (N) ds$

- $12 L$
- $h3$
- $h3$

what makes it possible to calculate:

- +
- ,
- , etc...

$12 (N_2 N_2) ds$

$NN ds$

y

Z

L

$X y$

$12 L$

-

summation of these quantities on (S) from where the calculation of:

-

$WITH = S$

- position

of

G

- tensor

of inertia

I

· knowing

G , elementary calculation on the elements of the list of the meshes of edges of

hulls of:

GM $\{X, y, \}$

Z

$NR ds;$

$xN ds;$

$yN ds;$

$Zn ds$

I

I

I

where:

=

elt

elt

elt

I

elt

$Nor =$ functions of form of the element

(It should simply be noticed that in this case, the integrals on the elements of edge are to be multiplied by the thickness of the hull:

$NR ds = H NR dl$

I

I where L represents the X -coordinate

elt

L

curvilinear of average fibre of the element of edge of hull).

$h3$

Moreover, one adds the terms additional coming from:

$(nQ) (N (Q) ds$

$12 L$

nx

X

While noting $\mathbf{N} = ny$ and $= y$ in the total reference mark one obtains:

nz

Z

$(N^2 + n^2$

y

$Z) - N N - N N$

X

$X there y$

$X Z Z$

$($

$N Q) (($

$N Q) = - N N$

2

2

$X there X$

$+ (N + N$

X

$Z) - N N$

y

$y Z Z$

$= \mathbf{A}$

$- N N - N N$

2

2

$X Z X$

$y zy + (N + N$

X

$y) Z$

then:

h3

h3

$(nQ) ((nQ) ds =$

With

12

12

$((S) NR (S) ds$

L

J

el

) *J*

el

· “assembly” of the terms calculated above to obtain of each node of the meshes of edge, coefficients of the terms of the linear relations.

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4 Use

4.1 Modeling

For each connection, the user must define under the key word factor LIAISON_ELEM of AFFE_CHAR_MECA:

S:

the trace of the cross-section of the beam on the hull: it does it by key words MAILLE_1 and/or GROUP_MA_1 i.e. it gives the list of the linear meshes (affected of elements “edge” of modeling hull) which represents this section geometrically.

P:

a node (key word NOEUD_1 or GROUP_NO_1) carrying the 6 ddl traditional of beam: DX, DY, DZ, DRX, DRY, DRZ

V:

the vector defining the axis of the beam, directed hull towards the beam, and defined by its co-ordinates using key word AXE_POUTRE: (v1, v2, v3)

Note:

- the node *P* can be a node of element of beam or discrete element,
- the list of the meshes of edge of hull, defined by MESH or GROUP_MA must represent **exactly the** cross-section of the beam. It is an important constraint for grid.

4.2

Examples and tests

4.2.1 Test

SSLX101

It is about a subjected right beam has unit efforts out of B (traction, moments bending and of torsion). One takes a mean section of tube thickness $H \ll R$.

y

H

M

R

Z

O

X

With

MX B

F

My

Embedding out of O is carried out using a connection between the edge of the hull and a specific element located out of O. This element is embedded (null translations and rotations).

This makes it possible to obtain in the hull a state of stresses very close to a solution “beam”: there is not no disturbance of the stress field. The solution differs from the analytical solution (solution RDM) of 3%, this being only due to the smoothness of the grid in elements of hulls.

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4.2.2 Inflection of a plate

Let us consider a sufficiently long thin section, length $2L$, of width B , thickness H , modelled by an element of hull OA and an element of beam on AB :

Z

y

O

L

B

D

With

L

B

X

H

C

.

1st condition of connection is written:

$B H U ()$

$With = H$

U

$(y) Dy$

CD

the displacement of point A (pertaining to the beam) is the average of displacements of the edge CD plate.

.

2nd condition of connection is written:

h^3

$I () = H$

$AQ U (Q) ds +$

12

$(Q) ds$

CD

CD

$h^3 B^2$

In the case of an inflection around y , the only term not no one is:

B

$\int(y) Dy$

$12 - 2$

B

Indeed, H

$AQ U$

$(Q) ds = H2 U ydy$

$B Z$

$.x 0$

CD

2

$=$

$-$

For an inflection around y , the connection is thus written:

$bh3$

$I ()$

With

$y y$

$=$

$y \text{ bus}$

12

y is constant on CD .

This application is implemented in test SSLX100B: mix 3D_coque_poutre.

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R3.03.07 document

Following pressure for the elements of hulls

voluminal

Summary:

We present in this document, the model used to calculate the loading of following the pressure type acting on the average surface of the finite elements of voluminal hulls corresponding to modeling COQUE_3D. Discretization of the loading led to a nodal vector of the external forces and to one nonsymmetrical contribution in the tangent matrix of rigidity. These finite elements objects are evaluated with each iteration of the algorithm of Newton of STAT_NON_LINE.

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1 Introduction

Our analysis leaves the weak formulation of balance under a loading of following the pressure type activated by key word TYPE_CHARGE: "SUIV" in order STAT_NON_LINE [U4.32.01]. difference compared to a traditional geometrical linear analysis is that the pressure acts on geometry deformed and either on the initial geometry. This new geometry is obtained to leave transform of the initial average surface subjected to great displacements and the large ones rotations [R3.07.05]. The notations are inspired by [R3.07.05].

This transform can be paramétrisée exactly as initial surface by using them reduced co-ordinates of the associated isoparametric element: Co-variable or counter-variable reference marks

build themselves in each point of deformed surface. The writing of the virtual work of the pressure with this parameterization is done in the configuration deformed by using the isoparametric elements associated. It results an independence from it from the field of integration with displacements that one use to express the variation of the virtual work of the efforts external of pressure compared to the known as ones

displacements. That has an important advantage compared to the method applied for pressure which follows the facets of the elements 3D [R3.03.04]. Indeed, this last method, based on

a brought up to date Lagrangian formulation, led to nonlinear terms difficult to linearize, coming from the transformation jacobienne compared to the configuration of reference. The finite elements objects obtained by linearization compared to incrémentaux displacements of virtual work of the efforts external of pressure are to be reactualized with each iteration of the algorithm of Newton of STAT_NON_LINE. We underline the fact that the contribution of the following pressure to stamp tangent rigidity is nonsymmetrical, and we point out that the geometrical part of stamp tangent is already nonsymmetrical [bib2].

2 Kinematics

For the elements of voluminal hull one defines a surface of reference, or surfaces average, left (of curvilinear co-ordinates

1 2 for example) and a thickness (

H,

1 2)

measured according to the normal on the average surface. The position of the points of the hull is given by

curvilinear co-ordinates (1,2) of average surface and rise 3 compared to this surface.

One points out the great transformation undergone by the hull:

(together of the points P with 3 = 0) is the transform of initial average surface

(together of the points P with 3 = 0).

The position of the point P on the deformed configuration can be established according to the position of not initial P as follows:

$$\mathbf{X}(1,2) = \mathbf{X}(1,2) + \mathbf{U}$$

P

P

$P(1,2)$.

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$$\mathbf{N}(1,2) = (1,2) \mathbf{N}(1,2)$$

$$\mathbf{N}(1,2)$$

$$\mathbf{uQ}(1,2,3)$$

$$Q(3,0)$$

$$Q(3,0)$$

.

.

(

)

$$P(3=0)$$

.

$$P(3=0)$$

.

H

$$\mathbf{U}P(1,2,3=0)$$

$$\mathbf{xQ}(1,2,3)$$

$$\mathbf{X}P(1,2,3=0)$$

\mathbf{X}

$$Q(1,2,3)$$

$$\mathbf{X}P(1,2,3=0)$$

$$\mathbf{E}_2, y$$

$$\mathbf{e}_1, X$$

$$\mathbf{E}_3, Z$$

Appear 2-a: Voluminal hull.

Great transformations of an initially normal fibre on the average surface

2.1

Parameterization of the transform of average surface

The transform can be paramétrisée in a way similar to parameterization of surface initial. Thus one can define the infinitesimal element of tangent vector in:

\mathbf{X}

\mathbf{X}

$d\mathbf{x}$

P

P

$$P(1,2) =$$

D +

D

1

2

1

1

dx

P (1,2) = *D* **has**

1

1 (1,2) + *D*

has

2

2 (1,2)

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where [**has** (,); **has**

1

1 2

2 (1

, 2

)] represent a nonorthogonal natural base (**has** .a

1

2)
0 and not

normalized (1 **has**; **has**

1
1)
1 tangent on the surface. The two basic vectors can be dependent on displacements via the following formula:

$$\begin{aligned}
 & \mathbf{X} \\
 & \mathbf{X} + \mathbf{U} \\
 & \mathbf{has} \\
 & P \\
 & p \\
 & p \\
 & 1 (1 \\
 & , 2 \\
 &) \\
 & (\\
 &) \\
 & = \\
 & = \\
 & 1 \\
 & 1
 \end{aligned}$$

$$\begin{aligned}
 & \mathbf{X} \\
 & \mathbf{X} + \mathbf{U} \\
 & \mathbf{has} \\
 & P \\
 & p \\
 & p \\
 & 2 (1 \\
 & , 2 \\
 &) \\
 & (\\
 &) \\
 & = \\
 & = \\
 & 2
 \end{aligned}$$

2

what makes it possible to connect them to the vectors of the natural base related to initial surface by relations:

U**has** p

1 (1

, 2

) = **a1** (1, 2) + 1**U****has** p

2 (1

, 2

) = **a2** (1, 2) + 2

It is important to note that these vectors are distinct from the vectors obtained by great rotation vectors **has** (,); **has**

1 1 2

2 (1

, 2

):

a1 (1

, 2

) (1

, 2

) **a1** (1

, 2

)

a2 (1

, 2

) (1

, 2

) **a2** (1

, 2

)

Indeed, because of deformation due to transverse shearing, the turned vectors are not any more tangent with. The illustration of that is given by [Figure 3.1-a].

With this parameterization, the infinitesimal vector element of surface which is perpendicular to

can be written:

D (,) = **has** (,) × **has**

1 2

1

1 2

2 (1,2) *d1d2*

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3 Formulation

variational

3.1 Work

virtual

N

(,

1)

2

p0

N

(,

N

1)

2

=

(,

1)

2

(,
1)
2

$$p = p_0$$

Appear 3.1-a: Voluminal hull.

Following pressure on initial average surface and its transform

The virtual work of a **following** pressure p (i.e. acting on transformed average surface and moving with) can be expressed in the form:

$$pressure = -$$

up.p D

following

If one uses the isoparametric element of surface corresponding to our modeling of hull voluminal, surface D is expressed directly according to the isoparametric co-ordinates

$D D$

1 2 and one obtains the following simple form of the equation above:

$$= -$$

U p. p (, 12)

a1 (,)

pressure

× **has (,)**

[- ,1+]1 [×- ,1+]

$D D$

following

1

1 2

2

1 2

1

2

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3.2 Operator

tangent

As the virtual work of the following pressure depends on the current configuration, its variation linear is not null and must be taken into account. The tangent operator associated this virtual work is written with the iteration ($I +$)

1 pennies the form:

(

$i+$)

1

(I)

(I)

L

$$pressure = pressure + pressure$$

following

following

following

(I)

where *pressure* is the increment between two iterations of the virtual work of the following pressure. If

following

pressure is given in the form:

$$p = p0$$

being the level of load which is fixed lasting the iterations (piloting = 0 charge some), one can to write:

$$\begin{aligned}
 & \text{pressure} = - [\\
 & \mathbf{U} \\
 & \text{has} \\
 & \text{has} \\
 & \text{has} \\
 & \text{has} \\
 & - , +] [\\
 & \times -, +] P p (1 \times 2 - 2 \times 1) D D \\
 & 1 2 \\
 & 1 1 \\
 & 1 1
 \end{aligned}$$

following

Incremental variations of the vectors of the tangent local base to the transform of surface average are given by:

$$\begin{aligned}
 & \mathbf{a1} = \\
 & \mathbf{U} P \\
 & 1
 \end{aligned}$$

$$\begin{aligned}
 & \mathbf{a2} = \\
 & \mathbf{U} P \\
 & 2
 \end{aligned}$$

since initial surface average “does not move” not during the iterations what involves $\mathbf{X} P = 0$. These calculations finally make it possible to establish the expression of the increment of the virtual work of pressure following in the form:

$$\begin{aligned}
 & \text{pressure} = - [, 1] 1 [, 1] \mathbf{uP.p} \text{ has} \\
 & \mathbf{U} \\
 & \text{has} \\
 & \mathbf{U} \\
 & 1 \\
 & 2
 \end{aligned}$$

$$\begin{aligned}
 & - + \times - + 1 \\
 & [
 \end{aligned}$$

×] P -
×
 $P D D$
1 2
following

[]
2
2

where [× **has**] and [**has**
1
2 ×] are respectively the antisymmetric matrices of the tangent vectors
has and **has**
1
2 respectively.

Note:

In the reference [bib2], an integration by part is undertaken on the expression above. It is shown that the tangent matrix can be broken up into a symmetrical part resulting of an integration on the field and an antisymmetric part resulting from integration on contour. It is as shown as the assembly of the antisymmetric parts of the matrices elementary tangents leads to a null matrix when the pressure is continuous of one finite element with another, because of existence of a potential associated with work with the pressure in this case there.

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4 Discretization

At the points P of average surface, the interpolation of virtual displacement is written:

$$U$$

$$1$$

$$NB$$

$$\mathbf{U} ($$

$$1$$

$$1, 2)$$

$$()$$

$$= \mathbf{NOR} (1,2) v$$

$$I 1$$

$$=$$

$$W I$$

and the interpolation of incremental displacement between two iterations is written:

$$U$$

$$1$$

$$NB$$

$$\mathbf{U} ($$

$$1$$

$$1, 2)$$

$$()$$

$$= \mathbf{NOR} (1,2) v$$

$$I 1$$

$$=$$

$$W I$$

We rewrite the two preceding equations in the matrix form:

$$\mathbf{U} ($$

$$E$$

$$1, 2) = [\mathbf{NR}] \{ \}$$

$$\mathbf{U}$$

$$\mathbf{U} ($$

$$E$$

$$1, 2) = [\mathbf{NR}] \{ \}$$

$$\mathbf{U}$$

where $[\mathbf{NR}]$ is the matrix of the functions of form of translation on the average surface, of which the expression

is:

1 0

0 0 0

0

0 0

0

[]

()1

NR =! *NR I 0 1*

0 0 0

0

!0 0

0

0 0 10 0

0

0 0 0

*I 1, NB1**NB2*

=

() 1

(2)

Functions of form *NR*and *NR**I**I*

(used thereafter are given in appendix of [R3.07.04].

The nodes $I = 1, NB1$ are the nodes tops and the mediums on the sides (for the quadrangle and it triangle). Node $NB2$ is with the barycentre of the element.

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The vector $\{ \}$

U.E. is the nodal vector of virtual displacements given by:

.

.

U

v

W

X

{

y

$\mathbf{u}^e\} =$

$Z I$

.

.

.

$I =,$
 $1 NB$

1

X

X

$X NB2$

The vector $\{\}$

U.E. is the nodal vector of displacements incremental between two iterations.

.

.

.

U

v

W

X

{

y

ue} =

ZI

.

.

.

I =,
1 *NB*

1

X

X

X NB2
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This discretization also enables us to establish the expression of derived from incremental displacement average surface compared to the surface isoparametric co-ordinates in the form:

U (

E

1,2) =

NR { }

U

1

1

U (

E
1,2) =
NR {}

U
2
2

where
and
are the matrices derived from the functions of forms of translation on the surface

NR
NR
1

2
average, whose expressions are:

() 1
0 0 0
0 0
0
0 0
1
NR I

= !

!
NR

0 1 0 0 0 0
0 0 0

1

1

0

0 1 0

0 0

0

0 0

I 1

=, *NR* 1

B

NB 2

() 1 0 0 0

0 0

0 0

0

1

NR

I

0 0 0

NR =

0 1 0 0 0 0

!

!

2

2

0 0 1 0

0 0

0 0 0

I 1, NB1

NB2

=

Thus one can express the virtual work of the following pressure in the following matric form:

E

E

pressure = {U} .f pressure

following

following

with $\mathbf{F} E$

pressure the nodal vector of the external forces which can be expressed in the following way:

following

$\mathbf{F} E$

=

$[\mathbf{NR}] T$

[

-1,+]

1 [

$\times -1,+]$

1

$(\mathbf{a1} \times \mathbf{a}$

pressure

1) $D D$

1

2

following

It is important to note that with our parameterization of the transform of average surface, it

jacobien $\det [(\mathbf{J} (3 =)$

0)] of this surface are not implied in the calculation of the finite elements objects.

It will be also noted that the pressure is discretized with an isoparametric interpolation of the values with NB2 nodes:

(

NB2

p

2

1,2)

()

= NR (1,2) p

I

I

I 1

=

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One can also express the increment between two iterations of the virtual work of the following pressure under

the matric form:

$$\begin{aligned}
 &E \\
 &E \\
 &E \\
 &pressure = - \{ \mathbf{U} \} \cdot \mathbf{K} T pressure \{ \\
 &\mathbf{U} \}
 \end{aligned}$$

following
following

where **K E**

T pressure is the contribution in the tangent matrix of rigidity of the external forces which can following

to be expressed in the form:

KE

=

[NR] *T*

***a1* ×**

NR

1

2

-

1, 1

1, 1

[NR] *T*

p

DD

[1,]1 [1,]

p

1

[*a2* ×]

[

][

]

[]

NR

T pressure

DD

- + × - +

- + × - +

1

2

following

2
1

Note:

It is noted that the finite elements formulations resulting from this approach do not make to intervene degrees of freedom of rotations. The treatment is thus also valid for facets of the finite elements of three-dimensional elasticity.

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Following pressure for the voluminal elements of hulls

Date:

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Author (S):

P.MASSIN, Mr. Al MIKDAD

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Page:

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R3.03 booklet: Boundary conditions and loadings

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J.P. LEFEBVRE, X. DESROCHES

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Organization (S): EDF/IMA/MMN

Handbook of Reference

R3.06 booklet: Machine elements and thermal for the continuous mediums

Document: R3.06.02

Linear modeling of the elements of medium

continuous in thermics

Summary:

One describes the expression of the elementary terms intervening in the linear modeling of the equation of

heat and in postprocessings. One gives the mathematical expression of the integral to be evaluated, and for

each element one provides the number of points of integration used.

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2.1 General notations	4
2.2 Elementary terms contributing a share	4
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2.2.2 Mass thermal	5
2.2.3 Rigidity due to the boundary conditions of exchange	5
2.2.4 Rigidity due to the conditions of exchange between walls	6
2.3 Elementary terms contributing a share to the second member	6
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1**Options of linear modeling**

In this document, one considers only the linear modeling of the physical phenomenon of evolution temperature in a continuous medium. All coefficients intervening in the equation of heat will be constants or many functions being able to depend on time or space.

boundary conditions could be only linear functions of the temperature.

By defect the material is supposed to be isotropic, the Fourier analysis connecting the heat flow to the gradient of

temperature utilizes a scalar coefficient thermal conductivity:

$$\mathbf{Q} = -T$$

In the case general, unspecified medium, this relation is expressed with a tensor of conductivity thermics. The definite associated matrix being positive, it is always possible to be brought back to one stamp diagonal in the reference mark associated with the clean directions. Treatment of the anisotropy thermics (see [bib 1]) is thus carried out in *Code_Aster* by providing the values of conductivity thermics for each principal direction and the clean reference mark. The evaluation of the elementary terms

be carried out then by recovering the various coefficients and while changing reference mark. Two types of anisotropy are treated in *Code_Aster*, it acts:

- of the Cartesian anisotropy where the privileged directions remain fixed in a reference mark Cartesian, the data of the three nautical angles, and makes it possible to pass from the reference mark total with the principal reference mark of anisotropy,

- of the cylindrical anisotropy where the privileged directions remain fixed in a reference mark cylindrical, the data of the two nautical angles and defining the direction of the axis and of the three punctual coordinates of this axis allows to pass from the total reference mark to the reference mark

the main thing of anisotropy.

The variational formulation of the linear equation of heat (cf [R5.02.01]) led to the evaluation of a certain number of expressions in the form of integrals which constitute finally a system matric. The matrix and the second member are built starting from various bricks: options of calculation which gathers one or more integrals. The options described here are common to the unit isoparametric finite elements. Their evaluation depends on the type of element: degree of the functions of form, numbers and family of points of integration used.

One will be able to refer to the documents [U1.23.01], [U1.23.02] and [U1.24.02] concerning the different ones

modelings (type of mesh supporting the finite elements).

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2

**Expression of the elementary terms for the different ones
options of calculation**

2.1 Notations

general

We indicate by:

open of α_3 or α_2 of border,

T

the variable representing time,

T

the step of time used,

R

the variable of space,

T

the temperature (unknown of the problem),

T_n

the temperature at the previous moment (known),

T^*

the function test,

density,

C

specific heat with constant pressure,

p

$C = C$

heat-storage capacity with constant pressure per unit of volume,

p

p

the parameter of - method for the transitory thermal analysis.

2.2

Elementary terms contributing a share

2.2.1 Rigidity

thermics

Term utilizing variations in temperatures and the coefficient of conduction in the case isotropic mediums (denomination used by analogy at the end of rigidity intervening in the equation of the modeling of the mechanical phenomenon of elasticity). The coefficient can depend time.

· mathematical expression:

() .

*

T

T T

D

,

when the medium is anisotropic, the evaluation of flow (*T*) *T* is carried out in the reference mark the main thing of anisotropy after a first change of reference mark (the tensor of conductivity thermics is diagonal there) then by a change locates opposite, one returns in the reference mark total,

· denomination of the option in the catalogues: RIGI_THER,

· a number of points of integration used: (first family of points of integration cf [R3.01.01]).

net support

a number of nodes

a number of points

triangle

3

1

6

3

quadrangle

4

4

8 or 9

9

tetrahedron

4

4

10

15

pentahedron

6

6

15

21

hexahedron

8

8

20

27

27

27

Table 2.2.1-1

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2.2.2 Mass

thermics

Term utilizing the coefficient of heat-storage capacity to constant pressure $C = C$

p

p

(denomination used by analogy at the end of mass intervening in the equation of modeling equations of dynamics). The coefficient CP can depend on time.

.

1

mathematical expression: (). *

T

CTTT

D

P

- denomination of the option in the catalogues: MASS_THER
- a number of points of integration used: (second family of points of integration)

net support

a number of nodes

a number of points

triangle

3

3

6

6

quadrangle

4

4

8 or 9

9

tetrahedron

4

4

10

15

pentahedron

6

6

15

21

hexahedron

8

8

20

27

27

27

Table 2.2.2-1

2.2.3 Rigidity due to the boundary conditions of exchange

Term utilizing the coefficient of exchange *H* having for origin a boundary condition modelling the convectifs exchanges with the edge of the field. The coefficient *H* can depend on time and space.

· mathematical expression: $H(R, T) T.T* D$

· denomination of the option in the catalogues: RIGI_THER_COEF_R or RIGI_THER_COEF_F

· a number of points of integration used:

net support

a number of nodes

a number of points

segment

2

4

3

4

triangle

3

3

6

4

quadrangle

4

4

8 or 9

9

Table 2.2.3-1

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2.2.4 Rigidity due to the conditions of exchange between walls

Term due to the boundary condition of the Neumann type bringing into play two pennies left the border in opposite and utilizing a single coefficient of exchange h . This type of boundary condition create new relations between the degrees of freedom of the border.

In this case, one uses a particular finite element whose mesh support is obtained by associating two meshes of identical edge or face, the functions of form used and the points of integration are

those of the starting mesh.

Into three-dimensional, the meshes support of the elements of face are of type TRIA3-TRIA3, QUAD4-QUAD4, TRIA6-TRIA6, QUAD8-QUAD8 or QUAD9-QUAD9.

Into two-dimensional, they are of type SEG2-SEG2 or SEG3-SEG3.

One will be able to refer to [U4.25.02 § 3.1.3] for the description of the algorithm of search for meshes in opposite.

· mathematical expression:

$((, +) (2 -)). *$

H R T

T

T

T

T D

1

and

$((, +) (1 -)). *$

H R T

T

T

T

T D

2

1

2

1

2

where 1 and 2 is two pennies left the border in opposite.

· denomination of the option in the catalogues: RIGI_THER_PARO_R or RIGI_THER_PARO_F

· a number of points of integration used: cf [Table 2.2.3-1].

2.3

Elementary terms contributing a share to the second member

2.3.1 Discretization in time

Term due:

· with the discretization of derived in time utilizing part of the term of mass with the coefficient of heat-storage capacity *CP*,

· with

- method utilizing part of rigidity in the second member with

coefficient of conduction,

· mathematical expression in the case of isotropic mediums:

1
N

. *
-
1
N
*

(-) .

T
C T T
D
T
T
D
P

when the medium is anisotropic, the evaluation of flow (T) T is carried out in the reference mark the main thing of anisotropy after a first change of reference mark (the tensor of conductivity thermics is diagonal there) then by a change locates opposite, one returns in the reference mark total,

- denomination of the option in the catalogues: CHAR_THER_EVOL,
- a number of points of integration used: cf [Table 2.2.2-1].

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2.3.2 Voluminal term of source

Term due to the voluminal source of heat.

· mathematical expression:

$$(S(R, T + T) + (1 -) S(R, T)). *$$

T D

,

· denomination of the option in the catalogues: CHAR_THER_SOUR_R or CHAR_THER_SOUR_F,

· a number of points of integration used: cf [Table 2.2.1-1].

2.3.3 Term of convectif exchange

Term due to the boundary condition of convectif exchange utilizing the coefficient of exchange H and the temperature of the “external” medium T_{ex} .

· mathematical expression:

$$(H(R, T) T(R, T) T) (1) H(R, T) (T(R, T) T N) + + - -)). *$$

T D

ex

ex

· denomination of the option in the catalogues: CHAR_THER_R or CHAR_THER_F,

· a number of points of integration used: cf [Table 2.2.3-1].

2.3.4 Term of imposed normal flow

Term due to the boundary condition of flow imposed according to the normal on the border, utilizing one function being able to depend on the variables R and T .

· mathematical expression:

$$(F(R, T + T) + (1 -) F(R, T)). *$$

TD

- denomination of the option in the catalogues: CHAR_THER_FLUN_R or CHAR_THER_FLUN_F,
- a number of points of integration used: cf [Table 2.2.3-1].

2.3.5 Term of exchange between walls

Term due to the boundary condition of the Neumann type bringing into play two pennies left the border in opposite and utilizing a single coefficient of exchange *h*.

- mathematical expression:

$$((, (-) (N$$

N

1

-

)). *

HRT

T

T

TD

N

N

2

1

1

and

$$((, (1-) ($$

-

)). *

HRT

T

T

TD

1

2

2

1

2

where 1 and 2 is two pennies left the border in opposite

· denomination of the option in the catalogues: CHAR_THER_PARO_R or CHAR_THER_PARO_F,

· a number of points of integration used: cf [Table 2.2.3-1].

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J.P. LEFEBVRE, X. DESROCHES

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Titrate:

Calculation of the constraints to the nodes by local smoothing

Date:

23/01/97

Author (S):

X. DESROCHES

Key:

R3.06.03-B

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Organization (S): EDF/IMA/MMN

Handbook of Reference

R3.06 booklet: Machine elements and thermal for the continuous mediums

Document: R3.06.03

Calculation of the constraints to the nodes by local smoothing

Summary:

One presents a local method of calculation of constraints at the nodes starting from the constraints at the points of

GAUSS. It is used in options SIGM_ELNO_DEPL and SIEF_ELNO_ELGA of the order CALC_ELEM [U4.61.02].

This method is summarized to calculate the constraints at the tops of an element by multiplying the constraints

at the points of GAUSS by a matrix of smoothing, constant for each type of element.

For the isoparametric elements of degree 2, the constraints with the nodes mediums are obtained by average of the values of the constraints at the 2 tops of the edge.

This method of smoothing has two advantages:

- the nodal constraints obtained have an order of precision moreover than by direct calculation with the nodes,
- the method is inexpensive in time CPU.

This method was generalized:

- with calculations of the deformations (option EPSI_ELNO_DEPL) and the variables intern (option VARI_ELNO_ELGA) with the nodes in mechanics,
- with the calculation of flows (option FLUX_ELNO_TEMP) to the nodes in thermics.

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1 Preliminaries

This method is based on the observation [bib1] which it exists points where the calculation of the constraints, with

to leave displacements in a primal formulation in displacements, is more precise.

In the case of isoparametric finite elements of order 2 (SEG3 in 1D, QUAD8 and QUAD9 in 2D, HEXA20

in 3D), one shows that points of GAUSS of the formula of squaring to $2n$ points (N : dimension of space) are such as one can hope, without that being formally shown, for the calculation of the same order of precision as for the calculation of the field of displacement \mathbf{U} .

The idea of the method is to calculate for each element the constraints! with the nodes from K at the points of GAUSS, these last being calculated on each element by the formula:

$N N 0$

K

K

K

$= \mathbf{dB} \mathbf{U} = D iB U_i$

$i=1$

where:

\mathbf{D} is the matrix of elasticity,

\mathbf{Bk} is the matrix connecting the deformations to displacements at the point of GAUSS K ,

\mathbf{U} are nodal displacements.

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X. DESROCHES*Key:**R3.06.03-B**Page:**3/10***2****Local method of minimization by least squares**

Generally, one wishes to approximate, within the meaning of least squares, a function (X, y) by a function polynômiale:

$! (X, y) = \sum_{i=0}^p \sum_{j=0}^Q a_{ij} x^i y^j$ has

*ij**i=0, p**i=0, Q*

The problem amounts finding the coefficients a_{ij} which minimize the functional calculus:

$$= \int \int \dots$$

2

-

!) $dx Dy$

The values of the function are known here only at the points of Gauss: $K = (x_k, y_k)$

The minimum will be reached if and only if:

$$I = 0, \dots, p$$

=

0

has

$$J = 0, \dots, Q$$
ij

Within the framework of the finite element method in displacement, the function of smoothing is written:

N

$$! (X, y) = \sum_{I=1}^N N_I(X, y)$$
I)! *I**i=1*

where:

N_i is related to form associated with node I ,

!

I is the value of the sought constraint to node I ,

N the number of nodes retained for smoothing.

One must thus solve the system:

$$= 0 \quad I =,$$
1, ..., N

éq 2-1

! I

One can choose between two methods of local smoothing: continuous smoothing or discrete smoothing.

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Methods of local smoothing (réf [bib2] and [bib3])

3.1

Continuous local smoothing

2

2

This type of smoothing led to solve the system [éq 2-1] with $= (-!) =$

NR

.

E

(- I! I)

E

What leads to M E

Fe

! =

with:

npg

M E = NR NR dx Dy = NR

ij

I J

() Nj (K) (det J)

E

I
 K
 K
 K
 $K = 1$
 npg
 and
 $F E = NR dx Dy =$

NR
 I
 I
 $I(K$
 $) K (\det J)$
 K
 K
 E
 $K = 1$

where
 K are the points of GAUSS in the element of reference
 $(\det J)$
 the jacobien of the geometrical transformation enters the element of reference and
 K
 the element running at the point K .
 K : the weight associated with the point K
 K : the constraint at the point K
 $Nor (K$
 $)$: the value of the function of form in the element of reference to the point K

\wedge
 \wedge
 \wedge
 3
 2
 \wedge
 1
 \wedge
 4
 3
 3
 2

2
4
4
1
1

direct calculation of the constraints

smoothed constraints

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Note:

· *If spaces of interpolation of and of! are the same ones, one has obviously =! .*

In practice, one retains for space of! a space smaller than that where is defined by the finite element.

· *One sees the bond between the approximation at the points of GAUSS of where thus converges better and this process of smoothing whose justification is on the contrary continuous.*

· *The way in which is calculated at the points of GAUSS does not intervene. Generalization with the nonlinear problems is thus obvious, although it cannot concern same justification.*

This method is however not adopted because it requires a resolution of system linear for each calculation of!

.

3.2

Discrete local smoothing

In this case, the functional calculus is replaced by the summation:

npg

npg

N

2

~
2

$$= ((K) -! (K) = \\ (K) - Ni! I (K) \\ K = 1 \\ K = 1 \\ i = 1$$

^

^

3
3

^
3

III
4
^
2
4

IV
II
2
4
2
^

1
I
1
1

constraints at the points of GAUSS
smoothed constraints

~
The system to be solved is written there still:
=

0 are:

! I

npg

npg

NR () NR ()! = NR () () I

I

K

J

K

J

I

K

K

K =1 J

K =1

maybe in matric form:

M {! node} = **P** {GAUSS}

The matrices **M** and **P** being then independent of the element running *E*.

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They can thus be calculated once and for all on the element of reference.

Note:

· *This method is more economic than the preceding one and gives results comparable [bib2],*

· *Là still, the manner thus *K* is calculated in each point of GAUSS is indifferent (since the number of points of GAUSS used for the calculation of and that of!*

is the same one). One will be able to thus use this method into nonlinear (option SIEF_ELNO_ELGA).

4**Application of the method to calculation of the constraints to nodes for various elements**

The local smoothing adopted in *Code_Aster* is the discrete local smoothing [§2.2], which makes it possible to avoid it calculation of integrals on the element.

On all the elements of continuous medium 2D and 3D, one chose a space of smoothing resting on functions of form relating to the tops of the element.

The method thus makes it possible to obtain the constraints at the tops. In the case of elements of order 2, one calculates the constraints with the nodes mediums by taking the average value of the two tops “framing” the node medium considered.

One gives hereafter the matrices of passage allowing to calculate the constraints with the nodes tops starting from the constraints at the points of GAUSS. These matrices can be square or rectangular. Indeed, matrices of passage **Mr. P**

1 is calculated once and for all with

the initialization of each type of finite element (in AFFE_MODELE). Two types of matrices exist:

· of

matrices

Square **M-1P**, which are to be used when the number of points of GAUSS used

for the calculation of the constraints at the points of GAUSS K is identical to the number of nodes tops,

· of

matrices

Rectangular **M-1P**, which are to be used when the number of points of GAUSS

K is different (in general higher) nodes tops.

4.1**Square matrices of passage**

These matrices are used by all the elements for option SIGM_ELNO_DEPL. The option calculates in first constraints in a number of points of GAUSS equal to the number of tops. Then them

matrices **M-1P** (given afterwards) are used to calculate the constraints with the nodes. These matrices are also used for option SIEF_ELNO_ELGA, in the elements for which the number

points of GAUSS of the calculation of SIEF_ELGA (in STAT_NON_LINE) is equal to the number of tops. They are the elements:

· in 2D: QUAD4, TRIA6, under-integrated QUAD8,

· in 3D: TETRA4, PENTA6, HEXA8, PYRAM5 and under-integrated HEXA20.

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4.1.1 Square matrices of passage for the elements 2D

4.1.1.1 Triangles

5 -1 -

1

1

1

Mr. P =

-1

5 -

1

3 -1 -1 5

4.1.1.2 Quadrangles

3

1

3

1

1+

-

1-

-

2

2

2

2

1

3

1

3

-
1+
-
1-

1

2
2
2
2
Mr. P =

3
1
3
1
1-
-
1+
-

2
2
2
2

1
3
1
3
-
1-
-
1+

2
2
2
2

4.1.2 Square matrices of passage for the elements 3D
4.1.2.1 Tetrahedrons

has

has

has -1

has

has has -

1

has

has

Mr. P

1

1

=

has - B has -1

has

has

has

has

has

has

has -

1

5 - 5

5 + 3 5

has =

B =

20

20

4.1.2.2 Pentahedrons

-

1 - -1 1 -

-

1 - 1 - -1

-

-1 1- 1 -

M-1P =

1 - -1 1 -

-

1 - 1 - -1

-

-1 1- 1 -

-

3 +1

= 2

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4.1.2.3 Hexahedrons

B B C B C C D has

5 + 3 3

has =

4

B C C D has B B C

(1+ 3)

C D B C B a.c. B

B =

4

B a.c. B C D B C

Mr. P

1
=

B has C B C B D C
(3 -) 1

C =
C B D C B has C B
4
D C C B C B B has

(5-3 3)

C B B has D C C B

D =

4

7

6

VIII

VI

8

5

VII

IV

3

V

II

Z

2

III

y

X

I

4

1

**Appear 4.1.2.3 - has: Classification of the points of GAUSS
on the hexahedron with 8 nodes**

4.2

Matrices of passage Mr. P

1 rectangular

Into nonlinear for certain types of elements (TRIA3, QUAD8 and QUAD9 in 2D, TETRA10, PENTA15 and

HEXA20 in 3D), the internal constraints and variables at the points of GAUSS are calculated on one family of points of richer GAUSS (9 points for the quadrangles, 15 points for the tetrahedrons, 21 points for the pentahedrons, 27 points for the hexahedrons).

Discrete local smoothing is then carried out starting from these fields and made transport with the nodes to intervene of the matrices different from the preceding ones. They are not square any more, because of dimension

(Nb tops, Nb points of GAUSS). Matrices of passage **Mr. P**

1 is not calculated

explicitly, in particular **M** is reversed by *Aster*.

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In the particular case of the triangle with 3 nodes, the fields are supposed to be constant by element (one only point of GAUSS) and:

1

1

Mr. P =

1

1

For example, the calculation carried out by option SIEF_ELNO_ELGA is then the following:

If the constraints were calculated (in STAT_NON_LINE for example) on a family

having a number of points of GAUSS higher than the number of tops (for the elements

Nb tops Nb pts Gauss

announced above). **Mr. P**

1 is then rectangular, and!

-1

K

$I =$

(**MR. P**).

ik

$i=1$

$K=1$

If not, if the number of points of GAUSS equal to the number of tops, (**Mr. P**

1) is then square.

One calculates!

-

K

$I =$ (**MR. P**

1) [§4.1].

ik

5

Other options of calculation using the same method

The method described previously is used in *Code_Aster* to calculate the deformations, them internal variables and flows with the nodes.

The list of modelings supporting these options is given below, with the numbers of routines of elementary terms YOU corresponding.

The produced fields are cham_elem with the nodes.

5.1 Phenomenon

:

“**MECHANICAL**”

MODELING

SIGM_ELNO_DEPL

EPSI_ELNO_DEPL

SIEF_ELNO_ELGA

VARI_ELNO_ELGA

AXIS

TE0086

TE0087

TE0098

AXIS_SI

TE0086

TE0087

TE0098

C_PLAN

TE0086

TE0087

TE0098

D_PLAN

TE0086
TE0087
TE0098
D_PLAN_SI
TE0086
TE0087
TE0098
AXIS_FOURIER
TE0116
TE0114
3D
TE0023
TE0025
TE0020
3D_SI
TE0023
TE0025
TE0020
AXIS_META
TE0352
TE0087
TE0098
3D_META
TE0357
TE0025
TE0020
AXIS_INCO
TE0448
TE0447
not disp
PLAN_INCO
TE0448
TE0447
not disp
3D_INCO
TE0454
TE0453
not disp
COQUE_AXIS
TE0230
TE0229
not disp
COQUE_C_PLAN

TE0230
TE0229
not disp
COQUE_D_PLAN

TE0230
TE0229
not disp

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5.2 Phenomenon

:

“THERMAL”

MODELING

FLUX_ELNO_TEMP

META_ELNO_TEMP

META_INIT_ELNO

AXIS

TE0069

TE0067

TE0320

PLAN

TE0069

TE0067

TE0320

AXIS_FOURIER

TE0265

not disp

not disp

3D

TE0063

TE0064

TE0321

HULL

not disp

not disp

not disp

6

Other methods of smoothing of constraints

There are two other methods of smoothing, relating only to the constraints, used by estimators of Zhu-Zienkiewicz version 1 and 2 [R4.10.01 §3].

The stress fields to the produced nodes are then cham_no.

The corresponding options of calculation are accessible by order CALC_ELEM [U4.61.02].

7 Bibliography

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Document: R3.06.04

Elements of Fourier for the structures

axisymmetric

Summary

The elements of Fourier are intended to calculate the response of structure for axisymmetric geometry solicited

by nonaxisymmetric loadings broken up into Fourier series.

One exposes in this document a general theory of Analysis of Fourier with coupling of the symmetrical modes

and antisymmetric in the anisotropic case. The case of isotropic, or orthotropic materials of axis OZ, where

modes are uncoupled, is studied separately.

The elements of Fourier are usable in *Code_Aster* starting from modeling AXIS_FOURIER.

meshes supports of these elements are triangles and quadrangles of degree 1 and 2.

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1 Introduction

The analysis of Fourier is intended to calculate the response of structures for axisymmetric geometries subjected to nonaxisymmetric loadings. In this case, it is necessary to develop them loadings in Fourier series. Generally convergence is reached for 4 or 5 harmonics, but the speed of this convergence depends on the nature of the loading: more the loading is regular and more the corresponding series converges quickly. The most unfavourable case is that of one force concentrated for which the practice shows that it is necessary to go to beyond (at least 7 harmonics).

In *Code_Aster*, the decomposition of the loading in Fourier series is supposed to be made au préalable by the user. *Code_Aster* makes it possible to calculate the answers to this loading, harmonic by harmonic (modeling `AXIS_FOURIER`), and overall after recombination of harmonics between them (operators `COMB_CHAM_NO` and `COMB_CHAM_ELEM`).

One will expose in a first chapter the framework general of the anisotropy, while insisting on decoupling modes in the orthotropic case. The second chapter clarifies the calculation of the matrix of rigidity in the isotropic case.

For the use of the elements of Fourier in *Code_Aster*, one returns to the note of use of modeling Fourier [U1.01.07].

2

Analyze of anisotropic Fourier

2.1 Theory

general

All the fields considered (forces, displacements, strains, stresses) are expressed in cylindrical co-ordinates with following convention on the order of the components:

1 radial component according to R

2 axial component according to Z

3 tangential component according to

Example: (ur, uz, U), (Fr, fz, F)

Z

R

uz

M

U

U

R

The grid is localised in plan (R, Z), the symmetry of revolution being done around axis OZ. trihedron (R, Z,) is directed in the direct direction.

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Z

rez

R

er

C

One breaks up displacement **U** (or the loading **F**) according to $\mathbf{U} = \mathbf{custom} + \mathbf{ua}$

custom

where

(resp. **ua**)

indicate the symmetrical part (resp. antisymmetric) of the development in Fourier series of **U** by report/ratio with the variable.

One obtains:

$custom = custom$

R

$L(R, Z) \cos l$

$l=0$

$custom =$

$v S$

S

Z

$L(R, Z) \cos l$ left symmetrical U

$l=0$
 $custom =$
 $ws,$
 \sin
 $L(R, Z) (-L)$
 $l=0$
 $ua = ua$
 R
 $L(R, Z) \sin L$
 $l=0$
 $ua = goes$
 has
 Z
 $L(R, Z) \sin L$ left antisymétriqu U
 E
 $l=0$
 $ua = wa, \cos$
 $L(R, Z)$
 L
 $l=0$

To note the choice of the sign for *custom*, which makes it possible to simplify later calculations. If one notes

 $\mathbf{Custom} = (S S S$
 has
 L
 U, lv, L
 $W) \text{ (resp. } \mathbf{U}$
 L

L) the L -ième component symmetrical (resp. antisymmetric) of development in Fourier series of \mathbf{U} , one obtains:

cosl

0

sinl

0

S

has

$\mathbf{U} =$

cosl

$\mathbf{U}l +$

sin L

$\mathbf{U}l$

éq 2.1-1

$l=0$

0

- sinl

0

cosl

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If one indicates by the vector deformation linearized, one realizes that can be broken up into following Fourier series:

$\cos l I$

0

$\sin l$

4

4 2

I

0

=

,

4

4,

S

2

has

$L +$

éq 2.1-2

0

- $\sin l$

2 4

I

0

$\cos l$

2

2 4

I

L

$l =$

,

,

2

0

with = $\{R, Z, rz, R, Z\}$

S

S

S

has

has

has

L =

L

B UI

L = L

B UI

with (see [bib1]):

0

0

R

0

0

Z

1

L

0

-

Bs

R

R

L =

0

Z R

L

1
0
-
 R
 $R R$

L

0

R
 Z

There is Ba

Bs

=

L

L

L

(this is due to the choice of the symmetrical development of \mathbf{U} in (cos, cos, sin) to the place of (cos, cos, sin)). One will omit starting from now the indices has and S and one will note Bl the operator

allowing to calculate the deformations corresponding to the harmonic L .

2.2

Coupling and decoupling of the symmetrical and antisymmetric modes

By taking again the preceding notations, one a:

$\cos l I$

0

\sin

2

2,

L

1

$I2$

02,1

$\mathbf{U} =$

custom +

ua

0

- $\sin l$

L

1 2

0

cos

L

,

,

L

1 2

L

L

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what is written, by introducing matrices $M S$ and M has

L

L:

$\mathbf{U} = (M sUs$

has

has

L

$L + MR. \mathbf{U}$

L

L)

L

U

S

S

has

has

$$L = MR. U$$

L

$$L + MR. U$$

L

L

One deduces from it that:

S S

has has

$$L = Me L L + Me L L$$

cosl I4

04,2

with

$$M L =$$

02 4

- *sinl I*

,

2

sinl I4

04 2

M has

,

$$L =$$

0

cosl

2 4

I

,

2

Calculation of the deformation energy

2

W

T

=

D dsd

with $ds = r dr dz$

L

L L

0 S

2

2

= *D*

T S T

M D M S S

'*ds + D*

T has T

M a' DM has has

'*ds*

L

L

L L

L

L

L

L

0

S

0

S

2

2

+ *D*

T has T

M a' DM S S

'*ds + D*

T S T

M DM is had has

'*ds*

L

L

L L

L

L

L

L

0

S

0

S

sinl *I4*

0

D

D

1

3 cosl *I4*

0

Since *M* has

'*DM S*

L

'*L =*

T

0

cosl *I2 D*

D

3

2

0

- sinl *I2*

2

has

S

D sin L cosl

1

- *D3 (sinl)*

Me DM

L

'*L =*

T D

2

3 (cos *L*)

- *D sinl cosl*

2

2

and that

sin *L cosl D =*

, if *D*

=

0

0

T has

S

T S

has

3

there is thus no term (L, L) or (L, L) in W .

0

There is then no coupling (*U has U.S.*) or (*U.S.U has*

,

,

).

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2.3

Calculation of the constraints

Just as, can be broken up into following Fourier series:

$= (M S$

has

has

$L + M$

L

$'L L)$

L

Law of Hooke = D , one deduces:

$L D I -$

L D3
sinl D
cosl
1
D3
= cos
sin

S
has
T
L +
T

cosl D3 - sinl D2
sinl D
cosl
3
D2 L
L

Maybe, while revealing the matrices *M S* and *M has*
'*L*
'*L*:

D1 04,2
0
D
4,4
3

= M S
S
has
'*L*

L +
T
L
0
0
L

D
2,4
2
- *D*³
2,2

04,4 - *D*³
*D*¹ 04,2

+ *M a'*
S
has
L
T

L +
L

*D*³
02,2
0
D
2,4
2

D
0
0
D
1
4,2
4,4
3

By posing *D**s*
and *D**a* =
, one deduces the parts from them symmetrical and

0
D
- *D**t*
2,4

2

3

02,2

antisymmetric of the constraint relating to the harmonic L :

S

$S S$

has has

S

S

has

has

$$L = D L + D L = D$$

L

$$B \mathbf{ul} + D L$$

$B \mathbf{ul}$

éq 2.3-1

has

S has

S.A.

has

S

S

has

$$L = - D L + D L = - D$$

L

$$B \mathbf{ul} + D L$$

$B \mathbf{ul}$

Note:

In the case of the orthotropism compared to OZ, there is $Da = 0$ and [éq 2.1-1] is reduced to:

S

S

S

$$L = D$$

L

$B \mathbf{ul}$

has

S

has

$L = D$

L

B ul

I.e. if displacements are symmetrical (or antisymmetric), the constraints are it too.

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Calculation of the matrix of rigidity

3.1 Case

general

Are U and two fields kinematically acceptable unspecified. By applying the principle of virtual work with the element of volume v , one can write:

$(T.) FD = (T U$

$. F) FD$

v

v

After decomposition in Fourier series and integration compared to, one obtains, for fields

$S.A.S.A.$

L, L, L

U, L

u.a. A. unspecified and for any harmonic L :

$(T S S T$ has has

T

S

$S T$

has

has

L

.1 + L .1) L

ds = (read.fl + read.fl) L

ds

S

S

L

L

Maybe, while using [éq 2.3-1] and while posing:

K S

T

=

B Ds B ds

L

L

L

L

sl

K has

T

=

B Ds B ds = K S = K

L

L

L

L

L

L

sl

K have

T

=

B Da B ds

L

L

L

L

sl

One obtains the system of equations according to:

K custom + Kas ua = F S

L L

L

L
L

éq 3-1

T Kasus

+ *K ua = F has*

L
L
L L
L

where *T*

have

have

Kl = - Kl one sees that if *Da* \neq 0, the decoupling of the modes in symmetrical harmonics and antisymmetric is not possible any more. On the other hand, if *Da* = 0 (orthotropism compared to OZ) then

Kas

L

= 0 and [éq 3-1] are reduced to:

K custom = F S

L
L
L

K ua = F has

L L
L

While taking for vectors displacement (resp. force) corresponding to the harmonic *L the* vectors:

U
S
S
S

has

has

has

L = {R

U, Z

U, U,

R

U, Z

U, U} L

F

S

S

S

has

has

has

$L = \{Fr, fz, F, F$

$R, F Z, F\} L$

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One has then:

have

G

G

K

K

$K \mathbf{U} = \mathbf{F}$

with K

L

L

L

L

L

$L = T$

Kas

K

L

L

3.2 Calculation

of

$K gl$ in the isotropic case

In this case there are thus K have

T

S

L

= 0. One details in the continuation the calculation of $K =$

$B D B ds$

L

S

L

L

L

L

In the isotropic case, one a:

$D1 D2 D2$

0

$D2 D1 D$

2

0

0

$D2 D2 D1$

0

$D Ds$

=

=

0

0

0

D

3

D

3

0

0

0

D

3

E (1 -)

with

$$D1 = (1+) (1 - 2)$$

E

$$D2 = (1+) (1 - 2)$$

E

$$D3 = (21+)$$

One can write:

R

Z

S

U

R

T

S

,

ur uz

U

ur uz

U

ur uz

U

= L

B U

Z = L

B

,

,

,

,

,

,

,

,

rz

R R R R R Z Z Z

S

U

R

Z

fcts of form

derived from the fcts of form

0

0

0

1 0

0

0 0

0

000000010

1
0
- L 000000
with $B' L =$

0
0
0
01
0
10
0

L
0
-1001000

0
 L
0
00
0
00
1

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While indicating by {*WJ*}

functions of form of the element considered, one a:

J = 1 with *N*

ur

node *J*

R

W

"#\$

J

!

0

0

!

U

R

Z

W

R

J

!

0

0

!

U

R

W

J

R

!

0
0

R
!

U

R
W

J

0
0
R

!

R

!

U
R (J)

U

W

J

U

Z
 $=$
 $= !$

0
 0
 $! uz (J)$

R
 R

W

U
 J

$U (J)$

0
 0

$!$
 R
 $!$

R

W
 J

U

0
0

R
!

Z
!

Z

W

J

U

!
0
0
!
Z

Z

Z

WJ

!
0
0

U
%&'
!
Z

block *P*

J
Z

One nodes (*P*) = (*P*

P

1,

,

! *NR*) where *NR* is the number of nodes of the element.

Then *K*

T

=

Pt B D B P ds

L

,

,

L

L

L

sl

I, J

Kl symmetrical and is made of blocks (*Kl*) 3 × 3:
(*I, J*

K)

T P you

,

= *I B D B P ds*

L

L

L

J

L

sl

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I, J

The calculation of the blocks (*KI*) is clarified below:

D1+12 D3

0

- *L (D1+ D)*

3

D2

0

ID3

0

D2

0

0

I2 D3

0

0

0

0

0

0

ID

3

- *L (D1+ D) 3*

0

L 2 D1+ D3

- *ID2*

0

- *D3 0*

- *ID2*

0

D2

0

- *ID2*

D1

0

0

0

D2

0

tB' D B'

L

L =

0

0

0

0

D3

0

D3

0

0

ID3

0

- *D3*

0

0

D3

0

0

0

0

0

0

0

D3

0

D3

0

0

D2

0

- *ID2*

D2

0

0

0

D1

0

0

ID3

0

0

0

0

0

0

D

3

I, J

I, J

I, J

K

K

K

I, J

11

12

13

,

,

I, J

I, J

I, J

T
T
L
I
P
L
B D L
B
J
 $P = (K_{ij})$

=
with
 $3 \times$

K
K
K
21
22
23
 $3 I, J$
I, J
I, J
K
K
K
31
32
33

2
I, J
 $D1 + L D3$
W

W
W W

D2
W

W
J
I

K11 =

I
J
I
J +
WI
+ W
2
I
W
J
W + D1
+ D3

R

R

R
ZZ
R
R
J

R

2
I, J
LD3
WW

WW
K22 =
WW

I
J
I
J

2
I
J + D3
+ D1

R

R

R

Z

Z

2

I, J

LDI + D3

W W

W

W

I

J

IJ

D3

W

W

J

I

K33 =

W W

2

I

J + D3

+

W

W

R

R R

Z

Z -

I

+
R
R
R
J

W

W

W

W
I, J
D2
W

K
= D

I
J
2
+ D

I
J
3
+
W
J
12

RZ
ZR
R
I
Z

W

W
W

W
I, J
D2
W
K
 = *D*
I
J
 3
 + *D*
I
J
 2
 +
W
I
 21
R
Z

ZR
R
JZ

I, J
L
L
W
L
W

K
 13 = -
 (*D1 + D*)
 3 *W W - D2W*
I + D W
J
 3
R 2
I
J
R
J

R

R

I

R

I, J

L

L

W

L

J

W

K

I

$31 = -$

$(DI + D)$

$3 W W - D2W$

$+ D W$

3

R 2

I

J

R

I

R

R

J R

I, J

L

W

L

I

W

$K23 = - D2$

$W + D W$

J

3

R

Z

J
R
I
Z
I, J
L
W
L
J
W
 $K_{32} = -D_{2W}$
 $+D$
I
3
W

R
I
Z
R
Z
J

The blocks K, I, J are not symmetrical except for $I = J$ (on the diagonal of K). One notices in fact that the blocks K, I, J can be written for any harmonic ($L = 0$ included/understood).

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D3

$K I, J = K I, J$

0

+ l2

W W

11

11

r2

I

J

D3

$K I, J = K I, J$

0

+ l2

W W

22

22

r2

I

J

D1

$K I, J = K I, J$

0

+ l2

W W

33

33

$r2 I J$
 KI, J

$= K I, J$
 12
 12
 0

$K I, J = K I, J$
 21
 021
 KI, J

$= - L K I, J$
 13
 13
 0

$KI, J = - L K I, J$
 31
 031
 $KI, J = - L K I, J$

23
 023

$KI, J = - L K I, J$
 32
 032

where the blocks $K I J$
 0, are independent of the harmonic L .

4 Loadings

It is supposed that the loading was broken up according to the same base which displacements, that is to say:

cosl
 0

sinl
 0

S
has
 $\mathbf{F} =$

 $\cos l$
 $\mathbf{F} l +$
 $\sin L$
 $\mathbf{F} l = 0$

 0
 $-\sin l$

 0
 $\cos l$

There is not coupling for the same harmonic between the parts symmetrical and antisymmetrical of loading because of orthogonality of the goniometrical functions $\sin L$ and $\cos l$, this for all types of loading. This wants to say in particular that the equivalent nodal forces are them same for the harmonics symmetrical and antisymmetrical if the amplitudes $F S$ and $F has L$ are them same.

For the nature of the acceptable loadings with Fourier modeling, one returns to the note of use [U1.01.07].

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Conclusion and Outlines

Currently, it is supposed that the decomposition of the loading was made as a preliminary by the user, i.e. $\{F S, F$

L

$L\}$

is known. This decomposition could be carried out by an operator

l_0

of *Code_Aster* which would project the loading on the modes of Fourier.

For the moment, only the nonanisotropic case is established, i.e. it y forever coupling of modes. The extension to the anisotropy can constitute a later development.

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Document: R3.06.06

**Functions of form and points of integration
elements pyramid at square base**

Summary:

The free maillor of CASTEM 2000 creating under certain conditions of the meshes of pyramidal form at base quadrangular, the associated finite elements were established in *Code_Aster*.

These elements have the characteristic to have rational functions of form, although guaranteeing a connection continuous with the traditional tetrahedrons or hexahedrons.

The expression of the functions of form and the formulas of numerical integration were communicated to us by CEA/DMT [bib1] and are thus those used by CASTEM 2000.

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1 General information

Two new finite elements pyramid at square base were established in Code_Aster for modelings three-dimensional mechanics and thermics:

.

pyramid with 5 nodes,

.

pyramid with 13 nodes.

The functions of form associated with these elements are rational functions which make it possible to have a continuous connection (C0) between these elements and traditional tetrahedrons and hexahedrons.

For example, the function of form associated with a top with the base with the pyramid is the product of equations of the plans passing by the other nodes, divided by the distance to the base of the pyramid. On a triangular face of the pyramid containing this top, the distance to the axis is simplified with the equation of the plan of the opposite face: the expression of the function form is then that of the triangles traditional.

The functions of form are not derivable at the top of the pyramid. Integration by points of Gauss cannot thus be exact even for the element of reference.

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2

Pyramid at square base

2.1 Denominations

The names of the finite elements are coded and respect following conventions

.

the characters in position 1 to 4 indicate the modelled phenomenon:

MECA: mechanics

THER: thermics

.

the character in position 5 is _,

.

starting from character 6, the name of the mesh support:

PYRAM5

: pyramid at square base with 5 nodes,

PYRAM13: pyramid at square base with 13 nodes.

Example:

MECA_PYRAM5: pyramid at square base with 5 nodes in mechanics.

2.2

Geometry, topology and functions of form

2.2.1 Pyramid with 5 nodes

Z

N5

N3

N4

N2

y

N1
X

The square base is consisted the quadrangle N1 N2 N3 N4 and N5 is the top of the pyramid.

X
y
Z

N1 1. 0.

0.

N2 0. 1.

0.

N3 1. 0. 0.

N4 0.

-1.

0.

N5 0. 0.

1.

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Functions of form:

(- X + y + Z -) 1 (- X - y + Z -) 1

w1 =

4 (1 - Z)

(- X - y + Z -) 1 (X - y + Z -) 1

$w_2 =$

$4(1 - Z)$

$(X + y + Z -) I (X - y + Z -) I$

$w_3 =$

$4(1 - Z)$

$(X + y + Z -) I (-X + y + Z -) I$

$w_4 =$

$4(1 - Z)$

$w = Z$

5

Formulate numerical integration at 5 points:

Not X y Z Weight

1 0.5

0.

$h_1 \ 2/15$

2 0.

0.5

$h_1 \ 2/15$

3 -0.5

0.

$h_1 \ 2/15$

4 0.

-0.5

$h_1 \ 2/15$

5 0.

0.

$H_2 \ 2/15$

with:

$h_1 = 0.1531754163448146$

$H_2 = 0.6372983346207416$

1 initialized family:

1st family: formulate at 5 points.

2.2.2 Pyramid with 13 nodes

Z

N5

N12

N3
N13
N8
N7
N11
N10
N4
N2
y
N9
N6
N1
X

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The square base is consisted mesh QUAD8 (N1 N2 N3 N4 N6 N7 N8 N9) and N5 is the top of the pyramid.

X

y

Z

N1 1. 0.

0.

N2 0. 1.

0.

N3 1. 0. 0.

N4 0.**-1.****0.****N5 0. 0.****1.****N6 0.5****0.5****0.****N7 0.5 0.5****0.****N8 0.5****-0.5****0.****N9 0.5****-0.5****0.****N10 0.5****0.****0.5****N11 0. 0.5****0.5****N12 0.5 0. 0.5****N13 0.****-0.5****0.5****Functions of form:** **$(-X + y + Z -) I (-X - y + Z -) I (X - 0.) 5$** **$w1 =$** **$2 (1 - Z)$** **$(-X - y + Z -) I (X - y + Z -) I (y - 0.) 5$** **$w2 =$** **$2 (1 - Z)$** **$(X - y + Z -) I (X + y + Z -) I (-X - 0.) 5$** **$W3 =$** **$2 (1 - Z)$** **$(X + y + Z -) I (-X + y + Z -) I (-y - 0.) 5$** **$w4 =$** **$2 (1 - Z)$** **$W = 2 Z (Z - 0.)$** **5****5**

$$(- X + y + Z -) I (- X - y + Z -) I (X - y + Z -) I$$

$$w6 = -$$

$$2 (1 - Z)$$

$$(- X - y + Z -) I (X - y + Z -) I (X + y + Z) I$$

$$w7 = -$$

$$2 (1 - Z)$$

$$(X - y + Z -) I (X + y + Z -) I (- X + y + Z -) I$$

$$W = -$$

$$8$$

$$2 (1 - Z)$$

(

$$X + y + Z -)$$

$$I (- X + y + Z -)$$

$$I (- X - y + Z -)$$

$$I$$

$$W = -$$

$$9$$

$$2 (1 - Z)$$

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$$Z (- X + y + Z -)$$

$$I (- X - y + Z -)$$

$$I$$

$$w10 =$$

$$1 - Z$$

Z (- X - y + Z -)

1 (X - y + Z -)

1

w11 =

1 - Z

Z (X - y + Z -)

1 (X + y + Z -)

1

w12 =

1 - Z

Z (X + y + Z -)

1 (- X + y + Z -)

1

w13 =

1 - Z

Formulate numerical integration: formulate at 6 points

Not X y Z Weight

1 A 0.

h1

p1

2 0.

has

h1

p1

3 -

0. h1 has

p1

4 0.

-

has

h1

p1

5 0.

0.

H2

p2

6 0.

0.

h3

p3

with:

p1 = 0.1024890634400000

p2 = 0.1100000000000000

p3 = 0.1467104129066667

= 0.5702963741068025 have

h1 = 0.1666666666666666

H2 = 0.08063183038464675

h3 = 0.6098484849057127

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Formulate at 27 points:

Not X y Z Weight

1 0.

0.

1/2 a1

2

B

B

1 (

1
1 - Z)
1/2
b6

1
(- Z)
2
2
3
B
B

- 1 1
(- Z)
1 1
(- Z)
1/2
b6
2
2
4
B
B

- 1 1
(- Z)
- 1 1
(- Z)
1/2
b6
2
2
5
B
B

1 (
1
1 - Z)
1/2
b6

-

1
(- Z)
2
2
6

0.
0.
1 - b1
B

6
2
7

0.
0.
1 + b1
B

6
2
8
C (
Z)
C
1 1 -

0.
(1-c) /2
8
1

9 0.
C (
Z)
C
1 1 -

(1-c) /2
8
1

10

- C (- Z)

C

1 1

0.

(1-c) /2

8

1

11 0.

- C (

)

C

1 1 - Z

(1-c) /2

8

1

12

C (

Z)

C

1 1 -

0.

(1+c)/2

8

1

13 0.

C (

Z)

C

1 1 -

(1+c)/2

8

1

14

- C (

)

C

1 1 - Z

0.

(1+c)/2

8

1

15 0.

- C (

)

C

1 1 - Z

(1+c)/2

8

1

16

d1

D

(

)

1

(- Z)

1 - D/2

1 1

(- Z)

1

d12

2

2

17

D

D

- 1

(

)

1

(- Z)

1 - D/2

1 1

(- Z)

1

d12

2

2

18

D

D

- 1

(

)

1

(- Z)

1 - D/2

- 1 1

(- Z)

1

d12

2

2

19

d1

D

(

)

1

(- Z)

1 - D/2

- 1 1

(- Z)

1

d12

2

2

20

D (

)

1 1 - Z

0. 1/2 d12

21 0.

D (

)
1 1 - Z
1/2 d12
22
- D (- Z)
1 1

0. 1/2 d12
23 0.
- D (
)

1 1 - Z
1/2 d12
24
d1
D
D
1

(- Z)
1 1
(- Z)
(1+d) /2
12
1

2
2
25
D
D
- 1
(
)

1
(- Z)
1 + D/2

1 1
(- Z)
1

d12
2
2

26

D

D

- 1

(

)

1

(- Z)

1 + D/2

- 1 1

(- Z)

1

d12

2

2

27

d1

D

(

)

1

(- Z)

1 + D/2

- 1 1

(- Z)

1

d12

2

2

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with:

$a1 = 0.788073483$

$b6 = 0.499369002$

$b1 = 0.848418011$

$c8 = 0.478508449$

$c1 = 0.652816472$

$d12 = 0.032303742$

$d1 = 1.106412899$

Note:

It proved in practice, in particular for the thermal elements, that the formula of integration at 6 points was not satisfactory. Only the formula at 27 points is thus used.

1 initialized family:

1st family: formulate at 27 points.

3 Bibliography

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R3.06.07 document

Diagonalisation of the thermal matrix of mass

Summary:

To improve the regularity of the solution in the problems of transitory thermics, one of the solutions consist with “lumper” (i.e.: to condense on the diagonal) the thermal matrix of mass (matrix of capacity).

This possibility is accessible by modelings PLAN_DIAG, AXIS_DIAG and 3D_DIAG for the phenomenon

THERMICS. It is activated at the time of the call to the orders of thermal calculation THER_LINEAIRE and

THER_NON_LINE.

When these modelings are used, only the linear finite elements (2D and 3D) have a matrix of mass lumpée. Indeed, the direct diagonalisation does not give satisfactory results for the finite elements quadratic. Consequently, for the quadratic finite elements 2D, one carries out a cutting in linear elements, which are lumpés. On the other hand, for the quadratic finite elements 3D, one does not make

diagonalisation of the matrix of mass.

The theoretical results are illustrated by the thermomechanical calculation of a cylinder subjected to a thermal shock.

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1 Introduction

One is interested in transitory thermal calculations where intervene of abrupt variations of loadings - for example, thermal shocks. In certain cases, one notes that the temperature oscillate spatially and temporally. Moreover, if one observes a profile of temperature at one moment given transient, the temperature can in certain nodes exceed the terminals min. and max. imposed by the initial conditions and the boundary conditions. This result physically unacceptable violate what is called the "principle of the maximum".

The diagonalisation of the matrix of mass can solve these problems of going beyond of maximum. This is detailed in the note [bib1]. One is satisfied here to recall the principal results of them. One points out the principle of the maximum in the continuous case, then sufficient conditions are expressed

who allow to check it for the discrete equations. It is shown in particular that the diagonalisation thermal matrix of mass is one of these sufficient conditions and one presents different methods for diagonaliser **Mr.**

Another sufficient condition depends on the thermal matrix of rigidity (conduction). One studies more particularly from this point of view finite elements of thermics used in *Code_Aster*.

It results from it that in the case of the linear elements, all the sufficient conditions to check it principle of the maximum are gathered. In particular, the diagonalisation of the mass allows indeed to obtain a regular solution. On the other hand, for the quadratic elements, one cannot to prevent the oscillations.

One thus describes the solution suggested in *Code_Aster*: the modelings developed in 2D (AXIS_DIAG, PLAN_DIAG) function with linear elements (if the mesh is of order 2, one cut out in linear elements for thermal calculation). In 3D, one treats only the linear elements. A numerical study of a thermal shock on a cylinder makes it possible to illustrate these results.

2

Principle of the maximum

2.1

Statement of the principle for the continuous case

One gives here one of the statements possible of the principle of the maximum for the operator of heat (in

the absence of terms of source, and in isotropic homogeneous linear thermics) [bib2].

That is to say open limited of \mathbb{R}^n of border, whose adherence is noted.

That is to say (

U, X, T) such as:

$U - 0$ on $\times 0$], [, ($>$

U
 T
 T
 0)
 T
of class 2 compared to X and U of class

1
 C
 C compared to T on $\times] 0, T [$
Then $\max U = \max U$, where $P = (\times \{ \}$
 $0) (\times [,$
 $0 T])$ is the border of the cylinder $\times] 0, T [$.
 $[$
 $\times 0, T]$
 P

This result thus ensures that the maximum of U is inevitably reached either at the time of the initial conditions maybe on an edge of the field during the transient.

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2.2

Respect of the principle of the maximum at the discrete level

The equation of heat (thermal conduction) is considered:

T

$$\text{div} (. T) + S (X, T) = C$$

0

P

=

+ **Conditions limit + initial condition** $T(T, X) T(X$

0

)

 T

with

 T

temperature

 S

heat per unit of volume (sources intern)

 T

variable of time

 X

variable of space

thermal coefficient of conductivity

 CP

voluminal heat with constant pressure

Limiting types of conditions:

· Imposed Temperature: condition limits of Dirichlet

 $T(X, T) = T(X, T$ imp) on imp

· Imposed normal Flux: condition of Neumann defining flow entering the field

- (

 $\mathbf{Q} X, T) \cdot \mathbf{n} = F(X, T)$ on $flow$

· Exchange: condition limits of Fourier modelling the convectifs exchanges on the edges of field

- (

 $\mathbf{Q} X, T) \cdot \mathbf{n} = ($ $H X, T) (T(X, T) - T(X, T$ $ext.$) on $éch$ $angel$

The variational formulation of the problem is as follows: [bib3]

 T C · $D + T$ · $D +$ $HT. D = S.D +$ $F. D +$ HT

ext.

D

p

T

exchange

flow

exchange

v checking $v = T(X, T$

imp

) on *imp*

After discretization in space of this equation, one obtains the system:

T

M

$(T) + \mathbf{K} ($

T T) = F

$(T).$

T

with

(

T T): vector of the nodal temperatures

M: stamp of thermal mass

M = C NN

T FD

E E

K

T

T

: stamp thermal rigidity

K = NR. NR

FD +
H N.N

D
E E
exchange

E

F: vector of the second member

F =
S Nd +
F NR D +
H T NR

D
ext.

E E
flow

E
echangee

NR: (functions of form)

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For the discretization in time, one applies one - method ([0]

1

), which leads to:

(M + K

T) T_{n+1} = (M + (-) tK) T_n + F_{n+}

1

1

+ (1 -) **F_n**

where **T_n T_n**

,

+1 is the vectors of the nodal temperatures at the moment T, T

N

n+1.

2.3 Conditions sufficient for the respect of the principle of the maximum with discrete level

One of the characteristics of nonrespect of the principle of the maximum is the appearance of oscillations

(temporal or space): if one observes the variation in the temperature in a node during time, it is noted that the solution oscillates and exceeds the values minimal and maximum determined by initial conditions and limiting conditions. Or, at a given moment, one observes oscillations space.

One thus seeks sufficient conditions on T, **K** and **M** so that the solution does not oscillate with run from time ([bib1], [bib4], [bib5]). Indeed, one cannot obtain conditions necessary and sufficient. One thus seeks conditions of nonoscillation of the solution in the course of time. If those are checked, it will be checked that the space oscillations also disappeared, and then it respect of the principle of the maximum is assured.

Assumptions:

To be able to express these sufficient conditions of nonoscillation, two should be added assumptions:

· *one is placed at the elementary level. The respect of the properties at the elementary level is enough for that the conditions of nonoscillation are checked for the assembled matrices.*

· *one considers that the **matrix of rigidity K** is formed only of the voluminal term*

K

NR NR

V =

. *TFD*

E

This assumption is valid whenever there is no condition of exchange or when it coefficient of exchange H is sufficiently large: one can then approach the condition of exchange by a condition in imposed temperature.

The sufficient conditions of nonoscillation amount expressing conditions on the step of time and on the diagonal and extra-diagonal terms of \mathbf{M} and \mathbf{K} so that certain properties of these matrices are checked (based on the monotony of the matrices) [bib1]:

$M + T$

$K \ 0 \ I \ J$

ij

ij

éq 2.3-1

$M + (-)$

$1 \ T$

$K \ 0 \ I \ J$

ij

ij

éq 2.3-2

$M + (-)$

$1 \ T$

$K \ 0 \ I$

II

II

éq 2.3-3

In the case general, the extra-diagonal terms can be of unspecified sign. A fast study allows to determine the conditions on T according to their signs so that the equations the preceding ones are checked:

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Kij 0

Kij 0

Mij

M

M

+ .

0

II

ij 0

M

tK

I

J

ij

ij

max

T

min

K

(1) K

[éq 2.3-1] unconditionally distorts

I J

ij

I

II

except $M = K$

ij

$ij = 0$

M

M

M

ij

II

$ij 0$

$M + (-)$

$1 T$

$K 0$

$I J$

ij

ij

max

T

min

$(1 -) K$

$(1) K$

[éq 2.3-2] unconditionally distorts

$I J$

ij

I

II

except $M = K$

ij

$ij = 0$

Some is T and the form of \mathbf{M} ,
Interval E to be respected on T .
there is **risk** of oscillations.
The diagonalisation of \mathbf{M} allows
to remove the lower limit.

The sufficient conditions to avoid the oscillations are then:

K

$0 I$

J

ij

T

$T T$

min

max

with:

M

.

H

$tmax = \min$

$I(1$

and

) K_{ii}

M_{ij}

M_{ij}

.

T

= max

,

min

$I J(1) K_{ij} - K$

,

ij

Consequently, it is necessary initially that the elementary matrices check $K_{ij} = 0$ (it is the case of **linear finite elements** studied further).

With regard to the interval on the step of time:

If the oscillations are due to a step of too large time ($T > tmax$), one can advise:

- **is to choose a diagram of integration in time of the implicit type (=)**

1, to eliminate

the upper limit of the interval.

- **is to decrease T .** (In practice, it is difficult to know an order of magnitude of

tmax).

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Enough often, the problem of the oscillations arises for steps of small times ($T < tmin$); in effect, to take into account the variations of the solution (for example, at the time of a thermal shock), one

is brought to choose a fine discretization in time. In this case, to avoid the oscillations, one can to suggest:

- **is to increase the step value of time.** In practice, this is not always possible because T can be imposed by the nature of the problem (fast variation of the loading). Moreover, it is difficult to have an order of magnitude of $tmin$.
- **is to decrease the size of the meshes** and thus to increase the number of elements. Indeed, value of $tmin$ depends directly on the space discretization:

The forms of the elementary matrices are indeed:

$M =$

$C \quad NR \quad NR \quad FD$

ij

$I \quad J$

E

$K =$

NR

NR

FD

ij

I

J

E

For the elements 2D, the terms of \mathbf{M} are thus of the form $C \times \text{Surface}$ whereas those of \mathbf{K} are related only to. This solution remains the best if one is not not limited by the cost calculation, because the thermal solution and especially mechanics will be of as much more precise.

· **Is of diagonaliser the matrix \mathbf{M}** , which removes the lower limit of the interval.

It is the solution suggested here.

In the continuation of the study, one is interested only in the problem of the oscillations which appear for steps of too small times: $T < t_{min}$. One presents the method more precisely of diagonalisation of the matrix \mathbf{M} chosen, and the various types of elements to which it applies.

3

Method of diagonalisation selected and types of elements

3.1

Elements such as the extra-diagonal terms of \mathbf{K} are negative

It was seen that the diagonalisation of \mathbf{M} is effective only when the extra-diagonal terms of stamp rigidity \mathbf{K} are negative. In the contrary case, one of the sufficient conditions of not-oscillation is unconditionally false, whatever the shape of \mathbf{M} .

For each finite element used in thermics in *Code_Aster*, one checks if the matrix elementary of rigidity of the element has negative extra-diagonal terms, while resting mainly on [bib11], which gives the analytical expressions of the elementary matrices for traditional finite elements. One summarizes here the observations made in [bib11].

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3.1.1 Elements

linear

3.1.1.1 Elements TRIA3, TETRA4, PENTA6

The elementary matrix \mathbf{K} is a function of the cotangents of the angles. If one of the angles is blunt, certain extra-diagonal terms of \mathbf{K} are positive. If all the angles are acute, the property is

checked.

One has the same type of result in 3D for the tetrahedron with 4 nodes and the pentahedron with 6 nodes.

3.1.1.2 Elements QUAD4 and HEXA8

Certain extra-diagonal terms of \mathbf{K} can be positive if the element is lengthened too much in one direction. If not, the property is checked.

One has the same type of result in 3D for element HEXA8.

3.1.1.3 Element 3D pyramid with 5 nodes

For this element, the functions of form are not any more of the polynomials but rational fractions in X, y, Z. For this type of element, one does not have the expression, even approximate of \mathbf{K} .

3.1.2 Elements

quadratic

3.1.2.1 Element

TRIA6

In \mathbf{K} , certain extra-diagonal terms are necessarily positive.

3.1.2.2 Element

QUAD9

In the same way, on the analytical expression of the terms of \mathbf{K} , one notes that some of the terms extra-diagonal are necessarily positive.

3.1.2.3 Element

QUAD8

For this element, there is not the complete expression of \mathbf{K} for the real element. But for the element of reference, one notes that certain extra-diagonal terms are positive.

3.1.3 Conclusion on the elements: properties of the matrices \mathbf{K}

For the linear elements, if the real element is not too irregular, extra-diagonal terms of \mathbf{K} are quite negative. For the quadratic elements (in 2D), certain extra-diagonal terms of \mathbf{K} are positive. Even by diagonalising \mathbf{M} , one cannot ensure that the solution will not oscillate.

In *Code_Aster*, to eliminate the problems from oscillation and going beyond of the maximum, one diagonalise only matrices of mass for the thermal calculations carried out on elements

linear. For the quadratic elements, one saw that one could not diagonalise directly stamp of mass. One thus cuts out these elements in linear elements which themselves are lumpés.

This is applied to the quadratic elements 2D in *Code_Aster*, but not to the elements quadratic 3D, not for reasons of method but because automatic cutting is difficult to implement in 3D.

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3.2**Method of diagonalisation: Integration with the nodes of the elements**

If the elementary matrix of mass is calculated by numerical integration, its terms are written under the form [bib8]

NR

M =

C

NR NR FD W (C

NR NR

ij

I

J

Q

I

J) Q

q=1

C

where NR

C

NR

I

J is evaluated at the q ème point of integration

and W_q is the weight of integration associated with this point.

Classically, the points of integration are the points of Gauss; the position of the NR points and their weight

are defined so that the diagram integrates the polynomials of $2N-1$ degree exactly. If one

chooses the **points of integration to the nodes of the element**, one obtains: $M_{ij} = 0$ for $I \neq J$. This

method of integration is also called method of Newton-Dimensions.

Notice 1: Axisymmetric problems:

If the points of integration are with the nodes, one will have, for any type of element, of the null masses on the axis of symmetry.

Indeed, $M =$

C NR NR

R

2

*Dr. dz**ij**I J**M*

2

*C**W Jac R (X**ij**ij**I**I)*

If the point of integration I is a node of the axis, $R(x_i) = 0$ and the corresponding mass is null.

*For the axisymmetric elements, the **method of integration to the nodes is thus not adapted close to the axis**. In this case, it is necessary to integrate into the points of Gauss the elements which touch*

the axis of revolution, by using usual modeling (AXIS).

Notice 2: other possible methods of diagonalisation:

Other methods are studied in [bib1], in particular to test diagonaliser them quadratic elements. In practice, it are not retained at present in Code_Aster.

- Mise on a diagonal term scale ([bib9], [bib10]): Hinton suggests the scaling of diagonal terms of the consistent matrix \mathbf{M} , so that total mass of the element is preserved. It is noted that the lumpées masses are always positive, even for the elements quadrangles to 8 and 9 nodes.*

- Sommaton by line ([bib10]): One summons the values of M_{ij} per line and one concentrates it result on the diagonal. Unfortunately, this process can lead to masses negative, in particular for the quadrangle with 8 nodes.*

Notice 3:

For the quadratic elements, one notes in [bib1] that, even while diagonalisant with method of scaling of the diagonal terms, one obtains oscillations. One thus cannot not to use these elements within the framework of the diagonalisation (i.e. for a grid relatively coarse with respect to the speed of the thermal transient).

One can of course use the quadratic elements in thermics, with the proviso of adapting the smoothness grid with the stiffness of the thermal shock.

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4

Implementation in Code_Aster

In order to eliminate the oscillations from the temperature in space and time, modelings AXIS_DIAG, PLAN_DIAG and 3D_DIAG carry out the diagonalisation of the matrices of mass during calculation thermal linear (THER_LINEAIRE). The diagonalisation is not operational for THER_NON_LINE currently. To guarantee the effectiveness of it, it was seen that it should be carried out on linear elements. If the grid is linear, one carries out simply a diagonalisation of the matrices of mass by integration with the nodes.

In the case of a quadratic grid, in 2D, one carries out a thermal calculation ISO-P2: calculation on one QUAD9 is brought back to a calculation on 4 QUAD4; in the same way, one passes from a TRIA6 to 4 TRIA3.

This makes it possible not to lose the smoothness of the discretization of the grid, as well for the solution of thermal problem that for that of the mechanical problem. Indeed, one shows in [bib1] that this solution is preferable with that which consists in carrying out thermal calculation on linear meshes who are based on the nodes tops of the quadratic meshes (what is normal since discretization is finer).

Modelings available are thus:

4.1 Modelings

2D

Modeling

PLAN_DIAG

AXIS_DIAG

Net

Element

Element

TRIA3

THPLTL3

THAXTL3

QUAD4

THPLQL4

THAXQL4

SEG2
THPLSL2
THAXSL2
TRIA6
THPLTL6
THAXTL6
QUAD9
THPLQL9
THAXQL9
SEG3
THPLSL3
THAXSL3

Comments on elementary calculations 2D:

For the linear elements: terms of mass (matrix to the first member and vector with the second member) are lumpés by integration with the nodes. The new elements have options of calculations elementary identical to the traditional elements. The only modified elementary options are thus MASS_THER and CHAR_THER_EVOL.

For the quadratic elements: calculation is ISO-P2. Calculation on an element QUAD9 (resp. TRIA6) is brought back has a calculation on 4 linear elements QUAD4 (resp. 4 TRIA3) whose terms of mass are lumpés by the preceding method. The matrices and vectors of each of the 4 linear elements are assembled on the level of the elementary routine of calculation. By homogeneity, on the elements of edges,

one calculates the elementary terms on 2 SEG2, then one assembles.

Elements THPLTL6, THAXTL6, THPLQL9, THAXQL9 have the functions of form of the elements linear in which they are cut out.

Caution:

There is no element associated with mesh QUAD8. Consequently, if the grid is composed of quadratic meshes, it is initially necessary to change the QUAD8 into QUAD9 using order CREA-MAILLAGE:

CREA_MAILLAGE (MODI_MAILLE: (OPTION: "QUAD8_9")).

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Into axisymmetric: if elements of the grid touch the axis, one should not integrate into the nodes which find on the axis. It is thus necessary to isolate this layer from elements and to affect modeling AXIS to him.

4.2 Modeling

3D

Modeling

3D_DIAG

Net

Element

HEXA8

THER_HEXA8_D

PENTA6

THER_PENTA6_D

TETRA4

THER_TETRA4_D

PYRAM5

THER_PYRAM5_D

QUAD4

THER_FACE4_D

TRIA3

THER_FACE3_D

Comments on elementary calculations 3D:

For the linear elements: as in 2D, terms of mass (matrix to the 1st member and vector with the 2nd member) are lumpés by integration with the nodes (3rd family of points of Gauss).

For the quadratic elements, it would be necessary to cut out those in linear elements. This cutting is delicate to implement, because it results in creating a new element (PENTA18) with nodes with medium of each quadrilateral face (and it would also be necessary to create a new element PYRAM14).

One diagonalise thus currently only the linear elements 3D.

With regard to the pyramids with 5 nodes, integration with the nodes was tested but do not function well. Cf [§ 3.1.1.3] (it is not known if all the extra-diagonal terms are negative).

modeling “3D_DIAG” exists for the pyramids with 5 nodes but it is identical to modeling “3D”. In any case these element are minority in a grid 3D: it are generated only by the voluminal free maillor of GIBI, which creates some with the need of them to supplement the grid hexaedric.

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5

Thermal calculation of a cylinder subjected to a cold shock

One illustrates on a numerical example what was shown previously; namely that diagonalisation is effective to check the principle of the maximum.

One takes as a starting point the the industrial example of the cooling of a moulded elbow: a shock is applied

cold thermics (289°C with 20°C) on a fissured elbow. During the transient of cooling, temperature calculated in certain nodes reached 310° without diagonalisation of the matrices of mass. For the example treated here, one restricts with a hollow roll of the same dimension than the elbow on which one

apply a cold thermal shock.

5.1 Data

A presumedly infinite hollow roll is studied. Like there is no dependence compared to Z (cylinder infinite), one limits the study to a plane calculation. By reason of symmetry, one nets only one portion of structure.

C

M2

D

Rint

Rint = 417 mm

45°

Rext = 496 mm

Rext

Z

In M1

B

Co-ordinates of the points:

X (mm)

y (mm)

Z (mm)

M1

436.75

0.

0.

M2

436.75 cos 45°

436.75 sin 45°

0.

Calculations are carried out on a linear grid (meshs TRIA3-QUAD4):

M2

M1

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Characteristics of the grid:

A number of nodes: 90

Numbers and type of meshes: 64 TRIA3, 32 QUAD4

Characteristics of material:

= 19.97 W/m °C

CP = 4.89488 106 J/m3 °C

Limiting conditions and loading:

To ensure invariance by rotation, one forces conditions of null heat flow on faces AB and CD. The external wall is supposed perfectly insulated. On internal skin AD, the transfer thermics between the cylinder and the fluid is modelled by a coefficient of high convectif exchange: $H = 40.000 \text{ W/m}^2 \text{ } ^\circ\text{C}$.

The cold thermal shock applied to the moulded elbows is represented by a linear variation of temperature of the fluid circulating in the pipe: $289^\circ 20^\circ$ in 12 S. In order to accentuate the problem of going beyond of the maximum and thus to better highlight the influence of the diagonalisation, one adopt a more brutal shock: $289^\circ 20^\circ$ in 1 S.

Tfluide

289°

20°

T

10 S 11 S

The following discretization in time is adopted:

T = 0 S

with T = 10 S,

1 step of time

T = 10 S

with T = 11 S,

2 steps of time

T = 11 S

with T = 25 S,

7 steps of time

T = 25 S

with T = 60 S,

10 steps of time

Numerically, the value retained for the parameter of the discretization in time is = 0,57.

5.2 Results

The following figures show the profiles of temperature in the thickness of the cylinder at the moment $t=15\text{s}$

(moment when the goings beyond of the maximum are largest) without diagonalisation of the matrices of mass.

One gives also the temporal evolution T (T) to the M1 nodes and m2 located at a quarter thickness of internal skin.

Without diagonalisation, one notes that the temperature oscillates in time and in space exceed the value maximum of 289° at the beginning of the transient.

With diagonalisation on the linear elements, one observes a regular solution without going beyond maximum.

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A similar study was undertaken on linear elements 3D (tetrahedron to 4 nodes, pentahedron with 6 nodes, hexahedron with 8 nodes). The results lead to the same conclusions: with diagonalisation, the oscillations of the temperature disappear for calculation on the linear elements 3D.

Complementary remark concerning thermomechanical calculation:

Another study was carried out in [bib1] estimating the consequences of the diagonalisation thermics on the mechanical results. It is noted that calculation ISO-P2 (elements quadratic divided into linear elements, whose matrices of mass are lumpées) provides results satisfactory. One eliminates the space oscillations from the temperature. But in the studied case, with one relatively coarse grid, the mechanical solution remains not very precise. Although the thermal solution that is to say correct, to improve the solution in constraints, the grid should in any case be refined. For meshes TRIA3, the diagonalisation leads to a regular solution without going beyond of maximum:

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from France

CHANGE OF the TEMPERATURE TO the QUARTER Of the THICKNESS - MESH TRIA3

320

Thermal calculation

of a subjected cylinder

has a cold shock

DT 289 --> 20 in 1s

310

1

M

D

300

HAVE

O

NR

WITH

E
R
290
U
WITHOUT LUMPING
AT
LUMPING
PER
M

E
T
280
270
260
10
12
14
16
18
20
22
24
26
28

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MOMENT T (S)

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PROFILE OF TEMPERATURE IN the THICKNESS - LINEAR GRID - ELEMENTS TRIA3

294.365

Thermal calculation

of a subjected cylinder

has a cold shock

DT 289 --> 20 in 1s

Moment t=15 S

250

200

T

RUE

T

With

R

E

WITHOUT LUMPING

P

M

LUMPING

E

150

T

100

50

35.6814

0

10

20

30

40

50

60

70

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SKIN INTERNS CURVILINEAR X-COORDINATE (m) EXTERNAL SKIN

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For meshes QUAD4, the diagonalisation leads to a regular solution without going beyond of maximum:

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CHANGE OF the TEMPERATURE TO the QUARTER Of the THICKNESS - MESH QUAD4

320

Thermal calculation

of a subjected cylinder

has a cold shock

DT 289 --> 20 in 1s

310

2

M

D

300

HAVE

O

NR

WITH

E

R

290

U

WITHOUT LUMPING

AT

LUMPING

PER

M

E
T
280
270
260
10
12
14
16
18
20
22
24
26
28

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MOMENT T (S)

EDF

Mechanical department and Digital Models

Electricity

from France

PROFILE OF TEMPERATURE IN the THICKNESS - LINEAR GRID - ELEMENTS QUAD4

298.594

Thermal calculation
of a subjected cylinder
has a cold shock
DT 289 --> 20 in 1s
Moment t=15 S

250

T
200

E

R

U

AT

WITHOUT LUMPING

PER

M

LUMPING

E

T

150

100

50
34.9119
0
10
20
30
40
50
60
70

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SKIN INTERNS CURVILINEAR X-COORDINATE (m) EXTERNAL SKIN

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6 Conclusion

Modelings AXIS_DIAG, PLAN_DIAG and 3D_DIAG are proposed in order to solve them problems of going beyond of the maximum with oscillation of the solution in space and time which appear during certain transitory thermal calculations with abrupt variation of the loading.

At the discrete level, the analysis leads to a sufficient condition of not-oscillation on the step of discretization in time which must belong with an interval:

T

$T T$

min

max

where the values of $tmin$ and $tmax$ depend on the coefficients of matrices of mass and rigidity thermics as well as parameter of the discretization in time.

In practice, if the oscillations come from a step of too large time ($T > tmax$), it is suggested choice of an implicit scheme in time (=)

1. If the steps of time are too small, the diagonalisation matrix of mass can make it possible to remove the oscillations.

For the linear elements, one shows that the diagonalisation makes it possible indeed to avoid them oscillations of the solution. For the quadratic elements, a direct diagonalisation is not enough with to avoid the oscillations. For this type of element, one cuts out them in linear elements, and one carries out one

diagonalisation of the linear elements resulting by integration with the nodes (this is carried out only in 2D for the moment).

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R3.06.08 document

Finite elements treating the quasi-incompressibility

Summary:

In certain situations, the mechanical behavior of material imposes that voluminal dilation remains null, in other words that the deformation is done with constant volume: isotropic elasticity with coefficient of POISSON equal to 0.5, perfect plastic flows analyzes limit of it...

One proposes here to treat this condition of “incompressibility” or “quasi-incompressibility” while using a valid formulation as well in the compressible case in the quasi-incompressible case. For that, one uses a variational formulation with 3 fields where the unknown factors are displacement, deformation voluminal and the multiplier of associated Lagrange (which would correspond to the pressure in the case incompressible). Two versions of this formulation are proposed: one for the small deformations, the other valid in the presence of great deformations. After some recalls on the difficulties which raise the resolution of the incompressible problems, one describes the mixed finite element established (in 3D and 2D, plan and axisymmetric), and one also present the large ones lines of integration in Code_Aster (modeling INCO).

This modeling is necessary to practise the limiting analyses and to model behaviors rubber bands for Poisson's ratios close to 0.5. It can also be useful in the case of modelings generating of strong plastic deformations and for which traditional modelings

can be insufficient and generate oscillations of constraints.

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matters

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1***Difficulties related to the treatment of the incompressibility***

In certain situations, the mechanical behavior of material imposes that the deformation makes with constant volume. The materials having this property of not-dilatancy are often qualified “incompressible” materials. We will see that these problems pose two types of difficulties. The first difficulty is related to the writing of the condition of incompressibility, second is dependent on the numerical problems which this constraint generates. These difficulties are found when it material is quasi-incompressible.

One reasons here in small disturbances but the problem remains the same one within the framework of finished transformations.

1.1***“Incompressible” and “quasi-incompressible” behaviors “***

Within the framework of the mechanics of the continuous mediums, deformation of an isochoric type is characterized by the fact that the gradient of the transformation \mathbf{F} is such as $\det \mathbf{F} = 1$. If one places oneself

within the framework of the small disturbances, the preceding condition is reduced to:

$$\operatorname{div} \mathbf{U} = 0 = \operatorname{tr} (\quad)$$

The tensor is thus only deviatoric:

 \mathbf{D} $= .$

It results from it that in the case of isotropic materials, the invariant tr (or $\det \mathbf{F}$) does not intervene in

the expression of the density of free energy; thus in the case of incompressible elasticity in HPP, one has simply:

D

D

$() = \mu.$

This density makes it possible to express only the deviatoric part of the tensor of the constraints:

D

D

$= 2\mu$

In fact, the constraint is defined in a constant close p , which is opposite average pressure:

$= 2\mu D + pId$

éq 1.1-1

Note:

· incompressible isotropic elasticity is of course a borderline case of isotropic elasticity with a Poisson's ratio $= E - 1$ tending towards 0.5.

2μ

· it does not have there that the elastic materials whose Poisson's ratio is equal or slightly lower than 0.5 which utilizes the condition of incompressibility. Thus, it

G

intervenes also in the case of plastic rigid material

$= 0$. *Indeed, one*

tr

G

has in this case: $\& =$

$; 0; G 0; G = 0$

What leads to the condition of incompressibility $tr\& = 0$.

In addition, in the case of elastoplasticity, when plastic deformations become largely higher than the elastic strain, one finds itself in one almost incompressible case with $tr 0$.

Lastly, materials checking a relation of behavior of the type NORTON-HOFF (law used for calculations of analysis limits [R7.07.01]) show also the characteristic

of incompressibility:

N

(

1

v) = (

D

eq)

.

with $N \geq 1$ and $\nu > 0$

3

where

=

. D.D

eq

is the equivalent constraint of Von Mises.

2

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1.2

Some possible numerical solutions

If one wants to treat the condition of incompressibility exactly, we saw it, the constraint is not completely given starting from the deformation (cf [éq 1.1-1]). It is thus necessary to use one mixed formulation, i.e. to introduce (at least) another unknown factor of the problem which will allow to determine the tensor of the constraints completely. Several alternatives are possible, more

simple consisting in imposing the condition of incompressibility using a multiplier of Lagrange, who is then the pressure p .

Note:

If one chooses a procedure of penalization, one is reduced to the quasi-incompressible case and thus with the difficulties evoked below.

One also can, in particular in the case of linear elasticity, to choose to return material slightly compressible. In this way, the constraint is entirely defined starting from displacement and the use of a mixed formulation is not essential any more. On the other hand, the resolution of these problems with the traditional finite elements in displacement, raises numerical difficulties. In effect, the kinematic constraint that a deformation with constant volume represents is very strong, even too much strong if the degrees of freedom of the element are not important enough. Thus, the triangle with

3 nodes can present phenomena of blocking, i.e. the “grid” cannot to deform. In a less extreme way, majority of the elements traditional, in particular linear, comprise in an abnormally rigid way. New elements must thus be used in order to “to slacken” the system. These elements can be based on various types of formulation:

*.
only in displacement*

*.
mixed: displacements/forced, displacements/pressures, deformations/forced, voluminal displacements/pressures/dilations...*

In all the cases, if guard there is not taken, one can have numerical difficulties. Several tracks are used to facilitate the deformation of the elements:

*.
to use under-integration makes it possible to improve the results but it presents one disadvantage: it can lead to the appearance of parasitic modes or hourglass. For to solve this problem, one can is to enrich the matrix by rigidity thanks to matrices of stabilization which comes to neutralize the hourglass modes, is to use methods of projection which consists in projecting in a smaller space the condition of incompressibility in order to eliminate the phenomena of blocking. Most known is the method B-Bar [bib1],*

*.
to enrich the element using additional degrees of freedom: one speaks then about methods with increased deformations, incompatible modes,] [bib2]...*

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1.3***Option selected and frameworks of application***

We chose here to choose a formulation which covers the incompressible one as well (until the incompressible one) that the compressible one. For that, the tr term is treated like a variable independent. With the multiplier of Lagrange associated, that led to a formulation with 3 fields. A version great deformations was also developed on the same principle. In this case, variable independent related to the condition of incompressibility is not any more tr but $\det \mathbf{F}$.

The advantage of this formulation is that it makes it possible to use in a transparent way all the laws of behavior elastoplastic available in Aster (not need to separate the deviatoric part and the spherical part of the tensor of the constraints). It is thus not limited to elasticity or to the elastoplasticity of Von Mises. On the other hand, one will not be able to treat the case where the coefficient of Poisson is strictly equal to 0.5, because one uses for the calculation of the elastic constraint the term

*E
 tr , whose denominator is null when $\nu = 0.5$.*

1

*(+ 1
) (- 2)*

Consequently, this formulation INCO must be used:

.

to deal with the limiting analysis problems for which one supposes that the flow is done with constant volume [R7.07.01],

.

to deal with elastic problems whose Poisson's ratio is higher than 0.45.

This formulation can also be used:

to deal with the problems where the plastic deformations are important, which generates oscillations on the level of the constraints (example: in the case of calculations on test-tubes notched). Of course, this formulation being more expensive than the formulation in displacement traditional, it is to be held for the case posing problem and where one is interested in the values of constraints.

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2

Mixed variational formulation of the problem

2.1

Formulation within the framework of the small deformations

That is to say a solid subjected to:

a field of displacement D

U on D

a field of effort G on NR

a voluminal field of effort \mathbf{F} on

In the traditional case of the finite elements in displacement (modeling 3D or D_PLAN or AXIS in Code_Aster), when the problem derives from an energy, the solved problem is as follows:

to find
 D
 U with checking the relation of behavior, which minimizes the potential energy:

1
 U
 $() =$
 $D - fu$
 $D - gud$
 2

NR

As we explained to [§1], this formulation is not appropriate when one seeks with to bring closer the incompressible solution, i.e. of the condition $\text{div } \mathbf{U} = 0$ or $\text{tr} = 0$. For to circumvent this difficulty, a solution is to separately treat the spherical part of the tensor of deformations (the part which poses numerical problems) and its deviatoric part. One will have thus:

G
 1
 $(\mathbf{U}, G) = D(\mathbf{U}) + \mathbf{I}$ where $D(\mathbf{U}) = (\mathbf{U})$
 D
 $-(\text{tr}(\mathbf{U}))\mathbf{I}$ and $G = \text{tr}(\mathbf{U})$
 D

éq 2.1-1

3
 3

The preceding problem is thus reduced to the resolution of a problem to 2 variables, U and G , under constraint $G = \text{tr}$. It can be brought back to the resolution of an unconstrained problem by introducing one multiplier of Lagrange p ; it is written:

to find
 D
 U , p and G (problem of point-saddle), such as:

(

G

LU, p, G)

D

=

.

(U) + I

D

(pdivu G) D F ud

gu D

éq 2.1-2

3

+

-

-

-

NR

This problem can be solved, by writing the conditions of optimality:

L

= (*D*

+ *pi*). *D - F U*

D - G U

D =

D

0

U

NR

L $= (\text{div} u - G) p D =$ 0 $\acute{e}q$ **2.1-3** p L $=$ $1 \text{ tr} - p$ $G D =$ 0 G 3 **Note:**

.

the first equation corresponds to the equilibrium equation,

.

the second equation translates the kinematic relation binding G to U ,

.

the third equation gives the expression of the multiplier of Lagrange p ,

.

when the problem does not derive from an energy, one can directly use the system equations [éq 2.1-3].

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2.2

Formulation in great deformations

It is possible to extend the preceding variational formulation [éq 2.1-2] to the great deformations. The principle is identical, but one is based in this case on the decomposition of the tensor gradient of transformation \mathbf{F} proposed by Flory [bib3]:

1

1

$\mathbf{F} = \mathbf{S}$

\mathbf{S}

-

\mathbf{F} with \mathbf{F}

3

$= \mathbf{J} \mathbf{I}$

and \mathbf{F}

3

$= \mathbf{J}$

\mathbf{F} and $\mathbf{J} = \det \mathbf{F}$

\mathbf{D}

*The problem is brought back there still to a problem of point saddle:
to find*

\mathbf{D}

\mathbf{U} , \mathbf{V} , \mathbf{G} and p not saddle of the Lagrangian one:

$L(\mathbf{U}, p, \mathbf{G}) = [W(13$

$\mathbf{G} \mathbf{F})] -$

\mathbf{D}

$+ p(\mathbf{J} - \mathbf{G})$

D

- *FUD*

U

G D

éq 2.2-1

0

-

0

-

0

0

NR

where W is the deformation energy expressed according to the variation of the gradient of transformation F.

The choice which was made here is to write the deformation energy on the configuration - i.e. with beginning of the step of time. One notes classically:

- F the gradient of the transformation of 0 with -*
- F the gradient of the transformation of - with.*

-

-

One has then: $F = F F$ and $J = J J$

This problem can be solved as in small deformations by writing the conditions of optimality.

Derivation does not raise difficulties particular to condition of only remembering:

W

1

T

= and = F, being the first tensor of the constraints of Piola-Kirchhoff.

F

J

The system to be solved is thus the following:

13

L

= *G*

D
+ *pi. U D - F ud - G U D =*

D
X

0
U
J

NR
L

= (*J - G*) *p*
D
=

0
0

p
0

-
23

L
JJ
l

=
Tr - p G D =

0
G

-

0
3 G G

0

With regard to obtaining the tangent matrix, it of course asks a little more calculation that in small deformations.

K
K
K

*uu**up**ug*

$$K = K_{up} K_{pp} K_{pg}$$
*K**K**K**ug**pg**gg*

For the K_{uu} term, the method used is the same one as that used in [bib4] or [R5.03.21]. principle consists in deriving with fixed configuration, then to choose as configuration that which coincides

with the current configuration at the moment of calculation, i.e. This matrix is not a priori symmetrical. But in practice, a symmetrical tangent matrix is used. Other derivations do not pose particular problems.

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To simplify the writing of the various terms, one is brought to define the following tensors:

=

H
the algorithmic tangent matrix given by the law of behavior

1
G
3

F
J

1
P = H - I
(I: H) which corresponds to some extent to the deviatoric part of the matrix

3D
D
algorithmic tangent

1
J 3

D
T = P: F

+

G

Finally, the tangent matrix is made up of the following terms:

K

=

uu
[div U

-

U

eq

eq

X]:

v

D

X

(

géométriqu

rigidity

E)

G 2/3

I

+

P: - U

- div U

T:

v

D

(

comporteme

of

rigidity

NT)

J

3

X

X

K

= tr

up

(U

X) p

D

1 G 2/3

K

=

ug

T: + U

G

D

3 J

X

K

=

pp

0

K

= - p

G

pg

DO

O

1 G 2/3 -

G

1/3

K

=

G

gg

- 2tr + Id: H F ×

D

-

9g J

J

J

3

Discretization by mixed finite elements

3.1

Choice of the discretization

When a mixed formulation is used, it is necessary to discretize at the same time the space of displacements, of the multiplier of Lagrange p and “swelling” G . The experience gained on

mixed elements, in particular 2 fields for the incompressible elements, makes it possible to know that discretization of these fields cannot be unspecified, under penalty of obtaining phenomena oscillations (in particular on the level of the pressures) or phenomena of blocking (elements not being able to become deformed or too rigid). Thus it is necessary to have a number of points of Gauss of pressure sufficiently important to check the condition of incompressibility almost everywhere and one a number of points of sufficiently low Gauss of pressure to have more degrees of freedom with to calculate that constraints to be checked. One of the conditions necessary to obtain results satisfactory is the checking by the finite element considered of condition LBB (LADYJENSKAIA, BREZZI, BABUSKA). One can find in [bib5] and [bib6] of the examples of elements satisfying condition LBB.

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Here the problem is a little different since the formulation contains 3 fields. We are inspired by the uses of this kind of formulation (e.g. [bib7]), by using an element of the type P2/P1/P1. In other words, displacement is quadratic, the pressure and swelling is both linear.

The finite elements used are thus the following:

in 2D:

U

triangle with 6 nodes

**
quadrilateral with 8 nodes

p, G
triangle with 3 nodes
\
quadrilateral with 4 nodes

in 3D:
 U
tetrahedron with 10 nodes
\
cubic with 20 nodes \ pentahedron with 15 nodes

p, G
tetrahedron with 4 nodes
\
cubic with 8 nodes \ pentahedron with 6 nodes

For each type of element, one uses only one family of points of Gauss:

- .
3 points for the triangles
- .
9 points for the quadrilaterals
- .
4 points for the tetrahedrons
- .
27 points for the cubes
- .
6 points for the pentahedrons

3.2 **Writing of the discrete problem**

That is to say $U.E.$, EP and Ge , vectors of the elementary nodal unknown factors (resp. displacement, pressure and swelling). If Nq and Nl are related to forms (respectively quadratic and linear) associated the finite element considered:

$$E$$
$$U = NR U$$
$$Q$$

E

$$p = NR p$$

 L E

$$G = NR G$$

 L

3.2.1 Writing in small deformations

B is the traditional matrix of derivation making it possible to pass from

 E

U with:

 E

$$= DRUNK$$

In the formulation, one distinguishes $Dev.$ and dil , which leads us to define the **Bdev** operators and

 tr B D

dil such as:

 E

$$= B$$

 U

$Dev.$

and

 E

$$= B U$$

 dil

3

The discretized form of the equations of the problem [éq 2-3] is written:

 T D U

$$F = B$$

 $($

$$+ pi$$

$$D) D =$$

$ext.$

 F

F =
T
NR (*dil*
B U - L
NR G) $D = 0$
p
L

F =
T
NR ($1 \text{ tr} - p$) $D = 0$
G
L

3

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The tangent matrix of the problem is symmetrical and is based on the following terms:

F
K
=
U =
T

uu

B

DB

E

Dev.

Dev.

D

U

F

K

=

U =

up

B

NR

E

T

dil

D

L

P

F

l

K

=

U =

T

ug

(B

D) NR

E

Tr Dev.

D

L

G

3

F

K

=

$p = 0$

pp

E

P

F

K

=

$p = -$

pg

NR NR

E

T

D

L

L

G

F

l

K

=

$G =$

gg

NR

(D) NR

E

Ttr

D

L

L

G

9

3.2.2 Writing in great transformations

One notes **D** the matrix of derived from the functions of form (quadratic) on the current configuration and **D** on the configuration -, that is to say:

$$\begin{aligned}
 & \mathbf{E} \\
 & \mathbf{E} \\
 & U = U \\
 & - \\
 & D \\
 & U D U \\
 & X \\
 & \text{and} \\
 & X \\
 & =
 \end{aligned}$$

In addition, one defines the constraint of balance eq and the size Q by the following relations:

$$\begin{aligned}
 & 1 \\
 & 2 \\
 & G \\
 & 3 \\
 & - \\
 & 3 \\
 & D \\
 & 1 J J \\
 & eq = \\
 & + p I d \text{ and } Q = \\
 & tr - p \\
 & J \\
 & -
 \end{aligned}$$

$$3 G G$$

is here the tensor of the constraints resulting from the law of behavior.

The vector of the interior forces is written in form the following discretized form:

$$\begin{aligned}
 & \mathbf{F} = \\
 & U \\
 & D D \\
 & eq
 \end{aligned}$$

F =
p
T
NR (*J - L*
NR G)
D
L
0
0
F =
G
T
NR Q
D
L
0
0

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4 Integration in Code_Aster of the finite elements incompressible

4.1

General presentation of the incompressible element

The finite elements are integrated in Code_Aster in 2D plane deformations, in axisymmetric 2D and in 3D. 3 modelings are accessible by using the following options for AFFE_MODELE:

- .
“3D_INCO” for the 3D,
- .
“D_PLAN_INCO” for the 2D in plane deformations,
- .
“AXIS_INCO” for the axisymmetric 2D.

In the catalogue of the elements, the incompressible elements can apply to the meshes:

Meshs

A number of nodes in displacements

A number of nodes in pressure

TRIA6 6

3

QUAD8 8

4

HEXA20 20

8

TETRA10 10

4

PENTA15 15

6

In the routines of initializations of the incompressible elements, one defines:

- .
1 only family of points of GAUSS (the first family of points of GAUSS) [R3.01.01],
- .
2 families of functions of forms respectively associated with displacements (functions with forms of degree 2) and under the terms of pressure (functions of forms of degree 1).

Let us take as example the element tetraedric with 10 nodes: degrees of freedom in displacement are carried by all the nodes, on the other hand, only the 4 nodes tops have the degrees of freedom p and G .

The accessible components for field DEPL are thus

.

displacements: DX, DY and DZ in 3D with all the nodes,

pressure: NEAR for the nodes top,

swelling: GONF for the nodes top.

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4.2

Use of modeling

By choice, modeling INCO is accessible only with STAT_NON_LINE and option COMP_INCR.

Under this key word, the version small deformations is accessible by using DEFORMATION=' PETIT', the version great deformations by using DEFORMATION=' SIMO_MIEHE'.

It is thus not possible to use modeling INCO with the orders:

MECA_STATIQUE

CALC_MATR_ELEM/CALC_VECT_ELEM/ASSE_MATRICE/ASSE_VECTEUR/RESO_LDLT

STAT_NON_LINE (COMP_ELAS =...)

Note:

For the moment, only the tangent matrix can be used for the phase of prediction. However, of new developments in Code_Aster, should return the matrix soon accessible rubber band.

4.3

Formulation of the elementary terms of the second member

The loads can be gravity, of the surface forces distributed, the pressures. Terms elementary are calculated in a traditional way for the degrees of freedom of displacement and one affects the zero value for the degrees of freedom of pressure and swelling.

4.4

Calculation of the strains and the stresses

In this formulation, it is advisable to distinguish the stress field resulting from the law from behavior

D
 =
ldc, of the stress field which checks balance and which is defined by the relation

Id
p
ldc +

it is the latter field which is stored in SIEF_ELGA as well as the relation binding the multiplier p and ldc.

In short, the components of SIEF_ELGA are:

· SIXX, SIYY, SIZZ, SIXY in 2D like SIXZ and SIYZ in 3D: components of the tensor

D
 =
Id
p
ldc +

,

1

· SIP which is equal to tr

- p
ldc

in small deformations,

3

2
1 J - J
3
or
tr
p in great transformations.
-
ldc -

3 G G

It is also possible to recompute EPSI_ELGA_DEPL, which is the field of deformation to the direction traditional.

One can also carry out a calculation of limiting load with POST_ELEM.

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5 Validation

5.1

Incompressible elastic case

Test SSLV130 (cf [V3.04.130]) makes it possible to check the validity of modeling in the case of one roll elastic incompressible subjected to an internal pressure. Its equivalent into large deformations also exist: test SSNV112 (cf [V6.04.112]).

5.2 Case

elastoplastic

The goal of this example is to illustrate the contribution of modeling INCO if the deformations plastics are important compared to the elastic strain. One studies for that one notched sample into axisymmetric, subjected to an imposed displacement. Geometry and it loading are represented on the figure below. The grid consists of 548 TRI6.

U0

E

D

F

C

With

B

Appear 5.2-a: Geometry and boundary conditions

The behavior of material is of elastoplastic type with isotropic work hardening linear (VMIS_ISOT_LINE). The parameters are as follows:

.
 $E = 200.000 \text{ MPa}$

.
 $\nu = 0.3$

.
 $\sigma_y = 200 \text{ MPa}$

.
 $\sigma_{AND} = 1000 \text{ MPa}$

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*On the figure [Figure 5.2-b], one compares the constraint σ_{yy} obtained on way FC (cf [Figure 5.2-a]) with traditional modeling *AXIS* and modeling *AXIS_INCO*.*

EDF

Mechanical department and Digital Models

Electricity

SIGMA_{yy}

from France

300

250

200

150

AXIS

AXIS_INCO

100

50

0

0

1

2

3

4

5

agraf 26/02/2002 (c) EDF/DER 1992-1999

Appear 5.2-b: yy along line FC

It is seen very clearly that the solution obtained with formulation INCO makes it possible to be freed from

parasitic oscillations.

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Author (S):

J. LAVERNE *Key*

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R3.06 booklet: Machine elements and thermal for the continuous mediums

Document: R3.06.09

Finite elements of joint in plane 2D

Summary:

Description of the finite element of joint plane 2D allowing to model the creation of a crack along one predetermined way.

Presentation of the geometry, definition of the jump of displacement in the element, change of reference mark: room with the total element/, calculation of the interior efforts as well as tangent matrix.

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1 Geometry

The element of joint is a quadrangle with four nodes (QUAD4) with two small sides and two large what makes it possible to define a local reference mark in the element: N is a normal unit vector at a large side

and T a tangent vector with this one.

The local classification of the nodes must be done obligatorily as on [Figure 1-a], the side [1,2] must correspond to a large side.

Option `MODI_MAILLAGE` key words `ORIE_CONTACT` initially developed for the elements of contacts makes it possible to impose this classification.

2

N

T

3

1

Y

4

X

Appear 1-a: Element of joint

The element of joint has two points of Gauss positioned as on the SEG2 of reference:

The first PG1 in - 3/3 and the second PG2 in 3/3 on the segment [- 1,1] with for weight 1 each one.

2

Change of reference mark

To be able to pass from the total reference mark (X, Y) to the local reference mark to the element (N, T) we introduce

stamp rotation R . This matrix applied to a vector expressed in the total reference mark gives sound expression in the local reference mark.

cos
sin
R =

where is the angle between the two reference marks.
- sin cos

y -
-
2
y1
x2 X
one has
1
cos =
and sin = -

L
L

with L = 12 and (1
X, 1
y) and (x2, y2) co-ordinates of nodes 1 and 2.

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3 *Jump of displacement in the element*

Let us note $U = (U, v)$

loc

loc

loc

U

=

I

I

I

and

(U

, v

)

I

I

I

displacements with node I respectively in

total reference mark (X, Y) and in the local reference mark (N, T).

With the change of reference mark one a:

loc

$U_i = R U_i$

One definite jumps of normal and tangent displacement in the element on each point of Gauss starting from the components of the displacement of the four nodes in the local reference mark:

$[U]_g = C U U$

I C

U

U

N

G (loc -

loc

I

4) + (-

G) (loc -

loc

2

3)

[

$$U] g = C v$$

v

1 C

v

v

T

G (loc - loc

1

4) + (-

G) (loc -

loc

2

3)

with g=1,2 the list of the points of Gauss and C1 and C2 coefficients:

1

3

1

3

C

C

1 =

1+

,

2 =

1-

2

3

2

3

One can rewrite the jump in matrix form:

G

[

U
U
G
[] N
loc
=

éq 3-1

[
= B D
U]
G
G
T

with loc

loc
loc
loc
loc T
D
= U
(
, v
,..., U
, v
)
1
1
4
4

C
0

1 - C

0

C -1

0

- C

0

G

G

G

G

and B =

G

0

C

0

1 - C

0

C -1

0

- C

G

G

G

G

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4 Efforts

interiors

~

That is to say R matrix 8×8 which makes it possible to express the components of displacement to the four nodes

in the local reference mark: loc

loc

loc

loc

loc T

D

= U

(

, v

,..., U

, v

)

1

1

4

4

starting from the components of displacement with the four nodes in the total reference mark:

T

D = U

(, v,..., U, v)

1

1

4

4

.

R O O O

loc

~

~

O R O O

There are D

= D

R with R =

O O R O

O O O R

where R is matrix 2 × 2 of change of reference mark defined into 2) and O matrix 2 × 2 null.

The interior efforts in the element of joint are defined by a vector with eight components noted int

F

and checking the relation:

E =.

D

S

int

F

where ES =

([U]) dl

is energy in the element of joint.

,

1

[2]

(see Doc. of the law of behavior Barenblatt [R7.02.11]).

one a:

([U])

ES =

U

,

1

[2]

[U] [] dl

loc

=

B D dl

according to the definition of (Doc. [R7.02.11]) and according to [éq 3-1].

,

1

[2]

=

~

loc

~

Br D dl

since D

= D

R

,

1

[2] ~T T

=

R B D dl

,

1

[2]

One deduces the interior efforts from them:

F =

~

RT BT dl

éq 4-1

int

,1[2]

one can evaluate this integral:

L

F =

~ T T

R B with the weights of the points of Gauss = =.

int

G

G

G

1

2

G =,

1 2

2

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5 Matrix

tangent

The term which it is necessary to calculate in the tangent matrix is the derivative of the interior efforts by report/ratio

with displacements (matrix 8×8).

~
The interior efforts are given by: F

=
 T
 T
int
 $[R B dl$
,
 $1 2]$

int
 F
 $\sim T T [U]$
From where

=
 $R B$
 dl
 D
 $[,12]$
 $[U] D$

$[U]$
loc
~
~
like $[U] = data base$
 $= B D$
 $R then$
 $= Br$
 D

and one obtains:

F
~
 T
 T
~
int

=

R B
Br dl

éq 5-1

,
1
[2]
D
[U]
F
~

T
T

~
that one can evaluate:

int = R B
B R
G
G

G
D
G =,
1 2
[
U] G
L

with the weights of the points of Gauss = =

.
1
2
2

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***Quadrangular element at a point of integration,
stabilized by the method “Assumed Strain)***

Summary:

Despite everything the potential of the linear isoparametric quadrilateral element for calculation by finite elements, the application of a traditional bilinear formulation to define its field of displacement led to poor results. If under integration of the element allows to improve its performances, it makes however to appear parasitic modes which make calculations unstable. This document shows them principal stages of a method of stabilization of calculations named "assumed strain method" and explains the way in which it was established in code Code_Aster the computer. Many results resting on the stabilized element are compared and commented on in order to conclude on the performances from the method.

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1

Setting at fault of the element 2D QUAD4:

In order to propose the need for a more powerful plane element, we carried out calculations being based on two cases tests, which make it possible to highlight blockings of the element isoparametric quadrilateral with 4 nodes.

1.1

Case test n°1

The first series of calculations results from the modeling of a beam fixed and subjected to an effort of shearing f_{sy} at its loose lead [Figure 1.1-a]. This beam has the characteristics

material following: $E = 100, = 0.4999$. This case test takes as a starting point that written in [bib4] (assumption of plane deformations).

y

Boundary conditions:

Displacements on AD:

**D
C**

$$U_x (A) = U_y (A) = 0$$

$$U_x (D) = 0$$

$$f_{sx} = 8 * L * y / B^2$$

f_{sx}

**X
O**

**f_{sy}
B**

Efforts on BC:

$$F_y = f_{sy} = 1 - 4 * y^2 / B^2$$

$$F_x = 0$$

With

B

Geometry:

L

L = 100

B = 50

Appear 1.1-a

We carried out this calculation seven times by multiplying the number of meshes on each edge by two each time:

Etc

1

2 X 2

4 X 4

(A Number of meshes on an edge)

With the exit of each calculation, we brought the arrow out of C with that closer to the solution “exact” of the theory of the beams of Timoshenko.

[Figure 1.1-b] shows us that the convergence of calculation towards the theoretical solution is largely insufficient taking into account the number of meshes used for modeling.

However, by carrying out calculations in plane constraints, we note that the results of calculation converge towards the theoretical solution satisfactorily ([Figure 1.1-c], convergence quadratic).

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Convergence of the Q4 element

(Deformation planes)

Plane deformations

Exact solution (Tim bone henko)

4,5

4

3,5

3

He Vc

2,5

ê

C

Fl

2

1,5

1

0,5

0

0

10

20

30

40

50

60

70

NR

Appear 1.1-b: convergence of plane element QUAD4 in deformation

CONVERGENCE OF the Q4 ELEMENT

(Plane Constraints)

Exact solution (Timoshenko)

5

4,5

4

3,5

ARROW Vc

3

2,5

0

2

4

6

8

10

NR

Appear 1.1-c: convergence of element QUAD4 in plane constraints

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This request is said to dominant inflection. Through the first series of calculations, we put in obviousness impossibility for the QUAD4 of representing the modes of deformation in inflection [Figure 1.1-d] in plane deformation and for a coefficient close to 0,5. This results in one excessive rigidity of the element due under the terms of shearing of the operator discretized gradient.

QUAD4
QUAD8

(important shearing)

Appear 1.1-d

1.2
Case test n°2

The second series of calculation is based on the modeling of a notched sample [Figure 1.2-a] solicited by an imposed displacement.

Imposed displacement: $Dy = 1$

Characteristics material:

= 0.4999

5

$E = 200 \text{ Gpa}$

$y = 0,1$

$AND = 10$

Criterion of plasticity:

5

***Von Mises, plasticity with
linear isotropic work hardening.***

I

E

T

y

E

Appear 1.2-a

For the posting of the results, each QUAD4 was cut out in 4 zones containing each one a point of Gauss. It is the value of the constraint in this point which we post.

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Reference solution: QUAD8_INCO

(element treating the incompressibility perfectly).

yy

Solution QUAD4:

yy

The results show that the QUAD4 converges only at the price of strong oscillations of constraints with centre of each element. If these oscillations make it possible the element to put in agreement its nodal displacements and its plastic deformation with constant volume, they return the results unrealistic persons.

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2

Under integration of the QUAD4

Despite everything the potential of the isoparametric quadrilateral element for calculation by elements

stop, the application of a traditional bilinear formulation to define its field of displacement, leads to poor results. This is explained by various reasons:

- it has an excessive rigidity (“lock”), at the time of a request of which the cross-bending part is important.*
- the traditional bilinear formulation of the field of displacement is very sensitive to the distortion grid and presents severe “locking” when one applies it to a material incompressible.*

A solution with these problems of numerical blocking consists in calculating the matrix of rigidity by the intermediary of a reduced integration. The principle of this method is to consider at the time of numerical diagram of integration less points of integrations than one usually should not any for to evaluate the matrices of exact rigidity of the element. On the basis of isoparametric element QUAD4

[R3.03.02], one modifies the number of points of integration as well as the weight and the co-ordinates of these the last, to create the under-integrated element which we will name in this document: QUAS4

QUAD4 QUAS4

A number of points of integration: 4

A number of points of integration: 1

Weight of each point: 1

Weight of the point: 4

1
1
Co-ordinates: (X, y) = (0,0)

Co-ordinates: (X, y) = (+

, +/-

)

3

3

2.1 Formulation

In the center, the operator discretized gradient used to calculate the matrix of rigidity is form following:

T
B
0
X

B = Bc =
T
0
B y

éq 2.1-1

T
T

B y B X

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With

T

bx =

NR

NR, X (0) =

= = 0

X

éq

2.1-2

T

by =

NR

NR, y (0) =

= = 0

y

and NR represents (N1, N2, N3, N4), the vector of the functions of form. Let us recall that the vectors B represent the simplest shape of the operator discretized gradient under-integrated introduced by Hallquist [bib1] and which is based on the evaluation of derived from the isoparametric functions of form with the origin of the reference frame (,)

That is to say:

btx = 1 ([y2 - y4), (y3 - 1

y), (y4 - y2), (1

y - y3)] = Constant on

element

2A

éq

2.1-3

bty = 1 ([x4 - x2), (1

X - x3), (x2 - x4), (x3 - 1

X)] = Constant on

element

2A

$With = (1 \ 2) \cdot (x2 - x4) \cdot (y3 - y)$
 $1 + (x3 - X)$
 $1 * (y4 - y2)$: Surface of the element
The matrix of rigidity is written:

T
 $KE = Kc = A \cdot Bc \cdot C \cdot Bc^T$ éq
2.1-4
C being:

- is the elastic matrix of behavior for calculations in elasticity
- is the tangent matrix for the plastic designs. Let us note that during such calculations, it is the integration of the law of behavior at the point of Gauss (in the center in our case) which determine the value of the coefficients of C.

Finally, the internal forces are written:

T
 F
 $= Ke \cdot U$
int
 $U = Bc \cdot C \cdot \epsilon$

2.1-5
U: Vector of nodal displacements.
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2.2

Failure of calculation in Aster: parasitic modes

The calculations launched on the case test n°1 with such an element fail. The stage of the calculation which consists with to reverse $K E$ to determine displacement nodal cannot be crossed. Indeed, in the center, stamp rigidity is singular. By posting the core of $K E$, we note that his dimension is not any more three but five. Two vectors were added to the core and returned the inversion of stamp impossible rigidity. Appearance of these two additional vectors in the core of $K E$ is directly related to the fact that we choose the center like only point of integration. In other words, there are two fields of nodal displacements others that the fields of displacement corresponding to the rigid movement of solid cancelling the internal forces. These modes represent modes out of sand glass of the QUAD4 [Figure 2.2-a]. Thereafter, one of the stages of stabilization will consist to enrich the operator gradient discretized in order to return $K E$ invertible.

*Modes of rigid solid
Modes out of sand glass*

(hourglass modes)

Dim de Ker [K

$E] = 3 + 2$

Appear 2.2-a

2.3

Graphic interpretation of the problem involved in under integration

The problem of the under-integration of the QUAD4 is related to its modes of nodal displacements out of sand glass (request in inflection). [Figure 2.3-a] shows us that, on such modes, the co-ordinates of center remain unchanged. If this is in agreement with the theory of the beams (i.e in pure inflection, $xx = 0$ on neutral fibre), the classical theory of the finite elements do not make it possible to differentiate deformed state and not deformed of an element in such a case. Therefore these modes are also called

modes with null energy, which on the level of Code_Aster, make calculations unrealizable.

Appear 2.3-a

This problem originates in a value of the nonsignificant deformation in the center of deformation of the QUAD4. The next chapter will describe the step which will allow us to calculate the deformation of the QUAD4 whatever the modes of displacement of these nodes.

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3

Stabilization of the QUAD4 at a point of Gauss

The stabilization of the QUAS4 takes place in two stages:

1) To enrich the operator gradient discretized Bc and to thus allow to calculate the energy of deformation related to the modes of displacements out of sand glass (hourglass modes);

2) To interpolate a field of deformation/constraints allowing to account for deformation/of the constraints on the whole of the element while integrating the QUAD4 into center (assumed strain stabilization).

3.1

Variational principle of the problem

That Ci is drawn from the weak form of the variational principle of Hu-Washizu:

$$\begin{aligned}
 & (\\
 & T \\
 & , \&,) = \\
 & \&. \\
 & D + \\
 & T
 \end{aligned}$$

.

$$\begin{aligned}
 & T \\
 & ext. \\
 & \& D \\
 & D \\
 & \& \\
 & F \\
 & \acute{e}q \\
 & 3.1-1
 \end{aligned}$$

$$\begin{aligned}
 & (S -) - \\
 & \cdot \\
 & = 0
 \end{aligned}$$

With
 $\& (X, T) = B (X) .d\& (T)$

Stabilization “Assumed strain” is based on the fact that the postulated constraint is selected orthogonal with the difference between the symmetrical part of the gradient speed and the rate of deformation.

Consequently, that enables us to write:

$$\begin{aligned}
 & T \\
 & D \\
 & \&. \\
 & T \\
 & B. \\
 & D -
 \end{aligned}$$

T
D
&. ext.
F
= 0

E

And thus that:

F int =
T
B. (&)
D

E

3.2 *Enrichment of the operator discretized gradient*

To enrich the operator gradient discretized Bc consists in making a new operator B of it while adding to him a third component which, like bx and by, is a vector of IR4. However, like the initial operator Bc correctly calculates the gradient of the linear fields of displacement, new component must be orthogonal with the latter. Stages of the calculation of this operator nouveau riche are detailed in the paragraph [§1.2] of the document joint entitled: “Report bibliographical”.

Note: In this report, the new operator B connects the tensor rate of deformation & and it vector nodal speeds d&i. This formulation allows us in the continuation of the report of to reason in term of increments of displacement, and consequently to deal with problems incrémentaux, carried out on several steps of time. For the elastic designs (solutions obtained in a step of time) we formulate the problem starting from nodal displacements: the operator B connects then tensor of the nodal deformations and displacements ui.

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The new operator *B* is based on the expression of the field of displacement of the QUAD4 written by Belytschko and Bachrach [bib2]:

***u_i* = (*T*
T
T
T
+ *X B*
. *X* + *y B*
. *y* + *H*) *u_i***

**éq
3.2-1**

And is written:

***T*
T
B
H
*0***

***X* +**

, *X*

***B* = *B_c* + *B_n* =**

***T*
*0***

***B y* +**

T

H, y

éq

3.2-2

T

T

T
T

B y + H, y
B X + H, X

With:

= 1/ [
4 T - (tT .x) bx - (tT .y) by], = 1 [
4 H - (T
H .x) bx - (T
H .y) by]

1
btx =
([y2 - y4), (y3 - y1), (y4 - y2), (y1 - y3)]
2A

1
bty =
([x4 - x2), (x1 - x3), (x2 - x4), (x3 - x1)]
2A
H =

, are the co-ordinates of reference. ui is the vectors of nodal displacements and H are them values taken by the function H with the four nodes.

T
D = [ux, uy]

Let us note what can be expressed directly according to the nodal co-ordinates:

X (
2 y3 - y)
4 + X (
3 y4 - y)
2 + X (
4 y2 - y)
3

1 X (
3
1
y - y)
4 + X (

$$\begin{aligned}
 & 4 y^3 - y) \\
 & 1 + X (\\
 & 1 y^4 - y) \\
 & 3 \\
 & = \\
 & \text{éq} \\
 & 3.2-3 \\
 & 4 X (\\
 & 4 \\
 & 1 \\
 & y - y) \\
 & 2 + X (\\
 & 1 y^2 - y) \\
 & 4 + X (\\
 & 2 y^4 - y)
 \end{aligned}$$

$$\begin{aligned}
 & 1 \\
 & X (\\
 & 1 y^3 - y) \\
 & 2 + X (\\
 & 2 \\
 & 1 \\
 & y - y) \\
 & 3 + X (\\
 & 3 y^2 - y) \\
 & 1
 \end{aligned}$$

The shape [éq 3.2-2] of the operator B is equivalent to that of the QUAD4. However this writing particular of the operator allows to differentiate the terms to be integrated into the center and the terms of stabilization. It is only while intervening on the value of these terms of stabilization that us let us can improve the performances of the element.

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3.3

Interpolation of the field of deformation:

The general shape of a field “assumed strain” within a QUAD4 is given by Belytschko and Bindeman [bib3]. It is following form:

Q E H

Q E H

+

X (center)

X 1, X + y 2, y

+

X (center)

X (stab)

assumed strain

=

+ Q E H

Q E H

y (*center*)

$X^2, X +$

y^1, y

=

+

éq 3.3-1

y (*center*)

y (*stab*)

2

+ $Q E H$

$Q E H$

2

2

xy (*center*)

$X^3, y + y^3, X$

+

xy (*center*)

xy (*stab*)

With:

$qx = . ux$

$qy = . uy$

and 1

$E, e2, 3$

E which vary according to the consideration physics of each author [Table 3.3-1].

Each triplet of values characterizes an element and gives place to a particular interpolation of deformation:

Element

$e1 e2 e3$

QUAD4

$1 0 1$

ASMD

$1/2 -1/2 1$

ASBQI

$1 - 0$

ASOI

1 -1 0

ASOI (1/2)

1/2 -1/2 0

Table 3.3-1

Thus we can deduce the expression from the operator discretized gradient rising from the field from deformation supposed ((stab)) element. We note this new **Bn** operator.

T

H X

0

,

QUAD4:

Bn = 0

T

,

H y

T

T

,

H y

,

H X

T

H, X

-

T

H

, y

ASBQI:

Bn = -

T

T

H, X

H, y

éq

3.3-2

0
0

1
T
1
T

,
HX
-, hy
2
2

ASOI (1/2):

Bn = - 1

T
1
T
H
X
Hy
2,
2,

0
0

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This writing enables us to test the various elements easily.

The matrix of rigidity is written then in the following way:

stab

$$\mathbf{K E} = \mathbf{Kc} + \mathbf{K}$$

éq

3.3-3

With:

T

$$\mathbf{K C} = \mathbf{B C B}$$

D

C

C

=

A.B

D.C. Bc éq

3.3-4

E

And

$$\mathbf{K stab} = \mathbf{T}$$

Bc

$$\mathbf{C Bn D} + \mathbf{T}$$

Bn

B

C C D +
D T
Bn
B

C N D

E
E
E

éq
3.3-5

4
=
(
JAC I) (T
.
B. C. B (I)
C
N
+ T
B (I). C.
N
Bc + T
B (I). C. B (I)
N
N
)
i=1

Finally we calculate the forces intern in the following way:

int
F
= B. (C. ((center) + (stab)))

4
éq
3.3-6

=
(
JAC I). ((

Bc + Bn (I)

)
(

.

C (
. C + stab (I))
i=1

Notes:

*Although the calculation of **K** stab requires a sum on the four points of Gauss, the integration of law of behavior which determines the value of the terms of **C**, is carried out in the center. In addition, them*

equations [éq 3.3-4] and [éq 3.3-6] shows us that calculations remain relatively bulky. One solution with this problem (not yet established in Code_Aster) consists in carrying out calculations in placing itself in a reference mark turning with the element (cf [§3.4] of the bibliographical report). This has for advantage:

- to remove the calculation of the cross terms: **Bc. C. Bn** and **Bn. C. Bc**;*
- better a treatment of blocking in transverse shearing;*
- a writing of the law of behavior adapted better to the problems including of geometrical non-linearities.*

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4***Integration of the element in Code_Aster***

Element QUAS4, has two families of points of Gauss. The first family consists of one not being in the center and whose weight is worth four. Elementary calculations are done naturally with the first family. The calculated options are identical to that calculated by the QUAD4. One thus calculate in the center of the element the internal constraints and variables, as well as the behavior tangent. Within the framework of a post treatment, each element presents constant fields.

The second family is identical to that of the traditional QUAD4 and comprises 4 points of Gauss. She is used to calculate the matrix of stabilization. To store the forces of stabilization, one adds to stress field at the point of Gauss 6 components.

This element is activated by choosing modelings “C_PLAN_SI” or “D_PLAN_SI” in AFFE_MODELE for meshes QUAD4. Only calculations in small deformations are possible. It remain an important limitation: in version 7.2, two types of stabilization are programmed: ASBQI and ASOI (1/2), but are not accessible by a key word in the command file. It is necessary to activate them to modify parameter PROJ in routine NMAS2D.

5***Description of the contribution of element QUAS4***

In order to evaluate the contribution of element QUAS4, we used it to carry out calculations of the cases tests number one and two (cf [§3]).

5.1***Case test n°1 (SSLP106)******Convergence of calculation towards the theoretical solution (calculation in elastic strain with $\nu = 0,4999$)****ASOI (1/2)**Timoshneko solution**QUAD4 and QUAS4*

4,5

4

3,5

Vc

3

2,5

2

0

1
2
3
4
5
6
7
8
9
10
NR

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Among the elements appearing in [Table 3.3-1], only element ASOI (1/2) could be established with success. Moreover calculations converge only for the case test of the right beam, modelled by one regular grid. Same the calculations carried out on a divergent voluntarily deformed grid [Figure 5.1-a].

Regular grid

Irregular grid

Convergence of calculation

Divergence of calculation

Appear 5.1-a

Concerning the calculations carried out on a regular grid, we note a clear improvement of results. The theoretical solution is reached with a network of sixty four elements. This us reasonably allows to say that the blocking of element QUAD4 disappeared. This test corresponds to test SSLP106 of the base of tests of Code_Aster.

5.2

Case test n°2 (SSNP123)

Recall of the problem: With a mesh of the type QUAD4, calculation reveals the important ones oscillations of constraints on the grid. We will compare the exits results of the grid QUAD4 with those of grid QUAS4:

Calculation in elasticity in the plan: isovaleurs of

yy

· QUAD4

· QUAS4

· = 0,4999

· = 0,4999

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Plastic designs in the plan: isovaleurs of
yy

- *QUAD4*
- *QUAS4*
- = 0,3
- = 0,3

Each computing time indicated is an average carried out on three launching.
For the visualization of the results, each QUAD4 was divided into four zones containing each one one not Gauss. The QUAS4 as for them are not redivisés.
The tests carried out are voluntarily severe (near to 0,5). Indeed the goal here is to reach them

limits of element QUAD4 in order to note the performances of the new element.

5.3 Comments

We note through this series of calculations that the results obtained with elements QUAD4 comprise strong oscillations of constraints. Results resulting from calculations resting on QUAS4 do not have almost any more oscillations. When they appear, they are very localised. That it is for the QUAD4 or the QUAS4, displacement calculated with node A are identical. Indeed, the operator gradient discretized of the QUAS4 is deduced from the field of deformation of the QUAD4 (cf [éq 3.2-2]). Consequently, that it is with a grid with one or the other of these elements, the structure have same rigidity.

Taking into account the severity of the test and in order to account to us for the quality of the provided results

by the QUAS4, we carried out calculations being pressed on quadratic meshes such as QUAD8, QUAS8 (QUAD8 under integrated with 4 points of Gauss) or the QUAD8_INCO, element treating the incompressibility perfectly. The comparison of elements with interpolation quadratic with elements with linear interpolation little direction has. Such tests were carried out in order to have a reference solution and they allowed us to purely make an assessment qualitative on element QUAS4.

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Results of reference: calculations in elasticity in the plan with quadratic elements

• ***QUAD8***

• ***QUAS8***

• = ***0,4999***

• = ***0,4999***

• ***Dy (A) = 6,0E-2***

• ***Dy (A) = 6,01E-2***

• ***Durée of calculation: 5,6 S***

• ***Durée of calculation: 3,9 S***

· *QUAD8_INCO*

· = 0,4999

· *Dy (A) = 6,01E-2*

· *Durée of calculation: 5,5 S*

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Plastic designs with quadratic elements

- *QUAD8*
- *QUAS8*
- = 0,3
- = 0,3
- *Dy (A) = 1,01E-1*
- *Dy (A) = 1,01E-1*
- *Durée of calculation: 17 S*
- *Durée of calculation: 13,1 S*
- *QUAD8_INCO*
- = 0,4999
- *Dy (A) = 1,01E-1*
- *Durée of calculation: 34,4 S*

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A refinement of the grid for each calculation at summer carried out. This refinement consists in doubling it

a many meshes present on each stop notch. We summarize the results in following tables:

Elastic design Ground ref.: Depl. deformation plan

6,01E-02

Node A
= 0,4999

Numbers

504

2016

8064

Meshs

Time computing Depl Node A Time computing Depl Node A Time computing Depl Node A

QUAD8_INCO

5,5 6,01E-02

18 6,01E-02

QUAD8

5,6 6,00E-02

15,5 6,01E-02

QUAD8 IF

3,9 6,01E-02

10,2 6,01E-02

QUAD4

3,3 2,70E-02

8,4 3,60E-02

QUAS4

2,72 2,70E-02

5,77 3,60E-02 17,3

4,77E-02

Table 5.3-1: Calculations in elasticity = 0,4999

**Calculation
deformation
Ground ref.: Depl.
plane plastic
Node A
1,01E-01**

= 0,3

Numbers

504

2016

8064

Meshs

Depl Node

Depl Node

Time computing Depl Node A Time computing

Computing time

With

With

QUAD8

17 1,01E-01 75,1 1,01E-01

QUAD8 IF

13,1 1,01E-01 62,8 1,01E-01

QUAD4

6,6 9,32E-02 22 1,00E-01

QUAS4

6,5 9,18E-02 17,33 1,00E-01 133,2 1,01E-01

Table 5.3-2: Plastic designs = 0,3

**Calculation
deformation**

**Ground ref.: Depl.
plane plastic
Node A
1,01E-01**

= 0,4999

Numbers

**504
2016
8064**

**Meshs
Depl Node**

**Time computing Depl Node A Time computing
Computing time Depl Node A
With**

**QUAD8_INCO
34,4 1,01E-01 240,8 1,01E-01**

**QUAD8
21,7 8,84E-02 88,17 1,01E-01**

**QUAD8 IF
20,6 1,01E-01 87,83 1,01E-01**

**QUAD4
5,1 2,06E-02 13,07 2,25E-02**

**QUAS4
4,28 2,06E-02 9,87 2,25E-02 43,75 2,79E-02**

Table 5.3-3: Plastic designs = 0,4999

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In addition, we traced the value of the constraint y_y along a segment AB crossing the notch for a basic grid with 504 meshes.

B

With

Value of the constraint along AB

Plastic designs (= 0,3)

350

300

250

] 200

Pa

**[
IM 150**

[

yy

100

50

0

0

1

2

3

4

5

6

-50

X-coordinate

QUAS4

QUAD4

QUAD8

QUAS8

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Value of the constraint along AB

Plastic designs (= 0,4999)

1500

1000

500

yy [Mpa]

0

0
1
2
3
4
5
6

-500

-1000

X-coordinate

QUAS4
QUAD4
QUAD8_INCO
QUAS8
QUAD8

Comments on the results

Qualitative aspects:

- In spite of the richness of the field of displacement of mesh QUAD8, the majority of the results oscillations present.*
- In spite of under integration of the QUAS8, the oscillations appears on meshes QUAS8 for certain plastic designs.*
- Whatever the element used (except for the QUAD8_INCO), under integration remains essential, particularly for the materials of which approaches 0,5.*
- These calculations lead us to establish the following report: the QUAS4 remains stable with respect to oscillations whatever the parameters of calculation used.*

Quantitative aspect:

We always note that the value of the displacement of node A resulting from a grid QUAS4 remains between 3 and 5 times weaker than the reference solution (slow convergence). This value remains nevertheless identical to that calculated with a grid QUAD4.

The profiles traced along the notch enable us to affirm that the values of constraints calculated with a grid QUAS4 are of much better quality than those calculated with one grid QUAD4. By taking the computing times of the QUAD4, mesh QUAS4 allows a reduction substantial of the duration of calculation:

Saving of time (time of ref. = time QUAD4)

Grid

504

2016

= 0,3

26% 35%

Elasticity

= 0,4999

18% 31%

= 0,3

16% 21%

Plasticity

= 0,4999

18% 24%

Let us notice in the passing that the savings of time seem to increase with the number of meshes. In plasticity in particular, this saving of time depends as a large majority on the iteration count of Newton

necessary to the convergence of calculation within each step of time.

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6 Conclusions

In elasticity as in plasticity, that it is for materials of which is worth 0,3 or 0,5, the new one element QUAS4 remains stable and the always realistic results, without any oscillations of constraints. This, we noted it, is far from being the case for the QUAD4. The stability of this new element vis-a-vis cases tests as severe as those presented in this report/ratio is comparable with that of quadratic element QUAD8 under integrated.

On the other hand, this element with the convergence of a linear element in terms of a number of DDL. It is necessary thus to net with a sufficient smoothness to collect the gradients of constraints of the solution sought. This refinement necessary must be put out of balance with the saving of time induced by under-integration.

On the treated examples, the QUAS4 allowed a saving of time of significant calculation of about 20% on average for laws of elastic and elastoplastic behaviors. Let us note that these laws are relatively inexpensive to integrate. Savings of time much more important are awaited for laws more difficult to integrate.

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R3.07.02 document

Numerical modeling of the mean structures:

axisymmetric thermoelastoplastic hulls

and 1D

Summary:

One presents a numerical formulation for the modeling of the structures at average surface of geometry

particular:

- hulls with symmetry of revolution around the axis Oy ,**
- invariant hulls with unspecified section along the axis Oz .**

One describes the isotropic thermoelastoplastic case completely, within the framework of the theories of

LOVE-KIRCHHOFF and of HENCKY-MINDLIN-REISSNER, as well as the various studied loadings, for

the selected isoparametric finite element.

The examples of validation suggested show qualities of the finite element.

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1 Introduction

One is interested in what follows to the mechanical modeling of mean structures to average surface

of particular geometry:

- hulls with symmetry of revolution around the axis Oy ,
- hulls with invariant unspecified sections along the axis Oz .

More particularly, one limits oneself if mechanical parameters (materials, loadings) are independent of a direction of space (the circumference for the hulls of revolution, the axis Oz for hulls C_PLAN and D_PLAN).

For the resolution of chained thermomechanical problems, one must use before the finite element of thermal hull describes in [R3.11.01] according to case's in its axisymmetric version, or its version plane invariant according to Oz .

One gives hereafter first of all a progress report on the description of the mechanical model: kinematics, law of

thermoelastoplastic behavior. Then one presents the selected finite element, the interpolation and method of integration.

One gives finally some numerical results of application, by comparison with solutions analytical.

2 Problem

continuous

The geometry is defined in a unidimensional way:

- by the meridian line in the plan (Oxy) for a hull of revolution,
- by the section of the hull in the plan (Oxy) for an invariant hull in Z .

In this last case, by analogy with the two-dimensional problems, one considers two cases:

- the case “forced plane”, i.e. that of a free hull according to the direction Oz , or that of an arc in the Oxy plan,
- the case “plane deformations”, i.e. when displacements according to the direction Oz are null.

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2.1

Description of the geometry, kinematics

One considers a hull of revolution of axis Oy , or an invariant hull according to the axis Oz . For all two, average surface is defined by the curve = AB in the Oxy plan: is a meridian line for the hull of revolution, or the section for the invariant hull according to Oz .

**y
O
X
Z
Appear 2.1-a: Hull of revolution**

**y
T
B
N
m**

**!!
E
S
y O
.**

**E
X
With
Z
ex**

**Appear 2.1-b: Meridian line
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y
O
X
Z

Appear 2.1-c: Hull with invariant section according to Oz

The curve = AB is parameterized by the curvilinear X-coordinate S. One will note the derivative partial S by: , S.

In a point m of one defines the local reference mark (N, T, ez) by:

*Om
T =
, S; N T = ez.*

*Om, S
One notes also the angle such as:
N = cos E*

*X + sin E Y.
The curve of is defined by:
I = - n.t =*

*R
, S
, S*

In the case of the hull of revolution, the position on the parallel passing by m is noted. tangent vector on this parallel is E. For the meridian line located in the 0xy plan, = 0 and E = - E

Z. The radius of curvature of the parallel in m is:

*R
R =
where R is X-coordinate X of the point m of.
cos*

On the other hand, for an invariant hull according to Z this parallel is a right generator, directed according to ez, of null curve.

The transformations kinematics of the hull are defined by displacement U of the point m of surface average, as by rotation S of normal N at the point Mr. the vector U can be expressed in local base:

*U () = U () .T () + (
W) .n
S*

S
S
S

(S).
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Or in Cartesian base:

$U() = U(S)E + U(S)E$

S

X

X

y

y.

The deformations of the hull associated with this transformation (U, S) are determined by:

- *a membrane tensor of deformation E,*
- *a tensor of variation of curve K,*
- *a vector of deformation of distortion tranverse.*

This last appears in the theory of hulls of HENCKY-MINDLIN-NAGHDI and not in that of COIL. According to displacement U and rotation S, these sizes are expressed (cf [bib1]):

Case

Hull of revolution

Invariant hull according to Oz

U expressed in

W

W

base local

$E = U$

SS

S +
 ,
E = U +
R
ss
 , ***S***
R
(N, T, ez)
I
E
= (- U sin + W cos)
R
Kss = S
 , ***S***
Kss = S
 , ***S***
sin
K = -

R
S
U
U
S = S + Ws -

,
S = + W -
R
S
 , ***S***
R

U expressed in
E = U

ss
y, S cos - ux, S sin
E = U

ss
y, S cos - ux, S sin
base total

(
U
E, E
 , ***E***
E

X
=
X
y
Z
)

R
K_{SS} = S
, S
K_{SS} = S
, S
sin
K = -

R
S
S = S + ux, S cos + uy, S sin
S = S + ux, S cos + uy, S sin

Note:
The change of direction of the curvilinear X-coordinate S does not modify the values of:
S, E_{SS}

E
,
,
but the sign changes of,

U,
W,
R,
K,
K
SS

Within the framework of the theory of COILS, the condition S = the 0 (normals with the hull remain it afterwards deformation) results in a direct relation between rotations S and the slope W, S. Them components of the tensor variation of curve are according to displacement in the local base:

U
R
K = - W
, S
+
- U

, *S*

SS

SS

,

R

R2

sin

U

K

=

W

-

R

, *S*

R

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If displacement is expressed in total base:

1

K

=

U

sin - U***cos - U******cos******- U******ss******sin******R (X, S******y, S******) X, ss******y, ss******sin******K =******(U cos + U******X S******y******S sin******,******,******)******R***

It is noticed that the expression of the variations of curve according to displacement in theory of COIL is rather complicated and that it utilizes derivative second. If one is required interpolation conforms i.e. here C1, this requires the use of finite elements of high degree. The tensors E, K

,

allow to express the three-dimensional deformation in the thickness.

H H

On [Figure 2.1-d], one indicates by x3 the position in the thickness -, compared to fibre

2 2

average, at the point m, of curvilinear X-coordinate S on.

S***!!******T******x3******N******!!******m***

H
+
2
R
H
- 2

Appear 2.1-d

In a point thickness, displacement is expressed in total reference mark:

U (S, X)
3
= (U (S)
-
(S) .x
sin
3
(S)
).e + (
U (S)
+ (S) .x cos
3
(S
X
S
X
y
S
)).ey

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In order to take account of the variation of metric in the thickness (due to the curve of surface average), one defines the functions:

X

X

3

3

S (x3) = 1 +

; (x3) = 1 +

.cos

R

R

For a sufficiently thin hull, this correction is negligible:

S

1

;

1

In practice this correction carried out if MODI_METRIQUE: "YES" in AFFE_CARA_ELEM [U4.42.01] is useless if the reports/ratios H R and H R, when they exist, are lower than 1

15.

In theory of HENCKY-MINDLIN-NAGHDI, the components of the tensor of deformation are:

1

ss (S, x3) =

(Ess + X K

3 ss)

S

1

(S, x3) =

(E + x3

K)

(only in the case hull of revolution)

I
sx
=
3 (S, x3)

2
S

S
2.2 Balance

thermoelastoplastic

It is considered that the material constitutive of the hull is thermoelastoplastic isotropic. One makes the usually allowed assumption that the transverse normal constraint is null: X X 0. The law of

3 3
behavior most general is written then:

HT

11
11
C 11
11
C 22
0
11 - 11

HT
22 = C2211
C2222
0
22 - 22

1
0

0**X****11****C X X 1****3****3 3****3****X**

where C (, μ) of Cijkl components is the local matrix of behavior in plane constraints and μ represent the whole of the internal variables when the behavior is nonlinear. In the continuation index 1 makes reference to the curvilinear X-coordinate and 2 with or Z. With the three-dimensional deformations

defined above, one associates the components of the tensor then forced:

· in the case of a hull of revolution:

=

HT**HT****ss****Cssss (ss -****+****ss)****Css (-****)**

=

HT**HT****C ss (ss - ss +****)****C****(-)****sx = C****ssx****X sx****3**

3 3

3

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· in the case hull invariant according to direction Z and free in Z (“forced plane”):

C

C

sszz zzss

HT

ss = (C_{ssss} -

) (ss - ss)

C_{zzzz}

zz = 0

sx = C

ssx

X sx

3

3 3

3

· in the case hull invariant according to direction Z and blocked in Z (“plane deformations”):

=

HT

ss

C_{ssss} (ss - ss)

=

HT

zz

Czzss (ss - ss)

$$sx = C_{ssx} X sx$$

3

3 3

3

One draws the expression from it from the elastic energy of deformation, which one will deduce the matrix from rigidity in

function of the kinematics of hull seen in the paragraph [§2.1]:

· in the case hull of revolution:

1

2 *H*

/2

$$W_{él} =$$

2

2

(
)

2

2

1 .

2

0

$$- /2 [C_{ssss} ss + C$$

$$+ C_{ss} + C_{ss} ss$$

$$+ C_{ssx} X sx$$

+

- $rdsd dx$

H

3 3

3] (S

)

3

· in the case invariant hull according to Z , in “plane constraints”:

$h/2$

1

C

C

=

$W \acute{e}l$

$sszz zzss$

(C

2

2

$ssss -$

) $ss + 2C_{ssx} X$

. $d_{sd}x$

3 3

sx

S

3

2

C

3

- 2

$zzzz$

H

· in the case invariant hull according to Z , in “plane deformations”:

H

1

/2

$W \acute{e}l =$

2

2

2

.

2

- /2 [C_{ssss} s_s + C_{ssx} X

3 3

sx

S dsdx₃

H

3]

Note:

In thermoelasticity, if one notes E the modulus YOUNG and the Poisson's ratio, one a:

E

E

E

C

=

; C

=

I (, J

iii

ijj

) {1, }

2; C

=

1 - v²

1 - v

X X

2

11 3 3

1 + v

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The following sizes are defined:

· the membrane rigidity of a hull of revolution:

[

H

2 + - 1

C

C

C]

S

ssss

ss

=

·

dx

ij

; who is worth:

C

C

3

- *h/2*

I

J

ss

Eh 1

1
2
-
1

in elasticity and absence of correction of metric in the thickness;

· the rigidity of coupling membrane-inflection of a hull of revolution:

H
[
2
+ -1 C
 C
 $B]$ =
 X
 S
 $SSSS$
 SS
·
·
 dx
 ij
3

, which is null in elasticity and in

H
 C
 C
3

I
 J
-
 SS

2
absence of correction of metric in the thickness;

· the rigidity of inflection of a hull of revolution:

H

[
2
+ - 1 C
C
D]
=
x2
S
SSSS
SS
.
.
dx
ij
3

, which is worth:

H

C
C
3

I
J
-

SS

2
Eh3
1
2

12 1
(-)

in elasticity and absence of correction of metric in the thickness;

1

· the transverse rigidity of distortion of a hull of revolution:

H

2

+ - 1

G

S

=

. *C*

dx

sx

, which is worth:

H

ssx X

3

3

2

3 3

-

S

2

Eh

1

+ in elasticity and absence of correction of metric in the thickness.

For an invariant hull according to direction *Z*, one considers in these expressions only the terms

ij = ss; moreover one must replace there (*S + -*)

1 by *S*: the coefficients thus are defined

C D, B D

, *D D*

C

C

C

ss

ss

ss

and *C, B*

, *D*

ss

ss

ss

for the case, respectively, plane deformations or of

plane constraints. In elasticity, the coefficients *DC, B C*

, *Cd*.

SS
SS
SS
, are the products of the coefficients

$C D, B D$

, $D D$

SS

SS

SS

by 1

2

-. Lastly, the coefficient of transverse rigidity of distortion G_{sx} is

3

identical for three modelings to the correction of metric near.

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One can thus express elastic energy according to the tensors of deformations of hull:

E, K

,

by:

· for a hull of revolution:

1

2

$W_{él} =$

2

2

2

$$\begin{aligned}
& 2 \\
& 2 \\
& 2 \\
& 2 \ 0 \ [C_{ss} \ E_{ss} + B_{ss} \ E_{ss} \ K_{ss} + D_{ss} \ K_{ss} + C \ E \\
& + B \ E \\
& K \\
& + D \ K
\end{aligned}$$

$$\begin{aligned}
& + 2 \ (C_s \ E \\
& ss \ E + B_s \ (E
\end{aligned}$$

$$\begin{aligned}
& ss \ K + E \\
& K \\
& ss) + D_s \ K \\
& ss. \ K) \\
& G_{sx} \ 3 \ 2
\end{aligned}$$

$$\begin{aligned}
& + \\
& S \ r. \ ds. \ d \\
& 2
\end{aligned}$$

· for an invariant hull according to Z in “plane constraints”:

$$\begin{aligned}
& 1 \\
& G_{sx}
\end{aligned}$$

$$\begin{aligned}
& W \ \acute{e}l \\
& C \\
& 2 \\
& C \\
& C
\end{aligned}$$

$$\begin{aligned}
& = \\
& C
\end{aligned}$$

$$\begin{aligned}
& 2 \\
& 3 \\
& 2 \\
& ss \ E \ ss + 2B_{ss} \ E \ ss. \ K_{ss} + D_{ss} \ K_{ss} + \\
& S \ . \ ds \\
& 2 \\
& 2
\end{aligned}$$

· for an invariant hull according to Z in “plane deformations”:

$$\begin{aligned}
& 1
\end{aligned}$$

G_{sx}

$W_{él}$

D

2

D

D

$=$

C

2

3

2

$ss E_{ss} + 2B_{ss} E_{ss} \cdot K_{ss} + D_{ss} K_{ss} +$

$S \cdot ds$

2

2

For these expressions, it is necessary to add the potential associated with the thermal stresses, which will be one

contribution to the second member (whom one will express below in total reference mark):

· in the case hull of revolution:

$2 H$

$/2$

$($

L_{th}

$réf$

$) =$

$T - T$

C

$+ C$

$+ C$

$+ C$

$rd dx ds$

V

3
0
- $H [($
) (ssss ss) ss (ss
)]

2
expression which for an isotropic elastic behavior becomes:

2
 H
/2
sin

 HT
 E
 $réf vx$

(
L

3

$\mathbf{V}) =$

sin
cos

$(T - T) - vx, S$
+ v_y, S
+ X
 $S, S -$
 S
 $rd dx ds$
1-

R

R

3

0

- $h/2$

· in the case invariant according to Z in “plane constraints”:

H

$/2$

HT

réf

$C_{sszz}C_{zzss}$

(

L

$\mathbf{V}) =$

$(T - T) C_{ssss} -$

$ss dx ds$

C

3

- $/2$

$zzzz$

H

expression which for an isotropic elastic behavior becomes:

$h/2$

HT

réf

(

$L \mathbf{V}) =$

sin cos 3

E T - T

- v,

+ v,

+ X

dx ds

3

-

,

h/2 [

(

) (*X S*

y S

S S)

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· in the case invariant according to Z in “plane deformations”:

H

/2

Lth

réf

(

=

T - T

C

dx ds

V)

$$\frac{3}{2} \left[\left(\frac{H}{E} \right)^2 \right]$$

expression which for an isotropic elastic behavior becomes:

$$\frac{H}{E}$$

$$\frac{HT}{E}$$

$$\frac{E}{\text{réf}}$$

$$\left(\frac{L}{H} \right)^3$$

$$\mathbf{V} =$$

$$\int_0^1 (T - T_0) (-v_x \sin + v_y \cos + X S, S) dx ds$$

$$\frac{3}{2}$$

-

$$\frac{1}{2} \frac{H}{E}$$

In these three expressions, one deliberately neglected the correction of metric in the thickness (terms in S , seen for rigidity). Moreover the temperature T which appears is defined by thermal model of hull with three fields (cf [R3.11.01]):

$$X^2$$

$$X$$

$$X$$

$$X$$

X

$$T(SX) = Tm(S)$$

3

1-

$$TS(S) 3$$

3

1

 TI

,

.

$$(S) 3$$

1 3

3

 $H +$

+

2h

 $H +$

-

+

2h

 H

From the whole of these expressions, one deduces the tensors from generalized efforts **NR** and **M** (efforts normal and bending moments) associated the generalized deformations **E** and **K** by the principle of virtual work. They are related to the tensor of the three-dimensional constraints by:

 $h/2$ NR

=

 dx $- H$

/

3

2

 $h/2$

M
 $=$
 $X.$
 3
 dx

$- H$

$/$
 3
 2

(where one neglected the variations of metric in the thickness).

Note:

Transverse energy of shearing

The model of hull presented above, said HENCKY-MINDLIN-NAGHDI, rests on one kinematic assumption: the parameters W and S indicate the normal displacement of the point \mathbf{m} of average surface and the rotation of normal vector \mathbf{N} .

One also frequently finds the model known as of REISSNER which rests on an assumption statics of the distribution of stresses shear transverse. Parameters kinematics deduced W and S in this model are weighted averages in the thickness of normal displacement and local rotations. If one wishes to place oneself in it tally, it is enough to affect the coefficient $= 5/6$ at the end of transverse energy of shearing (in 2s). (cf [bib7], [bib9]).

Lastly, if one wants, for a thin hull, to be located within the framework of the model of LOVE-KIRCHHOFF, one can neutralize the energy of shearing with a great value of (which penalizes the condition $S = 0$), for example $106 H/R$, where H is the thickness and R a characteristic radius of curvature or a distance characteristic of the loadings: (cf [bib 2]). In practice the user can inform the value of under key word A_CIS of order $AFFE_CARA_ELEM [U4.42.01]$.

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3

Formulation of the finite element. Discretization

3.1

Description of the selected finite element

3.1.1 Motivations

The choice of framework HENCKY-MINDLIN-NAGHDI to describe the kinematics of hull, presented to

paragraph [§2], led to expressions of the deformations where the derivative are limited to order 1, contrary to the model of LOVE-KIRCHHOFF. This offers the advantage of being able to use an element finished of a nature limited while ensuring conformity. The natural choice is the element of LAGRANGE P2,

isoparametric, which makes it possible to have a fine representation of a curved geometry and the maid estimates of the constraints.

The degrees of freedom are of course displacements and rotations.

As it is known as previously, the model of LOVE-KIRCHHOFF can be recovered by penalization for a very large parameter affecting the transverse energy of shearing.

This formulation joined the category of the finite elements of hulls known as “degenerated”, i.e. built by injecting the kinematics of hull in elements of three-dimensional continuous mediums: cf [bib10].

As for all the finite elements of hulls, of the particular aspects must be analyzed: the catch in account of the rigid modes and risks of blocking of membrane or shearing.

In the case of the axisymmetric hull of revolution, there is only one rigid mode: translation according to the axis of symmetry Oy .

On the other hand, in the case of the invariant hull according to the direction Oz , there are three rigid modes: two

translations in the plan ($X Oy$) and rotation around Oz .

So that the finite element is powerful, it is necessary that the approximations retained for description of displacement ensure an exact representation of the state of null deformations (mode rigid). In practice, as the concept of rigid mode is defined compared to the total reference mark one decides

thus to describe displacements in total base (\mathbf{E}, \mathbf{E}

X

y), in which rigid modes

(functions closely connected) are represented by the selected interpolation.

As for the risks of blocking out of membrane and transverse shearing, usual treatment consist in a selective numerical integration (cf [bib2]), but the practice reveals that these phenomena seldom appear for the hulls of revolution.

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3.1.2 General presentation of the element

The selected element of reference is quadratic, isoparametric with three nodes and three degrees of freedom

by node. These degrees of freedom are:

U, U

X

y :

components of displacement \mathbf{U} in total reference mark,

S :

rotation around \mathbf{ez} of normal \mathbf{N} .

See [Figure 3.1.2-a].

This element is a generalization of the element of plane beam curved. It is well adapted to discretization of the hulls with meridian curve R variable, cf [bib2].

U

y

$U X$

$N.1$

$N.3$

$N.2$

.

.

.

T-1

0

$+1$

!!

N

Appear 3.1.2-a: Element of reference

The functions of form (basic) are the polynomials of LAGRANGE:

-1 +

+

"

1

NR

2

1 () =

;"

NR

2 () =

;"

NR

3 () = 1-

2

2

3.1.3 Transformations finite element/finite element of reference

y

y

-1

0

+1

2

N2

.

.

.

y

N3

N1

N3

N2

3

y

N1

1

X

X

X

2

x3 1

The geometry is interpolated using the co-ordinates (X, y

I
D) of the three nodes *NI*, *NR* 3, *N2*:

3
3
 $X () = X NR "$

I
I ()

y

;
(
)

=
y NR

I
I

" ()
I =1
I =1

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In the same way using the ddl (*U*, *U*

X

y

,

S

on the nodes, one a:

I

I

I)

3

3

U

=

"

X ()

U NR

X

I;

=

"

I

() *uy* ()

U

NR

y

I

I

()

I=1

I=1

3

= "

S ()

NR

S

I

I

()

I=1

One also needs the jacobien of the transformation:

(

ds

2

2

m) =

() = (*X*,) + (*y*,)

D

And of the vectors of the local base:

1
T () =
X
y
m (

X +

) (
E
E

,

,

Z)
(
1
N) =
y
X
m (

X -

) (
E
E

,

,

Z)
Finally:

y,
- *X*,
cos

=
m (
=
)
;

sin

 $m()$

The meridian curve is obtained by:

1

$$= -(\mathbf{n.t.}) D$$

1

.

=

 $X. y$ - $y. X$

3

 R ds $m()$ (,

,

,

,)

Because of the P2 interpolation, the derivative second which appears below express with the assistance co-ordinates of the three nodes by:

 X = X 1 + X 2 - 2. $X y$

3

= y 1 + y 2 - 2. y

,

,

3

3.1.4 Surface numerical integration

For numerical integrations along the element one uses a numerical formula of integration with four points of GAUSS, single for all the terms to be integrated. This formula reveals them blockings mentioned in the paragraph [§3.1.1] in the event of extremely localised plasticization. One thus advise to avoid the use of these elements in plasticity for the moment. The formula of numerical integration at four points of Gauss will be replaced later on by a formula with two points of Gauss supposed to avoid these nuisances.

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3.1.5 Numerical integration in the thickness

For an elastic behavior, insofar as it is admitted that one limits oneself to characteristics uniform rubber bands in the thickness, rigidities $[C]$, $[B]$

, $[D]$ ij ij $ij]$ and G_{sx} defined in the paragraph

3

[§2.2] are calculated exactly.

For a non-linear behavior, one subdivides the initial thickness in N layers thicknesses identical numbered in the direction of the normal to the average surface of the element. For each layer one uses three points of integration. The points of integration are located in higher skin of layer, in the middle of the layer and in lower skin of layer. For N layers, the number of points of integration is of $2N+1$. One advises to use from 3 to 5 layers in the thickness for a number of points of integration being worth 7, 9 and 11 respectively.

For each layer, one calculates the state of the constraints (11,22,12) and the whole of the variables interns, in the middle of the layer and in skins higher and lower of layer, from

local plastic behavior and of the local field of deformation (11,22,12). The positioning of

points of integration enables us to have the rightest estimates, because not extrapolated, in skins lower and higher of layer, where it is known that the constraints are likely to be maximum.

plastic behavior does not include/understand for the moment the terms of transverse shearing which are treated in an elastic way, because transverse shearing is uncoupled from the behavior membrane in plane constraints.

Cordonnées of the points

Weight

1/3

1 = -1

4/3

2 = 0

1/3

3 = +1

1

N

$y() D =$

$I y (I)$

-1

$i=1$

Formulate numerical integration for a layer in the thickness in plasticity

For a thermoelastic behavior, one uses integration, by layer in the thickness

H

H

-, + described previously in the non-linear field, of the thermomechanical terms seen

2

2

in the paragraph [§2.2]. It is then necessary to use STAT_NON_LINE with a behavior rubber band.

Note:

One already mentioned with [§2.2]. and in [R3.07.04] that the value of the coefficient of correction in transverse shearing for the elements of hull was obtained by identification of elastic complementary energies after resolution of balance 3D. This method is not more usable in elastoplasticity and the choice of the coefficient of correction in shearing transverse is posed then. The transverse terms of shearing are thus not affected by plasticity and are treated elastically, for want of anything better. If one places oneself in theory of Coils-Kirchhoff for a value of this coefficient of $106 h/R$ (H being the thickness of the hull and R its average radius of curvature) transverse terms of shearing become negligible and the approach is more rigorous.

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3.2

Formulation of the elementary terms

3.2.1 Mass, centre of gravity, stamps inertia

In the case of the hulls of revolution, the mass is worth:

$$2 \int_0^h r^2 dx$$

2

2

3

=

$$hr^2 ds =$$

$$H R ds$$

-

1

2

H

S

)

0

/

0

where ρ is the presumed constant density of the element.

The position of the centre of inertia is given in the Oxyz reference mark of [§2.1] by:

$$x_G = 0$$

H2

$$1 \cos$$

$$y r ds +$$

$$\sin$$

$$r ds$$

$$+$$

$$12$$

$$R$$

$$R$$

$$y$$

$$G =$$

$$R ds$$

$$zG = 0$$

The terms of the matrix of inertia compared to O in the Oxyz reference mark of [§2.1] have then for expression:

$$x^2$$

$$h^3$$

$$\cos^2$$

$$I$$

$$= 2 H ($$

$$+ y^2) +$$

$$(\sin^2 +$$

$$+ X$$

$$\cos + 2 y$$

$$\sin) r ds$$

$$xx/O$$

$$2$$

$$12$$

$$2$$

$$h^3$$

$$1$$

$$\cos$$

$$I = 2 \int \int \int (x^2 + y^2) \cos^2 \theta \, r \, ds$$

where =

+

.

R

R

12

x^2

h^3

\cos^2

$$I = 2 \int \int \int (x^2 + y^2) \sin^2 \theta \, r \, ds$$

zz/O

2

12

2

In the case of the invariant hulls according to Oz , the mass is worth:

$$h/2$$

-

=

$$h/2 \int \int \int dx \, ds$$

3

$H \, ds$.

The position of the centre of inertia is defined in the Oxy reference mark of [§2.1] by:

H_2

\cos
 $X ds +$

ds

12

R
 X

$G =$
 ds

$H2$
 \sin
 $y ds +$

ds

12

R
 y

$G =$
 ds

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The terms of the matrix of inertia compared to O in the $Oxyz$ reference mark of [§2.1] have then for expression:

h3

I

$$= h y^2 + (\sin^2 + 2 y \sin) ds_{xx/O}$$

12

h3

I

$$= I = h x y + (\sin \cos + X \sin + y \cos) ds_{xy/O}$$

12

1

where =

.

h3

R

I

$$= H = x^2 + (\cos^2 + 2 X$$

$\cos) ds$
 yy/O

12

h^3

I
 $= H(x^2 + y^2) +$
 $(1 + 2 X$
 $\cos + 2 y$
 zz/O
 $\sin) ds$

12

H^2

h^3

Terms in
 for the centres of inertia and
 for the matrices of inertia are not taken in

12

12

count in the programming. That amounts neglecting the variation of metric with the curve in the calculation of these terms.

3.2.2 Stamp of mass

$2 h/2$

The term: v ,
 , of kinetic energy is treated while considering
 $- h/2$

$(S x^3) v (S x^3) r dx D$

$3 ds$

0

constant density in the thickness and the correction of metric due to the curve negligible. The intégrende is burst in three terms:

$H(U .u + U .u$
 kinetic energy of
 X

X
 y
 $y)$
translation
 h^3
.

kinetic energy of
 $S.S$
12
rotation
 h^3
.

sin
-
+
+
cos
+

kinetic energy of
12
 $((U U)$
 $(U U$
 $X S$
 $X S$
 $y S$
 $y S)$

coupling, with:
1
cos
= +
 R
 R

for the case axisymmetric hull of revolution.

1
= for the case invariant hull according to Oz (moreover in this case the integral disappears

R
 $2 rd$
).
0

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3.2.3 Second member of centrifugal force

In the case of the hulls of revolution, a vector rotation is considered: =.

2nd y, carried by

the axis of revolution. The term of the second corresponding member is:

2

$h/2$

2

$2 \cdot r U -$

X

.

X

S

$3 \sin$

$dx_{3rd} ds$

$- h/2$

(

)

0

$= 2 H^2 2$

2

$R \cdot u D$

X

ds

0

(one neglects the correction of metric in the thickness).

In the case of invariant hulls according to Oz , one considère a vector rotation: =.

3rd Z ,

perpendicular in the plan of the section.

The second member is then:

H

$2 \cdot$

$+$

$3 (X U$

$y U) ds$

X

y

3.2.4 Second member of gravity

In the case of the hulls of revolution, gravity is directed according to \mathbf{E}_Y .

The second member is:

$2 \int g h U R$

y

$D ds$

0

In the case of the invariant hulls according to Oz , this one is directed in the plan

$X O_y$: $\mathbf{G} = G \mathbf{E}_X + G \mathbf{E}_Y$

X

X

$y \mathbf{E}_Y$.

The second member is:

$H G \cdot e + G \cdot e D S$

$(X X$

y

$y)$

3.2.5 Second member of distributed loads

These distributed loads can be two forces in the plan $(X O_y)$ and couples it $M Z$ carried by the axis

Oz . Two forces, which one considers that they are applied to average surface, will be able

to be provided in total reference mark $(\mathbf{E}_X, \mathbf{E}_Y)$

X

$y)$ or room (\mathbf{T}, \mathbf{N}) . The second member is:

$2 \int (F U + F U + M) r dds$

X

X

y

y
Z
S
0
2

(in invariant hull according to Z, the integral
 $\int rd$

disappears).

0

Note:

The specific actions are treated as nodal forces where they are applied, since they work in the ddl of the finite element.

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3.3

Calculation of the strains and the stresses

After resolution, one with the possibility with operator CALC_ELEM [U4.81.01] of calculating with the nodes them

elementary fields according to:

· generalized deformations E

, K

: option DEGE_ELNO_DEPL,

· three-dimensional deformations on average fibre and in skins internal and external

(with or without correction of curve): option EPSI_ELNO_DEPL,

· three-dimensional constraints on average fibre and in skins internal and external

(with or without correction of curve): option SIGM_ELNO_DEPL in linear elasticity,

· generalized efforts NR

, M

(with or without correction of curve): option

EFGE_ELNO_DEPL in linear elasticity.

These values with the nodes are obtained by extrapolation starting from the values at the points of GAUSS of

the element, according to the exposed method in [bib4] [R3.06.03].

Lastly, one can have also the values NR

, M

at the points of GAUSS of the element: option

SIEF_ELGA_DEPL in linear elasticity.

No postprocessing of constraints or generalized efforts is for the moment available for nonlinear behaviors materials.

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4

Validation - Case test

One considers hereafter, to judge capacities of this formulation, some examples of application (cf [bib10]).

4.1

Roll under internal pressure

One studies a vertical roll subjected to a pressure interns p constant on the part $y < 0$, and null on $y > 0$: to see [Figure 4.1-a].

$L/2$

C

R

+ $L/10$

B2

B

X

- $L/10$

B1

p

- $L/2$

With

Figure 4.1 - a: Rolls under axisymmetric pressure

The ray is: $R = 4$ m, the thickness $T = 0.25$ m, the length $L = 10$ Mr. Celle-ci is selected so that the effects edge free in $y = \pm L/2$ are negligible on the solution (into axisymmetric, L must

1
to check:

$L > 3 Rt = 3$ m here).

2

The material is elastic (E

=

1 Pa

,

= .

0)

3 .

The boundary conditions are: $p = 1N/m^2$, vertical displacement of A null.

One chooses the solution obtained by model LOVE-KIRCHHOFF.

To reach it numerically, one takes as coefficient of shearing: $\mu = 10^6$, to inhibit the distortions S . *the* analytical solution is:

P

P

for y

0: $U(y)$

(2nd y

cos y

y

=

-

),

cos sin

4

$S(y) =$

E

3

(

y -

y

X

)

8 *D*

8 *D*

P

P

for *y*

0: *U* (*y*)

E y

cos *y*

- *y*

=

,

cos

sin

4

S (*y*) =

E

3

(

y +

y

X

)

8 *D*

8 *D*

And 3

And

with *D* = 12 1-2

(), 44=DR2.

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The efforts generalized are (sin =)

0 :

And

P

NR

= *U*

- *y*

X

(*y*) *M*

;

= *Of*

ss

X (*y*)

= *E*

sin *y*

R

4 2

The three-dimensional constraints are:

NR

M

X

M X

= +
3

; ss

=

ss 3

12

12

, D

'where:

T

T3

T3

Pr

E y

X

3

(y, X) = 1 -

cos y

3

+

2

sin y

3

T

2

T

1 - 2

for y 0:

Pr X

ss

3

(y, X

3

) = .

$E y$
 $\sin y$
 3

T
 T

1 - 2

$Pr E y$

X
 3

$(y, X) =$
 $\cos y$
 3
-

$\sin y$
 3

T
 2

T
1 - 2

for y 0:

$Pr X$
 ss

3
 $(y, X$
 3
 $) = .$

$E y$
 $\sin y$
 3

T

T

1 - 2

For a regular grid of one hundred meshes and two hundred nodes, one finds:

Reference

Aster

% difference

U_x displacement

Not A

63.9488

63.922

0.042

Not B

32.000

32.005

0.015

Not C

0.05120

0.08755

Rotation *S*

Not A

0.06583

0.04057

Not B

41.133

41.165

0.078

Normal effort *NR*

Not B

2.0000

2.0003

0.015

Not B1 (in *L/10*)

3.84429

3.8442

0.002

M_{ss} moment

Not B1

4.01497 102

4.013 102

0.05

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Appear 4.1-b: Arrow of the cylinder under pressure

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Appear 4.1-c: Rotation of the cylinder under pressure.

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Appear 4.1-d: Bending moments axial cylinder under pressure

4.2

Plate circular embedded under uniform pressure [V3.03.100]

One considers the plate of ray $R = 1$ m, thickness $T = 0,1$ m (see [Figure 4.2-a] below) embedded on its circumference.

y

R

p

p

0

X

D

With

$R/2$

Appear 4.2-a

The material is elastic (E

$= 1 P$

. *has*

,

= .

0)

3. The pressure is: p

$= 1 NR$

.

/m².

The boundary conditions are: in 0:

.,

S

$= 0$ in

A: U

$= U$

X

$y = 0. , S = 0.$

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One is interested in the solutions of the models of REISSNER =

and of LOVE-KIRCHHOFF (one

6

will take = 106).

The analytical solution is for the arrow:

pR4

X 2

X 2

U

1

1

$y(X) = -$

-

.

64D

R

-

R

+

And 3

16 *T 2*

1

5

with

D =

0 for the solution

12(

=

=

=

1 - 2

);

if

;

5

R

1-

6

LOVE-KIRCHHOFF.

pR2 X

The distortion is indeed: $S(X) = -$

.

16D 2

pR2

X 2

Rotation

1

S is: S(X) =

X

-

.

16D

R

The variations of curve are (sin

= +)

1 :

pR2

X 2

K

X

1 3

ss () =

-

-

16D

R

pR2

X 2

K (X) = -

1

-

16D

R

The bending moments are (sin

= +)

1 :

pR2

X^2

M

X

3

1

$ss() =$

(
+)
(
)

16

$R -$

+

pR^2

X^2

$M(X)$

=

(
1 + 3)
(1)

16

$R -$

+

The constraints are written:

E

$ss(X, X) =$

$X [K$

3

+

2

3

$ss(X)$

$K(X)$

1 -

E

$(X, X) =$

$X [$

3

+

2

3

$K(X) K(X)$

1 -

ss

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One notices independence in rotation, variations of curve and moments

bending. In center 0 of the plate:

$pR4$

$pR2$

$uy ()$

0 = -

$(1 +), M () 0 = M$

ss

()

0

= -

(1 +),

64D

16

pR2

K ()

$0 = K$

ss

()

0

= -

.

16D

2

E T Pr

ss (,

$0 \pm t/2) =$

(,

$0 \pm t/2) = \#$

.

1 - 2 16D

It is noticed that one is in compression in higher skin of plate.

pR2

pR2

With embedding *a*: $M_{ss}(R) =$

; $M(R) =$

.

8

8

S

uy

Appear 4.2-b: Arrow, rotation of an embedded circular plate

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For a regular grid of 10 meshes (21 nodes) one finds:

Reference

Aster

% difference

U_y displacement

5

101.827

101.7769

0.049

Not D =

6

LOVEKIRCHHOFF

95.9765

95.0395

0.978

5

178.424

178.368

0.031

Point 0 =

6

LOVEKIRCHHOFF

170.625

169.761

0.507

Rotation *S*

5

256.001

0.024

Not D =

6

LOVEKIRCHHOFF

255.94

257.123

0.462

Variation of K_{ss} curve

5

173.406

1.60

Not D =

6

LOVEKIRCHHOFF

170.625

162.765

4.61

Variation of curve K

5

514.001

0.024

Not D =

6

LOVEKIRCHHOFF

511.875

512.242

0.46

M_{ss} moment

5

0.081751

+0.617

Point 0 =

6

0.08125

LOVEKIRCHHOFF

0.081394

0.18

5

0.12373

1.02

Not A =

6

0.125

LOVEKIRCHHOFF

0.10717

14.3

Moment M

5

0.081751

0.617

Point 0 =

6

0.08125

LOVEKIRCHHOFF

0.081394

0.18

5

0.037121

1.01

Not A =

6

0.03750

LOVEKIRCHHOFF

0.032146

14.3

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It is noticed that solution LOVE-KIRCHHOFF (= 106) is less quite approximate than that by

5

REISSNER =

at the variations of curve and the time bending. On the other hand, them

6

displacements and rotations are well calculated.

These differences are due to the relative thickness of this plate, with respect to the coarseness of the grid chosen. The figures hereafter show the comparison of the solutions analytical and numerical, in case LOVE-KIRCHHOFF, on grids of 10 and 100 elements.

- K

K_{ss}

Figure 4.2 - C: Variations of curve of an embedded circular plate

The layout of the variations of curve K

and K

ss

illustrate the fact that these two components are not

not approximate in the same manner: first is linear since derived from a function of form P_2 , while second is constant per pieces.

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4.3 Axisymmetric modal analysis of a thin spherical envelope**[V2.03.007]**

One considers a sphere, of average radius $R_m = 2.5$ m, thickness $T = 0.10$ Mr.

The material is elastic ($E = 200000$ MPa, $\nu = 0,3$), of density = 7800 kg/m³.

y

R

X

Appear 4.3-a: Sphere

One studies his axisymmetric free vibrations within framework LOVE-KIRCHHOFF (= 106).

One uses a grid made up of 40 meshes and 81 nodes. One is interested in the frequencies included/understood

between 220 and 375 Hz. Compared to the reference solution [V2.03.007] one finds like 5 first frequencies:

N°

1**2****3****4****5**

Reference

237.25

282.85

305.2

324.2

346.8

Aster

237.32

282.78

304.95

323.7

346.2

Table 4.3-a: Frequencies of the axisymmetric modes*Handbook of Reference**R3.07 booklet: Machine elements on average surface**HI-75/00/006/A*

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5 Conclusion

The finite elements that we propose were selected with a quite particular aim: structural analysis thin axisymmetric, or of orthogonal sections of infinite hulls with independence in direction Z , with the concern of obtaining a good precision on the membrane and flexional solution all by having a simple element of establishment and not too expensive.

The choice of the degrees of freedom allows a good representation of the boundary conditions.

Moreover,

this displacement formulation and rotation lead to elements of smaller degree: elements are P2 out of membrane and P2 in inflection. It appears that they are easy to handle and that their formulation

allows to use a structure of pre and post simple processor, favours considerable for to carry out rather fine grids (unidimensional) and to easily display the results (on a simple curve). Selected kinematics: formulation of HENCKY-MINDLIN-NAGHDI, in displacements and rotations of average surface makes it possible to utilize the energy of shearing transverse (interesting for the hulls average thickness).

This energy can be affected of a factor of correction: if one wants to place oneself in theory of REISSNER, it is enough to choose $= 5/6$ instead of 1 (but of course, the arrow W and rotations in this theory only weighted averages in the thickness are). Moreover, the formulation of hull of LOVE-KIRCHHOFF (for the very mean structures) can be simulated by penalization of condition of nullity of the transverse distortion, by choosing a factor $= 106 \times H L$, H being the thickness and $L a$ characteristic distance (radius of curvature, zone of application of the loads...). The non-linear behaviors in plane constraints are available for these elements. One announces however that the constraints generated by the transverse distortion are treated elastically, fault of better. Indeed, the taking into account of a transverse shearing constant not no one on the thickness and

determination of the correction associated on rigidity with shearing compared to a model satisfying the boundary conditions are not possible and thus return the use of these elements, when transverse shearing is nonnull, rigorously impossible in plasticity. In any rigour, for nonlinear behaviors, it would thus be necessary to use these elements in tally of the theory of Coils-Kirchhoff.

Elements corresponding to the machine elements exist in thermics; chainings thermomechanical are thus available with finite elements of thermal hulls to three nodes described in [R3.11.01] according to case's in its axisymmetric version, or its invariant plane version according to

Oz.

In the case-test treated, the phenomena of blocking did not appear. Decomposition of the deformation energy will make it possible, where necessary, to integrate in a selective way the terms persons in charge for blocking, such a modification not having to raise particular difficulties. One more detailed study must of course be undertaken on this subject, as for the numerical methods to use to avoid this blocking when the thickness becomes low.

The possible developments are:

- anisotropy in order to be able to treat the multi-layer hulls,
- problems of buckling,
- decomposition in Fourier series to study nonaxisymmetric problems of hulls of revolution,
- the taking into account a variable thickness...

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Organization (S): EDF/MTI/MMN

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R3.07.03 document

Elements of plate DKT, DST, DKQ, DSQ and Q4g

Summary:

These elements of plate are intended for calculations in small deformations and small displacements of structures

thin curves or plane. In fact plane elements do not take into account the geometrical curve structures, contrary to the elements of hull which are curved: it results from it from the parasitic inflections which can be reduced by using more elements in order to be able to approach the geometries correctly curves. The formulation is thus simplified by it and numbers it degrees of freedom reduced. These elements are considered as being among most precise for the calculation of displacements and the modal analysis.

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1 Introduction

The elements of hulls and plates are particularly used to model structures thin where the relationship between dimensions (characteristic thickness/length) is with more than 1/10. They thus intervene particularly in fields like the civil engineering, the interns of heart REFERENCE MARK, vibratory analysis One limits oneself to the framework of small displacements and the small deformations.

Contrary to the elements of hull, the plane elements of plate do not make it possible to take in hope the geometrical curve of the structure to be represented and induce parasitic inflections. It is thus necessary to use a great number of these elements in order to approach correctly geometry of the structure, and this, more especially as it is curved. On the other hand, one gains in simplicity

of formulation and the number of degrees of freedom is reduced. In addition, the formulations "Discrete Shear " (DST, DSQ and Q4g) or "Discrete Kirchhoff" (DKT and DKQ) of kinematics, with or without transverse distortion respectively, allow good results in terms of displacements and of modal analysis.

The way in which these elements in *Code_Aster* like certain receipts are established of use are given to [§5] present note.

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2 Formulation

2.1

Geometry of the elements plates [bib1]

For the elements of plate one defines a surface of reference, or surfaces average, planes (plane X y for example) and a thickness H (X, y). This thickness must be small compared to the different one dimensions (extensions, radii of curvature) of the structure to be modelled. [Figure 2.1-a] below illustrate our matter.

Solid 3D

Z

H
Y
B
X
L
R1
R2
Thickness $H < L, B, R1, R2$

Z
y
H
N
X

B
L
Plate

Appear 2.1-a

One attaches to average surface a reference mark orthonormé local Oxyz associated with the tangent plan with structure different from total reference mark OXYZ. The position of the points of the plate is given by Cartesian co-ordinates (X, y) of average surface and rise Z compared to this surface.

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2.2

Theory of the plates

These elements are based on the theory of the plates in small displacements and small deformations according to

which:

2.2.1 Kinematics

The cross-sections which are the sections perpendicular to average surface remain right; material points located on a normal at not deformed average surface remain on a line in the deformed configuration. It results from this approach that the **fields of displacement vary linearly in the thickness of the plate**. If one indicates by U, v, W displacements of a point $Q(X, y, Z)$ according to X, y and Z , one has the kinematics of Hencky-Mindlin thus:

$U(X, y, Z) = U(X, y) + Z \theta_x(X, y)$

$v(X, y, Z) = v(X, y) + Z \theta_y(X, y)$

$w(X, y, Z) = w(X, y) + Z \theta_z(X, y)$

)

$U(X, y)$

$v(X, y)$

$w(X, y)$

$\theta_x(X, y)$

$\theta_y(X, y)$

)

$U(X, y, Z) = v(X, y) + Z \theta_x(X, y) = v(X, y) + Z \theta_x(X, y)$

$v(X, y, Z) = v(X, y) + Z \theta_y(X, y)$

$w(X, y, Z) = w(X, y) + Z \theta_z(X, y)$

)

)

$U(X, y, Z) = W(X, y) + Z \theta_x(X, y)$

0

$W(X, y)$

Z

)

0

where U, v, W are displacements of average surface and $\theta_x, \theta_y, \theta_z$ are rotations of this surface by report/ratio with two axes X and y respectively. One prefers to introduce two rotations

$$X(X, y) = y(X, y), y(X, y) = -X(X, y).$$

The three-dimensional deformations in any point, with kinematics introduced previously, are thus given by:

$$xx = exx + zxx$$

$$yy = eyy + Z yy$$

$$2xy = xy = 2exy + 2zxy$$

$$2xz = X$$

$$2yz = y$$

where E, E

and E

xx

yy

xy is the membrane deformations of average surface, X and *there* them

deformations associated with transverse shearings, and xx, yy, xy deformations of inflection

average surface, which is written:

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U

exx = X

v

eyy = y

v

U

2exy =

+

X

y

X

xx = X

y

yy = y

X

y

2 xy =

+

y

X

W

X = X

+ X

W

$$y = y + y$$

Note:

In the theories of plate the introduction of X and makes it possible it to symmetrize the formulations of deformations and will see we it thereafter the equilibrium equations. In the theories of hull one uses X and y rather and the associated couples M X and M y compared to X and Y.

2.2.2 Law of behavior

The behavior of the plates is a behavior 3D in “plane constraints”. **The constraint transversal zz is null** bus regarded as negligible compared to the other components of tensor of the constraints (assumption of the plane constraints). The most general law of behavior is written then as follows:

xx

xx

E

0

xx

xx

yy

yy

E

0

yy

yy

2nd, 2

and = 0

xy

$$= \mathbf{C} (,) xy = \mathbf{This} + zC + \mathbf{C}$$

with $\mathbf{E} =$

xy

$=$

\cdot

xy

xz

X

0

0

X

yz

y

0

0

y

where

(

\mathbf{C}_i) is the matrix of local tangent rigidity combining forced plane and distortion transverse and represents the whole of the internal variables when the behavior is nonlinear.

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For behaviors where the distortions are uncoupled from the deformations of membrane and from inflection, (

C), is put in the form:

H

0

C =

0 **H**

where

(**H**), is a matrix 3x3 and (

H), a matrix 2x2. One will remain within the framework of this assumption.

For an isotropic homogeneous linear behavior elastic, one has as follows:

1 ν

0

0

0

ν 1

0

0

0

1 -

ν

E

0 0

0

0

C =

2

1 - ν^2 K 1(- ν)

0 0

0

0

2

 K 1

(-

 ν)

0 0

0

0

2

where K is factor of transverse correction of shearing whose significance is given to following paragraph.

Note:

One does not describe the variation thickness nor that of the transverse deformation z_z that one can however calculate by using the preceding assumption of plane constraints. In addition no restriction is made on the type of behavior that one can represent.

2.2.3 Taking into account of transverse shearing [bib2]

The taking into account of transverse shearing depends on factors of correction determined a priori by energy equivalences with models 3D, so that rigidity in shearing transverse of the model of plate is nearest possible to that defined by the theory of elasticity three-dimensional. Two theories including the deformation due to the shearing action exist and are presented in [bib2].

2.2.3.1 the theory known as of Hencky

This theory as that of Coils-Kirchhoff which results from this immediately rests on kinematics presented to [§2.2.1]. The relation of behavior is usual and the factor of correction of shearing is worth $k=1$.

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Note:

The model of Coils-Kirchhoff (DKT and DKQ): When one does not take into account the distortions transverses X and y in the theory of Hencky, the model obtained is that of Coils-Kirchhoff.

Two rotations of average surface are then related to displacements of surface average by the following relation:

W

$X = - X$

W

$y = - y$

2.2.3.2 the theory known as of Reissner (DST, DSQ and Q4g)

The second theory, known as of Reissner, is developed starting from the constraints. Variation of stress membrane (xx , yy and xy) is supposed to be linear in the thickness like in the case of the theory of Hencky where that results from the linearity of the variation of the deformations of membrane with

the thickness. However, whereas one supposes, in the theory of Hencky, the constant distortion in the shear thickness and thus stresses, which violates the boundary conditions $xz=yz=0$ on faces higher and lower of the plate because of law of behavior stated than the §2.2.2., one use within the framework of the theory of Reissner the equilibrium equations to deduce the variation from it from

shear stresses in the thickness of the plate, by observing the conditions in particular of balance on the faces higher and lower of plate. Energy interns model afterwards obtained resolution of the equilibrium equations in 3D, for inflection only, with the variation of plane constraints according to Z , reveals, for an elastic material, a relation between the efforts resulting and rotations and the arrow averages. It is in this relation that the factor appears of correction of shearing of $k=5/6$ instead of 1 in the relation which binds the sharp effort to the distortion for a homogeneous and isotropic plate. Determination of the factors of correction of shearing for orthotropic plates or laminated plates is left in appendix.

2.2.3.3 Equivalence of the approaches Hencky-Coils-Kirchhoff and Reissner

If one assimilates the slopes of average surface X , y with the averages of the slopes in the thickness of the plate and the arrow W with the average arrow, the only difference between the theory of Hencky and that of

Reissner is the coefficient of transverse correction of shearing of $5/6$ instead of 1. This difference is due to the fact that the starting assumptions are of different nature and especially that the variables

chosen are not the same ones. Indeed, the arrow on average surface is not equal to average of the arrows on the thickness of the plate. It is thus normal that relations of behavior which utilizes different variables are not identical.

The fact of having to solve on the level finite elements of the problems in displacements rather than of problems in constraints by interpolation of displacements leads us to use the approach equivalent in displacements of the problem of Reissner formulated in constraints.

2.2.3.4 Remarks

Because of preceding equivalence one presents here only the model in displacement for all them elements. In the facts elements DKT and DKQ are based on the theory of Hencky-Coils-Kirchhoff and the elements DST, DSQ and Q4 are based on the theory of Reissner.

The determination of the factors of correction rests within the framework of an other theory, that of Mindlin,

on equivalences of Eigen frequency associated the mode of vibration by transverse shearing.

One then obtains $k=2/12$, value very close to $5/6$ for the DST elements, DSQ and Q4 in the case isotropic.

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Within the framework of plasticity the problem of the choice of the coefficient of correction of shearing transverse arises because the equivalent approach in displacements of the problem of Reissner formulated in

constraints utilizes the non-linearity of the behavior. One cannot thus deduce some, like it is the case for elastic materials a value of the coefficient of correction of shearing transverse. Plasticity is thus not developed for these elements.

3

Principle of virtual work

3.1

Work of deformation

The general expression of the work of deformation 3D for a plate is worth:

$h/2$
 W
 $(++++D$
 $)V$
 $def = xx xx$
 $yy yy$
 $xy xy$
 $X xz$
 $y yz$
 $S - h/2$

where S is average surface and the position in the thickness of the plate varies between $h/2$ and $+h/2$.

3.1.1 Expression of the resulting efforts

By adopting the kinematics of [§2.2.1], one identifies the work of the interior efforts:

W
 $E NR$
 $+ E NR + 2nd NR + M + M + 2 M + T + T) dS$
 $def = (xx xx$
 yy
 yy
 xy
 xy
 xx
 xx
 yy
 yy
 xy
 xy
 $X X$
 $y y$
 S

where:

NR

M

$/2$

$/2$

$xx +h xx$

$xx +h xx$

T

$h/2$

X

+
 xz
 $\mathbf{NR} = NR yy = yy dz; \mathbf{M} = Myy = yyzdz; \mathbf{T} = =$

$dz.$
 T

y
 yz
 $- h/2$

$- h/2$

NR
 $- h/2$
 xy
 xy
 Mxy
 xy
 NR, NR, NR

xx
 yy
 xy is the efforts resulting from membrane (in N/m);
 M, M, M

xx
 yy
 xy is the efforts resulting from inflection or moments (in NR);
 T, T

X
 $y,$ are the efforts resulting from shearing or sharp efforts (in N/m);

3.1.2 Relation efforts resulting-deformations

The expression of the work of deformation is also written:

$h/2$
 $h/2$
 W
 $[($
 $\mathbf{C},) D$
 $] V$
 $[eCe Z$

EC.

$z_{Ce Z}$
2 C C D
] V
def =
=
+
+
+
+

$S - h/2$
 $S - h/2$
where (
C,) is the local matrix of behavior.

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By using the expression obtained for W_{def} in the preceding paragraph one finds the relation following between the resulting efforts and the deformations:

$$\mathbf{NR} = \mathbf{H E} + \mathbf{H}$$

m

MF

+h/2

+h/2

+h/2

$$\mathbf{M} = \mathbf{H E} + \mathbf{H}$$

2

MF

F with **H**

=
Hdz, H
 =
Hdz, H =
H
m
MF
F
Z dz
T = H
 - *h/2*
 - *h/2*
 - *h/2*
ct
G11
 0
 and: **Hct =**
 0
G22
 and:
E

xx
xx
X
E = e_{yy}, = yy, =

y
 2*e_{xy}*
 2*xy*
 The matrices **H**, **H** and **H**
m
F

ct is the matrices of rigidity out of membrane, inflection and shearing transverse, respectively. **The Hmf** matrix is a matrix of rigidity of coupling between membrane and the inflection.
 For an isotropic homogeneous elastic behavior of plate these matrices have as an expression:

1 v
0
3
1 v
0
Eh
Eh
kEh 1

0
H
=

v 1
0
, **H** =

v 1
0
, **H**
m
F
ct =

, and
1- 2
v
1-
v
12 1
(- 2
v)
1-
v
2 1
(+ v) 0

1
0 0

0 0

2

2

$\mathbf{Hm} = 0$ but there is material symmetry compared to the $z=0$ plan.

For an orthotropic material, the behavior is given in appendix.

3.1.3 Energy interns elastic of plate

Taking into account the preceding remarks, energy interns elastic plate is expressed more usually for this kind of geometry in the following way:

1

int =

[E (H E

m + H

)

MF

+ (H E

MF + H)

F

+ H

] dS.

2

ct

S

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3.1.4 Remarks

Relations binding **H, H, H**

with **H** and **H**

with **H**

m

F

MF

ct

are valid whatever the law of

elastic behavior, with anelastic deformations (thermoelasticity, plasticity,....).

For a plate made up of NR orthotropic layers in elasticity, matrices

H, H, H

and **H**

m

F

MF

ct is written:

NR

NR

NR

NR

1

H = H H, H

3

3

I

= **H H, H**

I I

= (*z*_{*i*}+1 - *Z*) **H, H**

I

= **H H**

m

I

MF

I

F

I

ct

I

I

3

i=1

i=1

i=1

$i=1$

1

where: $H = Z + 1 - Z, = (Z + 1 + Z$

I

I

I

I

I

) and \mathbf{H}, \mathbf{H}

and

2

\mathbf{I}

\mathbf{I}

the matrices \mathbf{H} represent

\mathbf{H} for the layer

\mathbf{I} .

The homogenisation for multi-layer hulls can lead to matrices of rigidity of membrane and of inflection nonproportional of the type:

C

C

0

D

D

0

1111

1122

1111

1122

$G11$

0

$\mathbf{H} = C$

C

1122

2222

0, $\mathbf{H} = D$

D

1122

2222

0, \mathbf{H}

\mathbf{m}

\mathbf{F}

ct =

0

G22

0

0

C1212

0

0

D1212

for which one cannot find equivalent values of the Young modulus and thickness allowing to find the traditional expressions of rigidity, cf [bib7].

3.2

Work of the forces and couples external

The work of the forces and couples being exerted on the plate is expressed in the following way:

+h/2

+h/2

W

F. D

U V

F. D

U.S.

F. D

U zds

ext. =

v

+

S

+

C

S - h/2

S

C - h/2

where **F, F, F**

v

S

C are the voluminal, surface efforts and of contour being exerted on the plate, respectively. **C** is the part of the contour of the plate on which the efforts of contour **FC** are applied. With the kinematics of [§2.2.1], one determines as follows:

W

$$(F U + F v + F W + C + C) dS + (U + v + W + +) ds$$

ext. =

X

y

Z

X X

y y

X

y

Z

X X

y y

S

C

$$= (F U + F v + F W + C - C) dS + (U + v + W + -) ds$$

X

y

Z

y X

X y

X

y

Z

y X

X y

S

C

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· where are present on the plate:

F, F, F

X

y

Z : surface forces acting according to X, y and Z

$+h/2$

$F =$

$\cdot dz$

I

$+$

$\mathbf{F} \mathbf{E}$

$\mathbf{F} \cdot \mathbf{e}$

\mathbf{v}

\mathbf{I}

\mathbf{S}

\mathbf{I} where \mathbf{e}_x and \mathbf{E} are **there the** basic vectors of the tangent plan and $\mathbf{E} \mathbf{Z}$ them

$- h/2$

normal vector.

C, C

X

y : surface couples acting around axes X and Y .

$+h/2$

H

$C =$

$ze \mathbf{F} \mathbf{E} dz$

I

$+\pm \mathbf{E}$

(

).

(

$\mathbf{F}) \cdot \mathbf{e}$

\mathbf{Z}

\mathbf{v}

\mathbf{I}

\mathbf{Z}

\mathbf{S}

\mathbf{I} where $\mathbf{E}, \mathbf{E}, \mathbf{E}$ are the basic vectors

2

\mathbf{X}

y

\mathbf{Z}

$- h/2$

previously definite.

· and where are present on the contour of the plate:

X, y, Z : linear forces acting according to X, y and Z

$+h/2$

$I =$

$\int dz$

$\mathbf{F E}$

\mathbf{C}

\mathbf{I}

where $\mathbf{ex}, \mathbf{ey}, \mathbf{ez}$ are the basic vectors previously definite.

$- h/2$

X, y : linear couples around axes X and Y .

$+h/2$

$I =$

$\int_{-h/2}^{+h/2} \mathbf{F E} dz$

(

).

\mathbf{C}

\mathbf{I}

where $\mathbf{E}, \mathbf{E}, \mathbf{E}$

\mathbf{X}

\mathbf{y}

\mathbf{Z} are the basic vectors previously definite.

$- h/2$

Note:

The moments compared to Z are null.

3.3

Principle of virtual work

It is written in the following way: W

$= W$

ext.

def for all virtual displacements and rotations

acceptable.

3.3.1 Kinematics of Hencky

With this kinematics, it results after integration by parts of work of deformation them equilibrium equations static of the following plates:

NR

$+ NR$

$+ F$

xx, X

xy, y

$X = 0,$

· For the efforts: NR

+ NR
+ F
 yy, y
 xy, X
 $y = 0,$
 T
+ T
+ F
 X, X
 y, y
 $Z = 0.$
 M
+ M
 $- T + C$
 xx, X
 xy, y
 X
 $y = 0,$
· For the couples: M
+ M
 $- T - C$
 yy, y
 xy, X
 y
 $X = 0.$

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as well as the boundary conditions following on contour C of S:

$$NR N + NR N$$

$$xx X$$

$$xy y = X,$$

$$U = U,$$

$$NR N + NR N$$

$$yy y$$

$$xy X = y,$$

$$v = v,$$

$$TN + TN$$

$$XX$$

$$y y = Z$$

,

$$\text{or } W = W,$$

$$MN + MN$$

$$xx X$$

$$xy y = y,$$

$$X = y,$$

$$MN + MN$$

$$yy y$$

$$xy X = - X.$$

$$y = - X.$$

where n_x and n_y are the cosine directors of the normal with C directed towards the outside of the plate.

The physical interpretation of these efforts (NR, T and M) starting from the preceding equations is given below:

Z

Ty

Myy

P

P

Tx

Nyy

Myx

y

Nyx

Nxy

Mxx

X

Nxx

Mxy

Appear 3.3.1-a: Efforts resulting for an element from plate

Note:

NR, NR

xx

yy represents the tractive efforts and *NR xy* plane shearing. *M xx* and *M yy* the couples of inflection and *M xy* represent the torque. *Tx* and *Ty*, are the efforts of transverse shearing.

3.3.2 Kinematics of Coils-Kirchhoff

One recalls that within the framework of this kinematics, one with the following relation binding the derivative of

W

$X = - X$

arrow with rotations:

W. After a double integration by parts of the work of deformation,

$y = - y$

one obtains the following equilibrium equations static:

NR

+ *NR*

+ *F*

xx, X

xy, y

$X = 0,$

· For the efforts of membrane: *NR*

+ *NR*

+ *F*

yy, y

xy, X

$y = 0,$

· For the transverse shearing and bending stresses:

M

+ 2 *M*

+ *M*

+ *F* + *C*

- *C*

xx, xx

xy, xy

yy, yy

Z

y, X

$X, y = 0,$

M

+ *M*

- *T* + *C*

xx, X

xy, y

X

y = 0,

M

+ M

- T - C

yy, y

xy, X

y

X = 0.

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as well as the boundary conditions on contour C and with the angular points O of contour C of S:

NR N + NR N

xx X

xy y = X

,

U = U,

NR N + NR N

yy y

xy X = y,

v = v,

T + M

N

NS, S = Z

- N, S,

or *W = W,*

Mnn = S,

= - W

N
 $N = S$
 $,$
 $M(O+) - M(O) = - [(O+) - (O$
 NS
 NS
 N
 N
 $-)]$.

$$T = T N + T N$$

$$= M N^2 + 2 M N N + M N^2$$

nn
 $xx X$
 $xy X y$
 $yy y,$
 $.$
 M
 $= - M N N + M (N^2 - N^2) + M N N$
 NS
 $xx X y$
 xy
 X
 y
 $yy X Y$.

Surface average S
 S
 Contour
 Million (+)
 S Million (-)
 Discontinuity

Appear 3.3.2-a: Boundary condition with angular points for an element of plate

Note:
The kinematics of Coils-Kirchhoff implies that on the contour of the plate the shearing force transverse is related to the torque. It is noted that the order of the equilibrium equations of inflection is higher than with the kinematics of Hencky. Thus, to choose the kinematics of Coil-Kirchhoff come down to increase the degree of the functions of interpolation because a regularity is needed larger for the terms of arrow compared to the terms of membrane because of presence from derived seconds of the arrow in the expression of the work of the deformations. No element

of plate of Code_Aster does not use this kinematics. One can thus have differences between results obtained with the elements of Code_Aster and the analytical results obtained in using the kinematics of Coils-Kirchhoff for structures with angular contours.

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3.3.3 Principal boundary conditions met [bib1]

S

Surface average S

y

nx

Contour

X

ny

N

S, S

N, N

Appear 3.3.3-a: Boundary condition for an element of plate

The boundary conditions frequently met are gathered in the table which follows. They are given for the kinematics of Hencky in the reference mark defined by S and the normal external with plate:

Embedding

Simple support

Free edge

Symmetry compared to

Antisymetry by

an axis S

report/ratio with an axis S

$U = 0,$

one = 0,
one = 0,
custom = 0,
v = 0,
W = 0,

W = 0,
S = 0.
W = 0,
N
= 0.
N
= 0.
S
= 0,
N
= 0.

=
=
S = 0,
S
0 ,
S
0 ,
N = 0,

= 0,
= 0,
S = 0.
N
Z
S = 0.
Z = 0,
N = 0.
S = 0,
N = 0

U = *one* + *vn*; *U* = - *one* + *vn*
N
X
y
S
y
X,

$$= N + N; = - N + N$$

N

X X

y y

S

X y

y X,

$$= N^2 +$$

2

$$N N + N^2$$

N

X X

xy X y

y y,

$$\text{with: } = - N N + (N^2 - N^2) + N N$$

S

X X y

xy

X

y

y X y,

$$= N^2 + 2 N N + N^2$$

N

X X

xy X y

y y,

$$= - N N + (N^2 - N^2) + N N$$

S

X X y

xy

X

y

y X Y.

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S =

- ,

Note: one has

N.

$N = S$.

4

Numerical discretization of the variational formulation

exit of the principle of virtual work

4.1 Introduction

By exploiting the law of behavior, the virtual work of the interior efforts is written (with $\mathbf{Hmf} = 0$ until [§4.4], which does not remove anything with the general information following results, but allows to reduce them

notations):

$$W_{int} = \mathbf{eH E} + \mathbf{H} + \mathbf{H}$$

m

F

ct dS

(

)

S

U

, X

X, X

$W, X + X$

with: $\mathbf{E} =$

$v, y, =$

y, y

, =

.

$W, y + y$

$U, y + v, X$

$X, y + y, X$

It results from it that the elements of plate are elements with five degrees of freedom per node. These degrees of freedom are displacements in the plan of the element U and v , except plan W and both rotations X and Y .

The elements DKT and DST are triangular isoparametric elements. Elements DKQ, DSQ and Q4 are quadrilateral isoparametric elements. They are represented below:

4

3

3

1

y

2

1

2

X

Appear 4.1-a: Real elements

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The elements of reference are presented below:

(0,1)

(-1,1)

(1,1)

3

4

3

1

2

(0,0)

(1,0)

1

2

(1,-1)

(-1,-1)

Appear 4.1-b: Elements of reference triangle and quadrangle

One defines the reduced reference mark of the element as the reference mark (,) of the element of reference. The local reference mark

element, in its plan (X, y) is defined by the user. The X1 direction of this local reference mark is projection of a direction of reference D in the field of the element. This direction of reference D is chosen by the user who defines it by two nautical angles in the total reference mark. The normal NR with plan of the element (12 13 for a triangle numbered 123 and 12 14 for a numbered quadrangle 1234) fix the second direction. The vector product of the two vectors previously definite $Y1=N \times X1$ makes it possible to define the local trihedron in which will be expressed the generalized efforts

representing the state of stresses. The user will have to take care that the selected reference axis do not find parallel with the normal of certain elements of plate. By defect, direction of reference D is axis X of the total reference mark of definition of the grid.

The essential difference between elements DKT, DKQ on the one hand and DST, DSQ, Q4 on the other hand comes

owing to the fact that for the first the transverse distortion is null is still = 0. The difference between Q4 and elements DST and DSQ comes from a choice different of interpolation for the representation of transverse shearing.

4.2

Discretization of the field of displacement

If one discretizes the fields of displacement in the usual way for elements isoparametric i.e.:

NR

NR

*NR**NR**NR* $U = NR U, v = NR v, W = NR W, = NR, = NR$ *II**II**II**X**I xi**y**I yi,**i=1**i=1**i=1**i=1**i=1*

and that one introduces this discretization into the variational formulation of [§4.1] it a blocking results from it

in transverse shearing analyzed in [bib1] which returns the solution in inflection controlled by the effects of

transverse shearing, and not by the inflection, when the thickness of the plate becomes small by report/ ratio

with its characteristic dimension.

To cure this disadvantage the variational form presented in introduction is slightly modified so that:

*W**1**int =***eH E****m + Hf + Hct dS =****eH E****m + Hf + TH****T dS***-**(**)**(**)***ct***S**S*

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where are deformations of substitution checking = in a weak way (integral on the sides of J the element) and such as $\mathbf{T} = \mathbf{Hct}$. One checks thus that on the sides ij of the element ($S -$) $S ds =$

0

 I with $S = W, S + S$.

Two approaches are then possible; in the first, that of the element Q4, one uses bilinear discretization of the fields of displacement and the fact that is constant on the sides of the element. The relations on the sides ij then make it possible to express the values of on the sides in function of the degrees of freedom of inflection. In the second approach, which is that of the elements of the type DKT and DST, one uses the weak formulation of the preceding paragraph which makes it possible to bind the inflection to shearing forces to deduce the interpolation from it from the terms of inflection.

4.2.1 Approach**Q4g**

It rests on the linear discretization of the fields of displacement presented above:

 NR NR NR NR NR $U = NR U, v = NR v, W = NR W, = NR, = NR$ II

II

II

X

I xi

y

I yi,

i=1

i=1

i=1

i=1

i=1

where the functions *Nor* are given below.

Nor (i=1, N)

Q4

i=1 with 4

4

3

1

NI (,) =

1

(-) 1

(-)

4

1

NR (,) =

1

(+) 1

(-)

2

4

1

NR 3 (,) =

1

(+) 1

(+)

4

1

2

1

NR 4 (,) =

1

(-) 1

(+)

4

Functions Nor for the Q4 elements

Note:

1

One also notes Nor (,) = (1+ I

) (1+ I

) with (,

1 ,

2 ,

3) = (-, -) and

4

4

111 1

(,

1 ,

2 ,

3) = (- , - , ,)

4

1 111 .

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4.2.2 Approach DKT, DKQ, DST, DSQ

Like $T = M, + M$

and T

,

$= M, + M$

X $xx X$ $xy y$ y $yy y$

xy , X and $\mathbf{M} = \mathbf{Hf}$ one deduces from it that is defined

according to the derived seconds of X and y via two equilibrium equations internal and of the law of behavior in inflection. The discretization retained for X and y , such as S is quadratic on the sides and N linear, then utilizes of the quadratic functions of forms incomplete in the form:

 NR $2 NR$ NR $2 NR$ $X = Nk xk + xk$ $P K, y = Nk yk + yk$ $P K$ with $P = PC$ and $P = P S$ xk $K K$ yk $K K$ $K = 1$ $K = NR + 1$ $K = 1$ $K = NR + 1$

where Ck and Sk are the cosine and directing sines on the side ij to which belongs the node K defined by:

/

$$C = X/L = (X - X)/L; S = y/L = (y - y)/L; L = (x_2^2 + y_2^2)^{1/2}$$
 K ji K J I K K ij K J I K K ji ji)

Note:

To introduce the preceding discretization amounts adding like degrees of freedom to the element of rotations K in the middle of the sides K of the element. Indeed, rotations S and N such as:

C
 S
 S
 X
 $=$

$S - C$

N

y

are quadratic for S and linear for N with:

$$S = (1 - S)if + ssj + 4s(1 - S)$$

$$) K; N = (1 - S)nor + snj \text{ where } 0 \leq S \leq 1.$$

1

1

One observes thus that: $sk = S(S =) = (if + sj) + K.$

2

2

J

It is the relation ($S -$)

$$S ds =$$

0 with $S = W, S + S$ which will make it possible to eliminate the degrees from

I

freedom additional and to express them according to displacements and of nodal rotations.

sk

sj

K

nj

if

nor

$$1/2 (si+sj)$$

I

K

J

S

I

J

S

Variation of S

Variation of N

Appear 4.2.2-a: Variations of S and N

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Nor (i=1, N)

Pi (i=n+1,2n)

DKT, DST

i=1 with 3

i=4 with 6

NR (,) = = 1 - -

P (,) = 4

3

1

4

NR (,) =

P (,) = 4

5

2

5

6

NR 3 (,) =

P6 (,) = 4

1

4

2

DKQ, DSQ

i=1 with 4

i=5 with 8

1
1
4
7
3
NR
2
1 (,) =
1
(-) 1
(-)
P (,) =
1
(-) 1
(-)
4
5
2
1
1
NR (,) =
1
(+) 1
(-)
P
2
(,) =
1
(-) 1
(+)
8
6
2
6

4
2
1
1
NR
2

3 (,) =

1

(+) 1

(+)

P (,) =

1

(-) 1

(+)

4

7

2

1

5

2

1

1

NR

2

4 (,) =

1

(-) 1

(+)

P (,) =

1

(-) 1

(+)

4

8

2

Functions *N_i* and *p_i* for elements DKT, DST, DKQ, DSQ

4.3

Discretization of the field of deformation

The matrix jacobienne **J** (,) is:

NR

NR

NR X

NR y

X

y

I, I

,

,

I

I

J

J

i=1

i=1

11

12

J =

=

=

NR

NR

.

X

y

,

,

J

J

NR X

NR y

21

22

I, I

I, I

i=1

i=1

Moreover:

X

J

J

11

12

1

1 *J*²²

- *J*

12

-

=

J

front

EC. **J** =

= **J** =

or $J = \det \mathbf{J} = J J$

- $J J$

J

J

11 22

12 21

21

22

J - *J*

J

21

11

y

It is pointed out that the field of displacement is discretized by:

W

W

0

U

NR

the U.K.

NR
K
2 NR

= *NR*,
K (
)
and = *NR*
(,) + [*P*
(,)
], the term enters
v
X
K
xk
xk
K
=1
vk
K

k=1

K = NR +1

P
y
yk
yk
(,)

hooks being present for the elements of the type DKT, DST, but not for the Q4 elements.

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4.3.1 Discretization of the membrane field of deformation:

NR

NR

E

= U, = NR (,) the U.K.

,

= (J NR

11

, + J

NR

) the U.K.

xx

X

K X

K

12

K,

,

K =1

K =1

NR

NR

E

= v, = NR (,) vk

,

= (J NR

21

, + J

NR

) v K

yy

y

$K y$

K

22

$K,$

,

$K = 1$

$K = 1$

NR

2nd

$= U, + v, =$

NR

(,) the $U.K.$

,

$+ NR (,) vk$

xy

X

y

$K y$

K, X

$K = 1$

NR

$= (J NR$

K

K

21

$, + J$

NR

K

22

$K,) U + (J NR$

11

$K + J NR$

)

,

12

$K,$

v

$K = 1$

Maybe in matric form:

E

xx

NR

U

E

K

$yy = \mathbf{B}$

U

= is the membrane field of displacement to the node *K*

mk

K where the **U.K.**

v

2nd

K 1

K

=

xy

and:

J NR

11

K, + J NR

0

12

K,

Bmk =

0

J NR

21

K, + J

NR

22

K,

J NR

21

K, + J

NR

JNR
 22
K,
 11
K, + J
NR
 12
K,
u1

v
 1

The matrix of passage of the membrane deformations to the field of displacement **U**

m =!
U
NR

v
NR

in the plan of the element is written as follows: $\mathbf{B} [3 \times 2N = (\mathbf{B}$

B
m
]
m1 “
mn).

4.3.2 Discretization of the transverse distortion

4.3.2.1 For the Q4g elements

One linearly discretizes the constant field by side so that:

1-
 1
 12
 + 34

+

= = 2
 2

1-
1
23
-

41
+
2
2

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+1

(- (

W +))

D = 0;

,

-1

By using the relations then:

,

+1

(- (

W +))

,

$$D = 0$$

-1

1

$$ij = (wj - I$$

$$W + I +$$

J);

2

it is established that:

for $(ij) = (12,34)$ and $(kp) = (23,41)$.

1

kp

=

$$(wp - wk + p + K);$$

2

By deferring the two results above in the expression of, one establishes that:

wl

1

1

$$= = \mathbf{B} \mathbf{U} \text{ where } \mathbf{U} = ! \text{ and } \mathbf{B} = (\mathbf{B}, \mathbf{B}$$

1 "

)

NR

wN

NR

NR

$NR K, K NR K,$

0

with $\mathbf{B} K =$

.

$NR K,$

0

$K Nk,$

wI

xI

yI

I

J

J

11

12 xi

Like

=

one deduces from it that = $\mathbf{B} \mathbf{U}$

=! and

F where $\mathbf{U} F$

I

J

J

21

22 yi

W

NR

xN

yN

$NR K, K Nk J$

,

11

K Nk J

,
12

B = (B, B

)

1 "

NR with **B K =**

.
NR

K, K NR K J

,
21

K Nk J

,
22

X J

J

11

12

Finally:

=

= **B U** with **B**

= **jB**

=

C

F

C [2×3N]

.

y

J

J

21

22

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4.3.2.2 For the elements of the type DKT, DST

With regard to the transverse distortions one deduces from

$$T = M, + M$$

and T

$$, \\ = M, + M$$

X

$xx X$

$xy y$

y

$yy y$

xy, X with $\mathbf{M} = \mathbf{H} F$ that $\mathbf{T} = \mathbf{H} F$, xx where:

$T, xx = (X, xx X, yy X, xy y, xx y, yy y, xy)$ and

H

H

$2H$

H

H

H

11

33

13

13

23

$12 + H33$

$\mathbf{H} F =$

where \mathbf{H}

H

H

H

ij is the terms (*I*, *J*) of

13

23

12 + *H*

H

H

2*H*

33

33

22

23

Hf.

NR

2 *NR*

NR

NR

$$= NR (,) + P (,) = NR (,) + (j2 P$$

2

2

11

+ *J J P*

11 12

+ *J P*

)

,
X, xx

K, xx xk

xk, xx K

K, xx xk

xk,

xk

,
12 *xk*

,
K

K = 1

K = NR + 1

K = 1

K = NR + 1

NR

2 NR

NR

NR

$$= NR (,) + P (,) = NR (,) +$$

2

2

(J P

, + 2 J

J P, + J P

)

,

X, yy

K, yy xk

xk, yy K

K, yy xk

21 xk

21 22 xk

22 xk,

K

K = 1

K = NR + 1

K = 1

K = NR + 1

NR

2 NR

,

$$= NR (,)$$

,

+ P (,)

X xy

K xy

xk

xk, xy

K

K = 1

K = NR + 1

NR

NR

$$= NR (,)$$

,
 + ($J J P$, + [$J J + J J$] P , + $J J P$),
 $K xy$
 xk
 $11 21 xk$
 $11 22$
 $12 21$
 xk
 $11 21 xk$,
 K
 $K = 1$
 $K = NR + 1$
 NR
 $2 NR$
 NR
 NR

(\cdot) + P (\cdot) = NR (\cdot) + ($j2 P$
 2
 2
 11
 + $J J P$
 $11 12$
 + $J P$
)

12

,
 =
 NR
 $y xx$
 K, xx
 yk
 yk, xx
 K
 K, xx
 yk
 $yk,$
 $yk,$
 $yk,$

K

K = 1

K = NR + 1

K = 1

K = NR + 1

NR

2 NR

NR

NR

= NR (,) + P (,) = NR (,) + (j2 P

2

2

J J P

+ J P

),

21

+

y, yy

K, yy

yk

yk, yy

K

K, yy

yk

yk,

21 22

yk,

22

yk,

K

K = 1

K = NR + 1

K = 1

K = NR + 1

NR

2 NR

= NR (,) + P (,)

y, xy

K, xy

yk

yk, xy

K
 $K = 1$
 $K = NR + 1$
 NR
 NR
 $= NR (.) + (J J P + [J J + J J] P + J J P)$

K, xy
 yk
 $11 \ 21 \ yk,$
 $11 \ 22$
 $12 \ 21$
 $yk,$
 $11 \ 21 \ yk,$

K
 $K = 1$
 $K = NR + 1$

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that is to say still in matric form that:

X, xx

X, yy

T = H X, xy =

F y, xx

y, yy

y, xy

0

j2 NR

2 J J NR

j2 NR

0

11

K

*,
+ 11 12 K*

*,
+ 12 K,*

0

j2 NR

2 J J NR

j2 NR

0

21

K

*,
+ 21 22 K, + 22 K*

*,
W*

NR 0

J J NR

[J J

J J] NR

J J NR

0

K

11 21

K

*,
+ 11 22 + 12 21 K*

,

+

H

11 21

K

,

F

= 0

0

j2 NR

2 J J NR

+ j2 NR

xk

K 1

11

K

,

+ 11 12 *K*,

12

K,

0

0

j2 NR

+ 2 J J NR

+ j2 NR

yk

21

K,

21 22

K,

22

K,

0

0

J J NR

+ [J J + J J] NR

+ *J J N R*

11 21

K,

11 22

12 21

K,

11 21

K,

C (j2 P

+ 2 *J J P* + *j2 P*)

K

11 *K*,

11 12 *K*,

12 *K*

,

C (j2 P

2

2

)

K

21 *K*,

+ *J J P*

21 22 *K*,

+

J P

22 *K*

,

2 *NR*

C (J J P

[

]

)

K

11 21 *K*,

+ *J J*

11 22 + *J*

JP
12 21
K,
+ *JJP*
+ **H**

11 21 *K*
,
=
F *K*
S (*j*² *P*
2
2
)
K
11 *K*,
+ *JJP*
11 12 *K*,
+ *JP*
K = *NR* + 1

12 *K*
,

S (*j*² *P*
2
2
)

K
21 *K*,
+ *JJP*
21 22 *K*,
+

JP
22 *K*
,

S (J J P
[
+ J J] P
)
K, +

J J P
K
11 21 K,
+

J J
11 22
12 21
11 21 K,
C P
K
K,

C P
W
K K,
NR
K
2

NR
C P
NR
2 NR
H
K
K,
P

H T
+ H T T H P U
H T T

B U
B

F
2
ck
K =
F
F
F
+ F 2
= C F +
F
2

H
P U
F
F K
xk +

K =
S P
F
F K

F K

C
K = 1
K = NR 1

+
K K,

K

= 1
K = NR + 1

yk

SP
K
K,

SP
K
K,

2
2
J
J
2 J J

T
11
12
11 12

2
0
where **T**
(**T**
T
2
2
=
C NR +
)

(
)
1 “*c2 NR* and **T2** =

= *J*
J
2 J J

with **T**
.
0
T

2
21
22
21 22
2

$$j_{11} j_{21} j_{12} j_{22} j_{11} j_{22} + 12$$

$$J j_{21}$$

J

We use then the relation (*S* -)

$$S ds =$$

0 with $S = W$, $S + S$ for each side *ij* of

I

the element which makes it possible to obtain *K* since she is still written:

L

2

$$W - W$$

K

+

$$(C + S + C + S) + L = L$$

J

I

$$K x_i$$

$$K y_i$$

$$K x_j$$

$$K y_j$$

$$K K$$

$$K K$$

K sk where:

2

3

-*I*

-

$$= (C$$

$$S) = (C$$

$$S) \mathbf{H T} = (C$$

$$S) \mathbf{H 1} [\mathbf{B U} + \mathbf{B}$$

]

sk

K

K

K
K
ct
K
K
ct

C

F
C

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The relation above is still written in matric form: **WITH = A U**

W

F

L

0

0

NR 1

LN 1CN 1 LN 1SN 1

2

+

+

+

+

+

with: **With**

=
0
0

-

H -1B

!
!
ct

3
C

0
0
L2N L2 C

NR

2

L

NR

2 *S*

N2 NR

and:

- 2 *L C*

L

S

2

L

C

L

S

0

0

0

NR +1 NR +1

NR +1 NR +1

NR +1 NR +1

NR +1 NR +1

0

0

0

- 2 L C

L S

2

L C

L S

K +1 K +1

K +1 K +1

K +1 K +1

K +1 K +1

0

0

0

0

0

0

- 2 L

C

L

S

2 NR -1 2 NR -1

2 NR -1 2 NR -1

1 2

L C

L S

0

0

0

0

"

"

With

2 NR

2 NR

2 NR

2 NR

= -

W

2

0

0

0

"

0

0

0

"

2

L

C

L

"

S

2 NR -1 2 NR -1

2 NR -1 2 NR -1

“-2 L C

L S

2 NR

2 NR

2 NR

2 NR

L C

L

S

NR +1

NR +1

NR +1 NR +1

1-

+

!

!

H B

ct

C

L C

L S

2 *NR*

2 *NR*

2 *NR*

2 *NR*

Thus =

front

EC.

1

=

-

WITH U

With

WITH A

F

W, which implies $\mathbf{T} = [\mathbf{B} + \mathbf{B} \mathbf{A}$

\mathbf{U}

]

C

C

F.

Note:

For the DST elements, this expression is simplified a little since \mathbf{B}

= 0 because of linearity of

C

functions of form NR (k=1,2,3).

K

This expression is simpler for elements DKT and DKQ since they are without transverse distortion,

1 0

0

i.e. = 0, which implies $\mathbf{A} =$

0 #
0 and

0 0 1
- 2/LN

C
+1
NR

S
+1
NR

2/L
+1
NR

C
+1
NR

S
+1
NR +

0
0
0
1
"

0
0
0
- 2/L

K
C
+1
K
S
+1
K

2/L

+1

K

C

+1

K

S

+1

K +1

"

0

0

0

0

0

0

- *2/L*

2 NR

C

-1

2 NR

S

-1

2 NR -1

"

3

2/L

C

S

0

0

0

0

"

"

"

With

2 NR

2 NR

2 NR

W = - 4

0

0

0

"

0

0

0

"

2/L

"

2 NR

C

-1

2 NR

S

-1

2 NR -1

“- 2/L2N

C2N

S2N

It is also noticed that for elements DKT the expression of the efforts sharp is calculated from balance and not starting from the behavior (on the basis of the behavior one would find a value zero sharp efforts what would not make it possible to carry out balance!). It results from it according to the §3.1.1 from

transverse stress shear nonnull in the thickness of the plate that one is in formulation DKT or DST.

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4.3.3 Discretization of the field of deformation of inflection:

4.3.3.1 For the Q4g elements

The relation binding the deformations of inflection to the field of displacement of inflection is written:

NR

NR

$$xx = X, X = j11x, + j12x, = j11 Nk, xk + j12 Nk,$$

$xk,$

$K = 1$

$K = 1$

NR

NR

$$yy = y, y = j21y, + j22y, = j21 Nk, yk + j22 Nk,$$

$yk,$

$K = 1$

$K = 1$

NR

NR

$$2 xy = y, X + X, y = j11y, + j12y, + j21x, + j22x, = J NR$$

21

K

$$, xk + j22 NR K, xk$$

K 1

=

K 1

=
NR
NR
+ 11
J Nk, yk + 12
J Nk, yk.
K 1

=
K 1

=
That is to say still in matric form:

xx
W
NR

K

$yy = \mathbf{B}fk \mathbf{U}fk$ where $\mathbf{U}fk = xk$ represents the field of displacement of inflection to the node K,

K
2
1
=
xy
yk
with:
0 *J NR*
0

11
K, + J
NR
12
K

,

B $fk = 0$
0
J NR
21
K, + J
NR
22
K.
0 *J NR*

21
K, + J
NR
22
K
J NR

,
11
K, + J
NR
12
K,
wI

xI

y
1

The matrix of passage of the field of displacement of inflection $\mathbf{U} F =!$ with the deformations of
W
NR

xN

yN
inflection is written then: **B**
= B, B
F [× N]
(F “fN)

3 3

1

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4.3.3.2 For the elements of the type DKT, DST:

The relation binding the deformations of inflection to the field of displacement of inflection is written:

NR

2 NR

NR

2 NR

xx = X, X = jll,

12

,

(

11

,

,)

(

12

,

X + J X = J

NR K xk +

xk

P K + J

NR K xk +

xk

$P, K),$

$K = 1$

$K = NR + 1$

$K = 1$

$K = NR + 1$

NR

$2 NR$

NR

$2 NR$

,
21

,
22

,
(
21

,
,)
(
22

$yy = there y = J y + J y = J$

$NR K yk +$

yk

$P K + J$

$NR K, yk +$

$Pyk, K),$

$K = 1$

$K = NR + 1$

$K = 1$

$K = NR + 1$

$2 xy = y, X + X, y = jll y + J$

,
12 y
+ J

,
21x
+ J

,
22 X, =
NR
2 NR

NR
 $2 NR$
 NR
 $2 NR$
 $J ($
 $21 NR$
 $+$
 $+ 22$
 $+$
 $+ 11$
 $+$
 K, xk
 xk
 $P, K) J ($
 $NR K, xk$
 xk
 $P, K) J ($
 $NR K, yk$
 yk
 $P, K)$
 $K = 1$
 $K = NR + 1$
 $K = 1$
 $K = NR + 1$
 $K = 1$
 $K = NR + 1$
 NR
 $2 NR$
 $+ J ($
 $12 NR$
 $+ P$
 $K,$
 yk
 $yk, K).$

$K = 1$
 $K = NR + 1$

For elements DKT, DKQ:

In matric form the preceding relation is also written by introducing the relation $= \mathbf{A U}$
 $F:$

B
B

xx
 $j11$
 $X + j12$
 X

$yy =$
 $j21B$
 $y + j22B$
 y
 $\mathbf{U F} = \mathbf{B F} \ 3 \times 3 \mathbf{N U}$
[
]
 F where

$2 \ xy$
 J
 $11B$
 $y + j12By + j21B$
 $X + j22B$
 X
 $w1$

$x1$
 y
 1
 $\mathbf{U F} =!$ represent the field of displacement in inflection for the element with:
 W
 NR

xN

yN
 $6 \ NR$
 $P + 1C$

,

NR

6P

+1

2 NR C

,

2 NR

3

B

2

2

X = (

-

, NI, - (NR

P + 1C

, NR + 1 + P2 NR C

,

2 NR),

4 LN

4L

+1

2 NR

4

3

- (NR

P + 1C

,

NR

S

+1 NR + 1 + P2N C

,

2 NR S2 NR),

,

4

"

6 NR

P + K C

,

NR + k

6 NR

P + k - 1C

,

$NR + k$ 1
 3
 -
 -, NR
 2
 2
 K
 , -
 (NR
 $P + K C$
 $+ P$
 C
),
 4 L
 , $NR + k$
 $NR + k - 1$
 ,
 $NR + k - 1$
 $NR + k$
 4 $LN + k -$
 4
 1
 3
 - ($PN + k C$
 S
 ,
 $+ P$
 $NR + K NR + K$
 $NR + K - 1 C$
 S
 ,
 $NR + K - 1 NR + K - 1$),
 4
 "

($K = 2. , NR$)
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6 NR

P +1C

,
NR

6P

+1

2 NR C2

3

B

,
NR

2

2

X

= (

-

, NI, - (NR

P +1C

,
NR +1 + P2 NR C

,
2 NR),

4LN

4L

+1

2 NR

4

3

- (NR

P +1C

,
NR

S

+1 NR +1 + P2N C

,
2 NR S2 NR),

,
4
"

6 NR
P +K C

,
NR +k
6 NR
P +k-1C

,
NR +k 1
3

-
-, NR
2
2
K

, -
(NR
P +K C
+ P
C
)

4L
, NR +k
NR +k -1

,
NR +k -1
NR +k
4LN+k-

4
1
3
- (PN+k C
S

,
+ P
NR +K NR +K
NR +K -1 C
S

,
 $NR + K - 1$ $NR + K - 1$),
4
"
($K = 2$., NR))
6 NR
 $P + 1S$
,
 NR
6 P
+1
2 $NR S$
,
2 NR
3
 $\mathbf{B} y = ($
-
, - (NR
 $P + 1C$
,
 NR
 S
+1 $NR + 1 + P2 NR C$
,
2 $NR S2 NR$),
4 LN
4 L
+1
2 NR
4
3
 NR
2
2
1
, -
(NR
 $P + 1 S$
,
 $NR + 1 + P2 NR S$
,
2 NR),
,

4

"

6 NR

P +K S

,

NR +k

6 NR

P +k-1S

,

NR +k 1

3

-

-, - (NR

P +K C

,

NR

S

+ P

C

S

),

4 L

+k NR +k

NR +K -1, NR +K -1 NR +K -1

NR +k

4LN+k-

4

1

3

NR

2

2

K, -

(NR

P +K S

, NR +K + NR

P +k

S

-1, NR +k-1),

4

"

(K = 2. , NR))

6 NR

P +1S

,

NR

6P

+1

2 NR S2

3

B

,

NR

y = (

-

, - (NR

P +1C

,

NR

S

+1 NR +1 + P2 NR C

,

2 NR S2 NR),

4 LN

4 L

+1

2 NR

4

3

NR

2

2

1

,

-

(NR

P +1S

,

NR +1 + P2 NR S

,

2 NR),

,

4

"

6 NR

P +K S

,

$NR + k$
 $6 NR$
 $P + k - 1S$
 ,
 $NR + k 1$
 3
 -
 -, - (NR
 $P + K C$
 ,
 NR
 S
 $+ P$
 C
 S
),
 $4 L$
 $+k NR + k$
 $NR + K - 1, NR + K - 1 NR + K - 1$
 $NR + k$
 $4LN + k -$
 4
 1
 3
 NR
 2
 2
 $K, -$
 $(NR$
 $P + K S$
 , $NR + K + NR$
 $P + k$
 S
 $-1, NR + k - 1)$,
 4
 "

($K = 2. , NR$)

For elements DST, DSQ:

The relation binding the deformations of inflection to the field of displacement in inflection is also written under matric form:

xx
W
NR
2 NR

K

yy = **B** fk **U** fk + **B** fkU fk where **U F K** = xk and **U F K**
= K represent it

K =
K = NR
2
1
1
+
xy
yk

field of displacement of inflection to the node K, so that:

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0 J NR

0

11

K, + J

NR

12

K

,

B F K = 0

0

J NR

21

K, + J

NR

22

K

, and

0 *J NR*

21

K, + J

NR

22

K

J NR

,

11

K, + J

NR

12

K

,

J

11 *xk*

$P, + J$

12 xk

P

,

$\mathbf{B} F K =$

J

21 yk

$P, + J$

22 yk

$P,$

.

J

11 yk

$P, + J$

12 yk

$P, + J$

21 xk

$P, + J$

22 xk

$P,$

wI

xI

y

1

The matrix of passage of the field of displacement of inflection \mathbf{U}

$= (\mathbf{U}$

F

$F,)$

with $\mathbf{U} F =!$ and

W

NR

x_N

y_N

1
=! with the deformations of inflection is written then:

NR
B
= (**B**, **B**
, **B**
,
, **B**
)
"

"

= (**B**
, **B**
 $F [\times NR]$
 F
 $F NR$
 $F (NR +)$
 F
 NR
 $F [\times NR]$
 $F [\times NR]$)

3 4

1

1

2

3 3

3

.

4.4

Stamp rigidity

The principle of virtual work is written in the following way: W

= W

ext.

int is still in elasticity

$U^T K U = F U$ in matrix form where **K** is the matrix of rigidity coming from the assembly

in the total reference mark of the whole of the elementary matrices of rigidity.

4.4.1 Stamp elementary rigidity for the Q4g elements

$W_{eint} =$

$$\begin{aligned}
 & [E (\\
 & \mathbf{Hme} + \mathbf{Hmf}) + \\
 & (\mathbf{Hmf} \mathbf{E} + \mathbf{H} \mathbf{F}) + \\
 & \mathbf{Hct} \mathbf{D} \\
 &] S = \\
 & E \\
 & T T \\
 & T \\
 & T \\
 & T \\
 & T \\
 & T \\
 & T \\
 & (\\
 & \mathbf{UmBmHmBmUm} + \mathbf{UmBmHmf} \mathbf{B} \mathbf{F} \mathbf{U} \mathbf{F} + \mathbf{U} \mathbf{F} \mathbf{B} \mathbf{F} \mathbf{Hmf} \mathbf{BmUm} + \mathbf{U} \mathbf{F} \mathbf{B} \mathbf{F} \mathbf{H} \mathbf{F} \mathbf{B} \mathbf{F} \mathbf{U} \mathbf{F} \\
 & E \\
 & + T T \\
 & \mathbf{U} \mathbf{B} \mathbf{H} \mathbf{B} \mathbf{U}) dS \\
 & F \\
 & C \\
 & ct \\
 & C \\
 & F \\
 & = \\
 & T \\
 & T \\
 & \mathbf{U} (\mathbf{B} \mathbf{H} \mathbf{B} dS \\
 & T \\
 & T \\
 & T \\
 & T \\
 & m m m m) \mathbf{Um} + \mathbf{U} (\mathbf{B} \mathbf{H} \mathbf{B} dS \\
 & F F \\
 & F \\
 & F \\
 &) \mathbf{U} \mathbf{F} + \mathbf{U} (\mathbf{B} \mathbf{H} \mathbf{B} dS) \mathbf{U} \\
 & m m MF \\
 & F \\
 & F
 \end{aligned}$$

E

E

E

+ **UT (BT H B dS) U**

F F MF m

m

E

+ **UT (BT H B dS) U**

T

T

T

T

T

F C ct C

F = the U.K.U

m

m

m + the U.K.U

F

F

F + the U.K.

U

m

MF

F + the U.K.

U

F

Fm

m + the U.K.U

F

C

F

E

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with **K**= **K T****MF****Fm.****Um**This is still written: **W E**int = (**Um, U F**) **K****U**

where

F**K****K**

[

m2 NR × 2 NR]**MF [2 NR × 3N]****K****[5N × 5N] =**

is the matrix of rigidity of the element.

K T**K**+ **K****MF [3N × 2 NR]****F [3N × 3N]**

[

C 3N × 3N]**4.4.2 Stamp elementary rigidity for elements DKT, DKQ**

Since the relation = 0 is satisfied, one can write:

W eint =**E (****Hme + Hmf) +****(Hmf E + H F) dS =**

E

TT

T

T

T

T

T

T

(

UmBmHmBmUm +UmBmHmf B F U F +U F B F Hmf BmUm +U B H B U) dS

F

F

F

F

F

=

E

T

T

U (B H B dS

T

T

T

T

m m m m) Um + U (B H B dS

F F

F

F

) U F + U (B H B dS

m m MF

F

) U F

E

E

E

+ UT

T

T

T

T

T

F (B F Hmf B mdS) U m

U

$$mKUm$$

$$U$$

$$F K F U F$$

$$U$$

$$mKmf U F$$

$$U$$

$$F K fmU$$

$$=$$

$$+$$

$$+$$

$$+$$

$$m$$

$$E$$

with K

$$= K T$$

$$MF$$

$$Fm.$$

$$Um$$

$$K$$

$$K$$

$$[$$

$$m^2 NR \times 2 NR]$$

$$MF [2 NR \times 3N]$$

This is still written: $W E$

$$\text{int} = (Um, U F) K$$

$$=$$

$$U$$

where $K [5N \times 5N]$

$$T$$

$$F$$

$$K$$

$$K$$

$$MF [3N \times 2N]$$

$$F [3N \times 3N]$$

is the matrix of rigidity of the element.

4.4.3 Stamp elementary rigidity for elements DST, DSQ

$$W E =$$

$$E (H E + H$$

$$) +$$

(H E + H) +

-

TH

T

1

=

int

m

MF

MF

F

dS

ct

E

TT

T

T

T

T

T

T

(U B H B U + U B H B U + U B H B U +

U B H B U

m

m

m

m

m

m m MF F F F F MF m m F F F F F

E

+

T

T

-1

T

T

-1

T

T

U
B H
B U + U B H

B

+

-

B H

B

1

U + TBT

-1

) $dS =$

C H

B

ct

C

F

C

ct

C

F

F

C

ct

C

C ct C F

UT (BTH B dS) U

T

T

T

T

T

T

+

(

dS)

+

(

dS)

+

(

dS

)

m

m

m

m

m

U

B H B

U

F

F

F

F

F

U

B H B

U

m

m

MF

F

F

U

B H B

U

F

F

MF

m

m

E

E

E

E

E

+

T

T

-1

T

T
 -1
 T
 U
 T
 $($
 $dS)$
 $+$
 $($
 dS
 $-$
 $) +$

1
 $($
 $+$
 $\mathbf{B} dS \mathbf{U}$
 $\mathbf{T} \mathbf{B} \mathbf{T} \mathbf{H} -$
 $)$
 $($
 \mathbf{B}
 1
 dS

$) =$
 F
 $\mathbf{B} fH \mathbf{B}$
 ct

\mathbf{U}
 C
 F

$\mathbf{U} F \mathbf{B} fH \mathbf{B}$
 ct
 C

$\mathbf{B} \mathbf{H}$
 C
 ct
 C

F
C
ct
C
E
E
E
E
UTK U + UT K U + UTK U + UT K U + UT K U + UT K

T
+ K U
T
+ K

m
m
m
F
F
F
m
MF
F
F
Fm
m
F

F
F

F
C
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It is also known that **U**

= (**U**

F

F),

from where it results that:

K

= **BT H B**

F 11

F F F dS

;

K

K

S

F 11

F 12

K

T

F =

with: **K**

= **B H B dS**;

K T

K

F 12

F

F

F

F 12

22

S

K

= **BT H B**

F 22

F F F dS

.
S
K
= BT H B
MF 11
m MF F dS

;
K
= (K
K
S
MF
MF 11
MF 12) with: K
= BT H B
MF 12
m MF F dS

.
S
K
= K T
Fm
MF.

Using the fact that = **WITH** one deduces from it that:

F
Wint = T
T
T
T
UmK mUm + U fK F U F + **UmKmf**
U F + U fK Fm

Um where:

K
T
T
T
T
(
)
(
)

(
)
 $F = \mathbf{K} F_{11} + \mathbf{K} + \mathbf{A}$
 \mathbf{K}

$F_{22} + \mathbf{K}$
With
 C
 $+ \mathbf{K} F_{12} + \mathbf{K}$
With

$+ \mathbf{A}$
 \mathbf{K}

$F_{12} + \mathbf{K}$.
 $\mathbf{K}_{mf} = \mathbf{K}_{mf}_{11} + \mathbf{K}$
With
 MF_{12}

Um
 This is still written: $W E$
 $\text{int} = (\mathbf{U}_m, \mathbf{U} F) \mathbf{K}$

\mathbf{U}
 where
 F
 \mathbf{K}
 \mathbf{K}
 $[$
 $m^2 NR \times 2 NR]$
 m
 $F [2N \times 3N]$
 \mathbf{K}

$[5N \times 5N] =$
 is the elementary matrix of rigidity for an element
 $\mathbf{K} T$

\mathbf{K}
 $MF [3N \times 2 NR]$
 $F [3N \times 3N]$
 of plate.

4.4.4 Assembly of the elementary matrices

The principle of virtual work for the whole of the elements is written:

nbelem

$$W = We = T$$

int

int

U KU where U is the whole of the degrees of freedom of the discretized structure and

$e=1$

K comes from the assembly of the elementary matrices.

4.4.4.1 Degrees of freedom

The process of assembly of the elementary matrices implies that all the degrees of freedom are expressed in the total reference mark. In the total reference mark, the degrees of freedom are three displacements

compared to the three axes of the total Cartesian reference mark and three rotations compared to these three axes.

One thus uses matrices of passage of the local reference mark to the total reference mark for each element. However one

saw previously that the degrees of freedom of the elements of plate are two displacements in the plan of the plate, displacement except plan and two rotations. These rotations not being exactly rotations compared to the axes of the plate since

$$X(X, y) = y(X, y), y(X, y) =$$

- X(X, y) it is necessary to hold account of it with the level of the assembly for

to reveal the good degrees of freedom x_i, y_i .

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4.4.4.2 Rotations

fictitious

Rotation compared to the normal with the plate is regarded as not being a degree of freedom.

To ensure compatibility between the passage of the local mode the total mode, a degree is thus added

of additional freedom room of rotation to the plate which is that corresponding to rotation by report/ratio with the normal in the plan of the element. This implies an expansion of the blocks of dimension (5,5) matrix of local rigidity in blocks of dimension (6,6) by adding a line and a column correspondent with this rotation. These additional lines and these columns are a priori null. One then carry out the passage of the matrix of local rigidity extended to the matrix of total rigidity. In the preceding transformation, one was satisfied to add rotations compared to the normals in the plan of the elements without modifying the deformation energy. The contribution to the energy brought by these additional degrees of freedom is indeed null and no rigidity is associated for them. The matrix of total rigidity thus obtained presents the risk however to be noninvertible. For to avoid this nuisance it is allowed to allot a small rigidity to these additional degrees of freedom on the level of the matrix of widened local rigidity. Practically, one chooses it between 10⁻⁶ and 10⁻³ times it diagonal minor term of the matrix of rigidity of local inflection. The user can choose it multiplicative coefficient COEF_RIGI_DRZ itself in AFFE_CARA_ELEM; by defect it is worth 10⁻⁵.

4.5

Stamp of mass

The terms of the matrix of mass are obtained after discretization of the variational formulation following:

+h/2

W_{ac}

$\mathbf{U} \mathbf{U}$

\$

\$

mass =

$\int dz dS = (U$

$m \int U + \int v^2 + \int W W) + (U$

$MF \int + v$

X

$\int y + X U + y v)$

$- h/2 S$

S

+ \$

\$

$F (X X + y) dS$

y

+h/2

+h/2

+h/2

with

2

$m =$

$dz,$

$MF =$

$zdz,$ and

$F =$

$Z dz$

.

- $h/2$

- $h/2$

- $h/2$

Note:

If the plate is homogeneous or symmetrical compared to $z=0$ then $MF = 0$. One considers in continuation of the talk that it is always the case.

4.5.1 Stamp of traditional elementary mass

4.5.1.1 Element

Q4g

The discretization of displacement for this isoparametric element is:

the U.K.

NR

v_k

$\mathbf{U} =$

W

$NR \quad K \quad K = 1, \dots, NR$

K

$K = 1$

x_k

y_k

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The matrix of mass, in the base where the degrees of freedom are gathered according to the directions of translation and of rotation, has then as an expression:

M

0

0

M

m

MF

0

0

M

0

0

M

m

MF

M 0

0

M

=

0

0

m

T

M

0

0

M

MF

F

0

0

MT

0
0
M

MF
F

with: **M**
= **NTNdS**, **M**
= **NT dS** and
NR
M = NT NR

m
m
MF
MF
F
F

dS and **NR = (NR**
NR
1 “*K*).

S
S
S

4.5.1.2 Elements of the type DKT, DST

W
W

0

NR
K
2 *NR*

Like = *NR*
(,) + *P*
X
K
xk
xk
(,)

K where = **WITH** *F* one deduces from it that:

$$k=I$$

$$k=N+I$$

P

y

yk

yk

(,)

W

NR

(,)

0

0

W

NR

K

K

$$= NR$$

(,) *NR*

(,) *NR*

X

kxw

kxx

kxy

(,) *xk.*

$$k=I$$

NR

(,) *NR*

(,) *NR*

y
 kyw
 kyx
 kyy
 $(,) yk$

The membrane part of the elementary matrix of mass is the same one as for Q4 with $k=3$ with the place from 4 in **NR**. The inflection part is composed of the blocks kp (k th line and p ème column) following:

NR
 $/$
 $kxw NR pxw + NR kyw NR pyw + m NR K NR p$
 $F Nkxw NR pxx + Nkyw NR pyx Nkxw NR pxy + Nkyw NR pyy$

F
 $NR kxx NR pxw + Nkyx NR pyw$
 $NR kxx NR pxx + Nkyx NR pyx$
 $NR kxx NR pxy + Nkyx NR pyy$

NR

$kxy NR pxw + NR kyy NR pyw$
 $NR kxy NR pxx + Nkyy NR pyx$
 $NR kxy NR pxy + Nkyy NR pyy$

4.5.2 Stamp of improved elementary mass

As the arrow of a flexbeam can be represented with difficulty by an approximation linear, one can enrich the functions by form for the terms of inflection. This approach is used in *Code_Aster* for the elements of type DKT, DST and Q4 where functions of form used in the calculation of the matrix of mass of inflection are of order 3. The interpolation for W is written as follows:

NR
 $W = NR$
 $(,) W$
 $(-)$
 1
 $+1$
 $+ NR$
 $(,) W$
 $(-)$
 1
 $+2$
 $, + NR$
 $(,) W$

K

NR

K

K

NR

K

(K -)

1 NR +3

, K

K =1

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where the functions of form are given for the triangle and the quadrangle in the following table:

DKT, DST

DKQ, DSQ, Q4

3

4

7

3

5

6

1

4

2

8

6

1

5

2

Interpolation

= 1 - -

i=1 with 12

for W

i=1 with 9

1

NR

2

2

(,) =

1

(- 1

)(-)(2 - - - -)

NR

2

3

1

8

1 (,) = 3 - 2 + 2

1

NR

2

2

2 (,) = +/2

N2 (,) =

1

(- 1

)(- 1

)(-)

8

NR

2

3 (,) = +/2

1

NR

2

(,) =

1

(- 1
)(- 1
)(-)
 NR
 2
 3
 3
 4 (,) = 3
 - 2 + 2
 8
 NR
 2
 1
 2
 2
 5 (,) = (-1 +) -
 NR 4 (,) =
 1
 (+ 1
)(-)(2 - - + -)
 8
 NR
 2
 6 (,) = +/2
 1
 NR
 2
 (,) = -
 1
 (+ 1
)(- 1
)(-)
 NR
 2
 3
 5
 7 (,) = 3 - 2 + 2
 8
 NR
 2
 1
 2
 8 (,) = +/2

NR 6 (,) =

1

(+) 1

(-)(1-)

8

NR

2

9 (,) = (-1 +) -

1

NR (,) = (1+) (1+) (2

2

2

- - + +)

7

8

1

NR (,) = - (1+) (1+) (1

2

-)

8

8

1

NR (,) = - (1+) (1+) (1

2

-)

9

8

1

NR (,) = (1 -) (1+) (2

2

2

- - - +)

10

8

1

NR (,) = (1 -) (1+) (1

2

-)

11

8

1

NR (,)

12

= (1) (1) (1)
2
-
+
-)
8

Functions of interpolation for the arrow of elements DKT, DST, DKQ, DSQ and Q4G, in dynamics and modal.

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4.5.2.1 Elements of type DKT

One knows that in the approximation of one Coils-Kirchhoff has $X = -W$, X and $y = -W$, there of any point of the element.

Because of discretization stated above one a:

NR

$W = NR$

(,) W

(-)

1

+1

+ ($J NR$

11

(-)

1

+ (,)

2

+ $J NR$

(,)W

21

(-)

1

+3

,

+ (J NR

K

NR

K

K

NR

K

NR

xk

12

(K -)

1 NR + (,)

2

K =1

+ J NR

(,)W

22

(K -)

1 NR +3

, yk

W, K

J

J

11

12 W, xk

since:

=

W

.

, K

J

J

21

22 W

, yk

This is still written:

NR

$W = NR$

$(,) W$

$(-) +$

$+ NR (-) + (,) + NR$

K

NR

K

K

NR

xk

$(K$

$-) N+ (,)$

1

1

1

2

1

3

yk

$K = 1$

$NR (K) 1N+ (,)$

1

$= NR (K) 1N+ (,)$

1

where: $NR (K) 1N+ (,)$

2

$= - J NR$

11

$(K -)$

$1 NR + (,)$

2

$- J NR$

21

$(K -)$

$1 NR + (,)$

3

.

$NR (K) 1N+ (,)$

3

$= - J NR$

12

(K -)

1 NR + (,)

2

- J NR

22

(K -)

1 NR + (,)

3

By not taking account of the effects of inertia, the matrix of mass has the following form thus:

M

0

0

m

M = 0

Mm

0 where **M = NR NR**

F

m dS.

0

0

M

S

F

4.5.2.2 Elements of the DST type

It is known that for these elements one has $X = X - W$, X and $y = y - W$, *there* where the distortion is constant on the element.

Like:

NR

$W = NR$

(,) *W*

(-)

1

+1

+ (*J NR*)

11

(-)

1

+ (,)

2

+ *J NR*

(,)W

21

(-)

1

+3

,

+ (J NR

K

NR

K

K

NR

K

NR

xk

12

(K -)

1 NR + (,)

2

K =1

+ J NR

(,)W

22

(K -)

1 NR +3

, yk

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one can also write:

NR

$W = NR$

(,) W

(-)

1

+1

+ NR (-) 1 + (,)

2

+ NR

K

NR

K

K

NR

xk

(K

-) 1N+ (,)

3

yk

$K = 1$

+ (J11 + J) NR

12

(-)1 + (,)

2

+ (J21 + J) NR

X

y

K

NR

X

22 y (K -)

1 NR + (,)

3

NR (K) 1N + (,)

1

= NR (K) 1N+ (,)

1

where: NR (K -) 1N + (,)

2

= - J NR

11 (K -)

1 NR + (,)

2

- J NR

21 (K -)

1 NR + (,)

3

,

NR (K) 1N + (,)

3

= - J NR

12 (K -)

1 NR + (,)

2

- J NR

22 (K -)

1 NR + (,)

3

NR

NR

(,)

1

1

= NR

(K -) NR +

(K -)

1 NR

(,)

+1

K =1

NR

NR

(,)
 1
 2
 = NR
 (K -) NR +
 (K -)
 1 NR
 (,)
 +2
 K =1
 NR
 NR
 (,)
 1
 3
 =
 NR
 (K -) NR +
 (K) 1N (,)
 +3
 K =1
 w1
 w1

x1

x1

y1

y

1
 X
 and
 =
 -

**H 1 [B + B A
]! = T! .**

ct
C
C

W
y
W
W
NR
NR

xN
xN

yN
yN

One obtains the interpolation for *W* then:

NR
W = NR
(,) *W*
(-) +
+ *NR* (-) + (,) + *NR*

K
NR
K
K
NR

xk
(*K*
-) *N* + (,)

1
1
1
2
1
3

y_k
 $K = 1$
 $NR(K) 1N+ (,)$
 1
 $= NR(K) 1N+ (,)$
 1
 $+$
 $(JT (,$
 $1 (K$
 $11 W$

$-)$
 $1 NR +)$
 $1 + JT (2, (K$
 12
 W

$-)$
 $1 NR +)$
 $1) NR$
 $(J) 1N+ (,$
 2

$+$
 $(JT (,$
 $1 (K$
 $21 W$

$-)$
 $1 NR +)$
 $1 + JT (2, (K$
 $22 W$

$-)$
 $1 NR +)$
 $1) NR$
 $(J) 1N+ (,$

3
 $NR(K) 1N+ (,$
 2
 $= NR(K) 1N+ (,$
 2

$+$
 where:
 $(JT (,$
 $1 (K$

11 W

-)
1 NR + 2) + J T (2, (K
12
W

-)
1 NR + 2)) NR
(J) 1N+ (,
2

+
(J T (,
1 (K
21 W

-)
1 NR + 2) + J T (2, (K
22
W

-)
1 NR + 2)) NR
(J) 1N+ (,
3

NR (K) 1N+ (,
3
= NR (K) 1N+ (,
3

+
(J T
1
(, (K - 1) NR + 3) + J T (2, (K - 1) NR + 3)) NR
11 W

12 W
(J 1
-) NR + 2
(,) +
(J T
1
(, (K - 1) NR + 3) + J T (2, (K - 1) NR + 3)) NR
21 W
22
W

(J 1
-) NR +3
(,)
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By not taking account of the effects of inertia, the matrix of mass has the following form thus:

M

0

0

m

M = 0

M m

0 where **M =**

NN dS

.

F

m

0

0

M

S

F

4.5.2.3 Elements of the Q4g type

One proceeds in the same way that for the elements of the DST type but with:

wI

xI

yI

X

= \mathbf{B} ! where \mathbf{B}

C

C is the matrix established with [§ 4.3.2.1].

y

W

NR

xN

yN

4.5.2.4 Notices

One neglects in the form of the elementary matrix of mass the terms of inertia of rotation

$\$$

+ $\$$

$F(X X \text{ there } y dS$

)

because the latter are negligible [bib3] compared to the others. In

S

effect a multiplicative factor of $h^2/12$ the drops to the other terms and they become negligible for one thickness report/ratio over characteristic length lower than $1/20$.

4.5.3 Assembly of the elementary matrices of mass

The assembly of the matrices of mass follows same logic as that of the matrices of rigidity.

degrees of freedom are the same ones and one finds the treatment specific to normal rotations to plan of the plate. For modal calculations utilizing at the same time the calculation of the matrix of rigidity

and that of the matrix of mass, it is necessary to take a rigidity or a mass on the degree of rotation normal in the plan of the plate from 10^3 to 10^6 times smaller than the diagonal minor term of the matrix of rigidity or mass for the terms of inflection. That makes it possible to inhibit the modes being able to appear

on the additional degree of freedom of rotation around the normal in the plan of the plate. By

defect, one takes a rigidity or a mass on the degree of normal rotation in the plan of plate 105 time smaller than the diagonal minor term of the matrix of rigidity or mass for the terms of inflection

4.5.4 Stamp of lumpée diagonal mass

The use of a matrix of lumpée diagonal mass has two advantages: it is simpler with to implement numerically and it converges better. However the results are worse that with the traditional diagram for which the error is minimal [bib5].

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The matrix of mass is made diagonal, for the degrees of translation, according to a technique of lumping near to that developed by Hinton [bib6]. The advantage of this method is of always to produce positive coefficients of mass, contrary to other methods like the summation terms in column [bib5]. One uses the diagonal coefficients of the elementary matrix of mass in the total reference mark by making sure that the total mass is well represented in each of the three directions of translation:

dS

m

$m = M$

S

x_i

X

$x_i; X = NR$

Mx_j

$j=1$

dS

m

$m = M$

S

y_i
 y
 $y_i; ; y = NR$
 M_{y_j}
 $j=1$
 dS
 m
 $m = M$
 S
 z_i
 Z
 $z_i; Z = NR$
 M
 z_j
 $j=1$

where $M_{x_i}, M_{y_i}, M_{z_i}$ are the diagonal terms of the elementary matrix of mass in the total reference mark, **MT**
 0
 in each of the three directions of translation. The matrix of mass obtained: **m** =

0
m
m
 0
 0
X

with **m** = 0
m
T
y
 0 and **m** = 0 are diagonal.

0
 0
m
Z

Note:
The creation of a matrix of diagonal mass in the directions of rotation with a technique analogue with that of the terms of translation (coefficients I are then the coefficients previously defined in three directions X, y and Z) is inappropriate for the modal analysis and the results obtained are for the moment better with a matrix of reduced diagonal mass with the only degrees of translation. The difficulty of the use of this matrix arises then in

dynamics, when it is necessary to reverse the matrix of mass. One thus considers for the moment of to make diagonal only the terms of translation and to preserve a matrix of mass **m** supplements for the terms of rotation.

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4.6

Numerical integration for elasticity

For the triangular elements DKT, DST where [H] is constant, the matrices of rigidity are obtained exactly with three points of integration of Hammer since [B] is linear in.

Cordonnées of the points

Weight

1/6

1 = 1/6; 1 = 1/6

1/6

2 = 2/3; 2 = 1/6

1/6

3 = 1/6; 3 = 2/3

1 -

1

N

y (,) D D

=

iy (I, I)

0 0

i=1

Numerical formulas of integration on a triangle (Hammer)

For the elements quadrangle an integration of Gauss 2x2 is used.

Cordonnées of the points

Weight

1

1 = 1 /

3; 1 = 1 3

1

2 = 1 /

3; 2 = -1/3

1

3 = -1 /

3; 3 = 1 3

1

3 = -1 /

3; 3 = -1/3

1 -

1

N

$y(,) D D$

=

$iy(I, I)$

0 0

$i=1$

Numerical formulas of integration on a quadrangle (Gauss)

4.7

Numerical integration for plasticity

Integration on the surface of the element is supplemented by an integration on the thickness of behavior since:

+h/2

+h/2

+h/2

H =

Hdz H

,

=

Hdz, H =

H2

m

*MF**F**Z dz*

where **H** is the plastic matrix of behavior

*- h/2**- h/2**- h/2*

local.

The initial thickness is divided into NR identical layers thicknesses. There are three points of integration by

sleep. The points of integration are located in higher skin of layer, in the middle of the layer and

in lower skin of layer. For NR layers, the number of points of integration is of $2N+1$. One advise to use from 3 to 5 layers in the thickness for a number of points of integration being worth 7, 9 and 11 respectively.

For rigidity, one calculates for each layer, in plane constraints, the contribution to the matrices of rigidity of membrane, inflection and coupling membrane-inflection. These contributions are added and assemblies to obtain the matrix of total tangent rigidity.

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For each layer, one calculates the state of the constraints (xx , yy , xy) and the whole of the variables interns, in the middle of the layer and in skins higher and lower of layer, from

local plastic behavior and of the local field of deformation (xx , yy , xy). The positioning of points of integration enables us to have the rightest estimates, because not extrapolated, in skins lower and higher of layer, where it is known that the constraints are likely to be maximum.

Cordonnées of the points

Weight

1/3

1 = -1

4/3

2 = 0

1/3

3 = +1

1

N

y () D =

iy (I, I)

-1

i=1

Formulate numerical integration for a layer in the thickness

Note:

One already mentioned with [§2.2.3] that the value of the coefficient of correction in shearing transverse for the elements DST, DSQ and Q4 was obtained by identification of energies complémentaires elastic after resolution of balance 3D. This method is not usable any more in elastoplasticity and the choice of the coefficient of correction in transverse shearing is posed then. Plasticity is thus not developed for these elements.

4.8

Discretization of external work

The variational formulation of work external for the elements of plate is written

:

$$W = (F U + F v + F W + m + m) dS + (X$$

U + y

v + Z

W + X

 $\mu X +$

y

 $\mu) ds$

ext.

X

y

Z

X

X

y
y
y
S
C

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By taking account of a linear discretization of displacements, one can write for an element:

NR

$$W E = (F NR$$

ext.

$$X K (,) U + F NR$$

K

y

$$K (,)$$

$$v + F NR$$

K

Z

$$K (,)$$

wk

$$K = 1S$$

$$+ m NR$$

X

$$K (,)$$

$$+ m NR$$

xk
 y
 $K (,$
 $) dS$
 yk
 $+$
 $(NR$
 $X K (,$
 $the U.K. + NR$
 y
 $K (,$
 $vk + NR$
 Z
 $K (,$
 wk
 C
 $+ \mu NR$
 X
 $K (,$
 $) xk + \mu NR$
 y
 $K (,$
 $)) ds$
 yk
 NR
 $=$
 $F NR$

 $(,) dS + NR$
 $y K (,$
 X
 $K (,) dS + NR$
 $X K (,) ds F NR$
 y
 K
 ds
 $K =1 S$
 C
 S
 C
 $F NR$
 $Z K (,) dS + NR$

$Z K (,) ds m NR$

$X K (,) dS + \mu NR$

$X K (,) ds$

S

C

S

C

$m NR$

E

$y K (,) dS + \mu NR$

$y K (,) ds) \text{ the U.K.}$

S

C

NR

E

E

E

E

$= K$

F The U.K. = F U

$K = 1$

The variational formulation of the work of the efforts external for the unit of the elements is written then:

$Nb elem$

$W = We = \mathbf{F} \mathbf{U} = \mathbf{T} \mathbf{T}$

ext.

ext.

$\mathbf{U} \mathbf{F}$ where \mathbf{U} is the whole of the degrees of freedom of the structure

$e=1$

discretized and \mathbf{F} comes from the assembly of the vectors forces elementary.

As for the matrices of rigidity, the process of assembly of the vectors forces elementary

imply that all the degrees of freedom are expressed in the total reference mark. In the total reference mark,

the degrees of freedom are three displacements compared to the three axes of the total Cartesian reference mark

and three rotations by report/ratio these three axes. One thus uses matrices of passage of the local reference mark

with the total reference mark for each element.

Note:

The external efforts can also be defined in the reference mark user. One then is used

stamp passage of the reference mark user towards the local reference mark of the element to have the expression of

these efforts in the local reference mark of the element and to deduce the vector from it forces

elementary room

corresponding. For the assembly one passes then from the local reference mark of the element to the total reference mark.

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4.9

Taking into account of the thermal loadings

4.9.1 Thermoelasticity of the plates

The temperature is represented by the model of thermics to three fields according to [R3.11.01]:

$$T(X, x3) = T m ($$

$$X). P1(x3) + T S(X). P2(x3) + Ti$$

$$(X). P3(x3),$$

with: $P(X)$

$$- H/2, +h/2$$

J

3): three polynomials of LAGRANGE in the thickness:]

[:

2

X

X

$$P(X) = 1 - (2x/H); P$$

3

3

$$2(x3) =$$

$$(1+2x/h$$

3

$$); P_3(x) = - (1 - 2x/h)$$

1
3
3
3

);

H

H

From the representation of the temperature above, one obtains:

· the average temperature in the thickness:

$$1 + h/2$$

1

$$T(X) =$$

T

m

S

I

,

=

4

3

3

+

+

;

$$- h/2 (X$$

X

) *dx*

T

X

T X

T X

H

6 (

() () ()

· the average variation in temperature in the thickness:

%(

$$12 + 2$$

)

H

T X

=
T
;
2 - 2 (X, X X dx
3
3
3 = T S X
- T I X
H

)
() ()
H

Thus the temperature can be written in the following way:

~
 $T(X, x^3) = T(X) + T\% (X). X/H + T$
3
(X, X

3) such as:

h/2 ~
H2
T

(
/
~
X, X

3 = 0
= 0

.
- 2
);
X T X X
H
- H
3 (

,
/

3
2
)

If the temperature is indeed closely connected in the thickness one has, $\sim T = 0$.

Code_Aster treats three different thermoelastic situations, where characteristics thermoelastic E , depend only on the average temperature T in the thickness:

- the case where the material is thermoelastic isotropic homogeneous in the thickness;
- the case where the plate models an orthotropic grid (concrete reinforcing steels);
- the case where the behavior of the plate is deduced from a thermoelastic homogenisation, cf [bib7].

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For the elements of plate in thermoelasticity, the heating effects are taken into account by the intermediary of generalized efforts, out of membrane and inflection. Thus, in the case of a plate homogeneous, knowing the dilation coefficient, the generalized thermal efforts are defined in to leave the plane constraints in the thickness by:

+h/2

+h/2

NR ther =

C

ther

dx3 =

. C

réf

-

/2

- H

- H

$(T T) dx$

3

2

+ $h/2$

+ $h/2$

$M_{ther} =$

$X C$

$ther$

3

$dx^3 =$

$.x C$

$réf$

3

-

/2

- H

- H

$(T T) dx$

3

2

V_{ther}

= 0

Maybe in the homogeneous isotropic thermoelastic case in the thickness:

Eh

$NR_{ther} = . C$

..

-

H

$(T - T_{réf}) =$

\cdot
 $2 (T - T_{réf})$

;

1-
 H^2
 Eh^2
 $M_{ther} = C$

\cdot
 T

% = .

\cdot
 %

;

= .

0

12

(
 $12 \cdot 1 - 2) T$

V_{ther}

The thermal constraints of origin withdrawn from the usual mechanical constraints are calculated in three positions (sup., moy. and inf.) in the thickness:

$ther$

$\cdot E$
 =
 $2 ($
 $réf$
 $T - T$
 $+ T\% \cdot X/H$
 3
 $)$

1-

In the case deduced from the thermoelastic homogenisation, cf [bib7], the thermal efforts generalized are defined by the general relation, starting from the “correctors” of membrane, those of inflection, and that of dilation **udil**, like averages on representative elementary volume

(cell Z):

~

NR ther =

C

..

-

+ %

.

/

3

+

, 3

+

. **U**

;

(T Tréf T (X) Z H T (X X

)

dil

ij (

) Cijkl kl () Z

Z

~

M ther =

Z. C

.

3

.-

+ %

.

/

3

+

, 3

+

. **U**

;

$(T_{Tréf} T(X) Z H T(X X$

)
dil

ij (
) *Cijkl kl* () *Z*
Z
V ther

= 0

In this case when one limits oneself to the orthotropic situations without coupling inflection-membrane, one

~
neglect the role of $T(X, X$

3) on the corrector **udil**, and it is thus found that the thermal efforts which appear to the second member have as an expression:

$NR_{ther} = . H m$

.
-

;
 $(T_{Tréf})$
 $M_{ther} = . H F$
. T
%
; V_{ther}

= 0

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One cannot however go up with the complete three-dimensional constraints: it would be necessary of to know the “correctors” within the basic cell having been used for the determination of coefficients of homogenized behavior.

In the thermoelastoplastic situations, or for the hulls (elements of the family COQUE_3D), it is necessary to evaluate the three-dimensional constraints, of which constraints thermics, in each point of integration in the thickness.

Note:

To go up with the complete three-dimensional constraints is not immediate for the hulls multi-layer (laminated) because it is necessary to know layer by layer the state of stress; in elasticity, this one results from the state of deformation and the behavior on the level of each layer.

4.9.2 Chaining**thermomechanics**

For the resolution of chained thermomechanical problems, one must use for thermal calculation finite elements of thermal hull [R3.11.01] whose field of temperature is recovered like input datum of *Code_Aster* for mechanical calculation. It is necessary thus that there is compatibility between

thermal field given by the thermal hulls and that recovered by the mechanical plates. It the last is defined by the knowledge of 3 fields TEMP_SUP, TEMP and TEMP_INF given in skins lower, medium and higher of hull.

The table below indicates compatibilities between the elements of plate and the elements of hull thermics:

Modeling**Net****Finite element****to use****Net****Finite element****Modeling****THERMICS****with****MECHANICS****HULL**

QUAD4 THCOQU4

//////////

QUAD4

MEDKQU4

DKT

//////////

MEDSQU4

DST

//////////

MEQ4QU4

Q4

HULL

TRIA3

THCOTR3

//////////

TRIA3

MEDKTR3

DKT

//////////

MEDSTR3

DST

Note:

The nodes of the thermal elements of hulls and mechanical plates must to correspond. The grids will be identical.

The elements of thermal hulls surface are treated like plane elements by projection of the initial geometry on the level defined by the first 3 tops.

The chaining with definite multi-layer materials via order DEFI_COQU_MULT [U4.23.03] is not available for the moment.

The thermomechanical chaining is also possible if one knows by experimental measurements variation of the field of temperature in the thickness of the structure or certain parts of structure. In this case one works with a chart of temperature defined a priori; the field of temperature is not given any more by three values TEMP_INF, TEMP and TEMP_SUP of thermal calculation

obtained by EVOL_THER. It can be much richer and contain an arbitrary number of points of discretization in the thickness of the hull. Operator DEFI_NAPPE allows to create such profiles temperatures starting from the data provided by the user. These profiles are affected by order AFFE_CARTE (cf the case-test hsns100b). It will be noted that it is not necessary for calculation mechanics that the number of points of integration in the thickness is equal to the number of points of discretization of the field of temperature in the thickness. The field of temperature is automatically interpolated at the points of integration in the thickness of the elements of plates or of hulls.

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4.9.3 Case-test

The case-tests for the thermomechanical chaining enters of the thermal elements of hulls and of elements of plate are the hpla100e (elements DKT) and hpla100f (elements DKQ). It is about a cylinder heavy thermoelastic hollow in uniform rotation [V7.01.100] subjected to a phenomenon of dilation thermics where the fields of temperature are calculated with THER_LINEAIRE by a calculation stationary.

Z

IH

Re

Interior ray *IH = 19.5 mm*

External ray *Re = 20.5 mm*

Not F

R = 20.0 mm

Thickness

H = 1.0 mm

Height

L = 10.0 mm

R

Z

NR

K

Z

Q

J

P

D

C

y

H

M

+

L

X

With

B

R

F

Thermal dilation is worth: $T () - T_{ref} () = 05$

. $(T + T$

S

$I) + 2. (T + T$

S

$I) (R - R)/H$

with:

.

$T = 05$

. °C, $T = -05$

. °C, T

$= 0. °C$

S

I

ref.

.

$T = 01$

. °C, $T = 01$

. °C, T

$= 0. °C$

S

I

ref.

One tests the constraints, the efforts and bending moments in L and Mr. the results of reference are analytical. One obtains very good results whatever the type of element considered.

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5

Establishment of the elements of plate in Code_Aster

5.1 Description

:

These elements (of names MEDKTR3, MEDSTR3, MEDKQU4, MEDSQU4 and MEQ4QU4) rest on meshes TRIA3 and plane QUAD4. These elements are not exact with the nodes and it is necessary to net with several elements to obtain correct results.

5.2

Introduced use and developments:

These elements are used in the following way:

- AFFE_MODELE (MODELING: "DKT".) for the triangle and the quadrangle of the type DKT
- AFFE_MODELE (MODELING: "DST".) for the triangle and the quadrangle of the DST type
- AFFE_MODELE (MODELING: "Q4G".)

One calls upon routine INI079 for the position of the points of Hammer and Gauss on the surface of plate and the corresponding weights.

- AFFE_CARA_ELEM (HULL: (THICKNESS: "EP"

ANGL_REP: (" ")

COEF_RIGI_DRZ: "CTOR")

To make postprocessings (forced, generalized efforts,...) in a reference mark chosen by the user who is not the local reference mark of the element, one gives a direction of reference D defined by two angles nautical in the total reference mark. The projection of this direction of reference in the field of the fixed plate a X1 direction of reference. The normal in the plan into fixed one second and the vector product of both vectors previously definite makes it possible to define the local trihedron in which the efforts will be expressed

generalized and constraints. The user will have to take care that the selected reference axis is not found not parallel with the normal of certain elements of plate of the model. By defect this direction of reference is axis X of the total reference mark of definition of the grid.

Value CTOR corresponds to coefficient that the user can introduce for the treatment of the terms of rigidity and of mass according to normal rotation in the plan of the plate. This coefficient must be sufficiently

small not to disturb the energy balance of the element and not too small so that matrices of rigidity and of mass are invertible. A value of 105 is put by defect.

· ELAS: (E: NAKED Young: ALPHA:. RHO:.)

For a homogeneous isotropic thermoelastic behavior in the thickness one uses key word ELAS in DEFI_MATERIAU where the coefficients E are defined, Young modulus, Poisson's ratio, thermal dilation coefficient and RHO density.

· ELAS_ORTH (_FO): (

E_L: ygl. E_T: ygt. G_LT: glt. G_TZ: gtz. .NU_LT: nult.

ALPHA_L: L. ALPHA_T: T.)

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For an orthotropic thermoelastic behavior whose axes of orthotropism are L, T and Z with isotropy of axis L (fibres in the direction L coated by a matrix, for example) the seven should be given independent coefficients ygl, longitudinal modulus Young, ygt, transverse modulus Young, glt, modulus of rigidity in the plan LT, gtz, modulus of rigidity in plan TZ nult, coefficient of Poisson in plan LT and the dilation coefficients thermal L and T for thermal dilation longitudinal and transverse, respectively. **The orthotropic behavior is not available that associated the key word DEFI_COQU_MULT which makes it possible to define a multi-layer composite hull.**

For only one orthotropic material, one will thus use DEFI_COQU_MULT with only one layer. If one wish to use ELAS_ORTH with transverse shearing, it is necessarily necessary to employ DST modeling. If modeling DKT is used, transverse shearing is not taken into account.

· ELAS_COQUE (_FO): (

MEMB_L: C1111. MEMB_LT: C1122. MEMB_T: C2222. MEMB_G_LT: C1212.

FLEX_L: D1111. FLEX_LT: D1122. FLEX_T: D2222. FLEX_G_LT: D1212.

CISA_L: G11.... CISA_T: G22.... ALPHA:. RHO:.)

This behavior was added in DEFI_MATERIAU to take into account matrices of rigidity nonproportional in rigidity and inflection, obtained by homogenisation of a multi-layer material. The coefficients of the matrices of rigidity are then introduced with the hand by the user into the reference mark

user defined by key word ANGL_REP. The thickness given in AFFE_CARA_ELEM is only used with the density defined by RHO. ALPHA is thermal dilation. If one wishes to use ELAS_COQUE with transverse shearing, it is necessarily necessary to employ modeling DST. If modeling DKT is used, transverse shearing is not taken into account.

· DEFI_COQU_MULT: (LAYER: THICKNESS: "EP"

MATER: "material"

ORIENTATION: ("))

This key word makes it possible to define a multi-layer composite hull on the basis of the sub-base towards

roadbase starting from its characteristics sleeps by layer, thickness, type of material constitutive and orientation of fibres compared to a reference axis. The type of constitutive material is product by operator DEFI_MATERIAU under key word ELAS_ORTH. is the angle of the first direction of orthotropism (longitudinal direction or direction of fibres) in the tangent plan with the element by report/ratio

with the first direction of the reference mark of reference defined by ANGL_REP. By defect is null, if not it must be

provided in degrees and must lie between 90° and $+90^\circ$.

· AFFE_CHAR_MECA (DDL_IMPO: (

DX:. DY:. DZ:. DRX:. DRY:. DRZ:. DDL of plate in the reference mark total.

FORCE_COQUE: (FX:. FY:. FZ:. MX:. MY:. MZ:.) They are the efforts surface on elements of plate. These efforts can be given in the total reference mark or in the reference mark user defined by ANGL_REP.

· FORCE_NODALE: (FX:. FY:. FZ:. MX:. MY:. MZ:.) It is about efforts of hull in the total reference mark.

5.3

Calculation in linear elasticity:

The matrix of rigidity and the matrix of mass (respectively options RIGI_MECA and MASS_MECA) are integrated numerically. It is not checked if the mesh is plane or not. Calculation holds account owing to the fact that the terms corresponding to the DDL of plate are expressed in the local reference mark of

the element. A matrix of passage makes it possible to pass from the local DDL to the total DDL.

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Elementary calculations (CALC_ELEM) currently available correspond to the options:

- EPSI_ELNO_DEPL and SIGM_ELNO_DEPL which provide the strains and the stresses with the nodes in the reference mark user of the element in lower skin, with semi thickness and in higher skin of plate, the position being specified by the user. These values are stored in the following way: 6 components of strain or stresses:

- EPXX EPYY EPZZ EPXY EPXZ EPYZ or SIXX SIYY SIZZ SIXY SIXZ SIYZ

- DEGE_ELNO_DEPL: who gives the deformations generalized by `element to the nodes to leave displacements in the reference mark user: EXX, EYY, EXY, KXX, KYY, KXY, GAX, GAY.

- EFGE_ELNO_DEPL: who gives the efforts generalize by element with the nodes from displacements: NXX, NYY, NXY, MXX, MYY, MXY, QX, QY.

- SIEF_ELGA_DEPL: who gives the efforts generalize by element at the points of Gauss with to leave displacements: NXX, NYY, NXY, MXX, MYY, MXY, QX, QY.

- EPOT_ELEM_DEPL: who gives the elastic energy of deformation per element from displacements.

- ECIN_ELEM_DEPL: who gives the kinetic energy by element.

Finally one calculates also option FORC_NODA of calculation of the nodal forces for operator CALC_NO.

5.4

Plastic design

The matrix of rigidity is there too integrated numerically. One calls upon the option of calculation STAT_NON_LINE in which one defines in the level of the nonlinear behavior the number of layers to be used for numerical integration. All laws of plane constraints available in

Code_Aster can be used.

STAT_NON_LINE (...

COMP_INCR: (RELATION: "

COQUE_NCOU: "A NUMBER OF LAYERS")

....)

Elementary calculations (CALC_ELEM) currently available correspond to the options:

- EPSI_ELNO_DEPL which provides the deformations by element to the nodes in the reference mark user starting from displacements, in lower skin, with semi thickness and in higher skin of plate.

- SIGM_ELNO_COQU which makes it possible to obtain the stress field in the thickness by element with the nodes for a given layer and a position requested (in lower skin, in the medium or in higher skin of layer). These values are given in the reference mark user.
- SIEF_ELNO_ELGA which makes it possible to obtain the efforts generalized by element with the nodes in the reference mark user.
- VARI_ELNO_ELGA which calculates the field of internal variables and the constraints by element with the nodes for all the layers, in the local reference mark of the element.

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6 Conclusion

The finite elements of plate plans which we describe here are used in the structural analyses thin, in small displacements and deformations, of which the thickness report/ratio over characteristic length

is lower than 1/10. As these elements are plane, they do not take into account the curve of structures, and it is necessary to refine the grids if this one would be important.

It is elements for which strains and stresses in the plan of the element

vary linearly with the thickness of the plate. Moreover, the distortion associated with shearing

transverse is constant in the thickness of the element. Two types of elements of plate exist:

elements DKT, DKQ for which the transverse distortion is null and the elements DST, DSQ and Q4

for which it remains constant and nonnull in the thickness. One advises to use the second type

elements when the structure to be netted has a thickness report/ratio over characteristic length included/ understood

between 1/20 and 1/10 and first in the remainder of the cases. When the transverse distortion is not

null, the elements of DST plate, DSQ and Q4 do not satisfy the equilibrium conditions 3D and them

boundary conditions on the nullity of stresses shear transverse on the faces higher

and lower of plate, compatible with a constant transverse distortion in the thickness of

plate. It results from it thus that on the level from the behavior a coefficient from 5/6 for a plate

homogeneous the usual relation between the constraints and the distortion transverses corrects in order to ensure the equality enters energies of shearing of the model 3D and the model of plate to distortion constant. In this case, the arrow W has as an interpretation average transverse displacement in the thickness of the plate.

The non-linear behaviors in plane constraints are available for the elements of plate DKT and DKQ only. Indeed the rigorous taking into account of a constant transverse shearing not no one on the thickness and the determination of the correction associated on rigidity with shearing by report/ratio with a model satisfying the equilibrium conditions and the boundary conditions are not possible and thus the use of the DST elements, DSQ and rigorously impossible Q4 return in plasticity.

Elements corresponding to the machine elements exist in thermics; chainings thermomechanical are thus available except, for the moment, laminated materials.

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Appendix 1 orthotropic Plates

For an orthotropic material like that represented below, composed for example of fibres of direction L coated with a matrix, whose axes of orthotropism are L, T and Z with isotropy of axis L, the expression for

matrices **H** and **H** in the reference mark of orthotropism previously definite becomes:

H
H
0
LL
LT

GLZ
0

HL = *H*
H
LT
TT

0 and **HL** =

0
GTZ

0
0
GLT
E
E
E
H
L
=

; *H*
T
L
LL
TT =
G
=

1-
LZ
LTTL
1 - *LTTL*
2 1
(+ *LZ*)

with
and
.
E

E

E

H

T LT

L TL

T

LT =

=

G

=

1-

TZ

LTTL

1 - LTTL

2 1

(+ TZ)

The knowledge of the five independent coefficients *E*, *E*, *G*

, *G*

L

T

LT

TZ, *LT* is sufficient to determine

coefficients of the matrices **H** and **H** since:

AND

LT

TL =

and *G*

= G.

E

LZ

LT

L

If one indicates by the angle between the reference mark of orthotropism and the principal axis of the

reference mark defined by the user in

means of ANGL_REP one establishes that:

$$\mathbf{H} = \mathbf{T} \mathbf{H} \mathbf{T}$$

T

1

L 1 and **H**

$$= \mathbf{T} \mathbf{H} \mathbf{T}$$

2

L
2
*C*₂
*S*₂
*C*_S

C
S
with: **T**
₂
₂
1 = S
C
- *C*_S and **T**₂ =

= cos, = sin
X L

-
where *C*
S
and = (,) like
S C

- *C*_S *C*_S *C*₂ - *S*₂
₂
₂

indicated on the figure below.

Z
L

X
T

In the case of forced initial of thermal origin, we have moreover:

L T

T
HT = - T H
1

L T T

0

where L and T are the dilation coefficients thermal in the directions L and T and T the variation of temperature.

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Appendix 2 Factors of transverse correction of shearing for orthotropic or laminated plates

The **Hct** matrix is defined so that the surface density of transverse energy of shearing obtained in the case of the three-dimensional distribution of the constraints resulting from the resolution of balance is equal to that

model of plate based on the assumptions of Reissner, for a behavior in pure bending. One must thus to find **Hct** such as:

1 +h/2

1

1

+h/2

xz

H-1

-1

= **TH T**

ct

= **Hct** with =

and **T** =

$$dz = \mathbf{H}$$

.
2
2
2

ct

yz
- h/2
- h/2

To obtain **Hct** one uses the distribution of following Z obtained starting from the resolution of the equations of balance 3D without external couples:

$$Z$$

$$Z$$

$$= - ($$

with $xz =$
 $= 0$ for $z = \pm h/2$.

$$, +$$

$$) D;$$

$$,$$

$$= - (, +$$

$$) D$$

$$xz$$

$$xx X xy y yz$$

$$xy X yy, y$$

$$yz$$

$$- h/2$$

$$- h/2$$

If there is no coupling membrane inflection (symmetry compared to $z=0$), constraints in plan of element xx, yy, xy in the case of have as an expression a behavior of pure inflection:

$$= zA (Z) \mathbf{M} \text{ with } \mathbf{A} Z = \mathbf{H} Z \mathbf{H} \mathbf{1}$$

$$()$$

$$() - \mathbf{F}.$$

If (**H Z**) and **Hf** do not depend on X and y one can determine **Hct**. Indeed:

$$Mxx, X - Mxy, y$$

$$Tx Mxx, X + Mxy, y$$

$$M xy, X - M$$

yy, y
 $(Z) = \mathbf{D} (Z) \mathbf{T}$

1
 $+ \mathbf{D} (Z)$

2
where $\mathbf{T} =$
 $=$
and $=$
 T

M
 y
 $Mxy, X + M yy, y$
 yy, X

$M xx, y$

like:

Z
With
With
With
With

11 +
33
13 +
32
 $\mathbf{D} = -$

$D,$
1
2 A
With
With
With
 $h/2$
31 +
23

22 +

33

-

Z

With

With

With

With

With

With

11 -

33

13 -

2

2

32

12

31

D = -

D.

2

2 A

With

With

With

With

With

h/2

31 -

23

33 -

2

2

22

32

21

-

+h/2

C =

DTH 1

- **D**

11

1 1dz;

- $h/2$

1 + $h/2$

1

+ $h/2$

1

T C

C

11

12

T

It results from it that

-

H

T

-1

=

T

with: **C =**

D H D

$dz;$

2

2 **C**

C

12

1

2

12

22

- $h/2$

- $h/2$

+ $h/2$

C

=

DTH 1

- **D**

22

2 2dz

- $h/2$

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1 +h/2

1

As in addition

-1

-

H

1

1

-

=

TH T one proposes to take **H = C** to satisfy them as well as possible

2

2

ct

ct

11

- h/2

two equations whatever *T* and.

+h/2

By comparing **Hct** thus calculated with **H**

H

ct = dz one reveals the coefficients of correction of

- h/2

following transverse shearing: $K = H11/H 11$; K

$= H12/H 12$; $K = H22/H 22$

1

ct

ct

12

ct

ct

2

ct

ct.

For a homogeneous, isotropic or anisotropic plate, one finds as follows: $\mathbf{Hct} = kh \mathbf{H}$ with $k=5/6$.

Note:

This method is valid only when the composite plate is symmetrical compared to $z=0$.

· For a multi-layer material, one establishes that:

NR

i-1

I

H

1

1

1

C

= I (H AT

2

2

pp

- Z AT) H-1

I

(

H With

pp

- Z A

11

p

I

p

I

I) +

4

2

2

i=1

p=1

p=1

i-1

I

1

1
3
3
-
1
1
2
2
-
($z_{i+1} - Z$) [ATH 1
I
(

H With
p p
- Z A
I
) + (H AT
p p
- Z AT) H 1A
I
p
I
p
I
I

I]
24
2
2
 $p=1$
 $p=1$
1
+
(z_5
5
T
-1
 $i+1 - z_i$) A H
With
80
I

I

1

WITH + A

WITH + A

11

33

13

32

where: $H = Z + 1 - Z, = (Z + 1 + Z$

I

I

I

I

I

I) and **A**

for

2

I represents the matrix $A + A$

WITH + A

31

23

22

33

layer I.

· Validity of the choice **H**

= **C-1**

ct

11 can be examined a posteriori when one has an estimate of

solution (fields of displacements and plane constraints, in particular). One can then estimate the variation enters the two estimates on energy. A step of calculation in two stages for multi-layer plates and hulls (with **Hct** diagonal and two coefficients k1 and k2) was besides developed by Noor and Burton [bib11] [bib12].

· Danslecasde an isotropic or anisotropic homogeneous plate the equality between two energies is satisfied in a strict sense since **D2** = 0. The choice makes above is then valid and no examination has posteriori is not necessary.

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R3.07.04 document

Finite elements of voluminal hulls

Summary:

With an aim of supplementing the library of finite elements of plate plans [R3.07.03] currently available

in Code_Aster (DKT, DST, Q4G...), one proposes to introduce two finite elements of voluminal hull or three-dimensional [bib1]. This new modeling COQUE_3D [U1.12.03] makes it possible to carry out calculations of structures hull of an unspecified form with a better approximation of the geometry and kinematics.

One will limit oneself to the framework of linear kinematics. One thus remains in small displacements and small deformations. No restriction is made on the type of behavior in plane constraints.

The two elements which are introduced are the quadratic element quadrangle Hétérosis with 9 nodes and sound triangular equivalent with 7 nodes. The formulation of the continuous problem is done in Cartesian co-ordinates, it who allows to avoid explicit calculations of the curves. These two elements have as a correspondent the element linear of hull with 3 nodes presented in the document [R3.07.02].

These two new elements are validated on existing case-tests of plate, and on three new cases tests of hull developed in the documentation of validation and whose principal conclusions are presented briefly here.

This note also presents in appendix how to take into account the anisotropy of materials.

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1 Introduction

One introduces into Code_Aster two finite elements of hull voluminal with shearing transverse (the quadrangle with 9 nodes MEC3QU9H and the triangle with 7 nodes MEC3TR7H) in calculation of

structures hull of an unspecified form. To represent this type of structures, one used until present with Code_Aster of the elements of plate with plane facets which induced inflections parasites and of the too restrictive hulls of revolution on the type of structure [R3.07.02]. development was carried out for isotropic materials with linear kinematics. They cannot thus to be used that within the framework of small displacements and small deformations. This formulation can be extended to anisotropic materials [Appendix 1] and to nonlinear kinematics

[R3.07.05].

For the resolution of chained thermomechanical problems, one must use the elements before stop thermal hull with 7 and 9 nodes described in [R3.11.01].

One develops hereafter the mechanical continuous problem by describing the kinematics of hull of the type

Hencky-Mindlin-Naghdi (assumption of the cross-sections or plane) supplemented by a distortion transverse and the thermoelastoplastic law of behavior. Thanks to a parameter of penalization one can pass from a theory with shearing to a theory without shearing. One presents then the selected finite elements which are isoparametric quadratic elements making it possible to have one fine representation of a curved geometry and good estimates of the constraints. The interpolation and the method of integration are also described.

One validates finally the development on some cases of test.

The nonlinear kinematics of these hulls is treated in the reference material [R3.07.05].

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2 Formulation

2.1

Geometry of the hull

For the elements of voluminal hull one defines a surface of reference, or surfaces average, left (of curvilinear co-ordinates 1 2 for example) and a thickness (

H 1 2)

measured according to the normal on the average surface. This thickness must be small compared to other dimensions (extensions, radii of curvature) of the structure to be modelled. The figure [Figure 2.1- has] below illustrates our matter.

Solid 3D

Z

H

Y

B

X

L

R1

R2

Thickness $H < L, B, R1, R2$

2

N, 3

H

E, X

3

3

1

E, X

2

2

O

E, X

1

1

Appear 2.1-a

*The position of the points of the hull is given by the curvilinear co-ordinates (1 2) of surface average and rise 3 compared to this surface. (O, **ek**) is the total Cartesian reference mark, associated axes (xk).*

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2.1.1 Geometrical description of average surface

Local natural base and bases Cartesian local

That is to say P an unspecified point of the average surface of reference, one a:

$$CO_p = x_0^k (1, 2) e_k$$

One defines the vectors has natural local base of the tangent plan out of P with, attached to P by:

$$CO_p \text{ has } = \dots = CO_p$$

and one defines unit normal N by:

$$a_1 \text{ has } N = \dots a_1 a_2$$

3 is the variable of position in the thickness associated with N.

(has, has, has
1

2

3) constitutes the natural base attached to P.

The curvilinear frame of reference (1 2) not being inevitably orthogonal, the base (A) is not thus not inevitably orthogonal (and even orthonormée). A local base is thus defined orthonormée T K as follows:

has
 T
 1
 $=$
 $, T = N T$
 $T = N$
 1

has
 2
 1
 3
 1

and one notes (S, S
 1
 2) the frame of reference associated to (T, T
 1
 2) .

Calculation of the tensor of curve

The tensor of curve is related to the variation of the normal on. It is defined by its components mixed:

$N = - C.A.$
 ,

or by its components covariantes: C
 $= - .n$ has, $= n.a$

. This tensor is symmetrical
 since has
 $= has$

. *Its trace tr C is the average curve and its determinant the curve Gaussian.*

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2.1.2 Description of the geometry of the hull

That is to say Q an unspecified point of, volume of the hull thickness H considered constant, one a:

$$OQ + COp + PQ = COp + 3 H N$$

2

where 3 [- 1,]

1 .

3

1, 2 ,

H

2 constitutes a curvilinear frame of reference of.

One can also write OQ according to his components (x_k) in the total base (e_k) :

$$OQ = X E$$

$$K$$

$$K$$

Local natural base, bases orthonormée local and tensor metric

As for P , one defines the natural base of space $3D$ (g_k) attached to Q by:

$$OQ$$

$$H$$

$$g1 G$$

$$G =$$

$$= + N \text{ has}$$

$$G$$

$$2$$

$$3$$

$$,$$

$$,$$

$$3 =$$

$$= N$$

$$2$$

$$g1 g2$$

As (g_k) is not inevitably orthogonal, one defines a local base orthonormée (T_k) like follows:

$$G$$

$$T$$

$$1$$

$$=$$

$$, T = N T$$

$$, T = N$$

$$1$$

$$G$$

$$2$$

1
3
1

and one notes (\mathbf{xk}) the frame of reference associated with (\mathbf{Tk}) .

One will call (\mathbf{Tk}) the local orthonormée base, and (\mathbf{xk}) the co-ordinates in this base orthonormée local.

By definition, one a:

$$\begin{aligned}
 \mathbf{OQ} &= \mathbf{XJ} \\
 \mathbf{T} &= \\
 &= \\
 \mathbf{E} &= \\
 \mathbf{J} &= \mathbf{T E} \\
 \mathbf{K} &= \\
 &= \\
 \mathbf{J} &= \\
 \mathbf{K} &= \\
 \mathbf{J} &=
 \end{aligned}$$

xk
 xk

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 $X J$ J

with

 T components of $(\mathbf{T}k)$ in the total base $(\mathbf{E} J)$. (They are also them X $\sim = K$ K components of the matrix of passage of $(\mathbf{T}k)$ with $(\mathbf{E} J)$ since the matrix of passage is J orthogonal. Thus if $\mathbf{T} = \mathbf{T} \mathbf{E}$ K K K one has also \mathbf{E} $= \mathbf{T} \mathbf{T}$ K J J).

One defines the metric tensor \mathbf{G} associated with Q by his components deduced from the scalar products vectors of the local orthonormée base:

 $G_{ij} = I$ $\mathbf{T} \cdot \mathbf{T}_j$

This tensor \mathbf{G} is worth the \mathbf{Id} identity.

2.1.3 Notice

The figures [Figure 2.1.3-a] and [Figure 2.1.3-b] illustrate the geometrical magnitudes mentioned above.

2

 t^2 N $a_2 a_1, T_1$ P

1

Appear 2.1.3-a

T1
3
T1

N
1
Q
P

Appear 2.1.3-b

It should be noted that two local orthonormées bases, that associated average surface (tk) and the other with the volume of the hull (Tk) are confused only when the curve is null. In it cases the elements of hull are comparable to elements of plate

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2.2

Theory of the plates and the hulls

These elements are based on the theory of the plates and the hulls according to which:

2.2.1 Kinematics

2.2.1.1 Field of displacement

The cross-sections which are the sections perpendicular to average surface remain right; material points located on a normal at not deformed average surface remain on a line in the deformed configuration. It results from this approach that the fields of displacement vary linearly in the thickness of the hull.

If one notes Q' , the position of Q after deformation, one a:

$$\mathbf{OQ}' = \mathbf{OQ} + \mathbf{QQ}' = \mathbf{OQ} + \mathbf{U}(Q)$$

where the field of displacement chosen, corresponding to the kinematics of Hencky-Mindlin, is written:

$$\mathbf{U}(Q) = (\mathbf{U}(P) + 3\mathbf{H}(P) \cdot \mathbf{n} = 0)$$

where $\mathbf{U}(P)$ and $\mathbf{H}(P)$ are respectively the vector displacement and the vector rotation of P , projection of Q on the average surface of the hull. The fact that $\mathbf{H}(P) \cdot \mathbf{n} = 0$ indicates that one does not take in count in this kinematics rotations of the hull around its normal.

Notation:

One notes \sim the quantities expressed in the local Cartesian bases (\mathbf{tk}) or (\mathbf{Tk}) for the points P and Q respectively. It results from it that:

· the vector three-dimensional displacement \mathbf{U} can be written $\mathbf{U} = \sim$

$\mathbf{U} \mathbf{T}$

$\mathbf{K} \mathbf{K}$ or $\mathbf{U} = \mathbf{U} \cdot \mathbf{E}$.

$\mathbf{K} \mathbf{K}$, where it is expressed respectively in its local orthonormée base or the base Cartesian total,

· the vector displacement of average surface \mathbf{U} can be written $\mathbf{U} = \sim \mathbf{u} \mathbf{T}$

$\mathbf{K} \mathbf{K}$ or

$\mathbf{U} = \mathbf{U} \cdot \mathbf{E}$.

\mathbf{K}

\mathbf{K} according to whether it is expressed in its local orthonormée base or in the base Cartesian total,

· the vector rotation of average surface is written $= \sim$

\mathbf{T} in its orthonormée base

local. being the rotation of normal \mathbf{N} (on average surface), one also writes $= \mathbf{N}$

with, vector rotation of average surface, such as $= \sim$

\mathbf{T} . The equivalence of both

\sim

\sim

\sim

~
formulations shows that $1 =$,
2
2 =
- 1.

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2.2.1.2 Expression of the three-dimensional deformations

The tensor of deformation is calculated in the local orthonormée Cartesian base (Tk). It is defined like the half-difference of the metric tensors associated the local orthonormées bases afterwards and before deformation. The metric tensor associated this base in the not-deformed state is simply OQ' the Id identity, while the metric tensor of the deformed state is $G_{ij} = T'.T$
I
'J with You K = ~

.
xk

The components of the tensor of deformation in (Tk) are thus given by:

~
~
U
U

~

1

=

~ + ~

2

X

X

~

~

~

1 U U

3

3 =

~ + ~

2 x3

X

The equations above are linear relations deformation-displacements. Variables of

~

displacement are components the U.K.

The components ~

kl of the tensor ~ can be also expressed according to the components in

U p

total reference mark. Indeed as in the total reference mark

xm

= I

J

I

J

K

L

~

K

L

ije E

= ijT T T T

KL

= klT T one thus deduces from it immediately that:

~

I

J

kl = ijT T

KL. (ek) and (Tk) are the bases contravariantes associated (ek) and (Tk)

respectively such as: I.E.(internal excitation) .e

I

J = ij

and T. T

E

T

J = ij. As the bases (K) and (K) are

orthonormées, their associated bases contravariantes are confused with themselves. Thus of

J

J

even manner that one had T = T E

K

J

K

K

J one finds T

= T E

K

.

If one notes T = T I E

K

~

K

L

~

K I T then T T: = T

kl

T =. For the continuation one indicates by ~

the form of the tensor of the deformations in the local orthonormé reference mark and by the expression of

even tensor in the total reference mark. The relation of passage of the one with the other is given above

in
term of tensors.
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Note:

J

The Tk terms contain the terms of curve of the hull.

One notes in the relations deformation-displacements that the component ~

33 is not

determined by kinematics. This is to be associated the assumption nullity constraints
transverse normals ~

33 = 0 justified by the behavior of the hulls.

In the literature (see for example [bib3]), the modeling of the hulls by the approach based on
curvilinear components ~

U of displacement reveals explicitly the sizes of

K

curve on the level of the form of the tensor of deformation [bib5]. Like, in general,

geometry of the hull is not known explicitly, one must thus determine

*numerically the geometrical characteristics that are the vectors **has**, **G***

,... and them

curves C. With the finite element method it is necessary to derive them twice

functions of form (see page 20 of [bib5] and [R3.07.02]) to calculate C. This can return

their vague calculation according to the family of the functions of selected form. The made error depends on these last (linear, quadratic, cubic polynomials....) and becomes independent of refinement of the grid. A formulation utilizing derivation first functions of form (calculation of slopes) does not present this disadvantage. Thus the consequent error with calculations terms of curve in a formulation based on the curvilinear approach does not decrease with the refinement of the grid whereas for the formulation described above it becomes small by increasing the number of finite elements. Within sight of the preceding observations, approach known as curvilinear was not followed.

2.2.2 Law of behavior

The behavior of the hulls is a behavior 3D in “plane constraints”. It binds the components constraints and deformations, in the form of vectors, in the local orthonormée base. transverse constraint ~ 33 is null bus regarded as negligible compared to the others components of the tensor of the constraints (assumption of the plane constraints). The law of behavior most general is written then as follows:

~
~
~

HT
11
11 - 11

~
~
~

HT
22
22 - 22

~
~

12
= (
C, μ)

~
12

~

~
13

1

~

~

23

2

~

where (C, μ) is the local matrix of behavior in plane constraints and μ represents the unit variables intern when the behavior is nonlinear.

For behaviors where the transverse distortions are uncoupled from the deformations from membrane and of inflection, (C, μ) is put in the form:

~

~ **H**

0

C =

~

0 H

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~
~
where $H(\mu)$ is a matrix of behavior of membrane-inflection 3x3 and $H(\cdot)$
 μ a matrix of
transverse behavior of distortion 2x2. The two phenomena being uncoupled one can too
to write the behavior in the form:

~
~
MF ~
MF
~ = (
C,) ~ with:

μ

~
~
~

HT
11
11 -
11

~
~
~
~ ~

13

1

HT

~

~

~

$$MF = 22 = H(\mu) \sim$$

~

$$22 - 22 = H(\mu) \sim$$

$$MF \text{ and } \sim = \sim = H(,)$$

$$\sim = H(,)$$

~

~

μ

23

μ

2

12

12

One will remain from now on within the framework of this assumption.

For an isotropic homogeneous linear behavior elastic, one has as follows:

1 v

0

0

0

v 1

0

0

0

1-**v****~****E****0 0****0****0****C =****2****2****K (1 - v)****1-****v 0 0 0****0****2****K (1 - v)****0 0****0****0****2**

where K is factor of transverse correction of shearing whose significance is given in reference material of the elements of plate [R3.07.03], and [bib4] for more details. Its coefficient is worth 5/6 for a theory of the Reissner type and 1 within the framework of the theory of Hencky-Mindlin. Lastly, if one chooses K very large, one brings back oneself to a theory of the type Coils-Kirchhoff.

One neutralizes the transverse distortion by penalization of associated energy by taking $K = 106 h/R$ (H being the thickness of the hull and R its average radius of curvature).

Always in the isotropic case, the two only nonnull components of ~

HT are ~th

II for i=1,2,

such as:

~

HT

réf

II

= (T - T

)

where is the thermal dilation coefficient and T T réf

-

the difference in supposed temperature

known.

Note:

One does not describe the variation thickness nor that of the transverse deformation ~

33 that one

can however calculate by using the preceding assumption of plane constraints. In addition

no restriction is made on the type of behavior in plane constraints which one can

to represent.

Same manner as T T: = ~

one can deduce some ():

MF =

:

~

T

T

Tmf = MF and

() :

=

:

~

T T

T =, which makes it possible to find ~mf and ~ starting from the tensor of deformations in the total reference mark.

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3

Principle of virtual work

3.1

Work of deformation

In 3D the expression of the work of deformation is written:

$h/2$

$h/2$

$h/2$

~

W

=

(~ ~

D

) V

=

(~ ~

C ~ D

) V

=

(Pr PsC P p Pq

D

) V

def

ij ij
ij ijkl kl
rs I K ijkl K L pq
S - h/2
S - h/2
S - h/2

h/2
h/2
=
(C
D
) V

=
(D
) V
rs rspq pq
ij ij
S - h/2
S - h/2

It is checked that this expression is invariant compared to the base in which the tensors are expressed. One chooses for the continuation of this document all to express in the local base (Tk) in knowing that one passes from the local tensor of behavior to the total tensor of behavior by

P
Q
relation C
=
~
P R P Sc
P P
rspq
I
K
ijkl K
L.

The general expression of the work of deformation 3D for the element of hull is worth:

h/2
h/2
h/2
h/2

$$W = \int (\dots) V$$

$$= \int (\dots) V$$

$$= \int (\dots) V$$

$$+ \int (\dots) V$$

def
MF
MF

$$S - h/2$$

$$S - h/2$$

$$S - h/2$$

$$S - h/2$$

where *S* is average surface and the position in the thickness of the hull varies between *h/2* and *+h/2*. It in the expression of the work of deformation a contribution of deformation appears in membrane-inflexion and a contribution of transverse shearing strain.

3.1.1 Energy interns elastic hull

It is expressed in the following way:

- 1
- E
- 2

2
2
2
2
int =
[
(~
~
~ ~
+ 22 + 2)
11 22 + G (~
12 + K (~
~
1 +))]**FD**
2
1 - 2 11
2

S
E
where K is the factor of correction in transverse shearing defined in paragraph 2 and $G = 2 1 (+.$
)
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3.1.2 Expression of the resulting efforts

One notes:

NR

~

M

~

11 +h/2 11

11 +h/2 11

T +h/2 ~

1

13

NR = NR

~

~

22 = 22 dz; M = M 22 = 22 z dz; T =

=

dz

T

~

2

.

23

/

~

NR

/2

~

- H 2

- h/2

12

H

12

-

M12

12

NR, NR, NR

11

22

12 are the generalized efforts of membrane (in N/m);**M, M, M**

11

22

12 are the generalized efforts of inflection or moments (in NR);**T, T**

1

2, are the generalized efforts of shearing or sharp efforts (in N/m);**The expression of the resulting efforts that one gives here is an approximate expression which does not hold****count curve of the hull (cf p.316 of [bib3]). The error made on these efforts is then in H^2/R where $1/R$ is the average curve. When the hull becomes plane, expressions given above are exact and the significance of the resulting efforts can be found in [R3.07.03].****We will not develop more this aspect in addition documented well in [bib3] because the theory of hull used here does not rest on a resulting generalized deformations formulation/efforts but on three-dimensional/forced a deformations formulation.**

3.2

Work of the forces and couples external**The work of the forces being exerted on the voluminal hull is expressed in the following way:****+h/2****+h/2****W****F. D****U V****F. D****U.S.****F. D****U zds****ext. = v****+ S****+ C****S - h/2****S****C - h/2**

where F, F, F

v
 S

C are the voluminal, surface efforts and of contour being exerted on the hull, respectively. C is the part of the contour of the hull on which the efforts of contour FC are applied.

has) Loads given in the total reference mark:

With the kinematics of [§2.2.1], one determines as follows:

\sim
 \sim

W
 $(F U$

$+ C) dS + (U$

$+) ds = (F U$

$+ C (T$
 $2 I - T))dS$

ext. =

II
 II
 II
 II
 II
 I
 I
 $I 2i$

S
 C
 S

$+$
 \sim
 \sim
 \sim
 \sim
 $(U$

$+ (T$
 $2 I - T))ds$

I_2
 $= (F U$
 $+ C (T$
 $2 I - T)) dS$
 I_2
 $+ (U +) ds$
 $I I$
 I
 I
 I
 $I I$
 I
 I
 I
 C
 S
 C

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· *where are present on the hull:*

F, F, F
1
2

3 :

surface forces acting along the axes of the reference mark

Cartesian total

+h/2

F =

. dz

I

F E + F .e

v

I

S

I

where I.E.(internal excitation) are the vectors of the total Cartesian base.

- h/2

C, C, C

1

2

3 :

surface couples acting around the axes of the reference mark

total.

+h/2

H

C =

Z

. dz

I

F E ± F .e

v

I

S

I where E

2

I are the vectors of the total Cartesian base.

- h/2

· and where are present on the contour of the hull:

1,2,3 :

linear forces acting along the axes of the reference mark

Cartesian total.

+h/2

where E

I =
. dz
F E
C
I

I are the vectors of the total Cartesian base.

- h/2
1, 2 , 3 :
linear couples acting around the axes of the total reference mark.
+h/2

where E

I =
Z. dz
F E
C
I

I are the vectors of the total Cartesian base.

- h/2

Note:

One also notes and the linear distributions of force and moment applied to contour of the finite element.

b) Loads given in the local reference mark:

One has then:

3
~
~
~
3 ~
~
~

Wext = (F T u.a. ~

C) dS
(

T U

) ds

II + I

I +

2

2

+

II +

I

I +

2

2

=

S

i=1

C

i=1

3

3

(~

~

~

~

~

~

FTU

C

~

C

~) dS

(

T U

) ds

II + I 2 -

2 I

+

II +

1 2 -
2 1
S
i=1
C
i=1

~ ~ ~

Forms of F, F, F

1
2
3 and ~, ~, ~
C C C

1 2
3 are the analogues of the expressions obtained for
F, F, F

1
2
3 and C, C, C

1
2
3 by replacing I.E.(internal excitation) by Ti.

Note:

*For the couple C, the contribution ~
c3 associated N is null in theory of hull.*

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3.3

Work of the inertias

Work due to the quantities of acceleration is written:

..

$$W_{ac} = OQ'.OQ' FD$$

where is the density.

..
It is supposed that OQ' , the vector of acceleration of the Q' point are following form:

..

$$OQ' = \&$$

$$U.E.$$

$$0$$

$$K$$

$$K + [X E$$

$$K$$

$$K]$$

where one neglected the forces of Coriolis and the correction of metric in the thickness.

$$D 2U$$
One notes &

$$U$$

$$K$$

$$K =$$

O, E (by $dt 2$, and is the uniform vector of rotation of the total reference mark (K) report/ratio with a Galiléen reference mark which in the same beginning O as the total reference mark).

One expresses in the total base (ek):

$$= kek$$

For virtual displacement OQ' , one a:

$$OQ' = U.E.$$

K
 K

Work due to the quantities of acceleration becomes then:

$$W_{ac} = U \int_0^t \ddot{x} dt = U \dot{x} = U v$$

$$= \int_0^t U \ddot{x} dt = \int_0^t U \dot{v} dt = \int_0^t U dv$$

$$= \int_0^t U dv = U v = U \dot{x}$$

with:

$$W_{ac} = U \dot{x}$$

$$= \int_0^t U \dot{v} dt = \int_0^t U dv$$

and:

$$W_{ac} = U \dot{x}$$

$$= \int_0^t U \dot{v} dt = \int_0^t U dv$$

3.4 Principle of virtual work

For a static loading, he is written in the following way: $W_{ext} = \sum W_{ext}$

def where W_{ext} is the sum of various elementary work, corresponding to the various loadings.

In harmonic dynamics (calculations of clean modes), the principle of virtual work

give: W
+ W ac
ext.
mass = 0
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4
Numerical discretization of the variational formulation
exit of the principle of virtual work

4.1 Introduction

This chapter is devoted to the discretization of the various terms of energy introduced into the chapter precedent. The choice of framework HENCKY-MINDLIN-NAGHDI to describe the kinematics of hull, presented at the paragraph [§2] led to expressions of the deformations where the derivative are limited with order 1, contrary to the model of LOVE-KIRCHHOFF. One can thus use a finite element of a nature limited while ensuring conformity (see p.110 [bib7]).

The degrees of freedom are 3 displacements in the total reference mark and 2 rotations in local reference mark.

The selected elements are isoparametric quadrangles or triangles. The quadrangle is represented below. The quadrangles give the best results (see p.202 [bib8]). better choice consists in taking for these elements of the quadratic functions of interpolation (see p.224 of [bib8]) in order to correctly modelling the effects of membrane, inflection and shearing.

According to the results based on many case-tests of the literature, the best alternative is it quadratic isoparametric quadrangle, which makes it possible to have a fine representation of a geometry curve and of good estimates of the constraints. One chooses among the elements with functions quadratic the element hétérosis (Q9H) whose displacements are approached by the functions of interpolation of the Sérendip element and rotations by the functions of the element of Lagrange (cf Annexe3). This choice is justified hereafter.

2
2
3
4
=
7
3
1
3
3
7
3
8
4
= -
1
P
1
1
1
6
8
6
1
5
2
5
2 = 1
-
1
2

*Appear 4.1-a: Representations of the isoparametric quadrangle
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The figure [Figure 4.1-b] summarizes the three families of elements previously named.

Sérendip element**Element of Lagrange Hétérosis Element****~****U,****K****~**

Appear 4.1-b: Families of finite elements for the isoparametric quadrangle

Risks of bloquage or locking of membrane or shearing appear when the thickness of the hull becomes small compared to its radius of curvature and that functions of interpolation are of a too low nature. To solve them a selective numerical integration is used [bib6]. For certain types of boundary conditions (embedding) with the Sérendip element it locking persists in spite of selective integration. Moreover, for the element of Lagrange, this type of integration leads to singularities in the matrix of rigidity. The element Hétérosis Q9H with selective integration does not encounter the problems mentioned and seems being more powerful for the modeling of the very thin hulls (see p.224 [bib8]). It should be noted that this element has a mode of deformation without associated energy if it is used only. This mode disappears when one uses more than two elements [bib7].

For the elements triangle, the element Hétérosis T7H is essential for the same reasons but proves definitely less powerful (see paragraph 5 concerning the validation).

One decides to carry out all calculations of discretization in the total Cartesian base.

4.2

Discretization of the geometrical terms

The co-ordinates x_0k of a point P of average surface are interpolated by the functions of form in the following way:

N_{b1}

$x_0 = \sum_{I=1}^{N_{b1}} N_I(x_0) x_{0I}$

K

I

x_{0I}

$i=1$

()

where the N_{b1} number and functions of form N_I

0

I

depend on the type of element chosen, and x_{0I} is them co-ordinates with node I of the element.

The vectors covariants has (attached to the point P) are then given by:

$N_{b1} () 1$

= Nor $x_0 E$ has

x_{0I}

K

$i=1$

J

The calculation of the T_k vectors is avoided because the T_k components contain the sizes of curve whose calculation is often vague C_0

Mrs. it was shown in the paragraph [§2.2.1].

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In order to avoid the presence of the terms of curve, one writes:

Nb1

$N = NR () 1 N$

I

I

i=1

where nor is the normal vector with the nodes of the element.

4.3

Discretization of the field of displacement

One adopts the following writing for displacement at the point Q:

Nb1

Nb2

$U = NR () 1 U.E. + 3 NR (2)$

~

~

H

I

ik

I

I (T

i2 I -

T

K

I

$i1 i2)$

2

 $i=1$ $i=1$

(2)

where T are evaluated with the nodes, and where it is observed that the functions of interpolation N_i and them

~

N_{b2} for rotations numbers are a priori different from those used for displacements the U.K.

By expressing T_i according to their components in the total Cartesian base, one obtains:

 N_{b1} N_{b2}

$$U = NR () 1 U.E. + 3 NR (2)$$

~

~

 H I $I (T$ $i2 I K$ $1 + T$ I $ik K$ $i1 i2k) ek$

2

 $i=1$ $i=1$

One calculates then the various elementary terms, in order to obtain the complete discretized formulation.

In the continuation one uses the convention of summation of Einstein, while having with the spirit that the number

~

interpolations is N_{b1} for x_0, N, U

 K

K , and N_{b2} for, T .

4.3.1 Element Hétérosis Q9H

With this element, the number of interpolations for the geometry (x_{0k}, N) and displacements the U.K.

are

Nb1=8 (nodes tops and mediums on the sides), while the number of interpolations for T and

~

rotations is Nb2=9 (nodes tops and mediums on the sides + barycentre). The number of degrees of freedom total of the element is thus Nddle=3x8+2x9=42.

Functions of interpolation NR (1)

(2)

I

and Ni respectively for the geometry and displacements, and for rotations, can be found for example in [bib2] and are quoted in appendix 2.

The elementary vector of displacement can be put in the following form:

~

qe = (

~ ~

~ ~

~

~

U, U, U, ..., U, U, U, ...,

11

12

13

11

12

i1

i2

i3

i1

i2

91

9

2)

I =,

1 8

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4.3.2 Element triangle T7H

With this element Nb1=6 (nodes tops and mediums on the sides) and Nb2=7 (nodes tops and mediums sides + barycentre). The number of degrees of freedom total of the element is Nddle=3x6+2x7=32.

()
1
The 6 functions of interpolation Nor which are traditional can be found in [bib2] and are
(2)
quoted in appendix 4. On the other hand the 7 Ni are it much less and their expressions are
data in Appendix 3.

The elementary vector of displacement can be put in the following form:

~
qe = (
~ ~
~ ~
~
~
U, U, U, ..., U, U, U, ...,
11
12
13
11
12
i1
i2
i3
i1
i2

71
7
2)
I =,
1 6

4.3.3 Notice

*One notices on the level of the elementary vector ~
qe the presence of terms associated with the local base and
at the total base.*

**4.4
Discretization of the field of deformation**

The field of deformation is expressed like the symmetrized gradient of the field of displacement:

$$I = \frac{1}{2} (U + U^T)$$

2
Like:

$$U = [NR] qe$$

() ~
X
X

one thus has:

$$U = NR$$

qe
()
~
X

*where NR gathers the functions of form NR ()
I*

(2)

I
and Ni
and matrices of passage $T_i K$

, is
X
the reverse of the jacobien J and \sim
qe is the vector of the degrees of freedom to the nodes (translations the U.K. and
~
rotations).

Taking into account these relations and of \sim
= $T T$, one obtains the components of the tensor of
deformation in the local reference mark:

~
~ ~
= $B qe$

~
where B is the matrix of interpolation of \sim
, such as:

~
 $B = T T S J -$

I
(
NR)
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Note:

If one takes again the expression of

Nb1

Nb2

()

1

3

(2)

~

~

U (X) = NR U.E. +

NR H

I

I (T

i2 I K

1 + T

I

ik

K

i1 i2k) ek = U (X) + U (X)

T

R

one notices

2

i=1

i=1

that the terms of membrane are contained in the first part U T ()

X of U (X) and that them

terms of inflection are contained in the second part U R ()

X of U ()

X. Terms of

~

~ ~

E

m = B m Q

transverse shearing come from the two contributions. One obtains as follows: ~

~

~

E
F = B F Q where

~

~ ~

= B qe

~

B = T SJ-1 NR

m

MF

I ()

~

H

B = T SJ -1 [

NR

F

MF

3

2 ()) by simple decomposition of the expression ~

~ ~

= B qe. One calls

2

~

B = T SJ-

I

(

NR)

membrane part of the deformation projection on the membrane-inflection part of the field of deformation room of the symmetrized gradient of the translations in the total reference mark. Part is called

inflection of the deformation projection on the membrane-inflection part of the field of deformation room of the symmetrized gradient of rotations in the total reference mark. One calls transverse distortion

projection on the shearing part of the local field of deformation of the gradient symmetrized of total displacement.

4.5

Stamp rigidity

The principle of virtual work is written in the following way: W

= W

ext.

def is still

$UT K U UT$

=

F in matrix form where K is the matrix of rigidity coming from the assembly in the total reference mark of the whole of the elementary matrices of rigidity. At the elementary level discretization of the work of deformation is written with the preceding notations:

1

~ ~ ~

~

$W_{el} = \int_{\Omega} \sigma \cdot \epsilon \, d\Omega$

T

Q

$B^T C B \det J D$

$D D \sim e$

$\sim e t$

$E \sim e$

def

$Q = Q K Q$

1

2

3

$-1 A r$

where $A r$ is the area of reference of the element.

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4.5.1 Decomposition of the elementary matrices

This matrix of rigidity includes/understands three contributions due to the deformations of membrane, of inflection

~ E

~ E

~ E

~

and of transverse distortion. One has as follows: K

K + K + K E

= m

F

with:

1

~ E

~ T

~

K =

B H B det J D D

1 D

2

;

m

m

m

3

-1Ar

1

~ E

~ T

~

K =

B H B det J D D

1 D

2

;
 F
 F
 F
3

$-I A r$
1
 $\sim E$
 $\sim T$
 \sim
 $K =$
 $B H B \det J D$

D
 $1 D$
2
.
3
 $-I A r$

4.5.2 Assembly of the elementary matrices

The principle of virtual work for the whole of the elements is written:

nbelem
 W
 $= W e$
 T
def
def = $U K U$ where U is the whole of the degrees of freedom of the discretized structure
 $e=1$
and K comes from the assembly of the elementary matrices.

4.5.2.1 Degrees of freedom

The process of assembly of the elementary matrices implies that all the degrees of freedom are expressed in the total reference mark. In the total reference mark, the degrees of freedom are the three displacements compared to the three axes of the total Cartesian reference mark and three rotations compared to these three axes. One thus uses, for the degrees of freedom of rotation, of the matrices of passage of

locate local orthonormé T with the total reference mark for each element.

4.5.2.2 Rotations

fictitious

Rotation compared to the normal with the hull is not a true degree of freedom. To ensure compatibility between the passage of the local reference mark to the total reference mark, one thus adds a degree of freedom

additional room of rotation to the hull which is that corresponding to rotation compared to normal on the average surface of the element. This implies an expansion of the blocks of dimension (5,5)

matrix of local rigidity in blocks of dimension (6,6) by adding a line and a column correspondent with this rotation. These additional lines and these columns are a priori null. One then carry out the passage of the matrix of local rigidity extended to the matrix of total rigidity.

In the preceding transformation, one was satisfied to add rotations compared to normals on the surface of the elements without modifying the deformation energy. The contribution to energy

brought by these additional degrees of freedom is indeed null and no rigidity is to them associated.

The matrix of total rigidity thus obtained presents the risk however to be noninvertible. For to avoid this nuisance it is allowed to allot a small rigidity to these degrees of freedom additional on the level of the matrix of widened local rigidity. Practically, one chooses it between 10⁶ and 10³ times the diagonal minor term of the matrix of rigidity of local rotation. The user can to choose this multiplicative coefficient COEF_RIGI_DRZ itself in AFFE_CARA_ELEM; by defect it is worth 10⁵.

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4.6

Stamp of mass

The virtual work of the effects of inertia can be expressed in the form:

$$W_{ac} = \int_U (Q) \cdot U (Q) \text{ mass }) D$$

It is supposed that the deformations and displacements remain sufficiently small so that the normal on the average surface of the hull remains unchanged.

With these assumptions, we can write the field of virtual displacement:

$$H$$

$$U (Q$$

$$)(,$$

$$1 ,$$

$$2)$$

$$3 = U (P$$

$$)(,$$

$$1)$$

$$2 + 3$$

$$(,$$

$$1) N$$

$$2$$

(,
1)
2
2

and the field of acceleration:

&
H
U(Q) (
2)
1
3 = &u(P) (,
2 +
&
3
(,) N (
2
,)
1
1
1 2
2

In this expression, we neglected the gyroscopic terms.

4.6.1 Discretization
displacement for the matrix of mass

At the point Q, one takes as interpolation of the field of displacement:

U
N
N

Nb
11
0
-
Nb
13
12

I
2
II

H

U (

Q) (,

1

2

1,

2)

3 = NR I

(,

1)

2 U I 2 - 3

NOR (, 1) 2 nI3

0

- nII

I 2

2

I 1

=

I 1

=

U

N

N

I 3

- I2

II

0 I3

For the field of acceleration, the interpolation is written:

U
0
I
- N
N

1
Nb
&
&
1
Nb
I 3
I 2

2
II

H

&

U (Q) (,
1
2
2) = NR (
3
I,)
1
1
2 &uI 2 -
NR (
3
I,)
N
0
I
- nor &
1
2

3
1
I2
2
I1
=

I =

&
1
uI

- N
N
0
I
I
&
3
2
1
I3

We rewrite the two preceding equations in the matrix form:

$U(Q)$
 $NR ue$
 $(1\ 2\ 3 =$

$\&U(Q$
 $,) NR$
2
3 =
&
 ue
) (1

where **NR** is the matrix of interpolation, whose expression is:

1 0 0
0
- nI3 nI2
0
-

nNb23
nNb22

H

1
2

H

NR = NEITHER 0 1 0 - 3 NOR N

0
I 3
- nI1
-
2

3 NNb2 N

0
Nb23
- nNb21

2

2
0 0 1
- N

N
O

I 2
II

-

N
N
O
Nb22
Nb21

I =,
I
I
Nb

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The vector \mathbf{ue} is the elementary nodal vector of displacements in the total reference mark which is put in the following form:

$$E$$

$$U = (U, U, U, \dots, U, U, U, \dots,$$

,

,

$$I = Nb$$

11

12

13

11

12

13

1

1

i2

i3

1

1

i2

i3

Nb21

Nb22

Nb23)

,

1

1

4.6.2 Stamp of elementary mass

With the preceding notations, the virtual work of the effects of inertia is put in the matrix form following:

$W_{inertie}$

and

$E E$

mass

$$= U M u \&$$

with M_e the matrix of coherent mass which can be expressed in the form:

$M_e =$

$NTN \det (J$

() D D D

3

1

2

3

E

It is important to note that because of the curve, a coupling of the terms of translation with those of rotation is possible (indeed, $\det (J ())$

3

is not constant in the thickness).

4.6.3 Assembly of the elementary matrices of mass

The assembly of the matrices of mass follows same logic as that of the matrices of rigidity. degrees of freedom are the same ones and one finds the treatment specific to normal rotations to surface hull. Although the matrix of coherent mass is built in the total reference mark, it remain singular compared to the rotation of the normal in each node. We need to supply this matrix on the basis of the variational form:

Nb2

Winertie = m

N

I.E.(internal excitation) (N I N I

) &I

I =1

where is selected constant for me by element and calculated according to the formula:

m

Cm

E =

max

mmax being the major term due to rotations (in the local reference mark of the element) on the diagonal

matrix Me. It is thus to note that with this intention it was necessary to bring back the contribution the rotations initially expressed in the total reference mark of the element, the local reference mark of the element by change of reference mark.

For modal calculations utilizing at the same time the calculation of the matrix of rigidity and that of stamp of mass, it is necessary to take a mass on the degree of normal rotation on the surface of the

hull

being worth C time the diagonal minor term of the matrix of mass for the terms of rotation in locate local, where C is worth between 106 and 103. One chooses to confuse the values of this coefficient with

those of the COEF_RIGI_DRZ for the equivalent operation on the matrix of rigidity. By defect C is worth

thus 105. That makes it possible to inhibit, during a modal analysis, the modes being able to appear on

additional degree of freedom of rotation around the normal on the surface of the hull.

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4.7

Numerical integration for elasticity

4.7.1 Integration

surface

For the element Hétérosis Q9H the inflection part of the matrix of stiffness is integrated classically with 9 points of Gauss while the parts membrane and shearing are obtained by integration reduced with 4 points by Gauss.

For element T7H, by analogy with Q9H, the matrix of stiffness is obtained with 7 points of integration of Hammer for the inflection part and 3 points of integration of Hammer for the parts shearing and membrane.

Cordonnées of the points

Weight

$$1 = 1/3; 1 = 1/3$$

9/80

$$2 = has; 2 = has$$

$$155 + 15$$

With =

$$6 + 15$$

2400

has =

21

$$3 = 1 - 2a; 3 = has$$

With

$$4 = has; 4 = 1 - 2a$$

With

31/240 - With

$$5 = B; 5 = B$$

$$B = 4/7 - has$$

$$6 = 1 - 2b; 6 = B$$

31/240 - With

$$7 = B; 7 = 1 - 2b$$

31/240 - With

1 -

1

N

y (

), D D

$$= I y$$

(I, I)

0 0

i=1

Normal numerical formulas of integration on triangle T7H (Hammer)

X-coordinates of the points

Weight

Ordinates of the points

Weight μ

$$1 = - 3 / 5$$

5/9

$$1 = - 3 / 5$$

5/9

=
 $2 = 0$
 $8/9$
 $2 0$
 $8/9$
 $3 = + 3 / 5$
 $5/9$
 $2 = + 3 / 5$
 $5/9$
 $1 1$
 N
 N
 $y ($
 $), dd = i\mu_j y (I, J)$
 $--$
 $1 1$
 $i=1 j=1$

Normal numerical formulas of integration 3x3 on quadrangle Q9H (Gauss)

The principle of reduced integration consists in evaluating the membrane and shearing strains at the points of reduced integration and to extrapolate them at the points of traditional integration.

This returns to

to suppose that these deformations are bilinear on element Q9H and linear on the T7H.

functions of form chosen to make this extrapolation are related traditional to form

bilinear of the quadrangle with 4 nodes for the Q9H and linear of the triangle with 3 nodes for the being worth T7H

1 at the points of reduced integration.

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For more details on the principle of reduced or selective integration, one can refer to [bib6].

Cordonnées of the points**Weight**

$$1 = 1/6; 1 = 1/6$$

$$1/6$$

$$2 = 2/3; 2 = 1/6$$

$$1/6$$

$$3 = 1/6; 3 = 2/3$$

$$1/6$$

$$1 -$$

$$1$$

$$N$$

$$y(,) D D$$

$$=$$

$$I y$$

$$(I, I)$$

$$0 0$$

$$i=1$$

Numerical formulas of integration reduced on triangle T7H (Hammer)

For the elements quadrangle an integration of Gauss 2x2 is used.

Cordonnées of the points**Weight**

$$1 = 1/3; 1 = 1/3$$

$$1$$

$$2 = 1/3; 2 = -1/3$$

$$1$$

$$3 = -1/3; 3 = 1/3$$

$$1$$

$$3 = -1/3; 3 = -1/3$$

$$1$$

$$1 1$$

$$N$$

$$y ($$

,) $dd = I y (I, I)$

--
1 1
i=1

Reduced numerical formulas of integration 2x2 on quadrangle Q9H (Gauss)

4.7.2 Integration in the thickness

Integration in the thickness is made with three points for the two elements.

Cordonnées of the points

Weight

1 = -1
1/3
2 = 0
4/3
3 = +1
1/3
1
N
y (
) $D = I y (I)$
-1
i=1

Formulate numerical integration in the thickness in elasticity

4.8

Numerical integration for plasticity

The principle of surface integration remains the same one as in elasticity, but the initial thickness is divided into NR identical layers thicknesses. There are three points of integration per layer. Points of integration are located in higher skin of layer, in the middle of the layer and in lower skin of layer. For NR layers, the number of points of integration is of $2N+1$. One advises to use of 3 with 5 layers in the thickness for a number of points of integration being worth 7, 9 and 11 respectively.

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For rigidity, one calculates for each layer, in plane constraints, the contribution to the matrices of rigidity of membrane, inflection and transverse distortion. These contributions are added and assemblies to obtain the matrix of total tangent rigidity.

For each layer, one calculates the state of the constraints (11,22,12) and the whole of the variables interns, in the middle of the layer and in skins higher and lower of layer, from local plastic behavior and of the local field of deformation (11,22,12). The positioning of points of integration enables us to have the rightest estimates, because not extrapolated, in skins lower and higher of layer, where it is known that the constraints are likely to be maximum. plastic behavior does not include/understand for the moment the terms of transverse shearing which are treated in an elastic way, because transverse shearing is uncoupled from the behavior membrane in plane constraints.

Cordonnées of the points

Weight

1 = -1

1/3

2 = 0

4/3

3 = +1

1/3

1

N

y (

) D =

I y (I)

-1

i=1

Formulate numerical integration for a layer in the thickness in plasticity

Note:

One already mentioned with [§2.2.2] that the value of the coefficient of correction in shearing transverse for the elements of plate and hull was obtained by identification of elastic complementary energies after resolution of balance 3D. This method is not more usable in elastoplasticity and the choice of the coefficient of correction in shearing transverse is posed then. The transverse terms of shearing are thus not affected by plasticity and are treated elastically, for want of anything better. If one places oneself in theory of Coils-Kirchhoff for a value of this coefficient of $106 h/R$ (H being the thickness of the hull and R its average radius of curvature) transverse terms of shearing become negligible and the approach is more rigorous.

4.9

Discretization of elementary work for the loadings

4.9.1 Discretization

elementary of the work of the forces and external couples being exerted on average surface

According to the paragraph [§3.2], one recalls that one has for these efforts and couples:

$$W = (fu + C) dS$$

ext.

S

where S is the average surface of the hull.

For the first term of this expression one has as follows:

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4.9.1.1 Loads given in the total reference mark

**1
2
~
~
2
W
=
F NR
[
U + C NR
(
T
2 1 - T
)] det
1 2
J•
D 1
D
ext.
K
I
ik
K
J
J
J K
J
J K

H
2
Ar**

**with det J• = det J (3 =)
0**

4.9.1.2 Loads given in the local reference mark

1
 1
 2
 \sim
 \sim
 2
 W
 $= [$
 $F \ N \ R \ T \ N \ R \ U \ +$
 $C \ T \ N \ R$
 $($
 T
 $2 \ 1 - T$
 $)] \ det$
 $1 \ 2$
 $J \bullet$
 $D \ 1$
 D
 $ext.$
 $J \ J \ K$
 I
 ik
 K
 J
 J
 $J \ K$
 J
 $J \ K$

 H
 2
 Ar

4.9.2 Discretization

elementary of the work of the forces and external couples being exerted on contour

According to the paragraph [§3.2], one recalls that one has for these efforts and couples:

$$W = (U +) ds$$

ext.

C

where C is the average contour of the hull. and linear distributions of force and moment applied to the contour of the hull in the total reference mark.

1
2
~
~

The discretization gives then: W

$$= [NR U + NR (T 2 1 - T D)] S$$

ext.

K
I
ik
K
J
J
J K
j1 j2k

C

4.9.3 Discretization of the term of gravity

One has for this term:

$$1$$

$$3$$

$$2$$

$$\sim$$

$$\sim$$

$$\mathbf{Wpes} = \mathbf{G U (Q D)}$$

$$\mathbf{) V = gkU (Q D)}$$

\int_V
 K
 $= G [NR$
 K
 $I U +$
 NR
 ik
 J
 $($
 T
 $j_2 J K$
 $1 - T$
 D
 $)] V$
 $j_1 j_2 k$

2
 E
 E
 E

That is to say: W

$= G NR$
 $1 U FD$

pes
 K
 I
 ik

by supposing negligible the second term of expression Ci

E
above.

4.9.4 Discretization of the term of pressure

It is supposed that the pressure p is applied to the average surface of the hull. One has then:

W
 $= ep$

$NU (P D$
 $) S = ep$
(has has) U (P)
 I

2

D 1

D

near

2

With

With

R

R

where $e=\pm 1$ according to whether p is applied in internal or external skin.

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Like A

= E has

1

K K, this is still written: W

= epN

U v

D 1

D

near

I

ik K
2
Ar
v J • J • - J • J •
1
12 23
13 22

where v = J • J • - J • J • J •

,
= J
2
13 21
11 23
ij
ij (3
=)
0 .

v
• •
•
•
J J - J J
3
11 22
12 21

4.9.5 Discretization of the terms of centrifugal inertia

One adds with the expression of the field of accelerations of the paragraph [§4.6] the corresponding term with the accelerative forces centrifuges if the total reference mark (O, ek) is in uniform rotation by report/ratio with a Galiléen reference mark which in the same beginning O as the total reference mark. The expression of the field accelerations becomes as follows:

&
H
U (Q) (
2)
1

$3 = \&u (P) (,)$

2 +

&

3

(,) N (

2

,) + [COp]

1

1

1 2

2

where one neglected the forces of Coriolis and the correction of metric in the thickness.

One expresses in the total base (ek): = K ek.

By taking again the expression of: W inertia = &U (Q). U (Q D) the contribution of the terms is identified

of centrifugal inertia: W inertia

0

hundred

= U.E. [(X E) D] V

K K

K K

by neglecting the terms of rotation

E

in virtual displacement. The terms of mass are unchanged compared to [§4.6].

Like one a:

x0

=

x0

= x0

E

E

E

E

K

K

P

P

K
K
p
K
qpk E Q

where eqpk is the permutation of Lévi-Strauss.

One also writes:

(x0 E) = E
E
x0
K
K
qpk
srq
R
p
K E K

From where it results from it that:

1
W inertia =
U
NR () 1 E
E
x0 NR () 1 det J D D
1 D
hundred

is
I
qpk srq
R
p
jk
J
2 3
-1Ar

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Code_Aster ®**Version****7.4****Titrate:****Finite elements of voluminal hulls****Date:****14/04/05****Author (S):****X. DESROCHES Key****:****R3.07.04-B Page****: 30/42****4.9.6 Taking into account of the loadings of thermal dilation**

One treats only the case where the thermoelastic characteristics E , depend only on average temperature T in the thickness. Moreover, the material is thermoelastic isotropic homogeneous in the thickness.

The variational formulation of work due to thermal dilations is written:

l
+
 l
+
 HT
~
~
 $\sim th$
 $\sim T$
 $\sim T \sim \sim th$
 $\sim T$
 $\sim T \sim$
 $W = - ($
 C
-
) $FD = Q$
 $B C$
 J
 $= Q$

B C

-

Id

J

E

det

D

D

D

E

(T T réf) det

D

D

D

1

2

3

1

2

3

1

- A

1

- A

E

R

R

The temperature is represented by the model of thermics to three fields according to [R3.11.01]:

T (, 3) = T m (
). P1 (3) + T S (). P2 (3) + Ti

(). $P_3(x)$,

with: $P_j(x)$: three polynomials of LAGRANGE in the thickness: $x = 0, 1, 2$

2

$$P_0(x) = 1 - (x - 1)(x - 2); P_1(x) = 3(x - 1)(x - 2); P_2(x) = -6(x - 1)(x - 2)$$

3

3

2 (3

) = (1 + 3

); $P_1(x)$

1 3

3

3 (3

) = - (1 - 3

);

2

2

From the representation of the temperature above, one obtains:

• the average temperature in the thickness:

$\frac{1}{3} \int_0^3 T(x) dx$

1

$T(x) =$

$T_0(x) + T_1(x) + T_2(x)$

$(T_m(x) + T_s(x) + T_i(x))$

;

2 -1

$(T_m(x) + T_s(x) + T_i(x))$

()

6

• the average variation in temperature in the thickness:

$\frac{1}{3} \int_0^3 T(x) dx$

+1

$T(x) = 3 T_0(x) + 3 T_1(x) + 3 T_2(x)$

(,

3) D

$$3 = T S () - T I$$

();
-1

Thus the temperature can be written in the following way:

$$T (,) = T () + T\$ (). /2 + T (,$$

3
3
3) such as:

$$+1~
1
+
T
(
~
,
3) = 0 ;
T
3 (,
3) = 0 .$$

1
-
1
-

If the temperature is indeed closely connected in the thickness one has, T = 0.

It is necessary to evaluate the three-dimensional thermal stresses, in each point of integration in the thickness. These constraints of thermal origin withdrawn from the mechanical constraints usual are calculated at the points of integration in the thickness by:

~ther
.
E
réf
=

$(T - T + \$T. /2$
2
3
)
1 -

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4.9.7 Assembly

The variational formulation of the work of the efforts external for the unit of the elements is written then:

nbelem

$W = W_e = T$

ext.

ext.

U F where U is the whole of the degrees of freedom of the discretized structure and

e=1

F comes from the assembly of the vectors forces elementary.

As for the matrices of rigidity, the process of assembly of the vectors forces elementary imply that all the degrees of freedom are expressed in the total reference mark. In the total reference mark, the degrees of freedom are three displacements compared to the three axes of the total Cartesian reference mark

*and three rotations compared to these three axes. Matrices of passage of the reference mark are thus used
room with the total reference mark for rotations of each element.*

Note:

*The external efforts can also be defined in the reference mark user. One then is used
stamp passage of the reference mark user towards the local reference mark of the element to have the
expression
of these efforts in the local reference mark of the element and to deduce the vector from it elementary
room forces
corresponding. For the assembly one passes then from the local reference mark of the element to the
total reference mark.*

5 Validation

*To judge relevance of thick the hull formulation, few examples of application
according to relate to as well linear statics as the calculation of clean modes. Three new cases
tests relating to the two finite elements described in the preceding parts were integrated in
Code_Aster. They come to enrich the case-tests by the elements of plate already present in
environment of Code_Aster. Most of these case-tests were indexed in [bib10].*

*The three new case-tests, two in statics plus one in dynamics, are traditional examples
of validation drawn from [bib3]. Reference solutions, analytical or numerical, resulting from [bib3]
are compared with the numerical results given by Code_Aster. For more information on these
case-tests, one will refer to the documentation of validation indicated in reference.*

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5.1

Case test in linear statics

5.1.1 Static case test n° 1

The first case test is that of a cylindrical panel subjected to its own weight [V3.03.107].

This test makes it possible to highlight effects of membrane more important than those of inflection. It allows to measure the performance of the elements hulls compared to elements DKT or DKQ of which the interpolation out of membrane is linear.

5.1.2 Static case test n° 2

The second case test is that of a helicoid hull subjected to two concentrated types of loading [V3.03.108].

The helicoid shape of the hull makes it possible to study the geometrical representation of the finite elements.

The concentrated loadings can be:

- in the plan: the influence due to the effects of membrane is then not important and it behavior dominating is that due to the inflection,*
- except plan: the effects of membrane affect the behavior of the hull.*

5.2

Case test in dynamics

This case test is a simplified model of paddle of compressor, which is in fact a cylindrical panel [V2.03.102].

This test highlights the performances of the elements in dynamic behavior by the data frequencies and clean modes.

The frequencies and clean modes of the paddle are experimental values which are used as results of reference.

6 Chaining thermomechanics

6.1 Description

For the resolution of chained thermomechanical problems, one must use for thermal calculation finite elements of thermal hull [R3.11.01] whose field of temperature is recovered like input datum of Code_Aster for mechanical calculation. It is necessary thus that there is compatibility between thermal field given by the thermal hulls and that recovered by the mechanical hulls. It the last is defined by the knowledge of 3 fields TEMP_SUP, TEMP and TEMP_INF given in skins lower, medium and higher of hull.

The table below indicates compatibilities between the elements of mechanical hull and hull thermics.

Modeling Nets

Finite element

to use with Mesh Element

finished

Modeling

THERMICS

MECHANICS

HULL QUAD9

THCOQU9

//////////

QUAD9 MEC3QU9H COQUE_3D

HULL TRIA7

THCOTR7 ////////////

TRIA7 MEC3TR7H COQUE_3D

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Note:

· *The nodes of the thermal elements of hulls and mechanical hulls must to correspond. The grids for thermics and mechanics will thus have the same number and the same type of meshes.*

· *The elements of thermal hulls surface are treated like plane elements by projection of the initial geometry on the level defined by the first 3 tops.*

The thermomechanical chaining is also possible if one knows by experimental measurements variation of the field of temperature in the thickness of the structure or certain parts of structure. In this case one works with a chart of temperature defined a priori; the field of temperature is not given any more by three values TEMP_INF, TEMP and TEMP_SUP of thermal calculation

obtained by EVOL_THER. It can be much richer and contain an arbitrary number of points of discretization in the thickness of the hull. Operator DEFI_NAPPE allows to create of such profiles of temperatures starting from the data provided by the user. These profiles are affected by order AFFE_CARTE (cf case-test HSNS100B). It will be noted that it is not necessary for mechanical calculation that the number of points of integration in the thickness is equal to the number of points

of discretization of the field of temperature in the thickness. The field of temperature is automatically interpolated at the points of integration in the thickness of the elements of hulls.

6.2 Case-test

The case-tests for the thermomechanical chaining enters of the thermal elements of hulls and of mechanical elements of hulls are the HPLA100C (elements MEC3QU9H) and HPLA100D (elements MEC3TR7H). It is about a heavy thermoelastic hollow roll in uniform rotation [V7.01.100] subjected to a phenomenon of thermal dilation where the fields of temperature are calculated with

Ther LINEAIRE by a stationary calculation.

Z

IH

Re

Interior ray IH = 19.5 mm

External ray Re = 20.5 mm

Not F

R = 20.0 mm

Thickness

H = 1.0 mm

Height

L = 10.0 mm

R

Z

NR

K

Z

Q

P

J

y

D

C

H

M

+

L

R

With

B

X

F

Thermal dilation is worth: $T () - Tref () = 0.5$

. $(T + T$

S

I) + 2. $(T + T$

S

I) $(R - R)/H$

with:

.
T = 0 5
. °C, *T = -0 5*
. °C, *T*
= 0. °C
S
I
ref.

.
T = 01
. °C, *T = 01*
. °C, *T*
= 0. °C
S
I
ref.

One tests the constraints, the efforts and bending moments in L and Mr. the results of reference are analytical. One obtains very good results whatever the type of element considered.

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7

Establishment of the elements of hull in Code_Aster

7.1 Description

These elements (of names MEC3TR7H and MEC3QU9H) are pressed on meshes TRIA7 and curved QUAD9.

These elements are not exact with the nodes and it is necessary to net with several elements to obtain correct results.

7.2

Introduced use and developments

These elements are used in the following way:

*MA = CREA_MALLAGE (GRID: MAILINI
MODI_MAILLE: (OPTION: "QUAD8_9"
ALL: "YES")...)*

One calls upon a routine MODI_MAILLE of modification of the grid to pass from the elements quadrangles with 8 nodes with the elements quadrangles with 9 nodes or many elements triangles to 6 nodes with the elements triangles with 7 nodes.

AFFE_MODELE (MODELING: "COQUE_3D"...) for the triangle and the quadrangle

One calls upon routine INI080 for the position of the points of Hammer and Gauss on the surface of the corresponding hull and weights.

*AFFE_CARA_ELEM (HULL: (THICKNESS: "EP"
ANGL_REP: (" ")
COEF_RIGI_DRZ: "CTOR")*

To make postprocessings (forced, generalized efforts,...) in a reference mark chosen by the user who is not the local reference mark of the element, one defines the X1 direction of the reference mark user

like the projection of a direction of reference D on the surface of the element. This direction of reference D is chosen by the user who defines it by two nautical angles in the total reference mark. The normal NR on the surface of the element fixes the second direction at the point of observation concerned.

vector product of two vectors previously definite $Y1=N X1$ makes it possible to define the local trihedron in which will be expressed the generalized efforts representing the state of stresses. The user will have to take care that the selected reference axis is not found parallel with the normal of some elements of hull. By defect, the direction of reference D is axis X of the total reference mark of definition of grid.

Value CTOR corresponds to coefficient that the user can introduce for the treatment of the terms of rigidity and mass according to normal rotation on the surface of the hull. This coefficient must be

sufficient small not to disturb the energy balance of the element and not too small so that the matrices of rigidity and mass are invertible. A value of 105 is put by defect.

ELAS: (E: NAKED Young: ALPHA: RHO:)

For a homogeneous isotropic thermoelastic behavior in the thickness one uses the key word ELAS in DEFI_MATERIAU where the coefficients E are defined, Young modulus, coefficient of Poisson, thermal dilation coefficient and RHO density.

AFFE_CHAR_MECA (DDL_IMPO: (

DX: DY: DZ: DRX: DRY: DRZ: DDL of hull in the total reference mark.

FORCE_COQUE: (FX: FY: FZ: MX: MY: MZ:). They are the efforts surface on elements of hull. These efforts can be given in the total reference mark or in the reference mark user defined by ANGL_REP.

FORCE_NODALE: (FX: FY: FZ: MX: MY: MZ:). They are the efforts hull in the total reference mark.

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7.3

Calculation in linear elasticity

The matrix of rigidity and the matrix of mass (respectively options RIGI_MECA and MASS_MECA) are integrated numerically in the TE0401 and TE0406, respectively. Calculation takes account of fact that the terms corresponding to the DDL of rotation of hull are expressed in the local reference mark of

the element. A matrix of passage makes it possible to pass from the local DDL to the total DDL.

Elementary calculations (CALC_ELEM) currently available correspond to the options:

- *EPSI_ELNO_DEPL and SIGM_ELNO_DEPL which provide the strains and the stresses with the nodes in the reference mark user of the element in lower skin, with semi thickness and in higher skin of hull, the position being specified by the user. Calculation is carried out in the TE0410. One stores these values in the following way: 6 components of deformation or constraints,*
- *EPXX EPYY EPZZ EPXY EPXZ EPYZ or SIXX SIYY SIZZ SIXY SIXZ SIYZ,*
- *EFGE_ELNO_DEPL: who gives the efforts generalize by element with the nodes from displacements: NXX, NYY, NXY, MXX, MYY, MXY, QX, QY. This option is calculated in the TE0410,*
- *SIEF_ELGA_DEPL: who gives the constraints by element to the points of Gauss in locate local element starting from displacements: SIXX, SIYY, SIZZ, SIXY, SIXZ, SIYZ. This option is calculated in the TE0410,*
- *EPOT_ELEM_DEPL: who gives the elastic energy of deformation per element from displacements. This option is calculated in the TE0401,*
- *ECIN_ELEM_DEPL: who gives the kinetic energy by element. This option is calculated in the TE0401,*

*Finally the TE0416 calculates also option FORC_NODA of calculation of the nodal forces for the operator
CALC_NO.*

7.4

Plastic design

The matrix of rigidity is also integrated numerically, by layers, in the TE0414. One calls upon the option of calculation STAT_NON_LINE in which one defines in the level of the nonlinear behavior it a number of layers to be used for numerical integration. All laws of plane constraints available in Code_Aster can be used.

```
STAT_NON_LINE (...
COMP_INCR: (RELATION: "
COQUE_NCOU: "A NUMBER OF LAYERS")
....)
```

Elementary calculations (CALC_ELEM) currently available correspond to the options:

- *EPSI_ELNO_DEPL which provides the deformations by element to the nodes in the reference mark user starting from displacements, in lower skin, with semi thickness and in skin higher of hull. This option is calculated in the TE0410,*
- *SIGM_ELNO_COQU which makes it possible to obtain the stress field in the thickness by element with the nodes for a given layer and a position requested (in lower skin, with*

medium or in higher skin of layer). These values are given in the reference mark user. This option is calculated in the TE0415,

· SIEF_ELNO_ELGA which makes it possible to obtain the efforts generalized by element with the nodes in

the reference mark user. This option is calculated in the TE0415,

· VARI_ELNO_ELGA which calculates the field of internal variables and the constraints by element with the nodes for all the layers, in the local reference mark of the element. This option is calculated in the TE0415.

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8 Conclusion

The finite elements of hull curves which we describe here are used in the structural analyses thin curves whose thickness report/ratio over characteristic length is lower than 1/10. Two finite elements of voluminal hull being pressed on quadrangular and triangular meshes were introduced into Code_Aster. They were selected with a quite particular aim: to be able to represent one

complete behavior of curved structures whereas until now one could only use elements with plane facets which induced parasitic inflections and required to refine them grids.

It is elements for which strains and stresses in the plan of the element vary linearly with the thickness of the hull. Selected kinematics is a kinematics hull of Hencky-Mindlin-Naghdi type allowing to utilize the transverse energy of shearing. distortion associated with transverse shearing is constant in the thickness of the element.

variable correction on the coefficient K of shearing transverse offers a flexibility in use allowing to pass from the theory of HENCKY-MINDLIN-NAGHDI for $k=1$, with that of REISSNER for $k=5/6$ and with that of LOVE_KIRCHHOFF (for very mean structures) if a value is chosen K equalizes $\approx 106 \times H/L$ H being the thickness and L a characteristic distance (radius of curvature means, zone of application of the loads....). As in this last case, one uses a method of penalization to make small the terms of shearing transverse, one can, if a value is taken K too important, to make singular the numerical system. In this case, it is necessary to decrease the value of K .

The default value of K is $5/6$. It is generally used when the structure to be netted has one thickness report/ratio over characteristic length ranging between $1/20$ and $1/10$. For thicknesses more weak where the transverse distortion becomes low one can want to use a value of $k=106 \times H/L$ (to be able to make comparisons with elements of plate DKT for example). When transverse distortion is nonnull, the elements of hull do not satisfy the equilibrium conditions 3D and boundary conditions on the nullity of stresses shear transverse on the faces higher and lower of hull, compatible with a constant transverse distortion in the thickness of the hull. It results from it thus that on the level from the behavior a coefficient from $5/6$ for a homogeneous hull corrects the usual relation between the constraints and the transverse distortion of way to ensure the equality enters energies of shearing of the model 3D and the model of hull to constant distortion. In this case, the arrow $\sim u_3$ has as an interpretation average transverse displacement in the thickness of the hull and not the displacement of the average surface of the hull.

For structures low thickness in order to avoid the phenomena of blocking, one uses under-integration reduced for the parts membrane and shearing of the matrix of rigidity. The choice on the finite elements went on the elements quadrangle Hétérosis $Q9H$ and triangle $T7H$. Indeed, among the finite elements with quadratic functions of interpolation, the performance of the Hétérosis element $Q9H$ is known. It is in particular higher than that of the elements Sérendip $Q9S$ or the elements of Lagrange $Q9$. This performance rests however on the selective integration of the element with reduced integration of the terms of membrane and shearing on the one hand, and normal integration of terms of inflection in addition. By analogy with $Q9H$, one took the finite element $T7H$ like element of triangular form. However, as far as possible, one will use the $Q9H$ rather than the $T7H$ which is definitely less powerful.

The non-linear behaviors in plane constraints are available for these elements. One announce however that the constraints generated by the transverse distortion are treated elastically, for want of anything better. Indeed the rigorous taking into account of a transverse shearing constant not no one on the thickness and the determination of the correction associated on rigidity with shearing compared to a model satisfying the equilibrium conditions and the boundary conditions are not possible and thus return the use of these elements, when transverse shearing is nonnull, rigorously impossible in plasticity. Rigorously, for behaviors not

linear, it would thus be necessary to use these elements within the framework of the theory of Coils-Kirchhoff.

Elements corresponding to the machine elements exist in thermics; chainings thermomechanical are thus available with finite elements of thermal hulls to 7 and 9 nodes. Extensions of the preceding formulation presented in appendix allow also the catch in account of the anisotropy of materials and kinematic non-linearity. This second extension is operational in Code_Aster and is the subject of a reference material [R3.07.05].

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Appendix 1 Extension to anisotropic materials not programmed

It is considered that the hull consists of an orthotropic material, axes of orthotropism ~

*~
xk associated the base*

kk. The law of behavior in these axes is written:

~

~

~

~

~

~

= ***Sk***

(6x)

1

(6x6) (6x)

1

~

where S is the matrix of flexibility of the component K.

Are ~

and ~, tensors of strain and stresses in the axes ~xk, one a:

$$\begin{aligned} & \sim T \sim \\ & = \\ & \mathbf{Q} \mathbf{Q} \\ & \sim T \sim \\ & = \mathbf{Q} \mathbf{Q} \end{aligned}$$

where $\mathbf{Q} = [\mathbf{T}, \mathbf{T}, \mathbf{T}$

1
 2
 $3]$
 $(\mathbf{Q}$
 \cdot
 $)$ is the matrix of the cosine directors of \mathbf{T}

$$\begin{aligned} /k \\ ij = \mathbf{T} \mathbf{K} \\ I \\ J \\ K \text{ in the base } \mathbf{K} \mathbf{K}. \\ K \end{aligned}$$

In vectorial form, one a:

$$\begin{aligned} & \sim \\ & \sim \sim \\ & = \\ & \mathbf{T} \\ & \sim \\ & \sim \sim \\ & = \mathbf{T} \end{aligned}$$

where the components of \mathbf{T} are defined according to those of \mathbf{Q} .

Conversely, one a:

$$\begin{aligned} & \sim \\ & \sim \\ & \sim -1 \sim \\ & = \mathbf{T} \\ & \sim \\ & \sim \\ & \sim -1 \sim \\ & = \mathbf{T} \end{aligned}$$

therefore, one obtains:

$$\begin{aligned}
&\sim \\
&\sim \sim \sim -1\sim \\
&= \mathbf{TS T} \\
&\mathbf{K}
\end{aligned}$$

that one writes:

$$\begin{aligned}
&\sim \\
&\sim \sim \\
&= \mathbf{Sk}
\end{aligned}$$

*To be coherent with the assumption of plane constraint ~
33 = 0, one write:*

$$\begin{aligned}
&\sim \\
&\sim \\
&\sim \\
&R = \mathbf{Skr R} \\
&(5x) \\
&I \\
&(5x) \\
&5 (5x) \\
&I
\end{aligned}$$

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with the symbol R like tiny room, which gives:

~
~ ~
~
~

-1
 $R = Ck$

,
 R
 $C = S$
 K
 Kr

that one récrit by omitting the symbol R ,

~
~ ~
 $= Ck$

The elastic deformation energy W_{el} is:

1
1
~ ~ ~
 W_{el}
 $T \sim E$
 T
 Q
 $B C B \det J D D D \sim e$
=

Q
2
 K
1
2
3
 $-1Ar$

If the hull consists of N_c layers, each layer being regarded as a component K , then:

$2e+ H$
 Nc
 K
 1
 $\sim \sim \sim$
 $W el$
 $T \sim E$
 T
 $= Q$
 $B C B det J D D D$
 $\sim E$

K
 Q
 2
 1
 2
 3
 $K = 12e- H Ar$
 K

where E
 $+$
 K and e_k are the X -coordinates of the limits lower and higher of the layer K thickness
 $E = e+ - E$
 $-$
 $+$
 K
 K
 K , with $e1 = - h/2$ and $E Nc = h/2$.

While posing:
 $+$
 E
 K
 $K + -$
 E
 E
 K
 $3 =$
 $+$
 $,$

- 11,

3

3

[

]

H

H

one a:

Nc

1

E 1

~ ~ ~

W el

T ~ E

K

T

=

det (1, 2,

~

Q

B C B

J

Q

K

3) D

D

D

E

2

H

1

2

3

K = 1

-1Ar

In the same way, for work due to thermal dilations W HT, one a:

~

~k

K

$$\begin{matrix}
 K \\
 K \\
 HT = (T, \\
 1 \\
 T, \\
 2 \\
 T, \\
 3 \\
 0 0) \\
 0
 \end{matrix}$$

where the k_i are the dilation coefficients thermal of the layer K in the axes of orthotropism ($\sim\sim xk$).

With the relation:

$$\begin{matrix}
 \sim K \\
 \sim \sim \sim
 \end{matrix}$$

$$\begin{matrix}
 K \\
 HT = T HT \\
 one\ obtains:
 \end{matrix}$$

$$\begin{matrix}
 1 \\
 T \sim \\
 \sim \\
 W HT \\
 T \sim E \\
 \mathbf{Q B} (\\
 \sim K \\
 = - \\
 - Ck HT) \det \mathbf{J D D D} \\
 1 \\
 2 \\
 3
 \end{matrix}$$

-1Ar
That is to say:

$$\begin{matrix}
 Nc E 1 T \sim \sim \\
 W HT T \sim E \\
 K \\
 \sim K \\
 = \mathbf{Q} \\
 \mathbf{B C} \det \mathbf{J D D D}
 \end{matrix}$$

H

K
HT
1
2
3
h=1
-1Ar
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Appendix 2 Functions of form for element Q9H

These functions are given on page 174 of [bib8].

A2.1 Functions of form for the translations

8 functions of the shape of incomplete Lagrange of the element quadrangle Q9H [A2.2-a Figure] for the interpolation of displacements the U.K. are:

1
.
NR ()
1 (, 1 2) = (- 1+ +) (1+) (1+) I
I
I
1 1
2i 2

I
1 1
2i
 = ,
1 2, ,
2
3 4
4
1
 .
NR ()
1 (,
2
1 2
) = (1 -) (1+) I
I
1
2i
 = ,
2
5 7
2
1
 .
NR ()
1 (,
2
1
2
) = (1 -) (1+) I
I
2
I
 = ,
1 1
6 8
2

1 = 1
 -
I = 1 8
, 4
I

, ;
 $2 = -1 I = 15, 2$
 I
 , ;
with: $1 = 0 I = 5 7$
 I
 , ;
and $2 = 0 I = 6 8$
 I
 , ;
 .
 $1 = +1 I = 2 6, 3$
 I
 , .
 $2 = +1 I = 3 7, 4$
 I
 , .

A2.2 Functions of form for rotations

9 functions of the shape of Lagrange of the element quadrangle Q9H [A2.2-a Figure] for the interpolation of
 ~
rotations are:

-
 $NR (2) (,$
 P
 I
 2
 $) = NR () NR$
 I
 I
 I
 $I ()$
 2 where $Nor ()$
 Pr
 $P =$
for $p=1,2$ and where R describes the whole of both
laughed Pr
- Pi
nodes aligned with node I in the direction P .
 $I = I$

-
 $I = 18$
, 4
 I
, ;
 $2 = -1 I = 15, 2$
 I
, ;
One a: $1 = 0 I = 57$
 I
, ;
and $2 = 0 I = 68$
 I
, ;
.
 $1 = +1 I = 26, 3$
 I
, .
 $2 = +1 I = 37, 4$
 I
, .

2
2
(0,1)
4
(0,1)
7
3
4
7
3
(1,0)
8
9
(1,0)
8
(0,0)
6

(0,0)
6

1
1
1
5
2
1
2
5

Appear A2.2-a: Degrees of freedom for the translations and rotations of the element quadrangle Q9H
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Appendix 3 Functions of form for element T7H

A3.1 Functions of form for the translations

6 functions of form of triangular element T7H [A3.2-a Figure] for the interpolation of displacements the U.K. are given on page 175 of [bib8]:

.

NR 1

()

1 (1

, 2

) = 2

(2 2

-1)

.

NR 1

()

2 (1**, 2****) = (2 -1)**

.

NR 1

()

3 (1**, 2****) = 1(2 1 -1)**

.

NR 1

()

4 (1**, 2****) = 4 2**

.

NR 1

()

5 (1**, 2****) = 4 1**

.

NR 1

()

6 (1**, 2****) = 4 1 2****where: = 1 - 1 - 2****A3.2 Functions of form for rotations**

~

7 functions of form of triangular element T7H [A3.2-a Figure] for the interpolation of rotations are:

()

1**• N2 (****(2)****1**

, 2
) = 2
(2 2
-) 1+ NR
1
7

9
()
1
· N2 (
(2)
1
, 2
) = (1 - 1
- 2
) [(21 - 1 - 2) -] 1+ NR
2
7

9
()
1
· N2 (
(2)
1
, 2
) = 1 (2 1 -) 1 + NR
3
7

9
()
4
· N2 (
(2)
1
, 2
) = 4 2
(1 - 1
- 2
) - NR
4
7

9
 (
 4
 · N2 (
 (2)
 1
 , 2
) = 4 1
 (1 - 1
 - 2
) - NR
 5
 7
 9
 (
 4
 · N2 (
 (2)
 1
 , 2
) = 4 - NR
 6
 1 2
 7
 9
 with:
 · NR (2)
 7 (1
 , 2
) = 27 1 2
 (1 - 1 - 2
)

2
 2
 (0,1)
 (0,1)
 1
 1
 6
 6
 4

4
7
(1,0)
(1,0)
2
2
(0,0)
5
3

(0,0)
5
3

1
1

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Finite elements of voluminal hulls

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***Voluminal elements of hulls into nonlinear
geometrical***

Summary:

We present in this document the theoretical formulation and the numerical establishment of a finite element of voluminal hull for analyses into nonlinear geometrical. This approach must make it possible to take in count great displacements and great rotations of mean structures, of which the thickness report/ratio on characteristic length is lower than 1/10. One will take care that these rotations remain lower than 2.

This formulation is based on an approach of continuous medium 3D, degenerated by the introduction of kinematics of hull in plane constraints in the weak form of balance. The measurement of the deformations that we retain is that of Green-Lagrange, énergétiquement combined with the constraints of Piola-Kirchhoff of second species. The formulation of balance is thus Lagrangian total.

The geometrical entirely nonlinear problem is examined in first. The case of linear buckling is treaty like a borderline case of the first approach.

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1 Introduction

The great transformations of hull are characterized by great displacements of surface average and of great rotations of initially normal fibres on this surface. The transformation thus is represented exactly, at least in the continuous problem. The derivation of the objects finite elements associated the linearized system of equations resulting from the principle of virtual work is carried out without any simplifying assumption on displacements or rotations. Moreover, one new diagram of selective numerical integration is presented in order to solve the problem of blocking in membrane and in transverse shearing.

The degrees of freedom of rotation retained are the components of the vector of space iterative rotation.

Between two iterations, it is the vector of the infinitesimal rotation superimposed on the configuration deformation. This choice led to a tangent matrix of rigidity which is not symmetrical. This is due to nonvectorial character of great rotations which actually belong to the differential variety $SO(3)$. Rotations must remain lower than 2 because of the choice of update of large rotations established in Code_Aster, for which there is not bijection between the vector of full slewing and the orthogonal matrix of rotation.

An important difference compared to the linear analysis is to be announced. The finite elements objects are directly built in the total reference mark; displacements and rotations nodal are measured in the total reference mark.

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2 Formulation

In this chapter, we present the various equations controlling the problem of deformation hull within the framework of a theory of great transformations.

2.1

Geometry of the elements of voluminal hull

The voluminal hull is represented by volume (together points (

Q 3)

0) built around the average surface (together of the points (

P 3 =)

0). In any point Q of, one

built a local orthonormé reference mark [T (,): T (,): N

1 1 2

3

2

1 2

3

(1, 2)]. The vector (n1,2)

represent the normal on the surface.

N (,),

1

2

3

T (,)

0

2

1

2

3

T (, = 0)

2

1

2

3

2

Q (0) ·

3

P (= 0) ·

3

T (, 0)

1

1

2

3

T (, = 0)

1

1

2

3

H

1

E, y

2

E, X

1

E, Z

3

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In the initial configuration, the position of an unspecified point Q normal on the average surface can to be expressed, according to the position of the revolved center P of normal fibre, the manner following:

H

$$\mathbf{X}(1,2,3) = \mathbf{X}(1,2) + 3$$

(N

Q

P

1,2)

2

2.2

Kinematics of the voluminal hulls

N

(,

N

1)

2

=

(,

1)

2

(,

1)

2

N

(,

1)

2

U

Q (,

1

2

3

Q ()

0

3

.

Q ()

0

3

.

P (=)

0

3

.

P (=)

0.

3

H

uP (, =)

0

1

2

3

xQ (,

1

2

3

xP (, =)

0
1
2
3
X Q (,)
1
2
3
E, y
2

xP (, = 0)
1
2
3
E, X
1
E, Z
3

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Great transformations of an initially normal fibre on the average surface
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In the deformed configuration, the position of the point Q can also be expressed according to

position of the point P:

$$\mathbf{X}(1,2,3) = \mathbf{X}(1,2) + \mathbf{N}$$

Q

P

3

(1,2)

2

where N is the unit vector obtained by great rotation of normal N.

Vector N is not necessarily normal on the deformed average surface, because of transverse shearing strain. It is connected to the initial normal vector by the relation:

$$\mathbf{N} = (1,2) \mathbf{N}$$

is the orthogonal operator of the great rotation around the vector, angle, undergone by fibre who was initially normal on the average surface whose expression is given by:

sin

1 - cos

$$= \exp [\times] = \cos [I] +$$

[\times] +

[]

2

where [\times] is the antisymmetric operator of the vector of full slewing of which the matrix expression is:

0

- Z

y

$$[\times] =$$

0

Z

- X

- y

0
X

and $[]$ is the symmetrical operator given by $[] = T$.

More details on great rotations and their digital processing can be found in [bib1] or [R5.03.40]. One can also write:

$$\mathbf{T1} = \begin{pmatrix} 1 \\ , 2 \\ \end{pmatrix} \mathbf{T1}$$

$$\mathbf{t2} = \begin{pmatrix} 1 \\ , 2 \\ \end{pmatrix} \mathbf{t2}$$

One can express the virtual variation of the operator of great rotation in the form:

$$= [\mathbf{W} \times]$$

where $[\mathbf{W} \times]$ is the antisymmetric operator of the vector of space virtual rotation \mathbf{W} which is also rotation part of the functions tests:

$$[\mathbf{W} \times] \mathbf{B} = \mathbf{W} \mathbf{B} \mathbf{B} R3$$

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Its matrix expression is:

0

- wZ

wy

[

W ×] =

W

0

Z

- wx

- wy

W

0

X

One can also express the iterative variation of the operator of great rotation in the form:

= [**W** ×]

*where **W** is the vector of space iterative rotation, which is also the rotation part of the solution of system of linearized equations.*

This vector can be connected to the vector of total iterative rotation. There are thus the relations:

W = **T** () and **W** = **T** (

)

*where **T** () is the differential operator of rotation, of which the expression according to the vector of rotation*

total is given by:

sin

1 - cos

- sin

T () =

[**I**] -

[×] +

[]

2
3

This matrix has the same values and clean vectors that the matrix and checks the relation:

$$\mathbf{T}(\cdot) = (\cdot) \mathbf{T} \mathbf{T}(\cdot)$$

In addition, the iterative variation of the matrix of virtual rotation can be put in the form:

$$= [\mathbf{W} \times] [\mathbf{W} \times]$$

The total displacement of the point Q on fibre can be connected to the displacement of the centre of gravity P:

$$\begin{aligned} &H \\ \mathbf{U}(\cdot) &= \mathbf{U}(\cdot) + (\mathbf{N} \\ &1 \ 2 \\ &3 \\ &1 \ 2 \\ &3 \\ &(1,2) - (\mathbf{N} \\ &Q \\ &P \\ &1,2) \\ &2 \end{aligned}$$

In order to lead to a system of linearized equations, obtained starting from the weak form of balance, we need to calculate various differential variations of this total displacement.

virtual displacement has as an expression:

$$\begin{aligned} (\cdot) &= (\cdot) + H \\ &1 \ 2 \\ &3 \\ &1 \ 2 \\ &3 \\ &(1,2) \end{aligned}$$

$$\begin{aligned} &U \\ &U \\ &W \\ &N(1,2); N \\ &Q \end{aligned}$$

P
= 0
2

Iterative displacement has as an expression:

H
U (,) = U (,) + W (,) N
1 2
3
1 2
3
1 2
(1,2); N

Q
P
= 0
2

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The iterative variation of virtual displacement has as an expression:

H
(1,2,3) = 3 (1,2) ((1,2)
U

W
W
N
Q
 (1,2)
 2

Note: The formulation suggested remains limited to rotations lower than 2. This limit is had with the particular choice of update of the great rotations established in Code_Aster. This is had with nonthe bijection enters the vector of full slewing and the orthogonal matrix of rotation.

2.3

Law of behavior

We consider a linear law of behavior hyper elastic: local constraints of Piola-Kirchhoff of second species are proportional to the local deformations of Green-Lagrange:

~
 ~
S = OF

Hereafter, the symbol ~ indicates the quantities expressed in the orthonormé reference mark

[T (,): T (,): N

1 1 2

3

2

1 2

3

(1, 2)].

The matrix of elastic behavior linear in plane constraints is written as follows:

E

E

0

0

0

1 - 2

1 - 2

E

0
0
0

1 - 2

E

D =

(
0
0

2 I +)

Ek

sym

(
0
2 I +)

Ek

(

2 I +)

E being the Young modulus, the Poisson's ratio and K the coefficient of correction of transverse shearing.

~

*In the local reference mark, the state of Piola-Kirchhoff stress of second species is plane (*S_{nn}* =)*

*0 and
can be characterized by a vector with 5 components:*

~
S
T T
~ 1 1
St T
~2 2
~
S
S = T t12

~
St N
1
~
St N
2

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*The vector of the deformations of Green-Lagrange is also expressed him in the local reference mark by
one
vector with 5 components:*

~

*E**T T*

~ 1 1

And *T*

~ 2 2

~

$$\mathbf{E} = T t_{12}$$

~

*T N**1*

~

*T N**2*

~

Here, we were unaware of the Enn term which is normal on the average surface and which is not inevitably

no one. This is a consequence of the assumption of the plane constraints.

2.3.1 Taking into account of transverse shearing

The correction of the transverse shear stress is carried out by extension of equivalences energy given in the case of small deformations and of small displacements [R3.07.03].

3

Principle of virtual work

The principle of virtual work is the weak formulation of the static balance of the internal forces and external forces:

$$int - ext. = 0$$

The non-linearity of the equilibrium equations leads us to solve the system above way iterative by a method of Newton. We carry out thus the exact linearization of the principle of virtual work with each iteration, which leads to the equality:

- *ext.* = *ext.* -

int

int

3.1

Internal virtual work

The virtual work of the internal forces can be written on the initial configuration in the form:

~ ~

int = (.

E S) *D*

~

~

*where **E** and **S** are the vectors of deformation of Green-Lagrange and Piola-Kirchhoff constraint of second species respectively, expressed in the local reference mark. Indeed, like the state of stress is plane for Piola-Kirchhoff of second species, we use the formulation of the principle of work virtual in the local reference mark. However, to limit the passages of the local reference mark to the total reference mark and*

vice versa, the vectors of strains and local stresses are not calculated explicitly in the local reference mark but they are obtained by the rotation of their representation in the total reference mark.

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3.1.1 Form

incremental

internal virtual work

The iterative variation of the work of virtual work interns is written:

~ ~

~ ~

int = (.

E S +.

E S) D

In this equality, iterative variation of the vector of local constraints of Piola-Kirchhoff of second species is calculated by the iterative discrete form of the relation of behavior:

~ = ~

S

D E

3.1.2 Passage of the total reference mark to the local reference mark

In tensorial form one passes from the tensor of the total constraints to the tensor of the constraints local 3 × 3 (see [bib4] p. 111 for the constraints of Cauchy, the same relations applying to constraints of Piola-Kirchhoff of second species) while using:

~

[S]
[
P S] PT
=

and of the tensor of the local constraints to the tensor of the total constraints by the inversion of the relation

the preceding one:

T ~
[S] = P [S] P

In the two preceding expressions, the matrix of passage of the local reference mark to the total reference mark is

an orthogonal matrix P 1

- = Pt, and its expression clarifies according to the unit vectors of locate orthonormé local is:

tT

1 (1,2 ,3)

P (
T

1,2, 3) = t2 (1,2, 3)

NT

(1,2)

Within the framework of the conventional notation, one will be able to note:

T1 (1

, 2

, 3

) = E

0 1

t2 (1

, 2

, 3

) = E

0 2

**T3 (1
, 2
, 3
) = (
N 1
, 2
) = E
0 3**

with the orthogonal matrix of passage (initial rotation):

**0 (1
, 2
, 3
) = [T1
(1
, 2
, 3
): t2 (1, 2
, 3
): T3 (1, 2
, 3
)]**

It will be noticed that:

**=
0
Pt**

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The two relations of rotation of the constraints are also valid for the tensors of the deformations of Green-Lagrange. Nevertheless, a writing which connects the vectors of local and total deformation is

necessary. This relation makes it possible to pass from vector 6×1 of the total deformations to the vector

6×1 of the local deformations:

~

6 1

×

6 × 6.6 1

×

E = H E

with the form of the matrix of transformation of vectors 6×1 of deformation (see [bib2] p. 258):

2

2

2

1

L

1

m

1

N

1

L 1

m

1

m 1

N

1

N II

2

2
2

L
2
2

m
N2
l2 2
m
2
m N2
n2l2

6×6
2
2
2
L
m
N
L m
m N
N L

H
3
3
3
3 3
3 3
3 3
=

2 1l2 2 1
m2
m
2 1
N N2
l
L 2
m + l2 1
m
l

$m N^2 + 2$ m N 1 $N l^2 + N^2 1l$ $2l$ $2l^3$ 2^2 m 3 m $2n^2n^3 l^2 3$ $m + l^3 2$ m 2 $m n^3 + 3$ $m N^2 n^2 l^3 + n^3 l^2$ $2l L$ $2m m$ $2n N$ $L m$ $+ L m$ $m N + m N$ $N L + N L$ $3 1$ $3 1$ $3 1$ $3 1$ $1 3$ $3 1$ $1 3$ $3 1$ $1 3$

and components of the unit vectors of the local reference mark:

 $L = T . e$ $m = T . e$ N 1

1 1 1 1 2

$$1 = t1.e3$$

$$L = T .e$$

$$m = T .e$$

 N 2 2 1 2 2 2

$$2 = T 2 .e3$$

$$L = T .e$$

$$m = T .e$$

 N 3 3 1 3 3 2

$$3 = t3.e3$$

These expressions are general for the curvilinear reference marks. In the Cartesian total reference mark

[E: E: E

 1 2

3], these components are:

$$L = T$$

 1 $1()$ 1

$$m = T$$

 1 $1(2)$

$$N = T$$

 1

$I()$
 3
 $L = T$
 2
 $2()$
 1
 $m = T$
 2
 $2(2)$
 $N = T$
 2
 $2()$
 3
 $L = T$
 3
 $3()$
 1
 $m = T$
 3
 $3(2)$
 $N = T$
 3
 $3()$
 3

We have, actually, need for a writing which connects the vector of local deformation 5×1 and it vector of total deformation 6×1 :

\sim
 $5 \ 1$
 \times
 $5 \times 6.6 \ 1$
 \times
 $E = H \ E$

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6×6

For that, one forgets the third line of the expression of H (line associated with Snn):

(

2

2

2

T1 ()

1)

(T1 (2))

(T1 () 3)

2

2

2

5×6

(t2 () 1)

(t2 (2))

(t2 () 3)

H = 2

T1 ()

1 t2 ()

1

2t1 (2) t2 (2) 2t1 ()

3 t2 ()

3

2t2 () 1t3 () 1 2t2 (2) T3 (2) 2t2 () 3t3 () 3

2

T3 ()

1 T1 ()

1

2t3 (2) T1 (2) 2t3 ()

3 T1 ()

3

T1 ()

1 T1 (2)

T1 (2) T1 ()

3

T1 ()

3 T1 ()

1

t2 ()

1 t2 (2)

t2 (2) t2 ()

3

t2 ()

3 t2 ()

1

T

1()

1 t2 (2) + t2 ()

1 T1 (2) T

t2 ()

3 + t2 (2) T1 ()

3

T1 ()

3 t2 ()

1 + t2 ()

3 T1 ()

1(2)

1

t2 ()

1 T3 (2) + T3 ()

1 t2 (2) t2 (2) T3 ()

3 + T3 (2) t2 ()

3

t2 ()

3 T3 ()

1 + T3 ()

3 t2 ()

1
T

3()
1 T1 (2) + T1 ()
1 T3 (2) T3 (2) T1 ()
3 + T1 (2) T3 ()
3
T3 ()
3 T1 ()
1 + T1 ()
3 T3 ()
1

The same preceding relations can be applied for the passage of the vectors of total deformation with the local deformation.

3.1.3 Relation deformation-displacement

Tensor 3×3 of the total deformations of Green-Lagrange is defined by (see for example [bib2]):

1
[E] = (U
+ C + U
You)
2

with the tensor of the gradient of displacements:

U v W

X
X X X

U v W
U = U v W

y <

> = *there there y*

U v W

Z

Z Z Z

The tensor of deformation of Green-Lagrange can be also written:

$$[\\ 1 \\ E] = (FTF - I) \\ 2$$

with F the tensor gradient of deformations 3×3 which is not symmetrical:

$$F = X = I + U$$

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and I the tensor identity:

1 0

0

$I = 0 1$

0

0 0

1

Vector 6×1 of the total deformations of Green-Lagrange is ordered as follows (see [bib4] p 117):

1

2

2

2

E

, X

, X

, X

xx

U, X

(U + v + w)

2

1

E yy

v, y

2

2

2

2 (U, y + v, y + W, y)

E W

zz

, Z

E =

=
+ 1
2
2
2

xy U, y + v, X
(U, Z +v, Z +w, Z)
2

U
xz
, Z +

W
U U
, X, y + v v
, X, y + W W

, X
, X, y

U U + v v + W W
yz

v, Z + W, y
, X, Z
, X, Z
, X, Z

U
, y U, Z + v, y v, Z + W, y W, Z

It as follows is calculated:

I
U U
E = Q + A (
)

2
X
X

with:

1 0 0 0 0 0 0 0 0

0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 1
Q =

0 1 0 1 0 0 0 0 0
0 0 1 0 0 0 1 0 0

0 0 0 0 0 1 0 1 0

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and the vector of the gradient of displacements:

U, X

U, y

U

$, Z$

v, X

U

v

$=$

X

$, y$

v, Z

W, X

W, y

W, Z

and tensor A depend on the gradient of displacements:

U

0

0

v

0

0

W

$, X$

$, X$

$, X$

0

0

0

U

0

0

v

0
0
W
,y
,y
,y
0
U
0
0
U
0
0
v
0
0
W
,Z
,Z
,Z
With

X = U
U
0
v
v
0
W
W
,y
,X
,y
,X
,y
,X
0
U
0
U
v

0
v
W
0
W
, Z
, X
, Z
, X
, Z
, X

0
U
U
0
v
v
0
W
W
, Z
, y
, Z
, y
, Z
, y

The virtual variation, noted, from the deformations of Green-Lagrange is obtained by a calculation differential:

U

U
E = Q
+
With

XX

In this expression and that which follows, we took account of the following property (see [bib4] p 141):

1 U
U 1 U U

With
= A

2 X X
2 X X

The iterative variation is it also obtained by a differential calculus:

U
U
E = Q
+ A

X
X

The iterative variation of the virtual deformation of Green-Lagrange is put thus in the form:

U U
U
U
E = A
+ Q + A

X X

X X
traditional term
nontraditional term
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Whereas the first term of this expression is traditional for the continuous mediums 3D, it second, which translates the taking into account of great rotations, is less.

3.1.4 Calculation of the constraints of Cauchy

3.1.4.1 Case

general

Tensor 3×3 of the total constraints of Piola-Kirchhoff of second species is connected to the tensor 3×3 of the total constraints of Cauchy by the relation:

$$[S] = (F) F [] F$$

det***1******T***

Thus, knowing the state of the constraints of Piola-Kirchhoff of second species, one can calculate the state

constraints of Cauchy by the relation:

[]***1******=***

$$F S F T$$

$$\det (F) []$$

It should be noted that the state of stresses of Cauchy is not plane, in general, contrary to the state of constraints of Piola-Kirchhoff of second species. In addition, the choice of a local reference mark in which

to represent this tensor is not at all obvious. It will be however shown, in the following paragraph, that

within the framework of the small deformations, there is a local reference mark, easily identifiable, in which the state of stresses of Cauchy is him-also plane.

In the case of completely general laws, a detailed attention will have to relate to the diagrams of numerical integration allowing to calculate the values of substitution of the gradient F at the points of normal numerical integration.

3.1.4.2 Approximation in small deformations

It is pointed out [bib4] that the gradient F can be written thanks to the polar decomposition under two forms:

$$F = RU = VR$$

where $R = R - T$ is an orthogonal tensor, and where U and V are symmetrical matrices of elongation defined positive.

Into the geometrical nonlinear field, we can introduce an important simplification in the polar decomposition of the gradient of the deformations if the deformations remain small. This simplification is not introduced into nonlinear calculation but in postprocessing of the constraints.

Elongation at the point Q being minor in front of the great rotation of the section:

$$U V I$$

One can then write:

*$F R =$
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where is the tensor of great rotation which transforms normal N into N :

$$N = N$$

Simplification translates the fact that on a section, the transformation is reduced to a great rotation. With this approximation of the gradient of the deformations, one can write:

$$F R =$$

and thus, by exploiting the orthogonality of one obtains:

$$F^{-1} T$$

and:

$$\det(F) = 1.$$

These simplifications lead to the final relation:

$$[S] T$$

This relation translates the fact that the constraints of Cauchy are quite simply obtained by great rotation of the constraints of Piola-Kirchhoff of second species.

One can now rewrite the property of plane constraints of the tensor of Piola-Kirchhoff of second species N . $[S] N = 0$ pennies the new form:

$$n \cdot T [S] N = 0$$

who leads in addition to the property:

$$N \cdot [S] N = 0$$

That is to say still:

$$\sim = 0$$

NN

*Constraints of Cauchy [(1,2, 3)] are also plane in the local reference mark
[T (*

*1
, 2
, 3
) : T (1, 2
, 3
) : N (1, 2
) obtained by great rotation of the local reference mark on
1
2
]
initial configuration:*

*[T T N
:
:
] = [T: T: N
1
2
1
2
]*

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In this reference mark, we can write all the components of the tensor as follows []:

~
~
~

T.

1 [] T

T.

1

1 [] T

T.

2

1 [] N

T T

T T

T N

~1 1

~1 2

~1

= T.

2 [] T

T.

1

2 [] T

T.

2

2 [] N

TT

TT

TN

~2 1 ~2 1

2

0

N

T

N

T

N

N

.

[]

.

1

[]

.

2

[]

TN

TN

1
1

By taking again the relation [] [S] T, one obtains:

T

1. [] T

T

1

1. [] T

T

2

1. [] N

T

1. [S] T

T

1

1. [S] T

T

2

1. [S] N

T

2. [] T

T

1

2. [] T

T

2

2. [] N

= T
2. [S] T
T
1
2. [S] T
T
2
2. [S] N
N. [] T N

1
. [] T
N
2
. [] N
N.
[S] T
N
1
. [S] T
N
2
. [S] N

from where the final result:

~
 ~
 ~
 ~
 ~
 ~
 ~

S
S
S
TT
TT
TN
TT
TT
TN
~1 1
~1 2
~1
~1 1 ~1 2 ~1

= S
S
S
TT
TT
TN
TT
TT
TN
2 2
2 2
2
~2 1 ~2 1
2
~
~

0

S
S
0

TN
TN

T N
T N
1
2

1
1

In so far as the deformation remains small, components of the tensor of the constraints of Cauchy in the local reference mark attached to the deformed configuration are identical to the components tensor of the constraints of Piola-Kirchhoff of second species in the local reference mark attached to initial configuration.

We take the party in the continuation, to consider only the constraints of Piola-Kirchhoff of second species. We must note that within the framework of a more general constitutive law, one will be able to pass from a stress measurement to another as indicated in the preceding paragraph.

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4
Numerical discretization of the variational formulation
exit of the principle of virtual work

4.1 Elements**finished**

The three figures below summarize the finite elements choices concerning the voluminal hulls [R3.07.04]. The selected finite elements are isoparametric quadrangles or triangles. quadrangle is represented below. One chooses among the elements with functions of interpolation quadratic, the element hétérosis whose displacements are approached by the functions of interpolation of the Sérendip element and rotations by the functions of the element of Lagrange. All the justifications as for these choices are given in [R3.07.04].

NB1 = 8***NB2 = 9******Sérendip element******Element of Lagrange******Hétérosis element******~******the U.K.,******~***

Appear 4.1-a: Families of finite elements for the isoparametric quadrangle

2***3 3 =1******7******3******4******= -******1******1******1 8******6******1******5******2******2 = 1******-***

Appear 4.1-b: Voluminal element of reference

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2

4

7

3

3

8

P

1

6

5

1

2

Appear 41-c: Real voluminal element

4.2

Discretization of the field of displacement

With an aim of avoiding the explicit calculation of the curves, which becomes extremely heavy in the case of them

great rotations, we choose to interpolate the normal on the initial average surface with the place to interpolate rotations:

NB2

N (

2

1,2)

()
 = *NOR (1,2) nor*
I I
 =

(2)
where NR I (1
, 2
) the function of interpolation to node I indicates among the NB2 nodes of Lagrange.

The same interpolations are adopted for the transform of the initial normal:

NB2
N (
2
1, 2)
()
 = *NI (1, 2) N*

I
I I
 =

The interpolation of the initial position of a point on the average surface of the hull (not P) is given by:

X
(
1
NB

X
1

1,2)
()
 =
NR I (1,2) y
I I
 =

Z I

() I

where NR I (1

, 2

) the function of interpolation to node I among NB1 indicates = NB2 -1 nodes of Serendip.

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The interpolation of the initial position of an unspecified point of the hull (not Q) can then be written in the form:

X

N

NB

X

(

1

NB

2

H

X

1

2

NR

y

I

1,2 ,3)

()

=

(1,2)

()

+ 3

NR I (1,2) ny

2

I I

=

I I

Z

=

nz

I

The same interpolations are adopted for the deformed position of an unspecified point of fibre:

X

N

I

NB

NB2

H

X

X (

I

2

NR

y

I

1,2 ,3)

()

=

(1,2)

()

+ 3

NR I (1,2)

ny

2

I I

=

I I

=

Z

nz

I

The interpolations for the positions initial and deformation being the same ones, we can adopt them for the real displacement of an unspecified point of the hull:

U

N

N

I

NB

NB2

H

X

X

U (

1

2

NR

v

NR

N

N

I

1,2 ,3)

()

=

(1,2)

()

+ 3

I

(1,2)

y - y

2

I I

=

I I

=

W

I

nz

nz

I

I

Thus, the interpolation of virtual displacement becomes:

U

0

- N

N

NB

Z

y

W

1

NB

2

H

X

U (

1

2

NR

v

I

1,2,3)

()

=

(1,2)

()

- 3

NR I (1,2)

N

0

Z

- nx wy

2

I I

=

I I

=

W

-

N

N

0

y

X

I

wz

I

I

In the same way, the interpolation of iterative displacement becomes:

U

O
- N
N
NB
Z
y
W
1
NB
2

H

X
U (
1
2
NR
v
I

1,2,3)
()
=
(1,2)
()
- 3
NR I (1,2)

N
0
Z
- nx wy
2
I I
=

I I
=

W

-

N

N

O

y

X

I

wz

I

I

Moreover, the interpolation of the iterative variation of virtual displacement is:

NB2

H

(, ,)

(2)

=

NOR (1,2)

2

((

U

W

WN

1 2

3

3

)

I 1

=

I

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4.3

Discretization of the gradient of displacement

4.3.1 Gradient of total displacement

The vector of the gradient of real displacement can be connected to the isoparametric gradient of real displacement by the following relation:

$$U \sim -1 U$$

$$= J$$

X

The isoparametric gradient of displacement is organized as follows:

$U,$

1

$U, 2$

$U,$

3

$v, 1$

U

$v,$
=

2
 $v,$

3
 $W,$

1
 $W,$

2
 $W, 3$

~

The matrix jacobienne generalized $9 \times 9 J^{-1}$ can be expressed according to the matrix jacobienne isoparametric transformation 3×3 as follows:

J^{-1}
0
0
~

$J^{-1} =$
0
 J^{-1}
1
0
0
0
 J^{-1}

The isoparametric gradient of real displacement can be calculated as follows:

$U N R E$
=

p

1
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with the first matrix of derived from the functions of form:

2

()

()

1

NR

NR

3

1

0

0

0

0

,

I

I

,

I

()

(2)

I

0

0

NR

NR

3

I

0

0

,

I

2

,

2

(2)

0

0

0

NR

I
0
0

()

I
(2)
0
NR

0
0

NR
0

I
3
,
I

,

I

I

()
H

I

(2)

L
0

NR

0

0

NR

0

I

L = 1, NR 1

B

I

3

,

I

,

2

2

2

(2)

0

0

0

0

NR

0

()

I

0

0

NR 1

(2)

I

,

0

0

NR

1

3

I

,

()

1

0

0

NR 1

(2)

I

,

0

0

NR

2

3

I, 2

0

0

0

(

2)

NR

0

0

NR

I

=

(2)

I

NR

0

0

3

NB2,

I

(2)

NR

0

0

3

NB2,

2

(2)

NR

0

0

NB2

(

2)

0

NR

0

3

NB2,

1

H

(2)

0

NR

0

2

3

NB2,

2

0
(2)
NR

NB2
0

(
2)

0
0
NR

3
NB2, 1

(
2)

0
0
NR

3
NB2, 2

(
2)

0
0
NR

NB2

and the vector of “generalized nodal real displacement”:

M

U

v

W

nx - nx
N

y - ny

EP
N
=
Z - nz

I

M

I = 1, NB1

M

N

X - nx

N

y - N y

N

Z - nz NB2

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Finally, one will be able to write the gradient of real displacement in the form:

U ~

-I NR

$$E = J$$

$$p$$

$$X$$

$$1$$

4.3.2 Gradient of virtual displacement

While proceeding similarly to the gradient of real displacement, one can connect the two gradients of virtual displacement:

$$U \sim -1 U$$

$$= J$$

$$X$$

The isoparametric gradient of virtual displacement can be calculated as follows:

$$U NR$$

$$=$$

$$E$$

$$U$$

$$2$$

with the second matrix of derived from the functions of form:

$$(2)$$

$$(2)$$

$$() 1$$

$$0$$

$$NR N$$

$$3$$

$$,$$

$$Z$$

$$-$$

I
NR
3
I

I
N
NR
,
y
I

I

0
0
, 1

(2)

(2)

()

I
0
NR
N
3
,
Z
-

0
0
I
NR
3
I
2
N

NR

,

y

2

I

, **2**

0

NR (2)

(2)

0

0

0

IN

- NR

Z

I ny

()

(2)

(2)

0

NR I

- NR

N

0

NR N

I

0

,

3

I

,

Z

3

I

,

X

I

I

I

()

H

L

(2)

(2)

0

NR 1

0

- NR

N

0

NR

N LI = 1, NBI

I

,

3

I

,

Z

3

I

,

X

2

2

2

2

0

0

0

(2)

(2)

- NR N

0

NR

N

()

I

Z

I

X

0

0

NR I

(2)

(2)

I

**,
NR N**

- NR N

0

I

3

I

,

y

3

I

,

X

()

I

I

0

0

NR 1

(2)

(2)

I

,

NR

N

- NR

N

0

2

3

I

,

y

3

2

I, 2

X

0

0

0

(2)

(2)

NR

NR

I N

- NR

y

I N

0
X

=

(2)

(2)

2

0

NR

N

3

- NR

N

NB2, Z

3

NB2, y

1

1

(2)

(2)

0

NR

N

3

- NR

N

NB2,

Z

3
NB2,
y

2
2

(2)
(2)

0
NR
N

NB2 Z
- NR
N
NB2 y

(
2)

(2)
- NR
N
0
NR
N

3
NB2, Z
3
NB2, X

H
1
1

(2)

(2)

-

3 *NR*

N

0

,

Z

NR

N

2

NB2

3

NB2

2

,

X

2

(2)

(2)

- *NR*

NB2n

0

NR

Z

NB2nx

(2)

(2)

3 NR NB2 N

,

y

- 3N NB2 N

0

1

, 1 X

(2)

(2)

NR

N

NR

N

0

3

NB2

,

y

- 3 NB2

2

, 2 X

(

2)

(2)

NR

NB2ny

-

NR NB2n

0

X

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and the vector of the virtual nodal variables:

M

U

v

W

wx

W

y

ue = wz

I

M

I = 1, NB1

M

W

X

W

y

wz NB2

Finally, one will be able to write the gradient of virtual displacement in the form:

U ~

-I NR

J

E

=

U

X

2

4.3.3 Gradient of iterative displacement

The step here is similar to virtual calculation. It is enough to replace by:

$$\begin{aligned} U &\sim \\ -I NR \\ E \\ &= J \end{aligned}$$

$$\begin{aligned} U \\ X \\ 2 \end{aligned}$$

with the vector of the iterative nodal variables:

$$M$$

$$U$$

$$v$$

$$W$$

$$wx$$

$$W$$

$$y$$

$$ue = wz$$

$$I$$

$$M$$

$$I = 1, NB1$$

M

W
X

W
y

W

Z NB2
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4.3.4 Gradient of the iterative variation of virtual displacement

U ~
-1 NR
E

= J

Q
X
3

with:

(2)
NR

3
I

0
0
, 1

(2)

NR
0
0

3
I

, 2

(2)

NR
0
0

I

(2)

0

NR

3

0

,

I

NR

H

1

(2)

= L

0

0

,

,

L = 1

2

NR

I

NB

3

I

2

2

3

(2)

0
NR

I

0

(2)

0
0
NR

3
I

, I

(2)

0
0
NR

3
I

, 2
(

2)

0

0
NR

I

and the vector of the iterative variation “nodal virtual displacement” generalized:

.

.

.

(W
(WN

E
)
Q
I
=

.

.

.

**I =,
I NB2**

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4.4 Discretization of the variational formulation resulting from the principle of virtual work

We take again the iterative variation (between two iterations) of internal virtual work:

**~
~
~ ~
int = (.
E S +.
E S) D**

and iterative variation of the vector of local constraints of Piola-Kirchhoff of second:

**~ = ~
S
D E**

Then, the linearized form of the principle of virtual work of the §3 can be written for the finite element described above in the following matrix form:

$$\begin{matrix}
 u^e K E \\
 E \\
 E \\
 E \\
 E \\
 \cdot \\
 T U
 \end{matrix}$$

$$= U. (F - R)$$

where u^e is the nodal vector of the functions tests. One deduces the system from it from equations:

$$\begin{matrix}
 K E \\
 E \\
 E \\
 E \\
 T U
 \end{matrix}$$

$$= F - R$$

where:

$$\begin{matrix}
 KeT \\
 \text{is the tangent matrix of rigidity} \\
 u^e
 \end{matrix}$$

is the elementary vector of the solution of the linearized system of equations (nodal vector between two iterations)

is the external level of load

$$\begin{matrix}
 F E \\
 \text{is the nodal vector of the external forces (associate with } = 1) \\
 Re
 \end{matrix}$$

is the nodal vector of the internal forces

4.4.1 Vector of the internal forces

It is a vector (6 ×

1

Nb +)

3 × 1 entirely expressed in the total reference mark and which must be evaluated with each iteration by the relation:

Re =

BT S

Jd D D

~ ~ det

2

1

2

3

with the vector of the local constraints Piola-Kirchhoff of second species:

~

~

S = OF

It is pointed out that the symbol ~ indicates an object expressed in the local reference mark.

The local deformations of Green-Lagrange are updated to each iteration:

~

~

E = B EP

1

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where the operator of the total deflections (first operator of the deformations) is written:

~

$I U \sim NR$

$B = H Q$

+ A

$J I$

-

I

$2 X$

I

with the gradient of real displacement:

$U \sim$

$-I NR$

E

$= J$

p

X

I

The operator of the virtual deformations (second operator of the deformations):

~

$U \sim NR$

$B = H Q + A$

$J I$

-

2

 X

2

is highlighted by the relations:

~ ~

$$E = B$$

 E $2u$

~

~

$$E = B u e$$

2

4.4.2 Stamp tangent rigidity

The tangent matrix of rigidity which is serious ($6 \times$

 1 $NB +)$ $3 \times (6 \times$ 1 $NB +)$

3 is expressed too

entirely in the total reference mark. One must be able to evaluate it with each iteration if it is wanted that

convergence of the method of Newton is quadratic. In a traditional way into nonlinear geometrical, it takes the form:

$$K E = K E + K E$$

 T m G

where the first part represents the material part:

$$K E =$$

BT DB

J

m

DD D

~ ~ det

2

2

1

2

3

and the second part represents the geometrical part, it even made up of two parts:

KE = KE

+ KE

G

G

G

traditional

nontraditional

with the traditional part of the geometrical part (see [bib4] p. 141):

T

NR

NR

KE

=

J -1

SJ -

~

~ 1 det J

G

DD D

traditional

1

2

3

2

2

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where S the generalized tensor of the constraints expressed in the total reference mark is written:

3×3

[S]

0

0

9×9

S = 0

[S] 0

0

0

[S]

The nontraditional part of the geometrical part not represents terms uncoupled from rotation symmetrical which has as a form:

$$\begin{matrix}
 3 \times 3 \\
 \mathbf{K E} \\
 (\mathbf{I}, \mathbf{I}) = [\mathbf{Z} \times] [\mathbf{N} \\
 \mathbf{G} \\
 \mathbf{I} \\
 \mathbf{I} \times] \\
 \text{nontraditional}
 \end{matrix}$$

where $\mathbf{N I}$ is the transform of the initial normal to node \mathbf{I} and $\mathbf{z I}$ a vector 3×1 with the node $\mathbf{I} = 1, \mathbf{N B 2}$ of the nodal vector $3 \times (\mathbf{N B 2} \times)$
 $\mathbf{1 Z I}$

\mathbf{Z}
 \mathbf{Z}
 \mathbf{I}
 $\mathbf{I} =$

$$I =, \\ 1 NB \\ 2$$

Nodal vector Z I is similar to a vector of internal force and its expression is:

$$Z = \\ BT S \\ J \\ I \\ D D D \\ \sim \sim det \\ 3 \\ 1 \\ 2 \\ 3$$

with the operator of the iterative variation of the virtual deformations (third operator of deformations):

$$\sim \\ U \sim NR \\ B = H Q \\ + A \\ J I \\ - \\ 3$$

X

3

who is highlighted by the relation:

~ T ~

~

~

B S det Jd

D D =

E

. S

3

1

2

3

D

nontraditional

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*4.4.3 Diagrams
of integration*

The integration of the terms of rigidity in the thickness of the hull is identical to the method used in analyze linear geometrical [R3.07.04] for nonlinear behaviors. The initial thickness is divided into NR identical layers thicknesses. There are three points of integration per layer. Points of integration are located in higher skin of layer, in the middle of the layer and in lower skin of layer. A layer in the thickness of the hull appears sufficient in the majority of the cases.

In order to be able stage with the problem of blocking out of membrane of the curved hulls and to solve it problem of blocking in transverse shearing, it is necessary to modify the diagram of integration on average surface. If the technique is completely known in linear analysis, it is it less in geometrical nonlinear analysis.

The procedure is presented in the form of a generalization of the separation of the effects of membrane, of inflection and of transverse shearing if one uses the deformations of Green-Lagrange:

~

~

E**E = m**

~

ES

~

E**T T**

~

~

~ **11**

~

T Nwhere **E****1****m = And T represent the deformation of membrane-inflection and ES =**

~

deformation of

~ **2 2****T N****2****T t12**

transverse shearing.

During the numerical evaluation of the deformations at the points of normal numerical integration of Gauss

~

~

(9 points for the quadrilateral and 7 points for the triangle), one uses the relation $E = B EP$

1

.

modification is introduced on the level of the first operator of the deformations:

~

substitution

~

B

MF

B

1

1 =

~

B substitution

S

1

~

~

B

substitution

substitution

MF

and B

are the first operators of the deformations of substitution of

1

s1

membrane-inflection and of transverse shearing, respectively.

During the calculation of the nodal vector of the internal forces and material part of the tangent matrix of

rigidity, the modification is introduced in a way similar to the level of the second operator of deformations:

~

known

~

B

MF

B

2

2 =

~

B known

S

2

4.4.3.1 Operators of deformations of substitution

In what follows the points of normal and reduced numerical integration of Gauss, on surface average, are $NPGSN = 9$ and $NPGSR = 4$, respectively, for the element quadrilateral, and $NPGSN = 7$ and $NPGSR = 3$, respectively, for the triangular element.

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Membrane-inflexion part

At point INTSN among the NPGSN points of normal numerical integration of Gauss of surface average, one will calculate:

normal

normal

~

B *substitution*

complete

incomplete

MF

(INTSN) ~

= **Bmf**

(INTSN) ~

- **Bmf**

(INTSN) +

1

1

1

reduced

NR

incomplete

I (INTSN) ~

Bmf

(INTSR)

I
INTSR=,
I NPGSR

where INTSR is a point among the NPGSR points of reduced numerical integration of Gauss of surface average.

normal
~
complete
~

*In the expression above, **B MF***
*represent the first three lines of **B***

I
I calculated with
NR

points of normal numerical integration by considering the complete matrix
. The operator

I
normal
~
~

B
incomplete
MF

*represent the first three lines of **B***
I
I calculated at the points of numerical integration

NR
normal by considering a matrix
incomplete where the columns of rotation are cancelled:

I
()
NR I

I
0
0
0 0 0
,
0 0 0

I

()

NR I

0

0

00

000

I

0

, 2

0

0

0

00

0

000

()

I

Inc

0

NR

0

000

000

I

NR

,

I

()

= L

I

0

0

00

0L = I,

1000

NR

I

NB

I

,

1

2

0

0

0

00

0

000

()

0

0

NR 1

0 0 0

I

0 0 0

, 1

()

0 0 0

0

0

NR 1

0 0

I

0

, 2

0 0 0

0

0

0

0 0

0

reduced

~

incomplete

~

B MF

*(INTSR) the first three lines of **B** represent*

1

1 calculated at the points

NR Inc

of reduced numerical integration with the matrix

incomplete above definite. They are thus

1

stored to be extrapolated at each point of normal numerical integration.

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Transverse shearing part

For the transverse shearing part, one will calculate:

reduced

~

***B** substitution (INTSN) =*

NR (INTSN) ~B complete

S

1

I

S

(INTSR)

1

INTSR=,

1 NPGSR

reduced

~ complete

~

where **B S**
 (INTSR) represents the two last lines of **B**
 1
 1 calculated at the points of integration
 NR
 numerical reduced with a matrix
 complete. They are also stored to be extrapolated
 1
 at each point of normal numerical integration.

4.4.3.2 Substitution of the geometrical part of the tangent matrix of rigidity

The nontraditional part of the tangent matrix of rigidity **K eg**
 is numerically
 nontraditional
 integrated into the points of normal integration of Gauss. No operation of substitution is necessary.
 For the traditional part of the tangent matrix of rigidity, we use substitution:

normal
 normal
 reduced
 reduced
 complete
 incomplete
 incomplete
 complete
 membrane
 membrane
 membrane
 shearing
 substitution
 K E
 = K E inflection
 - K E inflection
 + K E inflection
 + K E transverse
 G
 G
 G
 G
 G

traditional
 traditional

traditional
traditional
traditional

where:

normal
complete
membrane
K E inflection
G

is numerically integrated on the points of normal integration with a matrix

traditional
NR

supplements, and the local constraints of membrane inflection only;

2

normal
incomplete
membrane
K E inflection
G

is numerically integrated on the points of normal integration with a matrix

traditional
NR

incomplete, and local constraints of membrane inflection only;

2

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**:
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**reduced
incomplete
membrane
K E inflection**

**G
is numerically summoned on the points of integration reduced with a matrix
traditional
NR**

**incomplete, and integrated local constraints of membrane inflection only;
2**

**reduced
complete
shearing
K E transverse**

**G
is numerically summoned on the points of integration reduced with a matrix
traditional
NR**

**supplements, and the integrated local constraints of transverse shearing only;
2**

**To be able to calculate the two last tangent matrices in the preceding equation, us
let us carry out the numerical integration of the local constraints on the NPGSN points of integration
normal:**

**~
S (INTSR) =
NR (INTSR) ~S (INTSN) det J D D
1**

**D
I
2
3
INTSN =,**

1 NPGSN

This equation contains the terms of weight of the points of Gauss.

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5

Rigidity around the transform of the normal

5.1

Singularity of the tangent matrix of rigidity

Although the finite elements objects of the hull are expressed directly in the total reference mark

[E: E: E

1

2

***3] (the degrees of freedom are displacements and rotations in the total reference mark),
the tangent matrix of rigidity presents a singularity compared to the component of rotation
around the transform of the normal in each node:***

K E N

T

= 0

W
W

I=1, NB2

The contributions (W N)

are null.

I =1, NB2

In the preceding equation, this matrix represents the rigidity of rotation in the total reference mark.

Its

structure is full:

K
K
K
[
11
12
13
K and] = K
K
K

I
12
22
23
W
W

K
K
K
31
32
33 I

it is a nonsymmetrical matrix.

This singularity is a direct consequence of the kinematics of hull. It is due to the product vectorial appearing in linearized displacements (virtual and incremental). Thus displacement between two iterations is given by:

H
 $U(1,2,3) = U(1,2) + 3W(1,2)N$

Q
p
(1,2)
2

H
It is noticed that the contribution
W(,)

3
1 2 N (1,2) is perpendicular to N. One
2

interpret this singularity in the following way: the rotation of an initially normal fibre on the surface average does not lead to an elongation of this one, and consequently does not induce deformation.

5.2 Principle of virtual work for the terms associated with rotation around the normal

We propose to define the full slewing around the transform of the normal in the hull like the projection of the vector of full slewing on the transform of the normal:

= .n
N
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It is pointed out that the vector of rotation is an invariant of the matrix of rotation = exp [×]

=

The vector of rotation is a clean vector of the matrix of rotation associated with the eigenvalue identity. So the first relation is rewritten:

= (). (N)

N

= .

N

= N

This relation translates an important result:

The projection of the vector of full slewing on the transform of the normal is equal to projection of the vector of full slewing on the initial normal

In discrete form, one defines a deformation energy associated with this rotation:

1

2

=

K

2

(

N

N) I

I = I, NB2

where K is a rigidity of torsion of which the determination of the value will be discussed further. One supposes

that this rigidity remains constant and undergoes neither virtual variation nor incremental variation.

The existence of the potential is supposed:

1

$$= \mathbf{K} \left(\begin{matrix} \mathbf{N} \\ \mathbf{N} \end{matrix} \right) \mathbf{N}^2 \mathbf{I} \quad \mathbf{I} = \mathbf{1}, \text{NB2}$$

that one can rewrite in a more elegant form:

$$\mathbf{I} = \mathbf{K} \left(\begin{matrix} \mathbf{I} \\ \mathbf{N} \mathbf{N} \end{matrix} \right) \mathbf{N}^2 \mathbf{I} \quad \mathbf{I} = \mathbf{1}, \text{NB2}$$

By exploiting the property of orthogonality $\mathbf{-1} = \mathbf{T}$ of the matrix of rotation:

$$\mathbf{T} \mathbf{N} \mathbf{N} \mathbf{N} = \mathbf{N} \mathbf{T} = \mathbf{nnT} \left(\begin{matrix} \mathbf{N} \end{matrix} \right) = \mathbf{N} \mathbf{N}$$

This property will be exploited in the double linearization of the potential energy.

One rewrites the potential in the form:

I

=

K ([
N N])

N

2

I

I =1, NB2

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The first linearization of, makes it possible to obtain the virtual variation:

N

I

=

K

(

[

N N] + [

N N

])

N

$$2$$

$$I$$

$$I = 1, NB2$$

$$= K$$

$$($$

$$[$$

$$N N]) I$$

$$I = 1, NB2$$

It is necessary to express this form according to the function tests of rotations retained in variational form.

$$= -$$

$$T I ($$

$$) W$$

with the form of the matrix reverses of the differential operator of rotation:

$$1$$

$$1$$

$$T-1 () = 2 [I] - [x] +$$

$$1- 2 []$$

$$2$$

$$2$$

$$\tan$$

$$\tan$$

$$2$$

$$2$$

From where the final form of the virtual work which makes it possible to deduce the vector from the interior forces:

$$- T$$

$$= K$$

$$(wT () [N N])$$

$$N$$

I
I =,
1 NB2

One carries out the second linearization of:
N

- T
- T
= K
W. T () [N N] + T () [N N]

N
((

)
=
I
I
,
1 NB2

with the particular choice of the ddls of rotation $W = 0$, and owing to the fact that the initial normal “does not move not “during the iterations $N = 0$.

The expression of the tangent operator who gives rise to the terms corresponding to the ddls of rotation around the transform of the normal of tangent matrix is as follows:

- T
-I
- T
= K
W. T () [N N T
J
() W
+ K
W. T
() [N N]

N
((

$\)) I$
 $(($
 $)$
 $I =,$
 $I NB2$
 $=$
 I
 I
 $,$
 $I NB2$

In this relation, the last term is a differential term due to the nonlinear relation between parameters of rotation. Its linearization is heavy to carry out and its contribution will be neglected in the expression of the tangent operator.

With the property: N
 N

= $N N$, we give the final expression:

$- T$

 $-I$
 K
 $W. T () [N N T$
 $] W$
 $() W$
 N
 $(($
 $))$
 $=$
 I
 I
 $,$
 $I NB2$

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The contribution of operator T-1 is noted () [I] in the tangent matrix of rigidity.

5.3 Notice

The potential energy brought by the terms of rotation around the transform of the normal is nonnull even for a rigid rotational movement. This energy does not correspond to one deformation. For this reason it must be nonsignificant. The default value of COEF_RIGI_DRZ must guarantee that.

5.4

Borderline case analysis geometrically linear

In the case of the analysis geometrically linear, initial configuration and configuration deformation are confused what leads us to confuse initial normal N with its transform N:

NN

Rotations become small in this case and the operator of great rotation becomes:

= exp [×] [I] + [×]

The differential operator of rotations becomes:

T () [I]

and the parameters of rotations become confused:

W and W

All these approximations introduced into virtual work lead to its simplification:

K
(
[
 $N N$]
 N
 I
 $K =,$
 $I NB2$

The same approximations introduced into the differential part of virtual work also lead to its simplification:

K
(
[$N N$
]
 N
 I
 $K =,$
 $I NB2$

The two last equations are those of the analysis geometrically linear. They show that contributions in the vector of the interior forces and the tangent matrix of rigidity cover the borderline case well with [R3.07.04].

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5.5

Determination of the coefficient K

The coefficient K is calculated with each iteration of each step of time. With each iteration of Newton of each step of time, one builds with the NB2 nodes the matrix of passage

***I = [T
: T
: N
1
2
]; I = 1, NB2
I***

who allows to pass from the vector W I, vector of iterative rotation to the node expressed in the reference mark

total [E: E: E

***1
2
3] with the vector ~
W I expressed in local reference mark [T: T: N
1
2
]: I***

***~w I = Iw; =,
I
I
1 NB2***

One can build in each node, the matrix [KeT] of size 3×3

***I
~
W W***

***[K E]
T***

**= [K E]
~ ~T I
I
T I
I
W W**

**W
W**

This matrix represents the rigidity of rotation in the local reference mark. Its structure, nonsymmetrical, is:

K

K

0

T T

tt

1 1

1 2

[K E] = K

K

0

~ ~T I

T T

T T

2 1

2 2

W W

K K

0

N T

N T

I

2
I

The coefficient K is then calculated according to the relation:

$$K = COEF_RIGI_DRZ \times KMIN$$

where COEF_RIGI_DRZ is a coefficient introduced like a mechanical characteristic of hull by the user. In traditional linear analysis of the hulls or plates, this coefficient is selected enters 0.001 and 0.000001. By defect it is worth 0.00001. In the case of great rotations calculated with great steps of load, one advise to use value 0.001.

~
KMIN is the minimum of the nonnull terms of rotation on the diagonal of K and.

KMIN =

MIN

K, K

I

, NB

T T

T T

=1

2

1 1

2 2

I

Note:

It would be undoubtedly more rigorous to calculate K with the first iteration of the first step of time and to store this value like invariant information during the iterations and the steps of following times. The experiment shows that this way of proceeding is often not optimal in measurement where the values of the coefficients of the matrices of rigidity can evolve/move in way important during a calculation in great displacements. The value of K determined initially can then become too small and the matrix rigidity to end up being singular.

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6 Buckling

linear

Linear buckling is presented in the form of a particular case of the geometrical nonlinear problem. It is based on the assumption of a linear dependence of the fields of displacements, deformations and constraints compared to the level of load, and this, before the critical load is not reached.

In a total Lagrangian formulation, one recalls that linearized balance can be written under variational form:

int - ext. = ext. - int

maybe in matrix form after discretization:

the U.K.U

T = U (F - R)

where the dependence of the tangent matrix of rigidity K T is nonlinear compared to the vector of nodal displacement generalized p =

U EP.

e=, Nel

1

If we suppose the linear dependence of displacement compared to the level of load, one can to write:

U = u0

where u_0 is the solution obtained following a linear analysis for $\lambda = 1$ by:

$$K U = F$$

where K_0 is the tangent matrix of initial rigidity. One can then develop the tangent matrix of rigidity in a linear way compared to the level of load:

$$K T = K_0 + (K_1 u_1 + K_2 u_2 + \dots + K_n u_n)$$

where K_1

K_2

U is the matrix of initial displacements depending on p_0 , traditionally neglected in Code_Aster, and K_1 the matrix of the initial constraints depending on the total tensor of the constraints

~

of Piola-Kirchhoff of second species $[S_0]$ and local vector S_0 . These constraints are voluntarily confused with the constraints of Cauchy. They are obtained by a postprocessing linear analysis.

For the rotation part of EP, the assumption of linearity of the deformations according to the level of load results in the equality of:

$$N = N$$

I

I

who leads us to confuse the initial normals n_0 with their transforms $N I$.

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The matrix of the initial constraints KE represents the constant part in geometrical part tangent matrix of rigidity. It is evaluated at the point pe_0 and $= 1$ with a transform of normal replaced by the initial normal:

$$KE = KE + KE$$

***traditional
nontraditional***

with the traditional part of the geometrical part (see [bib4] volume 1 p. 141):

T

$$E -1 NR -1 NR K = J \sim \sim$$

***SJ
det J***

D 1

D 2

D

traditional

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where the second matrix of derived from the functions of form becomes:

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and the generalized tensor of the total constraints:

3×3

[S]

0

0

9×9

$S = 0$

[S] 0

0

0

[S]

The nontraditional part of the geometrical part represents the terms uncoupled from rotation, not symmetrical. Since the current algorithm of resolution of the problem to the eigenvalues [R5.06.01]

$[K + (Ku + K$

0

$)] = 0$ (being the level of critical load) consider only matrices

symmetrical, one makes symmetrical, while dividing by two the sum with its transposed, the matrix:

1

K E

(I, I) =

×

×

nontraditional

[(zI) [N

I

)]

2

where N I is the normal with node I and zI a vector 3 × 3 with the node I = 1, NB2 of the nodal vector 3 × (3 × NB2) Z I:

.

.

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Z

Z

I

I =

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.

.

$I =,$
 1 NB
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Nodal vector Z I is similar to a vector of internal force and its expression is:

$Z =$
 $BT S$
 J
 I
 $DD D$
 $\sim \sim det$
 3
 1
 2
 3

with the operator of the iterative variation of the virtual deformations (third operator of deformations):

\sim
 $\sim NR$
 $B = HQJ 1$
 $-$
 3

3
who is highlighted by the relation:

$\sim T \sim$
 \sim
 \sim
 $B S det Jd$

$DD = Enon \text{ traditional } S$
 3
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Note:

For numerical integration in the thickness of the various terms of rigidity, we retain one diagram of Gauss at two points just like in elasticity for the geometrical linear hulls [R3.07.04].

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7

Establishment of the elements of hull in Code_Aster

7.1 Description

These elements (of names MEC3QU9H and MEC3TR7H) are pressed on meshes QUAD9 and TRIA7 which are of geometry curves [R3.07.04].

7.2 Use

These elements are used in the following way:

AFFE_MODELE (MODELING: "COQUE_3D"...) for the triangle and the quadrangle.

One calls upon routine INI080 for standard calculations of numerical integration.

AFFE_CARA_ELEM (HULL: (THICKNESS: "EP"

ANGL_REP

:
 (
 " ")
COEF_RIGI_DRZ
 :
 "CTOR")

to introduce the characteristics of hull.

ELAS: (E: NAKED Young: ALPHA.: RHO.:)

For a homogeneous isotropic thermoelastic behavior in the thickness one uses the key word ELAS in DEFI_MATERIAU where the coefficients E are defined, Young modulus, coefficient of Poisson, thermal dilation coefficient and density.

AFFE_CHAR_MECA (DDL_IMPO: (
DX.: DY.: DZ.: DRX.: DRY.: DRZ.: DDL of plate in the total reference mark.

FORCE_COQUE: (FX.: FY.: FZ.: MX.: MY.: MZ.:)

They are the surface efforts on elements of plate. These efforts can be given in total reference mark or in the reference mark user defined by ANGL_REP.

FORCE_NODALE: (FX.: FY.: FZ.: MX.: MY.: MZ.:)

They are the efforts of hull in the total reference mark.

7.3 **Calculation in geometrical nonlinear "elasticity"**

Calculation imposes the following instructions user:

COMP_ELAS: (RELATION: "elas"
COQUE_NCOU: 1 (or more)
DEFORMATION: "green_gr")

Numerical integration in the thickness is based on an approach multi-layer with 3 points of integration by layer. It is about the approach currently used in nonlinear material [R3.07.04]. Options of postprocessing SIEF_ELNO_ELGA of the constraints and VARI_ELNO_ELGA of variables intern (here null) by defect are activated with the convergence of each filed step.

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7.4 Establishment

Options FULL_MECA, RIGI_MECA_TANG, and RAPH_MECA are already active in the catalogues elementary mec3qu9h.cata and mec3tr7h.cata for material non-linearity. They direct it calculation towards /fort/te0414.f, then towards /fort/vdxnrl.f to calculate and store, inter alia, stamp tangent symmetrical rigidity in the address corresponding to mode MMATUUR PMATUUR.

For the geometrical nonlinear analysis, the calculation of the tangent matrix of rigidity is directed towards new routine VDG NLR. This matrix is not symmetrical and must be stored in the address corresponding to mode MMATUNS PMATUNS.

One defines the two local modes at the same time symmetrical and nonsymmetrical, at exit of the catalogues elementary. The tangent matrix of nonsymmetrical rigidity into nonlinear geometrical is stored with the address reserved for a nonsymmetrical matrix. On the other hand, if it is about nonmaterial linearity in small deformations, all the tangent matrix of rigidity is stored with the address corresponding to nonsymmetrical mode. The lower triangular part is duplicated starting from the triangular part higher. Material nonlinear calculation in small deformations thus proceeds also in not symmetrical.

7.4.1 Modification

TE0414

Calculation is directed towards /fort/vdgnrl.f when the type of behavior COMP_ELAS is checked, i.e. when the problem is nonlinear geometrical.

7.4.2 Addition of a routine VDGNLNR

According to the option, the /fort/vdgnlr.f routine has as a role of:

Options: FULL_MECA and RAPH_MECA:

To calculate the 6 components of the state of the local constraints of Cauchy (confused with the state of constraints of Piola-Kirchhoff of second species) at the points of normal numerical integration and its nodal vector of the internal forces. They are stored in local modes ECONT PG PCONTPR and MVECTUR PVECTUR respectively.

Options: FULL_MECA and RIGI_MECA_TANG:

To calculate and store the tangent matrix of nonsymmetrical rigidity in mode MMATUNS PMATUNS.

7.5

Calculation in linear buckling

Option RIGI_MECA_GE, inactive until now, is activated in the elementary catalogues mec3qu9h.cata and mec3tr7h.cata.

The new TE0402 is dedicated to the calculation of the matrix of geometrical rigidity due to the constraints initial for the buckling of Euler. One recovers the plane states of the local constraints of Cauchy (null Snn component) at the points of normal numerical integration of Gauss. These states of constraints must be obtained by postprocessing with the option of calculation SIEF_ELGA_DEPL following a linear analysis (mode ECONT PG PCONTRR).

In analysis of buckling of Euler, the constraints [] of Cauchy can be confused with constraints [S] of Piola-Kirchhoff of second species. Therefore we will keep notation [S].

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The matrix of rigidity of the initial constraints can be broken up into a traditional part symmetrical and a nonsymmetrical nontraditional part. First is calculated according to total tensor of constraints 3×3 , contrary to the second which, it, is calculated according to vector of the local constraints 5×1 .

Since the current algorithm of resolution of the problem to the eigenvalues [R5.06.01] does not consider that symmetrical matrices, we force the symmetry of the nontraditional part of the matrix geometrical before storing the higher triangular part of all the matrix in the mode MMATUUR PMATUUR.

8 Conclusion

The formulation that we have just described applies to the curved mean structural analyses in great displacements, whose thickness report/ratio over characteristic length is lower than 1/10. It comes in direct object from the finite elements described in the preceding reference material [R3.07.04] and used within the framework of small displacements and deformations. They rest on same meshes quadrangle and triangle.

Their formulation rests on an approach of continuous medium 3D into which one introduces one kinematics of hull of the Hencky-Mindlin-Naghdi type, in plane constraints, in the formulation weak of balance. The measurement of the deformations retained is that of Green-Lagrange, énergétiquement combined with the constraints of Piola-Kirchhoff of second species. The formulation balance is thus Lagrangian total. The transverse distortion is treated same manner that in [R3.07.04]. Rotations must remain lower than 2 because of the choice of update of great rotations established in Code_Aster for which there is not bijection between the vector of full slewing and the orthogonal matrix of rotation.

Because of singularity of the tangent matrix of rigidity compared to the component of rotation around the transform of the normal, one defines a fictitious deformation energy associated this rotation. With this rotation, one associates a rigidity of constant torsion. Interior efforts associated this potential energy are taken into account. This potential energy, nonnull, does not correspond with a physical deformation. One thus needs that it remains negligible, which the user can control in imposing a value of the being worth COEF_RIGI_DRZ from 103 to 105.

For the postprocessing of the constraints, one limits oneself to the framework of the small deformations. One then could to prove the identity enters the tensor of the constraints of Piola-Kirchhoff observed the initial geometry and the tensor of the constraints of Cauchy in the deformed geometry. Moreover, the state of the constraints being plane for the tensor of Piola-Kirchhoff, one finds this property for the state of stresses of Cauchy. It should be noted that in more general contexts, this property is not preserved.

Linear buckling is treated like a particular case of the geometrical nonlinear problem. It rest on the assumption of a linear dependence of the fields of displacements, deformations and constraints compared to the level of load, before the critical load. It results from it that one can to linearly develop the tangent matrix of rigidity compared to the level of load. One finds then the geometrical part of the matrices of nonlinear the geometrical general obtained while identifying deformed normal on the average surface and the initial normal, which is coherent with linearity of the deformations according to the level of load.

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Appendix 1: Flow chart of calculation in linear buckling

Local reference marks with the NB2 nodes [T: T: N

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Buckle on the points of normal numerical integration of Gauss

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~2 2

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2S

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recovery of the vector of the local constraints $S = 1 2 =$

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0

starting from the 6 components tensors stored in mode PCONTRR ~

St T

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~

· *formation of the symmetrical tensor 3×3 of the local constraints [S]*

tT

1 (1,2 ,3)

· *construction of the matrix of transformation P (*

T

1,2, 3) = t2 (1,2, 3) where T3 (

1 23) = (

N

1 2)

tT

3 (1,2 ,3)

T ~

· *calculation of the symmetrical tensor 3×3 of the total constraints [S] = P [S] P*

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3 9

[HSFM]

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for the nontraditional term, calculation of HQ = 2×9

[HSS]

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1)

(t2 (2))

(t2 () 3)

t2 ()

1 t2 (2)

t2 (2) t2 ()

3

t2 ()

3 t2 ()

1

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T3 ()

1)

(T3 (2))

(T3 () 3)

T3 ()

1 T3 (2)

T3 (2) T3 ()

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T3 ()

3 T3 ()

1

2t2 ()

1 T3 ()

1

2t2 (2) T3 (2) 2t2 ()

3 T3 ()

3

t2 ()

1 T3 (2) + T3 ()

1 t2 (2) t2 (2) T3 ()

3 + T3 (2) t2 ()

3

t2 ()

3 T3 ()

1 + T3 ()

3 T3 ()

1

2t

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3 T ()

3

T ()

1 T (2) + T ()

1 T (2)

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3 + T (2) T ()

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3 T

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· calculation of the matrix jacobienne opposite J -1 and the determinant det J

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· calculation of J -

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· calculation and numerical integration $Z =$

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· *calculation of the generalized tensor of the total constraints*

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· *calculation and numerical integration of the matrix of geometrical rigidity traditional*

T

E

-1 NR

-1 NR

K

=

J

~

~

SJ

det J

D D

1 D

2

traditional

3

2

2

Fine buckles on the points of integration

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Buckle on all the nodes of Lagrange with distinction of the super node

.

.

.

zI

· calculation of [zI ×] knowing that ZI =

.

.

.

I

=,

I NB2

· calculation of the vector normal N I and its antisymmetric matrix [nor ×]

· calculation of the nontraditional matrix of geometrical rigidity

3×3

K E

(I, I) =

,

nontraditional

[zI ×] [N

I ×]

I = I NB2

3×3

· addition of K E

(I, I) with distinction of the super node

nontraditional

Fine buckles on the nodes

Storage of the higher triangular part of K E

END

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Appendix 2: Flow chart of geometrical nonlinear calculation

Local reference marks with the NB2 nodes [T: T: N

1

2

] I

Beginning Buckles JN on the NB2 nodes

IF JN NB1

· recovery of the vector of total displacement already updated by MAJOUR:

U (II) ZR (IDEPLP IDEPLM 1 6* (JN

) 1 II); II 13; (JN 1, NB

I

=

+

- +

- +

=

=

) I

· recovery of the vector of full slewing already updated by MAJOUR

I (II) = ZR (IDEPLP -1+ 6* (JN -) 1 + II +)

3

; II = 1 3

,

; (JN = 1, NB)

1

ELSE JN

· recovery of the vector of full slewing

I (II) = ZR (IDEPLP -1+ 6* NB1+ II); II = 1 3; (JN = NB2)

END IF JN

· *calculation of the matrix of rotation I =*

[
exp I ×]

· *transformed normal N = N*

I

I

I

Fine Buckles on the NB2 nodes

M

U

v

W

nx - nx

N

y - ny

Calculation of EP

N

=

Z - nz I

M

I = 1, NB

l

M

N

X - nx

ny - ny

nz - nz

NB2

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Beginning Buckles INTSR on the points of normal reduced integration of Gauss

~

~

· construction of part of the operators B, B

1

2

with the J = 1, INTSR points of integrations to be able to extrapolate them

Fine INTSR Buckles on the points of normal reduced integration of Gauss

Beginning Buckles INTSN on the points of normal numerical integration of Gauss

· construction of the matrix of transformation:

< T

T

l(1,2,3) >

Tl (1,2, 3)

P (

T

l,2, 3) = < t2 (1,2, 3) > = t2 (1,2, 3)

< *T*

T

$3(1,2,3) >$

T

$3(1,2,3)$

where $T(,) = ($

N

$3 \ 1 \ 2$

3

1

$, 2$

)

· *calculation of the matrix jacobienne opposite J -1 and the determinant det J*

J-1

0

0

~

· *calculation of J -1 =*

0

J -

1

0

0

0

J -1

NR

· *calculation of the second matrix of derived from the functions of form*

1

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2

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()

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NR

NR

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1

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L = 1, NR 1
B

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U ~
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· calculation of
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· calculation of A

X = U

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, y
, X

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, Z
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, Z
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, y
, Z
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calculation of H Q
+ A

2 X

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· *calculation of the first operator of the deformations*

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$$1 U \sim NR$$

$$B = H Q$$

+ A

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2 X

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· *calculation of the vector of the local deformations $E = B EP$*

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· *calculation of the second matrix of derived from the functions of form*

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· calculation of the second operator of the deformations

~

U ~ NR

B = H Q + A

J 1

-

2

X

2

· calculation and numerical integration Re =

BT S

Jd D D

~ ~ det

2

1

2

3

· calculation of the matrix of behavior D

· calculation and numerical integration K E =

BT DB

J

m

DD D

~ ~ det

2

2

1

2

3

~

· construction of the symmetrical tensor 3×3 of the local constraints [S]

T ~

· calculation of the symmetrical tensor 3×3 of the total constraints $[S] = P [S] P$

~

T

~

NR

· calculation of $B = [$

H S] J 1

-

3

3

calculation and numerical integration $Z =$

BT S

J

I

DD D

~ ~ det

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3×3

[S]
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9×9

· calculation of the tensor generalized of the total constraints $S = 0$

[S] 0

0
0
[S]

· calculation and numerical integration of traditional rigidity

T

NR

NR

KE

=

J -1

SJ -

~

~ 1 det J

G

DD D

traditional

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Fine INTSN buckles on the points of integration

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Beginning Buckles JN on the NB2 nodes

.

.

.

zI

· calculation of [zI ×] knowing that ZI =

*I =,
1 NB
2*

· calculation of [nor ×]

3×3

· calculation of the matrix nonsymmetrical K E

(I, I) = [zI ×] [N

G

I ×

nontraditional

]

IF JN NB1

3×3

· addition of K eg

(I, I) with distinction of the extra-node

nontraditional

ELSE JN

3×3

· assignment of K eg

(I, I) with distinction of the extra-node

nontraditional

END IF JN

Storage of all the matrix nonsymmetrical K and

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Document: R3.07.06

**Treatment of offsetting for the elements of
plate DKT, DST, DKQ, DSQ and Q4G**

Summary:

The elements of plate [R3.07.03] are intended for the three-dimensional mean structural analyses. average layer of these structures always does not coincide with the plan of diagram or plan of grid. One introduces thus concept of offsetting of the average layer compared to the plan of diagram. It is usable for elements with taking into account of transverse shearing, or without this assumption.

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1 Introduction

With an aim of being able to analyze the behavior of slim structures of type plates, or surfaces curves approached by facets, whose average layer is excentré compared to the plan of load application, one introduces the concept of offsetting of the average layer compared to surface grid. Fields of displacement varying linearly in the thickness of the plate originate in the surface of grid, i.e. on the level of the surface of grid, only degrees of freedom of translation are necessary to the description of displacement.

The introduction of kinematics into the expression of the work of deformation makes it possible to obtain them rigidities of membrane, inflection and transverse shearing of the excentré element from those element are equivalent nonexcentré and of the distance from offsetting. The whole of calculations (out specific postprocessing) is thus made in a reference mark of diagram attached to the plan of the grid. By defect the results are thus obtained in the reference mark of the grid. For certain postprocessings, it is possible to have automatically these results in other reference marks insofar as the user indicate the position of the plan of postprocessing compared to the plan of the grid.

The distance from offsetting between the plan of the grid and the average layer of the plate is given in AFFE_CARA_ELEM on the same level as the thickness. A offsetting D positive means that surface average of the plate is actually at a distance dn of the element of plate with a grid, direction N being given by the normal to the element (see [§4.1] reference material [R3.07.03] of elements of plate for the construction of this normal). The adopted notations are those of the note [R3.07.03] on the elements of plates DKT, DST, DKQ, DSQ and Q4G.

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2 Formulation

2.1 Geometry

For the offset elements of plate, the surface of reference is given by the plan of diagram or plan grid (plane $X y$ for example). The average layer of the element is positioned compared to this surface reference. The thickness $H (X, y)$ must be small compared to other dimensions (extensions, radii of curvature) of the structure to be modelled. The figure [Figure 2.1-a] below illustrates our matter. Concerning the value of offsetting D , and the conditions of linearization of the inflection adopted in the theory, D will be taken so that an element thickness $d+h$ remains in the theory plates.

Solid 3D**Z****H****Y****B****X****L****R1****R2****Thickness $H < L, B, R1, R2$** **$Z = Z + D$** **H****H****Z** **$Z D -; D +$**

y

2

2

H

N

Plate

X

B

L

Z

offsetting $D > 0$

y

X

Grid

Appear 2.1-a

One attaches to the plan of diagram (the plan of the grid) a local reference mark orthonormé $Oxyz$ associated the plan of grid different from total reference mark $OXYZ$. The position of the points of the plate is given by Cartesian co-ordinates (X, y) in the plan of diagram (plane of the grid) and rise Z compared to it plan.

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2.2 Kinematics

The cross-sections which are the sections perpendicular to the average layer of the plate remain right-hand sides. The material points located on a normal at not deformed average surface remain on a line in the deformed configuration. It results from this approach that the fields of displacement vary linearly in the thickness of the plate. If one indicates by U, v, W them displacements of a point of the plan of diagram $Q(X, y, Z)$ according to X, y and Z , the kinematics of Hencky-Mindlin us gives:

$$U(X, y, Z) = U(X, y) + Z(X, y)$$

$$U(X, y, Z) = v(X, y) + Z(X, y)$$

$$U(X, y, Z)$$

Z

W (X, y)

0

W (X, y)

0

where: U, v, W are displacements of the plan of diagram;

and

X

y are respectively rotations of this plan compared to respectively axis X and axis Y.

One prefers to introduce two rotations $(X, y) = (X, y,) (X, y) = - (X, y)$. Deformations

X

y

y

X

three-dimensional in any point, with kinematics introduced previously, are thus given by:

= E + Z

xx

xx

xx

= E + Z

yy

yy

yy

2 = = 2nd + 2z

xy

xy

xy

xy

2 =

xz

X

2 =

yz

y

*where: ϵ_{xx} , ϵ_{yy} and ϵ_{xy} are the membrane deformations of average surface;
 X and y deformations associated with transverse shearings;
 κ_{xx} , κ_{yy} , κ_{xy} the deformations of inflection of average surface, which are written:*

U

$E =$

κ_{xx}

X

v

$E =$

κ_{yy}

y

v

U

$2nd =$

$+$

κ_{xy}

X

y

X

$=$

κ_{xx}

X

y

$=$

κ_{yy}

y

2

y

X

$=$

$+$

xy

y

X

W

= +

X

X

X

W

= +

y

y

y

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Note:

· in the theories of plate, the introduction of and makes it possible to symmetrize them

X

y

formulations of the deformations and the equilibrium equations [R3.07.03]. In the theories of hull, one uses rather

M

M

X and

and associated couples

and

compared to X

y

X

y

and y,

· the degrees of freedom which one chose are displacements and rotations of the plan of diagram and not those of the average layer. Indeed if one considers the superposition of several plates offset to carry out a material sandwich it cannot correspond to nodes of the grid that only one field of displacement and not the various fields displacements of the layers composing material.

2.3

Law of behavior

*The behavior of the plates is a behavior 3D in “plane constraints”. **The constraint transversal zz is taken null** because negligible compared to the other components of the tensor of constraints (assumption of the plane constraints). The most general law of behavior is written then as follows:*

E

0

xx

xx

xx

xx

E
O
yy

yy
yy
yy

E = 2nd

,
2
and

= 0
xy
= xy

xy
= C (,) xy = This + Z
C +

C with

O
O
X
xz

X 0
 0 yz
 y
 y

where: $C(,)$ is the matrix of local tangent rigidity in plane constraints;
represent the whole of the internal variables when the behavior is nonlinear.

For behaviors (for example of multi-layer) for which the distortions are coupled at the deformations of membrane and inflection, $C(,)$ is put in the form:

$$\begin{matrix} H \\ H C \\ C = \end{matrix}$$

$$\begin{matrix} H T \\ H \end{matrix}$$

$$C$$

where: $(,)$ is a symmetrical matrix 3×3 ;

$(,)$ a symmetrical matrix 2×2 ;

$C(,)$ a matrix 3×2 of coupling between the effects of membrane or inflection and of transverse shearing.

If it is uncoupled, there are $C(,) = 0$. Determination of $(,)$ within the framework of the theory of Reissner ([§ 2.2.3.2] of [R3.07.03]) is given in appendix. It is shown that it is equivalent to that not offset plates.

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3

Principle of virtual work

3.1

Work of deformation

The general expression of the work of deformation 3D for the element of excentré plate of the distance D compared to the datum-line is worth:

$d+h/2$

W

= (+

+ + +

FD

)

def

xx

xx

yy

yy

xy

xy

X
 xz
 y
 yz
 S
 $d-h/2$

where S is average surface, $dV=dx dy dz$ and where the position in the thickness of the plate varies between $d-h/2$ and $d+h/2$.

3.1.1 Expression of the resulting efforts

By adopting the kinematics of [R3.07.03], one identifies the work of the interior efforts:

$$W = \int_{def} (E \epsilon_{xx} + E \epsilon_{yy} + 2nd \epsilon_{xy} + M \kappa_x + M \kappa_y + 2 M \kappa_{xy} + T \gamma_x + T \gamma_y) dS$$

ϵ_{xx}
 ϵ_{yy}
 ϵ_{xy}
 κ_x
 κ_y
 κ_{xy}
 γ_x
 γ_y
 S

where:
 ϵ_{xx}

ϵ_{yy}
 $d+h/2$
 ϵ_{xy}

$$\mathbf{NR} = NR =$$

dz

yy

yy

$NR d-h/2$

xy

xy

M

xx

$d+h/2$

xx

$$\mathbf{M} = M =$$

dz

Z

yy

yy

$M d-h/2$

xy

xy

T

$d+h/2$

X

xz

$\mathbf{T} =$

$=$

dz

T

$y \text{ } d-h/2 \text{ } yz$

where: N_{xx} , N_{yy} , N_{xy} are the efforts resulting from membrane (in N/m);

M_{xx} , M_{yy} , M_{xy} are the efforts resulting from inflection or moments compared to the plan of diagram (in NR);

T_x , T_y are the efforts resulting from shearing or sharp efforts (in N/m).

3.1.2 Relation resulting efforts generalized deformations

The expression of the work of deformation is also written:

$$\begin{aligned}
 & d+h/2 \\
 & d+h/2 \\
 & W \\
 & = [(\\
 & C,) FD \\
 &] \\
 & = [eCe + Z \\
 & EC. + \\
 & EC. + zCe + z^2 \\
 & C + Z \\
 & C + (\\
 & CE + Z + dSdz \\
 &)]
 \end{aligned}$$

def

$S \text{ } d-h/2$

$S \text{ } d-h/2$

where: $C (,)$ is the matrix of local tangent rigidity (symmetrical matrix).

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This is still written:

H/2

W

= [eCe + (+ D)

EC. +

EC. + (+ D) This + (+ D) 2 C

C

C E

def

+ (+ D) + (+ (+ D) + dSd

)]

S - H/2

By using the expression obtained for Wdef in the preceding paragraph, one finds the relation following between the resulting efforts and the généraliées deformations:

NR = H E (H

dH)

H

m

+

MF +
m +
m
M = (**H**
dH
E
)
(H
2dH
D 2H)
(H
dH)
MF +
m
 +
F +
MF +
m +
F +
m
T = HT E (HT dHT)
H
m
 +
F +
m +
ct

with:

+h/2

+h/2

+h/2

H

m =

Hd Hmf = Hd Hf = H2d

- H/2

- H/2

- H/2

+h/2

+h/2

+h/2

H

ct =
H D Hm = H D H
H
D
C

F
=
C

- *H/2*
- *H/2*
- *H/2*

and:
E

xx

xx
X
E = E

,

,

yy
= *yy*
=

y
2nd
2
xy
xy

The matrices **Hm**, **Hf** and **Hct** are the matrices of rigidity out of membrane, inflection and shearing transverse, respectively, for the element of nonexcentré plate. **The Hmf** matrix is a matrix of rigidity of coupling between the membrane and the inflection for the element of nonexcentré plate. It is null if the element of plate is symmetrical compared to its average layer. **The Hm** matrix is one stamp rigidity of coupling between the membrane and the transverse distortion. The matrix **Hf** is one stamp rigidity of coupling between the inflection and the transverse distortion. These matrices are null for a null offsetting, except in the case of the multi-layer ones where they remain nonnull.

For an isotropic homogeneous elastic behavior, these matrices have as an expression:

$$\begin{matrix} 1 & \nu \\ 0 & \\ 1 & \nu \\ 0 & \\ E_h & \\ E_h^3 & \\ kE_h & 1 \ 0 \end{matrix}$$

$$\begin{matrix} H \\ H \\ H \end{matrix}$$

$$\begin{matrix} m = \\ \nu & 1 \\ 0 & \end{matrix}$$

$$\begin{matrix} , \\ F = \\ \nu & 1 \\ 0 & \end{matrix}$$

$$\begin{matrix} , \\ ct = \\ 2 & \end{matrix}$$

$$2$$

$1 - \nu$
 $1 - \nu$
 1
(
 $12 - \nu)$
 $1 - \nu$
 1
(
 $2 + \nu) 0 1$
 $0 0$

 $0 0$

2

2

and $\mathbf{Hmf} = \mathbf{Hm} = \mathbf{Hf} = 0$ bus there is a material symmetry compared to the $=0$ plan.

For the determination of the coefficient of shearing K one returns to [§2.2.3] of [R3.07.03].

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The system of relation between the resulting efforts and the generalized deformations can be also

written:

$$\mathbf{NR} = \mathbf{H E H}$$

\mathbf{H}

\mathbf{m}

$$+ \mathbf{MF} + \mathbf{m}$$

$$\mathbf{M} = \mathbf{H E H}$$

\mathbf{H}

\mathbf{MF}

$$+ \mathbf{F} + \mathbf{F}$$

$$\mathbf{T} = \mathbf{HT E H T}$$

\mathbf{H}

\mathbf{m}

$$+ \mathbf{F} + \mathbf{ct}$$

with:

$$\mathbf{Hmf} = \mathbf{Hmf} + \mathbf{H}$$

$\mathbf{D m}$

\mathbf{H}

2

$$\mathbf{F} = \mathbf{Hf} + 2 \mathbf{H}$$

$$\mathbf{D MF} + \mathbf{D Hm}$$

$$\mathbf{H} = \mathbf{H} + \mathbf{dH}$$

\mathbf{F}

\mathbf{F}

\mathbf{m}

Thus, in the case of a plate having material symmetry compared to the plan $=0$, one has $\mathbf{Hmf} = 0$ but $\mathbf{H} = \mathbf{dH}$. The offsetting of the plate involves a coupling between the terms of membrane

\mathbf{MF}

\mathbf{m}

and of inflection.

Note:

The relations binding \mathbf{Hm} , \mathbf{Hf} , \mathbf{Hmf} with \mathbf{H} and \mathbf{Hct} with \mathbf{H} are valid whatever the law of elastic behavior tangent, with anelastic deformations (thermoelasticity, plasticity, ...).

For a plate made up of NR orthotropic layers in elasticity, the matrices \mathbf{Hm} , \mathbf{Hf} , \mathbf{Hmf} and \mathbf{Hct} are written:

NR
NR
NR
NR
H
H
H
H
H
H
H
H
H
H
m =

,

(

,

)

(

2

,

)

semi

MF =

mfi + I

semi

F =

fi +

I mfi + semi 2i

ct =

I

ct

i=1

i=1

i=1

i=1

l

where: =

Z + + Z

I

(I I I)

2

H_{mi}, H_{fi}, H_{mfi}, H_i represent the matrices of membrane, inflection, coupling membrane inflection and of transverse shearing for layer I. One notices the analogy between these expressions with form established above:

H = H

+ H

D

MF

MF

m

H = H +

H

2

D

2

+ D H

F

F

MF

m

One from of deduces whereas offsetting for such a plate is obtained in substituent + D with.

I

I

3.1.3 Energy interns elastic of plate

Taking into account the preceding remarks, energy interns elastic plate is expressed more usually for this kind of geometry in the following way:

1

=

[E (H E H

H

H E H

H

H E H

H

.

m

+ **MF** +

m) + (
MF + **F** + **F**) + (**T**
T

m
+ **F** + **ct dS**

)]
int

2 S

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3.1.4 Notice

One can choose to express the efforts resulting from inflection or moments compared to the average layer

element and either compared to the datum-line. In this case one obtains:

NR

M

xx

d+h/2

xx

xx
d+h/2
xx

T
d+h/2

NR = *NR* =

dz
, ***M*** = *M* =
(*Z - D dz*
) ,
X

xz
T =

=
dz
yy

yy

yy
yy
T

NR d-h/2

-

y

yz

M

D H/2

d-h/2

xy

xy

xy

xy

and the expression of the work of the interior efforts becomes:

$$W = (E NR + E NR + 2nd NR + (M + dN) + (M + dN) + 2 (M + dN) + T + T) dS$$

def
xx xx yy yy

xy

xy

xx

xx

xx

yy

yy

yy

xy

xy

xy

X

X

y

y

S

One then deduces from it by using the expression 3D from work from deformation that:

$$NR = H E (H dH) H m$$

+
MF +
m +
m
M + $dN = (H$
 $dH E$
 $)$
(H
 $2dH$
 $D 2H)$
(H
 $dH)$
MF +
m
 +
F +
MF +
m +
F +
m
T = HT E (HT dHT)
H
m
 +
F +
m +
ct

That is to say still:

NR = H E (H
 $dH)$
H
m
 +
MF +
m +
m
M = H E (H
 $dH)$
H
 .
MF
 +

$$\begin{aligned}
 & \mathbf{F} + \\
 & \mathbf{MF} + \\
 & \mathbf{F} \\
 & \mathbf{T} = \mathbf{HT E} (\mathbf{HT} \\
 & d\mathbf{HT}) \\
 & \mathbf{H} \\
 & \mathbf{m} \\
 & + \\
 & \mathbf{F} + \\
 & \mathbf{m} + \\
 & \mathbf{ct}
 \end{aligned}$$

The expression of the internal energy of the plate remains unchanged of course as for it. In the case of elasticity, it is always written:

$$\begin{aligned}
 & 1 \\
 & = \\
 & [E (\mathbf{H E H} \\
 & \mathbf{H} \\
 & \mathbf{H E H} \\
 & \mathbf{H} \\
 & \mathbf{H E H} \\
 & \mathbf{H} \\
 & \mathbf{m} \\
 & + \mathbf{MF} + \\
 & \mathbf{m}) + (\\
 & \mathbf{MF} + \mathbf{F} + \mathbf{F}) + (\mathbf{T} \\
 & \mathbf{T} \\
 & \mathbf{m} \\
 & + \mathbf{F} + \mathbf{ct} dS \\
 &)] \\
 & \int \\
 & 2 S
 \end{aligned}$$

The question of the choice of the plan interesting to use for the expression of the moments can vary from one situation with another.

M
 M
 M
 M

In the case of the figure of right-hand side, the approach developed above is preferable because the expression of loadings is defined compared to the average layer of each plate. In the case of the figure of left, if one wishes to replace the multi-layer hull by two offset hulls, the axis of reference is the average layer of the multi-layer hull. One thus may find it beneficial with all to define compared to plan of diagram. It is this approach which is adopted in the code. All the loadings applied are regarded as being defined by defect in the reference mark of diagram or plan of the grid. If ever certain loadings are defined compared to other plans (average layer, higher layer or inferior) is with the user to make the adapted changes of reference mark, with the hand or by the means command file by specifying the plan of load application when that is possible (see [§5]), to bring back itself to a loading defined in the plan of the grid.

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3.2

Work of the forces and couples external

The work of the forces and couples being exerted on the plate is expressed in the following way:

$d+h/2$

$d+h/2$

W

$F \cdot FD$

U

$$\begin{aligned}
 & \mathbf{F} \cdot d\mathbf{S} \\
 & \mathbf{U} \\
 & \mathbf{F} \cdot \mathbf{U} \, ds \\
 & dz \\
 & \text{ext.} = \mathbf{v} \\
 & + \mathbf{S} \\
 & + \mathbf{C} \\
 & S \, d-h/2 \\
 & S \\
 & C \, d-h/2
 \end{aligned}$$

where: \mathbf{Fv} , \mathbf{Fs} , \mathbf{FC} are the voluminal, surface efforts and of contour being exerted on the plate, respectively.

C is the part of the contour of the plate to which the efforts of contour \mathbf{FC} are applied.

With the kinematics of [§2.2], one determines as follows:

$$\begin{aligned}
 & W \\
 & (\mathbf{F} \cdot \mathbf{U} \, \mathbf{F} \cdot \mathbf{v} \, \mathbf{F} \cdot \mathbf{W} \cdot \mathbf{C} \\
 & \mathbf{C} \\
 &) \, dS \\
 & (\mathbf{U} \\
 & \mathbf{v} \\
 & \mathbf{W} \\
 &) \, ds \\
 & \text{ext.} = \\
 & X \\
 & + y + Z + xx + yy \\
 & + X + y + z + xx + yy \\
 & S \\
 & C \\
 & = (\mathbf{F} \cdot \mathbf{U} \, \mathbf{F} \cdot \mathbf{v} \, \mathbf{F} \cdot \mathbf{W} \cdot \mathbf{C} \\
 & \mathbf{C} \\
 &) \, dS \\
 & (\mathbf{U} \\
 & \mathbf{v} \\
 & \mathbf{W} \\
 &) \, ds \\
 & X \\
 & + y + Z + yx - xy \\
 & + X + y + Z + yx - xy \\
 & S \\
 & C
 \end{aligned}$$

where are present on the plate:

$\cdot F, F, F$ surface forces acting according to X, y and Z ;

X

y

Z

$+h/2$

\cdot

$F =$

$\mathbf{F E}$

$\cdot dz + \mathbf{F E}$

\cdot where \mathbf{E}

I

$\mathbf{v I}$

\mathbf{S}

\mathbf{I}

X and \mathbf{e}_y are the basic vectors of the tangent plan and \mathbf{e}_z them

$- H/2$

normal vector;

$\cdot C, C$: surface couples acting around axes X and y ;

X

y

$+h/2$

H

\cdot

$C =$

Z

$[(+d) \mathbf{E}$

where \mathbf{E}

$\mathbf{Z F}] \cdot e dz$

\mathbf{v}

\mathbf{I}

$+ [(D \pm) \mathbf{e}_z \mathbf{F}] \cdot e$

I

\mathbf{S}

\mathbf{I}

2

$X, \mathbf{e}_y, \mathbf{e}_z$ are the basic vectors

$- h/2$

previously definite.

and where are present on the contour of the plate:

· X, y, Z linear forces acting according to X, y and Z ;

+ $h/2$

·

$\mathbf{F} \cdot \mathbf{e} dz$ where \mathbf{E}

$I =$

$\mathbf{C} \mathbf{I}$

$\mathbf{X}, \mathbf{e}_y, \mathbf{e}_z$ are the basic vectors previously definite;

- $H/2$

· X , there linear couples around axes X and y ;

+ $h/2$

·

$[(Z D) \mathbf{E}$

$\mathbf{F}] \cdot \mathbf{e} dz$ where \mathbf{E}

$I =$

+ $\mathbf{Z} \mathbf{C} \mathbf{I}$

$\mathbf{X}, \mathbf{e}_y, \mathbf{e}_z$ are the basic vectors previously definite.

- $H/2$

Note:

The moments compared to Z are null. The efforts and the couples are expressed in locate grid. All calculations are made by defect in the reference mark of diagram. If efforts or of the couples are expressed in another reference mark (that of the average layer of plate for example) the user will have to make conversions with the hand if it uses the options by defect or to specify the plan of load application (see the paragraph [[§ 5](#)]).

3.3 Principle of virtual work and equilibrium equations

This paragraph is unchanged compared to the paragraph [[§3.3](#)] of [[R3.07.03](#)].

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4

Numerical discretization of the variational formulation exit of the principle of virtual work

4.1 Introduction

The variational formulation for energy interns enables us to write:

W

= [E (H E H

H

H E H

H

H E H

H

m

+ MF +

m) + (

MF + F + F) + (T

T

m

+ F + ct dS

)]

int

S

with:

U

, X

X, X

$W, X + X$

$E =$

v

,

,

, y

=

y, y

=

$W, y + y$

U

v

, $y +$

, X

$X, y + y, X$

The five degrees of freedom are displacements in the plan of the grid U and v , except plan W and them two rotations X and Y .

The elements DKT and DST are triangular isoparametric elements. Elements DKQ , DSQ and $Q4$ are quadrilateral isoparametric elements. They are represented below:

4

3

3

1

y

2

1

2

X

Appear 4.1-a: Real elements

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The elements of reference are presented below:

(0,1)

(-1,1)

(1,1)

3

4

3

1

2

(0,0)

(1,0)

1

2

(1,-1)

(-1,-1)

Appear 4.1-b: Elements of reference triangle and quadrangle

One defines the reduced reference mark of the element as the reference mark (,) of the element of

reference. The reference mark

room of the element, in the plan of diagram (X, y) is defined by the user, by key word *ANGLE_REP*. $X1$ direction of this local reference mark is the projection of a direction of reference D in the field of the element.

This direction of reference D is chosen by the user who defines it by two nautical angles in the total reference mark. The normal NR in the plan of the element (12 13 for a triangle numbered 123 and 12 14

for a quadrangle numbered 1234) the second direction fixes. The vector product of the two vectors previously definite $Y1=N X1$ make it possible to define the local trihedron in which they will be expressed

generalized efforts representing the state of stresses. The user will have to take care that the axis of reference selected is not found parallel with the normal of certain elements of plate. By defect, the direction of reference D is axis X of the total reference mark of definition of the grid.

Note:

For the elements of plate *QUAD4*, the use of a noncoplanar element can lead to irregularities ([bib1]). In this case, the user is alerted.

4.2

Discretization of the field of displacement

The matrix jacobienne $\mathbf{J} (,)$ is:

NR

NR

X

y

$NR X$

$NR y$

$I,$

I

$I,$

I

J

J

,

,

11

12

i=1

i=1

J =

=

NR

NR

=

X

y

J

J

,

,

21

22

NR X

NR y

I,

I

I, I

i=1

i=1

Moreover:

J
J
X

-
1 J
- J
11
12
1
22
12

= J

with

J =
= J =
J
where

= det J = J J - J J

11 22
12 21

J
J

J - J

J

21

22

21

11

y

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The field of displacement is discretized by:

U

NR

K

=

U

NR,

K ()

K

v k=1

v

W

NR

wk

2 NR

0

$$= NR(,) + [$$

$P(,)$

X

K

xk

xk

$K]$

$$K = I$$

$$K = NR$$

$$+ I P$$

(,

y

yk

yk

)

In this last expression, the term between hooks is present for the elements of type DKT, DST, DKQ or DSQ, but not for the Q4 elements.

4.3 ***Taking into account of the transverse distortion***

It is pointed out that the essential difference between elements DKT, DKQ on the one hand and DST, DSQ, Q4 in addition comes owing to the fact that for the first the transverse distortion is null is still = 0. difference between Q4 and elements DST and DSQ comes from a choice different of interpolation for representation of transverse shearing. The introduction of offsetting leads to a treatment private individual of transverse shearing.

One replaces in the expression of the internal energy established with [§4.1] by where are them deformations of substitution checking = in a weak way (integral on the sides of the element), and such as:

$$NR = H E H$$

H

m

$$+ MF + m$$

$$M = H E H$$

$$H$$

$$MF$$

$$+ F + F$$

$$T = H T E H T$$

$$H$$

$$m$$

$$+ F + ct$$

$$J$$

One checks thus that on the sides ij of the element, one $a: () ds 0$ with $= W +$.

$$S - S$$

$$=$$

$$S$$

$$, S$$

$$S$$

$$I$$

4.3.1 For the Q4 elements

One linearly discretizes the constant field by side so that:

$$1 -$$

$$1$$

$$12$$

$$+ 34$$

$$+$$

$$=$$

$$= 2$$

$$2$$

$$1 -$$

$$1$$

23

-

41

+

2

2

By using the relations then:

+1

(- (
W +))

D = 0;

,

-1

+1

(- (
W +))

,

D = 0

-1

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it is established that:

ij
I
 $= (W - W + +)$

2
J
I
I

J

for (ij) = (12,34) and (kp) = (23,41).
kp
I
 $= (W - W + +)$

2
p
K
p

K

By deferring the two results above in the expression of
, it is established that:

$= = B U$

w1

I

I
Nk, K Nk,
0

where: U = M and B = (B, B

B

I L
)

NR with K = Nk,
0
K Nk,
wN

NR

NR

Moreover, like:

I J
J
11
12 xi

=

I J
J
21
22 yi
one deduces from it that
= Drunk F

w1

x1

y

1

where: U

M

F =

and B = (B, B)

1 L

NR

W

NR

xN

yN

Nk, K Nk J

,

11

K Nk J

,

12

with: B

K =

NR

K, K Nk J

,

21

K Nk J

,

22

Finally:

X J

J

11

12

=

= B U with B

= jB

=

C

F

C [2×3N]

y

J

J

21

22

Note:

This treatment is equivalent to that of the Q4 elements not offset of [§ 4.3.2.1] of [R3.07.03].

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4.3.2 For the elements of type DKT, DST, DKQ, DSQ

With regard to the transverse distortions, one knows that:

T = M
+ M
and T = M
+ M
with M = H E H
H

MF
+ F + F
X
xx, X
xy, y
y
yy, y
xy, X

One deduces from it that:

C
C
T = H U
H

m
+ F
, xx
, xx

Calculation of:

C
C
H H
m
F

where: T
=

, xx
(X, xx X, yy X, xy y, xx y, yy y, xy)

T
U
= U
U
U
v
v
v

, *xx*
(, *xx, yy, xy, xx, yy, xy*)

MF
MF
MF
MF
MF
MF
H
H
2H
H
H
H
H
C
11
33
13
13
23
12 +
MF
with: H

m =
33

MF
MF
MF
MF
MF
MF

MF

H

H

H

H

H

H

2H

13

23

12 +

33

33

22

23

F

F

F

F

F

F

H

H

2H

H

H

H

H

C

11

33

13

13

23

12 +

F

H

F =

33

F
F
F
F
F
F
F

H
H
H
H
H
H
2H
13
23
12 +
33
33
22
23

where them

MF

H are the terms (I, J) of H and where F

H are the terms (I, J) of H.

ij

MF

ij

F

Like:

NR

2N

NR

2N

2

2

$$\begin{aligned}
&= NR (,) \\
&+ P \\
&(, \\
&) \\
&= NR (,) \\
&+ (JP + 2j JP + JP \\
&)
\end{aligned}$$

X, xx
K, xx
xk
xk, xx
K
K, xx
xk
11 xk,
11 12
xk,
12
xk,
K
K 1
 =
K =N 1
 +
K 1
 =
K =N 1
 +
NR
2N
NR
2N
 2
 2

$$\begin{aligned}
&= NR (,) \\
&+ P \\
&(, \\
&) \\
&= NR (,) \\
&+ (JP + 2j JP + JP \\
&)
\end{aligned}$$

X, yy
 K, yy
 xk
 xk, yy
 K
 K, yy
 xk
 $21 xk,$
 $21 22$
 $xk,$
 22
 $xk,$
 K
 $K 1$
 $=$
 $k=N 1$
 $+$
 $K 1$
 $=$
 $k=N 1$
 $+$
 NR
 $2N$
 NR
 $2N$
 $= NR (,$
 $+ P$
 $(,$
 $)$
 $= NR (,$
 $+ (J J P + [J J + J J P$
 $]$
 $+ J J P$
 $)$

X, xy
 K, xy
 xk
 xk, xy
 K
 K, xy
 xk

11 21 xk,
11 22
12 21
xk,
11 21 xk,
K
K 1
=
K =N 1
+
K 1
=
K =N 1
+
NR
2N
NR
2N
2
2

= NR (,
+ P
(,
)
= NR (,
+ (JP + 2j JP + JP
)

y, xx
K, xx
yk
yk, xx
K
K, xx
yk
11 yk,
11 12
yk,
12
yk,
K
K 1

$$\begin{aligned}
&= \\
&\mathbf{K} = \mathbf{N} \mathbf{I} \\
&+ \\
&\mathbf{K} \mathbf{I} \\
&= \\
&\mathbf{K} = \mathbf{N} \mathbf{I} \\
&+ \\
&\mathbf{NR} \\
&2\mathbf{N} \\
&\mathbf{NR} \\
&2\mathbf{N} \\
&2 \\
&2
\end{aligned}$$

$$\begin{aligned}
&= \mathbf{NR} (,) \\
&+ \mathbf{P} \\
&(, \\
&) \\
&= \mathbf{NR} (,) \\
&+ (\mathbf{JP} + 2\mathbf{j} \mathbf{JP} + \mathbf{JP} \\
&)
\end{aligned}$$

$$\begin{aligned}
&y, yy \\
&\mathbf{K}, yy \\
&yk \\
&yk, yy \\
&\mathbf{K} \\
&\mathbf{K}, yy \\
&yk \\
&21 yk, \\
&21 22 \\
&yk, \\
&22 \\
&yk, \\
&\mathbf{K} \\
&\mathbf{K} \mathbf{I} \\
&= \\
&\mathbf{K} = \mathbf{N} \mathbf{I} \\
&+ \\
&\mathbf{K} \mathbf{I} \\
&= \\
&\mathbf{K} = \mathbf{N} \mathbf{I} \\
&+
\end{aligned}$$

NR

2N

NR

2N

= NR (,)

+ P

(,

)

= NR (,

+ (JJP + [JJ + JJP

]

+ JJP

)

y, xy

K, xy

yk

yk, xy

K

K, xy

yk

11 21 yk,

11 22

12 21

yk,

11 21 yk,

K

K 1

=

K =N 1

+

K 1

=

K =N 1

+

1 4

4 2

4

4

4 3

1

xx

with:

2

2

0

J NR

2j J NR

J NR

0

11

K, +

11 12

K, +

12

K,

2

2

0

J NR

2j J NR

J NR

0

21

K, +

21 22

K, + 22

K,

W

K

NR

0 J J NR

[J J

J J] NR

J J NR

0

1

11 21
K, +
11 22 + 12 21
K, +

, xx =
12 22
K,

2
2
xk
0
0
J NR
2j J NR
J NR
k=1
11
K, +
11 12
K, + 12
K,

2
2
yk
0
0
J NR
2j J NR
J NR
21
K, +
21 22
K, + 22
K,

0
0
J J NR
[J J
J J] NR
J J NR
11 21
K, +
11 22 + 12 21
K, + 12 22
K,
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the first contribution to in the expression above and:
, xx

2
J NR
2j J NR
J NR
0
11
K,

+ *11 12 K*, + 2

12
K,

2
J NR
2j J NR
J NR
0
21
K,
+ *21 22 K*, + *222 K*,

N

J J NR
[J J
J J] NR
J J NR
0
U
U

, *xx =*
11 21
K,
+ *11 22* + *12 21 K*, +

12 22
K,
K

2
2

0
J NR
2j J NR
J NR
v
K 1
11
K,
+ 11 12 K, +

=

12
K,
K

2
2

0
J NR
2j J NR
J NR
21
K,
+ 21 22 K, +

22
K,

0
J J NR
[J J
J J] NR
J J NR
11 21
K,

+ *11 22 + 12 21 K*, +

12 22

K,

that is to say still in matrix form that:

U, xx

X, xx

U, yy

X, yy

U

C

T

H, xy

C

H X, xy

= m

+

v

F

, xx

y, xx

v, yy

y, yy

v, xy

y, xy

2

2

C(JP
2j JP
JP
)
K
11 K, +
11 12 K, +

12 K,

2
2
C(JP
2j JP
JP
)
K
21 K, +
21 22 K, + 22 K,

W
NR
NR
K
2N

U
C(JJP
[JJ
JJP
]
JJP
)
C
K
K

C
K
C

K 11 21 K, + 11 22 + 12 21 K, + 11 21 K,

= Hm P

H

P

H

cm

+ F C xk

F

+

K

v

2

2

S (JP

2j JP

JP

)

k=1

K

k=1

k=N+1

K

11 K, +

11 12 K, +

12 K,

yk

2

2

S (JP

2j JP

JP

)

K
21 K, +
21 22 K, + 22 K,

S (J J P
[J J
J J P
]
J J P
)

K
11 21 K, +
11 22 + 12 21
K, + 11 21 K,
C P
K K,

C P
K K,
W
NR
NR
K
2N

U
C P
C
K
K

C
K
C
H
P
H
P
H T
= m cm

+ *F C xk*

K K,

+

F

2

v

S P

K 1

K

K 1

K NR 1

K K, K

=

=

= +

yk

S P

K K,

S P

K K,

W

NR

U

C

K

K

NR

K

2N
= Hm P
H
P
HT
T
cm
+ cf K
C
C
xk
F
2

+
ck

K
v
k=1
K
k=1

k=N+1
yk
= C
HPU
HPU
HTT
BU
BU
B
m cm
m +
C
C
F

C
F +

F
2 =
cm
m +

C
F +

C

Where:
u1

v
l
U =

m
M
one

v
NR

T
(T
T
=
C NR +
)
(
)
1 L c2 NR

2
2
J
J
2 J J

T
0
11
12

11 12

T
2
2
2
2 =
with T = J
J
2 J J

0 T
2
21
22
21 22
2

1j1 j21
12
J j22
11
J j22 + 12
J j21
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w1

x1

ly

U

F

= M

W NR

xN

yN

One can also write:

T = C

H U

H

B U

B U

B

m,

+ C

xx

F, xx =

cm

m +

C

F +

C

J

By using the relation () ds 0 with = W + for each side ij of the element, one

**S - S
=
S
, S
S
I**

can obtain K since this relation is still written:

**L
2
K
W - W +
(C + S + C + S) + L = L
J
I
K
xi
K
yi
K
xj
K
yj
K
K
sk
2
3**

**where:
= (C
S) = (C
S)
I
-
H [T - T
T
H E H
m
- F]**

sk
K
K
K
K
ct

= (*C*
S) *I*
H [(*B*
H B U
B
H B U
B
H B
m -
T
)
T
T
m
m
m + (
-

)
F
F
F + (
-

)
F
F]
K
K
ct
C
C
C

The relation above is still written in matrix form:

WITH = (A

W + A) U

+ A U

F

m

m

L

0

0 L

C

L

S

2

NR 1

+

NR 1

+

NR 1

+

NR 1

+

NR 1

+

with: To = 0

0

1

-

0

-

M

M

H (

T

B

- H B)

3

ct

C

F

F

0

0

L

LC

LS

2N

2N 2N

2N

2N

- 2LC

LS

2

L

C

LS

0

0

0

0

0

0

N+1 N+1

N+1 N+1

N+1 N+1

N+1 N+1

10

0

0

- 2LC

LS

2

LC

LS

0

0

0
K +1 K +1
K +1 K +1
K +1 K +1
K +1 K +1

Aw = -

2 0
0
0
0
0
0
- 2 L
C
L
S
2
L
C
L
S
2N-1 2N-1
2N-1 2N-1
2N-1 2N-1
2N-1 2N-1

2
LC
LS
0
0
0
0
2
LC
LS
2N
2N
2N 2N

L

L

-

2N

2N

2N 2N

L C

L

S

NR 1

+

NR 1

+

NR 1

+

NR 1

+

1-

With =

M

M

H (

T

B - H B)

ct

C

F

F

L C

L S

2N 2N

2N

2N

L C

L

S

NR 1

+

NR 1

+

NR 1

+

NR 1

+

I

With =

M

M

H (

T

B

- H B)

m

ct

cm

m

m

L C

L S

2N 2N

2N

2N

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As follows:
 $= P U + P U$

F
m
m

with:

-1
 $P = A (A + A)$

W

-1
 $P = A A$
m

m

what implies:

$T = (B$
 $B P) U$
 $(B$
 $B P) U$
 $cm +$
 C
m
 $m +$
 C
 $+$

C

F

Note:

For the elements of type DKT and DST, there is B . It results from it from the expressions

$$m = B = 0$$

C

C

simplified preceding equations.

4.4

Stamp elementary rigidity

4.4.1 Stamp elementary rigidity for the Q4 elements

One takes again the forms of the matrices of rigidity given to [§4.4.1] of the documentation of reference [R3.07.03] and one replace H by H, H by H and H per H. One will note that

MF

MF

F

F

F

F

in [R3.07.03] the results were presented without term of coupling membrane shearing transverse or transverse inflection shearing. They here are added.

4.4.2 Stamp elementary rigidity for elements DKT, DKQ

One takes again the forms of the matrices of rigidity given to [§4.4.1] of the documentation of reference [R3.07.03] and one replace H by H, H per H. Since the relation = 0 is

MF

MF

F

F

satisfied the couplings transverse membrane shearing or transverse inflection shearing are non-existent.

4.4.3 Stamp elementary rigidity for elements DST, DSQ

One a:

WE

$$=$$

$$E$$

$$($$

$$1$$

$$HE + H + H - HH - T) + ($$

$$1$$

$$HE + H + H - HH - T)$$

$$-1$$

$$+ TH$$

TdS =
int

m
MF
m
m
ct
MF
F
F
F
ct
ct
E

E
([
1
-
T
H - HH
H]E + [
1
-
T
H - HH
H]) +
([
1
-
T

H - H H
H] E + [
l
-
T
H - H H
H])
l
+ TH -

dS
T

m
m
ct
m
MF
m
ct
F
MF
F
ct
m
F
F
ct
F
ct
E

That is to say still:

We

=
E
(H E + H) + (T
H E + H)
l
+ TH -

TdS

int

m

MF

MF

F

ct

E

where:

l

-

T

H

H

H H H

m =

m -

m

ct

m

l

-

T

H

H

H H H

MF =

MF -

m

ct

F

l

-

T

H

H

H H H

F =

F -

F

ct

F
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From where:

E

W

(U B H B U

U B H B U

U B H B U

U B H B U

int

=

T T

m

m

m

m

m +

T

T

m

m

MF

F

F +
T
T
T
F
F
MF
m
m +
T
T

F
F
F
F
F
E
T
T
-1
T
T
-1
T
T
-1
T
T
-
+ B H B
B H
B U
U B H
B
U B H
B U
C
ct
C +
C
ct

cm
m +
m
cm
ct
C +
1

m
cm
ct
cm
m
T
T
-1
T
T
-1
T
T
-
+ B H B U
U B H
B
U B H
B U
C
ct
C
F
+ F C ct
C +
1

F

C
ct

C

F

T

T

-l

T

T

-

+ U B H B U

U B H

B U) dS

m

cm

ct

C

F

+

l

F

C

ct

cm

m

= T

T

U (B H B

)

dS U

U (B H B

)

dS U

U (B H B

)

dS U

U (B H B

)
dS U
m
m
m
m
m +
T
T

F
F
F
F
F +
T
T

m
m
MF
F
F +
T
T

F

T
F
MF
m
m
E
E
E
E
T
T
-I
T

T
-1
T
T
-1
T
T
-
+ (B H B
)
dS
(B H
B
)
dS U
U (B H
B
)
dS
U (B H
B
)
dS U
C
ct
C
+
C
ct
cm
m +
m
cm
ct
C
+
1

m cm

ct
cm
m
E
E
E
E
T
T
-1
T
T
-1
T
T
-
+(B H B
)
dS U
U (B H
B
)
dS
U (B H
B
)
dS U
C
ct
C
F
+ F
C
ct
C
+
1

F
C ct C

F
E
E
E
T
T
-1
T
T
-
+ U (B H B
)
dS U
U (B H
B
)
dS U
m
cm
ct
C
F
+
1

F
C ct cm
m
E
E
= T
THE U.K.U
THE U.K.U
THE U.K.U
THE U.K.U
K
The U.K.

K U

m

m

m +

T

F

F

F +

T

m

MF

F +

T

F

Fm

+ T

m

+ Tm m + T T

m

m

+ T

The U.K.

K U

THE U.K.U

THE U.K.U

THE U.K.U

F

+ T T F + Tm m F + Tf

m

m +

T

F

F

with:

K

[B H B

B H B

dS

]

m =

T

T

-

m

m

m +

l

cm

ct

cm

S

It is also known that $U = (U,$

) from where it results that:

F

F

K

B H B dS

F 11 = T

F

F

F

S

K

K

F 11

F 12

K

with: K

B H B dS

F 12 =

T

F =

T

K

K

F

F

F

F 12

22

S

K

B H B dS

F 22 = T

F

F

F

S

K

B H B dS

mf11 = T

m

MF

F

K

= K

K

with:

S

MF

(mf11
mf12)
K
B H B dS
mf12 = T

m
MF

F
S

T
K
= K
Fm
MF

Using the fact that = P U

+ P U one deduces from it that:

F
m
m

T
T
T
T
W

= U

K U + U

K U

+ U

K U + U

K U
int
m

m

m

F

F

F

m

MF

F

F

Fm

m

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where:

T

T

T

T

K = K + P (K

+ K) P

+ (K

+ K) P

+ P (K

+ K)

m
m
m
F 22
m
mf12
m
m
m
mf12
m
T
T
T
T
K = K
+ K + P (K

+ K) P

+ (K
+ K) P

+ P (K

+ K)
F
F 11
F 22
F 12
F 12

T
T
T
T
K = K
+ K + (K
+ K) P

+ P (K
+ K)
+ P (K

+ ***K***) ***P***

MF

mf11

m

mf12

m

m

F 12

m

F 22

T

K = K

Fm

MF

This is still written:

U

E

W

U, THE U.K.

int = (

m

m

F)

U F

K

K

m [2N×2N]

MF [2N×3N]

where: K

is the elementary matrix of rigidity for an element of

[5N×5N] =

T

K***K******MF [3N×2N]******F [3N×3N]******excentré plate DST.*****4.5*****Stamp of elementary mass******The terms of the matrix of mass are obtained after discretization of the variational formulation following:******d+h/2******W ac******=******U******& udzdS******mass******d-h/2 S******= (u& U******+ v& v******+ w& W******) + (+ D) (U******&******+ v******&******+ & U******+ & v******) + (+ D******2 + D 2) (& + &) dS******m******MF******m******X******y******X******y******F******MF***

m
X
X
y
y
S

+h/2
+h/2
+h/2
with
dz,
zdz and

,
Z dz.
m =
MF =
= 2
F
- H/2
- H/2
- H/2

Note:

If the plate is homogeneous or symmetrical compared to its average layer then $mf=0$.

4.5.1 Stamp of traditional elementary mass

4.5.1.1 Element

Q4

The discretization of displacement for this isoparametric element is:

U
K

v

NR
K

$U = NR W$

=

K

K

K

,...,

1

NR

$K 1$

=

xk

yk

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The matrix of mass, in the base where the degrees of freedom are gathered according to the directions of

translation and of rotation, has then as an expression:

M
0
0
M
0
m
MF

0
M
0
0
M
m
MF

M = 0
0
M
0
0

m

T
M
0
0
M
0
MF
F

T

0
M
0
0

M
MF
F

with:

T
M =
NR
dS
NR

m
m
S

M
= (+ D) T
NR
dS
NR

MF

MF
m
S

M = (+ D
2 + d2
) T
NR
dS
NR

F

F
MF
m
S

where: NR = (NR
NR

I
L
K).

For the continuation, one poses =

+ D and

2

= + D

2 + D.

MF

MF

m

F

F

MF

m

4.5.1.2 Elements of the type DKT, DST

Like:

W

W

0

NR

K

2N

= NR (,)

+ P (,)

X

K

xk

xk

K

K I=

$k = N I +$

$P ($
 $)$

y
 yk
 yk

where: $= P U +$
 $P U$
 m
 m
 F

one deduces from it that:
the U.K.

W

0
 0
 $NR (,)$
 0
 0
 v
 K

K

NR

NR
 $(, NR$
 $(, NR$
 $(, NR$
 $(, NR$
 $(, W$

.

X =

kxu

kxv

kxw

kxx

kxy

K

k=1

NR

(,) NR

(,) NR

(,) NR

(,) NR

(,)

y

kyu

kyv

kyw

kyx

kyy

xk

yk

The matrix of mass has then as an expression:

M

M

m

MF

M =

M

M
Fm
F
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The part membrane M of the elementary matrix of mass is composed of the blocks k_p (kth line m and pième column) following:

NR NR
0
NR NR
NR
NR
NR NR
NR NR
K
P
 $K_{pxu} + k_{xu} p$
K
 $pxv +$
 k_{yu}
p
m
+

MF

0

NR NR

NR NR

NR NR

NR NR

NR NR

K

p

K pyu + kxv p

K

pyv +

kyv

p

NR NR

NR NR

NR

NR

NR NR

kxu

pxu +

kyu

pyu

kxu

pxv +

kyu

pyv

+ F

NR NR

NR NR

NR NR

NR NR

pxu

kxv +

pyu

kyv

kxv
pxv +
kyv
pyv

The part inflection M is composed of the blocks kp (kth line and pième column) following:
F

NR NR
0 0
NR
NR
NR
NR
NR
NR
NR
NR
NR
NR
NR
NR
NR
K
P

kxw pxw + kyw pyw
kxw
pxx +
kyw
pyx
kxw
pxy +

kyw
pyy

0
0 0
NR
NR
NR NR

NR
NR
NR NR
NR
NR
NR NR

m
+ *F kxx pxw + kyx pyw*
kxx
pxx +
kyx
pyx
kxx
pxy +
kyx
pyy

0
0 0
NR NR
NR NR
NR NR
NR NR
NR NR
NR NR
kxy
pxw +
kyy
pyw
kxy
pxx +
kyy
pyx
kxy
pxy +
kyy
pyy

The coupling part between the membrane and the inflection M is composed of the blocks kp (kth line and

MF

pième column) following:

NR NR

NR NR

NR NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

K

pxw

K

pxx

K

pxy

kxu pxw + kyu pyw

kxu

pxx +

kyu

pyx

kxu

pxy +

kyu

pyy

MF

+

F

NR NR

NR NR

NR NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

K

pyw

K

pyx

K

pyy

$kxv pxw + kyv pyw$

kxv

pxx +

kyv

pyx

kxv

pxy +

kyv

pyy

The coupling part between the inflection and the membrane M is composed of the blocks k_p (k th line and

F_m

pième column) following:

NR

NR

NR

NR

NR NR

NR

NR

NR

NR

NR

NR

kxw

p

kyw

p

kxw

pxu +

kyw

pyu

kxw

pxv +

kyw

pyv

NR NR

NR NR

NR

NR

NR NR

NR

NR

NR NR

MF

kxx

p

kyx

p + F

kxx

pxu +

kyx

pyu

kxx

pxv +

kyx

pyv

NR NR

NR NR**NR NR****NR NR****NR NR****NR NR****kxy****p****kyy****p****kxy pxu + kyy pyu****kxy****pxv +****kyy****pyv**

4.5.2 Stamp of improved elementary mass

As the arrow of a flexbeam only can be represented with difficulty by one linear approximation, one can enrich the functions by form for the terms of inflection. This approach is used in Code_Aster for the elements of type DKT, DST and Q4G where functions of form used in the calculation of the matrix of mass of inflection are of order 3. The interpolation for W is written as follows:

NR**W = NR****(,) W****NR****(,) W****NR****(,) W****3 (K)****1 1****K +****3 (K)****1 2****, K +****3 (K)****1 3****- +****- +****- +**

, K
 $k=1$

4.5.2.1 Elements of type DKT

It is known that in the approximation of one Coils-Kirchhoff has $= - W$ and $= - W$ in any point of

X

, X

y

, y

the element.

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Because of discretization stated above one a:

NR

$W = NR$

(,) W

(J NR

(,) J NR

(,)W

(J NR

(,) J NR

(,)W

3 (K)

1 1

K +

11

3 (K)

1 2 +

21

3 (K)

1 3

, xk +

12

3 (K)

1 2 +

22

3 (K)

1 3

- +
- +
- +
- +
- +
- +
, y_k
 $k=1$

since:
 W
 J
 J
 W
, K

11
 12
, x_k
=

W
 J
 J
 W
, K
 21
 22
, y_k

This is still written:

NR
 $W = NR$
(,) W
 NR

$$\begin{aligned}
 & (,) \\
 & NR \\
 & (,) \\
 & 3 (K) \\
 & 1 1 \\
 & K + \\
 & 3 (K) 1 2 xk + 3 (K) 1 3 \\
 & - + \\
 & - + \\
 & - + \\
 & yk \\
 & k=1
 \end{aligned}$$

$$\begin{aligned}
 & NR \\
 & = NR (,) W NR (,) \\
 & NR \\
 & (,) \\
 & kww \\
 & K + \\
 & kwx xk + \\
 & kwy yk \\
 & k=1
 \end{aligned}$$

where:

$$\begin{aligned}
 & NR \\
 & (, \\
 & - + \\
 &) \\
 & = NR \\
 & (, \\
 & - + \\
 &)
 \end{aligned}$$

$$\begin{aligned}
 & 3 (K) \\
 & 1 1 \\
 & 3 (K) \\
 & 1 1 \\
 & NR \\
 & (, \\
 & - + \\
 &) \\
 & = - J NR \\
 & (, \\
 & - +
 \end{aligned}$$

)
- *J NR*

(,
- +
)

.
3 (K)
1 2
1 1

3 (K)
1 2
2 1

3 (K)
1 3
NR

(,
- +
)
= - *J NR*

(,
- +
)

- *J NR*
(,
- +
)

3 (K)
1 3
1 2

3 (K)
1 2
2 2

3 (K)
1 3

As follows:
the U.K.

W
0
0

NR
(,) *NR*
(,) *NR*
(,)
v
kww
kwx
kwy

K

NR

NR
(,) *NR*
(,) *NR*
(,)
NR
(,) *NR*
(,) *W*

X =

kxu
kxv
kxw
kxx
kxy

K

k=1

NR
(,) *NR*
(,) *NR*
(,)
NR
(,)
NR

(,)
y
kyu
kyv
kyw
kyx
kyy

xk

yk

By not taking account of the effects of inertia, the matrix of mass has the following form thus:

M
M
m
MF
M =

M
M
Fm
F

*The part membrane **M** of the elementary matrix of mass is composed of the blocks kp (kth line m and pième column) following:*

NR NR
0
NR NR
NR
NR
NR NR
NR NR
K
P

K pxu + kxu p

K

pxv +

kyu

p

m

+

MF

0

NR NR

NR NR

NR NR

NR NR

NR NR

K

p

K pyu + kxv p

K

pyv +

kyv

p

NR NR

NR NR

NR

NR

NR NR

kxu

pxu +

kyu

pyu

kxu

pxv +

kyu

pyv

+ F

NR NR

NR NR
NR NR
NR NR

pxu
kxv +
pyu
kyv
kxv
pxv +
kyv
pyv

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*The part membrane-inflection **M** is composed of the blocks kp (kth line and pième column) following:*
MF

NR NR
NR NR
NR NR
NR
NR
NR
NR
NR
NR

NR
NR
NR
NR
NR
NR
K
pxw
K
pxx
K
pxy
kxu pxw + kyu pyw
kxu
pxx +
kyu
pyx
kxu
pxy +
kyu
pyy
MF
+

F

NR NR
NR NR
NR NR
NR
NR
NR
NR
NR
NR
NR
NR
NR
NR
NR
NR

NR
K
pyw
K
pyx
K
pyy
kxv pxw + kyv pyw
kxv
pxx +
kyv
pyx
kxv
pxy +
kyv
pyy

*The part inflection-membrane **M** is composed of the blocks kp (kth line and pième column) following:*
Fm

NR
NR
NR
NR
NR NR
NR
NR
NR
NR
NR
NR
NR
kxw
p
kyw
p
kxw
pxu +
kyw
pyu
kxw
pxv +

kyw

pyv

NR NR

NR NR

NR

NR

NR NR

NR

NR

NR NR

MF

kxx

p

kyx

p + F

kxx

pxu +

kyx

pyu

kxx

pxv +

kyx

pyv

NR NR

NR NR

NR NR

NR NR

NR NR

NR NR

kxy

p

kyy

p

kxy pxu + kyy pyu

kxy

pxv +

kyy

pyv

*The term **M** of inflection is composed of the blocks kp (k th line and p ième column) following:
F*

*NR
NR
NR
NR
NR
NR*

*kww pww
kww
pwx
kww
pwy
NR NR
NR
NR
NR
NR
m
kwx
pww
kwx
pwx
kwx
pwy +*

*NR
NR
NR
NR
NR
NR
kwy
pww
kwy
pwx
kwy
pwy*

NR NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

kxw

pxw +

kyw

pyw

kxw

pxx +

kyw

pyx

kxw

pxy +

kyw

pyy

NR NR

NR NR

NR

NR

NR

NR

NR

NR

NR

NR

F

kxx

pxw +

kyx

pyw

kxx

pxx +

kyx

pyx
kxx
pxy +
kyx
pyy

NR NR
NR NR
NR NR
NR NR
NR NR
NR NR
kxy
pxw +
kyy
pyw
kxy
pxx +
kyy
pyx
kxy
pxy +
kyy
pyy

4.5.2.2 Elements of the DST type

It is known that for these elements one has $\epsilon = -W$ and $\nu = -W$ where the distortion is constant

X
X
, X
y
y
, y
on the element.

Like:

NR
W = NR
(,) W
(J NR

(,) J NR

(,) W

(J NR

(,) J NR

(,) W

3 (K)

1 1

K +

11

3 (K)

1 2 +

21

3 (K)

1 3

, xk +

12

3 (K)

1 2 +

22

3 (K)

1 3

- +

- +

- +

- +

- +

, yk

k=1

one can also write:

NR

W = NR

(,

- +

)

W + NR

(,

- +

)

+ NR

(,

- +

)

3 (K)

1 1

K

3 (K)

1 2

xk

3 (K)

1 3

yk

K 1

=

+ (**J + J**) **NR**

(,

- +

)

+ (**J + J**) **NR**

(,

- +

)

11 X

12 y

3 (K)

1 2

21 X

22 y

3 (K)

1 3

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where:

NR

(,

- +

)

= NR

(,

- +

)

3 (K)

1 1

3 (K)

1 1

NR

(,

- +

)

= - J NR

(,

- +

)

- J NR

(,

- +

)

3 (K)

1 2

1 1

3 (K)

1 2

2 1

3 (K)

1 3

NR

(,

- +

)

= - **J NR**

(,

- +

)

- **J NR**

(,

- +

)

3 (K)

1 3

1 2

3 (K)

1 2

2 2

3 (K)

1 3

NR

NR

(,)

NR

(,)

3 (K)

1 1 =

3 (K)

1 1

- +

- +

k=1

NR

NR

(,)

NR

(,)

3 (K)

1 2 =

3 (K)

1 2

- +

- +

k=1

NR

NR

(,)

NR

(,)

3 (K)

1 3 =

3 (K)

1 3

- +

- +

k=1

U

W

U

W

1

1

1

1

v

v

1

x1

1

x1

X

Iy

Iy

=

-I

H

[(B

B P)

(B

B P)

m

m

M

M] T

T

ct

C

+ C

+

C

+

C

= U M

+

M

W

y

W
W
NR

NR
U

U

NR
xN

NR
xN

v
v
NR
yN
NR
yN

One obtains the interpolation for W then:

NR
NR
W = NR
(,) U

NR

(,) v

NR

(,) W

NR

(,) v

NR

(,) v

5 (K)

1 1

K +

5 (K) 1 2 K +

5 (K) 1 3 K + 5 (K) 1 4 xk + 5 (K) 1 5

- +

- +

- +

- +

- +

yk

k=1

k=1

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where:

NR

(,
- +
)

= (**J T**

,
1

((
2 K

-)
1 +)
1 + J T (,
2 (
2 K

-)
1 +))
1
NR

(,
- +
)

5 (K)

1 1

1 1

U

1 2

U

3 (J)

1 2

+ (**J T**

,
1

((
2 K

-)
1 +)
1 + J T (,

2 2 (K

-)
1 +))
1 NR

(,
- +
)

21

U

22

U

3 (J)

1 3

NR

(,
- +
)

= (J T

,

1

((

2 K

-)
1 +)

2 + J T (,

2 (

2 K

-)
1 +))

2

NR

(,
- +
)

5 (K)

1 2

11

U

12

U

3 (J)

1 2

+ (J T

,

1

((

2 K

-)

1 +)

2 + J T (,

2 2 (K

-)

1 +))

2

NR

(,

- +

)

21

U

22

U

3 (J)

1 3

NR

(,

- +

)

= NR

(,

- +

)

5 (K)

1 3

3 (K)

1 1

+ (J T

(
3
,
1
(
K

-)
1 +)
1 + J T (
(
3
,
2 K

-)
1 +))
1 NR
(,
- +
)

11
W
12
W
3 (J)
1 2
+ (J T
(
3
,
1
(
K

-)
1 +)
1 + J T (
(
3
,
2 K

-)
I +))
I NR
(,
- +
)

2I
W
22
W
3 (J)
I 3
NR
(,
- +
)
= NR
(,
- +
)

5 (K)
I 4
3 (K)
I 2
+ (J T
(
3
,
I
(
K

-)
I +)
2 + J T (
(
3
,
2 K

-)
I + 2))NR

(,
- +
)

11
W
12
W

3 (J)
1 2
+ (J T

(
3
,
1
(
K

-)
1 +)
2 + J T (
(
3
,
2 K

-)
1 +))
2 NR

(,
- +
)

21
W
22
W
3 (J)
1 3
NR

(,
- +
)
= NR

(,
- +
)

5 (K)
1 5
3 (K)
1 3
+ (J T

(
3
,
1
(
K

-)
1 +)
3 + J T (

(
3
,
2 K

-)
1 +))
3 NR

(,
- +
)

11
W
12
W
3 (J)
1 2
+ (J T

(
3
,
1
(
K

-)
I +)
3 + J T (
(
3
,
2 K

-)
I +))
3 NR
(,
- +
)

21
W
22
W
3 (J)
1 3

This can be still written in the following way:

the U.K.

W
NR
(,) **NR**
(,) **NR**
(,) **NR**
(,) **NR**
(,)
v
kwu
kwv
kw w
kwx
kw y

K

NR

NR

(,)

NR

(,)

NR

(,)

NR

(,) NR

(,) W

X =

kxu

kxv

kxw

kxx

kxy

K

k=1

NR

(,)

NR

(,)

NR

(,)

NR

(,)

NR

(,)

y

kyu

kyv

kyw

kyx

kyy

xk

yk

The matrix of mass has the following form thus:

M

M

m

MF

M =

M

M

Fm

F

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The part membrane M of the elementary matrix of mass is composed of the blocks kp (kth line m

and pième column) following:

NR NR

NR

NR

NR

NR

NR NR

NR

NR

NR NR

NR NR

K

p +

kwu

pwu

kwu

pwv

K pxu + kxu p

K

pxv +

kyu

p

m

+

MF

NR

NR

NR NR

NR

NR

NR NR

NR NR

NR NR

NR NR

kwv

pwu

K
p +
kwv
pwv
K pyu + kxv p
K
pyv +
kyv
p
NR NR
NR NR
NR
NR
NR NR
kxu
pxu +
kyu
pyu
kxu
pxv +
kyu
pyv
+ F

NR NR
NR NR
NR NR
NR NR
pxu
kxv +
pyu
kyv
kxv
pxv +
kyv
pyv

The part membrane-inflection M is composed of the blocks kp (kth line and pième column) following:
MF

NR
NR

NR

NR

NR

NR

NR NR

NR NR

NR NR

kwu

pww

kwu

pwx

kwu

pwy

K pxw

K

pxx

K

pxy

m

+

MF

NR

NR

NR

NR

NR

NR

NR NR

NR NR

NR NR

kwv

pww

kwv

pwx

kwv

pwy

K pyw

K

pyx

K

pyy

NR NR

NR NR

NR

NR

NR NR

NR

NR

NR NR

kxu

pxw +

kyu

pyw

kxu

pxx +

kyu

pyx

kxu

pxy +

kyu

pyy

+ F

NR NR

NR NR

NR NR

NR NR

NR NR

NR NR

kxv

pxw +

kyv

pyw

kxv

pxx +

kyv

pyx

kxv

pxy +

kyv

pyy

*The part inflection-membrane M is composed of the blocks kp (kth line and pième column) following:
Fm*

NR

NR

NR

NR

NR NR

NR

NR

kww pwu

kww

pwv

kxw p

kyw

p

NR NR

NR

NR

NR

NR

NR NR

m

kwx

pwu

kwx

pwv + MF

kxx

p

kyx

p

NR

NR

NR

NR

NR NR

NR NR

kwy

pwu

kwy

pwv

kxy p

kyy

p

NR NR

NR

NR

NR

NR

NR

NR

kxw

pxu +

kyw

pyu

kxw

pxv +

kyw

pyv

+ NR NR

NR NR

NR

NR

NR NR

F

kxx

pxu +

kyx

pyu

kxx

pxv +

kyx

pyv

NR NR

NR NR

NR NR

NR NR

kxy

pxu +

kyy

pyu

kxy

pxv +

kyy

pyv

The term M of inflection is composed of the blocks kp (kth line and pième column) following:

F

NR

NR

NR

NR

NR

NR

kww pww

kww

pwx

kww

pyy

NR NR

NR

NR

NR

NR

m

kwx

pww

kwx

pwx

kwx

pyy +

NR

NR

NR

NR

NR

NR

kwy

pww

kwy

pwx

kwy

pwy

NR NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

NR

kxw

pxw +

kyw

pyw

kxw

pxx +

kyw

pyx

kxw

pxy +

kyw

pyy

NR NR

NR NR

NR

NR

NR

NR

NR

NR

NR

NR

F

kxx

pxw +

kyx

pyw

kxx

pxx +

kyx

pyx

kxx

pxy +

kyx

pyy

NR NR

NR NR

NR NR

NR NR

NR NR

NR NR

kxy

pxw +

kyy

pyw

kxy

pxx +

kyy

pyx

kxy

pxy +

kyy

pyy

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4.5.2.3 Elements of the Q4 type

One proceeds in the same way that for the elements of the DST type but with:

w1

x1

X

ly

= B M

C

y

W NR

xN

yN

where: Bc is the matrix established with [§4.3.1].

One deduces from it that:

the U.K.

W
0 0 NR
(,) NR
(,) NR
(,)
v
kww
kwx
kwy

K

NR

0 0
0
NR (,)
0
W

X =

K

K

k=1

0 0
0
0
NR (,)
y
K

xk

yk

The matrix of mass has the following form thus:

M
0
m

M =

0
MF

The part membrane M of the elementary matrix of mass is composed of the blocks kp (kth line m and pième column) following:

NR NR
0
K
P

m

0
NR NR
K
P

The term M of inflection is composed of the blocks kp (kth line and pième column) following:
F

NR
NR
NR
NR
NR
NR

0
0
0

kww pww
kww
pwx
kww
pwy

NR NR
NR
NR
NR
NR
0 NR NR
0

m
kwx
pww
kwx
pwx
kwx
pwy + F
K
p

NR NR
NR
NR
NR
NR
0
0
NR NR
kwy
pww
kwy

pwx

kwy

pwy

K

P

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4.5.2.4 Notices

One neglects in the form of the elementary matrix of mass without offsetting the terms of inertia of rotation (& + &

dS

)

because the latter are negligible compared to

F

X

X

y

y

S

others. Indeed a multiplicative factor of $h^2/12$ the drops to the other terms and they become negligible for a thickness report/ratio over characteristic length lower than 1/10. When offsetting is introduced, these terms of the form (+ D

2
+ d2) (& + &
dS
)

are not any more

F
MF
m
X
X
y
y
S

negligible and are introduced into the form of the matrix of mass.

5 *Implementation and postprocessings*

Offsetting is introduced by the optional key word OFFSETTING on the level of AFFE_CARA_ELEM of same manner as the thickness according to methods' defined in introduction. When this key word is not not present offsetting is worth zero per defect.

5.1 *Load application and couples*

All calculations are made in the reference mark of diagram (plane of the grid). If one defines forces or couples compared to another reference mark, the user will have to make for FORCE_ARETE and FORCE_NODALE transformations necessary to be reduced to the reference mark grid. For FORCE_COQUE the user will be able to specify the plan of load application and conversion towards the reference mark of calculation will be automatic.

One thus introduces into AFFE_CHAR_MECA the concept of plan of load application by the key word PLAN under FORCE_COQUE. This plan of application is different from the datum-line or plan from diagram

on which the grid rests. For this key word one will define the four following possibilities of application of the forces: “INF” “MOY” “SUP” “EMAIL”. “INF” “MOY” and “SUP” mean that one respectively apply the efforts in lower, average and higher skin of plate. “EMAIL” mean that one applies the efforts to the level of the datum-line or plan of the grid. By defect them efforts will be applied to the plan of the grid of the plate. The efforts of the type are concerned FORCE_COQUE of the TE0032.

In local reference mark with the element, when the forces and the couples are appliqués on “MOY” one uses

simple relation of passage:

$$C = C - df$$

X

X

y

$$C = C + df$$

y

y

X

to bring back the efforts and the couples in the reference mark of the grid where calculations are made.

In local reference mark with the element, when the forces and the couples are applied to “SUP” one uses

simple relation of passage:

$$C = C - (D + H/2) F$$

X

X

y

$$C = C + (D + H/2) F$$

y

y

X

In local reference mark with the element, when the forces and the couples are applied to “INF” one uses

simple relation of passage:

$$C = C - (D - H/2) F$$

X

X

y

$$C = C + (D - H/2) F$$

y

y
X
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If the efforts are given in the total reference mark of the element, one uses relations of passage of type: $C = C + (D + H$

/2) N F where C is defined compared to reference mark "INF" "MOY" "SUP" with equal to -1, 0 and 1, respectively. When there is no offsetting, the preceding formula is reduced to $C = C + H/N$

2 F.

Note:

For the loadings of the type FORCE_ARETE or FORCE_NODALE efforts and couples can be expressed that compared to the reference mark of the grid. If the user does not know them that compared to the average layer of the plate, it will have to carry out the change of reference mark with

the hand to have the expression of the efforts and the couples compared to the surface of grid.

The relation to be used is $C = C + dn F$ where D is the distance between the plan of calculation and the plan

of loading directed by the normal with the hull. It is obvious that the user has interest with it that the loading plan is the plan of the grid, but it is not always possible to make to coincide these two plans as one can see it on the left part of the figure of page 6.

5.2

Application of the boundary conditions in displacement

For the boundary conditions of the displacement type the user will have to pay attention to the fact that they can apply that to the reference mark of grid. Relations of passage compared to conditions given on the average layer are as follows:

*=
ref.
moy*

*U
= U
-
dn
ref.
moy
moy*

5.3 Postprocessings

For postprocessings, the results owing to lack of generalized efforts type are given in locate corresponding to the plan of diagram. To have them in the other reference marks, it will be necessary that the user indicate the plan of postprocessing and the changes of reference mark will be automatic.

For the postprocessing of the efforts generalized in the TE0033, one will define the four possibilities following of postprocessing of the efforts by the PLANE key word: "INF" "MOY" "SUP" "EMAIL" of orders CALC_ELEM and CALC_CHAM_ELEM with the same direction as previously. The defect is put at "EMAIL". All calculations are made in the plan "EMAIL" of the grid (in particular calculation nodal forces). When there is no offsetting it is the average layer of the plate: one thus find postprocessing by defect. To pass from the efforts results generalized of "EMAIL" to "MOY" one uses the simple relation of passage:

*NR = NR
M = M - NR
D
T = T*

To pass from the efforts results generalized of "EMAIL" to "SUP" one uses the simple relation of passage:

*NR = NR
M = M - (D + H/)*

2 NR

T = T

To pass from the efforts results generalized of “EMAIL” to “INF” one uses the simple relation of passage:

NR = NR

M = M - (D - H/)

2 NR

T = T

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6

Static and modal validation

6.1 Validation

initial

The first part of the validation consists in testing a simple plate subjected to forces and couples and whose plan of grid does not coincide with the plan of the average layer on which are applied efforts. For the plate subjected to forces and couples, results with and without offsetting must take account of the change of reference mark for the couples as indicated Ci below.

M

M

F

F

$M+dn^F$

D

$M+dn^F$

N

F

F

Displacements are in the following way dependent for a point located at a height Z compared to average layer:

$U = U$

+

N

$Z = U + (Z + D N$

)

moy

moy

ref.

ref.

what is still written:

=

moy

ref.

U

$= U + dn$

moy

ref.

ref.

what enables us to establish the relations of passage between displacements compared to the layer means and those compared to the datum-line.

For the generalized efforts, in the two preceding cases of figure, there are the same results on layers means, inferior and superior of plate.

6.2

Case-test SLS111: offsetting for simple plates

It is about a calculation in inflection of double-layered made up of two different isotropic materials. One studies

the coupling membrane-inflection. The calculation of reference is that of double-layered defined by DEFI_COQU_MULT made up of two different isotropic materials (not symmetry according to Z). The other modeling is made up of two plates offset compared to average fibre of the plate used with DEFI_COQU_MULT. The results, identical of one modeling to the other, are given in term of displacements and generalized efforts. Moreover one carries out on the geometry of this test one analyze modal for two modelings: the found Eigen frequencies are identical.

6.3

Case-test SSLS112: offsetting for composite plates

It is about a calculation in inflection of a quadricouche having a material not-symmetry compared to its average plan. The calculation of reference uses a definite quadricouches by DEFI_COQU_MULT. The other modeling uses two double-layered definite by DEFI_COQU_MULT but offset compared to fibre average of the quadricouche. The results, identical of one modeling to the other, are given in term of displacements.

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7 Conclusion

The finite elements of plate which we describe here are used in the mean structural analyses hurred whose thickness report/ratio over characteristic length is lower than 1/10. The average layer of these structures coincide not with the plan of the grid (plane of diagram). Offsetting corresponds thus with the distance from the average layer compared to the layer of diagram. A offsetting D

positive means

that the average surface of the plate is at a distance dn of the element of plate with a grid, the direction

N being given by the normal to the element.

The values of displacements and generalized efforts obtained are given by defect in the reference mark

grid. For the generalized efforts, one can however define a reference mark of postprocessing - locate associated with the average layer - different from the reference mark of diagram. Same manner, efforts

applied are regarded as being given by defect in the reference mark of diagram. In the case of FORCE_COQUE, one can however specify a reference mark of load application and couples - reference mark

associated the average layer - different from the reference mark of diagram.

Equivalent elements are not available in thermics; thermomechanical chainings are thus not available for the offset elements of plates.

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Appendix 1 Factors of transverse correction of shearing for orthotropic or laminated plates offset

*The **Hct** matrix is defined so that the surface density of transverse energy of shearing obtained in the case of the three-dimensional distribution of the constraints resulting from the resolution of balance is equal to that of the model of plate based on the assumptions of Reissner, for a behavior in pure bending. One must thus find **Hct** such as:*

$$1 + h/2$$

$$1$$

$$1$$

$$+h/2$$

$$xz$$

$$\mathbf{H-1}$$

$$-1$$

$$= \mathbf{TH T}$$

$$ct$$

$$= \mathbf{Hct}$$
 with =

$$\text{and } \mathbf{T} =$$

$$dz = \mathbf{H}$$

$$\cdot$$

$$2$$

$$2$$

$$2$$

ct

yz
 - h/2
 - h/2

To obtain **Hct** one uses the distribution of following **Z** obtained starting from the resolution of the equations of balance 3D without external couples:

Z
Z
 = - (
 with $xz =$
 = 0 for $z = \pm h/2$.
 , +
) **D**;
 ,
 = - (, +
) **D**
 xz
 $xx X xy y yz$
 $xy X yy, y$
 yz
 - h/2
 - h/2

If there is no coupling membrane inflection (symmetry compared to $z=0$), constraints in plan of element xx, yy, xy in the case of have as an expression a behavior of pure inflection:

= $zA (Z) \mathbf{M}$ with $\mathbf{A Z} = \mathbf{H Z H-1}$
 ()
 () **F**.

If (**H Z**) and **Hf** do not depend on **X** and **y** one can determine **Hct**. Indeed:

$M_{xx}, X - M_{xy}, y$

 $T_x M_{xx}, X + M_{xy}, y$
 $M_{xy}, X - M$

 yy, y

$$(Z) = D (Z) T$$

$$1 + D (Z)$$

2 where $T =$

= and =

T

M

$$y M xy, X + M yy, y$$

$$yy, X$$

$$M xx, y$$

like:

Z

With

With

With

With

$$11 +$$

$$33$$

$$13 +$$

$$32$$

$$D = -$$

$$D,$$

1

$$2 A$$

With

With

With

$$h/2$$

$$31 +$$

$$23$$

$$22 +$$

$$33$$

-

Z

With

With

With

With

With

With

11 -

33

13 -

2

2

32

12

31

D = -

D.

2

2 A

With

With

With

With

With

h/2

31 -

23

33 -

2

2

22

32

21

-

+h/2

C =

DTH 1

- D

11

1 1dz;

- h/2

1 +h/2

1
TC
C
T
+h/2
It results from it that

-
H 1
11
12
T
-1
=
T
with: C =
D H D

dz;
2
2 C
C
12
1

2
12
22
- h/2
- h/2
+h/2
C

=
DTH 1
- D
22
2 2dz
- h/2

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6.3

Titrate:
Treatment of offsetting for the elements of plate

Date:
15/07/03

Author (S):
P. MASSIN, J.M. PROIX, A. ASSIRE Key

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1 +h/2
1
As in addition

-1
-
H
1
-1
=
TH T one proposes to take $H = C$ to satisfy them as well as possible

2
2
ct
ct
11
- h/2
two equations whatever T and.

+h/2
By comparing Hct thus calculated with H
H
 $ct = dz$ one reveals the coefficients of correction of
- h/2
following transverse shearing: $K = H_{11}/H_{11}$; K
 $= H_{12}/H_{12}$; $K = H_{22}/H_{22}$

1
ct
ct
12
ct
ct

2

ct

ct.

For a homogeneous, isotropic or anisotropic plate, one finds as follows: $Hct = kh \mathbf{H}$ with $k=5/6$.

Note:

This method is valid only when the composite plate is symmetrical compared to $z=0$.

· For a multi-layer material, one establishes that:

NR

i-1

I

H

1

1

1

C

= I(HAT

2

2

pp

- ZAT) H-1

I

(

H With

pp

- ZA

II

p

I

p

I

I) +

4

2

2

i=1

p=1

p=1

i-1

I
1
1
3
3
 -
1
1
2
2
 -
 ($z_{i+1} - Z$) [*ATH 1*
I
 (

H With
pp
 - *Z A*
I
) + (*H AT*
pp
 - *Z AT*) *H 1A*
I
p
I
p
I
I

I]
24
2
2
p=1
p=1
1
 +
 (z_5
5
T
 -*1*
i+1 - z_i) *A H*
With

80

I

I

1

WITH + A

WITH + A

11

33

13

32

where: $H = Z + 1 - Z, = (Z + 1 + Z$

I

I

I

I

I

I) and A

for

2

I represents the matrix A + A

WITH + A

31

23

22

33

layer I.

· *Validity of the choice **H***

*= **C I***

-

ct

*II can be examined a posteriori when one has an estimate of the solution (fields of displacements and plane constraints, in particular). One can then estimate the variation enters the two estimates on energy. A step of calculation in two stages for multi-layer plates and hulls (with **Hct** diagonal and two coefficients k1 and k2) was besides developed by Noor and Burton [bib10] [bib11].*

· *Dans le cas d'une plaque isotrope ou anisotrope homogène l'égalité entre deux énergies est satisfaite au sens strict puisque **D2** = 0. Le choix fait ci-dessus est alors valide et aucune vérification a posteriori n'est nécessaire.*

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7.2

Titrate:

Voluminal element of hull SHB8

Date:

04/08/04

Author (S):

J.M. PROIX, S. BAGUET, A. COMBESCURE *Key*

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Code_Aster: Voluminal element of hull SHB8

Summary:

We present in this document the theoretical formulation of element SHB8PS and its establishment numerical for non-linear incremental analyses implicit (great displacements, small rotations, small deformations).

It is about a three-dimensional cubic element with 8 nodes with a called privileged direction thickness.

Thus, it can be used to represent mean structures while correctly taking into account them phenomena through the thickness (inflection, elastoplasticity), thanks a numerical integration to 5 points of

Gauss in this privileged direction.

In order to reduce the computing time considerably and to draw aside various likely blockings to appear, this element under-is integrated. It requires consequently a mechanism of stabilization in order to control the modes of deformation to null energy (modes of Hourglass).

In addition to its cost of relatively weak calculation and its good performances in elastoplasticity, this element

have another advantage. Since it is based on a three-dimensional formulation and that it only has degrees of freedom of translation, it is very easy to couple it with voluminal elements 3D, which is very useful in systems where voluminal hulls and elements must cohabit.

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1 Introduction

Many recent work proposed to use a voluminal formulation for the structures thin. Two principal families of methods, which rest all on the introduction of a field of postulated deformation (“assumed strain”), emerge. Methods of the first family consist in using a conventional numerical integration with an adequate control of all them modes of blocking and locking (volume, transverse shearing, membrane). Methods of the second family consist in under-integrating the elements to remove blockings and controlling the modes of Hourglass which rise from this under-integration (see [bib3] [bib4]). Both approaches were studied in details in the case of an elastic behavior. On the other hand, very little work treats elastoplastic case.

The element presented here rests on an under-integrated formulation especially developed for elastoplastic behavior of the structures in inflection. The basic idea consists first of all with to make sure that there are sufficient points of Gauss in the thickness to represent it correctly phenomenon of inflection, then to calculate rigidities of stabilization in an adaptive way according to the state plastic of the element. That represents an unquestionable improvement compared to the formulations traditional for the forces of stabilization, because these last rest on an elastic stabilization who becomes too rigid when the effects of plasticity dominate the response of the structure.

Element SHB8 is a continuous three-dimensional cube with eight nodes, in which a direction privileged, called thickness, was selected. It can thus be used to model the structures thin and to take into account the phenomena which develop in the thickness within the framework

mechanics of the continuous mediums three-dimensional. Since this element is under integrated, it exhibe of the modes of Hourglass which must be stabilized. We chose the method of stabilization introduced by Belytschko, Bindeman and Flanagan [bib3] [bib4]. This element and this method of stabilization were already implemented in an explicit formulation by Abed-Meraim and Combescure [bib2]. This documentation describes the formulation of this element, its implementation

numerical for the prediction of elastic and elastoplastic structural instabilities, like sound establishment in Code_Aster. For the non-linear problems, an incremental formulation implicit of Newton-Raphson type is used [R5.03.01]. The equilibrium equations are solved by method of Lagrangian the update. The control of the increments of load and displacement is based on a method of piloting close to the algorithm to Riks [bib5]. Implementation the numerical of this element within a non-linear framework was proposed by Legay and Combescure in [bib1].

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Kinematics of the element

Element SHB8 is a hexahedron with 8 nodes. The five points of integration are selected along direction in the reference mark of the local co-ordinates. The shape of the element of reference as well as

points of integration are represented on [Figure 2-a].

7

8

6

5

4

5

3

2

3

1

4

2

1

Appear 2-a: Geometry of the element of reference and points of integration

This element is isoparametric and has the same linear interpolation and same kinematics that hexaèdraux elements with 8 standard nodes.

3 Formulation variational

The formulation used for the construction of element SHB8PS differs from a formulation traditional simply by the choice of a postulated deformation &, therefore of an operator gradient discretized, allowing to avoid the induced modes parasitized by under integration.

Thus, the variational principle is written:

$$\begin{aligned} (v, \dot{\epsilon}) &= (\dot{\epsilon}): FD - \\ &ext. \\ u \dot{\epsilon} &= F \\ &= 0 \\ V \end{aligned}$$

where represents the total virtual power, the variation, the v field speed, $u \dot{\epsilon}$ speeds nodal, & the rate of postulated deformation (assumed strain misses), the constraint of Cauchy, V it updated volume and ext.

*F
external forces.*

The discretized equations thus require the only interpolation the speed v and rate of deformation postulated & in the element. We now will build element SHB8PS to be left of this equation. The complete developments and the demonstrations concerning this element are explained in details in [bib2].

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4 Discretization

4.1

Discretization of the field of displacement

The space co-ordinates X of the element are connected to the nodal co-ordinates X by means of

I

II

isoparametric functions of forms NR by the formulas:

I

8

$$X = X NR (,) = NR (,) X$$

I

II

I

I

II

I I

=

In the continuation, and except contrary mention, one will adopt the convention of summation for the indices

repeated. The indices in small letters I vary from one to three and represent the directions of

space co-ordinates. Those in capital letters I vary from one to eight and correspond to the nodes of the element.

The same functions of forms are used to define the field of displacement of the element U in

*I
function of nodal displacements U :*

II

$U = U NR ($

, ,)

I

II

I

Trilinear isoparametric functions of form are chosen:

I

$NR ($

, ,) = (1 +)(1 +)(1 +)

I

8

I

I

I

, [1

- , 1],

$I = 1, \dots, 8$

These functions of form transform a unit cube in space (,) in a hexahedron

unspecified in space (X, X, X) .

1

2

3

4.2

Operator discretized gradient

The gradient U

*U of node I in
I, of the field of displacement is a function of displacement*

*J
II
direction I:
U = U NR*

*I, J
II
I, J*

The linear tensor of deformation is given by the symmetrical part of the gradient of displacement:

*I
= (U + U)
ij
2 I, J
J, I*

Let us introduce the three vectors B, derived from the functions of form at the points of Gauss P:

*I
K
NR*

*T
B (P) =*

*I
K
X
I =0, =0, =k*

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Also let us introduce the following vectors:

T

S

= (1

1

1

1

1

1 1 1

)

T

H

= (1

1

-1

1

-

1

-

-1 1 1)

1

T

H

= (1

1

-

-1 1

1

-

1 1 -1)

2

T

H

= (1

1

-

1

1

-

1

1

-

1 -1)

3

T

H

=(
1

1

-

1

-1 1

1

1

-

1 -1)

4

T

X

=(
1

1

-

1

1

1

-

1

-

1 1 -1)

1

T

X

=(
1

1

-

1

-

1

1

$$\begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$

*The three vectors T
 X represent the nodal co-ordinates of the eight nodes. The four vectors T
 I*

*H
the functions H, H, H and H for each of the eight nodes represent respectively, which are
 1
 2
 3
 4
defined by:*

$H =$
 $H =$
 $H =$
 $H =$
 1
 2
 3
 4

Let us introduce finally the four following vectors:

3
 $= 1$
 $H -$

(T
H. X

J)

B

8

J

j=1

The gradient of the field of displacement can be now written in the form (without any approximation [bib3]):

4

T

T

T

T

U = B + H. U = B + H. U

I J

J

J

I

J

J

,

,

,

I

I

=

Or, in the form of vector:

U

X, X

U

y, y

U

Z, Z

U =

S

U + U

X y

y X

,

,

U + U

X Z

Z X

,

,

U + U

y, Z

Z, y

with U nodal displacement in direction I. The symmetrical operator gradient (noted)

I
S
discretized connecting the tensor of deformation to the vector of nodal displacements

U =.
B U

S
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takes the matric form then:

T
T
B + H
0
0

X
X

,

0
T

T
B + H
0

y
, y

0
0
T
T
B + H

Z
, Z
B =

T
T
T
T
B + h
B + h
0

y
, y
X
, X

T
T
B + H
0
T
T
B + H

Z
, Z
X
, X

0
T
T
T
T
B + H
B + h

Z
Z
y
y
,
,

The detailed formulation was presented by Belytschko in [bib3].

4.3 **Stamp rigidity**

The matrix of rigidity of the element is given by:

$$T$$
$$K =$$
$$B C B D$$
$$E$$
$$E$$

The five points of integration considered are on the same vertical line. Their co-ordinates are (,) and their weights of integration are the roots of the polynomial of Gauss-Legendre:

$$P(1)$$
$$0 0$$

= 0 9

. 1

= 0 2

. 4

1

1

P (2)

0 0

= 0 5

. 4

= 0 4

. 8

2

2

P (3)

0 0

0

0.57

P (4)

0 0

-

2

2

P (5)

0 0

-

1

1

Thus, the expression of rigidity K is:

E

5

K = () J () T

B () CB ()

E

J

J

J

J

J 1

=

where $J()$ is Jacobien, calculated at the point of Gauss J , of the transformation between the configuration

 J

unit of reference and an arbitrary hexahedron. The elastic matrix of behavior C chosen with following form:

+ 2μ

0

0

0

0

+ 2μ 0 0 0 0

0

0

 E 0

0

0

 $C =$

0

0

0 μ 0 0

0

0

0

0 μ 0

0

0

0

0

0 μ E

where E is the modulus Young, the Poisson's ratio, $\mu = ($
the modulus of rigidity and
 $2 \lambda +)$

$= E$

the coefficient of modified Lamé. This law is specific to element SHB8. It resembles

2

$1-$

with that which one would have in the case of the assumption of the plane constraints, put aside the
term (3,3). One

can note that this choice involves an artificial anisotropic behavior.

This choice makes it possible to satisfy all the tests without introducing blocking.

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4.4

Stamp geometrical rigidity K

By introducing the quadratic deformation Q

E :

Q

1

E

=
 $U U$

2
 $K, I K, J$

$K I 3$

=,

one can define this matrix of geometrical rigidity by:

T
 Q
 $U \cdot K \cdot u =$
 $: E (,$
 $U U) =$
 $:($
 T
 D

$U \cdot u) D$

0
0

In order to express this matrix in discretized space, let us introduce the operators gradient quadratic discretized Q B such as:

Q
 T
 Q
 Q
 T
 Q

Q
Q
E
E
E

U. B ()
T
. U U. B (). U U. B (). U
xx
yy
zz
xx
J

yy
J
zz
J

Q
Q
Q
Q
E
E
E +e
T
U. Q
B ()
T
. U U. Q
B ()
T

. U U. Q
B (). U

yy
zz
xy
yx
yy
J
zz
J
xy
J

T
Q
T
Q
T

Q

Q
Q
E
E + Q
Q
E
E + Q
E
U. B (). U U. B (). U U. B (). U

zz
xy

yx

xz

zx

zz

J

xy

J

xz

J

Q

E (U (), U ())

=

=

J

J

Q

Q

Q

Q

Q

Q

T

Q

T

Q

T

Q

E +e

E +e

E +e

U. B (). U U. B (). U U. B (). U

xy

yx

xz

zx

yz

zy
xy
J
xz
J
yz
J

Q
Q
Q
Q

E + e

E + e

T

U. Q

B ()

T

. U U. Q

B (). U

xz

zx

yz

zy

xz

J

yz

J

T
Q
Q
E + Q
E

U. B (). U

yz
zy

yz
J

The various terms Q
B are given by the following equations:

ij

B. T
B
0
0

X
X

Q
B ()
=
B. T
B

xx

0
0
J
X
X

0
0
T
B.b

X
X
B. T
B
0
0

y
y

Q
B ()
=
B. T
B

yy

0
0
J
y
y

0
0

T
B .b

y
y
B. T
B
0
0

Z
Z

Q
B ()
=
B. T
B

zz

0
0
J
Z
Z

0
0
T
B .b

Z
Z
B. T
B + B. T
B
0
0

X
y
y
X

Q
B ()
=
B. T
B + B. T
B

xy

0
0

J
X
y
y
X

0
0
T
T
B .b + B .b

X
y
y
X

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B. T

B + B. T

B

0

0

X

Z

Z

X

Q

B ()

=

B. T

B + B. T

B

xz

0

0

J

X

Z

Z

X

0

0

T

T

B .b + B .b

X

Z

Z

X

B. T

B + B. T

B

0

0

y

Z

Z

y

Q

B ()

=

B. T

B + B. T

B

yz

0

0

J

y

Z

Z

y

0

0

T

T

B .b + B .b

y

Z

Z

y

*With these notations, the geometrical matrix of rigidity **K***

at the point of Gauss

is given by:

J

K () = (). Q

B () + (). Q

B () + (). Q

B ()

J

xx

J

xx

J

yy

J

yy

J

zz

J

zz

J

+

$(\cdot) \cdot \mathbf{Q}$
 $\mathbf{B}(\cdot) + (\cdot) \cdot \mathbf{Q}$
 $\mathbf{B}(\cdot) + (\cdot) \cdot \mathbf{Q}$
 $\mathbf{B}(\cdot)$
 xy
 J
 xy
 J
 xz
 J
 xz
 J
 yz
 J
 yz
 J

and geometrical rigidity of the element stamps it is given by:

$$\mathbf{K} = (\cdot) \mathbf{J}(\cdot) \mathbf{K}(\cdot)$$

J
 J

 J
 $J 1$
 $=$

4.5 Stamp K_p pressure

The following compressive forces are present in the tangent matrix via the matrix \mathbf{K} , because them \mathbf{P} following external forces depend on displacement. The following compressive forces are written:

T
 1
 $p \cdot$
 $\mathbf{N} \mathbf{U} dS =$
 $p \text{ of } [$
 $\mathbf{T} \mathbf{F}(\mathbf{U})]$
 T

N. F

-

U

$$dS = p \mathbf{F} - p \mathbf{K}.$$

U

0

()

0

0

0

P

$$F(\mathbf{U}) = 1 + \mathbf{U}$$

by using the notations:

.

T

N = (N, N, N), normal on the surface external of the element in the configuration of

0

X

y

Z

reference

.

b%, vector of dimension 4, drift of the functions of form to the 4 nodes of the face of

I

the element charged in pressure

.

S surface of the face charged in pressure

0

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The preceding formulation leads to a not-symmetrical matrix. It is known that one can nevertheless to use a symmetrical formulation if the external forces due to the pressure derive from a potential. It is the case if the compressive forces do not work on the border of the modelled field. One thus consider that the symmetrical part of the matrix is enough. The symmetrized matrix takes the form following:

O
T
T
T
T
b%

y N-B
% N
N -
N
X
X
y
b%z X b%x Z

O
T
T
T
T
b%

y N-B
% N
N -

N
X
X
y
b%z X b%x Z

O
T
T
T
T
b%y n-b% N
N -
N
X
X
y
b%

Z
XB
% X Z

O
T
T
T
T
b%y n-b% N
N -
N
X
X
y
b%z X b%x Z
T
T
N -

%

% N

O

T

T

B

B

b% n-b%

X

y

Z

y N

y

X

y

Z

T

T

N -

N

O

T

T

b%

b%

b% n-b%

X

y

Z

y N

y

X

y

Z

K = S

P

O

T

T

N -

N

O

T
T
b%
b%
b% n-b%
X
y
Z
y N
y
X
y
Z

Tn
T
T
T
b%
-
N
O
b%
b% N - %
y
Z
by N
X
y
X
y
Z
T
T
T
T
N
N
N
N
O

-

-
b%
%
%
X
Z
bz X
by Z b%z y

T
T

T
T
b% N
b% N
N
N
O

-
-
X
Z
Z
X
b%y Z b%z y

T
T
T

T
b% N
b% N
N
N
O

-
-
X

Z
Z
 X
b%y Z b%z y

T
 T
 T

T
b% N
b% N
 N
 N
 0

-
 -
X
Z
Z
 X
b%y Z b%z y

It is a matrix (12×12), which it is necessary to multiply by displacements of the 4 nodes of the face on which one applies a pressure.

5 **Stabilization of the element**

5.1 Motivations

The under-integration of element SHB8 (5 points of Gauss only) aims at reducing considerably computing time (gradient displacement, law of behavior,...). It also makes it possible to draw aside the various blockings met in implementation the numerical of the finite elements.

However, this under-integration does not have only advantages: it introduces unfortunately parasitic modes associated a null energy (mode of Hourglass or sand glass). In statics, that can lead to a singularity of the matrix of total stiffness for certain boundary conditions. In transitory dynamics, on the other hand, that led to modes of sand glass which will deform it

unrealistic grid of way and which ends up exploding the solution. This deficiency of the matrix of stiffness, due to under-integration, must thus be compensated while adding to elementary rigidity a matrix of stabilization. The core of the new rigidity, thus obtained, must be reduced to only modes corresponding to the rigid movements of solids.

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5.2

Modes of "Hourglass"

Since the points of integration are on the same vertical line (privileged direction), the derivative of the functions H and H are cancelled in these points. The operator discretized gradient is thus

3

4

tiny room to:

2

T

T

B + H

0

0

X

X

,

1

=

2

0

T

T

B + H

0

y

, y

1

=

2

0

0

T

T

B + H

Z

, Z

I
=

B =

2
2

T
T
T
T
B +
H
B +
H
0

y
, *y*
X
, *X*

I
=

2
2

T
T

B + H

0

T

T

B + H

Z

, Z

X

, X

1

=

1

=

2

2

0

T

T

T

T

B + H

B +

H

Z

, Z

y

, y

1

=

1

=

The modes of Hourglass are modes of displacement to null energy, i.e they check $Bu = 0$. Six modes, others that rigid modes of solids, which check this equation are:

*H
0
0
H
0
0
3
4*

*0
H
0 0
H
0

3
4*

*0
0*

*H
0
0*

H

*3
4*

5.3

Stabilization of the type “Assumed Strain Method”

In this approach, inspired of work of Belytschko, Bindeman and Flanagan [bib3] [bib4], the functions of form are not calculated at the points of Gauss but are realised on

I
the element:

T
I
^ =
(,) D, I = 1, 2, 3

I
NR
B
, I
O C

Thus, the new operator discretized gradient can be written:

^
^
B = B + B

stab
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The expression of ^

B
is given by:
stab
4

H ^T

,
0
0
X

=3

4

0
H ^T

,
0
y

=3
^

B
=

stab

0
0
H ^T

,
4 Z
4

0
0
0

0
0
0

0
0
0

*and that of the vectors \hat{H}
by:*

3
1

$$\hat{H} = H - (T H \cdot X) \hat{H}$$

B
8

J
J

J 1
=

The new matrix of rigidity becomes:

T
T

^

$$K = B C B d + B C B D$$

stab

E
E

^
T
^
T
^

+
B
CBd +
B
CB D

stab

stab
stab
E
E

144424443

stab
K

The last term of the preceding equation (
stab
K
) is enough to stabilize the element. One can thus reduce
the matrix of rigidity stabilized with:

=
+ stab
K K
K

E

stab

^
T
^
K
=
B
CB D

stab
stab
E

The many cases which were studied showed that it is enough to calculate the diagonal terms of stamp stabilization

stab
K
, I = 1, 2,3, which is given by:

II
T
stab
1
K
=
(+ 2) [
T
H
μ +]
II
II
\$ \$
\$ \$
3
3
4
4
3

T
stab
1
K
=

(+ 2) [

T

H

μ +]

22

22

\$ \$

\$ \$

3

3

4

4

3

stab

H33

T

K

=

E

33

\$ \$

4

4

3

The Hii coefficients themselves are given by the following equation, in which there is not summation on the repeated indices:

1 T

T

X X

X X

J

J

K

K

H

=

II

3

T

XX

I

I

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6

Strategy for non-linear calculations

6.1 Non-linearities

geometrical

One treats here the case of great displacements, but weak rotations (see further) and small deformations. One adopts for that an updated Lagrangian formulation.

Into nonlinear we seek to write balance between internal forces and force external at the end of the increment of load (located by index 2):

int

extr

F
 $= F$

2
2

The expression of the internal forces is written:

int

T
 $F =$
 FD
2
 B

2
2
2

In the preceding equation the B2 operator is the operator allowing to pass from displacement to linear deformation calculated on the geometry at the end of the step, the constraint is the constraint of

2

Cauchy at the end of the step and integration is made on the volume deformed at the end of the step.

2

We chose this updated Lagrangienne formulation.

The element available to date in Aster is programmed in small rotations. Indeed the increment of deformation is calculated by using only the linear deformation:

1
 E
 $= (I (U) + T1 (U))$

2

The operator gradient is calculated on the geometry of beginning of step. This writing of the deformation is limited to small rotations (<5 degrees).

One can without difficulty of extending the formulation to great rotations by including in the deformation them terms of second order:

1

$$\begin{aligned}
 &T \\
 &T \\
 &E = \\
 &U + U + U U \\
 &2 (\\
 &\cdot \\
 &1 (\\
 &) 1 () 1 () 1 ()
 \end{aligned}$$

In elasticity, the law of behavior is written:

$$= \sigma = C \epsilon$$

where C is the matrix of Hooke. Let us notice that for the SHB8 this matrix is a matrix orthotropic transverse which is written in the axes of the lamina:

$$\begin{aligned}
 &+ 2\mu \\
 &\mu \\
 &0 \\
 &0 \\
 &0 \\
 &0 \\
 &\mu \\
 &2\mu \ 0 \ 0 \ 0 \ 0 \\
 &+
 \end{aligned}$$

$$\begin{aligned}
 &[C] \\
 &0 \\
 &E \ 0 \ 0 \\
 &0 \\
 &' =
 \end{aligned}$$

$$\begin{aligned}
 &0 \\
 &0 \\
 &0 \ \mu \ 0 \ 0
 \end{aligned}$$

$$\begin{aligned}
 &0 \\
 &0 \\
 &0 \\
 &0 \ \mu \ 0
 \end{aligned}$$

$$0$$

0

0

0 0 μ

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The formula allowing to calculate the constraint of Cauchy starting from the constraint of

2

Piola Kirchoff II is:

2

= +

2

1

1

T

=

F F

2

det

(F)**2** **$F = I +$** **I** **U**

The combination of the four last equations with the expression of the internal forces gives formulation of the element in great deformations into Lagrangian updated.

Let us notice that this updated Lagrangienne formulation is completely equivalent to total Lagrangienne formulation for which the internal forces are written:

int***T******NL******F =*** ***$B + B(U) FD$*** ***2******(******) 2******0******0***

In this case all integrations are made on the initial geometry the constraint used is

0***2***

the constraint of Piola Kirchoff II. This last method is probably preferable when it grid becomes deformed significantly and thus makes it possible to deal with the problems into large deformations but requires the development of operator NL

 $B(U)$.

The increment of deformation in Lagrangian total is expressed on the initial geometry of the structure.

1***T******T******E =*** ***$U + U + U U$*** ***2 (***

.
0 (
) 0 () 0 () 0 ()

The combination of the two preceding equations gives the formulation of the element into large deformations in linear behavior material.

6.2 Small displacements

In the case of small displacements one confuses geometry in beginning and end of step, constraint of Cauchy and of Piola Kirchoff II, moreover one uses the linear expression of the deformations.

6.3 Forces of stabilization

The forces of stabilization make it possible to avoid the modes of sand glass and are added in the calculation of residues to balance the contribution of the matrix of stiffness of stabilization to the first member. forces of stabilization stab

F
, to add to the forces intern int
F, are written:
2

stab = stab
F
K
U

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For reasons of effectiveness, one chooses not to assemble again

stab

K

to calculate stab

F

in

end of step, but rather to build stab

F

starting from \wedge

B

that one calculated previously. One must

stab

for that to place itself in the reference frame corotationnel of medium of step suggested in [bib3]. For this

reason, one does not obtain an exact expression of stab

F

, and some additional iterations are

generally necessary to converge. These some iterations are however unimportant by

report/ratio at the cost of calculation saved while not assembling

stab

K

.

6.4 Plasticity

The elastoplastic behavior of Von Mises, with isotropic work hardening, is calculated of each one of 5 points of integration. One thus uses quite simply the formulas and the usual programming of plasticity with a three-dimensional state of stresses, but the linear matrix of behavior It is orthotropic. We must quite simply slightly modify the usual algorithm of flow elastoplastic three-dimensional by replacing the usual matrix of Hooke C by the matrix of orthotropic behavior transverse It.

We must find the constraint at the end of the step which checks balance. In great displacements it problem is written:

= +

2

1

(
p
It
-
)

This equation is solved as soon as the increment of plastic deformation is known. This deformation is determined while forcing the final constraint to be plastically acceptable. This method is completely similar to the usual three-dimensional method except that there is no explicit solution with this problem if one uses for example the effective approximation of the radial return to calculate solution. We chose to solve this nonlinear problem by a method of Newton.

7 Establishment of element SHB8 in Code_Aster

7.1 Description

This element is pressed on the voluminal meshes 3D HEXA8.

7.2 Use

This element is used in the following way:

7.2.1 Grid

*Check the good orientation of the faces of the indicated elements (compatibility with the direction privileged) by using **ORIE_SHB8** of operator **MODI_MAILLAGE**.*

7.2.2 Modeling

To assign modeling SHB8 to indicated meshes HEXA8.

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7.2.3 Material

For a homogeneous isotropic elastic behavior in the thickness one uses key word ELAS in DEFI_MATERIAU where one defines the coefficients E, modulus Young and NAKED, Poisson's ratio.

To define a plastic behavior one uses the key word TRACTION in DEFI_MATERIAU where one the name of a traction diagram defines. Only this type of definition is available for the moment.

7.2.4 Boundary conditions and loading

DDL of volume 3D in the total reference mark. One imposes the boundary conditions on the ddl of volume

3D (AFFE_CHAR_MECA/DDL_IMPO), and efforts in the total reference mark (FORCE_NODALE).

They are the efforts in the total reference mark.

One defines the efforts of pressure distributed on the faces of the element (under key word PRES_REP). One

will have taken care as a preliminary to define meshes of skin QUAD4 and to suitably direct them outgoing normals with these meshes of skin using the order MODI_MAILLAGE key word ORIE_PEAU_3D

7.2.5 Calculation in linear elasticity

Order MECA_STATIQUE

The options of postprocessing available are SIEF_ELNO_ELGA and EQUI_ELNO_SIGM.

7.2.6 Calculation in linear buckling

Option RIGI_MECA_GE being activated in the catalogue of the element, it is possible to carry out one

traditional calculation of buckling after assembly of the matrices of elastic and geometrical rigidity.

7.2.7 Calculation in geometrical nonlinear "elasticity"

One chooses behavior ELAS under key word COMP_INCR of STAT_NON_LINE, into small deformations (“SMALL”) or in great deformations “GREEN” under the key word DEFORMATION.

Strategy used being based on the use of a matrix of tangent rigidity during iterations (reactualization at the beginning of step only), one will take care not to use another option that that which is activated by defect, namely REAC_ITER = 0 pennies NEWTON.

Numerical integration in the thickness is carried out with 5 points of Gauss, just like in not linear material.

7.2.8 Plastic nonlinear calculation

Only the criterion of Von Mises is available to date (RELATION = “VMIS_ISOT_TRAC” under COMP_INCR). One defines the mode of calculation of the deformations as in the case of elasticity not linear (DEFORMATION = “GREEN” or “SMALL”).

Strategy used being based on the use of a matrix of tangent rigidity during iterations (reactualization at the beginning of step only), one will take care not to use another option that that which is activated by defect, namely REAC_ITER = 0 pennies NEWTON.

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7.3 Establishment

Options RIGI_MECA, RIGI_MECA_GE, FORC_NODA, FULL_MECA, RIGI_MECA_TANG, RAPH_MECA,

SIEF_ELGA_DEPL, SIEF_ELNO_ELGA were activated in the catalogue gener_shb3d_3.cata. They direct all calculation towards te0520.f, then towards shb8.f.

The forces of stabilization of the element require the storage of a vector of size 12 for each not Gauss. We chose to store these terms like additional components of stress field.

No development was necessary for the compressive forces distributed and the forces of following pressures. Indeed, these loadings are pressed on meshes of skin identical to those voluminal elements 3D.

7.4 Validation

The tests validating this element are, in version 7.2 of Code_Aster:

- SSLS108 C and D: beam bored in inflection, test allowing to check the absence of blocking [V3.03.108],*
- SSLS105 C: hemisphere doubly pinch [V3.03.105] traditional test to check convergence of the element,*
- SSLS123 a: sphere under external pressure [V3.03.123] to validate the loadings of pressure and the orthotropic behavior particular to this element,*
- SSLS124 A and thin b: section in inflection with various twinges, to delimit the field of use of the element [V3.03.124]. The results are correct (less than 1% with the solution analytical) for reports/ratios of twinge (thickness/width) going from 1 to 5 10³,*
- SSLS125 a: buckling (modes of Euler) of a free cylinder under external pressure [V3.03.125] this test makes it possible to validate the geometrical nature of rigidity,*
- SSNS101 A, B and C: breakdown of a cylindrical roof [V6.03.101]. This test makes it possible to validate it geometrical nonlinear calculation and elastoplasticity,*
- SSNS102 a: buckling of a hull with stiffeners in great displacements and pressure following [V6.03.102].*

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“Exact” elements of beams (right and curved)

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Organization (S): EDF/IMA/MMN, IAT St CYR

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R3.08 booklet: Machine elements with average fibre

Document: R3.08.01

“Exact” elements of beams (right and curved)

Summary:

This document presents the elements of beam of *Code_Aster* based on an exact resolution of the equations of

continuous model carried out for each element of the grid.

The beams can be right (Elements POU_D_T and POU_D_E) or curves (Elements POU_C_T).

section, constant or variable over the length, can be of an unspecified form. The material is

homogeneous,

isotropic, elastic linear.

The assumptions selected are as follows:

· Hypothèse of Euler: transverse shearing is neglected, as well as the inertia of rotation.

This assumption is checked for strong twinges (element POU_D_E).

· Hypothèse of Timoshenko: transverse shearing and all the terms of inertia are taken into account.

This assumption is to be used for weak twinges (elements POU_D_T and POU_C_T).

· Hypothèse of Saint-Coming: torsion is free.

The treatment of the various loadings and the sizes awaited in result (forced - efforts) is

also presented.

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Foreword

This reference material of the elements of beam was carried out starting from a work carried out by m.t. Bourdeix, P. Hemon, O. Wilk of the Institute Aerotechnics of the National Academy of Arts and Trades, within the framework of an External Contract of Research and Development with this laboratory. The volume of this document is due at the same time to the required precision and the didactic character of the talk, which is voluntarily preserved.

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*Titrate:**“Exact” elements of beams (right and curved)**Date:*

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*Author (S):***J.M. PROIX, P. MIALON, m.t. BOURDEIX***Key:*

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Notations

The notations used here are not all identical to those used in [U1.04] and [U4.24.01], for reasons of compactness and homogeneity with [R3.08.03].

One gives the correspondence between this notation and that of the documentation of use.

DX, DY, DZ and DRX, DRY, DRZ are in fact the names of the degrees of freedom associated with the components

displacement U, v, W, X

, y, Z

.

C

constant of torsion

JX

$E E$

eccentricity of the center of torsion/shearing

EY, EZ

y, Z

E

Young modulus

E

Poisson's ratio

NAKED

G

E

modulate of Coulomb = (21+)

I, I

geometrical moments of inflection compared to the axes *y, Z*

IY, IZ

y

Z

I

polar geometrical moment

P

I X

polar moment of inertia around the longitudinal axis *X*

K, K

coefficients of shearing

y

Z

1

1

AY

AZ

K

stamp rigidity

M

stamp of mass

M, M, M moments around axes *X, y, Z*

MT, MFY, MFZ

X

y

Z

NR

normal effort with the section

NR

S

surface of the section

With

U, v, W

translations on axes X, y, Z

$DX DY DZ$

V, V

sharp efforts along axes y, Z

VY, VZ

y

Z

density

RHO

transverse shear stress

cT

rotations around axes X, y, Z

$DRX DRY DRZ$

X, y, Z

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Introduction

A beam is a solid generated by a surface of surface S of which the geometrical centre of inertia G followed a curve C called the average fibre or neutral fibre. The surface S is the cross-section (section transversal) or profile, and it is supposed that if it is evolutionary, its evolutions (size, form) are continuous and progressive when G describes the average line.

For the study of the beams in general, one makes the following assumptions:

- the cross-section of the beam is indeformable,
- transverse displacement is uniform on the cross-section.

These assumptions make it possible to express displacements of an unspecified point of the section, in function of displacements of the point corresponding located on the average line, and according to one

increase in displacement due to the rotation of the section around the transverse axes. This last can be neglected (POU_D_E) or to be the subject of a modeling (POU_D_T and POU_C_T). The discretization in "exact" elements of beam is carried out on a linear element with two nodes and six degrees of freedom by nodes. These degrees of freedom are the three translations U, v, W and the three rotations X, y, Z .

Z
y
1
2
X
U X
U
X
v y
v y
W Z
W Z

Waited until the deformations are local, it is built in each top of the grid a base local depending on the element on which one works. The continuity of the fields of displacements is ensured by a basic change, bringing back the data in the total base.

In the case of the right beams, one traditionally places the average line on axis X of the base local, transverse displacements being thus carried out in plan (y, Z).

Finally when we arrange sizes related to the degrees of freedom of an element in a vector or an elementary matrix (thus of dimension 12 or 122), one arranges initially the variables for top 1 then those of top 2. For each node, one stores initially the sizes related to the three translations, then those related to three rotations. For example, a vector displacement will be structured in the following way:

U, v, W

, ,

,

, $U, v,$

W

,

,

,

1

1

1

X

y

Z

2
2
2
X
y
Z

1
1
1
2
2
2
!### "

\$

!### "

\$

top 1
top 2

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Equations of the movement

In this chapter one presents the equations of the movement of the beams in traction and compression, in

torsion and in inflection in the elastic range. In each case, these equations are deduced by application of the equations of Lagrange, resulting from the principle of Hamilton, or by writing balance of a segment of beam. We chose to point out the two methods, the reader will be able to refer to that which is most familiar for him. One limits oneself here to the cases where the only loadings are loadings distributed (not of concentrated forces).

1.1

traction and compression

The traction and compression is the translatory movement on the longitudinal axis of the beam.

1.1.1 Local equilibrium equation

One considers a segment length dx subjected to an axial load NR [1.1.1-a] intern and a force external f_{ext} per unit of length.

X

$X + dx$

$NR (X)$

$NR (X)$

$NR (X + dx)$

O

X

f_{ext}

dx

Appear 1.1.1-a: Segment of beam charged axially

The beam has a section $S (X)$ and consists of a material of density (X) and module of Young $E (X)$. The fundamental principle of mechanics makes it possible to write:

2

$x+dx$

$X dx$

$U.S.$

$NR (X) + NR (X + dx) +$

$f_{ext} (S)$

$+$

ds

$(S) S (S)$

$()$

$-$

$=$

X

ds

X

t^2

where U is displacement on

X

center segment.

Thus:

$$NR(X + dx) - NR(X)$$

$$1 \ x+dx$$

$$1$$

$$2$$

$$x+dx$$

$$U$$

$$+$$

$$F$$

$$ext. (S) ds$$

$$=$$

$$S$$

$$ds$$

$$dx$$

$$dx X$$

$$dx X$$

$$t^2$$

While passing in extreme cases when $dx \rightarrow 0$, one obtains:

$$D NR(X)$$

$$2 U$$

$$+ F(X) =$$

$$S$$

$$(X$$

$$ext.$$

$$)$$

$$dx$$

$$t^2$$

éq 1.1.1-1

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Only the first order terms are preserved and one replaces in [éq 1.1.1-1], then one uses the law of Hooke and the assumption that the beam consists of longitudinal fibres working only in traction and compression to express the axial load by:

U

$NR(X) = ES$

éq 1.1.1. - 2

X

One obtains thus after simplification by dx :

U

$2 U$

ES

f_{ext}

S

X

$X +$

$=$

t^2

éq 1.1.1-3

who represents local balance with the first order of a beam, for a movement of traction and compression.

1.1.2 Method of the Lagrangian one

Taking again the segment of beam of the figure [Figure 1.1.1-a] total kinetic energy of the beam of length L is written:

1

2

L

U

E

$=$

S

dx

C

2 O

T

.

1

U 2

One will note for the continuation E

=

S

it

2

T elementary kinetic energy.

Energy interns deformation, thanks to the law of Hooke is written:

1

2

L

U

E

=

ES

dx

P

.

int

2 O

X

1

U 2

Of the same E will be noted

=

E

S

p

inte

2

X .

There is also the work of the external force given by:

L

E

=

F

$U dx$

p

$ext.$

$ext.$

O

and at the elementary level E

= F

U

p

$ext.$

.

$exte$

The Lagrangian one is given by:

$$L = E - E$$

- E

C

p

p

int

$ext.$

and Lagrangian density:

$$L = E - E$$

- E

C

p

p

.

E

inte

exte

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For the monodimensional continuous system, the equation of Lagrange is written in this case:

L

L

L

-

=

0

éq 1.1.2-1

U

X

$U -$

T

U

,

%

where U and %u respectively indicate the derivative compared to X and compared to time. Its application brings back for us obviously to the equation of the movement of a beam in traction and compression

[éq 1.1.1-3].

1.2

Pure torsion (torsion of Saint-Coming)

Torsion is the rotational movement around the longitudinal axis of the beam. It is supposed here that its centre of gravity is confused with the centre of rotation (of torsion) [R3.03.03], and it is neglected warping of the section. The case of the eccentricity of the center of torsion compared to the center of gravity is treated with [§3.1].

1.2.1 Local equilibrium equation

One considers a segment length dx put in rotation under the action of one moment M X [Figure 1.2.1-a] intern and of an external couple X per unit of length.

X
 $X + dx$
 $M (X)$
 X
 $M (X + dx)$
 $NR (X)$
 O
 X
 positive
 dx

Appear 1.2.1-a: Segment of beam in rotation around (OX)

The segment is turned of an angle X compared to the not deformed position. We have as follows:

1.2.1.1 circular Beam of section

2
 $x+dx$
 $x+$
 -
 dx
 $M (X) + M (X + dx) +$
 X
 X
 (S)

$ds =$
 I
 ds
 X
 X
 X
 X
 X

t_2
 with I

=

$R ds$

2 is the plane moment of inertia of the section S around the axis of rotation $(0, X)$.

X

S

As for traction, one obtains after division by dx and passage in extreme cases:

DM

2

X

+

= I

X

dx

X

X

t^2

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The law of behavior is introduced:

M

= $G I$

X

X

$p X$

where G is the module of Coulomb (or modulus of rigidity) and $I p$ the polar geometrical moment by

report/ratio in the centre of gravity of the section. (One has besides: I

= I for a material of mass

X

p

voluminal homogeneous).

We obtain the expression then:

2

X

X

+

=

$G I p$

I

X

X

X

X

t^2

éq 1.2.1.1 - 1

who represents local balance with the first order of a segment of beam for a movement of torsion.

1.2.1.2 unspecified Beam of section

To take account of warping while remaining on the free assumption of torsion, in the case of them

noncircular sections one is led to replace moment $I p$ by a constant of torsion C

(lower than $I p$) in the equation of torsion ([R3.03.03] for the calculation of C).

By definition, $M = G C$

X

X

. One obtains then:

X

2

 X $G C$ X $+ X =$ C X X t^2 **éq 1.2.1.2 - 1**

When the centre of gravity of the section is not the centre of rotation, this expression is not valid and the movements of torsion and inflection are coupled.

1.2.2 Method of the Lagrangian one

We have same manner as with [§1.1.2] the kinetic energy (for example for a beam of circular section):

2

 $L 1$ E

=

 I X dx C

,

 $O X$

2

 T

internal potential energy

2

 $L 1$ E

=

GI

X

dx

,

int

p

O

p

2

T
and the work of the external couple

L

E

=

dx

p

X

.

ext.

O

X

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By applying the equation of Lagrange [éq 1.1.2-1] to variable X , one leads naturally to [éq 1.2.1-1] giving the movement of a beam in pure torsion.

1.3

The pure bending

The inflection is the rotation and translatory movement around an axis perpendicular to the axis longitudinal of the beam. One speaks here about pure bending (around OY or OZ). One limits oneself to the case of

right beams. The curved beams are treated with [§5].

One describes the equation of inflection in plan (O, X, Z) , the extension to the plan (O, X, y) is immediate

[Figure 1.3-a].

Z

y

y

O

X

Appear 1.3-a: Inflection of a beam in plan (O, X, Z)

The translation along the axis (O, Z) is noted W and rotation around (O, y) is noted Y .

1.3.1 Local equilibrium equation

One considers a segment length dx subjected to the shearing action Vz , the moment bending $M y$, an external effort tz distributed uniformly per unit of length, and an external couple m distributed

ext.

yext

uniformly by unit of length [Figure 1.3.1-a].

$V(x+dx)$

$tzext$

$V(X)$

Z

$M(x+dx)$

y

Moments

y

positive

dx

X

$x+dx$

$M(X)$

X

myext

Appear 1.3.1-a: Segment of beam in inflection in plan (O, X, Z)

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The local balance of the forces and the moments (on the section of X-coordinate $X + dx$) gives for the forces:

2

 $x+dx$ $x+dx$

W

 $- V(X) + V$

Z

 $Z(X + dx) +$

T

 $ds =$

X

 z_{ext} $S ds$

X

 t_2

and for the moment:

2

 $x+dx$ $x+dx$ $x+dx$ $- M(X) + M$

y

y

 $y(X + dx) +$

m

 ds

y
-
 $V_z(X)$
)

=

ext.
X

ds
X
I ds

y
X
t2

The terms in dx^2 are neglected. While passing in extreme cases when dx tends towards 0, one obtains:

V
2
Z
W
+ t_z
=

S
X
ext.

t2
My
2
- $V + m$

y
Z
y
= +

I
.
X
y
ext.
t2

It is noted that the effort uniformly distributed t_z produces a term which is of the second order in balance *ext.*

moments and is thus neglected. One introduces then the relations of behavior of resistance materials.

$$M = + I.E.(\text{internal excitation})$$

$$y$$

$$y$$

$$y X$$

$$W$$

$$V$$

$$= K SG$$

$$Z$$

$$Z$$

+

$$X$$

$$y$$

éq 1.3.1-1

the expression [éq 1.3.1-1] of V_z is due to Timoshenko [bib4] where k_z is the coefficient of shearing in direction Z. It characterizes the model of beam of Timoshenko; it will be seen thereafter that it model of beam of Euler corresponds to a simplification of the model of Timoshenko. I_z is the moment geometrical of the section compared to the axis (O, y).

Consequently, one leads to the two equations coupled out of W and $there$ for the inflection in the plan (O, X, Z).

$$W$$

$$2 W$$

$$K SG$$

$$Z$$

$$+$$

$$+ T$$

$$y$$

$$Z$$

$$=$$

$$S$$

éq 1.3.1-2

$$X$$

X
ext.

T 2

2
y
W

I.E.(internal excitation)

-

K SG

y
y
Z
++ m
y
y
= +

I
X
X

X
y
ext.

t2
éq 1.3.1-3

When the beam is uniform, i.e. the section and the material are constant on the axis longitudinal, the equations [éq 1.3.1-2] and [éq 1.3.1-3] are reduced to only one equation out of *W*. For

that, one only once derives compared to X-coordinate X the equilibrium equation from the moments [éq 1.3.1-3].

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3

2

3

y

W

I.E.(internal excitation)

- K SG

y

+

= + I

y

y

.

x3

Z

x2

X

y

X t2

It will be noted that this handling eliminates the presence from the term resulting from an external couple uniformly distributed. Then, the equation [éq 1.3.1-2] can be put in the form:

2

2

y

W

I

W

=

-

+

T

.

X

2

Z

X

K

ext.

2

Z SG

kz G T

4 W

2 W

E 4 W

2I

4

y W

I.E.(internal excitation)

+ S

- I I+

+

- T

y

= 0 éq 1.3.1-4

x4

t2

y

K G

2

2

4

*zext**Z**X T**K G**Z**T*

There remains useful for this type of equation to point out the physical significance of the various terms, so at the time

simplifications to be aware of the neglected effects.

4 *W*

I.E.(internal excitation) $y x^4$ balance charging density in the direction of the translation due to the moment of inflection.

2 *W*

S t^2 is the term of inertia of translation.

4 *W**I*

$y x^2 t^2$ represents the inertia of rotation of inflection.

*E*4 *W*

$I_y K G^2$ is an additional term of the inertia of rotation due to the taking into account of

*Z**X T*

transverse shearing (assumption of Timoshenko).

2 I_y 4 *W*

result from the coupling between the inertia of rotation and the inertia of translation coming from

*K G**Z*

4

T

sharp effort.

The model of beam of Timoshenko (POU_D_T or POU_C_T), takes into account the whole of these terms, in particular those which relate to the sharp effort. One can thus model beams of weak twinge.

The model of beam of Euler (POU_D_E) is a simplification since the deformations in effort edge are neglected as well as the inertia of rotation (what is justified because it does not intervene in dynamic studies that for the high modes). These assumptions are justified in the case of one beam of twinge sufficiently large. So for the model of Euler, the equation of the movement of

inflection, in the case general of the beams with variable section is written:

$$2 W$$

$$2 W$$

I.E.(internal excitation)

+

-

$$= 0.$$

$$2$$

$$y$$

$$S$$

$$T$$

éq 1.3.1-5

$$X$$

$$x^2$$

$$t^2$$

zext

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In addition, it is indeed the shearing action which causes the rotation of the cross-sections by report/ratio with the neutral axis. To neglect this effect thus amounts writing that $V_z = 0$ what brings to [éq 1.3.1-1].

$$W$$

$$y = -.$$

éq 1.3.1-6

$$X$$

who is the translation of the assumption of Euler.

Concerning the inflection in the plan (O, X, y), the same step leads to [éq 1.3.1-7] for

beam of Timoshenko with:

v

$2v$

$K SG$

$y -$
 $+ T$

Z

y

$=$

S

X

X

ext.

t^2

éq 1.3.1-7

2

Z

v

I.E.(internal excitation)

$K SG$

Z

Z

y

$+ m$

Z

Z

$=$

I
X
X -
-

X
Z
ext.

t2
and when the section is constant:
4 v
2 v

E
4 v
2 I 4 v
I.E.(internal excitation)
- S

- I
Z
1+

+
+ T
Z
0.

éq 1.3.1-8

T 4
T 2
Z
K G
=
2
2
4
yext

y
X T
K G

y
T
v

The use of the assumption of Euler (i.e. in the plan (O, X, y) Z =) makes it possible to lead to X the equation of the movement of inflection for a beam of Euler according to [éq 1.3.1-9].

$$2 v$$
$$2 v$$
$$I.E.(internal\ excitation)$$
$$-$$
$$+$$
$$= 0.$$

2
Z
S
T
éq 1.3.1-9
X
x2
t2
yext

1.3.2 Method of the Lagrangian one

The kinetic energy is expressed by:

2
2
L 1
y
L 1
W
E
=

I

dx +

S
dx
C

O
y
2
T
O

2

T
according to displacements in rotation and translation.
The potential energy intern is worth:

2
L 1
y
L 1
W

E
=
I.E.(internal excitation)

dx +

+ dS dx
P

int
O2
X
O
C

2
S

X
y
T

W
where C is transverse shear stress and the term
+ deformation of

T
X
y

shearing. The model of beam of Euler neglects this term while the model of Timoshenko emits an assumption on the distribution of the constraints C in the section, compatible with the expression

T

[éq 1.3.1-1]. In the case general of the model of Timoshenko, the potential energy interns is written:

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2

2

L 1

y

L 1

W

E

=

I.E.(internal excitation)

$dx +$
 $K SG$

$+$
 dx
 p
 $.$

$O2$
 X
 O
 Z
 2

X
 y
 int

The potential of the external loads is expressed as for him by:

L
 L
 E
 $= - T$
 dx
 $- m dx$
 p
 Z

$.$
 O
 y
 O
 y
 $ext.$

*ext.**ext.*

The use of the equation of Lagrange [éq 1.1.2-1] applied once to the variable W then with the variable y brings back for us to the two equations [éq 1.3.1-2] and [éq 1.3.1-3] describing the movement in inflection of one segment of beam.

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2**Element of right beam**

One describes in this chapter obtaining the elementary matrices of rigidity and mass for the element of right beam, according to the model of Euler (POU_D_E) or Timoshenko (POU_D_T). Matrices of rigidity are calculated with option “RIGI_MECA”, and the matrices of mass with the option “MASS_MECA” for the coherent matrix, and option “MASS_MECA_DIAG” for the matrix of mass diagonalized.

2.1**Longitudinal movement of traction and compression**

A difficulty to write the variational formulation comes owing to the fact that there can be in the structures composed of beams of the concentrated loadings (assimilable to of the Dirac). The equilibrium equation [éq 1.1.1-1] must be replaced by:

 dN NR $(X) + F(X) + FCI(X)$ *ext.* I $) = 0$ dx $i=1$

One omitted by simplicity the inertias which would undergo the same treatment as the forces external f_{ext} .

C

I represents the function of Dirac located as in point I , the f_i are the concentrated forces applied with the beam.

For the application of the finite element method, the equilibrium equation must be written in the form principle of virtual work which is in this case:

FD

NR

NR

$dx =$

F

$v dx +$

$F C$

$I (v$

$ext.$

I

)

éq 2.1-1

dx

$i=1$

Any confusion being excluded, I indicates the measurement of Dirac associated with item I , v is a field of longitudinal displacement kinematically acceptable unspecified.

In practice, it is supposed that there is no force concentrated inside the elements of beam, but only with the nodes ends.

2.1.1 Determination of the matrix of rigidity

It corresponds to the expression of the virtual work of the interior forces according to a displacement given. I.e.:

$L FD$

NR

dx

for an element length L .

0

dx

The elastic relation of behavior is introduced:

$NR (X) = ES dx$

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While choosing by function test:

X

X

$$v(X) = 1 \quad (X) = 1$$

and

$$v(X) = 2 \quad (X) =$$

L

L

one obtains directly:

L D

L

ES of

ES

NR

$$1 \, dx =$$

-

dx

= -

$$[(uL) - (U) 0]$$

O

dx

O

L dx

L

and

L D

*L ES of
ES
NR
2 dx =
dx*

*=
[(uL) - U () 0]*

*O
dx
O
L dx
L*

The matrix of rigidity of the element is thus:

*ES 1
-
1
K =*

L -1 1

Note:

In the expression of the virtual work of the interior efforts, U intervenes only for (U)

0 and

(U L): U was not discretized inside the element. This is why the element is qualified of "exact": one obtains the exact solution with the nodes, but only with the nodes.

2.1.2 Determination of the second member

The second member is the expression of the virtual work of the efforts applied.

The second associate member with the loading distributed and the functions tests previously introduced is:

*f1
1*

*X
with
F
=
F*

*1
ext. (X) 1 - dx*

F
0

L

2

1

X

F

=

F

2

ext. (X) dx

0

L

Note:

Into AFFE_CHAR_MECA_F, one can introduce fext like an unspecified function of X.

On the level of the calculation of f1 and f2, on the other hand, integration is made by supposing that fext vary linearly between the values taken with the nodes ends. If one must model one radial force distributed nonlinear, it is then necessary to discretize more finely.

But let us insist on the fact that whatever the shape of F

(X

ext.

) (polynomial or different), if one

can calculate exactly the integrals f1 and f2, the solution of the static problem will be exact with the nodes of the problem.

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The virtual work of the concentrated forces (given by assumption to the nodes of the elements) does not intervene

not directly on the level of the element.

One introduces these forces concentrated in the form of nodal forces, directly in the vector assembled of the second member.

2.1.3 Calculation of the efforts to the nodes of the beam

The complete P.T.V [éq 2.2-1] is written indeed on the assembled system.

In addition, by writing the formula of integration per part on all the structure (beam $[X X O, 1]$):

$$NR$$

$$NR v dx$$

$$, X$$

$$= [NR(x1) v(x1) - NR(x0) v(x0)] + [NR] I(v)$$

$$I$$

$$i=1$$

éq 2.1.1-2

$$M$$

$$- NR v dx$$

$$, X$$

$$J$$

$$J$$

$$=1$$

J representing all the intervals without discontinuity of normal effort, therefore without concentrated force, and

$[NR]$ jumps of NR between these intervalles.

$$I$$

Indeed, by bringing this expression closer to the PTV, one finds, for each loading concentrated (in choosing suitable functions test v):

$$I 1, NR$$

$$[NR]$$

$$F C$$

$$=$$

$$=$$

$$I$$

$$I$$

Each finite element of beam is by assumption an interval without discontinuity. There can thus be discontinuity of the efforts intern NR from one element to another if there is a force concentrated on the node

connecting the two elements.

The internal efforts for an element are determined in the following way:

The equilibrium equation inside an element is:

$NR + F$ reference mark

, X

= 0

The formula of integration by parts [éq 2.1.1-2] on the element gives:

L

L

$NR(X) v dx$

, X

= [$NR(L) v(L) - NR()$

$0 v()$

0

] + F

$ext. (X) v (X) dx$

0

O

By regarding $NR(L)$ and $NR(O)$ as data, one could have obtained this formula directly

PTV [éq 2.1-1].

By still taking the functions test:

X

X

$v(X) = 1 (X) = 1$

and

$v(X) = 2 (X) =$

L

L

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One obtains:

ES

$$[(uL) - (uo)] = - NR (O) + F$$

L

$$ES [(uL) - (uo)] = NR (L) + F$$

L

2

- *NR (O)*

(

U O) f1

that is to say

K

$$NR (L) = [] ($$

*U L) -**f2*

I.e. the internal efforts are obtained by cutting off with the product **K U** the nodal forces equivalent to the distributed loads *fext*.

It is also observed that they are of opposite sign. So that the sign is the same one from one element to another, it

is thus necessary to change the sign of *NR (O)* calculated by this method. It is what is made by the calculation of

option EFGE_ELNO_DEPL.

2.1.4 Determination of the matrix of mass

The matrix of mass to be coherent with the matrix of rigidity is given from same functions test. However, it is not possible any more to calculate exactly the associated nodal forces without making assumption on the form of the solution. The calculation of the matrix of mass will involve one error of discretization.

A dynamic calculation will thus require a discretization of the structure of beam in small elements, what is not the case for a static calculation. It goes without saying that in the case of a dynamic calculation, it

calculation of the efforts which one will lead as to [§2.1.2] by cutting off the nodal forces from inertia is also approximate. The solution *U* is selected in the space generated by the functions tests (it be-with to say the polynomials of degree to most equal to 1):

$$U =$$

(
 U)
 $0 \ 1$
 $(X) + ($
 $U \ L) \ 2$
 (X)

The matrix of mass appears in the expression of virtual work due to the inertias:

U
 $W =$
 T
 \mathbf{V}
 $\mathbf{MR. U}$
 $\%, \mathbf{U} =$
 1

\cdot
 u_2
 Work is also written:

L
 $W =$
 $V(X) \ U$

$\% (X, T) \ dx$
 O
 m

with:
 $=$
 $dS = S$
 m

in the case of a homogeneous material.

S
 While taking
 $U(X, T) = (X) \ U(T) + (X) \ U(T)$, one a:

1
 1
 2
 2
 L
 $1(X)$
 $W = S$
 T
 \mathbf{V}
 (X)

1
X
2
) %U
O
2(

dx
X)
() ()

$L 1 (X)$

$W = T$
 $V S$

(X

1
X
2
) %U.
O
2(

dx
X)
() ()

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The matrix of mass is thus written:

L

L

2

dx

1

1 *dx*

2

M =

S

O

O

L

L

2

1 *dx*

2

dx

O

O2

and made calculations:

SL

2

1

 $\mathbf{M} =$

6 1

2

2.2**Free movement of torsion around the longitudinal axis**

The problem is similar to that of traction compression. For a beam, charged by torques distributed

 C

$X(X)$ and of the concentrated moments I , the principle of virtual work

is written:

 D NR M $dx =$ dx C X X

+

 I $I()$, dx $i=1$

The law of behavior is:

 D M X $X(X)$ $= G C dx$

Except for the variables, this equation with the same form as that of the movement of traction and compression.

By using the same reasoning, one obtains the same expressions for the matrices of mass and of stiffness elementary is:

$G C 1 -1$

$K =$

L

$-1 1$

$C L$

$2 1$

and

$M =$

\cdot
 6

$1 2$

The calculation of the matrix of mass like having previously required to discretize the field solution.

The second member, due to couple X distributed, is in the same way obtained that for the movement of traction and compression:

L

X

$1 (X)$

dx

X

$1 (X) = 1$

O

L

L

$X = X$

$2 (X)$

dx

X

$2 ()$

L

O

2.3

Movement of inflection

We place ourselves here within the framework of a right beam at constant section of Timoshenko type. Us

let us take account of the effects of transverse shearing. The beam of Euler-Bernoulli will be then treated by simplification of the equations of Timoshenko.

The description of the inflection is more complex than the preceding movements, but a judicious choice functions tests will enable us to obtain of the same results forms.

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2.3.1 Inflection in the plan (x0z)

With obvious notations and while not being interested initially, as in the cases precedents, with the efforts of inertia, the principle of virtual work is written for the movement of inflection

in the plan (x0z):

NR

V

(“+”) + $Me = (T + m + +)$, **éq 2.3.1-1**

ext.

ext.

) Tc

C

I ()

m

Z

y

Z

y

I

I

I ()

i=1

for all (,) kinematically acceptable.

The matrix of rigidity results from the expression of the virtual work of the interior forces which one goes

to clarify by using the relation of behavior then while integrating by parts:

V

,

Z (“+) + Me

=

K SG

y

Z

'+ '+ +

,

(W y) () E Iy y

= K S

Z

[G (wL) (“(L) + (L))- (W) 0 (“) () 0+ () 0)]

-

K SG

Z

(W " + ') + K SG

Z y ('+)

+ EIy [y (L) “(L) - y () 0” () 0] - I.E.(internal excitation)

y y “

The functions tests which one will choose “will check the equilibrium equations without second member”,

i.e. [éq 1.3.1-2] and [éq 1.3.1-3]:

"+ ' =

0

éq 2.3.1-2

I.E.(internal excitation) “-

K SG

y

Z

(+) = 0

Under these conditions, nodal forces, expression of the work of the interior forces in these virtual displacements given are expressed exactly, without assumption on the form of the solution, in function of displacements in end of beam as in the preceding cases:

V

Z (“+) + Me = K SG

y

Z

,

+

- 0 '0 + 0

[(wL) ((L) (L)) (W) (() ())] *éq 2.3.1-3*

+ EIy [y (L) “(L) - y () 0” () 0]

Note:

It is clear that the condition [éq 2.3.1-2] led to functions tests depending explicitly geometrical and material characteristics of the beam, but that A do not laugh at awkward.

The couples of functions selected tests are:

(,) = (, ii+) I

4

= ,

1..., 4

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12 I.E.(internal excitation) y

where, while having noted $y =$

, functions

K

2

I are defined by:

Z SGL

3

2

1

X

X

X

$1(X) =$

2

3

+ 1

1+

$L -$

L

y

-

+

$L ($

y)
y

6
X
X

5 (X) =
-
L (
1
1+

y) L
L
3
L
X
4 +
2
y X
2 + *there X*
2 (X) =

-
+
1+

L
2

L -
2

L
y

2
1
X
X

$$6(X) = 3$$

$$4 + 1 +$$

$$L(y) = 1 +$$

$$L - (y) = y$$

1

$$X^3 - X^2 - X + (X) =$$

$$3 - 2 + 3 +$$

$$1 +$$

L

L
y

L
y

- 6
X
X
(X) =
1
7
-
L (1+

y) L
L
L

X 3
2 -
2
y X
y X
(X) =

4
-
+
+
1+

L
2

L

2

L

y

éq 2.3.1-4

1

2

X

X

$(X) =$

3

+ -

2 +

1+

L

(

y)

8

L

y

One checks without difficulty that the couples $(I, i+4)$ check well [éq 2.3.1-2]. Moreover:

$(1, 5) = (1,)0 ,$

$= 0, 0$

(0)

$(1 5)$

$()$

(L)

$(2, 6) = (0,) 1 (,) = (0,) 0$

(0)

2

6

)
(L)
(
éq 2.3.1-5
3, 7) = (0,)
0
(,) = (1,)
0
(0
3
7
)
(L)
(4, 8) = (0,) 0 (,) = (0,) 1
(0
4
8
)
(L)

The matrix of rigidity results easily from [éq 2.3.1-3] (by ordering the columns according to ((wo), (O)

,
(
WL)
, (L
y
y
)
).

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L

L

1

-

-1

-

(2

2

4+

2

2

y

) L

L

(2 - y) L

12 I.E.(internal excitation)

K =

y

12

2

12

L3 (+

1

y

)

L

Sym

1

(2
4+
2
y) L

12

It is clear that the calculation of the efforts acts in the same way that to [§2.1.3].

2.3.2 Inflection in the plan (xOy)

The matrix of rigidity for a movement of inflection in the plan (xOy) is obtained in the same way that in the preceding case. The functions tests which lead to an exact expression of the nodal forces must this time check (equation similar to [éq 2.3.1-2]):

"- ' =

0

éq 2.3.2-1

I.E.(internal excitation) “-

K SG

Z

Z

('-) = 0

The couples of functions selected tests are:

(1, -5) ; (-2, 6) ; (3, -7) ; (-4, 8)

12 *I.E.(internal excitation)*

Z

I being given by [éq 2.3.1-4] while having replaced *y* by *Z* =

. The matrix of rigidity

K

2

y SGL

obtained is:

L

L

1

-1

(2

2

4+

2

2

Z) L

L (2 - Z

) L

12 I.E.(internal excitation)

-

K =

Z

12

2

12

L3 (+

1

Z

)

L

1

-

(2

4+

2

Z) L

sym

12

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2.3.3 Determination of the matrix of coherent mass with the matrix of rigidity

Option “MASS_MECA” of operator CALC_MATR_ELEM.

2.3.3.1 Inflection in the plan (xoz)

Let us consider the movement of inflection in the plan (X O Z), the work of the inertias is written:

L

$$W = (W w\%+ \%$$

m

y

$I y$

O

Z

) dx

with

$$= dS$$

and =

y

$$2 dS$$

m

.

S

Iz

S

In the case of a homogeneous material, we have:

$$m = S$$

and

$$I = I_z.$$

Z

(

$W(X, T)$ and $y(X, T)$ are discretized on the basis of function tests introduced for the calculation of stamp rigidity, that is to say:

$$W(X, T) = 1(X) w1(T)$$

$$+ 2(X)$$

(T

y

$$) + 3(X)$$

$$w2(T)$$

$$+ 4(X) (T$$

y

)

1

2

$$(X, T$$

y

$$) = 5(X) w1(T) + 6(X) (T$$

y

$$) + 7(X) w2(T) + 8(X) (T$$

y

)

1

2

$$w1$$

1

y

1

2

in other words:

$$W$$

$$T$$

$$= \mathbf{W}$$

with

W
W =
and

=

~
~

W
W

2
3

y

2
4
5

T
6

y = **W**
front

EC.

=
y
y

~

7
8

By integrating these notations in the expression of the work of the inertias, one a:

L
W
T

T

T

T

=

W W +

% W

W

% dx

m

O

W

W

Iz

y

y

~

~

~

~

L

or:

W

T

T

T

T

=

W W +

% W

W

% dx

m

.

O

W

W

Iz

y

y

~

~
~
~

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One deduces the form from it of the matrix of mass:

L

$\mathbf{M} = (m_{ij})$ *m =*

S

ij

(*X*)

I

(*X*) *J* (*X*) + *I*

Z (*X*) *i+4* (*X*)

j+4 (*X*) *dx*

O

for *I* from 1 to 4 and *J* from 1 to 4.

That is to say:

2

2

2

2

13L 7L

L

- 11L2

11L2

2

9
3

13 2
3 2
2

Z
Z
Z
L
L
L
L
L
L
L
L
Z
Z
Z
Z
Z

+
+
-
-
+
+
+
+

35
10
3
210
120
24
70
10
6
420
40
24

2

2

L3

L3

3

2

2

2

3

3

3

2

Z

L

- 13L

3L

L

- *L*

L

L

Z

Z

Z

Z

Z

S

+

+

-

-

-

-

140

60

M =

105

60
120
420
40
24
120
(
2
2
2
2
2
2
2
2
+
1
13L
7 L

L
- 11L
11L
L
Z
)
Z
Z
Z
Z

+
+
+
+

35
10
3
210
120
24

L3
L

3

L

3

2

Z

Z

+

+

105

60

120

6

-1 *Z*

- 6

-1 *Z*

+

+

5*L*

10

2

5*L*

10

2

2*L*

L

2

2

Z

Lz

1

Z - L L

L

Z

Z

I

+

+

-

-

+

Z

+

15

6

3

10

2

30

6

6

(

2

+

1

6

1

Z

Z)

-

5L

10

2

2L

L

L 2

+

$Z + Z$

15

6

3

It should well be noted, as in [§2.1.4], that in the dynamic case, one is not ensured to have one exact solution with the nodes, as it is the case in statics.

2.3.3.2 Movement of inflection around the axis (O Z)

In the same way, for the movement of inflection around the axis (O Z), in the plan (X O y), the work of the forces

of inertia is written:

 L $(v v +$ $\%$ $\%$ m Z I Z O y $) dx$

with:

 $=$ Z $dS =$ I I \cdot 2 y S y

This time $v(X, T)$ and $Z(X, T)$ are discretized in accordance with [§2.3.2] by:

$$v(X, T) = 1(X) v T -$$

$$1()$$

$$X$$

$$2()$$

$$T$$

$$Z() + 3(X) v^2(T) - X$$

$$4() Z$$

(T)

1

2

Z

$$(X, T) = -5 (X) v1 (T) +6 (X) Z$$

$$(T) - 7 (X) v2 (T) +8 (X) Z$$

(

T)

1

2

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We obtain the matrix of following mass then:

2

2

2

2

13L 7L

L

11L2

11L2

2

9

3

- 13 2

3 2

2
Z
Z
Z
L
L
L
L
L
L
L
Z
Z
Z
Z
Z

+
+
+
+
+
+
-
-

35
10
3
210
120
24
70
10
6
420
40
24

3
3
3
2

2

2

2

2

3

3

3 2

L

L Z

L

13L

3L

-

Z

Z

L

L

L

L

Z

Z

Z

S

+

+

+

+

-

-

M =

105

60

120

420

40

24

140

60

120

(

2

2

2

2

2

2

+

13L

7 L

L

- 11L

11L

1

L

Z

)

Z

Z

Z

Z

+

+

-

-

35

10

3

210

120

24

3

3

3 2

L

L

L

Z

Z

+

+

105

60

120

6

1

- 6

1

Z

Z

-

-

5L

10

2

5L

10

2

2

2

2L

L

Z

L

-1

Z

Z - L Lz

L Z

I

+
+
+
-
+
Z

15
6
3
10
2
30
6
6
+ (

-

1+
Z
Z) 2
6
1

+

5L
10
2

2
2L

L

Z

LZ

+
+

15

6

3

In the **model of beam of Euler-Bernoulli**, the effects of transverse shearing are neglected. It is thus enough, to obtain the matrices of mass and rigidity associated with this model, to cancel them variables y and Z contained in the matrices of mass and rigidity of the model of Timoshenko. (y and Z utilize the coefficients of form ky and kz , opposite of the coefficients of shearing). It will be noted that in the Euler-Bernoulli model programmed in Aster, the inertia of rotation is also neglected. It is thus necessary, for this model, to cancel the terms in “ I_z ” and “ I_y ” in stamp of mass of the model of Timoshenko.

2.4

Stamp of mass reduced by the technique of the concentrated masses

The matrix of mass is thus reduced to a diagonal matrix and is obtained by the option “**MASS_MEGA_DIAG**” of operator **CALC_MATR_ELEM**.

The element beam is considered with section constant S and constant density.

The technique known as of “Lumping” consists in summoning on the diagonal all the terms of the line of coherent matrix and to cancel all the extra-diagonal terms.

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With regard to the diagonal component related to the movement of traction and compression ($M11$) and that related to the movement of torsion ($M44$), we have:

L

M

= S

11

2

L

M

=

44

(

 $I + I$) I, I

y

Z

y

Z: geometrical moments.

2

One can consider that these components were obtained by dividing the element of beam into two

L

equal shares length

then by associating the mass and inertia obtained the node of

2

half-element. For $M44$, the preceding expression corresponds to a choice: one could also

L

to write: M

= C

44

·

2

Note: Comparison with the methods of numerical integration.

One can note that if one carries out a in the following way approached integration:

(E)

F

=

F

my (aei)

E

N

 $i=1, N$

(ae : nodes I of the élém

ent

E, N

I

: node of the element numbers)

one obtains an identical result (for a beam:

(

my $E) = L$ and $N = 2$).

The diagonal components related to the movements of inflection which are programmed are:

L
M
= *S*

,
22
2
L
M
= *S*
,
33
2

L3
L2
2*L*
M
= Min
S, S

+
I,
55
y

105
48
15

L3
L2
2*L*
M
= Min
S, S

+
I.
66
Z

105

48

15

One finds well the components M_{22} and M_{33} related to the translations of the movements of inflection by technique of the masses concentrated with the nodes. On the other hand, the origin of the formulas used for components M_{55} and M_{66} related to rotations, is unknown. One can simply notice that one find the values:

 L^3 $2L$ S $+ I$

,

 Z

105

15

 L^3 $2L$ S $+ I$

y

.

105

15

for the diagonal components of the matrix of equivalent mass [§2.3]. But this matrix is not not diagonal. Nevertheless, the results obtained by this method remain correct.

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3**Particular right beams**

It is a question in this chapter of taking into account right beams whose section has properties which were ignored until now, in particular the beams having a center of torsion excentré by report/ratio with the neutral axis (the section does not have 2 axes of symmetry), and those whose section evolves/moves continuously on their axis.

3.1**Eccentricity of the axis of torsion compared to the neutral axis**

The center of torsion is the point which remains fixed when the section is subjected to the only moment of torsion. It is also called center of shearing because an effort applied in this point does not produce rotation X .

F

M

O

C

C

O

(not of rotation)

(not of displacement in C)

At the point C, the effects of inflection and torsion are uncoupled, one can thus use the established results in the preceding chapter. One finds the components of displacement as in point O while considering rigid relation of body:

$$U(O) = U(C) + OC$$

 Q X

with

 $= \theta$ vector rotation θ θ

and

$$OC = ey$$

 EZ **Z**

E

C

Z

y

O

ey
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In fact, one obtains:

U

$= U$

X

C

X

U

$= U + E$

y

éq 3.1-1

C

y

Z

X

U

$= U - E.$

Z

C

Z

y

X

The change of variables given by [éq 3.1-1] is written matriciellement:

ux

ux

c1

1 0 0

0

0 0

1

U

y

uy

C

1

0 1 0 - *E*

0 0

1

U

Z

Z

U

c1

0 0 1 *E*

Z

y

0 0

0

1

X

X

c1

0 0 0

1
00

1

y

c1

000010
y1

Z

Z

c1
000
0
01

1

U

X
=
U
C

2
100
0
00
x2
U

y
U
c2

0 1 0 - E
0 0

y2

U
Z
Z

U
C
Z
2
0 0 1
ey
0 0

2

X
c2

0
0 0 0
1
0 0
x2

y

C

y
2
0 0 0
0
1 0
2

Z

C
2

0 0 0
0
0 1
Z
2

!##### "

\$

P

It is thus enough to determine the elementary matrices of mass (**M_c**) and stiffness (**K_c**) in locate (C, X, y, Z) where the movements of inflection and torsion are uncoupled then to be transported in the reference mark related to the neutral axis (O, X, y, Z) by the following transformations:

$$\mathbf{K} = \mathbf{P}^T \mathbf{K} \mathbf{P}$$

C

and

$$\mathbf{K} = \mathbf{P}^T \mathbf{K} \mathbf{P}$$

C

.

The values of e_y and e_z are to be provided to *Code_Aster* via the operand SECTION: "GENERAL" of operator AFFE_CARA_ELEM, default values being obviously zero values.

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3.2 Sections

variables

It is possible to take into account evolutionary sections in a continuous way for the beams right-hand sides of Timoshenko and Euler (POU_D_E and POU_D_T only). One distinguishes two types from

variation of section:

- linear or refines,
- quadratic or homothetic.

The distinction between the two types is conceived easily by taking the example of a beam rectangular:

- if only one of side dimensions varies, one supposes in a linear way, then the surface of cross-section varies linearly, and is given by:

S

X

$$S(X) = S + 2$$

1

-1

1

S

L

1

- when two side dimensions vary (in a linear way), the surface of section will evolve/move in a quadratic way.

2

S X

$$S(X) = S +$$

 2 $1 \ 1$ -1 S L 1

Code_Aster makes it possible to treat sections “RINGS”, “RIGHT-ANGLED” and “GENERAL”, but for obvious reasons of geometry, all these types of section cannot admit the two types of variation. The following table summarizes the existing possibilities.

Section**Constant****Linear****Quadratic**

ring

yes

not

yes

rectangle

yes

yes

yes

according to y

general

yes

not

yes

For the “RIGHT-ANGLED” section, it is the user who chooses the type of variation, while specifying “REFINES”

or “HOMOTHETIC” in AFFE_CARA_ELEM. It is necessary well to note that in the case “REFINES”, them

dimensions can vary only according to Y.

We consider generally that the section varies according to the formula [éq 3.2-1]:

 $X \ m$

$$S(X) = S + C$$

1 1

L

éq 3.2-1

SI is the initial section in $X = 0$

C is fixed by the knowledge of the final section *S2* in $X = L$.

m gives the degree of variation: $m = 1$ variation linear, $m = 2$ variation quadratic.

The section varying, it goes from there in the same way inertias $I(X$

y

), $I(X$

Z

) and $I(X$

p

).

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We will thus have:

X^{m+2}

$I(X) = I$

$1 + C$

y

y1

L

éq 3.2-2

X^{m+2}

$$I(X) = I \\ 1 + C$$

 $Z \\ z^1$ L

éq 3.2-3
 X^{m+2}

$$I(X) = I \\ 1 + C$$

 $p \\ p^1$ L

éq 3.2-4

C is given for each formula starting from the value for $X = L: I, I, I$

 y Z $p.$ 2 2 2

The Coulomb and Young moduli (E) (G) are supposed to be constant.

The principle adopted by *Code_Aster* consists in calculating equivalent characteristics of section, constants on the beam, starting from the real characteristics data at the two ends. These equivalent characteristics thus depend on the phenomenon to which they contribute, in particular, are distinct for the effects of rigidity or inertia.

3.2.1 Calculation of the matrix of rigidity

3.2.1.1 Determination of the equivalent section (Seq)

The determination of the equivalent section does not use nor the method taken with [§2.1.1] to obtain stamp exact rigidity nor an approximation of the solution by a polynomial function like described with [§2.1.4]. In fact, the method employed deviates from the finite element method and even of

the method of Galerkin, it consists in carrying out a resolution of the problem of the beam with section variable without efforts distributed imposed, which makes it possible to clarify the efforts at the ends in function

displacements. This method is “coherent” with that of [§2.1.1] because the definite functions tests in 2.1.1 1 or 0 is worth on the ends of the beam, therefore [éq 2.1-1] the nodal forces can be “comparable” with efforts.

In addition, this method makes it possible to obtain exact results for the static problem without force distributed and led as we will see it with a *Seq* value lain between *S1* and *S2* who, in the case general, guarantees the convergence of the solution approximated towards the exact solution (without however to know the order of convergence).

The section of the beam being variable, the equation of traction and compression in statics without effort distributed imposed is written:

$$U$$

$$E S (X)$$

$$= 0 0$$

$$X$$

$$L$$

$$X$$

$$X$$

éq 3.2.1-1

$$U$$

with

$$NR (X) = E S (X) X$$

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We determine the matrix of rigidity in the case general [éq 3.2-1], we deduce some thereafter values of the equivalent sections for the cases $m = 1$ (linear progression) and $m = 2$ (progression quadratic).

While integrating [éq 3.2.1-1], we have:

 U $E S (X)$ $= C$ X

1

or, by taking account of the expression of $S (X)$:

 $X m U$ $E S 1+ C$ $= C$

1

 $L X$

1

The constant of integration is given starting from the values of thrust loads to the nodes.

We integrate once again in order to obtain the efforts with the nodes according to displacements (

 U) $0 = u l$ and ($U L) = u 2$: U C $X - m$

1

=

+
1 C
X
E S

L
1
C
L

X
from where $U(X) =$

1
ln +
1 C
+ C if $m = 1$
E S
C

L
2

1
C
L
1
and
 $U(X)$

=
1
+ C if m_2
E S
C
 m_1

-
2
1
(

X
L -)

$m +$
 $1 C$

L
It is noted that the expression of (
 $U X$) is far from being polynomial.
By taking account of the fact that:

$$C = -NR$$
$$Po$$
$$ur X =$$
$$1$$
$$1$$
$$0$$
$$C = + NR \text{ for } X = L$$

1
 2
and that:
 $(1+c) - m 1$
 $+ U (0) - U (L)$

$$C =$$
$$2$$
$$(+$$
$$1 C) - m 1$$
$$+ -1$$

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we obtain:

E C M

NR

1

1 =

ln (

u1 - U

L 1+ c) (

2)

for *m =*

1

E C M

NR

1

2 =

ln (

u2 - U

L 1+ c) (

1)

E C M

1

1

1

(1-)

m

(1+ C) m

(1+ C) m

NR

1 =

- *U*

+ *u*

1

2

L

$1 (1+c) M-1$

$1 (1+c) M-1$

and for $m = 2$

E C M

$(1+ C) m 1 -$

$(1+ C) m 1$

-

1

$(1-)$

m

NR

U

$2 =$

U

.

L

1

$1 - (1+ C) m 1$

- - 2

$1 (1+c) m 1$

By replacing C by its value, that is to say:

· if $m = 1$

S

$C =$

$2 - ,$

1

SI

E

$$(S_2 - S_1)$$

$$N_1 =$$

$$(u_1 - u_2)$$

$$L$$

$$\ln S_2 - \ln S_1$$

$$E$$

$$(S_2 - S_1)$$

$$N_2 =$$

$$(u_2 - u_1)$$

$$L$$

$$\ln S_2 - \ln S_1$$

$$\cdot \text{if } m = 2$$

$$S$$

$$C =$$

$$2^{-1}$$

$$S_1$$

$$E$$

$$N_1 =$$

$$S S$$

$$1 \ 2 \ ($$

$$u_1 - u_2)$$

$$L$$

$$E$$

$$N_2 =$$

$$S S$$

$$1 \ 2 \ ($$

$$u_2 - u_1)$$

$$L$$

We note that the matrices of rigidity, in the two treated cases, will have the same form as for a constant section if one takes as equivalent section:

$$(S - S$$

$$2$$

$$1)$$

$$Seq =$$

for a section varying linearly

$$S -$$

$$\ln \ln S$$

$$2$$

$$1$$

$$S$$

=
S S
eq
for a section varying in a quadratic way

1 2
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3.2.1.2 Determination of a constant of equivalent torsion (Ceq)

The equation of pure torsion of a beam with variable section, is written:

$$Q$$
$$G C (X) X$$

0

X

X =

éq 3.2.1-2

X m+2

with: C (X) = C I

1

+ C

(m or

1

2)

L

=

The method is the same one as for the calculation of the equivalent section: it is a question of integrating the equation

the preceding one in order to obtain the efforts (torques X

, X) according to displacements

1

2

with the nodes (X ,

X and to deduce some, by comparison with the formulas with constant section,

1

2)

expression the one geometrical moment polar are equivalent.

By integration of [éq 3.2.1-2], we have:

 X^{m+2} $G C 1 + C$ X D

1

 L

=

,

 X

1

D the constant of integration is determined by the torques applied to the nodes.

- $(m+2)$

 D X

Of:

 X

=

1

+

1 C

,

 X $G C$

L

1

we deduce:

- (*m* +)

D

- *L*

X

1

1

X (*X*)

=

1 + *C*

D.

G C

2

+

1

(

C m +)

1

L

We determine *D*2 starting from the system:

- *D L*

1

X =

+ *D*

1

G C C

2

1

(*m* +) 1

- *D L*

1

- m 1

+

X

=

$1 + C$

+ D

2

$G C C$

2

1

$(m+)$ (

) ()

1

- m 1

+

$X (1 + c)$ (

) + x

that is to say: D

1

2

2 =

(

.

$1 + C) - (m+) 1 - 1$

By taking account of the fact that:

D

= - for X

=

1

1

0

D

= for X

=

L

1

2

,

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we have finally:

for $m = 1$

2

2

G

(2C C213 - C C123)

1

=

2

2

-

L

1

2

(

X

X

C2 3 - C13)

(

)

2
2

G
(2C C213 - C I1p 32)

2

=

2
2
-
L
2
1

(
X
X
C 3 C
2
13)
(
)

We thus take in the case of a section varying linearly, a polar geometrical moment *Ceq* equivalent of the following form:

(2
2
2 *C C*
2
13 - *C C*
1
2 3)

Ceq =
(
linear variation

2
2
C2 3 - C13)
for *m* = 2

3
3
G

(C C214 - C C124)

1
=
3

3
3
-

L
1
2

(
X
X
C24 - C14) (
)

3
3
G
(C C214 - C C124)
2

=
3

3
3
-
L
2
1

(
X

X
C 4 C
2
14)
(
)

In the case of a section varying in a quadratic way, the polar geometrical moment is written:

(3
3
C C
2
14 - C C
1
2 4)
Ceq = 3
(
quadratic variation
3
3
C24 - C14)

3.2.1.3 Determination of the equivalent geometrical moments

In fact, it does not seem possible to find, as we did for the section or the moment geometrical polar, of equivalent geometrical moments (I

I
y and Z
who would come to substitute themselves

eq
eq)
at the geometrical moments (I
and I

y
Z) in the expression of the terms of the matrix of rigidity.

We expose here the method suggested by J.R. BANERJEE and F.W. WILLIAMS [bib3] which clarifies the matrix of rigidity in the case of a movement of inflection of a Euler-Bernoulli beam to section variable (linear or quadratic).

The results correspond to those programmed in *Code_Aster* for a beam of the type Euler-Bernoulli with variable section (linear or quadratic). By extension, the same step is there applied for the beams of the Timoshenko type. The results are not here detailed.

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Let us consider the inflection in the plan (X O y).

On the basis of the static equation of the movement of inflection of a beam of the Euler-Bernoulli type:

2

2 v

v

I.E.(internal excitation) (X)

= 0

Z

, and

=

2

X

2

X

Z

X

X

v (X) is expressed according to four constants of integration (C, C

, C

, C

1

2

3

4

). These constants

are determined by the values of displacements to the nodes:

v (O) = v v (L)

1

= v2

(O) = (L) =,

Z

Z

Z

Z

1

2

that is to say:

v1

C1

Z

C

1

= B 2, B

v

stamp (4 X 4).

2

C

3

Z

2

C4

2

v

Efforts: V (X) =

I.E.(internal excitation) (X)

y

2

X

Z

X

2

v
and moments: $M(X) =$
 $I.E.(internal\ excitation)(X)$

Z
 Z

,
 x_2
also express themselves according to these constants of integration, and one can write:

T_y
 C
 1
 1

M

Z
 C
 1
 $= D^2$

, D

T

stamp (4 X 4).

y
 C
 2
 3

M
 Z
 C
 2
 4

The matrix of rigidity corresponds to the product $D B^{-1}$. The terms of this matrix are clarified in following tables.

X_{m+2}
Let us recall that $I(X) =$
 $I I + C$

, *m*

1 or 2,

Z

z1

L

=

C

C 2

C 3

Let us pose

W1 = I.E.(internal excitation)

, *W2 =*

I.E.(internal excitation)

, *W*

I.E.(internal excitation)

.

z1 L

z1

L

3

Z

=

1

L

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Variation closely connected

Quadratic variation

$m = 1$

$m = 2$

1

1

I

3

I^4

$C =$

2

-1

$C =$

2

-1

I

I

1

1

$C + 2$

4

2

1

$(C + 3 c+) 3$

C

$2 C$

2

$(c+)$

3

3

(

$C C +)$

1

$2 C (c+)$

1 (2 c+)
3

2 C
2
4
(c+)
1 (2 c+)
3
4 C

= C -
2 2
C
5
2
4
(c+) 1

= C -
4 2
C
2
6
3
5
(c+) 1
3

(
C
c+ 2) ln (c+)
1 - 2 C (c+) 1

The matrix **K** is written then:

W W
- W
W
3
1
2
2
3
1
2 3

1
W

- W

+W

K =

1
4
2
2
1
5
Sym
W
- W
3
1
2 3

W
1 6

Now let us consider the inflection in the plan (X O Z).

For the sections with quadratic variation, the step is identical. But it differs for the sections with linear variation (according to y only).

One calculates the terms of the matrix of rigidity corresponding to the inflection in plan (0, X, Z) by values given in the following table.

Variation closely connected: inflection in the plan (0xz)

I
C
= 2 -1
II
(
ln C +)
1

2
1
C
1
2 - 1

1
(C +) 1 1
- 2
1
1
+ C - 2
5 = C 2 - 4
6 = C 3 - 5

(C + 2) ln (C +) 1 - 2c
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In the case of the beams of Timoshenko, for the coefficients of shearing, one applies to the section reduced $K S$ relations used for the section, namely:

(
 $K S_2 - K S$

y

y

1

$K S$

2

1

y
)
(
)

=

eq
(
ln K S

y
2 -
1
1
2
) L (N K S

y
)
(
if the variation is closely connected
K S2 - K S

Z
Z
1

2
1
)
(K S
Z
) =

eq
ln (K S
z2 2) - ln (K S
z1 1)
(K S

y
) = S S K K

eq
1 2
y
y

1
 2
 (
 if the variation is quadratic

$$K S$$

$$Z$$

$$) = S S K K$$

eq
 1 2 Z

Z
 1
 2

and one in the same way introduces the additional terms into *K* that for a constant section. Calculations are not here detailed. One obtains a matrix *K* of the same forms than previously with for principal modification the value of:

· variation
 closely connected
 2
 = (C + 2) ln (C +)

$$1 - 2c +$$

$$c^3 (C + 2)$$

$$12$$

· variation
 quadratic

$$c^3$$

$$2$$

$$=$$

$$+$$

$$c^3 (c^2 + C$$

$$3 +)$$

$$3$$

$$C + 1$$

$$3$$

3.2.2 Calculation of the matrix of mass

3.2.2.1 By the method of the equivalent masses

“Average” values are calculated for the section, the reduced section, and the moments, namely:

$$S + S$$

$$S =$$

$$1$$

$$2 \text{ if the variation is closely connected}$$

$$2$$

$$1$$

L

$$S + S + S S$$

$$S =$$

$$S (X) dx =$$

1
2
1 2 if the variation is quadratic

$$L O$$

$$3
I + I$$

$$y$$

$$y$$

$$I$$

$$1$$

$$2$$

$$y$$

$$=$$

$$2$$

$$I + I$$

$$Z$$

$$Z$$

$$1$$

$$2$$

$$I_z =$$

whatever the variation

$$2$$

$$I + I + I + I$$

$$y$$

$$y$$

$$Z$$

$$Z$$

$$I$$

$$=$$

$$1$$

$$2$$

$$1$$

$$2$$

$$X$$

The matrix of mass is then calculated like that of a beam having these characteristics.

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3.2.2.2 By the method of the masses concentrated (diagonal matrix)

· If the section varies in a way closely connected, the programmed matrices correspond, with regard to movements of traction and compression and torsion, with those of the beams prismatic, by using sections and equivalent inertias of torsion:

S

$3 + S$

1

2

0

8

-

for traction compression: L

S

3

+ S

0

2

1

8

$(L I + I + I + I$

y

Z

y

Z

1 0

1

1

2

2)

X

-

for torsion:

1

4

0 1 .

x^2

-

for the movements of inflection:

$S L$

1

$v I$

$(w I)$

2

M

5,5 ($M6,6$)

0

$z I (y I)$

$S L$

2

2

v2 (w2)

0

M

11,

11

(*M*12,12) *Z*

2 (y2)

*S L*3

2

eq

S L

eq

L

with *M*

= *M*

= min

,

5 5

,

11 11

,

105

48

+

+

15 (*I*

I

y

y

1

1)

S L3

2

eq

S L

eq

L

M

= *M*

= min

,

48

+

+

15 (*I*

I

z1

z2)

6,6

12 12

,

105

S + S

with *S*

1

2

eq =

2

· If the section varies in a homothetic way, the matrices are programmed, for the different ones movements, in the following way:

-

for the traction and compression:

(5

1
 $S + S2)$

L
0

12
1
 U

(
 $S + 5$
 U
1
 $S2)$
2

0
 L

12

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-

for torsion:

$(I + I + I + I$

y

y

Z

Z

$1 0$

1

2

1

$2)$

X

L

1

4

$0 1 x^2$

$-$

for the inflection in each of the two plans:

$(5$

$v W$

1

$S + S^2)$

$1(1)$

L

12

M

Z

y

$5 5$

$, (M6,6)$

0

1

(1)

(

$S + 5$

1

S_2)

L

v

2 (2

W)

12

11

M 11

,

$(M1212,) z_2 (2y)$

(

3

3

1

$S + S_2) L$

(1

$S + S_2) L$

L

with M

$= M$

$= \min$

,

+

5 5

,

11 11

,

I + I
2
105
2
48
(1y 2y) 15

(
3
3
1
S + S2) L

(1
S + 2
S) L
L
and: *M*
=
12 = min

,
+
I + I
.
2
105
2
48
(1z 2z)
6,6
M12,
15

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4

Geometrical rigidity - prestressed Structure

Option: “RIGI_MECA_GE”

In the case of a prestressed structure, therefore subjected to initial efforts (known and independent time), one cannot neglect in the equilibrium equation the terms introduced by the change of geometry of the virgin state of stress in a prestressed state [bib2].

V_0

V^*

V

o_{ij}

ij

virgin state

state

state

of constraint

prestressed

deformed

Appear 4-a: Various states

This change of geometry does not modify the equilibrium equation, within the framework of the assumption of small

disturbances (HPP) around V_0 (and of V^*), that by the addition of a linear term in displacements whose associated matrix is called geometrical matrix of rigidity and who expresses himself by:

u^{3D}

v^{3D}

W

K

O

K

=

FD

G

X ij X

I

V

J

O

where **u3D** is displacement (resp. **v3D** virtual displacement kinematically acceptable) taken with to start from V^* (but compared to V_0 within the framework of the HPP) and *O* the prestressing (of Cauchy if one

wants) since one is within the framework of the HPP.

WG being a symmetrical bilinear form in **u3D** and **v3D**, it can be interpreted like the variation of a potentiel *UG*.

W

= *U*

G

G

One a:

3D

3

U

UD

2 *U*

K

O

K

G

=

X

ij X

I

V

J

O

For a model of beam 3D, the tensor of constraints is reduced in the local axes of the beam to components *xx*, *xy* and *xx*, from where:

3D

3D

3D

3D

3D

3

U

U

U

U

U

UD

2U

O

I

I

=

+ 2 *O*

I

I

+ 2 *O*

I

I

G

xx

X

X

xy X

y

xz X

Z

Vo

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2

u3D

3D

3D

3D

3D

X

ux ux

ux U

Terms

X

,
and

are neglected [bib5]. Moreover, in the reference mark

X

X y

X

Z

room of the center of torsion of the beam:

u3D

$$X(X, y, Z) = (ux) + zy(X) - y Z(X)$$

u3D

$$y(X, y, Z) = v(X) - Z y(X)$$

u3D

$$Z(X, y, Z) =$$

$$\begin{pmatrix} W X \\ (X) \\ u' \\ , \\ , \\ +z - y \end{pmatrix}$$

$$\begin{pmatrix} y \\ Z \\ - Z \\ y \end{pmatrix}$$

and $U =$

$$\begin{pmatrix} v' \\ , \\ - Z X \\ 0 \\ - X \end{pmatrix}$$

$$\begin{pmatrix} w' \\ , \\ + \textit{there} X \end{pmatrix}$$

X

0

from where one draws, according to the preceding assumption:

2

2

$2 U$

O

$=$

$(v' - z')$

$,$

O

$,$

O

$X) + (w' + \text{there } X)$

$2 xy (w' \text{ there } X)$

2

X

$xz (v' Z$

G

xx

$X) ($

$X)$

$+$

$+$

$+$

$-$

$-$

V_0

However, the generalized efforts are connected to the constraints by the expressions:

NR°

O

$= V$

O

$=$

V

O

xx

y

xy

$Z = xz$

S

S

S

$M = Z M = -y$

y

xx

Z

xx

S

S

One deduces some:

L

2 U

=

NR °

O

,

O

,

O

O

G

(' + ' -

' -

' +

' -

,

O

O

(v) 2 (W) 2) 2 M v 2 M W 2V W 2V v

y

X

Z

X

y

X

Z

X

O

,

O

O

+

,

xx (2 2

y + Z) (X

) 2 + (2 y + Z

xy

xz) X

X

V_0

By supposing, moreover, that σ_{xx} is constant in the discretized element (what is inaccurate for example for a vertical beam subjected to its actual weight) and that X varies linearly compared to

X

X

1

1

,

1

2

X

X

= 1-

1

2, from where

, it comes:

$L +$

= -

+

L

X

L

L

O

O

NR

$I_y + I_z$

NR

$I_y + I_z$

-

1
O2
2
2
,
L
S
L
S

,
xx (y + Z) (X)
= (1 2)
O
O

NR
I + I
NR
I + I
V
y
Z
y
Z
O
-

2
L
S
L
S

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By neglecting in particular the terms which had with the influence of the shearing action on the mode of buckling

or of vibration, and by supposing that the distributed loads are null on an element, one a:

$NR(X)$

X

= const

,

handle

$M_y = (M - M$

y^2

$y^1) + M$

L

y^1

$V(X)$

y

= constant,

X

$V(X)$

M

= constant,

$Z = (M$

$- M$

z^2

$z^1)$

$+ M$

Z

L

$z1$

Under this assumption and for the model of Euler-Bernoulli (for the model of Timoshenko, one uses even matrix), one obtains the following matrix:

With

With

With =

1

2

0

A3

Higher triangular part of the geometrical matrix of rigidity with:

1

2

3

4

5

6

1

U

1

v

1

W

X

1

1y

1z

1

$M o l$

y

O

-

NR

2L

$NR O$

2

12 L

O

O

10

$M y2 V$

-

+ Z

2L

2

M o l

Z

O

-

NR

2L

NR O

3

12

-

With

O

1

=

L

O

V

10

Mz2

-

- y

2L

2

O

O

O

O

(

- *Mz* + *Mz*

+ *My* - *My*

I

O

y + *Iz*)

1

2

1

2

NR

12

12

4

(SL)

(LVo

O

y)

(LVz)

-

-

12

12

2 L NR O

5

15

2 L NR O

6

15

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7

8

9

10

11

12

2

U

v2

2
W
X
2
y2
Z
2
1
M o l
y
NR O
2L
NR O
2
-12 L
O
O
10
M y2 V
+
+ Z
2L
2
M o l
Z
NR O
2L
NR O
3
-12
-
L
O
Original version
10
Mz2
+
- y
2L
2
M O
O
O

O
 O
(
 M_{oy2})
 y
 M
 $M_z - M_z$
 $- My +$
With
1
1
1
2
 Z
1
2
=
 O
 $2L$
 $2L$
 $(I_y + I_z)$
(
)
 NR
12
4
-
12
 MO
 O
 O
Original version
 SL
 O
 O
 y^2
 V
 M
 (LV_z)
 Z
 z^2
 y
()

(LVy)

+

-

+

+

2L

2

2L

2

-

-

12

(

12

MO - MO

z1

z2)

NR O

12

(*L No*)

5

-

-

10

(*LVo*

30

y)

+ (12

- *MO + MO*

y1

y2)

NR O

12

(*L No*)

6

-

-

10

(*LVo*

30

y)

+ 12

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7

8

9

10

11

12

2

U

v2

2

W

X

2

y2

Z

2

7

M oyl

O

-

NR

2L

NR O

8

12

-

L

O

O

10

M_y² V

-

- *Z*

2*L*

2

M_ol

Z

O

-

NR

2*L*

NR O

9

12

With

O

O

3

=

L

V

10

M_z²

-

+ *y*

2*L*

2

O

O

O

O

(

- *M_z* + *M_z*

+ *M_y* - *M_y*

I

O

y + *I_z*)

(1

2)

(1 2)

NR

12

10

12

+

(SL)

(LV_o

O

y)

(LV_z)

-

-

12

12

2 L NR O

11

15

2 L NR O

12

15

My

M

By using the equalities

-

Z

V = 0 and

+V = 0, one finds the programmed matrix.

X

Z

X

y

Moreover, to be able to deal with the problems of discharge of thin beams, requested primarily by moments bending and efforts normal, it is necessary to add the assumption of rotations moderated in torsion [bib 4], [bib 5].

This results in the following shape of the field of displacements:

U (X, y, Z) =

(U X) + Z ((X) + (X) (X))- y ((X) - (X) (X

y

X

Z

Z

X
 y
 $)$

In addition, if the center of torsion C is not confused with the centre of gravity, it is necessary to write:

V

$$(X, y, Z) = v(X, C) - (Z - z_C) X$$

W

$$(X, y, Z) = (W X, C) + (y - y_C) X$$

These two modifications bring additional terms in the geometrical matrix of rigidity:

The assumption of moderate rotations results in adding with 2UG the term:

$$L_{21} U = -M O$$

$$O(X, y) M X y(X, Z) V V G Z X y X y Z X y$$

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The terms of matrix *A* to be added are:

O

O

(

M

M

4 -)

5

1

: +

Z (10 -)

11

z2

: -

2

2

O

O

(

M

M

4 -)

6

y1

: -

(10 -12)

y²

: +

2

2

With regard to the eccentricity of the center of torsion, it is necessary to add the terms corresponding to:

*L**L**L**U*²*= NR O Z v'**- NR O y W'**O**O*

'

*G**C*

' -

+

*O**X**C O**X**(y V Z V**C**y**C Z)**X**O**X*

+ (

L

2

y M O - Z M O

'

*C**Z**C**y) (X*

)

O

Moreover, it is necessary to carry out a change of reference mark as with [§3.1].

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5 Beam

curve

To calculate the matrix of rigidity for a curved element of beam, we make calculation while passing by various stages.

We leave the equilibrium equations which integrated will give us a matrix (noted J)

allowing to determine the efforts in a point of the beam knowing the efforts in another point.

This matrix will take into account the local basic change.

Then by writing the potential energy of the element and by noticing the decoupling of the inflection in the plan of the element of the inflection out of this plan, one determines the two matrices of flexibility.

Finally the matrices of flexibility being calculated, one obtains the matrix of local rigidity by using it principle of Castigliano, which must be recomputed in the total base to be assembled.

z2

Z

y

z0

x2

arc

y2

Nj

M

NR

y0

I

C

0

x1

x0

Appear 5-a

Cf: Instruction manual of *Code_Aster* (booklet [U4.2]: modeling index C p26/30).

To attach the efforts applied in a point P of the structure to the efforts obtained in another point Q of the structure, one integrates the equilibrium equations static of a curved beam (without effort distributed).

We here will limit we to study the curved beam with constant section (with taking into account of transverse shearing) and with constant radius of curvature.

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'&

&&y

'&

'&

Ni

&&z

&&x

Nj

C

'& '& '&

&& (X, y, Z) bases local curved beam

Appear 5-b: Locate average (local reference mark)

The equilibrium equations static are:

M1

NR - V =

M

-
= 0
, S
1
0
T, S
R

NR

M
V

+
= 0
1
M + T

1
- V
= 0
, S
R
, S
R
2

V
= 0
M
+V
=
2, S
2, S

1
0

this for:
Q

'&
&&s
Y
M
'&
&&n 1

P
X
Z

Appear 5-c: Curvilinear reference mark

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To integrate, the conditions out of P are used:

$NR = F$

M

$= M$

y

T

y

V

$= - F M$

$= - M$

1

X

1

X

V

$= F$

$M = M$

2

Z

2

Z

While integrating and while passing in the system of following axis:

Q

X

'&

&&s

'&

&&n I

y

Z

P

Appear 5-d: System of axis chosen by integration

One obtains:

NR

\cos

$- \sin$

0

0

0

$0 Fx$

V

1

\sin

\cos

0

0

0

0

Fy

V

2

0

0

1

0

0

0 *Fz*

=

M

0

0

R

T

(

cos -)

1

cos - sin 0

MX

M

0

0

R sin

sin

cos

1

0
My

M
RR
2
(cos -1
) -
sin

0
0
0
1

Mz
!##### "

\$
###

J
We now will take into account the mechanical characteristics by using energy potential:

~
~
~
~
~
~
~
1
2
2
2
2
2
2
S
2

2
NR
V
V
M
M
M
Ep
1
2
T
1
2
=

+
+
+
+
+
ds

2 s1 ES K SG K SG
I.E.(internal excitation)
I.E.(internal excitation)
I.E.(internal excitation)

1
2
1
2
~

(the sign “~” means that we use the efforts intern) (note: $F = - F$ by the principle of action-reaction).

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**Recall of the relations of behavior
within the framework of the model of Timoshenko:**

U

$$NR = ES S$$

~

W

V

1

1

$$= K SG$$

1

-

S

2

~

W

V

2

2

$$= K SG$$

2

+

S

1

~

M

= *I.E.(internal excitation)*

T

S

~

M

1

1

= - *I.E.(internal excitation)*

S

~

M

2

2

= - *I.E.(internal excitation)*

2

S

Torque of the interior efforts:

Kinematic torque:

~

~

S

NR + T

s+ N +

1

1 **N**

{ }

2

T

2

~

~

{ }

C

MS + M

U.S. + WN + WN

T

1

1

2

2

from where:

1

0

0

ES

NR

1 s2

1

Ep =

0

0

2 (

NR, V, V

1

2)

VI

ds

s1

K SG

1

V

1

2

0

0

K SG

2

1

0

0

I.E.(internal excitation)

M

1

T

S

1

+ 2

(M, M, M

T

1

2) 0

0 *M ds*

2

S

1

I.E.(internal excitation)

1

1

1 *m2*

0

0

EI2
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or:

1

ES

1

Fx T

Fx

K SG

1

Fy

Fy

1

0

1

B Fz

K SG

Fz

Ep

T

=

J

2

J

Rd

2 *O MX*

Q

1

MX

My

GC

My

1

Mz

0

Mz

P

I.E.(internal excitation)

P

1

1

I.E.(internal excitation)

2

J being the matrix obtained previously.

One can thus calculate the matrix of flexibility []

C:

1

ES

1

K SG

1

1

0

K SG

[]

C

=

JT

2

J

1

Rd

0

GC

1

0

I.E.(internal excitation)

1

1

I.E.(internal excitation)

2

We can notice that the matrix **J** can break up into two pennies matrices independent, a part concerning the inflection in the plan of the element, the other concerning the inflection out of the plan of the element.

Note:

This decomposition will make it possible to reverse the matrices more easily than we go to obtain a little further.

J J

J

1

,

2

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Inflection in the plan of the element:

NR

cos

- sin

0 F_x

V

=

sin

cos

0 F_y

1

M

$R (\cos -) 1 - R \sin 1 M_z$

2

!#####"

#####\$

#

1

J

Inflection out of the plan of the element:

M
cos
-
sin

R (cos
-)
MX
T
1

M
= sin
cos

R sin
1
My
V

Fz
2
0
0

1

!##### "

##

\$

#####

J2

5.1

Stamp flexibility for the inflection in the plan of the beam [C1]

1

cos

$\sin R$

$(\cos -1)$
 $) ES$

[

1
 $CI] =$
- $\sin \cos$
- $R \sin$
J Rd

O

K SG

1

1

0

0

1

1

I.E.(internal excitation)

2

2

2

$\cos \sin R2 (\cos -) 12$

\cos

$\sin \cos$

$\sin R2 (\cos-)$

1 \sin

$R (\cos 1$

-)

+
+
-
+
-

ES K SG
I.E.(internal excitation)
ES
K SG
I.E.(internal excitation)

1
2
1
2
EI2

2
2
2
2

=

sin cos *R* sin
R sin
+
+
-
Rd
O

ES
K SG
I.E.(internal excitation)
I.E.(internal excitation)

1
2
2
1

sym.

I.E.(internal excitation)

2

Appendix:

$\cos^2 D +$
 S
 $\sin^2 D =$

O
 O

1
1
 $\cos^2 D$

=
(+sin cos

) =
(2 - sin (2))

O
2
4

1
1
 $\sin^2 D$

=
(+sin C bone) =
(2 - sin (2))

O
2
4

sin²

sin cos D =

O

2

from where:

R

R

R³

C111 =

(2 + sin (2)) +

(2 - S (

in 2)

) +

(6 - 8 sin + s (in2),

4ES

4 K SG

4 I.E.(internal excitation)

1

2

R

R

R³

C122 =

(2+ (

sin 2) +

(2 - sin (2)) +

(2 - (

sin 2),

4ES

4 K SG

4 I.E.(internal excitation)

1

2

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R

C133 =

*,
EI2*

R

R3

R3

R3

C1

2

2

2

12

= -

sin +

sin -

sin -

cos -

[2

2],

2ES

2k SG

2EI

2EI

1

2

2

R2

C113 =

(sin -),

EI2

R2

$C123 =$

(cos -)

1 .

$EI2$

5.2

Stamp flexibility for the inflection out of the plan of the beam [C2]

1

0

0

cos

sin

0 *I.E.(internal excitation)*

[

1

$C2] =$

- sin

cos

0 0

0 **J** Rd

O

I.E.(internal excitation)

2

R (cos

-)

1

R

1

sin 1

1

0

0

K SG

2

2

2

cos sin

cos

sin cos

sin

R (cos -)

1 cos *R*

2

sin

+

-

+

+

I.E.(internal excitation)

I.E.(internal excitation)

I.E.(internal excitation)

I.E.(internal excitation)

I.E.(internal excitation)

I.E.(internal excitation)

1

1

1

2

2

sin cos

R (cos-)

1

=

sin

R cos

sin

+

-

+

Rd

O

I.E.(internal excitation)

I.E.(internal excitation)

I.E.(internal excitation)

1

EII

R2 (cos-) 2

1

R2 sin2

1

sy.

m

+

+

I.E.(internal excitation)

I.E.(internal excitation)

K SG

1

2

R

R

C211 = -

(2+ (

sin 2) +

(2+s (in2)

4EI

4EI1

R

R

C222 =

(2

$$\begin{aligned}
 & - \\
 & (\\
 & \sin 2) + \\
 & (2 \\
 & + S (\\
 & \sin 2) \\
 & 4EI \\
 & 4EI1 \\
 & R3 \\
 & R3 \\
 & Rb \\
 & C233 = - \\
 & [6 - 8 \sin + s (\\
 & \sin 2)] + \\
 & (2 + (\\
 & \sin 2) + \\
 & 2EI \\
 & 4EI \\
 & K SG \\
 & 1 \\
 & 2 \\
 & R \\
 & R \\
 & C2 \\
 & 2 \\
 & 2 \\
 & 12 \\
 & = - \\
 & \sin + \\
 & \sin \\
 & 2EI \\
 & 2EI1 \\
 & R2 \\
 & R2 \\
 & C213 = \\
 & (2 + (\\
 & \sin 2) - 4\sin) + \\
 & (2 - (\\
 & \sin 2) \\
 & 4EI \\
 & 4EI1
 \end{aligned}$$

R2
2
R
C2
(sin² + 2cos - 2)

2
23
= -

+
sin
2EI
2EI1

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Having determined the matrix of flexibility, we will be able to calculate the matrix of rigidity.

One thus has:

1

T

E

E

=

T

C

T, this

T, (

: for outside)

P [
] *E*

E
E

P
P
P
2

Like one a:

Te
J Te

=
Q
P

(Recall: **Te** and **Te**
are not described in the same base).

P
Q

One also has:

T
1
T
E
E
1
-
1

-
E
E

P
=

T
J
C J T

,
T
Q

[] *Q*

Q

2

Using the theorem of Castigliano, one can obtain displacements associated with the external efforts

Te.

$$U = [C] T_e$$

P

in the local base of the point P

,

P

(S_N, N

1

2) P

T

and U

$$= J_1$$

-

- 1 in the local base of $P_0 Q$

int

,

Q

[]

$$C J T_e$$

Q

(S_N, N

1

2) Q

Te

and

Te.

P

Q

By breaking up the problem into two subproblems, and using the principle of superposition, one can write:

Te

=

E

+

E

=

I

I

-
-
I: for intern,

P

T

T

T

T

total

PP

PQ

PP

PQ

(

)

effort

effort

coming

coming

point

point

P

Q

and **Te**

=

Te

+ **Te**

=

I

I

-

-

Q

QQ

T

T

total

QP

QP

QQ

effort

effort
coming
coming

point

point

P

Q

(one applies two torques of effort (independent one of the other) to the point *P* and the point *Q*).

Action coming on a side: **Ti = - Te = - K Up**

-1

PP

P

(K = [] C)

Action coming on other side: **Ti = +**

1

J Te =

1

J

T

T

PQ

Q

[J

K

J

] U = K

J

U

Q

Q

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We obtain as follows:

Te

=

-

K U

K JT U

Ptotal

P

Q

and the same: **Te**

= - **J K U + J K JT U**

Qtotal

P

Q

The matrix of rigidity for “displacements” **U P** (given in its local base) and for “displacements” **U Q** (given in its local base) is:

K - K J

T

K =

- **J K J K JT.**

Moreover, in the case of beams with hollow circular section (elbows of pipings), one divides *I* and *I*

y

Z

by coefficients of flexibility given by the user, to take into account the variation of rigidity had with ovalization (cf [§7.2]).

For the matrix of mass, one considers only the reduced matrix (concentrated masses), and one makes the simplifying assumption that the expression given for the right beams [§2.4] remains valid in considering a right element length R .

One obtains:

$S R$.

$$M = M = M = M = M = M$$

11

22

33

77

88

99 =

2

$(I + I$

y

$Z) R$.

M

= M

44

10 10 =

2

2 $I R$

3

2

.

y

$S (R) S (R)$

$M = M$

55

11 11 =

+ min

,

15

105

48

3

2

2 I R

.
S

Z

(R)

S (R)

M

= M

66

12 12 =

+ min

,
15

105

48

This assumption is restrictive and does not allow to take well into account the inertia of inflection or of torsion which had with the curve. It is thus in this case to better model a beam curves by several elements POU_C_T.

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6 Loadings

Various types of loading developed for the elements of beam “MECA_POU_D_E”, “MECA_POU_D_T”, “MECA_POU_C_T” in linear elasticity are:

Types or options

MECA_POU_D_E

MECA_POU_D_T

MECA_POU_C_T

* CHAR_MECA_EPSI_R: loading by

developed

developed

developed

imposition of a deformation (of type

thermal stratification)

* CHAR_MECA_PESA_R: loading due to

developed

developed

developed

gravity

* CHAR_MECA_FR1D1D: loading distributed

developed

developed

developed

by actual values

* CHAR_MECA_FF1D1D: loading distributed

developed

developed

developed

by function

* CHAR_MECA_TEMP_R: loading

developed

developed

developed

“thermal”

* CHAR_MECA_FRELEC: loading “forces

developed

developed

not

electric " created by a secondary driver

right

* CHAR_MECA_FRLAPL: loading “forces

developed

developed

not

electric " created by a secondary driver

unspecified

6.1

Loading by deformation

OPTION: "CHAR_MECA_EPSI_R"

One calculates the loading starting from a state of deformation (this option was developed to take in account the thermal stratification in pipings). The deformation is given by the user to assistance of key word EPSI_INIT in AFFE_CHAR_MECA.

6.1.1 For the right beam of Euler and the right beam of Timoshenko

The model takes into account only one work in traction and compression and pure inflection (not of effort

edge, not of torque).

The following relations of behavior are used:

U

$$NR = ES, X$$

M

$$= EI$$

y

y

$y,$

X

M

$$= EI$$

Z

Z

$Z.$

X

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Thus, one obtains the second elementary member associated with this loading:

U

at item 1: *F*

=

E S

,

x1

1 X

M

=

E I

y

,

y

y

1

1 X

M

=

E I

Z

,

Z

Z

1

1 X

U

at item 2: *F*

=

E S

,

x2

2 X

M

=

E I

y

,

y

y

2

2 X

M

=

E I

Z

,

Z

Z

2

2 X

“U

y

”

While being given

Z

,

and

for a beam, one can affect a loading to him.

X

X

X

6.1.2 For the beam curves of Timoshenko

y

X

The method used can be compared with that previously presented. One uses to obtain it loading with the nodes, the matrix of local rigidity **K** which multiplied by displacements with the nodes *U* gives the efforts **F** applied to the nodes:

$\mathbf{F} = \mathbf{K} U$.

The method takes into account only the deformation related to the length of the beam. But one does not take

in account that the length projected on \mathbf{X} , i.e. the shortest distance connecting the two points extremes of the curved beam: $2R \sin$ (R : radius of curvature). This length is multiplied by one rate of deformation, which gives the state of deformation of the beam. Then one projects on \mathbf{X} and \mathbf{Y} .

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One has thus for U :

- $2R \sin EPX \cos$

$2R \sin EPX$

\sin

point 1

0

0

$U =$

0

0

point 2
 $2R \sin EPX$

cos

 $2R \sin EPX \sin$
 U

with $EPX = X$

6.2

Loading due to gravity

OPTION: "CHAR_MECA_PESA_R"

The force of gravity is given by the module of acceleration G and a normalized vector indicating direction \mathbf{N} .

The principle to distribute the loading on the two nodes of the beam is to take account of functions of form (X) associated each degree of freedom of the element [§2]. We thus have for

a loading in gravity an equivalent nodal force Q :

 L
 $Q =$
 $(X) S G N dx$
 O

Note:

The functions of form used are (simplifying assumption) those of the model Euler-Bernoulli.

It is necessary, of course, to be placed in the local reference mark of the beam to make this calculation.

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Axial load (in X):

L

F

=

G X

S

dx

X

I

I

O

X

X

=

1

1-

, =

L

2

L

S

S

from where: *F*

= **G**

X L 1

2

X

+

at item 1,

1

3

6

S

S

F

= **G**

X L 1

2

X

+

at item 2,

2

6

3

X

for a section varying *S* linearly = *S* +

1

(*S*₂ - *S*₁)

,

L

3 *S* + 2

S S + *S*

1

1 2

2

and F

$$= \mathbf{g} \mathbf{x} L$$

X

,

1

12

$S +$

$S S + S$

2

2

1 2

3 2

F

=

\mathbf{X}

X

$\mathbf{G} L$

,

2

12

2

X

for a section varying homothétiquement

S

$S +$

S

S

1

(2 - 1)

L

Torque:

Without taking into account of warping, it is non-existent.

· In the plan (X O Z):

L

F

=

G Z S dx

Z

1

O 1

L

M

=

G Z S dx

y

1

O 2

L

F

= **G Z**

S dx

Z

2

O 3

L

M

=

G Z S dx

y

2

O 4

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3

2

3

2

X

X

X

X

X

1 = 2

L

3

+1 , =

+ 2

,

2

L -

L

-

L

L - L

3
2
3
2
X
X

X
X

3 = -

2
3
, = *L*
+
4

L +
L
-
L

L

7 *S*
3 *S*
from where: *F*
=
2
Z

1

G

Z L

+

,

1

20

20

2

1

S

2

S

M

= - **G**

Z

L

+

,

1

y

20 30

3 S

7

1

S2

F

=

G

Z L

+

,

2

Z

20

20

2

1

S

2

S

M

=

G

Z L

+

,

2

y

30 20

for a section varying linearly,

$8 S + 5 S S$

+ 2

1

1 2

S²

and: F

=

G zL

,

1

Z

30

$- 2 S - 2 S S - S$

M

=

G zL2

1

1 2

2

y

,

1

60

2 *S* + 5 *S S*

+ 8 *S*

F

=

G zL

1

1 2

2

Z

,

2

30

S + 2 *S S* + 2 *S*

M

=

G zL2 1

1 2

2

y

,

2

60

for a section varying homothétiquement.

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· In the plan (X O Z):

L

F

=

G y S

D

X

y

1

O 1

L

M

=

G y - S D

X

Z

1

O

2

L

F

=

G y S

D

X

y

2

O 3

L

M

=

G y - S

D

X

Z

2

O

4

7 S

3 S

from where: *F*

=

G

y L

1 +

2 ,

yl

20

20

S

S

M

=

G

y L2

1 + 2 ,

z1

20 30

3 *S*

7 *S*

F

=

G

y L

1 +

2 ,

y2

20

20

S

S

M

= - **G**

y L2

1 + 2 ,

z2

30 20

for a section varying linearly,

$$8 S + 5 S$$

1

S

$$1 2 + 2 S^2$$

and: *F*

=

G y L

y

,

1

30

$$2 S + 2 S S + S$$

2

1

1 2

2

M

=

gy L

,

1

Z

60

$$2 S + 5 S S$$

8

+

1

1 2

*S*²

F

=

gy L

,

2

y

30

- S - 2 S S - 2 S

2

1

1 2

2

M

=

gy L

,

2

Z

60

for a section varying homothétiquement.

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6.3 Loadings

distributed

OPTIONS:

“CHAR_MECA_FR1D1D”,

“CHAR_MECA_FF1D1D”,

The loadings are given under key word FORCE_POUTRE, that is to say by actual values in AFFE_CHAR_MECA (option CHAR_MECA_FR1D1D), is by functions in AFFE_CHAR_MECA (option CHAR_MECA_FF1D1D).

The various possibilities are:

constant loading

variable loading

linearly

right beam with constant section

developed

developed

right beam with section varying linearly

developed

not

right beam with section varying

developed

not

homothétiquement

curved beam

developed

not

The loading is given only by forces distributed, not by moments distributed.

The method used to calculate the loading to be imposed on the nodes is that presented to [§2.1.2].

6.3.1 Right beam with constant section

For a loading constant or varying linearly, one obtains:

N
 N
 F
 $= L$
1
2
 X
+

3
6
 N
 N
 F
 $= L$
1
2
 X
+

6
3
 N and N
1

2 are the components of the axial loading as in points 1 and 2 coming from the data of the user replaced in the local reference mark.

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T, T

, T

and *T*

y

y

Z

Z are those of the sharp effort. And one a:

1

2

1

2

7 *T*

3 *T*

y

y

F

= *L*

1

2

y

+

,

1

20

20

T

T

y

y

M

= *L2*

1

2

Z

+
,
1
20 30
3 *T*
7 *T*
y
y
F
= *L*
1
2
y

+
,
2
20
20
T
T
y
y
M
= - *L2*
1
2
Z

+
,
2
30 20
7 *T*
3 *T*
Z
Z
F
= *L*
1
2
Z

+

,

1

20

20

T

T

Z

Z

M

= - *L2*

1

2

y

+

,

1

20 30

3 *T*

7 *T*

Z

Z

F

1

2

Z

=

L

+

,

2

20

20

T

T

Z

Z

M

= *L2*

1

+ 2 .

y²

30 20

6.3.2 Right beams with variable section

The provided loading must be constant. One uses a method similar to that used by the weight clean [§6.2].

6.3.3 Beam**curve**

The provided loading must be constant along the element. The loading reduced to the nodes is equivalent with that which one can obtain by taking again the results of [§6.3.1]. With a load constant, that becomes:

 N N F $= L$ $, F$ $= L$ X X

,

1

2

2

2

 T T F $= L y$ $, F$ $= L y$ y y

,

1

2

2

2

 T T M

= *L*

y

2

, *M*

= - *L*

y

2

Z

Z

,

1

2

12

12

T

T

F

= *LZ*

, *F*

= *LZ*

Z

Z

,

1

2

2

2

T

T

M

= - *L2Z*

, *M*

= *L2Z*

y

y

.

1

2

12

12

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6.4 Loading

thermics

OPTION: “CHAR_MECA_TEMP_R”

To obtain this loading, it is necessary to calculate the deformation induced by the difference in temperature

T - Tréférence:

U

= -

L

1

(T - Tréférence)

U

=

L

2

(T - Tréférence)

(: thermal dilation coefficient).

Then, the induced forces are calculated:

F =

K U

as **K** is the matrix of local rigidity to the element, one must then carry out a change of base to obtain the values of the components of the loading in the total reference mark.

6.5 Loading

electric

OPTION:

“CHAR_MECA_FRELEC”,

“CHAR_MECA_FRLAPL”.

This type of loading makes it possible to take into account the force of Laplace acting on a driver the main thing, due to the presence of a secondary driver.

The secondary driver is right and it is not based on part of the *Aster* grid if one uses option “CHAR_MECA_FRELEC”.

The secondary driver is not necessarily right and it can be based on part of the grid *Aster* if one uses option “CHAR_MECA_FRLAPL”.

Linear density of the force of Laplace exerted in a point *M* of the principal driver by secondary driver is written:

1

E R**F** (*M*) =**E**

2

ds

2 1

R 3

2

Note:

To obtain the force of Laplace, the vector turned over following the calculation carried out by one of these two options must be multiplied by the temporal function of intensity specified by operator “DEFI_FONC_ELEC”.

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6.5.1 Secondary driver finished or infinite right

OPTION: "CHAR_MECA_FRELEC"

For a finished secondary driver, we have:

,

,

,

E

N

$F(M) =$

$1 (\sin - \sin$

1

2

)

2

D

P

P

1

2

\mathbf{D}

$\mathbf{E} 2$

$\mathbf{E} 2D$

with

$\mathbf{N} =$

D

1

2

$D = \mathbf{D}$

$\mathbf{E} 1$

&&

M

1

For an infinite secondary driver, we have:

\mathbf{N}

$\mathbf{F}(M) = \mathbf{e1} D$

Three types of loading are possible:

- conducting infinite parallel right,
- conducting multiple infinite parallel rights,
- conducting right in unspecified position finished or infinite.

In the case of a **driver infinite parallel right**, its position can be given in two manners:

- is by a vector translation of the principal driver to the secondary driver,
- is by the distance between the two drivers and by a point of the secondary driver.

One thus obtains a constant loading on all the principal element. One can thus use the technique presented to [§6.3] [§6.4] to calculate the loading with the nodes.

L

\mathbf{F}

$= \mathbf{F}$

1

$(M) 2$

L

\mathbf{F}

$= \mathbf{F}$

2

$(M) 2$

2

L

\mathbf{M}

$=$

\mathbf{E}

\mathbf{F}

1

1

$(M) 12$

2

L

\mathbf{M}

$= - \mathbf{E} \mathbf{F} (M)$

, \mathbf{F}

2

1

$(M) = \text{constant } M$

.

12

For the case **of the parallel drivers right infinite multiple**, one must give it directly vector “forces of normalized Laplace”. This one being usually given in the total reference mark, it is necessary

to determine the vector “forces of Laplace” in the local reference mark of the principal element. Thus of same

manner that previously, one calculates the loading with the nodes.

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In the case of a **right driver in unspecified position finished or infinite**, its position is given by two points $P1$ and $P2$ such as the current circulates of $P1$ towards $P2$. The loading is not obligatorily constant along the principal element.

The selected method to calculate the loading reduced to the nodes is obviously the same one that before. But here, integration is done numerically by discretizing the element in some numbers (in practice: 100 points between $P1$ and $P2$).

One integrates as follows:

3

2

L

X

X

F

=

F

1

M

() 2

3

+1 dx

O

$L -$

L

3

2

L

X

X

F

=

F

2

M

() -

2

+ 3

dx

O

L

L

,

3

2

L

,

,

X

X

X

M

=

- *E*

F (M) L

-

+ 2

dx

1

O

1

L

L - L

,

3

2

L

,

,

X

X

M

=

- *E*

F (M) L

-

+

dx

2

O

1

L

L

Note:

Like $\mathbf{F}(M) \mathbf{X} = 0$

and

($\mathbf{e1 F}(M)$

) $\mathbf{X} = 0$ ($\mathbf{X} = \mathbf{E}$

1)

one uses only the functions of form associated with the problem with inflection.

6.5.2 Secondary driver describes by part of grid ASTER

OPTION: "CHAR_MECA_FRLAPL"

The secondary driver as a whole is not necessarily right. But it is described only by right elements. Its length is obligatorily finished.

Its position can be supplemented by the use of a vector translation or a symmetry plan (by one not and a normalized normal vector of the symmetry plane) compared to the principal driver.

Except the fact that it is necessary to summon the interaction of the various elements of the secondary driver on

principal driver, the method is the same one as previously (case (III)).

Note:

For numerical integration, one uses only 5 points ranging between P1 and P2.

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7

Torque of the efforts - Torque of the constraints (or efforts generalized) - nodal Forces and reactions

OPTIONS:

"EFGE_ELNO_DEPL",

"SIEF_ELGA_DEPL",

"SIGM_ELNO_DEPL",

"SIPO_ELNO_DEPL",

“FORC_NODA”,

“REAC_NODA”.

Option “EFGE_ELNO_DEPL” makes it possible to calculate the torque of the efforts to the 2 nodes of each element of “beam”.

Options “SIGM_ELNO_DEPL” and “SIPO_ELNO_DEPL” make it possible to calculate the maximum values

components of the tensor of the constraints intervening in a model of “beam”.

Option “SIEF_ELGA_DEPL” allows the calculation of the nodal forces (option “FORC_NODA” and of reactions “REAC_NODA”).

Note:

When one of these options end in “_C”, that means that the values of displacements (and thus of the efforts) are complex.

7.1

The torque of the efforts

OPTION: “EFGE_ELNO_DEPL”

One seeks to calculate with the two nodes of each element “beam” constituting the grid of studied structure, efforts exerted on the element “beam” by the remainder of the structure. The values are given in the local base of each element.

By integrating the equilibrium equations, one obtains [§2.1.3] the efforts in the local reference mark of the element:

R

= **Ke U**

+ **Me u%**

- **F E**

LOC

LOC

LOC

LOC

LOC

LOC

where: **R**

= - NR

,

1 - V1, - V1, - M1, - M1, - M1, NR

2, V

2, V

2, M

2, M

2, M

2

LOC

(

Y

Z

T

Y

Z

Y

Z

T

Y

)

Ke

elementary matrix of rigidity of the “exact” element of beam,

LOC

Me

elementary matrix of mass of the element beam,

LOC

F E

vector of the efforts “distributed” on the element beam,

LOC

U

vector “degree of freedom” limited to the element beam,

LOC

%

U

vector “acceleration” limited to the element beam.

LOC

One changes then the signs of the efforts to node 1 [§2.1.3].

OPTION: “SIEF_ELGA_DEPL”

Option “SIEF_ELGA_DEPL” is established for reasons of compatibility with other options.

It is used only for calculation of the nodal forces and the reactions.

It is calculated by: **R**

Ke

=

U

LOC

LOC

LOC

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7.2

The tensor of the constraints

OPTIONS:

“SIGM_ELNO_DEPL”,

“SIPO_ELNO_DEPL”.

One seeks to calculate the maximum values of components xx , xy

and xz which is connected to

efforts by:

$$NR = dS$$

xx ,

S

$$V = dS$$

y

xy ,

S

$$V = dS$$

Z

xz ,

S

$$MT = (y - Z$$

xz

xy ,

S

) dS

$$M = Z dS$$

y

,
 S_{xx}
 $M = -y D$
 S
 Z

.
 S
 xx
With option "SIGM_ELNO_DEPL", one calculates the maximum effect of the whole of the efforts on xx , xy and xz .

· For
 xy , one a:
 I
 I
 V
 M
 I_{Max}

Y
 T
 I
 xz
 $=$
 $+$
 R , for

node II
(= ,
1 2)
 I
 I
 T
 Y

With
 $J X$
with: *With K there A air*

: E of the section,
 ky : constant of shearing,
 $J X$: constant of torsion,
 RT : ray of torsion.

· For
 xz , one a:
 I

I

V

M

I Max

Z

T

I

xz

=

+

R,

I

I

T

Z

With

J X

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· For

xx, one a:

-

For a rectangular section:

I

I

I

I

I

NR

M

H

M

H

I Max

Y

Z

Z

Z

xz

= + +

+

if *NR*

0

,

I

I

I

With

I

Y

2

IZ

2

I

I

I

I

I

NR

M

Y HZ

MZ HZ

-

+
+
if
0

$NR <$

,
 I
 I
 I
With
 I

Y
2
 IZ
2
with IY, IZ : geometrical moments,
 Y
 H
 HZ

,
: sides of the rectangle.

-
For a circular section:

I
 I
 NR
2
2 R
 $I Max$
 I
 I

xx
= +
+
 M
+ M

if *NR*

0

,

I

(*Y*) (*Z*) *I*

With

I

Y

I

I

NR

2

2 *R*

I

I

-

+

M

+ *M*

if

0

NR <

,

I

(*Y*) (*Z*) *I*

With

I

Y

with *R*: ray of the section.

-

For a general section:

I

I

I

NR

M

M

I Max

Y

I

Z

I

xx

=

+

R

-

R.

I

I

Z

I

Y

With

IY

IZ

In this last case, one makes calculation at the point (*R I, R I*

I

I

I

Y

Z). R

R

Y

Y

and

and

RZ are them

distances to neutral fibre.

To find these formulas, one uses the relations between the constraints and the deformations then those between the internal efforts and (*U, X; xy; xz; X*

X; y X;

,

,

Z

, *X*):

$xx = E \ xx$

$xy =$
 $G \ 2 \ xy$

$xz =$
 $G \ 2 \ xz,$

$xx = U, \ x + Z \ y, \ X$
 $- y \ Z, \ X$

2

$xy = xy - Z \ X, \ X \ (xy = V, \ X - Z)$
2

$xy = xz + \textit{there} \ X, \ X$

$(xz = W, \ x + y),$

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NR

EA

U, X

V
K GA

Y

Y
0
xy

V
K GA

Z
Z
xz

=

M
GJ

T

X, X
M

0
I.E.(internal excitation)

Y
Y
y, X

M

I.E.(internal excitation)

Z

Z Z, X

(The various types of movement are uncoupled when one works with the principal axes (by definition)).

One obtains:

NR

M

M

Y

Z

xx (X, y, Z) =

E

+ Z

- y

*,
EA*

I.E.(internal excitation)

Y

EIZ

V

M

Y

T

xy (X, y, Z)

= G

- Z

*,
K*

Y GA

G J

X
V
M

Y
T
xy (X, y, Z) =
G
- Z

,

K

Y GA
G J

X
With option "SIPO_ELNO_DEPL", calculation is somewhat different. The effects are sought maxima of each effort (*NR, V, V, M, M, M*

Y
Z
T
X

Y) on components *xx, xy* and *xz*.
One finds the preceding results in broken up form. The vector result is written:

(1 1 1 1 1 1
1, 2
3
, 4
, 5
, 6

2
2
2
2
2
2
2
1, 2
3
, 4
5
, 6
)
I

NR

with: *I*

1

=

for node *II* =

I

(1, 2)

With

I

NR

iI =

I

With

I

V

I

Z

3

=

,

I

Z

With

I

M

I

T

I

4

=

R,

I

T

JX

for the rectangular sections:

I

I

M

H

I

Y

Z
5
=

,
I
I
2
Y
I
I
M
H
I
Z
Y
6
= -

,
I
I
2
Z

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I

M

I
Y
I
5
=

R,
I
IY
I
M
I
Z
I
6
= -

R,
I
IY

for the unspecified sections at the point (*I*

I
Y
R, Z
R):
I
M
I
Y
I
5
=

R,
I
Z
IY
I
M
I
Z
I
6
= -

*R.**I**Y**IZ*

In the case of beam with hollow circular section (pipes), the flexibility of the thin hulls not being not represented well, certain sizes should be corrected. Two coefficients are used for that:

- a coefficient of flexibility “flex” (also used by rigidity [§5]),
- a coefficient of intensification of the constraints “isigm”.

In particular, one can take as a starting point the rules **RCC_M**.

I traditional*I* traditional =

(cflex)

1 only in inflection.

cflex

MY

isigm

=

R

,

5

*I**Z**Y* corr é

ig

cflex

MZ

isigm

=

R

6

Y

(isigm)

1 .

Corrected *IZ*

cflex

7.3**Calculation of the nodal forces and the reactions**

OPTION: “FORC_NODA”

This option calculates a vector of nodal forces on all the structure. It produces a field with nodes in order **CALC_NO** by assembly of the elementary terms.

For this calculation, one usually uses in 3D [R5.03.01] for example the principle of virtual work and one writes:

$$\mathbf{F} = \mathbf{QT}$$

where \mathbf{QT} represents the matrix symbolically associated with the operator divergence. For an element, one

writing agricultural work of virtual deformations:

$$(\mathbf{QT}) H = (\mathbf{U})$$

(wh)

For the elements of beam, one calculates simply the nodal forces by assembly of the forces nodal elementary calculated by option SIEF_ELGA_DEPL, which is expressed by:

$$[\mathbf{F}] = [\mathbf{K}] [\mathbf{U}]$$

[

]

§7.1

LOC

LOC

LOC]

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OPTION: “REAC_NODA”

This option, called by CALC_NO, makes it possible to obtain the reactions **R** with the supports starting from the forces

nodal **F** by:

$$\mathbf{R} = \mathbf{F} - \mathbf{Fchar} + \mathbf{Finer}$$

Fchar

Finer

and

being nodal forces associated the loadings given (specific and distributed) and with the efforts of inertia.

8

Element of bar

Key word “BARS”

A bar is a right beam of constant section comprising only the degrees of freedom of traction and compression only. The equation of the movement, matrices of mass and rigidity, and the efforts are thus those of the beams (right of constant section) relating to the traction and compression. Concerning the characteristics of the section, the surface of the cross section constitutes the only data useful [U4.24.01 §11].

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Mr. AUFAURE, G. DEVESA

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*Organization (S): EDF/IMA/MMN, EP/AMV***Handbook of Reference****R3.08 booklet: Machine elements with average fibre****Document: R3.08.02****Modeling of the cables in *Code_Aster*****Summary:**

The cables are flexible structures which can undergo great displacements. Their analysis is thus not linear. From the mechanical point of view, a cable cannot support any moment and is not the seat that of an effort

normal called tension. The expression of virtual work and its differentiation compared to displacements lead to modeling in finite elements: stamp rigidity depending on the displacement of the nodes and stamp of constant mass. One presents the iterative algorithms statics and dynamics. Two examples are given: one, static, is the research of the figure of balance of a cable subjected to a horizontal tension data; the other, dynamic, is the comparison between calculations by finite elements and of the test results of

short-circuits. Finally four appendices treat: calculation of the forces of Laplace, change of the temperature

of a cable subjected to the Joule effect, force exerted by the wind and of the modeling of the operation of installation.

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1 Notations

With

surface of the cross-section of the cable.

B

stamp deformation.

C

voluminal heat of conducting metal.

v

E

Young modulus to traction.

E

current module of the cable.

has

E

modulate with compression.

C

F

external forces data.

ext.

F

inertias.

iner

F

internal forces.

int

G

measure of Green of relative lengthening compared to the situation of reference.

H

thermal convection coefficient of a cable with outside.

I

instantaneous intensity of current.

K

coefficient of variation of the resistivity with the temperature.

L

function of form relating to node I .

$I()$

$[L]$

$[L() \mathbf{1}, L$

I

$J() \mathbf{1}$

$]!$

$[It]$

$dL D$

$dLj D$

I

$\mathbf{1},$

1

D ds

D ds

!

O

O

NR

tension of the cable.

S

curvilinear X-coordinate on the cable in situation of reference.

O

T, T

temperature in current situation and situation of reference.

O

(custom, T

vector displacement at the moment T compared to the situation of reference.

O

)

(xs

vector position in situation of reference.

O)

thermal dilation coefficient.

,
parameters of Newmark.

density.

resistivity.

1

stamp unit of order 3.

stamp diagonal

,

,

.

1

so

ds

ds

ds

O

O

O

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2 Introduction

The essential characteristic of the cables is that these are **flexible structures**, of which analysis mechanics is **nonlinear** because they are likely to undergo **great displacements**. They in undergo during the operation of **installation**, when one adjusts arrows to respect constraints of environment.

Thereafter, the cables can be animated **movements of great amplitude** under the impulse **blast** pressure, fall of **sleeves of white frost** or, in the event of incident, the forces of Laplace resulting from the currents of **short-circuit**. They exert then on their supports of the efforts much higher than the static efforts. One must hold account in the design of it of works.

For the old works, which can be subjected to intensities of short-circuit increased because extension of the network, it should be checked that reliability is always assured.

3 Assumptions

mechanics

The cables are regarded as perfectly flexible wire, which cannot support any moment, neither bending, nor of torsion, and are the seat only of one normal tension. This tension plays the role of a generalized constraint.

One wants to calculate the field of displacement (

U.S., T

O

) at the moment T compared to the situation of reference. This one is a static configuration of the subjected cable, for example, with gravity and with *To* temperature; it is defined by the field of vectors position (

X *so*).

X

U

X + U +

ds

ds

S

O +

O

O

so

ds

X

+ U

X

X + ds

S

O

O

dso

X

Appear 3-a: Length of cable in situations of reference and moved

As [bib1], one takes for deformation the measurement of Green of relative lengthening compared to situation of reference [Figure 3-a]:

ds2 - ds2

G

O

=

.

ds2

2 O

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G must remain small. The second member lends himself to calculation because it comprises only squares of elementary lengths. One sees on [Figure 3-a] that:

X U

1 U 2

G =

.

+

.

éq 3-1

S

S

2 S

O

O

O

The relation of behavior is:

$NR = E$

[Ag - (T - T

has

O)]

éq 3-2

with:

E if $NR > 0$

$Ea = E$ if $NR 0$

C

.

4

Application of the Principle of Virtual Work

If one does not hold depreciation account, the virtual work of the whole of the forces applied to

a length of cable during virtual displacement \mathbf{U} is:

$$W(\mathbf{U}, \mathbf{U}) = W(\mathbf{U}, \mathbf{U}) - W(\mathbf{U}^0, \mathbf{U}) - W(\mathbf{U}, \mathbf{U}^0)$$

int
iner
ext.

\mathbf{U}),
éq 4-1

by distinguishing work from the interior forces, the inertias and the external forces.

According to [éq 3-1]:

$$\int_S \mathbf{X} + \mathbf{U} \cdot \mathbf{U} = \int_S W(\cdot) = \int_S (N \cdot g) = \int_S ds = NR \cdot ds = \int_S \mathbf{U} \cdot \mathbf{U}$$

=

(N.B \mathbf{U}) *ds*
int
, **éq 4-2**

S
S
l
l
S
S
O
O

$s1$

where:

$$\mathbf{T} = \mathbf{1} (\mathbf{X} + \mathbf{U})$$

1,
 éq 4-3
 S
 S
 O
 O

by indicating by the superscript T transposed of a matrix.

S
 W
 $(,)$
 2
 $\mathbf{U} \mathbf{U}$

$$= - (A \mathbf{U} \mathbf{U}) ds$$

iner “
 ” .
 •

éq 4-4

$s1$

In all the cases, we regard W_{ext} work as independent of \mathbf{U} during a step of time, bus:

- or it is it really, in the case of conservative forces like gravity;
- or, in the case of the forces of Laplace, the force applied to an element of cable depends not only of the displacement of this element (conventional following force), but still of displacements of the whole of the cables. It is considered whereas, during a step of time, the force is constant and equal to its value at the end of the step of previous time.

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5 Linearization

With balance:

$$W(\mathbf{U}, \mathbf{U}) = 0.$$

éq 5-1

If, short of a tolerance, the preceding equation is not satisfied, one seeks a correction \mathbf{U} of \mathbf{U} such as:

$$W(\mathbf{U}, \mathbf{U}) + DW(\mathbf{U}, \mathbf{U}). \mathbf{U}$$

$$= 0,$$

$$DW(\mathbf{U}, \mathbf{U}$$

$$). \mathbf{U}$$

being the directional derivative of $W(\mathbf{U}, \mathbf{U})$ in the direction \mathbf{U} [bib2] and [bib3].

According to [éq 3-2], one has obviously:

$$DN. \mathbf{U} = E A Dg. \mathbf{U} = E A$$

has

has

$$\mathbf{B} \mathbf{U}.$$

According to [éq 4-3]:

$$T$$

$$D$$

$$D$$

$$DB. \mathbf{U}$$

$$=$$

$$\mathbf{1}$$

$$\mathbf{U}$$

$$\mathbf{1}$$

$$ds$$

$$ds.$$

$$O$$

$$O$$

Thus:

s_2

T

$D \text{ Wint } (\mathbf{U}, \mathbf{U})$

$).$ \mathbf{U}

$=$ $($

$1 \{ \mathbf{B}$

\mathbf{U}

$\} E A$

has

$\mathbf{B} \mathbf{U}$

S

$\} ds$

T

S

éq 5-2

2

D

D

$+$

$1 \mathbf{U}$

NR

1

\mathbf{U}

$ds.$

$s_1 ds$

ds

O

O

According to [éq 4-4]:

S

D W
(“U, U). U

= - 2 U

"
U
.

éq 5-3

S (
T
With
) ds
iner
1

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6 **Numerical realization by the finite elements**

One notes by the subscript *H* the matrices discretized in finite elements.

If \mathbf{v} are a vector defined on the cable (position, displacement, acceleration,...) one has, at the current point

of a finite element of nodes *I, J, ...*:

$\mathbf{v} = [\mathbf{L}] \mathbf{ve}$,

\mathbf{ve} being the vector made up of the components of \mathbf{v} with the nodes.

In the same way:

1

v

$$= [\text{'}$$

L].

v

S

E

O

H

According to [éq 4-3]:

T

B

$$= (\mathbf{X} + \mathbf{U}) \mathbf{L} \text{ You } \mathbf{L}$$

H

' .

E

The internal forces **Feint** of a finite element *E* of structure are the forces which it is necessary to exert in its

nodes to maintain it in its current deformed configuration. According to the theorem of work virtual for the continuous mediums, the work of these specific forces is equal to the work of the constraints

in the element, i.e. in *Wint*, for any field of virtual displacement. One thus has, according to [éq 4-2]:

S

S

Fe

=

$$2 NR \mathbf{B} T ds =$$

$$2 NR \mathbf{L} \text{ You } \mathbf{L}$$

,

.

S

ds

H

$$(\mathbf{X} + \mathbf{U}$$

int

)

S

E

1

1

In addition, one replaces the inertias distributed in the element by specific forces with **Feiner** nodes such as their work is equal to that of the real inertias for any field of virtual displacement. According to [éq 4-4], one thus has:

 S **Fe** $= - 2 \mathbf{L} \mathbf{T}$ *WITH L**iner**ds*“*E**U*”.*sI*

In the same way, the external forces distributed are replaced by concentrated nodal forces **Feext** equivalent within the meaning of virtual work.

The differential of the virtual work of the interior forces of a finite element of cable is written, according to

[éq 5-2]:

 T $Dh \text{ Wint} (\mathbf{U}, \mathbf{U}$ $).$ \mathbf{U} $= (\mathbf{U}$ $E) (\mathbf{K} \mathbf{M} + \mathbf{K} \mathbf{G}) \mathbf{U}$ $E,$ *Handbook of Reference**R3.08 booklet: Machine elements with average fibre**HI-75/96/024/A***Code_Aster** ®*Version*

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with:

S

\mathbf{K}

=

$2 \mathbf{B}^T \mathbf{E} \mathbf{A} \mathbf{B}$

M

H has $H ds$

$s I s$

\mathbf{K}

=

$2 \mathbf{L} \mathbf{Y} \mathbf{N} \mathbf{R} \mathbf{L}$

G

' ds

.

$s I$

\mathbf{K}_M and \mathbf{K}_G are called matrices of material and geometrical rigidity of the element.

$(\mathbf{K}_M + \mathbf{K}_G) \mathbf{U}$

M

$G E$ is the principal part of the \mathbf{F}_{int} variation of the interior forces to the nodes due with the correction of \mathbf{u}_e displacements.

The differential of the virtual work of the inertias results from [éq 5-3]:

T

$D W$

H

$iner(\mathbf{U}, \mathbf{U}$

$).$ \mathbf{U}

= - $(\mathbf{u}_e) \mathbf{M} \mathbf{u}_e$

with:

S

$\mathbf{M} =$

$2 \mathbf{L} \mathbf{A} \mathbf{L}$

T

$ds.$

$s I$

\mathbf{M} is the matrix of mass of the element. - $\mathbf{M} \mathbf{U}$

\mathbf{u}_e is the variation \mathbf{F}_{iner} of the inertias

with the nodes due to the correction of acceleration \mathbf{u}_e .

7

Particular case of the elements of cable with two nodes

These elements are 1st degree: they are thus right in position of reference and remain right in deformed position.

No moment is applied in their ends and they are the seat only of one uniaxial constraint.

They are thus **elements of bar**.

In other words: to model a cable by elements with two nodes comes down comparing it to a **chain** whose **links** (elements of cable) **would be articulated perfectly** between them.

On the other hand, the elements of cable having more than two nodes have a curve in general variable with the deformation. One cannot thus treat them as elements of bar.

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8

Use in Code_Aster

This paragraph indicates how one introduces the cables into the orders concerned of *Aster*.

Order

Key word factor

Key word

Argument

AFFE_MODELE

AFFE

GROUP_MA

Name of the group of meshes supporting a cable.

PHENOMENON

“MECHANICAL”.

MODELING

“CABLE”.

DEFI_MATERIAU

CABLE

E

Value of the module of material.

EC_SUR_E

Report/ratio of the module of compression (very weak and being able to be null) on modulate.

AFFE_CARA_ELEM

CABLE

GROUP_MA

Name of the group of meshes supporting a cable.

SECTION

Value of the section of the cable.

STAT_NON_LINE

COMP_ELAS

GROUP_MA

Name of the group of meshes supporting a cable.

RELATION

“CABLE”

DEFORMATION

“GREEN”

9 Problem

statics

This problem is that of the research of the balance of a structure of cables in unspecified position and subjected to a system of forces given.

9.1 Algorithm

iterative

The equilibrium equation, forms discretized [éq 5-1] and [éq 4-1] without the term of inertia, which must be satisfied in each node, is:

F

F

int

=

ext.

éq 9.1-1

Let us suppose that one has just calculated the field of displacement of the cables, **one** (*O* *S*), with iteration *N*:

· if this field makes it possible to satisfy, except for a tolerance, with [éq 9.1-1], one considers that line:

(xs) + one (S

O

O)

is the figure of balance of the cables;

· if not, one calculates corrections of displacement **one + 1** by the linearized system:

[KN kN n+

N

+ G] U

1

= F - F

M

ext.

int.

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Displacement with the iteration ($N +$)

1 is:

one + 1

one

one +

=

+

1

.

It is looked at if [éq 9.1-1] is satisfied by the field **one + 1** and so on.

9.2 Example

One wants to calculate the figure of balance of a heavy cable [9.2-a] whose end A is fixed and of which other B , of level with A , is subjected to a given horizontal force.

This problem is dealt with in [bib4], where it is regarded as highly nonlinear.

0

46,4 m

61,0

B

With

B_0

F = 25,7 NR

C

-17,7 m

Extensionnelle rigidity (E.A): 4,45 X 105 NR

Linear weight: 1,46 N/m

Appear 9.2-a: Balance of a heavy cable subjected to a horizontal tension

At the beginning, the cable, modelled by 10 elements of the 1st degree, is supposed in weightlessness and has one

rectilinear position horizontal $A B_0$. One simultaneously subjects it to the action of gravity and to horizontal force F applied in B_0 . The static position of balance $A C B$ is reached in 8 iterations only.

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10 Problem dynamics

This problem is that of the calculation of the evolution of a structure of cables.

10.1 Iterative algorithm of temporal integration

The form discretized of [éq 5-1] and [éq 4-1] supplements, which must be satisfied in each node and with each moment is:

F () - F

() = F

int T

T

(T

iner

ext.

)

éq 10.1-1

The algorithm of temporal integration is of Newmark type [bib1] and [bib5]. Let us suppose that the state of

cable (fields U , U'' and U''' with the nodes) that is to say known at the moment T and which one has just calculated a value

approached these fields to the n th iteration of the moment $T + T$

- If these values satisfy [éq 10.1-1], except for a tolerance, one takes them for values of fields at the moment $T + T$
- If not, one seeks the correction of displacement $one + 1$, to which correspond, according to the algorithm of Newmark, corrections speed and acceleration:

“**one + 1**

one +

=

1

T

and

1

“**one + 1**

one +

=

1,

t_2

such as:

1

K N

+ **kN +**

M

1

N

int

2

one +

= **F**

N

G

ext. $(T + T) - \mathbf{F} (T + T) + \mathbf{F}$

M

iner $(T + T)$.

T

In the analysis of the movement of the cables, the algorithm of Newmark can be unstable. This is why we use the algorithm says HHT, defined in [bib7], in which two parameters of Newmark are related to a third parameter:

$$\begin{aligned} &1 \\ &= \\ &- \\ &2 \\ &(\ \\ &2 \\ &1 -) \end{aligned}$$

=
4
0.

For $\gamma = 0$, the algorithm are that of Newmark, known as “regulates trapezoid”. But for slightly negative ($\gamma < 0$)

γ , it appears numerical damping which stabilizes calculation.

Determination of initial acceleration and the initialization of the fields at the beginning of a new step of times are indicated in [bib5].

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10.2 Comparison of calculations and tests of short-circuits

To validate this modeling of the cables, we compared dynamic calculations by

Code_Aster with test results of short-circuits [bib8]. Those were carried out at the Laboratory of

Electric genius of EDF on an experimental structure representative of the configurations of station [Figure 10.2-a]. Three cables tended between two gantries apart 102 m are shorted-circuit, with foreground, by a shunt laid out on insulating columns.

On the level of the other gantry, they are fed by a three-phase current of 35 kA during 250 ms. One recorded the evolution:

- of the tension of the cables to their anchoring on the gantries, using dynamometers;
- of the displacement of the points mediums of the ranges, located by spheres of indication, with assistance of fast cameras. One sees the cage of glass of the one of these cameras assembled on one gantry, on the left of [Figure 10.2-a].

[Figure 10.2-b] the comparison for a tension of anchoring and the displacement of the medium gives of a cable.

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Appear 10.2-a: General sight of the testing facility of short-circuits

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A-evolution of a tension of anchoring

B-trajectory of a point medium of cable

**Appear 10.2-b: Comparisons of calculations by *Code_Aster*
and of tests of short-circuits**

11 Conclusion

The modeling of the cables presented above is powerful (a reasonable number of iterations by no time or to reach a balance static) and precise: it is adapted to the analysis of long cables. For the short cables, on the other hand, flexional rigidity is not negligible, especially with anchorings, and modeling must be done by elements of beam in great displacements and great rotations [R5.03.40].

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Appendix 1 Calculation of the forces of Laplace between drivers

Any driver traversed by a current creates a magnetic field in its vicinity. This field magnetic, acting on the current conveyed by another driver, induced on this one a force known as of Laplace.

ds

I

2

2 (*T*)

e2

R

I

1 (T)

E

1

*P***Appear A1-a: Provision of two close drivers**Let us take a driver! traversed by current $I T$ 1 () [A1-a Figure], located in the vicinity of the driver “
traversed by current $I T$ 2 (). At the point P of the driver! , where the unit tangent directed in the direction of
current is \mathbf{e}_1 , the linear force of Laplace induced by the driver “is:**R****F** (*P*) =

-

 $10^{-7} I(T) I(T) \mathbf{E}$

1

2

 $\mathbf{e}_1 \times$ $\mathbf{e}_2 \times$ ds

.

 r^3

2

One is interested only in the forces due to the very intense currents of short-circuit, the forces of Laplace
in

normal mode being negligible.

F (*P*) can be obviously put in the shape of the product of a function of time by a function of
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A1.1 Function of the time of the forces of Laplace

This function, $G(T)$, is, except for a factor, the product of the intensities in the drivers! and “:

$G(T) =$

-

$2 \cdot 10^7 I(T) I(T)$

1

2

éq A1.1-1

where:

$I(T) =$

$2 I [Co (S T$

+) - and

J

e_j

J

$\cos J]$

éq A1.1-2

with:

I_{ej} : effective intensity of the current J ;

:

pulsation of the current (= 100 for a current of 50 Hz);

J : phase depending on the moment when the short-circuit occurs;

:

time-constant of the shorted-circuit line dependent on its characteristics

electric (coil, capacity and resistance).

Very often, one replaces the function supplements $G(T)$ [éq A1.1-1] and [éq A1.1-2] by his average -

that the part is called continues - by neglecting the \cos terms ($T + \dots$) and $\cos(2T + \dots)$. The catch in

count these terms would require a step of very small time and the corresponding forces, to 50 and

100 Hz, are almost without effect on the cables whose frequency of oscillation is about the hertz.

Thus:

T

T

-

+

1

G

$(T) = 2 I I$

1

2

1

2

(

cos 1

- 2

) + E

continuous

E

E

cos cos

2

1

2

A1.2 Function of space

This function is:

1

R

H (P) =

E ×

e2 ×

ds.

2 1

*r*³

2

The integral is calculated analytically when one cuts out the driver “in rectilinear elements.

length of such an element *MR*. *M*

1

2 [A1.2-a Figure], there is an effect:

3

*r*³ = (y² + *r*² 2

m)

;

e2 × **R** = **e2** × *m*

R;

ds

= *Dy*

2

.
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M2

e2

M1

y

2

1

rm

R

P

Appear A1.2-a: Force of Laplace induced by a rectilinear element of driver

Like:

y2

y2

Dy

1

y

=

,

y

1 (
3
2
1
 $y^2 + r^2)^2$
 $rm (y^2 + r^2)^2$
 m
 $m)$

y^1
one a:
2

y
1
 M^2
R
1

y
E ×
E ×
 $Dy =$
E × **E** × **R**

·
2 1
2
3
 M
 m

1
 R
2 2 1
2
 rm

(
1
2
2
 $y + rm)^2 1y$

The hook of the second member is also equal to:

$\sin - \sin$

2

1.

A1.3 Realization in *Code_Aster*

The function of space $\mathbf{H}(P)$ previously definite is calculated by an elementary routine which evaluates for each element of the driver!, the contribution of all the elements of the driver “which act on him.

This contribution is evaluated to the points of Gauss (1 only for the elements with 2 nodes) of the element driver!.

The elementary routine has 2 parameters of entry:

- the load card of the element of the driver! including/understanding the list of the meshes of driver “acting on him;
- the name of the geometry, variable in the course of the time, which at every moment makes it possible to evaluate them

rm quantities, \sin , \sin

1

2.

The function of time $G(T)$ is calculated by a specific operator of *Code_Aster* which produces one concept of the function type.

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A1.4 Use in Code_Aster

· Définition of the function of time $G(T)$

Order

Key word factor

Key word

Argument

“COMPLETE”

DEFI_FONC_ELEC

SIGNAL

or

“CONTINUOUS”

COURT

INTE_CC_1

Ie1

TAU_CC_1

1

PHI_CC_1

1

INTE_CC_2

Ie2

TAU_CC_2

2

PHI_CC_2

2

INST_CC_INIT

Moment of beginning of short-circuit.

INST_CC_FIN

Moment of end of short-circuit.

· Définition of the function of space $\mathbf{H}(P)$

Order

Key word factor

Key word

Argument

AFFE_CHAR_MECA

MODEL

Name of the model.

INTE_ELEC

GROUP_MA

Name of the group of meshes of driver!.

GROUP_MA_2

Name of the group of meshes of driver “.

· Prise in account of the forces of Laplace

Order

Key word factor

Key word

Argument

DYNA_NON_LINE

EXCIT

CHARGE

name of \mathbf{H} (P)

FONC_MULT

name of G (T)

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Appendix 2 Calculation of the temperature of the cables

When the cables are traversed by a current of short-circuit, they strongly warm up by Joule effect.

When the current is tripped, their temperature drops because of the thermal losses towards outside.

A2.1 Formulation

One supposes:

- that at a moment given the temperature is uniform in any length of cable traversed by even running;
- that any node common to several cables is at the average temperature of these cables.

The change of the temperature is given by the equation of heat [bib1]:

$$dT$$

$$H ($$

$$p T - T$$

$$2$$

$$ext.)$$

$$J$$

$$+$$

$$=$$

éq A2.1-1

$$dt$$

$$WITH C$$

$$C$$

$$v$$

$$v$$

where p is the perimeter of the cable:

$$p = 2 A,$$

J the density of current, is, by neglecting the effect of skin:

$$I$$

$$J = A$$

and:

$$= O [1 + K (T - rTef)].$$

The expression of the current is given by [éq A1.1-2].

Thus:

$$T$$

$$2 T$$

$$1$$

$$-$$

$$1$$

$$-$$

$$i2 =$$

$$I 2$$

$$($$

$$\cos 2 T + 2) - 2nd \cos$$

$$($$

$$\cos T$$

+) + + E

2

2

cos

E

.

2

2

If one neglects the last term of the second member, who decreases exponentially in the time and which

is even null if =

, the integral of i_2 over one duration T , much higher than the period of the current,

2

is practically equal to:

$I_2 T$

E .

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In other words, under this simplification, the instantaneous voluminal heat source to the second member of [éq A2.1-1] is:

I_2

$j_2 = E$.

A_2

The differential equation [éq A2.1-1] is then with constant coefficients and is integrated analytically. It is written:

dT

B

has

+ *C* -

T

D

dt

=

+

2

2

with:

2 *I* 2

has

O

=

1 - *K T*

E

C (

ref.)

2

v

With

2 *I* 2

B = K O

E

C

2

v

With

H p

C = A C_v

D = C Text.

Its solution, summons general solution of the equation without second member and a solution particular of the equation with second member is:

· with the heating, during the existence of the current of short-circuit:

B
+ 2*D has*

B

$T = T \exp$
 $- C (T T)$
 \exp
 $- C (T T$
I
I
I)

1
2

-

B - 2 C

+

2

-

-

T_i being the temperature at the moment *T_i*;
· with cooling, as from the moment *T FCC* of end of short-circuit (= *B has* =)

0 :
 $T = T$
[
 $\exp - ($
 $C T - T)] + T \{ 1 - E [$
 $xp - ($
 $C T - T$

FCC

FCC

ext.

FCC)]}.

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A2.2 Use in Code_Aster

· Calcul of the function *T*

Order

Key word factor

Key word

Argument

DEFI_THER_JOULE

LIST_INST

List moments of calculation.

INST_CC_INIT

Moment of beginning of short-circuit.

INST_CC_FIN

Moment of end of short-circuit.

TEMP_EXT

Text

TEMP_RESI_REF

Tref

PARA_COND_1D

INTE_CC

IE

With

With

RESI_R

O

RESI_R1

K

CP

C_v

COEF_H

H

TEMP_INIT

T_i

· Affectation of the function *T* to the nodes subjected to the Joule effect

Order

Key word factor

Key word

Argument

AFFE_CHAM_NO

GRID

Name of the grid.

SIZE

“TEMP_F”

AFFE

GROUP_NO

Name of the group of nodes
subjected to the Joule effect.

NOM_CMP

“TEMP”

FUNCTION

Name of the function *T*.

· Affectation

of one

EVOL_THER with the preceding nodes

Order

Key word factor

Key word

Argument

CREA_RESU

TYPE_RESU

“EVOL_THER”

NOM_CHAM

“TEMP”

CHAM_GD

LIST_INST

List moments of calculation.

CHAM_NO

Name of the nodal field of function

T.

· Affection of the corresponding thermal load

Order

Key word factor

Key word

Argument

AFFE_CHAR_MECA

MODEL

Name of the model.

TEMP_CALCULEE

Name of the EVOL_THER.

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Appendix 3 Calculation of the force exerted by the wind

A3.1 Formulation

It is admitted that a wind speed \mathbf{V} exerts in the vicinity of the point P of a cable [A3.1-a Figure] one linear force aerodynamic \mathbf{F} having the following characteristics:

·

\mathbf{F} with the direction and the direction of the \mathbf{V}_n component the speed of the wind in the normal plan of cable;

·

\mathbf{F} has a module proportional squared of that of \mathbf{V}_n .

\mathbf{V}_n

\mathbf{V}

\mathbf{F}

cable

P Vt **Appear A3.1-a: speed of the wind in the vicinity of a cable**

The payments of calculation of the lines define the force of a wind by the pressure p which it exerts on a plane surface normal with its direction. For a cable, placed normally at the direction of the wind, these payments prescribe to take for linear force:

 F $= p$

,

2

being the diameter of the cable. That amounts considering that the cable offers to the wind a plane surface

equalize with its Master-couple. An increase of the force thus is obtained because the cable, cylindrical, have a less resistance to the air than a plane surface.

If the speed V of the wind forms an angle with the cable, its component in the plan perpendicular to cable has as a module:

 V V N

=

sin.

Therefore, the linear force is:

 F $= p$

sin2.

Of course, the linear force exerted by the wind depends on the position of the cable: it is “following”.

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A3.2 Use in *Code_Aster*

Here how one introduces the force of the wind into *Code_Aster*. The unit vector having the direction and the direction the speed of the wind has as components v_x, v_y, v_z

X

y

Z.

Order

Key word factor

Key word

Argument

AFFE_CHAR_MECA

FORCE_POUTRE

TYPE_CHARGE

“WIND”

FX

$p \ v_x$

FY

$p \ v_y$

FZ

$p \ v_z$

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Appendix 4 Modeling of the installation of the cables

A cable in the course of installation in a canton (several ranges between posts) [A4-a Figure] is fixed at the one of

supports of stop. It rests on pulleys placed at the bottom of the insulators suspension and it is retained by

one

force on the level of the second support of stop.

Appear A4-a: Pose of a cable in a canton with two ranges

While exploiting this force - or by moving its point of application - one adjusts the arrow of the one of the ranges, that

who is subjected to constraints of environment. Then one removes the pulleys and one fixes the cable at insulators: the length of the cable in the various ranges is then fixed. It is on this configuration that one assemble possibly additional components: spacers, descents on equipment, them specific masses,... to give to the canton its final form.

This scenario is carried out by *Code_Aster* in the following way.

One constitutes a first grid of the line, supposed in weightlessness (rectilinear drivers), comprising cable-pulleys [bib9] and also of the meshes representing the additional components. These last meshes will not be taken into account in calculation intended to determine the lengths of cable, but they will be useful

in procedure POSE_CABLE [U4.66.01] called later on. Meshes of cable-pulleys, meshes cables related to cable-pulleys and the meshes of the additional components must belong to groups respecting a specific nomenclature [bib10] to be able to be recognized and treated by POSE_CABLE.

Operator STAT_NON_LINE [U4.32.01] calculates the preceding structure subjected to gravity, with control

arrow, and constitutes a structure of data of the evol_noli type.

Procedure POSE_CABLE constitutes then a new grid, not comprising more cable-pulleys but containing the additional components and where the length of the cables is deduced from displacements from

precedent evol_noli.

One analyzes finally this real structure, that is to say by STAT_NON_LINE if one is interested only in the effect of the wind and/or

white frost, is by DYNA_NON_LINE [U4.32.02] if one wants to know the evolution caused by the forces of

Laplace due to currents of short-circuit, or by the fall of sleeves of ice.

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01/09/05

Author (S):

J.M. PROIX, NR. LAURENT, P. HEMON, G. BERTRAND *Clé: R3.08.03-C Page*

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Organization (S): EDF-R & D /AMA, IAT St CYR, CS-SI

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***Calculation of the characteristics of a beam of section
unspecified transversal***

Summary:

One presents the principle of the calculation of the various sizes characteristic of the sections of beams. Those are established starting from the geometrical characteristics of the cross section of the beam.

These values are to be provided to the operand SECTION: "GENERAL" of operator AFFE_CARA_ELEM [U4.42.01].

To determine them, of the numerical methods are presented, and implemented in the order MACR_CARA_POUTRE.

In the case of the sections "RECTANGLE" and "CIRCLE", one calculate directly in AFFE_CARA_ELEM them characteristics using simplified formulas which one clarifies here.

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1 **Geometrical characteristics**

Assumption:

One treats here only the cross sections of homogeneous and isotropic beams (same material characteristics for all the points and in all the directions). The order MACR_CARA_POUTRE can also calculate the geometrical characteristics of a whole of sections disjointed.

1.1 Section unspecified

1.1.1 Principle

That is to say a section (S) of surface S in the plan (0, y, Z) whose origin O is the centre of gravity G of section, [Figure 1.1.1-a].

Z
(S)
O
G
y
Z
R

Appear 1.1.1-a: section in plan (0, y, Z)

*Geometrical moment of inertia of (S) compared to the axis (OY) (which passes by the centre of gravity)
express yourself by:*

$$I = z^2 dS$$

with

$$[I] = \int y^2 dS$$

y

S

S

One defines in a similar way the geometrical moment compared to (OZ) by:

$$I = \int y^2 dS$$

Z

2

S

When centrifugal geometrical moment (often called produced inertia of surface) defined by

$$I = \int yz dS$$

yz

is null, axes (OY) and (OZ) are principal axes of the section (S). One

S

place for the continuation on this assumption; I_y and I_z are then called the geometrical moments principal.

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Generally, we must place ourselves in the principal axes of a section of beam for all that relates to its characteristics since the elements of beam of Code_Aster are formulated in this reference mark. On the basis of an origin located at the centre of gravity, it is enough, to pass from one

unspecified system of axes (G, y', z') to the system of principal axes (G, y, Z), to carry out a rotation of angle such as [Figure 1.1.1-b]:

I
2 I

= Arctg
y' z'

2
I

z' - I y'

Z
z'
Z

P
y
G
y

(G, y, Z) principal Axes
(G, y', z') unspecified Axes
y'

Appear 1.1.1-b: Principal and unspecified axes

The polar geometrical moment compared to the centre of gravity is given by:

$$I = R dS$$

P
2 where R is the distance from the element dS in the centre of gravity [Figure 1.1.1-a].
S

One deduces I naturally from it = I + I

P
y
Z

The polar geometrical moment intervenes in the calculation of the rigidity of torsion of the beams of section

circular (torsion of Coming Saint). For the other forms of sections, one will define a constant of the same torsion dimension.

Moreover, the geometrical moments can be calculated in another reference mark (P, y, Z), of origin P unspecified different from the centre of gravity G (formula of Huygens):

$$I_P = I_G + (G.P.Z)^2 S = \int_S z^2 dS$$

$$I_P = I_G + (G.P.Z)^2 S = \int_S z^2 dS$$

$$I_P = I_G + (G.P.Y)^2 S = \int_S y^2 dS$$

$$I_P = I_G + (G.P.Y)^2 S = \int_S y^2 dS$$

$$I_P = I_G + (G.P.Y)(G.P.Z) S = \int_S yz dS$$

$$I_P = I_G + (G.P.Y)(G.P.Z) S = \int_S yz dS$$

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R3.08 booklet: Machine elements with average fibre**HT-66/05/002/A****Code_Aster ®****Version****7.4****Titrate:****Calculation of the characteristics of a beam of cross section unspecified Dates:****01/09/05****Author (S):****J.M. PROIX, NR. LAURENT, P. HEMON, G. BERTRAND Clé: R3.08.03-C Page****: 5/42****in a general way, the formula of Huygens gives:**

$$[I] = (PG + GM) (PG + GM)$$

$$S$$

$$= PG PG + GM GM$$

S**S**

$$+ 2 PG GM$$

$$S$$

$$= S (PG PG) + GM GM$$

S**1.1.2 Calculation of the geometrical characteristics using MACR_CARA_POUTRE**

This macro-order allows the determination of the characteristics of a cross section of beam starting from a grid 2D of the section [U4.42.02]. It makes it possible to build a table of values, usable in order AFFE_CARA_ELEM (SECTION: "GENERAL" [U4.42.01]).

The geometrical characteristics can be calculated on the complete grid, half grid with symmetry compared to X or with Y, quarter of grid with two symmetries compared to X and with Y [Figure 1.1.2-a].

These characteristics are calculated in the table for all the grid and each group of meshes of the list specified by the user (case of a network of beams).

The data correspond to a half or a quarter of the section if key words SYME_X or SYME_Y are present.

Z (principal)**Y**

Y
y (principal)
CDG_X
R_MAX
Z_MAX
Y_MAX
G
X
Y_MIN
CDG_Y
Z_MIN
O
ALPHA
X

Appear 1.1.2-a: Definition of the geometrical characteristics
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The results are gathered in four groups:

- 1) In reference mark OXY of description of the grid 2D for the grid provided by the user**
 - surface: AIRE_M**
 - position of the centre of gravity: CDG_X_M, CDG_Y_M**
 - moments and product of inertia of surface, in the centre of gravity G in reference mark GXY: IX_G_M, IY_G_M, IXY_G_M**
- 2) In the same total reference mark, for the grid obtained by symmetrization if SYME_X or SYME_Y:**
 - surface: SURFACE**
 - position of the centre of gravity: CDG_X, CDG_Y**
 - moments and product of inertia of surface, in the centre of gravity G in reference mark GXY: IX_G,**

*IY_G,
IXY_G*

3) In the principal reference mark of Gyz inertia. cross-section, whose denomination corresponds to that

used with the description of the elements of neutral fibre beam Gx [U4.24.01].

· principal moments of inertia of surface in the Gyz reference mark, usable for the calculation of rigidity

of inflection of the beam: IY_PRIN_G and IZ_PRIN_G

· angle of flow of reference mark GXY to the principal reference mark of Gyz inertia: ALPHA

· characteristic distances, compared to the centre of gravity G of the section for calculations of maximum constraints: Y_MAX, Y_MIN, Z_MAX, Z_MIN and R_MAX.

4) In the total reference mark, in a point P provided by the user:

· X_P, Y_P: not calculation of the moments of inertia

· IX_P, IY_P, IXY_P: moments of inertia in reference mark PXY

· IY_PRIN_P, IZ_PRIN_P: moments of inertia in the Pyz reference mark.

1.1.3 Calculations

carried out

The list of the orders called by MACR_CARA_POUTRE is indicated in the document [U4.42.02].

The preceding quantities are obtained by the call to POST_ELEM, for option "CARA_GEOM". Of more, one can add to it key words SYME_X, SYME_Y, and ORIG_INER which defines the P. point.

Calculations are carried out in POST_ELEM, for all the grid, then possibly for each group meshes, in the following way:

1) Buckle on the elements 2D (modeling D_PLAN), with call of the elementary option "MASS_INER". One obtains a CHAM_ELEM with a value by element (1 point of Gauss) containing the components:

dS,

X dS,

y dS,

x2 dS,

y2

dS, xy dS

element

element

element

S

S

S

2) Summation of the preceding elementary quantities to obtain: AIRE_M, CDG_X_M, CDG_Y_M, IX_G_M, IY_G_M, IXY_G_M

3) Calculation

of

SURFACE, CDG_X, CDG_Y, IX_G, IY_G, IXY_G (taken into account of SYME_X, SYME_Y)

4) Calculation

of

IY_PRIN_G, IZ_PRIN_G, ALPHA

5) Calculation

of

Y_MAX, Z_MAX, Y_MIN, Z_MIN, R_MAX

6) If one clarifies a point P particular (key word ORIG_INER), one calculates also the characteristics in the total reference mark of origin P: PXY

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1.1.4 Examples

of use

: Full rectangle (treaty by test ZZZZ105G)

y

B

B

B = 0.01

GR2

H

H = 0.025

0
X
GR1
H

Geometrical characteristics obtained

PLACE
AIRE_M
CDG_X_M
CDG_Y_M
IX_G_M
IY_G_M
IXY_G_M

0.000003
1.00E-03
4.24E-18
-3.39E-18
2.08E-07
3.33E-08
2.65E-23
GR1
5.00E-04

2.20E-17 -1.25E-02 2.60E-08
1.67E-08
3.97E-23

GR2
5.00E-04
-8.47E-18 1.25E-02
2.60E-08
1.67E-08
5.62E-23

PLACE

SURFACE
CDG_X CDG_Y IX_G IY_G IXY_G IY_PRIN_G IZ_PRIN_G ALPHA
0.000003 1.00E-03 4.24E-18 -3.39E-18 2.08E-07 3.33E-08 2.65E-23 3.33E-08 2.08E-07 9.00E+01
GR1
5.00E-04 2.20E-17 -1.25E-02 2.60E-08 1.67E-08 3.97E-23 1.67E-08 2.60E-08 9.00E+01
GR2
5.00E-04 -8.47E-18 1.25E-02 2.60E-08 1.67E-08 5.62E-23 1.67E-08 2.60E-08 9.00E+01

PLACE
X_P

Y_P
IX_P
IY_P
IXY_P

IY_PRIN_P
IZ_PRIN_P
0.000003
0.00E+00 0.00E+00
2.08E-07 3.33E-08
2.65E-23
3.33E-08
2.08E-07
GR1

0.00E+00 0.00E+00
1.04E-07 1.67E-08
-9.79E-23
1.67E-08
1.04E-07
GR2

0.00E+00 0.00E+00
1.04E-07 1.67E-08
3.31E-24
1.67E-08
1.04E-07

PLACE
Y_MAX
Z_MAX
Y_MIN
Z_MIN
R_MAX
0.000003
2.50E-02
1.00E-02
-2.50E-02
1.00E-02

2.69E-02

GR1

1.25E-02

1.00E-02 1.25E-02

1.00E-02

3.36E-02

GR2

-1.25E-02 -1.00E-02 -1.25E-02 -1.00E-02 3.36E-02

1.2

Particular case of the sections rectangular and circular

The geometrical characteristics are directly calculated in AFFE_CARA_ELEM from data of the user.

y

epy

Z

G

hy

epz

Hz

Appear 1.2-a: rectangular section

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In the case of the rectangular beam (Operand SECTION: "RIGHT-ANGLED"), calculation gives:

I

I

3

3

y =

H H

- H - 2 ep

H

- 2 ep

12 [

y Z

(y

y) (Z

Z)]

1

3

I = H H 3 - (H

-

2 ep

)

- 2

12

(H ep

Z

Z

y

Z

Z

y

y)

y

R

G
Z
ep

Appear 1.2-b: circular section

For the circular section (Operand SECTION: "CIRCLE"), one obtains:

4
4

I = I
=
4
4
-
-
-
-

4 [R
(R
ep
)]
I = 2 [R
(R
ep
y
Z
p
)]

2
Coefficients of shearing and the center of shearing

1
1
It is a question of evaluating the coefficients A =
, A
y
=
intervening in the models of beams of
K

Z
K
y
Z

Timoshenko with taking into account of the shearing strains. For the beams of EULER, these coefficients do not intervene [U4.42.01 §7.4.2] and [R3.08.01 §2.3.1]. These coefficients are obtained for a linear elastic behavior.

In the case of the unspecified sections, the coefficients of shearing are to be provided by the user in AFFE_CARA_ELEM, if the selected element is a beam of TIMOSHENKO (model POU_D_T, POU_C_T, POU_D_TG and POU_D_TG_M).

In the case of the circular or rectangular sections, the coefficients of shearing are calculated by analytical methods of [§2.1].

In all the cases, they can be calculated by MACR_CARA_POUTRE, starting from the plane grid of section. The numerical method used is exposed to [§2.3]. This method applies to unspecified sections (of homogeneous and isotropic material). In appendix 2, one describes an extension of this method with the case of a network of parallel beams maintained between two rigid floors.

The position of the center of torsion (or shearing centers) is obtained only by methods numerical (cf [§2.3]). For the rectangular and circular sections, as for all the sections in 2 symmetry planes, the center of torsion is confused with the centre of gravity of the section.

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2.1 Methods

analytical

One describes three analytical methods allowing to calculate coefficients of shearing,

applicable to the unspecified sections.

The first two methods differ by the definition which they propose of the coefficient of shearing, but rest on the same assumption which consists in postulating the form of the distribution shear stresses in the section.

2.1.1 Assumption of distribution of shearings: formulate JOURAWSKI

Let us consider for example the case of a beam of cross-section S , subjected to a sharp effort

$$V = dS$$

y

xy. One writes the balance of a prismatic part of the beam, ranging between the sections

S

right-hand sides S_x and S_{x+a} and between the plan of cut located at the ordinate y and y_{max} (ref.

[bib8]). Efforts

acting on this part of beam are the vectors forced on the faces S_x and S_{x+a} , and those

acting on the face located in Y .

y_{max}

$B(Z)$

y

X

$x+a$

Z

S_x

S_{x+a}

Appear 2.1.1-a

By applying the theorem of the resultant, one obtains:

$B(y)$

$x+a$

2

$\iint (X, y, Z) Dy D$

$Z -$

$(X, y, Z) Dy dz$

$= NR(X + has, y) - NR(X, y) =$

$B(y)$

$(, y, Z)$

$D dz$

S

xx
S
xy
X
x+a
X
- 2

To evaluate the term of right-hand side, JOURAWSKI proposed to consider only the average of shearings according to Z:

B

1
2
xy (X, y) = (
X, y, Z dz

B y) B xy (
)
-2
then

x+
B (y)
has
X has
2
B (y) xy (, y, Z)
+
D D
Z
B (y), y D
X

=
xy (
)
-
X
2

and while making tend has towards 0,

**NR = (by) (X, y
xy
)**

**X
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The equilibrium equations of beam and the distribution of bending stresses (in elasticity) give:

**M (X). y
M (X)
y
NR (X, y) = (X, y, Z) dydz
Z
=
dydz =
Z
m (y)
max
with m (y)
xx**

**=
T B (T) dt
S
S
X
X**

I_z
I_z
y

thus
m (y) M
()

Z
m y
x_y (X, y) =
=
V_y
I_z B (y) X
I_z B (y)

The distribution of shearings following there is thus given by the formula of JOURAWSKI:
()

m y
y_{max}
x_y (X, y) =
V_y
with
m (y) =
T B (T) dt

éq
2.1.1-1
I_z B (y)

y
in accordance with [U4.24.01], with the notations of [Figure 2.1.1-b]. The quantity m (y) represents its static moment on behalf of section (hatched) ranging between y and y_{max}:

y
B (y)
y_{max}
(
m y\$) = y Dy dz
sy\$
G
Z

Appear 2.1.1-b: section of beam

This distribution checks the boundary conditions well following three-dimensional problem there: shearing is quite null on bottom fibres and higher ($y = y_{\min}$, or $y = y_{\max}$). But it account holds that average of shearings according to Z.

By applying this formula to a full rectangular section, one finds a distribution parabolic according to Y. By applying it to a beam of circular section, one finds a distribution parabolic in y and in Z, which varies more slowly according to Z than according to Y.

This remains valid for the other full sections. For sections comprising of the holes, it is necessary to take guard to consider only the matter in the calculation of B (y).

2.1.2 Method of TIMOSHENKO

With the origin, TIMOSHENKO (ref. [bib9]) proposed a simple definition of the coefficient of shearing,

as being the relationship between the transverse shear stress average in the noted section CT and its maximum value (CT

. Owing to the fact that we always have the shearing action by: Max)

$V =$
 dS

éq
2.1.2-1

S
CT

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we deduce:

$$V$$

$$CT =$$

$$\cdot$$

$$S$$

Knowing that TIMOSHENKO proposes to write:

$$K$$

$$CT$$

$$=$$

$$\int_{S} \tau \, dS$$

2.1.2-2

$$CT_{Max}$$

to determine K, it is enough to express the shearing action $V = \int_{S} \tau \, dS$

[$\int_{S} \tau \, dS$ 2.1.2-1] according to

S

$$CT$$

CT. In the case general of the unspecified sections, there will be naturally two coefficients K_{Max}

y

and k_z , for each of the two principal axes.

It remains to determine CT

. For that, TIMOSHENKO makes an assumption on the distribution of τ_{Max}

stresses shear transverse: the transverse shear stress has a distribution parabolic in the direction of the shearing action which produces it, with its maximum value in the center and zero values at the edges. This is true according to the formula of JOURAWSKI for a section rectangular. By extension, the method extends this assumption of parabolic distribution to one unspecified section

This method is not applied in Code_Aster, except for the hollow rectangular sections. One uses the following method in the other cases.

2.1.3 Method "energy"

Actually, the definition suggested by TIMOSHENKO proves little used in practice today; one him prefer a formulation based on internal energy due to shearing in the section. This one is written:

2

U
 CT
 $=$

dS
 CT
 1

S^2
 G

where G is the modulus of rigidity (equal to μ).

The new definition of the coefficient of shearing is sometimes allotted to MINDLIN and is expressed by:

$1 V^2$
 $UCT =$

ϵq
 2.1.3-1
 $2 K S$
 G

So by substitution, one thus defines for a homogeneous material section the coefficient of shearing by:

[
 2
 dS
 CT
 2
 S
 $] V$
 $K =$
 $=$

ϵq
 2.1.3-2
 $S^2 D$
 S
 $S^2 dS$
 CT

S
CT
S
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By making an assumption on the distribution of constraint in the section, one can thus estimate value of K. From the formula of JOURAWSKI [éq 2.1.1-1], the preceding expression can be written (ref. [bib5]):

I 2
K =

éq
2.1.3-3
m2 (y)
S
dS
S b2 (y)

2.1.4 Method of COWPER

One can also take into account the three-dimensional effects to determine the coefficient K; various formulations were proposed, by COWPER [bib3] and were taken again in particular by BLEVINS [bib2], while being based on the resolution of the three-dimensional problem of Saint-Coming.

In this case, the coefficient K is a function of the Poisson's ratio, in general one approximation with the first order. COWPER uses the three-dimensional equations of elasticity in the dynamic case to propose an expression of K giving of good results in statics and in low frequency dynamics. The approximation which makes it possible to lead to the formula suggested

consists with

to consider a distribution of not parabolic constraint, but resulting from the static problem (solved analytically) of the beam cantilever transversely charged at its loose lead. It is with to note that the distribution obtained is strictly identical to the problem with a transverse loading uniformly distributed.

2.2

Particular case of the sections rectangular and circular

One distinguishes the full beams and the tubes.

For the full rectangular section, the coefficient of shearing is determined by the method based on energy shearing with parabolic distribution of the constraints interns

I^2

K

Z

$y =$

[éq 2.1.3-3]. Applied to the rectangular section, one obtains

$m^2(y)$

S

Z

Dy

$S b^2(y)$

Z

5

$K = K$

y

$Z =$

. It should be noted that this value also corresponds to the method of COWPER when

6

the Poisson's ratio is taken equal to zero.

For the rectangular tube, Code_Aster uses the method of TIMOSHENKO which leads to

2

$K = K$

y

$Z =$

\cdot

3

In the case of the beams with full circular section, one uses the energy method which leads to

9

$K = K$

y

$Z =$

. This value is also obtained by the method of COWPER when it

10

1

Poisson's ratio is equal to

.

2

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For the circular tubes, one distinguishes the tubes with fine wall and those with thick wall. If one notes

R

m

l

= R the report/ratio of the internal ray to the external ray, a tube is with fine wall when m 0.9, if not

E

it is with thick wall.

The coefficient of shearing of the circular tube with fine wall is given by the method of COWPER, in

l

considering that m = 1 and for a null Poisson's ratio, is K = K

y

Z =

.

2

For the circular tubes with thick wall, one uses an approximate formula of the method of COWPER which is written:

l

K =

.

l

0.93 + ,

0.634 m + ,

1156 m2,

0

905 m3

,

-

Let us notice that this formula does not ensure continuity with the borderline cases of the full cylinder

$(m = 0)$

and of the cylinder with infinitely thin wall ($m = 1$).

If preceding choices (carried out by *AFFE_CARA_ELEM* in the case of circular sections and rectangular) are not appropriate, it is always possible to calculate the coefficients numerically of shearing using *MACR_CARA_POUTRE*, of which the method is specified in the § according to.

2.3 Numerical method of calculation of the coefficients of shearing and of center shearing

2.3.1 Calculation of the coefficients of shearing:

This method takes as a starting point the the ref. [bib1], page 62. It allows the simultaneous determination of constants of shearing and the center of torsion. It is implemented in MACR_CARA_POUTRE, starting from a plane grid of the section. It functions currently only for sections homogeneous and isotropic (for nonhomogeneous sections, the method is similar [bib1] but not available in Code_Aster).

*As for the energy method, one compares for a shearing action V_z energy interns U_1 had with shearing in the section with U_2 energy associated with the model with *MINDLIN*:*

$$U = \frac{1}{2} \int_V \sigma_{xy} + \sigma_{xz} \, dV$$

$$dS = U$$

Z

1

$=$

S^2

G

2

$2 K S G$

Z

$1 V^2$

The coefficient of shearing is expressed by: K

Z

Z =

2 SGU1

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It is thus necessary to calculate $U1$ and thus stresses shear (in elasticity) in the section for to estimate the value of K . One places oneself in the principal reference mark of inertia (G, y, Z), and one supposes that

beam is subjected only to one sharp effort Vz . It results from it that:

$M y (X)$

$xx = Z Iy$

xx

Z

V

$= Z$

X

$I y$

The equilibrium equations make it possible to write:

xx

xy

xz

xy

xz
Z
V

+
+
= 0 =
+
+ Z

X
y
Z
y
Z
I y

In addition, the kinematics of the beam in inflection/shearing is:

$$(u_x, y, Z) = (u_x) + Z$$

$$\sim$$

$$y(X) + U(y, Z)$$

$$v(X, y, Z) = 0$$

$$($$

$$W X, y, Z) = ($$

$$W X)$$

\sim
U(y, Z) representing axial displacement due to the warping of the section. Deformations are written:

$$($$

$$U X)$$

$$y(X)$$

$$xx =$$

$$+$$

$$Z$$

$$X$$

$$X$$

$$u \sim (y, Z)$$

$$2xy =$$

$$\begin{aligned}
 & y \\
 & u \sim (y, Z) \\
 & 2 \\
 & W X \\
 & xz = y (X) \\
 & () \\
 & + \\
 & + \\
 & Z \\
 & X
 \end{aligned}$$

By using the relation of behavior of linear elasticity, the constraints are written:

$$\begin{aligned}
 & u \sim (y, Z) \\
 & xy = 2\mu xy = \mu y
 \end{aligned}$$

$$u \sim (y, Z)$$

$$\begin{aligned}
 & W X \\
 & 2
 \end{aligned}$$

$$\begin{aligned}
 & xz = \mu xz = \mu y (X) \\
 & () \\
 & + \\
 & + \\
 & Z \\
 & X
 \end{aligned}$$

The components of shearing thus check:

$$\begin{aligned}
 & xy \ xz \\
 & - \\
 & = 0 \\
 & Z \\
 & y
 \end{aligned}$$

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This relation makes it possible to introduce the function of constraints Z such as shear stresses in the section are written:

Z

$$xy = \mu y$$

Z

$$xz = \mu Z$$

The equilibrium equation then makes it possible to obtain function Z by resolution of a problem quasi-harmonic which is written:

zV

$$G + F = 0 \text{ in } S$$

Z

Z

$$\text{with } F = Iy$$

G

$$Z = 0 \text{ on } S$$

N

$$Z = 0 \text{ in a point}$$

This makes it possible to calculate Z then shearings. In practice, in MACR_CARA_POUTRE, one uses

THER_LINEAIRE to solve the problem, by comparing Z to the temperature. One chooses $Vz = 1$ and $G=1$ (G does not intervene any more in the expression of the coefficient of shearing). Boundary conditions of

this problem of stationary thermics are:

zV

· source F being worth

Z

Iy

· null flow on S

· null temperature in a point of S

1

2

2

xy +xz

1

2

One can then determine U Z =

dS =

G

by a calculation

1

(Z)

S 2

G

2 S

elementary on all the elements of the section, with option "CARA_CISA" (calculation of the gradient), then

1 V 2

summation on these elements. K then is calculated

Z

Z = 2 SGUz

1

1 V 2y

Same calculation is carried out with Vy = 1 to determine K y =

y

2 SGU1

1

1

The provided result is A =

, A

y

=

•
K
Z
K
y
Z

2.3.2 Calculation of the co-ordinates of the center of shearing

The center of shearing C is the point of the section where shear stresses due to an effort edge generate one null torque. This point is also called center of torsion, because it remain fixed when the section is only subjected to one torque.

The torque compared to the point G is worth: $M_{xG} = (\int y - z) dS$

xz
xy

S

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The torque compared to the point C sought is:

$$M_{xC} = (\int (y - y) - (Z - Z) dS = M - y V + Z V$$

xz
C
xy
C

S

xG

C Z

C y

To determine the co-ordinates of the center of shearing, preceding calculation is used [bib1]:

From the shear stresses determined for $V_z = 1$ and $V_y = 0$, one calculates:

$M_Z = -$.

x_G

(y

Z

xz

xy

S

) dS

M_Z

One obtains: y

x_G

=

= Z

C

M

x_G

V_z

M_y

y

For V

x_G

$y = 1$ and $V_z = 0$, one obtains: $Z = -$

= -

C

M

x_G

V_y

2.3.3 Example

Let us take again the example of the rectangular section [§1.1.4].

y

B

B

$B = 0.01$

GR2

H

$H = 0.025$

0

X

GR1

H

The coefficients of shearing obtained are identical to the analytical value (6/5).

PLACE AY AZ EY

EZ

all 1.20E+00 1.20E+00 8.72E-19 3.16E18

Co-ordinates of the center of torsion EY and EZ (in the reference mark principal (G, y, Z)) are null: center shearing/torsion is actually confused with the centre of gravity.

2.4

Calculation of the coefficients of shearing of a network

The method described in appendix 2 makes it possible to calculate coefficients of shearing of a beam equivalent to a whole of parallel beams embedded on a rigid floor and embedded or rotulées on another.

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3

Constants related to torsion

The constant of noted torsion C must make it possible to take account of the warping of the sections right-hand sides (not circulars) at the time of a deformation in torsion. It is used in the models of

beams

right-hand sides treated by Aster (EULER, TIMOSHENKO and warped TIMOSHENKO or POU_D_E, POU_D_T, POU_C_T and POU_D_TG). In the case of the circular sections, the sections are not warped and constant of torsion is equal to the geometrical moment polar I_p . the constant of torsion C is defined as the moment necessary to produce a rotation of 1 radian per unit of length divided by the modulus of rigidity, that is to say:

**M
 C
 X
 $=$**

éq 3-1

X

μX

A.c. same dimension as the geometrical moments of inertia I and I

y

Z is M^4 .

For a circular section, the definition [éq 3-1] is coherent since we have:

$M =$

μI

X

X

$p.$

X

The determination of C in the case general is made in a numerical way (MACR_CARA_POUTRE) and

tiny room to a calculation of Laplacian in 2D. The method presented here is detailed in the ref. [bib1] [§3.6.3]

for the simply related sections. An original method for calculation of the constants of torsion with perforated sections is detailed in appendix. The results here are given.

3.1

Calculation of C in the case of unspecified sections

The complete resolution of the problem is in appendix. One gives here simply the results.

According to the assumptions of the theory of the pure torsion of Saint-Venant, there is no deformation of the average line and not of lengthening along the longitudinal axis. Torsion is free, i.e. that it does not generate axial stresses. In other words, the sections can warp freely. If one remains in small displacements, one admits that the swing angle of the cross-sections is worth:

X
 $X ()$
 $X = X = .x$

$\acute{e}q$
3.1-1
 X
 Z
 $()$
 (S)
 $M (y, Z)$
 G
 y
 C

Appear 3.1-a
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If the point C is the center of torsion (which by definition remains motionless when the beam is subjected to a torsion), the field of displacement $U(M)$ is given by [bib1]:

X

(,
 $y Z$)

U
 X

X

X
 X

($y Z$)
,

X
 X

X

$U(M) = v =$

0

X

(y - y
= -

X (Z -
C
Z)
c) +
0

X

W

0

(Z - Z
c)

0

X

X (y - yc)

X

where (
)

,
y Z is related to warping.

The law of HOOKE is written:

$$= 2\mu$$
$$(\text{Trace } E - 2\mu) I$$

I
where I is the matrix unit and the tensor of the deformations is worth =
(grad (U) + T
U
grad (U))
2
2

By neglecting the terms of the second order, in
X
x2, one leads to:

0
-
Z
+
y
0
-
y

Z

Z

y

X

X

=

μ

-

μ

Z

0

0

=

0

0

éq

3.1-2

X y

X Z

+

-

0

0

y

0
 0
 Z

y

One posed: (
)

*,
y Z function of constraint. It is noted that the relation of balance $div = 0$ is then checked.*

While deriving, one obtains:

2
 2
 $= -$
 $- 1$

2
 y
 $Z y$

2
 2
 $=$
 $- 1$

\cdot
 2
 Z
 $y Z$

By adding the two equations, one leads to:

$= -2$

éq 3.1-3
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It remains to establish the boundary conditions. One notes N the normal directed towards outside at the border

() which can be multiplement related:

0

N

N

I

Without external loading, one must have $N = 0$, which can be written

:

N

-

y

nz

Z

y

μ

X

0

=

0 where N

X

y and nz are the two components of the normal.

0

This writing can be thus put in the form $N \text{ grad} = 0$ which implies that vectors N and grad is colinéaires. It thus follows that (y, Z) is constant on each related component border (). One can impose for example that (y, Z) that is to say null on external contour:

= 0 out of 0 =

= I on I

If the sections behavior of the holes, constants I are unspecified. For to allow the resolution of the complete problem, it is necessary to add equations. Those are obtained with to leave the circulation of the function of warping on each closed contour. They are obtained following conditions:

dl

= 2 (

I)

I

N

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where (Γ) is the surface surrounded by the border (I). These conditions are brought back to conditions traditional of imposed flow (where $L(I)$ represents the length of border I):

$$2 \int_{\Gamma} \frac{\partial u}{\partial n} ds = N$$

$$L(I)$$

Finally, the problem to be solved is written:

$$= -2 \text{ on } \Gamma$$

$$= 0 \text{ out of } \Gamma =$$

$$= I \text{ on } I$$

$$2 \int_{\Gamma} \frac{\partial u}{\partial n} ds = N$$

$$L(I)$$

$n-1$
Once solved this problem, one obtains the constant of torsion by: $C = 2 \int_{\Gamma} ds +$

$$2 \int_{\Gamma} ds +$$

$$(I).$$

$$i=1$$

3.2 Calculation of the constant of torsion in MACR_CARA_POUTRE

This calculation is carried out in MACR_CARA_POUTRE by the resolution of a problem of thermics. It is necessary for that that the user specifies with MACR_CARA_POUTRE the group of mesh which defines the edge outside, and if the section comprises holes, groups of meshes which define the contour of

each one of them.

One then solves a linear problem of thermics (THER_LINEAIRE) on a plane grid of section to find the function. One places oneself first of all in the principal reference mark of inertia (CREA_MAILLAGE), starting from the co-ordinates of the centre of gravity and the orientation of the principal reference mark calculated previously.

One defines then the boundary conditions in AFFE_CHAR_THER:

- The source term is worth 2*
- The temperature of the external edge is imposed and is worth 0 (TEMP_IMPO)*
- And if the section comprises holes (presence of one or more groups of meshes them defining):*
 - On each group of mesh defining a hole, the temperature is constant (TEMP_UNIF)*

-
Flow is worth 2 times the surface of the hole divided by the length of its edge. These quantities are calculated before.

- The calculation of C is carried out in POST_ELEM by key word CARA_TORSION of the key word factor*

CARA_POUTRE. In this case, one calculates on each element the integral of, (option CARA_TORSION on the plane thermal elements), then one carries out the sum on all them elements.

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Example:

Still let us take again the example of the rectangular section [§1.1.4].

The coefficients of shearing obtained are:

PLACE**CT [éq 3.3.2] CT (Aster)****all 9.9805E-08****9.9681E-08****3.3****Calculation of the ray of torsion in an unspecified section**

The ray of torsion is calculated thanks to the calculation of the function of constraints on the grid of section. R_t is added in the table produced by MACR_CARA_POUTRE [U4.42.02].

The resolution of a stationary thermal problem of unknown factor makes it possible to determine the constant of torsion and stresses shear.

The determination of the ray of R_t torsion is the resolution of:

$$R_t = \text{grad} () \cdot n$$

(or N represents the normal vector external with the edge considered of the section)

R_t varies along external contour; indeed, for an unspecified section, shearings due to torsion vary on the edge. One chooses to take the value of R_t leading to shearings maximum on the external edge, i.e. the maximum value of R_t (in absolute value) on contour external.

Moreover, if the section is alveolate, there are several “several rays of torsion”: $R_t = 2 \cdot A (K) / L (K)$ (or

With (K) represents the surface of the cell K and $L (K)$ its perimeter). If one is satisfied to seek the value

maximum of shearing, it is necessary to take the maximum of the R_t values obtained on the external edge and on the cells.

The ray of torsion is given in MACR_CARA_POUTRE only by orders python. During the unfolding of MACR_CARA_POUTRE order POST_ELEM is called, one new parameter RT is thus created for this order.

3.4**Constant of torsion of the sections circular and rectangular**

Expressions simplified for these two types of sections are described here. The calculation of constants of torsion is then directly carried out in AFFE_CARA_ELEM.

For the circular section the preceding expressions remain valid. By taking a function of

*1
torsion of the form (
y Z)*

*,
= (R2
x2
y2*

*-
-
) one finds indeed:*

$$C = I = (R^4 - R^4) \frac{p}{1} \frac{2}{0}$$

For the rectangular section, calculation is naturally more complex but can be carried out in choosing a function which is cancelled indeed at the edges, of the form:

*y

(
Z
,
y Z)
=*

*With
ij cos (
I
2 +)
I cos (
2 J +)
I
.
H*

H

$i=0 j=0$

y

Z

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The resolution involves with a constant of torsion which is written:

H

3

3

C

y

H H

H

y

Z

y

H

C =

C

= ***H H***

Z

3

éq

3.4-1

H2 + H2

H

y

Z

2

y

Z

Z

H

Z

1 +

H

y

hy

where C

express yourself in the form of a series which takes the following values:

H_z

hy

1

2

4

8

H_z

H

C

y

0 281

,

0 286

,
0 299

,
0 312

,
1 3
Hz

In fact, Code_Aster employs a formula simplified (ref. [bib1]) for the full rectangular section who is written:

H H 3
5

y Z
16
H
H
C

Z
Z
=

- ,
3 36
+ ,
0 280

16
3
H
H
éq
3.4-2
y
y

It is valid if $H > H$

y

Z; in the other case it is enough to exchange the respective places of H and H

y

Z.

agreement between the two expressions is very good as indicates it the following table:

H

1 2 4 8

y

Hz

C

0,1405

0,2288

0,2814

0,3072

1/3

H H

y 3 according to [éq 3.3-1]

Z

C

0,1408

0,2289

0,2809

0,3071

1/3

H H

y 3 according to Aster [éq 3.3-2]

Z

For the hollow rectangular beam, there is an approximate solution which is written (ref. [bib2] and [bib6]):

2

2

2 ep ep

y
Z (H - E
p
y
y) (H - ep
Z
Z)
C =

H ep - ep 2 + H ep - ep 2

y
y
y
Z
Z
Z

with the notations of [Figure 1.2-a]: section in plan (0, y, Z).

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3.5***The effective ray of torsion***

The effective ray of torsion RT makes it possible to calculate the transverse shear stress of torsion maximum T according to the torque. One will be able to consult on this subject the drafting of M

MASSONET on this aspect (ref. [bib5]). We have as follows:

 T R T $= MX$ M C

In the case of the circular cylinders, RT is equal to the ray (external if it is a tube) of the section.

For the rectangular sections, the problem is definitely more complex. Code_Aster imposes it ray of torsion of the full section by:

 $C 4 (3 H +, 18 H$ y $Z)$ $RT =$ $H^2 + H^2$ y Z

This approximate expression remains valid if the beam is not flattened too much. DHATT and BATOZ (ref. [bib1]) give an expression having a field of validity more extended, but actually it acts

in any rigour of a series whose numerical values are given by MASSONET (ref. [bib5]).

For the hollow rectangular beam, Code_Aster imposes an expression which is valid only if wall is thin and constant thickness epz , is:

$$C$$
$$RT =$$
$$epz (H - 2ep$$
$$y$$
$$y) (H - 2ep$$
$$Z$$
$$Z)$$

It is about a “adaptation” of the formula:

$$C$$
$$RT =$$

2nd With

where E is the thickness of the wall (constant) and A the surface contained inside the average line. This last expression is known under the name of first formula of BREDT (cf ref. [bib1] and [bib5]).

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4

Calculation of the constant of warping

The constant of warping is used by the model of beam with warping (modeling POU_D_TG and POU_D_TGM), that it is important to take into account for the beams with open mean sections (cf [R3.08.04]).

This coefficient (noted I in [R3.08.04]) intervenes in the expression of the virtual work of the efforts interiors on the terms of torsion:

2
W
*
*
int =

0 (
μ.C. +
. E.I
,
,
,
.) dx
X X
X X
X xx
X xx

*I is expressed in the same unit as the geometrical moments of inertia I, I
y
Z, is M 4.*

By taking again the approach of [§3.1], and while placing itself in a reference mark related to the center of torsion C, kinematics of the torsion of an unspecified section is:

X
(,
y Z)
U

X

X

X

X

(*y Z*)

,

X

X

X

U (M) = v =

0

y +

X

0

= -

X Z

X

W

0

Z

0

X

X y

X

where (

)

,

y Z is related to warping (which cancels only in the case of a section circular).

The expression of the stress field is (in elasticity):

2

X

xx =

E xx =

E (y, Z) x²

,

X

(y Z)

2

xy = μxy = μ

- Z

X y

X
 $(y Z)$

2

$$xz = \mu xz = \mu$$

$$- y$$

$$X Z$$

2

With the difference of [4], terms of the second order in X are not neglected any more.

2

X

The first relation of balance (div) =, +, +, = 0 implies the condition then

X

$xx X$

$xy y$

$xz Z$

following on the function of warping:

$$= 0$$

In addition, without loading external on contour of the section, one must have $N = 0$, which

can be written:

$N +$

$$N = z.n - y.n, \text{ where } N$$

$y y$

$Z Z$

y

Z

y and n_z are the two components of

normal, or in vectorial form: $\text{grad}.n =$

$$= (N \mathbf{CM}) .x$$

N
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This determines the function of warping except for a constant. To raise this indetermination, one writes for example the expression of the normal effort (for a section where torsion produces warping):

2

NR

ds

E

X

=

=

ds

xx

=

0

x2

S

S

thus the additional condition on the function of warping is: ds

= 0

S

In practice, in MACR_CARA_POUTRE, one places oneself above all in a reference mark related to the

center of torsion

C.

One calculates then who must check:

= 0

*grad .n =
= (N CM)*

*.x
N
=
ds 0
S*

*2
The inertia of warping I is obtained then by: I =
ds*

S

MACR_CARA_POUTRE calls upon the following elementary orders:

· Translation of the co-ordinates of the nodes in the reference mark related to the center of torsion (calculated previously in table TCARS):

*CREA_MAILLAGE (GRID:
LOCATE: (TABLE: TCARS NOM_ORIG: "TORSION"));*

· Affectation of a model (thermal plan), of a material field:

*AFFE_MODELE (GRID:
AFFE: (ALL: "YES" PHENOMENON: "THERMAL" MODELING: "PLANE"));
AFFE_MATERIAU (GRID
AFFE: (ALL: "YES" MATER:));*

· Boundary conditions on external contour G0:

N +

$N = z.n - y.n$

y y

Z Z

y

Z

F1=DEFI_FONCTION (NOM_PARA: VALE: (0. , 0. , 10. , -10.)) ;

F2=DEFI_FONCTION (NOM_PARA: VALE: (0. , 0. , 10. , 10.)) ;

CH1 = AFFE_CHAR_THER_F (MODEL:

FLUX_REP: (GROUP_MA: G0 FLUX_X: F1 FLUX_Y: F2)) ;

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· *Condition on the field solution: ds*

= 0: creation of a unit source term on all it

S

grid, and of the vector second associate member. LIAISON_CHAMNO then makes it possible to impose desired condition.

CHS = AFFE_CHAR_THER (MODEL:

SOURCE: (ALL: "YES" SOUR: 1.));

VS = CALC_VECT_ELEM (OPTION: "CHAR_THER" CHARGES: CHS...);

MS = CALC_MATR_ELEM (MODEL: ... OPTION: "RIGI_THER");

NUM = NUME_DDL (MATR_RIGI: Ms);

= ASSE_VECTEUR (VECT_ELEM GOES: VS NUME_DDL: NUM);

CH2 = AFFE_CHAR_THER (

LIAISON_CHAMNO: (CHAM_NO: COEF_IMPO GOES: 0.));

· *Calcul of the function of warping*

Ther_LINEAIRE (MODEL:

EXCIT: (LOAD: CH1)

EXCIT: (LOAD: CH2)

);

2

· *Calcul of the constant of warping I =*
ds

and enrichment of the table:

S

TCARS = POST_ELEM (MODEL: ...
CARA_POUTRE: (CARA_GEOM: TCARS
LAPL_PHI: KSI OPTION: "CARA_GAUCHI"));
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Appendix 1 Determination of the constant of torsion for sections has borders multiplement related

That is to say a beam elastic, isotropic, length L and of unspecified section which can be not simply related. One notes contour external of and

= $IK - 1$, possible contours

0

I, for I

N

n-1

interiors. One notes =

I

U

its total border.

i=0

One chooses the axis x_1 according to the line of the centres of gravity of the cross-sections. One supposes to simplify

demonstration which the center of torsion is confused with the centre of gravity, which makes it possible to uncouple them

effects of torsion and inflection. The axes x_2 and x_3 are selected following the principal directions of inertia.

x_2

0

G 0

G1

x_1

I

x_3

Appear A1-a: Beam with unspecified section

The beam is charged on its section X

L

I =

by one torque M

= M X

G

T

.

1

1

In addition, the side surface of the cylinder is not charged and the forces of volume are null.

One deduces from it immediately that the torque of the interior forces at the G0 point is M

= M X

G

T

0

1

The problem of elasticity posed previously seems incompletely definite. Indeed, conditions with the limits on the cross-sections X

L

$I =$

and $x_1 = 0$ is incomplete because there is not a condition in each not, but on average. There is thus, a priori, an infinity of solutions. The assumption of Saint-Coming consists with to seek a solution such as the tensor of the constraints is form

$I_1 I_2$

I_3

$=$

I_2

0

0

$I_3 0$

0

The principle of Saint-Coming is valid far from the sections of application of the forces. Indeed, except in cases particular loadings, the four presumed null terms diminish exponentially with x_1 .

To solve this problem of elasticity, a formulation in constraints is chosen. The equations to be written are thus those of balance and those of compatibility.

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The equilibrium equations $\text{div} = 0$ lead to the three following scalar equations:

$$111+2 12 +313 = 0$$

éq A1-1

$$112 = 0$$

$$113 = \text{éq A1-2}$$

0

jk
(with the simplified notation: $I jk =$)

I

X

The equations of Beltrami, which take account of the equations of compatibility are written:

-

0

éq A1-3

11 -

$$11 11 =$$

1

-

0

éq A1-4

12 -

12

$$11 =$$

1+

1

-
0

éq A1-5

**13 -
13 11 =
1+**

-
0

éq A1-6

2211 + 11 =

-
0

éq A1-7

**23
11 =**

-
0

éq A1-8
 $33 \ 11 + 11 =$

The equations [éq A1-3], [éq A1-6] and [éq A1-8] show that,
 $= 0$ *and are solutions of one*

22
 $11 \ 11$
 22
 $33 \ 11$

homogeneous linear system and thus that =
 $=$

= 0. With the equation [éq A1-7], one deduces some

$11 \ 11$
 2211
 $33 \ 11$

that = X has + has + B X + B X + C X + C X. By taking account of the fact that one deals with the problem of

11
 $1 \ 1$
 0
 $(1 \ 1 \ 0) \ 2 \ (1 \ 1 \ 0) \ 3$

free torsion, one will take no one from now.
 11

The equations [éq A1-2] and [éq A1-5] show that and do not depend on X. The equation [éq A1-1]

12
 13
 1

is written:

$+ F X$
 $=$
 $-$
 $- G X$
 $2 [12$
 $(3)] \ 3[13 (2)]$

where F and G are two functions arbitrary. According to the theorem of Schwartz, there exists (X, X such as:

2
 $3)$

F X

$$\begin{aligned} 2 &= \\ -13 &- (2) \\ 12 &= 3 - F(x3) \end{aligned}$$

=

$$3 = 12 + G(x3)$$

$$13 = -2 + G(x2)$$

The equations [éq A1-4] and [éq A1-5] give:

$$\begin{aligned} &= 0 \\ &3 \end{aligned}$$

$$\begin{aligned} &= 0 \\ &2 \end{aligned}$$

that is to say

$$= 3F - 2F + K$$

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where K is a constant of integration. As F and G are arbitrary, they will be taken identically null.

3
- 2

The problem to be solved is thus a problem of Laplacian: $\Delta u = K$ on then =

0
0
3

- 2 0
0

It remains to write the boundary conditions, who will allow us to write conditions on for and on K

The boundary conditions are to be written on all the border.

On the cross-sections $X = L$ (and the same in $X = -L$), one a:

1
1
0

$$u_2 ds = 0$$

$$u_3 ds = 0$$

$$X - X ds = M$$

$$2 \quad 13 \quad 3 \quad 12$$

T

éq A1-9

That is to say N the normal external with. One has $N = 0$. One poses $= X$

X. One can also write

12

2 + 13

3

=

grad x1; the tangential part of the constraint in the cross-section is called. That is to say NR the point I

running of contour for $I = 0$ N

K -1. The condition, on side surface, stated higher, can

I

to be written dN

0 .

I =

0

N

N

I

Appear A1-b: Definition of the normals

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Thus, on the side surface of a beam, the vector forced tangential is tangent with contour.

• The equation dN

0 conduit in a condition which must observe on contour: $D = 0$

I =

• The equation [éq A1-9] led to M

$X X ds$

$T =$

-

-

$2 2$

$3 3$

that one can also write:

M

$2 ds$

$(X dx$

$X dx$

$T =$

+

3

$2 -$

2

$3)$

The problem to solve to obtain is thus:

$= K on$

$D = 0 on$

with the constraint M

$2 ds$

$(X dx X dx$

$T =$

+

3

$2 -$

2

$3)$

It remains to identify the constant of torsion C.

The law of behavior of the beams in torsion is: $M = CG$

X

T

(cf [§3]).

X

To solve the preceding problem more easily, one poses =

X

and $K = -2C$, the problem with

X

C

X

X

to solve becomes then:

= -2 on

$D = 0$ on

M

2C

X

= -

ds +

(X dx

3

2 - X dx

T

2

3)

X

With such a notation, one obtains $C = 2$

D
S +

(X dx
3
2 - X dx
2
3)

Contour consists of several contours: an external contour 0 and N -1 interior contours

I. The condition $D = 0$ conduit with N following conditions: $= I$ on I for $I = 0 N -1$

K

. I

are constant unknown factors. By noting that, and thus, is defined except for a constant, one can fix one

I. One will thus take $0 = 0$. Remain to determine I for $I = 1 N -1$

K

.

For that, one will study the warping of the cross-section of X-coordinate $x1$. Let us recall that the tensor of constraints is written (cf [§4]):

0

-

Z

+

y

0

-

y

Z

Z
y

=
G
X
- Z

X

0
0
=
G

0
0

X y

X Z

+ y

-
0
0

0
0
Z

y
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One poses =

grad xI ; is (except for a constant) the tangential part of the vector forced in cross-section.

One thus has: grad = - GM xI

One makes circulate this equation along I , one a:

0 = dl -

$X dx$

$X dx$

$3^2 - 2^3$

I

I

One was useful oneself here owing to the fact that the circulation of the gradient on a closed curve is null.

It is noted that the first integral can be written in term of flow of beam I . In addition, the second integral is equal to 2 (

I).

Finally, the problem is written:

= -2 on

= 0 out of 0

= I on I

dl

(

= 2

I)

I

N

n-1

Once solved this problem, one a: J = 2

D

S +

2 I (

I)

i=1

The last condition:

dl

(

= 2

I) is difficult to treat in a numerical way. Actually, both

I

N

conditions on each border bordering a hole are written:

= I on I

2 (

I)

=

on

N

L (

I

I)

Variational formulation:

v H1 () such as v

= 0

0
μ L2 (I)
v D +
iv dl = 2 v D +

2

v dl

(I)

I
I

I

μ dl =

dl

I

μ

I
I

I dl =

0

I

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One considers the function such as I on I . Matriciellement, one a: $=t [] []$ $v T$ $= [v] []$ $=t [] []$ *Where $[]$ is the vector whose components are related to form.**Under these conditions, the variational approximation of the weak formulation gives:*

$$\begin{aligned} & [\\ & T \\ & K] [] + [Bi] [I] = [F] + [T] \end{aligned}$$

 I $[B]$

$$\begin{aligned} & [] \\ & = [] \\ & 0 \end{aligned}$$

Where one posed:

$$T [F] [v] = 2 v D$$

$$\begin{aligned} & T \\ & 2 (\\ & I) \\ & [T] [v] = \\ & v dl \end{aligned}$$

I
I
[B] = [Bi]
I

One can check that the condition on flow is checked:

dl =

dl =
D +

D

IN
I

= -

T
2D
T

+ [F] [] T
+ [T] [] T
- []

[Bi] []
144 2
4
3
444
1 2
4 3
4
0
0

2 (
=
=
I)
=
dl

I
=
= 2 (I
I)

It is seen, with this new formulation, that the posed mathematical problem returns to a problem of linear thermics with a particular loading. This is easily programmable in Code_Aster.

By applying the preceding method to a beam whose cross-section is the crown ranging between rays R1 and R0 with $R < R$

I
0 .
R0
R1
G

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There is the following problem to solve:

$$= -2$$

$$(R = R_0) = 0 = 0$$

$$(R = R_1) = 1$$

$$dl \ 2R_2$$

=

$$0$$

$$1 \ N$$

r_2
 The general solution of the problem is written $= (R) = -$
 $+ A ($
 $\ln R) + B$
 2

Here, one has
 2
 2

= -
 E. From where $2R - 2$

1
 With (
 $\ln 1$
 $R) =$
 $2R$
 $To = 0.$ In addition, one
 N

$$R$$

$$1$$

$$R$$

$$1$$

$$2$$

$$($$

$$R$$

$$R$$

$$0$$

$$0) = 0$$

$$B =$$

. Finally:

2

2

2

(

R

R

R) = -

+ 0 .

2

2

2

2

R

R

One can now calculate $J = 2$

D

S +

2

1

0

2

1 (

1

). One has = -

+

and (

1) = R. All

1

2

2

1

made calculations, there is the traditional result:

$$J = (R4 - R41)$$

2

0

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Appendix 2 Determination of the constant of shearing of one beam equivalent to a whole of parallel beams

A2.1 Position of the problem:

One clarifies here a method developed in order MACR_CARA_POUTRE to obtain the coefficients of shearing AY and AZ of a beam equivalent to a whole of disjointed beams (e.g. embedded posts between two floors). This makes it possible for example to carry out models “skewers” of buildings, it is with to say condensed in only one beam.

For only one beam, the definition of the coefficients of shearing rests on the energy method [§2.1.3]: the formulation is based on complementary energy due to shearing in the section.

$$\int_0^L \int_{CT} dS$$

The coefficient of shearing is: $K =$

$$\frac{\int_0^L \int_{CT} dS}{2D S G}$$

Note:

This expression is still valid in the case of a heterogeneous beam (G variable).

The distribution of shear stress in the section, for only one beam, is based on the formula of Jourawski, [§2.1.1] which provides the distribution of shear stresses due to a shearing action in one direction and only the average of shearings in the other direction.

The formula of Jourawski is written:

m (y)
y_{max}
CT =
V with m (y) =
T B (T) dt

I B (y)
y

Quantity (
m y) represents the static moment on behalf of section A ranging between y and y_{max}

y
With
B (y)
y_{max}
B
G
Z

I 2
Then K can be written in the form: K =

2
y
m (y)

S G max
Dy
y
Gb² (y)

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The subjacent idea is that the section supports normal constraints resulting from the theory of the beams of Euler, and whom one evaluates the effort of slip of A on B.

For a nonrelated section, like the section out of cut of a building, the assumption of Jourawski cannot to be made (except considering that all the section becomes deformed axially like the same beam with each X-coordinate X). One cannot know a priori the distribution of shearings nor of the normal constraints in each post. The following figure gives an idea of the cross-section of a building engine:

***Appear A2.1-a: Section of a building engine
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A2.2 Expression simplified of the coefficients of shearing

Assumption: the posts are embedded in the floors: the building seen on side can be represented like a whole of parallel posts embedded between two floors:

F

H

Appear A2.2-a: together of posts between two boards

This particular case is calculated: each post is a rectangular beam of section (the principal axes of inertia of the various posts are not colinéaires). $H=3$ with $4m$.

The beams are embedded at the two ends. It is then necessary to seek a relation between an effort F imposed on higher floor, and the displacement of this floor in the same direction, i.e. to calculate the rigidity of this system in this direction.

A2.2.1 For a beam:

The method used is exposed for example in [bib11].

Beam fixed at an end and free with the other:

The system is isostatic and elastic. One wants to express displacement $U(H)$ according to F and. Principle of Virtual Work is written:

H

$$F(H) \cdot v(H) = M F. (v)$$

$$+V F. (v) dl$$

O

$$M(v)$$

F

$$(v) = E.I$$

H

v, y

$$V(v)$$

$$(v) =$$

U, X

$G.Sr$

for any virtual displacement v , and a specific effort F in $y=H$, (here the normal effort is null).

$F=1$ is chosen, and one calculates successively displacements due to an effort F and a couple in $y=H$. In

integrating the preceding expression, it is found that, under the effect F , displacement U real and rotation are worth:

$$F.H^3$$

$$F.H$$

$$F.H^3$$

12EI

F.H2

U (H) =

+

=

4 +

with S = k.S and (H) = -

2

3rd. I

G.S

12th. I

G.H S

R

2nd. I

R

R

Under the effect of the moment, one obtains:

. H2

. H

U (H = -

=

)

(H)

2nd. I

E.I

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If the beam has a rectangular section of width B and thickness H , one obtains: $S = bhk = bh$

R

6

F.H 3

12 H2

6F. H2

for F imposed: $U(H) =$

4

$1(H) = -$

3

+

(

2

+)

E.bh

5 H

E.bh3

6. H2

12. H

for imposed: $U(H) = -$

=

)

(H)

E.bh3

E.bh3

Beam fixed at the two ends: (embedding slipping in $y=H$)

The system is hyperstatic degree 1.

One expresses displacement $U(H)$ according to the hyperstatic unknown factors F and, using the preceding results.

Under the effect F and, displacement U real and rotation (null because of embedding) are worth:

F.H 3

F.H

. H2

F.H 3

12 H2

6.H2

U (H) =

+

-

=

4

1

3

+

(

2

+) -

E.I

GS

E.I

E.bh

5 H

E.bh3

3

2

R

F

2

F.H

. H

6

2

F.H

12. H

H

0 = -

+

= -

+

2

3

2
I.E.(internal excitation)
E.I
E.bh
E.bh

The resolution of this system makes it possible to obtain U (H) according to F:

F.H 3
12EI
F.H 3
12 H2

U (H) =
1+
1
1
2
=
3
+
(
2
+))
12EI
GH S
E.bh
5 H

R

12EI
1

E.bh3
1

$$\begin{aligned}
 F &= \\
 U(H) & \\
 3 & \\
 &= \\
 U(H) & \\
 3 & \\
 &= K.u(H) \\
 H & \\
 12EI & \\
 H \ 12 \ H^2 &
 \end{aligned}$$

$$\begin{aligned}
 1 &+ \\
 1 & \\
 1 & \\
 2 &
 \end{aligned}$$

$$\begin{aligned}
 &+ \\
 (& \\
 2 & \\
 +) &
 \end{aligned}$$

$$GH \ S$$

$$5 \ H$$

$$R$$

One also finds this result by considering the matrix of rigidity of an “exact” element of beam with 2 nodes ([bib1] or [R3.08.01]). The term above corresponds exactly at the end of rigidity of sharp effort only according to direction X:

$$\begin{aligned}
 12EI & \\
 1 &
 \end{aligned}$$

$$\begin{aligned}
 12EI & \\
 12EI & \\
 K = &
 \end{aligned}$$

xx

H3 (1+)

=

2

=

GH S

GH 2

kS

R

Note:

The embed-free situation differs only from one coefficient (4 instead of 1):

F.H 3

12 H2

U (H) =

4

1

3

+

(

2

+))

E.bh

5 H

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The two possibilities are offered in MACR_CARA_POUTRE:

- the two ends are embedded (actually one is embedded, the other is embedded in a floor mobile: slipping embedding)*
- the higher end is free (makes some in connection rotulée with the higher floor).*

One can thus propose to express rigidity with the shearing action of each post in the form:

$12EI$

1

1 embedded - embedded

$F =$

$U(H) \mathbf{K} \cdot () \text{ with } =$

3

$=$

$U H$

H

$12EI$

4 embedded - free

$+$

2

$GH S$

R

A2.2.2 For a whole of beams

The method consists in calculating the rigidity of each post in the preceding way, and to compare rigidity of the whole to that of an equivalent beam embedded between two floors. For that one expresses the effort total edge applied to the whole of the posts (for example in the direction y):

$T =$

~

 $T = K \cdot U$ y I y y

Each post having an unspecified orientation compared to the total axes, it is necessary for all to express them

efforts T in the total reference mark:

 I Z x^2

x_1 and x_2 indicating the principal axes of inertia of

 T_y

post I , the shearing action T in this reference mark is:

 I T_z

$$T_i = T_i \cos ()$$

$$\sin ()$$
 I $+T_i$ y I 2 I y

$$T_i = - T_i \sin ()$$

$$\cos ()$$
 I I $+T_i$ Z I 2 I x_1

Moreover, one supposes that total displacement U of the whole of the posts is uniform (components U

y
 and U) and must be colinéaire with the shearing action T . (what is not certain: couplings are possible if it

y
 does not have there particular symmetries). This involves for the direction y :

$$u_i = U \cos (\theta)$$

$$T_i = K_i \cdot u_i$$

I

y

I

I

I

I

and

$$u_i = U \sin (\theta)$$

I

I

I

2

y

I

$$T = K \cdot U$$

2

2

2

one obtains:

$$T_i$$

I

2

I

2

I

=

$$\cos (\theta)$$

$$\sin (\theta)$$

I

+

y

$$(K$$

K

I

2

I) uy

~

12EI

I

~

with K =

y

T = Ti

I

2

I

=

2

cos ()

sin ()

H 3

12EI

I

+ 2

=

y

y

(K

K

I

I) U

K U

y

y y

+

2

GH S

R

in the same way in direction Z:

ui = - U sin ()

I

Z

I

~

thus T =

TI

I

2

I

=

2

sin ()

cos ()

I

+ 2

=

Z

Z

(K

K

I

I) U

K U

ui = U cos ()

Z

Z Z

2

Z

I

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In addition, for the equivalent beam, one makes the assumption that rigidity with the shearing action S

`expresses

even way that that of each beam:

eq

eq

eq

12EI

12EI

T = K U

Z

=

U

Z

y

y

y

with

y =

y

H 3 (

eq

2

eq

l + y)

S H Gk y

In fact, it would have to be checked that energies due to the inflection and the normal effort are quite negligible.

The two expressions of the shearing action lead to the expression of the coefficient of equivalent shearing:

12EI eq

12EI eq

K eq

Z

=

and in direction Z: K eq

y

=

y

12EI eq

Z

$12EI$ eq

GS eq H2

Z

GS eq H

y

2

3 ~

- 1

3 ~

- 1

H K

H K

y

Z

A2.3 Method used in MACR_CARA_POUTRE

By using the assumptions described previously, namely:

- only rigidity due to the shearing action is taken into account in the calculation of the coefficients of shearing***
- the equivalent beam is embedded on the two floors***
- two design assumptions are to be envisaged concerning each post (embedded-rotulé and embedded embedded).***

one can propose a method of calculation in MACR_CARA_POUTRE to obtain coefficients of shearing are equivalent to a whole of beams of parallel axes, embedded in a floor with one of their ends, and free with the other, or embedded at the other end.

Restrictions of use:

- it is reasonable to place the equivalent beam on the centre of gravity of the whole of the posts, and in the principal reference mark of inertia of the unit, to avoid the parasitic couplings***
- it is necessary to ensure the continuity of all the DDL of the equivalent beam (translation and rotation) with the DDL floors (what models the embedding of the beam in the floor), which forces to model it***

floor in elements of hulls or, if it is with a grid in 3D, to connect it using beams or of plates.

A2.3.1 the method of calculation is as follows:

1) For each post, to make usual calculation by MACR_CARA_POUTRE of the geometrical characteristics and coefficients of shearing of the section, in the principal reference mark of inertia of each section (already available)

2) Always for each section, calculation of rigidity to shearing (the user must provide H, distance enters floors):

***12EI I
12EI I
Ki
2
2
1 =
with =
H 3 (
1
2
1 + 1)
If H Gk i1
12EI I
12EI I
Ki
1
1
2 =
with =
H 3 (
2
2
1 + 2)
If H Gk i2***

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3) Calculation of rigidity equivalent to the whole of the beams:

~

~

K

I

2

I

=

2

K

I

2

I

=

2

I

+

Z

(K sin () K cos ()

I

2

I)

I

+

y

(K cos () K sin ()

I

2

I)

4) Calculation of the coefficients of shearing are equivalent:

$12EI$ eq

$12EI$ eq

K eq

Z

=

K eq

y

=

y

$12EI$ eq

Z

$12EI$ eq

GS eq H^2

Z

GS eq H

y

2

$3 \sim$

$- 1$

$3 \sim$

$- 1$

$H K$

$H K$

y

Z

knowing that S eq, I eq and I eq are already calculated by

y

Z

MACR_CARA_POUTRE.

For the key words of order MACR_CARA_POUTRE, one needs that the user provides H , by the key

word

LENGTH, the characteristics (constant) of the material (key word MATERIAL) and chooses the conditions with limits by the key word CONNECTION:

CONNECTION: KNEECAP or CONNECTION: EMBEDDING

This calculation is of course activated only if several GROUP_MA are defined by the user (indicating that the section is made up of under disjointed parts).

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Organization (S): EDF-R & D /AMA

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R3.08.04 document

Element of beam with 7 ddl for the taking into account warping

Summary:

This document presents the element POU_D_TG which is a finite element of right beam with taking

into account of warping of the sections. It allows the calculation of the beams mean transverse sections and opened profile, with constrained or free torsion.

With regard to the inflection, the normal and sharp efforts, this element is based on element POU_D_T, who is an element of right beam with transverse shearing (model of Timoshenko).

For element POU_D_TG, the section is supposed to be constant (of an unspecified form) and the material is homogeneous and isotropic, of linear or elastoplastic elastic behavior (behaviors VMIS_POU_LINE and VMIS_POU_FLEJOU).

This reference material is based on the general reference material of the beams, in linear elasticity [R3.08.01] and in élasto - plasticity [R5.03.30]. It describes specificities of the element of beam right-hand side with warping POU_D_TG.

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of use**

The development of the elements of beam of Timoshenko with warping (modeling POU_D_TG) in Code_Aster was carried out initially with an aim of calculating the behavior pylons. The first development was made within the framework of a thesis at Department MMN. It mainly acted to calculate formed structures of beams with open mean profile (angles), for which warping is important. Plasticity was introduced into the element POU_D_TG [R5.03.30], but the nonlinear behavior relates only to traction, inflection and torsion. Shearing due to the sharp effort, as well as warping and the Bi-moment (effort related to warping) remain dependent by an elastic behavior, fault of being able to express one nonlinear behavior on these sizes. This is why the description of torsion with warping is valid for the use of element POU_D_TG with the linear operators (MECA_STATIQUE, DYNA_LINE_TRAN,...) or not linear (STAT_NON_LINE, DYNA_NON_LINE,...).

2 Notations

The notations used here correspond to those used in [R3.08.01] and [R3.08.03]. One gives here correspondence between this notation and that of the documentation of use.

DX, DY, DZ, DRX, DRY, DRZ and GRX are the names of the degrees of freedom associated with components of displacement U_v

, W, X, y, Z, X, X. They are expressed in total reference mark, except the degree of freedom associated with the warping GRX, which is expressed in local reference mark.

Notation used**Significance****Notation****of****documentation****of use****S****surface of the section**

With

I

geometrical moments of inflection compared to axes IY, IZ

y, IZ

X and Y.

C

constant of torsion

JX

I

constant of warping

JG

K

coefficients of shearing

1 1

y K

, Z

AY AZ

E

EY, EZ

y E

, Z

eccentricity of the center of torsion/shearing by

report/ratio in the centre of gravity of the cross-section

NR

normal effort with the section

NR

V

sharp efforts along axes y and Z

VY VZ

,

y V

, Z

M

moments around axes X, y and Z

MT, MFY, MFZ

X, M y, M Z

M

Bi-moment

BX

U, V, W
translations on axes X, y and Z
 DX, DY, DZ

rotations around axes X, y and Z
 DRX, DRY, DRZ
 X, y, Z

rotary derivative of torsion according to X
 GRX
 X, X
 E
Young modulus
 E

Poisson's ratio

NAKED
modulate of Coulomb (identical to the coefficient of Lamé) G
 $=$
 E

G
(
 $2 I +) = \mu$

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3

Kinematics specific to torsion with warping

Kinematics used to represent the displacement of the sections of beam is identical to that right beams of Timoshenko [R3.08.01] with regard to the traction and compression, and inflection - shearing. Only torsion here is detailed.

Two possibilities are to be considered for the modeling of behaviour in torsion of the sections not circulars [bib1], which always produces a warping of the cross-section.

torsion is free (torsion of Saint-Coming): the warping of the cross-sections is not no one (it can even be important for an open mean section), but it is independent of the position on axis X of the beam, (constant according to X) and it does not have there a constraint axial due to torsion.

Torsion is constrained (Vlassov): warping is nonnull, and moreover of the constraints axial not uniforms (from which the effort resulting Bi-moment is called) exist in the beam.

Element POU_D_TG makes it possible to treat these two configurations: torsion can be free or constrained.

The user will have access to warping in both cases, on the other hand the Bi-moment will not be nonnull that in the case of constrained torsion. It should be noted that at the place of the connection of the beams,

transmission of warping depends on the type of connection. In general, torsion in an assembly beams is constrained. Warping can then be blocked at the points of connection.

Note:

With elements without modeling of warping (POU_D_T, POU_D_E), one can to treat the case of free torsion (displacements other than warping will be correct), but not the case of constrained torsion.

One can uncouple the effects of torsion and inflection in a local reference mark (relocated principal reference mark of inertia) having for origin the center of torsion. The center of torsion is the point which remains fixed when the section is subjected to the only torque. It is also called center of shearing because one

effort applied in this point does not produce rotation around X.

Displacements in the plan of the section will thus be expressed in this reference mark. Displacements axial remain expressed in the principal reference mark of inertia related to the centre of gravity G, to keep one decoupling of displacements of inflection and traction and compression.

The displacement of an unspecified point of the cross-section is written then in general form (torsion free or constrained):

$$U(X, y, Z) = u_G(X) + Z \cdot y'(X) - yz'(X) + (y, Z) \cdot X, X(X)$$

$$v(X, y, Z)$$

$$= 0 + 0 + v(X) + -(Z - z_c) X(X)$$

$$W(X, y, Z)$$

$$0 \cdot W(X)$$

$$0$$

$$(y - y_c) X(X)$$

displacement = membrane + flexion/y + flexion/z + with

torsion

warping

The components are expressed in the principal reference mark of inertia (centered in G): X is directed according to

the axis of the beam, y and Z are the two other principal axes of inertia.

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Term $(y, Z) X, X (X)$ represents axial displacement due to the warping of the cross-section. (y, Z) is related to warping (expressed in m^2 , but which does not have physical interpretation obvious).

The deformations of an unspecified point of the section are then:

, ,

,

,

,

-

,

$xx (X y Z)$

$uG (X)$

Z

X

y

X

$y Z$

X

X

$y X ()$

$Z X ()$

$() X, xx ()$

$$2xy(X, y, Z) = 0 + 0 + xy(X) + (y - (Z - zc))X, X(X)$$

$$2xz(X, y, Z) = 0$$

$$xz(X) = 0$$

$$(Z + (y - yc)X, X(X))$$

$$xy(X) = v, X - Z$$

$$xz(X) = W, X + y$$

Deformation = membrane + inflection/y + inflection/Z + torsion with warping

Term $(y, Z)X, xx(X)$ is null in the case of free torsion: there are indeed $X(X)$
 xx
 $)0$

, since

warping is independent of X . It is considerable in the case of constrained torsion.

The law of elastic behavior isotropic is written (by making the assumption of the plane constraints in directions y and Z):

$$xx(X, y, Z) = E \cdot xx(X, y, Z)$$

$$xy(X, y, Z) = G \cdot 2xy(X, y, Z)$$

$$xz(X, y, Z) = G \cdot 2xz(X, y, Z)$$

The efforts generalized in the section are expressed according to the constraints for a section homogeneous by [bib1]:

$$NR(X) = xx(X, y, Z) ds$$

normal

effort

S

$$V(X) =$$

y

$$xy(X, y, Z) ds$$

y

according to

edge

effort

S

$$V(X) =$$

Z

$$xz(X, y, Z) ds$$

Z

according to

edge

effort

S

$$M(X) =$$

y

$$Z. xx(X, y, Z) ds$$

y

of

around

inflection

of

moment

S

$$M(X) = -$$

Z
y. xx (X, y, Z) ds
Z

of
around
inflection

of
moment

S
M (X) =
X
(
(y - yc) .xz (X, y, Z) - (Z - zc) .xy (X, y, Z))ds
torsion

of
moment

S
M (X) =
.

xx (X, y, Z) ds
Bi -
ent)
gauchissem

with
(associate

moment
S

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M (X)

represent the generalized effort associated with warping. It is expressed in N.m2. One can in to give an illustration as in [bib1] for a beam to I-section (the Bi-moment acts here according to Z only):

*Z**Z**xx+**Y**xx -**Y**M**X**X**xx -**xx +*

For an isotropic and homogeneous elastic behavior in the section, the efforts generalized thus express themselves directly according to displacements by the following relations:

$$NR(X) = E S$$

. *U*. , *X*

$$Vy(X) = GkyS (v, X - Z)$$

$$Vz(X) = GkzS (W, X + y)$$

$$M_y(X) = E I_{yy} X$$

$$M_Z(X) = E I_{zz} X$$

$$M X (X) = G.J .x, X$$

$$M (X) = E I$$

. .x, xx

where $K y K$

, Z are the coefficients of shearing. Warping does not intervene on the level of the efforts edges, because those are expressed in the reference mark related to the center of shearing. Indeed, the function

of warping is such as:

$$(y, Z) ds = 0$$

S

$$y (y, Z) ds = 0$$

S

$$Z. (y, Z) ds = 0$$

S

And the constant of warping is expressed according to by: $2 (y Z) ds = I$

,

S

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Element of right beam with warping: matrices of rigidity and of mass

The elementary matrices of rigidity and mass for element POU_D_TG are identical to those element of right beam of Timoshenko (POU_D_T) with regard to the terms of traction - compression and of inflection - shearing [R3.08.01]. The step is identical, one recalls simply the result.

This implies that, in the case of free torsion, one preserves the properties of exactitude of the solution with the nodes for the degrees of freedom of inflection and traction and compression.

On the other hand, we will see that with regard to obstructed torsion, one carries out an approximation which does not allow to find this property in the case general.

The matrices of rigidity are always calculated with option "RIGI_MECA", and the matrices of mass with option "MASS_MECA". But option "MASS_MECA_DIAG" (matrix of diagonalized mass) does not have not realized for this element (this option is especially useful for the problem of dynamics rapid, which is not the preferential field of application of this element).

The degrees of freedom of the element are those of the beams of Timoshenko, plus a degree of freedom by node allowing to calculate the terms relating to warping:

In each of the two nodes of the element, the degrees of freedom are:

*U v
, W
translations on axes X, y, Z
DX DY DZ*

*rotations around axes X, y, Z
DRX DRY DRZ
X
, y
, Z*

*rotary derivative of torsion according to X
GRX
X, X*

The local co-ordinates are expressed in the principal reference mark of inertia. Element POU_D_TG thus comprise 14 degrees of freedom. The element of reference is defined by: $0 < X < L$

Z
Z

Z
W
y
v
Y

Y
X
X
X
U

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4.1
Traction and compression the degree of freedom are U or DX

*ES 1
-
1*

The matrix of rigidity of the element is: $\mathbf{K} =$

L^{-1}

$2L$

The matrix of mass (coherent) is written: $\mathbf{M} =$

$6L$

4.2 Inflection in the plan (Gxz) the degrees of freedom concerned are W, y or DZ, DRY

The matrix of rigidity is written for the movement of inflection in the principal plan of inertia (Gxz):

L

L

1

$-$

-1

$-$

2

2

$(4+y) 2L L (2-y)$

2

L

$12 I.E. (internal excitation)$

$=$

y

\mathbf{K}

3

$L (1+y)$

12

2

12

L

Sym

1

2

(4+y)

2

L

12

12 I.E.(internal excitation) y

Transverse shearing is taken into account by the term: y =

2

K SGL

Z

For the matrix of mass,

(

W T

X,) and y (T

X,) are discretized on the basis of function tests

introduced for the calculation of the matrix of rigidity, that is to say:

(

W X, T)

= 1 (X) 1

W (T)

+ 2 (X)

(T)

y

+ 3 (X) 2

W (T) + 4 (X)

(T)

1
y2

(X, T)
y

= 5 (X) 1
W (T)

+ 6 (X)

(T)
y
+ 7 (X) 2
W (T) + 8 (X)

(T)
1
y2

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The function of interpolation used for the translations (1

to 4) are polynomials of Hermit of

degree 3, that which is used for rotations (5 to 8) are degree 2: for $0 < X < L$, they are

defined by [R3.08.01]:

3
2

1
6

=

(X)

1

X

X

X

X

X

2 -

3 -

+

y

(1+y)

=

(X)

1

+

1

5

yL

L

L

(L +

1 y) -

LL

L

X 3 4 +

2

2

y

X
 $2+$

1
 $=$
 (X)

y
 $-$
 X
 X
 X
 2
 $+$
 $-$

$=$
 (X)

6

$3 - (4+y)$
 $+(+$
 $1 y)$
 $+$
 $1 y L$
 $2 L$
 $2 L$
 $+$
 $1 y L$
 L

3
 2

1
X
X
X
-6
X
X
=
(X)
3

-
2 +
3 +y

=
(X)
1
+
1
7
y
L
L
L

(L +
1 y) -
L L

L
X 3 2 -
2

2
1
=
(X)
4

y X

y

-

X

X

X

+

+

=

(X)

8

3 + (2+y)

-

+

1 y L

2 L

2 L

+

1 y L

L

éq 4.2-1

The form of the matrix of mass is:

2

2

2

2

2

2

2

13L 7L

2

2

9

3

13 2

3

y

L

11L

11L

y

y

L

L

L

L

L

L

L

+

+

y

-

-

-

+

y +

y

+

y +

y

35

10

3

210

120
24
70
10
6
420
40
24

2
3
3
2
2
3
3
2

L3
L

13 2
2
3

y
L
L
3L

y
y
L
L
L
L
y
y
y
S
+

+

-

-

-

-

-

-

-

M = (

105

60

120

420

40

24

140

60

120

2

2

2

2

2

2

1+

13L

7L

L

11L2

11L

L

y

y

y

y

y

y

y)

+

+

-

+

+

35
10
3
210
120
24
3
3
2

L3
L
L

y
y

sym
+
+

105
60
120

6
1

6
1

-
+ y
-
-
+ y

5L
10
2

5L
10
2

2L
L
L 2
2

y
y
l
y
L
L
L
y
y
I
+
+
-
-
-
+

+
y
15
6
3
10
2
30
6
6
(

2

1+

6

1

y

y)

-

5L

10

2

2L

L

L 2

sym

+

y +

y

15

6

3

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4.3

Inflection in the plan (Gxy) the degrees of freedom concerned are, Z or DY, DRZ

In the same way, for the movement of inflection around the axis (Gz), in the principal plan of inertia (Gxy), stamp rigidity is written:

L

L

1

-1

2

2

(4+z) 2

L

L (2 - Z)

2

L

12 I.E.(internal excitation)

-

Z

12

2

12

=

K

3

L (1+z)

L

1

-

2

$(4+z)^2$

L

sym

12

Transverse shearing is taken into account by the term:

12 I.E.(internal excitation) Z

$Z =$

2

$K SGL$

y

To calculate the matrix of mass, $v(T$

X,) and $(T$

X,

Z

) are discretized by:

v

$(T$

$X,)$

$= I(X) v$

$1(T) -$

$2(X)$

$Z(T)$

$+ 3(X) v$

$2(T) - 4(X)$

$Z(T)$

1

2

$Z(T$

$$X_1 = -5(X_2) v$$

$$1(T) + 6(X)$$

$$Z(T) - 7(X) v$$

$$2(T) + 8(X)$$

$$Z(T)$$

$$1$$

$$2$$

We obtain the matrix of following mass then:

$$2$$

$$2$$

$$2$$

$$2$$

$$2$$

$$2$$

$$2$$

$$2$$

$$2$$

$$2$$

$$2$$

$$13L$$

$$7L$$

$$L$$

$$11L$$

$$11L$$

$$L$$

$$9L$$

$$3L$$

$$L$$

$$13L$$

$$3L$$

$$L$$

$$+$$

$$Z +$$

$$Z$$

$$+$$

$$Z +$$

$$Z$$

$$+$$

$$Z +$$

$$Z$$

-

-

Z -

Z

35

10

3

210

120

24

70

10

6

420

40

24

3

3

3

2

2

2

2

2

3

3

3 2

L

L

L

13L

3L

L

L

L

L

Z

+

+

Z

+
Z +
Z
-
-
Z -
Z

S

105
60
120
420
40
24
140
60
120

M = (
2
2
2
2
2
2
1+

Z)

2
13L
7L
L
11L
11L
L
Z
Z
Z
Z

+
+
-
-

35
10
3
210
120
24
3
3
3 2

L
L
L
sym
+
Z +
Z

105
60
120

6
1
6
1
Z
Z

-
-
-

5*L*
10

2

5L

10

2

2

2

2L L

L

L

L

L

Z

Z

+

+

- 1 + Z -

-

Z

Z

+

I

Z

15

6

3

10

2

30

6

6

+

I+

Z)

2

6

- 1 + Z

5L
10
2

2
2L L
L
Z
Z

sym
+
+
15
6
3

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4.4 Torsion and warping the degrees of freedom are X, X, X or DRX, GRX

With regard to torsion, the formulation is obviously different from that of the beams without warping of the reference [R3.08.01]. The virtual work of the interior efforts is written for torsion [bib1]:

L

W

=

* G

. J

.

* E

. I

.

int

+

.

dx

X, X

X, xx

X, xx

O

X, X

The functions of interpolation of the rotation of torsion must be of C2 class, since they must allow to interpolate the derivative second rotation.

By using the equilibrium equations, one shows in [bib1] that the analytical solution utilizes function of interpolation hyperbolic in X. This then makes it possible to obtain exact results with nodes. It is not the choice made for *Code_Aster*: one chose, by preoccupation with a simplicity for numerical integration like avoiding the numerical problems of evaluation of the function hyperbolic, of the polynomials of degree 3 of Hermit type, of the same kind as those used for inflection [éq 4.2-1]. One writes them here on the element of reference [- 1,1] according to [bib1] (instead of $0 < x < L$ previously):

$$\frac{2}{L} = X - 1$$

$$\frac{-1}{L} = \frac{1}{2} - X$$

$$\frac{1}{4} = \frac{2 + X}{L}$$

$$\frac{(1 - X)^2}{8} = \frac{1}{2} - X$$

$$\frac{NR(X)}{L} = \frac{(1 + X)^2}{8}$$

$$\frac{(1 + X)^2}{8} = \frac{1}{2} + X$$

The interpolation for the rotation of torsion and its derivative is:

$() = NR$
 X
 $() 1$
 1
 $+ NR () 1$
 2
 $+ NR () 2$
 3
 $+ NR () 2$
 4
 X
 X, X
 X
 X, X
 $X, X () = NR X () 1$
 $,$
 1
 $+ N2 X () 1$
 $,$
 $+ NR X () 2$
 $,$
 3
 $+ N4 X () 2$
 $,$
 X
 X, X
 X
 X, X

The reference [bib1] note which this approximation corresponds to a borderline case of the interpolation GJ

hyperbolic, obtained for

0. However, this parameter not being without dimension, it is

I.E.(internal excitation)

difficult to define a priori the values for which the approximation is acceptable. Tests numerical carried out show that one converges quickly towards the solution when the size of elements decreases.

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The matrix of rigidity corresponding to this approximation is written then:

36
3L
- 36
6L
12
6L
-12
6L

GJ

2
4L
- 3L - 2

L I.E.(INTERNAL EXCITATION)

2
4L

-

2
6L 2L

K = K + K =

+
T

3

30L

36

- 3L

L

12

- 6L

2

2

sym

4L

sym

4L

The matrix of mass can be obtained in several ways [bib1]:

.

the most complete method would consist in calculating the terms of inertia with the functions of interpolation above, by taking account of the additional term:

L

.

W

= -

*. *I.* && *dx**iner**O*

X, X

X, X

.

in *Code_Aster*, the simplest method was selected: the matrix of mass is identical to that of element POU_D_T. One preserves the already definite terms for traction - compression and the inflection - shearing and one use a linear approximation for torsion.

The coefficients of the matrix of mass associated with warping are null with this approach.

4.5

Eccentricity of the axis of torsion compared to the neutral axis

In the center of torsion C , the effects of inflection and torsion are uncoupled, one can thus use them results established in the preceding chapter.

The co-ordinates of the point C are to be provided to AFFE_CARA_ELEM: one gives the components of vector \mathbf{GC} (G being the centre of gravity of the cross-section) in the principal reference mark of inertia:

0

$$\mathbf{GC} = ey$$

ez

One can numerically determine them starting from the plane grid of the section using the operator MACR_CARA_POUTRE [R3.08.03].

Once the point C determined, one finds as in [R3.08.01] the components of displacement with centre of gravity G by considering the rigid relation of body:

\mathbf{U}

$=$

(G)

$$\mathbf{U}(C) + \mathbf{GC}$$

X

with

$= 0$ vector

rotation

0

U

U

$=$

G
C
v
v

=

+ *E*
G
C
Z X

W
W
=

- *E.*

G
C
y
X

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The change of variables is in the same way written that for POU_D_T, with 2 degrees of freedom

additional:

U

ux

X

1

C

1

U

Uy 1 0 0

0

0 0 0

y

1

C

1

U

E

U

Z

0 1 0 -

0 0 0

Z

C

Z

1

1

0 0 1

E

0 0 0

0

X
y
X
1
C

1

000
1
000

y
y
1
C

1

000
0
100

Z
Z
1
C

1

000
0
010

X, X

C

1

0 0 0

0

0 0 1

$X, x1$

ux

=

C

U

2

X

1 0 0

0

0 0 0

U

2

y

C

U

2

0 1 0 - E

0 0 0

Z

$y2$

uz

C

uz

2

0 0 1

E

0 0 0

y

2

X

c2

0

0 0 0

1

0 0 0 *x2*

y

c2

0 0 0

0

1 0 0 *y2*

Z

C

0 0 0

0

0 1 0

2

z2

X, X

C

0 0 0

0

0 0 1

2 1

4

4

4

4

4

4

4

4

4

4

2

4

4

4

4

4

4

4

4

4

4

3 X, x2

P

From the elementary matrices of mass and rigidity calculated previously in the reference mark (C, X, y, Z) where the movements of inflection and torsion are uncoupled, one obtains these matrices in reference mark related to the neutral axis (G, X, y, Z) by the following transformations:

$$\mathbf{K} = \mathbf{P}^T \mathbf{K} \mathbf{P}$$

C

$$\mathbf{M} = \mathbf{P}^T \mathbf{M}$$

.

P

C*Handbook of Reference**R3.08 booklet: Machine elements with average fibre**HT-66/05/002/A***Code_Aster** ®*Version**7.4**Titrate:**Element of beam with 7 ddl for the taking into account of warping**Date:**02/05/05**Author (S):****J.L. Key FLEJOU, J.M. PROIX***

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*R3.08.04-B**Page:**14/24***5****Geometrical rigidity - prestressed Structure**

This matrix is calculated by the option: "RIGI_GEOM". It is used to deal with problems of buckling or of the vibrations of prestressed structures. In the case of a prestressed structure, thus subjected to initial efforts (known and independent of time), one cannot neglect in the equilibrium equation terms introduced by the change of geometry of the unconstrained state with the prestressed state. This change of geometry modifies the equilibrium equation only by the addition of one function term of displacements and prestressed with which the matrix associated is called matrix of geometrical rigidity and which is expressed by:

*u3D**v3D**W**K**O**K**=**FD*

G

X

ij

X

I

J

Vo

where *O*

ij indicates the tensor of prestressing. This term appears naturally if the tensor is introduced deformations of GREEN-LAGRANGE in the virtual work of the deformation:

2

2

3D

3D

3D

2

U

1

3

X

U

U

X

y

UD

E

= + =

+

+

+
Z

xx
xx
xx

2

X
X
X

X

u3D

u3D

3 3
3 3
3
3
X

y
U D U D
U D U D
D
D

2nd
=
2
+
2
=

+
+ *X*
X
+
y
y
+ *U*
U
Z
Z

xy
xy
xy

y
X
X
y
X
y
X
y

3D
3D

U
U

u3D
u3D
u3D

u3D

3D

3D

2nd

=

2

+

2

=

X

+

Z

+ *X*

X

+

y

y

+ *U*

U

Z

Z

xz

xz

xz

Z

X

X

Z

X

Z

X

Z

In the expression of these deformations, the terms quadratic

2

u^3D

u^3D

u^3D

u^3D

u^3D

X

X

X

X

X

,

and

are neglected here, according to the assumption usually carried out

X

X

y

X

Z

by the majority of the authors [bib.3]. For a model of beam, the tensor of initial constraints
tiny room in the local axes of the beam to components xx

, xy

and

xz . Kinematics is used
introduced with [§2]:

$$u3D$$

$$X(X, y, Z) = uG(X) + Z y(X) - y Z(X) + (y, Z) X, X(X)$$

$$u3D$$

$$y(X, y, Z) = vC(X) - (Z - zc) X(X)$$

$$u3D$$

$$Z(X, y, Z) = WC(X) + (y - yc) X(X)$$

and the expression of the efforts generalized according to the constraints:

$$NR 0 = O$$

$$ds$$

$$V 0 =$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$xx$$

$$y$$

$$O ds V =$$

$$xy$$

$$Z$$

$$O ds$$

$$M =$$

$$xz$$

$$y$$

$$Z ds M =$$

$$xx$$

$$Z$$

$$- y ds$$

$$xx$$

$$S$$

$$S$$

$$S$$

S

S

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It is supposed, moreover, that

0

NR, 0

V, 0

V are constant in the discretized element (what is inaccurate

y

Z

for example for a vertical beam subjected to its actual weight). The moments are supposed to vary linearly:

0

M

0

X

M

y

0

y = (

0

0

M y² - M 1

y)

$$\begin{aligned}
&0 \\
&+ M 1 \\
&y \\
&- V = 0 \\
&L \\
&X \\
&Z
\end{aligned}$$

$$\begin{aligned}
&0 \\
&0 \\
&X \\
&M
\end{aligned}$$

$$\begin{aligned}
&M \\
&Z \\
&0 \\
&Z = (\\
&0 \\
&0 \\
&M z^2 - M 1z) \\
&0 \\
&+ M 1z \\
&+ V = 0 \\
&L \\
&X \\
&y
\end{aligned}$$

These assumptions make it possible to express G
 W for a right beam with warping in the way
following:

$$\begin{aligned}
&L \\
&I + I
\end{aligned}$$

$$\begin{aligned}
&0 \\
&0 \\
&2 \\
&2 \\
&=
\end{aligned}$$

$$\begin{aligned}
&+ \\
&+
\end{aligned}$$

+
+

G
W
NR

(*v*, *X v*, *X W*, *X W*, *X*)

y
Z
NR
yc zc X, X X, X
O

With

+ *Z*
0
0
C NR (v, X X, X + X, X v
, X) - ycN (W, xx, X +x, X W
, X)
- M 0 (W
0
, X, X +, X W
, X) - M (v, X, X +, X v
, X)

y
Z
- y 0
0
0
cV
- Z
o
cV° (X, X X + X X, X) + V (W, X X + X W
, X)

y
Z

y

- V 0 (

I 2

0

I

v

+

+

-

+

2

0

-

+

, X X

X v

, X)

yr

Zr

2yc

M

2zc

M

X, xx, X

Z

I

Z

y

I

y

Z

0

0

I

DM

DM

yr2

I

Z

zr2

y

+ -

+

(*X*, *xx* + *xx*, *X*)

I Z

dx

I y

dx

with the terms

I

= *y* 2

2

yr2

(*y* + *Z*) *ds*

S

I

= *Z* 2

2

zr2

(*y* + *Z*) *ds*

S

who represent not - symmetry of the section. If the section has two axes of symmetry (thus C is confused with G), these terms are null.

Attention, these terms (which name IYR2 and IZR2 in order AFFE_CARA_ELEM) are not currently not calculated by MACR_CARA_POUTRE. The user must thus inform them from values tabulées for each type of section (corner, right-angled,...).

Moreover, to be able to deal with the problems of discharge of thin beams, requested primarily by moments bending and efforts normal, it is necessary to add the assumption of rotations moderated in torsion [bib2], [bib3].

This results in the following modification of the field of displacements (only for the calculation of geometrical rigidity):

u3D
 $X(X, y, Z) U$
 $= G(X) + Z(y(X) + x(X) Z(X)) - y(Z(X) - X(X) y(X)) + (y, Z) X, X(X)$
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The origin of this expression cannot be here detailed. It is the subject of the thesis of CITY OF GOYET [bib2] on the buckling of the beams with open mean sections. The assumption of rotations of torsion moderate (and not infinitesimal) allows to model discharge correctly of a thin beam of section in torsion (coupling torsion - inflection).

The assumption of moderate rotations results in adding with

0

W term 1

G

G

W

1 *L*

1

O

G

W =

- *M*

Z (*X* *y*, *X* + *there X*, *X*)

O

+ *M* *y* (*xz*, *X* + *zx*, *X*)

O

+ *V*_{*y*} (*X* *y*)

O

+ *V*_{*z*} (*xz*)

O

2

Finally, one obtains the geometrical matrix of rigidity while discretizing

0

1

W = *W* + *W* using

G

G

G

same functions of interpolation as the matrix of rigidity of [§4.4]. After having calculated these matrices, it

is necessary to carry out a change of reference mark as with [§4.5]. A matrix of rigidity then is obtained geometrical of the form:

A1 A2

K G =

A2 A3

The blocks of the matrix are clarified hereafter. One uses to simplify the expressions:

O

O

NR E

0

y

NR E

0

Z

Ney = 2

.

1

Nose = 2

.

1

L

L

M 0

0

0

0

+

0

y

M

1

y2

M

0

Z + M

1

Z

M

2

y =

MZ =

L

L

M 0

0

0

0

-

0

y

M

1

y2

M

0

Z - M

1

Z

M

2

y =

M

Z =

2

2

~

I + I

O

y

Z

K = NR

+ e2

2

y + ez

S

I

~

yr2

~

I Zr

I

2

y = -

+ E

2 there I Z =

- E

2 Z

I Z

I y

A1

2 1

v 3 1

W 4

1x 5

1y

6 z1 7

X, x1

0

0

0

2

0

0

+

-

1

v

NR O

NR O

E NR

LM

M

1.2

M

M

0

y

Z

y

y2

L

NR

y

ez +

+ 1.2

2

L
10
10

3
0
0
O
0
0
0
1

W
NR O

- *E NR + LM - M*

.
1 2
0
M
M
Z
-
+
+
NR

-

y
Z
*z*²
L
NR
Z
ey
1 2
.

2
L
10

10

0

0

0

0

0

0

~

0 ~

0 ~

4

.

1 2 (~

0~

0~

K - M

eyN + LM Z + M z2

E NR - LM - M

K + MR. I + MR. I

Z I y - M there I Z)

1

X

Z

y

y2

z2 y

y2 Z

L

10

10

10

I

0

yr2

0

I
M
0
-
+
+
zr2
0
E
+

1
Z
-
M

y
M
E
M
1
y
2

Z
Z
I

2
y

+

2
Z
I

y
2

5
2 LNR O
2
0
(3 0 0
E LN
LM -
1
M
y
Z
z2)
1
y

15
-
15
30
sym

6
2 LNR O
2
0
(3 0
0
E LN
LM -
1
M
Z
y
y2)
1
Z

-
15
15
30
7

~
~
0
0
~
0
0
X, 1
X

4k L - LI (3M -

-
-
1
M2) LI (3M 1 M

y
Z
Z
Z
y
y2)
30
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A2 9: v2 10

:

w2 11

:

x2

12: y2

13: z2 14

:

X, x2

2

NR O

M 0

M 0

0

0

0

0

y

y

NR O

E NR - LM + M

Z

y

y1

v

- 12

.

- *NR +*

-

ez

12

.

1

L

2

L

10

10

3

NR O

M 0

M 0

- *E NR 0 - LM 0 +*

0

0

Z

Z

NR O

M

y

Z

zI

W

- 12

.

NR +

-

ey

12

.

-

1

L

2

L

10

10

4

M 0

M 0

0

0

0

0

0
0
0
0
+
+
~
0 ~
0 ~
0
M
M
-
-
E NR
LM
M
K - MR. I - MR. I
- NO - y -
y
NR
Z
Z
-
- 12
.

- A1 (4,4)
E NR
LM
M
y
Z
z1
Z
y
y1
y1 Z
z1 y
ez
12
.
ey

x1
2
L
2
L
10
10
10
5

NR O
- E NR O - LM O - M O

- E LN O + LM O

2
0
y
Z
LM

y
Z
z2
- LN O

+
Z
y1
10
10
30
30
60

6
NR O
- E NR O + LM O + M O

E LN O
LM O

L2 M O

-

Z

y

y²

- LN O

-

-

Z

y -

y

z¹

10

10

30

30

60

7

-

~

0 ~

0 ~

E NR 0 - LM 0 + M 0

0

0

0

-

+

- K - MR. I - MR. I

~

E LN 0

LM 0

- E LN 0 - LM 0

- kL +

0 ~

LM I

Z

y

y²

E NR

LM

M

y

Z

z2

y2 Z

z2 y

-

+

y

Z

Z

y

y Z

X, X

10

10

10

30

30

30

L2 M 0

L2 M 0

0 ~

-

Z

+

y

L MR. I

Z

y

60

60

+

30

A3

9: v2 10:

11: x2

12: y2

13: z2 14

:

X, x2

w2

2

NR O

0

0

- E NR 0 + LM 0 - M 0

12

.

M

M

NR O

Z

y

y1

v

0

y

y

L

NR -

+12

.

2

ez

-

2

L

10

10

3

NR O

0

0

0

0

0

12

.

0

M

M

Z

Z

NR O

E NR + LM - M

W

- *NR* -

+

y

Z

z1

L

ey

12

.

2

2

L

10

10

4

~

~

0 ~

0 ~

0

0

M

- *E NR 0 + LM 0 + M 0*

- *E NR 0 - LM 0 - M 0*

- $K + MR. I + MR. I$

K

$Mz \sim$

$y \sim$

y

Z

$z1$

Z

y

$y1$

12

.

- 12

.

$I +$

I

$y1 Z$

$z1 y$

$x2$

L

$L y$

L

Z

10

10

10

$M 0$

0

$z2$

M

I

-

+

y2

yr2

0

I_{zr2}

0

+ E +

M - E +

M

2

2

y

2I

Z

Z

2I

y

Z

y

5

2 L NR O

2

0

(0 -3 0

E LN

L M 1

M

y

Z

z2)

-

y2

15

15

30

6

2 LNR O

0

0

E LN

LM - 3 0

M

2 Z

(y1

y2)

+

z2

15

15

30

7

sym

~

~

~

4k L - LI (0

M - 3

0

0

0

-

-

1

M2) LI (M

3

1

M

y

Z

Z

Z
y
y2)

X, x2
30

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6 Loadings

The various types of loading available for element POU_D_TG are:

Types or options

CHAR_MECA_FR1D1D

loading broken down by actual values

CHAR_MECA_FF1D1D

loading broken down by function

CHAR_MECA_PESA_R

loading due to gravity

CHAR_MECA_TEMP_R

“thermal” loading

CHAR_MECA_EPSI_R

loading by imposition of a deformation (of stratification type thermics)

The loadings are in the same way calculated that for the elements without warping [R3.08.01]. There is thus nothing in particular to element POU_D_TG. Other types of loading described in [R3.08.01] are not available for this element.

With regard to warping, it is possible to give boundary conditions making to intervene the degree of freedom GRX (what makes it possible to model constrained torsion: $GRX=0$), but by against, nothing is designed to affect a loading of the Bi-moment type, of which physical interpretation is difficult to establish.

Concerning connection between elements, the transmission of warping is an open question as the reference [bib1] announces it: the continuity of variable GRX from one element to another (of which warping depends directly) depends in fact on technology on the connection between various beams (welding in the axis, in which case warping can be transmitted completely, connection by bracket,...).

For an assembled structure such as a lattice, it seems more reasonable to suppose than torsion is obstructed, therefore that warping is null at the ends. To determine the influence of this assumption, one will be able to refer to the test SSSL102 (beam of corner section) of which modelings C and D use element POU_D_TG, with free torsion for modeling C, and torsion obstructed for modeling D [V3.01.102B].

It is noted that for the loading of inflection, the variation on displacement is weak (2.5%), but for a loading in torsion, one obtains for this section a side displacement not no one (discharge) from which the value differs notably according to the assumption taken:

5

 U $2.2 \cdot 10^{-}$

=

-

 Z

for free torsion and

5

 $U = 2.62 \cdot 10^{-}$ Z

for constrained torsion.

In the same way, rotation strongly varies:

4

79

,

3

 10^{-}

=
-
X
for free torsion and
4
= 39
,
6
10
X
for constrained torsion (GRX is null with
ends).

6.1 Loadings distributed : options CHAR_MECA_FR1D1D and CHAR_MECA_FF1D1D

The loadings are given under key word FORCE_POUTRE, that is to say by actual values in AFFE_CHAR_MECA (option CHAR_MECA_FR1D1D), is by functions in AFFE_CHAR_MECA_F (option CHAR_MECA_FF1D1D). The loading is given only by forces distributed, not by moments distributed.

The second associate member with the loading distributed with traction and compression is:

$f1$
1

with
 $F = F$

1 ext. ()
X
X 1 - dx
F
2
0

L
1
F = F
2 ext. () X
X dx
0
L

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For a loading constant or varying linearly, one obtains:

1
N
*N*²
F
=

L
X

+
,
1
3
6
1
N
*N*²
F
=

L
X

+
.
2
6
3

1
N
N

and

2 are the components of the axial loading as in points 1 and 2 coming from the data of the user replaced in the local reference mark.

If *T*
,
*T*_{1y}
, *T*_{1z}
and

*T*₂ are those of the shearing action, one a:
1
Z

7 *T*

3
y
*T*_{*y*}
*T*_{*y*}
*T*_{*y*}
F =
1
L
+
2
2
M = *L*
1 + 2
1
y

1
Z

20
20
20
30

3 *T*

7
y
Ty
Ty
Ty
F =

1
L
+
2
M
- 2
= *L*
1 + 2 ,
*y*²

*z*²

20
20
30
20

7 T

3

Z

tz

tz

tz

F =

1

L

+

2

,

M

- 2

= L

1 + 2 ,

1

Z

1

y

20

20

20 30

3 T

7

Z

tz

tz

tz

F =

1

L

+

2

,
2
 $M = L$
 $1 + 2 .$
 z^2

y^2

20
20
30 20

6.2 Loading of gravity: option “CHAR_MECA_PESA_R”

The force of gravity is given by the module of acceleration G and a vector normalized \mathbf{N} indicating direction of the loading.

Remarks (*simplifying assumption*):

The functions of form used for this calculation are those of the Euler-Bernoulli model.

The step is similar to that used for the forces distributed, with the proviso of transforming initially the vector loading due to gravity in the local reference mark with the element. One obtains in the reference mark

room of the beam:

S
 S
 L
 F
 $=$
 $S \mathbf{G}$
 F
 $= \mathbf{G}$
 $\mathbf{X} L$
 $+$
not

with

1,

X

X

X

I

dx

O

I

1

3

6

X

X

from where:

=1-

, =

S

S

1

2

L

L

F

= **G**

X *L*

+

not

with

2

2

X

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Inflection in the plan (Gxz):

7 S

3 S

F

= **G**

Z L

+

1

Z

20

20

S

S

M

= - **G**

2

Z L

+

1

y

20 30

3 S

7 S

F

= **G**

Z L

Z

+

2

20

20

S

S

M

= **G**

2

Z L

y

+

2

30

20

Inflection in the plan (Gxy):

7 S

3 S

F

= **G**

y L
+
1
y

20
20
S
S
M
= **G**
2

y L
+
1
Z

20 30
3 S
7 S
F
= **G**
y L
y
+
2

20
20
S
S
M
= - **G**
2

y L
Z
+
2

30 20

6.3 Thermal loading: option: "CHAR_MECA_TEMP_R"

To obtain this loading, it is necessary to calculate axial displacements induced by the difference of temperature T - reference

T
:
 U
= -
1
 $L (T - \text{reference}$
 T
)
 U
=

2
 $L (T - \text{reference}$
 T
)

(coefficient
:

thermics

dilation

of
 T
)

Then, one calculates simply the forces induced by \mathbf{F}
 $= \mathbf{K U}$

As \mathbf{K} is the matrix of local rigidity to the element, one must then carry out a change of locate to obtain the values of the components of the loading in the total reference mark.

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6.4

Loading by imposed deformation option “CHAR_MECA_EPSI_R”

One calculates as for elements POU_D_T the loading starting from a state of deformation (this option was developed to take into account the thermal stratification in pipings).

model takes into account only one work in traction and compression and pure inflection (not of effort edge, not of torque).

The deformation is given by the user using key word EPSI_INIT in AFFE_CHAR_MECA. In

U

y

being given

Z

,

and

on the beam, one obtains the second elementary member associated with it

X

X

X

loading:

U

:

1

node

with

F

$= E S$

X

,

1

1 X

M

$= E I$

y

y

y

,

1

1

X

M

$= E I$

Z

Z

Z

,

1

1

X

U

:

2

node

with

F

$= E S$

X

,

2
2 X

M
= E I
y
y
y
,
2
2
X

M
= E I
Z
Z
Z
2
2
X

7 Torque of the efforts - nodal Forces and reactions

7.1 Options available

The various options of postprocessing available for element POU_D_TG are:

Types or options

EFGE_ELNO_DEPL
torque of the efforts to the 2 nodes of each element
SIEF_ELGA_DEPL
field of efforts necessary to the calculation of the nodal forces (option
"FORC_NODA") and of the reactions (option "REAC_NODA").
FORC_NODA
nodal forces expressed in the total reference mark
REAC_NODA

nodal reactions

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7.2

The torque of the efforts

7.2.1 Generalized efforts, option: “EFGE_ELNO_DEPL”

One seeks to calculate with the two nodes of each element “beam” constituting the grid of studied structure, efforts exerted on the element “beam” by the remainder of the structure. Values are given in the local base of each element. By integrating the equilibrium equations, one obtains efforts in the local reference mark of the element:

R

=

E

E

E

K

U

+ **M**

u&

- **F**

LOC

LOC

LOC
LOC
LOC
LOC

where: R

=

-

,-

-

-

-

-

,-

,

LOC

(1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2

NR

V, V, M, M, M

M

NR, V, V, M, M, M,

Y

Z

T

Y

Z

M

Y

Z

T

Y

Z

)

E

K

elementary matrix of rigidity of the element beam,

*LOC**E***M**

elementary matrix of mass of the element beam,

*LOC**E***F**

vector of the efforts “distributed” on the element beam,

*LOC***U**

vector “degree of freedom” limited to the element beam,

*LOC***u&**

vector “acceleration” limited to the element beam.

LOC

One changes then the signs of the efforts to node 1.

Indeed, by taking for example the case of the traction and compression, one shows [R3.08.01] that them efforts in the element (option EFGE_ELNO_DEPL) are obtained by:

$$-NR(O)$$

$$UOfI$$
K

$$NR(L)$$

$$=$$

$$[]()$$

$$U(L) -$$

$$f2$$

7.2.2 Generalized efforts, option: “SIEF_ELGA_DEPL”

Option “SIEF_ELGA_DEPL” is established for reasons of compatibility with other options. It is used only for calculation of the nodal forces. It produces fields of efforts by elements.

It is calculated by:

E

R
= K
U

LOC
LOC
LOC

7.3 Calculation of the nodal forces and the reactions

7.3.1 Nodal forces, option: "FORC_NODA"

This option calculates a vector of nodal forces on all the structure, expressed in total reference mark.

It produces a field with the nodes in order CALC_NO by assembly of the terms elementary.

For this calculation, one uses the principle of virtual work and one writes [R5.03.01]:

T
F = Q

where

T
Q symbolically represents the matrix associated with the operator divergence. For an element, one writing agricultural work of virtual deformations:

(*T*
Q) *
U = (U)

(*
U)
 *

U
 kinematically acceptable
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For the elements of beam, one calculates simply the nodal forces by assembly of the forces nodal elementary calculated by option SIEF_ELGA_DEPL, which is expressed by:

$$\begin{aligned}
 &[\mathbf{F}] \\
 &= \mathbf{K} \\
 &\mathbf{U}
 \end{aligned}$$

LOC]***[LOC] [LOC]***

7.3.2 Nodal reactions, option: “REAC_NODA”

This option, called by CALC_NO, makes it possible to obtain the reactions \mathbf{R} with the supports, expressed in total reference mark, starting from the nodal forces \mathbf{F} by:

$$\begin{aligned}
 &\mathbf{R} = \mathbf{F} - \mathbf{F} \\
 &+ \mathbf{F}
 \end{aligned}$$

tank***iner*** **\mathbf{F}** **and \mathbf{F}**

being nodal forces respectively associated with the loadings given (specific and distributed) and with the efforts of inertia.

8 Bibliography

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Document: R3.08.05

A finite element of cable-pulley

Summary:

The pulleys play an important part in the structures of cables such as the air powerlines. It is thus useful to know to model them in one at the same time realistic and powerful way. The formulation is presented

finite element of a length of cable passing by a pulley: expressions of the internal forces and the matrix of

rigidity. The pulley can fixed or be supported by a flexible structure. Its position on the cable with balance is not

not known a priori: this position is that for which on both sides the tension of the cable is the same one.

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1 Notations

With

surface of the cross-section of the cable.

E

Young modulus.

F

vector of the forces intern element.

F

force intern with node N .

N

H

horizontal component of the tension [$\$An1$].

I

stamp unit of order 3.

3

K

stamp rigidity of the element.

L

current length of the element.

L

length at rest.

0

L

initial length.

I

L

1

$NR NR$

3

1

l_1

euclidian norm of l_1

L

2

$NR NR$

3

2

***l*₂**

euclidian norm of *l*₂.

NR

current tension of the cable constituting the element.

NR

initial tension (prevoltage).

I

S

arrow of a range of cable [*SAn*₁].

S

length of a range [*SAn*₁].

T

current temperature.

T

initial temperature.

I

U

vector-displacement of the nodes compared to the initial position.

U

displacement of node *N* compared to its initial position.

N

W

weight per unit of length.

X

vector-position of node *N* in initial configuration.

N

thermal dilation coefficient.

current deformation compared to the initial configuration.

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2 Introduction

One uses pulleys, during the construction of the air powerlines, for the operation of installation cables. The cable in the course of installation [Figure 2-a] is fixed at the one of the supports of stop of the canton, it

rest on pulleys placed at the bottom of the insulators of the supports of alignment and it is retained by one

force on the level of the second support of stop. While exploiting this force - or by moving its point of application - one adjusts the arrow of the one of the ranges, that which is subjected to constraints

of environment. Then one removes the pulleys and one fixes the cable at the insulators. The length of the cable

in the various ranges is then fixed and it determines the later behavior of the line under static stresses (wind, overload of white frost) and in dynamic mode (movement due to forces of Laplace created by the currents of short-circuit).

Appear 2-a: Pose of a cable in a canton with two ranges

The finite element of cable-pulley presented here makes it possible to model the operation of installation and thus to calculate,

in a natural way, the length of cable in the various ranges.

The idea of this finite element came to us some time after the conversation [bib1] and we have presented its formulation in [bib2].

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3

Assumptions and definition of a finite element of cable-pulley

N3

N1

N2

Appear 3-a: Length of cable passing by a pulley - reality

Let us take a length of cable *NR NR*

1

2 passer by by the *N3* pulley [Figure 3-a]. This pulley is not inevitably fixes and can, for example and as it is the case in the example of [§8], being gone up to the end of a cable.

N3

N1

N2

Appear 3-b: Length of cable passing by a pulley - modelled

The pulley is supposed to be specific [Figure 3-b]. One makes moreover following assumptions:

- The position of balance of *N3* is not known, but it is necessarily on section *N1 N2* deformed starting from its initial position.

In modeling of the lines, the horizontal movements are of low amplitude and this assumption is generally not restrictive;

- The two bits *N3 N1* and *N3 N2* are always rectilinear, like elements of cable 1st order.

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It rises from this assumption that:

L

= X + U - X - U

1

1

$$\begin{aligned}
 &1 \\
 &3 \\
 &3 \\
 &\text{éq 3-1} \\
 &L \\
 &= X + U - X - U
 \end{aligned}$$

2

2

2

3

3

 L T

1

=

1

 $L l_1$

éq 3-2

 L T

2

=

 $l_2 l_2$

The overall length of the two bits is:

-

in the current position, where the temperature is T :

$$L = l_1 + l_2;$$

-

in the initial position, indicated by index I , where the tension is N_i and temperature T_i :

 L

$$= (l_1) + (L$$

 I

;

 I 2) I

-

in a not forced position, indicated by index 0 , where the temperature is T_0 :

 L

$$= (L) + (L$$

 0

1

.

0

2) 0

• The pulley is without friction and thus the tension is the same one in the two bits.

It rises from this assumption that the deformation is also the same one and one takes for value of this one the measurement of the relative lengthening of the section compared to the initial state:

$$L - L$$

=

I

éq 3-3

l0

must remain small, so that section A is regarded as constant.

It will be noted that, within the framework of the finite element method, the linear loads do not prevent the tension from being constant of $N1$ with $N2$. These forces are indeed concentrated with the $N1$ nodes and $N2$ and on the axis of the $N3$ pulley.

• The relation of behavior is elastic:

$$NR = E [$$

$$WITH - (T - T)] + NR$$

I

I

éq 3-4

One calls finite element of cable-pulley, a length of cable $N1$ $N2$ satisfactory $N3$ with the preceding assumptions.

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Forces intern of a finite element of cable-pulley

Let us point out the following definition: one calls internal forces of a finite element of structure the forces

that it is necessary to exert in its nodes to maintain it in its current deformed configuration.

In the case of a finite element of cable-pulley, the internal forces result immediately from statics. One has indeed [Figure 4-a]:

NR

F

=

L

1

1

éq 4-1

1

L

NR

F

=

L

2

2

2

L

and, to ensure balance:

F

= - (**F** + **F**

3

1

2).

F1 and **F2** having even module, **F3** is directed according to the bisectrix of the angle ($NR NR NR$

1

3

2) .

$$\mathbf{F3} = - (\mathbf{F1} + \mathbf{F2})$$

$N3$

$N1$

$N2$

F1

$$\mathbf{F1} = \mathbf{F2}$$

F2

Appear 4-a: Forces intern of an element of cable-pulley

F3 is applied to the **axis** of the pulley.

The system **F** of the internal forces of the element is thus:

F

1

$$\mathbf{F} = \mathbf{F2}.$$

F

3

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Stamp rigidity

The matrix of rigidity **K** of the element is derived from Fréchet of **F** in the direction of displacement **U** of the nodes:

$$\mathbf{F} = \mathbf{K} \mathbf{U}.$$

K is calculated by the following traditional formula, used intensively in [bib3], [bib4] and [bib5]:

D

$$\mathbf{F} = \lim (\mathbf{F}_u + \mathbf{U}).$$

éq 5-1

$0 D$

The detail of calculations is given in [§An1] and the final expression of \mathbf{K} is as follows:

$$\begin{aligned}
&NR \\
&NR \\
&\mathbf{K}_{11} + \\
&\mathbf{I} \\
&\mathbf{K} \\
&3 \\
&12 \\
&- \mathbf{K}_{11} - \mathbf{K}_{12} - \\
&\mathbf{I}
\end{aligned}$$

$$\begin{aligned}
&3 \\
&1 \\
&L \\
&1 \\
&L
\end{aligned}$$

$$\begin{aligned}
&NR \\
&NR \\
&\mathbf{K} = \\
&\mathbf{K}_T \\
&\mathbf{K} \\
&T \\
&12 \\
&22 + \\
&\mathbf{I}_3 \\
&- \mathbf{K}_{22} - \mathbf{K}_{12} - \\
&\mathbf{I}_3
\end{aligned}$$

$$\begin{aligned}
&2 \\
&L \\
&2 \\
&L
\end{aligned}$$

NR
NR
1
1
T
T

- **K11** - **K12** -
I3 - **K22** - **K12** -

I
K
3
11 + **K22** + **K12** + **K12** +
+ *NI3*

1
L
2
L
11
2
L

éq 5-2
NR is given by [éq 3-4] and [éq 3-3];
EA NR 1

K
=
-
L IT
11
;
2 1 1
01
1
L 11
EA
K
=
L IT
12

1 2 ;

0

L 11 21

EA NR 1

K

=

-

L IT

22

.

2 2 2

01

2

L 21

K is **symmetrical**, because of the symmetry of **K11** and **K22** and of total symmetry per blocks.

But **K** depends on displacements on *NR*, *NR* and *NR*

1

2

3 via **L**, **L**

1

2 and *NR*: the element

finished cable-pulley is thus a **nonlinear** element.

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Stamp of mass

This matrix intervenes obviously only in the dynamic problems. The element of cable-pulley is not used in *Code_Aster* that for the quasi-static problems of installation of cables [§7].

Note:

One presents nevertheless in [bib2] the example of a dynamic problem comprising a cable pulley.

The matrix of mass of the element N1 N2 N3 is obtained by assembling the matrices of mass “coherent” of the elements of cable with two nodes N3 N1 and N3 N2 [bib6] and while adding specific mass of the pulley.

It should be noted that, during a dynamic analysis, this matrix of mass must be updated because the lengths l1 and l2 vary.

7 Introduction of the element of cable-pulley into**Code_Aster**

The element of cable-pulley is supported by a mesh SEG3.

In order AFFE_MODELE, under the key word factor AFFE, one must define them as follows arguments of the key words:

key word

GROUP_MA

PHENOMENON

MODELING

argument

group meshes

“MECHANICAL”

“CABLE-POULIE”

of cable-pulley

The constitutive material must be elastic.

In order AFFE_CARA_ELEM, the cable-pulleys are treated like cables.

As the element of cable-pulley is nonlinear [§5] and that, for the moment, it is used only in statics [§6], it is accessible only by operator STAT_NON_LINE. Under the key word factor COMP_ELAS, them

arguments of the key words are as follows:

key word

GROUP_MA

RELATION

DEFORMATION

argument

group meshes

“ELAS”

“GREEN”

of cable-pulley

Lastly, the force of gravity acting on the nodes NR and NR

1

2d' an element of cable-pulley [3-b]

is following because it depends on the length of the bits NR NR and NR NR

3

1

3

2. For a structure

comprising at least an element of cable-pulley, one must specify it in STAT_NON_LINE under key word factor EXCIT:

EXCIT:

(LOAD: charge of gravity TYPE_CHARGE: "SUIV")

One finds an example of application in test SSNL100 A [V6.02.100].

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8 Example

of application

This example is that of the installation of a cable with two ranges and imposed tension of adjustment. On [Figure 7-a], *O* is the anchoring of the cable on the support of stop of left. *P1* is a first pulley placed at the foot of the lifting chain *PC*

1

, fixed out of *C* at the support of alignment. *P2* is one

second pulley placed on the support of stop of right-hand side. *O*, *P1* and *P2* are, to simplify, located on horizontal.

Z

C

X

Q

P

NR

R

1

R

Q

P
 1
 2 2
 2
 O
 'P
 Q 1
 '
 '
 R'
 1
 Q
 R
 2
 2
 1

Appear 7-a: Balance of a cable with two ranges, subjected to a tension of adjustment given.

$OP1 = P1$ $P2 = 100$ m; $W = 30$ N/m; $E \times A = 5 \times 10^7$ NR; $NR p = 5.000$ NR

In initial position, the cable at rest, supposed in weightlessness, is right: line in feature of axis of [Figure 7-a]. For modeling in finite elements, this line is cut out in:

- ten elements of cable with two nodes between O and $Q1$;
- an element of cable-pulley $Q1 R1 P1$;
- nine elements of cable between $R1$ and $Q2$;
- an element of cable-pulley $Q2 R2 P2$.

One simultaneously subjects the cable to gravity and the tension of adjustment $NR p$ exerted in $R2$. position of balance (line in feature full with [Figure 7-a]) is reached in 11 iterations by the operator STAT_NON_LINE of *Code_Aster*. The arrows are 7.955 m and 7.867 m, respectively in range of left and that of right-hand side.

For an **inextensible** cable of a range of 100 m, weight linear 30 N/m and subjected to a tension in end of 5.000 NR, the theoretical arrow is 7.941 m [§An2].

While exploiting the tension of adjustment, one can adjust one of the two arrows to a value fixed at advance.

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9 Conclusion

The finite element of cable-pulley presented in this note, of a very simple mechanical formulation, has performances comparable with those of an element of ordinary cable. It is very convenient and even essential for a realistic modeling of the air powerlines. It should find other applications, in particular in Robotics.

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Appendix 1 Calculation of the matrix of rigidity

One shows here how applies the formula [éq 5-1] to the calculation of the first three lines of the matrix **K** [éq 5-2], those which relate to the **F1** force. The other lines are obtained is by permutation indices, is by summation of two preceding lines.

The first lines are thus obtained by calculating the derivative:

D

lim

1

F (U + U)

éq An1-1

0 D

and by putting in factor the vector **U**.

According to the relations [éq 4-1], [éq 3-4] and [éq 3-3], one a:

$L \mathbf{U} + \mathbf{U} + L \mathbf{U} + \mathbf{U}$

- L

I

L U + U

F

1

2

1

1 (**U + U**

)

(

)(

)

= EA

- ($T - Ti$)

(

)

+ Ni

0

L

1

 $L(\mathbf{U} + \mathbf{U})$

with, according to [éq 3-1]:

 $\mathbf{L}(\mathbf{U} + \mathbf{U})$ $) = \mathbf{X} + \mathbf{U} + \mathbf{U}$ $- \mathbf{X} - \mathbf{U} - \mathbf{U}$

1

1

1

1

3

3

3

and, according to [éq 3-2]:

 $L(\mathbf{U} + \mathbf{U})$ $)$ T

1

=

1

 $\mathbf{L}(\mathbf{U} + \mathbf{U})$ $) \parallel (\mathbf{U} + \mathbf{U})$ $)$.

Consequently:

 D

lim

1

 $\mathbf{L}(\mathbf{U} + \mathbf{U}) = 1$ $\mathbf{U} - \mathbf{U}$

3

 $0 D$ D

1

lim

 T

1

 $L(\mathbf{U} + \mathbf{U}) =$

1
L U 1
U - U

3
0 *D*
1
L (U)
()(
)

and by permutation of indices 1 and 2:

D
1
lim
T
2
L (U + U) =
12 U u2 - U

3
0 *D*
2
L (U)
()(
)

Finally:

D
1
1
lim
= -
T
L U
U - U
0
3
1
1
3
D
1
L (U +

U)
()(
)
1
L (U)
.

While carrying the preceding expressions in [éq An1-1] and by putting in factor the vector **U**, one easily obtains the first three lines of **K**.

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Appendix

2 Figure of balance of an inextensible cable weighing subjected to a tension given in end

Let us take a cable [An2-a Figure] whose end is fixed at the point *O* and of which the other end *P* is subjected to the tension *NR p*. *O* and *P* is on same horizontal and distant ones of *S*. *the weight linear is W*. The arrow *S*. is sought.

Z

O (0, 0)

X

P (S, 0)

NR p

S

Appear An2-a: Heavy cable in balance

One finds in [bib7], p 6, the following well-known formulas:

· figure of balance of the cable:

H

$W S$

$W S$

$Z(X) =$

\cosh

$- X$

\cosh

;

éq An2-1

W

H

2

-

$2 H$

· tension

:

$W S$

$NR(X) = H \cosh$

$- X.$

éq An2-2

H

2

H is the horizontal, constant tension along the cable since the external force distributed - the weight - is vertical.

H is calculated by [éq An2-2] written out of P :

$W S$

$2 NR p W S$

\cosh

-

=

·

0

$2 H$

$W S 2 H$

$W S$ is thus root of the transcendent equation:

$2 H$

$2 NR$

$\cosh X$

p

=

X .

$W S$

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This equation has two roots [An2-b Figure] if:

$2 NR p > 0p,$

$W S$

with:

p

= $\sinh X$

0

0

and:

X

= $\cotanh X$

X

0

0

$0 > 0.$

$2 Np$

$W S$

1

$\cosh X$

$W S$

$2 H$

X

W S

Appear An2-b: Calculation of 2 A.m.

The smallest root, which corresponds to the greatest tension of the cable, is only useful. The other root corresponds to a sag of the considerable cable, of about size of its range.

W S being calculated, the arrow results from [éq An2-1]:

2 H

S 2 H

W S

S =

cosh

-

1 .

2 W S

2 H

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R3.08.06 document

***Finite elements of right pipe and curve
with ovalization, swelling and warping
in elastoplasticity***

Summary:

This document presents the modeling of a finite element of pipe usable in calculations of pipings in elasticity or in plasticity. The pipes, curves or rights, can be relatively thick (thickness report/ratio on ray of the transverse section up to 0.2) and are subjected to various combined loadings - internal pressure, cross-bendings and anti-plane, torsion, extension - and can have a nonlinear behavior.

This linear element combines at the same time properties of hulls and beams. The average fibre of the pipe comprise like a beam and the surface of the pipe like a hull. The element carried out is an element of right pipe or curve in small rotations and deformations, with an elastoplastic behavior in plane constraints.

Three modelings, corresponding to three various types of elements, are available:

- TUYAU_3M, which takes into account 3 modes of Fourier to the maximum, and which can rest on meshes with 3 nodes or 4 nodes.***
- TUYAU_6M, which takes as a count up to 6 modes of Fourier, and is pressed on meshes with 3 nodes.***

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1 Introduction

There is an important bibliography on the modeling of pipings and many elements stop pipes right and bent are available in the great codes of finite elements. syntheses were already realized [bib1], [bib5], [bib6], in the past that one supplemented in incorporating the last developments known in the field [bib11]. Important effects with to take into account are swelling due to the internal pressure and the ovalization of the transverse sections by combined inflections plane and anti-plane. One places oneself on the assumption of small rotations and deformations within the framework of this document.

It is about a linear element with 3 or 4 nodes, of curved or right beam type with local plasticity taking into account ovalization, warping and swelling. The kinematics of beam is enriched by a kinematics of hull for the description of the behavior of the transverse sections. This kinematics is discretized in M modes of Fourier of which the number M must at the same time be sufficient to obtain good results in plasticity and not too large to limit the computing time. literature encourages us to use $M=6$ [bib9], [bib13] in plasticity. In elasticity, for thick pipes, one can be satisfied with $M=2$ or 3.

2 Various theories of hulls and beams for finite elements of right or bent pipes

One presents in this chapter the elements of kinematics in three-dimensional curvilinear geometry, as their restrictions within the framework of the models of beam and hull. Indeed, to build the finite element of piping enriched which answers the schedule of conditions defined in introduction, one exploits

a technique of decomposition of three-dimensional kinematics. The kinematics of hull y bring the description of ovalization, swelling and warping, while kinematics of beam described there the movement general of the line of piping.

The various theories of hulls and beams used for each element translate them assumptions chosen a priori on the type of deformations and behaviors.

2.1

The pipe in theory of beam

2.1.1 Case of a bent pipe

A first approaches relatively simple come down to consider the elbow represented below like a beam digs circular section. The beam is obtained by rotation of angle of circular section around OZ. A point of the beam is located by its distance R compared to the axis beam and by the two angles, where is the longitudinal angle with OY indicated above and the trigonometrical angle with OZ measured on the circular section.

Z

Z

Z

X

2

2

.

X

ux

.

()

X

E

uz

R

E

U.

y

.

1

.
l
er
Z
.
Z
y
Z
Z
Y
R
y
Y
R
y

X

X
O
O

*Appear 2.1.1-a: Geometry and kinematics of the elbow in theory of beam
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In the curvilinear frame of reference (R), the relations between displacements U of points of the elbow of position R = OM = - E

R (0 y) + Re (R,) and deformations of Green-Lagrange are given by the following tensor in the natural base (R,):

$$\begin{aligned}
 & (0 \\
 & R + U) (0 \\
 & R + U) 0 \\
 & R 0 \\
 & R \\
 & 2 F = \\
 & \cdot \\
 & - \\
 & \cdot \\
 & , (,) \{R, \}.
 \end{aligned}$$

The vectors units in the directions (R,) are:

$$\begin{aligned}
 & 0 \\
 & R \\
 & 1 0 \\
 & R \\
 & 1 0 \\
 & R \\
 & 0 \\
 & R 0 \\
 & R \\
 & 0 \\
 & R R \\
 & E \\
 & 0 \\
 & R = \\
 & E \\
 & , \\
 & = \\
 & , E =
 \end{aligned}$$

where A =

.

and B =

.

.

R

With

B

If one expresses the position of a point of the elbow in the local toric orthonormée base (E E R, E)

by (yr,

y,

y) one has the following relations:

0

R

0

R

0

R

er =

E

, =

, E =

.

yr

y

y

The form of the tensor of the deformations of Green-Lagrange in this base is then:

(0

R + U) (0

R + U) 0

R

0

R
2 =
.
-
.
.
y
y
y y

Relations of passage between the expression of the deformations of Green-Lagrange in the system of co-ordinates curvilinear and in the local toric base previously definite are:

= F
rr
rr
F
= A2
F
= B2
F
= AB
F R

R = A
F R

R = B
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The use of this base is particularly interesting because the relations of behavior in base toric orthonormée are simple of use. For elbow Ci above, if it is considered that them deformations remain small, one obtains then [bib4] after linearization of the deformations of Green-Lagrange:

U
R
rr = R
1 U

U

With

ur
=
+
+
With
AB R
1 U

ur
=
+
B
R

1 U

1 U

U With

2 = =
+
-

B
With
AB
1 U

U
U
R

2 R
= R
=
-
+
With
R
R

1 U

U
U

R

2 R
= R
=
-
+
B
R
R

with:
R + R sin
With = R + R sin, B = R, R =
, R = R.

sin

The expressions of the deformations established above are written then:

U

R

rr = R

1

U

=

(

+ *U*

cos + U sin)

R + R

R

sin

1 U

= (

+ *U*

)

R

R

1

U

1 U

2 = =

(

- *U*

cos

) +

R + R sin

R
1
U

U
R

2 =
=
(
- U

R
R
sin) +
R + R sin
R

1 U

U

R

2 = = (
- U

R
R
) +
R

R

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Ur displacement,

U,

U of a point of the elbow in the toric base associated the transverse section

of observation can easily express itself according to displacements and rotations associated with center transverse section. Indeed, if 1 is noted

U, u2, u3 displacement in the curvilinear base

local (O (), X (), y (), Z ()) associated the transverse section as indicated on [Figure 2.1.1-a]

there are the following, valid relations within the framework of the kinematics of the beams of Timoshenko

[R3.08.01]:

U (R,

1

),) = ux

() + Z

() R sin - y

() R cos

U (R,

2

),) = U y

() + X

() R cos

U (R,

3

),) = uz

() - X

() R sin

where ux, U y, uz are the displacement of translation of the section and X, y, Z the rotation of its center

O. The expression of the components of displacement in the local toric orthonormée base

(E E

R, E) are obtained by change of reference mark:

$$U(R, y) = U(R, X) + (Z) R \sin - (y) R \cos$$

$$U(R, y) = U(R, X) \sin - U(R, y) \cos = U(Z) \sin - U(y) \cos - (X) 3 2 R U(R, R y) = [- U(R, y) \cos + U(R, X) \sin] = [- U(Z) \cos + U(y) \sin] 3 2$$

The introduction of this field of displacement into the expression of the linearized deformations us allows to obtain the expression of the three-dimensional deformations associated the kinematics of beam:

$$rr = 0 1 = (U X - U y - R cos X + R sin Z - R cos y) R + R sin$$

,
 ,
 ,
 = 0
 1
 2 =
 (- U cos
 X
 - U
 cos
 y
 + U
 sin
 Z
 - R X + R cos2
 y
 - R sin
 Z
 cos)
 R + R sin
 ,
 ,
 ,

+ (cos
 Z
 + sin
 y
)
 1
 2 R =
 (- U sin
 X
 - U
 sin
 y
 - U
 cos
 Z
 + R sin
 y
 cos - R sin 2

Z
)
R + R sin
,
,
+ (sin
Z
- cos
y
)
2 R = 0

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2.1.2 Case of the right pipe

The expressions of the deformations established above also apply to the case of the right pipe, where one replace by S where S is the curvilinear X-coordinate along average fibre of the pipe, with:

With =,
1 B = R 1
,/R =, 0 R = R.

The expressions given for the elbow are written then for the right pipe:

U
R
rr = R
U
X
xx = X
I U

= (
+ *U*
)
R
R
U

I U
X
2 X = X =
+
X

R
U
U
R
X

2 X-ray = X-ray =
+
X

R
I U
U

R

2 = = (
- U

R

R

) +

R

R

2

Z

Z

.

U

U R

X

With

Z

X

U

X

ux=u1

U 2

O

U

.

3

X

Z

U

1

U

y

y

y

y

Z

Z

Y
Z
X
Transverse section: sight of 1 towards 2
O

Appear 2.1.2-a: Geometry and kinematics of a right pipe in theory of beam
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As previously, ur displacement, U U
X, of a point of the pipe in the associated toric base
with the transverse section of observation can easily express itself according to displacements and
rotations associated with the center with the transverse section. Indeed, if 1 is noted
U, u2, u3 displacement
in the local curvilinear base (O, X, y, Z) associated the transverse section as indicated on the figure
below there are the following relations:

U (R, X,
I
) = U (X)
X
+ (X) R sin
Z
- (X) R cos
y

$$U(R, X, y) = U(X) + (X) R \cos X$$

$$U(R, X, Z) = U(X) - (X) R \sin X$$

and:

$$U(R, X, y) = U(R, X, Z) = U(X) + (X) R \sin Z - (X) R \cos I y$$

$$U(R, X, y) = U(R, X, Z) \sin - U(R, X, Z) \cos = U(X) \sin Z - U(X) \cos y - (X) R \cos 3 2 R X$$

$$U(R, X, R) = [- U(R, X, Z) \cos + U(R, X, Z) \sin] = [$$

$$\begin{aligned}
 & - U(X) \cos \\
 & Z \\
 & + U(X) \sin \\
 & y \\
 &] \\
 & 3 \\
 & 2
 \end{aligned}$$

The introduction of this field of displacement into the expression of the deformations given below us allows to obtain the expression of the deformations associated with kinematics with beam:

$$\begin{aligned}
 rr &= 0 \\
 xx &= ux X + R \sin \\
 Z X \\
 & - R \cos \\
 & , \\
 & , \\
 & y, X \\
 & = 0 \\
 & 2 \\
 & X \\
 & = - R X X + (y + U) \sin \\
 Z X \\
 & + (Z - U) \cos \\
 & , \\
 & , \\
 & y, X \\
 2 X\text{-ray} &= (Z - U) \sin \\
 y X \\
 & - (y + U) \cos \\
 & , \\
 & Z, X
 \end{aligned}$$

$$2 R = 0$$

2.1.3 Remarks

The fact that rr , and R are simultaneously null shows that the kinematics of beam cannot to represent the deformations of the transverse sections to average fibre of the pipe. Indeed, them transverse sections are actuated by a rigid movement of body, which prohibits to model it warping, swelling and ovalization.

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2.2

The pipe in linearized theory of hull

2.2.1 Case

general

The bent pipe is regarded as a thin hull of revolution (portion of torus). Surface average is obtained by rotation of angle of a circle of radius h whose center is at a distance R of the axis of revolution OZ . One indicates by H the thickness of the elbow. One imposes on this thickness

to remain constant like with the section of the elbow being perfectly circular. A point on surface average is characterized by the two angles, and its position - $H/2 + h/2$ compared to surface average, where is the longitudinal, variable angle between 0 and, and the angle measured on transverse section.

v

H

W

R

O

U : Axial displacement of average surface

U
y
v: Orthoradial displacement of average surface

E
R
Z
W: Radial displacement of average surface
er
: Rotation of average surface compared to E

E
: Rotation of average surface compared to E

O
Z
E
X
R: Radius of curvature
R: Ray of the cross section
H: Thickness of the elbow
: With
ngle longitudinal
:
Angle of cross section
X

Appear 2.2.1-a: Geometry and kinematics of the elbow in theory of hull

One places oneself first of all within the framework of the linearized theory of the hulls with shearing transverse such as it was described for example in Washizu [bib14]. This choice had already was made for the linear elements of hulls [R3.07.01]. It limits our study to the framework of small deformations. Moreover, great rotations of average surface are not taken into account. Displacements and rotations are thus defined compared to the initial geometry of the elbow. If them displacements of the points of average surface in the three directions axial, orthoradiale and radial is noted U, v and W those of any point of the elbow are written in the following way:

U = U (
,) + (

,)
 $U = v ($

,) - (
,)
 $U = ($

W,)

*where and are rotations compared to the vectors
E and
E respectively.*

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The deformations in any point are thus given by [bib14]:

E
+
 $= I + R$
 $E +$
 $= I + R$
2
 E
+
2

$2 = = ($

$1+ /$
 $R) (1+ /$
 $R)$
 2
 E

$2 = = (1+ / R)$
 $2nd$

$2 = = (1+ / R)$

with:
 $R + has sin$
 $With = R + has sin, B = has, R =$
 $, R =.$
 has
 sin

where
 E, E and
 E are the membrane deformations of average surface, them
deformations of inflection of average surface and
 E, E transverse distortions.

deformations of average surface are connected to displacements of average surface in
replacing the field of displacement of the preceding paragraph by that given above. One finds
then:

$1 U$
 $v A$
 W

E
 $=$
 $+$
 $+$
With
 AB

R
 $I v$
 W
 $E =$

+

B

R

1 U

1 v

U With

2

E

=

+

-

B

With

AB

1

With

=

-

With

AB

1

= - **B**

1

To 1

1 1 U

1

1 v

U

With

2 =

-

-

+ [

+

(

-

)]

B

AB A

R B

R

With

AB

1 W

U

2

E

= +

-

With

R

1 W

v

2nd = - +

-

B

R

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That is to say still:

I
U

E =
(
+ v cos + w sin)

R + has sin
I
v

E = (
+))
W
has
I
v

I U

=
(
- U cos) +
R + has sin
has
I

=
(

- cos

)
R + has sin
I
= -

has
I
I

sin
I U

I
 I
 v

 $=$
 $-$
 $($
 $+ \cos$
 $) + [$
 $+$
 $($
 $- U \cos)]$
has
 $R + \sin$
 $R + \sin \text{ has } R + \sin \text{ has}$
 I
 W

 $= +$
 $($
 $- U \sin)$
 $R + \sin$
 I
 W

 $= - + (- v)$
has

In this theory there are thus five unknown factors; 3 displacements U, v and W like two rotations

, If the assumption of Coils-Kirchhoff is applied (thin tube) shearings transverses are null and there are nothing any more but 3 displacements U, v and W since:

I
 W

 $= -$
 $($
 $- U \sin)$
 $R + \sin$

I

W

= (- v)

has

2.2.2 Case of the right pipe

If one applies these equations to the case of the right pipe with:

With =,

1 B = has 1

,/R =,

0 R =.

has

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One finds the more usual expression for this kind of geometry:

U

E_{xx} = X

I

v

E = (

+)

W

has

v

I U

2E_x =

+

X

has

xx =

X

I X

= - has

I
 X
 $I v$
 $2 =$
 $-$
 $+ [$
 $]$
 X
has
 X

X has

W

$2Ex = + X$
 I
 W

$2nd = - + ($
 $- v)$
 X
has

In this theory there are thus five unknown factors; 3 displacements U, v and W like two rotations,
 X

. If the assumption of Coils-Kirchhoff is applied (thin tube) shearings transverses are null and there are nothing any more but 3 displacements U, v and W since:

W

$= - X$

I
 W

$= ($
 $v)$
 X
 $-$
has

2.2.3 Notice

One can directly introduce the kinematics of hull into the field of deformation 3D. In it case one a:

=

E

+

= E +

2 = = 2nd +

2

2 = = 2nd

2 = = 2nd

where expressions

E, E and

E for the membrane deformations, for

deformations of inflection and

E, E for the transverse distortions are given by the expression

following in the case general:

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1

U

E =

(
+ v cos + w sin)

R + R sin

I

v

E = (

+))

W

R

I U

I

v

2nd =

+

(
- U cos)

R R + R sin

I

=

(

- cos

)

R + R sin

I

= -

R

I

I

2 =

-

(

+ cos

)
R
R + R sin
R + has sin
I
W

2nd =
+
(
- U sin)
R + R sin
R + R sin
I W has

2nd = - + (
- v)
R
R

It is noticed that to order 1 in the two ways of proceeding give identical results. It is the definition of the deformation of membrane or inflection which changes. In the first case it is independent of the position in the thickness and is calculated for the average radius of the section transverse of the pipe, whereas it depends on it in the case on the approach 3D. The term between hook in the expression of [§2.2.1]. represent a coupling between the inflection and the membrane which appears when one expresses $R + R \sin$ and R according to $R + has \sin$ and A . In the continuation of our analysis we will use this expression 3D degenerated of the kinematics of hull. If moreover we use the assumption of Coils-Kirchhoff for transverse shearings,

E
=
E
= 0 one find well the following expressions of rotations:

I
W

= -
(

- U sin)
R + has sin

1
W

= (- v)
has

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and:
1
1
2
W U
cos

W
=
[-
(
-
sin

) -
(

- v)]
R + R sin
R + has sin
2

has

1
2
W

v
= -
(
-
)

2
rear

W
cos
cos has
2
= (
- U sin

)
+
]

(R + R sin) (R + has sin) R (R + sin has) 2

2
W

1
1
-

[
+
]

(R + R sin) R has (R + has sin)

v
1

U
 I
 $+$
 $+$ (
 $\sin + U \cos$)
 $has (R + R \sin)$

$R (R + has \sin)$

expressions for
 $E,$
 E
and

E remaining unchanged.

One can easily extend this remark to the case of the right pipe.

2.3

Analyze right and bent pipes

In conclusion of the two preceding analyses one can model the pipe like an element of beam whose section is a thin hull. This interpretation is made in the majority of the codes ([bib2], [bib8], [bib9], [bib10], [bib12], etc...). In the absence of warping of the transverse sections (i.e the transverse sections remain plane) the axial displacement of beam gives the new position transverse section and displacements of ovalization (it is enough to take then $u=0$ in mean equations of hulls) make it possible to know how this one becomes deformed. Total deflection is obtained like superposition of the deformations of beam and the deformations of ovalization. field of displacement which one represents on the figure below writes:

p
 S
 $U = U + U.$

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In the first field of displacement the image of the transverse section is a transverse section identical obtained by translation and rotation of the first. In the second field of displacement, transverse section is deformed.

M

M

inflection-torsion of a right beam

In theory of the beams-Euler

In theory of the hulls

U

v

W

Cross

Transverse section

Cross

Transverse section

warping

ovalization

Appear 2.3-a: Decomposition of displacement in fields of beam and hull

Modeling finite element must thus give an account of two different mechanical answers: that beam and that of the hull for ovalization, swelling and warping. These three last modelings utilize degrees of freedom which are not nodal (decomposition in Fourier series for example).

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Finite elements of right pipe and curve***Date:******12/12/03******Author (S):******P. MASSIN, J.M. PROIX, A. Key BEN HAJ YEDDER******:******R3.08.06-B Page******: 17/54******3******Mixed elements hull-beam for the right pipes and curves******3.1 Kinematics***

One breaks up the field of displacement into a macroscopic part of “beam” and a part additional local of “hull”. V is the useful space of the fields of displacements three-dimensional definite on an unspecified section of pipe.

For the beam part, as in [R3.03.03], one introduces space T of the fields associated with a torque (defined by two vectors):

$$T = \{v \in V / (T,)$$

that***such***

$$v(M) = T + GM\}$$

For the fields of displacement of T , T is the translation of the section (or the point G), infinitesimal rotation and fields v are displacements preserving the section S plane and not deformation there (One uses still the assumptions of NAVIER-BERNOULLI).

T is a vectorial subspace of finished size equalizes to 6. It has additional orthogonal for the scalar product on V :

$$T = v \cdot V / v \cdot W$$

. =

$$0 \cdot W$$

T.

S

Any field U of V breaks up then in a single way all in all of an element of T and one

element of T :

p
 $S p$
 S

$$U = U + U U T U$$

,

T.

One postulates then for displacements of surface of the pipe defined in [§2.2] the decomposition in following Fourier series who check the preceding principle of orthogonality with displacements of beam until order 3 in the thickness of the pipe:

$$M$$

$$U(X) = I$$

$$\sum_{m=2}^{\infty} u_m(X) \cos m$$

$$M$$

$$+ O$$

$$\sum_{m=2}^{\infty} u_m(X) \sin m$$

$$M$$

$$v(X) = I$$

$$\sum_{n=1}^{\infty} v_n(X) \sin n$$

$$m=2$$

$$M$$

$$\dots - O$$

$$\sum_{n=1}^{\infty} v_n(X) \cos n$$

$$m=2$$

$$($$

$$W(X) = w_o(X)$$

N
(uniform radial expansion)

M
..... + I
 $wnm(X) \cos m$
 $m=1$

M
..... + O
 $wnm(X) \sin m$
 $m=1$

where X is the curvilinear X -coordinate along the elbow or of the right pipe, indifferently, and M the number of modes of Fourier. Rotations (X , X) and (X) result from $U(X)$, $v(X)$ and ($W X$) by the relations of Coils-Kirchhoff [§2.2.1].
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Note:
One can note that in the decomposition of $v(X)$ and ($W X$) terms in cos and sin

are not completely independent because of orthogonality with displacements of beam. This makes it possible moreover to avoid the movements of rigid body, because if

I

O

I

O

vn, v

I

n1 and

N

W 1, N

W 1 are independent, one can find a solution nonnull giving

null deformations. In addition in the expression of U (X,) one notes the absence of the terms in cos and sin already present in the beam part.

If one neglects the variation of metric with the thickness of the pipe the conditions of orthogonality rigorous between displacements of beam and those of the surface of the pipe are satisfied.

In the contrary case, to satisfy this condition rigorously one would need one development in Fourier series of rotations (X,

X

) and (,

X) starting with

order 2. This is incompatible with the assumptions of Love_Kirchhoff for these rotations.

3.2

Law of behavior

The behavior of the new element is a behavior 3D in plane constraints, because it total behavior of the structure is that of a thin hull. It results from it that $\epsilon = 0$ and the law from behavior is written in a general way in the following way:

xx

pxx + S

xx xx

p

S

+

P
S

X =
C
+

X

X =

C
X
P
S

+

R

X
P
S

xr +

X

X

In our case one will neglect transverse shearings for the hull part of our field of displacement. It thus results from it that S

***X = S
= 0. As in addition [§2.1.2] it was shown that***

***p
= 0***

=

***R
it results from it that***

0

. For an elastic behavior one has as follows:

1

0

0

1

0

0

xx

xx

1

0

0

1

0

0

E

1 - ν

E
I - v

=
0 0
0

and C =
0 0
0

.
2

X I - v
2

X
1
2
v
2

-

I - v

I - v

X
0 0
0

X

0 0

0

2

2

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3.3

Work of deformation

The general expression of the work of deformation 3D for the element of elbow with the type of above mentioned behavior is worth:

L 2 H/2

W

=

(

+++ FD

def

xx xx

X

X

X

X)

$0 \leq D \leq H/2$

where L is the curvilinear X-coordinate which is worth $L = (R + R \sin \theta)$ for an elbow where θ is the traversed angle to describe the elbow. In the case of an elbow, one has thus $FD = (R + R \sin \theta) D$

rd
D and for one
right pipe $FD =$

$dxrd$
D where x is the position in the thickness of the elbow which varies between $-h/2$ and $+h/2$. In the continuation, in order to reduce the notations, one will employ the second expression.

3.4 Energy interns elastic elbow

In the case of an elastic behavior, energy interns elastic elbow is expressed way following:

$L \geq H/2$
 E
 $int =$
 $(2 + 2) + G (2 + 2) FD$

xx

xx
X
X

2

2

-

0 0 - H

1

/2

This energy can be broken up into part of energy of beam, part of energy for H/2 L 2

surface pipe and terms of coupling of the type p

. S

xx xx FD.

- H/2 0 0

3.5

Work of the forces and couples external

With the decomposition of displacements stated at the head of paragraph, the work of the forces being exerted on the pipe expresses itself in the following way:

L +h/2

2

L

2

+h/2

2

P

S

P

S

P

S

W

=

F

ext.

v. (U + U FD

)

+ S

F. (U + U) (± H has/)

2D dx

+ C

F. (U + U) rd D

=

0 - H/2 0

0 0

- H/2 0

L + h/2

2

L

2

+h/2

2

p

p

p

F

v U

.

FD + S

F U

.

(± H has/)

2D dx

+ C

F U

.

rd D

+

0 - H/2 0

0 0

- H/2 0

L + h/2

2

L

2

+h/2

2

S

S

S

p

S

F

v U

.

FD + S

F U

.

$(\pm H \text{ has/})$

$2D dx$

$+ C$

$F U$

.

$rd D$

$= Wext + Wext$

$0 - H/2 0$

$0 0$

$- H/2 0$

by simple linear decomposition, where v

F, S

F, C

F are the voluminal, surface efforts and of contour

being exerted on the pipe, respectively. One determines as follows:

$L + h/2$

2

L

2

$+h/2$

2

$W p$

p

$=$

$F U$

.

FD

p

$+$

$F U$

.

$(\pm H \text{ has}$

p

$ext.$

v

S

$/)$

2

$D dx + C$

$F U$

.

rd

D

0 - H/2 0

0 0

- H/2 0

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and:

L +h/2

2

L

2

+h/2

2

W S

S

=

F U

.

FD

S

+

F U

.

(± *H has*
S
ext.
v
S
/)
2
D dx + C
F U
.

rd
D
0 - H/2 0
0 0
- H/2 0

The work of the external forces can thus be separate in two contributions distinct from same forces, on the kinematics of beam and its additional.

3.6
Principle of virtual work

It is written in the following way:

p
S
ext.
W
= ext.
W
+ ext.
W
= int
W with:

L 2 H/2
W
=
(+
+
+) FD
int
xx xx

X
X
X
X

0 0 - H/2

3.6.1 External efforts part and couples for the beam part

The discretization of the principle of virtual work for the external efforts gives:

$$\begin{aligned}
 &L \\
 &W \\
 &p_{ext} = (F \\
 &X U \\
 &X + F y U \\
 &y + F Z U \\
 &Z + m_{xx} + m_y y + m_{zz} dx \\
 &)+ \\
 &0 \\
 &[X U \\
 &X + y U \\
 &y + z U \\
 &Z + \mu_{xx} + \mu_{yy} + \mu_{zz}] 0, L
 \end{aligned}$$

F X, F y, F Z: linear forces acting according to X, y and Z passing by the centre of gravity of transverse sections:

+h/2
2

2
f_i =
F.
O C I

rd
D + F .e (has
S
I
± H/)
2
D

where e_x, e_y, e_z are the vectors of the base

- $H/2$ 0

0

curvilinear local.

M_x, m_y, m_z : linear couples acting around axes X, y and Z :

+ $h/2$

2

2

semi =

($r \times F$.)

v I.E.(internal excitation)

rd

$D + (r \times F.)E$ (has

v

I

$\pm H/$

2

D

where e_x, e_y, e_z are the vectors of the base

- $H/2$ 0

0

curvilinear local.

X, y, Z : concentrated forces acting according to X, y and Z passing by the centre of gravity of transverse sections:

+ $h/2$

2

$I =$

F .

E.C.I.

rd

D

where e_x, e_y, e_z are the vectors of the local curvilinear base.

- $H/2$ 0

μ_x, μ_y, μ_z : moments concentrated around axes X, y and Z :

+ $h/2$ 2

$\mu_i = (r \times F.) rd D$

C I.E.(internal excitation)

where e_x, e_y, e_z are the vectors of the local curvilinear base.

- H/2 0

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3.6.2 External efforts part and couples for the hull part

It is supposed that the external efforts applied to the elbow are independent the thickness of bend:

L

2

W S =

(F

ext.

xu + Fv + Frw+ MX + M

X

)

D

D

0 0

2

+ [xu + v + rw + +] 0 L,

D

0

where:

$F_x, F,$

R

F : surface forces acting according to X , and R :

$+h/2$

$F =$

$F \cdot e \, rd$

v

I

$+ F \cdot e$ (H has/)

2

I

S

I

\pm

where e_x, E, E

R are the vectors of the local toric base.

$- H/2$

$MR. M$

$X,$

$:$ surface couples acting around X and:

$+h/2$

$M =$

$(E \times F) \cdot e \, rd$

R

v

I

$+ (\pm H/2nd \times F) \cdot e$ (H has/)

2

I

R

S

I

\pm

where e_x, E, E

R are the vectors of the base

$- H/2$

toric local.

$X,$

R : linear forces acting according to X , and R :

+h/2

$I = F \cdot rd$

$C I.E.(internal\ excitation)$

where ex, E, E

R are the vectors of the local toric base.

- H/2

**,
 X : linear couples acting around X and:**

+h/2

$I = (E \times F) \cdot e rd$

R

C

I

where ex, E, E

R are the vectors of the local toric base.

- H/2

Note:

When the external forces applied are independent of external work on kinematics of hull is null except that of the compressive forces corresponding to the forces according to er . It is also noticed that the expressions of the moments linear and concentrated by report/ratio with R are null. One finds although exerted moment ago perpendicular to the plan of the hull.

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3.7 Efforts generalized

If S is the surface of the transverse section S of the pipe, one poses:

$$NR = dS$$

xx : normal effort in the centre of gravity of the transverse section.

$$T = dS = -(\sin \alpha + \cos \alpha) dy$$

xy

X
 X

and

$$T = dS = (\sin \alpha - \cos \alpha) dz$$

Z
 xz

X
 X
sharp efforts following y and Z .

$$M = (y - Z) dS = -dS$$

X
 xz
 xy

X : torque around X .

$$M = Z dS = -R \cos \alpha dS$$

y
 xx

xx
: bending moment around Y .

S

S

$$M = - y dS = R \sin dS$$

Z

xx

xx

: bending moment around Z.

S

S

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4

Numerical discretization of the variational formulations

4.1 Discretization of the fields of displacement and deformation for the beam part

In a point of average fibre, the field of displacement of beam is in the curvilinear reference mark

U

X

U_y

room defined in [§2.1]:

P

U

$U = Z$

X

y

Z

This field can be discretized in the following way:

NR

NR

U = H

K

K

K

K () K

U

[X xk + K

U there y K + K

uz zk] and = HK () [X xk + y yk + Z zk]

K = 1

K = 1

It should be noted that the nodal values are given in the local reference marks attached to the nodes and that U and must be expressed in the local reference mark associated the current point.

4.1.1 Beam curve

One obtains then:

U

U (X .x) U (y .x)

(X .x) (y .x)

X

kx K + K

X

K

X

K

+ K

NR

y

K

NR

y
K

U

y =
H K

K
X (xk .y) + K

(y .y)
y = H K () K

ux (xk .y) + K
U (y .y)

y
K

and
()

y
K

K =

U
1

K
U Z

K = 1
K
Z

Z

Z K

Z

Z

K

According to the kinematics of beam presented higher to [§2.1]:

$$r r = 0$$

1

=

$$(U X - U y - R \cos$$

X

$$+ R \sin$$

Z

$$- R \cos$$

y

)

$$R + R \sin$$

,

,

,

$$= 0$$

1

$$2 =$$

$$(- U \cos$$

X

$$- U$$

$$\cos$$

y

$$+ U$$

$$\sin$$

Z

$$- R X + R \cos^2$$

y

$$- R \sin$$

Z

$$\cos)$$

$$R + R \sin$$

,

,
 ,
 + (cos
 Z
 + sin
 y
)
 1
 2 R =
 (- U sin
 X
 - U
 sin
 y
 - U
 cos
 Z
 + R sin
 y
 cos - R sin 2
 Z
)
 R + R sin

,
 ,
 + (sin
 Z
 - cos
 y
)
 2 R = 0

Knowing that X, = - yety, = X with moreover

.
 X xk =.
 y y K = cos (- K) = Ck and.
 y xk = -.
 X y K = sin (- K) = Sk.

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That implies for the field of deformation:

NR

1

K

K

K

K

=

[H (kux

cos (-)

K - U y

sin (-)) + H (

K

K - U X

sin (-)

K - U y

cos (-))

R + R sin

K

K 1

=

- $H(K)$
 $K U X$

$\sin(-)$
 K
 $K + U y$

$\cos(-) - Rh \cos$
 K
 K

$(kx$

$\cos(-)$
 $K - ky$

$\sin(-)$
 K
 $- Rh \cos$
 K

$(kx$

$\sin(-)$
 $K + ky$

$\cos(-) - Rh \cos$
 K
 K

$(kx$

$\cos(-)$
 $K - ky$

$\cos(-)$
 K
 $+ Rh \sin$
 K
 $K]$
 Z
 $= 0$
 NR

I

K

K

=

[- H cos

K

(ux

cos (-)

K - U y

sin (-))

R + R sin

K

K 1

=

- H cos

K

(K

ux

sin (-)

K

K + U y

cos (-)) - H cos

K

K

(K

U X

cos (-)

K

K - U y

sin (-))

K

K

+ H U sin -

K Z

rHk

(kx

cos (-)

$K - ky$

$\sin (-) - Rh ($

K

K

K

$- X$

$\sin (-)$

$K - ky$

$\cos (-)$

K

$+ Rh \cos^2$

K

$(kx$

$\sin (-)$

$K + ky$

$\cos (-)$

K

$- Rh K K \sin$

Z

$\cos]$

$+ H \sin$

K

$(kx$

$\sin (-)$

$K + ky$

$\cos (-)$

K

$+ H K K \cos$

Z

NR

I

K

K

$=$

$[- H \sin$

R

K

$(ux$

$\cos (-)$

$K - U y$

$\sin (-)$

$R + R \sin$

K

$K 1$

$=$

$- H \sin$

K

$(K$

ux

$\sin (-)$

K

$K + U y$

$\cos (-) - H \sin$

K

K

$(K$

ux

$\cos (-)$

K

$K - U y$

$\sin (-)$

K

K

$- H U \cos$

$K Z$

$+ Rh \sin$

K

\cos

$(kx$

$\sin (-)$

$K + ky$

$\cos(-)$
 K
 $- Rh K K \sin 2$
 Z
 J
 $- H \cos$
 K

$(kx$

$\sin(-)$
 $K + ky$

$\cos(-)$
 K
 $+ H K K \sin$
 Z

Maybe in matrix form:

K
 U
 X
 K
 $U y$
 P

NR
 K
 U
 P

p
 $U = Z$
 $= P P$

Bk The U.K. where K
is the field of displacement to the node K

K
 P
 $K =$

I
X

K

y
K
Z

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and

H

K Ck

- HK Sk

- R HK Sk cos

- R HkCk cos

$R + R \sin$
 $R + R \sin$
 $R + R \sin$
 $R + R \sin$
 $R H$

$K \sin$

-
 0
 $2H$

$K Sk$
 $- 2HkCk$
 $- 2rH$
 \cos
 2
 \cos
 R

$+ R \sin$

$K Ck$
 $R HK Sk$

$R + R \sin$
 $R + R$
 $+$
 \sin
 $R + R \sin$
 $R + R \sin$

0
 0
 0
 0
 0
 0

$- H$

$+ 2$

K Sk cos
- HkC cos
H S (R sin
R)
H C (R sin + 2r
K
K K
K
K
)

B P = R + R sin
R + R sin
H sin
R + R sin
R + R sin
RH cos
K
K
K
- 2H

2

K Ck cos
H S
K K cos
R + R sin
R H

C
R H S
R + R sin
K
K
K K
+

R + R sin
R + R sin
-

+
 $R + R \sin$
 $R + R \sin$

- $HS \sin$
- HC

$K K$
 K
 $K \sin$

$R + R \sin$
 $R + R \sin$
- $H \cos$
 K

- $RH S \cos$
 $K K$

- $RH C \cos$
 K
 K

$RH \sin$
 K

- $2H C \sin$
 $2H S$

K
 K
 $K K \sin$
 $R + R \sin$
 $R + R \sin$
 $R + R \sin$
 $R + R \sin$

+
 $R + R \sin$
 $R + R \sin$

The matrix of passage of the deformations to the field of displacement is written as follows: P

$$\mathbf{B} = (P$$

P

\mathbf{B}

\mathbf{B}

$1 L$

$NR)$

4.1.2 Beam

right-hand side

U

K

U

K

X

X

X

X

NR

NR

U

K

y

$$= HK (X) K$$

$U y$ and $y = H K (X)$

y

$$K =$$

1

K

$$K =$$

1

K
uz
uz
Z
Z

According to the kinematics of beam presented higher [§2.1]:

$$\begin{aligned}
 rr &= 0 \\
 xx &= ux X + R \sin \\
 ZX & \\
 &- R \cos \\
 & , \\
 & , \\
 y, X & \\
 &= 0 \\
 & 2 \\
 X & \\
 &= - R X X + (y + U) \sin \\
 ZX & \\
 &+ (Z - U) \cos \\
 & , \\
 & , \\
 y, X &
 \end{aligned}$$

$$\begin{aligned}
 2 X\text{-ray} &= (Z - U) \sin \\
 y X & \\
 &- (y + U) \cos \\
 & , \\
 Z, X &
 \end{aligned}$$

$$2 R = 0$$

that implies for the field of deformation:

$$\begin{aligned}
 NR & \\
 & , \\
 & , \\
 & , \\
 xx &= \\
 K & \\
 H kux - H K R \cos () K &
 \end{aligned}$$

$y + H K r \sin () K$
 Z

$k=1$
 $= 0$
 NR

,

$X = - H K \cos () K$
 $U y + H K \sin () K$
 $uz -$
 K

$H K R X + H K \sin () K$
 $y + H K \cos () K$
 Z

$k=1$
 NR

,

$X\text{-ray} = (- H K \sin () K$
 $U y - H K \cos () K$
 $uz - H K \cos () K$

$y + H K \sin () kz$
 $k=1$

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Maybe in matric form:

K
U
X

K
P

U

y
X

P
NR

K
U

=
B U

p
U = Z

P
P P

K
K where

is the field of displacement to the node K

K
K

X
K = I

X
P

X

K

y

K

Z

and:

H K

0

0

0

- r cos () H r sin () H

K

K

0

0

0

0

0

0

P

B

K =

0

- cos () H

sin () H

K

K

- Rh K

sin () H K

cos () H K

0

- sin () H K - cos () H K

0

- cos () H K

sin () H K

The matrix of passage of the deformations to the field of displacement is written as follows: P

B = (P

P
B
B

1 L
NR)

4.2 Discretization of the fields of displacement and deformation for the additional part

NR
One discretizes the field of displacement for the surface of the pipe in the form:

S
$$U = HK (X) S$$

The U.K.

K = 1
with:

I
U
I

U
m
K m

I
v
VI
m
K m

I
W
wi
m
K m
O
O
U m
ukm

S
U = O
v and S
U =

K
vo
m =,
2 Mr.
m
K m

O
W
O

m
wkm

I
I
wI
W
kI
O
W
O

I
wkI

O
W
O

W
K

One has as follows:

$U(X,) \cos(m)$

0

0

$\sin(m)$

0

0

0

0

0

$v(X,) =$

0

$\sin(m)$

0

0

$\cos(m)$

0

$\sin() - \cos() 0 S$

U

$W(X,) 0$

0

$\cos(m)$

0

0

$\sin(m) \cos()$

$\sin()$

1

1

4

4

4

4

4

4

4

4

4

4

4

4
2
4
4
4
4
4
4
4
4
4
4
4
4
4
4
3

$m=$,
2 M

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if the indices m of S
The U.K. are ordered in the following way:

I
 U

$km=2$

I

v

km=2

I

wkm=2

O

ukm=2

O

v

km=2

O

wkm=2

M

I

U

S

The U.K. = km=M

I

vkM=M

I

wkm=M

O

U

km=M

O

vkM=M

O

wkm=M

I

W

k1

O
wk1

O

wk

The kinematics of hull presented higher to [§2.2] is:

$$\begin{aligned}
&= \\
&E + X \\
&= E + \\
&= 2 \\
&E \\
&+ 2 \\
&= 2 \\
&E \\
&= 0 \\
&= 2nd = 0
\end{aligned}$$

4.2.1 Bend

With:

$$\begin{aligned}
&I \\
&U \\
&E \\
&= \\
&(\\
&+ v \cos \\
&+ w \sin) \\
&R + R \sin \\
&I v \\
&E \\
&= (\\
&+) \\
&W \\
&R \\
&I U
\end{aligned}$$

1
v
2nd
=
+
(
- U cos
)
R R + R sin
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and:

1

1

2

W U

cos

W

=

[-

(

-

sin

) -
(
- v)]
R + R sin
R + has sin
2

has

1
2
W

v
= -
(
-
)

2
rear

W
cos
cos has
2
= (
- *U sin*

) [
+
]

(R + R sin) (R + has sin) R (R + sin has) 2
2
W
1
1
-
[
+
]

$(R + R \sin) R \text{ has } (R + \text{has } \sin)$

v

1

U

1

$+$

$+$ ($\sin + U \cos$)

$\text{has } (R + R \sin)$

$R (R + \text{has } \sin)$

$R (R + \text{has } \sin)$

allows to break up the field of deformation of hull on the modes of Fourier in the way following:

S

xx

S

NR

$=$

$B U$

S

$S S$

K

$K \text{ with:}$

$X k=1$

S

X

S

$Bk = (if$

if

so

so

sg

Bkm

B

$=2$

L

$K m=M$

$B km$

B

$=2$

L
K m=M
Bk)

where

H

K cos m sin
HK cosm

-

H

cos
sin
sin cos

R + R sin
(R + R sin) (R + has sin)

K
m
HK
m
(I+
)
I+

R + R sin
(R + has sin)
R + R sin

has
H

K m sin m cos

+

has (R + R sin)

m

1

m2

0

H

1

cos

1

K

+

m

HK

cosm

R

has

+

R

has

Bsi

km =

m

H cosm cos

-

H

K

K sin m -

R

R + R sin

mH

K sin m

mHk sinm

H 1+

+

K

sin m

cos sin

H

+

+

K cos m

aHk cosm

has

R (R has sin)

has (R R sin)

-

[

+
]

(R + has sin) (R + R sin) R (R + sin has)
R + R sin
cos

cosm H
cosm H has

+

[
K
K
+
]

(cosm cos - msin m sin)
R + has sin (R + R sin) R (R + has sin)
+ H
K

R (R + has sin)

0
0
0

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H

K sin m sin

HK sinm

-

H

sin

sin

cos

R + R sin

(R + R sin) (R + has sin)

K

m

HK

m

[1+

]

1+

cos

R + R sin

(R + has sin)

R + R sin

has

H

K m cos m

-
cos

(*R + R sin has*
)

m

l

m2

0

-
H l
sin

l

K
+
m
HK
sin m

R

has
+
R

has

Bso

km = m
H sin m cos

H

K

K cosm -

R

R + R sin

mH

K cosm

mHk cosm

H 1+

-

+

K

cosm

cos sin

H

+

+

K sin m

aHk sinm

has

R (R has sin)

has (R R sin)

-

[

+
]

R + has sin R + R sin
R (R + has sin)
R + R sin
cos
H

aHk sin
K sin m

+

m
[
+

]
(mcosm sin + sin m cos)
R + has sin
sin
(
sin)
+

R + R
R R + has

HK

R (R + has sin)

0
0
0

and
2H
2

2

K cos sin
Hkcos
HK (sin
- cos)
Hksin
HK sin

-
-

R + R sin
(R + R sin) (R + has sin)
R + R sin
(R + R sin) (R + has sin) (R + R sin)

2 *H*
2

K cos sin
2 *H*
cos
H

K
K
+
-

(R + R has

-

sin)

(R + R has

(R

sin)

+ sin has) (R + R sin)

2

2

H

I+

H

I+

H

K

K

K

cos

sin

Bsg

R

has

R

has

R

K =

2

2

I+
H cos
K

+

has

I

has

H sin
(R + R sin) (R + has sin)

K

[
]

- *H cos*

K

[
]

R +

+

R sin R (R + has sin)

R +

+

R sin R (R + has sin)

Ha cos

K

H cos

K

cos

cos has

H sin

K

cos

cos has

+

+

[

+

]

+

[

+

]

R (R + has sin) 2

R + has sin R + R sin R (R + has sin)

R + has sin R + R sin R (R + has sin)

0

0

0

4.2.2 Pipe

right

With:

U

Exx = X

1

v

E

= (

+ W

)
R
v

1 U

2Ex =
+
X
R

xx =
X

1 X
= - has
1
X

2 X =
-
R
X

W

= - X

1
W

= (
v)
X
-

has
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the field of deformation of hull breaks up on the modes of Fourier in the following way:

NR M

xx =

H (X)

N

(cos I

m un_m + sin O

m un_m)

X N 1=m=2

2

NR

M

-

H (X)

N

O

wn +

I

m w_nm +

O

m w_nm

2

(cos

sin

)
X N I
=

m I
=

I N R

M

I
O
I
O
=
H (X) sin
N

wn - cos
I
W In + (sin
m vnm + cos
m vnm)
R N I
=

m=2

I N R

M

+ *H (X)*
N
O
wn + (cos I
m wnm + sin O
m wnm)
R N I

=

m 1

=

NR

M

+

H (X) sin

N

I

wn - cos O

I

O

I

W In + (sin

m vnm + cos

m vnm)

rear N 1

=

m=2

2

NR

M

-

H (X)

N

O

wn +

I

m wnm +

O

m wnm

2

(cos

sin

)

rear N 1

=

m 1

=

NR

M

I

O

I

O

X =

H (X)

N

sinwn - cos

I

W In + (sin

m vnm + cos

m vnm)

X N 1=

m=2

I NR M

+

H (X)

N

(cos I

m unm + sin O

m unm)

R N 1

= m=2

NR

M

+
H (X) sin
N

I
wn - cos O

I
O
I
W In + (sin
m vnm + cos
m vnm)
X has

N I
=

m=2

2

NR

M

- +
H (X)
N
O
wn + (cos I
m wnm + sin O
m wnm)

R
R
X N I
=

m 1

=

X = 0

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That is to say still:

NR M

xx = H (X)

N

(cos I

m un m + sin O

m un m)

N 1

= m=2

NR

M

- H (X)

N

O

wn + (cos I

m wnm + sin O

m wnm)

N I

=

m I

=

I NR

M

I

O

I

O

= H (X)

N

coswn + sin

I

W In + (mcos

m vnm - msin

m vnm)

R N I

=

m=2

I NR

M

+ H (X)

N

O

wn + (cos I

m wnm + sin O

m wnm)

R N I

=

m 1

=

NR

M

+

H (X)

N

cos I

wn + sin

O

I

O

1

W In + (mcos

m vnm - sin

m

m vnm)

rear N 1

=

m=2

NR

M

+

H (X)

N

(2

m cos

I

2

m wnm + m sin O

m wnm)

rear N 1

=

m 1

=

NR

M

I

O

I

O

X = H (X) sin

N

wn - cos

1

W 1n (sin

m vnm + cos

m vnm)

N 1

=

m=2

1 NR M

+ H (X)

N

(- sin

m

I

m unm + mcos O

m unm)

R N 1

= m=2

NR

M

+ H (X) sin

N

I

wn - cos

O

I

O

I

W In (sin

m vnm + cos

m vnm)

has

N I

=

m=2

NR

M

- + H (X)

N

(- sin

m

I

m wnm + mcos O

m wnm)

R

N I has

=

m I

=

X = 0

This gives in matric form:

S

xx

S

NR

=

B U

S

S S

K

K with:

X k=1

S

X

S

Bk = (if

if

so

so

sg

Bkm

B

=2

L

K m=M

B km

B

=2

L

K m=M

Bk)

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where:

**$H \cos ($
 K**

$m)$

0

**$- H \cos ($
 K**

$m)$

m

1

2

m

0

**$H \cos ($
 K**

$m) 1$

**$H \cos (m$
 K**

$) 1$

$+$

$+$

if

Bkm

R

has

=

R

has,

m

- H sin (

K

m)

H sin (

K

m

) I +

+ H

m sin (

K

m)

R

has

R has

0

0

0

H sin (

K

m)

0

- H sin (

K

m)

m

l

2

m

0

H sin (
K

m) l

H sin (m
K
) l

-

+

+

so

Bkm

R

has

=

R

has

m

H cos (
K

m)

H cos (
K

m

) 1+

- + H

m cos (
K

m)

R

has

R

has

0

0

0

and

- H cos (

K

)

- H sin (

K

)

- H K

2

2

H K

1 + H cos (

K

)

1 + H sin (

K

)

sg

R

has

R

has

R

Bk

=

2

2

1 +

+ H sin (

K

) - 1 +

+ H cos (

K

)

0

has

R

has

R

0

0

0

4.3

Discretization of the field of total deflection

P

S

xx xx xx

P

S

NR

NR

NR

= + =

B U

B U

B U

B U

P

S

P P

K

K + S

S

K

K =

=

with

X

X X

K

K

K = 1

K = 1

K = 1

P S

X X X

P

U

B = (P S

B

K
U =

K B K) K,
1
= NR and

S
The U.K.K, 1
= NR

4.4
Stamp rigidity

The variational formulation of the work of deformation is:

xx

L
2 H/2

W
=

def
(xx

X

X) rd

dxd

0 0 - H/2

X

X

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that is to say still:

xx

L

2

H/2

W

=

C

def

(xx

X

X)

rd

dxd

0 0 - H/2

X

X

L

T

2

H/2 NR

NR

W

=

B U

C

B U

rd dxd

def

K K

K K

0 0 - H/2 k=1

k=1

L
2
H/2 NR
T
T
NR

= U B C B U

rd dxd

K
K
K K

0 0 - H/2 k=1
k=1

2
/2

L H
U

1

T
T T

= U
U
B

CB

I

L

M

NR

rd dxd

0 0 - H/2

U

NR

U

I

T

T

= U

U

K

I

L

NR

M

U

NR
*The principle of virtual work is written **UT** then*

$$\mathbf{K} \mathbf{U} = \mathbf{F} \mathbf{U}$$
*where **K** is the matrix of rigidity which*
is worth:

L
2
H/2

$$\mathbf{K} = \{\mathbf{B} \mathbf{T}\}$$
CB rd

dxd
0 0 - H/2

Note:

One makes no assumption on the law of behavior. This expression is thus in private individual validates the nonlinear behaviors in the case of (plasticity).

4.5 **Stamp of mass**

The terms of the matrix of mass are obtained after discretization of the variational formulation following of the noncentrifugal terms of inertia:

u1 (X, R)

u2 (X, R)

L

2 + h/2

u3 (X, R)

W ac

U =

mass =

u&.v

rdxd

D

with

.

U (X, R)

0 0 - H/2

$v(X, R)$

(
 $W X, R)$

The notations used are those of [§2.1]: 1
 U, u_2 and u_3 sont displacements of beam in a point
section and U, v and W are displacements of average fibre of this section in this same
not.

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The discretization gives then:
the U.K.
 X

the U.K.
 y

the U.K.
 Z
 K
 X

K
y
K
Z

ui
NR
km
U = H

K NR K v I
km
K I
=
wi
km

m =,
2 M
U okm
vo
km

wo

km
wi
kI

wo
kI

wo
K

where the *Nk* matrices have as an expression:

$$\mathbf{X} \cdot \mathbf{x} \cdot \mathbf{y} \cdot \mathbf{x} \cdot \mathbf{0} - R \cos(\mathbf{X} \cdot \mathbf{y}) - R \cos(\mathbf{y} \cdot \mathbf{y})$$

K
K
K
K
) *R sin*
0
0
0
0
0
L

X .y y .y 0
R cos

(*X .x*)
R cos (y .x

K
K
K
K
)
0
0
0
0
0
0
0
L

0
0
1 - R sin (X .x) - R sin (y .x)
0
0
0
0
0
0
L

NR

K
K

$$K = 0$$

0

$\cos (m$

L

)

0

0

$\sin (m$

0

0

0

0

0

0

0

0

$\sin (m$

L

)

0

0

$\cos (m$

0

$\sin () - \cos () 0$

0

0

0

0

L

$\cos (m$

0

0

$\sin (m) \cos ()$

$\sin ()$

1

144444444444 2

4 4

3

444444444444

$m=2, M$

The matrix of mass has then as an expression:

L

$2 + h/2$

$M =$

T

$NR NR$

$rdxd$

D

.

$0 0 - H/2$

with

$NR = (H K Nk) K, 1$

$= NR.$

Note:

In the case of the right pipe, one has $X .x$

K

$= y .y$

K

$= 1$ and $X .y$

K

$= y .x$

K

$= 0 .$

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4.6

Functions of form

One chooses at least quadratic functions of form for the part beam (displacements and rotations) in order to avoid the phenomena of numerical blocking [bib3]. This choice implies the use of one

finite element with three or four nodes. In the case of an element with 3 nodes, the functions of form are quadratic, and for an element with 4 nodes, the functions of form will be cubic. For the part additional, one chooses to take the same functions of form as for the beam part.

The quadratic functions of form (element with 3 nodes) are as follows:

X

X

H1 (X) 2

=

-1 -1

L

L

X

X

H2 (X) 2

=

-1

L L

X X

H3 (X)

= - 4 -

1

L L

The cubic functions of form (element with 4 nodes) are related to Lagrange of order 3:

$H_1(X)$ (

2) (

3) (

4)

$$= (1 - 2)(1 - 3)(1 - 4)$$

$H_2(X)$ (

1) (

3) (

4)

$$= (2 - 1)(2 - 3)(2 - 4)$$

$H_3(X)$ (

1) (

2) (

4)

= (

$$3 - 1)(3 - 2)(3 - 4)$$

$H_4(X)$ (

1) (

2) (

3)

$$= (4 - 1)(4 - 2)(4 - 3)$$

-1 1

4.7 Integration numerical

Integration is done by the method of Gauss along average fibre, the method of Simpson in the thickness and on the circumference. For the integration of Gauss, one uses 3 points of integration for the elements with 3 nodes, as for the elements with 4 nodes (those under-are thus integrated).

Integration in the thickness is an integration by layers of which the number could be fixed later on by the user. For each layer one takes 3 points of Simpson, the 2 points ends being common with the close layers. Thus for N layers one uses $2n+1$ points. a many sectors for integration on the circumference, could also be fixed later on by the user. Currently the numbers of layers and sectors are fixed at their maximum value: 3 layers (7 points) and 16 sectors (33 points), which gives 693 points of integration on the whole. The integration of Simpson amounts calculating the sum of the values of the function at the points integrations (ends and medium of each layer or sector) affected of the weights given by the table below.

Cordonnées of the points

Weight

- 3 / 5 = -0,77459 66692 41483

5/9=0,55555 55555 55556

0

8/9=0,88888 88888 88889

3 / 5 =0,77459 66692 41483

5/9=0,55555 55555 55556

Integration of Gauss on average fibre

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1

4

2

4

2
2
4
2
4
1

3
3
3
3
3
..... 3 3 3 3 3

*Weight of the points of integration for the method of Simpson
Thus for a function $F(X,)$ on a right geometry one $a:$*

$L \ 2 \ H/2$
 $1 \ 2 \ H/2$

~
 $F(X,)$ $rdxd = L$
 D
 $F x~$
(, $R X$
 $d~$
)
 dd
2
 $0 \ 0 - H/2$
 $-1 \ 0 - H/2$

$NPG \ 2NCOU +1 \ 2NSECT +1$

$= L$
 H
2

[
 ~
 W
 ~
 K wn wm RN
 F (xNPG, NR
 , NR
)
 NECK
 SECT
 NECK
]
 2 2NCOU 2N SECT k=1 n=1
 m=1

K
 W
 N
 W
 m
 W being respectively weights of integration over the length, the circumference and in the thickness, ordered as the two tables show it above.

4.8
Discretization of external work

The variational formulation of external energy for the beam part is written

:
 L
 W
 $p_{ext} = (F$
 $X U$
 $X + F y U$
 $y + F Z U$
 $Z + m_{xx} + m_y y + m_{zz} dx$
 $) +$
 0
 $[X U$
 $X + y U$
 $y + z U$
 $Z + \mu_{xx} + \mu_{yy} + \mu_{zz}] 0, L$

and for the additional part she is written by taking into account only the loading of

L

pressure: W

sext = R

F dx

W + [R W

] 0, L

0

By taking account of the discretization of displacements, one can write:

NR L

Wp = (F H (X

K

K

K

K

K

ext.

X K) (xk.xux +yk.xu) + F H (X

y

y

K

) (xk.

y ux + yk .yu) + F H (X

y

Z

K

) uz +

K =1 0

m H (X

K

K

K

K

K

X

K

) (xk.

X X + yk.x) + m H (X

y

y

K

) (xk .yx + yk.

y

) + m H (X

y

Z

K

)

) dx

Z

+

[H (X

K

K

K

K

K

K

y HK (X

) uy + Z HK (X

) uz + μx HK (X

) X + μy HK (X

) y + μz HK (X

)

X

K

) ux +

Z] 0, L

NR

L

L

L

= H

K (X) (F X (X .x)

K

+ F y (X .y)

K

) dx+ [xHk (X)]0, L H

K (X) (fx (y .x)

K

+ F y (y .y)

K
 $) dx + [yHk(X)]0, L fzHk(X) dx + [\mu xHk(X)]0, L$
K 1
 $= 0$
0
0
L
L
L
H(X)
P

μ
 μ
 μ

K
 $(m(X .x)$
K
 $+ m(X .y)$
X

y
K
 $) dx + [H(X)] H(X)$
X
K
0, L

K(m(y .x)
K
 $+ m(y .y)$
X

y
K
 $) dx + [H(X)] m H(X) dx + [H(X)]$
y
K
0, L

ZK
Z
K
0, L
The U.K.

O
O
O

NR
P
p
P
p
=

K
F The U.K. = F U
K 1
=
and
NR
L

WS =
FH(XO
O

ext.
R
K

) W dx + [H (X
K
R
K

) wk] 0, L

K = 1 0

NR
L

=

0 0 0 0 0 0

F H (X D

) X

+ [*H (X*

S

R

K

R

K

)]0, *L the U.K.*

K = 1

0

NR

S

S

S

S

=

F

kUk = F U

K = 1

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Note:

For the extreme nodes of the elbow there are $X .x$

K= $y .y$ **K**= 1 and $X .y$ **K**= $y .x$ **K**

= 0. In the case of it

right pipe, one has $X .x$ **K**= $y .y$ **K**= 1 and $X .y$ **K**= $y .x$ **K**

= 0 for all the element.

5**Geometrical characteristics of the pipe section**

One has in this chapter some useful results to characterize the element pipe and which are calculated by the option of calculation MASS_INER of Code_Aster. In the continuation the index D indicates them

results for the right pipe and the index C for the curved pipe.

 L $2 H/2$ $V =$ $rdxdd = 2 lah$ D

.

 $0 0 - H/2$ **Volume:**

\cdot
 L
 $2 H/2$
 $2 H/2$
 $V =$
 $rdxdd =$
 $[R + (has +) \sin ($
 $] +) D dd has = 2 R$
 ah
 C

$0 0 - H/2$

$0 0 - H/2$

\cdot **Centre of gravity:** The position of this last is calculated starting from the point medium to both extreme nodes of the pipe section, in the reference mark associated with the node interns element (cf [§2.1.1]). In this reference mark, the co-ordinates of the centre of gravity are:

X

0

Gd

0

xGc

2

2

1

2

H

y

$yGc = - R \sin$

1

$(+$

$[has +$

$] - \cos$

$Gd = 0$ and

2
2 2
 R
4
2

$zGd 0$
 Z

Gc

0

· **Matrice of inertia:** It is relatively easy to calculate the matrix of inertia in the center of curve of the elbow O in the reference mark defined above. To have his expression one uses then the fact that:

2
 $I(G) = I(O) - mb$

xx

xx

$I(G) = I(O)$

yy

yy

2

$I(G) = I(O) - mb$

zz

zz

where B is the distance between the centre of gravity and the center of curve which is worth:

2

1

2

H

$B = R$

\sin

1

(+

[2
has +
]).

2
2 2
R
4

In the case of a right pipe, the concept of center of curve does not have a direction. O and G are confused

with the node interns element and the medium of the segment joining the two nodes top.

If one notes A the surface of the transverse section and I his inertia compared to the center of the section one can

to write:

C
I
2
I I sin
I (O) = R
(+ [AR + 3] [+
])

xx
I D (O) = Li

2
2 2
4

xx
C
I
2
I I sin
I D (O) = L

(I/2
2
+ Al /12)

yy
and I
(O) = R
(+ [AR + 3] [-
])

yy
.
2

2 2

4

$ID(O) = L(I/2$

2

+ AI/)

12

zz

C

I

$I(O) = R$

(

2

AR + 3)

zz

2

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6 Connection pipe-pipe

In order to be able to represent a line of piping correctly where the elbows are not coplanar, it is necessary to choose an origin common Des. Ainsì for two elbows belonging to two perpendicular plans between them, it is necessary to be able to take into account the fact that displacements in the plan of the first elbow are equal to displacements except plan of the second in the cross-section of

connection.

..

Generating line

2

.

.

..

1

Appear 6-a: Representation of two noncoplanar elbows connected by a right pipe

In [bib12], this common origin is defined by a generating line continues along piping as indicated above. This generator intersects each transverse section in one not. The angle between Z defined on [Figure 2.1.1-a] and the line passing by the center of the section transverse and this point are worth.

6.1

Construction of a particular generator

For a transverse section end of the line of piping, one defines a vector origin $z1$ unit in the plan of this section. The intersection enters the direction of this vector and surface average of the elbow determines the trace of the generator on this section. One calls $x1, y1, z1$ it direct trihedron associated this section where $x1$ is the unit vector perpendicular to the section transverse built with [Figure 2.1.1-a]. For the whole of the other transverse sections, the trihedron xk, yk, zk are obtained either by rotation of the trihedron $xk 1, y$

-

$K 1, Z$

-

$K 1$

- in the case of bent parts,
maybe by translation of the trihedron $xk 1, y$

-

$K 1, Z$

-

$K 1$

- for the right parts of piping. The intersection between the transverse section and the line resulting from the center of this section directed by zk is the trace of a generator represented below.

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Z

Z

B

z2

x2

generator

y2

O

Y

Z

l

Z

x1

With

y1

X

Appear 6.1-a: Representation of the generator of reference

The origin of the commune to all the elements is defined compared to the trace of this generator on the transverse section. The angle enters the trace of the generator and the current position on the section transverse is then called.

6.2

Connection from one element to another

The kinematics of [§3.1] is given in the plan of the elbow. This one is determined by the arc of circle

generated by the axis of the elbow. The origin of the angles is the normal in the plan chosen as with [§2.1].

To define the origin starting from a generator makes it possible to raise the problems of continuity of displacements of an element to another. Indeed if one postulates that relative displacements of M

transverse sections are of type I

$$U_p \cos p + O$$

$U_p \sin p$ where p is the angle with the trace of

$$p=1$$

generator on the transverse section, the continuity of displacements is automatically assured from one element to another.

One notes \mathbf{Z} the vector perpendicular to the plan of the elbow corresponding at the selected origin of the angles

up to now. It is noticed that vectors \mathbf{Z} and \mathbf{z}_k are in the plan of the section K . α is the angle defined compared to \mathbf{Z} . If one introduces the angle counted starting from the trace of the generator on transverse section (thus compared to \mathbf{z}_k) one with the following relation: $\alpha = -K$ where $\alpha = (\mathbf{Z},$

)

\mathbf{K}

\mathbf{Z}_K

angle between \mathbf{Z} and \mathbf{z}_k in the plan of the transverse section. Thus displacements are from now on

M

type I

$$U_p \cos p (-K) + O$$

$U_p \sin p (-K)$. It should be noted that for an elbow given the angle K

$$p=1$$

whatever the selected transverse section is identical. It is at the time of the passage from one elbow to another

that the value of K changes.

Note:

When piping consists of right elements colinéaires, one chooses arbitrarily

$\alpha = 0$.

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6.3

Numerical establishment

The line of piping is with a grid by right or curved elements to order. The first element indicate the beginning of the line of piping. One determines for this element the associated trihedron $x1, y1, z1$. If this element is right, one chooses

0

1 =

, if not one calculates 1 as indicated in

preceding paragraph. If the first element is right the trihedron associated with the first section

transverse of the second element $x2, y2, z2$ is obtained by translation of $x1, y1, z1$. If the first

element is curved, the associated trihedron $x2, y2, z2$ is obtained by rotation of $x1, y1, z1$ in the plan

elbow. In this case

0

2

2 =

if the second element is right and $= (Z,$

)

2

$z2$ if the second

element is curved where 2

Z is built like Z of [Figure 2.1.1-a]. The continuation of construction

results easily by recurrence from the preceding diagram.

7

Connections hull-pipe and 3D-pipe

7.1 Step

followed

One adopts here a step similar to the cases 3D-beam [R3.03.03], and hull-beam [R3.03.06]: it acts to characterize the connection between a node end of an element pipe and a group of meshes of edge of elements of hulls or 3D. This makes it possible to net part of piping (for example one

bend) in hulls or elements 3D, and the remainder in right pipes.

Appear 7.1-a: Connection between a right grid COQUE_3D and pipes [HI75-98/001]

Thanks to the kinematics introduced into the element pipe, the connections hull - pipe and 3D - pipe must allow to net in elements of hulls or 3D only the elbow, without right parts, since the damping of ovalization (and warping) is taken into account in the element pipe.

The connection results in relations kinematics between the degrees of freedom of the nodes of S (which represent the section of connection, modelled by elements of edge of hull or 3D), and it node NR of pipe.

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So that the connection is effective, it is necessary [R3.03.03] that it checks the following properties:

- 1) to be able to transmit efforts of beam to the grid hulls or 3D, and to be able to transmit all degrees of freedom of the element pipe (or duaux efforts of those),*
- 2) not to generate in the elements of hulls or 3D of secondary stresses,*
- 3) not to support the static relations kinematics or conditions ones by report/ratio with the others,*
- 4) to admit unspecified behaviors and to function in dynamics.*

The linear relations will have the same form as in the case hull-beam, with equations

additional specific to the degrees of freedom of the pipe.

One already introduced with [§3.1] space \mathbf{T} of the fields associated with a torque (defined by two vectors):

$$\mathbf{T} = \{\mathbf{vV}/(T,)$$

that

such

$$\mathbf{v}(M) = T + \mathbf{GM}\}$$

where for the fields of displacement of \mathbf{T} , T is the translation of the section (or the point G), infinitesimal rotation and fields \mathbf{v} are displacements preserving the section S plane and not deformation there (One uses still the assumptions of NAVIER-BERNOULLI).

The displacement of the pipe is worth then:

\mathbf{T}

\mathbf{p}

\mathbf{S}

\mathbf{p}

\mathbf{S}

$$\mathbf{U} = \mathbf{U} + \mathbf{U}$$

$\mathbf{U T U}$

,

\mathbf{T}

where:

$$\mathbf{T} = \mathbf{v V}/\mathbf{v W}$$

.

$\mathbf{0 W}$

\mathbf{T}

\mathbf{S}

The step consists in breaking up the field of displacement of hull C \mathbf{U} or the field of

three-dimensional displacement 3D

U in three fields:

C

P

S

$$U = U + U + U$$

· a field of displacement following a kinematics of beam p

U (torque),

· a field of local displacement of the section according to the kinematics of pipe S

U (series of

Fourier) defined in [§3.1],

· an additional field

U orthogonal with the two first within the meaning of the scalar product.

Note:

When the decomposition in Fourier series of [§3.1] is infinite one has

$$U = 0.$$

To translate the equation above into linear relations, it is shown that the integrals should be calculated following, for the hull (or the 3D) and the pipe:

· average displacement: C

$$U dS$$

S

· average rotation: GM C

$$U dS$$

S

GM

· average swelling:

C

U

. dS

S GM

GM

· modes of Fourier

: C

$$U \cos p dS$$

,

C

$$U \sin p dS$$

, CPU

$$\cos p dS,$$

S
S
S
GM
GM
GM
GM
CPU
sin p dS

,
. *C*
U cos p dS,
. *C*
U sin p dS
. *For*
S
GM
S GM
S GM

modes of Fourier one will choose the relations simplest to exploit since some are redundant (see remark of [§6.7]).

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Note:

One passes easily from the analytical expressions of the connection 3D - pipe to those of the connection hull - pipe. It is enough to substitute 3D

U with C

U in the whole of the integrals of

connection proposed above. One will thus speak again of this connection only to [§7.8] treating numerical establishment.

7.2

Kinematics of the pipe.

In the curvilinear base (O, X, y, Z) associated the transverse section of [§2.1] one notes displacements

1
U, u2 and u3 where:

$$\begin{aligned}
 &U(R, X, \\
 &1 \\
 &)= U(X) \\
 &X \\
 &- (X) R \cos \\
 &y \\
 &+ (X) R \sin \\
 &Z \\
 &+ U(X,) + (X,) \\
 &U(R, X, \\
 &2 \\
 &)= U(X) \\
 &y \\
 &+ (X) R \cos \\
 &X \\
 &- v(X,) \cos - (\\
 &W X,) \sin + (X,) \cos.
 \end{aligned}$$

$$\begin{aligned}
 &U(R, X, \\
 &3 \\
 &)= U(X) \\
 &Z \\
 &- (X) R \sin \\
 &X \\
 &+ v(X,) \sin - (\\
 &W X,) \cos - (X,) \sin
 \end{aligned}$$

Once discretized this expression becomes:

NR

U(R, X,) = H(X) (X .x) the U.K.

K
+ $H(X) (y .x) \text{ the U.K.}$

K
- $H(X)$
 K
) $(X .y)$

K
 $R \cos - H(X)$
 K
) $(y .y)$

K
 $R \cos + H(X K)$
) $R \sin + U(X,) + (X$

I
 K
 X
 K
 y
 K
 y
 K
 y
 K
 Z
,)
 $K = I$
 NR

$U(R, X,) = H(X) (X .y) \text{ the U.K.}$

K
 K
 XR
+ $HK X$
 $y R$
- $v X$
- $W X$
+ X

K
+ $H(X) (y .y) \text{ the U.K.}$
K
+ $H(X)$

2

 K X K y K $) (x_k .x)$ \cos $() (y_k .x)$ \cos $(,) \cos$ $(,) \sin$ $(,) \cos$ $K = 1$ NR $U (R, X,$

3

 $) = H (X) K$ $U - H (X) (X$ K K K Z K $X R - H K X$ $y R + v X$ $- W X$ $- X$ $K .x)$ \sin $() (y_k .x)$ \sin $(,) \sin$ $(,) \cos$ $(,) \sin$ $K = 1$

where $U (X,)$, $v (X,)$, ($W X,)$, $(X,)$ and $(X,)$ are discretized as with [§3.1].

Displacement in a node K of X -coordinate x_k end of pipe is written then:

 M $U (R, X,) = \text{the } U.K.$

K
- R
K
cos + R sin + (ui cosm + uo sinm) + (X

l
K
X
y
Z
km
km
K,)
m=2
M
M
U (R, X,) = the U.K.

K
+ R cos - cos (VI sinm + vo cosm) - sin (wi cosm + wo sinm

2
K
y
X
km
km
km
km
)
m=2
m=2
- wi sin2 + wo cos2 - wo
+

k1
k1
K sin
(
xk,) cos

M
M
U (R, X,

3
) = the U.K.
 K
 - R sin + sin
 (VI sin m + vo cosm) - cos
 (wi cosm + wo sin m

K

Z

X

km

km

km

km

)

m=2

m=2

- wi cos

1

2 - wo sin

1

2 - wo cos - (X

K

K

K

K,) sin

GM

For the pipe the kinetic vector moment **GM U (M)** and swelling

.u (M) has for

GM

respective expressions:

- U (X) R sin

Z

+ U (X) R cos

y

+ 2

R (X)

X

- $rv(X,)$

$GM U(M) =$

- $Ru(R, X,$

1

) cos

,

$Ru(R, X,$

1

) sin

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and:

GM .u (M) = U

- (X) cos

Z

- U (X) sin

y

+ (

W X,).

GM

Note:

The first component of the field of displacement utilizes U (X,) in an isolated way. It in goes in the same way for the first component of the vector rotation with respect to v (X,) and of swelling with respect to (W X,). This remark will be used with [§5.6] to bind the modes of Fourier with the degrees of freedom of the edges of hull.

7.3

Kinematics of hull

The kinematics of hull of Coils-Kirchhoff or of Naghdi-Mindlin is written in the thickness:

CPU (M) = **CPU** (Q) + (C

(Q) N

).y3

.

C*U (Q) constitutes the vector displacement of average surface in Q,*

.

C*(Q) constitutes the vector rotation in Q of the normal according to directions' T1 and t2 of the plan tangent with Q.* x^3 y^3 **N****M** y^3 **H****Q** Q_y **T = e****2****E****1****1****3****E***Section of hull:***2****G****E** $S = L \times I$ **1***x1 L: line of the points Q on the average layer***H****H H***I = -, interval describing the thickness.***2 2**

This displacement and this rotation are calculated in the total reference mark. It is possible by change of reference mark to have their expressions in the curvilinear base (O, X, y, Z) of [§2.1] associated the section

transverse of the junction enters the hull and the pipe.

For each node, the program calculates the coefficients of the $9+6(M-1)$ linear relations which connect:

- 6 ddl of the node of beam P of the pipe,
- $2+3 \times 2(M-1)$ ddl of Fourier of the pipe,
- the ddl of swelling of the pipe,
- with the ddl of all the nodes of the list of the meshes of the edge of hull.

These linear relations will be dualisées, like all the linear relations resulting for example from key word `LIAISON_DDL` of `AFFE_CHAR_MECA`. They are built as for the connection 3D-beam has to leave the assembly of elementary terms.

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7.4

Calculation of average displacement on the section S

It is a question of calculating the integral \int_C

$U dS$, where C

U is the displacement of hull (comprising 6 ddl by

S

node), S is the edge of hull.

Average displacement on the section S is written:

CPU (

/2

$M) dS =$

H
H CPU Q dl
C
Q N
y Dy dl
S
() +
L
(())

L

3
3

- h/2

that is to say C
U (M) dS = H C
U Q dl.

S
()
L
In addition one also has for the pipe part:

K
U
K
U
X
X
CPU (M) dS =
p
S
[U M
U M dS
]
U dS
S U
.
S

() + () = K
y

= K
S

y
S K
U

K
U
Z
Z

One establishes as well as the average displacement of the section of the pipe to the node K is equal to displacement of beam of the node K. One can thus linearly bind the degrees of freedom of beam of translation with the node K with the average of the degrees of freedom of displacement of the edge of the hull.

One neglects in this expression the variation of metric in the thickness of the hull.

7.5
Calculation of the average rotation of the section S

$$\begin{aligned}
 & \mathbf{GM} \text{ CPU } (\\
 & /2 \\
 & M) dS = \\
 & H \\
 & \mathbf{GQ} \text{ y } N \text{ Q} \\
 & \text{u.a. } Q \\
 & C \\
 & Q \text{ N } Q \text{ y } dldy \\
 & S \\
 & L (+ 3 () (() + () () 3) \\
 & - H/2 \\
 & 3 \\
 & = \\
 & /2 \\
 & h\mathbf{GQ} \text{ CPU } (Q) dl + \\
 & H \\
 & \mathbf{GQ} \\
 & C \\
 & Q \text{ N } Q dl \\
 & y Dy \\
 & L
 \end{aligned}$$

(() ())

L

- $h/2$ 3 3

H

+ *N* (

/2

Q) *CPU* (*Q*)

H

y Dy dl

N Q

C

Q

N Q 2 *y*2 *Dy dl*

.

L

3

3

+

- $h/2$

() (() ())

L

3

3

- *H*2

3

C

C

H

that is to say *GMu* (*M*) *dS*

.

= *hGQu* (*Q*) *dl* + *C*

(*Q*) *dl*.

S

L

L

12

In addition one also has for the pipe part:

$r^2 K$

K

X

X

GM CPU

$(M) dS = \mathbf{GM}$

2

2

K

K

$[$

$]$

\cos

\cdot

S

$\mathbf{up} (M) + \mathbf{custom} (M) ds =$

S

R

$y dS = I y$

$S r^2$

2

K

K

\sin

Z

Z

where I is the tensor of inertia of the beam. One establishes as well as the average rotation of the section of the pipe

with the node K is equal to the rotation of beam to the node K . One can thus bind the degrees linearly of freedom of beam of rotation to the node K with the degrees of freedom of rotation of the edge of the hull.

One neglects in this expression the variation of metric in the thickness of the hull.

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7.6

Calculation of the average swelling of the section S

C

C GM

GM

GQ

It is a question of calculating the integral $U N$

. $dS = U$.

dS, where $N =$

=

is the normal with

S

S

GM

GM

GQ

surface average hull.

Average displacement on the section S is written:

/2

CPU (M) N

. dS =

H

H CPU Q N

. dl

C

Q N N

.

y Dy dl

H CPU Q N

. dl.

S

() +

L

(())

L

3

3

=

- H/2

()

L

In addition one also has for the pipe part:

GM C

GM

.u (M) dS

p

=

. [U (M)

S

+ U (M) dS

J

= wodS

K

S

S

GM**GM****S**

One establishes as well as the average swelling of the section of the pipe to the node K is equal to the degree of

freedom of swelling of the pipe to the node K. One can thus linearly bind the degree of freedom of swelling of pipe to the node K with the degrees of freedom of displacement of the edge of the hull.

One neglects in this expression the variation of metric in the thickness of the hull.

7.7

Calculation of the coefficients of Fourier on the section S

GM

It is a question of calculating the six integrals C

$U \cos p dS$

, C

$U \sin p dS$

, CPU

$\cos p dS,$

S

S

S

GM**GM****GM****GM****CPU**

$\sin p dS$

,

. C

$U \cos p dS$ and

. C

$U \sin p dS$

, where C

U is it

S

GM**S GM****S GM**

displacement of hull (comprising 6 ddl by node), S is the edge of hull.

One with the following relation for displacements on the section S:

$$\begin{aligned}
 & \mathbf{CPU} (\\
 & /2 \\
 & M) \cos p \, dS \\
 & = \\
 & H \\
 & H \mathbf{CPU} \, Q \cos p \, dl
 \end{aligned}$$

$$\begin{aligned}
 & \mathbf{C} \, \mathbf{Q} \, \mathbf{N} \\
 & y \, Dy \cos p \, dl \\
 & 3 \\
 & 3
 \end{aligned}$$

$$\begin{aligned}
 & S \\
 & () \\
 & + \\
 & L \\
 & (())
 \end{aligned}$$

L

- h/2

that is to say C

$$\begin{aligned}
 & U (M) \cos p \, dS \\
 & = H \, C \\
 & U \, Q \cos p \, dl \\
 & \text{and } C \\
 & U (M) \sin p \, dS \\
 & = H \, C \\
 & U \, Q \sin p \, dl
 \end{aligned}$$

·
S
()
S
()
L
L

In addition one also has for the pipe part:
U (R, X

1
K,)

$$\int_C U(M) \cos p \, dS = \int_K U(R, X, y) \cos p \, dS$$

$$\int_S U(R, X, y) \cos p \, dS$$

3
K,)

The first component of this relation then enables us to connect the coefficient linearly of Fourier I u_{kp} with the components of displacements of the edge of hull in the following way:
- $R K \cos^2 dS$

$$\int_C H U(Q) \cos p \, dl = \int_S U(R, X, y) \cos p \, dS$$

$$\int_S U(R, X, y) \cos^2 p \, dS$$

if p
kp

l

S
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U (R, X

l

K,)

Of the same C

U (M) sin p dS

= U (R, X,

2

) sin p dS

K

from where one deduces from it that:

S

S U (R, X

3
K,) $R K$
 $\sin^2 dS$

if p

1
=
Z
C

$H U (Q) \sin$
 S
1
 $p dl$
 $= U (R, X,) \sin$
1
 $p dS$
=

S
 K
 L
 $u_0 \sin 2 p dS$

if p

kp

1

S

One with the following relation for rotations on the section S :

$+h/2$
CPU (
GM
C
GQ

$$\begin{aligned}
 &M) \\
 &\cos p \, dS \\
 &= H U (Q) \\
 &\cos p \, dl \\
 &+ (C (Q) N) N (y Dy) \cos p \\
 &3 \\
 &3 \\
 &dl \\
 &. \\
 &S \\
 &L \\
 &L \\
 &\mathbf{GM} \\
 &\mathbf{GQ} \\
 &- H/2
 \end{aligned}$$

The first component of this relation then enables us to connect the coefficient linearly of Fourier O vkp with the components of displacements and rotations of the edge of hull in the manner following:

$$\begin{aligned}
 &- [K \\
 &Ru \cos^2 \\
 &y \\
 &+ \\
 &O \\
 &rw \cos^2] \, dS \text{ if } p \\
 &1 \\
 &k1 \\
 &= \\
 &\mathbf{GQ} \\
 &H [CPU (Q)
 \end{aligned}$$

$$\begin{aligned}
 &] \cos \\
 &S \\
 &1 \\
 &p \, dl \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 &L \\
 &\mathbf{GQ}
 \end{aligned}$$

O

rv cos2 (p) dS

if p

kp

l

S

In the same way one a:

[K

Ru sin 2

Z

+

I

rw sin 2] dS

if p l

kl

=

GQ

H [CPU (Q)

J sin

S

l

p dl

=

L

GQ

I

rv sin 2 (p) dS

if p

kp

l

S

One with the following relation for swelling on the section S :

GM

GQ

. **C**

$U (M) . \cos p \, dS$

$= H$

. **C**

$U Q \cos p \, dl$

.

S

()

L

GM

GQ

This relation enables us to connect linearly the coefficient of Fourier ikp W with the components of displacements of the edge of hull in the following way:

[- K

$U \cos^2$

Z

+

I

$W \cos^2] \, dS$ if p

I

kI

=

GQ

H

. **C**

$U (Q)$

$\cos p \, dl$

$= S$

.

$L \, \mathbf{GQ}$

I

$W \cos^2 (p) \, dS$

if p

kp

l

S

In the same way, one a:

$-K$

$U \sin 2$

y

$+$

O

$W \sin 2 dS$

if p l

kl

$=$

GQ

H

$.C$

$U(Q)$

$\sin p dl$

$= S$

$L GQ$

O

$W \sin 2 (p) dS$

if p

kp

l

S

One uses for all these relations the fact that $\cos p \cos q dS$

$= 0$

if p Q.

S

One neglects in this expression the variation of metric in the thickness of the hull.

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Note:

One can note that some of the relations established in this paragraph for $p=1$ are redundant with those established in the paragraphs [§7.4] and [§7.5]. On the six relations established starting from the calculation of the integral forms

CPU cosdS

*,
CPU sindS*

*,
S*

S

C

GM

C

GM

GM

GM

U

cos dS,

U

sindS,
CPU

·
cos dS

and
CPU

·
sindS

, *only*

S
GM

S
GM

S GM

S GM

two among the four last are linearly independent of the others. Thus both first were already established with [the §7.4] and of the combinations of the four last give again those of [§7.5].

7.8 ***Establishment of the method***

The calculation of the coefficients of the linear relations is done in three times:

· *calculation of elementary quantities on the elements of the list of the meshes of edges of hulls (mesh of type SEG2):*

- *surface*

=

l ;

X;

y;

Z.

elt

elt

elt

elt

-

summation of these quantities on (S) from where the calculation of:

-

WITH = S

- *position*

of

G.

· knowing G, elementary calculation on the elements of the list of the meshes of edges of hulls of:

$$\mathbf{GM} = \{X, y, Z\}$$

N_i ;

xN_i ;

yN_i ;

zN_i

:

where

elt

elt

elt

elt

Nor =

of

form

of

functions

element

It should be noticed that in the case of the connection hull - pipe, the integrals on the elements of edge are to be multiplied by the thickness of the hull: $NR H NR$ where L represents average fibre of

$I =$

I

elt

L

h^3

the element of edge of hull. Moreover, one adds the additional term:

$NR.$

I

$12 L$

· “assembly” of the terms calculated above to obtain of each node of the section of connection, coefficients of the terms of the linear relations,

· connection between the modes of Fourier and displacements of hull as shown at the beginning [§7].

More precisely:

· for the connection hull - pipe, one carries out elementary calculations on all the elements of edge of the section of connection S of the type:

u.a.

C

N

U

2

2 X

Nb

elements S 2

X

U

U

P

cm = 1

C cos (

1

C

1

m) D =

U cos

C

(m) D =

cos (m) U D

y

C

N

0

0

=

U

1

N

C

1

U

R

Z

and

u.a.

C

N

U

2

2 X

Nb

elements S 2

X

U

U

P

Sm = 1

C sin (

1

C

1

m) D =

U sin

C

(m) D =

sin (m) U D

y

C

N

0

0

=

U
I
N

C
I

U
R
Z

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if $m > 1$ where \mathbf{P} is the matrix of passage of the local reference mark of the element to the total reference mark and

I

position orthoradiale of the element. By expressing displacement according to the degrees of freedom nodal:

I
U
N

N
U
m

Nbél

éments S 2

Nb _ nodes

X

U

cm =

l

O

v

cos m P

NR U

D

m =

() N () N

I

y

n=1

N

n=

W

l

l

N

U

m

Z

and

O

U

N

N

U

m

Nb

elements S 2

Nb _ nodes

X

U

Sm =

l

I

v

sin m P

NR U

D

m =

() N () N

O

y

n=1

N

n=

W

l

l

N

U

m

Z

where the NR are related to form of the element, one obtains for each calculation twice 9 coefficients with the nodes of the element running of S:

I

U

has

has

has

U

m

11

12

13

N

Nb _ nodes

X

O

ucm = *v*

has

has

has

U

.

m =

21 22 23 N

I

y

elements S

n=1

W

has

has

has

N

U

m

31

32

33

Z

2

L

1

1

N
has
 $\cos m P NR D$
 $\cos MX P X NR X dx$
 $ij =$
 $() ij () N () = () ij () N ()$

R

0
 1

and an equivalent expression for U
where I is the length of the element of edge of hull.

Sm

· for the connection 3D - pipe, one carries out elementary calculations on all the elements of edge of the section of connection S of the type:

$U 3D$
 X
 U
 $= 2 u3d$
 cm
 $($
 2
 $\cos m) dS = U 3D \cos$

$(m) rd D$
 S
 S
 S
 $S U 3D$
 R

3
 NN

D
 UN
 Nb

elements SH
 $2 2$

Nb _
X
nodes

= 2
cos (m) P NR (,) U3 rdd
N

D
y
S
n=1
NN

n=
H
1
3D
1 1

U Z
and
U 3D

X
usm = 2 3D
2
u.a. sin (m) dS = U 3D sin

(m) rd D
S
S
S

S U 3D
R

3
NN

D
U N
Nb

elements S H
2 2
Nb _
X

nodes

= 2

sin (m) P NR (,) U3 rdd

N

D

y

S

n=1

NN

n=

H

1

3D

1 1

U Z

if m > 1 where P is the matrix of passage of the local reference mark of the element to the total reference mark, the position

1

orthoradiale of the element, H its position radial and the NR are related to form of the element.

1

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8

Establishment of the element PIPE in Code_Aster

8.1 Description

This new element (of name METUSEG3) is pressed on a mesh SEG3 or curvilinear SEG4. It supposes that the section of the pipe is circular. Contrary to elements POU_D_E, POU_D_T, [R3.08.01] this element is not <<exact>> with the nodes for loadings or torques concentrated with ends. It is thus necessary to net with several elements to obtain correct results.

8.2

Introduced use and developments

The element is used in the following way:

AFFE_MODELE (MODELING = "TUYAU_3M"...)

The meshes with 4 nodes are generated starting from the meshes with 3 nodes using:

***EMAIL =CREA_MAILLAGE (MAILLAGE=MAIL,
MODI_MAILLE=_F (OPTION = "SEG3_4", ALL = "YES")
)***

One calls upon routine INI090 for the functions of form, their derivative and their derivative seconds (for the hull part) at the points of Gauss, as well as the corresponding weights.

The characteristics of the section are defined in AFFE_CARA_ELEM

***AFFE_CARA_ELEM (BEAM = _F (SECTION = "CIRCLE",
CARA = ("R" "EP"),
VALE = (.....),),
ORIENTATION=_F (GROUP_NO=D, CARA=' GENE_TUYAU', VALE= (X Y Z)),***

***TUYAU_NCOU = ' A NUMBER OF COUCHES', TUYAU_NSEC = ' A NUMBER OF
SECTEURS',),
)***

R and EP represent, as for the elements of beams traditional, respectively the ray external and the thickness of the section. One also defines on one of the nodes end of the line of piping the vector whose projection on the transverse section is the origin of the angles for decomposition in Fourier series. This vector should not be colinéaire with the average line of the elbow

with the node end considered.

One also defines in this level the number of layers and angular sectors to use for integration numerical.

AFFE_CHAR_MECA (DDL_IMPO = _F (
DX =. , DY =. , DZ =. , DRX =. , DRY =. , DRZ =. , DDL of beam
UI2 =. , VI2 =. , WI2 =. , UO2 =. , VO2 =. , WO2 =. , DDL related to mode 2
UI3 =. , VI3 =. , WI3 =. , UO3 =. , VO3 =. , WO3 =. , DDL related to mode 3
WO =. , WI1 =. , WO1 =. ,

DDL of swelling and mode 1 on W

FORCE_NODALE = _F (FX =. , FY =. , FZ =. , MX =. , MY =. , MZ =.)

They are the conventional forces of beam, which work only on displacements of beam.

Addition of a key word in AFFE_CHAR_MECA (FORCE_TUYAU = _F (CLOSE =.)) for the calculation of work of the internal pressure.

The pressure works on the DDL of swelling WO, one calculates then:

NR
L
2
L
NR
2
L

O
O

2
O
W
=
pw R ddx =
P
H W R ddx =

O O

int
0 0
int
H
Pr D dx W
near
K
K
K
0
K

0
int

K = 1
K = 1
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8.3

Calculation in linear elasticity

The matrix of rigidity and the matrix of mass (respectively options RIGI_MECA and MASS_MECA) are integrated numerically in the TE0582. Calculation holds account owing to the fact that the terms corresponding to the DDL of beam are expressed classically in total reference mark, and that DDL of

Fourier are in the local reference mark with the element. In the case or the element does not belong to any elbow, it

locate local is defined by the generator and the directing vector carried by average fibre of the element as indicated on [Figure 8.3-a]. If the element belongs to an elbow, the local reference mark is defined starting from the plan of the elbow as mentioned with [§2.1].

Z

y

Z

X

Z

O

Y

generator

X

Appear 8.3-a: Locate local for a right pipe

Elementary calculations (CALC_ELEM) currently available correspond to the options:

· EPSI_ELGA_DEPL and SIEF_ELGA_DEPL which provide the strains and the stresses at the points of integration in the local reference mark of the element. Calculation is carried out in the TE0584,

and currently gives the values to the 693 points of integration (for an element to 3 modes of Fourier). These fields are called fields at “under-points” of integration. One stores these values in the following way:

-

for each point of Gauss in the length, ($n=1, 3$)

-

for each point of integration in the thickness, ($n=1, 2NCOU+1=7$)

-

for each point of integration on the circumference, ($n=1, 2NSECT+1=33$)

-

6 components of strain or stresses:

EPXX EPYY EPZZ EPXY EPXZ EPYZ or SIXX SIYY SIZZ SIXY SIXZ SIYZ

where X indicates the direction given by the two nodes tops of the element, Y represents the angle describing the circumference and Z represents the ray. EPZZ and EPYZ corresponding to,

in the case of them

rr

R

deformations and SIZZ and SIYZ corresponding to,

in the case of them

rr R

constraints are taken equal to zero.

· *EFGE_ELNO_DEPL*: who gives the efforts generalize of beam traditional: NR, VY, VZ, MT, MFY, MFZ. These efforts are given in the local curvilinear reference mark of the element. This option is calculated in the TE0585.

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· *SIGM_ELNO_TUYO*, and *VARI_ELNO_TUYO* allow, starting from fields *SIEF_ELGA* or *VARI_ELGA*, and of the data of an angular position and a position in the thickness, of to calculate the constraints or variables intern with the nodes of the elements, with this position. One then obtains a field without under-points, i.e. a traditional field, usable for operations of postprocessing (for example the statement of a component of constraints length one line of piping).

· Options *EQUI_ELGA_SIGM* and *EQUI_ELGA_EPSI* allow the calculation of the invariants, in each point of integration (fields at “under-points”)

· Option *VALE_NCOU_MAXI* makes it possible to extract, of each of the 3 linear points of Gauss of an element, the values maximum and minimum of a component of a field.

· In version 7, one can extract from other fields in a point of the section: *EPSI_ELNO_TUYO* for the deformations, *SIEQ_ELNO_TUYO* for the equivalent constraints, *EPEQ_ELNO_TUYO* for the equivalent deformations.

Finally the TE0585 calculates also option *FORC_NODA* for operator *CALC_NO*.

8.4

Plastic design

The matrix of tangent rigidity (options RIGI_MECA_TANG and FULL_MECA) as well as projection plastic (options FULL_MECA and RAPH_MECA) are integrated numerically in the TE0586. One makes call to the option of calculation STAT_NON_LINE. All laws of plane constraints available in Code_Aster can be used: if they are not integrated directly, it is always possible to use a law of behavior formulated in plane deformation, and to treat the assumption of plane constraints using the method of Borst:

STAT_NON_LINE (...

*COMP_INCR = _F (RELATION = ' ', ALGO_C_PLAN=' DEBORST'),
.....)*

Elementary calculations (CALC_ELEM) currently available correspond to the options:

- SIEF_ELNO_ELGA which makes it possible to obtain the efforts generalized by element with the nodes in locate beam. This option is calculated in the TE0587.*
- VARI_ELNO_ELGA which calculates the field of internal variables by element with the nodes for all layers and all sectors, in the local reference mark of the element. This option is calculated in the TE0587.*

8.5

Test: SLL106A

It is about a right pipe of directing vector (4, 3, 0) fixed in its end O and which is with a grid with 18 elements PIPE.

y

B

X

Y

3

L

Z

O

X

O

4

L = 5

Z

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The pipe is subjected to different types of load:

- *a tractive effort,*
- *2 sharp efforts,*
- *2 bending moments,*
- *1 torque,*
- *an internal pressure.*

Displacements at the point B, the strain and the stresses in certain points are calculated of integration of the section containing B, as well as the first clean modes.

This makes it possible to test the DDL of beam, the DDL of swelling and modes 1 of the development in Fourier series.

9 Conclusion

The finite elements of elbow which we describe here are usable for calculations of piping in elasticity or in plasticity. Pippings can be subjected to various combined loadings - internal pressure, cross-bendings and anti-plane, torsion, extension.

For the moment, the element carried out is a linear element of beam type, right or curve, to three nodes, in small rotations and deformations, with a local elastoplastic behavior in plane constraints. It makes it possible to take into account ovalization, warping and swelling. It combine the properties of hulls and beams. The kinematics of beam for the axis of the elbow is increased by a kinematics of hull, of type Coils-Kirchhoff without transverse shearing, for

description of the behavior of the transverse sections. This last kinematics is discretized in M modes of Fourier, of which the number M , which the literature encourages us to choose equal to 6 [bib8], [bib13], must at the same time be sufficient to obtain good results in plasticity and not too large to limit its computing time. In elasticity, for relatively thick pipings (the thickness ratio on ray of the transverse section higher than 0.1), one can be satisfied with $M=2$ or 3.

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Elements of grid of reinforcement GRILLE_MEMBRANE

Summary:

This document describes the formulation and the establishment in Code_Aster of finite elements dedicated to the representation

steel reinforcements in a solid mass (for applications of Civil Engineering standard reinforced concrete). The principal ones

characteristics of these elements are as follows:

- *elements of membrane, without rigidity of torsion;*
- *not of unknown factors of rotation, therefore not of additional unknown factors to modeling “solid mass”, but*
- n the other hand not of possibility of offsetting;*
- *geometrical support: any mesh surface (triangle, quadrangle; linear or quadratic);*
- *only one direction of reinforcements.*

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1 Introduction

The elements of the type GRILLE_MEMBRANE come to supplement the possibilities of modeling of reinforcement in Code_Aster, in complement of modeling ROASTS. They below are presented differences between modelings GRILLE_MEMBRANE and GRID.

It is pointed out that modeling GRID is based on a kinematics of hull DKT with only one sleep in the thickness. This base DKT implies the presence of degrees of freedom of rotation with the nodes of the elements ROASTS: if it allows the concept of offsetting, it is useless when one does not have not need for offsetting (in this case it weighs down the useless model of way, because not only them degrees of freedom of rotation lengthen the vector of unknown factors, but it is necessary moreover block one number considerable of these degrees of freedom by double multiplier of Lagrange). Modeling GRILLE_MEMBRANE is a modeling based on a “surface” kinematics, it does not require not other degrees of freedom that usual displacements (on the other hand, obviously, this modeling does not make it possible to use the concept of offsetting).

Modeling ROASTS, based on DKT, requires geometric standards of support of the type linear triangle ; modeling GRILLE_MEMBRANE is developed starting from the supports geometrical surface triangle or quadrangle, linear or quadratic.

For the two types of modeling, on the other hand, only a direction of reinforcement is available by finite elements. That makes it possible to model any type of reinforcement to several directions, while superimposing

an element by direction; the cost of calculation generated by these duplications is weak: no duplication degrees of freedom (thus constant cost of inversion of matrix), duplication of the computation functions elementary (but they remain simple, numbers some reduced in 3D elements of surfaces against elements of volume and elementary calculations for structures with great numbers of degrees of freedom are of cost weak compared with the cost of inversion).

2

Formulation of the elements of GRILLE_MEMBRANE

For a tablecloth of uniaxial reinforcement, the deformation energy can be put in the form:

$$= S ds$$

with S the section of reinforcement per unit of length, the constraint (scalar) and deformation (scalar).

The only difficulty is to obtain an expression of the type = DRUNK

nodal

For that, it is necessary to use a little differential geometry. One leaves the expression of the derivative contravariante:

$$I$$

$$U$$

$$I$$

$$J$$

$$I$$

$$jk$$

$$U =$$

$$= U \text{ has has} = U G \text{ has has}$$

I

I

K

J

J

J

with {J} an acceptable parameter setting of surface and I = X has

I

while noting at the natural base (nonorthogonal, only the 3rd vector, normal on the surface, is normalized)

plan of the reinforcement and G metric the contravariante associated this base (cf [R3.07.04] .pour more details).

One then defines the direction of reinforcement by the vector normalized E (which one supplements, for the facility of

I the talk in a orthonormée base {E} and one calls R the operator of passage such as

*P
= R E has.*

I}

I

I

P

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One has then:

= (

U

U)

I

1

1

jk

=

R R G has

11

J

I K

By definition of R: 1

R = 0 and per definition of G: 13

23

G = G = 0

3

One notes in Greek the indices taking only the values in (, 1 2), one obtains:

U

= (*U*)

1

1

=

R R G has

11

If one now notes B^ the derivative of the functions of form at the point of Gauss considered, it comes:

R1

=
R1 G B^ has U

N
(
) *I in*

from where B sought. It will be noted that it with the shape of a vector, due to the scalar nature of the deformation sought.

B
R1
=
R1 G B^ has
in

N
(
) *I*

From B, there are then all the traditional expressions of the deformation, the nodal forces and of the tangent matrix which is written:

= *DRUNK*
F = T
B

K
T
= *B*
B

It will be noted that in fact the laws of behavior 1D are used to obtain the constraint to be left deformation.

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Author (S):

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Organization (S): EDF-R & D /AMA, LMT Cachan

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Document: R3.08.08

Multifibre element of beam (right)

Summary:

This document presents the elements of multifibre beam of Code_Aster based on a resolution of a problem of beam for which each section of a beam is divided into several fibres. Each fibre behaves

then like a beam of Euler.

The beams are right (Element POU_D_EM). The section can be of an unspecified form.

The assumptions selected are as follows:

- assumption of Euler: transverse shearing is neglected (this assumption is checked for forts twinges),*
- the elements of beam introduces here do not make it possible to make correct calculation in torsion.*

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Notations

One gives the correspondence between this notation and that of the documentation of use.

DX, DY, DZ and DRX, DRY, DRZ are in fact the names of the degrees of freedom associated with the components

displacement U, v, W, X, y, Z.

E

Young modulus

E

Poisson's ratio

NAKED

G

E

G

modulate of Coulomb = (

2 I+)

I

y,

y, I Z

geometrical moments of inflection compared to the axes

Z

IY, IZ

J X

constant of torsion

JX

K

stamp rigidity

M

stamp of mass

M

X, y,

X, M y, M Z

moments around the axes

Z

MT, MFY, MFZ

NR

normal effort with the section

NR

S

surface of the section

With

U, v, W

translations on axes X, y, Z

DX DY DZ

V

sharp efforts along axes y, Z

y, Vz
VY, VZ

density
RHO

rotations around axes X, y, Z
X, y, Z
DRX DRY DRZ

qx, qy, qz
External linear efforts

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1 Introduction

The analysis of the structures subjected to a dynamic loading requires models of behavior able to represent non-linearities of material.

Many analytical models were proposed. They can be classified according to two groups: has) detailed models founded on the mechanics of the solid and their description of the local behavior of material (microscopic approach) and b) of the models based on a total modeling of

behavior (macroscopic approach). In the first type of models, we can find them traditional models E.F as well as the models of the type “fibre” (having an element of the beam type how support).

While “traditional” models E.F are powerful tools for the simulation of nonlinear behavior of the complex parts of the structures (joined, assemblies,...), their application to the totality of a structure can prove not very practical because of a computing time prohibitory or of the size memory necessary to the realization of this calculation. On the other hand, a modeling of multifibre beam type (see [Figure 1-a]), has the advantages of the simplifying assumptions of a kinematics of the beam type of Euler - Bernoulli while offering a practical and effective solution for a nonlinear analysis complexes composite elements of structures such as those which one can meet for example out of reinforced concrete. Moreover, this “intermediate” modeling is relatively robust and inexpensive in time calculation because of use of nonlinear models of behavior 1D.

Appear 1-a: Description of a modeling of the multifibre beam type

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2

Element of theory of the beams (recalls)

One takes again here the elements developed within the framework of the elements of beam of Euler ([bib4]).

A beam is a solid generated by a surface of surface S of which the geometrical centre of inertia G followed a curve C called the average fibre or neutral fibre. The surface S is the cross-section (section transversal) or profile, and it is supposed that if it is evolutionary, its evolutions (size, form) are continuous and progressive when G describes the average line.

For the study of the beams in general, one makes the following assumptions:

- the cross-section of the beam is indeformable,*
- transverse displacement is uniform on the cross-section.*

These assumptions make it possible to express displacements of an unspecified point of the section, in function of displacements of the point corresponding located on the average line, and according to one increase in displacement due to the rotation of the section around the transverse axes.

The discretization in “exact” elements of beam is carried out on a linear element with two nodes and six degrees of freedom by nodes. These degrees of freedom are the three translations U, v, W and the three rotations X, y, Z [Figure 2-a]).

Z
y
1
2
X
U X
U X
v y
v y
W Z
W Z

Appear 2-a: Element beam

Waited until the deformations are local, it is built in each top of the grid a base local depending on the element on which one works. The continuity of the fields of displacements is ensured by a basic change, bringing back the data in the total base.

In the case of the right beams, one traditionally places the average line on axis X of the base local, transverse displacements being thus carried out in plan (y, Z).

Finally when we arrange sizes related to the degrees of freedom of an element in a vector or an elementary matrix (thus of dimension 12 or 122), one arranges initially the variables for top 1 then those of top 2. For each node, one stores initially the sizes related to three translations, then those related to three rotations. For example, a vector displacement will be structured in the following way:

U, v, W, U, v, W,
11 1 4
4
4 1 21x y 4
4
4 1 31z 12 2 4
4
4
4
2 22x y2 4
4
4
4
3
z2
top 1

top 2

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3

Equations of the movement of the beams

We will not include in this document all the equations of the movement of the beams. For more complements concerning this part one can refer to documentation concerning elements POU_D_E and POU_D_T ([bib4]).

4

Element of right beam multifibre

One describes in this chapter obtaining the elementary matrices of rigidity and mass for the element of multifibre right beam, according to the model of Euler. The matrices of rigidity are calculated with options “RIGI_MECA” or “RIGI_MECA_TANG”, and matrices of mass with option “MASS_MECA” for the coherent matrix, and option “MASS_MECA_DIAG” for the matrix of diagonalized mass. We present here a generalization [bib3] where the reference axis chosen for the beam is independent of any geometrical, inertial or mechanical consideration. The element functions for an unspecified section (heterogeneous is without symmetry) and is thus adapted to an evolution not linear of the behavior of fibres.

One also describes the calculation of the nodal forces for the nonlinear algorithms: “FORC_NODA”

and

“RAPH_MECA”.

4.1

Element beam of reference

[Figure 4.1-a] the change of variable shows us realized to pass from the real finite element [Figure 2-a] with the finite element of reference.

y
y
1
2
1
2
X
0
L
X
u1 1 X
u2 2 X
Z
Z
v1 1 y
v2 2 y
w1
1 Z
w2 2 Z

Appear 4.1-a: Element of reference vs real Elément

One will then consider the continuous field of displacements in any point of the average line by report/ ratio

with the field of displacements discretized in the following way:

$$\{Custom\} = [NR] \{U\}.$$

The index S indicates the quantities attached to average fibre.

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By using the functions of forms of the element of reference, the discretization of the variables custom (X), vs (X), ws (X), sx (X), sy (X), sz (X) becomes:

I
U

I
v
W
I
U

S (X)

N1
0
0
0
0
0
0
N2
0
0
0
0
0 1
X
v

$S(X)$

0

$N3$

0

0

0

$N4$

0

$N5$

0

0

0

$N6$ 1y

$ws(X)$

0

0

$N3$

0

- $N4$

0

0

0

$N5$

0

- NR

0

6

=

1

Z

sx 0

0

0

NI

0

0

0

0

0

N2

0

0 u2

0

0

sy

- NR, 3

0

X

N4,

0

0

0

X

- NR

0

NR

0 v

,

5 X

6, X

2

NR

NR

NR

NR

W

sz

0

,

3

0

0
0
X
4,
0
X
,
5
0
0
0
X
6, X 2

x2
y2

z2

éq 4.1-1

With the following functions of interpolation:

X
1
N1 = 1
; NR, 1 = -
L
X
L
X
1
N2 =
; N2, =
L
X
L
2
3
X
X
6
X
N3 = 1 - 3

+ 2
 ; NR, 3 = -
 +12
 2
 3
 xx
 2
 3
 L
 L
 L
 L
 2
 3
 X
 X
 4
 X

éq
4.1-2
 $N4 = X - 2$
 +
 ; N4, = - + 6
 2
 xx
 2
 L
 L
 L
 L
 2
 3
 X
 X
 6
 X
 $N5 = 3$
 - 2
 ; NR, 5 =
 -12
 2

3
xx
2
3
L
L
L
L
2
3
X
X
2
X
N6 = -
+
; N6, = - + 6
2
xx
2
L
L
L
L

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4.2***Determination of the matrix of rigidity of the multifibre element*****4.2.1 Case general (beam of Euler)**

Let us consider a beam of Euler, right-hand side directed in direction X, subjected to efforts distributed

qx, qy, qz [Figure 4.2.1-a].

Y, v

X, U

Z, W

Appear 4.2.1-a: Beam of Euler 3D

The fields of displacements and deformations take the following form then when it is written displacement of an unspecified point of the section according to the displacement (custom) of the line of average:

$$U(X, y, Z) = \text{custom}(X) - ysz(X) + zsy(X) \quad \text{éq}$$

4.2.1-1

$$v(X, y, Z) = vs(X)$$

éq

4.2.1-2

$$(W(X, y, Z) = ws(X)$$

éq

4.2.1-3

$$= u'$$

,

,

xx

$$X(X) - ysz(X) + Zsy(X)$$

éq

4.2.1-4

$$xy = xz = 0$$

éq

4.2.1-5

Note:

- *Torsion is treated overall separately, one does not calculate yz here.*
- *f' (X) indicates the derivative of F (X) compared to X.*

By introducing the equations [éq 4.2.1-4] and [éq 4.2.1-5] into the principle of virtual work one obtains:

L
xx xx 0
FD =
U
X Q
v
X Q
W

X Q dx
éq
4.2.1-6

V
(S () X + S () y + S () Z)
0
0

qx, qy, qz indicating the linear efforts applied.
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What gives by using the equation [éq 4.2.1-1]:

L (NR u'

,

,

,

S (X) + M xsx (X) + M ysy (X) + M zsz (X))dx =

0

L (custom (X) qx + vs (X) qy + ws (X) qz) dx

0

éq 4.2.1-7

With:

NR = xxdS; M y = Z dS; M

y dS éq

4.2.1-8

S

xx

Z =

S

-

S

xx

Note:

- **The torque M X is not calculated by integration but is not calculated directly with to leave the stiffness in torsion (see [éq 4.2.2-4]).**
- **The theory of the beam associated with an elastic material gives: xx = E xx**

4.2.2 Case of the multifibre beam

We suppose now that the section S is not homogeneous [Figure 4.2.2-a].

Without adopting particular assumption on the intersection of axis X with the section S or on

orientation of the axes Y, Z, the relation between the “generalized” constraints and deformation “generalized” Ds becomes [bib2]:

$$S$$
$$F = KS Ds$$

éq 4.2.2-1

with:

$$S$$
$$F = (NR, M y, M Z, M) T$$
$$X$$

éq
4.2.2-2

$$D$$
$$,$$
$$,$$
$$,$$
$$,$$
$$S = (custom (X), sy (X), sz (X), (X))T$$

sx
Center
Cross-section
Material 1
Material 3
Material 2

Appear 4.2.2-a: Section S unspecified - multifibre beam
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The matrix $K S$ can then be put in the following form:

K_{11}

S

K_{12}

S

K_{13}

0

S

$K_{s22} K_{s23}$

0

K_s

éq

4.2.2-3

K

0

s_{33}

sym

K_{s44}

with:

$K_{s11} = EdS; K_{s12} = Ezds; K$

$Eyds$

S

$s_{13} = -$

S

S

éq

4.2.2-4

$$K$$

$$2$$

$$2$$

$$s_{22} = E_z dS; K_{s23} = - E_{yz} ds; K$$

$$E_y ds$$

$$S$$

$$s_{33} =$$

$$S$$

$$S$$

where E can vary according to y and Z . Indeed, it may be that in modeling planes section [Figure 4.2.2-a]), several materials cohabit. For example, in a section concrete reinforced, there are at the same time concrete and reinforcements.

The discretization of the fibre section makes it possible to calculate the integrals of the equations [éq 4.2.2-4].

The calculation of the coefficients of the matrix $K S$ is detailed in the paragraph [§4.2.3] according to.

Note:

The term of torsion $K s_{44} = GJ X$ is given by the user using the data of $J X$.

The introduction of the equations [éq 4.2.2-1] to [éq 4.2.2-4] in the principle of virtual work leads to:

$$L T$$

$$D$$

$$S K_s D_s dx -$$

$$0 \text{ éq}$$

4.2.2-5

$$0$$

$$L (custom (X) q_x + v_s (X) q_y + w_s (X) q_z) dx =$$

$$0$$

The generalized deformations are calculated by (D_s is given to the equation [éq 4.2.2-2]):

$$D_s = \{$$

$$B U\}$$

$$\text{éq}$$

4.2.2-6

With the following matrix B :

$$NR$$

0
0
0
0
0
NR
0
0
0
0
0
,
1 X
2, X

0
0
- NR
0
NR
0
0
0
,
3 xx
4, xx
- NR
0
NR
0
,
5 xx
6, xx

B =

0
NR
0
0
0
NR

0
NR
0
0
0
NR

,
3 xx
4, xx

,
5 xx
6, xx

0
0
0
NR
0
0
0
0
0
NR
0
0

,
1 X
2, X

éq 4.2.2-7

Discretization of space [
0 L] with elements and the use of the equations [éq 4.2.2-5] returns
the equation [éq 4.2.1-6] equivalent to the resolution of a traditional linear system:

$KU = F$ éq

4.2.2-8

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The matrix of rigidity of the element [Figure 4.2.2-b] and the vector of the efforts results are finally given by:

L

K

=

BT K B dx

elem

S

0

éq

4.2.2-9

L

F =

NR T Q dx

0

Center

Cross-section

Points of integration

Under-points of integration

L

E ds

E y ds

T

S
S

K
B K B dx
elem =
K S =

S

2

0

E y ds
E y ds
S
S

Appear 4.2.2-b: Multifibre beam Calculation of Kelem

With the vector Q which depends on the external loading: $Q = (q_x \ q_y \ Q \ 0 \ 0 \ 0) \ T$
Z

If we consider that the efforts distributed q_x, q_y, q_z are constant, we obtain the vector nodal forces according to:

T
Lq Lq Lq
L2q L2q
Lq
Lq
Lq
L2q L2q

X

y

Z

Z

y

X

y

Z

Z

y

F

=

0 -

0

éq

4.2.2-10

2

2

2

12

12

2

2

2

12

12

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4.2.3 Discretization of the fibre Calculation section of Ks

The discretization of the fibre section makes it possible to calculate the various integrals which intervene in the matrix of rigidity. Thus, if we have a section which comprises N fibres we will have them approximation following of the integrals:

$$\begin{aligned}
 &N \\
 &N \\
 &N \\
 &K_{s11} = E_i S_i; K_{s12} = E_i z_i S_i; K_{s13} = I.E.(\text{internal excitation}) y_i S_i \\
 &i=1 \\
 &i=1 \\
 &i=1
 \end{aligned}$$

éq 4.2.3-1

$$\begin{aligned}
 &N \\
 &N \\
 &N \\
 &K \\
 &2 \\
 &2 \\
 &s_{22} = I.E.(\text{internal excitation}) z_i I_f \\
 &; K_{s23} = - I.E.(\text{internal excitation}) y_i z_i S_i; K_{s33} = I.E.(\text{internal excitation}) y_i I_f \\
 &i=1 \\
 &i=1 \\
 &i=1
 \end{aligned}$$

with I.E.(internal excitation) and I_f the initial or tangent module and the section of each fibre. The state of stress is constant by fibre.

Each fibre is also located using y_i and z_i the co-ordinates of the centre of gravity of fibre compared to the axis of the section defined by order "COO_AXE_POUTRE" (document "AFFE_SECT_MULTI").

4.2.4 Integration in the linear elastic case (RIGI_MECA)

When the behavior of material is linear, the element beam is homogeneous in its length, the integration of the equation [éq 4.2.2-9] can be made analytically. One obtains the matrix of following rigidity then:

K
K
K

s11

s12

s13

- Ks11

- Ks12

- Ks13

0

0

0

0

0

0

L

L

L

L

L

L

12Ks33 -12K

6K

6K

s23

s23

s33

-12K

12K

6K

6K

s33

s23

s23

s33

0

0

0

L3

L3

L2

L2
L3
L3
L2
L2

12Ks22
- 6Ks22 - 6K
12K
s23
s23
-12Ks22
- 6Ks22 - 6Ks23

0
0
0
3
2
2
3
3
2
2

L
L
L
L
L
L
L

Ks44
0
0
0
0
0
0
- Ks44
0

0

L

L

4K

4K

s22

s23

- Ks12 - 6K

6K

2K

2K

s23

s22

s22

s23

0

2

2

L

L

L

L

L

L

L

4Ks33

- Ks13 - 6K

6K

2K

2K

s33

s23

s23

s33

0

L
L
L2
L2
L
L

Kelem =
K
K
K

s11
s12
s13
0
0
0

L
L
L

12Ks33
-12Ks23
- 6Ks23 - 6Ks33
0

L3
L3
L2
L2

12K
6K
6K

SYM

s22

s22

s23

0

3

2

2

L

L

L

Ks44

0

0

L

4K

4K

s22

s23

L

L

4Ks33

L

éq 4.2.4-1

with the following terms: K 11,

S

K 12,

S

K 13,

S

Ks22, Ks33, Ks23, Ks44 are given to the equation

[éq 4.2.2-4].

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4.2.5 Integration in the non-linear case (RIGI_MECA_TANG)

When the behavior of material is nonlinear, to allow a correct integration of efforts intern (see paragraph [§4.4]), it is necessary to have at least two points of integration it length of the beam. We chose to use two points of Gauss.

The integral of Kelem [éq 4.2.2-9] is calculated in numerical form:

L

2

T

Kelem =

B K

S B dx = J I

W B (X) T

I

Ks (xi) B (xi) éq

4.2.5-1

0

i=1

*where x_i are the position of the point of Gauss I in an element of reference length 1,
 $1 \pm 0,57735026918963)/2$*

I

W is the weight of the point of Gauss I. I here is taken

W = 0,5 for each of the 2 points

*J is Jacobien One takes J here = L, the real element having a length L and the function of form
X*

to pass to the element of reference being

.

L

*Ks (x_i) is calculated using the equations [éq 4.2.2-3], [éq 4.2.2-4] (see paragraph [§4.2.3] for
the numerical integration of these equations)*

The analytical calculation of B (X) T

I

Ks (x_i) B (x_i) gives:

21

B K 11

S

- 1

B 2

B K 13

S

1

B 2

B K 12

0

S

- 1

B 3

B K 12

S

- 1

B 3

B K 13

S

- 21

B K 11

S

1

B 2

B K 13

S

- 1

B 2

B K 12

0

S

- 1

B 4

B K 12

S

- B B K

1 4

13

S

2

2

2

B Ks33

2

B Ks23

0

2

B 3

B Ks23

2

B 3

B Ks33

1

B 2

B K 13

S

- 2

2

2

B Ks33

2

B Ks23

0

2

B 4

B Ks23

2

B 4

B Ks33

2

2

2

2

B Ks22

0

- 2

B 3

B Ks22 - 2

B 3

B Ks23 - 1

B 2

B K 12

S

2

B Ks23

- 2

B Ks22

0

- 2

B 4

B Ks22 -

2

B 4

B Ks23

2

1

B Ks44

0

0

0

0

0

- 21

B Ks44

0

0

2
2

3
B Ks22

3
B Ks23

1
B 3
B K 12

S
- 2
B 3
B Ks23

2
B 3
B Ks22

0
3
B 4
B Ks22

3
B 4
B Ks23

2
3
B Ks33

1
B 3
B K 13

S
- B B K
B B K

0
B B K
B B K

2 3 s33
2 3 s23

3 4 s23

3 4 s33

2

1

B K 11

S

- 1

B 2

B K 13

S

1

B 2

B K 12

0

S

1

B 4

B K 12

S

1

B 4

B K 13

S

2

2

B Ks33

- 22

B Ks23

0

- 2

B 4

B Ks23 - B B K

2 4 s33

2

2

B Ks22

0

2

B 4

B Ks22

2

B 4

B Ks23

2

B K

0

0

1

s44

2

2

4

B Ks22

4

B Ks23

2

4

B Ks33

éq 4.2.5-2

where I

B are calculated with X-coordinate xi of the element of reference with:

1

B = - NR

= NR

1

,

1 X

2, X = L

6

X

B = NR

= - NR

I

2

,

3 xx

,

5 xx = -

+12

L2

L2 *éq*

4.2.5-3

4

X

$B = NR$

I

3

4, xx = -

+ 6

L

L

2

X

$B = NR$

I

4

6, xx = -

+ 6

L

L

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4.3 *Determination of the matrix of mass of the multifibre element*

4.3.1 *Determination of Melem*

In the same way, the virtual work of the efforts of inertia becomes [bib2]:

$$\begin{aligned}
&L \\
&D^2 u_{X, y} \\
&D^2 v_{X, y} \\
&D^2 w_{X, y} \\
&\text{inert} \\
&W \\
&= , \\
& , \\
& , \\
&0 \\
&U(X, y) \\
&(\) + v(X, y) (\) + (wx, y) (\) dS dx \\
&S \\
&2 \\
&2 \\
&2 \\
&dt \\
&dt \\
&dt \\
&2 \\
&= L \\
&D U.S. \\
&U \\
&Sm S \\
&dx
\end{aligned}$$

0
dt 2
éq 4.3.1-1

with $U.S.$ the vector of “generalized” displacements.

What gives for the matrix of mass:

M_{11}
0
0
 S
 M_{12}
 S
 M_{13}
0
 S

M_{11}
0
0
0
 S
- M_{12}
 S

M_{11}
0
0
 S
- M

$M S =$
13
 S
éq
4.3.1-2

$M_{s22} M_{s23}$
0

M
0

s33

sym
M s22 + M s33

with:

M s11 = ds; Ms12 =
zds

; M
yds

S

s13 = -
S
S

éq
4.3.1-3

M
2
2
s22 = Z
ds; M s23 = -
yzds

; M
y
ds
S

s33 =
S
S

with which can vary according to y and Z.
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As for the matrix of rigidity, we take into account the generalized deformations and discretization of space [0, L]. What gives finally for the elementary matrix of mass:

1

M

elem

2

M

elem

3

M elem

4

M elem

5

M elem

6

M elem

M elem =

7

M elem

8

M

elem

9

M

elem

10

M elem

11

M elem

12

M elem

with:

LM

M

M

LM

LM

LM

M

M

LM

LM

1

11

S

-

13

S

12

S

12

S

13

S

11

S

13

S

-
12
S
-
12
S
-
13
S

M elem =
0
0

3
2
2
2
12
6
2
2
12
12

LM
M
M
LM
M
LM
M
M
LM
M
M
LM
M
LM
M
2
13
6

11

S

s33 - 6

s23 - 7

11 2

12

S

s23

11

S

s33 -

9

6

6

13

S

11

S

s33

s23 - 3

12

S

s23 - 13 2

M

sym

elem =

+

+

-

11

S

+ s33

35

5L

5L

20

10

210

10

2

70

5L
5L
20
10
420
10

LM
M
LM
LM
M
M
M
M
LM
M
LM
LM

M
M
3
13
6
11
S
s22 - 7
S

-11 2
13
11
S

s22 -
6
9
6
s23

12
S
s23
11
S
S 22 - 3

13 2

13

S

11

S

s22 -

M

sym sym

elem =

+

-

-

-

s23

35

5L

20

210

10

10

2

5L

70

5L

20

420

10

10

LM

LM

L M

L M

LM

LM

LM

LM

L M

L M

4

s22 +

2

s33

S

- 2

13

12

S

- 3

12

S

- 3

13

S

s22 +

s33 -

2

2

M

sym sym sym

elem =

0

13

S

12

S

3

20

20

20

20

6

30

30

3

LM

LM

LM

LM

M

LM

M

LM

LM

LM

LM

5

2

2

11

S

S 22

s23 -

12

S

- s23 -13 2

2

11

S

s22

S

- 3

13

11

S

S 22 -

M

sym sym sym sym

elem =

+

+

-

-

s23

105

15

15

12

10

420

10

30

140

30

30

3

LM

LM

LM

LM

M

M

LM

LM

LM

LM

6

2

11

S

s33 -

13 2

13

S

11

S

s33

S

- 2

23

12

S

-

s23 -

3

M

sym sym sym sym sym

elem =

+

-

11

S

-

s33

105

15

12

420

10

10

30

30

140

30

LM

M

M

LM

LM

7

11

S

13

S

-

12

S

12

S

13

S

M

sym sym sym sym sym sym

elem =

0

3

2

2

12

12

LM

M

M
LM
M
LM
M
8
13
6
11
S
s33 - 6
S 23 - 7
12
S
- s23 -11 2

M
sym sym sym sym sym sym sym
elem =
+
11
S
- s33

35
5L
5L
20
10
210
10

LM
M
LM
LM
M
M
9
13
6
11
S
s22 - 7

11 2

M

sym sym sym sym sym sym sym sym

elem =

+

13

S

11

S

+

s22

s23

35

5L

20

210

10

10

LM

LM

L M

L M

10

s22 +

s33 -

2

2

M

sym sym sym sym sym sym sym sym

elem =

13

S

12

S

3

20

20

3

LM

LM

LM

11

2

2

M

=

11

S

sym sym sym sym sym sym sym sym sym sym

elem

+

S 22

*s*23

105

15

15

3

LM

LM

12

2

M

=

11

S

sym sym sym sym sym sym sym sym sym sym

elem

+

*s*33

105

15

éq 4.3.1-4

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with the following terms: M

, M, M, M

, M, M

who are given to the equation

11

S

12

S

13

S

s22

s33

s23

[éq 4.3.1-3].

Note:

*The matrix of mass is reduced by the technique of the concentrated masses ([bib4]). This stamp of diagonal mass is obtained by option “**MASS_MECA_DIAG**” of the operator **CALC_MATR_ELEM**.*

4.3.2 Discretization of the fibre section - Calculation of ms

The discretization of the fibre section makes it possible to calculate the various integrals which intervene

in the matrix of mass. Thus, if we have a section which comprises N fibres we will have their approximation following of the integrals:

N

N

N

M

$S; M$

$Z S; M$

$y S$

$s_{11} = I I$

$s_{12} = I I I$

$s_{13} = - I I I$

$i=1$

$i=1$

$i=1$

$\acute{e}q$

4.3.2-1

N

N

N

M

$Z 2S; M$

$y Z S; M$

$y 2S$

$s_{22} = I I$

I

$s_{23} = - I I I I$

$s_{33} = I I$

I

$i=1$

$i=1$

$i=1$

with S density and the section of each fibre. y and Z are the co-ordinates of

I

I

I

I

centre of gravity of fibre defined as previously.

4.4

Calculation of the internal forces

The calculation of the nodal forces F due in a state of internal stresses given is done by
int
the integral:

L

F

éq

4.4-1

int =

$BT F dx$

S

0

where B is the matrix giving the generalized deformations according to nodal displacements
[éq 4.2.2-6] and where F is the vector of the generalized constraints given to the equation [éq 4.2.2-2],
 S

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Center Section

Points of integration

Under-points of integration

L

T

ds

F

B F

dx

int =

S int

F

S

= S

int

0

y ds

S

Appear 4.4-a: Multifibre beam Calculation of Fint

T

F = NR M

M

M

éq

4.4-2

S

[

y

Z

X]

The normal effort NR and the bending moments M and M are calculated by integration of the constraints

y

Z

on the section [éq 4.2.1-8].

Behaviour in linear torsion remaining, the torque is calculated with displacements

nodal:

-

$$M = GJ$$

x^2

x^1

éq.

4.4-3

X

X

L

The equation [éq 4.4-1] is integrated numerically:

2

$$F = L T$$

$B F dx$

J

$W B X$

$F X \text{ éq}$

4.4-4

I

S

=

0

$I () T$

I

$S (I)$

$i=1$

The positions and weights of the points of Gauss as well as Jacobien are given in the paragraph [§4.2.5].

The analytical calculation of $B (X) T F X$ gives:

I

$S (I)$

$$[B (X) T F X = - B N R B M - B M 0 B M R. B M R. B N - B M R. B M 0 B M R. B M$$

I

$S ($

$)] T$

I

$[1 2 Z 2 y$

3

y

3

Z
1
2
Z
2
y
4
y
4
Z]

éq 4.4-5

where B are given to the equation [éq 4.2.4-1].

I

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4.5

Nonlinear models of behavior usable

The supported models are on the one hand the relations of behavior 1D of the type ECRO_LINE and PINTO_MENEGOTTO [R5.03.09], in addition the model Labord_1D [R7.01.07] dedicated to the behavior of concrete into cyclic. In addition all the laws 3D are usable thanks to a routine “shunting” which puts all the deformations other than the axial deformation (along fibre) at zero.

Note:

The internal, constant variables by fibre, are stored in the attached under-points at the point of integration considered.

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Version

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Titrate:

Model of thermics for the thin hulls

Date:

03/11/93

Author (S):

F. VOLDOIRE, S. ANDRIEUX

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Page:

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Organization (S): EDF/IMA/MMN

Handbook of Reference

R3.11 booklet: Thermal elements on average surface

Document: R3.11.01

Formulation of a model of thermics

for the thin hulls

Summary:

The model presented here results from the asymptotic analysis of the equations of thermics when the thickness from the structure tends towards zero.

The temperature is described by 3 fields defined on the average surface of the hull.

One shows on some examples, the capacities of the model by reference to 3D solutions.

The applications concerned are thermomechanical calculations of hulls, the thermal restitution of wall for

thermohydraulics of pipings, problems of identification.

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1 Introduction

The mechanical models of mean structures (hulls and plates) arrived at a stage of extremely at least advanced development for the homogeneous elastic structures in the thickness. The problem is known since strong a long time and various theories were born, generally dedicated to specific problems (thick hulls, buckling etc...). However one basic model, that of LOVE-KIRCHHOFF, achieve the unanimity in the most current applications.

The difficulties lie rather in the numerical calculation of this one of the fact, on the one hand, of the need

to approach correctly the surface of the hull (in particular its curve), and in addition of the order raised partial derivative equations which should be solved (4th order).

In thermics on the other hand, the situation is much less clear and a great number of approaches coexist. It is indeed only recently that the problem arose with the possibilities (and need) for thermomechanical calculations. The first models neglect conduction in parallel on average surface to retain only the thermal transfers in the thickness of the hull, this step is completely paradoxical among that of the mean structures where, on the contrary, the low thickness

structure leads to simplifying assumptions on the variation in the thickness of the fields physical sizes.

The most recent modelings take as a starting point the the mechanical ideas of thin hulls being attached

with the second approach, one can classify them according to a completely similar order.

1) Models utilizing a more or less thorough development polynômial of the temperature in the thickness [bib2], [bib9], [bib10]. It is primarily about formulation of finite elements.

2) Models associated with the theories with surfaces to directors (Surfaces of COSSERAT) [bib5], [bib8].

The director is here the gradient of the temperature in the thickness. The problem of these approaches reside in the law of behavior to introduce. Coherence with the three-dimensional law led with choices which are interpreted like an assumption of linear distribution of the temperature in the thickness. This formalism thus joined practically the preceding models (the introduction of several directors being identified with various orders of development of the polynomials).

3) Models of degenerated finite elements [bib11]: on the basis of a three-dimensional finite element, the introduction

constraints between the degrees of freedom located on the same normal at average surface allows by condensation to deduce an element from "thermal hull". Practically, there still, the basic element using a parabolic interpolation according to the thickness, the element of hull corresponds to a linear distribution in the thickness.

Parallel to these approaches numerical (1) and (3) or based on assumptions a priori (2), results on the shape of the field of temperature of a thin section and problem of which it is solution were obtained by asymptotic methods [bib3], [bib1].

As for the mechanical model, those make it possible to justify the assumptions made a priori in mean theories of hulls, to even obtain the equations of the problem of hull. Results of [bib 1] are recalled low and will be used as a basis for the model suggested. Let us note simply here that the idea

subjacent with any step of the asymptotic type is to introduce a small parameter (here the report/ratio thickness of the plate on a dimension characteristic of this one), then having obtained the problem limit when tends towards zero starting from the three-dimensional problem, to approach in the applications

(where takes obviously a nonnull value) the solution by its limit.

From a practical point of view, the limit obtained for the equations of stationary thermics seems

to be too “poor” to be of a real interest, (one will give of it an illustration in [β2.2.2]). More precisely the values to reach to identify the solution with its limit are very small in real situations met.

This is why one proposes in this note to keep the form of the limiting solution (parabolic distribution in the thickness) but to lay out about it like assumption a priori on the three-dimensional solution allowing to bring back itself to a problem arising on average surface.

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One thus has an approximate model of mean structure converging towards the model limits three-dimensional equations. In this direction, it is “optimal” since a linear assumption of distribution in the thickness leads to a model not converging towards the limiting solution and only one model based on a richer development in the thickness sees its terms of a nature higher than two to converge towards zero when the hull is thin.

The plan of the note is as follows:

- one starts by pointing out the equations of the stationary thermal problem for the solid three-dimensional and their expressions in a frame of reference adapted to the cases where the solid is a “thin hull”,*
- then, having pointed out the results of an asymptotic study of these equations carried out in the case of a plate, one gives the complete description of the model suggested,*
- one then applies the model to a certain number of geometries and thermal loadings and a comparison is made compared to analytical solutions or numerical calculations three-dimensional,*
- finally, one gives some indications on the numerical aspects of the use of the model in one calculation by surface and linear finite elements.*

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Code_Aster ®**Version****2.6****Titrate:****Model of thermics for the thin hulls****Date:****03/11/93****Author (S):****F. VOLDOIRE, S. ANDRIEUX****Key:****R3.11.01-A****Page:****5/44****2****Presentation of the model****2.1****Position of the thermal problem in the hulls**

In this paragraph, we first of all will point out the description of the geometry of the hulls, sights like thin three-dimensional solids. One will pose then the thermal problem of conduction.

2.1.1 Description of the geometry

A hull is defined as being a solid, thin perpendicular to an average surface.

One notes $2h$ the thickness of the hull; one chooses a frame of reference (x_1, x_2) on surface.

One notes G the associated metric tensor, N the normal vector, C the tensor curve of.

 N **x_1** **x_2** **+** **$2h$** **-** **$I =] - H, H$** **Appear: 2.1.1-a**

The hull is described by the frame of reference (X, x_3) , x_3 according to $N: = X] - H, H [$ (The Greek indices, β , are dedicated to the surface co-ordinates on).

This description is appropriate of course for a hull thickness $2h$ lower than the smallest ray of curve of.

In an unspecified point (X, x_3) of the hull, the metric tensor G is expressed according to fundamental tensors G and C of average surface by:

$$G_{X, x_3} = G_X - 2 x_3 C_X$$

$$G_3 = 0, G_{33} = 1$$

éq 2.1.1-1

and $\det G = \det G_1 - x_3 \operatorname{tr} C$

$= \det G \mathbf{1} + x^3 \mathbf{1} + \mathbf{1}$

R_1

R_2

where R_1, R_2 are the principal radii of curvature of as in point X considered.

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Note:

It is known indeed that the trace (tr) of a tensor is an invariant (by basic change). One has the practice however to write the sizes in orthonormée physical base i.e. As follows:

phy

phy

$G_X, x^3 = - 2x^3 C_X.$

phy

1

$C = -$

And if the base is principal of curve:

R

(without summation).

$\mathbf{1} + \mathbf{1} = H$

1

1

$-\mathbf{1} = H_2$

One will note: R

.

1

R_2

and R1

R 2

One limited oneself here under the first order in x^3 tr C; as it subsequently will be done. In practical, indeed, the thinness of the hull allows such a simplification. There will be advantage also with

to place in a principal reference mark of curve, orthonormé. The tensor G is then the identity, C is diagonal. It is what one will do henceforth.

2.1.2 Equation of heat

The equations of three-dimensional thermal conduction are written (for a rigid driver):

- div

K grad T + C T =

T

éq 2.1.2-1

where K indicates the tensor of conductivity, C the heat-storage capacity and R the possible sources.

There is advantage to write the expression of the differential operator according to metric G's generated by

surface average. One will consider indeed tensors of conductivity K isotropic transverse according to these axes of co-ordinates (multi-layer cf materials).

I

K

0

K =,

K

,

X,

X.

J

and K which can vary with

X

1

2

3

0

K

The expression of the operator:

1/2

I

I J

- div

K

grad T =

- det G.

- 1/2 .

det G. K G T

I

J

J

is written then with the first order in $x_3/tr C$, for an orthotropic conductivity according to directions' of curve principal:

- 1 - X H.

1 - X H K T + 1 + X H K T + H K T

3

1

1

3

2

11

1

2

3

2

22

2

1

3

-

K T

3

3

éq 2.1.2-2

If the curves are constant, this becomes:

- 1 - 2x₃/R₁.

1 k₁₁ 1 T-1 - 2x₃/R₂. 2 k₂₂ 2 T - H₁ 1 - x₃ H₁ K 3 T-3 K 3 T

The effect of the curve is thus in the same way standard than a modified distribution of conductivity in the thickness.

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2.1.3 Thermics for a mean structure

The equations of stationary thermics on the hull can be written in the form of one problem of minimization.

It is supposed in particular that the boundary conditions on ends $X I$ of the hull are of even standard on all thickness I . One partitionne $X I$ in:

$X I$ (zone at imposed temperature),

T

and $X I$ (zone in condition of exchange or imposed flow).

To find the field of temperature T :

1

$T =$

$\text{Arg Min } J ()$, with $J ($

$) = A (,) - F ()$, with:

\mathbf{V}

2

WITH $(T,$

$) = \mathbf{K}$.

T .

$D +$

T .

$d_{\pm} +$

T .

dS

$+ -$

$X I$

$F () =$

.

$d_{\pm} +$

·
 dS.
 + -
 X I
éq 2.1.3-1

One notes:

1
 ·
 $\mathbf{V} = H(\mathbf{X} \cdot \mathbf{I})$,
 $= 0$ on
 X I

·
 T
 · Boundary conditions on + - (X I) are of the type exchanges or imposed flow:

· $N = T - = - (\mathbf{K} \cdot \mathbf{T})$.

N being a coefficient of exchange.

The term of conductivity in A (T,) is written:

\mathbf{K}
 T.
 D
 $= K_1 - X_1 + 1$

T.
 $1 + X H dx dx dx,$

3
 T.
 $+ K_3$

3
 3
 1
 1
 2
 3
 I

R
 R

this, in a orthonormé principal reference mark of curve of (K and K are then the components physiques of the tensor of conduction \mathbf{K}).

The terms of exchange on surfaces + and - are:

\pm
 $T \cdot D \pm = \pm T \pm \cdot 1 \pm h \cdot H_1 dx_1 \cdot dx_2$

+ -

The object of a thermal model of hull is thus to bring back from three to two variables of space dependence of the field of temperature T in the expression of the differential operator corresponding to [eq 2.1.2-2] or [eq 2.1.3-1], with the help of the choice and the justification of suitable assumptions. The model suggested in [§2.3] rests on the results of the asymptotic development of the equations thermics presented in [§2.2] hereafter.

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2.2

Recall of the results resulting from the asymptotic development

2.2.1 The limiting model obtained

One summarizes here the principal results obtained in [bib1] by a technique of development asymptotic. The case of a plate is considered: $X I$, thickness $2 h$. the temperature is fixed at 0 on edge $X I$, and flows $+$, $-$ on the faces $+$ and $-$.

One seeks to study the dependence of the solution T of the thermal problem [éq 2.1.3-1] with respect to the thickness of the plate $2 h$. One uses for that a technique of change of open which brings back it problem with a field fixes $X I$, with $I =] - H, + H [$. The parameter appears then explicitly in equations of the transported problem (P), $X I$ to $X I$.

On $X I$, the initial problem is written in variational form:

1

To find:

$T \mathbf{V} = H X$

$I, = 0$ on

$X I$

such as:

K .

T .

, $+ K T$.

, = + + + - - ,

V.

33

3

3

X I

éq 2.2.1-1

The results of the asymptotic development [bib 1] consist of the checked following properties by $T(\cdot)$, the solution of the transported problem (P), posed on $X I$:

1

(I) 1

$T(\cdot)$ tends towards $T_1 X$; $x_3 = T_1 X$ in $H(X I)$.

$T_1 X$ appears COM m E a tem pérature m oyenne on thickness I, at item X.

(II) $T, 3(\cdot)$, which is the derivative of $T(\cdot)$ according to the variable thickness $x_3 I$, tends towards

2

1

1

derived according to x_3 from the champ $p X$; x_3 in $L(\cdot) X H m(I)$, where $H m(I)$ indicates the space of

1

functions of $H(I)$ with m oyenne null.

éq 2.2.1-2

In conclusion, the solution T of the initial problem on $X I$ can be represented by the two first terms of its development:

$T X, X_3 = 1$

$T_1 X + X, x_3 = X_3 + \dots$

éq 2.2.1-3

However the gradient of T is not represented by the gradient of the representation of T . This situation is generic problems of singular disturbances encountered in the study of mean structures (plates, beams...):

$T X, X = 1 T$

/ ,

3

$1 X. E + X, x_3 = X_3$

3

e3.

éq 2.2.1-4

The field of the “gradient of T” is thus not a field of gradient!

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Fields T1 and are calculated on average surface. If conductivity is homogeneous in the thickness, one a:

1
T1 H 0 (), solution of:

1
H K T
1
1, . , =
+ + -, H 0 ()

éq 2.2.1-5

2
+ X + - X
X
+ X - - X
X, X
3
3
=
.
- H +
. x3.

4 K X

H

3

2 K X

éq 2.2.1-6

It is noted that T1 is the solution of a problem arising on, whereas is obtained explicitly in function of imposed flows. These two equations constitute the “limiting” model obtained by asymptotic development.

Note:

· *In a language more coloured and more blur, the preceding results are interpreted while saying that for a thin section, the average temperature is governed by received average flow and conduction in the plan of the plate. The distribution in the thickness is not a function, in one not given, that the flows imposed in this point on the faces higher and lower, it is not affected by the presence of the close points.*

· *The distribution of temperature in the thickness is “parabolic” according to the representation [éq 2.2.1-6].*

2.2.2 One**application**

One can illustrate the results of the asymptotic development for a simple example, which shows too limitations of the model obtained by using the representation of the temperature [éq 2.2.1-3] using fields T1 and, [éq 2.2.1-5] and [éq 2.2.1-6].

One considers an infinite plate subjected on his half $x_2 < 0$ to a couple of constant flows (+ =, - = -) balanced, and insulated on other half $x_2 > 0$.

x3

+ =

= 0

x1

X 2

- = -

= 0

T (0, 0) = 0

I

Appear: 2.2.2-a

The problem [eq 2.2.1-5] of determination of average temperature T1 is an equation here

2

differential in X

2 T1 (x2) = 0

2: x2

since average flow $j_+ + J$ is null. The solution is then $T_1 = 0$ everywhere.

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The field (x2, x3) is calculated easily by [éq 2.2.1-6]: `

X

for X

2,

x3 =. x3

2 < 0,

K

= 0

for x2 > 0.

The discontinuity of the boundary condition of NEUMANN on \pm thus refers directly on field of temperature: opposite the higher temperature T is as follows:

+

T

h/K

0

X 2

Appear: 2.2.2-b

This discontinuity appears of more independent the thickness H in this limiting model, once it flow brought standardized per h.

This limitation of the limiting model obtained by asymptotic development is inherent in purely local determination of the parabolic complementary term (X, x3). Discontinuities induced will be awkward for the applications, in particular in thermomechanics.

One is thus brought to differently formulate the model of thermics of hull, while keeping them results of this asymptotic development.

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2.3

Formulation of the stationary model of thermics of hull

One saw on the results of the asymptotic study of the three-dimensional equations on the solid = X I, that the limiting model obtained comprised an average temperature solution of a problem of 2nd order posed on, and that the additional parabolic term was given only locally (not by point on). This thus had the disadvantage of providing discontinuous solutions when them thermal “loadings” are it.

One thus presents in this paragraph a representation of the temperature, always parabolic in the thickness, but avoiding the preceding pitfall. One describes the equations obtained, and their properties.

2.3.1 Equations of the model

Following the results of the asymptotic development, one **chooses the** representation in the thickness following on = X I:

T X, X =

T X

X

X +

T X

X

3

1

+

T2

.

W 2

3

3

.

W3

3

éq 2.3.1-1

with $(w_1 = 1, w_2, w_3)$ a given base of the polynomials of degree 2.

One thus replaces the determination of the field T to three variables of space by that of three fields scalars T_1, T_2, T_3 with 2 surface variables on. This decomposition [éq 2.3.1-1] is practical for to show its bond with the asymptotic model. But one will use another representation for digital model: to see [§ 2.3.5].

One will inject this representation of the temperature $T(X, x_3)$ directly in the thermal problem [éq 2.1.3-1] posed on $\Omega = X \cup \Gamma$.

From the definition of space \mathbf{v} in [éq 2.1.3 - 1], one adopts for the fields T_i :

$$\begin{aligned} &1 \\ &3 \\ \mathbf{W} &= \mathbf{V} = 1, \\ &2, 3 \text{ H } (\cdot), \text{ I} = 0 \text{ on} \\ &\Gamma. \end{aligned}$$

By posing $\mathbf{T} = (T_1, T_2, T_3)$ the formulation of the thermal problem on becomes:

To find

$$\mathbf{T} \text{ W},$$

$$\begin{aligned} &1 \\ \mathbf{T} &= \end{aligned}$$

Arg Min $J(\cdot)$, with

$$J(\cdot) = A(\cdot) - F(\cdot), \text{ and}$$

$$\begin{aligned} &2 \\ \mathbf{V} \text{ W} \end{aligned}$$

$$\begin{aligned} &\Gamma \\ &\Gamma \\ \text{WITH } (\mathbf{T}, \\ &)\text{ W} = \mathbf{T}. \end{aligned}$$

$$\begin{aligned} &\mathbf{A}. \\ &+ \mathbf{T}. \end{aligned}$$

$$\begin{aligned} &\mathbf{B}. \\ &\Gamma \end{aligned}$$

$$\begin{aligned} &\Gamma \\ &\Gamma \\ \mathbf{F}(\cdot) &= \mathbf{C}. \end{aligned}$$

$$\begin{aligned} &\Gamma + \\ &\mathbf{D}. \\ &ds \end{aligned}$$

éq 2.3.1-2

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Indeed, from [éq 2.3.1-1] one deduces the expressions:

1

(T) =

T. W 2

W3

0

T =

T.

3

w'2

w'3

Tensor **A** of order 4 corresponds to surface average conductivities:

With

X

K

.

W.

1 -

X. 1 + 1

1 + **X H**

dx

I J =

W I

J

3

3

1
3
I
R
R

éq 2.3.1-3

(by using the metric sight in [éq 2.1.1-1]).

The dependence of A following (X) comes from that of $K \beta$ and that of the average curve $H1$ from surface.

The tensor B of order 2 described transverse conduction as well as the exchanges on the faces $+$ and $-$:

,

B

$$X = K W.$$

$$W 1 + X H$$

$$dx +$$

$$+ W (H).$$

$$W (H) 1 + H$$

H

ij

I

J

3

1

3

I

J

1

I

$$+ - W ($$

(

$$I - H).$$

$$W J - H) 1 - H H 1$$

éq 2.3.1-4

With regard to the second member F , the vector C is:

1

1

$$C X =$$

$$1 + H$$

H

+

$$1 - H$$

H

+

W (H)

W (- H)

2

1

-

2

1

W (H)

3

W (- H)

éq 2.3.1-5

3

(One supposes the absence of heat sources in the thickness to simplify.)

Finally:

1

$$D X = W X$$

.

1 + X H

dx, for

X

2

3

3

1

3

I

W X

éq 2.3.1-6

3

3

With the examination of the formulation [éq 2.3.1-2] obtained for the thermics of hull, one notes that the differential operator remains of order 2, contrary to mechanics where this one passes to 4. In thermics the curve of average surface intervenes only in one modification of metric, and not directly in the operators, as an inhomogeneousness of conductivities would do it in the thickness.

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2.3.2 Case of a homogeneous plate

If a plate is considered, or if one neglects the variation of metric in the thickness of the hull ($1 \gg hH1$) and by supposing homogeneous material in the thickness to simplify, one can propose the choice of a base $(1, w_2, W_3)$ of the polynomials of degree 2 (polynomials of Legendre), of kind

that the tensors of conduction **A** and **B** are diagonalised on indices I, J (in U_i, V_j):

x2

W

X =

X /h;

W

X =

3 3 - 1

2

3

3

3

3

2

2

H

3

éq 2.3.2-1

that is to say: $W(H) = 1,$

I;

$W(-H) =$

$-1 =$

$-W(-H)$

I

2

3

and:

$$W = 0 = W = W.$$

$$W = W'.$$

$$W'$$

$$2$$

$$3$$

$$2$$

$$3$$

$$2$$

$$3$$

$$I$$

$$I$$

$$I$$

$$I$$

$$W^2 =$$

$$2h; W^2 = 2h; W'^2 = 2; W'^2 = 6$$

$$2$$

$$3$$

$$2$$

$$3$$

$$3$$

$$5$$

$$H$$

$$H$$

$$I$$

$$I$$

$$I$$

$$I$$

Thus T1 will be the average temperature, T2 will be associated the gradient in the thickness.

One finds then:

$$1 1$$

$$2 2$$

$$3 3$$

$$I J$$

With

$$= 2 KH$$

$$=$$

$$2 KH$$

$$=$$

$$2 KH$$

= 0 if I

J

;

With

;

With

;

With

3

5

0 0 0

1 0 1

0 1 0

B =

2K 0 1 0 + + + - 0 1 0 + + - - 1 0 1

H

Moreover:

0 0 3

1 0 1

0 1 0

1

0

C =

+ + - 0 + + - - 1

1

0

I

D

=

.

X /h

3

I

2

3 x2/h - 1/3

3

2

I

, on

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By writing the variational formulation of the problem [éq 2.3.1-2]:

T1

3

3

To find U =

T

H () such as,

V H ():

2

1

1

T3

T

T

T

T

U.

A.

V + U.

B. V

dx dx = C. V

dx dx +

D.V

1

2

1
2

one establishes the local equations to solve in:

$$\begin{aligned}
 & - 2 KH \\
 & T + \\
 & + + - T + T + + - - T = \\
 & + + -
 \end{aligned}$$

1
1
3
2

$$\begin{aligned}
 & - 2 KH \\
 & T + 2 K T + + + -. \\
 & T + \\
 & + - - T + T = + - -
 \end{aligned}$$

2
2
2
1
3
3
H

$$\begin{aligned}
 & - 2 KH \\
 & T + 6K T + + + - T + T + + - - T = \\
 & + + -
 \end{aligned}$$

3
3
1
3
2
5
H

éq 2.3.2-2

with the boundary conditions following:

T1, T2, T3 given on T
T

1,
= 12 2

4k H

I

T

X /h

on

2,

= 9

3

2 2

4k H

I

2

T

.
3 .

x²/h - 1/3

3,

= 25

3

2 2

2

4k H

I

The equations [éq 2.3.2-2] are thus valid for the thin plates and hulls which one neglects terms of curve in the metric one ($1 \gg hH1$), and for a homogeneous material in the thickness.

General solutions [Ti] of [éq 2.3.2-2] comprise the exponential ones of type E X has

-/! with

K

$\pm H$

lengths of damping! has depend on the values on K and K. For example, in the absence of

conditions of the type exchanges on the walls + - ($\pm = 0$), one obtains for fields T2 and T3 them respective lengths of damping:

= H

K

= H

K

.

! has

has

2

3K

! 3

15K

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It in practice frequently arrives that to neglect the terms of curve ($hH1 \ll 1$) in the operator deteriorate that little the solution; on the other hand, one often may find it beneficial to keep the expression supplements in second member. Indeed this makes it possible to calculate the true quantity of heat brought by flows applied to the faces \pm (cf example in [§3.1]). In this case, it is necessary to take C in [éq 2.3.1-2] and [éq 2.3.2-2].

1

H H 1

 $C =$

+ + - H H + + - -

1

1

1

H H 1

éq 2.3.2-3**2.3.3 Bond with the asymptotic model**

One can check easily that the model suggested here has well as a limit when the thickness H tends towards 0

results of the asymptotic development presented in [éq 2.2.1-5] and [éq 2.2.1-6].

Indeed the thickness H intervenes here explicitly in the coefficients of the differential operator in local equations [éq 2.3.2-2], which are solved on average surface.

In the case without heat exchange ($+ = - = 0$) considered in the asymptotic study, these equations

[éq 2.3.2-2] have the form:

$$- 2 K T1 = 1 + + -$$

H

$$- 2 KH$$

2

$$2. T2 + 2 K T2 = H + - -$$

3

$$- 2 KH$$

$$2. 2 T3 + 6 K T3 = H + + -$$

5

After a formal asymptotic development of the solution (Ti) according to the thickness in these equations, it is checked well that:

· T1 is solution of the problem [éq 2.2.1-5] giving the principal term of the development asymptotic (cf [§2.2]).

1

and 1

are:

.

T2 T3

1

(+

H (+ + -)

T

- -); 1 T

, which corresponds well to the definition

$$2 = H$$

3

=

$$2 K$$

$$6 K$$

[éq 2.2.1-6] of the complementary field.

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The model [éq 2.3.1-1] to three scalar fields T1, T2, T3, parabolic in the thickness, appears in some kind the optimal model with respect to the asymptotic behavior of the equations of stationary thermics in the mean structures. The following diagram indicates the overlap of various possible models, with their behavior when the thickness tends towards zero (arrows):

Model with

Limiting model

Model with 3 fields

Models more

2 fields (refines)

asymptotic

(parabolic)

rich person

T

1

1 X

X

T

T

1 X

T1

1 X

+

+

+

+

T2 X W 2 x3

T2 X W 2 x3

X

X

T X

3
 X
 3
 +
 +
 2
 ,
 H

T3 X W3 x3
 T3 X W3 x3
 +
 +
 2 ... + ...
 T X
 X
 I
 W I
 3
 +
 :

One saw the interest of the additional term to describe the evolutions of temperature in the thickness x_3 (whereas $T(X_1)$ is constant on the thickness).

However the preceding result proves that the T_2 term of the model with 2 fields does not converge towards: it is necessary

at least a representation with 3 fields for that. However, knowing that mechanical models of hulls consider thermal deformations closely connected in the thickness, one could have believed sufficient

a thermal model with 2 fields. One will see in [§3.3] an example illustrating (for a thickness data) the effect of parabolic term T_3 on average temperature T_1 enters the various models.

Other authors propose richer models of thermics (cf for example [bib9], [bib10], [bib2]), probably interesting for thick hulls, but of which terms higher than the order 2 become useless for the mean structures.

Indeed, as shows it the preceding diagram, the terms of a higher nature only come to correct (when 0) expressions of which the principal parts are given by T_1 on the one hand, T_2 and T_3 in addition. Qualitatively, they thus do not bring anything (contrary to T_2 and T_3), quantitatively their contribution quickly becomes negligible in general in front of the principal parts.

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2.3.4 Generalization with the problems of thermal evolution

The model of thermics in the hulls presented previously was justified starting from the results of asymptotic development of the three-dimensional equations of stationary thermics. One do not have however results on the problem of evolution, except the convergence of the term in average temperature $\langle T \rangle$ (cf [bib3]) (see also the remark made hereafter in [éq 2.3.4-5]).

One can however give some indications on the resolution of the problem of evolution, in particular within the framework of a modal approach (contrary to a direct integration in time).

The three-dimensional equations are:

- **K** $T + C T = R$ on

T

with:

T =

T on

,

- **K**

T =

on

D

T

N

0

T (X, T = 0) =

T (X) on

éq 2.3.4-1

One notes: $(\mu, T$

Q

Q) eigenvalues and clean vectors of the following problem:

$K T + \mu C T = 0$ on

;

$T = 0$ on

, $T = 0$ on

T

N

éq 2.3.4-2

The solution (three-dimensional) of [eq 2.3.4 - 1] is then given by:

T

0

$- \mu T$

$- \mu (T - S)$

Q

Q

$T(X, T) =$

$T. T$

Q

E

$+$

$R(S).$

$Tq +$

$(S).$

$Tq E$

$ds.$

$Tq(X)$

$Q = 1$

0

éq 2.3.4-3

The μq , opposite of relaxation time, are characteristic of the space modes of the problem [éq 2.3.4-2]. To solve the equations [éq 2.3.4-1] on a thin hull, one can adopt as in stationary the representation [éq 2.3.1-1] for the field of temperature in the hull:

3

$T X; x3, T = T_i X. f_i (T). W I x3.$

$I = 1$

One then obtains the problem of clean modes, posed on average surface, in form variational:

3

3

3

To find μ, T

$R X$

$H ()$ such as,

$H ():$

Q

Q

+

1

1

$\mu 1 0 0$

Q

T

T

T

.

A.

+ $T. B - 2h$

C.

2

$D = 0$

Q

Q

$0 \mu 0$

Q

$0 0 \mu 3q$

éq 2.3.4-4

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Note: the operator T (.).

A.

(.) + (.) .

B. (.) is quite elliptic; it is pointed out that **B** described

transverse conduction (coefficient **K**) as well as the exchanges on the two walls of the hull, then that **A** corresponds to surface conduction (coefficient **K**). It was supposed here that **C** was homogeneous in the thickness.

For example, if one neglects the effect of curve in the thickness, in the absence of condition of exchange on

walls +, -, and with a homogeneous material, one obtains the partial derivative equations following, to solve on (cf [éq 2.3.2-2]):

C

$$T1 + \mu1 T1 = 0$$

K

C

$$T2 + 3 - K + \mu2 T2 = 0$$

K

2

H C

C

$$T3 + 5 - 3 K + \mu3 T3 = 0 \text{ with } \mu I > 0$$

K

2

H C

éq 2.3.4-5

It is noted here that the thickness **H** does not affect the modes of average temperature **T1**. On the other hand, one

relative increase in transverse conductivity **K/k** or a reduction thickness **H** causes of to decrease times characteristic for the modes of “temperatures” **T2** and **T3**.

The complete solution according to this representation thus appears in the form:

T

1

1

0

- μ T

- μ (T - S)

T X

Q

Q

; X 3, T =

2h T1. T1q E

+

+ (S) + - (S). T1q E

ds. T1q X

$q=1$

0

T

2

2

0

- μ T

- μ (T - S)

X

+ 2h T

Q

Q

3

2. T2q E

+

+ (S) - - (S). T2q E

ds. T2q X

$q=1$

3

H

0

T

3

3

2

0

- μ T

- μ (T - S)

X

+ 2h T

Q

Q

3

3. T 3q E

+

+ (S) + - (S). T3q E

ds. T 3q X 3

- 1

2

$q=1$

5

2

3

0

H

éq 2.3.4-6

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where an initial temperature was considered:

$$T_0(X; x_3) = T_{0i}(X) \cdot w_i(x_3)$$

and where one supposed the absence of heat sources in the thickness.

By comparing the 3D solution [éq 2.3.4-3] and the model of hull [éq 2.3.4-6], one notes that in it the last the transverse modes T_q according to x_3 are represented only by the functions $w_i(x_3)$ given; what amounts truncating the T_q series. But another limitation appears in the product of convolution

1
for the relaxation times μ_q characteristic of the transverse modes in [éq 2.3.4-3] which disappear in the model [éq 2.3.4-6] beyond from a parabolic “mode”.

In a diffusion of purely transverse heat (described by (T, μ)

i_0

i_0 in the model

[éq 2.3.4-6]), the eigenvalue lowest being $K/h^2 C$ one can hope for a correct solution with model of hull if the relaxation times T_c of the loadings applied are such as:

C^2

$$T_c > H$$

K

éq 2.3.4-7

This inequality can be used as practical limit of application of the model.

2.3.5 Equations of the model with usual variables

The choice of variables T_1, T_2, T_3 of the representation [éq 2.3.1-1] corresponded to the development of the temperature according to the thickness.

For the applications, it is however more convenient to replace them by the variables: T_m, T_s, T_i :

T_m indicates the temperature on the average surface of the hull,

T_s the temperature on “external” surface ($x_3 = + H$),

T_i the temperature on “interior” surface ($x_3 = - H$).

The representation in the thickness uses the polynomials of LAGRANGE then: P_1, P_2, P_3 :

m

S

I

$T X;$

X

$=$

$T X$

X

$+$

$T X$

X

$+ T X$

X

3

\cdot

P_1

3

.

P 2

3

.

P 3

3

with:

2

P X

= 1 -

X /h

1

3

3

X

P X

= 3 1 + X /h

2

3

3

2hx

P X

= - 3 1 - X /h

3

3

3

2h

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The formulation of the thermal problem on is similar to [éq 2.3.1-2], but where one considers:

m

S

I

$T = T,$

T,

T

P

,

1

P 1

T

$= T.$

;

T =

$T.$

,

P 2

3

P 2

P

,

3

P 3

Tensor A of order 4 is written then:

With X

$P_1 + X.H$

$1 - X_1 + 1$

dx

ij

$=$

$K_{Pi J}$

3

1

3

3

R

R

I

The tensor **B** of order 2 is:

''

$$B_{ij} = K P P 1 + X H dx + +P H P H 1 + hH + - P - H P - H 1 - hH$$

ij

I J

3

1

3

I

J

1

I

J

1

I

For the second member, **C** becomes:

0

0

$$C_{X} = + 1 1 + hH + -$$

1 - H

H

1

0

1

0

1

And **D**:

P X

1

3

$$D_{X} = P X$$

.

1 + X H

, for

X

2

3

3

1

dx3

.

I

P X

3
3

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2.3.5.1 Case of a homogeneous plate

Various integrals on $I =] - H, H$ [necessary to the calculation of **A** and **B** are gathered hereafter:

2

16h

2

2

4h

P

dx =

; P

dx =

P

dx =

1

3

2

3

3

3

15

15

I

I

I

2h

H

P. P

dx =

P. P

dx =

; P. P

dx = -

1

2

3

1

3

3

2

3

3

15

15

I

I

I

2

2

2

,

8

,

,

7

P

dx =

;

P

=

P

=

1

3

2

3

3h

6h

I
I
I
,
,
,
,
4
,
,
1
P. P
dx =
P. P
dx = -; P. P
dx =

1
2
3
1
3
3
2
3
3
3h
6h
I
I
I

One finds then (by neglecting the correction of curve):

To 11
22
33
=
16hk
=
With
=
4hk

; **With**

15

15

To 12

21

13

31

23

32

=

With

=

With

= **A**

= 2hk

=

With

= -

HK

;

With

15

15

Then:

16

-8

-8

0 0 0

B = K -8

7

1

+ 0 + 0

6h

-8

1

7

0 0 -

0

$C = +$
 $-$
 2
 $1 - X / h$
 $.dx$
 3
 3
 I
 x^3
 D
 $=$
 \dots
 $1 + X / h dx$
 3
 3
 on
 $2h$
 I
 $- X$
 $. 3 1 - X / h dx$
 3
 3
 $2h$
 I

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2.3.5.2 Relation between the variables of the two representations

m

$$T X = T X$$

T X

1

- 1

3

2

S

$$T X = T X$$

X

X

1

- T2

+ T3

I

$$T X = T X$$

X

X

1

+ T2

+ T3

and:

m

S

I

T X

4 T X

X

X

1

= 1

+ T

+ T

6

S

I

T X

T X

X
2
= 1
+ T

2
m
S
I
T X
- 2 T X
X
X
3
= 1
+ T
+ T

3
2.3.6 Synthesis

The problem to be solved on the hull, thickness 2h is written:

m
S
I
1
3
To find
 $T = T,$
T,
 $T W = m,$
S, I H (), $m = S = I = 0$ on

T
such as:

T
T
T
T
T.
A.
+ T.
B.
. D
= C.

D +
D.
ds,
W

with:

H
With
X
K

.
P 1 + X H

1 -
X 1 + 1

dx
ij = P I

J
3
1
3
3
- H

R
R

H
B
,

X
K P.
P 1 + X H

dx +
± .
P (± H).
P (± H) 1 ± H H
ij =

I
J
3
1

3

I

J

1

- H

H

C X

(± H) 1 ± H H +

r.

P 1 + X H

dx

I =

± .

P I

1

I

3

1

3

- H

H

D X

.

P 1 + X H

dx, for

X

I =

I

3

1

3

- H

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and:

m

S

I

T X;

X =

T X

X +

T X

X +

T X

X

3

.

P 1

3

.

P 2

3

.

P 3

3

Pi (x3):

three polynomials of LAGRANGE in the thickness [- H, H]:

2

X

X

P X

= 1 -

X /h

;

P X

= 3 1 + X /h;

P X

$= -3 \frac{1-X}{h}$

1

3

3

2

3

3

3

3

3

2h

2h

H

+ 1

H

1 = 1

1 :

average curve:

R 1

R 2;

(x1, x2):

frame of reference orthonormées according to principal curves' of; $D = dx_1 \cdot dx_2$;

k_{β} :

surface components of the tensor \mathbf{K} of conductivity;

\mathbf{K} :

transverse component of the tensor \mathbf{K} of conductivity;

\pm :

coefficients of exchange on the faces + and -;

\pm :

flows applied to the faces + and -;

R:

sources distributed in the thickness;

:

flow imposed on the end of the hull.

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*Model of thermics for the thin hulls**Date:**03/11/93**Author (S):****F. VOLDOIRE, S. ANDRIEUX****Key:**R3.11.01-A**Page:**24/44***3****Validation of the model on some examples**

One presents here applications on cylinders and plates. First draft in fact of a case unidimensional in the thickness and allows to evaluate the effect of the terms of curve, in particular in the second member of the equations. The others make it possible to judge the capacity of the model to treat the case

discontinuous thermal loadings, by reference to 3D solutions.

3.1**The infinite cylinder subjected to a uniform interior flow**

One considère an infinite cylinder (ray R , thickness $2h$), subjected to a uniform flow inside: I , and with a condition of exchange in external skin $+(T - T_{ext}) = +T - E$.

One notes K the coefficient of transverse conductivity.

The analytical solution of this axisymmetric problem 1D is:

$$T(x) =$$

$$T_1 + T_0 \ln$$

$$1 + x^2/R^2$$

$$X^2$$

$$R$$

$$R - H$$

$$R + H$$

$$X$$

$$+$$

$$1$$

$$T -$$

$$R$$

$$E = -KT, \quad 3$$

$$Z$$

$$-KT, \quad 3 = I > 0$$

$$x^2$$

$$2h$$

with:

$$R$$

$$T = -I \ln$$

$$H$$

0
K
R
R I
H
H
1 - h/R
1
T = 1 - ln
1 + +

+
.
1
I
E
K
R
R
1 + h/R
+

A development limited to the 2nd order in x^3/R is:

2
+
2
X
X
T (X
I
+ E
I
R
3
3
3)

-
. H 1 - H -
1 - 3 H -
1 - H
-
. H +...

I H

+

+

R

R

2 K

2 R

K

R

H

2

2 H

R

Now let us use the model with 3 fields $T = (T1, T2, T3)$ defined in [§ 2.3.2]. Because of independence in $x1$ and $x2$ of the solution, one is reduced to the resolution of: $B T = C$.

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2

For the representation

X

X

T (X

3

3

one a:

3)

= T1 + T2

+ 3. T3

- 1

H

2

2

H

3

0

0

0

1 1 1

B =

2K 0

1

h/R

+

+ 1 + H 1 1 1

H

R

0

h/R

3

1 1 1

1

1

C =

1 - H

+

1 + H

for the second member.

I

- 1

E

1

R

R

1

1

H

If one neglects the intervention of the curve in the metric one, one removes the terms in R in the preceding expressions.

1 >> H

The solution is, if the curve completely is neglected

R:

X
T X
I
+ E
I
. H
3
3
=
+
1 -

i.e. the solution of the plane "wall".

+
K
H

If one takes account of the curve in the second member like in the terms of exchange + (true surfaces of application of flows):

+
2
+ R
X
T X =
I
E - I.
H 1 - H
-
1 - H
- H 1 - h.
3 + 0
3
+
+
I
R
R
2K
R
K
R
H

One finds the analytical solution developed with the 1st order in x_3/R . the taking into account of the curve

in the terms of conductivity in B would intervene on the level of the terms in $(x_3/R)^2$.

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3.2

The infinite plate under a couple of antisymmetric flows

Let us take again the case of the infinite plate subjected on its half $x_2 < 0$ to a couple of constant flows

(+ =

, - = -) balanced, and adiabatic on other half $x_2 > 0$.

The antisymmetry of the loading imposes that: $T(x_1, x_2, 0) = 0$. One can also show that T is linear

in the thickness in $x_2 = -, 0, +$.

The equations [éq 2.3.2-2] are reduced here to:

"

1

0

0

T

T

1

0 0 0

1

0

- 2kh 0

1/3

0

T

+ 2K

$T =$

2

0 1 0

2

2

H

0

0

1/5

T

T

3

0 0 3

3

0

x3

= 0

x1

X 2

= 0

$T(0, 0) = 0$

I

The derivative T_i , 2 being cancelled ad infinitum, T_1 and T_3 are identically null everywhere. It remains to determine T_2

(depending only on x_2) such as:

$3K$

3

$-T''_2 +$

$.T =$

whose solution is form:

kh^2_2

KH

$T_2(x_2) = \frac{E 3K}{k} \cdot \frac{x_2^2}{h} + H$ if $x_2 < 0$,

K

$T_2(x_2) = \frac{E 3K}{k} \cdot \frac{x_2^2}{h}$ if $x_2 > 0$.

The continuity of T

,

2 and T_2 in 0 gives:

$T_2(x_2) = H - \frac{E 3K}{k} \cdot \frac{x_2^2}{h}$ if $x_2 < 0$,

$2K$

$T_2 x_2 = h \cdot E \cdot 3K/k \cdot x_2/h$ if $x_2 > 0$.

2K

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After change for the variables Tm, Ts, Ti, one finds:

m

S

I

T X

= 0 ;

T X

=

T X

;

T X

= -

T X

2

2

2

2

2

2

2

The temperature of the plate, calculated within the framework of this model is thus linear in the thickness and

express yourself with T2 (x2), or Ts (x2) and Ti (x2) by:

X

S

X

I

X

T X,

X,

$X =$
 $T X.$
 $3 =$
 $T X. 3 1 + X /h -$
 $T X. 3 1 -$
 X /h
1
2
3
2
2
2
3
2
3
H
2h
2h

[Figure 3.2-a] allows to compare the temperatures in higher skin ($x_3 = + H$) of the plate under an external flow standardized ($= K/h$, with $K = K = 1, H = 1$), obtained by a numerical calculation 3D (*Code Aster*), the model hull, and the asymptotic limiting model (with the discontinuity observed in [§2.2]).

One notes the good capacity of the model to describe the boundary layer appearing in the vicinity of one discontinuity of external flow.

Temperature in higher skin of the plate pb1
Homogeneous plate
 $H = 1; K = K = 1$

Unit flows
Opposed on $X < 0$
Null flows on $X > 0$

Caption:
0 = model 3D
= model of Hull
X = Asymptot model.

Appear 3.2-a: Temperature compared in higher skin of the plate subjected to antisymmetric flows.

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3.3**The infinite plate under a couple of symmetrical flows**

In the preceding example, the antisymmetry of the loading involved the nullity of the even terms in x_3 ($T_1 = T_3 = 0$). One treats now another case of loading, symmetrical, (compared to $x_3 = 0$) allowing to judge the effect of term T_3 , in particular on T_1 , which requires to take conditions in extreme cases of the type exchanges, for dédiagonaliser \mathbf{B} [§ 2.3.2].

In $x_3 = +H$, one has like condition:

- K_T = $T +$ if $x_2 < 0$ = $T -$ if $x_2 > 0$

In $x_3 = -H$, one a:

- K_T

=

 $T +$ if $x_2 < 0$ = $T -$ if $x_2 > 0$ X_3 $T +$ $T -$ X_2 $T +$ $T -$ $T(0, 0) = 0$

I

The conditions of symmetry and antisymmetry force the solution to check:

- $T(x_1, -x_2, x_3) = T(x_1, x_2, x_3) = T(x_1, x_2, -x_3)$

from where: $T(x_1, 0, x_3) = 0$, and $3 T(x_1, x_2, 0) = 0$.

The equations (18) are written in our case (T_i depends only on x_2):

- $K_H \cdot T'' + T_1 + T_3$

= for $x_2 < 0$ or - for $x_2 >$

0

K

- $KH/3 \cdot T''^2 + +$

T

0

H

2

=

3K

- $KH/5 \cdot T''^3 + T1 +$

+

T

for X

0 or

for X

0

H

3 =

2 <

-

2 >

T2 is thus identically null (what is coherent with the conditions of symmetry).

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Solutions T1 and T3 are given by (cf [Year 1]):

2

2

/kh - S

2

- S X

/kh - S

X

1

2

1

- s2 2

T1 x1,

x2 =

- 1 -

.

E

+

.

E

sgn

X

2

2

2

2

2

s1 - s2

s1 - s2

2

2

KH

/kh - s1 /kh - s2

- S X

X

1

2

- s2 2

T3 x1,

x2 =

- . . .

E

-

E

sgnx

2

2

2

s1 - s2

s1 and s2 being positive roots of the characteristic polynomial.

After change for the variables Tm, Ts, Ti, one finds:

m

T X,

X

=

T X,

X

1

2

1

1

2

S

T X,

X

=

T X,

X

+ T X,

X

1

2

1
 1
 2
 3
 1
 2
 I
 S
 T X,
 X
 =
 T X,
 X
 1
 2
 1
 2

If one adopts to solve the thermal problem a model with 2 fields (T, T)

1
 2
 , with one
 representation closely connected in the thickness, one obtains as solution:

T
 /kh. x2
 1 x1,
 x2 =/1 - E
 if x2 < 0

= -/1 - E /kh. x2 if x2 > 0

T2 x1, x2 = 0

In such a model the temperature appears constant in the thickness. The asymptotic limiting model product the same solution.

One compares the numerical solution 3D and that of a model with 2 fields (T1). The latter comparison makes it possible to judge effect of the parabolic term on the distribution of the temperature average. Indeed it is the latter which, in mechanical theory of the hulls, generates one membrane deformation.

These comparisons are made for unit values of K, K, H, in units If. They are visualized isovaleurs 3D in the thickness on [Figure 3.3-a].

Average temperatures T1 and T1 are represented [Figure 3.3-b]. Finally it [Figure 3.3-c] shows change of the temperature in higher skin (x3 = + H) of the plate, for the three solutions considered, like by that of the asymptotic limiting model; [Figure 3.3-d] the same one presents

comparison for the average layer of the plate.

One notes on these results the good adequacy between the solution supplements 3D (items 0) and that obtained with the model of hulls with 3 fields (points), whereas the model with 2 fields (points +) appears insufficient.

linear

These observations remain valid for other choices of K , K , H , since the problem is

in, and that the variable of space x_2 appears normalisable by KH in the equations.

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Appear: 3.3-a: Isovaleurs of temperature by numerical calculation 3D

model of hull

model with 2 fields

Appear: 3.3-b: Comparison of the average temperatures: effect of the parabolic term

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model with 2 fields

model 3D

model of hull

asymptotic model

Appear: 3.3-c: Temperature compared in higher skin of the plate subjected to symmetrical exchanges

model 3D

model of hull

model with 2 cham PS

model asymptotic

Appear: 3.3-d: Temperature compared on the average layer plate subjected to symmetrical exchanges

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3.4

The infinite cylinder subjected to a horizontal stratification

One is interested in this paragraph in a situation more industrial than the preceding cases. It is about one thermal problem of stratification in a horizontal pipe [bib12]. Under certain conditions thermohydraulics, the temperature of the fluid can vary very quickly with dimension Z (cf appears below). One can practically consider that there are two zones at constant temperatures of leaves and other of a horizontal interface.

Z

0

y
0

· Geometrical Caractéristiques:

$R = 1.0m$

$H = 0.075m$

$\theta = - 30^\circ$

· Caractéristiques physics:

Conductivity

$K = 17 W/m/^\circ C$

Exchanges:

outside (air)

$= E$

$= 12 W/m^2/^\circ C$

interior (hot water)

$= C$

$= 1000 W/m^2/^\circ C$

interior (cold water)

$= H$

$= 1000 W/m^2/^\circ C$

Temperatures: external: $25^\circ C$

interior: heat $250^\circ C$

cold $50^\circ C$

The determination of the temperature in the pipe is of two interests, first is to be able to lead to the distribution of constraint in the vicinity of the stratification, second is to estimate them heat transfers enters the “cold” water zone and the “hot” water zone via conduction tube.

The problem being independent of variable X, it becomes unidimensional within the framework of the model of

hull. To solve it, one first of all seeks the general solutions of the equation without second

]-, 0 [and] 0, [
member on each segment

2

2

:

T

T

1

1

- A T

+

B

T = 0

2

2

T

T

3

3

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Solving for that numerically a cubic equation (characteristic polynomial in S), one writes then the conditions of continuity of the fields tangential T_i and their derivative with the interface, by expressing those by the combination of the general solutions and the solutions particular in each field. The linear system to solve (12 X 12) is brought back to one system of dimension 6 X 6 by considerations of symmetries, then solved numerically.

One has an semi-analytical solution thus [bib4] (numerical resolution of the equation of the third degree and of the linear system) although the situation is complex. The comparison with a calculation 2D by finite elements is given on [Figure 3.4-a] and [Figure 3.4-b]: the difference between the two solutions is indistinguishable.

Extreme values

47.3717

247.6520

Appear 3.4-a: Laminated piping: analytical solution by the thermal model of hull

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extreme values

47.1831

247.2360

Appear 3.4-b: Laminated piping: solution finite elements thermal 2D Aster

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4

Remarks on the numerical discretization

In this paragraph one limits oneself to some observations as for the numerical resolution of equations of the thermal model of hull: first of all on the use of a method finite elements and then on numerical blocking appearing when the thickness $2h$ is low. This last comes intervention of H to powers different in the coefficients from the equations.

4.1

Resolution by finite elements

The model of thermics of hull describes in [§ 2.3] shows the following characteristics:

- it leads to an operator of a nature 2 acting out of the three scalar fields $T = (T_m, T_s, T_i)$;
- these three fields are defined on a surface field, plunged in \mathbb{R}^3 ;

· the curve of surface intervenes, possibly, only in the expression of the coefficients

With, B, C, D .

In the case general of a hull of an unspecified form plunged in IR3, one can discretize geometry of its average surface by a grid in plane triangular elements (this method present certainly the defect not to be able to explicitly take into account the curve of).

The thermal problem (see [§ 2.3.6]) being scalar, with 3 fields, of the second order, one proposes them usual finite elements: the plane triangles P1 (with 3 nodes) or P2 (with 6 nodes).

Their formulation is the same one, that plane or is curved: one neglects the corrections thus of metric in the operators of rigidity A and B , (one saw in the cases of validation that that had little of effect in practice). On the other hand the user, if it connait the expression of the curve, will have interest with in

to hold account in the values of the coefficients \pm and flows \pm , as in the expressions [éq 2.3.1-4] and [éq 2.3.1-5].

In the case of materials composite (just as if one wanted to take account of the curve), one has to envisage a preprocessing providing the coefficients A, B, C, D , as well as a postprocessing allowing to reconstitute the temperature and flows in any point thickness.

There are situations where the problem does not depend any more but on one variable of space: they are the hulls

of axisymmetric revolution loading, or “sections of hulls”, axis e_3 .

The geometry is then represented by a meridian line: (see [Figure 4.1-a]).

Y

The average curve is then:

$1 \cos$

B

S

- case revolution: $H1 =$

+

X

R

X

3

1

- case “slices”, or arc: $H1 =$

T

T

R

N

T

y

where R indicates the radius of curvature of the line meridian $A B$.

With

R

X

X

Z

Appear: 4.1-a

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For these types of problems, one proposes also a finite element P2 with 3 nodes, using the same one formulation, where one neglects the correction of metric in the thickness, for the coefficients A and B .

One

use a formula of squaring at 4 points of GAUSS.

This element is associated exactly that proposed in mechanics for studies thermomechanical chained [R3.07.02].

4.2

Numerical blocking of a finite element of thermal hull

Blocking is a phenomenon appearing in the numerical resolution by finite elements of certain problems such as that of the thin hulls or the arcs when the element is curved (blocking of membrane), that of the hulls or beams with taking into account of shearing (blocking of shearing), or that of plasticity (plastic blocking of incompressibility [bib7]). It was met initially in mechanics of the incompressible fluids and it is within this framework that its study theoretical began [bib6].

This phenomenon of blocking appears by a very great loss of precision and oscillations important on certain calculated quantities when a physical parameter of the model becomes “small”. The illustration of these nuisances is given in note HI-71/7131, (§4.2). The origin of these problems lies in the difference in order of magnitude which appears between certain components of

the bilinear form of “rigidity” when the physical or geometrical parameter tends towards zero (thickness hull for the blocking of membrane, reverses tangent module of compressibility for plastic blocking for example). Here, it is the thickness of the hull which will play the part of small parameter.

Let us take again the equations of the stationary thermal problem posed on a plate in form variational; let us note $2H$ its thickness (real without dimension):

3

1

To find

$T = T,$

$T,$

T

W

$= H ()$

such as

1

2

3

0

1

WITH $(T,$

$) + B (T,) = F ()$

$= (,) \mathbf{W}$

1

2

3

éq 4.2-1

with:

1 0 0

WITH $(T,$

$) = 2kh T.$

0 1 0

.

D

3

0 0 1

5

. indicating the usual scalar product,

0 0 0

$B(T,$
 $) = 2K T.$

0 1 0

.

D

H

0 0 3

=, (gradients surfaciqu

1

2

3

An equivalent mixed formulation of this problem is obtained with the variables q_2 and q_3 , flow of heat in the thickness (cf [Year 2]): one notes $= (L_2 ())^2$.

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To find (

T, p)

W X Q such as

WITH (**T**,

) - M (, p) = F (

)

W

- B (p, Q)

- M (, Q) = 0

Q Q

éq 4.2-2

where:

Q = (

Q,

Q)

2

3

M (, Q) = Q + Q D

2

2

3

3

1 0

B (p, Q) =

H p

Q D

2K

0 1

3

On this formulation, numerical blocking appears clearly (at least formally). Indeed, discretization of $\mathbf{W} \times \mathbf{Q}$ being carried out, (it is noted $\mathbf{W}_d \times \mathbf{Q}_d$), the problem tends formally when tends towards zero towards the following problem:

WITH $(T,$

)

) = F (

) \mathbf{W}

D

- M (,

Pd

D

 $M(T, Q) = 0$

Q Q

D

D

What amounts solving on the core of M:

WITH $(T,$

) = F ()

 \mathbf{W}

D

D

éq 4.2-3

Blocking appears when the discretized core of M is too small or reduced to zero: the resolution of [éq 4.2-3] is done on a very small space even reduced to zero. Even if the grid is fine, the solution is then of very bad quality.

The core of M in \mathbf{W}_d being by definition, space:

 $\text{Ker } M = \mathbf{W}$

D

D

 $| M(, Q) = 0$

Q

 \mathbf{Q}_d

It is seen that the choice of the discretization of Q is not innocent and strongly conditions it behavior of the solution when tends towards zero. There is a condition of convergence carrying on spaces \mathbf{W}_d and \mathbf{Q}_d which ensures the good numerical behavior of the solution with small, it is condition known as LBB discrete, version adapted to the discrete case of continuous condition LBB (LADYJENSKAIA - BREZZI - BABUCHKA). We return to [bib 7] for a case study (plasticity) and a bibliography on this subject.

Parallel to the theoretical studies quickly mentioned above, a remedy practises with

blocking, appearing once the selected discretization in **Wd** if this choice were unhappy, consists with under-to integrate the term “blocking” in the construction of rigidity, i.e. the term **B** here. Some choices of under-integration, in the primal formulation [éq 4.2-1] are interpreted like choices of interpolation of **Wd** and **Qd** in the mixed formulation and can thus, via the checking (sometimes hard) of discrete condition **LBB**, being justified on the theoretical level.

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Let us consider indeed, a triangular finite element with 3 nodes and P1 interpolation to solve it problem [éq 4.2-2]. Then let us choose for discretization of **Q** a discontinuous P0 interpolation, it is with-statement a representation of **[Q]** constant by element.

The second equation of [éq 4.2-2] is then a local equation, i.e to be solved on each element separately since **p** is unspecified on each element **E**.

$$- H p Q + 1 p Q - T Q + T Q = 0$$

(Q,

Q)

2

2

3

3

2

2

3

3

2

3

2K

3

E

E
from where the immediate solution if $|E|$ is the surface of E.

p =
- 2K
1 T
2
2
H
E
E

p =
- 6K
1 T
3
3
H
E
E

By deferring these results in the form M, one has on the element E:

2K
1
3
M(, p) =
T +
T for all
W

2
2
3
3
D
H
E
E
E
E
E
E

having thus eliminated p, one is brought back to a primal formulation on T only:

$$2K$$

$$1$$

$$3$$

$$M(, p) =$$

$$T$$

T

$$2$$

$$2 +$$

$$3$$

$$3 \text{ for all}$$

W D

H

E E

E

E E

E

who corresponds very exactly to the formulation [éq 4.2-1] in which the elementary term:

$$B(T,$$

$$) = 2K T + 3 T$$

$$E$$

2

2

3

3

H

E

under-is integrated by a diagram into a point of GAUSS:

F. G

1. F. G

E

E

E

E

The examination of discrete condition LBB remains to be made for this discretization in order to conclude with its

convergence (cf [bib7]).

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5 Conclusion

An asymptotic analysis of the equations of thermics in a mean structure when the thickness tends towards zero leads to a limiting model characterized by an average temperature, solution of one problem in extreme cases, and a parabolic complementary term in the thickness, definite locally. One deduced the formulation from it from a model with 3 scalar fields definite on average surface from hull, giving a parabolic representation of the temperature in the thickness. The operator differential obtained is of order 2; the thickness of the hull appears in its coefficients.

This model seems “optimal” for the mean structures:

- its limit when the thickness tends towards zero is identical to the asymptotic limiting model;
- of the possible additional terms would tend towards zero with the thickness.

In a standard version the curve of the average surface of the hull does not intervene directly.

Test examples show a good adequacy of the temperature obtained with solutions three-dimensional complete.

This model thus appears completely entitled with:

- to be used in a finite elements formulation to calculate the temperature in a hull thin of an unspecified form; the solution obtained being able to be easily injected into one thermomechanical calculation of the hull; one proposes surface and linear elements thus for the cases where a variable of space does not intervene;
- to be introduced directly (or by coupling) into a method of resolution of the equations governing the thermohydraulic state of a piping for example, in order to take account of thermal restitution of the wall on the fluid;
- to be used as model integrated in the resolution of problems of identification (problem opposite) starting from experimental measurements (for example for laminated conduits);
- to seek analytical solutions in cases with simple geometry.

The model describes here can also be used in the problems of thermal evolution, provided that

the thermal loadings do not vary too quickly.

Lastly, it remains to study the numerical methods to use to avoid the blocking which could appear in a calculation by finite elements, when the thickness becomes low.

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Appendix 1 Plates infinite under a couple of symmetrical flows

In $x_3 = +H$, the boundary conditions are:

- $K_3 T$

= T_+

if $x_2 < 0$

= T_-

if $x_2 > 0$

In $x_3 = -H$, one a:

- K 3 T
 = T +
 if $x_2 < 0$
 = T -
 if $x_2 > 0$
 X 3
 T +

 T -

 X 2
 T +

 T -

$T(0, 0) = 0$

I
 The conditions of symmetry and antisymmetry force the solution to check:

$T(x_1, -x_2, x_3) = T(x_1, x_2, x_3) = T(x_1, x_2, -x_3)$
 and thus: $T(x_1, 0, x_3) = 0, 3T(x_1, x_2, 0) = 0.$

The equations [éq 2.3.2-2] are written in our case:

"
 - $KH T_1 + T_1 + T_3 = \text{for } x_2 < 0 \text{ or } - \text{ for } x_2 > 0$

"
 - $kh/3. T_2 + K + T_2 = 0$

H
 "
 - $kh/5. T_3 + T_1 + 3 K + T_3 = \text{for } x_2 < 0 \text{ or } - \text{ for } x_2 > 0$

H
 T2 is thus identically null (what is coherent with the conditions of symmetry). The system precedent admits as particular solution:

-
 T_p
 $1(X, X$
 1
 $2) =$
 if $X < 0$ and
 if X
 2
 $2 > 0$

T p
2

3 (X, X
1
2) = 0
on R

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The polynomial characteristic in S of the homogeneous system is:

2
2
 $K H s^4 - K 6 H + 3K s^2 + 3 K = 0$

, including 4 roots if are:

5
5
H

2
2

if = ± 1.

3 .

H + 5 K ±

$2 H + 25K + 10K H, s_1 > s_2 > 0 > s_3 > s_4$

H
K

2
4
3

Solutions T1 (x1, x2) and T3 (x1, x2), finished in x2 =, are thus expressed:

T
X
X
1 x1,
x2 = + es1 2 + es2 2
for x2 > 0

= - - E S X
X
1
2 - E s2 2
for x2 > 0

T
X
X
3 x1,
x2 = es1 2 + es2 2
for x2 > 0

= - E S X
X
1
2 - E s2 2
for x2 > 0

The conditions of connection in x2 = 0 are naturally expressed by the conditions of antisymetry of T, already used above. The four constants, β, are determined by:

+ = - /

nullity of T in x2 = 0
+ = 0

$$2$$
$$- KH s1 + = 0$$

m odes T1 - T3 associated with s1, s2

$$2$$
$$- KH s2 + = 0$$

From where:

2

$$/kh - S$$
$$=$$
$$- .$$
$$2$$

$$s2 - s2$$
$$1$$
$$2$$

2

$$/kh - S$$
$$=$$
$$.$$
$$1$$

$$s2 - s2$$
$$1$$
$$2$$

2 .

2

$$. KH$$
$$/kh - S$$
$$/kh - S$$
$$=$$
$$.$$
$$1$$

2

2

s2 - s2

1

2

2 .

2

. KH

/kh - S

/kh - S

=

.

1

2

2

s2 - s2

1

2

Solutions T1 and T3 are thus written:

2

2

/kh - S

2

/kh - s1

T

X

X

1 x1,

x2 = -. 1 -

E s1 2 +

E s2 2 sgn X

2

2

2

2

2

s1 - s2

s1 - s2

2

2

KH

/kh - s1 /kh - s2

T

X

X

3 x1,

x2 = -

. E s1 2 - E s2 2 sgn x2

2

2

2

s1 - s2

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Mixed appendix 2 Formulation of the stationary problem for plate

In the case of the plate, the variational problem [éq 4.2-1] is equivalent to a problem of minimization (while revealing explicitly the thickness H in B) of the functional calculus:

J () =

1 A (

,) + 1 B (,) - F ().

2

2

To obtain the mixed formulation, let us notice that:

Proposal:

1

2

2

B (,) =

Sup

-

$$q_2^2 + q_3^3 - H Q_1 + 1 Q_3$$

2

2

2

4

K

3

q₂, q₃ L ()

Demonstration:

Let us write the condition of extremality of the functional calculus between hooks (its opposite is strictly convex, coercive and semi-continuous in a lower position) and let us note p the couple where the sup is reached:

Q

H

2

$$2 + q_3^3 + q_2 p_2 + 1 q_3 p_3 = 0 Q$$

2 K

3

$$p_2 = - 2 K_2$$

H

$$p_3 = - 6 K_3$$

from where:

H

The value of the functional calculus in this point is thus:

2

2

$$+ 2 K_2$$

2
H
2
2
2
2
2
+ 6 K 3 -
4 K
2 + 3 6 K 3 = 1 2 K 2 + 3 K 3

H
H
4 K
2 2
2
H
3 2 H
2
H
H

that is to say indeed the announced result.

There is thus a formulation equivalent to the minimization of J on W:

1
Min max A (,) -
B (Q, Q) - M (
, Q)
- F ().

W
Q Q
2
2

while noting:

M (, Q) = Q + Q,
B (Q, Q) = H p Q +

1
p Q
2
2
3
3
2

2
3
3

2K
3

The condition of point-saddle of this Lagrangian led to the formulation [éq 4.2 - 2]:

WITH (T ,
) - M (, p) = F (
)
 W ,

-
 B (p , Q)
- M (T , Q) = 0
 Q Q

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Organization (S): EDF/AMA, SEPTEN

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R4.01.01 document

*Pre and Postprocessing for the thin hulls
out of “composite” materials*

Summary:

One extends the results of the theory of the elements of plates exposed in documentation [R3.07.03] to the case multi-layer orthotropic materials. Documentation suggested gathers the thermal aspects and thermo-élasto-mechanics. The use of these materials is theoretically valid only in the case of one geometrical symmetry compared to the average layer of the plate. It is thus necessary that the

coupling
membrane-inflection is null.
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1 Introduction

The modeling of the thermomechanical behavior by a theory of hulls of the structures composed of laminated composite materials present compared to the homogeneous case isotropic one certain number of characteristics:

- coefficients intervening in the relations of linear behavior connecting the sizes mechanics and thermics defined on the average surface of the hull must be calculated starting from the space distribution in the thickness of various materials,*
- the materials constitutive of the hull are in general orthotropic:*
 - it is necessary to define, in each point of the average surface of the hull, a direction material fixing the reference mark in which the relations of behavior are described,*
 -
- the form of the anisotropy produced on the total behavior of the hull can be unspecified,*
- finally of the couplings between sizes characterizing of the symmetrical phenomena and antisymmetric compared to average surface can appear (coupling inflection-membrane, coupling temperature average average-gradient in the thickness). In thermo_mecanic the results presented are however theoretically valid only when the coupling membrane-inflection is null,*
- the analysis of the rupture or the damage of these structures requires to return to one level of description finer than that provided by the models of hulls: the criteria are formulated, layer by layer in the thickness, according to the constraints “three-dimensional”.*

The preprocessing makes it possible the user “to build” the sizes intervening in the theories of hulls starting from a simple space description of the distribution of the various materials (position,

thickness, orientation).

Postprocessing intervenes once the structural analysis completed to provide, layer by layer, an evaluation of some criteria of rupture or damage.

The party taken here is to specify pre and postprocessings so that they are independent, in the framework of the models of hull selected, the type of element chosen by the user to make the calculation of structure. Indeed, numerical difficulties of the calculation of the hulls and the representation of their geometry results in proposing according to the situations, several types of finite elements of hull or of plate.

The note is divided into three parts. The first briefly points out the assumptions of the theory of hull used for thermomechanical calculations and the expressions of the coefficients homogenized with to introduce. The second specifies the choices retained for the description of the orientation of materials by report/ratio with the elements like some notations. The last part details the application of these choices with the case of the hulls made up of homogeneous layers.

To allow the use of the options of calculations available in Code_Aster, it is thus necessary to define orders the pre one and postprocessing for composite materials laminates compatible with the existing orders.

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2

Homogenized characteristics of a thin hull in thermoelasticity and in thermics

2.1

Notations - Assumptions

The hull is made up various layers of orthotropic materials parallel to laid out surface average (cf [Figure 2.1-a]).

N

2h

x2

X

1

I =] - H, [

H

Appear 2.1-a

By noting (X) the co-ordinates (X, X out of and X

1

2)

3 the normal co-ordinate on the surface

X] - H, H, one can define the various characteristics of materials intervening in Thermics

3

[

and in Thermoelasticity. One will suppose moreover than one of the axes of orthotropism coincides with

normal N at the item (X) with the hull.

• Conductivité: K

(X, X

3), k33 (X, X

3)

• Voluminal Chaleur: C (X, X

3)

• Dilation coefficients: D

(X X

- 3)
- *Elastic Rigidité (plane constraint): μ (X, X*
- 3)
- *Rigidité of shearing:*
- X, X
- 3 3 (
- 3)
- *Density: (X, X*
- 3)

The Greek indices traverse {1, 2}. The system (X) necessarily does not correspond to the axes of orthotropism of materials in the tangent plan.

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2.2 Thermics

One places oneself within the framework of the thermal model of hull describes in [R3.11.01] and [bib1].

A field of temperature “hull” is represented by the three fields (Tm T S Ti

,

),) definite on

the following way in the thickness:

3
 $T(X, X =$
 $=$
 $+$
 $+$
éq 2.2-1
 $3)$
 $T J(X) P$
 m
 S
 I

$J(x^3)$
 $T(X) P1(x^3) T(X) P2(x^3) T(X) P3(x^3)$
 $J = 1$

2
where the P are the polynomials of LAGRANGE

$I =] - H, [$

H:

P

1

$1(x^3) =$

$-(X/H$

3

)

J

X

P

3

1

$2(x^3) =$

$(+ X/h$

3

)

2 H

X

P

3

1

$3(x^3) = -$

(- X/h
 3
 $)$
 $2H$

The interpretation of the fields T_j is then the following one:

$$T_m(X) = T(X, 0)$$

(temperature on the average surface of the hull),

$$T_S(X) = T(X, +h)$$

(temperature on the upper surface of the hull),

$$T_i(X) = T(X, -H)$$

(temperature on the lower surface of the hull).

Thanks to the representation [éq 2.2-1], one calculates bilinear form KT

*m
 S
 I*

of (T, T, T)

T to be left

form of the 3D problem (the indices ij take the values m, S, I):

$$K T(T) = (A_{ji} T_i$$

J
 \cdot

$$+ B_{ji} T_i J$$

\cdot
 $,$

(summation on the repeated indices),

$,$
 $) D$

where is a virtual field of temperature and where

$$A_{ij} = A_{ji} = A_{ij}, B_{ij} = B_{ji}$$

$$A_{ij}(X) = K$$

,

éq
2.2-2

I(X)
X

$$3) P_i(x_3) P_j(x_3) dx_3$$

P

P

B_{ij}

$$(X) = K$$

,

I₃₃(X)
X

$$3) I(x_3) J(x_3) dx$$

X
3
3
x₃

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The bilinear form related to voluminal heat in the problem of evolution is written:

M (T,

ij

= C. Ti J

.

éq

2.2-3

ij

C (X) = C

,

I

(X X

3) Pi (x3) pi (x3) dx3

2.3 Thermomechanical

One places oneself within the framework of the modeling of hull of LOVE-KIRCHHOFF (hull thin)

or

REISSNER-MINDLIN (thick hull). In both cases, the sections are supposed to remain plane.

The deformations of the tangent plan with are thus expressed, in the thickness, using the tensors of deformations E

(X

), of variation of curve K

(X

) and of distortion (X) of surface [bib2]:

*X
(
X, X =
+
=
éq
2.3-1
3)*

*E (X) x3
K (X) 3 (X, x3) ()*

2

The material undergoing a local deformation of thermal origin given by (T réf is the temperature of reference):

*HT
réf
(
X, X =
-*

*3)
(T (X, x3) T) D (X, x3)*

The local stress field is given by the thermoelastic law in plane constraints:

*=
HT
μ (μ - μ)*

maybe with the preceding model for T:

*HT
(X,*

$$\begin{aligned}
 & X = \\
 & + \\
 & - \\
 & 3) \\
 & \mu (X, \\
 & x3) [E\mu (X) X K \\
 & 3 \\
 & \mu (\\
 & X) \mu (X, x3)] \\
 & 3
 \end{aligned}$$

éq
 2.3-2
 with HT
 J
 réf

$$\begin{aligned}
 & \mu (X, \\
 & X = \\
 & - \\
 & 3) \\
 & T (X). Pj (x3) T \\
 & d\mu (X, x3) \\
 & j=1
 \end{aligned}$$

The generalized efforts (inflection M and membrane NR) are related to by:

$$\begin{aligned}
 & M \\
 & (X \\
 & = \\
 &) \\
 & , \\
 & , \\
 & I \\
 & (X X \\
 & 3) X dx
 \end{aligned}$$

3

3

éq

2.3-3

NR

(X

=

)

,

,

I

(X X

3) dx3

so that the law of behavior of the hull is written as in point X:

$$M = P\mu K + Q\mu E + M$$

μ

μ

HT

éq

2.3-4

NR

$$= R\mu E + Q\mu K + NR$$

μ

μ

HT

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where

$P\mu$

μ

= +

(X 2

3) X

dx

I

3

3

$Q\mu$

μ

= +

(x3) X dx

I

3

3

$R\mu$

μ

= +

(x3) dx éq

2.3-5

I

3

NR

μ

HT

= -

dx

HT

I

μ

3

M

μ

HT

= -

X dx

HT

I

μ

3

3

When the temperature is calculated by the model of Thermics one can express them directly “thermal” efforts according to the three “components” (Tm T S Ti , ,) :

M

J

réf

J

réf

= -

3

3

3

3

-

=

-

HT

[

D

μ

μ (X) P (X) X dx

I

J

] (T T) DMj (T T)

éq 2.3-6

NR

J

réf

J

réf

= -

3

3

3

-

=

-

HT

[

D

μ

μ (X) P (X) dx

I

J

] (T T) DNj (T T)

The quantities DN and DM depend only on materials constitutive of the hull and of their distribution.

Note:

When the provision of materials is symmetrical compared to, certain integrals, being summon odd terms, cancel themselves:

μ

Q

= ,

0 DM

= DM = 0; DN = 0.

1

3

2

The sharp efforts and stresses shear transverse are obtained by writing of

local equilibrium equations without voluminal force:

$ij = 0$ where $\{I, J\} \{1, 2\}$

3
, J

what makes it possible to write:

$V(X$

) = M, (X)

3 (
X
X, X
3

3
= -

)
,
,
-
(X Z

) dz
H

by using the fact that 3 (

3

$X, + H) = (X, - H) = 0.$

The role of the preprocessing is to calculate various sizes A, B, C, P, Q, R, DM, DN, to leave description of the material (a number, orientation and thickness of the various layers, local characteristics C, K

,
, D
).

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**3 Reference marks in the tangent plan with the hull. Notation
matric**

3.1 Reference mark

**One considers the total reference mark of the structure (X, Y
 $, Z$**

): to see figure [Figure 3.1-a]. In the case of them

**laminated composites the orientation of full-course is defined compared to a direction of reference
 E in the tangent plan (**

réf

T).

This vector E

is determined by the projection of a vector X , given by the user under the key word

réf

1

**ANGL_REP of AFFE_CARA_ELEM [U4.24.01], on the tangent level (T) in an unspecified point of
hull.**

Z

$X1$

$X1$

Z

2 Y

0

1

(T)
X
eréf
Y
Tangent plan (T)
XYZ locates total
X

Appear 3.1-a

Vector X is defined by the user by two directed angles:

1
: between 0X and X

1
1 proj (X, Y)
: between X
and X

2
1 proj (X, Y)
1

: fact of passing from the direction 0X to projection in plan X0Y of vector X.

1
1
: fact of passing from this projection to X itself: to see figure [Figure 3.1-a].

2
1

Whenever in a given zone of the hull, (T) is orthogonal with X, the user will have

1
to define another vector (in practice for certain meshes).

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For a finite element of type facets planes, contained in the tangent plan (T), one defines the reference mark

orthonormé (V, V room with the element using the classification of the tops. For example for

1

2)

triangle:

N3

N2

V1

V2

0

E réf

(T)

N1

Appear 3.1-b: Locate local element (V, V

1

2)

The directed angle

= V,

E

allows to pass from the local reference mark to the element to the reference mark of

0

(1 réf)

reference.

3.2 Notation

matric

In thermics as into thermomechanical, the programming of the elements requires to express them operators of elasticity and conduction in the local reference mark of the finite element (V, V. There is the practice

1

2)

to simplify the representation of the tensorial sizes as follows.

3.2.1 Thermics

One represents the tensorial sizes in the reference mark (V, V:

1
2)

(
2
2

Aij
I, J m, S
, I
,

() {
} () { , 1 } 2)

in a vectorial form with 6 vectors by taking account of symmetries [§2.2]:

ij
With
K
11

11
H

Aij
ij
= A
22
=
I
P (x3). Pj (x3). k22 dx

3
ij
- H

A12

K

12

K

11

0

K

where $K = K$

indicate the thermal vector conductivity built using the tensor

22

0

K

0 0 K

12

33

(cf [§2.1]),

and of $P(X)$, polynomials of LAGRANGE in the thickness. One makes in the same way for B_{ij} C_{ij}

,

.

I

3)

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While being placed in the reference mark of the element (V, V, one uses the matrix of passage (m)

P of the tensor

1

2)

K

K

11

of conductivity $K = K$

of (V, V towards the reference mark associated with E [bib3]:

1

2)

22

réf

k12

C2

S2

2CS

(

2

2

m)

C = cos

P

= S

C

- 2CS

where

(0)

K

$$S = \sin(\theta)$$

$$-CS \quad C^2 - S^2$$

It results from it that the matrix from passage P (m) - I of the tensor of conductivity of the reference mark associated with E

K

réf

towards (V, V is given by:

1
2)

C2

S2

- 2CS

(
1

$$C = \cos(\theta)$$

2
2

m)

$$P - = S$$

C

2CS

where

K

$$S = \sin(\theta)$$

CS

$$-CS \quad C^2 - S^2$$

3.2.2 Thermomechanics

One also represents in a vectorial form in the reference mark (V, V:

1
2)

· *on the one hand, normal constraints,
, shearing in the plan and it*

11 22

12

transverse shearing and:

13

23

11

13

=,

22

=

23

12

· *in addition, corresponding deformations:*

11

1

13

=,

2

22

=

12 =

2

12

23

12

who break up with the generalized deformations of membrane E and inflection K:

(X =

-
3)
(U) (x3)
HT (x3)
with (U) (X = E + K)

3)
x3
HT (X = D)

-
3)
(x3)
réf

(
T X

3)
T
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for an ordinate X] - H, H [

3

, and:

E

K

D

11

11

11

E = E

,

,

22

K = K22

D = d22

E11

K11

d11

where D is the vector associated with the dilation coefficients thermal.

The vector forced is obtained using the matrix of rigidity (3 X 3):

=

R ((

. U)

HT

-

) with R, matrix of flexibility reverses (see in [§4.3]).

While being placed in the reference mark of the element (V, V, one uses the matrix of passage ()

P m of the tensor

1

2)

11

deformations =

of (V, V towards the reference mark associated with E [bib3]:

1

2)

22

réf

12

C2

S2

CS

()

C = cos (0)

2

2

P m =

S

C

- CS

where

S = sin (0)

- CS

CS C2 - S2

2

2

While being placed in the reference mark of the element (V, V, one uses the matrix of passage (m)

P of the tensor

1

2)

2

1

13

deformations

= of (V, V towards the reference mark associated with E:

1

2)

2

réf

23

C

S

$C = \cos (\theta)$

(

m)

P

=

where

2

- *C M*

$S = \sin (\theta)$

In the same way, while placing itself in the reference mark of the element (V, V, the matrix of passage

(m)

P of the tensor

1

2)

11

constraints =

of (V, V towards the reference mark associated with E is worth:

1

2)

22

réf

12

C2

S2

2CS

(

$C = \cos (\theta)$

2

2

m)

P

where

= *S*

C

- *2CS*

S = sin (0)

- *CS CS C2 - S2*

It results from it that the form of the matrix of passage of the reference mark associated with E towards the reference mark of

réf

-1

the element (V, V for the constraints above is such as: (m)

(m)

P

= *P (*

T

-)

. This

0

= *P (m)*

1

2)

property will be particularly useful in the continuation of the talk.

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4

Hulls made up of homogeneous layers

4.1

Description of the layers

One considers the hull made up of a stacking of NR layers (parallel with the tangent plan) in couch

the thickness] - H , H [made up each one of one of the M orthotropic homogeneous materials (hull to subdue laminated [Figure 4.1-a]).

x_3

+ H

$2h$

E

- H

N

Appear 4.1-a

A layer N is defined by:

· its thickness E with the ordinates of the interfaces lower and higher:

N

$N-1$

-

$x_{N-1} = -H + E; x_N = x_{N-1} + E;$

3

J

3

3

N

J = 1

- *the constitutive material m, and its physical characteristics,*
- *the angle of the first direction of orthotropism (noted L) in the tangent plan (*

N

T) by

report/ratio with the direction of reference E

(see figure [Figure 4.1-b]).

réf

Note:

In the case of a layer made up of fibres in a matrix of resin, first direction of orthotropism corresponds to the direction of fibres.

L

VI

N

0

T

L

V

E

2

réf

(T)

Appear 4.1-b: On orthotropic layer

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4.2 Thermics

2

The expression of the vectors \mathbf{A}_{ij} (I, J) $\{m, S, I\}, I J$ defined in [§3.2.1] is obtained from

conductivities \mathbf{k}_m of the material m constituting layers N .

In the cases of orthotropism (L, T) of the material m , the coefficients of conductivity are:

K

L

\mathbf{K} (

= K

L, T)

T

0

In the case of a transverse isotropic material the coefficient \mathbf{K} is equal to \mathbf{K} .

33

T

To have the expression of \mathbf{A}_{ij} in the reference mark of the element (\mathbf{V}, \mathbf{V} one must apply rotation

1

2)

following, of the reference mark of orthotropism towards the reference mark of the element, as clarified

with [§3]:

$K C^2$

S^2

with

C

=

(

\cos

+

I

0)

11

K

(

$K m$

L

= K

2

2

S

=

(

\sin

+

I

0)

$22 = S$

C

K

$T(L, T)$

$k_{12} CS - CS$

C

K

$T(L, T)$

$k_{12} CS - CS$

C

K

$T(L, T)$

$k_{12} CS - CS$

C

K

$T(L, T)$

$k_{12} CS - CS$

C

K

The A_{ij} vectors can then be expressed by integration in the thickness of the contributions of

sleep:

Ncouch

N

ij

With = $x^3 P$

.

X P

.

X .k

dx

.

éq

4.2-1

n-1

I(3) J(3) ()

X

m

3

n=1

3

The ij terms ((I, J) {

,

2 }

3 2

B

, I J) are:

Ncouch

N

X

3

P X

P X

ij

I(3)

J(3)

B =

.

.k

dx

.

n-1
(
33
)
X
m
3
X
X
3
n=1
3
3

In the same way for Cij:

Ncouch
N
ij
C = x3 P X. P X. C dx

.

n-1
I (3)
J (3)
()
X
m
3
n=1
3

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4.3 Thermomechanics

4.3.1 Relation of behavior

In the case of the laminated hulls, it is shown that the relation between the strains and the stress in layer “N” depends on the constants of orthotropic material” m “:

That is to say:

(m) (m) (m) (m) (m) (m)

ELL,

T

E T, T

L, G T

L, GLZ, T

G Z

elastic coefficients

(

m)

(m)

dilation coefficients

DLL, dTT

In the axes of orthotropism (L, T) of the material m, the matrix of flexibility S is expressed by:

1

LT

-
0
E
LL
T
ET

TL
1

S (m) (L T,) =
0

T
ET
T
ET

1
0
0

G

T
L

(m)

with

TL
LT =

ELL
ETT

Rigidity (m)

1
-
=S (being:
m)

E

LL

TL .ELL

0

-

1

.

-

1

.

TL

T

L

TL

T

L

LT. ETT

ETT

(m) (L T,) =

0

-

1 H T.

-

T

L

1 H T. T

L

0

0

G

T

L

(m)

Rigidity in transverse shearing is expressed for its part in the following way:

G
LZ
0
(m) (L T,) =

0
T
G Z (m)
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While being placed in the reference mark of the element (V, V, one uses the matrix of passage ()
P m of the tensor

1
2)
deformations defined in [§3] of (V, V towards the reference mark of orthotropism:

1
2)

C^2
 S^2
 $2CS$

()

$C = \cos (+$
 I
 $0)$
 2
 2

$P m =$
 S
 C
 $- 2CS$
where

$S = \sin (+$
 I
 $0)$

$- CS \quad CS \quad C^2 - S^2$

In the same way the vector dilation is expressed in the reference mark (V, V:

I
 $2)$

D
 D
 C^2
 S^2
 11
 LL

D
 (m)
 D

= *D*

1

2

2

22 = Pm-

LL

D

= *S*

C

TT

D

TT (L, T)

D

0

2

2

12

(

CS - CS

L, T)

One thus has in layer N (material: m), in X:

3

(m) - 1

(m)

HT

T

m

m

HT

HT

= *P*

.

. P

.

U -

= **P.**

. P

.

U -

=

U -

N

L, T

(()) ()

()

L, T

(())

m ()

)

()

()

()

()

with:

E

K

D

11

11

11

(*U*) *E*

and

.

$22 + x^3 K22$

$HT = d22 (T(x3) - Tréf$

=

)

E12
K12
d12

Note:

In the code, one chose to carry out the passage of the reference mark of orthotropism to the reference mark of the element in

two stages. A first stage relates to the passage of the reference mark of orthotropism to the definite reference mark

by ANGL_REP. The data of DEFI_MATERIAU are thus transformed at the time of this first passage. One treats then equivalent material as one would do it with elements of traditional plates.

The treatment of thermal dilation is made in the form of a contribution to the second member of the matric equation to solve resulting from the principle of virtual work. This contribution

D

T

is written:

T

(m)

HT(N) = - P. (.d T

.

L, T)

TT

0

**4.3.2 Shearing
transverse**

Rigidity in transverse shearing of each layer is written in the reference mark (V, V of same

1

2)

way that dilation:

T

(m)

(*m*)
(
= *P*.
. *P*

m) (*V*, *V*
2
2
1
2)

(*m*)
C
S
with (*m*)
P
=
V, *V* towards the reference mark of orthotropism.
2

stamp passage vectorial from (1
2)
- *C M*
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Rigidity in transverse shearing total of the hull [\mathbf{R} calculated so as to be equal to

C]

that given by the law of three-dimensional elasticity [bib2], the matrix [\mathbf{R} is defined so that

C]

surface density of transverse energy of shearing U_2 obtained for a distribution

three-dimensional of the constraints and is identical to that associated the model of plate of

13

23

Noted REISSNER-MINDLIN U .

2

1 H

-1

$U =$

=

1

2 -

[m

H

] $\{ \}$

()

d^3

13

23

1

-1

1

H

-1

H

$U =$

$\mathbf{V} \mathbf{R}$

$\mathbf{V} =$

\mathbf{H}

2

[C]

$\{ \} D$

- H
3
[C]
{ } D

- H
3
2
2

x^3

= -
 D
,
+
13

- $H(11 1$
12,2) 3
with the equilibrium equations:
 X

3
= -
 D
,
+
23

- $H(12 1$
22,2) 3

and conditions: $0 = =$ for $X = \pm h$.
13
23
3

Plane constraints,

*,
express themselves according to the resulting efforts while making*

11

22 12

the assumption of pure inflection and absence of coupling membrane/inflection. It results from it that:

(X

1

=

-

. () P.M and A (X

1

=

-

()

3)

X P

3)

X

X

3

(m)

3

(m)

3

where P is the matrix of rigidity of inflection of the whole of multi-layer defined by [éq 2.3-5].

These calculations, as well as the following are to be carried out in a single reference mark. One chooses in

Code_Aster the intrinsic reference mark with the element. It is thus necessary to transform matrix A in this reference mark.

One has then: $\{(X)\} = D (X) V + D (X$

3

1

3

2

3) {}

with $V = M$

;

11,1 + M

M

12,2

12,1 + M22,2

= M

;

;

;

11,1 - M

M

12,2

12,1 - M

M

M

22,2

22,1

11,2

H

ZA + A

WITH + A

11

33

13

32

and D = -

dz

1

- x3

2 A + A

WITH + A

31

23

22

33

H

ZA - With

WITH - WITH

2 A

2 A

D

11

33

13

32

12

31

= -

dz

2

- X

WITH - WITH

With -

3

2

With

2 A

2 A

31

23

33

22

32

21

1

C

C V

U

11

12

is thus written: U =

V

1

1

T

2

C
C

12
22

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l

H

-

with

T

11

C = 1

D ()

l

D d3

-

2 2

H

m

×

l

H

-
T
12
C =
1
D () D2 d3

-
2 4
H
m
×
1
H

-
T
C22 =
D
2 () D2d3

-
4 4
H
m
×

1
C

- *H* -
C
C
V

from where $U_1 = U_2 V$
11
12

=0V, {}
C T
C

12
22

one thus proposes the solution $\mathbf{H} = \mathbf{C}^{-1}$.

\mathbf{C}
11

The coefficients of transverse correction of shearing correspond to the report/ratio of the terms of $\mathbf{H}\mathbf{c}$ with the integral on the thickness of the laminate of the terms of (m) .

4.3.3 Efforts generalized

The efforts generalized defined in [§1.3] and put in a vectorial form are obtained by integration in the thickness of the hull by summoning the contributions of the layers (thickness

N
 l
 E
-
 $N =$
 N
 3
 X
- 3
 X
):

M_{11}

N_{couch}
 N
 $\mathbf{M} =$
 X
 M
 3
 $22 = \int_{-x}^x dx$
 $3.$
 $3 =$
 \dots
 l
 $X dx$
 L
 $N() 3 3$

X

N

$n =$

3

$M12$

1

$N11$

N_{couch}

N

$N =$

X

NR

3

$22 = dx$

$. 3 =$

$.$

1

dx

L

$N () 3$

X

N

$n =$

3

$N12$

1

If one expresses like previously (with m material of layer N):

$N = Mr. (E + x3.K - d m ($

réf

$T x3 - T)$

$()$

$()$

$()()$

one can note the efforts generalized in the form: (cf [§1.3])

$M - Mth = P K$

$. + Q E$

·
 $N-Nth = Q K$
 · $+R E$
 ·

with P, Q, R of the matrices 3×3 being expressed by:

N_{couch}
 N
 N_{couch}
 3
 1
 3
 3
 $P =$
 X
 2
 N
 $N 1$

(m) $1 \times dx$

·
 $. X$
 X
 N
 3
 3
 $= (m) (3) - (-$
 3
 $)$
 x^3
 3
 $n=1$
 $n=$

1
 N_{couch}
 N
 N_{couch}
 3
 1
 2
 2

Q =
X
N
N 1

(m) 1 X dx

.
. X
X
N
3
3
= (m) (3) - (-
3
)
x3
2
n=1
n=

1
Ncouch
Ncouch
R =
N
N
(
1
m) (
. x3 - x3) = (m) e.n
n=1
n=1

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the shearing action V is obtained by derivation of the moment [§4.3.2].

The generalized efforts of thermal origin are calculated directly:

Ncouch

xn

Mth

3

réf

= . 1 .

.

.

-

3

3

-

N

X

T X

T

dx

m

X

(()

)

()

D (m)

3

N = 1

3

Ncouch

xn

Nth

3

réf

= 1

·

·

-

3

-

*N**T X**T**dx**m**X*

(()

)

()

D (*m*)

3

N = 1

3

4.3.4 Localization of the constraints (postprocessing)

Conversely, following a calculation by finite element and of obtaining the deformations ***E*** and variations of curve ***K***, one can then calculate the stress field

N = 1, NR

in each

N (

couch)

()

lay down element.

It is necessary to calculate in each layer (*N*), the matrix (and the terms

m)

(*T* (*X* -

· ***D*** (cf [§3.2]) (*m* = chechmate represents the characteristics material of the layer

3)

T réf) (*m*)

N

N).

Constraints

N 1

N

with an ordinate X

-,

in layer (N) are then:

3

J X X

3

3 [

X =

. E + X. K - D

T X - T

N (3)

m

[3

m (

(3) réf)]

()

()

()

and transverse shearing:

X = D X. V + D X.

éq

4.3.4-1

N (3)

1 (3)

2 (3)

()

Note:

In the code postprocessings of the elements of plates are generally defined in locate associated with ANGL_REP. Constraints in the reference mark in

trinsèque of the element are thus

brought back in the reference mark of the variety. One a:

2

2

$$C$$
$$S$$
$$+ 2CS$$

where $C = \cos(\theta)$

11

11

$$2$$
$$2$$
$$= S$$
$$C$$
$$- 2CS$$

$$S = \sin(\theta)$$

cf § 4.1

$$0)$$
$$[])$$

22

22

$$2$$
$$2$$
$$- CS + CS C - S$$

12

12

where θ is the angle between \mathbf{V} and \mathbf{E}

eref

N

0

1

réf

4.3.5 Calculation of the criteria of rupture in the layers (postprocessing)

The limiting values of breaking stresses depend on material of the layer, the direction and of feel request (for a group of elements corresponding to the same field material):

X

L

feel

in

traction

in

limit

:

orthotropi

direction

(1st

fibres)

feel

:

E

X

compressio

in

limit

:

L

feel

in

N

orthotropi

direction

*(1st
fibres)*

feel

:

E

chechmate

Y

T

feel

in

traction

in

limit

:

orthogonal

direction

(2nd

èr

1

with

E

E)

N

Y

compressio

in

limit

:

T

feel

in

N

orthogonal

direction

(2nd

èr

1

with

E

E)

S

cisailleme

in

limit

:

LT

feel

in

NT

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13/09/02

Author (S):

P. MASSIN, F. NAGOT, F. VOLDOIRE, J.M.PROIX *Key*

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It is necessary to calculate the constraints in the reference mark of the layer (defined by the axes of orthotropism) starting from the constraints in the reference mark of the element:

*the angle between **V** and **E***

*is, and that between **E** and the reference mark of orthotropism is:*

1

réf

0

réf

N

C2

S2

+ 2CS

where

$$C = \cos \left(\frac{\pi}{2N} \right)$$

11

$$= S^2 - 2CS$$

$$S = \sin \left(\frac{\pi}{2N} \right)$$

2
2

$$-CS + CS^2 - S^2$$

N
N

Maximum criterion of constraint:

The 5 following criteria are calculated by layer: (N = 1, NR - couch)

$$(LN)$$

if L > 0

if

< 0

N

)

(*N*)

(*Ln*)

X (chechmate

X

N)

()

(*matn*)

()

(

T N) (

T

if

T > 0

if

< 0

N

) (*N*) (*Tn*)

(

Y chechmate

Y

N)

()

(*matn*)

()

(*T*

L N)

S (matn)

Criterion of TSAI-HILL:

This criterion is written in each layer in the following way:

2

$L(N) L(N). T(N)$

2

$T(N)$

2

$LT(N)$

$CTH =$

-

+

+

$X(2mat$

X

Y

S

$N)$

$(2matn)$

$(2matn)$

$(2matn)$

The material is broken when C

1.

TH

Values X and Y are replaced by X and Y when constraints (

,

$L N$

$T N)$

()

()

corresponding are negative.

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Organization (S): EDF-R & D /AMA

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R4.01 booklet: Composite materials
Document: R4.01.02***

Anisotropic elasticity

Summary

This document treats anisotropic elasticity.

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1 Introduction

The objective of this document is to give the form of the matrices of flexibility and Hooke for elastic materials orthotropic, isotropic transverse and isotropic in the cases 3Dn 2D-constraints, plane 2D-deformations and axisymetry.

We speak about “matrices” of Hooke because, by preoccupation with a simplification, we did not adopt notation of a tensor of order 4.

In any rigour, for linear elastic materials, the constraints are linear functions deformations.

One writes: $ij = Hijkl. kl$

The symmetrical nature of ${}_{ij}$ and ${}_{kl}$ and adoption for these tensors of order 2d' a vectorial form allows to write:

$$\{ \} = [H] \{ \}$$

or $\{ \}$ and $\{ \}$ are the vectorial representation of the tensors of a nature 2 $\{ \}$ and ${}_{ij}$ and where $[H]$ is one stamp 6 X 6.

2

Topology of the matrices of Hooke

2.1 Orthotropism

One can show the symmetry of the matrix of Hooke H .

We thus have twenty and one independent components in the case 3D.

**H_{11}
 $H_{12} H_{13} H_{14} H_{15} H_{16}$
 $H_{22} H_{23} H_{24} H_{25} H_{26}$
 $[H]$
 $H_{33} H_{34} H_{35} H_{36}$
=**

**SYM
 $H_{44} H_{45} H_{46}$
 $H_{55} H_{56}$
 H_{66}**

An orthotropic material has two orthogonal plans of elastic symmetry.

*This wants to say that if one calls $[H']$ the matrix $[H]$ after symmetry (S)
 $[H'] = [H]$.*

The relations obtained between the coefficients make it possible to write that $[H]$ is defined by new independent components.

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In the axes of orthotropism:

H

H

H

0

0

0

11

12

13

H

H

0

0

0

22

23

[H]

H

0

0

0

33

=

SYM

H

0

0

44

H

0

55

H66

9 coefficients thus should be provided.

2.2 Isotropy

transverse

The transverse isotropy is a restriction of the orthotropism in where one has the isotropy in one of both orthogonal plans of elastic symmetry.

The matrix [H] will have the same form as for the orthotropism but with additional relations between the components.

5 components are enough to determine [H].

2.3 Isotropy

The material is isotropic if [H] remains invariant in any change of reference mark.

Two coefficients are enough to determine [H].

3 Stamp of Hooke and flexibility

3.1 Notations

Instead of using indices 1, 2 and 3 to locate the axes, one will use the corresponding indices L, T and NR:

L for longitudinal

T for transverse

NR for normal

NR

T

L

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The coefficients which intervene are as follows:

E_L

: Longitudinal Young modulus

E_T

: Transverse Young modulus

E_N

: Normal Young modulus

G_{LT}

: Modulus of rigidity in the plan (L, T)

G_{TN}

: Modulus of rigidity in the plan (T, NR)

G_{LN}

: Modulus of rigidity in the plan (L, NR)

ν_{LT} : Poisson's ratio in the plan (L, T)

ν_{TN} : Poisson's ratio in the plan (T, NR)

ν_{LN} : Poisson's ratio in the plan (L, NR)

Very important remark:

ν_{LT} is different from ν_{TL} :

If one applies a traction according to L

ΔL

$\Delta L =$

(law of Hooke following a direction).

ΔL

This traction is accompanied, proportionally, of a contraction according to

ΔL

T, - ν_{LT} .

ΔL

and of a contraction according to

LL

NR, - Naked _ LN

.

EL

The first index indicates the axis where the effect of the loading is exerted and the second index indicates direction of the loading.

Then one exerts a traction according to T, then a traction according to NR; one obtains:

LL

=

- Naked _ LT TT - Naked _ LN NN

LL

E

E

E

L

T

NR

= - Naked _ TL LL

TT

+

- Naked _ TN NN

TT

(

S)

E

E

E

L

T
NR

= - Naked _NL LL - Naked _NT TT

NN
NN

+

E
E
E
L
T
NR

The matrix of flexibility [H] 1 is symmetrical; one deduces some:

U _LT
Naked _TL

=

E
E
L
T

Naked _LN
Naked _NL

=

E
E
L
NR

Naked _TN
Naked _NT

=

E

E
T
NR

In all that follows NAKED will be noted.
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3.2

Case 3D

3.2.1 Orthotropie

3.2.1.1 Stamps flexibility

l

-

-

LL

LT

LN

0
0
0

E
E
E
LL

L
NR

-
I
-
TL
TN

TT
0
0
0

E
E
E

L
T

NR
TT

-
-
I

NN

NL
NT
O
O
O

E
E
E

L
T
NR

=
NN

I

O
O

LT

GLT

LT

l

SYM

0

LN

G

LN

LN

l

G

TN

TN

TN

H1 Orthotropie

3.2.1.2 Stamps of Hooke

l

LL
- TN NT

(+

LT
LN
NT)

(
+
LN
LT
TN)
LL

0
0
0

EE
EE
.
E. E

T
NR
T
NR
T
NR

LL
(+
I

.

TL
TN
NL)
(- *NL LN*)
(
+
TN
LN
TL)

TT

O
O
O

E E
E. E
E E

.

L
NR
L
NR

L
NR

(+ .

.
l
.

NL
NT
TL)
(
+
NT
LT
NL)
(- *LT TL*)

NN

EEE
O
O
O
NN
=

L
T
NR

EE
.
E. E
EE
.

L
T
L
T
L
T

I -
TN
NT

GLT

LT

0
0

-
LT

NL
LN

*

-

GLN

LT
TL

LN

SYM

0

-

2

*

LN

TN

NL

LT

GTN

1

TN =

* *TN*

H Orthotropism with TL

LT

NL

LN

NT

TN

=

;
=
;
=

E
E
E
E
E
E
L
T
L
NR
T
N

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3.2.2 Isotropy
transverse

3.2.2.1 Stamps flexibility

NR
T
L

The H1 matrix can be deduced directly from the matrix H1-Orthotropism by using the properties transverse isotropy.

In the plan (L, T):

E = E
L
T
=

TL
TL EL
G
=
LT
(
2 1+ LT)

In the plans (L, NR) and (T, NR):

=
NT
NL

=
LN
TN
G
= G
TN
LN

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NR

T

L

E = E

L

T

=

LT

TL

EL

G

=

LT

(

2 1+ LT)

=

NT

NL

=

LN

TN

G
= G
TN
LN

NT
LN
=
E
E
L
NR

1
-
-
LL
LT
LN

0
0
0

E
E
E
LL

L
L
NR

-

1

-

TL

LN

TT

0

0

0

E

E

E

L

L

NR

TT

-

-

1

NN

NL

NT

0

0

0

E
E
E

L
L
NR

=
NN

(
2 1+ LT)

0
0

LT

EL

LT

1

SYM
0

LN

G

LN

LN

I

G

TN

TN

TN

***H1 - Isotropy transverse
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3.2.2.2 Stamps of Hooke

The matrix $[H]$ has same symmetries as $[H] 1$

NR

T

L

LL

1 - .

+

+

NL

LN

LT

NL

LN

LN

LT

LN

0

0

0

LL

E. E

E. E

E E

.

L

NR

L

NR

L

NR

TT

E 2.E

+

1 -

.

+

L

NR

TL

NL

LN

NL

LN

LN

LT

LN

0

0

0

TT

1

2

.

E. E

E. E

E E

.

-

NL

LN

L

NR

L

NR

L

NR

2

NN

2

+ .

+ .

1 -

- LT

NL

LT

NL

NL

LT

NL

LT

=

0

0

0

2

2

2

NN

E

E

E

-

2

NL

LN

LT

L

L

L

LT

E

'

.

1

L

=

2 (1 + LT)
LT

LN

G

.

LN
LN

TN
G

.

LN

TN

H Transverse isotropy
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3.2.3 Isotropy

3.2.3.1 Stamps flexibility according to E and

LL

1

-

-

LL

0

0

0

E

E

E

1

-

TT

0

0

0

TT

E

E

1

0

0

0

NN

E

NN

=

1

1

(

2 +)

LT

=

0

0

LT

G

E

1

(

2 1+)

SYM

=

0

LN

G

E

LN

1

(

2 1+)

=

G

E

TN
TN

H1 Isotropie complete

3.2.3.2 Stamps of Hooke according to E and

LL
1-

0
0
0
LL

TT
1-

0
0
0
TT

NN

1-
0
0
0

NN

E
1-

2

LT
SYM
0
0
LT
=

*(1+)**(1-*
2)
2

1-
2

0

LN

LN

2

1-

2

2

TN

TN

***H Complete isotropy
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3.2.3.3 Stamps flexibility according to the coefficients of Lamé and μ

The law of Hooke takes the following form with the coefficients of Lamé and μ .

$$ij = kk$$
$$ij + 2\mu ij$$

By using the system of equations (S), one obtains:

E
E
.
0
0

LL

L
TL
T
LL

1
E
.
E
0
0

TT
LT L
T
TT
=

1-

.

0

0

0

0

NN

LT

TL

NN

0

0

0 *G*

LT

LT LT

H Orthotropism planes in plane constraints

3.2.3.4 Stamps of Hooke according to the coefficients of Lamé and μ

2μ

0

0

0

LL

+

LL

2μ

0

0

0

TT

+
TT

+ ***2μ 0 0 0***
NN =
NN

SYM
μ 0 0
LN

LN

μ 0
LT
LT

μ
TN

TN

H Isotropy supplements with the coefficients of Lamé

3.3
Orthotropic in plane deformations and axisymmetric case 2D

3.3.1 Stamp flexibility

1
1

1
.

.
0
0

LL
(- NL LN)

-

(+
TL
TN
NL)

LL

E
E
L
L

TT
- 1 (+
I

.

0

0

LT
LN
NT)

(- **TN NT**)
TT

= **E**
E

T
T

0

0

0

0 NN

1

0

0

0

LT

LT

GLT

H-1 Orthotropic planes in plane deformations and axisymetry

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3.3.2 Stamp of Hooke

E

E

E

I

.

.

0

LL

L (

TN

NT)

-

(+

LT

LN

)

NR

NT

(+

+

NL

NT

TL)

LL

,

,

,

TT

E

E

E

T (+

+

1

.

1

0

TL

TN

)

T

NL

(- NL

)

T

LN

(-

+

+

NT

LT

NL)

TT

=

,

,

,

E

E

0

NN

NR (

+ .

0

0

NL

NT

)

NR

TL

(

+

+

NT

LT

NL)

,

,

LT

LT

0

0

0

GLT

'=1 - .

- .

- . -

2

TN

NT

NL

LN

LT

TL

TN

NL

LT

H Orthotropism planes in plane deformations and axisymetry

3.4

Case orthotropic 2D in plane constraints

3.4.1 Stamp flexibility

l

LT

0

0

LL

-

LL

E

E

L

T

TT

l

TL

TT

-

=

0

0

E

E

L

T

0
0
0
0

NN

NN

1
0
0
0

LT
LT

GLT
H1 Orthotropic planes in plane constraints

3.4.2 Stamp of Hooke

E
E
0
0

LL

L
TL
T
LL

1
E
E
0
0

TT
LT L
T
TT
=

0 1-
.
0
0
0
0

LT
TL
NN

0
0
0 G

LT

LT LT
H Orthotropism in plane constraints

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4 Use

in

Code_Aster

In Aster, the definition of the constant orthotropic elastic characteristics or functions of temperature are carried out by the order `DEFI_MATERIAU`, key word `ELAS_ORTH` or `ELAS_ORTH_FO`

for the elements of hull and the solid elements isoparametric or the constitutive layers of a composite (see order `DEFI_COQU_MULT`).

To define the reference mark of orthotropism (L, T, NR) related to the elements, one can refer to documentations

[U4.42.03] `DEFI_COQU_MULT` and [U4.42.01] `AFFE_CARA_ELEM`.

NR

T

L

L, T and NR: directions of orthotropism

longitudinal, transverse and normal

/ELAS_ORTH = _F

(E_L = ygl longitudinal Modulus Young.

$E_T = ygt$ *transverse Modulus Young.*

$E_N = ygn$ *normal Modulus Young.*

$GL_T = glt$ *Modulus of rigidity in plan LT.*

$G_{TN} = gtn$ *Modulus of rigidity in plan TN.*

$G_{LN} = gln$ *Modulus of rigidity in plan LN.*

NU_LT = nult Poisson's ratio in plan LT.

NU_TN = nutn Poisson's ratio in plan TN.

NU_LN = nuln Poisson's ratio in plan LN.

Important remark:

The talk of this note of reference is based on the convention of the books of J.L.Batoz and D.Gay. Documentation U of DEFI_MATERIAU describes these choices, and coefficient NU_LT is interpreted the following way in Aster:

if one exerts a traction according to the axis L giving place to a deformation according to this axis equalizes with

L

L

L =

, there is a deformation according to the axis T equalizes with: $T = - nult^$*

*.
ygl
ygl*

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Finite elements in accoustics

Summary:

This document describes in low frequency stationary accoustics the equations used, the formulations variational which results from this as well as the corresponding translation in finite elements, for each one of

two methods used in Code_Aster: traditional " formulation " with an unknown factor p (acoustic pressure),

and "mixed" formulation with two unknown factors p, v (pressure and speed acoustics).

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1 Introduction

Options of modeling were developed in Code_Aster, making it possible to study low frequency stationary acoustic propagation, in closed medium, for fields of propagation with complex topology, i.e. to solve there under the quoted conditions the equation of Helmholtz.

The solution by finite elements of this equation can be carried out according to two methods:

- a first method consists in being fixed like unknown factors of the problem, only them nodal complex acoustic pressures, is 1 degree of freedom per node [bib1]; it is that that one qualifies formulation with the finite elements "traditional",*
- in the second method, called to the finite elements "mixed", one is fixed like unknown factors at time nodal acoustic pressures and 3 components nodal vibratory speed, that is to say on the whole 4 degrees of freedom per node [bib5].*

To know the ways of propagation of energy in the fluid, the acoustics expert has 2 sizes: the acoustic intensity activates I and the acoustic intensity reactivates J ; these two sizes are defined like:

I

J

$I = \text{Re} [\text{statement}]$

*** J**

and =

Im [statement]

éq 1-1

2

2

*where v * indicate the combined complex one vibratory speed. The knowledge of these sizes bring a very important further information in the resolution of problems of all kinds, such as for example the measurement of the powers radiated by the machines, recognition and localization of the sources.*

The calculation of the acoustic intensity by the finite element method mixed must provide values more precise that traditional method; indeed in the mixed case one ensures the continuity of the derivative first of the pressure and not simply the continuity of the latter.

However if it is more precise, the mixed formulation consumes on the other hand more size memory and of time CPU, while keeping the advantage of having, with a number of degrees of freedom per length

of wave equal, a relative error increasingly weaker on the calculation of the acoustic intensity.

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Equations and boundary conditions of the problem

2.1

Equations and boundary conditions

The equation to be solved is the equation of Helmholtz [bib2]:

(+k²) p = S

éq 2.1-1

.

K indicates the number of wave of the dealt with problem; it can be complex or real, according to whether

propagation is carried out or not in a porous field [bib6]:

K =
éq 2.1-2
C

C indicates the speed of sound, which can be complex in the case of a propagation in medium porous.

p is a complex size indicating the acoustic pressure and S, also complex, represent the sources terms of the problem.

is a reality in all the cases, which indicates the pulsation:

= 2 F
éq 2.1-3

F is the work frequency of the harmonic problem.

We represent on the figure [Figure 2.1-a] the unspecified confined field where the equation applies of Helmholtz [éq 2.1-1] and conditions at the borders.

is open limited of R3 of regular border, partitionnée into v and Z;

= v
Z
N
Z Border absorban
of impedance locali

Z
Fluid
Vibratory source
Z
monochromatic
of normal amplitude

y
V
v
N
X

Appear 2.1-a: Configuration of the problem
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The equation [éq 2.1-1] is to be solved in a closed field. Boundary conditions to take in count on the border of the field express themselves in their most general form like:

 p **p^+** **=****éq 2.1-4** **N**

$/N$ appoints the operator of normal derivative.

, are complex operators, who can be scalars, or integral operators

according to whether the border of application of the boundary condition is with local reaction or reaction not

local (case of the interaction fluid-structure).

The developments currently carried out in Code_Aster relate to only conditions

with the limit with local reaction, for which, are scalars; the cases spécifiables are the following:

.

= 0, 0, 0 which indicates a border of the field at imposed vibratory speed. In

effect, there exists a relation connecting the acoustic gradient of pressure complexes at the speed vibratory particulate complex.

 $p = - J$ **éq 2.1-5** **V** **N** **ρN**

ρ indicate the density of the fluid considered, and one imposes V_n , vibratory speed

normal with the wall ($V_n = v \cdot n$ where N indicates the unit vector of the normal external with border).

0, 0, = 0 relate to a border with acoustic impedance Z imposed.

Acoustic impedance Z is defined like the report/ratio of the pressure at the vibratory speed particulate in the vicinity of the wall with imposed impedance:

$$Z = \frac{p}{V_n}$$

éq 2.1-6

0, = 0, 0 represent the case where one imposes the acoustic pressure p on one border (generally = 0, corresponding to $p = 0$).

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3

Traditional formulation in pressure

3.1

Mathematical expression of the problem

The standard procedure aiming at posing the problem with the traditional finite elements is as follows:

· one supposes the sufficiently regular solution of the problem, $p \in H^2(\Omega)$. One multiplies the equation:

$$(\Delta + k^2) p = 0$$

éq 3.1-1

by a function test. One integrates on Ω and one uses the formula of Green. The border

field, is subdivided in 2 zones, a zone at imposed vibratory speed, v and one

zone with imposed acoustic impedance, Z . The equation obtained can be rewritten under form:

2

0

*grad (p). grad () -**p. FD + J**p. dS + J**0 V.**N dS*

0

2

C

=

Z

Z

v

éq 3.1-2

·

*FD represents an element of differential volume in and dS represents an element of surface on.**· particulate vibratory speed is then determined by:*

J

v =

*grad (p)**éq 3.1-3*

0

3.2

*Discretization by finite elements**In the case of the traditional finite elements, the elementary integrals are four**Ke Me This Ue*

,

,

,

according to the decomposition indicated in [éq 3.2-3] (K E is the matrix of rigidity, Me the matrix of mass, the This matrix of damping and Ue the vector source). Two of them come from voluminal integrals, the two others are the result of integrals respectively on

a vibrating surface and on a surface with imposed impedance.

It will be supposed that the total co-ordinates of an element can be written thanks to the data of m elementary functions of form H_i :

m

$$OM = H OM$$

I

I

éq 3.2-1

$i=1$

One is given moreover, of basic functions N_i , to describe the elementary pressure.

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The pressure inside an element will be able to be written:

m

$$EP (,) = NR (,$$

E

$X y Z$

I

) IP

éq 3.2-2

$i=1$

where EP

I is the pressure with node I of the element E .

In the case of the isoparametric finite elements, the basic functions N_{or} are equal to the functions of H_i form.

On each element of the field, the problem with the finite elements in pressure is written:

(

Q

$$Q$$

$$Q$$

$$K_e - 2M_e + T_{his} (EP) 1 = - J (U_e$$

$$J$$

Q
 $) 1$
 éq 3.2-3

Q
 where (EP) 1 is the matrix column of the nodal values of the pressure on the element.

3.2.1 The matrix of rigidity

The matrix of rigidity $K E$ corresponds to the calculation of:

(p) .
 $() FD$
 E

$grad$
 $grad$
 It admits like general term:

$$K_e =$$

$$FD$$

$$ij$$

$$éq 3.2.1-1$$

NR
 NR
 I
 J
 E

3.2.2 The matrix of mass

1
 The matrix of mass corresponds to M_e with the calculation of:

FD
 2

p .
 E

C

It admits like general term:

I

ME =

NR NR FD

ij

2

I

J

éq 3.2.2-1

E

C

3.2.3 The matrix of damping

The matrix of damping This corresponds to the calculation of:

0

dS

E

p.

Z

Z

It admits like general term:

This =

0 NR NR dS

ij

I

J

éq 3.2.3-1

E Z

Z

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3.2.4 The vector source

The vector U_e source corresponds to the calculation of:

$\int_V dS$

N

0

ev

It admits like general term:

$U.E. =$

$\int_V NR dS$

I

$0 N$

I

éq 3.2.4-1

E

v

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Mixed formulation pressure-speed

4.1

Mathematical expression of the problem

4.1.1 Formulation

local

The equation of Helmholtz [éq 1-1] with the boundary conditions [éq 2.1-3] result in fact from local equations below:

$$I p + \text{div } \mathbf{v} = 0$$

in

éq 4.1. -

1 1

$I \mathbf{v}$

0

$$+ \text{grad } p = 0$$

in

éq 4.1. -

1 2

1

$$\mathbf{v} \cdot \mathbf{N} =$$

p

on

éq 4.1. -

1 3

Z

Z

$$\mathbf{v} \cdot \mathbf{N} = Vn$$

on v

éq 4.1. -

1 4

where $\rho = 1$ 2

ρ C is the adiabatic coefficient of compressibility of the fluid.

The mathematical problem is as follows: being given functions Z L (Z) and

1

V

2

N H (V)

V

), to find functions p and \mathbf{v} defined in and with values in C checking these

equations. They describe, in harmonic mode of pulsation, the small fluctuations in pressure

p and speed \mathbf{v} starting from at-rest state (c.à.d. acoustic pressure and particulate speed

acoustics) of a fluid compressible homogeneous, isotropic, nonviscous, confined in and subjected to a distribution of normal velocity V_n on V .

ρ , and C represent the density respectively, the coefficient of compressibility

adiabat and the speed of sound relating to the fluid, in acoustic absence of disturbance; the coefficient

$= 1$ Z is the localised admittance of material constituting V to the pulsation considered.

To build a method of approximation by finite elements of this problem, it is necessary of

to put in a variational form.

4.1.2 Mixed variational formulation

One takes the scalar product of the equation [éq 4.1.1-1] in 2

L (ρ) with an unspecified function Q

in

1

H (ρ) (it is the function-test).

The formula of Green and the fact that \mathbf{v} check the boundary conditions [éq 4.1.1-3] and [éq 4.1.1-4] us

allow to lead to:

$\int \rho q^*$

ρq^*

$\mathbf{v} \cdot \text{grad} q^*$

$V_n q^*$

éq 4.1.2-1

+

-

= -

Z

v

One proceeds in the same way with the equation [éq 4.1.1-1] by taking his scalar product in 2
 L () with
 an unspecified function-test **U** in (2
 3
 L ()) one obtains:

$$I \mathbf{v} \cdot \mathbf{u} * \\ \mathbf{grad} p \cdot \mathbf{u} * 0 \\ \text{éq 4.1.2-2}$$

0

+

=

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Now we multiply [éq 4.1.2-1] by $J 0$ and [éq 4.1.2-2] by $- J 0$, then we do it
change of function:

$J \mathbf{v}$

\mathbf{v}

!

Thus we obtain the mixed variational formulation [éq 4.1.2-3]:

To find $(p, \mathbf{v}) \in X \times M$ such as:

2
2

$$- \text{div}(\mathbf{v} \cdot \text{grad} q^*) - \frac{1}{C} \rho q^* + J_0 \rho q^* = -$$

J
 $V q^*$
 Q
 X

$0 N$

V
 Z

éq 4.1.2-3

2

$$\text{div}(\mathbf{v} \cdot \mathbf{u}^*) + \text{grad} \cdot \rho \mathbf{u}^* = 0$$

M

where: $X =$

$$X = \begin{pmatrix} 1 & & & \\ & 2 & & \\ & & 2 & \\ & & & \dots \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & & & \\ & 2 & & \\ & & 2 & \\ & & & \dots \end{pmatrix}$$

, , }

and: $M = \begin{pmatrix} 2 & & & \\ & 3 & & \\ & & 2 & \\ & & & \dots \end{pmatrix}$

$$L = \begin{pmatrix} 1 & & & \\ & 2 & & \\ & & 3 & \\ & & & \dots \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & & & \\ & 2 & & \\ & & 3 & \\ & & & \dots \end{pmatrix}$$

, 3

; I

v L }

4.2

Discretization by finite elements.

The field and its borders V and Z are cut out in fields and borders elementary:

E, E

E

V, Z

on which are calculated elementary integrals.

To represent the fields of EP and \mathbf{ve} inside the element the same functions are used basic N_i .

Inside each element (comprising m nodes) one writes:

m

$$\mathbf{OMe} = \text{Nor } (,) \mathbf{OMei}$$

$i=1$

m

E

$$p = \text{Nor } (,) I.E. (\text{internal excitation})$$

p

$i=1$

m

$$\mathbf{ve} = \text{Nor } (,) \mathbf{vei}$$

$i=1$

, are the curvilinear co-ordinates of a three-dimensional element;

$\mathbf{O}M_i$ is the vector position of the M_i node of the element E with m nodes;

N_R , =,

$I I$

1 m are the basic functions on the element E ;

\mathbf{v}_i is the vector “acceleration” with the M_i node of the element E .

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In this case the system of equations [éq 4.1.2-3] is written matriciellement for each element E :

q_e^*

q_e^*

q_e^*

q_e^*

$EP E$

E

$\mathbf{v K}$

$- 2 EP E$

E

$\mathbf{v M}$

$+ J EP E E$

$\mathbf{v C}$

$= - J$

E

S

éq 4.2-1

e^*
 e^*
 e^*
 E
 U
 U
 U
 U^*

where:

T
 EP
 $EP E$
 E
 E
 E
 E
 E
 E
 E
 E
 $\mathbf{v} = =$

,

is the vector solution in

E
{statement v v
statement
 v
 v
1
1x
1y

Z
 1
 $"$
 m
 MX
 my
 $mz\}$
 \mathbf{v}

the element E ;

4.2.1 The matrix of rigidity

\mathbf{K}_e is the matrix of elementary “rigidity”, corresponding to the calculation of the following part of [éq 4.1.2-3]:

$\mathbf{v} \cdot \text{grad } q^*$

- 0

E
 2

$0 \mathbf{v} \cdot \mathbf{u}^* + - \text{grad } p \cdot \mathbf{u}^*$

0

E
 E

One can write it by breaking up it into $m \times m$ pennies matrices \mathbf{K}_{eij} of dimensions 4 X 4 like below:

\mathbf{K}_e
 \mathbf{K}_E
 \mathbf{K}_E
 11
 $“1j” m$

1
 $\mathbf{K}_e = \mathbf{K}_e$
 \mathbf{K}_e
 \mathbf{K}_e

il
“*ij*”
“*im* for $I, J = 1, m$ ”
#

Ke
Ke
Ke
ml
“*mj*” *mm*

with the following terms for **K** *eij*:

Ni
Ni
Ni

0
-

0

NR *J*
0

NR *J*
0

NR *J*

-
X

-
y

Z

E
E

E

NR *J*

-
2

0

Ni
0 NiN *J*

0
0

X

E
E

Keij = NR *J*

-
2
0

Ni
0
0 NiN *J*
0

y
E

E

NR *J*

-

2

0

Ni

0

0

0 NiN *J*

Z

E

E

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4.2.2 The matrix of mass

Is to **me the** matrix of elementary “mass”, corresponding to the calculation of:

1 2
/C pq*

Its coefficients are as follows:

for $I, J = 1,$
4
, R

- 3, 4
, m

- 3
 $ME =$
1 c^2
 ij
NiN J

with R
= 1, m
#

The other terms are **null**

4.2.3 The matrix of damping

It is the matrix of elementary “damping”, coming from calculation from:

0 pq*
 eV

Its coefficients are as follows:

for $I, J = 1,$
4
, R

- 3, 4
, m

- 3
 $C_{ij} =$
0 NiN J

with R
= 1, m

 E
 V

The other terms are **null**.

4.2.4 The vector source

Is the vector elementary “source”, representing the calculation of the terms:

V_n

$0 q^*$
 eZ

Its components are as follows:

for I
, $J = 1,$
4
, R

- 3, 4
, m

- 3
=
 V
 I
 $0 N N_i$

with R
= 1, m

 E
 Z

The other terms are **null**.

After having obtained the field p, v on the field by resolution of the equation [éq 4.2-1] assembled, one returns to the field p, v by the opposite change of function; one can calculate the intensities acoustics defined by [éq 1-1] which are in this case continuous in all the field.

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5

Orders specific to acoustic modeling

At the time of a study by modeling in acoustic finite elements with *Code_Aster* one uses general orders and of the orders which are specific to accoustics, or of which key words and options are particular with this discipline; we present the list below of it.

Definition of the characteristics of the propagation mediums

It is necessary to give the density (actual value) and the celerity of propagation (value complex); one uses for that the order:

DEFI_MATERIAU with the following key words:

key word factor:

FLUID

key words:

RHO

(density ρ)

CELE_C

(celerity c)

Example:

air = DEFI_MATERIAU (FLUID: (

RHO: 1.3

CELE_C: IH 343. 0.));

In this case $0 = 343. + j0$.

Assignment of the model

It is obligatorily necessary to specify that it is about the “acoustic” **phenomenon** and to choose one of the 3

possible **modelings** of accoustics; the order is thus used:

AFFE_MODELE with the following key words for which one specifies the values of assignment

possible:

key word:

PHENOMENON: "ACOUSTIC"

MODELING: "3D" or "PLAN" or "3D_MIXTE"

Example:

guide = AFFE_MODELE (GRID: email

AFFE: (

ALL: "yes"

MODELING: "3d_mixte"

PHENOMENON: "acoustic"));

Boundary conditions

One must affect values normal vibratory speed per face (or edge into two-dimensional) to meshes defining the borders sources, and also values of acoustic impedance per face (edge into two-dimensional) with the meshes defining the borders in imposed impedance.

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One uses the order specific to accoustics:

AFFE_CHAR_ACOU with the following key words:

key word:

MODEL

key word factor:

VITE_FACE

key word:

NET

GROUP_MA

VNOR

(normal vibratory speed V_n)

key word factor:

IMPE_FACE

key word:

NET

GROUP_MA

IMPE

(acoustic impedance Z)

key word factor:

PRES_IMPO

NODE

GROUP_NO

NEAR

(pressure p imposed on the nodes)**Example:**

characou = AFFE_CHAR_ACOU (

MODEL: guide

VITE_FACE: (

GROUP_MA: entry

VNOR: IH 0.0135 0.));

In this case $V_n = 0.0135 + j0$.**Calculation of the elementary matrices**

The various elementary matrices (rigidity, mass and damping) are calculated by options specific. The order is employed:

CALC_MATR_ELEM with the key word OPTION for which one specifies the possible values of assignment:

key words:

OPTION:

"RIGI_ACOU"

"MASS_ACOU"

"AMOR_ACOU"

Example:

matele_k = CALC_MATR_ELEM (

MODEL: guide

CHAM_MATER: chamat

OPTION: "rigi_acou").

Foot-note:

the assembled matrices can be obtained directly with the macro order

MACRO_MATR_ASSE and same options.

Calculation of the elementary vector source

The elementary vector is calculated by a specific option; it is obligatorily necessary to indicate it loading. The order is employed:

CALC_VECT_ELEM with the key word OPTION for which one specifies the only value of assignment possible:

key words:

OPTION:

"CHAR_ACOU"

key words:

CHARGE

Example:

vectelem = CALC_VECT_ELEM (

MODEL: guide

CHAM_MATER: chamat

OPTION: "char_acou"

CHARGE: characou);

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Calculation of the solution

After assembly of the elementary matrices and vector the harmonic solution can be calculated directly with the order:

DYNA_LINE_HARM

Example:

presarm = DYNA_LINE_HARM (

MATR_MASS: matasm

MATR_RIGI: matask

MATR_AMOR: matasi

FREQ: F

EXCIT: (VECT_ASSE: vectass COEF_MULT: - 1.);

PUIS_PULS: 1 PHAS_DEG: 90.);

Postprocessings

From the result of the resolution of the matric transcription of the equations [éq 3.1-2] or [éq 4.1.2-3], of the orders of postprocessing allow to obtain the nodal fields of sizes following acoustics:

P

· level

Acoustic LP of pressure P in dB : $LP = 20 \log_{10}$

5

·

-

2 0 10

· real part of the acoustic pressure

· imaginary part of the acoustic pressure

1

· acoustic intensity activates $\mathbf{I} =$

[

Re statement]

*

2

1

· acoustic intensity reactivates $\mathbf{J} = \text{Im}$ [statement]

*

2

These fields are calculated by the use of the ordering of postprocessing `CALC_ELEM` (the concept result is of type “`ACOU_HARMO`” or “`MODE_ACOU`”):

`CALC_ELEM` with the key words `RESULT` and `OPTION` for which one specifies the values of assignment

possible:

key word:

`RESULT`

key word:

`OPTION:`

“`PRES_ELNO_DBEL`”

(level of pressure in dB)

“`PRES_ELNO_REEL`”

(real part of the pressure)

“`PRES_ELNO_IMAG`”

(imaginary part of the pressure)

“`INTE_ELNO_ACTI`”

(intensity activates)

“`INTE_ELNO_REAC`”

(intensity reactivates)

Example:

`&presharm = CALC_ELEM (`

`MODEL: guide`

`CHAM_MATER: chammat`

RESULT: presharm
OPTION: "pres_elfno_dbel");
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6 Conclusion

Modules were thus integrated in *Code_Aster*, making it possible to make calculations of acoustics interior in low frequencies for complex geometries by two methods: finite elements traditional acoustics and mixed acoustic finite elements.

The two formulations were validated by comparison with the same analytical solution; cases tests are presented in the handbook of V7 validation under coding AHLV100.

As it was envisaged, the precision, with identical grid, is higher in the mixed case; if one holds count obstruction memory this superiority is advantageous only if we want to obtain it field of intensity: one should use the mixed E.F only in this case there.

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Vibroacoustic elements

Summary:

The elements described here make it possible to carry out calculations of the frequencies and clean modes of a structure

coupled to a fluid. They also allow the acoustic calculation of answer.

After the formulation of the problem of coupling fluid-structure, this document describes the step followed for

to implement in *Aster* the new finite elements.

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1 Introduction

The vibratory behavior of a structure is often modified if this one is in the presence of a fluid: it is what is called the vibroacoustic coupling. One distinguishes the cases from coupling into two categories: either the fluid is infinite (it is the case of the immersed structures), or the fluid is contained in a limited medium (it is the case of the tanks more or less filled with fluid).

The finite elements described here make it possible to solve the problems of coupling with a fluid of finished dimension.

General notations:

P:

instantaneous total pressure in a point of the fluid,

p0:

pressure at rest,

p:

acoustic pressure,

:

instantaneous total density in a point of the fluid,

T

0 :

density at rest,

:

acoustic density,

:

density of the structure,

S

X:

acoustic displacement,

U:

displacement of the structure,

:

gradient of acoustic displacements,

, F: pulsation, frequency,
 C:
 speed of sound in the fluid,
 , K: wavelength, numbers wave,
 :
 tensor of the constraints of the structure,
 :
 tensor of the structural deformations,
 C:
 tensor of elasticity of the structure,
 T:
 tensor of the constraints of the fluid.

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2

Vibroacoustic coupling

2.1 Presentation

That is to say an elastic structure defined in a field S which vibrates in the presence of a true fluid, not weighing, compressible, in isentropic evolution defined in a field F. One indicates by

=

F

S, their common surface.

One notes N, the normal external with the fluid field F.

NS

Sf

N

F

Known

S

At a given moment, the state of the fluid is defined by its field of pressure P and that of the structure by its field of displacement U .

It is considered that the coupled system is subjected to small disturbances around its state of balance where the fluid and the structure are at rest.

As follows:

$$P = p_0 + p$$

$$U = U (u_0 = 0)$$

The problem of interaction fluid-structure then consists in solving two problems simultaneously:

- one in the structure subjected, on, to a field of pressure p imposed by the fluid
- the other in the fluid subjected to a field of displacement U of the wall.

2.2

Formulation of the vibroacoustic problem

2.2.1 Description of the structure

Assumption:

The structure is homogeneous and obeys the laws of linear elasticity.

Taking into account this assumption, one can write the various following equations controlling the state of

the structure [bib2].

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2.2.1.1 Conservation equation of the momentum

The conservation equation of the momentum is written, in the absence of voluminal forces others than inertias:

$d^2 U$

-

$$I = 0$$

ij, J

S

dt²

[**éq 2.2.1.1 - 1**]

where:

S

is the density of the structure,

U

is displacement,

ij

is the tensor of the constraints.

2.2.1.2 Relation of compatibility

$$= 1 U + U$$

kl

K, L

L, K

2

[**éq 2.2.1.2 - 1**]

where kl is the tensor of the deformations

2.2.1.3 Law of behavior in isotropic linear elasticity

$$= C$$

ij

ijkl

kl

[**éq 2.2.1.3 - 1**]

with the moduli of elasticity Cijkl checking the identities: Cijkl = Cklij = Cjikl = Cijlk
C being the tensor of elasticity.

2.2.2 Description of the fluid

Assumption: the fluid obeys the laws of linear acoustics.

Taking into account this assumption, the equations controlling the state of the fluid are:

2.2.2.1 Conservation equation of the momentum

The conservation equation of the momentum is written, in the absence of sources:

2

D X

T

-

$$I = 0$$

ij, J

0

2

dt

[éq 2.2.2.1 - 1]

where:

T

is the tensor of the constraints in the fluid,

kl

is the density of the fluid in a natural state,

0

X

is the field of displacement of a particle of fluid.

2.2.2.2 Conservation equation of the mass

With the first order and in the absence of acoustic sources, it is expressed by the relation:

p

+

X

0 div

0

T

T =

[éq 2.2.2.2 - 1]

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2.2.2.3 Law of behavior

T

= - p

ij

ij

[éq 2.2.2.3 - 1]

The fluid is supposed in evolution barotrope (the pressure p is, for the fluid given, a function known only density) $p = C$

20 ;

where c_0 is the speed of sound in the fluid at rest.

2.2.2.4 Equation of propagation waves or equation of Helmholtz

One deduces it by combination from the conservation equations from the mass [éq 2.2.2.2 - 1] and from momentum [éq 2.2.2.1 - 1] written in harmonic mode, with the pulsation:

2

 $p + K p = 0$ **[éq 2.2.2.4 - 1]**

where $K = \rho_0 c_0^2$ is the number of wave.

2.2.3 Description of the interaction fluid-structure

With the interface fluid-structure (Γ) , the fluid being nonviscous, it does not adhere to the wall. One thus writes:

- the continuity of the normal constraints:

 $\mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n} = - p \cdot \mathbf{n}$

ij

I

ij

I

ij

I

[éq 2.2.3-1]

- the continuity normal speeds:

dx

i.n =

i.n

I

I

dt

dt

[éq 2.2.3-2]**2.2.4 Formulation of the coupled problem**

Ultimately, the formulation of the problem of vibroacoustic in terms of displacements for structure and of pressure in the fluid led to the equations of the harmonic problem (P):

C

.u

+ 2 U = 0

ijkl

K, lj

S I

in S

[éq 2.2.4-1]

$p + k^2 p = 0$ in F

C

.u

.n = - p .n

ijkl

K, L

I

ij

I

on

1

p

U .n

I

I =

2

N on

0

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2.3

Variational equations associated the problem

One solves the problem coupled by using the finite element method starting from the weak formulation problem.

2.3.1 Variational equations associated the structure

Either U , kinematically acceptable in S , the equation [éq 2.2.4-1] can be written in the form integral:

$$\int_C \mathbf{u} \cdot \mathbf{U} + 2 \int_C \mathbf{U} \cdot \mathbf{U} \mathbf{F} \mathbf{D} = 0$$

$ijkl$
 K, Li

I

S

I

I

S

After integration by part, one obtains the weak formulation:

$$\int_C \mathbf{u} \cdot \mathbf{U} - 2 \int_C \mathbf{U} \cdot \mathbf{U} \mathbf{F} \mathbf{D} -$$

$$\int_C \mathbf{U} \cdot \mathbf{U} \mathbf{n} \cdot \mathbf{dS} = 0$$

$ijkl$
 K, L

I, J

S

I

I

$ijkl$

I

K, L

I

S

Maybe, if one takes into account the boundary condition [eq 2.2.3 - 1]:

$$\int_C \mathbf{u} \cdot \mathbf{U} - 2 \int_C \mathbf{U} \cdot \mathbf{U} \mathbf{F} \mathbf{D} - \int_C p \mathbf{U} \cdot \mathbf{n} \mathbf{dS} = 0$$

ijkl
 K, L
 I, J
 S
 I
 I

I
 I
 S

[éq 2.3.1-1]

2.3.2 Variational equation associated the equation of the fluid

That is to say p , kinematically acceptable in F . One writes in variational form the equation [éq 2.2.2.4 - 1]:

$$2$$

$$p p + K p p FD = 0$$

$$F$$

$$div (p$$

$$.p) FD -$$

$$p. p$$

$$FD + K 2$$

$$p p$$

$$FD = 0$$

F
 F
 F
 dp

$$p$$

$$\cdot$$

$$ds -$$

$$p. p$$

$$FD + K 2$$

$$p p$$

$$FD =$$

0
 F
 N

F

F

Maybe, if one takes into account the boundary condition [éq 2.2.3-1]

1

$$p.p - k_2 p.p \text{ FD} - U .p \text{ dS} = 0$$

2

N

0

F

[éq 2.3.2-1]

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2.4

Discretization by finite elements

The approximation by finished parts of the complete problem leads then to the system:

U

K 2M

- C

= 0

Q

- CT

H

-

P

.2
.c2
0
0
maybe,
K
- C
M
0
U
U

- 2

= 0

p
Q
p
0
H
.CT
0
c2

where:

K and M are the matrices of stiffness and mass of the structure,
H and Q are the fluid matrices obtained respectively starting from the bilinear forms:

p.p FD
p.p FD

F
and F

C is the matrix of coupling obtained starting from the bilinear form

p.u dS
N
F

The choice of the formulation led to a nonsymmetrical system matrix what does not make it possible to use traditional algorithms of resolution.

2.5

Choice of an additional variable for the description of the fluid

2.5.1 Formulation of the new problem

To obtain a symmetrical problem, one associates the variable pressure, a variable additional. This new variable is, that is to say the potential of displacement of the fluid such as $X =$

grad [bib1], [bib4], [bib5], that is to say the variable [bib3] such as ρ . The variable makes it possible to take

in account directly fluids with variable density. However, it does not represent anything physically. This is why, the potential of displacements is preferred to him.

One thus replaces displacement X of the fluid by grad in the equations of the problem (P) [§ 2.2.4].

One thus obtains the new problem to be solved (P'):

C

.u

$$+ 2 U = 0$$

ijkl

K, lj

S

I

in S

$$2 + k^2 p = 0$$

0

in F

[éq 2.5.1-1]

$$p = 2$$

0

in F

[éq 2.5.1-2]

C

.u

$$.n = - 2 .n$$

ijkl

K, L

I

0

ij

I on

[éq 2.5.1-3]

$U .n$

I

$I = \text{on}$

[éq 2.5.1-4]

N

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2.5.2 Variational formulation associated the problem (P')

One applies to the equation [éq 2.5.1-1] the formula of GREEN:

2

2

$$+ K p FD = 0$$

a.c.

0

F

grad grad

a.c.

$$[K 2 p -$$

2

2

0

$$] FD +$$

ds

0

=

0

F

F

N

Maybe, if one takes into account the boundary condition [éq 2.5.1-3]

p

$$\text{FD} - \text{grad grad FD} + \int_U dS = 0$$

a.c.

[éq 2.5.2 - 1]

N

c2

F

0

F

Moreover, one writes in weak form the equation [éq 2.5.1 - 2] for Q a.c.,

pq

Q

$$p - 2 Q \text{FD} = 0 \text{ Q a.c.}$$

FD - 2

$$\text{FD} = 0$$

0

c2

c2

F

F

0

F

[éq 2.5.2 - 2]

By summoning the equations [éq 2.5.2 - 1] and [éq 2.5.2 - 2], one obtains the associated variational equation

with the fluid:

pq

$$Q + p$$

FD - 2

FD -

$$\text{grad grad FD} +$$

$$\int_U dS = 0$$

0

N
c2

c2

F
0
F
0
F

(Q,) a.c.
[éq 2.5.2 - 3]

2.6
Discretization by finite elements

While proceeding with the same step as that used in [§ 2.3], one is led to the system matrix according to:

U
U
K
O
0
M
0
. M

0

M
M
0

F
0

P
- 2
0
0
fl

P
= 0
c2

c2
0

T

T
M

0
0
0

. M
fl
. H
0

0
c2

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K and M being respectively matrices of stiffness and mass of the structure,
M being the matrix of coupling fluid-structure obtained starting from the bilinear form:
U dS and

M,

,

F Mfl H being fluid matrices, respectively obtained starting from the bilinear forms:

p2 FD,
p FD and of

grad 2 FD

F
F
F

2.7

Calculations of acoustic answer

2.7.1 Speed imposed on the fluid

On a part v of the fluid border F , one can impose a limiting condition of normal speed type v_0 .

The term of edge of the fluid is written then:

- 2

$$\int_S \mathbf{u} \cdot d\mathbf{S} = - 2$$

$$\int_S \mathbf{u} \cdot d\mathbf{S} + I$$

$$\int_S \mathbf{v} \cdot d\mathbf{S}$$

0

N

0

N

0

0

-

F

F

v

v

2.7.2 Impedance imposed on a wall of the fluid

On a part Z of the fluid border F , one can impose a limiting condition of type impedance Z .

$$p = Z v_n$$

where v_n is the outgoing normal speed of the fluid.

By deferring this condition in the equation translating the conservation of the momentum

[$\acute{e}q$ 2.2.2.1 - 1] and by taking account of the law of behavior of the fluid [$\acute{e}q$ 2.2.2.3 - 1], one a:

0

$$\text{grad } p + \rho \mathbf{v} = 0$$

Z

[$\acute{e}q$ 2.7.2 - 1]

To preserve the symmetry of the system, one expresses the equation [$\acute{e}q$ 2.7.2 - 1] according to the

potential of
displacement of the fluid, one a:

$$\begin{aligned} &3 \\ &0 \\ &\text{grad}! + \\ &= 0 \\ &Z^3 \\ &T \end{aligned}$$

Maybe, in harmonic:

$$\begin{aligned} &0 \\ &\text{grad} + I \\ &= 0 \\ &Z \end{aligned}$$

The term of edge of the fluid is written then:

$$\begin{aligned} &2 \\ &2 \\ &2 \end{aligned}$$

$$\begin{aligned} &3 \\ &0 \\ &0 \\ &\cdot \\ &ds = \end{aligned}$$

$$\begin{aligned} &0 \\ &\cdot \\ &ds + I \\ &ds \\ &- \end{aligned}$$

$$\begin{aligned} &F \\ &N \\ &F Z \end{aligned}$$

$$\begin{aligned} &N \\ &v Z \end{aligned}$$

Ultimately, to impose an impedance on a wall of the fluid amounts introducing into the system one term of damping.

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2.7.3 Discretization by finite elements

If one imposes limiting conditions of imposed speed type or impedance of wall imposed on the fluid, one is led to solve the following matric system:

K
0
0
M
0
. M
0
S
0
0
0
0
M
- 2
M
- I 3

0
F
0
0
0
fl
0

0

0

= I V

.c2

c2

0

2

0

T

0

0

0

0

.

M

0

0

Q

MT

fl

. H

0

S

0

Z

c2

Q being the matrix obtained starting from the bilinear form:

$2 \int dS$ and V , the vector obtained from

\int_Z

$\int v dS$

0

0

v

.

3 Integration

in

Aster

The elements described previously belong, for the fluid part, with modeling

“3D_FLUIDE” of the MECHANICAL phenomenon and, for the interface fluid-structure, with modeling

“FLUI_STRU” of the same phenomenon.

They lead to voluminal or surface elements, for the fluid part, in pressure-potential of

displacement and with surface elements for the interface fluid-structure in potential of

displacement of the fluid-displacement of the structure.

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Beam élasto - acoustic

Summary:

One presents an element of coupling élasto - acoustic right which applies to an element of structure of the type beam of Timoshenko. This element makes it possible to carry out, into vibroacoustic, the modal analysis of a piping right-hand side containing of the fluid under pressure (water, vapor...). One can also carry out calculations of answer to fluid sources (flow masses, volume flow rate, pressure) by modal recombination. Boundary conditions applicable to the nodes of these elements are of Dirichlet type: displacement, pressure or potential can y to be imposed.

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1 Notations

P

:

instantaneous total pressure in a point of the fluid

p

:
pressure realised on a cross-section
~
 P
:
fluctuating pressure
 U
:
displacement of the structure

:
potential of displacements of the fluid

:
density of the acoustic fluid
 0

:
density of the structure
 S
, F
:
pulsation, frequency
 C
:
speed of sound in the fluid
, K
:
wavelength, numbers wave

:
tensor of the constraints of the structure

:
tensor of the structural deformations
 FD
:
element of volume
 dA
:
element of surface

:
surface interaction between piping and the fluid

S
:
cross-section of the fluid

F
S
:
cross-section of piping

2 Introduction

In order to be able to carry out calculations of dynamic response of structures filled of fluid to fluids, of the elements of coupling fluid-structure 3D were developed in *Code_Aster* (cf [bib2]).

These voluminal elements have the advantage of allowing a fine description of the structure in particular places like, for example, connection enters a principal piping and one pricking of instrumentation. On the other hand, their systematic use for the analysis of ramified networks and complexes would lead to costs of modeling (realization of grid) and of calculation prohibitory. For this reason, and in order to facilitate simplified studies of dynamic behavior of pipings, one developed an element of beam right élasto-acoustics allowing to realize, with lower cost of calculation, and calculation labour of overall behavior of the parts right-hand sides of pipings in low frequency.

One finds hereafter a presentation of the finite elements of pipings of the type beam élasto-acoustics. The vibratory behavior of the networks of pipings is conditioned by the flow of the fluid which traverses.

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The model of beam élasto-acoustics

3.1 Assumptions

One studies low frequency the vibrations of a piping elastic, linear homogeneous and isotropic coupled to a compressible fluid.

The effects due to viscosity and the flow of the fluid are neglected.

Pipings are lengthened bodies. Indeed, their transverse dimensions are much lower than their length: $D \ll L$, and the thicknesses are such as one can neglect the modes of swelling and of ovalization of the pipe. One can use a model of beam.

Low frequency the acoustic wavelengths associated the studied problems are large compared to transverse and small dimensions compared to the longitudinal dimension of the circuit: $L/C > 1$ and $D/C \ll 1$. Compressibility acts indeed mainly on displacements longitudinal. Transversely, it is considered that the fluid moves like a solid indeformable, i.e. it acts like an added mass. Pressure in a cross-section of pipe being then constant, it is said that the acoustic wave is plane.

3.2

Functional calculus of the coupled problem

One can write the variational formulation of the problem of the pipings filled with fluid from and behavior equilibrium equations of the fluid and the pipe as well as boundary conditions. With to leave the general functional calculus of the three-dimensional coupled problem ([bib1], [bib2]), one can write

functional calculus applied to the particular case of the beams.

The variational formulation of the 3D problem amounts minimizing the functional calculus:

$$F(U, p) =$$

$$\int_S [(\cdot) : (\cdot)] FD$$

$$+ \int_S P^2 2P$$

$$- \int_S$$

$$\mathbf{u} \cdot \mathbf{n} \int_C (grad) FD \} - 0 dA$$

$$C$$

$$0$$

C

0

F

with:

S, the field of the structure

F, the field of the fluid

, the fluid surface of interaction - structure.

3.2.1 Contribution of piping

The model of beam used is that of Timoshenko with deformations of shearing action and inertia of rotation of the cross section. It corresponds to the modeling POU_D_T of which it takes again calculations

elementary. One does not take into account the effects of ovalization [bib3].

The terms associated with piping in the variational formulation are written then:

$$[(U): (U) - S^2 u^2] S ds$$

L

S

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with: L, the average fibre of piping and Ss, the section of piping to the X-coordinate S (cf [Figure 3.2.1-a]).

N

N

J

S

S

D
M (S)
M (S)
J
I
L
K
S (S)
F
F

S (s+ds)

Appear 3.2.1-a: geometry of piping

3.2.2 Contribution of the fluid

In this paragraph, one is interested only in the fluid part of the functional calculus, i.e., term of coupling put aside, at the end which is written in 3D:

P^2
 $2P$
 2
 2
 $-$
 $-$
 2
 0
 2
 $($
 $) FD$

grad
éq 3.2.2-1

C
 C
 0
 0

F

It is supposed that the pressure breaks up into two terms:

$$P(M(S), T) = p(S, T) \sim$$

$$+ p(M(S), T)$$

where p is the value realised on a cross-section of the pressure:

$$p(S, T) =$$

$$\frac{1}{P} \int_S \rho \frac{\partial^2 F(M, S, T)}{\partial t^2} ds$$

and \sim

p is a term of fluctuating pressure which corresponds to the contribution of the transverse modes.

According to the assumptions of the paragraph [§1], p checks the equation 1-D of Helmholtz and \sim p the equation of

Laplace (incompressible). The integral [éq 3.2.2-1] thus breaks up into two terms.

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3.2.2.1 Term corresponding to the contribution of \sim

p

In the movements perpendicular to the axis of the pipe, one considers that the fluid intervenes only by its added mass [bib4], the term related to \sim

p is thus a term of inertia:

2 2

0 (C) S ds

L

F

C being transverse components of the vector displacement of the structure and $S F$ the section of fluid with the X-coordinate S .

3.2.2.2 Term corresponding to the contribution of p

 $P2$ $2P$ 2 2 $- 0$ $-$ $S ds$ $c2$ $c2$ S F $L 0$ 0

3.2.3 Term of coupling

3.2.3.1 Section

current

According to the references [bib4] and [bib5], one shows that the term of coupling C :

 dS 0 $C = -$ $\mathbf{u.n} dA$ 0 $= -$ $\mathbf{U. J} S ds$ F $+$ $\mathbf{u.i}$ ds

R
 F
 0

L

ds
 L

Equilibrium equations of the structure and the equation of propagation plane waves (Helmholtz) in the fluid are thus coupled with the level of the bent parts and the right parts where there is one change of section of piping. In the case of a pipe with constant cross-section:

dS

R

F

and

$= 0$ thus $C = 0$

ds

There is thus no coupling between the movements of beam of the structure and displacements longitudinal of the fluid in the right parts of the circuit. In this case, the fluid is characterized only by its added mass related to transverse displacements.

3.2.3.2 Melts of pipe

S_f

In the case of a bottom of pipe, one notes $T = + S F$, the total surface of interaction between the fluid and piping.

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In the case of a right piping with closed constant section, the term of coupling C is worth then:

$$C = -$$

$$\int_S \mathbf{u} \cdot \mathbf{n} \, dA$$

$$0$$

$$= - \int_S \mathbf{u} \cdot \mathbf{n} \, dA$$

$$S$$

$$+S$$

$$F$$

$$0$$

$$F$$

It is the basic effect.

This term is added at the end of coupling of a current section. Thus, a free node which carries out one condition of null flow through the section [bib6] carries out an acoustic basic condition. Indeed, one incidental plane wave is completely considered on the bottom: the acoustic pressure in the conduit obeys the equation of Helmholtz with normal gradient of pressure no one (fluid displacements and solid being null).

$$2 p$$

$$2$$

$$+ K p = 0$$

$$2$$

$$X$$

$$p$$

$$($$

$$)$$

$$= 0$$

$$X S f$$

One seeks the solution in the form: $p = A \cos (T$

$$- kx) + B \cos (T$$

$$+ kx)$$

i.e. in the form of a linear combination of an incidental plane acoustic wave and one considered wave.

The condition of null gradient on the bottom, checked for every moment, imposes:

$$W_{ITH} = B$$

The considered wave is thus "equal" to the incidental wave (coefficient of reflexion equal to the unit).

3.2.4 Functional calculus of the system coupled in the case of pipings

In the particular case that we treat of a not bent piping, with constant section, functional calculus of the coupled problem is thus written in the following form:

$$1$$

$$2$$

$$F (U, p,) =$$

(U): (U) S ds

2

2

S

-

+

U

2

0

T

L

L [S U

S

S S

F (

)] ds

P2

2P

2

2

+

-

-

S D

2

u.n

2

0

2

F

} $S \ 0 \ dA$

C
 0

C
 $S \ F$
 0

L

3.3

Discretization by finite elements

The solution $(U, p,)$ sought minimizes the functional calculus F . the approximation by finite elements of complete problem leads then to the symmetrical system:

K

0
 $0 \ U$

M + M
 0
M

F

U

K

M
F
 2
fl

0
 $0 \ p -$
 0
 0
 $p = 0$

2

2

0

.c

C

0

0

0

MT

MT

fl

. **H**

2

0

C

K and **M** being respectively matrices of rigidity and mass of the structure,

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KF, Mfl, H being fluid matrices, respectively obtained starting from the quadratic forms:

2

$p^2 S ds, p S ds,$

$S ds$

F

L

L

F

S

F

L

2

MF being the fluid matrix obtained starting from the quadratic form: $p S () ds$

F

$L 0$

C

M being the matrix of coupling obtained starting from the bilinear form: p

dA

S F 0 U N

.

By discretizing p linearly and, one thus has:

$L X$

X

$L X$

X

$p = p$

$+ p$

1

2

and = 1

+

, L being the longor of the element considered.

 L L L $2L$

In this case, the elementary matrix of stiffness of the fluid is written:

 $S L$

1

 $1 \ 2p$ **K**

F

1

 $F =$ $\{1p \ p2\}$

3

1/

2

1

p

2

The elementary matrix of coupling is written:

-1

0 $u1$ $\mathbf{M} = 0 \ S_f \ {1 \ 2}$

0

1 $u2$

The various matrices of fluid mass elementary are written:

 $S L$

1

 $1 \ 2p$ **M**

F

1

 $fl =$ $\{1$

2}

3

1/

2

1

P

2

S

1 -

1

H

F

= -

{

1

1

2}

L

-1 1 2

3.4

Establishment in *Code_Aster*

On the principles which we have just described, a vibroacoustic element of beam, Timoshenko for the piping part, right to constant section, was established in *Code_Aster*. It belongs to modeling "FLUI_STRU" of the "MECHANICAL" phenomenon.

This element has 8 ddl by node: displacements and rotations of piping, pressure and potential of displacement of the fluid (cf [Figure 3.4-a]). The formulation is written for displacements **buildings** in the local reference mark with the element made up of neutral fibre (axis X) and of the principal axes

of inertia (axis Y, axis Z) of the section. The two scalars p and (pressure and potential of fluid displacements) are invariants by change of reference mark.

On each node of this element, one can impose boundary conditions of the Dirichlet type in pressure, potential of fluid displacements and displacements (translation or rotation).

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Y

Uy

Z

Uz

Ury

Y

Urz

Uz

X

Urz

Z

Uy

Ux

Urx

p,

Ury

Ux

Urx

p,

Appear 3.4-a: element of beam filled with fluid

This element to date makes it possible only to calculate the clean modes of a right piping filled of fluid and to make harmonic calculation of answer. Effects of curve or abrupt widening of section are not taken into account for the moment, but these fluid-structure effects, lorqu' one has business with not very dense fluids like the vapor of a piping of admission, do not seem to have a determining importance on the calculation of the first modes: the correct mechanical representation elbow (coefficient of flexibility) seems sufficient to calculate these frequencies [bib7].

In modal analysis, one can quote the case of a right piping filled with fluid with loose lead:

Fluid

Structure

Appear 3.4-b: beam filled with fluid embedded - free

the Eigen frequency of the mode of traction and compression of this fluid coupled system/structure is data by the relation:

L
S
E
tg (
)
S
=
C
S
2
F C
0

One indicates by:

E: Young modulus of solid material

SS: section of the solid

S F: section of the fluid

It is supposed here that the speed of speed of sound in the fluid is equal to the speed of sound in

E
solid $C_s =$

S
[bib7].

The transitory calculation of response for this type of finite element ($\mathbf{U}, p,$) is not yet available in *Code_Aster*.

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Fluid coupling - Structure with Free Face

Summary:

One has here the fluid coupling/structure if the fluid has a free face. Elements of free face were established in *Code_Aster* to calculate the modes of ballotement of a coupled fluid with an elastic structure for a three-dimensional problem.

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3.2.3 Introduction of an additional variable 8

4 Establishment in *Code_Aster* 9

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1 Notations

- P*
:
stationary pressure in the fluid
- p*
:
fluctuating pressure in the fluid,
- X**
:
displacements in the fluid,
- F**
- X**
:
the field of displacements in the structure,
- S**
- G*
:
gravity,
- :
potential of displacements of the fluid,
F, S: density of the fluid, the structure,
- T**
:
the tensor of the constraints in the fluid,
- :
the tensor of the constraints in the structure,
- :
the tensor of the deformations in the structure,
- C**

:
the tensor of elasticity,
 C
:
the speed of sound in the fluid,
 H
:
the height of the fluid (or average height),
 N
:
the normal external of the fluid.

2 Introduction

In order to study the behavior of structures filled of fluid, one can be led to take in hope the phenomena of shaking i.e. to add to the system coupled fluid-structure, the effect gravity on the level of the free face of the liquid. The structures concerned are, for example, tanks of nuclear thermal power stations of the rapid system, swimming pools of fuel storage [bib4]. One thus supplemented the developments already carried out in coupling fluid-structure [bib3] by the introduction of new surface elements which take into account, in their formulation, the effect gravity.

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3

Theoretical formulation of the problem

The problem of interaction heavy structure-fluid amounts solving three problems simultaneously:

- the structure is subjected to a field of pressure P imposed by the fluid on the wall;
- the fluid is subjected to a field of \mathbf{X} s displacement imposed by the structure on;
- gravity acts on the free face where $p = G Z$.

It will be considered initially that the fluid is nonheavy before introducing gravity with

paragraph [§3.2].

3.1

Recalls on the coupling fluid-structure

In order to account for the interaction fluid-structure well, we will analyze them separately equations governing the behavior of the fluid and those which govern that of the structure, without to consider in this chapter the boundary conditions concerning the free face.

3.1.1 Description of the fluid

It is considered that the studied system is subjected to small disturbances around its state of balance where the fluid and the structure are at rest: thus, $P = p_0 + p$ and $X = X_0 + X$

S

S

$0 =)$

0. What allows

to write [bib2]:

$= -$

2

$F \operatorname{div}(\mathbf{X})$ of or p

\mathbf{F}

$= - F C \operatorname{div}(\mathbf{X})$

\mathbf{F} .

With:

.

p fluctuating pressure of the fluid,

.

the disturbance of density of the fluid, F density of the fluid at rest,

.

X (,

$F R T$) the field of displacement of a particle of fluid.

The fluid is:

· perfect (i.e. nonviscous)

· barotrope

:

$p = c^2$

éq 3.1.1-1

2

.

and irrotational: there is a potential of displacements, such as $p = F t^2$

The behavior of the volume of fluid eulérien is thus described by the following equations:

· law of behavior:

$T = - p$

ij

ij

· conservation equation of the momentum in the fluid in the absence of source:

2

X

div (**T**)

F

= *F*

2

éq 3.1.1-2

T

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· conservation equation of the mass:

xs

+

div

0

éq 3.1.1-3

T

F

T =

By combining the conservation equations of the momentum [éq 3.1.1-2] and of the mass [éq 3.1.1-3] written in harmonic mode with the pulsation, one obtains, thanks to [éq 3.1.1-1], the equation

of Helmholtz:

2

p +

$$p = 0$$

c_2

3.1.2 Description of the structure

It is considered that the structure is elastic linear and that one remains in the field of the small ones disturbances. Taking into account these assumptions, one writes:

· the law of behavior in linear elasticity:

$$\epsilon_{ij} = \frac{1}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij}$$

C_{ijkl}

· the conservation equation of the momentum in the structure in the absence of voluminal forces others than inertias:

$\rho \ddot{\mathbf{x}}$

$$\text{div}(\mathbf{S}) = \mathbf{f}$$

\mathbf{S}

$\mathbf{S} \cdot \mathbf{n}$

· the equation of compatibility on the tensor of deformation:

$$\epsilon_{kl} = \frac{1}{2} (\partial_k u_l + \partial_l u_k)$$

ϵ_{kl}

3.1.3 Description of the interface fluid-structure

With the interface Γ between the fluid and the structure, like the fluid is not viscous, there is continuity of

normal constraints and normal speeds to the wall, and nullity of the tangential constraint (absence of viscous friction). These boundary conditions are written:

$$\mathbf{u} \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{n} = -p \mathbf{n} \cdot \mathbf{I} \mathbf{n}$$

$\mathbf{u} \cdot \mathbf{t} = \mathbf{v} \cdot \mathbf{t} = 0$

$\mathbf{x} \cdot \mathbf{n}$

\mathbf{X}

$\mathbf{n} \cdot \mathbf{n} = 1$

$\mathbf{S} \cdot \mathbf{n}$

\mathbf{T}

\mathbf{T}

3.1.3.1 Formulation of the coupled problem

Finally, the equation of the problem coupled fluid-structure is written, by taking p like variable describing the field of pressure in the fluid and \mathbf{x} the field of displacements in the structure:

C

X

$$+ 2 \mathbf{X} = 0$$

dans V

$ijkl$

S
S
S
S
K, lj
I

2
p
+
p = 0
dans V
c2
F
N

= C
X
N = - p N
ij I
ijkl
S
I
ij I
on
K, L
p

= 2 X N
on
N
F
F
I
I

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Fields of \mathbf{x} s displacements for the structure and pressure p for the fluid sought minimize the functional calculus:

1
1
1
 p^2
 $L(\mathbf{X}, p, Z)$
2
2
2
S
=

2 [
 (\mathbf{X})
 $(\mathbf{X}) -$
 \mathbf{X}
S
S
 $S] - p \mathbf{X} \mathbf{N} D +$

2
 $(\text{grad } p)$
S
-
2
2 FD
 ij
 ij
S

C
 V
 F
 V

S
 F

3.2

Action of gravity on the free face

3.2.1 Formulation of the problem

One points out the linearized dynamic equations here describing the small movements of a fluid perfect. One chooses a description eulérienne fluid:

$$\text{grad } P = \left(\begin{array}{c} 2x \\ S \end{array} \right)$$

$\mathbf{G} -$

)

V

F

t^2

in

F

With balance the particle of fluid was in \mathbf{M}_0 and thus: $\text{grad } P$

\mathbf{G}

in V

$$0 = F$$

F .

One considers movements of low amplitude around the state of balance (it is the assumption of small disturbances): then $\mathbf{M} = \mathbf{M}_0 + \mathbf{X}$ (\mathbf{M}

\mathbf{F}

$0, T$)

Are p the fluctuation in pressure eulérienne and p_L the fluctuation in Lagrangian pressure, then:

$$(p_M, T) = P(\mathbf{M}, T$$

$$0) - P_0(\mathbf{M}_0)$$

$$p = P$$

L

$$(\mathbf{M}, T) - P_0(\mathbf{M}_0)$$

Taking into account the assumption of small displacements:

p

$p \text{ grad} P(\mathbf{M}, T) \mathbf{X}(\mathbf{M}, T)$

L -

=

O

F

O

= - $G \mathbf{X}(\mathbf{M}, T)$

éq 3.2.1-1

F

F

O

If one considers the case of a heavy fluid having a free face in contact with a medium to constant pressure $Patm$, one can write, by neglecting the effects of surface tension:

$P(\mathbf{M}, T) = Patm$ on free face SL i.e.: $pL = 0$. Maybe, with [éq 3.2.1-1],

$p = G$

F

($\mathbf{X} F$.)

Z

Taking into account the assumption of the small movements, the instantaneous slope of the tangent plan is one

infinitely small first order. $\mathbf{X} \cdot \mathbf{Z}$

F

thus merges with the second order near with rise vertical h .

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M

H

xf
G
SL O
Mo
SL
P (M) = P
O
O
atm

Appear 3.2.1-a: approximation on the free face

Thus, if one adds in the boundary conditions the condition of gravity on the free face, that returns to consider in $Z = H$ the linearized condition:

$$p = G Z$$

F

éq

3.2.1-2

The equations of the total problem become:

C

X

$$+ 2 X = 0$$

dans *V*

ijkl

S

S

S

S

K, lj

I

2

p

+

$$p = 0$$

dans *V*

c2

F

$$N = C$$

X

$$N = - p N$$

ij I

ijkl

S

I

ij I

on

K, L

p

$$= 2 X N$$

on and *SL*

N

F

F

I

I

$$p = G Z$$

on *SL*

F

To express the functional calculus, one uses the law of behavior on the free face. By considering one acceptable field of displacement *Z* one obtains [bib2]:

$$G Z Z ds = p Z ds$$

SL

SL

Maybe, finally, the functional calculus of the total system fluid structure subjected to gravity:

1

1

1

*p*²

L (**X**, *p*, *Z*)

2

2

2

S

=

2 [
(X)
(X) -
X
ij
S
ij
S
S
S] - p X .n D +

2
(grad p)
S
-
2
2 FD

C
V
F
V

S
F
1
+
G z2ds - p Z ds
2

SL
SL
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This taking into account of gravity implies two additional terms in the functional calculus describing the fluid:

1

· a term of potential energy related to the free face:

2

$G Z ds$

2 SL

· a term due to the work of the hydrodynamic pressure in the displacement of the free face

:

$p Z ds$

SL

However it should be noted that it is not the single effect of gravity since in any point of the wall be exerted a permanent pressure - $G Z$ (or Z is the altitude of the point **M** considered: one supposes that $Z = 0$ on the level of the free face to balance). The point **M** is actuated by a **Xs** movement infinitesimal, the element of surface D thus varies and the effort due to the permanent pressure too. This effort is responsible for an additional term of rigidity being added to the rigidity of structure in system. It could cause a buckling of the structure by cancelling structural rigidity. This effect is negligible on the vibratory characteristics ([bib2], [bib1]), one thus does not take it into account.

3.2.2 Discretization by finite elements

To obtain the discretized form of the functional calculus, one replaces each integral by a sum integrals on each element I of the discretized system, then one uses an approximation by elements stop unknown functions of displacement and pressure on each element I [bib18].

The unknown factors are **Xs** (U, v, W), p, Z , one has then by posing Nor the functions of forms (or functions

of interpolation nodal on element I):

$\mathbf{x}_s = \mathbf{NR} \mathbf{U}$

$= \mathbf{D}$

I

X. N

S

$= \mathbf{NR} \mathbf{U}$

$\mathbf{p} (X, y, Z) = \mathbf{NR} \mathbf{p}$

I

I

$$= \mathbf{B} \mathbf{U}$$

$$\mathbf{Z} = \mathbf{NR} \mathbf{Z}$$

if

$$\mathbf{p} = \mathbf{NR} \mathbf{p}$$

I

where, p , are the unknown factors with the nodes structures and the fluid nodes, and Z *the* unknown factors with free face.

From where the discretized expression of the functional calculus associated with the problem:

H**Q**

$$\mathbf{L} = \mathbf{C} (\mathbf{K} - 2 \mathbf{M}) \mathbf{U} + \mathbf{P} \mathbf{t} ($$

-

$$) \mathbf{p} + \mathbf{z} \mathbf{t} \mathbf{G} \mathbf{K} \mathbf{Z}$$

T**T**

2

2

2

2

*F***Z**

$$- \mathbf{p} \mathbf{M} \mathbf{Z}$$

Z

$$- \mathbf{PC} \mathbf{U}$$

*F**F C*

with

$$\mathbf{K} = \mathbf{NT} \mathbf{L} \mathbf{T} \mathbf{D} \mathbf{B} \mathbf{NR}$$

I I I I I*FD*

stamp stiffness of the structure

I VI

$$\mathbf{M} = \mathbf{NT} \mathbf{NR}$$

I F I I*FD*

stamp mass of the structure

I If

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and

Q = NT NR

T

I

I

I

FD

M = NR NR

Z

I I I

dS

I VI

I

fl

I

K = NT NR

T

Z

If If I

dS

M = NR NR

Z

I If I

dS

I If

*I If***H = NT NR****III***FD**I Vijl*

where C is the speed of sound in the fluid, F the density of the fluid and where **KF** corresponds to potential energy of the fluid, **K Z** with the potential energy of the free face, **H** with the kinetic energy of fluid, **M** with the coupling fluid-solid and **Mz** with the coupling $p - Z$ on the free face.

The approximation by finished parts of the complete problem leads then to the following matrix system:

K- **C**0 **U****M**

0

0

U**Q**

2

0

H0 $p - C$ **M** p F

2

 F **Z = 0** C 0 - **MR. K****Z****Z Z**

0

0

0

Z

The first equation corresponds to the movement of the structure subjected to the forces of pressure, second with that of the movement of the fluid coupled with the structure and the free face, third is the free equation of face.

However the written problem of the kind has matrices masses and rigidity nonsymmetrical what prevent the use of the traditional algorithms of resolution of *Code_Aster*.

3.2.3 Introduction of an additional variable

To make the problem symmetrical and to be able to use the traditional methods of resolution, one introduced an additional variable: potential of displacements in the fluid [bib2].

X

2

$$\mathbf{F} = \text{grad i.e.} = p$$

This additional unknown factor is related to the unknown factors of the problem, which leads to a matrix of singular rigidity.

One reformulates the problem coupled heavy structure-fluid:

C

X

$$+ 2 X = 0$$

dansV

ijkl

S

S

S

S

K, lj

I

2

+

$$p = 0$$

dansV

c2

F

F

$$p = 2$$

dansV

F

F

$N = C X N = - 2 N$

ij I

ijkl

S

I

F

ij I

on

K, L

$= X N$

on

N

F

I

I

$p = G Z$

on *SL*

F

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What leads to the functional calculus of the coupled system:

1

1

p2

1
L (**X**, *p*, *Z*)

2
2
2
S
= 2 [(**X**) (**X**) - **X**

ij
S
ij
S
S
S] +
FD +
G Z ds

2 2
2
C
V
V
SL
S
F

F
p
2
2

- **X N D** + *Z ds* +

(grad)

F
S
+ 2 *FD*
F

2
C

SL

V

F

Maybe while discretizing:

$$\begin{aligned}
&1 \\
&L = T (K - 2 M) + \\
&T \\
&p Q p + \\
&T \\
&G Z K Z
\end{aligned}$$

2

F

Z

F C

F

1

- 2 2

$$T H + F T C + F T M Z$$

Z

+

T

p Q

2

2

C

What is written, in matric form:

M

0

C

0

K

0

0

0

F

Q

Q

0
0
0
0
0
0
0

2
2
C

P
P

2
C

F

-

Qt
= 0
0
0
0
0

Ct
F
H

F
M

2
F
Z

0
0
0 **G K Z**
C
Z

F
Z

0
0
MT
F
Z
0

4 **Establishment in *Code_Aster***

The library of the finite elements of *Code_Aster* was enriched by five surface elements isoparametric having like degrees of freedom the deflection of the free face and the potential of displacements of the fluid on the free face. They are compatible with the éléments 3D which treat it fluid problem of coupling/structure [bib3]

One names:

MEFP_FACE3 and MEFP_FACE6 respectively triangles with 3 or 6 nodes,
MEFP_FACE4, MEFP_FACE8 and MEFP_FACE9 respectively quadrangles with 4, 8 or with 9 nodes.
These elements belong to modeling 2D_FLUI_PESA of the MECHANICAL phenomenon.

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Elements of absorbing border

Summary

This document describes the establishment in *Code_Aster* of the elements of absorbing border. These elements of type paraxial, which one describes the theory here, are assigned to borders of elastic ranges or fluid for to deal with problems 2D or 3D of interaction ground-structure or ground-fluid-structure. They make it possible to satisfy condition of Sommerfeld checking the assumption of anechoicity: the elimination of the elastic plane waves or diffracted acoustics and not physiques coming from the infinite one.

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1 Introduction

1.1

Problems of a semi-infinite medium for the ISS

Standard problems of seismic answer and interaction ground-structure or ground-fluid-structure bring to consider infinite or supposed fields such. For example, in the case of stoppings subjected to the seism, there is often business with reserves of big size which enable us to make the assumption of anechoicity: the waves which leave towards the bottom reserve “do not return” not. This has

for goal to reduce the size of the structure to be netted and to allow to pass from complex calculations with average data processing current. One proposes on [Figure 1.1-a] below diagram which described the type of situations considered.

Fields modelled with the finite elements:

F

fluid field (for example retained stopping)

B

structure field (for example vault of stopping)

S

non-linear ground field

Surface absorbing

fluid

'S linear ground field

F

B

S

'S

Surface absorbing

rubber band

Appear 1.1-a: Field for the interaction ground-fluid-structure

In all the document, one considers that the border of the grid finite elements of the ground is in a field with the elastic behavior.

The elliptic system theory ensures simply the existence and the unicity of the solution of acoustic or elastoplastic problems in the limited fields, under the assumption of conditions with the limits ensuring the closing of the problem. It goes from there differently for the infinite fields.

One must

to have recourse to a condition particular, known as of Sommerfeld, formulated in the infinite directions of

problem. This condition ensures in particular, in the case of the diffraction of a wave planes (elastic or acoustics) by a structure, the elimination of the diffracted waves not physiques coming from the infinite one

that the traditional conditions on the edges of the field remotely finished are not enough to ensure.

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1.2

State of the art of the numerical approaches

The privileged method to treat infinite fields is that of the finite elements of border (or integral equations). The fundamental solution used checks the condition automatically of Sommerfeld. Only, the use of this method is conditioned by knowledge of this fundamental solution, which is impossible in the case of a ground with complex geometry, for example, or when the ground or the structure is nonlinear. It is thus necessary then to have recourse to the finite elements.

Consequently, of the conditions particular to the border of the grid finite elements are necessary for to prohibit the reflexion of the outgoing diffracted waves and to reproduce the condition thus artificially of Sommerfeld.

Several methods make it possible to identify boundary conditions answering our requirements. Some lead to an exact resolution of the problem: they are called “consistent borders”.

They are founded on a precise taking into account of the wave propagation in the infinite field.

For example, if this field can be supposed elastic and with a simple stratigraphy far from structure, one can consider a coupling finite elements - integral equations. One of the problems of this solution is that it is not local in space: it is necessary to make a separating assessment on all the border

the finished field of the infinite field, which obligatorily leads us to a problem of under-structuring. This not-locality in space is characteristic of the consistent borders.

To lead in the local terms of border in space, one can use the theory of the infinite elements [bib1]. They are elements of infinite size whose basic functions reproduce as well as possible

elastic or acoustic wave propagation ad infinitum. These functions must be close to solution because the traditional mathematical theorems do not ensure any more convergence of the result of

calculation towards the solution with such elements. In fact, one can find an analogy between the search for

satisfactory basic functions and that of a fundamental solution for the integral equations. geometrical constraints are rather close but especially, this research presents a disadvantage of size: it depends on the frequency. Consequently, such borders, local or not in space, can be used only in the field of Fourier, which prohibits a certain category of problems, with non-linearities of behavior or great displacements for example. One thus arrives at having to find borders absorbing powerful who are local in space and in time to treat with the finished parts of the transitory problems posed on fields infinite.

We will present in the continuation the theory of the paraxial elements which carry out absorption sought with an effectiveness inversely proportional to their simplicity of implementation as well as description of the constraints of implementation in *Code_Aster*. The developments are presented to deal with 3D problems. Those for the cases 2D were carried out and their theory results simply of modeling 3D.

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Theory of the paraxial elements

One presents in this part the principle of the paraxial approximation in the case of the elastodynamic one linear. Two theoretical approaches make it possible to determine the spirit and the practical application elements

paraxial rubber bands: one owes the first in Cohen and Jennings [bib2] and the second with MODARESSI

[bib3]. The application of the theory of the paraxial elements to the fluid case will be made in the part following.

Subsequently, as presented on [Figure 1.1-a], one supposes that the border of the grid of ground is located in a field at the elastic behavior.

The approach of Modaressi established in *Code_Aster* at the same time makes it possible to build borders absorbing and to introduce the incidental seismic field.

2.1

Spectral impedance of the border

To obtain the paraxial equation, we should initially determine the shape of the field of displacement diffracted in the vicinity of the border. For that, one leaves the equations of the elastodynamic 3D:

2

2

2

2

U2 **U**

2

U2 **U**- **E C**- **E C**- **E C**

= 0

2

11

T

2

12

*x'**x'*

22

2

*x*³*x*³*u'*

2

1 **C**

0

1

0 1

With: **U = E =***P***E**

2

2

12 =

CP - C

2 (
S)

U
11
2
2
3
C 0

C
C
1 0
S

2
1 C
0
E
S
22 =
2
2
C 0

CP

Constant C , homogeneous at a speed, is introduced to return certain quantities adimensional. The equations and their solutions are of course independent of this constant.

One calls x' and u' the directions and the components of displacement in the tangent plan and x_3 and u_3 according to \mathbf{e}_3 , normal direction at the border.

One proceeds to two transforms of Fourier, one compared to time, the other compared to variables of space in the plan at the border. One limits oneself to the case of a plane border and without corner:

The equations are written then:

(
2

! U

$c_2 - c_2$
3

2

2

2

P

S) - . ! **U** + *I*

C - +

! **U** + ! **U** = 0

X + *S*

2

3

X

3

(

2

2

! **U** **U**

!

*c*² - *c*²

3

2

2

2

P

S) - *I*.

+

+ *C* - +

! **U** + ! **U** =

X

2

2

3

3

0

3

X

S

X

3

3

where! **U** and! **u3** indicates the transforms of Fourier and the vector of wave associated with x' .

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It is about a differential connection in x^3 which one can solve by diagonalisant it. One deduces some:

(! **U**) .e3 = A (

exp - I X

S

3)

! **U**. = [A exp (- I x3) + A

(

exp - I X

P

P

S

S 3)]

! **u3** = - A

(

exp - I 3

X) - A E (

xp - I 3

X

$P P$ P $S S$ S

)

2

2

2

2

With: $P =$

- and =

-

 c^2 S

2

 P S C To determine constant A , A and A S P , one supposes known! (U'),

0 on the border of the field

finite elements. One expresses them according to! (',)

0 = !'

 u' U and! U (',)

0

0

3

=! u_3 .One now will evaluate the vector forced on a facet of normal e_3 in $x_3 = 0$, which usthe impedance of the border will give. One subjects to $T(x', x_3)$ the same transform of Fourier in space that for the equations of elastodynamic, so that:

!

 U U $T(x, X)$

! 3

$$= I \mathbf{U} + (+ 2\mu)$$

E
3 +

μ
+ I
3

! u3
 x^3
 x^3

One wishes to free oneself in $x^3 = 0$ from the terms containing from derived in x^3 . The system obtained previously allows it to us according to!

'
 U and! U
0
30 :
! u0. = I 2 **U**

! 30
3
 X
! U

$$0 .e^3 = - iS (! \mathbf{u0}) .e$$

3
3
 X
! U

30
=
 $I P ! \mathbf{U}.$
0
 S
+
+

2

(P S) U

! 30

3

X

One thus obtains the spectral impedance of the border:

$$\mathbf{T} = a_0 \mathbf{E} + b_0 + c_0 \mathbf{E}$$

0

3

3

where a_0 , b_0

c_0

and

are related to and which depends linearly on! '

\mathbf{U} and! \mathbf{U}

0

30

One can then write: $\mathbf{T} = ($

With,)! \mathbf{U}

0

0(,)

where A appoints the operator total spectral impedance. One returns to physical space by two transforms of Fourier opposite.

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2.2

Paraxial approximation of the impedance

The spectral impedance calculated previously is local neither spaces some nor in time since it makes to intervene! u_0 (“), transformed of Fourier of $u_0(X, T)$ for all x' and any T .

The idea is then to develop P and S according to powers' of. This approximation will be good either high frequency, or for small.

Let us examine the dependence in x^3 , for example of! u_3 : one will have, for $u_3(x', 3 X, T)$ of the terms of form:

$$[\exp I (X + T - X P^3)]$$

2
C

With the development of

P
1

P :

$$P = - + \dots CP$$

It is shown that for small, one will have waves being propagated according to directions' close to normal e_3 at the border, because the exponential one is written:

X

3

$$\exp I$$

$T -$
 $+ IO$

C

P

Consequently, with an asymptotic development of P and S , while multiplying by a power suitable to remove this quantity with the denominator, one obtains:

$A_0(\omega) \mathbf{T} = A$

0

$1(\omega) !$

u0

where A_0 and A_1 are polynomial functions in ω and ω^{-1} .

Maybe, after the two transforms of Fourier opposite:

With

$\mathbf{T} = A$

0

0

1

u0

$X T$

$X T$

One thus obtains the final form of the approximate transitory local impedance according to the last term

in retained. One can find the calculation detailed of Have in [bib5].

For example, for order 0:

U

U

T

3

$= C$

E

0

P

3 +

C

T

S T

This corresponds to viscous shock absorbers distributed along the border of the elements field finished.

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With order 1:

T

2

2

2

2

0 = U

U

U

U

3

C

E

P

+

Cs

+

3

C 2c - *C*

+ *C* -

2c

2

3

2

S (

S

)

e3 (*P*

)

T

T

T

X

T

X

T

2

2

.

+ 2

CP U.E.

3 3

C

C C

S

P

S -

+ 2

U

C C

-

2 2

X

S

P

2 2

X

One sees appearing the derivative compared to the time of the vector forced. In the digital processing, it will be necessary to have recourse to an integration of this term on the elements of the border.

To conclude, it will be retained that the paraxial approximation led to a transitory local impedance utilizing that derivative in time and in the tangent plan at the border. In way symbolic system, one writes:

U**t0 =**

A0

with order 0

T

T

2

U U

0 = A

,

, **U**

1

2

with order 1

T

T

T

2.3

Taking into account of the incidental seismic field

It is pointed out that the behavior of the ground is supposed to be elastic at least in the vicinity of the border. With

the infinite one, the total field U must be equal to the incidental field \mathbf{u}_i (one of the consequences of the condition of

radiation of Sommerfeld). One thus introduces the field diffracted \mathbf{u}_r such as:

$$\mathbf{U} = \mathbf{u}_i + \mathbf{u}_r$$

$$\lim_{X \rightarrow 0} \mathbf{u}_r = 0$$

X

At the border of the grid finite elements, one writes the condition of absorption for the diffracted field:

\mathbf{U}

\mathbf{T}

R

$$0(\mathbf{u}_r) =$$

A_0

with order 0

T

\mathbf{T}

2

0 (

\mathbf{U}

\mathbf{U}

\mathbf{U}

R

R

$$R) = A$$

,

\mathbf{U}

1

2

R

with 'order 1

T

T

T

One deduces from it the total vector forced on the border from the grid finite elements:

\mathbf{U}

\mathbf{U}

\mathbf{T}

I

$$0(\mathbf{U}) = \mathbf{t}_0(\mathbf{u}_i) + \mathbf{t}_0(\mathbf{u}_r) = \mathbf{t}_0(\mathbf{u}_i) + A_0$$

$$- A_0$$

T

with order 0

T

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One thus obtains the variational formulation of the problem in the vicinity of the border for order 0:

2u

U

U

$v + U: v -$ With

$v = \mathbf{T} \mathbf{U}$

I

- A

v

2 () () 0

(*I*)

T

0

T

T

For any field v kinematically acceptable

For order 1, one preserves the traditional formulation:

2

$$\mathbf{U} v + \mathbf{U} : \mathbf{v} - T \mathbf{U} v = 0$$

2 () () ()

T

where (

$\mathbf{T} \mathbf{U}$) follows the law of following evolution:

$\mathbf{T} (\mathbf{U}) \mathbf{T} (\mathbf{U}$

2

2

D)

\mathbf{U}

\mathbf{U}

\mathbf{U}

\mathbf{U}

=

+ A

,

\mathbf{U}

I

I

1

- A

,

\mathbf{U}

T

T

$t^2 T$

1 $t^2 T I$

The request due to the incidental field appears explicitly in the case of order 0, but it is contained in the law of evolution of (

$\mathbf{T} \mathbf{U}$) for order 1.

3

Anechoic fluid elements in transient

This part presents the main part of the general constraints of implementation of fluid elements anechoic of border absorbents with the paraxial approximation of order 0 in *Code_Aster*.

For reasons of simplicity related to the handling of scalar sizes such as the pressure or potential of displacement, in opposition to the vector quantities like displacement, one be interested initially in the fluid elements.

3.1 Formulation

standard

One takes again here the reasoning of Modaressi by adapting it to an acoustic fluid field. In one the first time, one is interested in the only data of the size pressure in this fluid. One will return then on this modeling to adapt to the constraints of *Code_Aster*, by underlining them adjustments to be made.

That is to say thus following configuration, by taking again conventions of the preceding part in the vicinity

border:

Border locally orthogonal with the axis x_3

x_2

Grid elements

Fluid field

x_3

finished

infinite

x_1

The definition of a local reference mark on the level of the element makes it possible to bring back for us systematically in such a situation.

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3.1.1 Finite elements formulation

The pressure p checks the equation of Helmholtz in all the field modelled with the finite elements, it who gives, for any virtual field of pressure Q :

1

2

 p $- p \cdot q -$ $p q +$ $Q =$

2

2

0

 $C T$ N

represent the border of the field.

 p

The size to be estimated on thanks to the paraxial approximation is here.

 N **3.1.2 Approximation****paraxial** p p

In the configuration suggested, the term corresponds to

.

 N x^3

Let us consider a wave consequently planes harmonic being propagated in the fluid:

 $p = A$

[

 $\exp I (K X$ $1 1 + K X$ $2 2 + K X$ $3 3 - T)]$

While replacing in the equation of Helmholtz, one obtains:

 $C 2$ K

$$2 \\ 2 \\ = \\ 1- \\ + \\ 2 (K \\ K \\ 3 \\ 1 \\ 2) \\ C$$

One obtains the following development then, for high frequencies (large) or in the vicinity of the border (k_1 and k_2 small):

$$C^2 \\ K \\ 2 \\ 2 \\ = 1- \\ + \\ 2 (K \\ K \\ 3 \\ 1 \\ 2) \\ C \\ 2$$

Maybe, while multiplying by making disappear this quantity with the denominator and after one transform of Fourier reverses in space and time:

$$2 p \\ 1 2 p 1 2 p 2 p \\ = - \\ + \\ +$$

$$C \\ X$$

2

2

2

3 *T*

CT

2 1

X

x2

As had presented it Modaressi, this equation utilizes the derivative compared to the time of surface term. Within the framework of this part, one is interested only at the end of order 0, that is to say, after one integration in time, which makes disappear the awkward derivative:

p

1 *p*

p

1 *p*

= -

or more generally:

= -

X

CT

3

N

CT

It is this relation of impedance which we will discretize on the border of the finite elements field.

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Note:

Taking into account the disappearance of the term of order 1 in the development of the square root, the order

minimal of approximation for the paraxial fluids is in fact 1 and not 0. We will preserve the name of elements of order 0 for coherence with the solid. However, one speaks about elements fluids of order 2 at the time to consider elements of a strictly positive nature.

3.2**Impedance of the vibroacoustic elements in *Code_Aster***

Code_Aster has vibroacoustic elements. One recalls in this paragraph the choices of formulation made at the time of their implementation. One is inspired to present existing it of reference material of *Code_Aster* [bib6].

3.2.1 Limits of the formulation out of p

In the framework of the interaction fluid-structure in harmonic, the formulation in pressure only of acoustic fluid led to nonsymmetrical matrices. Indeed, the total system is expressed, under variational form, in the following way:

$$\begin{aligned}
 & C \cdot u \\
 & v \\
 & - 2 U v - \text{statement} \cdot n \\
 & ijkl K L, I, J \\
 & S I I I I = 0 \text{ for the structure} \\
 & S \\
 & S \\
 & 1 \\
 & 2 \\
 & p \cdot Q \\
 & - K \\
 & p Q - U \cdot n Q = 0 \\
 & 2
 \end{aligned}$$

for the fluid

$$\begin{aligned}
 & I \\
 & I \\
 & F \\
 & F \\
 & F
 \end{aligned}$$

with $K =$

, a number of wave for the fluid, v and Q two virtual fields in the structure and in

C
fluid respectively.

After discretization by finite elements, one obtains the following matrix system:

$$\mathbf{K} - \mathbf{C} U$$

\mathbf{M}

$0 U$

2

T

\mathbf{Q}

0

0

$\mathbf{H} -$

\mathbf{C}

$=$

p

F

2

p

C

where:

\mathbf{K} and \mathbf{M} are the matrices of rigidity and mass of the structure

H and Q are the fluid matrices obtained respectively starting from the bilinear forms:

.

$p Q$ and $p q$

F

F

\mathbf{C} is the matrix of coupling obtained starting from the bilinear form: $p U .n$

II

The nonsymmetrical character of this system does not make it possible to use the algorithm of resolution traditional of *Code_Aster*. This justifies the introduction of an additional variable into description fluid.

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3.2.2 Symmetrical formulation out of p and phi

The new introduced size is the potential of displacements, such as $X =$. According to [bib6], one obtains the new variational form of the system coupled fluid-structure:

$C .u$

v

$- 2 U v - 2 v .n$

$ijkl KL, I, J$

$S I I F I I = 0$ for the structure

S

S

1

1

2

$pq -$

F

$Q + p -$

.

$+ U N$

2

(

)

0

2

for the fluid

$$II = FC$$

$$FC$$
$$F$$
$$F$$
$$F$$

With: $p = 2$

F in the fluid and a field of potential of virtual displacement

This leads us to the symmetrical matrix system:

K

M

0

M

0

0

F

U

U

M

M

0

F

0

2

$p - 0$

0

$$fl p = 0$$

2

2

F C
C

0
0
0
MT

MT
fl

H
F
2
F

C

where: **K** and **M** are the matrices of rigidity and mass of the structure
M is the matrix of coupling obtained starting from the bilinear form *U N*
I I

M, M
F

fl and **H** are the fluid matrices obtained starting from the bilinear forms:

pq
, p

F
F

(or
 Q
) and.

F
 F

3.2.3 Imposition of an impedance with the formulation out of p and ϕ

Generally, a relation of impedance at the border of the fluid is expressed as follows:

$$p = Z v.n$$

where:

Z is the imposed impedance

$v.n$ is the outgoing normal speed of the fluid particles

One deduces some, according to the law of behavior of the fluid, which connects the pressure to the displacement of

$2u$
fluid particles for an acoustic fluid $p - F$

$= 0 :$

t^2

$F p p$

$=$

$Z T$

N

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The discretization of such an equation leads to a nonsymmetrical term in a formulation out of p and ϕ . One prefers to formulate the condition compared to the potential of displacement, that is to say:

F

+

= 0

 $Z T$

One obtains then like expression for the term of edge associated with the relation with impedance:

2

3 2

 F F

=

 $T 2$ N $T 3 Z$

One then notes the appearance (somewhat artificial) of a term in derived third compared to time. In harmonic, which is the privileged applicability of the vibroacoustic elements in *Code_Aster*, that does not pose a problem. One treats a term into 3 without difficulty. For calculation transient, rather than to introduce an approximation of a derivative third into the diagram of Newmark implemented in the operators of direct integration in dynamics in *Code_Aster* DYNA_LINE_TRAN [U4.53.02] and DYNA_NON_LINE [U4.53.01], one prefers to operate a simple correction

of the second member, which returns in fact to consider the impedance explicitly. Conditions of stability of the diagram of Newmark are not rigorously any more the same ones, but the experiment has us

shown that it is simple to arrive at convergence starting from the old conditions.

This choice of an explicit correction of the second member will be also justified at the time of implementation of paraxial elements of order 1, which it makes easier definitely.

3.2.4 Formulation

detailed

One proposes here the precise formulation for an acoustic fluid modelled on a field with one anechoic condition on a part *has* border of the field. Apart from that, one break up the border into a free face and a part in contact with a rigid solid. The introduction requests external or the presence of an elastic structure is modelled easily by current methods. The elements of volume and surface are formulated out of p and.

The equations in the fluid are:

1

 F

+

 $p = 0$

c_2

in volume

éq 3.2.4-1

2

$p = F t_2$ in volume

éq 3.2.4-2

$p = 0$ on the free face

éq 3.2.4-3

= 0

on the rigid wall

éq 3.2.4-4

N

p

1 p

= -

on the part of the border with anechoic condition

éq 3.2.4-5

N

$C T$

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One multiplies the equation [éq 3.2.4-1] by a field of virtual potential and one integrates in:

1

2

2

$$p + F$$

.

+

=

according to the formula of Green

$$2 ($$

)

0

2

$$F^2$$

C

T

$$TN$$

$$F$$

Maybe, with the boundary conditions on and the equation [éq 3.2.4-2]:

1

2

$$p$$

$$p + F$$

.

+

=

$$2 ($$

)

0

2

C

T

F

N

F
has

One can consequently apply the condition of impedance formulated in pressure:

p
1
p

F
= -

F
N
C
T
has
has

Moreover, to arrive to a symmetrical formulation of the terms of volume, one multiplies the equation [éq 3.2.4-2] by a virtual field of pressure *Q* and one integrate in:

pq
2
Q
-
= 0

c2 T 2
c2

F
F
F

By summoning the two variational equations, one obtains:

1
2

1
p
pq +

F
Q + p -.

=
2
2

(
)
1
0
2

-
T

F
C
C
C
T
F

F
F
F
F
has

Matriciellement:

M *fl*
M
Op
0
1 0 **A**

F
p "

$c_2 p$

“0

0 0 -

$C_0 0 +$

”

MT

=

fl

”

H

2

F

C

where the submatrices **M**, **M**
and **H**

F

fl

the same bilinear forms discretize as previously.

p

Submatrix **A** discretizes the term

F . The matrix of damping obtained is not

T

has

symmetrical, as one had predicted higher. This is why one rejects this term with the second member.

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3.2.5 Direct temporal integration

In our case, because of nonthe symmetry of the matrix of impedance, one chooses to consider it anechoic term explicitly as we evoked before. That amounts calculating it at the moment T and to place it among the requests at the time of the expression of dynamic balance at the moment

$T + T$

.
One solves:

M_{fl}

M

$0p$

0

2

$+$

$p'' + 10 A$

F

T

T

C

T

T

“ tp

0

0

$+$

MT

$=$

”

”

éq 3.2.5-1

T +t

fl

C 0 0

H *t*+*t*

T

2

F

C

Instead of:

M *fl*

M

Op

0

+

1 0

With

F

T

T

"*tp* + *T*

*c*2 "*T*

p " +t

0

0 0

-

C 0

0

+

"

MT

=

"

 $T + T$ $t + t_{fl}$ $\mathbf{H} t + T$

2

 F C

Thus, there is not a nonsymmetrical matrix to treat in the system giving \mathbf{X} at the moment $T + T$

.

Note:

In a nonlinear calculation, one reactualizes the second member with each internal iteration. Calculation can thus prove more exact and more stable in this case.

3.3**Use in Code_Aster**

The taking into account of anechoic fluid elements and the calculation of their impedance require one specific modeling on the absorbing borders:

- in 2D with modeling "2D_FLUI_ABSO" on the finite elements of type MEFASEn (n=2 or 3) on the absorbing edges with N nodes.
- in 3D with modeling "3D_FLUI_ABSO" on the finite elements of MEFA_FACE type (n=3, 4, 6, 8 or 9) on the absorbing faces with N nodes.

In harmonic analysis with operator DYNA_LINE_HARM [U4.53.11], one calculates one as a preliminary

mechanical impedance by option IMPE_MECA of operator CALC_MATR_ELEM [U4.61.01] and one inform in DYNA_LINE_HARM (key word MATR_IMPE_PHI).

In transitory analysis, the taking into account of the correct force due under the terms of impedance is automatic with modelings of elements absorbents in operators DYNA_LINE_TRAN [U4.53.02] and DYNA_NON_LINE [U4.53.01].

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4**Elastic elements absorbents in *Code_Aster***

This part presents the main part of the general constraints of implementation of elastic elements of border absorbents with the paraxial approximation of order 0 in *Code_Aster*. The relation is pointed out of paraxial impedance of order 0 such as it was established by Modaressi for an elastic range linear:

(

U**U****T U) =****C**

//

P+ **Cs****T****T****U becomes U and U**

/

Tbecomes **U****3**

//

4.1**Adaptation of the seismic loading to the paraxial elements**

One presented in the first part the principle of taking into account of the incidental field thanks to paraxial elements. It is advisable here to present the methods of modeling of the seismic loading in *Code_Aster* to be able to adapt the data to the requirements of the paraxial elements.

The fundamental equation of dynamics associated with an unspecified model 2D or 3D discretized in finite elements of continuous medium or structure and in the absence of external loading is written in locate absolute:

MX

“+ **CX**

has

“+ **KX**

has

= 0 *have*

One breaks up the movement of the structures into a movement of drive **Xe** and a movement relative **Xr**.

X

X

S

has

Xe

Xr

X

X

S: imposed displacement of the supports

S

Teststemxà: absolute movement

Xe: movement of drive

(

Xr: relative movement

has)

Appear 4.1-a: Decomposition of the movement of the structures

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Thus, **X = X + X**

has

R

E

.

Teststem \mathbf{x}_a is the vector of displacements in the absolute reference mark,

.

Xr is the vector of relative displacements, i.e. the vector of displacements of structure compared to the deformation which it would have under the static action of displacements imposed on the level of the **Xs** supports. **Xr** is thus null at the points of anchoring,

.

Xe is the vector of displacements of drive of the structure produces statically by the imposed displacement of the **Xs** supports: $\mathbf{X} = \mathbf{X}$

E

S,

.

is the matrix of the static modes. The static modes represent the answer of structure with a unit displacement imposed on each degree of freedom of connection (others being blocked), in the absence of external forces. Thus, $\mathbf{K} = 0$, i.e., $\mathbf{K} \mathbf{Xe} = 0$.

In the case of the mono-support (all the supports undergo the same imposed movement), is a rigid mode of body.

Assumption in Code_Aster:

It is supposed that the damping dissipated by the structure is of viscous type i.e. force damping is proportional to the relative speed of the structure. Thus, \mathbf{CX}
"E = 0.

The fundamental equation of dynamics in the relative reference mark is written then:

MX"+ **CX**

R

"+ **KX = - MR. X**

R

R

"S

Operator CALC_CHAR_SEISME [U4.63.01] calculates the term - **M**, or more exactly - **M D**

,

where **D** is an unit vector such as $\mathbf{X} = \mathbf{D}$

S

.f (T) with F a scalar function of time.

One distinguishes two types of seismic loadings introduced into *Code_Aster* thanks to the operator CALC_CHAR_SEISME:

1) The loading of the type MONO_APPUI, for which is the matrix identity (the modes statics are modes of rigid body),

2) The loading of the type MULTI_APPUI, for which is unspecified.

According to the method of taking into account of the incidental field with the paraxial elements presented

in the first part, it is necessary for us to know on the border displacement and the constraints due to

incidental field. For the loading of the type MULTI_APPUI, only displacement is directly accessible at any moment. It thus seems difficult to allow the use of such a load pattern with paraxial elements in the ground. Moreover, if such a loading models displacements imposed supports, it does not require a modeling of the ground since all the influence is taken in count to these displacements.

Case MONO_APPUI can be perceived differently. It represents an overall acceleration applied to the model. Consequently, the propagation of wave in the ground can have a role to play in behavior of the structure, since the movements of the interface ground-structure are not imposed. Moreover, the paraxial elements are usable with this type of loading because it does not create no constraints at the border of the grid (a rigid mode of body does not create deformations). Consequently, one has all the data necessary to calculation of the impedance absorbing on border.

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Notice 1:

In the case of a seismic request MONO_APPUI, dynamic calculation is done in relative reference mark. If one amounts on the term discretizing on the paraxial elements (see first part), one notices that u_i corresponds exactly to the displacement of drive X_e presented higher. Thus, $U - u_i$ corresponds to the relative displacement calculated during calculation. Consequently, the relation to be taken into account on the paraxial elements in such configuration is simply:

U

T (U) = A0

T

Notice 2:

In the case of a calculation of interaction ground-fluid-structure with infinite fluid, pressure to be taken in account for the calculation of the anechoic impedance in the fluid is well the pressure

*absolute, if there is not an incidental field in the fluid (what is often the case).
correction which one could exempt to make for the ground must then be made for
fluid paraxial elements.*

4.2

Implementation of the elements in transient and harmonic

4.2.1 Implementation in transient

The mode of implementation of the elastic paraxial elements in transient is very close to that presented for the fluid elements. The difference comes primarily from the need for breaking up displacement in a component according to the normal with the element, corresponding to a wave P, and one

component in the plan of the element, corresponding to a wave S. One is then capable to discretize the relation of impedance introduced into the first part:

U

/

3

U

T (U) = CP

+ C

T

S T

One does not reconsider the diagram of temporal integration which one already described in the part the preceding one, knowing that one always considers the relation of impedance explicitly by one correction of the second member.

4.2.2 Implementation in harmonic

The fluid acoustic elements of *Code_Aster* propose already the possibility of taking into account an impedance imposed on the border of the grid in harmonic. That corresponds to the treatment of one term in 3 in the equations, like city higher. It is trying to introduce the possibility to impose an impedance absorbing for an elastic problem in harmonic.

For a harmonic calculation of response of an infinite structure, the taking into account of the impedance absorbing as a correction of the second member is obviously not applicable. However, relation of impedance to order 0 expresses the surface terms according to the speed of the nodes element. One can thus build a pseudo-matrix of viscous damping translating presence of the infinite field.

Decomposition of the relation of impedance according to components' normal or tangential of displacement on the constrained element us to build the matrix of impedance in a local reference mark on the element. One defines this local reference mark in the elementary routine as well as the matrix of passage which the return to the total base allows.

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Note:

In the case of elastic paraxial elements of order 0, to create a matrix of damping could have allowed us to solve the problem in transient without deteriorating the stability of diagram of Newmark, contrary to the taking into account clarifies that we retained. However, we showed the problems that that created for the fluid elements and we wished to keep the homogeneity of the modes of implementation. Moreover, treatment elements of a nature 1 making the way explicit obligatory, the whole seems coherent. That also allows to take into account at the same time an infinite field with elements paraxial and a damping of modal the damping type for the structure.

4.3

Seismic load pattern per plane wave

In complement of the methods of taking into account of the seismic loading already available and in reason of the inadequacy of mode MULTI_APPUI with the paraxial elements, it seems interesting to introduce a principle of loading per plane wave. That corresponds to the loadings classically met during calculations of interaction ground-structure by the integral equations.

4.3.1 Characterization of a wave planes in transient

In harmonic, a wave planes elastic is characterized by its direction, its pulsation and its type (wave P for the waves of compression, waves SV or HS for the waves of shearing). In transient, the data of the pulsation, corresponding to a standing wave in time, must be replaced by the data of a profile of displacement which one will take into account the propagation with run from time in the direction of the wave.

More precisely, one will consider a plane wave in the form:

(
 $\mathbf{U}(\mathbf{X}, T) = F(\mathbf{k} \cdot \mathbf{x} - C t) \mathbf{K}$ for a wave P (with \mathbf{K} unit)

$\mathbf{U}(\mathbf{X}, T) = F(\mathbf{k} \cdot \mathbf{x} - C t) \mathbf{K}$ for a wave S (with always \mathbf{K} unit)

F then represents the profile of the wave given according to the direction \mathbf{K} .

O

“Principal” face of wave

K

corresponding at the origin
 profile

H

Function F

H is the distance from the origin to the principal face of wave. To initialize calculation, the distance should be given

H_0 which separates the principal face of wave from the origin at moment 0.

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4.3.2 User data for the loading by plane wave

In accordance with the theory exposed in first part, it is necessary for us to calculate the constraint at the border

grid due to the incidental wave and the term of impedance corresponding to incidental displacement, that is to say:

U

T (U

I

) and A_0

T

To express the constraints, it is necessary for us to have the deformations due to the incidental wave, the law of

behavior of material allowing us to pass from the ones to the others.

On the elements of border, one can express the tensor of the deformations linearized in each node by the traditional formula:

1

$(X T) = [U (X T) T$

,

, + $U (X, T)]$

2

Finally, to consider the constraints due to the incidental field, we thus should determine them

(ui) J

derived

for J and K traversing the three directions of space. One obtains these quantities with

xk

to leave the definition of the incidental plane wave:

(ui) $J = kk F(\mathbf{k} \cdot \mathbf{x} - Cmt) K$

J with $m = S$ or P

xk

U

With regard to the term of impedance, it is necessary for us

I

. One also obtains it starting from the wave

T

plane: **ui** = $mCf(k \cdot x - mCt)$

K always with $m = S$ or P

T

It is seen whereas the important function for a loading by wave planes with elements paraxial of order 0 is not the profile of the wave F , but its derivative f' . It is thus a this function which the user must enter like data of calculation.

One can consequently recapitulate the parameters to enter for the definition of a loading per plane wave in transient:

Type of the wave

: P, SV or HS

Direction of the wave

: k_x, k_y, k_z

Outdistance principal face of wave with

the origin at the initial moment

: H_0

Derived from the profile of the wave

: $f'(X)$ for $X] -, + [$

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4.4

Use in *Code_Aster*

The taking into account of elastic elements absorbents and the calculation of their impedance requires one

specific modeling on the absorbing borders:

- in 2D with modeling “D_PLAN_ABSO” on the finite elements of type MEPASEn (n=2 or 3) on the absorbing edges with N nodes.

- in 3D with modeling “3D_ABSO” on the finite elements of type MEAB_FACEn (n=3, 4, 6, 8 or 9) on the absorbing faces with N nodes.

In harmonic analysis with operator DYNA_LINE_HARM [U4.53.11], one calculates one as a preliminary

mechanical cushioning by option AMOR_MECA of operator CALC_MATR_ELEM [U4.61.01] and one informs in DYNA_LINE_HARM (key word MATR_AMOR).

In transitory analysis, the taking into account of the correct force due under the terms of impedance is automatic with modelings of elements absorbents in operators DYNA_LINE_TRAN [U4.53.02] and DYNA_NON_LINE [U4.53.01].

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Organization (S): EDF/MTI/MMN

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Document: R4.03.01

Sensitivity of the thermomechanical fields

with a variation of the field

Summary

To know the influence of a variation of the field on the thermomechanical fields, the traditional approach consist in making several calculations and evaluating, by difference, the sensitivity. Method described in it

document makes it possible to obtain in only one calculation with *Code_Aster* the value of the fields of temperatures,

displacements and forced and their derivative compared to the variation of the field.

The method is initially exposed in its general information: thermics and linear static mechanics, 2D and 3D,

variation of an unspecified edge. Then various calculations in the case 2D and for some are detailed loadings.

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1 Introduction

The developments presented in this document aim at allowing probabilistic studies of brutal rupture by a coupling mechanic-reliability engineer. The geometry of the field is treated like one random field. The evaluation of the probability of starting of the rupture is ensured by coupling with software PROBAN, with methods FORM/SORM. This evolution supposes to know them variations of the constraints and the temperatures by report/ratio geometry. Thus, a first application industrial seeks to determine the probability of rupture of the nuclear reactor vessel, of which the thickness of the lining is regarded as a random variable. These constraints and temperatures being calculated by *Code_Aster*, the traditional technique consists in carrying out series of calculation for several values thickness of the coating. Then by difference, one deduces some the influence thickness on these fields.

This technique has limits; in particular:

- precision: how to choose the values of the parameter thickness so that the difference enters does two calculations represent its influence well?
- performance: for a value of the parameter, at least two calculations with *Code_Aster* are necessary to calculate the influence.

The method developed in this work makes it possible to obtain in only one calculation with *Code_Aster* the value

constraints and temperatures and their derivative compared to the thickness of the coating.

The technique selected is based on a direct derivation of the equations expressed in form variational. It takes again the method as under the name of "method" already introduced for calculation of the rate of refund of energy in *Code_Aster*. This fact a certain number of results of base are not redémontrés but are the subject only of references in appendix.

The first part relates to the derivation of the temperature, in stationary regimes and transitory, in linear thermics. The principal loadings are studied: convectif exchange, temperature imposed, internal source.

Then, we expose the derivation of the field of displacement in linear static mechanics.

loadings taken into account are limited to imposed displacements and pressures distributed.

derivation of the stress field is summarized then with a postprocessing of derived from the field from displacement.

The method is presented in its general information: 2D or 3D, influence of an unspecified variation of

one

edge of the field. In practice, the functionality is currently available only in 2D, for loadings mentioned herebefore.

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2

Determination of the gradient of the temperature

2.1

problem

A first stage of the calculation of the gradients of the rate of refund of energy G is the calculation of gradient of the temperature compared to a real parameter. This parameter controls the variation of field of calculation: starting from the area of reference, one studies a transformed field, where is the parameter symbolizing the transformation. The required gradients are those which are expressed at the place of the resolution of reference, i.e. for $\theta = 0$. One will refer to appendix 1 for notations employed.

We leave the variational equation governing it thermal of the problem on the field transformed. While following [R5.02.01], we define the borders by:

1: imposed temperature

2: imposed normal flow

3: convectif exchange

what gives the following variational equation:

T

C

$T^* +$

$T.T^*$

$+ H T T^* =$

$$S T^* + Q T^* + H T T^*$$

P

ext.

T

3

2

3

with:

T

Temperature

C

voluminal heat with constant pressure

P

thermal conductivity

H

coefficient of convectif exchange

S

thermal voluminal source

Q

normal heat flow entering imposed on edge 2

T^*

function of test in $H1$ (), null on 1

Note:

We present here only the problem with boundary conditions of temperature imposed, of imposed normal flow and convectif exchange. The taking into account of the conditions of exchange between wall or radiation will be done later on.

2.2

Derivation of the variational equation

We will derive each integral successively forming the equation. Each time, us

F

will use the formula of Reynolds, after having defined the vector from for transformation F (cf Appendix 2). represent the direction of variation of the field.

We choose functions of test T^* which are independent of the parameter. In addition, one is here in a case where derivations Lagrangienne and temporal commute (cf Annexe 3).

In all the formulas presented, we note with a point the Lagrangian derivative of size: !

T is the Lagrangian derivative of T .

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2.2.1 Integral of the temporal variation

T

$I =$

C

*

T

1

p

T

.

dI

T

T
 1
 $*$

$=$

$*$
 C

$T + C$

T
 div

D
 P

T
 P

T

We suppose that voluminal heat CP is independent of the parameter i.e. purely Lagrangian (attached to the material point).

By using proposal 2 of appendix 2!

$=$
 $+$
 \cdot
 $=$

, we have here!

. From where
the expression:

\cdot
 \cdot

T

.

T

T

C

p

= *CP*

C

T

+

T

pT

!

= (

T

T

CP).

+ *C*

T

pT

dII

T! *

T

*

=

T
 C
 T

*

p
 $+ C$
 $div T$

p
 $+$

$(CP). T$
 D
 T
 T
 T

2.2.2 Integral of thermal conductivity

$I =$

.

*

$T T$
 2

dI

.

2

$= T.T$
 $* + T.T$
 $* div$

D

[
]

We suppose that thermal conductivity is also independent of the parameter. Thus, we have:

.

*

*

.

.

$$T.T =! T.T + T. T* T.$$

*

T

+

With the general result of appendix 2, we have:

.

!

$$T = T - T.$$

.

T*

T! *

T* .

T* .

=

-

= -

!

= .

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From where the result:

$D I 2 =$

T

$T^* +$

!

(

) T

. T

*

D

- (T). T

* - T

. (T

*)

+ $div T$

. T

*

2.2.3 Integral of the convectif exchange - Part 1

$I =$

hTT

3

*

3

We use this time the proposal 4 (cf Annexe 2) which establishes derivation for an integral surface.

dI

$$\cdot$$

$$3 =$$

$$hTT^*$$

$$hTT^* \text{div}$$

$$D$$

$$+$$

$$S$$

$$3$$

We suppose that the coefficient of heat exchange by convection is him still independent of parameter. Thus! $H =$.

$$h.$$

$$D I 3 = H TT^* + (H$$

$$) TT^* + hT \text{div } T$$

$$!$$

$$\cdot$$

$$*$$

$$D$$

$$S$$

$$3$$

2.2.4 Integral of the internal thermal sources

$$I =$$

$$St^*$$

$$4$$

$$dI$$

$$\cdot$$

$$4$$

$$=$$

$$sT^* + sT^* \text{div}$$

$$D$$

$$[]$$

We suppose that the thermal voluminal source is independent of the parameter. Thus,

$$\Delta S = 0$$

$$D I 4 = ((S) \cdot) T^* + S \operatorname{div} T^*$$

D

2.2.5 Integral of the boundary conditions with imposed flow

$$I = \int_{\Omega} Q T^* \operatorname{div} T^* + Q T^* \operatorname{div} T^* d\Omega$$

$$= \int_{\Omega} Q T^* \operatorname{div} T^* d\Omega$$

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We suppose that the external heat flow is independent of the parameter. Thus! $Q = Q$.

$D I 5 = (Q$

$) T^* + Q T^*$

div

D

S

2

2.2.6 Integral of the convectif exchange - Part 2

$I =$

$H T$

T^*

6

$ext.$

3

dI

.

6

$= H T T^* + H T T^* div$

D

$ext.$

$ext.$

S

3

We suppose that the coefficient of heat exchange by convection and the outside temperature are independent of the parameter.

$D I 6 = (H) T T^* + H$

.

*

*

$ext.$

$(T$

ext.) T + HT T div

D

ext.

S

3

2.2.7 Assessment

In all these expressions of derivations of integrals, only the Lagrangian derivative of temperature is unknown. We can thus form a new variational equation of which!

T is

the unknown factor.

!

T

C

*T **

*

*

p

+ *T*

!. *T + hTT*

!

=

T

3

-

-

(

*T **

T

C

*

p).

T

C div

T

T

P

T

- (

) *T*

. *T**

+

[*T*

]. *T**

+ *T*

-

-

[. *T**] *div T.T**

+

(*H*

.)

(*T - T **

*

*

ext.

) *T + H*

(*T - T*

ext.

) *div T + H*

S

(*T*

ext.) *T*

3

3

3

+ (*S.*) *T** +

*S div T**

+ (*Q.*) *T** +

*Q div T**

S
2
2

Border 1 of boundary conditions of Dirichlet for the calculation of T corresponds to the same type of boundary conditions: !

T is imposed on a zero value along this border.

2.3

Comments on this equation

One can notice that the first member of this equation is, formally, identical to that of the variational equation which allows the calculation of the temperature. It is thus a question of solving the same one

equation, with a second modified member.

The solution of this equation provides the Lagrangian derivative of T . To have the derivative which us T

interest, it remains to achieve the last operation:

$$T$$
$$!$$
$$= T - T$$

.

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2.4

Discretization in time

The temporal resolution will be done by using the method known as of - diagram, as for the equation who controls the change of the temperature. To know the size at the moment $T + T$

, us

will pose:

! $T+$

! T (T
 T
) and! T
 =
 +
 =! T (T)
 ! T
 ! T + -! T -

The derivative in time approaches thus by:

~
 -
 T
 T

At the current moment we will use the approximation:

! T ~ -! T + + (1 -)! T
 I
 I

We will apply this technique for the principal variables of the problem: !

T, T, H, Q, S. All them

fields at the moment T being known, the equation discretized in time can be written:

CP T+! T* +

+
 *
 + + *
 I
 T
 ! . T

+
 HT! T

=
 T
 I

3
 CP T! T

* - (1-
 -

*

--*

D) T! . T

- (1-i) *H T! T*

T

3

+

-

+

-

-

*

-

-

T

T

T

T

C div

T

*

p

-

.

(CP)

T

T

T

+

+

-

+ 1-

..*

([*T*
I
(*I T*)] *T*
+
(*T*+ (
-
*
I) *T*
). [*T*
.]
I
+ 1-
- (.)

1

(+
-
*
I T
+ (- *I*) *T*
). *T*

-
div

1

[+
-
*
I T
+ (- *I*) *T*]. *T*

+
1
(
+
-
*
I S
+ (- *I*) *S*

). *T*
+ *div*

1

(+
-
*

IS
+ (- *I S*) *T*

+
1

(
+
-
*

IQ

+ (- *I Q*). *T*

2

+
1

(
+
-
*

IQ
+ (- *I Q*) *divs T*

2

+

With div

*

ST

3

+ (*h* + + (

-

+

+

-

-

1 -

*

I

) H . $(I(T - T$

ext.

) + $(1 - I)(T - T$

ext.

)

I

T

3

+ $(h+$

-

+

-

*

I

+ $(1 - I)H(T$

I

ext. + $(1 - I$

) T

ext.). T

3

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The term of flow of convectif exchange, A , takes two expressions distinct according to the dependence from H

and $Text$ with respect to time.

If H and $Text$ are independent of time, only the temperature is with impliciter. From where:

H

$$(T - T_+ - (1 - i) T)$$

) $div T^*$

ext.

I

I

S

3

If not, the whole of flow undergoes it - diagram:

*

3

[+

+

+

-

-

-

$I H (ext.$

T

$- T) + (1-i) H (ext.$

T

$- T)]divs T$

Note:

One will have noted the nuisance to have to treat with “Dupond and Dupont of numerical”, with to know it - diagram and method... to keep coherence with the remainder of documentation of Code_Aster, we chose to preserve the notations for both parameters of these methods. Insofar as it is the “method” which interests us it more in this work, of the “diagram” was affected of an index I , like “implication”. Let us hope that that will have been clearly...

2.5 Discretization

space

The space discretization of this equation is copied exactly on that employed for resolution of thermics. We return to [R5.02.01] for his description.

3

Calculation of the gradient of displacement

3.1

problem

A second stage necessary to the calculation of the gradients of the rate of refund of energy is calculation gradient of the field of displacement compared to the variation of the field. We begin again exactly same conventions as in the preceding chapter for the calculation of the gradient of

temperature.

Only certain loadings are taken into account. The extension to other types of change would be done while following the principles which will be stated. We place ourselves in the case of linear elasticity isotropic, in two dimensions. The relation between the tensor of the constraints and the tensor of deformations is then of the type:

$$= + K (T - T_{ref}) \mathbf{Id}$$

The field of calculation is noted, where is the real parameter of piloting of the variations of the field.

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For this application, we retain only three types of behavior at the edge of:

- 1: border with imposed displacement
- 2: border with “uniform connection”
- 3: border with imposed external pressure

what gives the following variational equation:

$$(\mathbf{U}): (\mathbf{v}) = \mathbf{v}$$

$p \cdot n$

3

with:

\mathbf{U}

field of displacement

(\mathbf{U})

tensor of the constraints related to displacement U

(\mathbf{v})

tensor of the deformations associated with displacement v

p

pressure distributed on edge 3

3.2

Derivation of the variational equation

We will derive the two integrals which constitute the variational equation by applying it theorem of Reynolds (cf Appendix 2 and [§2.2]).

3.2.1 Integral

voluminal

$I = \mathbf{U}:$

$(\)$

(\mathbf{v})

Because of isotropy of the problem, we have the equality of the scalar products:

$:\mathbf{v}) =: S(\mathbf{v})$

where $S(\mathbf{v})$ is the symmetrized gradient of \mathbf{v} .

The integral and its derivative are thus written:

I

$=$

$\mathbf{U}:$

$(\) S(\mathbf{v})$

dI

.

$= (\mathbf{U}) S$

$:\mathbf{v})$

S

$+ \mathbf{U}:\mathbf{v} \operatorname{div}$

D

$[(\) (\)]$

We will use the property of derivation of a gradient of vector, where FT is a tensorial function, described in appendix 2:

.

\mathbf{v}

\mathbf{v}

$($

FT $\mathbf{v},$)

= -

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We have:

.

$(\mathbf{U}): S(\mathbf{v}) = !(\mathbf{U}): S$

S

$\mathbf{v} + (\mathbf{U}): !$

(\mathbf{v})

$=! (\mathbf{U}): S$

S

S

$\mathbf{v} + (\mathbf{U}): [(\mathbf{v}!) - ($

$T \mathbf{v},)]$

$=! (\mathbf{U}): S$

S

$\mathbf{v} - (\mathbf{U}): F ($

$T(\mathbf{v}),)$

because functions \mathbf{v} are supposed to be independent of the variations of the field, therefore $\mathbf{v} = 0$. For to calculate the derivative not, we notice that is a function of the deformation and of temperature:

$! (\mathbf{U}) =$

$! \mathbf{E} +$

$! T$

\mathbf{E}

T

In the particular case of isotropic linear elasticity:

$$(\mathbf{E}, T) = \mathbf{E} + K (T - T_{ref}) \mathbf{Id}$$

$$= \mathbf{E}$$

$$= K \mathbf{Id}$$

T

from where: $\dot{\mathbf{U}} = \dot{\mathbf{E}} + K \dot{T} \mathbf{Id}$

It remains to express the derivative of the deformation \mathbf{E} , starting from its expression according to displacement \mathbf{U} .

$$\dot{\mathbf{E}} = \frac{1}{2} [\dot{\mathbf{U}} + \dot{\mathbf{U}}^T]$$

$$\dot{\mathbf{E}} = \dot{\mathbf{U}}$$

2

$$\dot{\mathbf{E}} = [\dot{\mathbf{U}} - (FT \mathbf{U},)] + [\dot{\mathbf{U}} - (FT \mathbf{U},)] T$$

2
2

$$\begin{aligned}
 &1 \\
 &1 \\
 &\mathbf{E}! = \\
 &T \\
 &T \\
 &! \\
 &! \\
 &FT \\
 &, \\
 &FT \\
 &, \\
 &2 [\mathbf{U} \\
 &+ \mathbf{U} \\
 &] - 2 [(\mathbf{U}) + (\mathbf{U} \\
 &)]
 \end{aligned}$$

The final expression of the derivation of the integral is then:

$$\begin{aligned}
 &dI \\
 &T \\
 &S \\
 &1 \\
 &T \\
 &S \\
 &= \\
 &!\mathbf{U} + !\mathbf{U}: \mathbf{v} - FT \mathbf{U}, + FT \mathbf{U}, \\
 &: \\
 &1 \\
 &\mathbf{v} \\
 &D \\
 &2 [\\
 &] (\cdot) \\
 &2 [[\\
 & (\\
 &) \\
 & (\\
 &)] \\
 & (\\
 & + K T \\
 & S \\
 & ! \mathbf{Id}: (\mathbf{v}) - (\mathbf{U}): F (S \\
 & T (\mathbf{v}), \\
 &) \\
 & + [(\mathbf{U}) S
 \end{aligned}$$

: (v)] div

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3.2.2 Integral surface

I =

p

3

v

3

D I 3

=

statement

statement div

.

D

S

3

+

By choice of functions **v** we have! $\mathbf{v} = 0$. We suppose that the external pressure is independent of the parameter. As follows:

.

statement

[*p.*] \mathbf{v}

=

$D I 3 = [p$
 $.] \mathbf{v} + p \operatorname{div} \mathbf{v}$

D

S

3

3.2.3 Assessment

In all these expressions of derivations of integrals, only the Lagrangian derivative of displacement is unknown. We can thus form a new variational equation of which \mathbf{U} is the unknown factor and where \mathbf{v} are the symmetrized gradient of \mathbf{v} .

1

$\delta \mathbf{U} + \delta \mathbf{C} : \mathbf{v} =$

[

]

2

1

$\delta \mathbf{T} \mathbf{U}, + \delta \mathbf{T} \mathbf{U}, T : \mathbf{v}$

2 [[

(

)

(

)]

- $K! T \mathbf{Id} : \mathbf{v}$

+ (\mathbf{U})

: $F ($

$T \mathbf{v},)$

- $[(\mathbf{U}) : \mathbf{v}] \operatorname{div}$

+ $[.p] \mathbf{v}$

3

+ $p \operatorname{div}$

3

On border 1, displacement is imposed. Whatever the position of this border,

boundary condition follows the matter, which involves! $\mathbf{U} = 0$.

On border 2, with uniform connection, the degrees of freedom are identical, but free: $u_y =$ constant for example. It is the same for the derivative! \mathbf{U} .

3.3

Comments on the equation to be solved

We will note that, as for the problem in temperature, the problem to be solved here is formally the same one as that of the determination of displacement \mathbf{U} . The matrix is the same one. Only the second member evolves/moves.

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If we set out again of the initial variational equation:

$(\mathbf{U}): (\mathbf{v}) =$

\mathbf{v}

$p \cdot n$

3

we can transform it by using the expressions of and of: .

$(\mathbf{U}): \mathbf{v}$

=

statement

3

(\mathbf{U})

[

+ $K (T - T_{ref}) \mathbf{I}$ $\mathbf{D}: \mathbf{v} = \text{statement}$

3

[(\mathbf{U}): \mathbf{v}

= -

K

$(T - T_{ref}) \mathbf{I} \mathbf{d}: \mathbf{v}$

+

statement

3

1

Like (\mathbf{U}) = [\mathbf{U}

+ \mathbf{U}

T], we find well the same expression with the first member as

2

for the transformed equation.

4

Determination of the gradient of the constraints

The following stage aims at determining the gradient of the constraints. It will be calculated from knowledge of the gradients of the temperature and displacement.

We saw that in the particular case of isotropic linear elasticity! could express itself under form (cf [§3.2.1]):

1

1

! (\mathbf{U}) =

T

T

$K T$

2 [$! \mathbf{U} + ! \mathbf{U}$] - 2 [

(

$\mathbf{F} \mathbf{T} \mathbf{U}$

), + \mathbf{F} (

$\mathbf{T} \mathbf{U}$),] + $\mathbf{I} \mathbf{d}$

This stage, all the quantities on the right of the sign = are known; there is not any more but to do one postprocessing to obtain! .

In the same way, knowing that:

= ! -

,

the derivative eulérienne of the constraints is expressed in postprocessing of the quantities previously calculated.

Note:

This phase of derivation imposes that calculation took place with quadratic elements.

5 Conclusion

To continue this work, it remains to make the calculation of derived from the rate of refund of energy.

That

is envisaged in a later version.

The use of this functionality, planned initially for probabilistic mechanics, can extend to other fields: optimization of forms, identification.

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Appendix 1 the transformation of the field

The technique used to calculate the various gradients during a variation of the field is that of method known as “method”. This method was developed for calculation of the rate of refund of energy G ; it is described in [bib1] and [R7.02.03]. We give here the various expressions which are used in this document.

The field of calculation of reference is noted. It is transformed into a noted field, where is one

real parameter. The whole of the transformations is represented by the functions F . We agree that F_0 corresponds to the identity.

In a general way, the only sizes which interest us are the gradients expressed at the point of resolution. They are thus the derivative compared to the parameter expressed for $= 0$. This is why, for to reduce the notations, in all the document we write:

instead of $=0$

F

The field of vectors is noted.

We will use the following deduced sizes:

· field

scalar

div, voluminal divergence, and *divs*, surface divergence,

· tensor

.

Appendix 2 Form

We will recall here, the principal formulas useful for calculations of derivation. One will refer to [bib1] for

their demonstrations.

That is to say (F, M) an unspecified field. We note:

$(M) = ($

,

$F (M))$

Lagrangian derivative: !

=

()

(

) - 1

F

F

(

det F)

Proposal 1: (I)

= (II)

= - (III)

= *div*

Proposal 2: (I)!

=

+

!

is the Lagrangian derivative for the movement F

is the derivative eulérienne

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· if

is a scalar field,

is a scalar product, which gives

!

=

+

$X K$

K

K

· if

is a field of vector, is a tensor, which gives:

I

!

I

$I =$

+

K

X

K

K

· if

is a tensor, the same formula applied to each component of the tensor gives:

I, J

I,

!

J

I, J =

+

K

X

K

K

Note:

The analytical expression of this formula is the same one in plane 2D or axisymmetric 2D. Indeed, it term complementary to if is a vector is R/R. It would be to multiply by the component orthoradiale of the vector; that one being null, there is no modification of the expression. One passes thus of the formula plane 2D to the axisymmetric formula 2D by the formal analogy (X, y) (R, Z).

Proposal 2: (II) (!

) =! - FT (,)

Operator FT is the matric operator which is connected formally with the matric product.

· if

is a scalar field, in plane 2D:

On the basis of the expression (2i) and by deriving it compared to X:

! =

+

X +

y

X

y

!

2

=

+

X +

y
X

X

X X

y
!

2

2

2
X

=
+
X +
+
y +
y
X
X
X
X X
X y
y X

X
=

y
X

X +

X

X +

y

X y +

+

X X

y X

By applying the formula (2.i) to the first three terms of this sum, we have:

.

! X y

=

+

+

X X X X there X

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The same technique used while deriving (2.i) compared to makes it possible it to establish:

.

!

X
 y

+

X
 X
 XX
 yX

= -
!

X
 y

yy
+

Xy
 yy

.

!
(
FT,)

= -

· if
is a scalar field, in axisymmetric 2D, its gradient in axisymmetric 2D is worth the vector:

=

er +

E

R

Z Z

The starting point is still the expression (2.i):

!

=

+

R +

R

Z Z

By deriving this formula compared to *R* or with *Z*, we find the same expression formally

that in plane 2D, for the terms in and

.

R

Z

From where the expression:

.

! *R*

Z

+

R

R

R R

Z R

= -

!

R

Z

+
Z
Z R Z
Z Z

Summary for a scalar:

.

!
K
=
-
X
I
xi
X
K
K xi

front
I
K
EC. and {X,}
y
{R
or,}
Z

· when
is a vector or a tensor, we apply the same reasoning to each one of its
components in Cartesian co-ordinates:

.

J! J
J K

=
-

X

I

X

I

X

X

K

K I

.

J L! J L,

J L, K

=

-

X

I

X

I

X

X

K

K I

with *I, J, K, L* {*X,*}

y

· for a vector or a tensor into axisymmetric, it is necessary to take account of the characteristics of the gradient.

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We have indeed:

R

R

0

R

Z

=

R

R er + ze Z =

0

0

R

Z

Z

0

R

R

J

Derivations of the terms in are obtained as considering previously. It is necessary from now on

I

X

to apply the expression (2.i) at the end exchange *R*:

R

R

R

R

.

R

R

R

R

=

+

R +

R

R

Z

Z

.

1

R 1

R

R

1 *R*

R

1

=

+

R

R

+

R -

2 *R* +

R R

R R

R

R

Z

Z

1

The derivative eulérienne of is null by construction. Terms 1, 3 and 5 of the sum are

R

the expression (2.i) applied to *R*. What gives:

.

R R

= !

1

-

2

rr

R

R

R

From where the expression

.

R

R R!

R!

0

0

R

Z

R

Z

R
R

!
0
0
= 0
0

R

R

Z
Z Z!
! Z
0

0

R
Z R
Z

R R R Z
R R R Z

+
0
+

R R
Z R
R Z
Z Z

R

-
0

-
R

0
R 2

Z R Z Z
Z R Z

+
0
+
Z
R R
Z R
Z Z
Z Z
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Proposal 3 (theorem of Reynolds)

With $I =$

D

field of R

3 :

$$\begin{aligned}
 &(D I \\
 &I) \\
 &= (! + div) D \\
 &D
 \end{aligned}$$

$$\begin{aligned}
 &(D I \\
 &II) \\
 &= \\
 &D + \\
 &\cdot \\
 &Nd S
 \end{aligned}$$

D

Proposal 4:
 With $J =$
 $D S, S$

surface of R^3 and by noting N the normal external with S :
 S

$$\begin{aligned}
 &(D J \\
 &I) \\
 &= [! + divs] ds \\
 &D \\
 &S
 \end{aligned}$$

Appendix

3

Lagrangian commutation of derivations and temporal

Proposal:

.

!
 =

T

T

Proof:

For the transformation F , we pose classically:

$$(M, T) = ($$

,

,

$$, F(M), T)$$

By definition, the Lagrangian derivative is worth:

!

$$(M, T) =$$

If the transformation applied is the same one at every moment, the moment, T , and the parameter of transformation are independent one of the other. Derivations compared to T and can thus commute.

!

=

=

$$($$

$$, F(M), T)$$

T

T

T

=

(

$$, F(M), T)$$

T

.

=

$$T = T$$

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Appendix 4 Implementation numerical

A4.1 Calculation of the variation in temperature

A4.1.1 Principle general

We saw with [§2.2] whom the equation to solve is the same one as that which controls calculation thermics, except for the second member. That encourages us to insert the calculation of derived from the temperature

in the calculation of the temperature itself (operator THER_LINEAIRE). It will be thus possible with each moment to re-use the assembled matrices and to treat all the loadings of the problem thermics.

A4.1.2 total Algorithm

More precisely, calculations of T and!

T are imbricated in the following way:

· Initialisation of the field of temperatures T , and its gradient!

T , with two possibilities:

-

zero setting

-

resumption of a field previously calculated

· Boucle in time:

1)

Calculation of the elementary matrices, then assembly

2)

Calculation of the second member of the equation of thermics

3)

Resolution

This stage, one knows T and T

N

$n+1$. One can connect on the calculation of!

T .

4)

Calculation of the second member of the calculation of!

T

-

term due to the thermal source and the boundary conditions of flow

-

term due to the derivation of the equation

-

term due to the method of implicitation. One uses the same program as for the calculation of T , while having replaced the field T by the field!

T

5)

Resolution of the system to know the new value of!

T

6)

Rock of the values of T

and T

$n+1$

N

! +1 in T and T

N

N

!

A4.1.3 Boundary conditions of Dirichlet

Everywhere where one has boundary conditions of Dirichlet on the thermal problem, one finds boundary conditions of Dirichlet for the calculation of!

T . In these points, T being imposed, T are independent of the variations of the field:

T

= 0

As we have the relation:

T

! $T =$

+ T .

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we deduce the values from them from the boundary conditions of Dirichlet:

$\! T$

T .

$di =$

This value is thus calculated in each node of border 1.

A4.1.4 Detail of the various terms of the second member

We will gather under the same integral the obligatory terms due to the derivation of the equation, then to examine each possible change. The result will be written in the form of contribution of node I to the point of Gauss pg the element running in the calculation of the integrals by formulate Gauss, knowing that all its contributions are to be cumulated.

Term due to derivation

It is necessary to calculate the contribution of:

$I = I1 + I2 + I3 + I4 + I5 + I6 + I7 + I8$, with

C

I

p

*

$1 =$

$T -$

$\! T$

T

I

*

$2 = - (1 -$

-

I
 $) T$
 $! T$

$T + - T -$
 I
 $*$
 $3 =$
 $- \text{div } C$

T

p

T

$T + - T -$
 I
 $*$
 $4 =$
 $-$

$(CP).$
 T
 T

I
 $*$
 $5 =$

$([T+$
 $-$
 I
 $+ (1 - I) T$
 $)]. T$

I
 $*$
 $6 =$

$(T+$

$$\begin{aligned}
& - \\
& I \\
& + (1 - I) T [T] \\
& I \\
& (\cdot) \\
& (I + T + (1 - i) - T) * \\
& 7 =
\end{aligned}$$

$$\begin{aligned}
& - \\
& . T \\
& I = div
\end{aligned}$$

$$\begin{aligned}
& [I + T + (1 \\
& 8 \\
& - D) - \\
& T] * \\
& T
\end{aligned}$$

· Calcul
of
II

In stationary regime, this term does not exist. In transient, one will have expressed!

T -, derived

Lagrangian of T at the previous moment at the points of Gauss. CP is supposed to be constant by element. From where the contribution:

C

$$\begin{aligned}
& I \\
& p \\
& = \\
& T - \\
& 1, , \\
& ! (pg) ui (pg) \\
& I pg \\
&) \\
& T \\
& pg
\end{aligned}$$

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· Calcul

of

I2

In stationary regime, this term does not exist. In transient, one will have expressed the gradient of!

T with

points of Gauss. is supposed to be constant by element. From where the contribution:

u_i

I

-

$2 I p_g = (1$

, ,

- I ($T!$)

p_g, J

x_j

p_g

J

· Calcul

of

I3

In stationary regime, this term does not appear. CP is supposed to be constant by element.

$div, T +$

$T -$

and

will have been calculated at the points of Gauss as quoted in appendix 5. From where

contribution:

C

I

$p div$

+
 -
 3, ,
 = -
 $(pg) (T (pg) - T (pg) u_i (pg)$
 $I pg$
)
 T
 pg

· Calcul
 of
 $I4$
 CP is supposed to be constant by element. Its gradient is thus null by element. From where:
 $I4 I pg 0$

, ,
 =
 · Calcul
 of
 $I5$
 is supposed to be constant by element. +, -
 T
 T and will have been calculated at the points of Gauss.
 One starts by calculating the quantity

+
 -
 $I T$
 + (1-i) T
 . The result is a vector of which them
 components at the point of Gauss are:
 u_i
 With $(pg) =$
 $[T+ (I) + (1 -) T$

$(I)] (pg$
 J
 I
 I
)
 x_j
 I

The tensorial product contracted **A**

is written: $(A) =$

K
 $A_{j,k}$
 J

y

Example: $(A) =$

$X +$
 Z
With

A_y
 $+$
 X
 X
 Z

With
 X
 X
 X

etc, from where the formula:

u_i
 $I_5 = ($
With (p, g)
 (p, g)
 $I(p, g)$

K
)
 x_k
 p, g
 K

In axisymmetric 2D, product **A**

is written:

(

With $=$

R
With
 $+$
 Z

Z
With
+ A
R
R R
X

X
(

With) =
R
With
+
Z

Z
With
+ A
Z
R Z
Z
Z
(

With) =
R
With

R
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Here, component A is always null. We thus find the same expression as in Cartesian co-ordinates 2D.

· Calcul

of

 I_6

is supposed to be constant in the element. $+$, $-$

 T

T and div will have been calculated at the points of

Gauss. The vector

+

-

 $I T$ + (1-i) T

\mathbf{A} will be noted, like previously. The product

tensorial contracted T^* has on the knot slip I for component:

(

 U $T^*) = I (pg)$ I, K X $J K$ J J

Ex: (

 U U y U $T^*) = I (pg) X + I (pg)$ + $I (pg) Z$ I, X X X

y

X

Z

X

from where the formula:

I

= A

*

6, ,

(pg) (T

I pg

K

).

I, K

pg

K

As for the preceding integral in axisymmetric 2D, component A is null.

The expression is thus the same one in Cartesian 2D or axisymmetric.

· Calcul

of

I7

is supposed to be constant in the element. Its gradient is thus null there. From where:

I7 I pg 0

, ,

=

· Calcul

of

I8

is supposed to be constant in the element. +, -

T

T and div will have been calculated at the points of

Gauss. The vector

+

-

I T

+ (1-i) T

A is noted, like previously. We have then:

U

I

div
 $8, ,$
 $=$
 $(pg) A (pg) I (pg)$
 $I pg$
 K
 $)$
 X
 pg
 K
 K

Source term of energy

We calculate the two integrals:

$I =$
 div
 $+$
 $-$
 $*$
 1
 $(if + (1 - I) S) T$
 I
 $+$
 $-$
 $*$
 $2 = (S$
 I
 $+ (1 - I) S$
 $). T$
 $s+$
 S

and
 are known at the points of Gauss. div was calculated at the point of Gauss. From where
 contribution:

I
 div
 $+$
 $-$
 1
 $(pg) (S (pg) (1$
 $, ,$
 $=$
 $+ -) S (pg) U (pg$
 $I pg$

I
I
I

) pg

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The source S being constant by element, its gradient is null. From where:

$I2 I pg 0$

, ,
=

Term of the boundary conditions of imposed flow

I

+

-

*

$$1 = (Q + (1 -) Q) div T$$

I

I

S

q^+

Q

and

are known at the points of Gauss. $div ()$ is calculated at the point of Gauss.

I

+

-

1

$[Q (pg) (1$

$$\begin{aligned} & \cdot \cdot \\ & = \\ & + \cdot) Q (pg)] \times \text{div} (pg) U (pg) \\ & I pg \\ & I \\ & I \\ & S \\ & I \\ &) pg \end{aligned}$$

Note:
 The calculation of the J/xk is done at the points of Gauss of the element of edge, for example on one segment for a calculation 2D. However on this segment, one knows only the derivative curvilinear of functions of form, i.e. the derivative tangential. It is thus necessary to calculate them as a preliminary quantities J/xk while basing itself on the elements of volume and that with the nodes of elements of edge. Then, one evaluates their values at the points of Gauss of the element of edge with the functions of this element of edge.

The expression is the same one in axisymmetric 2D as into Cartesian because the complementary term R of

, is multiplied by the component orthoradiale of normal N . However this R component is null.

$$\begin{aligned} & I \\ & + \\ & - \\ & * \\ & 2 = (Q \\ & + (1 \cdot) Q \\ &). T \\ & I \\ & I \\ & 2 \end{aligned}$$

Q is supposed to be constant by element. Its gradient is thus null there. From where:
 $I2 I pg 0$

Term of the boundary conditions of convectif exchange

If H and $Text$ are independent of time, we calculate the following expression:
 $I = -$
 $(1 \cdot) H T! T^* + H$

$$(T - T + - (1 -) T -$$

) div T*

I
ext.

I
I
S
3
3

What gives:

$$I - I, pg = ([1 - I] H T! (pg)) + (HT - T+$$

-
ext.

$$I (pg) - (1 - I) T (pg) \times divs (pg) ui (pg) pg$$

If H or Text is independent of time, it is then necessary to calculate:

$$I = - (1 - - *) H T! T + 3$$

+
*

3
[h+
+
+
-
-
-

$$I (T - T$$

ext.
) + (1 - I) H (T - T

ext.
)]div T
S

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What gives:

I

-

-

I, pg =

[(1 - I) H T (pg)]

+ [h+ +

+

-

-

-

I

(T - T

ext.

(pg) + (1 - I) H (T - T

ext.

(pg)] ×

div (

pg) ui (pg) pg

Same remarks that to the preceding paragraph apply to the calculation of the quantities J/xk .
The two integrals utilizing H and $Text$ are

presumedly constant by element.

A4.2 Calculation of the gradient of displacements and the constraints

A4.2.1 Principle general

As for thermics, the calculation of the gradient of displacement is inserted in the calculation of displacement, i.e. operator MECA_STATIQUE.

Then the calculation of ϵ and will be done in postprocessing, in orders CALC_ELEM and CALC_NO.

A4.2.2 Boundary conditions of Dirichlet and uniform connection

Nothing is to be made for these two types of boundary conditions. Their treatment is ensured by standard operation of the calculation of static mechanics linear.

A4.2.3 Detail of the various terms of the second member

We will gather under the same integral the obligatory terms which had with the derivation of the equation, then to examine each possible loading. The result will be written in the form of contribution of node I and the point of Gauss pg for the current element.

Term due to derivation

It is necessary to calculate the contribution of:

$$I = \int_{\Omega} \mathbf{FT} \mathbf{U},$$

$$\mathbf{FT} \mathbf{U},$$

$$:$$

$$2 \left[\left(\mathbf{U} + \mathbf{U}^T \right) \mathbf{S} \right]$$

$$\mathbf{v}$$

$$- \mathbf{K} \mathbf{T}$$

$$\mathbf{S}$$

$$! \mathbf{Id}:$$

\mathbf{v}

(\mathbf{U})

$$: \mathbf{F} (\mathbf{S}$$

$$\mathbf{T} \mathbf{v},)$$

-

$$[(\mathbf{U})$$

$$\mathbf{S}$$

$$: \mathbf{v}] \text{ div}$$

where sv is the symmetrized gradient of \mathbf{v} is $1 + \mathbf{T}$

$$2 (\mathbf{v}$$

$$\mathbf{v}).$$

To format this writing symbolic system, we will leave the broken up analytical form and to write derivations on the scalar terms.

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One will remember (cf [§3.2.1]) that this integral results from derivation from:

(U): sv

and that we broke up it into:

.

[

(U): sv] + **(U):**

sv +

! **(U):** S

div

v

We start by clarifying the first integral because that will make it possible to set up them various terms according to the mode: plan or axisymmetric.

· In plane deformations:

v

vy

v

v

: sv =

X

X

y

xx

+ yy

+ xy

+

X

y

y

X

The divergence of the field is a data, calculated at the points of Gauss.

The tensor of the constraints (**U**) is known at the points of Gauss. Tensor **v** is known.

From where contributions:

v

v

I

I

I

1 I, pg, X = - xx (pg)

+ xy (pg)

div

() (pg).

X

y

pg

v

v

I

I
 I
 $1 I, pg, y = -xy (pg)$
 $+ yy (pg)$
 div
 $() (pg).$
 X

y
 pg

The term of the integral second breaks up into:

\cdot
 \cdot
 \cdot

v
 v
 v
 X
 y
 v

X
 y
 xx
 $+$

yy
 $+$
 $+$
 X

xy
 y
 y

X

It is then enough to use the formulas shown in appendix 2 for function FT and to establish the following expression, knowing that $v = ! v$

X

y = 0:

vx X v

-

X

y

xx

+

X X

y X

vy

v

+

X

y

y

yy

+

X y

y y

v

v

v

X X

v

X

y

y

+
 X
 y
 y
 xy
 +
 +
 +

 X y
 y y
 X X
 y X
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While distributing on the functions of form, we have:

v

y

y v

I

X

X

I

I
2 I, pg, X = - xx
+

X
xy
y
+

X
xx
X
xy

X

y
pg

v

y
y v

I
X
X
I
I

2 I, pg, y = - yy
+

y
xy

X
+
+
 X
 yy

 y
 xy

 X

 y
 pg

The third integral is worth:

v
 v

 v
 X
 y
 v

!
 X
 y
 xx
+! yy
+ !
+

X

 xy
 y
 y
 X

In the isotropic elastic case, we have:

U

U

X

y

$xx = 1$

+ 2

+ $K (T - ref.$

$T)$

X

y

U

U

X

y

$yy = 2$

+ 1

+ $K (T - ref.$

$T)$

X

y

U

U

X

y

$zz = 2$

+ 2

+ $K (T - ref.$

$T)$

X

y

1

U

U

X

y

$xy =$

3

+

2

y

X

E (1 -)

with

$$1 = (1+)(1- 2)$$

E

$$2 = (1+)(1- 2)$$

E

$$3 = 1+$$

Let us detail the calculation of!

xx:

.

.

uy

uy

!

$$= 1$$

$$+ 2$$

$$+ K!$$

xx

T

y

y

! uy

! uy

=

$$1$$

$$+$$

$$2$$

$$+ K!$$

T

X
y
U

U
U

XX
U

X
y
y

X
y
y
- 1
+

- 2
+

XX
yX
Xy
yy

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How it was seen with [§3.2], the terms in! U and! U

X

y are to the first member. The same one

technique of derivation applied to yy , zz and xy encourages to pose notation LAGUGT for

1

T

the expression

$FT U, + FT U,$

2 [

(

)

(

)]

$ux X U$

U

X

y

y

U

LAGUG (

T)

1 =

X

y

y

1

+

+ 2

+

+

XX

yX

Xy

yy

$ux X U$

U

X

y
y
U
LAGUG (
T 2) =

X
y
y
2
+
+ 1
+

X X
y X
X y
y y
U

U

X X
U
U
LAGUG (
T)
3 =

X
y
y
X
y
y
2
+
+ 2
+

X X
y X
X

y
y y

1

LAGUG (
T 4)
U

U
U

X
X
ux

y
y
X
y
y
=

+
+
+

2
3
X
y
y

y

X
X
y

X

what gives:

$! U$

U

X

$!$

$!$

y

$= 1$

$+ 2$

- LAGU

xx

(

WP)

$1 + !$

kT

X

y

$! U$

U

X

$!$

$!$

y

$= 2$

$+ 1$

- LAGU

yy

(

WP 2) +!

kT

X

y

$! U$

U

X

$!$

$!$

y

$= 2$

$+ 2$

- LAGU

zz
(
WP)
3 + !
kT
X
y
1
! U
U
X
!
!

y
= 3
+
- LAGU

xy
(
WP 4)
2
y
X

The contribution to the second member is thus:

v
v
I
I
I
3 I, pg, X = [LAGUG

(
T)
1 - kT!
+ LAGUG (
T 4)

X

y pg

v
v
I
I
I
 $3 I, pg, y = \text{LAGUG}$

(
T 4)
+ [LAGUG (
T 2) - $kT!$]

X
y pg

· In axisymmetric 2D
The starting expression is:

v
v
v
v
v

S
R
Z
R
R
Z
: $\mathbf{v} = rr$
+ zz
+
+ $r\zeta$
+
R
Z

R

Z
R

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We thus find the same formal writing as in plane 2D, increased by a term complementary:

v
 v
 v
 I
 I
 I
 I
 $1 I, pg R$
 $, = - rr (pg)$
 $+ rz (pg)$
 $+ (pg)$
 $div () (pg).$

R
Z

R
pg

v
 v
 I
 I
 I
 $1 I, pg, Z = - rz (pg)$
 $+ zz (pg)$
 $div () (pg).$

R
 Z
 pg

The second integral breaks up into:

R
 $v!$
 $v! Z$
 R
 $v!$
 $vr! v$

$! Z$
 rr
 zz

$R +$
 rz
 $Z +$

$R +$
 $+$

$Z R$

In appendix 2, we established the Lagrangian expressions of each one of the derivative. It is enough to defer them here, by sorting them by type of component:

v

v
I
R
R
I
Z
Z
I
2 I, pg R
, = - rr
+

R
rz
Z

+
+
R
rr

R
rz
Z
Z

R
+

v

Weight
R 2 R

v

v

I
R
R
I
Z
Z
I
 $2 I, pg, Z = - zz$
+

Z
rz

Z

+

+

R
zz

Z
rz

R Z
pg

The third integral is worth:

R
v
vz
vr
vr v

!

Z
rr
+! *zz*
+ !
+ !

+

R

rz

Z

R

Z R

In the isotropic elastic case, we have:

ur

U

U

Z

R

$rr = 1$

+ 2

+ 2

+ $K T - T$

R

Z

R

(*ref.*)

ur

U

U

Z

R

$zz = 2$

+ 1

+ 2

+ $K T - T$

R

Z

R

(*ref.*)

ur

U

U

Z
R
 $= 2$
 $+ 2$
 $+ 1$
 $+ K T - T$
R
Z
R
(*ref.*)

1
R
U
U

Z
 $r_z =$
3
 $+$
2

Z R
where 1, 2, 3 is worth like previously.
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The detail of the calculation of!

rr gives:

.
.
.

ur
U
U
!

R
R
= 1
+ 2
+ 2
+ !
rr

kT
R

Z

R

! *ur*
! *uz*
! *ur*
= 1
2

!

kT
R +

Z + 2 *R* +
ur *R* *R*
U *Z*
uz *R* *uz*

U
Z
R
-
R
1
2
+
+

+
R R
Z R -

R Z Z Z
R 2

By taking again the axisymmetric equivalent of LAGUGT:

ur R ur Z
uz R uz
U
Z
rr

LAGUG (
T)
1 = 1
+
2
2

R R
Z R +
+
+

R Z Z Z
R

ur R ur
U
Z
rr
uz R uz Z

LAGUG (

T 2) = 2

+

+

2

1

RR

ZR

R

+

+

RZ

ZZ

ur R ur

U

U

U

LAGUG (

T)

3 =

Z

Z

R

Z

Z

RR

2

+

+

+

RR

ZR

R

Z

Z

Z

+
 1 r^2
 LAGUG (
 1
 T 4)
 U
 R R

 U
 R Z
 U
 Z R

U
 Z Z
 =
 3
 +
 +
 +

2

 R Z
 Z
 Z
 R
 R

Z

 R

we have the same expression symbolic system:

! ur
 ! uz
 ! U
 !

 R
 = 1
 + 2
 + 2
 - LAGUG

rr
(
T)
1 + !
kT
R
Z
R
! *ur*
! *uz*
! *U*
!

R
= 2
+ 1
+ 2
- LAGUG

zz
(
T 2) +!
kT
R
Z
R
! *ur*
! *uz*
! *U*
!

R
= 2
+ 2
+ 1
- LAGUG

(
T)
3 + !
kT
R
Z
R
1

! ur! U

!

Z

= 3

+

LAGUG

rz

(

T 4)

2

Z

R -

The contribution to the second member is thus:

v

v

I

I

I

3 I, pg R

, = [LAGUG (

T)

1 - kT!]

+ [LAGUG (

T)

3 - kT!]

R

R

v

+

LAGUGT (4) I

Z

pg

v

v

I

I

I
3 *I*, *pg*, $Z = \text{LAGUG} ($
 $T 4)$
+ [$\text{LAGUG} ($
 $T 2) - kT!$]

R

Z
pg

Term of the loading in pressure

$II = [p.] \mathbf{v} + p \text{divs } \mathbf{v}$
3

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The loading in pressure is supposed to be known, therefore the tensor which expresses its gradient is calculable

easily:

p

p

X

X

X

y

p =

p

p

y

y

X y

p

p

X

X

X

+

y

X

y

p. =

p

p

y

y

X

+

y

X

y

[

p

p

X

X

p
p
y
y

p
.] $\mathbf{v} =$
X
+
y
 $\mathbf{v} X +$
X
+
y
 \mathbf{v}
X
y

X
y
y

The calculation of the *divs* term is done as in the case of thermics. From where contributions:

p
p

I
X
X

1 *I, pg, X =*
(pg) X
(pg) +
(pg) y
(pg) + px (pg) divs (
pg)]ui
X

y
pg

p

p
I

y
 y
 $1 I, pg, y =$
 $(pg) X$
 $(pg) +$
 $(pg) y$
 $(pg) + py (pg) \text{ divs } ($
 $pg) ui$
 X
 y
 $] pg$

In axisymmetric 2D, the gradient of P comprises a complementary term out of Pr/R . This component would be to multiply by the component orthoradiale field. That one being null, there is not no the particular contribution and we thus use formally the same expression as in 2D Cartesian.

A4.2.4 Passage to the gradient of the constraints

Knowing the Lagrangian derivative of the field of displacement \mathbf{U} and temperature, us let us calculate the Lagrangian derivative of the tensor of the constraints by (cf [§4]).

1
 1
 $! = \mathbf{U}$
 \mathbf{C}
 \mathbf{U}
 \mathbf{U}
 T
 $kT \mathbf{Id}$
 $2 [! + !] -$
 $2 ($
 $($
 $FT,) + F ($
 $T,)) +!$

Analytical expressions of the various components of the tensor! were seen in the paragraph precedent. It is enough to apply them in postprocessing.

A4.2.5 Calculation of derived the eulérienne from the constraints

The last stage of the treatment is conversion Lagrangian/eulérien for the derivative of the tensor of constraints. It is enough to apply the formula:

$= ! -$

Like the vector does not have component orthoradiale, the expression of the product is the same one

in plane 2D or axisymmetric 2D. We have as follows:

I, J

I, J

I

!, J

X

y

=

-

+

X

y

I, J

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Date:

01/07/03

Author (S):

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Organization (S): EDF-R & D /SINETICS

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Document: R4.03.02

Calculation of sensitivities in thermics

Summary:

During digital simulations obtaining a rough result is not sufficient any more. The user is moreover in petitioning of calculation of sensitivity compared to the data input of the problem. That allows him to estimate the uncertainty which the field result according to the law of variation of the data answers. This derived is also the basic substrate of problems opposite (retiming of parameters...) and of problems of optimization. This sensitivity can be obtained "manually", but the experiment shows that these parametric studies are often expensive, little mutualisables and less reliable than an analytical calculation established in the software of calculation. In this note, one places oneself in the perimeter of use of the standard thermal operators of Code_Aster and one are interested in this analytical sensitivity of the field of temperature and its flow by report/ratio with the characteristics material and the loadings. One described the process allowing there to exhume the linear system which this derivative checks. In order to minimize the overcost calculation, a particular effort was brought to bind its resolution to that of the initial problem. One details theoretical, numerical work and data processing which governed the establishment of these calculations of sensitivity in the code. One specifies their properties and their limitations while connecting these considerations with a precise parameter setting of the accused operators and with the choices of modeling of the code. One has tried constantly to bind different the items approached while detailing, has minimum, the demonstrations a little techniques. Required environment, the parameter setting and the perimeter of use of this new functionality are described. An example extracted from an official case-test is clarified.

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1 Problems

During digital simulation, obtaining a result starting from a data file is not sufficient any more. Taking into account uncertainties which weigh on the evaluation of the loadings, the geometries and of characteristics material, taking into account also the numerical approximations due to modelings employees, with their space-time discretizations and the algorithms of resolution, the user is increasingly petitioning of calculations of sensitivity [bib5]. One then seeks to evaluate sensitivity of a variable compared to an input datum of the problem. It makes it possible to estimate the uncertainty (the probabilistic function) to which answers the field result according to the law of variation of certain data.

This derivative can be estimated “manually”, but these of parametric studies are often expensive, little mutualisables and less reliable than an analytical calculation established in the code.

Note:

- The sensitivities by finished differences are of course dependent on the parameters on shift and of the grid, but into non-linear, another worsening factor is superimposed: degree of convergence of the solution. In any rigour, this one also intervenes on quality analytical sensitivities, because one uses the field of temperature solution for to assemble the linear system “derived”.***
- The third way gathers the techniques of automatic differentiation (ODYSSEY [bib7], [bib9]...) but they are not plantable in Code_Aster because of sound software architecture (transmission of arguments between the routines by pointer...). Of any way these products are still “relatively embryonic” and their use seem fixed quotas for with model problems or parts of software well specific. The ideal would be of course to incorporate these problems as of the compiler...***

Recently, the introduction of calculations of sensitivity of thermomechanical fields [R4.03.01] and of rate of refund of energy [R7.02.01] compared to a variation of field, showed relevance and the feasibility of this type of approach in Code_Aster. By coupling this last with software PROBAN, one can thus know the probability of starting of the rupture for a distribution of variation of field given. This type of studies mechanic-reliability engineers, for example, was carried out in the framework of project PROMETE [bib4] to determine the probability of rupture of a tank REFERENCE MARK in considering the variability the thickness of its lining. These sensitivities can also intervene in a crucial way in the resolution of problems opposite (retiming of parameters...) and in many problems of optimization.

In this document, one restricts oneself with the linear and non-linear thermal problems

Code_Aster and thus with the analytical sensitivities of the field of temperature T (and its flow) by report/ratio with the characteristics material and the loadings. One places oneself in the perimeter of use of the standard thermal operators (the loadings are supposed to be fixed, one do not thus be interested in the phenomena of convection-diffusion in pointer of THER_NON_LINE_MO [R5.02.04]) for isoparametric finite elements (one does not treat it thermal problem for the thin hulls [R3.11.01] (modeling COQUE_) and for elements of Fourier (resp. AXIS-FOURIER)) (THER_LINEAIRE [R5.02.01] and THER_NON_LINE [R5.02.02]) and also in that of the operators of preprocessing of data (DEFI_MATERIAU [U4.44.01], AFFE_MATERIAU [U4.44.03] and AFFE_CHAR_THER [U4.44.02]).*

One is interested only in derivation of T and his flow, fields depending on the variables of space X and of time T and parameters materials and loadings, compared to one of these parameters (which must be a constant scalar by geometrical zones (these under-parts are supposed to be distinct and motionless, one neglects thus in particular the phenomena of dilation)). Thus let us consider, by example, a Bi-material whose isotropic thermal conductivity is a constant reality by zones: 1 on 1 and 2 out of 2

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1

2

1
2

Appear 1-a: Designation of thermal conductivities for a Bi-material

One thus models total thermal conductivity in the form:

(X): = I X + I X

,
éq
1-1
1 1 (
2 2 (
(1)
2
2

***1 if X
with I
I
I the indicating function of the ième part (I X
) . One is interested in the sensitivity***

***I () =
: 0 if not
T***

field of temperature compared to one of the two parameters

(, xt)

. The problem

***I
0
i= I
formulate same manner for a loading or a limiting condition.***

By parameterizing advisedly the “derived loadings and materials” (cf [§6.2], [§6.4]) calculations developed thereafter and the data-processing developments that they underlie can too to take into account the modelings more sophisticated with several space dependences and temporal. For example, considering a thermal source,

$S(X, T) = S I X X + X X$

éq
1-2
1 1 ()
1 (, T)
S I
2 2 () 2 (, T)

T

S
on
1

one can calculate
1

(, xt) while parameterizing
= I
and
1
1 =

S

S

0 on
1

I

S
0
2
I = si

S
0 on
= I

in the definition of the derived sources. In the remainder of the document,

2

2 =

1

S

on

2

2

2

we restrict ourselves with the first modeling [éq 1-1], in order not to overload them later theoretical developments, but also because it appears closer to the real needs users. On a case-by-case basis, we specify however more sophisticated derivations which are accessible taking into account the new introduced functionalities and the perimeter from use of the code.

In order to more easily be able to commute space or temporal derivation with derivation by report/ratio with one of the parameters, one works with a derivation “within the meaning of the distributions” on initial parabolic problem (derivation clarified in appendix 1). But the same exercise could to be carried out starting from its version semi-discretized in time, of the variational formulation or of linear system (one then takes the “discrete derivative” i.e. compared to the components of discretized parameter) resulting from its discretization. In linear thermics, one shows that these derivations, to each stage of the numerical process, lead to the same result. The problem derived discretized being identical to the discrete derived problem, theoretical results exhumed on continuous problem can apply to the problem actually implemented.

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The approach adopted for the calculation of the sensitivity is thus purely analytical. Him could to prefer an semi-analytical method (it is the step partly retained by the codes MSC/NASTRAN, FEMtools and ABAQUS (cf [§9] [bib8])) : broadly analytical for the determination derived problem and using differences finished at the local level to determine the derivative of matrices and of the second elementary derived members. But the latter, although easier with to establish and to maintain, is also more expensive and it introduces a dependence with respect to parameter of shift.

For linear thermics, the “derived” problem is very similar to the initial problem. first member of the resulting linear system is preserved. He does not have thus to be reassembled, only the second member is to be packed by suitable a source term. Theoretical results of existence, of unicity and convergence of the solution are not appreciably modified. Obtaining the derivative in temperature require the same processes numerical (dualisation and inversion of the system linear resulting).

On the other hand, in non-linear thermics, the derived problem is metamorphosed: the operator parabolic is modified. He became linear, just like the limiting conditions. These last are more than of two types: Dirichlet or Robin, exit conditions of Neumann and radiation. Because of its linear character, the usual theoretical results are thus much easier with to exhume. In addition the resolution of the derived problem is faster and more robust than that of initial problem. One does not need to have recourse to an algorithm of Newton-Raphson for to determine the increment of temperature between two contiguous moments. A linear solvor is enough: no one is not need to assemble a tangent matrix with each under-iteration.

This time the two members of this equation are fundamentally different from those of the problem in temperature. However, after each step of time, once determined T_+ starting from T , one does not have to reassemble all the matrix of the linear system and its second associate member. It is enough to to supplement the first tangent matrix of the step of time following by the term due to the non-linearity of thermal conductivity. One also leaves the second member of the problem in temperature for to constitute that which interests us: one packs it by the terms of implicitation of non-linearities of thermal conductivity and of the limiting conditions.

One details theoretical, numerical work and data processing which governed the establishment of these calculations of sensitivities in the code. One specifies their properties and their limitations while connecting these considerations with a precise parameter setting of the accused operators and with the choices of modeling of code. One tried constantly to bind different the items approached while detailing, has minimum, them a little technical demonstrations.

In short, the perimeter of use of this functionality gathers it or not thermal, linear, isotropic or anisotropic, stationary or transitory, being pressed on isoparametric finite elements lumpés or not. Within this framework there, it covers the same perimeter as that with the operators accused thermics.

The request for one or more sensitivities does nothing but enrich the structure of data thermics (EVOL_THER) and provides also the thermal field of which they are the derivative. In term of performance, the calculation of an analytical sensitivity is much less expensive than a calculation standard since the same factorized matrix is re-used.

In addition to the calculation of sensitivities in thermics, Code_Aster proposes their hanging in mechanics statics or quasi-static [R4.03.03] and in dynamics [R4.03.04]. All these functionalities and theirs postprocessings associated (impressions, tests...) are included in the user's documentation [U4.50.02] and belong to deliverable project "Uncertainties of numerical calculations" [bib5].
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This document is articulated around the following parts:

- initially, one is interested in the various problems resulting from derivation of the linear problem of thermics (THER_LINEAIRE) compared to the parameters characteristics material and loadings,*
- then one reiterates this process on the nonlinear problem of thermics (THER_NON_LINE),*
- various linear systems "derived", - directly plantable in Code_Aster to determine such or such sensitivity -, are recapitulated in the third part,*
- in the following paragraph, one describes necessary postprocessings to obtain the sensitivities*

heat flow (CALC_ELEM/CALC_NO),

· one concludes by approaching the practical difficulties from implementation, the environment, its parameter setting and the perimeter of use. An example of use extracted from an official case-test (SENST04A) is also detailed.

Warning:

The reader in a hurry and/or not very interested by the theoretical springs genesis of these sensitivities and the details of modeling of the code can, from the start, to jump to [§4] and [§6] which the principal theoretical and practical contributions recapitulate preceding chapters.

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2 Thermics

linear

*One considers a limited open body occupying related of R_q ($q=2$ or 3) of border lipschitzienne characterized by its voluminal heat with pressure constant $CP(X)$ (the variable vectorial X symbolizes here the couple (X, y) (resp. (X, y, Z)) for $q=2$ (resp. $q=3$)) and its coefficient of isotropic thermal conductivity (X) . These data materials are supposed to be independent of time (modeling *THER* of *Code_Aster*) and constants by element ($P0$ discretization).*

Note:

*With modeling *THER_FO* these characteristics can depend on time. As of first versions of the code and before the installation of *THER_NON_LINE*, it allowed*

to simulate “pseudo” non-linearities. Taking into account its rather marginal use, us we will not interest, initially, with its derivation.

One is interested in the changes of the temperature in any item X of opened and at any moment T [0 [> 0), when the body is subjected to limiting conditions and loadings independent of the temperature but being able to depend on time. It is about voluminal source S (X , T), boundary conditions of imposed the temperature type F (X , T) (on the external portion of surface), normal flow imposed G (X , T) (on) and exchanges convectif H (X , T) and T).

ext. (X , T) (on 3

*One places oneself thus within the framework of application of operator **THER_LINEAIRE** [R5.02.01] of*

Code_Aster by retaining only the conductive aspects of this linear thermal problem.

This problem in extreme cases interfered (type Cauchy-Dirichlet-Neumann-Robin (also called condition of

Fourier) inhomogenous, linear and with variable coefficients) is formulated

*T
 C*

*p
- $\text{div} (T) = S$
 $\times]$,
 $0 [$*

*T
 $T = F$*

*$1 \times]$,
 $0 [$*

*T
 $= G$*

*éq
2-1
2 $\times]$,*

0 [
N
T

+ HT = HT

ext.
3 ×],
0 [
N

T (X)
0
, = 0
T (X)

Note:

• The condition of Robin modelling the convectif exchange (key word EXCHANGE) on a portion of edges of the field, can be duplicated to take account of exchanges between two pennies parts of the border in opposite (key word ECHANGE_PAROI). This limiting condition model a thermal resistance of interface

T

1 + HT

HT

1 =

2

12 ×],

0 [

With

N

T

T

has

one

éq

2-2

3 = 12

,

21

I =

ij

T2 + HT

HT

2 =

1

21 ×],

0 [

N

· The condition of Dirichlet can spread in the form of linear relations between the ddls (key word LIAISON_) to simulate, in particular, of geometrical symmetries of structure.*

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With =, T

has

one

$I = T$

ROUP)

(LIAISON_G

1

12

21

ij

TIXT

X

X

éq

2-3

I

+

TJ

T

J

=

T

on ×

1

1 (,)

2 2 (,) (,)

I
 $], 0[$
 I
 J
or
simply
more
 $T X$
 X
 I
 $I(, T)$
 $=(, T) \text{ on } \times$
 I
 $], 0[$
 $DL)$
 $(LIAISON_D$
 I

- *One will not speak about functionalities **LIAISON_UNIF** and **LAISON_CHAMNO** which allow to impose the same temperature (unknown) on a whole of nodes, because they are not that a surcouche of the preceding conditions imposing of the couples (,) particular.*
 - *When the material is anisotropic (modeling **THER_ORTH**), conductivity is modelled by a diagonal matrix expressed in the reference mark of orthotropism of material. That basically does not change calculations according to which only hold account isotropic case. Guard should just be taken not to commutate more, under the conditions limit Neumann and of Robin, the scalar product with the normal and multiplication by conductivity. In practice, in elementary calculations, one is not interested in the derivative normals. The problem thus arises only in the preliminary theoretical part. **The sensitivity compared to one of the components of anisotropic conductivity is not not yet available.** These calculations were set up in the accused subroutines (elementary calculations **YOU.**), they await nothing any more but the software evolution consisting with to extend the taking into account of the anisotropy to the functions (a modeling **THER_ORTH_FO**). Indeed, from a point of view structures (cf **DEFI_PARA_SENSI** [bib6]), variable **ASTER** representing the significant parameter must be a data-processing object of function type.*
 - ***In all following calculations of sensitivities, one calculates only it derived by report/ratio with a constant parameter by zone.** If not, it would be necessary to introduce a concept of derivative directional!*
- This does not exclude a temporal or space dependence from characteristics material or loadings. By parameterizing advisedly the “derived” loadings and materials in the command file, one can also have access to some derivatives made up (cf [§6.2]/[§6.4]).*
- *For a transitory calculation, the initial temperature can be selected in three manners different: by carrying out a stationary calculation over the first moment, by fixing it at one*

*uniform or unspecified value created by a **AFFE_CHAM_NO** and by carrying out a recovery with to start from a preceding transitory calculation. This choice will affect the initialization of derived problem.*

*· We will not treat the case where (almost) all the loadings are multiplied by one even function dependent on time (option **FONC_MULT** (this well adapted functionality for certain mechanical problems is disadvised in thermics, because it can return in conflict with the temporal dependence of the loadings and, in addition, it applies selectively with each one of them. It was not included besides in **THER_NON_LINE**)).*

In order to be able to consider different the derivative from the temperature in configurations multimatériau and multichargement one introduces the following notations:

=

U I

I

I = ij (I = 1, 2 ou 3)

U_i

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The characteristics material and the loadings will be subscripted to notify their membership of such or such opened or portion of border. Thus, if one is interested in a Bi-material, 1 models heat voluminal of material occupying the open first and

1

2 that of material occupying the second

opened.

2

1

2

1

2

Appear 2-a: Designation of voluminal heats for a Bi-material

2.1

Derived compared to voluminal heat

2.1.1 Elements

theoretical

One models total voluminal heat penny the form C X

I X

. All them

p () = I

I ()

(I)

I

C

opened being solidified, one has

p = I the indicating function of the ième part. The derivation of

I

I

I

[éq 2-1] then leads us “trivialement” (cf Appendix 1) to the new problem in extreme cases of which is

T

solution sought sensitivity, noted U =

,

I

U

~

C

$$\begin{aligned}
 &P \\
 &- \operatorname{div}(U) = S \\
 &\times], \\
 &0 [
 \end{aligned}$$

$$\begin{aligned}
 &\sim T \\
 &U = F
 \end{aligned}$$

$$\begin{aligned}
 &, \\
 &0 \\
 &1 \times] \\
 &[
 \end{aligned}$$

$$\begin{aligned}
 &U \\
 &= \sim g
 \end{aligned}$$

$$\begin{aligned}
 &, \\
 &0 \\
 &\acute{e}q \\
 &\mathbf{2.1.1-1} \\
 &2 \times] \\
 &[\\
 &N \\
 &U
 \end{aligned}$$

$$+ hu = \sim h$$

$$\begin{aligned}
 &, \\
 &0 \\
 &3 \times] \\
 &[\\
 &N
 \end{aligned}$$

$$\begin{aligned}
 &U(\mathbf{X}) \\
 &= \sim
 \end{aligned}$$

)
0
,
u0

with the new source voluminal and the new limiting conditions and initial

~
~
~
~
T
~

$$F = G = H =,$$

0

$$S = - I$$

and

U

éq

2.1.1-2

I

$$= 0$$

0

T

One is thus brought to solve a homogeneous problem out of U similar to that which T answers. One fixes

a step of time T

such as

*that is to say an entirety NR. **Semi-discretization in times** of [éq 2.1.1-1],*

T

[éq 2.1.1-2] by - method leads to the following problem: to find a continuation.

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Note:

By using an adaptation of the theorem of Lax-Milgram to the parabolic problems ([R4.10.03 §1] or [bib1] theorems 1 & 2 chap XVIII pp615-620 or [bib3] pp220-241) one show, under certain conditions of regularity on opened, materials, the loadings and the initial condition, that this problem admits a single solution.

(U

éq

2.1.1-3

N)

$V = U H U =$

$0nN$

{1

0

()/ 1 }

0

This space comprises also the conditions of “generalized” Dirichlet of linear relations type between ddls when they exist.

such as:

$n+1$

U

- N

$\sim n+1 \sim$

U

N

$n+1$

N

S

- S

C

$$\begin{aligned}
 &P \\
 &- \operatorname{div}(U) - (1 -) \operatorname{div}(U) = \\
 &0 \ N \ NR \ -1
 \end{aligned}$$

T

N

~

T

+1

U

= *n+1*

F

0 *N NR 1*

1

-

n+1

U

= ~*n+1*

G

0 *N NR 1*

2

-

N

n+1

U

n+

N

~

1

+1

n+1

+ *H U = H*

0 *N NR 1*

3

-

N
 0
 $U(\mathbf{X} = \sim$
 $) u_0$

éq 2.1.1-4
while posing:

\sim
 \sim
 \sim
 \sim

N

C
 $U = U,$
 $\mathbf{X} N$
 $, S N = -$
 $p(\mathbf{X})$

N
 N
 N
 N

$T,$
 $\mathbf{X} N$
 $, F = G = H = 0 \text{ and } H =$
 $H,$
 $\mathbf{X} N$

NR
 I

NR

NR
éq 2.1.1-5

By applying the theorem of Green to [éq 2.1.1-3], [éq 2.1.1-4], [éq 2.1.1-5] and by introducing them following notations

+

-

= ,

$\mathbf{X} (N +)$

I

and = ,

$\mathbf{X} N$

with $\{U, T,\}$

H and $0 N NR -I$

NR

NR

one is brought to solve the following variational problem:

\pm

\pm

-

Being donn

és H

, T and U

+

To calculate $U V$

that

such

éq

2.1.1-6

0

v

+

V

has

0

$$(+u, v) = \pm l(v)$$

with the bilinear form depending on the current moment (via $h+$)

+

has (+

$$U v) = 1$$

,

+

$$C U v dx$$

éq

2.1.1-7

p

+

+

$$U v dx + + +$$

$$H U v$$

D

T

3

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and the linear form parameterized by the moments running and precedent (via H , U and T)

\pm

l

$L(v) =$

-

$C U v dx l$

l

p

$+ (-)$

-

$U v dx + (-) - -$

$H U v D +$

T

3

éq 2.1.1-8

$l II (- +$

$T - T) v dx$

T

Note:

- Contrairement to the initial problem, the unknown field and the function test belong to same functional space, which is more comfortable from a theoretical and numerical point of view.
- The first member of this equation is formally identical to that of the equation in temperature. After each step of time, once determined T_+ starting from T , one rests also on H , h_+ and U to determine u_+ . The matrix of the linear system corresponding does not have to be reassembled. Only the second member is to be packed by suitable the source term.
- By deriving the variational formulation (cf [§5.1.3]) from the problem in temperature [R5.01.02] one finds well [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8].
- For a condition of exchange between walls, the term of usual exchange is of course replaced by (by taking again the notations of [éq 2-2]) \pm

$H (\pm$

\pm

U

.

$I - U J) v D$

ij

- These problems of suitable initialization of the problem derived is recurring. It was found in the calculation of the derivative Lagrangian of the field of temperature compared to one variation of field (cf [R4.03.01] operand **DEUL_INIT**).
- The reader interested by a theoretical study of the thermal problem actually put in place in the code, which underlines these holding and bordering and their bonds with the choices of

modeling, will be able to refer to [§1] Doc. R: "Indicating of error in residue for transitory thermics" [R4.10.03]. It relates to a related field of improvement and of calibration of the studies, that of the space errors due to the grids finite elements.

To solve this problem numerically one spatially discretizes it by considering one subspace H

V of V of dimension finished

0

0

N

$U_{\pm} = u_{\pm} NR V H$

0

$= \pm$

:

/

0

\pm

H

I

I

$\{U V K$

U

H

H

$H K$

$K (K)\}$

$i=1$

by noting $(Th) H$ a regular family of triangulations of the polygonal or polyhedral field discretized

, P

H

$K (K)$ the space of the polynomials of degree $< (k+1)$ on K and N_i the function of form associated with node $n^{\circ}i$. From where the **discretized variational problem**

Note:

This property of total continuity of the elements and maximization of their characteristics geometrical (which ensures the convergence of the finite element method) is checked for all isoparametric elements of the code: segment, triangle, quadrangle, tetrahedron, pentahedron and hexaèdre.

$\pm \pm$

-
Being donn

és H, T and U

H
H
H

+
To calculate U H
V
that

such

éq
2.1.1-9

H
0
v V has U, v L v

H
H
+
0
H (+
H
H) = ±
H (H)

leading to the linear system of order N.

In U+ = L

éq
2.1.1-10

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The p boundary conditions of the Dirichlet type are taken into account in Code_Aster by one technique of double dualisation [R3.03.01] via “ ddl s (degrees of freedom)” of Lagrange

$= ($

. That is to say J the whole of the nodes belonging to the portion of border on which

$I) I,$

1

$= p$

apply the condition of Dirichlet ($card(J) = p$), one considers the new unknown vector

\sim

$U [+$

$= U$

$] T$

and the operator B (of order $p \times N$) checking

$(+$

$B U) = u+$ with $I J$

I

I

The homogeneous condition of Dirichlet of [éq 2.1.1-1], [éq 2.1.1-2] is realized while imposing

$B U+ = C = 0$

The dualized problem then consists in reversing the system of order $n+2p$

With LT

$LT +$

U

L

$\sim \sim +$

\sim

WITH $U = L$

B - Id

Id

=

0

éq

2.1.1-11

B Id - Id 0

Note:

· One carries out the possible taking into account of limiting conditions of Dirichlet generalized like in the problem in temperature (here $p=1$) but with a second null member

+

B U =

+

U

with J

J and C

J

J

=

= 0

J

I

We now will see how these calculations are declined in the code.

2.1.2 Establishment in Code_Aster

The matrix of this system results from the assembly of the following elementary terms, due to contribution of the nodes (I, J) to the point of gauss (of weight (this weight gathers in fact “truth

G

G

weight” of the formula of squaring multiplied by the jacobien of the element considered and possibly by the ray of the point of gauss rg (in modeling AXIS or AXIS_DIAG))) element running K

(in Code_Aster the conditions limit are affected on particular elements of skin of dimension $q-1$. No confusion not being really possible, one will make formal distinction here between those and the elements of volume which support them).

WITH K

WITH K

ϵq

2.1.2-1

$ij ($

$, G)$

3

$= iij (, G)$

$i=1$

with

· the thermal term of mass (calculated by option MASS_THER)

A1 K, =

C K NR NR

$ij ($

$) G$

G

p (

) J (G) I (G)

T

· the thermal term of rigidity (RIGI_THER)

A2 K, = K NR. NR

$ij ($

$G)$

G

() J (G) I (G)

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· the term of rigidity due to the conditions limit of exchange ($RIGI_THER_COEF_F/R$)

A3 K

,

=

+

3

H

K

NR

3

NR

ij (

G)

G

(

) J (G) I (G)

In the event of exchange between walls this term is replaced by ($RIGI_THER_PARO_F/R$)

A3 K

,

3

= h+ K

NR

3

- NR

NR

ij (

G)

G

(

) (J (G) F J (G) I (G)

()

by noting F the bijection putting in opposite the two walls.

The second member is written, with the same notations,

$L K$

$L K$

$\dot{e}q$

2.1.2-2

$J($

$, G)$

3

$= ij(, G)$

$i=1$

where

· the term resulting from the implicitation of the matrix of rigidity and mass (new option CHAR_SENS_EVOL, copy of CHAR_THER_EVOL with U instead of T, the field material derived and standard and the new source term)

$L1 K, =$

$C K U NR + -1 K U$

$. NR$

$J($

$) G$

G

$p($

$) (G) J (G) () G () (G)$

$J (G)$

T

· the term resulting from the implicitation of the conditions limit of exchange (CHAR_THER_TEXT_F/R with Text=0 and U instead of T)

$L2 K$

,

1

3

$= - H K$

U

3

NR

$J($

$G)$

$G($

$)($

$) (G) J (G)$

In the event of exchange between walls this term is replaced by (CHAR_THER_PARO_F/R with U instead of T)

L2 K

,
1

3
= - H K

U
3

- U F

NR

J (

G)

G (

)(

)((G) ((G)) J (G)

· the term due to the “new source” comprising the field derived material (cf. CHAR_SENS_EVOL above)

L3 K, = -

I K T + - T -

NR

J (

)

G

G

I (

)((G)

(G) J (G)

T

As one already specified all the elementary terms of the matrix are the subject of an option of calculation and will already have been evaluated for the calculation of T+. It thus remains to estimate the second member

by re-using (with a different parameter setting) the existing options of calculation or by introducing one news (CHAR_SENS_EVOL). This new option is common with the other derived material (thermal conductivity) and it redirects towards the same routine of elementary calculation (TE.). The chain

of character power station (SENS instead of THER) joined to a detection of the nullity of the field material

derived, allows to parameterize this routine towards one of its three possible orientations: calculation of term of mass and rigidity standard implicities, idem of sensitivity compared to one of both characteristics material which thus adds a new source term.

In accordance with the principles of architecture set up in the code to treat calculations of sensitivity [bib6], the assembly and the resolution of [éq 2.1.1-11] are started by the analysis of table of correspondence associated with the significant variable. It was seen that this calculation is very close to one standard linear thermal calculation, only the initial condition and the loadings are modified

$F =$

T

,
 0

$G =,$

$0 T$

and

ext. =

,
 0

0

$S = - I$

U

I

= 0

T

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This feedback is carried out via the succession of orders

v = *DEFI_PARA_SENSI* (VALE = < value of I >)

my = *DEFI_MATERIAU* (THER = _F (RHO_CP = v))

affe = *AFFE_MATERIAU* (AFFE = _F (GROUP_MA = < definition of I >, MATER = my))

...

one = *DEFI_CONSTANTE* (VALE = 1.)

MEMO_NON_SENSI (NAME = _F (NOM_SD = "my", PARA_SENSI = "v", NOM_COMPOSE = "ma_v"))

ma_v = *DEFI_MATERIAU* (THER = _F (RHO_CP = one))

MEMO_NON_SENSI (NAME = _F (NOM_SD = "affe", PARA_SENSI = "v", NOM_COMPOSE = "affe_v"))

affe_v = *AFFE_MATERIAU* (AFFE = _F (GROUP_MA = < I >, MATER = ma_v))

...

resu = *THER_LINEAIRE* (CHAM_MATER = affe,

SENSITIVITY

=

(

v)

...)

Note:

· In its command file, the user will not have soon any more but to specify the first and it third blocks of instruction. The block of the medium will be generated automatically by supervisor thanks to the tree of dependence which it builds between the various orders.

C

· The essential data of this calculation, derived voluminal heat

p

I =

, is provided by

I

I

ma_v.

· To take into account a more sophisticated modeling of voluminal heat

C X

I X X

$p () = I$

$I ()$

$I ()$

(I)

I

in these calculations of sensitivity, it is enough to substitute for the indicating function the new term

C

source

p = I in the definition of the field derived material

I

I

ma_v.

I

We will unroll the same process for the various sensitivities, to start with that concerning the other characteristic material: thermal conductivity.

2.2

Derived compared to thermal conductivity

2.2.1 Elements

theoretical

T

One poses $(X) = I X$

and $U =$

required sensitivity. All the open ones

I

$I ()$

$($

I

$)$

I
I

being solidified, one has
= I the indicating function of the ième part. The derivation of [éq 2-1] us
I

I
I

conduit with a problem in extreme cases identical to [éq 2.1.1-1] but with a nonnull voluminal source
and of the new conditions of Neumann and Robin

~
T
~

G = H = - I
and
s~ = div

éq
2.2.1-1

I
(I T
I
)
N

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It is a homogeneous problem of Cauchy-Dirichlet and inhomogenous Neumann-Robin similar to that which T answers. Its semi-discretization in time led to seek a continuation (U

N)

V

OnN

0

checking a system similar to [éq 2.1.1-4] whose first relation is rewritten

N 1

+

U

- one

C

P

- div (

N 1

+

U

) - (1 -) (one) ~

div

N 1

=

+

S

+ (-)~

I

S N

0 N NR -1

T

éq 2.2.1-2

with the new source term

~sm = div (I T m

éq

2.2.1-3

I
 $) m \{, nn+\} 1$
and new limiting conditions

$+$
 \sim
 ~ 1
 $+1$
 N
 N
 $T N$
 G
 $= H$
 $= - I$

2.2.1-4

I
 N

*From where a variational problem identical to [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] comprising even bilinear form [éq 2.1.1-8] joined to the linear form [éq 2.1.1-7] of which **only the fourth integral is modified** to adapt to the new source*

\pm
 $L(v) =$
 $- I$
 I
 L

2.2.1-5

$I($
 $+$
 $T + (-) -$
 $T) v dx$

Note:

· The first member of this equation is formally identical to that of the equation in temperature. After each step of time, once determined T_+ starting from T , one rests on H , h_+ and U to determine u_+ . The matrix of the linear system corresponding does not have to be

reassembled. Only the second member is to be packed by suitable the source term.

· By deriving the variational formulation (cf [§5.1.3]) from the problem in temperature [R5.01.02] one finds well [éq 2.2.1-5].

· In stationary regime this complementary source term is tiny room to

$$L(v) =$$

$$- I$$

.

$$L$$

$$I T$$

$$v dx$$

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

2.2.2 Establishment in Code_Aster

Compared to [§2.1.2], only the term due to the new source is to be modified

$$L3 K, =$$

$$- I K T + + I - T -$$

$$NR$$

éq

2.2.2-1

$$J ($$

$$G)$$

$$G I ($$

$$)($$

$$(G) () (G)$$

$$J (G)$$

What is made in the new option of calculation CHAR_SENS_EVOL with the field derived material and standard.

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. It was seen that this calculation is very close to a calculation

standard linear thermics, only the initial condition and the loadings are modified

$$F =$$

$$T$$

$$,$$

$$0$$

$G = HT$

and

$ext. = - I$

, S

I

$= div (iT)$

0

$U = 0$

N

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This feedback is carried out via the same succession of orders as with [§2.1.2] in substituent LAMBDA (or LAMBDA_L/T/N into orthotropic) with RHO_CP in the DEFI_MATERIAU.

Note:

· *The essential data of this calculation, derived thermal conductivity $I =$, is provided*

I

I

by **ma_v**.

· *When the material is anisotropic the thermal matrix of conductivity is expressed in*

locate orthotropism of material: it is thus diagonal (in our $q=2$ example)

0
1
=

0 2

One can then dissociate derivation compared to a value of one of his diagonal terms derivation compared to a value of this diagonal. The formulas detailed here are

identical whatever the configuration selected. Only the evaluation of

*ij
ij
or
must*

*K
kl*

to take account of these characteristics.

· In practice, one has access to the sensitivity compared to an isotropic conductivity constant by zone. Sensitivity compared to a component of conductivity anisotropic is not yet available. These calculations were set up in subroutines accused (elementary calculations YOU.), they await nothing any more but the evolution software consisting in extending the taking into account of the anisotropy to the functions (one modeling

THER_ORTH_FO). Indeed, from a point of view structures (cf DEFI_PARA_SENSI [bib6]), the variable ASTER representing the significant parameter must be a data-processing object of function type.

· One reasoned here as if the condition of initial Cauchy of the problem had been fixed uniform or unspecified. If it is given by carrying out a stationary calculation on first moment it is necessary to reiterate this process with the derived problem. On the other hand, if it results of a recovery starting from a preceding transitory calculation, the derived problem must be initialized with to leave the value of the same derivative at the same moment of recovery.

In short, two initializations (that of the problem in temperature and that of the problem derived) must be homogeneous. On the other hand, contrary to a calculation of thermics standard, one cannot thus modify the conditions limit and it is necessary to carry out a recovery with to start from a calculation of comparable nature.

· *To take into account a more sophisticated modeling of thermal conductivity*

$(X) = I X X$

I

I ()

I ()

(I)

I

in these calculations of sensitivity, it is enough to substitute for the indicating function the new one

source term

= I in the definition of the field derived material

I

I

ma_v.

I

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2.3

Derived compared to the source

2.3.1 Elements

theoretical

T

One poses $S(\mathbf{X}) = S I \mathbf{X}$

S

and $U =$

***required sensitivity.** All the open ones being*

I

I()

(I)

I

if

S

solidified, one has

= I the indicating function of the ième part. The derivation of [éq 2-1] leads us

I

S

I

I

with a problem in extreme cases identical to [éq 2.1.1-1] but with another voluminal source

$s \sim I$

éq

2.3.1-1

I

It is a homogeneous problem similar to that which T answers. Its semi-discretization in time

conduit to seek a continuation (U

checking a system similar to [éq 2.1.1-4] of which

N)

V

OnN

0

first relation is rewritten

N I

+

U

- one

C

éq

2.3.1-2

$$\begin{aligned}
 &P \\
 &- \operatorname{div} (\\
 &N I \\
 &+ \\
 &U \\
 &)- (1 -) (one) \sim \\
 &\operatorname{div} \\
 &= S \\
 &0 N NR -1 \\
 &T
 \end{aligned}$$

From where a variational problem identical to [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] comprising even bilinear form [éq 2.1.1-7] joined to the linear form [éq 2.1.1-8] of which *only the fourth integral is modified* to adapt to the new source

$$\begin{aligned}
 &\pm \\
 &L(v) = L + I v dx \\
 &\text{éq} \\
 &\mathbf{2.3.1-3} \\
 &I
 \end{aligned}$$

Note:

- The first member of this equation is formally identical to that of the equation in temperature. After each step of time, once determined T_+ starting from T , one rests also on H , h_+ and U to determine u_+ . The matrix of the linear system corresponding does not have to be reassembled. Only the second member is to be packed by suitable the source term.
- By deriving the variational formulation (cf [§5.1.3]) from the problem in temperature [R5.01.02] one finds well [éq 2.3.1-3].
- In stationary regime this complementary source term is not modified.

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

2.3.2 Establishment in Code Aster

Compared to [§2.1.2], only the term due to the new source is to be modified

$$L3 K, = I K NR$$

$$\begin{aligned}
 &\text{éq} \\
 &\mathbf{2.3.2-1} \\
 &J(\\
 &G) \\
 &GI(
 \end{aligned}$$

) **J (G)**

It is enough to re-use standard option CHAR_THER_SOUR_F/R with the derived field source. The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. It was seen that this calculation is very close to a calculation

standard linear thermics, only the initial condition and the loadings are modified

$F = G = T$

and

ext. =

,
0

0

$S = I$

U

I

= 0

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This feedback is carried out via the succession of orders

$v = \text{DEFI_PARA_SENSI} (\text{VALE} = \langle \text{value of } v \rangle)$

$\text{chth} = \text{AFFE_CHAR_THER_F} (\text{SOURCE} = _F (\text{GROUP_MA} = \langle \text{definition of } I \rangle, \text{SOUR} = v))$

...

$\text{one} = \text{DEFI_CONSTANTE} (\text{VALE} = 1.)$

$\text{MEMO_NON_SENSI} (\text{NAME} = _F (\text{NOM_SD} = \text{“chth”}, \text{PARA_SENSI} = \text{“v”}, \text{NOM_COMPOSE} = \text{“chth_v”}))$

$\text{chth_v} = \text{AFFE_CHAR_THER_F} (\text{SOURCE} = _F (\text{GROUP_MA} = \langle I \rangle, \text{SOUR} = \text{one}))$

...

$\text{resu} = \text{THER_LINEAIRE} (\text{EXCIT} = \text{chth},$

SENSITIVITY

=

(

v)

...)

Note:

S

· The essential data of this calculation, the derived field source $I =$

, is provided by

I

chth_v.

S

I

· This calculation is independent of the three types of modeling of the source: constant by mesh (**AFFE_CHAR_THER + SOUR**), constant by point of Gauss (**AFFE_CHAR_THER + SOUR_CALCULEE**) and constant by mesh and dependent on time (**AFFE_CHAR_THER_F + SOUR**). These considerations do not even return in line of account during the effective calculation of

S

in **chth_v**, because this size represents the derivation of a function parameterized by

S

I

one of its constant parameters. One is not interested here in derivative of the type

S

S

X,

O

X.

S

X,

I(jg) (

T) U if (tj) (T)

· To take into account a more sophisticated modeling of the source

S (,

X T) = S I X,

X T

S

I

I ()

I (

) (I)

I

in these calculations of sensitivity, it is enough to substitute for the indicating function the new term

S

source

= I in the definition of the derived loading

I

I

chth_v.

if

2.4

Derived compared to the imposed temperature

2.4.1 Elements

theoretical

T

One poses $F(X) = F I X$

F

and $U =$

required sensitivity. Portions of

I

I()

(I)

I

fi

F

external border being solidified, one has

= I the indicating function of the ième portion.

I J

I

F

I

I

I

derivation of [éq 2-1] leads us to a problem in extreme cases identical to [éq 2.1.1-1] but with one another voluminal source and a new condition of Dirichlet

~

~

F = I

and

S

éq

2.4.1-1

I

= 0

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It is an inhomogenous problem of Dirichlet and Cauchy- Neumann-Robin homogeneous similar to that which T answers. Its semi-discretization in time led to seek a continuation (this space can to comprise, if necessary, also conditions of “generalized” Dirichlet of linear relations type between ddls)

(U

$V = U H1$

éq

2.4.1-2

0

1

/u = I

N) nN

{

() I

1

}

checking a system similar to [éq 2.1.1-3] whose first relation is rewritten

N I

+

U

- one

C

éq**2.4.1-3**

$$\begin{aligned}
 & p \\
 & - \operatorname{div} (\\
 & N I \\
 & + \\
 & U \\
 &) - (1 -) \operatorname{div} (\text{one}) = 0 \\
 & 0 N N R -1 \\
 & T
 \end{aligned}$$

with the new limiting condition

~n

$$F + 1 = I$$

éq**2.4.1-4**

I

From where a variational problem identical to [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] with the same form

bilinear [éq 2.1.1-7] joined to the linear form [éq 2.1.1-8] whose fourth integral is null.

Note:

- The first member of this equation is formally identical to that of the equation in temperature. After each step of time, once determined T_+ starting from T , one rests also on H , h_+ and U to determine u_+ . The matrix of the linear system corresponding does not have not with being reassembled. On the other hand this time it is necessary to constitute the Lagrangian part of second member dualized in order to approximate new functional space V_1 .
- By deriving the variational formulation (cf [§5.1.3]) from the problem in temperature [R5.01.02] one finds well [éq 2.4.1-3], [éq 2.4.1-4].

Space discretization in a subspace H

V and the taking into account of the condition of Dirichlet

I

inhomogenous lead to a **linear system dualized similar** to [éq 2.1.1-11]

With LT **$LT +$** **U**

L

~ ~ +

~

WITH $U = L$

$B - Id$

Id

=

C

$\acute{e}q$

2.4.1-5

$B Id - Id C$

with

F

J

$C =$

$= \acute{e}q$

2.4.1-6

K

ij

F

I

by noting f_j the value of the condition of Dirichlet to the node $n^{\circ}k$ (classification room) of 1.

2.4.2 Establishment in Code_Aster

Compared to [§2.1.2], only the second member is modified since

3

$L K$

$\acute{e}q$

2.4.2-1

$J ($

$, G) = 0$

There is not thus an option of calculation particular to envisage, it is just necessary to assemble the linear system

dualized associated the derived imposed temperatures.

The assembly and the resolution of the linear system are started by the analysis of the table of

correspondence associated with the significant variable. It was seen that this calculation is very close to a calculation

standard linear thermics, only the initial condition and the loadings are modified

F = I, G

and

I

= Text = S = 0

0

U = 0

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This feedback is carried out via the succession of orders

v = DEFI_PARA_SENSI (VALE = < value of fi >)

chth = AFFE_CHAR_THER_F (TEMP_IMPO = _F (GROUP_MA = < definition of il >, TEMP = v))

...

one = DEFI_CONSTANTE (VALE = 1.)

MEMO_NON_SENSI (NAME = _F (NOM_SD = "chth", PARA_SENSI = "v", NOM_COMPOSE = "chth_v"))

chth_v = AFFE_CHAR_THER_F (TEMP_IMPO = _F (GROUP_MA = < il >, TEMP = one))

...

resu = THER_LINEAIRE (EXCIT = chth,

SENSITIVITY

=
(
v)

...)

Note:

F

· *The essential data of this calculation, the field imposed temperature derived I =*
, is

I

F

I

provided by chth_v.

· *This calculation is independent of the three types of modeling of the imposed temperature:*
constant by mesh (AFFE_CHAR_THER + TEMP), plus the dependence in time provided by
a function (AFFE_CHAR_THER_F + TEMP) or provided by a structure of data
“EVOL_THER” (AFFE_CHAR_THER_F + EVOL_THER + “TEMP”). These considerations

F

do not even return in line of account during the effective calculation of
in chth_v, because this

F

I

size represents the derivation of a function parameterized by one of its parameters

F

constant. One is not interested here in the derivative

X.

F

X,

I (

T J) (T)

· *The calculation of derived from T compared to one of the parameters of the relations of Dirichlet*
generalized [éq 2-3] same manner would be carried out. A distinction only appears
on the level of the Lagrangian components of the dualized system:

T

+

With U =

B U =

+

U

with J

J and C

T éq

2.4.2-2

J

J

= - J +

J

I

J

J

I

T

+

+

With U =

B U = U

with J J and C =

éq

2.4.2-3

J

J

I

J

I

The taking into data-processing account of these calculations would be carried out, like above, via the word

keys COEF_MULT_1/2 and COEF_IMPO of the key words factors LAISON_GROUP and LIAISON_CHAMNO.

· Sensitivity compared to a multiplying coefficient of this condition of Dirichlet

generalized [éq 2.4.2-2 is not available] because it has little direction with coefficients often discrete. One does not have access, by parameterizing advisedly the condition of Dirichlet generalized derived, that with derivation compared to the total coefficient [éq 2.4.2-3].

· To take into account a more sophisticated modeling of a condition of Dirichlet

*F (,
X T) = F I X,
X T
F*

*I
I (
I (
) (I)*

*I
in these calculations of sensitivity, it is enough to substitute for the indicating function the new one
F*

*source term
= I in the definition of the derived loading*

*I
I*

chth_v.

*fi
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2.5 *Derived compared to imposed normal flow*

2.5.1 Elements *theoretical*

T

One poses $G(X) = G I X$

G

and $U =$

required sensitivity. Portions

I

I ()

(I)

2 J

I

gi

G

being solidified, one has

= I the indicating function of the ième portion. The derivation of [éq 2-1]

I

G

2i

I

us leads to a problem in extreme cases identical to [éq 2.1.1-1] but with another voluminal source and a new condition of Neumann

~

~

G = I

and

S

éq

2.5.1-1

I

= 0

It is a problem of inhomogenous Neumann and homogeneous Cauchy-Dirichlet-Robin similar to that which T answers. Its semi-discretization in time led to seek a continuation (U

N)

V

0nN

0

checking a system similar to [éq 2.1.1-3] whose first relation is rewritten

N I

+

U

- one

C

éq

2.5.1-2

p

- div (

N I

+

U

) - (1 -) div (one) = 0

0 N NR -1

T

with the new limiting condition

~n

G +1 = I

éq

2.5.1-3

I

From where a variational problem identical to [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] comprising even bilinear form [éq 2.1.1-7] joined to the linear form [éq 2.1.1-8] of which only the fourth integral is modified to adapt to the new "surface" source

±

$$L(v) = L + I v dx$$

éq

2.5.1-4

I

2

Note:

· The first member of this equation is formally identical to that of the equation in temperature. After each step of time, once determined T_+ starting from T , one rests also on H , h_+ and U to determine u_+ . The matrix of the linear system corresponding does not have not with being reassembled. Only the second member is to be packed by suitable the source term.

- *By deriving the variational formulation (cf [§5.1.3]) from the problem in temperature [R5.01.02] one finds well [éq 2.5.1-4]*
- *In stationary regime this source term is not modified.*

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

2.5.2 Establishment in Code_Aster

Compared to [§2.1.2], only the term due to the new source is to be modified

L3 K
,

éq
2.5.2-1

2
I K
NR
2

J (
G)
G I (
) J (G)

It is enough to re-use standard option CHAR_THER_FLUN_F/R with the field derived flow. The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. It was seen that this calculation is very close to a calculation

standard linear thermics, only the initial condition and the loadings are modified

F = S = T
and

ext. =

,
0
0
G = I
U
I
= 0

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This feedback is carried out via the succession of orders

v = DEFI_PARA_SENSI (VALE = < value of gi >)

chth = AFFE_CHAR_THER_F (FLUX_REP = _F (GROUP_MA = < definition of 2i >, FLUN = v))

...

one = DEFI_CONSTANTE (VALE = 1.)

MEMO_NON_SENSI (NAME = _F (NOM_SD = "chth", PARA_SENSI = "v", NOM_COMPOSE = "chth_v"))

chth_v = AFFE_CHAR_THER_F (FLUX_REP = _F (GROUP_MA = < 2i >, FLUN = one))

...

resu = THER_LINEAIRE (EXCIT = chth,

SENSITIVITY

=

(

v)

...)

Note:

G

· The essential data of this calculation, the field normal flow derived I = , is provided by

I

G

I

chth_y.

· This calculation is independent of the three types of modeling of the condition of Neumann: constant by mesh (AFFE_CHAR_THER + FLUN), plus the dependence in time provided by a function (AFFE_CHAR_THER_F + FLUN) or components of vectorial flow dependent on time and constants by meshes (AFFE_CHAR_THER_F + FLUN_X/Y/Z). One can then dissociate derivation compared to the q-uplet components of derivation compared to one of its components. The formulas detailed here are identical whatever that is to say configuration selected. Only possibly the evaluation of

G L

G L

I

I

or

, ,

must take account of these characteristics.

K

(L {X y Z})

G

G

J

J

· The dependence in time of the condition of Neumann is not taken into account at the time of

G

calculation of

because this size represents the derivation of a function parameterized by one

G

I

G

of its constant parameters. One is not interested here in the derivative

X.

G
X,
I (
T J) (**T**)
T

· In practice, one thus has access to the sensitivity compared to a scalar flow or vector, constant by zone. By parameterizing advisedly the vector derived flow, one can too

to obtain the sensitivity compared to one of its components

,
X
J (
T)
G
I

· To take into account a more sophisticated modeling of a normal flow

G (
X T) = **G I X**,
X T
G

I I (
I (
) (**I**)
I

in these calculations of sensitivity, it is enough to substitute for the indicating function the new one

source term
= I in the definition of the derived loading
I
I

chth_y.
gi

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2.6
Derived compared to the coefficient from convectif exchange

2.6.1 Elements
theoretical

T

One poses $H(X) = H I X$
H
and $U =$
required sensitivity. Portions being
I
I ()
(I)

3 J
I
hi
H

solidified, one has
= I the indicating function of the ième portion. The derivation of [éq 2-1] us
I
H
3i
I
conduit with a problem in extreme cases identical to [éq 2.1.1-1] but with another voluminal source
and
a new condition of Robin

~

H = I

and

éq

2.6.1-1

I (Text - T)

~s = 0

It is an inhomogenous problem of Robin and homogeneous Cauchy-Dirichlet-Neumann similar to that

which T answers. Its semi-discretization in time led to seek a continuation (U

N)

V

0nN

0

checking a system similar to [éq 2.1.1-3] whose first relation is rewritten

N I

+

U

- one

C

éq

2.6.1-2

p

- div (

N I

+

U

) - (1 -) div (one) = 0

0 N NR -1

T

with the new limiting condition

~n I+

H

= I T

T

éq

2.6.1-3

I (N I

+
N I
 +
ext.
 -
)

From where a variational problem identical to [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] comprising even bilinear form [éq 2.1.1-7] joined to the linear form [éq 2.1.1-8] of which only the fourth integral is modified to adapt to the new “surface” source

±
 $L(v) = L + I$

éq
2.6.1-4
 $I \{ (+$
 +
 $Text - T) + (1 -) (-$
 -
 $Text - T) \} v dx$
 3

Note:

- *The first member of this equation is formally identical to that of the equation in temperature. After each step of time, once determined + T from - T, one rests also on - H, + H and - U to determine + U. The matrix of the linear system corresponding does not have to be reassembled. Only the second member is to be packed by suitable the source term.*
- *By deriving the variational formulation (cf [§5.1.3]) from the problem in temperature [R5.01.02] one finds well [éq 2.6.1-4].*
- *These calculations spread without sorrow in the condition of exchange between walls of [éq 2-2]. It is enough to replace the ± terms*

(±
 ±
 T
 by
ext. - T
) v D

3i

$\pm (u_{\pm} - u_{\pm} v D \text{ with } H, I, 1 - I.$

I

J)

{

I (

) I}

ij

· In hover this complementary source term is tiny room to

$L(v) = L + I$

$I(\text{Text} - T) v dx$

3

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to

dualized linear system [éq 2.1.1-11].

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2.6.2 Establishment in Code_Aster

Compared to [§2.1.2], only the term due to the new source is to be modified

T + K

3 - T

3

++

L

K

**,
éq2.6.2-1**

3

= I K

3

NR

J (

G)

(ext. (

)

(G)

GI (

)

-

-

(I -) (T K

3 - T

ext. (

)

(G) J (G)

What is made in the new option of calculation CHAR_SENS_TEXT_F with the fields coefficient of standard exchange and derived.

The assembly and the resolution of the linear system are started by the analysis of the table of correspondance associated with the significant variable. It was seen that this calculation is very close to a calculation

standard linear thermics, only the initial condition and the loadings are modified

F = S = G =,

0

HT

and

ext. = I I (Text - T)

0

$U = 0$

This feedback is carried out via the succession of orders

$v = \text{DEFI_PARA_SENSI (VALE = < value of hi >)}$

$\text{chth} = \text{AFFE_CHAR_THER_F (EXCHANGE = _F (GROUP_MA = < definition of 3i >, COEF_H = v, TEMP_EXT = W))}$

...

$\text{one} = \text{DEFI_CONSTANTE (VALE = 1.)}$

$\text{zero} = \text{DEFI_CONSTANTE (VALE = 0.)}$

$\text{MEMO_NON_SENSI (NAME = _F (NOM_SD = "chth", PARA_SENSI = "v", NOM_COMPOSE = "chth_v"))}$

$\text{chth_v} = \text{AFFE_CHAR_THER_F (EXCHANGE = _F (GROUP_MA = < 3i >, COEF_H = one, TEMP_EXT = zero))}$

...

$\text{resu} = \text{THER_LINEAIRE (EXCIT = chth,}$

SENSITIVITY

=

(

v)

...)

Note:

H

· *The essential data of this calculation, the field coefficient of derived exchange I =*

I

H

I

provided by chth_v.

· *To take into account a more sophisticated modeling of a coefficient of exchange*

H (,

X T) = H I X,

X T

H

I I ()

I (

) (I)

I
in these calculations of sensitivity, it is enough to substitute, with the indicating function, the new one
H

source term
= I in the definition of the derived loading

I
I

chth_v.
hi

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2.7
Derived compared to the outside temperature

2.7.1 Elements
theoretical

In the case of convectif exchange with the external medium, one poses T
X
T
I X
T

ext. () = iext

I ()

(iext)

I

T

T

and U =

required sensitivity. As previously

ext. = I the indicating function of

I

T

I

I

T

ext.

ext.

the ième portion. The derivation of [éq 2-1] leads us to a problem in extreme cases identical to

3i

[éq 2.1.1-1] but with another voluminal source and a new condition of Robin

~

~

H = H I

and

S

éq

2.7.1-1

I

= 0

It is an inhomogenous problem of Robin and homogeneous Cauchy-Dirichlet-Neumann similar to that

which T answers. Its semi-discretization in time led to seek a continuation (U

N)

V

0nN

0

checking a system similar to [éq 2.1.1-3] whose first relation is rewritten

N I

+

U

- one

C

éq

2.7.1-2

p

- div (

N I

+

U

) - (1 -) div (one) = 0

0 N NR -1

T

with the new limiting condition

~n I+

N

H

= H I+ I

éq

2.7.1-3

I

From where a variational problem identical to [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] comprising even bilinear form [éq 2.1.1-7] joined to the linear form [éq 2.1.1-8] of which only the fourth integral is modified to adapt to the new “surface” source

±

L (v) = L + I

éq

2.7.1-4

I {

+

H + (1 -) -

H} vdx

3

Note:

• The first member of this equation is formally identical to that of the equation in temperature. After each step of time, once determined + T from -

T, one

also rest on -

H, +

H and -

U to determine +

U. The matrix of the linear system

corresponding does not have to be reassembled. Only the second member is to be packed by the term suitable source.

· By deriving the variational formulation (cf [§5.1.3]) from the problem in temperature [R5.01.02] one finds well [éq 2.7.1-4].

· In stationary regime this source term is tiny room to

$L(v) = L + I H v dx$

I

3

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to

dualized linear system [éq 2.1.1-11].

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2.7.2 Establishment in Code_Aster

Compared to [§2.1.2], only the term due to the new source is to be modified

L3 K

,

=

++ 1-

-

éq

2.7.2-1

3

I K

3

H K

3

H K

NR

3

J (

G)

G I (

) { (

) (

) (

)} J (G)

It is enough to re-use standard option CHAR_THER_TEXT_F with the fields coefficient of exchange

-

standard and derived, and while it “bluffant” with a field T =0.

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. It was seen that this calculation is very close to a calculation

standard linear thermics, only the initial condition and the loadings are modified

F = S = G =,

0

0

T

and

ext. = I

U

I

= 0

This feedback is carried out via the same succession of orders as with [2.6.2] in substituent TEMP_EXT with COEF_H in the AFFE_CHAR_THER_F.

Note:

T

• *The essential data of this calculation, the field derived outside temperature ext.*

I =

, *is*

I

I

T

ext.

provided by chth_v.

• *To take into account a more sophisticated modeling of the outside temperature*

T

,

X T

T I X

,

X T

T

ext. (

) = iext I () I () (iext)

I

in these calculations of sensitivity, it is enough to substitute for the indicating function the new one

T

source term

ext. = I in the definition of the derived loading chth_v.

I

I

I

Text

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Calculation of sensitivities in thermics**Date:****01/07/03****Author (S):****O. BOITEAU Key****:****R4.03.02-A Page****: 28/66****3****Nonlinear thermics**

In nonlinear thermics the characteristics materials $CP(X, T)$ and (X, T) can depend on temperature (in non-linear thermics, one cannot define anisotropic materials. There does not exist of modeling $THER_NL_ORTH$. Modeling $THER_NL$). The body is subjected to the same types of limiting conditions and of loadings which the linear problem to which are added two conditions non-linear: normal flow imposed $I(X, T)$ (on) and radiation ad infinitum of a gray body (on).

4**5**

This last condition is modelled (P0) by its emissivity (X, T) , the constant of Stefan-Boltzmann (X, T) and the temperature ad infinitum $T(X, T)$.

The operator of Code_Aster dedicated to this type of problem is $THER_NON_LINE$ [R5.02.02]. It allows to solve the problem in extreme cases interfered according to (Cauchy-Dirichlet-Neumann-Robin type

Inhomogenous, non-linear radiation and with variable coefficients)

C **$p() T$** **T** **$- \text{div}((T) T)$** **$= S$** **$\times]$,** **$0 [$** **T** **$T = F$** **$I \times]$,**

0 [

() T

T

= G

2 ×],

0 [

N

() T

T

+ HT = HT

ext.

3 ×],

0 [

N

éq

3-1

() T

T

= I (T)

4 ×],

0 [

N

() T

T

= (I

4

4

T +

15

.

273

) - (T +
15
.
273
)] 5 ×], 0 [

N

T (X)
0
, = 0
T (X)

Non-linearities pose theoretical problems to show the existence and the unicity of the solution [bib2]. They can be also prejudicial with the numerical resolution itself. Thus, with regard to the modeling of voluminal heat CP (X, T), during an iteration, either because the thermal transient is violent, or because the beach of phase shift is very small (for example, for a pure substance), two the reiterated successive ones of the temperature can to locate on both sides one of its discontinuities. One then missed a large part of information relating to the phase shift.

To free itself from this type of problem one rewrites the first equation of [éq 3-1] while introducing a voluminal function enthalpy which will smooth these non-linearities (dependent on T (X, T) and while noting

T = T (X, t*) the value of the temperature at one moment t* < T arbitrary)*

(T) - div ((T) T) = S

×],

0 [

T

T

éq

3-2

with

(T) = C D

p ()

*T **

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Note:

- *Of share its definition (difference in a primitive (in temperature) of voluminal heat between the temperature considered and a temperature T^* at one arbitrary moment), the function enthalpy is known except for a constant of integration. It will thus have to be taken care that this constant is eliminated in all the handled expressions.*
- *The generalized conditions of type exchanges linear walls [éq 2-2] or relations between ddls [éq 2-3] are also usable. As in [§2] one will not be interested in derived by report/ratio with the parameters of functionalities LIAISON_*.*
- *For a transitory calculation, as the problem of thermics linear, three strategies can govern the choice of the field of initial temperature and they have an incidence on the initialization of the derived problem.*
- *Implicitemment, THER_NON_LINE must tolerate conductivities rather badly strongly non-linear. Because the tangent matrices and the initial predictive phase does not comprise it term representing their derivative compared to the temperature. Moreover this term complementary would have the bad taste to make nonsymmetrical the tangent matrix of standard system and the matrix of the derived system! What is problematic to take in count by means of computer in the handling of the structures of data.*
- ***In an any state of cause, with respect to the non-linear characteristics material generally used, the perimeter of use of calculations of sensitivity is the same one that that of the standard problem. It does not take into account rigorously nonlinear thermal conductivity.***
- ***In addition, as in linear thermics, one has calculates only the sensitivity by report/ratio with a constant parameter by geometrical zone. What does not exclude a dependence temporal, space or non-linear of characteristics material or loadings nonconcerned by derivation. By parameterizing advisedly the loadings and “derived” materials in the command file, one can also have access to***

some derivatives made up (cf [§6.4]).

- *Into non-linear, obtaining a sensitivity by finished differences is even less reliable than into linear, because it can be very sensitive to the degree of convergence of the solution. In any rigour, that also influences the quality of the analytical sensitivity, the field of temperature solution intervening in the assembly of the linear system “derived”.*
- *Within the framework of non-linear thermics, the derivative compared to the enthalpy will not thus have no the direction (a enthalpy is not constant!).*
- *On the other hand, knowing the sensitivity U of the field of temperature compared to a parameter, one reaches easily that of enthalpy v compared to this same parameter, via the formula*

(T)

T

T

$v =$

$:$

$=$

$= C$

$= C U$

T

P

P

3.1

Derived compared to voluminal heat

3.1.1 Elements

theoretical

That is to say $C X$

$I X$

, one will derive compared to the parameter the formulation

$p () = I$

I (
I)
I
I
T

[*éq 3-2*]. **The required sensitivity is noted $U =$**
. *By noticing that the enthalpy can*

I
to model like a function of time T and J (considered makes some like functions
indicatrixes of the type of material)

$T(T, X, J)$
: (T, X

=
-

J)
C

p (
J) *D*
C p (
*

T T)
*

T (X, J)
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its derivative is written

T

C

*

P

T

T

=

D +

CP

-

*

I

T

I

I

I

*

=

*

T

T

ITT

C

I(

-)

+
p
-

I
I

C
Indeed quantity

p
I =
is independent of the temperature (and thus time) because one
I

I
suppose that the portions are fixed (the under-parts of the body are supposed to be motionless, one
I
neglect in particular the phenomena of dilation). The derivative in time of the first term in enthalpy
of [éq 3-2] is thus worth

(T)

U

I
T

T

= I

+

éq
3.1.1-1
T
I

T

T

In addition one has

(

T (T, X,

X

J)

T T

T

=

+ *T*

J

+
=

U with {,}

I

I

T

T

I

I

I

J

T

{

{

J {I

0
0

ij

(

$T + 273.15$
.) 4

X

= 4 ($T + 273.15$)
.) 3 T T
T

+ T
J

+
= 4 ($T + 273.15$)
.) 3u

T

I

{I
{I
J
J {I

0
0

ij
éq 3.1.1-2

The derivation of [éq 3-2] leads us to the new problem in extreme cases out of U

U
 T

-

$C + U = \sim$
 div
 S
 $\times]$,
 $0 [$

T
 T

$U = \sim f$

,
 0
 $1 \times]$
 $[$

+
 U
 U
 $= \sim g$

,
 0
 $2 \times]$
 $[$
 N
 N

$U \sim$

+ hu +
 $= H$

,

0 éq

3.1.1-3

3 ×]

[

N

N

I

U ~

-

U +

= I

,

0

4 ×]

[

NT

N

U

+ 4 (T +

) 3

U +

= ~

15

.

273

,

0

5 ×]

[

N

N

U (X

= ~
)
0
,
u0

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with the new source voluminal and the new limiting conditions and initial

T ~

~

~

~

~

~

~

~

~

~

~

~

~

~

~

~

~

~

~

~

S = - I

, F

and

éq

3.1.1-4

I

= G = H = I = = 0

U = 0

O
T

Note:

· The derivation of the formulation in voluminal heat [éq 3-1] led of course to same result because voluminal heat is a function of the J (considered in fact indicating functions of the type of material) and the temperature (depending itself on time, of the variable of space and the J!)

C, T T,
X

p (J
(
J)

Its derivative compared to I is thus written

C
C
C
X

p
p
J
p T
T
T

=
+
+ T
J
+
= I +

I

J
T
T
I
J
{I

{I
{I
J
J {I

0
0

ij
ij

from where

U
C

P
T
U
T
CP
U

T

T

+ C
= I
+
U + C
=
+ I

T
P
T
I
T
T
P
T
T
I
T
I

one finds well the formulation [éq 3.1.1-1] of the only term which distinguishes these two modelings.

Contrary to linear thermics, the derived problem is completely metamorphosed. The parabolic operator is modified and it became linear out of U . **All the problem is besides become linear because the conditions limit underwent the same treatment.** The condition of Dirichlet is from now on homogeneous and those of Neumann and radiation left room to conditions of Robin. The condition of Cauchy became homogeneous. Contrary to the problem in temperature, the theoretical results of existence and unicity of the solution U are thus easier with to exhume.

From a practical point of view, one does not need to have recourse, as in THER_NON_LINE, with one algorithm of crossbred Newton-Raphson of a predictive phase to determine the increment of temperature between two contiguous moments. A linear solvor is enough. No one is not need to assemble one

stamp tangent with each under-iteration. **The resolution of the “derived” problem is thus more rapid and more robust than that of the initial problem.**

Semi-discretization in times of [éq 3.1.1-3] by - method leads to the following problem: to find a continuation (U such as (with the notations of [§2.1.1])

N)
 V
 $0nN$

0
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n+1

N

n+1

U

-

U N

T

T

n+1

-

n+1

div

U n+1

T

+ n+1

n+1

U

T
T

N
~ N
~

+I
N
- (
N
N
N
N
S
S
I -)

-

div
U T + U
=
0 N NR -

I
T

T
N
~

+I
U
= n+I
F

0 N NR 1

1

-

n+1

N 1

n+

N

U

1

+1

+

~

U

+

= n+1

G

0 N NR 1

2

-

N

N

n+1

N 1

n+

N

N

U

~

1

+1

1

+

+

n+1

+ H U +

= H

0 N NR 1

3

-

N

N

n+1

n+1

I

N I

n+

N

U

I

+I

+

~

-

U

+

= n+1

I

0 N NR I

4

-

N

T

N

n+1

n+1

3

N

N

N

N

N

U

+

$+1$
 $+1$
 4
 $(+1$
 T
 $+ 273.15) +1$
 $+1$
 U
 $+$
 $= \sim n+1$

$0 N NR 1$
 5
 $-$
 N

 N

 0
 $U (0 = \sim$
 $) u0$

éq 3.1.1-5
while posing

\sim
 \sim
 N

 N

 \sim
 \sim
 \sim
 $U = U,$
 $X N$
 $, T = T,$
 $X N$
 $, S N = - I X$
 $,$
 \sim

,
0
I()TN
FN = GN = HN = IN = N =

NR

NR

in = I(
XTN)
N

, = (
XTN)
N

, = (
XTN)
N

, H =
H,
XN

NR

N

N

N

=,
XN
, = X, N,

T =
TX, N

NR

NR

NR

éq 3.1.1-6

By applying the theorem of Green to [éq 3.1.1-5], [éq 3.1.1-6] and by introducing the notations following

+

-

=

X, (N +)

1

and = **X**, N with {U, T, H, T, T

0

-1

ext.

}

N

NR

NR

NR

+

-

=

X, T **X**,

(N +) 1 and =, **xT**, **xn**

with

{, I}

NR

NR

one is brought to solve the following variational problem

\pm

$\pm \pm \pm \pm$

\pm

\pm

-

Being donn

és, I, H, T and U

+

*To calculate U V
that*

such

éq

3.1.1-7

0

v

+

V

has

0

$(+u, v) = \pm l(v)$

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with the bilinear form depending on the current moment

l

+

has (+

U, v)

+

+

=

+

U v dx +

+

+

+

T U + U

. v dx +

T T

T

+
 + +
 $H U v D - I +$
 $U v$
 $D + 4 + +$
 (+
 $T +$
 15

.
 273
) 3 +uv
 D
 T
 3
 4
 4

éq 3.1.1-8

and the linear form parameterized by the moments running and precedent
1

\pm
 $L(v)$
 -
 -
 =
 -
 $U v dx + (-)$
 I
 - -
 -
 -
 $T U + U$
 $. v dx - I$

$II(+)$

-
 $T - T) v dx$
 $T T$
 T

 T

+ (-)
 $I - -$
 $H U v D$

3
+ (
 I
 $I -)$
-

-
 $U v D + ($
 $4 -)$
 $I - -$
(-
 $T +$
15

.
273
) 3 - $U v$
 D
 T
4
5

éq 3.1.1-9

Note:

· While posing in [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9]

±
±
=

 $C,$
=
±

$0, = \text{and}$

$4 = 5 =$
 T
 p
 T

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] of the linear problem.

In addition, by deriving the variational formulation [éq 4.2-1] from the problem in temperature [R5.02.02] one finds well [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9].

· Contrairement to the initial problem, the unknown field and the function test belong to even functional space, which is more comfortable from a theoretical point of view and numerical.

· This time the two members of this equation are fundamentally different from those from problem in temperature. However, after each step of time, once determined T_+ with to start from T , one does not have to reassemble all the matrix of the linear system and his second associate member. It is enough to supplement the first tangent matrix (by taking them again notations of [R5.02.02]) of the step of next time (allowing to pass from $+$

$++$
 $T = T$ with $++$

$T)$

1

2

by the term due to the non-linearity of thermal conductivity. One also leaves the second member of the problem in temperature L ($- \pm$

$T, T)$ to constitute that which interests us: one

fabric by the terms of implicitation of non-linearities of the thermal conductivity and of limiting conditions.

· Concerning the initialization of the problem derived the remarks from the linear case apply in extenso (cf [§2.2.2]).

· Contrairement to calculation in temperature (at the time of the predictive phase of **THE_NON_LINE**,

the elimination of this constant imposes a suitable reformulation of the elementary term

CHAR_THER_EVOLNI), the constant of integration of the enthalpy does not appear here because one handle that its derivative in temperature.

· For a condition of exchange between walls, the term of usual exchange is of course replaced by (by taking again the notations of [éq 2-2]) \pm

$H (\pm$

\pm

U

$I - U J) \vee D$

ij

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*Space discretization of [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] provides the **variational problem discretized***

±

± ± ± ±

±

±

-

Being donn

és, I, H, T and U

H

H

H

H

H

H

H

H

+

To calculate U H

V

*that**such**éq***3.1.1-10**

H

0

v V

has U, v

L v

H

H

+

0

H (+

H

H) = ±

H (H)

The taking into account of the conditions of Dirichlet leads then to the dualized linear system [éq 2.1.1-11].

We now will see how these calculations are declined in the code.

3.1.2 Establishment in Code_Aster

With the same notations that with [§2.1.1] one can break up the elementary matrix into

WITH K**WITH K***éq***3.1.2-1**

ij (

, G)

5

= $\sum_{i=1}^5 ij (, G)$ *i=1**with*

· the term of mass and thermal rigidity (option of calculation MTAN_RIGI_MASS by estimating them

- characteristics material in T)

I

+

**WITH K, =
NR NR + + NR**

. NR

ij (
) G

G
(G) J (G) I (G) G (G) J (G) I (G)

T

T

· a term of thermal rigidity due to the non-linearity of thermal conductivity (not introduced because one supposes, in the perimeter of use of the sensitivities, which is independent of T)

2

+

**WITH K, =
T + NR. NR**

ij (
G)

G
(G)

(G) J (G) I (G)
T

· the term of rigidity due to the conditions limit of exchange (MTAN_THER_COEF_F/R)

A3 K

,

=

+

3

H
K

NR

3

NR

ij (
G)

G
(
) $J(G) I(G)$

In the event of exchange between walls this term is replaced by (RIGI_THER_PARO_F/R)
 $A_3 K$

,
 3
 $= h + K$
 NR
 3
 $- NR$
 NR
 ij (
 G)
 G
(
) $(J(G) F J(G) I(G))$
()

by noting F the bijection putting in opposite the two walls.

· the term of rigidity due to the condition of non-linear Neumann (MTAN_THER_FLUXNL in
-
considering flow non-linear in T)

4
 I
 $+$
With
 K

,
 4
 $= -$
 $NR NR$
 ij (
 G)
 G
 $(G) J(G) I(G)$
 T

· the term of rigidity due to the condition of radiation (MTAN_THER_RAYO_F/R)
 $A_5 K$

,
= 4
+
+
+ 273.15

5

K

T

5

3 *NR NR*

ij (

G)

G

() (

) (*G*)

) *J* (*G*) *I* (*G*)

The second member is written, with the same notations,

L K

L K

éq

3.1.2-2

J (

, *G*) 6

= *ij* (, *G*)

i=1

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where

· the term resulting from the implicitation of the matrix of rigidity and mass (new option CHAR_SENS_EVOLNI copies CHAR_THER_EVOLNI with U instead of T, the fields derived and standard material and the new source term)

1
 -
 $L K, =$
 $U NR + -I - U$
 $. NR$
 $J($
 $) G$
 G
 $(G) (G) J (G) G () (G) (G) J (G)$
 T
 T

· a term resulting from the implicitation of the non-linearity of thermal conductivity (not introduced because one supposes, in the perimeter of use of the sensitivities, which is independent T)
 2

-
 $L K, = -I$
 $U T -. NR$
 $J($
 $G)$
 $G ($
 $)$
 $(G) (G)$
 $(G) I (G)$
 T

· the term due to the “new source” (cf CHAR_SENS_EVOLNI)

$L_3 K, = -$
 $I K T + - T -$
 NR

$J($
 $)$
 G
 G
 $I($
 $)(G)$
 $(G) J(G)$
 T

· the term resulting from the implicitation of the conditions limit of exchange (CHAR_THER_TEXT_F/R with Text=0 and U instead of T)

$L_4 K$

,

3
 $= - H K$

U
 3
 NR

$J($
 $G)$
 $G($
 $)($
 $)(G) J(G)$

In the event of exchange between walls this term is replaced by (CHAR_THER_PARO_F/R with U with place of T)

$L_4 K$

,

3
 $= - H K$

U
 3
 $- U F$

NR
 $J($
 $G)$

G (
)(
)((G) ((G)) J (G)

· the term resulting from the implicitation of the condition of non-linear Neumann (new option CHAR_SENS_FLUNL)

5
I

L
K

,

4

= I-
U NR

J(
G)

G (
)

(G) (G) J (G)
T

· the term resulting from the implicitation of the condition of radiation (new option CHAR_SENS_RAYO_F)

L6 K

,

4

1

273.15

5

= -
- K

T -

5

+

3 U NR

J(
G)

G (

)() (

)((G)

) (G) J (G)

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Note:

*· The elementary terms of the matrix of the “derived” problem are identical to those of stamp tangent initial problem. Indeed the problem in temperature can be written in revealing a vector residue **R** [R5.02.02] and a vectorial function test **V***

T

***V. R** (+ +*

T, T) = T

***V. L** (- ±*

T, T)

While deriving compared to a parameter from calculation from the residue (for example a characteristic material) noted this vectorial relation and by translating it in indicielle form, one has

(+

V R =

V L±

K

K)

(K K)

R

+ *T*
+
L
±
K
J
K
V
= *V*

K
K
T

{*J*
Kkj

T

+

+

+

L

+

K T

=

with *V* =

ij (

) *J*

I

(

K

ki)

4 4

3

2

1

With

{ {

ij
U +
L
J
I

One thus finds well the indicielle formulation of the derived linear system [éq 2.1.1-10].

· Compared to the “derived” problem linear, the elementary matrix and the second member are supplemented by the terms incorporating non-linearities of the thermal conductivity and of limiting conditions. On the other hand, the new source is identical in both cases (one will be able thus mutualiser the option of calculation).

· To take into account a more sophisticated modeling of voluminal heat

C
,
X T
I X,
X T

p (
) = I I () I () (I)
I

in these calculations of sensitivity, it is enough to substitute for the indicating function the new term

C
source
p = I in the definition of the field derived material
I
I

ma_v.
I

As one already specified, all the elementary terms (except the second) of the matrix make the object of an option of calculation and will already have been evaluated for the calculation of ++ T. It thus remains with

2
to estimate the second member by re-using (with a different parameter setting) the options of calculation existing or by introducing news (CHAR_SENS_EVOLNI/FLUNL/RAYO_F). These last redirect towards the same routine of elementary calculation (TE.) that their associated options standards (CHAR_THER_EVOLNI/FLUNL/RAYO_F).

The central character string (SENS instead of THER) joined to a detection of the nullity of the field derived material, makes it possible to parameterize these routines towards one or the other their orientations

possible: calculation of an elementary term derived (new source term) or of a term “spectator” had with the presence of a condition of exchange, radiation or a non-linear flow.

In accordance with the principles of architecture set up in the code to treat calculations of sensitivity [bib6], the assembly and the resolution of [éq 2.1.1-11] are started by the analysis of table of correspondence associated with the significant variable. This feedback is carried out via even succession of orders that with [§2.1.2] in substituent THER_NL with THER in DEF1_MATERIAU and by replacing of course operator THER_LINEAIRE by THER_NON_LINE.

We will unroll the same process for the various sensitivities, to start with that concerning the other characteristic material.

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3.2

Derived compared to thermal conductivity

3.2.1 Elements

theoretical

By taking again the notations of [§2.2.1], one carries out derivation compared to the parameter, with

I

T

U =

sought sensitivity,

I

T

C

P

T

C

p

T

U

(C up)

=

U

+ *C*

=

T

T

P

T

T

I

(T)

(T)

U

T

T

=

T

I

The derivation of [éq 3-2] delivers to us a problem in extreme cases identical to [éq 3.1.1-1] but with one

another voluminal source and of new conditions of Robin

~

~

~

~

$$S = \text{div} (I T,$$

$$= = = \sim = -$$

éq

3.2.1-1

I

)

T

GH

I

II N

It is a homogeneous problem of Dirichlet-Cauchy and inhomogenous Robin. One can thus take back same remarks concerning the linear character of the derived problem and simplifications theoretical and numerical that that implies. Its semi-discretization in time results in seeking one continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6] of which the first relation

N)

V

OnN

0

rewrites itself

N I

+

N

n+

I

U

-

one

N I

T

T

+

- *div*

N I

+

N I

+

N I

+

N I

+

U T

+ *U*

T

T

éq

3.2.1-2

N

- (

I-)

N

N

N

N

~

div

N I

U T + U =

+

S +

N

(-)~

I

S

0 N NR -1

T

with the new source term

$$\sim sm = \text{div} (I T m$$

éq

3.2.1-3

I

) m {, nn+} I

and new limiting conditions

I

+

~

~ I

+I

~ +I

~ +I

+

N

N

N

N

T N

G

= H

= I

=

= - I

éq

3.2.1-4

I

N

From where a variational problem identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose only third integral is modified for to adapt to the new source

±

$$L (v) = L - I$$

1

éq

3.2.1-5

I (

+

T + (-) -

T) vdxL

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Note:

· While posing in the new variational problem [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.2.1-5]

±

±

=

C,

=

±

0, = and

4 = 5 =

T
P
T

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] + [éq 2.2.1-5] of linear problem. In addition, by deriving the variational formulation [éq 4.2-1] from problem in temperature [R5.02.02] one finds well [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.2.1-5].

The new source term is identical into linear and non-linear.

· For the moment, operator **DEFI_MATERIAU** does not make it possible to model a conductivity orthotropic non-linear thermics. There are not thus the particular cases of the linear problem.

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

3.2.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified. The same one is taken L3 K, that for the linear case (cf [éq 2.2.2-1]). What is made in the new option of calculation

J (

G)

CHAR_SENS_EVOLNI with the fields derived and standard material.

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via the same one succession of orders that with [§2.1.2] in substituent **THER_NL/LAMBDA** with **THER/RHO_CP** in **DEFI_MATERIAU** and by replacing of course operator **THER_LINEAIRE** by **THER_NON_LINE**.

Note:

· To take into account a more sophisticated modeling of thermal conductivity

(,

X T) = I X,

X T

I

I ()

I (

) (I)

I

in these calculations of sensitivity, it is enough to substitute for the indicating function the new term

source

= I in the definition of the field derived material

I

I

ma_v.

I

3.3

Derived compared to the source

3.3.1 Elements

theoretical

By applying the formulas of the preceding paragraphs, the derivation of [éq 3-2] compared to parameter S (cf [§2.3.1]) delivers to us a problem in extreme cases identical to [éq 3.1.1-1] but with one

I

another voluminal source

s~ = I

éq

3.3.1-1

I

One can thus renew the same remarks concerning the linear character of the derived problem and theoretical and numerical simplifications that that implies. Its semi-discretization in time conduit to seek a continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6]

N)

V

0nN

0

T

whose first relation is rewritten, by noting U = sought sensitivity,

S

I

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N 1

+

N

n+

l

U

-

one

N 1

T

T

+

- *div*

N 1

+

N 1

+

N 1

+

N 1

+

U T
 + U
 T
 T

éq 3.3.1-2

N
 - (

1 -) div
 unT N + N
 one = II 0 N NR -1

T

The application of the theorem of Green to [éq 3.3.1-2] led to solve a **variational problem identical to [éq 3.1.1-7]** comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose only **third integral is modified** to adapt to the new source

±
 $L(v) = L + I v dx$

éq 3.3.1-3

I
 L

Note:

· While posing in the new variational problem [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.3.1-3]

±

±

=

C,

=

±

0, = and

4 = 5 =

*T**p**T*

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] + [éq 2.3.1-3] of linear problem. In addition, by deriving the variational formulation [éq 4.2-1] from problem in temperature [R5.02.02] one finds well [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.3.1-3].

· The new source term is identical into linear and non-linear.

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

3.3.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified. The same one is taken L3 K, that for the linear case (cf [éq 2.3.2-1]) and the same option of calculation.

*J (**G)*

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via the same one succession of orders that with [§2.3.2] by replacing operator THER_LINEAIRE by THER_NON_LINE.

Note:

The taking into account of more sophisticated modeling of the source term is carried out as in linear.

3.4

Derived compared to the imposed temperature

3.4.1 Elements

theoretical

By applying the formulas of the preceding paragraphs, the derivation of [éq 3-2] compared to parameter F (cf [§2.4.1]) delivers to us a problem in extreme cases identical to [éq 3.1.1-1] but with one

I

null voluminal source and another condition of Dirichlet

*s~ =**~**0 and**F = I*

éq
3.4.1-1
I
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It is an inhomogenous problem of Dirichlet-Robin and homogeneous Cauchy. One can thus take back same remarks concerning the linear character of the derived problem and simplifications theoretical and numerical that that implies. Its semi-discretization in time results in seeking one continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6] of which the first relation

N)

V

0nN

I

T

rewrites itself, by noting U = sought sensitivity,

F

I

N I

+

N

n+

l

U

-

one

N 1

T

T

+

- *div*

N 1

+

N 1

+

N 1

+

N 1

+

U T

+
U

T

T

éq

3.4.1-2

N

- (

l -) div

unT N + N

a =0 0 N NR -1

T

with the new limiting condition

~n

$$F + 1 = I$$

éq

3.4.1-3

I

The application of the theorem of Green to [éq 3.4.1-2], [éq 3.4.1-3] resulted in solving a problem variational identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the form linear [éq 3.1.1-9] whose only third integral is modified to adapt to the news source: here this integral is null.

Note:

• While posing in the new variational problem [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + “null source” + [éq 3.4.1-3]

±

±

=

C,

=

±

0, = and

4 = 5 =

T

P

T

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] + [éq 2.4.1-2], [éq 2.4.1-3], [éq 2.4.1-4] of the linear problem. In addition, by deriving the formulation variational [éq 4.2-1] of the problem in temperature [R5.02.02] one finds [éq 3.1.1- well 7], [éq 3.1.1-8], [éq 3.1.1-9] + “null source” + [éq 3.4.1-3].

• The new source term is identical into linear and non-linear.

• One finds the same linear system not dualized as for the derivative in enthalpy. Only taken into account of the condition of Dirichlet via of Lagranges will make the difference.

The space discretization and the taking into account of the inhomogenous condition of Dirichlet lead to

dualized linear system [éq 2.4.1-5]. Components of its second associate member with Lagranges are nonnull.

3.4.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified. The same one is taken

3

L K

that for the linear case (cf [§2.4.2]) and the same option of calculation to assemble it

J (

, G) = 0

dualized linear system associated the “derived” conditions of Dirichlet.

*The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via the same one succession of orders that with [§2.4.2] by replacing operator **HER_LINEAIRE** by **HER_NON_LINE**.*

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Note:

- Concerning various modelings of the imposed temperature and the conditions of Dirichlet generalized, there is no difference between the non-linear one and the linear one. For more information one can thus refer to [§2.4.2].*
- The taking into account of more sophisticated modeling of the imposed temperature is carried out as into linear.*

3.5

Derived compared to linear imposed normal flow

3.5.1 Elements

theoretical

By applying the formulas of the preceding paragraphs, the derivation of [éq 3-2] compared to parameter G (cf [§2.5.1]) delivers to us a problem in extreme cases identical to [éq 3.1.1-1] but with one

*I
null voluminal source and another condition of Robin on*

*2
s~ =
and*

*g~
0*

= I

éq

3.5.1-1

I

It is a homogeneous problem of Dirichlet-Cauchy and inhomogenous Robin. One can thus take back same remarks concerning the linear character of the derived problem and simplifications theoretical and numerical that that implies. Its semi-discretization in time results in seeking one continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6] of which the first relation

N)

V

0nN

0

T

*rewrites itself, by noting U =
sought sensitivity,*

G

I

N I

+

N

n+

I

U

-

one

N I

T

T

+

- div

N I

+

N I

+

N I

+

N I

+

U T

+

U

T

T

éq

3.5.1-2

N

- (

1 -) div

unT N + N

a =0 0 N NR -1

T

with the new limiting condition

~n

G +1 = I

éq

3.5.1-3

I

From where a variational problem identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose only third integral is modified for to adapt to the new "surface" source

±

$$L(v) = L + I v dx$$

éq

3.5.1-4

I

L

2

Note:

· While posing in the new variational problem [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.5.1-4]

±

±

=

C,

=

±

0, = and

4 = 5 =

T

p

T

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] + [éq 2.5.1-4] of linear problem. In addition, by deriving the variational formulation [éq 4.2-1] from problem in temperature [R5.02.02] one finds well [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.5.1-4].

· The new source term is identical into linear and non-linear.

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to

dualized linear system [éq 2.1.1-11].

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3.5.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified. The same one is taken
L3 K

, that for the linear case (cf [§2.5.2]) and the same option of calculation.

2

J (

G)

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via the same one succession of orders that with [§2.5.2] by replacing operator THER_LINEAIRE by THER_NON_LINE.

Note:

- Concerning various modelings of this condition of linear Neumann it does not have there no difference between the linear and non-linear derived problem. For more information one can thus refer to [§2.5.2].*
- The taking into account of more sophisticated modeling of imposed normal flow is carried out as into linear.*

3.6

Derived compared to non-linear imposed normal flow

3.6.1 Elements

theoretical

T

One poses $I(X) = I I X$

I

and $U =$

required sensitivity. Portions of border

I
I ()
(I)

I
II
I

*external being solidified, one has
= I the indicating function of the ième portion. By applying them*

4 J
I
I
4i
I

formulas of the preceding paragraphs, the derivation of [éq 3-2] compared to parameter I delivers to us

I
a problem in extreme cases identical to [éq 3.1.1-1] but with a null voluminal source and another condition of Robin on

4
 $s \sim =$
 \sim
0 and I = I

éq
3.6.1-1

I
It is a homogeneous problem of Dirichlet-Cauchy and inhomogenous Robin. One can thus take back same remarks concerning the linear character of the derived problem and simplifications theoretical and numerical that that implies. Its semi-discretization in time results in seeking one continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6] of which the first relation

N)
V
0nN
0

rewrites itself

N I
+
N

n+

I
U
-
one
N I

T
T
+

- div
N I
+
N I
+
N I
+
N I
+

U T
+ U
T
T

éq
3.6.1-2
N
- (

I -) div
unT N + N
a =0 0 N NR -1

T

with the new limiting condition
~n
I +1 = I

éq
3.6.1-3
I
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From where a variational problem identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose only third integral is modified for to adapt to the new “surface” source

±
 $L(v) = L + I v dx$

éq
3.6.1-4
I
L
4

Note:

· *While posing in the new variational problem [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.6.1-4]*

±

±

±

±

=
I
G
C,
= 0, = ,
=
±
0, I =,
and

4 = 2
5 =
T
p
T
T
2

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] + [éq 2.5.1-4] of linear problem. In addition, by deriving the variational formulation [éq 4.2-1] from problem in temperature [R5.02.02] one finds well [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.6.1-4].

- *This new source term is identical, except for the borders, at the end source of the derivative compared to linear normal flow.*
- *In stationary regime, it is of course unchanged.*

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

3.6.2 Establishment in Code Aster

Compared to [§3.1.2], only the term due to the new source is to be modified. The same one is taken L3 K

, that for the linear case by replacing the border by (cf [éq 2.5.2-1]). What

4
J (
G)
2
4

is made in the new option of calculation CHAR_SENS_FLUNL with the field derived non-linear flow.

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via the same one

succession of orders that with [§2.5.2] in substituent FLUX_NL with FLUX_REP in AFFE_CHAR_THER_F and by replacing of course operator THER_LINEAIRE by THER_NON_LINE.

Note:

To take into account a truly non-linear modeling

$$I(XT) = IIX, \\ XT \\ I$$

$$I \\ I() \\ I(\\)(I)$$

in these calculations of sensitivity, it is enough to substitute for the indicating function the new term

source = I in the definition of the derived loading

chth_v. \\ II

3.7 Derived compared to the coefficient from convectif exchange

3.7.1 Elements theoretical

By applying the formulas of the preceding paragraphs, the derivation of [éq 3-2] compared to parameter H (cf [§2.6.1]) delivers to us a problem in extreme cases identical to [éq 3.1.1-1] but with one

I null voluminal source and another condition of Robin on

$$3 \\ s\sim = \\ \sim \\ 0 \text{ and } H = I$$

- \\ éq

3.7.1-1

I (T

T

ext.

)

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It is a homogeneous problem of Dirichlet-Cauchy and inhomogenous Robin. One can thus take back same remarks concerning the linear character of the derived problem and simplifications theoretical and numerical that that implies. Its semi-discretization in time results in seeking one continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6] of which the first relation

N)

V

OnN

0

T

rewrites itself, by noting U =

sought sensitivity,

H

I

N I

+

N

$n+$

l

U

-

one

$N l$

T

T

+

- *div*

$N l$

+

$N l$

+

$N l$

+

$N l$

+

$U T$

+ U

T

T

éq

3.7.1-2

N

- (

$l -) div$

$unT N + N$

$a = 0 0 N NR -1$

T

with the new limiting condition

$\sim n I +$

H

$= I T$

T

éq

3.7.1-3

$I(N I$

$+$

$N I$

$+$

$ext.$

$-$

)

From where a variational problem identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose *only third integral is modified* for to adapt to the new source

\pm

$L(v) = L + I$

éq

3.7.1-4

$I \{ (+$

$+$

$Text - T) + (I -) (-$

$-$

$Text - T) \} v dx L$

3

Note:

· While posing in the new variational problem [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.7.1-4]

\pm

\pm

$=$

$C,$

$=$

\pm

$0, = and$

4 = 5 =
T
P
T

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] + [éq 2.6.1-4] of linear problem. In addition, by deriving the variational formulation [éq 4.2-1] from problem in temperature [R5.02.02] one finds well [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.7.1-4].

· The new source term is identical into linear and non-linear.

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

3.7.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified. The same one is taken L3 K

,
that for the linear case (cf [éq 2.6.2-1]) and the same option of calculation.

3
J (
G)

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via the same one succession of orders that with [§2.6.2] by replacing operator THER_LINEAIRE by THER_NON_LINE.

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Note:

- *The taking into account of more sophisticated modeling of the coefficient of exchange is carried out as into linear.*
- *The condition of imposed normal flow non-linear (cf [§3.6]) makes it possible to model an exchange convectif with a non-linear coefficient of exchange via an adequate DEF1_FONCTION*

I (,
X T) = J (,
X T) (T

,
X
-
,
X

ext. (
T) T (T)
T

Same manner, knowing U =
one could have access “easily” to

I
I
T

W =
. Indeed,
J
I

T T I
I T
=

+
J
I J

T J

{

{

T

T

- J

ext. -

from where

U (T - T)

W

ext.

= (

l+ju)

3.8

Derived compared to the outside temperature

3.8.1 Elements

theoretical

By applying the formulas of the preceding paragraphs, the derivation of [éq 3-2] compared to parameter I

T (cf [§2.7.1]) delivers to us a problem in extreme cases identical to [éq 3.1.1-1] but with one ext.

null voluminal source and another condition of Robin on

3

s~ =

~

0 and H = hI

éq

3.8.1-1

I

It is a homogeneous problem of Dirichlet-Cauchy and inhomogenous Robin. One can thus take back same remarks concerning the linear character of the derived problem and simplifications

theoretical and numerical that that implies. Its semi-discretization in time results in seeking one continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6] of which the first relation

N)

V

OnN

0

T

***rewrites itself, by noting U =
sought sensitivity,***

I

T

ext.

N I

+

N

n+

1

U

-

one

N I

T

T

+

- div

N I

+

N I

+

N I

+

N I

+

UT

+ U

T

T

éq

3.8.1-2

N
- (

1 -) div
unT N + N
a =0 0 N NR -1

T

with the new limiting condition

~n 1+
N
H
= H 1+I
éq
3.8.1-3

I
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From where a variational problem identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose only third integral is modified for to adapt to the new “surface” source

±

$$L(v) = L + I$$

éq

3.8.1-4

I {

+

 $H + (1 -) -$ $H\} v dx L$

3

Note:

• While posing in the new variational problem [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.8.1-4]

±

±

=

C,

=

±

0, = and

4 = 5 =

T

p

T

one finds well the formulation [éq 2.1.1-6], [éq 2.1.1-7], [éq 2.1.1-8] + [éq 2.6.1-4] of linear problem. In addition, by deriving the variational formulation [éq 4.2-1] from problem in temperature [R5.02.02] one finds well [éq 3.1.1-7], [éq 3.1.1-8], [éq 3.1.1-9] + [éq 3.8.1-4].

• The new source term is identical into linear and non-linear.

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to

dualized linear system [éq 2.1.1-11].

3.8.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified. The same one is taken

L3 K

,
that for the linear case (cf [éq 2.7.2-1]) and the same option of calculation.

3

J (

G)

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via the same one succession of orders that with [§2.6.2] in substituent TEMP_EXT with COEF_H in AFFE_CHAR_THER_F and while replacing, of course, operator THER_LINEAIRE by THER_NON_LINE.

Note:

The taking into account of a more sophisticated modeling of the outside temperature be carried out as into linear.

3.9

Derived compared to emissivity/constant of Stefan-Boltzmann

3.9.1 Elements

theoretical

One poses $(X) = I X$

(resp. $(X) = I X$

) the parameter and

I

I ()

(I)

I

I ()

(I)

I

I

T

T

U =

(resp. U =

) required sensitivity. The portions being solidified, one has

= I

5 J

I

I

I

I

(resp. = I) the indicating function of the ième portion. By applying the formulas of

I

5i

I

preceding paragraphs, the derivation of [éq 3-2] delivers a problem in extreme cases identical to us to [éq 3.1.1-3] but with another voluminal source and another condition of Robin on (resp. in

5

inverting the role of and of)

~

~

S = 0 and I = I

éq

3.9.1-1

I

{ T + 273.15) 4 - (T + 273.15) 4}

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One can thus renew the same remarks concerning the linear character of the derived problem and theoretical and numerical simplifications that that implies. Its semi-discretization in led time to seek a continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6] of which

N)

V

0nN

0

first relation is rewritten

N I

+

N

n+

1

U

-

one

N I

T

T

+

- div

N I

+

N I

+

N I

+

N I

+

U T
+ U
T
T

éq
3.9.1-2
N
- (

1 -) div
unT N + N
one = 0 0 N NR -1

T

with the new condition (resp limits. by inverting the role of and of)

~ N I

+

N I

I

=

+

I

T

T

éq
3.9.1-3

I

{n+

N

+

15

.

273

) 4

I

- (+ +

15

·
273
) 4
1
}

From where a variational problem identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose only third integral is modified for to adapt to the new “surface” source (resp. by inverting the role of and of)

$$L \pm (v) = L + I$$

+
v
4
4
+

+ 273.15
- + + 273.15 -
I
({T
) (T
) }
5

éq 3.9.1-4
I
4
4
-1 -
-

+ 273.15
- - + 273.15
I (
) v ({T
) (T
) } dx

Note:

· *Derivation compared to the constant of Stefan-Boltzmann has certainly only one interest practical minor. But its overcost of numerical establishment was moderate and it allows cross validations.*

· *In stationary regime, this source term is reduced to (resp. by inverting the role of and of)*

$$L(v) = L + I$$

$$I$$

$$\left(\begin{array}{c} T + \\ 4 \\ 15 \\ \cdot \\ 273 \\ - T + \\ 4 \end{array} \right) \left(\begin{array}{c} 15 \\ \cdot \\ 273 \end{array} \right) v dx L$$

$$5$$

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

3.9.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified (resp. by inverting it role of and of)

L3 K

$$,$$

$$=$$

$$+$$

$$+$$

$$+$$

$$15$$

$$\cdot$$

$$273$$

$$- +$$

$$+$$

$$15$$

$$\cdot$$

$$273$$

$$+$$

$$5$$

I
K
T
K
4
T
5
5

4 NR
J (
G)
G (I
) (
) ((
)
) ((G)
) J (G)
(1 -) I - K

T - K
4

+
15
.
273
- T -
+
15
.
273
5
5

4 NR
G (I
) (
) ((
)
) ((G)
) J (G)
éq 3.9.2-1

What is made in the new option of calculation CHAR_SENS_RAYO_F with the fields of

standard and derived radiation.

The assembly and the resolution of the linear system are started by the analysis of the table of correspondence associated with the significant variable. This feedback is carried out via succession of orders.

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v = DEFI_VALEUR_SENSI (VALE = < value of I >) (resp. I)

chth = AFFE_CHAR_THER_F (RAYONNEMENT= _F (GROUP_MA = < definition of 5i >, SIGMA = v, EPSILON = W, TEMP_EXT = Z))

...

one = DEFI_CONSTANTE (VALE = 1.)

zero = DEFI_CONSTANTE (VALE = 0.)

MEMO_NON_SENSI (NAME = _F (NOM_SD = "chth", PARA_SENSI = "v", NOM_COMPOSE = "chth_v"))

chth_v = AFFE_CHAR_THER_F (RADIATION = _F (GROUP_MA = < 5i >, SIGMA = one, EPSILON = zero, TEMP_EXT = zero))

...

resu = THER_NON_LINE (EXCIT = chth,

SENSITIVITY

=

(

v)

...)

Note:

· *The essential data of this calculation, the field derived radiation*

=
T

I,
(resp.
= I), is provided by

I
=,
0
= 0

I

chth_y.

I
I
I
I

· *To take into account a more sophisticated modeling of emissivity (resp. constant of Stefan-Boltzmann)*

(,
X T) = I X,
X T

I
I ()
I (
) (I)

I
in these calculations of sensitivity, it is enough to substitute for the indicating function the new term

source

= I in the definition of the derived loading

I

I

chth_v.

I

3.10 Derived compared to the temperature ad infinitum

3.10.1 Theoretical elements

T

T X is posed

I

I

=

() =

T I X

T

the parameter and U

required sensitivity.

I ()

()

I

I

T

T

portions being solidified, one has

= I the indicating function of the ième portion. By applying them

5 J

I

I

T

5i

formulas of the preceding paragraphs, the derivation of [éq 3-2] delivers a problem in extreme cases to us

identical to [éq 3.1.1-3] but with another voluminal source and another condition of Robin on

5

~

~

S = 0 and I = 4I

éq

3.10.1-1

I

(

T +

.

273

) 3

15

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One can thus renew the same remarks concerning the linear character of the derived problem and theoretical and numerical simplifications that that implies. Its semi-discretization in time conduit to seek a continuation (U

checking a system similar to [éq 3.1.1-5], [éq 3.1.1-6]

N)

V

OnN

0

whose first relation is rewritten

N 1

+

N

n+

1

U

-

one

N 1

T

T

+

- div

N I

+

N I

+

N I

+

N I

+

U T

+

T

T

éq

3.10.1-2

N

- (

I -) div

unT N + N

one = 0 0 N NR -1

T

with the new limiting condition

~n I

+

I

= 4I

éq

3.10.1-3

I

(

T + 273.15) 3

From where a variational problem identical to [éq 3.1.1-7] comprising the same bilinear form [éq 3.1.1-8] joined to the linear form [éq 3.1.1-9] whose only third integral is modified for to adapt to the new “surface” source

±

$$L(v) =$$

$$3$$

$$3$$

$$L + 4I$$

$$I\{$$

$$)+ (+$$

$$T +$$

$$15$$

$$\cdot$$

$$273$$

$$+ 1 -$$

$$T +$$

$$)() () - (-$$

$$15$$

$$\cdot$$

$$273$$

$$)\} vdxL$$

$$5$$

éq 3.10.1-4

Note:

In stationary regime, this source term is unchanged.

The space discretization and the taking into account of the homogeneous condition of Dirichlet lead to dualized linear system [éq 2.1.1-11].

3.10.2 Establishment in Code_Aster

Compared to [§3.1.2], only the term due to the new source is to be modified

L3 K

$$,$$

$$= 4$$

+

+

+

+

15

.

273

+

5

I

K

T

K

3 NR

5

5

J (

G)

G

(I

)(

)((

)

) J (G) éq 3.10.2-1

4

1 - I - - K

T - K

3

+

15

.
273

NR

5

5

G (

)(I

)(

)((

)

) J (G)

What is made in the new option of calculation CHAR_SENS_RAYO_F with the fields of standard and derived radiation.

The assembly and the resolution of the linear system are started by the analysis of the table of correspondance associated with the significant variable. This feedback is carried out via the same one succession of orders that with [§3.9.2] in substituent TEMP_EXT with SIGMA.

Note:

Ad infinitum to take into account a more sophisticated modeling of the temperature

T

X T

I
I

(
)=

T I X

,
X T
T

I ()
I (
) ()

in these calculations of sensitivity, it is enough to substitute for the indicating function the new term

T

source
= I in the definition of the derived loading chth_v.

I
I
I
T

We now will recapitulate all the linear systems “derived” to assemble according to desired sensitivities.

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Summary of the sensitivities of the temperature

That one is in or not linear linear thermics, the sensitivity required to the moment running $U+$ is solution of a dualized system of the type (with the notations of [§2.1])

With LT

$LT +$

U

L

$\sim \sim +$

\sim

WITH $U = L$

$B - Id$

Id

$=$

C

$B Id - Id C$

where

WITH $K,$

WITH $K,$

$ij ($

$G) = ij ($

$G)$

I

$L K,$

$L K,$

$J ($

$G) = ij ($
 $G)$

I

One will draw up the nomenclature of the potential elementary terms of this system and of their option of calculation in Code_Aster. In order to be more synthetic, one agrees here on a classification different from those practised in the preceding paragraphs.

For the matrix, there are the seven following possible terms:

· *MASS_THER* (cf [§2.1.2])

$A1 K, =$
 $C K NR NR$

$ij ($
 $) G$
 G
 $p ($
 $) J (G) I (G)$
 T

· *RIGI_THER* (cf [§2.1.2])

$A2 K, = K NR. NR$

$ij ($
 $G)$
 G
 $() J (G) I (G)$

· *RIGI_THER_COEF_F/R or MTAN_THER_COEF_F/R* (cf [§2.1.2], [§3.1.2])

$A3 K$

,

=

+

3

H

K

NR

3

NR

$ij ($

$G)$

G

$($

$) J (G) I (G)$

In the event of exchange between walls this term is replaced by RIGI_THER_PARO_F/R or MTAN_THER_PARO_F/R (cf [§2.1.2], [§3.1.2])

A3 K

,

3

= h+ K

NR

3

- NR

NR

ij (

G)

G

(

) (J (G) F J (G) I (G)

()

· MTAN_RIGI_MASS (cf [§3.1.2])

4

WITH K, =

T +

NR NR + T +

NR

. NR

ij (

) G

G

((G) J (G) I (G) G ((G) J (G) I (G)

T

T

· Pas coded. (cf [§3.1.2])

5

WITH K, =

T +

T + NR. NR

ij (

G)

G

((G)

(G) J (G)

I (G)

T

· MTAN_THER_FLUXNL (cf [§3.1.2])

6

I

With

K

,

4

= -

T +

NR NR

ij (

G)

G

((G) J (G) I (G)

T

· MTAN_THER_RAYO_F/R (cf [§3.1.2])

A7 K

,

= 4

+

+

+ 273.15

5

K

T

5

3 NR NR

ij (

G)

G

() (

) ((G)

) **J (G) I (G)**

For the second member, there are sixteen possibilities:

· **CHAR_SENS_EVOL** (cf [§2.1.2])

**L1 K, =
C K U NR + -1 K U
. NR**

**J (
) G
G
p (
) (G) J (G) () G () (G)
J (G)
T**

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· **CHAR_THER_TEXT_F/R** (cf [§2.1.2])

L2 K

,
1

3

= - *H K*

U

3

NR

J (

G)

G (

) (

) (G) J (G)

In the event of exchange between walls this term is replaced by CHAR_THER_PARO_F/R (cf. [§2.1.2])

L2 K

,

1

3

= - *H K*

U

3

- *U F*

NR

J (

G)

G (

) (

) ((G) ((G)) J (G)

· CHAR_SENS_EVOL (cf [§2.1.2])

L3 K, = -

I K T + - T -

NR

J (

)

G

G

I (

) ((G)

(G) J (G)

T

· CHAR_SENS_EVOL (cf [§2.1.2])

L4 K, =

- I K T + + 1 - T -

NR

J
G
GI
)
(G) () (G)
J (G)
· CHAR_THER_SOUR_F/R (cf [§2.3.2], [§3.3.2])
L5 K, = I K NR

J
G
GI
) J (G)
· CHAR_THER_FLUN_F/R (cf [§2.5.2], [§3.5.2])
L6 K

,
 =

2
IK
NR
 2

J
G
GI
) J (G)
· CHAR_SENS_TEXT_F (cf [§2.6.2], [§3.7.2])

T + K
3 - T
 7

++

L
K
 ,

3
= IK
 3
NR

J (
G)

(ext. (
)

(G)
GI (
)

-
-

(1 -) (T K
3 - T

ext. (
)
(G) J (G)

· CHAR_THER_TEXT_F (cf [§2.7.2], [§3.8.2])
L8 K

,
=

++ I-
-

3
IK
3
HK
3
HK
NR
3

J (
G)
GI (
){ (
)(
)(
)} J (G)

· CHAR_THER_TEXT_F (cf [§2.7.2], [§3.8.2])

9

L K, =
T -
U - NR + - 1 T -
U -

. NR
J (
) G
G
((G) (G) J (G) G () ((G)
(G) J (G)
T
T

· CHAR_SENS_EVOLNI (cf [§3.1.2])
10

L K, = - 1
T -
U - T -. NR

J (
G)
G (
) ((G) (G)
(G)
I (G)
T

· CHAR_SENS_FLUNL (cf [§3.1.2])
11
I

L
K
,

4
= 1-
T -
U - NR
J (
G)

G (
) ((G) (G) J (G)
T

· CHAR_SENS_RAYO_F (cf [§3.1.2])
L12 K

,
4
1
273.15

5
= -
- K
T -

5
+
3 U NR

J (
G)
G (
) () (
) ((G)
) (G) J (G)

· CHAR_SENS_FLUNL (cf [§3.6.2])
L13 K

,
=

4
I K
NR
4

J (
G)
GI (
) J (G)
· CHAR_SENS_RAYO (cf [§3.9.2])

L14 K

,
=
+

+

+

15

.

273

- +

+

15

.

273

+

5

I

K

T

K

4

T

5

5

4 NR

J (

G)

G (I

) (

) ({ (

)

) ((G)

) } J (G)

(1 -) I - K

T - K

4

+

15

.

273

- T -

+

15

.

273

5

5

4 NR

G (I

) (

) ({ (

)

) ((G)

) } J (G)

L15 K

,

=

+

+

+

15

.

273

- +

+

15

.

273

+

5

I

K

T

K

4

T

5

5

4 NR

J (

G)

G (I

) (

) ({ (

)

) ((G)
)} J (G)
(1 -) I - K

T - K
4

+
15

.
273
- T -

+
15

.
273
5
5

4 NR
G (I

)(
) (((

) ((G)
)} J (G)

· CHAR_SENS_RAYO (cf [§3.10.2])
L16 K

,
= 4

+
+
+

+
15

.
273

+
5

I
K
T
K

3 NR

5

5

J (

G)

G

(I

)(

)((

)

) J (G)

4

1 - I - - K

T - K

3

+

15

.

273

NR

5

5

G (

) (I

)(

)((

)

) J (G)

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The operator B allowing to check the condition of Dirichlet is written

*(+
B U) = u+ with I J*

I

I

with its second associate member

C = 0

Note:

· The possible taking into account of limiting conditions of Dirichlet generalized requires to rewrite this operator in the form

*+
B U = u+ with J J*

J

J

J

· As one rigorously does not take into account nonlinear thermal conductivity, the terms A5 and L10 are not programmed yet.

The eighteen dualized systems exhumed in the preceding paragraphs can then gather in the following table:

*Type of
Variable
Stamp
Second
Key word
Characteristics
sensitivity
member
Thermics
Stamp*

THER_LINEAIRE Sensitivity compared to

linear
identical to
a constant parameter
that of
by zone
problem
(possibly derived
direct
composed of [§6.4])
Heat
T

A1+A2+A3
L1+L2+L3
THER/RHO_CP

voluminal
U =

I
Conductivity
T

Idem
L1+L2+L4
THER/LAMBDA
Sensitivity compared to
thermics
U =

a characteristic
I
orthotropic not
accessible
Source
T

Idem
L1+L2+L5
SOURCE
Three types of
U =

modeling

S

I

Temperature

T

Idem

L1+L2

TEMP_IMPO

F

imposed

U =

J

C =

=

F

K

ij

I

F

I

***Three types of
modeling.***

***Sensitivity compared to
a coefficient***

multiplier of one

condition of Dirichlet

generalized not

accessible

Normal flow

T

Idem

L1+L2+L6

FLUX_REP

Three types of

imposed

U =

modeling.

G

I

*Sensitivity compared to
a component of flow
vectorial accessible.*

Exchange

T

Idem

L1+L2+L7

ECHANGE/COEF_H No ECHANGE_PAROI

convectif

U =

or

H

in lumpé.

I

ECHANGE_PAROI

Temperature

T

Idem

L1+L2+L8

EXCHANGE

external

U =

TEMP_EXT

I

T

ext.

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Type of

Variable

Stamp

Second

Key word

Characteristics

sensitivity

member

Thermics

Stamp the new THER_NON_LINE Sensibilité compared to

nonlinear

near to the sources terms

a constant parameter

stamp

are similar

by zone

tangent of into linear and in

(possibly derived

problem

nonlinear.

composed cf [§6.4]).

direct.

No sensitivity by

report/ratio with the enthalpy.

Account is not held

of a conductivity not

linear

A5 and L10 not taken in

count

Heat

T

A3+A4+A5

L2+L3+L9

THER_NL/RHO_CP

voluminal

U =

+A6+A7

+L10+L11+L12

I

Conductivity

T

Idem

L2+L4+L9

Linear THER_NL/LAMBDA Idem case

thermics

U =

+L10+L11+L12

I

Source

T

Idem

L2+L5+L9

SOURCE

Idem linear case

U =

S

+L10+L11+L12

I

Temperature

T

Idem
L2+ L9
TEMP_IMPO
Idem linear case
imposed
U =

F

+L10+L11+L12
I
Normal flow
T

Idem
L2+L6+L9
FLUX_REP
Idem linear case.
imposed
U =

+L10+L11+L12
linear
G
I
Normal flow
T

Idem
L2+L13+L9
FLUX_NL
Allows to model
imposed not
U =

+L10+L11+L12
sensitivity compared to
linear
I
I
a convectif exchange
nonlinear
Exchange
T

Idem
L2+L7+L9
ECHANGE/COEF_H No ECHANGE_PAROI
convectif
U =

+L10+L11+L12
or
in lumpé.
linear
H
I
ECHANGE_PAROI
Temperature
T

Idem
L2+L8+L9
EXCHANGE

external
U =
TEMP_EXT
I
T

+L10+L11+L12
ext.
Emissivity
T

Idem
L2+L15+L9
RADIATION

U =

EPSILON

+L10+L11+L12
I
Constant of

T

Idem

L2+L14+L9

RADIATION

Stefan-

U =

+L10+L11+L12

SIGMA

Boltzmann

I

Temperature with

T

Idem

L2+L16+L9

RADIATION

the infinite one

U =

TEMP_EXT

I

T

+L10+L11+L12

Table 4-1: Summary of the linear systems “derived”

We will now close the theoretical part this document by calculating the sensitivities of one related size with the field of temperature: heat flow.

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Sensitivity of the heat flow

After having carried out a thermal calculation, one has access to the heat flow (calculation is carried out either with nodes (field with the nodes by elements via $OPTION='FLUX_ELNO_TEMP'$), is at the points of Gauss (field at the points of Gauss by elements via $OPTION='FLUX_ELGA_TEMP'$)) via the operators $CALC_ELEM/CALC_NO$. It is given starting from the field of temperature by using the Fourier analysis

who is written in linear thermics with isotropic materials

$$Q(X, T) = - (X) T(X, T)$$

With anisotropic materials, thermal conductivity is modelled by a diagonal matrix (expressed in its reference mark of orthotropism). Into nonlinear (inevitably isotropic in Code_Aster) one

has on the other hand

$$Q(X, T) = - (X, T) T(X, T)$$

By taking again the notations and the step developed in the preceding paragraphs, one synthesize the calculation of derived from the heat flow in the following table. The column entitled "formula" expresses the relation to be set up in operators $CALC_ELEM/CALC_NO$ for to determine the sensitivity of the heat flow required. It depends on the field of temperature and of its sensitivity compared to the same parameter, all the two resulting one from a thermal calculation (via $THER_LINEAIRE$ or $THER_NON_LINE$).

Note:

In practice, calculations of sensitivity of the heat flow do not take account of the possible one non-linearity of thermal conductivity. In non-linear thermics, the first term is not thus not programmed.

Type of

Variables

Sensitivity

Formulate/

sensitivity

of exit of

sought

Options of calculation in Code_Aster

the operator

thermics

Thermics

linear

Heat

T

$$v(X, T) = - (X) U(X, T)$$

voluminal

U =

, T

= Q

v

FLUX_ELGA/NO_TEMP

I

I

Conductivity

T

v (,

X T) = - I X

,

X - X

,

X

I () T (

T)

() U (T)

thermics

U =

, T

= Q

v

FLUX_ELGA/NO_SENS

I

I

Source

T

v (X, T) = - (X) U (X, T)

U =

, T

= Q

v

S

S

FLUX_ELGA/NO_TEMP

I

I

Temperature

T

Idem

imposed

U =

, T

= Q

v

F

F

I

I

Normal flow

T

Idem

imposed

U =

, T

= Q

v

G

G

I

I

Exchange

T

Idem

convectif

U =

, T

= Q

v

H

H

I

I

Temperature

T

Idem

external

U =

, *T*
= *Q*
v

T I

I
T

ext.
ext.

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Type of
Variables
Sensitivity
Formulate/
sensitivity
of exit of
sought

Options of calculation in Code_Aster

the operator

thermics

Thermics

nonlinear

Heat

T

voluminal

U =

, T

= Q

v

v (X, T) = -

(X, T) (U T) (X, T) -

T

I

I

(X, T) U (X, T)

FLUX_ELGA/NO_TEMP

Conductivity

T

thermics

U =

, T

= Q

v

v (,

X T) = -

$(, xT) U (, xt) + I, X, X$

$I ($

$T) T (T)$

I

I

T

$- (,$

$X T) U (,$

$X T)$

FLUX_ELGA/NO_SENS

Source

T

$U =$

$, T$

$= Q$

v

$v (X, T) = -$

$(X, T) (U T) (X, T) -$

S

S

T

I

I

$(X, T) U (X, T)$

FLUX_ELGA/NO_TEMP

Temperature

T

Idem

imposed

$U =$

, T

$= Q$

v

F

F

I

I

Normal flow

T

Idem

imposed

$U =$

, T

$= Q$

v

linear

G

G

I

I

Normal flow

T

Idem

imposed not

$U =$

, T

$= Q$

v

linear

I

I

I
I
Exchange
T

Idem
convectif
U =
, T
= Q
v

linear
H

H

I
I
Temperature
T

Idem
external
U =
, T
= Q
v

TI

I
T

ext.
ext.
Emissivity
T

Idem
U =

, T
= Q
v

I
I
Constant
T

Idem
of Stefan-
U =
, T
= Q
v

Boltzmann

I
I
Temperature
T

Idem
ad infinitum
U =
, T
= Q
v

TI

I

T

Table 5-1 Sensitivities of the heat flow

Note:

While posing in the formulas of nonlinear thermics

$(X, T) = 0$ and $(X, T) = (X)$

T

one finds the formulations of linear thermics well.

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6

Implementation in Code_Aster

6.1 Difficulties

particular

The principal difficulty of these calculations of sensitivity is to detect the presence of parameters sensitive in the standard loadings and materials, and, to associate the fields to them suitable derivatives. To be done, a whole architecture (cf [bib6] and [§6.2], [§6.4]) has being put in place in order to notify to the supervisor the affiliation of a derived field with such or such significant variable. Via

the orders `DEFI_PARA_SENSI` and `MEMO_NOM_SENSI` and their utilities `FORTTRAN` associated, one

can thus make the joint within a thermal operator, between a significant parameter, the field where it intervenes and the associated derived field.

With a particular mention for the materials which are known only in one coded form. One detect the significant characteristic of the aforesaid material by testing the nullity of derived material (a problem related is posed for the characteristics of the radiation).

Inter alia installations, it A was also necessary:

- Insérer, within the thermal operators, a loop on the sensitivities requested, wedged between the temporal loops and those on the loadings.*
- Mettre in place the resolution of a linear system in the non-linear process of THER_NON_LINE.*
- Organiser and to manage the mutualisation of the matrices between the standard problem and it (or them) problem (S) derived (S).*
- Prendre in account, in robust and fast manner, the insertion of possible parameters insensitive (creation of a CHAM_NO of null components for the thermal operators and of a CHAM_ELEM of null components for postprocessings of calculation of flow).*
- Eviter an inopportune proliferation of option of calculations (and their TE000 FORTRAN associated) while mutualisant and “bluffant” existing it (one can visualize the exact advance of calculation (at the macroscopic level of the options) and easy ways deployed to re-use existing the cf [§6.2]).*

Beyond these fastidious developments, a large effort of validation “numérico-data processing” was deployed on all the meshes supports, all modelings, all the loadings, all them types of initialization of the thermal solveurs and for all the sensitivities. These hard tests on small cases model tests (TPLL01A/H for the PLANE 2D and 3D and TPNA01A for the 2D AXIS) are revealed profitable (including for standard thermal calculation in lumpé!) and essential. Because one seldom has theoretical values allowing to validate a thermal calculation complicated: “nothing resembles more one sensitivity... than another value of sensitivity! ”. In crossing with converged finished differences, one thus tried to release a maximum of confidence in all building blocs constituting the derived system.

To be exhaustive on the aspect validation, let us note that several case-tests were delivered (SENST<00> [V1.01.15]) including one analytical (SENST04 cf [§6.4]).*

6.2 Environments necessary/parameter settings

*C
T*

*C
p=10*

$p=v=5$

2

v

1

*Appear 6.2-a Désignation of thermal conductivities for a Bi-material
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*To calculate the sensitivity of the thermal field compared to a constant parameter by zone (here one particular value of conductivity in one of the zones of a Bi-material during a stationary calculation), it is first of all necessary to notify this significant parameter in the command file via one **DEFI_PARA_SENSI***

v = DEFI_PARA_SENSI (VALE = 5.)

as well as the constant functions equal to zeros and the unit which will be used to define the essential one indicating function

zero = DEFI_CONSTANTE (VALE = 0.)

one = DEFI_CONSTANTE (VALE = 1.)

Then, as for a standard problem, one definite the fields material (or the conditions limit) associated. One does not have to modify the other standard loadings.

ma1 = DEFI_MATERIAU (THER = _F (RHO_CP = v))

ma2 = DEFI_MATERIAU (THER = _F (RHO_CP = 10.))

*affe = AFFE_MATERIAU (AFFE = (_F (GROUP_MA = < 1 >, MATER = ma1),
_F (GROUP_MA = < 2 >, MATER = ma2))*

....

other insensitive data material and limiting conditions with respect to v

It is then necessary to define the “derived” fields material (or the conditions limit “derived”) and only those concerned with this derivation. It is them which will provide information essential with the assembly of the derived problem: the indicating function of the derived field CP 1 on

I =

=

1 .

1

v

0 out of 2

One notifies to the supervisor their affiliation with the initial field via order MEMO_NON_SENSI (it is it

who filled a structure of data suitable sensitivity allowing to make the joint, within thermal operators, between the significant parameter, the field where it intervenes and the derived field

associated)

MEMO_NON_SENSI (NAME = _F (NOM_SD = “my”,

PARA_SENSI = “v”,

NOM_COMPOSE = “ma_v”))

ma_v = DEFI_MATERIAU (THER = _F (RHO_CP = one))

mazero = DEFI_MATERIAU (THER = _F (RHO_CP = zero))

MEMO_NON_SENSI (NAME = _F (NOM_SD = “affe”,

PARA_SENSI = “v”,

NOM_COMPOSE = “affe_v”))

affe_v = AFFE_MATERIAU (AFFE = (_F (GROUP_MA = < 1 >, MATER = ma_v),

_F (GROUP_MA = < 2 >, MATER = mazero))

One concludes by requesting from the thermal operator the aforementioned calculation via the key word sensitivity and notifying to supervisor the presence of a field sensitivity in the EVOL_THER resu (accessible in the field symbolic system resu_v of the SD result)

MEMO_NON_SENSI (**NAME** = **_F** (**NOM_SD** = “**resu**”,
PARA_SENSI = “**v**”,
NOM_COMPOSE = “**resu_v**”))
resu = **THER_LINEAIRE** (**CHAM_MATER** = **affe**,

....

SENSITIVITY = (**v**)

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*One can of course make same postprocessings on the derived field as on the standard field, it is enough to add with the suitable orders (**CALC_ELEM** for the calculation of the flow of this **CHAM_ELEM** with nodes, **CALC_NO** for its transformation into a **CHAM_NO**, **POST_RELEVE** for the statement of value, **IMPR_RESU** for the impression and **TEST_RESU** for the comparison of values) the key word **SENSITIVITY** and to inform it good value of parameter of derivation. The operator the good field sensitivity dividing will select then the same **SD** as the standard field. By example, one filled here the table **rlresuv** with certain components of derived from **T** compared to **v***

rlresuv = **POST_RELEVE_T** (**ACTION**=**_F** (**RESULTAT**=**resu**,

....

SENSITIVITY = (**v**)

By printing the table one can then visualize the components of the sensitivity specifically wished.

**ASTER 6.02.24 CONCEPT RLRESUV CALCULATES THE 27/03/2002 A 14:56: 43 OF TYPE
 TABL_POST_RELE**
ENTITLE NODE RESU NOM_CHAM PAR_SENS NUME_ORDRE INST COOR_X/Y/Z TEMP
TEMPERAT NO1 RLRESUV TEMP v 0 0. 0. 1.1 -1.5 0. 1.69334E+01
TEMPERAT NO2 RLRESUV TEMP v 0 0. 0. 1.1 -1.0 0. 1.88532E+01
TEMPERAT NO3 RLRESUV TEMP v 0 0. 0. 1.1 -0.5 0. 1.19433E+01
TEMPERAT NO4 RLRESUV TEMP v 0 0. 0. 1.1 -0.0 0. 2.15978E+01

Example 6.2-1: Layout, via IMPR_TABLE, in the file result

For further information, the reader will be able to consult [bib6], [U4.50.02] or the case-tests of type SENST<00> [V1.01.15].*

Note:

- *By parameterizing advisedly the loadings and materials “derived” in the file from order, one can also have access to some derivatives made up. Cf the remark of [§6.3].*
- *The syntax of the specific orders of sensitivity (DEFI_PARA_SENSI, MEMO_NOM_SENSI) can be brought to change according to the architectural choices which will be taken (automation of the process of detection and parameter setting of the fields derived by supervisor or not). Methodology will not be on the other hand modified and one will always be able to pass in “manual”. The user then remaining only judge of the relevance of these calculations “except limits”.*
- *During the construction of the derived fields, in AFFE_MATERIAU or AFFE_CHAR_THER, one need does not have to specify the “nullity” of the insensitive fields. They are initialized by defect to zero.*

To be completely exhaustive on the parameter setting, let us conclude by a functionality which can to interest of future mainteneurs/developers or the “pointed” users. Unfolding exact of calculation (options calculated with their field IN and OUT, loadings and materials taken in count...) as well as the easy ways deployed to re-use existing it are traced in the file message if one notifies INFO=2 in the accused thermal solvor and in CALC_ELEM (for options FLUX_ELGA/NO_TEMP).

THERMAL CALCULATION OF SECOND MEMBER: NXACMV

TYPESE/INST: 3 5.000000000000

LINEAR CALCULATION: F

.....

--> COMPLEMENTARY CALCULATION IN SENSITIVITY

--> ***BLUFF OF THE OPTION: T EAST REPLACES BY (DT/DS) -***

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--> AND ADDITION Of a NEW SOURCE TERM

--> OPTION: CHAR_SENS_EVOLNI

LPAIN/LCHIN: PGEOMER EMAIL .COORDO

LPAIN/LCHIN: PTEMPER &&OP0186VAR____001

.....

NO. LOADS: 6

BUCKLE ON THE LOADS OF THE TYPE NEUMANN FLAX

CHARGE: CH3

EXICHA/NOMCHS: 1?????????

CHARGE: CH4

EXICHA/NOMCHS: 1?????????

--> COMPLEMENTARY CALCULATION IN SENSITIVITY

--> ECHANGE/NUMCHM: 2

--> BLUFF OF THE OPTION: NULL CHART TEXT+/-

--> K: 1

--> OPTION: CHAR_THER_TEXT_F

LPAIN/LCHIN: PT_EXTF &&VECHTH.T_EXTNUL

LPAIN/LCHIN: PGEOMER EMAIL .COORDO

**Example 6.2-2: Layout of THER_LINAIRE or THER_NON_LINE,
via INFO=2, in the file message.**

CALCULATION OF HEAT FLUXES

OPTION OF CALCULATION FLUX_ELGA_TEMP

MODEL MOTH

SD EVOL_THER GIVEN TH

RESULT TH

*MATERIAL TAKEN INTO ACCOUNT CMAT
SEQUENCE NUMBER 1 NUMBERS
A NUMBER OF SIGNIFICANT PARAMETERS 7

*OP0058 *****
INST/IAUX/IORDR 0. 1 0
NRPASS/TYPESE/NOPASE 1 3 PS1
CHTEMP/CHTESETH_PS1 .001.000000/TH .001.000000
--> OPTION: FLUX_ELGA_TEMP
LPAIN/LCHIN: PGEOMER EMAIL .COORDO
LPAIN/LCHIN: PMATERC CMAT .MATE_CODE
LPAIN/LCHIN: PTEMPER TH_PS1 .001.000000
LPAIN/LCHIN: PTEMPSR &&MECHTI.CH_INST_R
LPAIN/LCHIN: PTEREF
LPAIN/LCHIN: PMATSEN CMAT_PS1.MATE_CODE
LPAIN/LCHIN: PTEMSEN TH .001.000000*

*Example 6.2-3: Layout of CALC_ELEM,
via INFO=2, in the file message.
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*6.3 Perimeter
of use*

The perimeter of use of thermal calculations of sensitivities can be formulated in some points:

- 1. Thermal calculations of sensitivity is pressed on operators *THER_LINEAIRE* and *THER_NON_LINE*. They deal with thermal, linear problems or not, isotropic or anisotropic, stationary or transitory.**
- 2. They were not still extended to the problems of drying and hydration which are too treaty by *THER_NON_LINE* (option *THER_HYDR* or *SECH_** of *COMP_THER_NL/RELATION*). If this type of problems were retained for a calculation of sensitivity, calculation stops in *ERREUR_FATALE* after having meant the reason of it.**
- 3. In an exhaustive way, these calculations of sensitivities relate to all the meshes supports (*TRIA3/6*, *QUAD4/8/9*, *TETRA4/10*, *PENTA6/13/15*, *PYRAM5/10* and *HEXA8/20/27*) and all modelings isoparametric (*PLAN*, *PLAN_DIAG*, *AXIS*, *AXIS_DIAG*, *3D* and *3D_DIAG*). If other modelings (only *THER_LINEAIRE* accepts other modelings that the usual isoparametric elements) are present in grid (*COQUE_* or *AXIS_FOURIER*) calculation stops (option 1 front the line of the suitable options of the catalogue elements) in *ERREUR-FATALE*, after having meant the reason of it.**
- 4. While being pressed on the perimeter of use of the code, one did not envisage the calculation of sensitivity (of the temperature and its flow) in the presence of non-linear conductivity.**
- 5. One has access to the sensitivity of the field of temperature (and its flow cf [§5]) compared to all parameters of loadings and all the characteristics material. Those having to depend that variables on space.**
- 6. The only exceptions are the entropy (a constant entropy according to the temperature, that would not have any direction!), multiplying coefficients of the conditions of Dirichlet generalized (calculation of sensitivity not established because it has little direction with coefficients often discrete) and conductivities anisotropic (data-processing limitations, absence of modeling *THER_ORTH_FO*).**
- 7. One can, on the other hand, calculate the sensitivity compared to one of the components of a normal flow vectorial.**
- 8. Calculation provides, at the same time, the field of standard temperature and the sensitivities of this same field compared to the parameters provided to the key word *SENSITIVITY* of the thermal operator. On the other hand, for the calculation of flow (via *CALC_ELEM* and/or *CALC_NO*), one calculates only the part sensitivity. To also obtain that of the field of standard temperature, it is necessary to reiterate the operation without the key word *SENSITIVITY*.**
- 9. If the user, involuntarily or not, request a calculation of sensitivity compared to one parameter known as “insensitive”, i.e. nonconcerned by thermal calculation in progress, one message of alarm prevents it. No calculation is produced, the structure of data planned for the reception of this *CHAMNO* is initialized to zero, and calculation continues with the other sensitivities demandées.**
- 10. The request for one or more sensitivities does nothing but enrich the structure of data**

thermics (EVOL_THER) and provides also the field of temperature of which they are the derivative (cf n°8). In term of performance, the calculation of an analytical sensitivity is much less expensive that a standard calculation since the same factorized matrix is re-used.

11. During the calculation of the sensitivity of the heat flux via CALC_ELEM, it is necessary to specify not

only the field material used (as for the standard problem) but also them loadings accused by the parameters of derivation.

12. To avoid any confusion (at the supervisory level, but also on the level user!), it is not better to re-use several times a significant parameter in loadings or different materials.

13. The calculation of the sensitivity of the heat flux was not developed, as for the problem standard, in COQUE_PLAN and COQUE_AXIS.

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Note:

By parameterizing advisedly the loadings and materials “derived” in the file from order, one can also have access to some derivatives made up. Thus if one loading or a characteristic depends, explicitly or implicitly, of the time or of space and that one is able to exhume this dependence, one can then calculate the derivative T (and of its flow) compared to this coefficient of dependence.

The main thing being which it also does not depend on the calculated solution (the field of temperature).

In which case, it will be necessary to develop a true made up calculation of sensitivity.

In short, the perimeter of use of this functionality gathers it or not thermal, linear, isotropic or anisotropic, stationary or transitory, being pressed on finite elements isoparametric lumpés or not. Within this framework there, it covers the same perimeter as that accused thermal operators.

6.4 Example of use

To familiarize oneself with the use of this new functionality, one can take as a starting point this expurgée version of the case test SENST04A [V1.01.154]. In this case analytical test, it is about to make sure of the validity of derived compared to the coefficient from exchange-wall and conductivity, in

a calculation of transitory thermal response linear of two plates separated by a play in which a transfer of heat is carried out.

The problem is two-dimensional, but the limiting conditions make that the field of temperature reached

quickly the stationary state and depends analytically only on the X-coordinate and the data. One in then deduced easily the analytical expressions from the sensitivities of the field of temperature and sound

flow compared to the thermal parameters which interest us.

1. Definition/memorizing of the constant functions

```
PS_UN=DEFI_CONSTANTE (VALE=1.0,);
MEMO_NOM_SENSI (NOM_UN = PS_UN);
PS_ZERO=DEFI_CONSTANTE (VALE=0.0,);
MEMO_NOM_SENSI (NOM_ZERO = PS_ZERO);
```

.....

2. Definition of the significant parameters and other parameters

```
PS1=DEFI_PARA_SENSI (VALE=80.0,);
PS2=DEFI_PARA_SENSI (VALE=40.0,);
```

.....

3.1. Installation of the materials (std and sensitive)

```
MAT=DEFI_MATERIAU (THER_FO=_F (LAMBDA=PS2, RHO_CP=A5))
MEMO_NOM_SENSI (NOM=_F (NOM_SD=' MAT',
PARA_SENSI=PS2,
NOM_COMPOSE=' MAT_PS2'));
MAT_PS2=DEFI_MATERIAU (THER_FO=_F (LAMBDA = PS_UN,
RHO_CP = PS_ZERO))
CMAT=AFFE_MATERIAU (MAILLAGE=MAIL, AFFE=_F (TOUT=' OUI', MATER=MAT))
MEMO_NOM_SENSI (NOM=_F (NOM_SD=' CMAT',
PARA_SENSI=PS2,
NOM_COMPOSE=' CMAT_PS2'));
```

**CMAT_PS2=AFFE_MATERIAU (MAILLAGE=MAIL, AFFE=_F (TOUT=' OUI',
MATER=MAT_PS2))**

.....

3.2.2 Loadings significant (echange_pari)
CH_1=AFFE_CHAR_THER_F (MODELE=MODTHER,
ECHANGE_PAROI=_F (GROUP_MA_1=' INTERG',
GROUP_MA_2=' INTERD', COEF_H=PS1));
MEMO_NOM_SENSI (NOM=_F (NOM_SD=' CH_1',
PARA_SENSI=PS1,
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NOM_COMPOSE=' CH_1_PSI');
CH_1_PSI=AFFE_CHAR_THER_F (MODELE=MODTHER,
ECHANGE_PAROI=_F (GROUP_MA_1=' INTERG',
GROUP_MA_2=' INTERD', COEF_H=PS_UN));

.....

4.1 standard Calculation + 2 calculations of sensitivity
MEMO_NOM_SENSI (NOM= (_F (NOM_SD=' RESU',
PARA_SENSI=PS1,
NOM_COMPOSE=' RESU_PSI'),
_F (NOM_SD=' RESU',
PARA_SENSI=PS2,
NOM_COMPOSE=' RESU_PS2')));
RESU=THER_LINEAIRE (MODELE=MODTHER, CHAM_MATER=CMAT,
EXCIT= (_F (CHARGE=CH_0), _F (CHARGE=CH_1)),
TEMP_INIT=_F (CHAM_NO=TEMPINIT),

***SENSIBILITE= (PS1, PS2),
INCREMENT=_F (LIST_INST=LINST))***

4.2 Calculation of flow to the nodes

***RESU=CALC_ELEM (reuse=RESU, MODELE=MODTHER, CHAM_MATER=CMAT,
RESULTAT=RESU, OPTION=' FLUX_ELNO_TEMP',
EXCIT= (_F (CHARGE=CH_0), _F (CHARGE=CH_1)),
SENSIBILITE= (PS1, PS2))***

***RESU=CALC_NO (reuse=RESU,
RESULTAT=RESU, OPTION=' FLUX_NOEU_TEMP',
SENSIBILITE= (PS1, PS2))***

Example 6.4-1: Installation of a calculation of sensitivity in linear thermics

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7 Outline Conclusion/

During digital simulations obtaining a rough result is not sufficient any more. The user is of more in petitioning of calculation of sensitivity compared to the data input of the problem. That he makes it possible to estimate the uncertainty which the field result according to the law of variation answers

data. This derivative is also the basic substrate of problems opposite (retiming of parameters...) and of problems of optimization.

This sensitivity can be obtained “manually”, but the experiment shows that these studies parametric are often expensive, little mutualisables and less reliable than an analytical calculation established in the computation software.

In this note, one places oneself in the perimeter of use of the thermal operators standards of Code_Aster and one is interested in this analytical sensitivity of the field of temperature and of its flow compared to the characteristics material and the loadings. One y described the process allowing to exhume the linear system which this derivative checks. In order to to minimize the overcost calculation, a particular effort was brought to bind its resolution to that of initial problem.

One details theoretical, numerical work and data processing which governed the establishment of these calculations of sensitivity in the code. One specifies their properties and their limitations all in

connecting these considerations to a precise parameter setting of the accused operators and to the choices of

modeling of the code. One tried constantly to bind different the items approached while detailing, has minimum, the a little technical demonstrations.

Required environment, the parameter setting and the perimeter of use of this news functionality are described. An example extracted from an official case-test is clarified.

Thereafter, the prospects for this work are several orders:

- From a functional point of view, one could extend truly the perimeter of use thermal calculations of sensitivity (and also that of the standard problem on which they lean) with non-linear and/or anisotropic conductivities and the conditions of exchange wall in lumpé. With few expenses, one could also deal with the problems of hydration, of drying and of convection-diffusion.*

- In addition, of the developments still remains to be established to be able to treat suitably a broad spectrum of derivative made up and, in particular, to carry out chainings thermomechanical (which are the genuine target of the current developments). One will be able to then obtain the mechanical sensitivities of variables (displacements, deformations and constraints) compared to loadings or characteristics material of the problem thermics.*

- From a theoretical point of view, it remains to carry out a study “numérico-functional calculus” similar to that of this document, to exhume the same thermal sensitivities in modeling HULL and FOURIER. That will open another field of investigation then: sensitivities compared to the geometrical characteristics of the structural elements.*

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Appendix 1 Concept of derived “within the meaning of the distributions”

Let us place within the framework of a Bi-material and a derivation compared to the voluminal heat of one of these materials, in linear thermics. The same reasoning can be led into non-linear and compared to do not matter that it another characteristic material, loading or limiting condition.

One considers the modeling of following total voluminal heat (I the indicating function of ième

**I
part)**

I

C

éq

A1.1

p (X): =

I X + I X

**,
1 1 ()**

2 2 ()

(I

)

2

2

I

2

1

2

Appear A1-a: Designation of voluminal heats for a Bi-material

That is to say the family of distributions parameterized by

(

*

Of \times ,

0

+

T)

(11)

éq A1.2

T

such as

T is the solution of the problem in extreme cases

(

T

II

T

S

1 (1 +

) 1 + 2 2)

- div () =

×],

0 [

T
T

= **F**
 $1 \times]$,
0 [

T
(P
)
= **G**

éq
A1.3
 $2 \times]$,
0 [
N
T

+ **HT**
= **hText**
 $3 \times]$,
0 [
N

T (X)
0
,
= 0
T (X)

*One of course recognizes the solutions of as many slightly shifted problems of linear thermics (cf [éq 2
T
1] in order to be able to approximate gradually, when the small parameter tends towards zero,
.*

*1
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By writing that T_0 and T check, respectively, (P_0) and (P) , and by withdrawing members from members both

EDP one obtains

$(T - T$

T

$0)$

$- \text{div} ((T - T$

I

$0) = -$

I

I

$\times J,$

$0 [$

T

T

T

$- T_0 = 0$

$I \times J,$

$0 [$

$(T - T_0)$

(P P0**= 0****éq****A1.4****2 ×],****0 [****)****N****(T - T0)****+ H (T****- T0) = 0****3 ×],****0 [****(N****T - T X****0)(****)****0****, = 0***It any more but does not remain to divide by**1 this new EDP and to make tend towards zero the parameter. The problem**in extreme cases then becomes a simple problem interfered similar homogeneous Cauchy-Dirichlet-Neumann-Robin type**with the initial problem and comprising however a different source term**U**T**- div (U) = -**0**I**1**×],**0 [**T**T**U = 0*

$1 \times]$,

$0 [$

1

U

lim

$(P - P = 0$

éq

A1.5

$2 \times]$,

$0 [$

$0)$

0

1

N

U

$+ hu = 0$

$3 \times]$,

$0 [$

N

$U(\mathbf{X})$

0

$, = 0$

1

which one notes $U: = lim$

$(T - T$ the solution. This one exists and is single because it inherits all them

$0)$

0

1

good properties of the initial problem. By definition, one then indicates it under the term of “distribution

derived compared to characteristic 1 from voluminal heat”: $= T$

U

1

*Taking into account the process of obtaining of this type of derived (translation, passage in extreme cases then definition of a distribution derived as soon as one is ensured of his existence and his unicity), **it appears that that returns** “formally” to directly derive the terms from the initial problem in extreme cases [éq A1-2], [éq A1-3]. It is in this way that one proceeded for all the derivative exhumed in this document. Moreover, for each one of them, one checked that this “formal” derivation produced the same result as it process describes previously.*

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Calculation of sensitivity in mechanics

Summary

Classically, the digital simulations provide the response of a system to a request. It appears currently an important evolution aiming at providing in addition to this answer the tendency of the answer to a modification of parameters of entry of simulation (material, loading, geometry,...). These tendencies are obtained by calculating the derivative of the response compared to parameters given.

The object of this note is thus the determination of the sensitivity of the results of a calculation of mechanics of solids with various data input by the method of direct differentiation. These data input are them data material and loadings. Moreover will be detailed the linear calculation cases (operator MECA_STATIQUE) and nonlinear (operators STAT_NON_LINE and DYNA_NON_LINE).

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1 Introduction

Classically, the digital simulations provide the response of a system to a request. It currently appears an important evolution aiming at providing in addition to this answer the tendency response to a modification of parameters of entry of simulation (material, loading, geometry,...). The possible applications are important:

probabilistic calculations related to the existence of an uncertainty on the value of a parameter,

problems opposite, retiming, optimization for which the knowledge of derived from one field can be capital in term of effectiveness,

reliability of the studies (which credit to bring to a simulation where the response of the system can to strongly vary following a small variation of a parameter?).

These tendencies are generally obtained by calculating the derivative of the answer, which can be done

various manners: finished differences, direct differentiation or method of the assistant state. finished differences are to be excluded from their weak precision and their important numerical cost. method of the associated state, though powerful and specifies, requires developments particular to each study and it will not be retained. We will thus concentrate here on the method of differentiation direct, powerful, specifies, general and very adapted to nonlinear calculations.

The object of this note is thus the determination of the sensitivity of the results of a calculation of mechanics

solids with various data input by the method of direct differentiation. These data of entry will be the data material and the loadings.

Moreover, the choice was made carry out our reasoning on the equations of the mechanical problem discretized. This choice seems important to us insofar as it ensures the coherence of derived calculation compared to direct calculation. We insist on the fact that this coherence is essential to precision of the results obtained.

Lastly, we will treat two model cases within the framework of the calculation of sensitivity of linear problems and nonlinear: linear elasticity and the plasticity of Von Mises with linear isotropic work hardening. The goal of these examples is well to clarify the difference in nature of the problems derived in both preceding cases. In the linear case, the derived problem is very similar to the direct problem in measurement where only the second member of the equations is modified. In the nonlinear case, the problem derived is appreciably different from the direct problem: the second member but also the law of behavior are modified. Nevertheless, in these two cases, the derived problem preserves one extremely interesting property insofar as it consists of a succession of linear problems whose matrices already were calculated and factorized (in the case of the direct use of solveurs).

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2

Sensitivity to the data materials

2.1

The direct problem

We place ourselves in this part within the framework of the resolution of non-linear calculations. Us let us use the notations of [bib1] and return to the reading of this document for any precision on technique of resolution approached in the continuation.

In Code_Aster, any non-linear static calculation is solved incrémentalement. It thus requires with each step of load I ,

*I
{ I } the resolution of the non-linear system of equation:*

$$\begin{aligned} &R U \\ &(\cdot) + T \\ &B \\ &= L \end{aligned}$$

*$I T_i$
 I
 I
éq
2.1-1*

*Drunk
=
 D
 I
 u_i*

*with
($R(U, T)$) =
 I
 I
 K
(U): (W)
 D
éq
2.1-2*

*I
 K*

w_k is related to form of the k th degree of freedom of the modelled structure,

$(R(U,))$

$I T_i$

is the vector of the nodal forces.

The resolution of this system is done by the method of Newton-Raphson:

N

K_{n+1}

U

$+ T$

B_{n+1}

$= L - R(N$

$U, T) + T N$

B

I

I

I

I

I

I

I

$I \acute{e}q$

2.1-3

$n+$

B

I

$=$

0

I

R

R

where

N

$K I =$

is the tangent matrix with the step of load I and the iteration of Newton N .

U

$(N$

$U, T)$

$I I$

The solution is thus given by:

NR

U

= U

+

N

I

i-1

U

I

n=0

NR

=

+

N

I

i-1

I

n=0

with NR, the iteration count of Newton which was necessary to reach convergence.

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2.2

Derived calculation

2.2.1 Preliminaries

Within the framework of the calculation of sensitivity, it is necessary to insist on the dependences of a size

compared to the others. We thus will clarify that the results of preceding calculation depend of a given parameter (modulus elastic Young, limit, density,...) and that of following manner:

$U =$

$()$

I

ui

$, =$

$()$

I

I

.

But that is not sufficient. Also we within the framework of an incremental calculation with law place ourselves

of elastoplastic nonlinear behaviour with linear isotropic work hardening [bib2]. If one considers the interdependences of the parameters on an algorithmic level, one can write [bib3]:

$R = R ($

$), p$

$), U ()$

$I I$

-

$I I$

-

=

$() + ($

$), p (), U (),$

I

I I

-

I I

-

I I

-

$$p = p () + p ($$
$$(), p (), U (,))$$

I

I I

-

I I

-

I I

-

Where U

is the increment of displacement to convergence with the step of load I.

Let us specify the direction of the notations which we will use for the derivative:

X

indicate the explicit derivative partial of X compared to Y,

Y

X, Y indicates the total variation of X compared to Y.

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2.2.2 Derivation of balance

Taking into account the preceding remarks, let us express the total variation of [éq 2.1-1] compared to:

R

R

R

R

+

U, +

, +

p, +Bt,

= 0

U

i-1

i-1

I

i-1

p

éq

2.2.2-1

I

I

Drunk,

=

0

R

Let us notice that here

= 0: R does not depend explicitly on but implicitly like us

will see in detail in the continuation.

That is to say:

$K NR U, +Bt,$

$= - R,$

I

I

uu ()

éq

2.2.2-2

Drunk,

=

0

Where

.

NR

K I is the last tangent matrix used to reach convergence in iterations of Newton,

.

R,

is the total variation of R, without taking account of the dependence of U

by

uu ()

report/ratio with.

The problem lies now in the calculation of R,

.

uu ()

Note:

R
U
(, **T**)
In [éq 2.2.2-2], one used the fact that $K NR =$

I
I
I
whereas in [éq 2.1-3] one has it
U

R
U
(, **T**)
defined by
NR
I
I
K I =
. There is well equivalence of these two definitions in measurement

NR
U
I
where $U = U$
+ U

I
i-1
*and that **R** depends indeed on **U***
*(and as well sure of **I 1***
- and
pi 1
-).

Note:
If one derives compared to directly [éq 2.1-3], one finds

N 1
+
N
U
T
N

N I
K =
+ B, = - R,
+
/uu/- K,

U
. What is the same thing

U
with convergence and reveals that the error on
depends on
-1
K K.

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2.2.3 Calculation of derived from the law of behavior

In the continuation, by preoccupation with a clearness, we will give up the indices $I = 1$.
According to [éq 2.1-2], one can rewrite R,
in the form:

$uu ()$

$R,$
 $=$
 $, +,$
 $: (W$

D

$éq$
 2.2.3-1

$uu ()$
 $($
 $uu ())$

$K)$

One must thus calculate,
. With this intention, we will use the expressions which
 $uu ()$
intervene in the numerical integration of the law of behavior.

2.2.3.1 Case of linear elasticity

Within the framework of linear elasticity, the law of behavior is expressed by:

$\sim = \mu \sim$
 2. (U)

$Tr () = 3K.Tr ((U))$

or:

$= 2\mu \sim$
 $. (U$
 $) + K.Tr$
 $((U$

Id
 $)). éq$
 2.2.3.1-1

where Id is the tensor identity of order 2.

Then, by calculating the total variation of [éq 2.2.3.1 - 1] compared to, one obtains:

$$\begin{aligned}
 & , = 2\mu, \sim \\
 & \cdot (U \\
 &) + K \cdot Tr \\
 & ((U \\
 & Id \\
 &)), + 2\mu \sim \\
 & \cdot (U \\
 & ,) + K \cdot Tr \\
 & ((U \\
 & , \\
 & Id \\
 &)),
 \end{aligned}$$

**éq
2.2.3.1-2**

That is to say:

$$\begin{aligned}
 & , / \\
 & = 2 , \\
 & \mu \\
 & \sim \\
 & \cdot (U \\
 &) + K \cdot Tr \\
 & ((U \\
 & Id \\
 & U \\
 & U \\
 &)), \\
 & ()
 \end{aligned}$$

2.2.3.2 Case of elastoplasticity with linear isotropic work hardening

The elastoplastic law of behaviour to linear isotropic work hardening is written:

$$\begin{aligned}
 & \sim \\
 & 3 \\
 & + \sim \\
 & (U) - S: = \\
 & p \\
 & 2 \\
 & (+) eq \\
 & (+) \\
 & R'. (p + p) \\
 & eq
 \end{aligned}$$

where S is the tensor of the elastic flexibilities and R' is the slope of work hardening defined by:

$$\begin{aligned}
 & E.ET \\
 & R' =
 \end{aligned}$$

where

E - AND

T
E

E

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In numerical terms, this law of behavior is integrated using an algorithm of return radial: one makes an elastic prediction (noted

E

) that one corrects if the threshold is violated. With

notations of [bib4], one thus writes:

E - y - R'. p

~

=

μ ~

2. (U) - μ

3 eq

~e

(R +

'μ).

3

E

eq

Tr () =

3K.Tr ((U))

éq

2.2.3.2-1

E - y - R'. p

p

=

eq

R +

' μ

3

We will distinguish two cases.

1st case: $p = 0$

What comes down to saying that at the time of these step of load, the point of Gauss considered did not see of increase in its plasticization. One finds oneself then in the case of linear elasticity:

,
= 2μ , ~
. (U
) + K .

((U

I_d

)).

Tr

U

$U ()$

2nd case: $p > 0$

Taking into account the dependences between variables in [éq 2.2.3.2 - 1], one can write:

,
=
+

, +
p, +
(*U*)

p
(
U)

éq 2.2.3.2 - 2

p

p

p

p

p,

=

+

, +

p, +

(*U*),

p

(*U*)

Moreover, in agreement with the algorithmic integration of the law, we will separate parts deviatoric and hydrostatic.

~
1 Tr ()

,
=
+

Id
uu ()

3

~

1 Tr (
+

, +
) Id,

3

~
1 Tr ()

+
p, +
Id

p,

p
3
p

p

p

p

p,
=
+
, +

p,

uu ()

p
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And thus, one calculates:

eeq

y
R'
E
y
~
-
-

μ
2
μ
3
--
p
eq
R'. p
~
~ E

=
(U) -

- μ
3
~e

$(R + \mu)^3 .e$

$(R + \mu)^3 .e$
 eq

$E - y - R' . p$

eq
 $R' \mu$
 3

E

E

eq

$+$

μ

3

$+$

$+(R + \mu)^3$

$\sim e$

2

$((R + \mu)^3)$

eq
 E

eq

E - y - '.

~

eq

R p E

-

μ

3

(R +

'μ) .e

3

eq

Tr (

) 3

=

K Tr ((U

))

E

y

E

E

y

~

- μ

3

- - R'. p

--

eq

R'. p

=

1-

eq ~e - μ

3 eq

J

E

E

(R +

'μ).

3

(R +

'μ) .e

eq

eq

3

eq

where J is the operator deviatoric defined by: J

~

:

=

Tr () = 0

p

~

3 R

$\mu \cdot '$

$\sim E$

=

E

p

(R' 3

+ μ) .eq

Tr () = 0

p

p,

The fact is used that:

y

eq = R' . (p + p

) +

~

~

~

~

y

1 3 (, + ,) : (+

) R'

p, +p, =

(

)

R'

-

p + p -

2

eq

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***Note:
In these calculations were or must be used the following results:***

***μ
2
~
~
~
~***

***(
(U)) : (+ μ.
2 (
E
2μ
E***

U

~

)

=

3

(U

)

eq

=

E

2

eq

Tensor of order 2

Scalar

E

~e

eq = 3

~e

E

2

= J

eq

Tensor of order 2

Tensor of order 4

2.2.3.3 Calculation of derived from displacement

**Once,
R,**

*calculated, one can constitute the second member
while using*

uu ()

uu ()

[éq 2.2.3-1]. One solves then the system [éq 2.2.2-2] and one obtains the increment of derived displacement compared to.

2.2.3.4 Calculation of derived from the other sizes

Now that one has U, one must calculate the derivative of the other sizes. One separates still two cases:

Linear elasticity

According to [éq 2.2.3.2 - 1], one as follows calculates the derivative of the increment of constraint:

, =

,

+ 2μ ~

. (U

,) + K.Tr

((U

,

Id

)).

U

U

()

The increment of cumulated plastic deformation, as for him, does not see evolution:

p, = 0

Elastoplasticity with linear isotropic work hardening

If $p = 0$, one finds the preceding case.

If not, one obtains according to [éq 2.2.3.2 - 2]:

$$\sigma = \sigma_0 + K \epsilon^n$$

$$u = u_0 + \frac{1}{2} \epsilon^2$$

And for the cumulated plastic deformation:

$$\epsilon_p = \frac{1}{n} \ln \left(\frac{\sigma}{\sigma_0} \right)$$

$$\sigma = \sigma_0 \exp(n \epsilon_p)$$

$$p + \Delta p = \frac{1}{n} \ln \left(\frac{\sigma}{\sigma_0} \right)$$

$$\left(\frac{\sigma}{\sigma_0} \right)^{\frac{1}{n}} = p + \Delta p$$

$$R' = \frac{1}{n} \ln \left(\frac{\sigma}{\sigma_0} \right)$$

$$p + \Delta p = \frac{1}{2} \epsilon^2$$

eq

Once all these calculations are finished, all the derived sizes are reactualized and one passes with the step of load according to.

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2.2.3.5 Synthesis

To summarize the preceding paragraphs, one represents the various stages of calculation by following diagram:

Convergence of the step of

Calculation of the terms

Assembly of

charge N of direct calculation

,

R,

uu ()
uu ()

Passage to the step
Calculation of
Resolution of the system

[éq 2.2.2-2]
of load $N + 1$
, and p ,

U,

2.3 Establishment *data processing*

For each finite element of Code_Aster, it is necessary to create two new options of calculations to allow calculations above:

.
for the calculation of,

,
uu ()

.
for the calculation of, and p .

Derived calculation is controlled by a routine which launches elementary calculations of,

,
uu ()

carry out the assembly of these terms to create the second member, solves the linear system [éq 2.2.2-2], then lance the calculation of, and p . This routine is called by OP0070 after convergence was detected.

Note:

For the user, the sequence of various calculations will be transparent. The definition of

significant variables will be done according to the diagram [bib5]:

```
...
v = DEFI_PARA_SENSI (VALE = < value of the parameter >)
= DEFI_MATERIAU subdue (VMIS_ISOT_LINE = _F (SY = v),...)
chmat = AFFE_MATERIAU (AFFE = _F (GROUP_MA = < group (S) >,
MATER
=
matt
))
...
resu = STAT_NON_LINE (CHAM_MATER = chmat,
SENSITIVITY
=
(
v),
...
...)
```

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3 *Sensitivity to the loading*

3.1 *The direct problem: expression of the loading*

Until now we expressed the direct problem in the form:

$$\begin{matrix} R U \\ (,) + T \\ B \\ = L \end{matrix}$$

*I Ti
I
I
éq
3.1-1*

*Drunk
=
D
I
ui*

The loadings are gathered with the second member and include/understand the imposed forces L_i and them imposed displacements D_{ui} .

Let us suppose that the loading in imposed force L_i depends on a scalar parameter on following manner:

$$\begin{matrix} L () \\ 1 \\ 2 \\ = L + L () \end{matrix}$$

*I
I
I
éq
3.1-2*

Where

•
1
Li is a vector independent of,

•
2
Li depends linearly on.

One wishes to calculate the sensitivity of the results of direct calculation to a variation of the parameter.

3.2
The derived problem

3.2.1 Derivation of balance

As in the preceding chapter, by taking account of the dependences between the various fields, one derives balance [éq 3.1-1] by report/ratio:

R
R
R
R

+
U, +
, +
p, + T
B,
=
2
L)
1
(

U

i-1

i-1
I

I
p
éq
3.2.1-1
i-1
I

l

Drunk,
= - Drunk

I,
l

One used the fact that 2
Li depends linearly on.

That is to say:

NR
K U, + T
B,
=
2
L)
l
(- R,
I

I
I
uu ()

Drunk,
=
- Drunk

I,
l
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Where

.

NR

***K** I is the last tangent matrix used to reach convergence in iterations of Newton,*

.

R,

*is the total variation of **R**, without taking account of the dependence of **U** by **uu** () report/ratio with.*

*The problem lies like previously in the calculation of **R**,*

.

uu ()

Note:

If one seeks to calculate the sensitivity compared to the loading of Dirichlet, the system with to solve becomes:

NR

K U, + T

B,

=

- **R,**

I

I

uu ()

D

Drunk,

=

2

U

)

1

(- *Drunk*

I

I,

1

D

Where *2*

U is the part of the loading of Dirichlet which depends linearly on.

I

3.2.2 Calculation of derived from the law of behavior

According to [éq 2.1-2], one can rewrite **R**,

in the form:

uu ()

R,

=

, +,

: (**W**

D

éq

3.2.2-1

uu ()

(

uu ())

K)

With this intention, we will use the expressions which intervene in the numerical integration of law of behavior to calculate,

$uu ()$

3.2.2.1 Case of linear elasticity

Within the framework of linear elasticity, the law of behavior is expressed by:

$$= 2\mu \sim$$

$$\cdot (U$$

$$) + K.Tr$$

$$((U$$

Id
)). *éq*

3.2.2.1-1

where Id is the tensor identity of order 2.

Then, by calculating the total variation of [éq 3.2.2.1 - 1] compared to, one obtains:

$$, = 2 ,$$

$$\mu \sim$$

$$\cdot (U$$

$$) + K.Tr$$

$$((U$$

$$Id$$

$$)). + 2\mu \sim$$

$$\cdot (U$$

$$,) + K.Tr$$

$$((U$$

$$,$$

$$Id$$

$$)).$$

éq 3.2.2.1 - 2

= .

0
 $+$
 0
 $+ 2\mu \sim$
 $\cdot (U$
 $,) + K.Tr$
 $((U$
 $,$
 Id
 $)).$

That is to say:

$= 0.$

uu ()
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3.2.2.2 Case of elastoplasticity with linear isotropic work hardening

As previously, we will distinguish two cases.

Ist case: $p = 0$

What comes down to saying that at the time of these step of load, the point of Gauss considered did not see

of increase in its plasticization. One finds oneself then in the case of linear elasticity:

$$\begin{aligned} & , \\ & = 0. \end{aligned}$$

uu (*)*

2nd case: $p > 0$

Taking into account the dependences between variables, one can write:

$$\begin{aligned} & , \\ & = \\ & + \\ & , + \\ & p, + \\ & (\mathbf{U},) \end{aligned}$$

p
(
U)

p

p

p

p

$p,$

$=$

$+$

$, +$

$p, +$

$(\mathbf{U}),$

p

(\mathbf{U})

Moreover, in agreement with the algorithmic integration of the law, we will separate parts deviatoric and hydrostatic.

\sim

$1 \text{ Tr } ()$

$,$

$=$

$+$

Id

$uu ()$

3

~

1 Tr (

+

, +

) Id,

3

~

1 Tr (

+

p, +

Id

p,

p

3

p

p

p

p

p,

=

+

, +

p ,

$uu ()$

p

And thus, one calculates:

Insofar as there is not explicit dependence from report/ratio with, one obtains:

\sim

$= 0.$

$Tr () = .0$

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E
y
E
E
y

~

- μ
3
- - *R'. p*

- -

eq

R'. p
=
1-

eq ~ $e - \mu$
3 *eq*
J

(*R* +
'3rd
E

μ).

(*R* +
' μ) .*e*
eq
eq

3

eq

where **J** is the operator deviatoric.

$$Tr () = 0$$

p

~

$$3\mu.R'$$

$$\sim E$$

=

E

p

(*R'* 3

$$+ \mu) .eq$$

$$Tr () = 0$$

p

p,

The fact is used that:

y

$$eq = R'. (p + p$$

) +

~

$$I 3 (, +~$$

~

$$,) : (+ ~$$

)

$$p, + p$$

, =

R' 2

eq

3.2.2.3 Calculation of derived from displacement

Once,

R,

calculated, one can constitute the second member

. One solves

uu ()

uu ()

then the system [éq 3.2.1-1] and one obtain the increment of derived displacement compared to.

3.2.2.4 Calculation of derived from the other sizes

Now that one has **U**, one must calculate the derivative of the other sizes. One separates still two cases:

Linear elasticity

According to [éq 3.2.2.1 - 1], one as follows calculates the derivative of the increment of constraint:

, = .

$0 + 2\mu \sim$

. (**U**

,) + $K.Tr$

((**U**

,

Id

)).

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The increment of cumulated plastic deformation, as for him, does not see evolution:

$$p, = 0$$

Elastoplasticity with linear isotropic work hardening

If $p = 0$, one finds the preceding case.

If not, one obtains:

$$\begin{aligned} &, = , \\ &+ \\ &: (\mathbf{U},) \end{aligned}$$

$$\begin{aligned} &uu () \\ &(\mathbf{U}) \end{aligned}$$

And for the cumulated plastic deformation:

$$\begin{aligned} &\sim \\ &I 3 (, + \sim \\ &\sim \\ &,) : (+ \sim \\ &) \\ &p, + p \\ &, = \end{aligned}$$

$R' 2$
 eq

Once all these calculations are finished, all the derived sizes are reactualized and one passes with the step of load according to.

3.2.2.5 Synthesis

To summarize the preceding paragraphs, one represents the various stages of calculation by following diagram:

Convergence of the step of

Calculation of the terms

Assembly of
charge N of direct calculation

,

$uu ()$
 $uu ()$

Passage to the step

Calculation of

Resolution of the system

[$\acute{e}q$ 3.2.1-1]
of load $N + 1$

, and p ,
 U ,

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3.3 Establishment data processing

The data-processing establishment is in any point similar to the preceding one insofar as one adds two options:

.
for the calculation of,

,
uu ()

.
for the calculation of, and p.

Let us notice that the organization of the calculation of the loadings (where all the contributions are summoned in only one vector) obliges with the revaluation of the force compared to which one calculates them sensitivities.

Note:

The definition of the significant variables will be done like previously:

...
 $v = \text{DEFI_PARA_SENSI}$ ($\text{VALE} = \langle \text{value of the parameter} \rangle$)
 $\text{force} = \text{AFFE_CHAR_MECA}$ (
 $\text{PRES_REP} = _F$ ($\text{GROUP_MA} = \langle \text{groups (S)} \rangle$,
 $\text{PRES} =$
 v),...))
...
 $\text{resu} = \text{STAT_NON_LINE}$ ($\text{EXCIT} = \text{force}$,
 SENSITIVITY
 $=$
(
 v),
...)
...)

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4 **Availabilities within Code_Aster**

The following table recapitulates the derivative available in Code_Aster. It is updated at the fur and at

measure developments of the derivatives.

***Linear mechanics (operator
Nonlinear mechanics
MECA_STATIQUE)
(operators STAT_NON_LINE
and DYNA_NON_LINE)***

***Sensitivity to the data
materials in elasticity
All the elements continuous mediums
Not
linear isotropic***

***Sensitivity to the data
materials in elasticity
All the elements continuous mediums
Not
linear orthotropic***

***Sensitivity to the data
materials in elasticity
All the elements continuous mediums
Not
linear isotropic transverse***

Sensitivity to
.
***nodal forces
loadings***
.
force divided voluminal into 3D
.
force divided surface into 3D
.
force divided linear into 3D
.
force divided surface into 2D
.
force divided linear into 2D
.
force divided linear into 1D
Not
.

force distributed for the hulls

“2D”

.

force distributed for the hulls

“3D”

.

pressure distributed

Without restriction of finite elements.

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Organization (S): EDF-R & D /AMA

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Document: R4.03.04

Calculation of sensitivity in dynamics

Summary

Obtaining the sensitivities is important in dynamics where small disturbances of the parameters can to have a great influence on the sizes of interest. The objective of the document is to clarify the calculation of

sensitivities (i.e derived) of the principal sizes: modes and Eigen frequencies, dynamic responses in frequential and temporal, into linear and nonlinear). For each size, the “derived problem”, i.e. the problem whose derivative is solution, is written (in variational and/or matric form). numerical algorithms of calculation of these derivative are then detailed and discussed. All establishments

corresponding data processing in Code_Aster were not carried out. A table thus gives the current state developments.

Derivation compared to the parameters of the matrices of mass, stiffness, damping, the loading and also the particular case of derivation compared to a variation of field is presented.

calculations by the finite element method, which is specific to the derivative, are clarified on the level of elementary calculations.

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1 Introduction

In a mechanical context of calculation on a model of structure, one understands by “sensitivities” them derived from a size result of the model compared to parameters of model entry of the aforesaid. In dynamics, of small disturbances of the parameters can have a great influence on dynamic and vibratory sizes [bib1]. The calculation of the sensitivities proves thus paramount, because it allows to carry out analyses of influences of the parameters, analyses probabilistic, of identification of parameters (or retiming), optimization...

This documentation develops obtaining the sensitivities of the dynamic sizes compared to mechanical parameters, in the boundary conditions, with the loading, and a variation of field. The dynamic sizes treated are the dynamic responses in displacement, the frequencies clean and clean modes. Sensitivities of the other dynamic sizes (forced, efforts....) which is in general calculated by postprocessing can result some a posteriori by ad hoc calculations.

The general method of calculation retained is a method of exact direct differentiation (by opposition to the methods of the associated state, the methods of finished differences or other methods semi-analytical (the calculation of derived by an semi-analytical method consists in calculating them derived from the elementary matrices by finished differences and then to calculate them analytically derived from the sizes results. This method is easier to establish than the analytical method complete and was retained in Nastran or Abaqus for example (cf [bib2])).

This method has as principal advantage of being exact. One avoids the problems thus of convergence of the algorithms of optimization based on the gradient or the problems of precise details of finished differences (when the results result from an iterative algorithm, the finished differences can to be particularly vague). It is also more general than the method of the assistant state where a functional calculus to be derived must be specified before calculation (it thus makes it possible for example to obtain directly fields derived).

This method can be very economic in computing times if the algorithms of resolution of derived problems - i.e. the problems whose derivative are solutions - are well chosen and if the programming is neat. In the most favorable cases, the computing time of a derivative represent that a very small percentage of the computing time of the standard size.

Its principal disadvantage is the extent of the data-processing developments necessary to sound establishment (more especially as the architecture of Code_Aster does not make it possible to use the differentiation automatic).

The document attempts to present the various “derived problems”. They are written in form

variational and/or matrix according to the need. These derived problems often have the very same properties. For example, in linear dynamics, the first members of the systems to be solved are preserved. In nonlinear dynamics, the derived problem is a linear problem.

The methods of resolution which meet a priori more the needs for the users of Code_Aster are presented. In order to minimize the computing time, the calculation algorithm of derived must exploit as much as possible intermediate results of the resolution of the standard problem. In addition, the effectiveness of certain algorithms depends on the properties of the matrices of the problems to solve, of a number of parameters per report/ratio for which it is necessary to derive, and of the required precision.

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2 **Derived from dynamic responses**

2.1 Problem **standard**

In the frequential field, equations of linearized dynamics and the dualisation of the conditions to the limits lead to the following matrix equation (it is the discretization by the method of finite elements of the variational formulation which makes it possible to obtain this equation. During document, the variational formulations and with the method of the elements are clarified when necessary):

$$(-2M(p) + iC(p) + K(p)) \dot{X}(p) = F(p)$$

éq

2.1-1

where:

$X(p)$ RN

($N = n + 2$) is, for the pulsation, the vector made up of N ddl physical structures discretized and of the 2 multipliers of Lagrange [R3.03.01],

p is a scalar parameter of mechanical property, loading or of geometry which intervenes directly explicitly in the matrices of mass, stiffness or of damping, or in the loading. Let us notice that that can be a parameter of geometry for the elements of structure (for example a thickness of hull) but not a geometrical parameter of the grid for which the field of integration is modified and for which thus it is necessary to return to a variational formulation of the problem and to establish one paramétrage of the field (cf [§4]).

$M(p)$, $C(p)$ and $K(p)$ are the real symmetrical matrices of mass, damping and of stiffness “generalized” (thus not necessarily definite positive),

$F(p)$ is the vector of the external forces.

Code_Aster solves a more general equation for the systems fluid-structures [R5.05.03].

The writing of the derived problem and its resolution are not basically modified by it. For more of clearness and concision, one thus limits oneself in the continuation of this Doc. to the equation [éq 2.1-1].

In Code_Aster, the resolution of [éq 2.1-1] is carried out by the direct methods multifrontale and “LDLT” [R6.02.02]. These two methods proceed in three stages:

renumerotation of the unknown factors,

factorization of the matrix,

gone up descent/(resolution of two triangular systems).

If several linear systems, of the same matrix, are to be solved, only them gone up descents/are to be carried out several times. The same if several of the same matrices structure are to be factorized, the renumerotation of the unknown factors will not be to remake (this phase preliminary has a considerable cost, even if its relative weight in computing times decreases with

cut matrices to be factorized). These two properties are very important and will allow to calculate the derivative at numerical lower cost.

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2.2

Derived from a harmonic dynamic response

2.2.1 Method

direct

The derivation of [éq 2.1-1] gives (after handling for exhiber a “good” second member):

$$\begin{aligned}
 & (\\
 & \mathbf{X} () \\
 & \mathbf{F} () \\
 & \mathbf{M} \\
 & \mathbf{C} \mathbf{K} \\
 & - 2 + I +) \\
 & = \\
 & - - 2 \\
 & \mathbf{M} \\
 & \mathbf{C} \mathbf{K} \\
 & + I \\
 & + \\
 & \mathbf{X} ()
 \end{aligned}$$

éq

2.2.1-1

P

P

P

p
 p
 $\mathbf{X}()$
 The resolution of the linear system [éq 2.2.1-1] gives the size

sought.

p

One can observe that the system [éq 2.2.1-1] has the same matrix coefficient as [éq 2.1-1], and a second different member but whom one can obtain without inversion starting from elementary calculations.

This second member is called in the literature “pseudo-load” giving to the direct method its name also of “method of the pseudo-loads”.

There are as many resolutions of [éq 2.2.1-1] that parameters by report/ratio to which one derives. For to carry out these resolutions by minimizing the computing time, one uses the properties of the methods direct multifrontale and “LDLT” recalled in [§2.1]. The renumerotation of the unknown factors and factorization of the matrix are thus carried out only once for all the resolutions.

Also let us notice that the majority of the terms of the second member are null. Indeed, if p is one

$\mathbf{F}()$

nodal force only remainder the term

, if p is a density only remainder the term

p

$2 \mathbf{M} \mathbf{X}()$, etc... This remark subsequently remain valid of Doc., for all them

p

terms containing of the derivative.

2.2.2 Simple modal method

To accelerate the resolutions of [éq 2.1-1] frequency per frequency, it is usual to project the operator on a truncated modal basis $[\mathbf{I}] = [$,

\mathbf{L}

\mathbf{I}

$\mathbf{2}$,

,

$\mathbf{L} \mathbf{L}]$ where \mathbf{I} is the i -ième vector

clean (cf [§3.1]) and with $L < n$ [R5.06.01]. The new system to be solved becomes:

$[(\mathbf{I}) \mathbf{T}$

$\mathbf{2}$

\mathbf{T}

$\mathbf{L} (- \mathbf{M} + i\mathbf{C} + \mathbf{K}) [(\mathbf{I}) \mathbf{L}] = [(\mathbf{I}) \mathbf{L}] \mathbf{F}()$ éq

2.2.2-1

with the projection of $X()$ on the incomplete modal basis which is written $X = []$

L

L().

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The derivation of [éq 2.2.2-1] gives:

T

[(

2

] T -

M +

C + K

=

F

+

F

L (

I

) [] () [] **L**

L

() [] **T** ()

P

P

L

P

- [**I**

2 M

C K

] **T** -

+ **I**

+

L

[]

éq

2.2.2-2

P

P

P

L

[] **T**

L

+

(**2**

T

- **M + iC + K**) [] +

- **2M + C** +

L

[] L (

I

K) [] L (

)

p

p

()

The resolution of [éq 2.2.2-2] provides

. One obtains then the derivative of projection on the basis

p

XL

modal of X (), i.e. by:

p

X

[

]

(

L

L

)

=

() + []

L

éq

2.2.2-3

p

p

p

The remark made for the direct method is valid for the simple modal method: the pre ones treatments of the matrix coefficient in the linear system [éq 2.2.2-1] (factorization primarily) can advantageously be re-used for the resolution of [éq 2.2.2-2].

In this method, only truncation carried out is that of the projection of X () on the basis modal incomplete. The calculation of the derivative is then carried out without another approximation. In

counterpart, the resolution of [éq 2.2.2-2] and the calculation of [éq 2.2.2-3] require calculation as a preliminary

[
]

of

L , for example by one of the methods presented at the chapter [§3.3].

p

2.2.3 Double modal method

$X()$

In the same way that $X()$ is projected on truncated modal basis,

can be projected on

p

this same base. Applying this projection to [éq 2.2.1-1], one obtains:

[$() T$

2

μ

$L(-M + iC + K) [L] () =$

éq

2.2.3-1

$= [T F$

$() -$

2

L

[$T M$

-

$+ I C$

K

+

X

L

$()$

p

P
P
P

X ()
with projection on the basis of modal
 μ

who is written []

P

L ()
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In practice, in [éq 2.2.3-1], X () is known only after the first modal projection. Method by double projection thus consists in carrying out two projections successively, and thus writing (substitution of X () by XL in [éq 2.2.3-1]):

[(] T - 2 +
+
^ μ
=
L (M
iC K) [(] L) ()

= [] T F
()

2 M

C

K

-

-

+

+

L

[] T

I

L

X

L ()

p

p

p

p

éq 2.2.3-2

X ()

It should well be seen that the resolution of [éq 2.2.3-2] gives an approximation of

after one

p

double truncation. The first truncation is usual truncation carried out to obtain

X ()

projection of X (). The second truncation is of comparable nature but concerns

.

p

$X ()$

Techniques of acceleration of the convergence of the series giving

were proposed in

 p

literature with [bib3]. The user of Code_Aster will use the simple modal method preferentially to obtain more precision in the results.

2.2.4 Establishment by the finite element method

For the calculation of the second members of [éq 2.2.1-1], [éq 2.2.2-2] and [éq 2.2.3-2], several choices are possible according to whether the matric products are calculated on an elementary level or not.

For reasons of performance, it was selected to carry out the matric products of the second member of [éq 2.2.1-1] on an elementary level. In Code_Aster, the second member of [éq 2.2.1-1] is thus calculated directly by the assembly of the following elementary matrices:

 F (G) 2 E T E E E T E E $- G$ $((G)) (G) (G) X + gi$ $(Ca (G)) (G) (G) X$ p p p $+ (E$ $G (G))T$ E $(H (G))(E$ E

(G) X)

p

éq 2.2.4-1

where:

.
same diagrams of integration (same functions of forms, points of Gauss,...) who have allowed to obtain the standard linear system are used,

.
G are the parametric co-ordinates of the point of Gauss of weight G,

.
(G) is the matrix of the functions of form of the element E,

.
Be (G) is the matrix connecting the deformations to the nodal variables for the element E.

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Thus, the assembled vector - 2 M

C

K

+ I

+

X ()

is obtained without calculation and

P

P

P

MR. K

C

*the assembly preconditions of the matrices,
and*

. This procedure is thus economic in

P

P

P

assembly and storage of assembled matrices. It has on the other hand the disadvantage of to require the calculation of the elementary matrices derived at each frequency since the product by $X()$ is carried out on an elementary level and thus prevents the storage of these matrices elementary derived (at least with the architecture of Code_Aster).

The second members of [éq 2.2.2-2] and [éq 2.2.3-2] will result them from a shandy between operations elementary and matric operations.

Once again, one can notice that many terms of [éq 2.2.4-1] are null. For example if p is Young the modulus of certain elements, only remainder the term

E

T

((

E

E

G))

(H ()) () X (

G

) for these elements and [éq 2.2.4-1] is null everywhere

G (

G

G

P

)

elsewhere.

2.3

Derived from a temporal dynamic response into linear

One limits oneself in this chapter and the following if one does not derive compared to parameters intervening under the initial conditions. This case raises theoretical difficulties and practical out of the field of this Doc.

2.3.1 Method direct

Let us leave the differential equation of dynamics in matrix form:

$$M(p) \dot{X}(T, p) + C(p) X(T, p) + K(T, p) X(T, p) = F(T, p)$$

éq

2.3.1-1

where:

$X(T)$ RN

($N = n+2$) is, at the moment T , the vector made up of N ddl physical of structures discretized and of the 2 multipliers of Lagrange [R3.03.01],

$F(T, p)$ is the vector of the external forces, with, to calculate the response in transient linear in Code_Aster [U4.53.02], the loading which must be written in the form

$$F = (T F$$

) (X

I

I

).

I

X

The differential equation governing the derivative compared to p of the vector displacement $X, Y =$, is

obtained while deriving directly [éq 2.3.1-1] compared to p . After change about

derivation compared to time and p (the sufficient regularity is supposed), and after rearrangement terms, one obtains:

$$M(p) \dot{Y}(T, p) + ($$

$C(p) Y(T, p) + K(p) Y(T, p) =$

F

(T, p)

M

(p)

éq 2.3.1-2

-

$X&$ (

C

K

$T, p)$

(p)

-

$X&$ (T, p)

(p)

-

X (T, p)

p

p

p

p

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The vector of displacements and its derivatives are obtained while solving successively [éq 2.3.1-1] then [éq 2.3.1-2]. For that, one uses a diagram of numerical integration (implicit or clarifies in

Code_Aster). On the same principle as that of the derivation of a harmonic answer, one exploits intermediate results of the resolution of the standard problem in order to minimize calculations related to resolution of the derived problems.

Let us specify this in the case of an implicit scheme, the identical remaining principle in the case of one

explicit diagram: For the standard problem, the vectors accelerations and speeds, with the step from integration t_{n+1} , are obtained by the linear approximations of the following form:

$$\begin{aligned} & \mathbf{X} \\ & = \mathbf{L}(\mathbf{X} \\ & , \mathbf{X}, \\ & n+1 \\ & 1 \\ & n+1 \\ & \mathbf{N} \& \\ & \mathbf{X}, \\ & \mathbf{N} \& \mathbf{X}) \\ & \mathbf{N} \\ & \text{éq} \end{aligned}$$

2.3.1-3

$$\begin{aligned} & \mathbf{X} \\ & = \mathbf{L}(\mathbf{X} \\ & , \mathbf{X}, \\ & n+1 \\ & 2 \\ & n+1 \\ & \mathbf{N} \& \\ & \mathbf{X}, \\ & \mathbf{N} \& \mathbf{X}) \\ & \mathbf{N} \text{ éq} \end{aligned}$$

2.3.1-4

One solves the standard problem then, i.e one obtains \mathbf{X}_{n+1} , by the substitution of [éq 2.3.1-3] and [éq 2.3.1-4] in [éq 2.3.1-1]. This gives a linear system to solve.

The idea for the problems derived is to use the same diagram of integration:

$$\begin{aligned} & \mathbf{Y} \\ & = \mathbf{L}(\mathbf{Y} \\ & , \mathbf{Y}, \\ & +1 \\ & 1 \end{aligned}$$

+
 &Y, &Y)
 N
 N 1
 N
 N
 N
 N
 éq
 2.3.1-5
 &Y
 = L (Y
 , Y,
 +1
 2
 +
 &Y, &Y)
 N
 N 1
 N
 N
 N éq
 2.3.1-6

Thus, factorization used to solve the system can be also used to obtain Y_{n+1} .

Indeed, since the matrices coefficient of [éq 2.3.1-1] and [éq 2.3.1-2] are identical, and if one uses the same diagram of integration for the calculation of X_{n+1} and Y_{n+1} , then the matrix coefficient is even in the two linear systems to solve after substitution of [éq 2.3.1-3] and [éq 2.3.1-4] in [éq 2.3.1-1] and of [éq 2.3.1-5] and [éq 2.3.1-6] in [éq 2.3.1-2].

The computing time of derived can then be negligible compared to the computing time associated with standard problem. It should however be noted that, in the case of a resolution by explicit diagram, each calculation of derived can be as expensive as standard calculation. Indeed, in this case, the main part of time CPU for the standard problem is associated at the stage of calculation of the second member with each step of time; and it is also necessary to carry out this stage for the derivative (calculation of the second member of [éq 2.3.1-2]).

2.3.2 Method modal

Two modal methods simple and double for calculation of derived from a dynamic response [frequential \(\[§2.2.2\] and \[§2.2.3\]\)](#) are easily transposable with the calculation of the dynamic response in

temporal. One obtains a linear system of the same type as [éq 2.3.1-2] with second members different according to whether one uses the simple or double modal method. In the case, method modal double, one obtains:

[] T

^

^

^

&

+

&

+

=

L M [] L (p) Y (T, p)

[] T

L

(

C p) [] L Y (T, p) [] L K (p) [] L Y (T, p)

[

] T F

(T, p)

M

C

K

-

&

-

&

-

L

[] T

(p)

L

XL (T, p) [] T

(p)

L

XL (T, p) [] T

(p)

L
 $XL(T, p)$
 p
 p
 p
 p
éq 2.3.2-1
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The response in generalized co-ordinates is obtained by solving this system by a diagram of numerical integration of the same type as [éq 2.3.1-3] and [éq 2.3.1-4]. And in the same way, them factorizations used for the calculation of the answer can be used for the calculation of the derivative.

2.3.3 Establishment by the finite element method

In a way similar to the preceding cases [§2.2.4], the second member of [éq 2.3.1-2] can be calculated
MR. K
C
after assembly of the matrices,
and
, or in a more powerful way directly to one

p

p

p

elementary level.

The temporal integration of X to the step tn+1 is carried out the temporal integration before of its derivative Y. With the step of integration tn+1 of Y, terms X (T

, p)

n+1

, &

X (T

, p)

n+1

, and &X (T

, p)

n+1

are thus known (the last two terms are calculable by [éq 2.3.1-3] and [éq 2.3.1-4] by example).

M (p)

C (p)

K (p)

With the step of integration tn+1 of Y,

X

& (T

, p) +

X

n+1

& (T

, p) +

X (T

, p)

p

p

n+1

p

n+1

result thus from the assembly of the following elementary matrices (cf [§3.2.3] for the others notations):

(()) E

NR () T NR () E

X& (T

) +

(C ()) E

NR () T NR () X& (, T

)

G

G

G

G

N I

+

G

has

G

G

G

G

N I

+

p

p

éq

2.3.3-1

+

E

B () T

(E

H ()) E

B () T E

X& (T

)

G

G

G

G

N I

+

p

2.4

Derived from a temporal dynamic response into nonlinear

Let us consider the class of the problems of nonlinear dynamics of which the equation discretized of movement is form:

$$M(p) \ddot{X}(T, p) + C(p) \dot{X}(T, p) + [G(X(T, p), p)] = F(T, p) \quad \text{2.4-1}$$

where
 $[G(X(T, p), p)]$ *is a function (nonlinear) of* $X(T, p)$ *and of* p . *It represents the forces interns of the system and all the other forces which are dependent on* $X(T, p)$.

$[G(X(T, p), p)]$
 Noting the tangent matrix $K(X(T, p), p) =$

, the equation of derived compared to p
 X
vector displacement is obtained:

$$M(p) \ddot{Y}(T, p) + C(p) \dot{Y}(T, p) + K(X(T, p), p) Y(T, p) = F(T, p)$$

T, p, p
 -

$$\text{2.4-2}$$

$$X \& (T, p)$$

$$(C)$$

$$-$$

$$X \& (T, p)$$

$G(X)$
)

-
 p

p

p

p

X fixed
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$G(p)$
With share the term
, all the terms derived from [éq 2.4-2] are calculated in a traditional way
 p X fixed
 $G(p)$
(i.e by assembly of the matrices or elementary vectors derived). The term
can
 p X fixed
to require a particular treatment. Indeed, in [éq 2.4-1], dependence between G and the value of
 $X(T)$,
 p is explicit at the moment T ; however G also depends on the history in plasticity. This
 $G(p)$

dependence complicates calculations clearly of that we will not approach here.

p X fixed

What is remarkable, it is that the derived problems [éq 2.4-2] are linear problems then that the standard problem is nonlinear. This property allows a calculation of really derived very economic compared to the cost calculation of the standard problem. This is all the more true as, for the resolution of [éq 2.4-1] and [éq 2.4-2], factorizations of matrices used for the calculation of vector displacement can be re-used for the calculation of the derived vector; as it is the case in the case of a linear standard problem. Indeed, the resolution of [éq 2.4-1] by an algorithm of newton type requires the resolution of a linear system whose matrix coefficient is the same one as in [éq 2.4-2]. Once the algorithm of the Newton type converged to obtain X (T

*)
n+1, the vector
gradient Y (T*

*)
n+1 can thus be obtained directly not a simple resolution of two systems triangular without convergence of an iterative algorithm.*

3

Derived from the frequencies and clean modes from vibration

**3.1 Problem
standard**

The problem with the eigenvalues (problem known as quadratic) associated [éq 2.1-1] or [éq 2.3.1-1] is written:

$$((p) 2M (p) + (p) C (p) + K (p))(p) = 0$$

éq

3.1-1

where (p) and (p) are complex values and clean vectors.

In the continuation, to gain in compactness of the equations, we will omit the dependence of the sizes compared to the parameter p and/or time T, except when the clarification of this dependence will be able to avoid ambiguities.

The reduction of the quadratic problem [éq 3.1-1] in a problem generalized equivalent [R5.01.02] give the system of size $NR \times NR = (N$

2) $\times (N$

2) according to:

- K O

CM

=

éq
3.1-2

O
M
MR. O

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One can standardize the clean vectors so that they check:

T
- K 0
J
J

=

éq

3.1-3

0

M

J

J

J

T

CM

J

J

=

1

éq

3.1-4

M

0

J

J

3.2

Derived from the Eigen frequencies

The multiple eigenvalues are not differentiable with the direction running of derived (i.e within the meaning of

Fréchet). We must thus distinguish the case from the simple eigenvalues of the case of the eigenvalues

multiples for which some precautions must be taken.

3.2.1 The case of the simple eigenvalues

The derivation of the equation [éq 3.1-1] written for the $j^{\text{ème}}$ mode compared to a scalar parameter p give:

$$\begin{aligned}
 & \left(\right. \\
 & J \\
 & J \\
 & M \\
 & C K \\
 & 2M + \\
 & 2 \\
 & J \\
 & jC + K) \\
 & = - (2 M + C) \\
 & - \\
 & + \\
 & +
 \end{aligned}$$

$$\begin{aligned}
 & J \\
 & J \\
 & J \\
 & \\
 & J
 \end{aligned}$$

$$\begin{aligned}
 & J \text{ éq 3.2.1-1} \\
 & p \\
 & p \\
 & p \\
 & p \\
 & p
 \end{aligned}$$

While multiplying on the left per T
 J two members of [éq 3.2.1-1] and while using [éq 3.1-1] and
[éq 3.1-4], the derivative of the eigenvalue can be obtained from:

$$J$$

T
= - 2 M
C
K
+
+

J
J

J

J
éq
3.2.1-2

p
p
p
p

The equation [éq 3.2.1-2] above requires the calculation of the clean vector J associated with J. One cannot thus not to exempt calculation of the clean vectors for the calculation of derived from eigenvalues.

3.2.2 The case of the multiple eigenvalues

In a clean subspace whose associated eigenvalue is multiple of order m, any combination linear of vectors is clean vector. One cannot however apply the formula directly [éq 3.2.1-2] with any of these vectors, because the clean subspace can be divided into m distinct clean subspaces when a parameter p varies.

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On the theoretical level, the multiple eigenvalues are not differentiable as a function with several variables, only of the derivative directional exist (this can pose problems of convergence with many algorithms of optimization which are based on the differentiability of function objectifies as a function with several variables). As example, let us consider the values clean of the following matrix:

 $4 + 2y$ X**2****2** **$1 = 4 + y +$** **$X + y$** **With =****,****X****4****2****2** **$2 = 4 + y -$** **$X + y$** **For $(X, y) = ($** **$0)$** **0 , the eigenvalue of A are multiple. And, one a:****1****= -1****1****= 1****2****= 1**

$$2$$

$$= -1$$

,

,

,

.

X

$$(0-, 0)$$

$$X(0+, 0)$$

$$X(0-, 0)$$

$$X(0+, 0)$$

It is possible “to connect”, 1 for $X < 0$ with 2 for $X > 0$, and “to connect” 2 for $X < 0$ with 1 for $X > 0$ (let us take for example $y = 0$. Then, “the smallest eigenvalue of A” $(2 - X)$ is not differentiable in 0, and “the largest either” $(2 + X)$. On the other hand, “the eigenvalue of A corresponding to the clean vector (,

1) T
1 “ $(2 + X)$ and “the eigenvalue of A corresponding to
clean vector (,

1 -) T
1 “ $(2 - X)$ is differentiable into 0). If one does not consider independently
the two Eigen frequencies of A, one could thus write $X = \pm 1$. (One can proceed of
even with the derivative in y and to write $y = 0$ or 2). However, eigenvalues of A
are thus not differentiable as functions with two variables (i.e one cannot

to write $D =$
 $D X +$
 $D y$

), and have only derivative directional. For of
 X

y
to convince, it is enough to notice that:

$$(\cos, \sin)$$

$$(\cos, \sin) - (,$$

$$0)$$

$$0)$$

$$= \lim$$

$$= \sin \pm 1$$

=

0

0

On the plan practises, for the calculation of these derivative directional, the choice of the clean vector J

associated J in [éq 3.2.1-2] is ambiguous when the eigenvalue is multiple. This choice thus must to be specified.

In the case of an eigenvalue m of order of multiplicity m , the problem with the eigenvalues [éq 3.1-1] can rewrite itself in the following matric form:

[
 $M] 2 + [$
 $C] + K [$
 m
 m
 m
 m
 $m] = 0$
 éq
 3.2.2-1

with $m = m I_m$ and $[] = [$,

m
 $i+1$
 $i+2,$

*,
 $L i+m]$ where the $i+k$ are the clean vectors associated Mr .*

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In any rigour and in the case general, the matrices M and K not being definite positive (with cause dualisation of the boundary conditions), it does not exist inevitably m vectors clean generating a clean subspace of dimension equalizes with the multiplicity of the value clean. This point is never approached in the literature. By convenience, we will not approach either this pathological point which becomes delicate when it is about calculation of derivative.

The conditions of orthogonality of the clean vectors are written with the matrix notations above:

[] T (2 m **M +)** **C [** **m** **m] = Im $éq$** **3.2.2-2**

Let us note $[X]$ the m vectors clean of the same multiple subspace clean which become m clean vectors $[X(p)]$ m subspaces two to two distinct when when a parameter p vary. When p tightens towards its face value P_0 , $[X(p)]$ tends towards $[X]$ and the m eigenvalues distinct tighten towards the eigenvalue multiple Mr . In the literature, such vectors clean are called clean vectors “adjacent”. It should be noted that these vectors depend on the parameter which varies

i.e. parameter by report/ratio to which one derives. In other words, if parameter is changed, it is then necessary to change adjacent vectors.

The adjacent vectors $[X]$ can be connected to $[m]$ by an orthogonal transformation T ($[X]$ and $[m]$ two bases of the same clean subspace constitute):

[] **$X = [m] T$** **$éq$** **3.2.2-3**

with $TTT = Im.$

m

The procedure consists in finding the transformation T for then deducing some $[X]$ and. It is it

p
that we develop below.

The adjacent vectors check the problem with the eigenvalues [éq 3.2.2-1]:

[
 $MR. X] 2$

$m + [$
 $C X] + K$

m
 $[X] = 0$

éq
3.2.2-4

The derivation of [éq 3.2.2-4] compared to p gives:

(
 X

M
 $C K$
 $2M +$

m
 2
 m
 $mC + K) [] = - (2mM +)$
 $C [X]$

-
+
+
 $[X]$

m

m

p
 p
 p

p
p
éq 3.2.2-5
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T

While multiplying [éq 3.2.2-5] on the left by [m], using [éq 3.2.2-2] and substituent []

$X = [m] T$, one

the new problem with the eigenvalues obtains:

$$DT = T$$

m

éq

3.2.2-6

p

T

M

C K

with $D = []$ 2

+

+

m

m

[

m

m]

.

p
p
p

m

The resolution of the problem to the eigenvalue [éq 3.2.2-6] gives the derivative of the eigenvalues

p
and stamps it orthogonal transformation T.

It should be stressed that there will be a subproblem with the eigenvalues of size m to be solved for each eigenvalue of multiplicity m to be derived and for each parameter.

3.2.3 Establishment by the finite element method

In a traditional way, several choices are possible for the calculation of the second member of [éq 3.2.1-2] and

matrix D of [éq 3.2.2-6] according to whether the matric products are calculated on an elementary level

or not.

The calculation of the second member of [éq 3.2.1-2] on an elementary level is a priori most powerful.

T 2 M

C

K

+

+

can be written directly on an elementary level by the scalar:

p

p

p

T

T

2

((

E

G

G))((G)) ((G)) +

(C (
G
has
G))((G)) ((G))

p
p
éq
3.2.3-1

T
+
E
E
E
G (B () (

G
G))
(H (G) () B () (
G
G))

p
In particular, a procedure not to start the calculation of each elementary term of

[éq 3.2.3-1] that if quantities

()
E
(H ())
(C ())

,
and
are nonnull allows

p
p
G
p has G
to minimize time CPU.

*In a way similar to the case of the simple eigenvalues, the matrix D of the system [éq 3.2.3-1] can
to be calculated directly on an elementary level or after effective assembly of the matrices*

MR. K
C

,

and

.

p

p

p

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3.3

Derived from the clean modes of vibration

3.3.1 The case of the simple eigenvalues

One cannot directly obtain the derivative of the clean vectors by solving the linear system

[éq 3.2.1-1] because it is a noninvertible system since $2M +$

J

J C + K

is singular (by definition of J).

Many methods were proposed in the literature to solve or circumvent this

difficulty. Among them, two types of methods emergent: methods of projection on basis

modal incomplete [bib4], [bib5] and algebraic methods [bib6], each one of them with theirs

alternatives. These methods are presented in [bib1]. The algebraic methods are methods

exact which, moreover, does not require the other clean vectors to calculate the derivative of one clean vector given. One presents only one algebraic method here consisting in adding one equation with [éq 3.2.1-1] to obtain a regular and symmetrical system to solve [bib6].

The rewriting of the condition of orthonormality [éq 3.1-4] gives:

$T(2jM +)$

C

J

$J = 1$

éq

3.3.1-1

The derivation of this equation compared to the parameter p leads to:

1

MR. C

$T($

J

T

J

2

$jM +)$

C

$+$

2

$M +$

$+$

J

J

J

$J = 0$

éq

3.3.1-2

p

2

p

$p p$

The equations [éq 3.2.1-1] and [éq 3.3.1-2] can be rewritten together:

(2

M + C +

J

J

K) (

2 M +

J

C)

J

J

p

T

J (

2 M +

J

C)

=

0

0

2 M

C

- (

2 M +

K

J

C)

J

-
+
+

J

J

J

J

P

P

P

P

I

T

J

MR. C

- 2

M +

+

J

J

J

2

P

P

P

éq 3.3.1-3

The resolution of the system [éq 3.3.1-3] is the key idea of the algebraic method. It can be shown that J

this system is always regular. The derivative of the clean vectors can be obtained

directly by solving this system of dimensions (

$N +$)

$I \times ($

$N +)$

I with a factorization **LDLT**

for example.

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The algebraic method preserves the structure in band and the symmetry of the matrices. Moreover, for to calculate the derivative of a clean vector, only the knowledge of this clean vector and value clean corresponding is necessary. Indeed, [éq 3.3.1-3] does not utilize the other vectors clean that that to derive.

However, the algebraic method requires the resolution of a system of size (

$N +$)

$I \times ($

$N +)$

1

different for each vector suitable to derive and each parameter.

3.3.2 The case of the multiple eigenvalues

[

X]

Taking again the notations of the paragraph [§3.2.2], the derivative of the clean vectors “adjacent”

p

[X] = [m] T are solutions of the equation [éq 3.2.2-5].

The matrix (2M +

m

mC + K) being singular, one cannot solve [éq 3.2.2-5] directly.

[

X]

The idea, still, is to add an additional equation for.

p

The condition of standardization of [X] is written:

T (2 jM +)

C

J

J = 1

éq

3.3.2-1

The derivative of [éq 3.3.2-1] gives the sought additional equation:

[

X

M

1

C

X] T (2 M

T

T

m

T

m

+) []

C

$$= [$$

$$- X]$$

$$[X] - [X]$$

$$[$$

$$MR. X]$$

$$- [X]$$

$$[X]$$

$$m$$

éq 3.3.2-2

p

[éq 3.2.2-5] and [éq 3.3.2-2] form the following system to solve:

(2

$$M + C + K$$

$$m$$

$$m$$

$$) (2 M+C$$

$$m$$

$$) [X] [X]$$

p

$$[X] T ($$

$$2$$

$$M + C$$

$$m$$

$$)$$

=

0

2

- (
2
M + C
m
)[]

m -
M
X

+
C

+ *K*
m
m
[*X*]

p

p
p
p

T M
T
m 1 T C
- [*X*]
[*X*] -
m
[*X*] *M* [*X*]
- [*X*]
[*X*]

p
p

2

p

éq 3.3.2-3

The system [éq 3.3.2-3] to solve is of dimension (

N +)

m × (

N +)

Mr. to calculate the derivative of one

clean vector, the knowledge of the clean vectors associated the eigenvalue is necessary.

As for the case of the nonmultiple eigenvalues, the structure in band and the symmetry of matrices are preserved and it can be shown that the system to be solved is always regular. Of more, the system must be solved for each vector suitable to derive and each parameter.

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3.3.3 Establishment by the finite element method

In the equations [éq 3.3.1-3] and [éq 3.3.2-3], the second members can be calculated either in MR. K

C

assembling the matrices initially,

and

then by carrying out matrix operations, that is to say

p

P

P

by directly calculating the various terms on an elementary level.

A direct calculation at the elementary level being more powerful, 2 M

C

K

+

+

, by

P

P

P

example, will be calculated on an elementary level [by \(cf \[§3.2.3\] for the notations\):](#)

2

((

E

))(T

)() +

(C (

E

))(T

)(

G

)

G

G

G

G

P

has

G

G

G**p****éq****3.3.3-1****T****+****E****E****E****G ((G))****(H (G) () () (****G****G))****p****4*****Derived compared to a variation from field***

The matrix equation [éq 3.1-1] uses an explicit dependence of the matrices of mass, of damping, and stiffness compared to the variable parameter p. When it is a question of deriving by report/ratio with a variation of field, one cannot in the case general write this dependence directly. In this case, obtaining derived from the dynamic sizes requires a work precondition. It is necessary on the one hand to describe the variation of field in a mathematical form which one can to handle, and in addition to return to the variational formulation of the equations of dynamics that one derives directly.

This chapter discusses these various items. One gives in particular the derivative of the various operators of mass, stiffness necessary to the calculations of derived from all the dynamic sizes. One limit then with the calculation of derived from the Eigen frequencies and clean vectors. One limits oneself to conservative problem and thus with the real clean modes.

4.1***Variational formulation of the standard problem***

One considers an elastic solid on a field, subjected to surface static loadings F and voluminal F. In variational form, the problem of balance is written:

To find U***V0***

:

that

such

éq

4.1-1

With (U): (v)

D = f.v

D + F.v

D

v

V0

F

where A is the tensor of elasticity, (U) the linearized traditional tensor of the deformations and V0 space

3

of Sobolev of (H1 ()) displacements kinematically acceptable.

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The clean modes around this balance are obtained by the resolution of the following spectral problem:

To find

real

clean

frequency

> 0

W

mode

and

V

:

that

such

no one

not

éq

4.1-2

With (W): (v)

D + W (U)

2

: v

D -

w.v

D = 0

v

V0

(notation: if A and B are two tensors of order 2, A B is the contracted product of A and B)

where is a parameter of possible amplification of the solution of [éq 4.1-1] for the loadings statics above, is the tensor of the constraints given by $(U) =$

With (U) and masses it voluminal of material considered.

This problem can be rewritten by using the following traditional multilinear operators:

.
elastic operator of stiffness has $(W, v) =$
With (W) : $(v) D$

.
geometrical operator of stiffness $B (U, W, v) =$
 W
 $(U): vd$

=
 $(U) W$

ik
 $L I,$
 $kl (v) D$

.
operator of mass $C (W, v) =$
 $W. vd$

in the form:

To find

reality

> 0 and $W V0$

:

that

such

no one

not

has $(W, v) + B$

$(U, W, v) - 2$

$C(W v$

$,) = 0 v V$

éq

4.1-3

0

$C(W W$

$,) = 1$

4.2

Description of the variation of field and form of derivation

One considers now that the field is likely to change form, by a variation of its edge (not included/understood the edge where are applied imposed displacements). To describe this

variation, one uses a bijective transformation making correspond the area of reference (that that one nets) with the field modified (representation known as Lagrangienne of variation of field, cf [R7.02.04]).

The edge of the field () is controlled by the scalar (such as () 0 =), according to the field of vectors.

The derivative of a field of vector X compared to a variation of field can be now written like the Lagrangian derivative, which one will note with a point (cf [R4.03.01]):

*&
X
D*

X
X =
=
+ X
éq
4.2-1
D

=0
=0

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And, the derivative of the gradient of X is written:

D ()
X
dX
=
-
X
éq
4.2-2
D
D

The derivation of the operators of mass and stiffnesses [\[§4.1\]](#) requires the derivation of sizes integrals whose field of integration depends on the scalar. These integral sizes are form:

$$I(U) = \int (E(U)) \quad (4.2-3)$$

where U is the solution of the problem of balance [eq 4.1-1] on the field Ω .

This integral for derived (theorem of Reynolds):

$$dI(U) = \int (E(U))_{,i} dx_i + \dots \quad (4.2-4)$$

$$dI = 0$$

The derivative intervening in the integrande is expressed:

$$\frac{\partial}{\partial U} (E(U)) = \dots$$

D

D

with

D (U)

()

T

T

=

- U - U éq

4.2-5

D

D

Note:

Implicitly, in the continuation the derivative will be taken in = 0.

4.3

Derivation of the variational formulation

4.3.1 Derivation of the elastic operator of stiffness

Using the symmetry of the tensor of elasticity, has (W, v) is also written:

has (W, v) =

With (

W): (

v) D

éq

4.3.1-1

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The derivation of this expression and the use of [éq 4.2-4] give:

D (

dA W
D v
has (W, v))
(
(
= A (
W): (
v) div +
:(
v) + A (
W):
D éq
4.3.1-2
D

D
D

With (
W)
(
W)
v
By [éq 4.2-5], and knowing
= A
= 0

and

, [éq 4.3.1-2] becomes:

$D ($
 W
has (W, v)

=

With (W): (v)
 D
div +
With
 : (v) -
With (W): (v)

D

D
éq
 4.3.1-3
 - $A (W)$: (v)
 $) D$

that one can rewrite, by using amongst other things again the symmetry of the tensor of elasticity:

$D ($
 W
has (W, v) =
 D
has (
 $, v) + a^{\wedge} (W, v)$
 D
 D
éq
 4.3.1.-4
 with $a^{\wedge} (W, v) =$
 (
With (W): (v) *div* - (W)
 : $v - (W)$: v
 $)$

D

4.3.2 Derivation of the geometrical operator of stiffness

In the same way, one obtains:

D

dw

B U

(, W, v) = (W U

): v) div

U

(): v

W

U

(): v

+

-

D

D

+ W

With

: v - W

With U

: v - W U

(): v

D

D

éq 4.3.2-1

that one can rewrite:

D

dw

$$B(U, W, v) = B(U, W, v) + B(U, v) + b(U, W, v) \text{ éq}$$

4.3.2-2

D

D

D

with:

b^U

$(, W, v) =$

$($

$(W U$

$): v) \text{ div} - W U$

$): v - W$

With U

:

v

$- W U$

$): v$

)

D

éq 4.3.2-3

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In [éq 4.3.2-2], one sees that the calculation of derived from the geometrical operator of stiffness requires

au préalable the derivative D of the solution of the problem of prestressed [éq 4.1-1].

This problem of prestressing, which is thus to derive, is written:

To find U V such as: has (,

$U v) = F. D$

$v + F.D$

$v = F (v) v$

V

0 éq

4.3.2-4

F

The derivation of this variational equation gives:

D

has

, $v =$

$F (v) - a\$ (,$

$U v)$

éq

4.3.2-5

D***D******with, by supposing that the forces F, F are independent of and:***

$$D F (v) = (f.v. (div) + (F.) .v) d+ \\ F.v div - \\ .n + F. .v$$

$$((, N) ()) dS$$

D***F******éq 4.3.2-6******(The formula of derivation of the surface integral is given by proposal 4 of appendix 2 of [R4.03.01]).******The resolution of the linear system associated [éq 4.3.2-5] giving D has the same matrix of rigidity that the problem of standard prestressing, only the second member changes. Code_Aster solves this problem in deformations plane and 2D-axisymmetric, cf [R4.03.01].******4.3.3 Derivation of the operator of mass******The derivation of the operator of mass gives:******D (******dw******C,******v W) =******v.w******+ . (******v.w) +******div******v.******D******D******D******that one can rewrite:***

$D ($
 W
 $C (W, v)) = D$
 $C ($
 $, v) + c^{\wedge} (W, v)$

D
 D
 $\acute{e}q$
4.3.3-1
with $c^{\wedge} ($,
 $v W) =$
 $(v.w$

div +. $(v.w)$

D

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4.3.4 Derivation of the variational formulation of the problem of modal analysis

Derivation in $= 0$ of the problem [éq 4.1-3] of calculation of the clean modes on (
), brought back on
(

)
0 =, the parameter of initial loading being fixed, gives now:

To find

reality

&

W

and

& V

:

that

such

has $(w \& v) + ^a (W v) + (B (\&, uw v) + B (U w \& v) + ^$

*,
 $B ($
 $U W v$
 $,)$*

*éq
 4.3.4-1*

*-
 $(2 \&c$
 $(W v$
 $,) + (C (w \& v$
 $,) + ^c (W v$
 $,) = 0v V$
 $2c (w \& W$
 $,) + ^c (W W$
 $,)$*

= 0

4.3.5 Obtaining derived from the Eigen frequencies and the clean modes

In a preoccupation with a simplification, we suppose here who the eigenvalue of the mode which one

drift is

simple. The case of a multiple eigenvalue results from the adaptation of the developments from [paragraphs \[§3.2.2\] and \[§3.3.2\]](#).

has

$$\begin{aligned} & (, \\ & W w) + (\\ & B U, \\ & W w) - 2 (\\ & C, \\ & W w) = 0 \end{aligned}$$

While using

and while taking $v = W$ in [éq 4.3.4-1], one

$$\begin{aligned} & (C, \\ & W W) = 1 \end{aligned}$$

obtains the following expression of derived from the own pulsation:

$$\begin{aligned} & \$a (, \\ & W W) + ((\\ & B \&u, \\ & W W) + \$b (U, \\ & W W) - 2 \$c (, \\ & W W) \end{aligned}$$

$\& =$

*éq
4.3.5-1
2*

Obtaining [éq 4.3.5-1] is to be brought closer that of [éq 3.2.1-2], except for standardization. In cases of eigenvalues multiple, the choice $v = W$ condition the calculated directional derivative. [éq 4.3.4-1] gives the following linear system:

$$\begin{aligned} & \text{has } (\&, \\ & W v) + (\\ & B U, \&, \\ & W v) - 2 (\\ & C \&, \\ & W v) = - a\$ (, \\ & W v) - ((\\ & B \&u, \\ & W v) + b\$ (U, \end{aligned}$$

*W v) éq
4.3.5-2
+ (2 & (
C,
W v) + c\$ (
W v))*

This system is same form as the system [éq 3.2.1-1]. It must be solved same manner, for example by using the addition of the equality (

*C &
W W) = - 1 c\$ (
W W).
2*

4.3.6 Establishment by the finite element method

In a way similar to the preceding chapters, terms of the second members [éq 4.3.5-1] and [éq 4.3.5-2] can be calculated, either directly on an elementary level, or after assembly matrices of mass, stiffness and damping.

In addition, of new procedures of elementary calculations must be developed for calculations of \$a (,

W v) (this term A already was developed in deformations plane and 2D-axisymmetric, D

cf [R4.03.01]), \$

B (U,

W v), \$c (,

W v), and

F (v).

D

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Effective establishment in Code_Aster

The numerical calculation of derived from the clean modes or the dynamic responses proceeds in two principal stages:

stage 1: calculation of the elementary terms constituting the new terms (often in second member) associated the calculation of the derivative,

stage 2: resolution of the derived problem. The calculation of derived from the clean modes requires the use of specific algorithms different from those used to solve the problem standard. The calculation of derived from the other sizes, i.e. Eigen frequencies and the dynamic responses, re-uses the standard algorithm by modifying some terms (in general second members of the systems to be solved).

Sensitivities of the other dynamic sizes (forced, efforts...) require a third stage which consists of the derivation of calculations of “postprocessing”. We do not treat these sizes in this document.

The following table recapitulates the derivative available in Code_Aster. It is updated at the fur and at measure developments of the derivatives.

Sizes and methods

Operators, and operands private individuals (i.e. Established data necessary to the method of calculation)

Derived from a dynamic response

harmonic

Direct method

DYNA_LINE_HARM

YES

Simple modal method

DYNA_LINE_HARM

NOT

clean vectors and derived clean vectors

Double modal method

DYNA_LINE_HARM

NOT

clean vectors

Derived from a dynamic response

temporal into linear

Direct method

DYNA_LINE_TRAN

YES

Simple modal method

DYNA_TRAN_MODAL

NOT

clean vectors and derived clean vectors

Double modal method

DYNA_TRAN_MODAL

NOT

clean vectors

Derived from a dynamic response

standard loading Dirichlet

NOT

temporal into nonlinear

DYNA_NON_LINE

for some laws of behavior

YES

Derived from an Eigen frequency

simple modes

YES

MODE_ITER_SIMULT, MODE_ITER_INV

multiple modes

NOT

Derived from a clean mode

simple modes

YES

MODE_ITER_SIMULT, MODE_ITER_INV

multiple modes

NOT

Derived compared to a variation

**** NOT***

of field

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***Parametric probabilistic models and not
parametric in dynamics***

Summary:

This document describes two probabilistic approaches, one parametric and the other nonparametric in order to take into account uncertainties of model and modeling for the dynamic systems in mechanics structures. The not-parametric approach is specific to the resolution on modal basis of the systems linear (in transient or harmonic) and with the resolution about modal base of the systems with localised non-linearities (in transient). The parametric approach is a priori universal, but it is more particularly presented if it is combined with the nonparametric approach.

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1 General information

Uncertainties, even when they are reduced, can change the prediction significantly of vibratory behavior of the structure ([bib13], [bib16]). It is thus necessary to take them in hope in a quantifiable and explicit way to increase the realism and the robustness of forecasts. In this context, a probabilistic model of uncertainties contributes to the realism of step.

The probabilistic approach traditional, known as parametric, makes it possible to incorporate in the mechanical analysis them uncertainties on the data, i.e parametric uncertainties on the geometry, conditions with limits or properties of materials. In this approach, each parameter identified like source of random uncertainties is modelled by a random variable. Parameters of entry of model being thus characterized, the probabilistic numerical methods seek to characterize probabilistic way the results quantities of the model. For complex structures, for which the vibratory behavior depends on a great number of parameters, this type of analysis probabilist is limited by the great quantity of information necessary to characterize them parameters of entry and difficulties of implementation of the propagation of variability.

A new approach, known as nonparametric probabilistic approach of random uncertainties in dynamics of the structures was recently proposed by C. Soize ([bib20] with [bib24]). This approach allows to take into account uncertainties of model (uncertainties on the geometry for example) and uncertainties of modeling (uncertainties on the kinematics of beam or plate for example). It is based on the construction of random matrices of the linear dynamic systems, afterwards projection on modal basis.

These two probabilistic approaches, one parametric and the other nonparametric, are complementary. Thus a mixed, parametric and nonparametric approach, can be is developed (original method having given place to publications ([bib6] and [bib11])). In particular, this mixed method is well adapted to the taking into account of uncertainties in the analysis of a system nonlinear dynamics composed of a structure linairre reduced on modal basis and of non-linearities located. Indeed, uncertainties on the level of the linear structure can be treated naturally by the nonparametric approach and uncertainties on non-linearities can be treated naturally by the parametric approach.

The basic digital model of the nonlinear dynamic system is a model finite elements which will be called “model fine elements means”. That it is about the parametric approach or not parametric, the laws of probability must be defined in an adequate way and most objectively

possible starting from this average model. A Gaussian model of the matrices random is not adapted to dynamics in low frequency (negative Eigen frequencies). In order to build the law of corresponding probability, one uses the principle of the maximum of entropy of Jayne ([bib14], [bib15], [bib17]) as well as information available (model fine elements means, algebraic properties of matrices, etc)

In this document, we present the nonparametric approach for transient resolutions or harmonic of the dynamic system. The parametric approach is more particularly presented if it combined with the non-parametric method.

The readers seeking the fundamental results of stochastic dynamics will be able to refer to [18] and readers seeking the theoretical details of the probabilistic approach presented in it document will be able to refer to [bib19]. Examples of uses of the approach are given in [bib5], and [bib7]. In [bib10], experimental tests made it possible to show the predictive character of approach.

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2

Modelings of the dynamic system

2.1

Average model finite elements

2.1.1 Transitory resolution in absolute co-ordinates

In the absolute reference mark, the mechanical system is modelled by the method with the finite elements. It

basic model (in general that which would have been used in the deterministic study) is indicated under name of “average model finite elements”. All the sizes relating to the average models are underlined.

That is to say T

$y(T)$

has

the transitory response in the absolute reference mark of the “model finite elements average” definite on the interval of study $[0, T]$ and at value in K

R where K is the number of d.d.l. Matrices of mass,

of damping and rigidity are respectively noted $[M]$, $[D]$ and $[K]$.

The transitory answer $y(T)$ of the “average model finite elements” checks the differential equation not linear discretized following:

$$[M] \ddot{y}(T) + [D] \dot{y}(T) + [K] y(T) + F(T, y(T), \dot{y}(T); W) = F(T), T \in [0, T], \text{ \acute{e}q}$$

2.1.1-1

C

with the initial conditions,

$$y(0) = \dot{y}(0) = 0, \text{ \acute{e}q}$$

2.1.1-2

1) $F(T, m)$

R represents the discretization by finite elements of the external forces.

2) $F(T, y(T), \dot{y}(T), W)$

C

$\&$

m

R corresponds to nonthe localised linearities (for example due to butted elastic of shock). The elements W ,

, W of the vector W

R represent a play

$1 L$

parameters defining these nonlinearities (for example play, stiffness of shock, damping of shock, etc).

2.1.2 Transitory resolution in relative co-ordinates (seism)

As in the transitory case in absolute co-ordinates, the mechanical system is modelled by one basic model, the “average model finite elements”.

T is noted

(T)

Z has

the transitory response in absolute co-ordinates of this model on the interval of study $[0, T]$ with value in K

R (attention: notice the change of notation compared to preceding paragraph.).

Transitory answer $Z(T) = (Z(T), Z(T))$ “average model finite elements” checks the equation S

following discretized nonlinear differential:

$[M]$

$[M] Z ($

$\&\& T) [D]$

$[D] z\& (T) [K] [K] Z (T)$

ls

ls

ls

+

+

$[M] T$

$[M] z\&\& (T) [$

$D] T [D] z\& (T)$

[

$K] T [K]$

$Z (T)$

ls

S

S

ls

S

S

ls

S

S

$F (T, Z (T), z\& (T); W) G (T)$

C

+

=

, $T [0, T]$, éq

2.1.2-1

0

$G(T)$

D

S

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with the initial conditions,

$Z(0) = z\&(0), Z(0) = z\&(0),$

éq 2.1.2-2

S

S

1) $G(T) m$

R represents the discretization by finite elements of the external forces and

$G(T) D$

R corresponds to the discretization of the forces of reaction due to D conditions of

S

Dirichlet $Z(T)$.

S

2) $F(T, Z(T), z\&(T); W) m$

R corresponds to nonthe linearities located with like

C

previously *W*

R representing a set of parameters defining these nonlinearities.

After static raising, matric equations [éq 2.1.2-1] and [éq 2.1.2-2] in the absolute reference mark are rewritten in "relative" co-ordinates:

$$[M] \dot{y}(T) + [D] y(T) + [K] y(T) + F(T, y(T), y(T); W) = F(T), T \in [0, T], \text{ éq 2.1.2-3}$$

C

$$y(0) = y(0) = 0,$$

éq

2.1.2-4

1) $y(T) \in m$

R is the vector of the free d.d.l in the "relative" frame of reference such as

$$Z(T) = y(T) + [R] Z(T) \text{ with}$$

l

$[R]$

$[]-$

$$= -K [K].$$

S

ls

2)

function *T*

$F(T)$

has

defined on $[0, T]$ and in value in *m*

R and the nonlinear application

$(X, y) \in F$

has

m

$R \times m$

R in *m*

R are such as

$C(T, X, y; W)$

$$F(T) = G(T) - ([M] [R] + [M]) Z(T) - ([D] [R] + [D]) z(T), \text{ éq}$$

$$2.1.2-5$$

2.1.2-5

ls

ls

F

= *F*

+

+

éq

2.1.2-6

$C(T, X, y; W)$

$C(T, X$

$[R] z_s(T), y [R] z_s(T); W).$

Note:

1) *In the continuation, according to whether or not a static raising were carried out, $y(T)$ corresponds*

either with the transitory response in absolute co-ordinates defined by [§ 2.1.1], or transitory response in “relative” co-ordinates defined by [§ 2.1.2].

2) *It is supposed that if D conditions of Dirichlet were homogeneous no movement rigid body could not occur. Consequently, $[K]$ is symmetrical defined positive and its reverse $[K] - 1$ is defined, which makes it possible to introduce the real matrix*

1
 $[R]$
 $[]-$
 $= - K [K]$ of dimension $(m \times D).$

ls

3) *In Code_Aster the term of damping in [éq 2.1.2-5] is neglected.*

2.1.3 Resolution

harmonic

As in the transitory case, the mechanical system is modelled by a basic model, it “average model finite elements”. On a frequential tape l , the harmonic answer $Q()$

1

$2]$

linear “model finite elements average” checks the following equation:

$(- 2$

$[M] + I [D] + [K]) Q () = F (), [, éq$

2.1.3-1

1

$2]$

with $F()$ representing the discretization by finite elements of the external forces.

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It is supposed that the energy of vibration of the dynamic response is mainly localised in field of the low frequencies. One can thus build the average matrix model reduced in projecting $y(T)$ or $y()$ on under clean space generated by N first modes of the system dynamics linear (infinite plays) conservative homogeneous (blocked supports) associated which is written,

$$[K] = [M]. \text{ éq}$$

2.2-1

The matrices $[M]$ and $[K]$ being definite positive (for $[K]$ cf notices 2 [§ 2.1.2]), values clean,

L are real and positive,

1 **N**

$$0 < L. \text{ éq}$$

2.2-2**1****2** **N**

The clean modes of vibration associated $\{L\}$ check the properties of orthogonality,

1**2**

$$\langle [M], \rangle = \mu, \text{ éq}$$

2.2-3

$\langle [K], \rangle =$
2
 $\mu,$
 $\acute{e}q$
2.2-4

with
=
.

éq 2.2-5

One respectively notes the matrix of generalized mass, the matrix of generalized stiffness and stamp damping generalized by:

$[M]$

$[M] []$, *éq*
2.2-6

$N = [$
 $] T$
 N
 N

$[K]$

$[K] []$, *éq*
2.2-7

$N = [$
 $] T$
 N
 N

$[D]$

[D] [],
éq
2.2-8
N = [
] T
N
N

2.2.1 Resolution in transient

Projection N

y (T) of y (T) on under space generated by N first modes of the system

homogeneous dynamics linear conservative associated is written:

N
N
y (T) = [] N
Q (T)
N
= Q (T),

éq
2.2.1-1
N

1
=
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Generalized displacements N **$Q(T)$ are solutions of the average reduced matrix model (system****dynamics not linaires),** **$[M] N$** **Q** **$\&\& (T) + [D] N$** **$q\& (T) + [K] N$** **$Q(T) + N$** **$F(T, N$** **$Q(T), N$** **$q\& (T); W) = N$** **$F(T)$, éq 2.2.1-2** **N** **N** **N** **C** **N** **$q\& (0)$** **N** **$= Q(0) = 0,$** **éq****2.2.1-3****with** **N** **$F(T) = [] T F(T),$** **éq****2.2.1-4** **N** **F_n** **$= [] T F(T, [] Q, [] p; W)$. éq****2.2.1-5** **$C(T, Q, p; W)$** **N** **C** **N** **N** **2.2.2 Resolution in harmonic****Projection N**

y () of y () on under space generated by N first modes of the system

homogeneous dynamics linear conservative associated is written

N

$$Q () = [] T F (),$$

éq

2.2.2-1

N

Generalized displacements N

Q () are solutions of the model matrix average tiny room

(- 2

$$[M] + I [D] [K] N$$

$$Q () = N$$

F () éq 2.2.2-2

N

N +

J

N

with

N

$$F () = [] T F (), \text{ éq}$$

2.2.2-3

N

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3 Model probabilist

3.1

Introduction of the probabilistic model into the dynamic problem

In order to take into account uncertainties of modeling and uncertainties on the data, one parametric nonparametric mixed formulation probabilistic is used. For that, the vector of N d.d.l generalized N

$Q(T)$ (resp. N

$Q()$) is replaced by a random variable N

$Q(T)$

(resp.

N

$Q()$).

In transient, the stochastic process

N

T

$Q(T)$

has

indexed by $[0, T]$ and with value in N

R is

solution of the nonlinear dynamic system,

$[M] N$

$Q&$

+ $D Q&$

+ $K Q$

+ F

Q

$Q&$

$W = F$

$\acute{e}q$

3.1-1

N

$(T) [] N$

N

$(T) [] N$

N

$(T) nc (T, N (T), N (T);) N (T),$

N

$Q (0)$

N

$= Q& (0) = 0,$

$\acute{e}q$ **3.1-2**

and in harmonic, the stochastic process T

N

$Q ($

has

) indexed on $[, and with value in $N$$

R

1

$2]$

is solution of the system:

$(- 2$

$[M + I [D + [K) N$

$Q () = N$

$F (),$

$\acute{e}q$

3.1-3

$N]$

$N]$

$N]$

where, in the two transitory and harmonic cases, $[M], [D]$ and $[K]$ are real matrices

N
N
N

symmetrical definite positive full random and where W is a random variable with value in R .

The introduction of random matrices into the equations [éq 3.1-1] and [éq 3.1-3] makes it possible to model

the random uncertainties associated the linear part of the dynamic system. The random variable W with vectorial value introduced into the equation [éq 3.1-1] makes it possible to model uncertainties random concerning the parameters of nonthe linearities of shock.

The parametric probabilistic approach and the nonparametric probabilistic approach introduce random matrices ($[M]$, $[D]$ and $[K]$) and a random variable W of which laws of probability

N
N
N

are a priori nonknown. The choice of a probabilistic model rather than another must rest only on information available (algebraic properties of the generalized matrices, values averages of the parameters and the generalized matrices, etc). In order to build the laws objectively of probability of the probabilistic model of uncertainties, ([bib20] with [bib24]), the principle of the maximum

of entropy ([bib14], [bib15], [bib17]) is used with a system of constraints defined by this information available. Information available and the probabilistic model which results from this are presented

in the next paragraph.

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3.2 Probabilistic model for the matrices of the dynamic system (nonparametric uncertainties)

3.2.1 Information available on the matrices of the dynamic system

The nonparametric probabilistic model is built in substituent the matrices [M], [K] and

N

N

[D] by respectively noted random matrices [M], [K] and [D]. So that the system

N

N

N

N

probabilistic dynamics thus built either mechanically and statistically correct, construction random matrices [M], [K] and [D] must be such as:

N

N

N

1) [M], [K] and [D] is of the random variables of the second order with values in

N

N

N

the whole of the positive definite real matrices symmetrical and dimension $(N \times N)$.

[M], [K] and [D]

+

M P.S. (almost surely),

éq

3.2.1-1

N

N

N

N

where

+

M is the whole of the positive definite symmetrical matrices real of dimension

N

$(N \times N)$.

This algebraic property is absolutely required to have a random model of equation which corresponds to that of a dynamic system of the second deadened order.

2) Average values of the random matrices $[M]$, $[K]$ and $[D]$ are respectively

N

N

N

$[M]$, $[K]$ and $[D]$:

N

N

N

$E [$

$\{M = M, E \{K = K \text{ and } E \{D = D, \text{ éq 3.2.1-2}$

$N\}$

$[N]$

$N\}$

$[N]$

$N\}$

$[N]$

where E indicates the expectation.

3) So that the solution of the probabilistic dynamic system is also a variable of the second order, one imposes on the moments of the second order standards of Frobenius of the matrices opposite

1

$[M]$ -,

1

$[K]$ - and

1

$[D]$ - to be finished:

N

N

N

$E \{ \| [M] - 1 \| 2 \} < +, E \{ \| [K] - 1 \| 2 \} < +, E \{ \| [D] - 1 \| 2 \} < +, \text{ éq 3.2.1-3}$

N

F

N

F

N

F

$1/$

with $[\]$

With

$= (tr \{ \{$

$WITH \} T$

With

.

F

} 2

Note:

The only property of positivity of the matrices is not enough and only their opposite should be secured are of the second order, from where (3 (a random variable of the second order almost surely invertible in the case general a random variable does not have reverses of the second order). For details to see more [bib19].

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3.2.2 Construction of the probabilistic model by the principle of the maximum of entropy

The entropy “measures” the level of uncertainty of a law of probability. Thus, if p is related to

[A]

density of probability corresponding to a random matrix [A] (representing the matrices [M],

N

[K] or [D]) of law given, then the entropy (or probabilistic uncertainty) S (p) of p is defined

N

N

[A]

[A]

by:

~

$S(p)$

,

éq**3.2.2-1****With**

= -

 $p([A]) \ln(p([A])) dA$

[]

 $M + [A]$ **$[A]$** **N**

The principle of the maximum of entropy of Jayne consists in building the function of density of probability

who maximizes the probabilistic entropy $S(p)$

) while checking a system of constraints. In the case

 $[A]$

present, the system of constraints is defined by information available corresponding to the equations [éq 3.2.1-1] with [éq 3.2.1-3]. For the random matrix $[A]$, this system of constraints is written

+

 $[A] M P.S., E\{[A]$ **$\} = []$** **WITH, $E\{\| [A] - 1 \|^2\} < +.$** **éq 3.2.2-2** **$N$** **$F$**

It is shown whereas the random matrix $[A]$ is such as (see [bib20] with [bib24])

 $[A] = [L] T [G] [L],$ **éq****3.2.2-3****With****With****With**

where $[L]$

A is the lower triangular matrix resulting from the factorization of Cholesky of the matrix average []

With and where the function of density of probability of the random matrix $[G]$ is defined on

With

~

the unit

+

M compared to dG measurement such as:

 N

~

$N(N$

dG

-

=

)

$1/4$

2

$dG,$

éq

3.2.2-4

$1 I jn$

ij

1

(

2

-) ($2 2$

) 1

- ($n+$)

1

- ($n+$) (

$1 2 2$

) 1

- $tr [G]$

With

With

With

P

$([G]) = I$

, **éq**

3.2.2-5

G

×

×

×

+ ($[G]$)

C

$(det [G])$

E

[

]

G

With

M

With

N

with

2

1

$N(n+)$ (

1 2)-

With

-

-

+

$N(N)$

1/4

$N 1$

(2)

2

2

With

C

=

,

éq

3.2.2-6

GA

$N +$

- *J*

N

=

J 1

(1 1

+

)

2 2

2

With

where

$$(Z) = + z-1 - T$$

T

E dt,

éq

3.2.2-7

0

and where **1**

M

+ is the indicating function of

+, and where the parameter

controlling the dispersion of

M

N

With

N

random matrix [A] is defined by:

{[G

G

With] - [

To] 2

E

F} 1 2

, éq

3.2.2-8

With =

[GA]

2

F

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The theoretical construction of the model provides an acceptable terminal for the level of uncertainty introduced.

A must be selected so that

 N 1 0 0 $+$ $<$ $<$

,

*éq***3.2.2-9***With* $N + 5$ 0

where N NR is a constant of the probabilistic model selected so that $N < N$.

 0 0

One shows moreover than, under the only constraints of the equations [éq 3.2.1-1] with [éq 3.2.1-3], it principle of the maximum of entropy leads so that random matrices $[\mathbf{M}]$, $[\mathbf{K}]$ or $[\mathbf{D}]$ are

 N N N

statistically independent as a whole.

This probabilistic model for the positive definite symmetrical matrices random real differs from more traditional models of the random matrices based on the Gaussian Sets and them circular sets (references in [bib19]). The orthogonal Gaussian unit used in addition for fields high frequencies would lead in the field low frequencies (in which one

places itself) at negative eigenvalues, which one cannot admit for the systems considered. Moreover, one matrix of the orthogonal Gaussian unit does not have in the case general an opposite matrix of the second order, which would lead to a solution of the dynamic system of infinite variance, which one cannot admit either.

3.3 Probabilistic model for the real variables (uncertainties parametric)

3.3.1 Information available on the real variables

In the mixed probabilistic approach, parametric probabilistic modeling consists in substituting it parameter \mathbf{W} of non-linearities in the nonlinear dynamic systems given by [éq 2.1.1-1] or [éq 2.1.2-3] by a noted random variable $\mathbf{W} = (W_1, \dots, W_K)$

\mathbf{W} ,
,
) . In an approach purely parametric (i.e without making random the matrices of the system dynamic), modeling parametric probabilist consists in substituting certain parameters \mathbf{W} of the matrices $[M, N(\mathbf{W})], [K, D_n(\mathbf{W})], N(\mathbf{W})]$ and $[\dots]$ of the average dynamic system reduced by a random variable \mathbf{W} . These parameters can be for example parameters of material.

It is supposed that the components of \mathbf{W} are independent real random variables enters they and independent of the random matrices of the dynamic system. In the continuation, to reduce the writing, one notes W an unspecified co-ordinate W_j . The construction of the probabilistic model require to define the information available, which constitutes a system of constraint under which the entropy of the density of probability of the random variable W is maximized.

Information available is as follows:

1) The support of the random variable W is an interval D of \mathbb{R} .
 $W \in D$.

p .
 S .
éq

3.3.1-1

2) The average value of the random variable W is W :

$E \{W\} = W.$

éq

3.3.1-2

3) Possibly, according to information indeed available, the moment of the second order of
1

-
the random variable W
is finished:

$E \{2$

-

W
 $\} < +.$

éq

3.3.1-3

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3.3.2 Construction of the probabilistic model by the principle of the maximum of entropy

If p is related to density of probability corresponding to the random variable W then the entropy
 W

(probabilistic uncertainty) $S(p)$ of p is defined by:

W

W

$S(p) = -$

$\int p(W) \ln(p(W)) dw,$

éq

3.3.2-1**W****+ W****W****-**

By using the principle of the maximum of entropy, one obtains three densities of probability according to nature

support D and according to whether the constraint corresponding to the equation [éq 3.2.2-6] is considered or not.

3.3.3 Closed support limited without information on the reverse

If there are two real has and B such as $D = [has, B]$ and Si information available is given by equations [éq 3.3.1-1] and [éq 3.2.2-5], then the random variable W follows a truncated exponential law

whose function of density of probability is:

 K **$p()$** **$W = \mathbf{1}$** **$()$** **W** **$\exp(-)$** **kw** **éq 3.3.3-1** **W** **$[has, +]$** **(K)** *where $\mathbf{1}$* *is the indicating function of $[has, B]$* *and K are such as* *$[has, B]$* *and where (K)* **(K)** **$W -)$** **$\mathbf{1}(K) - K(K) = 0$, éq****3.3.3-2***with*

- ak

- B K

(K) = E - E,

éq

3.3.3-3

and

- ak

- B K

(K) = E - Be has. éq

3.3.3-4

3.3.4 Closed semi support not limited without information on the reverse

If there is real A such as D = [has, + [

and if information available is given by the equations

[éq 3.3.1-1] and [éq 3.3.1-2] then the random variable W follows an exponential law of which the function of

density of probability is:

1

W - has

p ()

W = 1

()

W

exp (-

), éq

3.3.4-1

W

[has, +]

W - has

W - has

where 1

is the indicating function of [has, + [

.

[0, +[

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3.3.5 Closed semi support not limited with information on the reverse

*If there is real A such as $D = [has, + [$
and if information available is given by the equations.*

*[éq 3.3.1-1], [éq 3.3.1-2] and [éq 3.3.1-3], then the random variable W follows a law gamma of which
function of density of probability is,*

2
(
W
-
2 -1/
has)
2
2
2
-
(1)/
W - has
p ()
W
1
,
éq 3.3.5-1
W
=
()
W
(W has)
exp -
[has, +]
-
2

2

1

(/)

(W - has)

where is a parameter controlling the level of uncertainty of the random variable which is written W (way similar to the nonparametric case [éq 3.2.2-8]):

*({W - W) 2**E**} 1/2*

=

*, éq**3.3.5-2*

2

W

3.4 Construction of the stochastic answer and the statistics associated

3.4.1 Case transient

Stochastic 3.4.1.1 transitory Answer

The excitations of the dynamic system are supposed to be deterministic, but in the paragraph [§3.1], of matrices and of the random parameters were introduced into the reduced matrix model. Therefore, transitory answer

*N**T**Q (T)**has*

is a nonstationary stochastic process indexed by [0, T] with value in

N

R (by using some additional assumptions of existence, unicity and of regularity of the deterministic solution, cf [bib19]).

Consequently, with the vector of the m d.d.l free N y (T) corresponds the stochastic process

N

Y (T) indexed by [0, T] and with value in m

R such as

N

Y (T) = [] N

N Q (T),

éq

3.4.1.1-1

In the case of the passage in relative co-ordinates, with the stochastic process

N

T

Y (T)

has

indexed by

[0, T] and with value in m

R defined by the equation [éq 3.4.1.1 - 1] corresponds the stochastic process

N

T

Z (T)

has

free d.d.l of the structure in absolute co-ordinates indexed by [0, T] and with value in m

R such as

N

Z (T)

N

= Y (T) + [R] Z (T).

éq

3.4.1.1-2

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3.4.1.2 elastic Spectrum of answer

N is noted

Z (T) the component jème of vector N

Z (T) correspondent with a random realization of

J

stochastic response of the free jème d.d.l of the structure. N

Z (T) perhaps characterized by its spectrum

J

of elastic answer (also called spectrum of oscillator in Doc. of Code_Aster) that one

note S (,) where and

J

are respectively the associated rate of depreciation and the pulsation.

With reasonable assumptions, in particular on the regularity of the nonlinear application

(T, Q, p; W) Fn

has

N

C (T, Q, p; W), one can show that Z (T) is a process of the second order of which

the trajectories are almost surely continuous. Consequently, for very fixed in an interval

J

has

given,

S (,) is a stochastic process indexed on the tape of analysis J

J

with value

in +

R. It is admitted that this process is of the second order, i.e.:

2

{

E S (,)} < +,

J. éq

3.4.1.2-1

J

3.4.2 Case

harmonic

In the paragraph [§3.1], random matrices and parameters were introduced into the model matrix tiny room. The harmonic answer T

*N
 Q (
has
) is thus a stochastic process indexed on
 $]$, with value in N
 R .
 1
 $2]$*

*Consequently, with the vector of the m d.d.l free N
 y () corresponds the process
stochastic
 N
 Y ()*

*indexed on $]$
, and with value in m
 R such as
 1
 $2]$
 N
 Y () = $]$ N
 N Q ()*

*éq
3.4.2-1
 N
 Y (
the component j ème of the vector
 N
 Y () is a random variable which one will admit of the second
 J
)
order.*

3.4.3 Construction of the stochastic response by the Monte Carlo method**3.4.3.1 Choice and implementation of the Monte Carlo method**

The answers and the spectra of answer correspond to strongly nonlinear transformations

random matrices and random parameters which result from probabilistic modeling from uncertainties. Moreover, one can of course build only numerical approximations as of these answers and of these spectra of answer. The statistics (first moments statistical, probability of going beyond of a threshold,...) are written formally like multiple integrals of very large dimension because the number of random variables of the probabilistic model is by construction high. Lastly, the number of sizes observed is very large (several ddl for several frequencies). For all these reasons, method the most adapted to calculate the probabilistic solution (answer stochastic and associated statistics) is the method of digital simulation of Monte Carlo.

The method of simulation of Monte Carlo has the advantage of giving results which one can to control the precision (checking of convergence, cf [§ 4.1]), contrary to the majority of methods based on approximations. It can be expensive in computing times, but the use of the techniques of reduction of the variance can make it possible to reduce the number of simulations necessary (cf [bib8] or [bib9]).

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The implementation of the Monte Carlo method consists for the problem which concerns us with to generate NS achievements of the random matrices $[M]$, $[K]$ and $[D]$ of the dynamic system and/or

N

N

N

NS achievements of the vectorial random variable W . Resolutions of the dynamic system determinist for each NS achievements of $([M], [K], [D], W)$ produce N achievements

N

N

N

S

stochastic process solution

N

T

Q (T)

has

(resp. T

N

Q (

has

)) and in consequence of

N

T

Y (T)

has

, of

N

T

Z (T)

has

has

N

and of

S (,) (resp.

Y (

has

). The generation of the random matrices is

J

)

J

treated in the following paragraph; the generation of the random variable W is more traditional and is not not recalled.

3.4.3.2 Generation of the pseudo-random matrices

In order to generate the achievements of the random matrix [G A], the algebraic representation is used

following of the random matrix [G A] of which the law of probability is defined by the equations [éq 3.2.1-2], [éq 3.2.2-1]:

[G] [L] T

With =

[L], éq 3.4.3.2 - 1

the triangular random matrix [L] being such as:

1) The random variables $\{[L]_{ij}, I, J\}$ are independent.

2) For

$I < J$, the real random variables $[L]_{ij}$ are written $[L]_{ij} = N U_{ij}$ where $N = A(n+1) - 1/2$ and where U_{ij} is a Gaussian real random variable of average 0 and variance 1.

3) For

$I = J$, the real random variables $[L]_{jj}$ are written $[L]_{jj} = N (2 V_j)^{1/2}$ where N is defined previously and where V_j is variable real positive random of law gamma of which the function of density of probability $p(v)$ compared to measurement FD is written:

v^J

1

(v

$[0, +$

)

[

$(n+)$

$1/(2^2$

$) + -$

1

(

$J)/2 - v$

With

$p(v) =$

v

$E,$

éq 3.4.3.2 - 2

v^J

$((N +)$

$1/(2^2$

$) + 1$

$(- J)/$

2

With

where 1

is the indicating function of [,

0

[

+ .

$[0, +[$

3.4.4 Statistics on the spectra

In this chapter, one presents the definition of the statistics of the spectra of elastic answer

S (
, in the case of a transitory resolution. In the harmonic case, the statistics on
 J),
random variables N
 Y (
are defined in the same way and are thus not presented.
 J
)

3.4.4.1 Estimate of the quantiles

For all
 $(,) J$
 S
is a random variable with value in $+ R$. *One seeks with*
 J
 $\times J$,
 $(,)$
to consider the quantile associated with the probability noted S (
and defined by:

J),
 ,
 S (
 ϵq

3.4.4.1-1

J),
 I
 $= -$
 F
 I
 $($
 $-)$
 ,
 ,

where
 S
 .
 J

F is related to unknown distribution of
 $(,)$

,
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That is to say (S (
,..., S (;),..., S (;)) the sample made up of N achievements of
J;)

I
J
R
J
S
S
N
S (
and (S (
,..., S (;),..., S (;
)) the associated ordered sample.

J;
)
J,)
)
I
(
J
(R)
J
(

)
S
N

R
A natural estimator of the quantile S

(
for =
, I R N is:
J,)

,
N
S
S

R
S J, (,) = S (;) éq
3.4.4.1-2

J
(R)
NS

To obtain a more robust estimator of the quantile, one can “realise” the estimator on several series of NS achievements. If the desired probability is such as $< 1/N$, or Si one wishes to reduce S the number of simulations, it is possible to use more sophisticated estimators, for example in supposing that the function of distribution F belongs to a field of attraction given (theory of extreme values) or for example by using a method of regularization bayésienne (cf [bib12]).

Extreme 3.4.4.2 Values of sample

For a sample of NS achievements of S (noted S (,..., S (;), one defines

J;)
J,)
I
J
NS

extreme values of sample by:

dB
(
has
; ***N***)
S

éq
3.4.4.2-1
J, min
S
= log
min
(, ;)
10
J
R

r=, ...,
1
NS

dB
(
has
; ***N***)
S

éq
3.4.4.2-2
J, max
S
= log
max
(, ;)
10
J
R

r=, ...,
1
NS

3.4.4.3 “Field of confidence” established starting from the inequality of Tchebychev

For a sample of NS achievements of the process

S (

has

,) noted

S (

has

, ...,

J ;))

J

1

S (

has

;), one can build the “field of confidence” of the random variable

J

NS

dB (,) = \log (S (,)) for all (,) $J \times J$, by using the inequality of Tchebychev

J

10

J

associated a level of probability P :

C

Proba $\{dB$ (,) < dB (,) < $dB+$ (,) P ,

éq

3.4.4.3 - 1

J

J

J

} C

where the lower envelope

dB (

has

,) and the higher envelope

$dB+$ (

has

,) are defined

J

J
by:
(,
+
dB (,) log
J
m (,
,

éq 3.4.4.3 - 2
J
=
10 1 J
+

1 - PC
-
dB (
m
dB
. **éq**
3.4.4.3-3
J,) = 2 log
J
+
-
J
10 (
(,)
1
)
(,)

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with $m(\cdot)$ the average and (\cdot) the standard deviation of dB (

:

J,

1 J

J

***$m(\cdot) = \{$
 $E S(\cdot)\}$, éq***

3.4.4.3-4

1 J

J

=

S

- m

.

éq

3.4.4.3 - 5

J (,

E ({J (,

J (,) 2

1

} 1/2

The “field of confidence” thus built proved to be a good approximation of the values extremes of sample for the case treated in [bib21]. However, this “field of confidence” does not utilize that the first two moments whose consistent estimators more quickly with respect to the number NS of simulations that extreme values of sample. It can thus be interesting to use this construction of the “field of confidence” rather than a construction based on the estimate of the quantiles more expensive in a number of simulations.

Note:

The term “field of confidence”, can be considered by certain an abuse language. One should rather use the less intuitive terminology “inter-quantiles field”. In effect, in the statistical literature, a confidence interval is theoretically the interval in which the true value of a parameter of a random variable is (for example its average) with a given probability. This terminology is employed within the framework very precis of the theory of the ensemblist estimate. The confidence interval is not one characterization of the variability of a random variable, contrary to a standard deviation or to quantiles. One nevertheless uses " field of confidence " with parsimony in the continuation, because it is certainly a little more speaking for the non-specialists of the statistics.

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4

Implementation in Code_Aster

4.1

Study of the stochastic convergence of the digital model

4.1.1 Case

transient

The convergence of the stochastic solution must be studied compared to number N of modes and with N numbers

N

S of simulations of Monte Carlo. As stochastic solution $\mathbf{Z}(T)$ is a process of second order (by assumption, cf [§ 3.4.1.2]), its convergence can be analyzed by studying them applications N

/// N

\mathbf{Z} & & ///

has

such as:

J

2

T

$\mathbf{Z} N$

&

=

2

E & ()

,

éq

4.1.1-1

J

{Z N T

J

} dt

0

where

N

T

Z&& (T)

has

is a stochastic process of the second order indexed by [0, T]

J

and with value in R

representing the acceleration of the jème d.d.l of the structure.

In the framework of simulations of Monte Carlo, this standard ||| N

Z&& ||| is estimated for N fixed to leave

J

of a whole of N

N

N

S random achievements Z

&& (T;), Z&& (T;

L

) by the approximation

J

1

J

S

N

||| N

Z&& ||| conv (,

NN) with

J

J

S

2

T

NS

1

conv (N, N2

) =

$Z N$
& (T;) dt éq
4.1.1-2

J
S

0
N
J
I
S I I =

The stochastic convergence of the model is thus analyzed according to the dimension of the small-scale model

(i.e. the number of mode N of under clean space of the average model finite elements on which the stochastic nonlinear dynamic system was projected in the paragraph [§ 2.2]) and numbers it NS simulations of Monte Carlo by studying function N

conv (,
N N)
S.A.

·
J
S

4.1.2 Case harmonic

Convergence in the case of a transitory resolution can be transposed directly in the case of a harmonic resolution, with the standard:

2

$Z N$
=
2
2
E
()

,
éq
4.1.2-1

J
{Z nj} D

1

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4.2

Choice of the parameters of dispersion

To use the method, the parameters of dispersion must be fixed. Two approaches can be used a priori to fix the value of these parameters.

The first approach consists in identifying the value of the parameters for a given structure or for a class of structure using suitable methods. For that, one can use results experimental of the dynamic responses of the structure. One can also use simulations numerical built by using a parametric approach of uncertainties. In this last case, it should be noted that only the errors on the data of the model are taken into account, since them errors of modeling cannot be taken into account by the parametric approach.

The second approach consists in not fixing a priori a fixed value of the parameters but at to vary in a beach given (only 3 scalars to vary for the matrices of mass, of stiffness and damping on the not-parametric part in comparison with the very large one a number of parameters to vary simultaneously in a traditional parametric study). This approach makes it possible to carry out a total analysis of sensitivity to uncertainties. In the case of absence of objective information on the parameters dispersion to be chosen, it is preferable to use such approach. The non-parametric method suggested seems a robust approach then and simple of analysis of sensitivity to uncertainties.

4.3 Principal

stages

The implementation in *Code_Aster* is made up of three principal stages: the construction of average reduced matrix model, the generation of the achievements of the answer seen like a process stochastic, and finally the statistical postprocessing of these achievements. Two last stages constitute in fact the method of digital simulation of direct Monte Carlo.

Stage 1: construction of the average reduced matrix model

The average reduced matrix model is built using a traditional sequence operators depending on the precise analysis carried out whose principal ones can be: *ASSE_MATRICE*, *MODE_ITER_SIMULT*, *MODE_STATIQUE*, *CALC_CHAR_SEISME*, *MACRO_PROJ_BASE*...

Stage 2: generation of the achievements of the transitory answer

The NS achievements of the stochastic transitory answer are calculated in a loop in language Python made up of:

1) Generation

pième achievements of the random generalized matrices of mass, stiffness and of damping by *GENE_MATR_ALEA* (Doc. [U4.36.06]). These matrices are not diagonals and thus require a full storage.

2) Generation

pième achievements of the random variables of the parameters of non-linearities by *GENE_VARI_ALEA* (Doc. [U4.36.07]).

3) Calculation of the pième realization

N

$Q(T; p)$ or N

$Q(; p)$ solution of the matrix system

stochastic S . This realization is the solution of the traditional matrix system of which them matrices and the second members are the achievements previously generated. Calculation is thus carried out by *DYNA_TRAN_MODAL* or *DYNA_LINE_HARM* (with *matr_asse_GENE_R* and *vect_asse_GENE* in entry).

4) 1-

Extraction of the temporal observations of the preset physical d.d.l (by example $\&ZN(T; p)$ or $YN(; p)$, but also possibly fields of displacement,

I

J

speed, constraints, etc) via *RECU_FONCTION* (after a *REST_BASE_PHYS* for $YN(; p)$).

J

2-

Calculation of the spectra corresponding (by *CALC_FONCTION* (*SPEC_OSCI*)) for S has

$(; p)$ and

YN (; p).

J
CALC_FONCTION (MODULE) for
)

J
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5) Evaluation, via CALC_FONCTION key words COMB or POWER or ENVELOPE, of contributions to the estimators of the averages, the moments of order two, the values max. extremes and min. of sample for the standardized spectra:

$$m(p) = S(p) +$$

$p - 2$

$p =$

$p +$

$p - 1,$

J

J

J

$m(;$

$)$

1

1

$1 J$

$m(;) S(;$

J

J

$m(;$

$)$

2
 2 J
 \$S
 (; p) = My
 p
 p - 1, \$S
 (; p) = Semi
 p
 p - 1.
 J, min
 {NS (;), \$S (;
 J
 J, min
 })
 J
 {xS (;), \$S (;
 J
 J
 })
 , max
 , max

Stage 3: statistical postprocessings

Averages, standard deviations, max. extreme values and min. of sample for the spectra standardized can be evaluated via CALC_FONCTION (COMB):

1
 1
 m (,) =
 N, m (,) =
 N.
 J
 \$m (;)
 1
 1 J
 S
 N
 J
 \$m (;)
 2
 2 J
 S
 N
 S

S

The confidence intervals can then be traced starting from the extreme values of sample or terminals obtained by Tchebychev cf [§ 3.4.4].

In the transitory case, an example is given by a case test of a flexbeam with not linearities of shock, cf Doc. [V5.06.001] [bib1]. Other details are given in Doc. [U2.08.05] [bib2].

4.4

Numerical effectiveness of the nonparametric approach

The nonparametric approach is more economic in computing times than an approach purely parametric in which the parameters of geometry, materials, etc are random variables.

In the purely parametric approach, the model finite elements depends on the dubious parameters. For each simulation of Monte Carlo, the model finite elements is different. It is thus necessary, for each simulation, to calculate the elementary matrices, to carry out the assemblies, to pass in relative co-ordinates, to solve the problem with the eigenvalues, to project on modal basis, to solve the reduced system and to return in physical base then in relative co-ordinates.

In the nonparametric approach, only the reduced system is different with each simulation. It is thus simply necessary, with each simulation, to solve the reduced system and to return in base physics then in relative co-ordinates. In particular, the resolution of the problem to the eigenvalues model average finite elements is carried out once and for all, before simulations of Monte Carlo.

The saving of time of calculation which results from it is variable, but it can be important. In first approximation, this saving of time calculation depends on the ratio between time CPU necessary to the resolution

at the eigenvalues and time CPU necessary to the resolution of the reduced system. More this ratio is large, more the nonparametric approach is advantageous compared to the approach purely parametric. In particular, the saving of time of calculation can be very important for structures with a very great number of degrees of freedom and a low-size modal base.

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Algorithm of retiming

Summary:

In this document the algorithm of retiming of MACR_RECAL is presented. It is about an algorithm of

Levenberg-Marquardt with terminals.

One initially describes the general method before specifying certain elements of them. Are detailed it calculation of the functional calculus, the Jacobienne matrix, determination of the initial parameter of regularization thus

that its evolution, the management of the terminals and the criterion of convergence.

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1 Introduction

Before approaching the problems of retiming strictly speaking, it is useful to recall some elements on the identification of parameters. Let us suppose that one wishes to identify N parameters to be left

of a given mechanical test. Within the framework of this identification, one defines the sizes:

.

C , the vector of N parameters to be identified, pertaining to O , convex closed N

R .

.

D , the vector of the sizes calculated during a simulation of the test by using them parameters C , in opposition to exp

D , the vector of the sizes measured during a test

experimental. Both belong to space L of the observable sizes.

simulation of the experimental test, parameterized by the vector C , can be carried out by various methods: finished differences, finite elements, elements of border,.... It is it that we will call the direct problem.

The goal of the identification is to determine the set of parameters C reducing the difference enters sizes measured and experimental (by strongly hoping that the reduction of this variation is sufficient to obtain the set of parameters wished...). A cost functional calculus is thus introduced noted J dependent on C and measuring the distance between D and exp

D .

$J(c)$

exp

$= D - D$

éq 1-1

where $\| \cdot \|$ indicates a standard on L .

The identification is thus expressed in the form of the problem of minimization according to:

To determine C

** O such as $J(c)$*

** = $\text{Min } J(c)$*

$C O$

Lastly, one defines retiming as the minimization of a particular type of said functional calculuses "least squares" which are expressed in the form:

NR

$J(c) = \sum_{j=1}^n j^2 \text{éq 1-2}$

$N(c)$

$n=1$

It is commonly allowed that the most effective algorithm of the minimization for this type of functional calculuses is the algorithm of Levenberg-Marquardt. It is the latter which is established in order MACR_RECAL of Code_Aster and that we present in the continuation.

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2 *Algorithm of Levenberg-Marquardt*

2.1 *Position of the problem*

There are several families of algorithms of minimization [bib1]. For the problems relatively regular, the most used are the methods of descent. Their principle is to generate in manner iterative a continuation (ck)

defined by:

K **N**

$$(k+1) K K K \\ = C + G$$

éq 2.1-1

such as, for $F(X) = J(K$

$C + X K$

G

, $X R +$

.

$F(X)$ is decreasing in the vicinity of +

0

.

$F(K$

) = Min $F(X)$

$x > 0$

K

G is the direction of descent to the step K . It is the method of determination of K

G which conditions

nature thus effectiveness of the algorithm used, knowing that these techniques are mainly based on approximations of J to order 1 or order 2. For the algorithm of Levenberg-Marquardt, one

handle an approximation with order 2 of the functional calculus.

2.2 Resolution

Within the framework of retiming, one handles square least cost functional calculuses of the type:

NR

$J(c) = j^2 \text{ \acute{e}q 2.2-1}$

$N(c)$

$n=1$

where for example $J C = F$

$C - F$

, with obvious notations.

$N ()$

$(calc$

N

$()$

exp

N

$)$

The characteristic of these cost functional calculuses lies in the fact that one knows the form of theirs derived first and seconds:

$($

NR

J

$\acute{e}q$

$2.2-2$

$C(c)$

J

2

$J C$

I

$()$

$=$

N

N

$n=1$

Ci

$(H(c))$

NR

2

=

J

J

J

2

.

J C

éq

2.2-3

ij

N

N + N ()

N

n=1 C C

C C

I

J

I

J

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Then, by supposing that the second term of the preceding equation is negligible in front of the first (what is true when the J are linear out of C : this term is null), one can rewrite:

K

$(H(c))$

NR

J

J

2

.

éq

2.2-4

ij

N

N

$n=1 C$

C

I

J

It is interesting on this level to introduce the matrix of sensitivity or Jacobienne matrix defined by:

J

J

J

1
1
1

...

C
C
C
1
2
N
J
J
2
2

...

...

C
C

To = 1
2
éq
2.2-5

...

...

...

...

...

...

...

...

J
J

NR
NR
jN

...

C
C
C
1
2
N

One can thus express the gradient and Hessian by:

$$\mathbf{J}^T \mathbf{K} = \mathbf{T}$$

éq 2.2-6

$\mathbf{C}(\mathbf{c})$

With \mathbf{J}

$$\mathbf{H}(\mathbf{c}) = 2\mathbf{A}^T \mathbf{A}$$

éq 2.2-7

with $\mathbf{J} = [\mathbf{J}_1, \dots, \mathbf{J}_n]$

\mathbf{J}_1
 \mathbf{J}_2
 \mathbf{J}_n

Then let us write the development limited to order 2 of \mathbf{J} :

\mathbf{J}_1

$$\mathbf{J}(\mathbf{c}) \mathbf{J}(\mathbf{K})$$

$$\mathbf{c}) + (\mathbf{C} - \mathbf{C}) \mathbf{T}$$

$$\mathbf{K}$$

$\cdot \mathbf{J}$
 $+$
 $-$
 $\cdot -$

éq 2.2-8

$$\mathbf{C}(\mathbf{K})$$

$$\mathbf{c})$$

$$(\mathbf{C} \mathbf{C}) \mathbf{T}$$

$$\mathbf{K}$$

$$\mathbf{H}(\mathbf{K})$$

$$\mathbf{c})$$

$$\mathbf{K}$$

$$\mathbf{C} \mathbf{c})$$

$$2$$

That is to say \mathbf{K}
 \mathbf{K}

$\mathbf{G} = \mathbf{C} - \mathbf{C}$, the step of descent at the point \mathbf{K}
 \mathbf{C} , it must check the condition of stationnarity of
the quadratic approximation:

\mathbf{J}

éq
2.2-9
 $\mathbf{C}(\mathbf{K})$
 $\mathbf{c}) + \mathbf{H}(\mathbf{K})$
 $\mathbf{c}) \mathbf{K}$
 $\mathbf{G} = 0$

According to the expression of the gradient and Hessien of \mathbf{J} , one can write:

$$(\mathbf{ATA}) \mathbf{gk} = - \mathbf{AT} \mathbf{J}$$

éq
2.2-10

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The solution of this equation leads to an algorithm known under the name of Gauss-Newton, very effective but which presents nevertheless some disadvantages:

*.
(ATA) can be almost singular and cause the non-existence of solution.
.*

*There is no control on K
G, which can be too large and thus leave the parameters
acceptable space.*

*To mitigate these disadvantages, one prefers to use the algorithm of Levenberg-Marquardt which
proposes
a regularization of the algorithm of Gauss-Newton:*

$$(ATA + I) \mathbf{g}_k = -AT \mathbf{J} \mathbf{é}_q$$

2.2-11

*where is a scalar and **I** the matrix identity.*

*It is noticed that if = 0, one finds the direction given by Gauss-Newton and if +, one
find the direction given by the opposite one of the gradient of **J** i.e the greatest slope.*

*The algorithm of Levenberg-Marquardt thus consists, on the basis of a value of “raised enough”,
to decrease it by a factor 10 for example, with each decrease of **J**. One passes thus
gradually of an algorithm of greater slope to the algorithm of Gauss-Newton. One thus can
to present this procedure in the form:*

Choice of a starting point C and of an initial value of

With the iteration K , to solve

(T With $A + I$) K

*T
 $G = -A J$*

$K + 1$

K

K

C

$= C + G$

If $J(K$

$C + 1) < J(K$

*c), then $= /10$ if not $= *10$*

Test of convergence

Note:

We considered above the algorithm of Levenberg-Marquardt under the angle of regularization of the algorithm of Gauss-Newton. It is possible to give a lighting different with this algorithm by regarding it as an algorithm from area of confidence [bib2]. Indeed, one can show easily that the system [éq 2.2-11] is the condition of stationnarity of the problem of minimization:

1

K

$K T T$

$K T$

T

K

To determine K

G such as $G = ArgMing$

With

$J + G A A G$ subjected to K

K

$G D$

2

1

-
Where

K

$D = - (AT A + I) AT J$ and 0.

It is a very simple establishment of the algorithm of Levenberg-Marquardt within which various questions are not tackled:

.

How to define the functional calculus J when one has several tests?

.

How to choose the initial value of?

.

How to make evolve/move in a finer way?

.

How to define the field of evolutions of each parameters?

We will clarify these various points in the continuation.

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Implementation practical

3.1

Definition of the functional calculus

At the time of a retiming, the user often has several different measurements; it is about discrete physical sizes, possibly of different nature, measured during one or several tests. They are related to a given parameter noted T (time, X-coordinate,...) that one can thus represent by:

exp
T
F
(T)

exp
T
F
(T)

exp
T
F
(T)

L

exp
1
1
1

exp
1
2
1

exp
1
1
F
=
F
=
F
=

éq 3.1-1

1
M
M

2

M

M

L

M

M

exp

T

F

(T)

exp

T

F

(T)

exp

T

F

(T)

NR

1

NR

M

2

M

P

L

P

Each one of these experimental measurements has sound during

~

calc

K ~

T

F

(C, T)

~

calc

K ~

T

F

(C, T)

~

calc

K ~

T

F

(C, T)

calc

K

1

1

1 calc K

1

2

1 calc K

1

L

1

F

(c) =

F

(c) =

F

(c) =

1

M

M

2

M

M

L

M

M

~

calc

K ~

T

F

(C, T)

~

calc

K ~

T

F

(C, T)

~

calc

K ~

T

F

(C, T)

I

l

I

J

2

J

K

L

K

éq 3.1-2

calculated for a play of parameter K

C given. Let us notice that the calculated sizes are not

inevitably in same number as the measured sizes nor evaluated for the same value of

parameter T. One can then define the functional calculus least squares to be minimized by:

2

2

2

NR

exp

F

(T) - F calc (ck, T)

M

exp

F

(T) - F calc (ck, T)

P

exp

F

(T) - F calc (ck, T)

I

I

I

I

2

I

2

+

I

L

I

L

I

+ ... +

exp

II

=

F

(T)

exp

F

T

F

T

K

I

I

II

=
 (
exp
 2
I
I I
 =
 (
J (c)

L
I
 =
J (0
c)
éq
3.1-3

It is important to notice that:

.
if a calculated measurement calc
F
is not defined in one moment T, then one interpolates linearly
J
I
its value
 .
if an experimental measurement exp
F
is null, one does not divide the quantity
J
exp
F
(T)
F calc
 -
(K
C, T) and it are present just as it is in the expression of the functional calculus
J
I

J
I
.

the functional calculus J is standardized so as to be worth 1. at the beginning of the iterations of retiming

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3.2 **Form of the Jacobienne matrix**

For the calculation of the Jacobienne matrix, one defines the vector J of the errors by:

exp
F
(T) - F calc (K
C, T)
1
1
1
1
exp

F
(T)

1
1

M

exp
F
(T) - F calc (K
C, T)

1
NR
1
NR

exp

F
(T)
1
NR

J = exp
F
(T) - F calc (K
C, T)

2
1
2
1
éq
3.2-1

exp

F
(T)
2
1

M

M
exp
F
(T) - F calc (K
C, T)
P
L
P
L

exp
F
(T)
P
L

That is to say:

exp
F
(T)
F calc
-
(K
C, T)
K
K
I
K
I
J =

éq
3.2-2
I
exp
F

(T)

K

I

One finds the form of the Jacobienne matrix then of [éq 2.2-5]:

j1

j1

j1

1

1

L

1

C

C

C

1

2

N

L

L L L

j1

j1

j1

NR

NR

NR

L

C

C

C

1

2

N

= j2

j2

éq

3.2-3

1

1

LL
C

C
1
2

L
LLL

L
LLL
jPJP
JP

L
L
L
L

C
C
C
1
2
N

Where the terms are calculated by direct finished differences:

j1
j1 (C,..., C + C,..., c) - j1 (C,..., C,..., c)
1
1
1
1
1
1
N
1
1
1
N

(C, \dots, C, \dots, c)

$\acute{e}q$

3.2-4

I

I

N

C

C

I

I

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3.3

Regularization of the linear system

We tackle here the problem of the determination and the evolution of the parameter of regularization

. One defines with this intention:

.

= max (Eigenvalues of $AT A$),

= Min (Eigenvalues of $AT A$),

max

min

cond =

/
 if
 0
 max
 min
 min
 T
 1
 T
 .
 $Q(c) = J(K$
 $c) + ($
 K
 $C - c)$
 T
With
 .
 $J + ($
 K
 $C - c) (T$
. WITH $A + I) ($
 K
. $C - c)$
 2

3.3.1 Initial value of

Knowing the sizes above, the following algorithm is defined:

.
 If
 = 0, then = 1.E-3

 min
 max
 .
 If not
 - If cond < 1.E5, then = 1.E-16

 max
 - If not = ABS (1.E5
 -
)/10001
 min

max

Note:

In the last case, the value allotted to A for effect to bring back the conditioning of ATA with $1.E5$

3.3.2 Evolution of the value of

$J(K)$
 $c) - J(K 1)$

+

$c)$

To make evolve/move, the ratio K is defined

$R =$

, which makes it possible to evaluate the validity of

$Q(K)$
 $c) - Q(K 1)$

+

$c)$

the quadratic approximation of J : the closer it is to 1, plus this approximation is valid. One in deduced the following algorithm [bib2]:

.

If K
 $R < 0.25$, then $= *10$

.

If K
 $R > 0.75$, then $= /15$

3.4 Limitations of the field of evolution of the parameters

For various reasons such as guaranteeing the physical validity of the parameters (module of strictly positive Young, Poisson's ratio ranging between 0 and 0.5,...), it is necessary of to limit their field of evolution. One thus imposes that C remains in an acceptable field O , convex closed

N
R. This thus imposes constraints on the parameters:

C
 $> ck + gk > C$

sup

inf

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After dualisation of these conditions by introduction of the multipliers of Lagrange μ and μ

, one

inf

sup

solves the system:

To find K

G μ μ *such as*

inf

sup

(T

With $A + I$) K

$G + \mu + \mu$

$= - T$

With J

inf

sup

K

$C + K$

$G > C$

inf

$$\mu > 0$$

inf

(K

C + K

$$G - c) (\mu) = 0 I =,$$

I

[N]

inf I

inf I

K

C + K

$$G < csup$$

$$\mu < 0$$

sup

K

K

$$(C + G - c) (\mu) = 0 I =,$$

I

[N]

sup I

sup I

This resolution is carried out using an algorithm of active constraints. For any precision on this algorithm, to refer to [bib3] or [bib4].

3.5 Adimensionnement

One is often brought to identify parameters of various physical nature. Orders of size of these parameters can be extremely different. This can generate the very strong ones differences in the orders of magnitude of the components of the gradient and Hessian of functional calculus cost and to compromise the resolution.

To mitigate this difficulty, it is imperative of adimensionner the unknown factors before beginning resolution. Here a simple and effective procedure.

T

That is to say 0

$$C = [0 \ 0$$

0

C, C, ..., C, the initial vector of the sizes to be rebuilt. The matrix is defined

1
2
N]
adimensionnement D:
c0

1
0

c2

D =

éq
3.5-1

c0n-1

c0n

Then, if 0
C all are nonnull, one can define the adimensional unknown factors by:
I

~0
1
-
0
C = D.C éq
3.5-2

In the same way, an adimensional cost functional calculus is introduced:

~ ~
~
J (c) = J (.
D c) = J (c) éq
3.5-3

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Like its gradient:

~ ~

~

~

J (c) J (.

D c)

J (c)

~

C

~**J (c) =**

=

=

.

= .

D J (c)

C

~

~

~

éq

3.5-4

C

C

C C

C*And its Jacobienne matrix:*

~

0

*WITH = A *C**éq***3.5-5***ij**ij**J*

*From an algorithmic point of view, the calculation of the Jacobienne matrix is done classically with functional calculus **J**, then it is adimensionnée as well as the current parameters **C**, before being transmitted to the algorithm of minimization. At the exit of this last, the parameters **C***

~ is

*redimensionnés to allow the calculation of the functional calculus **J**.*

3.6***Criterion of convergence***

The criterion of convergence used in MACR_RECAL consists in testing the decrease of the gradient of the functional calculus. It is pointed out that the use of this criterion is naturally justified by the fact that the objective of the algorithm of retiming is to cancel this gradient.

// **J (K****c) //****C**

<

Prec.*éq***3.6-1**// **J (0****c) //****C**

Where Prec is by defect taken equal to 1.E-3.

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4 Algorithm

total

So as to clarify the sequence of the various operations described above, one presents formally the algorithm of retiming:

.

Initializations

- $K = 0$

*- calculation of **A**, adimensionnement of **A***

- Calculation of initial

- Total Iterations

*- Adimensionnement of **K***

C

- Resolution of the equation of Levenberg-Marquardt

- Imposition of the respect of the terminals

*- Redimensioning of **K 1***

+

C

*- Calculation of **J** (**K 1+***

C

)

- Actualization of

*- Calculation of **A**, adimensionnalisation of **A***

- Test of convergence

- $K = K + 1$

.

End

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Postprocessing of sensitivity

Summary:

Classically, the digital simulations provide the response of a structure to a request. One seek here to determine in addition to this answer the tendency of the response to a modification of parameters of entry of simulation (material, loading...). This tendency is obtained by calculating the derivative of response compared to a parameter p given.

In this document, one places oneself in the case of linear elasticity and one supposes that the basic variable of calculation (displacement U) was calculated thus that its derivative U

$/p$. After having given some indications on this calculation of U and U

$/p$ (for more detail to see [R4.03.03]), one will be interested in derived from variables which result from this (strains and stresses) like with derived from the rate of refund from energy G .

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1

Recall of the calculation of sensitivity for the basic variable (displacement)

One is interested in this paragraph in the sensitivity of displacement U compared to a parameter p given: loading (imposed displacement or forces imposed) or characteristic material (module of Young, Poisson's ratio, or characteristics anisotropic).

1.1 Problem

direct

In the case of elasticity, the direct problem is written in a simplified way (cf [bib1]):

$R U$

$() = L$

with:

$[R U ()] =$

K

With $U ()$: (W)

D

K

where:

A is the matrix of elasticity

K *W is related to form of the kth degree of freedom of the modelled structure**That is written in matrix form:*

$$KU = L$$

1.2 Problem***derived****The derivation of the matrix writing above gives:*

$$\begin{aligned}
 & (K \\
 & /p \\
 &) U + K (U \\
 & /p \\
 &) = L \\
 & /p
 \end{aligned}$$

From where:

$$\begin{aligned}
 & K (U \\
 & /p \\
 &) = L \\
 & /p \\
 & - (K \\
 & /p \\
 &) U
 \end{aligned}$$

*If p is of loading type, one a: $K/p = 0$ and the second member is reduced to L**/p**.**If p is of material type, there is $L/p = 0$ and the second member is worth: - (K**/p**) U. One has**in particular in a more precise way:* **K** **-*****With***

U
= -
 U
(.): (W) D

p

p
 K

K

Term A

/p
is calculated in routine *DMATMC*.
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Calculation of derived from the strains and the stresses

One considers in this paragraph derivation compared to a parameter p given, p which can be a loading or a characteristic of material. It is also supposed that the field of

displacement U and its derivative U

/p

, all the two resulting one from a mechanical calculation (via MECA_STATIQUE), are known.

2.1

Derived from the deformations

In this case, the step is always the same one, whatever the nature of p. Indeed, in the assumption of the small disturbances, one a:

$$= (U + C)/2 = Drunk$$

From where while deriving compared to p:

l

U

U

T

U

=

+

(

)

= B

p

2

p

p

p

It is thus enough in the routine to operator CALC_ELEM to provide in entry to the subroutine

calculating the deformations the field U

/p

in the place of the field U .

2.2

Derived from the constraints if p does not depend on material

By noting A the tensor of elasticity, one a:

$$= ABu$$

Knowing that A and B do not depend on p , one a:

/p

$$= AB (U$$

/p

)

As to the preceding paragraph, in operator `CALC_ELEM`, one provides in entry to subroutine calculating the constraints the field U

/p

in the place of the field U .

2.3

Derived from the constraints if p depends on material

B not depending on p , one a:

/p

$$= (A$$

/p

$$) Drunk + AB (U$$

/p

)

Term $AB (U$

/p

) is in the same way calculated that in the paragraph [§2.2].

For the other term, it is necessary to calculate A

/p

, i.e. to re-use the routine `DMATMC` which had been developed for the calculation of U

/p

if p depends on the material (cf [§1.2]).

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3 **Calculation of derived from G**

3.1 **Recall of the formulation of G**

One considers a fissured elastic solid occupying the field of space 2 or 3. That is to say:

.
U *the field of displacement,*

.
T *the field of temperature,*

.
F *the field of voluminal forces applied to,*

.
G *the field of surface forces applied to a part S of*

,
.
U *the field of displacements imposed on a Sd part of*

.

F
S

G
Sd

Appear 3.1-a: Fissured elastic solid

*To simplify, one places oneself in **linear elasticity and small deformations**, but this approach generalize without sorrow with plasticity, the great deformations....*

One indicates by:

.
the tensor of the deformations,

.
0
the tensor of the initial deformations,

.
HT
the tensor of the deformations of thermal origin,

.
the tensor of the constraints,

.
0
the tensor of the initial constraints,

.
(, 0
, 0
, T) density of free energy.

Then the rate of refund of energy associated with a virtual field of propagation with the crack is written:

G ()
 =
 I
 I
 (
 U
 -)
 D
 -
 T D +

I(
 - 0 0

)
-
(
- *HT*
- *00*
)
]
ij I, p p, J
K, K

, *K*
K

D
T
ij
ij
ij, K
K
ij
ij
ij
ij, K
K
2
2

+ (*F U + F U*)
D
+ [*G U + G U*
-
N]
D
- *NU*
IIK, KI, KKI
I, K K I I I K, K
K

D

N
ij
J
I, K
K

S

K

Sd

The last term associated with a boundary condition with Dirichlet is not established in Code_Aster. One will thus not take it into account.

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3.2

Derived from G

3.2.1 Derived from G compared to the Young modulus

One supposes known the derivative of displacements, the strains and the stresses by report/ratio with E (see [§1] and [§2]).

G is in fact the sum of 5 terms: $G () = CLA$

T

+ THER

T

+ *TEPSINI* + *TFVOL* + *TFSUR*

5 terms being given in the order of [§3.1].

dG

dT

dT

dT

dT

dT

CLA

THER

EPSINI

FVOL

FSUR

=

+

+

+

+

of

of

of

of

of

of

3.2.1.1 Derived from the traditional term

dT

D

CLA

ij

I, p

D

T

= (*U*

-)

CLA

D

ij I, p p, J

K, K

= (
U
+
-
)

D
of
of I, p p, J
ij
of
p, J
K, K

with in 3D and axi:

((
9
U), T) =
2
2
kk + μijj - HT
with HT = 3K (T - réf
T
) kk - K (T - réf
T
)
2
2

E
E
E
where 3K

=
; =
; 2
=

1-
2
(1+)(1- 2)
 μ
1+

it comes:

D
D
 μ
D
(*T - T*)
2
kk
ij
ref.
D kk
3
=
+
+ + 2μ
-
(
+ *E*
- (*T - T*))
of
2
kk
kk
ij ij
ij
E
of
E
of
1 - 2
kk

of

2

ref.

in plane deformations:

(,)

$(1 -) E$

2

2

$T =$

(

E

E

+ +

+

2 -

xx

yy)

(

$2 (1 +) (1 -$

2)

$(1 +) (1 -$

2) $xx yy$

$(1 +) xy$

HT

that is to say:

D

I

$(-) E$

D

D

E

D

xx

yy

I

D

=

(
+
+
(2
2
+)) +
(
yy
xx
xx yy

+
+
)
of
1
(+ *1*
)(- 2)

xx
yy
of
of
2
xx

yy
E
1
(+ *1*
)(- 2)

xx
yy
of
of
E
2nd

D xy
1
(*T - T*)
2

ref.
D kk
3

+

+
-
(
+ *E*
- (*T* - *T*))
l
xy
+
of
l
xy
+
l - *2*
kk

of
2
ref.

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in plane constraints:

(,
E

2
2

T =

(
E
E
++
+
2 -
2
xx
yy)
(
2 *I*-)
(*I* -) 2 *xx yy (I*+) *xy*
HT

D
E
D
D
E
D

xx
yy
I
D
:

that is to say

=
(
+
+
(2
2
+)) +
(
yy
xx
xx yy

+
+
)
xx
yy
xx
yy
xx
yy
of
1
(-) 2
of
of
2nd
1
(-) 2
of
of
E
2nd
D xy
1
(T - T)
2
ref.
D kk
3
+
+
-
(
+ **E**
- **(T - T)**)
1
xy
+
of
1
xy
+

1 - 2

kk

of

2

ref.

3.2.1.2 Derived from the thermal term

T

=-

T

THER

D

T, K K

((

1

U), T)

dK (T)

=

(

D T

-

3 T - T

- 3K T

(

) +

T - T

-

3 T - T

kk

(

réf)

() (réf) (kk (réf)

T

2 dT

dT

dTTHER

**=-
D (
T
)**

**D
of
of
T
, K K**

in 3D and axi:

**D
1
(+ 2 2
)
2
D
D**

+

**kk
kk
of
E 1
(
2 2)
D
(
)=
+
(**

+
)
T
1
 (+) 2 1
 (- 2) 2 dT 2
kk
of
1
 (+ 1
) (- 2) dT
1
 (+) 2 1
 (- 2) 2 dT

D
1
dij 1
of
ED
 -
 +
 (
 -
)
ij ij dT 1
 (
 2 +) 2
ij of 1+ dT 1+ dT

1
D
D
(T - T)

kk
ref.
2
D
of
2nd D
D

-
 (+
 (T - T))(+ E
) -
 (
 + (
 +
)
 kk)
 1 - 2
 dT
 ref.
 kk
 of
 1 - 2
 1 - 2 dT kk
 dT
 1 - 2 dT of

3
 D

 D
 +
 (T - T) (+ (T - T)
 +
 (T - T)
)
 1 - 2
 ref.
 ref.

dT
 1 - 2
 ref.

dT

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in plane deformations:

D

(2 -)

D

D

D

-

of

- E

D

2

2

1

2 (2

)

(

)=

(

+) + (

XX

YY

+

)(

+

)

T

1

(+) 2 1
(- 2) 2 dT
XX
YY
XX
of
YY
of
1
(+ 1
) (- 2) dT
1
(+) 2 1
(- 2) 2 dT

1 + 2 2

D
D
D

+

YY
XX
of
1
(
2 2) E
D

+(
+
)(
+
)

1
(+) 2 1
(- 2) 2 dT XX YY
XX
of
YY
of
1
(+ 1

) (- 2) dT

1

(+) 2 1

(- 2) 2 dT

1

D

D

XY

of

E

D

2

1

-

+ 2

(

-

)

1

(+) 2 dT XY

XY

of

1

(+) dT

1

(+) 2 dT

1

D

D

(T - T)

kk

ref.

2

D

of

2nd D

D

-

(+
(*T - T*))(+ *E*

) -

(

+ (

+

)

kk)

1 - 2

dT

ref.

kk

of

1 - 2

1 - 2 dT kk

dT

1 - 2 dT of

3

D

D

+

(*T - T*) (+ (*T - T*)

+

(*T - T*)

)

1 - 2

ref.

ref.

dT

1 - 2

ref.

dT

in plane constraints:

D

D

D

D

XX
YY
of
E
D
2
2
1
2
(
)=
(
+) + (
+
)(
+
)
T
1
(
2
-) 2 dT
XX
YY
XX
of
YY
of
1
(
2
-) dT
1
(
2
-) 2 dT

1
2
+
D
D

D

+

YY

XX

of

1

(

2) E D

+(

+

)(

+

)

1

(

2

-) 2 dT XX YY

XX

of

YY

of

1

(

2

-) dT

1

(

2

-) 2 dT

1

D

D

XY

of

E

D

2

1

-

+ 2
 (
 -
)
 1
 (+) 2 dT XY
 XY
 of
 1
 (+) dT
 1
 (+) 2 dT

 1
 D
 D
 (T - T)

kk
 ref.
 2
 D
 of
 2nd D
 D
 -
 (+
 (T - T))(+ E
) -
 (
 + (
 +
)
 kk)
 1 - 2
 dT
 ref.
 kk
 of
 1 - 2
 1 - 2 dT kk
 dT

1 - 2 dT of

3

D

D

+

(T - T) (+ (T - T)

+

(T - T)

)

1 - 2

ref.

ref.

dT

1 - 2

ref.

dT

3.2.1.3 Derived from the deformations term and initial constraints

1 0 0

1

T

= [(-) - (- HT

-

0

) 0

)]

EPSINI

D

ij

ij

ij, K K

ij

ij

ij

ij, K K

2

2

dT
D
D
EPSINI =
ij
(
0 -
ij 0
)

D
of
of ij, K
of
ij, K
K

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3.2.1.4 Dérivée from the term forces voluminal

T

= (F U + F U)
FVOL

D
I
I
K, K
I, K
K
I

dT

FVOL = (F
I + F
I
)

D
of
I of K, K
I, K
K of

3.2.1.5 Dérivée from the term forces surface

T
= [G U + G U
-
N]
FSUR
I, K K I I I K, K
K D

S
nk
dT

***FSUR = [G
I***

***+ G
I***

***-
N]***

***I, K
K
I***

***K, K
K D
of
of
of***

S

nk

3.2.1.6 Notices

***From the formula of Irwin G =
(2 2
K I + KII)
E***

***with = 1
2***

***-
déformatio***

***in
plane***

NS

*=1
constraint*

*in
plane*

S

*where KI and KII are the coefficients of intensity of constraints and by taking account of the fact that
G*

2

2

G

KI and KII do not depend on material, it comes

= -

(K + K

I

II) = -

.

E

E 2

E

*This formula is interesting as checking; nevertheless it was not retained in
programming because it requires the calculation of G which a priori is uncoupled from the
calculation of sensitivity.*

3.2.2 Derived from G compared to the loading

*The significant parameter can be one (or several) component of forces F voluminal, surface or
I
nodal, and (or) one (or several) pressure on an edge, which returns to a surface force.*

In all the cases one can write:

G

ncha N

dim

G

F ichta

I

=

additivity coming owing to the fact that contributions from the loadings from

PS

ichta

=1 =1

ichta

I

F

PS

I

G cumulate

F ichta

with

I

= 1

if the significant parameter intervenes in component I of

PS

loading ichta

F ichta

I

= 0

if not

PS

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Example:

The user defines a significant parameter being worth 1. This significant parameter is used to define a force

voluminal F , a surface force of components (F, F) and a pressure p .

Z

X

Y

G

G

G

G

G

then

=

+

+

+

PS

F

F

F

p

Z

X
Y

In the same way that for the derivative of G compared to E, one derives term in the long term. The derivative of the terms traditional and thermal includes/understands less terms because the coefficients of Lamé

and K do not depend on the loading (whereas they depend on E).

On the other hand the voluminal terms surface forces and forces comprise a term moreover if it significant parameter intervenes in the corresponding loading.

To simplify, one will note F the component of the loading by report/ratio to which one derives.

3.2.2.1 Derived from the traditional term

dT
D

CLA

ij

I, p

D

T

= (U

-) D

= (

U

+

-

) D

CLA

ij I, p p, J

K, K

df

df

I, p

p, J

ij

df

p, J

df K, K

with in 3D and axi:

((

U), T)

2

=

+ μ - 3K T - T + K T - T

kk

ij ij

(

réf)

9

2

kk

(

réf) 2

2

2

E

E

E

where 3K

=

; =

; 2μ =

1-

2

(1+)(1-

2)

1+

D

D

D

kk

ij

D

it comes:

=

+ 2μ

- 3K

kk

ij

(T - Tréf) kk

df

df

df

df

in plane deformations:

(,)

(1 -) E

2

2

T =

(

E

E

+ +

+

2 -

xx

yy)

(

2 1+)(1-

2)

(1+)(1-

2) xx yy

(1+) xy

HT

D

l

(-) E

D

D

xx

yy

E

D yy

D xx
that is to say

=

(
+
) +

(
+
)
df
xx
1
(+ ***1***
) (-
2)

df
yy
df
xx
1
(+ ***1***
) (-
2)
df
yy
df

2nd
D
T
(- ***T***
xy
ref.)
D
+

-
E
kk
xy
1 +
df

1 -
2
df

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in plane constraints:

(,
E
2
2

T =

(
E
E

++

+

2 -

2

xx

yy)

(

2 I-)

(I -) 2 xx yy (I+) xy

HT

D

E

D

D

xx

yy

E

D yy

D xx

that is to say

=

(

+

) +

(

+

)

df

xx

1

(-) 2

df

yy

df

xx

1

(-) 2

df

yy

df

2nd

D

T

(- T

xy

ref.)

D

+

-

E

kk

xy

l +

df

l -

2

df

3.2.2.2 Derived from the thermal term

T

=-

T

THER

D

T, K K

(

1 dK T

D T

(U), T)

()

=

(- 3 (T - T)

()

- 3K +

(T - T)

- 3

-

2

(

(T T
kk
réf
réf
kk
réf)
T
dT
dT

dT

THER = - D (
T
) D

df
df T, K K

in 3D and axi:

D
D

+

kk
of
E 1
(
2 2)
D
(
)=
(
+
)
df T
kk
df
1

(+ 1
) (- 2) dT
1
(+) 2 1
(- 2) 2 dT

dij 1
of
E D
+
(
-
)
ij df 1+ dT 1+ dT

1
D
D
(T - T)

kk
ref.
2
D
of
2nd D
D
-
(+
(T - T))(+ E
) -
(
+ (
+
)
kk)
1 - 2
dT
ref.
kk
df

1 - 2
1 - 2 dT kk
dT
1 - 2 dT
df

3
D

D
+
(T - T) (+ (T - T)
+
(T - T)
)
1 - 2
ref.
ref.

dT
1 - 2
ref.

dT

in plane deformations:

D
D
D
1-
of
2 (2 -) E
D
(
)= (
XX
YY
+
)(
+
)
df T

XX
df
YY
df
1
(+ *1*
) (- *2*) *dT*
1
(+) *2 1*
(- *2*) *2 dT*

D
D

+

YY
XX
of
1
(
2 2) *E*

D
+ (
+
)(
+
)

XX
df
YY
df
1
(+ *1*
) (- *2*) *dT*
1
(+) *2 1*
(- *2*) *2 dT*

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D

XY

1

of

E

D

+ 2

(

-

)

XY

df

1

(+) dT

1

(+) 2 dT

1

D

D

(T - T)

kk

ref.

2

D

of

2nd D

D

-

(+
(*T - T*))(+ *E*

) -

(

+ (

+

)

kk)

1 - 2

dT

ref.

kk

df

1 - 2

1 - 2 dT kk

dT

1 - 2 dT

df

3

D

D

+

(*T - T*) (+ (*T - T*)

+

(*T - T*)

)

1 - 2

ref.

ref.

dT

1 - 2

ref.

dT

in plane constraints:

D

D
D

XX
YY
1
of
2nd

D
(
)= (
+
)(
+
)

df T
XX

df
YY
df
1

(
2
-)
dT
1
(
2
-)
2 dT

D
D

+

YY
XX
of
1

(
2)
ED
+ (
+
)

+
)
XX
df
YY
df
1
(
2
) *dT*
1
(
2
) *2 dT*

D

XY
1
of
E
D
+ 2

(
-
)

XY
df
1
(+) *dT*
1
(+) *2 dT*

1
D
D
(*T - T*)

kk
ref.
2
D

of
2nd D
D
 -
 (+
 (T - T))(+ E
) -
 (
 + (
 +
)
 kk)
 1 - 2
 dT
 ref.
 kk
 df
 1 - 2
 1 - 2 dT kk
 dT
 1 - 2 dT
 df
 3
 D

 D
 +
 (T - T) (+ (T - T)
 +
 (T - T)
)
 1 - 2
 ref.
 ref.

 dT
 1 - 2
 ref.

 dT

3.2.2.3 Derived from the deformations term and initial constraints

1 0 0

1

T

= [(-) - (- HT

-

0

) 0

)]

EPSINI

D

ij

ij

ij, K

K

ij

ij

ij

ij, K

K

2

2

dT

D

D

EPSINI =

ij

(

0 -

ij 0

)

D

df

df

ij, K

df

ij, K

K

3.2.2.4 Dérivée from the term forces voluminal

T
= (F U + F U)
FVOL

D
I
I
K, K
I, K
K
I

if F = fl

is a voluminal component of force:

dT

FVOL = ((F
I + U) + F
I

)

D
df
I df
L
K, K
I, K
K df
L

L
L
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if not:

dT

$FVOL = (F$

$I + F$

I

)

D

df

$I df K, K$

I, K

$K df$

3.2.2.5 Dérivée from the term forces surface

T

$= [G U + G U$

-

$N]$

FSUR
I, K K I I I K, K
K D

S

nk

if $F = gl$

is a surface component of force:

dT

FSUR = [G
I

+ (G
I + U)
-
N]

I, K
K
I
L K, K
K D
dg
dg
dg

L
S
L
L

nk

if not:

dT

FSUR = [G

I

+ G

I

-

N]

I, K K

I

K, K

K D

df

df

df

S

nk

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Date:
29/03/01
Author (S):
F. WAECKEL, A. RAZAKANAIVO

Key:
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Organization (S): EDF/MTI/MMN
Handbook of Reference
R4.04 booklet: Metallurgical behavior
Document: R4.04.01
Models of metallurgical behavior
steels in Code_Aster

Summary:
This document presents the models of metallurgical behavior at the heating and cooling allowing to describe structure transformations of steels at the time of cycles thermal exceeding of temperatures about 800°C.

For the two types of transformations (with the heating and cooling) a detailed description of models available is made and of the methods of identification are given.

Lastly, one presents the model of calculation of hardness associated with the metallurgy.

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1 Introduction

Operations of assembly and thermomechanical treatment which the components undergo metal of the power stations REFERENCE MARK, can generate, in the materials which constitute them, of the fields

constraints, known as residual, which exist before any loading of service. Transformations metallurgical and mainly structure transformations are an important cause of these stress fields because they modify the behavior (by modifying the characteristics physics) and generates thermomechanical requests within the materials which undergo them (latent heats, deformations due to the differences in density of the various phases metallurgical).

These structure transformations are due essentially to the succession of a heating (often with beyond 800 °C) and of a more or less fast cooling of the parts during their manufacture. These thermal “cycles” can desired (case of the heat treatments) or “be undergone” (case of welding). In all the cases, they are very variable of a point to another of the part.

This document relates to the modeling of these structure transformations to the heating and with cooling for low alloy steels and this, on a scale which, while remaining “reasonable” for the metallurgist, that is to say easily usable by the mechanic.

This type of modeling is realizable within Code_Aster for the whole of the elements (PLANE, AXIS, 3D) of the “THERMAL” PHENOMENON by the use of operator CALC_META in “post-treatment” of a thermal calculation of evolution. The relation of behavior dedicated to steel, by

difference with that dedicated to the zircalloys, to use under key word COMP_INCR of the operator is relation "STEEL". For the definition of the metallurgical behavior of steel the information of order DEFI_TRC and key word factor META_ACIER under order DEFI_MATERIAU is necessary. Lastly, the definition of the initial metallurgical state is realizable using the order CREA_CHAMP, under the key word factor ETAT_INIT of operator CALC_META. The metallurgy calculation

is necessary to the realization of mechanical calculations which take account of the consequences mechanics of these metallurgical phenomena [R4.04.02]

The models presented (with the heating and cooling) are formulated within the framework of the relations

of behavior with internal variables (or mémoratrices), and authorize a simple identification and rapid based on the experimental diagrams (diagrams TRC of Transformations into Continuous cooling). The choice of the variables and the forms of the laws of evolution selected are given and the description of the implementation of the models (method of identification) is also presented.

Lastly, one presents the model of calculation of hardness which can if necessary come to supplement calculations

metallurgical.

Bene foot-note:

· Basic metallurgical notions necessary to comprehension of the problem general and of the adopted step are gathered in [bib1] and [bib2] where one will find also a bibliographical study of the problem.

· This document is extracted from [bib3] and [bib4] where one makes a more detailed presentation of models and of some elements of validation. More complete elements of validation can also be found in [bib5] for the model of cooling and in [bib14] for the model of hardness.

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2

Metallurgical model of behavior to cooling

2.1 Introduction

On the basis of test of dilatometry [Figure 2.1-a], only knowledge, at a given moment, temperature of an undergoing steel of structure transformations does not make it possible to know its state

of deformation. On the other hand, the behavior of such a steel seems to be able to be described within the framework

models of behavior to variables mémoratives or internes [bib6]. Indeed, if one introduces:

.

$$\mathbf{Z} = \{Z_1 =$$

\}

I;

,

1

the p-uplet of the proportions of the possible metallurgical components

present in a point M and at one moment T given (here, Z1, Z2, Z3, Z4 will be the proportions of ferrite, pearlite, bainite and martensite and the proportion of austenite in M will be equal to:

$$1 - Z$$

(

)

$$1 + Z_2 + Z_3 + Z_4);$$

.

HT

HT

(T) = (T - T) and (T) = (T - T) + (T) thermal deformations

austenite and phases ferritic, perlitic, bainitic and martensitic; while noting:

-

the thermal dilation coefficient average of austenite;

-

T the temperature of reference to which one considers HT

null;

-

the thermal dilation coefficient average presumedly identical for ferrite, pearlite, the bainite and martensite;

-

deformation, at the temperature T, of the phases ferritic, perlitic, bainitic and martensitic compared to austenite (by taking the latter like the phase of

reference);

· if it is considered, moreover, that the deformation of a multiphase mixture can be obtained with to leave the deformations of each phase by a linear law of mixture, one can then describe

evolution of the state of deformation during a dilatometric test by:

$$I = 4$$

I

$$= 4$$

$$HT (Z, T)$$

HT

HT

$$= 1 - Z$$

I

$$(T) Zi (T)$$

+

$$I = 1$$

$$I = 1$$

éq 2.1-1

$$I = 4$$

I

$$= 4$$

$$= 1 - Z$$

I

[

$$(T - T)] + Zi$$

$$[(T - T) +].$$

$$I = 1$$

$$I = 1$$

The problem lies then in the determination of Z or, more precisely and within the framework of simple materials with variables mémoratrices, in the determination of the function of evolution F such

that: !

$Z = F(T, Z)$

... .

To give an account of an effect the speed of cooling on the evolution of the transformations structural, we propose, within the framework of simple materials with variables mémoratrices, one modeling of the metallurgical behavior of steels to the cooling which includes, a priori!

T among

its variables of state.

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Austenitization

1

2

3

T

1: speed of cooling = 1 °C/min

2: speed of cooling = 10 °C/s

3: speed of cooling = 100 °C/s

Appear 2.1-Error! Argument of unknown switch. : Diagrammatic curves of dilatometry

2.2 Assumptions

H1:

A steel likely to undergo structure transformations is a simple material with variables mémoratrices among which one can choose characterizing quadruplet Z

metallurgical structure in a given point and at one moment.

One thus models structure transformations on a scale where the material point can be multiphase. This scale of modeling which can appear métallurgiquement coarse is in conformity with the concept of material point used in mechanics of the mediums continuous and the test-tube of dilatometry, presumedly homogeneous, is representative.

H2:

Supplemented diagrams TRC of the martensitic kinetics of transformation of Koistinen-Marburger [bib7] completely characterize the metallurgical behavior of one steel austenitized during a continuous cooling.

This assumption results directly from the metallurgical practice and specifies the first of objectives to be fixed at the model: to be compatible with the whole of the experimental data relating to the metallurgical behavior which accompanies cooling by steels austenitized. In addition, this assumption also generates a “natural” choice and restrictions as for the variables to be introduced into the model.

H3:

The transformations ferritic, perlitic and (especially) bainitic are impossible in on this side martensitic initial temperature of transformation m_s .

This assumption, in conformity with the representation of diagrams TRC, makes it possible to uncouple transformations by diffusion of the martensitic transformation.

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2.3

Choice of the variables of state

Pilot variables of state

Into thermomechanical of the continuous mediums, the variables of state control are generally temperature and the state of constraints or deformations. However, because of the assumption H2, temperature is only the variable pilot retained. Indeed, the influence of the state of stresses on structure transformations does not appear in diagrams TRC. Moreover, there does not exist (except

an effect of the type Châtelier) of ideal model even if experimental data relative to this influence in isothermal conditions were obtained for certain steels [bib8].

Variables of state mémoratrices

The first variable mémoratrice to be introduced is quadruplet Z characterizing the structure metallurgical and to which knowledge is enough, a priori, to describe from a mechanical point of view a test

dilatometric [§2.2].

In addition to the temperature T, its derivative!

T and the state of stresses, austenitic size of grain D and

percentage of carbon C of austenite changing also influence the behavior

metallurgical of steels to cooling. However, always because of the assumption H2, one

chooses not to introduce C like variable mémoratrice. Indeed, diffusion of carbon

does not appear explicitly on diagrams TRC, although it is implicitly taken into account,

at least partially, in the concept even of component metallurgical. In addition Giusti showed

that if the taking into account of C were theoretically possible, it led to equations

of evolution coupled between C and Z whose experimental identification "seems very difficult, for not to say impossible " [bib9]. Nevertheless, an effect of the percentage of carbon on the decomposition of austenite

with cooling appears indirectly on diagrams TRC. It is the phenomenon of

stabilization of the austenite which results in a decrease of the temperature of transformation martensitic m_s [Figure 2.3-a].

Contrary to the percentage of carbon, the austenitic size of grain D appears on the diagrams

TRC which relate to conditions of austenitization to which correspond a value of D.

We thus choose to introduce D like variable mémoratrice. However, size of grain

austenitic, which results from the thermal history undergone with the heating does not evolve/move more with cooling and

D intervenes only as a parameter in the model of behavior to cooling.

In addition, the martensitic temperature of transformation m_s , which depends on the thermo history metallurgical undergone, intervenes in the law of Koistinen-Marburger adopted on the assumption H2 for

to describe the martensitic transformation. One thus chooses to introduce m_s like variable mémoratrice.

The character memorator of the variables mémoratrices introduced here in addition to Z appears clearly:

D characterizes the thermal history undergone at the time of the passage in austenitic phase and m_s connects

decomposition of austenite in the conditions of its transformation into martensite.

The relation "STEEL" of operator CALC_META thus comprises 7 internal variables:

V1: Z1, proportion of ferrite,

V2: Z2, proportion of the pearlite,

V3: Z3, proportion of bainite,

V4: Z4, proportion of martensite,

V5: D, austenitic size of grain,

V6: Ms martensitic temperature of transformation,

V7: temperature at the points of Gauss.

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Appear 2.3-Error! Argument of unknown switch. : Example of diagram TRC

It is also necessary to model the whole of the phenomena brought into play at the time of one operation of welding to introduce other variables mémoratrices such as the tensors of anelastic deformations which can correspond to the plastic deformations, of plasticity of transformation or of viscosity. But, in accordance with the assumption H2, one considers that these variables

do not intervene in the functions of evolutions of Z and ms.

Lastly, the following assumptions make it possible to simplify and specify more the general form of model.

H4:

T intervenes only in the relation of behavior expressing the current vector of

.

heat Q; its temporal derivative first T is not a variable of state and the relation of behavior expressing the current vector of heat is the Fourier analysis:

$Q = - (T, Z, D) T.$

H5:

I = 3

A diagram TRC makes it possible to identify an empirical relation between ms, D and Zi:

I = 1

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=

+

(

I

3

Ms Z, Z, Z; D

I

2

3

) = Ms0 (D) + AM (D) Z - Zs (D**I****).****éq 2.3-1****I = I**

The H5 assumption means that the martensitic initial temperature of transformation is constant (for a size of grain given) and equal to Ms0 as long as the proportion of austenite transform is lower than a threshold Z S and than its variation is a linear function of quantity of transformed austenite. This assumption seems checked relatively well in experiments [2.3 - has]. It makes it possible to exclude ms from all the relations of behavior other than that expressing Z and Z4.

With Z = {Z, Z, Z which one will distinguish well from Z = {Z I

I;**= ,****1 }****p defined in the § 2.1.**

1

2

3 }

Finally, and taking into account the assumptions H2 and H3 the relations defining the model are written

thus:

+

.

$T - Ms$

!(

$Z T) = F (T! T, Z ms; D)$

[

]

$= F T, T, Z; D$

with $Z = \{Z, Z, Z$

éq 2.3-2

1

2

3 }

$T - Ms$

$I = 3$

+

Z

4 (T,

$Z ms; D) = 1 -$

Z

-

-

éq 2.3-3

{1 ex (

$p (D) [ms T$

I

])}

$I = 1$

and

+
 I
 = 3

(
 $M_s T) = M_s 0 (D) + A (D).$
 Z
 - $Z_s (D)$
 M
 I
) *éq*
 2.3-4

I
 = 1

*where: is a characteristic of the material ($^{\circ}C^{-1}$) (possibly function of D);
 and $[X]^+$ the positive part of X indicates.*

*Lastly, as it seems difficult to propose a simple form of dependence of the model with respect to these variables, one chose not to impose of form particular to the functions of evolution f_i [bib2].
 The step to calculate speeds of evolution of the metallurgical variables uses then techniques of interpolation and rests on the fact that any thermometallurgic history in experiments known (dilatometric test for example) is a particular solution of the differential equation of evolution [éq 2.3-2].*

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3

Identification and implementation of the model with cooling

3.1 Principle

Identification of the model and the use of the experimental data which the diagrams constitute TRC to determine the value taken by the function F in a thermo-metallurgical state $(T, T, \mathbf{Z}; D)$ given are founded on the following observation and the assumption:

· the thermo-metallurgical stories being reproduced on a diagram TRC are all of the solutions particular of the differential equation [éq 2.3-2]. They thus make it possible to calculate in each thermodynamic state met in experiments and present in a diagram TRC value taken by the function F.

·

function

F is regular; i.e. if two points E and E

K

J are close

$(E_k = \{T(tK), T!(tK), (\mathbf{z}t\mathbf{K}); D(tK)\})$, their speeds of evolution in \mathbf{Z} are also neighbors is:

T - Ms +

T - Ms +

E

E F (E) [

] =! ($\mathbf{z}E$) F (E) [

] =! ($\mathbf{z}E$

K

J

K

K

J

J).

T - Ms

T - Ms

One determines then speeds of structure transformations of an unspecified state by interpolation among all the “couples” (E, F (E

K

K) defined by diagrams TRC.

3.2

Integration of the experimental data to the model

3.2.1 Principle

In general, a diagram TRC defines in a reference mark [(

In *T*) - *T*] structure transformations

associated a series of thermal stories traced on this diagram [Figure 2.3-a]. The integration of experimental data then consists in recording for each history of these diagrams the values successive of *T*, *T*! , *Z* so that for any temperature *T* the model knows

values taken by the function *F* in (*T*, *T*! (*T*), (

Z *T*). In order to be able, starting from a reduced number of

numerical data, to reconstitute the thermometallurgic evolutions continuously, one formulates some assumptions on the thermal evolutions and the metallurgical behavior of steels.

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3.2.2 Rules of interpretation of diagrams TRC

Thermal evolutions

To define the thermo-metallurgical stories present in a diagram TRC it is necessary to characterize their thermal evolutions. One can notice that, in a reference mark [1 (

N *T*) - *T*] and for

temperatures lower than 820 °C, the thermal stories of diagrams TRC can, with one enough good approximation, to result from/to each other by a horizontal adjustment

[Figure 2.3-a]. It is thus possible to define a thermal history *T* *I* (*T*) starting from the data of one curve controls *T* *p* *T*

(*t*) and of the moment (in second) for which this history crosses the isotherm 820 °C

by:

$$\begin{aligned}
 & TI(T) \\
 & \{ \\
 & \exp L [\\
 & N (\\
 & tp T)] L [\\
 & N T I \\
 & = \\
 & + \\
 & (\\
 &) \\
 & 820] - L [\\
 & N (\\
 & tp \\
 &) \\
 & 820]}
 \end{aligned}$$

éq 3.2.2-1

where: $T I (T)$ and $tp (T)$ indicate the reciprocal functions of $T I T$ () and $Tp T$ ().

In fact, one has more easily information relating to speeds of cooling of thermo-metallurgical stories of diagrams TRC that at moments of crossing of the isotherm 820 °C. It is in particular the case of steels of welding, whose diagrams TRC are layouts in a reference mark “speed of cooling with 700 °C-temperature”. Taking into account [éq 3.2.2-1],

one can then express the moment of crossing of the isotherm 820 °C according to $Tp T$ () and of

$Ti (700)$ and one obtain like $Ti T$ characterization ():

$$\begin{aligned}
 & TI(T) \\
 & [\\
 & \exp F (T) \\
 & F (\\
 &) \\
 & 700 \\
 & L (\\
 & N T I \\
 & = \\
 & - \\
 & - \\
 & ! (\\
 &) \\
 & 700 F (\\
 &) \\
 & 700)]
 \end{aligned}$$

éq 3.2.2-2

1

With $F(T) = 1$ [$N T ($ $p T)]$ and, in particular! $T_i(T(T)) = F(T) T_i(T)$

Concretely, one interpolates the function $F(T)$ by a polynomial of degree 5. A thermal evolution experimental thus is completely defined by the data of the coefficients of the characterizing polynomial its pilot curve and by its speed of cooling with 700 °C. Validation of this method of parameterization of the thermal stories “read” on diagrams TRC is presented in [bib2]. In the unit, and taking into account the relative inaccuracies of the layout of diagrams TRC, the reading of TIT

() and of the determination of! $T_i()$

700, the agreement between the thermal stories read and recomputed

seem very sufficient.

If one has the recordings of the thermal evolutions of diagrams TRC, one can define each experimental thermal evolution by considering that it is its clean curved pilot. By

elsewhere, if dilatometric tests defining diagram TRC used for

the identification of the model are carried out with constant speeds of cooling, one characterizes

these kinetics of cooling only by their speeds of cooling to 700 °C and one

identically null function F .*Handbook of Reference**R4.04 booklet: Metallurgical behavior**HI-75/01/001/A***Code_Aster** ®

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Initial temperatures and end of transformation

A diagram TRC provides, for a series of known thermal stories, the proportions of various metallurgical components which were formed during cooling as well as temperatures for which one observes on a swelling behaviour a significant variation of total coefficient of dilation of the test-tube [Figure 2.1-a]. These temperatures are then considered like and the end initial temperatures of the transformations. More precisely:

- initial temperatures of transformation indicated on diagrams TRC

correspond to 1% of component already formed;

- the temperatures of end of transformation correspond to the final proportion of the component in the course of formation minus 1%.

Kinetics of the ferritic, perlitic and bainitic transformations

The observation of a swelling behaviour shows that, except in the vicinity of initial temperatures and of end of transformation, the evolution of the deformation according to the temperature is almost linear. Taking into account the equation [éq 2.1-1] evolution of the quantity of phase transformed into function of the temperature is then not very distant from a function refines and one thus supposes only:

- for the ferritic, perlitic and bainitic transformations, the speed of transformation is, between the experimental temperatures of beginning and end of transformation, a function linear of the temperature;

- speeds of these transformations are twice slower at the beginning (from 0 to 1% of component transformed) and into end of transformation (of Z

- 1% with Z

final
final) that enters them
experimental temperatures of beginning and end of transformation.

Martensitic transformations

It is supposed that the martensitic transformations are described by the law of Koistinen-Marburger [éq 2.3-3] and the phenomenologic equation [éq 2.3-4] expressing ms . One uses then each diagram TRC to determine the coefficients, A and Zs as well as the $Ms0$ temperature. Lastly, to prevent that the model systematically transforms into martensite remaining austenite when one reached temperature ms , one introduces an additional parameter, called TPLM, characterizing (by its slowest speed of cooling with 700 °C) of the kinetics of cooling which generates a martensitic transformation. More precisely [Figure 3.2.2-a]:

$Ms0$ is regarded as the martensitic initial temperature of transformation when this one is total;

- is supposed to be constant and calculated in order to check, in the case of a transformation martensitic total:

$Z (MF$

4

) = 0 9,9

where MF is the experimental temperature of end of transformation;

- finally, A and Zs are determined by linear regression starting from the stories thermo-metallurgical experimental leading to a martensitic transformation partial.

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$M_{s0} = 415^{\circ}\text{C}$; $M_{s0} - M_F = 185^{\circ}\text{C}$

$M_{s0} - m_s$ according to $(Z_f + Z_p + Z_b)$

from where a value of (supposed

$M_{s0} - m_s$

constant for a size of grain

20

Linear ($M_{s0} - m_s$)

data) of -0.0249

$y = 30,086x - 11,437$

15

$R^2 = 0,8648$

With = -30,086

10

$Z_s = 0,38$

$M_{s0} - m_s$ ($^{\circ}\text{C}$)

5

and $T_{PLM} = -9^{\circ}\text{C/s}$

0

0,3

0,4

0,5

0,6

0,7

0,8

0,9

1

$Z_f + Z_p + Z_b$

Appear 3.2.2-a: Evolution of ($M_{s0} - m_s$) according to ($Z_1 + Z_2 + Z_3$);

for steel 16MND5 austenitized 5 minutes with 900°C .

3.2.3 Effect of the austenitic size of grain on the kinetics of the transformations with cooling.

The transformations of phase proceed by germination and growth. The stage of germination is done primarily on the grain boundaries. The size of grain of austenite thus plays an important part on

transformations with cooling. For this reason diagrams TRC are established for conditions of austenitization given and should not in any rigour be used only for similar conditions of austenitization. The experimental results tend to show that the size of austenitic grain modifies more the kinetics of transformation than the end and initial temperature transformations, which results relatively well in a translation of diagram TRC according to the axis of times. With each point M of a diagram TRC corresponds the tuple $(T, T! ,)$

Z. To relocate it

TRC according to the axis of times amounts multiplying!

T by a coefficient different from the unit (the axis of times

is given in logarithmic scale) [bib15]. One thus defines a speed of cooling “effective”

T_{eff} :

$T_{eff} = T \exp(\text{has}(D - D))$

ref.

with

$dref$: cut austenitic of reference of diagram TRC, homogeneous grain with a longor.

a : homogeneous coefficient material contrary a length.

The law of evolution retained is thus written:

+

.

$T - Ms$

!(

$Z T) = F(T! T, Z ms)$

[

]

$= F T, T, Z$

with $Z = \{Z, Z, Z$

EFF

EFF

1

2

3}

$T - Ms$

This writing has the advantage of limiting the interpolation to only one diagram TRC, of reference.

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Note:

Assumption: the germination and the growth of martensite are regarded as instantaneous and the density of the sites of nucleation has little influence on this transformation. The effect of the size of grain thus does not relate to the evolution of the martensitic phase and carries here only on the evolution of the phases ferritic, perlitic and bainitic.

3.2.4 Seizure of diagrams TRC

Taking into account the preceding assumptions, experimental acquisition data contained in diagrams TRC thus includes/understands:

· for diagram TRC:

- the value of the austenitic size of grain d_{ref} of the diagram and which will be the size of grain of reference,
- the coefficient of translation has for the taking into account of the effect of the size of grain austenitic,
- the initial temperature of total martensitic transformation $Ms0$,
- the value of the coefficient of the law of Koistinen-Marburger,
- the value of coefficients A and ZS intervening in the equation [éq 2.3-4],
- values of the six coefficients of the polynomial of degree five interpolating the function [$\ln (tp T)$] (if the thermal stories explicitly are known, each one of them is regarded as being its clean curved pilot and the definition of his six coefficients is to renew for each history);
- for each thermal history of a diagram TRC:
 - the speed of cooling with $700\text{ }^{\circ}\text{C}$,
 - final proportions of ferrite, pearlite and bainite (Z_{ff} , Z_{fp} , Z_{fb}),

*Tdf**(, Tdp, Tdb)*

-

initial temperatures of each transformation

,

*Tff**(, Tfp, Tfb)*

-

and temperatures of end of each transformation

.

The seizure of a diagram TRC is realizable by a software of seizure (available on workstation).

It is a simple and fast operation (approximately an hour for the seizure of about fifty stories).

The result of this procedure of seizure of diagrams TRC (cf [§An1]) is directly insertable

in a command file of *Code_Aster* as an order `DEFI_TRC`, order which

thus contains the data identifying the metallurgical behavior of steel.

The complete definition of the models of metallurgical behavior (values of the *Ar3* parameters

“quasi static” temperature of ferritic transformation, *Ms0*, and the complete definition of the model

with the heating and of austenitic growth of grain) is realized within the order

`DEFI_MATERIAU` under the key word factor `META_ACIER`.

An example of the procedure of seizure is presented in appendix [§An1].

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3.3 Evaluation of the function of evolution starting from the data experimental

3.3.1 Evaluation of the function of evolution for the experimental stories

Taking into account the assumptions concerning the evolution of the structure transformations associated

thermo-metallurgical stories *Hi* of a diagram TRC, one thus has a whole of solutions

particular parameterized by *dref* of the differential equation (for *T ms*):

!(

$Z(T) = F(T, Z; dref)$

who allow for any thermo-metallurgical state $E_k = \{T, Z; dref\}$ of an experimental history H_i to calculate:

!
 $Z(E(T)) = F(E$

K
 $K)$

Indeed:

Z
 D
 !
 $Z(E(T)) =$
 $(E) T(E$

K
 K

$K)$

dT

however, taking into account the assumptions of linearity on the evolutions of $Z_i(T)$ between two consecutive states

I

$I + 1$

E and E

of the same discretized history:

K

K

$dz($

I

I

Z

$+1$

$E_k - Z(E_k$

$E_k)$

$() ($

$)$

$=$

dT

$T(I$

$I + 1$

$E_k) - T(E_k$

$)$

where $T(E_k)$ can be estimated by derivation of the analytical expression selected to represent $T_i(T)$.

Thus, one can, for any temperature T , to know the values taken by the function F in the states thermo-metallurgical $I.E.(internal\ excitation) = \{T, iT! (T), \mathbf{z}_i (T); dref\}$ where index I refers to the stories

known in experiments.

3.3.2 Calculation of the advance of the transformations for an unspecified state

It acts, knowing $T, T!$,

\mathbf{Z} ms and D at one moment T given, to determine the values of the variables metallurgical at the moment $(T + T)$ according to. More precisely:

· If

$T(T) Ar3$ or if! $T > 0$,

the metallurgical model of transformation to cooling is inactive [§4.4].

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· If

Ar

$T(T)$

(

$Ms T$

$3 >$

),

!(

$\mathbf{Z} T) = F (T! T, \mathbf{Z}; D) = F (T! T, \mathbf{Z}; D$

EFF

$\mathbf{Z} T + T$

$= \mathbf{Z} T + \mathbf{Z}! T T$

$ref.)$ and (

)

() ()

then:

$I =$

+

3

(

$M_s T$

$T)$

M_s

With

Z

0

$(T + T) - Zs$

+

=

+

I

I

= 1

and, if $T (T + T) M ($

$S T + T): Z (T + T) = Z$

4

4 (T)

or, if not:

$I = 3$

+

Z

4 ($T +$

$T) = 1 -$

$Z (T + T) 1$

- exp [$M (St + T) - T (T + T$

I

)]

$I = 1$

· If

$T(T) < M(S T)$

$(z t + T) = (z t); M(S t + T) = M(S t)$

and

$I = 3$

+

$Z(T + T) = 1$

- $Z(T + T) 1$

-

exp (cd.

4

) [$M(S t + T) - T(T + T)$

I

)] .

$I = 1$

If $A r$

$T(T)$

(

$M s T$

$3 >$

), one determines (thanks to the assumption of regularity of F) the value catch by F in $(T, T!, Z; D)$ starting from knowledge for any temperature T of the values taken

by F in thermo-metallurgical states *I.E.(internal excitation)* $\{T, iT! (T), \mathbf{z}i (T); dref (T)\}$ known stories

in experiments, where $T (T$

I

) is the speed of cooling for the *Hi* history at the temperature T

(obtained by interpolation). More precisely, one will determine a linear approximation of F with

vicinity of $(T, T! , \mathbf{Z}; D)$. F is related to 5 (because the dependence compared to the parameter D

is included in the possible modification the current speed of cooling [\S 3.2.3]) in,

to determine a linear approximation of F in the vicinity of $(T, T! , \mathbf{Z}; D)$ amounts determining

the equation of a hyperplane in 6 and thus to have the value taken by F in six points

$\{E, F (E)$ “close” to $(T, T! , \mathbf{Z}; D)$.

I

$I\}$

Concretely, stages of this interpolation of the values of F in $(T, T! , \mathbf{Z}; D)$ are them

following:

· one calculates an “effective” temperature!

Teff allowing to take account of the effect of the size

of austenitic grain if it is different from that of the diagram, and the value then is sought

catch by F in $(T, T! , \mathbf{Z}; D$

EFF

ref.)

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· one calculates for all the experimental stories *Hi* known the values taken by

function F in the following thermo-metallurgical states (in order to know a whole of

values of F in a vicinity of $(T, T! , \mathbf{Z}; D$

EFF

ref.) rather dense in temperature):

1

E

T

=

T, T! T, Z T; D

T

I ()

{I () I () ref. ()}

2

E

T

=

T + 5 °C, T! T + 5 °C, Z T + 5 °C; D

T + 5 °C;

I ()

{

I (

) I (

) ref. (

)}

3

E

T

=

T - 5 °C, T! T - 5 °C, Z T - 5 °C; D

T - 5 °C;

I ()

{

I (

) I (

) ref. (

)}

· one determines the six closer neighbors of $E (T) = \{T (T), T! (T), (zt); D (T$

EFF

ref.

)} among all

J

I.E.(internal excitation) (T) ($J = 1$)

3

, defining the metallurgical behavior of material in the vicinity of

J

the temperature $T (T)$ by minimizing the distance from $E (T)$ to each one of I.E.(internal excitation) (T);

· one calculates the barycentric co-ordinates of $E (T)$ compared to its closer neighbors

v

$E(T) (v = 1)$

6

. For that, one solves the linear system associated with this calculation within the meaning of least squares and by choosing the solution of minimal standard if sound determinant is null (it is the case when the closest neighbors belong to a variety refine of size lower than six - [R6.03.01]);

· one retains only the neighbors W

$E(T) (W)$

6 such as all the barycentric co-ordinates

W of $E(T)$ are positive (so that $E(T)$ is located inside the convex polyhedron being based on these points);

· one calculates then:

!

Z (

W

$E(T) = F(E) = F[E$

K

K

W

$(T)] / W$

;

W

· finally, one calculates Z with the step of time following Z

$(T + T)$ according to the diagram clarifies according to:

$(zt + T) = (zt) +! (zt) T.$

Note:

The definition of a distance used in the criterion of proximity is not obvious, account held of the nonadimensional character of the space of the $\{T, T!, Z; D\}$. Currently, seek closer neighbors is carried out by adimensionnalisant each one simply of variables but one could plan to introduce weighting coefficients into each "direction" ($T, T!,$ or)

Z in order to return account of a dominating part played by such or such variable.

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4**Metallurgical model of behavior to the heating****4.1 Assumptions**

During the heating, the only transformation likely to occur is the transformation into austenite, which one supposes speed independent the heating rate. In addition, one also suppose that the whole of the phases ferritic, perlitic, bainitic and martensitic transform in an identical way into austenite. These assumptions are generally common to the whole of the models of austenitization [bib9], [bib10] and [bib11]. Consequently the model selected is

form:

!

Z $= F(T, \mathbf{Z})$

).

It is pointed out that the metallurgical model of transformation proposed by Leblond and Devaux and established

in the code Sysweld [bib11] is form (for the transformations with the heating and with cooling):

Z $T - \mathbf{Z}$

!

eq $Z(T, \mathbf{Z})$

()

=

 (T)

where, for the austenitic transformation, the parameter is taken constant.

Comparative data to the experiment presented in [bib11], [bib12] and [bib13] show that, with the help of the identification of functions $Z(T$

eq

) and (T) starting from tests at various speeds of

heating, this model allows a completely satisfactory description of the austenitic transformation steels. Nevertheless, it seems that the identification of the function (T) remains difficult [bib4].

In *Code_Aster*, the austenitic model of transformation is form:

Z $T - \mathbf{Z}$

eq

!

Z (*T*, **Z**)

()

=

(*T*)

but with a simple form for the function *T* (*T*), in order to keep a whole of models metallurgical of easy and fast identification.

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4.2

Form model selected

In its continuous form, the model selected is such as:

Z

T - Z

eq

!

Z (*T*, **Z**)

()

=

(

éq 4.2-1

T)

where:

.

Z indicates the proportion of austenite;

.

$Z(T)$
 eq
) is the function (with Ac_1 et Ac_3 positive constants):

0
 if $T < ac$

1
 $T - Ac$

Z
 =

1
 if

éq 4.2-2
 1

$eq(T)$
 Ac

T
 Ac

Ac
 3

$3 - Ac$
 1

1
 if T

Ac_3
 .

T
 () is the function (avec 1 et 3 positive constants):

if T
 1

Ac
 1

$T - Ac$
 $(T) =$

1
 1 +
 (3 - 1) if $Ac1 T ac$
 Ac
 3
éq 4.2-3
 3 - Ac

1
 if T

3
 $Ac3$

Notice 1: definition of the function $Z_{eq} T$

()

The definition of the function $Z_{eq} T$

() is identical to that given by Leblond and Devaux in

[bib11] and [bib12]. It corresponds to the evolution of the austenite rate transformed for very low heating rates. Indeed, in T fixed, $Z_{eq} T$

() is the asymptotic solution

towards which the solution of the differential equation [éq 4.2-1] tends with the time-constant T

(). For low heating rates, the asymptotic solution can be considered like attack at every moment and $Z_{eq} T$

() thus corresponds to the evolution of the austenite rate

transform during “quasi-static” evolutions. The function $Z_{eq} T$

() is thus entirely

defined by the data of $Ac1$ and $Ac3$ which is done under single-ended spanner words $AC1$ and $AC3$ under the key word factor $META_ACIER$ of order $DEFI_MATERIAU$.

Notice 2: form function T

()

In the model suggested by Leblond and Devaux, the form of the function T

() is not

specified and this function is identified in order to obtain a satisfactory agreement between

initial temperatures and end of transformation experimental and calculated. In order to obtain a model of identification simple and rapid we chose a simple form for the function

T

(). More precisely, to be able to integrate the equation of evolution [éq 4.2-1] there is all initially considered the case where the function T

() is constant. In this case, one can then

to propose two possibilities of simple identification of this constant function. The first possibility consists in identifying a value 1 of making it possible to describe it correctly

beginning of the transformations whereas the second consists in identifying a value 3 of allowing to describe the end of the transformations correctly. The model was then tested obtained with a function T

() refines definite from values 1 and 3 above definite.

Results obtained being completely satisfactory and comparable with those obtained with model available in Sysweld, one chose to introduce into Code_Aster a model where function T

() is closely connected and is defined by 1 and 3 which is indicated with AC1 and AC3.

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4.3**Integration of the equation of evolution**

In *Code_Aster*, one chose to integrate the equation of evolution [éq 4.2-1] exactly in **Z** and

explicitly in *T* and on each step of time (i.e while considering!

T and constants on the step and

equal to their values at the beginning of step of time). One obtains then:

- $T T! T$ $T T! T$ T $\mathbf{Z} (T + T)$ $() ()$

=

+ $Zeq (T + T)$ $() ()$

- -

+ $Zeq (T) - \mathbf{Z}$ $(T) \exp-$

.

 $Ac^3 - ac$ Ac

1

 $3 - Ac$

1

 (T)

The consequent evolution of the proportions of all the other metallurgical components is then defined

by:

$Z T$

+ T

- $Z T$

Z

$I(T + T$

)

(

)

()

= $Z_i(T) \cdot 1 -$

.

$1 - Z(T)$

In other words, each phase present is transformed into austenite to the amount of its proportion at the beginning of step of time.

4.4

Evolution of the austenitic size of grain to the heating

Once austenized, steel sees its size grain to increase more or less quickly according to temperature, but this growth always takes place since austenite appears with a size of grain null. The austenitic growth of grain is a thermally activated process. The model of growth selected is that of Grey and Higgins, adapted to treat material in the course of transformation [bib15]:

Model of growth:

D

$1 1$

1

$(D) = -$

dt

$D dlim$

Growth in the course of transformation, austenite appearing with a null size of grain:

D

$1 1$

1

dz/dt

$(D) = -$

-

D

dt

$D D$

Z

lim

with

Q_{app}

$= 0$

exp (

)

RT

W_{app}

$dlim = d10 \exp (-$

)

RT

with

Z : proportion of the austenitic phase

D : diameter of austenitic grain homogeneous to a length

$dlim$: cut limiting grain, dependent on D homogeneous parameter material to a length

10

Q and W : homogeneous parameters materials with energies of activation (J.mol⁻¹)

app

app

R : constant of perfect gases (8.314 J.K⁻¹.mol⁻¹)

D : homogeneous parameter material at seconds per unit of length

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Note:

The parameters materials are to be informed under key word META_ACIER of DEFI_MATERIAU.

4.4.1 Treatment**numerical**

The calculation of the size of grain is carried out after the calculation of proportion of phase and the integration of

the equation of evolution is made according to an implicit scheme in D. From where:

1 1

1

Z

 $D = -$

T -

D

D dlim

Z +

1 1

1

Z

 $D = D - + D$ $= D - + -$

T

-

D

D Dlim

Z +

A quadratic equation in D. is solved.

4.5**Feel metallurgical evolution**

In a structural analysis, certain zones can undergo a heating while others cool. Moreover, under certain conditions, an austenitic transformation initiated at the time of heating can continue at the beginning of cooling. There thus does not exist, strictly speaking, one austenitic model of transformation and a model of transformation to cooling but only one model of metallurgical transformations which according to the temperature considered and signs its speed of thermal evolution is described either by the model of decomposition of austenite, or by the model of formation of austenite.

With regard to the model introduces into *Code_Aster*, the direction of the metallurgical evolution (i.e. formation or decomposition of austenite) is given as follows:

T T

$(+ T) < Ac1$

[*Ac*

]

$> Ar$

1; *Ar3*

3

.

T T

$() > 0$

AUST

.

T T

$() = 0$

REFR

if **Z** *Zeq REFR* if **Z** < *Zeq AUST*

AUST

.

T T

$() < 0$

REFR

AUST

where *REFR* means that the metallurgical evolution is determined by the model of decomposition of the austenite and where *AUST* means that the metallurgical evolution is determined by the model of formation austenite.

Note:

AR3 is also a characteristic of the metallurgical behavior to cooling already defined by the model of transformation in cooling.

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5

Identification of the model to the heating

5.1

Determination of the function $Z_{eq}(T)$

$Z_{eq}(T)$

($Z_{eq}(T)$) can be regarded as the quasi static solution of the differential equation [éq 4.2-1] and one chooses to define it (as in [bib11]) by the relation [éq 4.2-2].

In this expression, temperatures Ac_1 and Ac_3 are the quasi static temperatures “theoretical” of beginning and austenitic end of transformation which correspond to an austenite rate still formed equal to zero or already equal to one.

In fact, these temperatures are difficult to determine in experiments and one generally considers that quasi static temperatures of beginning and given end of austenitic transformation in experiments correspond, respectively, to 5 and 95% of formed austenite. In other words, if one notes Ac'_1 and Ac'_3 these temperatures, they check:

$Z(Ac'_1) =$

0.05 and $Z(Ac'_3) =$

0.95

éq 5.1-1

')

1

3

0.95

éq 5.1-1

To determine Ac'_1 and Ac'_3 one can use tests of dilatometry at low heating rate or to apply formulas of the literature connecting the quasi static temperatures of beginning and end of austenitic transformation with the composition of steels. In general these temperatures are also indicated on diagrams TRC used for the identification of the model of transformation to cooling or can be considered using formulas knowing the composition of steel [bib4].

Lastly, knowing Ac'_1 and Ac'_3 , one can then determine the temperatures Ac_1 and defining Ac_3 function $Z_{eq}(T)$

($Z_{eq}(T)$) starting from the two equations [éq 5.1-1] above. A complete example of identification austenitic model of transformation is presented in [bib4].

5.2

Determination of the function RATE (T)

In a general way, it is not easy to release means of a simple and fast identification of function T

(T). This is why one proposes to adopt for this function the simplified form below [éq 5.2-1] if $ac(T)$:

1

3

 $T - Ac$ $(T) =$

1

1 +

 $(3 - 1)$ **éq 5.2-1** $Ac^3 - Ac^1$

where 1 and 3 is positive constants.

For the phase of identification, one considers the particular case initially where is constant enters Ac and ac . One proposes two types of identification then allowing to determine is a value

1

3

1

of coherent with the experimental temperatures of beginning of transformation, that is to say a value 3 of coherent with the experimental temperatures of end of transformation.

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One presents in [bib4] the results obtained by these two identifications and one shows (without another form

of theoretical justification) that the function T

() refines definite with values 1 and 3 previously

determined allows to obtain an agreement with the experiment completely comparable with that obtained with

model of Leblond.

5.2.1 Identification of TAUX_3 starting from AC'3

For!

T and constants and initial condition $Z (Ac^1) = 0$, the solution of the equation

of evolution [éq 4.2-1] is (as long as $Z(T$

eq

) is constant, i.e. as long as $T Ac3$):

$Ac1 - T$

$Z(T) = Z$

.

. !. 1 exp

$eq(T) -$

$Ze q(T)$

T

-

. $T!$

In particular, one thus has, for T

$= A$

$C 3:$

$Ac - Ac$

$0.95 = Zeq(ac) - Zeq(T). ! T. 1$

1

3

,

3

- exp

. !

T

A test of dilatometry at heating rate constant (and not very low) allows then to determine value 3 of allowing to reach the agreement between the experimental values and calculated of $Ac3$. One presents in [bib4] comparisons between experiment and calculation obtained in thus identifying the function considered as constant.

5.2.2 Identification of TAUX_1 starting from AC'1

In the same way that previously, one can also write, for $T = ac$:

1

Ac - Ac

$0.05 = Z_{eq}(ac) - Z_{eq}(T) \cdot T$

1

1

,

1

- exp

éq 5.2.2-1

. !

T

There still, having a test at constant heating rate, the equation [éq 5.2.2-1] allows to determine a value 1 of allowing to obtain a good agreement on the *Ac1* temperatures calculated and experimental.

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6

Model of calculation of hardness

Metallurgical calculations can be supplemented by a calculation of hardness of “hardening” associated with metallurgical structure.

The selected model uses the assumption according to which the hardness of a polyphase material point is well

represented by a linear law of mixture of the microhardnesses of the components (here phases austenite, ferrite, pearlite, bainite and martensite). The microhardnesses are taken as being constants of

material and of the phase considered.

The model is written then: $HV = \sum_k z_k HV_k$

K

K

K

HV : hardness (here Vickers for example) of the polyphase point,

z_k : proportion of the phase K ,

HV_k : hardness of the phase K .

Although enough simple, this model gives very correct results [bib14].

In *Code_Aster* the calculation of hardness is done via the operator of postprocessing

CALC_ELEM; option "DURT_ELGA_META" for calculations of hardness at the points of Gauss and option

"DURT_ELNO_META" for calculations with the nodes by elements.

Hardnesses of the various metallurgical phases are data materials provided by the user under the key word factor DURT_META of operator DEFI_MATERIAU.

DURT_META

(

F1_DURT: HVf

F2_DURT: HVp

F3_DURT: HVf

F4_DURT: HVf

C_DURT: HV

)

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Appendix 1

One presents below an example of the procedure of seizure of a thermo-metallurgical history experimental (at nonconstant speed of cooling). Provided information is first of all shown by the user with the software of seizure: speed of cooling with 700 °C, composition with the ambient one,

experimental temperatures of beginning and end of transformation,.... This information is registered in heavy and Italic type; the temperatures are indicated in °C and speeds of cooling in °C/s.

The whole of the thermo-metallurgical states defining this experimental history and their storage data processing are then presented and the figure [Annexe1-a] represents, always for this history and in function of the temperature, supposed metallurgical evolution solution of the differential equation [éq 2.3-2] thus

that the recall of the data provided during the seizure.

SEIZURE OF THE TRC

SEIZURE OF THE STORIES THERMOMETALLURGIQUES OF WHICH ONE A
THE EXPERIMENTAL KNOWLEDGE FOR A STEEL GIVES

Enter the name of steel (8 alphabetical characters maximum)

tracier

Enter the value of the Ar3 temperature

836

To enter the value of the temperature in lower part of which all them transformations are finished

200

THE TRC EAST CHARACTERIZES BY:

1 - The number of stories which composes it;

2 - The coefficients A, B, C, D, E, F of the polynomial: WITH + BT + CT2 + DT3 + ET4 + FT5

defining the curve cooling F (T) controls such as:

$T(T) = \exp \{F(T) - F(700) - \ln [Tp(700) F'(700) 1]\}$

where: Tp is the derivative of T (T) and F' that of F;

3 - The value of the austenitic size of grain dref of the diagram;

CAUTION! THE FIRST SEIZED HISTORY MUST BE SLOWEST;

I.E. NEAREST TO THERMODYNAMIC BALANCE

Enter the number of sets which you want to seize?

1

CAUTION! you will seize 1

together (S) of thermo-metallurgical stories

(OK = 0; Not = 1)

0

Enter the number of stories of unit 1

1

Enter the value of coefficients A, B, C, D, E, F and D

8.563 -0.0276 1.22D-4 -2.955D-7 3.492D-10 -1.517D-13 11D-6

CAUTION! the significant minimal proportion for a component with the ambient one is 0.03

History number 1

Enter the value of Tpoint 700

-0.00542

Enter the final proportions of ferrite, pearlite and bainite (Zff, Zfp and Zfb)

for history 1

0.764 0.199 0.037

Entry of and the end initial temperatures of transformation for history 1

Enter and the ferritic end initial temperatures of transformation Tdf and Tff

792 657.5

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Enter and the perlitic end initial temperatures of transformation Tdp and Tfp

657.5 615

Enter and the bainitic end initial temperatures of transformation Tdb and Tfb

490 420

VALIDATE you the HISTORY NUMBER? (YES = 1 NOT = 0) 1

TPOINT A 700 DEGREES

-5.420D-03

Zff

Tdf

Tff

7.640D-1

7.920D+2

6.575D+2

Zfp

Tdp

Tfp

1.990D-01

6.575D+2

6.150D+2

Zfb

Tdb

Tfb

3.700D-02

4.900D+2

4.200D+2

1

DEFINITION of the coefficient of translation used to calculate!

Teff, modelling

the influence of the size of grain on the kinetics of transforamtion:

$! T_{eff} = ! T \exp (has (D - D))$

ref.

The value of A. is thus defined.

Enter the value of the coefficient of translation has for the effect cuts grain:

11200.

DEFINITION OF the VARIATION OF m_s According to $Z_f + Z_p + Z_b$

It is considered that the martensitic transformation is described by the law of Koistinen-Marburger:

$$I = 3$$

Z

+

$4(T)$

=

$1 -$

Z

$I - \exp$

-

{1

$([M_s T])$ }

$I = 1$

in which the martensitic initial temperature of transformation m_s is, with beyond certain threshold, function of $Z_f + Z_p + Z_b$:

+

I

= 3

(

$M_s T)$

M_s

With

Z

0

- Z_s

=

+

I

.

I

= 1

One thus defines the values of M_s0 and, as well as the values of Z_s and A .
Enter the number of laws of variation of m_s according to $Z_f + Z_p + Z_b$ that you
please seize

1

Enter the value of the Z_s threshold and A for law 1
as well as the value, TPLM, speed of cooling with 700 °C of
the slowest history leading to a martensitic transformation
partial and of

0.47 -32.76 -3,497 14.06

You validate the law such as (YES = 1 NOT = 0): $Z_s = 0.47$

$AM = - 32.76$

$TPLM = - 3,497$

$= 14.06$

1

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Example of procedure of seizure of diagrams TRC

tracier = DEFI_TRC

(- 5.420D-03

1.100D+01

8.563D+00

-2.760D-02

(HIST_EXP:

VALE:

1.220D-04

-2.955D-07

3.492D-10

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0.000D+00
0.000D+00
0.000D+00
8.360D+02
0.000D+00
0.000D+00
0.000D+00
7.956D+02
1.000D-02
0.000D+00
0.000D+00
7.920D+02
7.277D-01
0.000D+00
0.000D+00
6.622D+02
7.540D-01
1.000D-02
0.000D+00
6.575D+02
7.640D-01
2.523D-02
0.000D+00
6.539D+02
7.640D-01
1.890D-01
0.000D+00
6.150D+02
7.640D-01
1.990D-01
0.000D+00
6.103D+02
7.640D-01
1.990D-01
0.000D+00
5.665D+02
7.640D-01
1.990D-01
1.000D-02
4.900D+02
7.640D-01
1.990D-01
2.700D-02

4.250D+02
7.640D-01
1.990D-01
3.700D-02
3.485D+02)
TEMP_MS: (P: 1.100D+01
THRESHOLD: 4.700D-01
AKM: -3.276D+01
BKM: 1.406D+01
TPLM: -3.497D+00)
GRAIN_AUST: (DREF: 11.D-6
A: 11200.)

;
**Result of the operation of seizure above providing in language
order *Aster the* definition of a metallurgical behavior with
cooling.**

*Visualization of the points seized and calculated for a history
of a TRC*

80
70
Zf (T) built
60
Zp (T) built
50
Zb (T) built
40
30
pts Zf-T seized
20
pts Zp-T seized
10
pts Zb-T seized
0
200
300
400
500
600
700
800
900

Temperature (°C)

Appear Annexe1-a: Example of thermo-metallurgical history resulting from a TRC

and integrated into the model

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Document: R4.04.02

***Modeling élasto- (visco) plastic fascinating
in account of the metallurgical transformations***

Summary:

This document presents the modeling installation in Code_Aster for the mechanical analysis operations generating of the metallurgical transformations. The various mechanical effects are

presented
resulting from structure transformations to take into account and their modelings.

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1 Introduction

Certain materials undergo structure transformations when they are subjected to evolutions thermics particular [bib1], [bib2], [bib3]. It is for example the case of the low alloy steels with run of operations of the welding type and heat treatment or the alloys of zircaloy of fuel sheaths for certain cases of accidental situation (APRP).

These transformations have a more or less strong influence on the thermal evolutions and mechanics.

From a thermal point of view, structure transformations are accompanied by a modification by thermal characteristics (voluminal heat-storage capacity, thermal conductivity) of the material which the sudden one, as well as production or of an energy absorption (latent heats of transformation) [bib2]. However, the latent heats of transformation in a solid state are relatively weak compared with the latent heats of change of state liquid-solid and one can therefore, at first approximation, to regard the thermal and structural evolutions as uncoupled. C`is currently the case of the established options of thermal and metallurgical calculations in Code_Aster. [bib16]

From a mechanical point of view, the consequences of structure transformations (at the solid state) are of four types [bib2]:

.

the mechanical characteristics of the material which undergoes them are modified. More precisely, the elastic characteristics (YOUNG modulus and Poisson's ratio) are little affected whereas plastic characteristics (elastic limit in particular) and it thermal dilation coefficient are it strongly,

.

the expansion or the voluminal contraction which accompanies structure transformations translated by a deformation (spherical) "of transformation" which is superimposed on the deformation of purely thermal origin. This effect is highlighted on a test of dilatometry and, in general, one gathers it with that due to the modification of the one and dilation coefficient speak overall about the influence of the transformations on the thermal deformation,

.

a transformation proceeding under constraints can give rise to a deformation irreversible and this, even for levels of constraints much lower than the elastic limit material (at the temperature and in the structural state considered). One calls "plasticity of transformation " this phenomenon,

one can have at the time of the metallurgical transformation a phenomenon of restoration of work hardening. The work hardening of the mother phase is not transmitted to the phases lately created. Those can then be born with a virgin state of work hardening or only inherit of a part, possibly of totality, work hardening of the mother phase.

In addition, the mechanical state also influences the metallurgical behavior. The state of constraints can in particular accelerate or slow down the kinetics of the transformations and modify them

temperatures to which they occur. However, the experimental characterization of this influence, in particular in the case of complex situations (three-dimensional, under temperature and state of variable stresses) remains very delicate and it is very frequent to consider the evolution structural like independent of the mechanical state. C`is the case of the model of transformations structural established in Code_Aster.

If one neglects the various couplings of mechanical origin, the determination of the mechanical evolution associated a process bringing into play structure transformations thus requires two calculations successive and uncoupled:

a thermo-metallurgical calculation (uncoupled) allowing the determination of the evolutions thermics then structural,

a mechanical calculation (élasto-viscoplastic) taking account of the effects due to the evolutions thermics and structural.

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This document presents the mechanical modeling established in Code_Aster. Modeling is available for two materials:

*.
the steel which undergoes around 850° a austénite-ferritic transformation (passage of cold phases of cubic structure face centered (CFC) with a hot phase of centered cubic structure (DC)). Steel presents 4 possible ferritic phases; ferrite, pearlite, the bainite and martensite,*

*.
the alloys of Zircaloy which undergo around 800°C a transformation of phase with cold of hexagonal structure compacts with a hot phase of structure DC.*

The models are identical for two materials, only the number of phase changes.

The model thus comprises 5 phases for steel and 3 phases for the zircaloy. The modeling of behavior of the zircaloy indeed requires to consider 2 cold phases of behavior mechanics different; a phase considered as pure and a phase mixed with [bib16], [bib17]. The various characteristics relating to the various phases are noted:

Zircaloy steels

*Ferrite: F1_ ****

*Alpha pure: F1_ ****

*Pearlite: F2_ ****

*Mixed alpha: F2_ ****

*Bainite: F3_ ****

*Beta: C_ ****

*Martensite: F4_ ****

*Austenite: C_ ****

Bene foot-note:

Metallurgical concepts of bases necessary to the comprehension of the problem general are gathered in [bib1].

The elastoplastic algorithm of resolution, without taking into account of the effects due to structure transformations is clarified into [bib4].

This document to some extent is extracted from [bib5] and [bib14] where one makes a presentation more detailed model and of some elements of validation.

The presentation of the models which one makes in this document is mainly illustrated with the case steel.

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2 Influence of structure transformations on thermal deformation

A test of dilatometry consists in measuring the deformation (homogeneous) of a test-tube of small dimension according to the temperature (or of time) at the time of an imposed thermal cycle (supposed identical in all the points of the test-tube). One presents [Figure 2-a] a test of dilatometry of one steel. The thermal cycle comprises a heating beyond the temperature of austenitization (either 850°C approximately), then a maintenance at this temperature and, finally, a cooling controlled until ambient temperature. One then obtains an evolution of the deformation (variable according to the kinetics of cooling imposed) as represented on the figure.

D

B

C

F

E

G

With

H

T0

T

Tref

F

Appear 2-a: Diagrammatic statement of dilatometry

2.1

Zones of thermal deformation

The various zones highlighted on the figure [Figure 2-a] can be interpreted as follows:

A-B:

thermal dilation of metal in its initial metallurgical structure (of type ferrito-perlitic (F + P), bainitic (B) and/or martensitic (M)) until initial temperature of austenitization T (B),

B-C:

austenitization and contraction of the test-tube (volume specific of the phase austenitic () smaller),

CD:

thermal dilation of austenite (with a dilation coefficient different from that phases known as ““(F), (P), (B), (M)),

OF:

thermal contraction of austenite,

E-F:

first transformation (partial) of the austenite (for example F + P) which be accompanied by a voluminal expansion,

F-G:

zone without transformation with thermal contraction of remaining the austenite mixture - formed phase (with a certain thermal dilation coefficient apparent),

G-H:

second transformation of the remaining austenite (for example M) which be accompanied by a voluminal expansion,

Ha:

thermal contraction of the final structure (with the same dilation coefficient that with the heating).

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2.2

Assumptions and notations

.

The structures ferritic, perlitic, bainitic and martensitic have a coefficient of identical thermal dilation (noted F) different from that of austenite (noted).

One defines a state of reference for which one considers that the thermal deformation is null: one for that a metallurgical phase of reference (phase austenitic or ferritic phase) and one choose temperature of T_{ref} reference.

.

That is to say HT

HT

thermal deformation of the austenitic phase, and F thermal deformation phases ferritic, perlitic, bainitic and martensitic, we will take:

HT

R

T_{ref}

$= (T) (T - T_{ref}) - (1 - Z)$

F

$HT =$

T

(T)

R

ref.

F

F

$(T - T_{ref}) + Z F$

where:

$T_{ref}:$

Temperature of reference,

(T) : average dilation coefficient of the austenitic phase at the current temperature

T , compared to the temperature of reference.

$F(T)$: dilation coefficient average of the phases ferritic, perlitic, bainitic and martensitic at the current temperature T , compared to the temperature of reference.

Z_R :

characterize the metallurgical phase of reference;

$Z_R = 1$ when the phase of reference is the austenitic phase,

$Z_R = 0$ when the phase of reference is the ferritic phase.

T_{ref}

H_T

$= T$

H_T

$- T$

F

$F(\text{ref.})$

(ref.) translated the difference in compactness between the structures crystallographic cubic with centered faces (austenite) and cubic centered (ferrite) with temperature of T_{ref} reference.

That is to say $Z(M, T) = \{Z_1, Z_2, Z_3, Z_4\}$

1

2

3

4) respective proportions of ferrite, pearlite, bainite and martensite present in a material point M at the moment T . With the help of the assumption of a law of mixture to define the thermal deformation of a multiphase mixture (characterized by Z) one a:

$I = 4$

$I = 4$

H_T (

T

T

$Z, T) 1$

$Z.$

R

ref.

R
ref.
=
-

I

$$(T - T_{ref}) - (1 - Z)$$

Z
F
I
F

$$(T - T_{ref}) + Z$$

F

+

$$I = 1$$

I = 1
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For the calculation of the thermal deformation it is thus necessary to be given:

- .
the dilation coefficient of the cold phases,
- .
the dilation coefficient of the hot phase,
- .
a metallurgical phase of reference and a temperature of reference,
- .
the difference in compactness between the hot and cold phase at the temperature of reference.

These data are provided by the user in operator DEF1_MATERIAU [U4.23.01] under the word key ELAS_META_FO except the temperature of reference which one defines in AFFE_MATERIAU.

Depend on the temperature and are calculated for the temperature of the point of current Gauss.

3 ***Plasticity of transformation***

In experiments, it is noted that the dilatometric statement of a test-tube in the course of structure transformation is strongly influenced by the state of stresses and that the application of one constraint even lower than the elastic limit of material can nevertheless cause one unrecoverable deformation (cf [Figure 3-a]).

Transformation
bainitic

= 0 MPa
T
Pt
= - 42 MPa
= - 85 MPa

Pt
Application of
the constraint

Appear 3-a: Swelling behaviours under uniaxial constraints of compression for a steel 16 MND5

One calls **plasticity of transformation** this phenomenon and one notes P_t the **unrecoverable deformation** corresponding.

The model of plasticity of transformation most frequently used is, at the origin, generalization three-dimensional of the unidimensional phenomenologic model established by DESALOS [bib12]. If, with to start from a dilatometric test, one traces the difference between the lengthening obtained for a constraint applied different from zero and that obtained for a null constraint according to the advance of the transformation, one notes that:

$$P_t(\epsilon, b) = (\epsilon, b) - (\epsilon, 0) \\ \Delta \epsilon = K F(b)$$

where:

K is a homogeneous constant contrary to a constraint,

F is a standardized function ($F(0) = 0$ and $F(1) = 1$),

$\epsilon = 0$ and $F(1) = 1$,

$\epsilon = 1$),

and

B is the proportion of the transformed phase.

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A three-dimensional and temporal generalization of the preceding experimental model, for only one transformation, was proposed by LEBLOND [bib8], [bib9], [bib10], [bib11], in the form:

3

&pt

~

=

K

F' (b) &

I J

I J

B

2

On the basis of experimental tests and for transformation of a bainitic type of a steel 16MND5 for example: K is taken equalizes at 10⁻⁴

-1

MPa and F (b) = B (2 - b).

It is based on the following heuristic considerations:

.

the relation must be “incremental”, i.e. to connect the rate of plastic deformation to the rate of transformation,

.

the speed of plastic deformation of transformation must be, as for plasticity traditional, proportional to the deviatoric part ~ tensor forced

~

1

= -

T R

ID, (the plasticity of transformation occurs without change of

3

volume, from where a dependence compared to the diverter of the constraints rather than to the field of constraints itself),

.

the rate of plastic deformation of transformation must be null apart from the beaches of transformations,

the integration of this relation in the uniaxial case with constant constraint must give again experimental relation.

The phenomenon of plasticity of transformation can exist at the time of structure transformations under constraints of type the ferritic, perlitic, bainitic and martensitic, which possibly can to appear simultaneously. On the other hand, it is considered that this phenomenon does not exist at the time of austenitic transformation. The model general established in Code_Aster is thus:

$$I = 4$$

$$I = 4$$

$$I = 4$$

$$3$$

$$\& Pt (, \mathbf{Z}) = \& Pt$$

$$I (, \mathbf{Z})$$

$$\sim$$

$$=$$

$$K F '$$

$$I$$

$$I$$

$$Z_i$$

$$\& Z_i$$

$$<$$

$$>$$

$$2$$

$$I = 1$$

$$I = 1$$

$$I = 1$$

where: $\langle X \rangle$ indicates the positive part of a size.

The K_i data and

F

are provided by the user in `DEFI_MATERIAU` under key word `META_PT`.

In Code_Aster it is possible not to take into account the phenomenon of plasticity of transformation. If this phenomenon is taken into account, it appears as soon as there is transformation and that even if the structure plasticizes. The model is more particularly dedicated to steel.

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**4 Restoration
of work hardening**

In a usual way the state of work hardening of a phase I is characterized by its plastic history. Thus by example in the case of plasticity with linear isotropic work hardening, one generally takes like variable of work hardening noted cumulated plastic deformation p. the term of work hardening is written then: $R = R p$

I

I

0

where R_{i0} is the linear coefficient of work hardening of phase I.

At the time of the metallurgical transformations, there exists within material of displacements of atoms more or less important. These displacements of atoms can destroy dislocations which are with the origin of work hardening. In these cases, the work hardening of the mother phase is not transmitted to the phase

produced, it is the restoration of work hardening. The new phase can then be born with a plastic state virgin or to inherit only one part, possibly totality, the work hardening of the mother phase.

The cumulated plastic deformation p is not characteristic any more of the state of work hardening and it is necessary to define

other variables of work hardening for each phase, noted R_i which take account of the restoration of work hardening.

The term of work hardening of phase I is written $R_{I then} = R R$

I

*I**O I.***4.1*****Model with 2 phases with a direction of transformation****To define the variables laughed, one chooses the model suggested by LEBLOND [bib11].**One considers an element of two-phase volume V which undergoes a metallurgical transformation and one plastic deformation.**F**voluminal raction V1,**Phase 1 is the mother phase characterized by Pr**oportion of phase (1 - Z),**variable décroissance**1**R**F**voluminal raction V2,**Phase 2 is the phase produced characterized by Pr**oportion of phase Z,**variable décroissance**2**R**The equations of evolution of laughed obtained by derivation compared to time are written:*

$$\dot{r}_1 = \dot{p}$$

*éq***4.1-1***&z**&z*

$$\dot{r}_2 = \dot{p} - R + R$$

Z 2

Z
1

characterize the proportion of work hardening transmitted of the mother phase to the produced phase.
&p is the rate of equivalent plastic deformation.

Note:

p here is not any more one variable internal of the problem as such. The only significance of &p is here to be the plastic multiplier and it is equal to the rate of equivalent plastic deformation.

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SJÖSTRÖM obtains the same equations by using a phenomenologic reasoning as one defer here to clarify the model [bib13].

That is to say an increment of time T, such as between T and t+t:

.

a V2 fraction of the mother phase is transformed into phase 2 and thus comes to be added with V2 volume of this phase produced,

.

the element of volume V undergoes a plastic deformation p.

1
1
2
2

T

V

, $R(T) + p$

V

, $R(T$

$2 2$

$2 2$

)

p

V

, $R(T) + p$

$V, R(T$

2

1

$1 1)$

V

V

, $R(T) + p$

2

$1 1$

It is supposed that at the time of the metallurgical transformation, the transformed fraction V_2 inherits only one

r_1 part of the work hardening of the mother phase ($0 < r_1 < 1$).

Then the variables of work hardening laughed at the moment $t+t$ are such as:

$$R(T + T) = R(T) + p$$

1

1

$$V(R(T) + p$$

$$) + V(R(T) + p)$$

$$R$$

$$(T + T$$

$$2$$

$$2$$

$$2$$

$$1$$

$$2$$

$$) =$$

$$V + V$$

$$2$$

$$2$$

Maybe, by considering that $R(T + T)$
 $) = R(T) + R$

$$I$$

$$I$$

$$I$$

$$R = p$$

$$I$$

éq
4.1-2

$$V$$

$$V$$

$$R = p$$

$$2$$

$$+$$

$$R$$

$$2$$

$$-$$

$$R$$

$$2$$

$$V + V$$

$$I$$

$V + V^2$

2
2
2
2

One obtains the equations [éq 4.1-1] while passing in extreme cases.

For the discretization of the laws of evolutions of laughed, one chooses a diagram of integration clarifies in

using directly the equations [éq 4.1-2].

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4.2

Generalization of the model with N phases with transformations with double feel

In the case of steel the existing phases are: Ferrite, Pearlite, Bainite, Martensite and Austenite, of

$K = 4$

respective proportions Z, Z, Z, Z

and $1 - Z$

1

2

3

4

K.

K 1

=

.

.

In the case of a cooling, the metallurgical transformations to consider are them transformations of () in (F), (P)

, (B)

or (M).

.

In the case of a heating one considers the transformations in the other direction: (F), (P)

,

(B), (M) in ().

One can thus write in a case general (where X indicates the positive part of X.

4

4

< - Z

> R - - < - Z

> R -

K

K K

K

= 1

=

If Z > 0

R

= p K

K 1

+

4

1 - Zk

K = 1

if not

R - = 0

and

R -

= 0

éq

4.2-1

< Z

> R - - < Z

> R -

K
K

K
K

If $Z > 0$

R

= p

K

K

+

Zk

if not

R - = 0

and

R -

K

K = 0

K: proportion of restoration of work hardening at the time of the transformation into K

K: proportion of restoration of work hardening at the time of the transformation K into

For transformations with diffusion (ex: out of F, P, B) implying of important displacements

atoms one will be able to take = 0; dislocations at the origin of plastic work hardening are

completely destroyed by the transformation. For transformations without diffusion

(ex: martensitic transformation), one will be able to take = 1, work hardening being completely

transmitted.

Are provided by the user in operator DEFI_MATERIAU under key word META_RE.

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Models of deformation (visco) plastic

The principal characteristic of the thermal evolutions concerned in this type of analysis is that they sweep a broad temperature range, which has an important effect on the behavior mechanics of the material which undergoes the thermal evolution. One is in particular in fields of temperature where the phenomena of viscosity can not be negligible more. It can thus be necessary to use a élasto-viscoplastic model of behavior especially when one remains in these fields for one important length of time; for example during the treatments of detensioning associated welding.

A viscoplastic model is thus chosen whose characteristics are such as it makes it possible to describe with the same formalism, therefore without changing model:

*·
a traditional plastic behavior; to model the cases at low temperature when the viscous effects are still negligible or to model the processes at speed raised (welding),*

*·
a hammer-hardenable viscoplastic behavior at high temperature, to model the effects of creep and relieving associated for example with the treatment with detensioning or with multirun weldings,*

*·
a behavior of the fluid type viscous for the temperatures higher than the temperature of fusion, in order to have a reasonable description of the molten zone.*

The selected viscoplastic model degenerates indeed for certain borderline cases in model of plasticity independent of time, or in model of viscous fluid.

One places oneself here within the framework of the plasticity of von Mises with additive isotropic work hardening.

The use of a kinematic work hardening being also possible (version 6.1.6).

Function threshold:

$$F =$$

$$- R (R; T, Z) - (T, Z$$

$$eq$$

$$C$$

$$)$$

$$3$$

$$1/2$$

~ ~

eq

equivalent constraint of von Mises,

:

$$E Q =$$

$$2$$

$R (R; T, Z)$:

isotropic term of work hardening,

(T, Z)

initial critical stress; corresponds to the minimal constraint

:

C

initial to apply to have a viscoplastic flow.

Plastic rate of flow:

$$F$$

$$\sim$$

$$3$$

$$\&vp = \&$$

$$=$$

$$\&$$

$$p$$

2 eq

The cumulated plastic deformation ϵ_p is viscous and is written:

$$\dot{\epsilon}_p = -R(R; T, Z) - (T; Z)$$

N

eq

C

>

$\epsilon_p =$

éq

5-1

, N : coefficients materials of viscosity.

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Note:

1

One can rewrite the equation [éq 5-1] in the form:

$$-R(R; T, Z) - (T, Z) -$$

N

$$p\dot{\epsilon} = 0$$

eq

C

,

i.e. in this model, the constraint can be interpreted as the sum of a constraint limit of flow (which breaks up it even into an initial ultimate stress and a term of work hardening) and a constraint “viscous” depending on the speed of deformation and null with null speed.

Viscous restoration of work hardening

One also introduces into modeling the phenomenon of viscous restoration of work hardening who leads to a évanescence partial of work hardening. Under the action of thermal agitation, it product a slow restoration of the crystalline structure of metal by annihilation of dislocations and internal stress relaxation. The model used to describe this phenomenon is as follows:

$$\dot{R} = R \dot{\epsilon} - \frac{R}{C}$$

0

$$r\dot{\epsilon} = p\dot{\epsilon} - \frac{r}{C}$$

m

C

(R)

The term of evolution of the variable of work hardening R thus comprises a term of work hardening due to plastic deformation and a term of restoration.

The model thus makes it possible to describe the primary education phenomenon of creep (work hardening) and creep secondary (stabilization of work hardening).

Case of linear kinematic work hardening:

In a way equivalent to the case with isotropic work hardening the equations are written;

function threshold:

$$F = \sim$$

(- X) -

eq

C

law of flow

Writing of the rate of deformation (visco) plastic

3
(~
~
- X)
< F N
>
&vp = &p
with =

2
(- X)
&p
eq

2
X = H
0
3

, ~: tensor of constraint and its diverter
: variable tensor of kinematic work hardening,
X: tensor of work hardening associated with the variable tensor with work hardening,
H: kinematic coefficient of work hardening
0

model of evolution of the tensor of work hardening of a material with N phases

(< z& >) - (z&)
K
K
K
< >

K

3

& = vp
& + K
K
+
m
C

()

eq
Z
2

eq

z& - z&
3

& = vp
& + K K K
K
K +

m
C

()
K
eq

Z
2

K
eq

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:

By preoccupation with simplification and a same manner that into isotropic, one takes for the term of viscous restoration:

$$= Z$$

I

I

I

$$C = Z C$$

I

I

I

$$m = Z m$$

I

I

I

ij : coefficients of metallurgical restoration at the time of transformation I

J

$C m$

,

: coefficients of viscous restoration of phase I .

I

I

5.1

Borderline case: Plastic model independent of time

One wants to describe an instantaneous elastoplastic behavior and to cancel the viscous effects. For that the viscous parameters and C will be taken equal to zero. To be free from the numerical problems what can pose the taking into account in and C null, and a way similar to the treatment carried out for the viscoplastic model of Taheri [bib15], one rewrites the equation [éq 5-1] in the form:

I

F

$p \leq N$

- & 0 éq 5.1-1

$F < 0$

the strict inequality being obtained in the case
(elastic mode).

$p \leq =$

0

In the purely plastic field of behavior (0) the inequality [éq 5.1-1] is then reduced
with: $F =$

$- R - (T$

eq

C

) 0 and p can be given more only by the equation of consistency

$f = 0$.

One thus finds oneself well within the framework of instantaneous plasticity independent of time, with
one

digital processing identical to that classically used for the treatment of this one.

Note:

It will be noted that C corresponds then to the traditional definition of the yield stress.

y

The elastic limit will be noted C in viscoplasticity and plasticity independent of time

y

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5.2

Borderline case: Model of behavior of viscous fluid

R 0

At very high temperature one a:

C 0

3

if one takes N 1, then: ϵ_{vp}

eq

=

maybe into unidimensional: $\epsilon_{vp} =$

. A model thus is obtained

2

of behavior of the fluid type viscous Newtonian, viscosity.

Note:

In Code_Aster, the relations of behavior available are is models completely plastic independent of time, that is to say models with viscous effect. (cf [§6]).

5.3 Plasticity multiphase

The metallurgical transformations involve modifications of the mechanical characteristics of material.

The elastic characteristics (YOUNG modulus and Poisson's ratio) are affected little by metallurgical changes of structures. Only their dependence compared to the temperature is thus taken into account.

On the other hand, the plastic characteristics (elastic limit in particular) strongly depend on metallurgical structure. It is thus necessary to take into account the differences in characteristics plastic for each possible phase. In modeling the strain and the stress are defined in the scale of the material point (macroscopic) which can be multiphase. One seeks to define it plastic behavior are equivalent of material when it has a multiphase structure, with in particular a single criterion of plasticity. The definition of the behavior of material are equivalent fact using a law of the mixtures on the characteristics of the phases. More precisely the definition this material equivalent would correspond in 1D to a rheological model of I bars in parallel such that:

$\epsilon_{vp} = \epsilon_{vp}$

I

=

= +

$R + \nu p$

$z_i I$

I

C_i

I

&i

with

I

More precisely, in the case of the plasticity of von Mises with isotropic work hardening;

.
the function threshold is expressed by:

$$F (, R; T, \mathbf{Z}) =$$

- R

eq

$$(T, \mathbf{Z}, R) - C (T, \mathbf{Z})$$

where:

$$R (T, \mathbf{Z}, R) = z_i I$$

$R (T, ir)$

I

is the work hardening of multiphase material, I_H being that of phase I .

where

$$C (T, \mathbf{Z}) = z_i C_i$$

I

is the elastic limit of multiphase material, that of phase I .

C_i

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.

and the rate of plastic deformation checks the condition of consistency $F = 0$ data by this law of mixture. I.e. when one is in load, \dot{p} is such as:

$$\dot{f} = - R (; rT, Z) - (T, Z)$$

I

-

I

N

Z p

eq

C

$$\dot{\epsilon} = 0$$

I I

I

One also gives the possibility of using a nonlinear law of the mixtures [bib9] such as one has in

$$D = (1 - F (Z)) + F (Z)$$

H

H

. One has then:

$$R = 1$$

$$(- F (Z))R + F (Z) R$$

H

H

.

= 1
(- F (Z
))
+ F (Z
)
C
H

C
H

C

y is the elastic limit of the austenitic phase,

4

Z = Z_k is the total proportion of the phases “” ” (F, P

, B

, M

)

K = 1

4Z kck

K

=

= 1

is the equivalent elastic limit of the cold phases “” ”

C

Z

4

Z R

K K

R

K 1

= =

is the average work hardening of the cold phases.

Z

~

1

I
Z
.
and &p of it checks $F = - R$ charges (;
R T, Z) - (T, Z) - 1
(- F (Z))

N
p
& -
K
N
p
.
eq
C
H
K & = 0
K
K
Z

F (Z)
H
is a function defined by the user under operand SY_MELANGE of the key word factor
ELAS_META_FO.

Parameters I, I
N, I
C and I
m are defined in DEFI_MATERIAU under the key word factor
META_VISC. The limit elastic parameters are defined under the key word factor ELAS_META_FO;
*key word *_SY for the plastic models independent of time and key word *_S_VP for*
viscoplastic models.

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6 Relations of behavior

6.1 Partition of the deformation:

The deformation is written as the sum of four components:

$$= E + T H + \nu p + P t$$

where: E T H νp

*,
,
and p T are respectively the elastic strain, thermics,
viscoplastic and of plasticity of transformation,*

6.2 Laws of behavior

6.2.1 Case with isotropic work hardening

$$= E
+ H T
+ \nu p
+ P t$$

$$= A (T) E$$

$$4
P t
& = 3 \sim
K ' F
I$$

I (1 - Z)

< &Z >

I

2 i=1

4

HT

(Z, T)

=Z.

(T -

ref.

ref.

ref.

T

)

T

R

- 1

(- Z)

+

Z

T T

Z

F

I

F (- ref.)

T

R

+

F

i=1

~

vp

3

& = p&

2 eq

5

F = - R (T, Z, R) - (T, Z) with R (T, Z, R) =

eq

C

Z R (T, R)

II

I

i=1

p& = 0si F < 0

-

- ~

1

p& 0 if F = 0 and checks F = - R (T, Z, R) - (T, Z) -

I

N

eq

C

Z p&

= 0

II

I

4

4

< - Z & > R -

K

K K

< - Z & >

K

R

k=1

k=

r& = p

& +

1

- (C moy

R

) m

if Z >,

0 R

& = 0 si Z = 0

4

1 - Z

K

k=

1

&

&

< Z >

R - < Z > R

r& = p& +

K

K

K

K -

K
(C moy
R
) m if Z >, Or& = 0siZ = 0

K
K
K
Z

K

5
with R

=

moy
Ir Zi
i=1

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F
the function threshold,

R, Z R
variables intern work hardening and their forces

I

II

thermodynamic associated,

WITH = (A

the tensor of elastic rigidity, depend on the temperature,

I J K L)

T (T) and Z (T)

the temperature and the metallurgical structure.

6.2.2 Case with kinematic work hardening

= **E**

+ **HT**

+ **v_p**

+ **Pt**

= **A (T) E**

4

Pt

& = 3 ~

K 'F

I

I (1 - Z)

< &Z >

I

2 i=1

4

HT

(Z, T)

=Z.

(T -

ref.

ref.

ref.

T

)
T
R

- 1
(- Z)

+
Z
T T
Z

F
I
F (- ref.)
T
R

+

F

i=1

~
vp
3
(- ~
X)
& = p&

2 (- X) eq

5
F = (- X) - (T, Z) with X (T, Z,) =

eq
C
Z X (T,)

I
I
I

i=1

p& = 0 if F < 0

-
- ~
I

p& 0 if F = 0 and checks F = - R (T, Z, R) - (T, Z) -

I
N

eq
C
Z p&
= 0
II

I

(< z&k >kk) - (< z&k >)

3

vp
K
K
m
& =

& +
+
C
(eq)

if
Z >

0 & = 0 if not

Z
2

eq

z& - z&
3

vp
K
K K
K K

m
&k = & +

+
C
(eq)

if
Zk > 0 &k = 0 if not

Z

K
2
eq

with
=

eq
Zi I

I
eq

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F

the function threshold,

R, Z R

variables intern work hardening and their forces

I

II

thermodynamic associated,

WITH = (A

the tensor of elastic rigidity, depend on the temperature,

I J K L)

T (T) and Z (T)

the temperature and the metallurgical structure.

In term of relations of behavior of STAT_NON_LINE available, the modeling put in place offers several possibility:

.

choice of the type of behavior for the plastic deformation; plastic independent of time or with taking into account of the viscous effects,

.

choice of a work hardening isotropic linear, isotropic nonlinear or kinematic,

.

taking into account or not of the plasticity of transformation,

.

taking into account or not of the metallurgical restoration of work hardening.

The choice of the material (steel or zircaloy) and thus of the number of phase is done by informing the key word

KIT of STAT_NON_LINE. "STEEL" for steel with 5 phases and "ZIRC" for the zircaloy with 3 phases.

6.3 Various relations of elastoplastic behavior

*META_P_****

There are 12 relations of elastoplastic behavior independent of time META_P.*

.

8 relations with isotropic work hardening according to whether a linear isotropic work hardening is considered

or not linear, that one takes into account or not the plasticity of transformations, that one takes into account or not the metallurgical restoration of work hardening.

.

4 relations with linear kinematic work hardening according to whether one takes into account or not plasticity of transformations and/or metallurgical restoration of work hardening.

For these 12 relations of behavior one informs under key word ELAS_META_FO or ELAS_META elastic parameters E and Naked, dilation coefficients, as well as the elastic limits.

/ELAS_META_FO

: (E: E

NAKED:

F_ALPHA:

F

C_ALPHA:

C

PHASE_REFE:

“HOT”

“COLD”

Tref

EPSF_EPSC_TREF:

F C

F1_SY

:

yf1

F2_SY

:

yf2

F3_SY: yf3

F4_SY: yf4

A_SY:

yc

SY_MELANGE

:

F)

with for steel:

F: F

C:

Tref

Tref

F C: F

yfi: elastic limit of phase I

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6.3.1 Relation

META_P_IL

This relation makes it possible to treat the relation of behavior in the case of the plasticity of Von Mises

with linear isotropic work hardening, applied to a material which undergoes phase shifts metallurgical. The phenomena of plasticity of transformation and restoration of work hardening are neglected. The coefficients of work hardening are provided under key word META_ECRO_LINE of the operator

DEFI_MATERIAU.

/META_ECRO_LINE:

F1_D_SIGM_EPSI

:

H0f1
F2_D_SIGM_EPSI
:
H0f2
F3_D_SIGM_EPSI
:
H0f3
F4_D_SIGM_EPSI
:
H0f4
C_D_SIGM_EPSI
:
H0c

with for steel:

H0fi: *Linear coefficient of work hardening of phase I.*
F: *function of Z defining the law of mixture for the plastic behavior.*

6.3.2 Relation **META_P_INL**

This relation makes it possible to treat the relation of behavior in the case of the plasticity of von Mises with nonlinear isotropic work hardening, applied to a material which undergoes phase shifts metallurgical. In DEFI_MATERAU in addition to ELAS_META_FO one returns under the key word META_TRACTION the curves R (R).

META_TRACTION:

F1_SIGM: R (R)

1

F2_SIGM: R (R)

2

F3_SIGM: R (R)

3

F4_SIGM: R (R)

4

C_SIGM: R (R)

C

6.3.3 Relation

META_P_CL

This relation makes it possible to treat the relation of behavior in the case of the plasticity of Von Mises

with linear kinematic work hardening, applied to a material which undergoes phase shifts metallurgical. The phenomena of plasticity of transformation and restoration of work hardening are neglected. The coefficients of work hardening are provided under key word META_ECRO_LINE of the operator

DEFI_MATERIAU.

/META_ECRO_LINE:

F1_D_SIGM_EPSI

:

H0f1

F2_D_SIGM_EPSI

:

H0f2

F3_D_SIGM_EPSI

:

H0f3

F4_D_SIGM_EPSI

:

H0f4

C_D_SIGM_EPSI

:

H0c

with for steel:

H0fi: Coefficient of kinematic work hardening linear of phase I.

F: function of Z defining the law of mixture for the plastic behavior.

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6.3.4 Relation

META_P_IL_PT, META_P_INL_PT, META_P_CL_PT

Compared to META_P_IL, META_P_INL or META_P_CL one holds account in addition to plasticity of transformation but one always neglects the restoration of work hardening. In addition to the data of the key word

factor ELAS_META_FO and of the key word relating to the data of work hardening, one must inform too

those relating to the plasticity of transformation which are provided under the key word factor META_PT.

/META_PT

:

(

F1_D_F_META:

F'1 F1_K

:

f1

F2_D_F_META:

F'2 F2_K

: f2

F3_D_F_META:

F'3 F3_K

: f3

F4_D_F_META:

F'4 F4_K

: f4)

with for steel:

$$F' f1 = F F$$

$$f1 = K F$$

$$F' f2 = F p$$

$$f2 = K p$$

$$F' f3 = F$$

B

$$f3 = KB$$

$$F' f4 = F$$

m

$$f4 = km$$

6.3.5 Relation

META_P_IL_RE, META_P_INL_RE and META_P_CL_RE

One takes account of the restoration of work hardening but the plasticity of transformation is neglected. data relating to the restoration of work hardening are provided under the key word factor META_RE of operator DEFI_MATERIAU.

/META_RE

: (

C_F1_THETA

: cf 1

F1_C_THETA

: cf 2

C_F2_THETA

: cf 2

F2_C_THETA

: cf 2

C_F2_THETA

: *cf 2*

F3_C_THETA

: *cf 3*

C_F2_THETA

: *cf 2*

F4_C_THETA

: *cf 4*)

with for steel:

CF1 = F

F1C = F

CF2 = P

F2C = P

CF3 = B

F3C = B

CF4 = M

F4C = M

6.3.6 Relation

META_P_IL_PT_RE, META_P_INL_PT_RE and META_P_CL_PT_RE

One holds account at the same time phenomena of plasticity of transformation and restoration of work hardening. The data of the key words factors ELAS_META_FO, META_PT and META_RE must to be well informed.

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6.4 Various relations of élasto-viscoplastic behavior

META_V_***

One has in the same way that in traditional plasticity, 12 relations of behavior which are available according to the type of work hardening and according to whether one holds account or not phenomena of plasticity of transformation and/or of metallurgical restoration of work hardening. One uses the same terminology as in the case of traditional plasticity to differentiate the 12 élasto-viscoplastic relations. For each relation one must inform in ELAS_META or ELAS_META_FO the yield stresses of flow viscous, in the place of the traditional apparent elastic limits.

F1_SC: cf 1

F2_SC: cf 2

F3_SC: cf 3

F4_SC: cf 4

C_SC: DC

SC_MELANGE: function for the law of the mixtures

*instead of the *_SY for the plastic case.*

6.4.1 Relation

META_V_IL and META_V_INL

Élasto-viscoplastic relation of behavior applied to a material which undergoes transformations metallurgical with or not linear linear work hardening. One does not take account of the phenomena of plasticity of transformation and metallurgical restoration of work hardening.

6.4.2 Relation

META_V_CL

Élasto-viscoplastic relation of behavior applied to a material which undergoes transformations metallurgical with linear kinematic work hardening. One does not take account of the phenomena of plasticity of transformation and metallurgical restoration of work hardening.

6.4.3 Relation

META_V_IL_PT, META_V_INL_PT and META_V_CL_PT

Idem that META_P_IL_PT, META_P_INL_PT and META_V_CL_PT but in viscoplasticity.

6.4.4 Relation

META_V_IL_RE, META_V_INL_RE and META_V_CL_RE

*Idem that META_P_IL_RE, META_P_INL_RE and META_V_CL_RE
but in viscoplasticity*

6.4.5 Relation

META_V_IL_PT_RE, META_V_INL_PT_RE and META_V_CL_PT_RE

Idem that META_P_IL_PT_RE, META_P_INL_PT_RE and META_V_CL_PT_RE but in viscoplasticity

Note:

.
*For the whole of relations META_**, the internal variables produced in
Code_Aster are:*

*laughed: variables of effective work hardening for I phases,
D: indicator of plasticity (0 if the last calculated increment is elastic; 1 if not),
R: the term of work hardening of the function threshold*

.
*In addition, these modelings can be carried out with the functionality of
geometrical reactualization PETIT_REAC.*

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7 Formulation numerical

One will treat the viscoplastic law of behaviour with isotropic work hardening.

7.1 Discretization

Knowing the fields, U and p at the moment T , one chooses an implicit scheme to discretize in time equations of the continuous problem, except for the parameters of work hardening where they are used equations [éq 4.2-1].

It is noticed that with an implicit discretization, only two points differentiate the two types from viscoplastic behavior and plastic independent of time:

· the form of the function of load, for which one has a complementary term in the case of viscosity,

· the presence of the term of restoration of work hardening in the evolution of the variable of work hardening for the viscoplastic case.

Moreover, incremental traditional plasticity seems the borderline case (without numerical difficulty

0 associated) of incremental viscoplasticity when $C \rightarrow 0$

*C
y*

This type of treatment was already carried out by LORENTZ [bib15].

*l
~
p
N*

If one poses $F = F -$

T

$$= E + HT + p + Pt$$

$$= A$$

$$(T) E$$

4

$$HT (Z, T) =$$

Z

.

$$(T - T) - (1 - R$$

Tref

R

Z)

F

Z

T - T

+

+

I [F (

) Z F]

I = 1

3

4

Pt

=

~

$iK F' I (1 - Z) < Z$

>

I

2

$I = 1$

~

3

p

=

p

2

eq

~

elastic mode:

$F < 0$ and

p

= 0

~

D

gime (visco) plastic $F = 0$ and

p

> 0

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4

4

< - Z > R - < - Z > R

K

K K

K

m

R = p k=1

K = 1

-

+

-

if

0,

0 if

0

4

(Crmoy)

Z >

R =

Z

=

1 - Zk

K =

1

$$\langle Z \rangle R - - \langle Z \rangle R -$$

m

$$\text{if } Z > 0, R = 0 \text{ if } Z = 0$$

K

K

K

K

K

K

$$R = p$$

-

K

+

- (C_{rmoy})

Z

K

with:

$$X = X(T + T)$$

$$X - = X(T)$$

$$X = X(T + T) - X(T)$$

7.2

Algorithm of resolution of the quasi-static problem

The incremental problem posed on the structure is a non-linear problem. Its formulation variational, in the case of the small deformations, is form:

To find U such as:

$$((U + U), T) (v) D$$

*= $L(T) v$ kinematically acceptable
and T*

$$\mathbf{B} U = \mathbf{u}d(T)$$

where:

U indicates the field of displacement

B U = ud T

() corresponds to the boundary conditions in displacement (connections kinematics)

and

(

L T) =

F v D +

g.v D

.

is the virtual work of the mechanical loadings at the moment

T.

In Code_Aster, this non-linear problem is solved by a method of NEWTON [bib6], [bib7]. The algorithm of resolution comprises:

.

a phase of prediction at the beginning of each step of time,

.

iterations of Newton inside a step of time.

We do not detail here the algorithm implemented (one will refer for that to the documents of reference [R5.03.01] and [R5.03.02]), but we endeavour to highlight them modifications made to the diagram of integration by the taking into account of the metallurgical evolution

Z (T) and of the plasticity of transformation.

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7.2.1 Integration of relations META__ ***

One gives the expression of according to;

.
(or U) unknown of the problem,
.
*known terms such variables calculated with the preceding step (-, variables intern...),
the characteristics materials, HT...*

= (
WITH T) E
E = early - HT - vp - Pt
One poses: = early - HT

μ
 $\sim = 2\mu \sim e =$
 $- + 2\mu \sim$
 $\sim vp$
 $\sim Pt$

-
(- -)

μ

3K
tr = 3K tr (E) =
-
tr
+ 3K tr

-
3K

μ

~

~

-

3

3

=

+ 2 μ ~

- ~

-

F (Z, Z) -

p

μ

2

2

eq

from where

~

~

1

μ

~ -

~

=

+ 2 μ

- μ

3

p

1 + μ

3 F (Z, Z

) μ -

eq
with:

~

.

expression of
eq

$$\sim = \sim e + \sim Pt + \sim$$

vp

~

~-

~

3

3

~

~

=

-

$$+ F(Z, Z) + p$$

μ

$\mu-$

2

2

2

2

eq

μ

~

2μ

~ +

~- =

-

$$(1 (+3MF(Z, Z)) + 3\mu$$

eq

p)

μ

eq

μ

one poses: 2μ

~

-- ~

+

= E

μ -

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one a: $E = (1 + \mu$

$3 F (Z, Z$

$)) + \mu$

eq

eq

3

p

and

~e

~

=

E

eq

eq

.

expression of p

E

eq

That is to say the function of load: F =

- R (R -; T, Z) - (T, Z)

1 + μ

3 F (Z, Z)

C

)

R (R -; T, Z) is the term of work hardening R (R; T Z) calculated for p = 0.

-

If F < 0 then one is in elastic mode and p = 0

-

If not one is in load and p checks;

1

p

E

N

- 3μ p

eq

=

- R (R -; T, Z) - R p

- (T, Z)

T

1+ 3μ F (Z, Z)

)

0

C

E
In
 \sim
 $- 3\mu p$

eq
 $-$
p

That is to say the function $F =$

$- R (R T$
 $; , Z) - R p$
 $- T$
 $(, Z) -$
 $, p$ is thus
 $1 + 3\mu F (Z, Z$
 0
 C
 $)$
 T

solution of the scalar equation nonlinear $F = 0$.

The resolution is made in Code_Aster by a method of the secants with interval of research [bib15].

Note:

*Whenever the plasticity of transformation is not taken into account, expressions obtained are the same ones by taking $F Z$
 $(, Z) = 0$.*

Whenever it is the restoration of work hardening which is neglected then one also has them same expressions but by taking all the equal ones to 1.

*H is the slope of work hardening of the traction diagram. In the case of isotropic work hardening not
 0
linear where the traction diagram is linear per piece, H is defined for the segment to which
 0
p belongs. **Stamp tangent***

7.2.2.1 Phase of prediction - Option RIGI_MECA_TANG

One linearizes the continuous problem compared to time, and one determines u_0 as solution of

problem of speed:

&
u0 T
v D
= L T
v kinematically acceptable

((,) () & (
where & (
L T) =
&f.v D +

&g.v D

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The problem from of speed is obtained by deriving compared to time the equations from the problem continuous:

~&
= 2μ (~& - &vp - &pt)

3
&PT = F (Z, Z

) ~&

2

In the case of the élasto-viscoplastic models, one uses, for the phase of prediction, the matrix “elastic” in the direction where one will not take account of the &vp term. As for the plastic case one a:

~

3

&p

if eq - R (T, Z, R) - y (T, Z) = 0

&vp = 2

eq

0

if eq - R (T, Z, R) - y (T, Z)

0

Derivation compared to the time of the equation

- R (T, Z, R) -

, Z =

E Q

y (T

) 0 give

the expression of &p (relation of consistency).

D

~

~vp

~ Pt

:

~

With & - &

- &

eq

D R

D y

3

(

) *D R Dy*

-
-
=
-
-

D T
D T
D T
2

D T
D T
eq

Note:

The derivation of A was neglected in this phase of prediction

i=5
i=5
i=5
R& T
(, Z
EFF
,) = Z& R R +
0
Z R& R +
0
Z R R
I
I
I
II
I
II
I
0i I
&
i=1
i=1
i=1
i=5

i=5
i=5
 = **Z& R R +**
0
Z R& R +
0
Z R p
I
II
I
II
I
0i &
i=1
i=1
i=1
K =4
K =4
 + < **Z& > R R +**
0

< - **Z& > R R**
K
K
K
K
0
KK
K =1
K =1
i=5
 - < **Z& > R R**

I
0i I
i=1
 = **R p**
0 & + B

y
&
 =
T

y
& +
Z& = C
y
T
Z
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from where:

~ ~
I = 4
D

&
eq
D R D

y
:
-
-
= 3μ
-3MK 'F -
<Z& >

$-\mu +$
 $\& - - =$
 I
 $I(1$
 $Z)$
 I
 eq
 $(3 R p B C 0$
 $0)$
 dt
 dt
 dt
 eq
 $I = 1$

$\sim \sim$
 $I = 4$
 $: \&$
 $< 3\mu$
 $- 3MK 'F -$
 $< Z\& > - - >$
 I
 $I(1$
 $Z)$
 $B C$
 I
 eq
 eq
 $I = 1$
 $p\&=$
 $3\mu + R0$

From where, finally the expression of $\& v_p$:

$\sim \sim$
 $\&$
 v_p
 3
 $:$
 $I = 4$
 $\& =$
 3
 $< \mu$
 3

**- $\mu K 'F$
&
 I
 $I (1 - Z)$
~
<Z >
- B C>
 $2 (3\mu + H$
 I
eq**

**0)
eq
 $I = 1$
eq**

if - R

**eq
 $(T, Z, effi) - y (T, Z) = 0$
vp
& = 0
if - R**

**eq
 $(T, Z, effi) - y (T, Z) 0$**

***Taking into account the variations of H and
0i***

***y according to the temperature and of the structure
metallurgical, one chooses by convenience to neglect the term (B+C) and one thus leads to one
expression of ~&
form:***

**~
 $I = 4$
~
~
&**

& =
3
:
~
 μ ~
2 & -
<3 μ
-3MK 'F
Z
Z&

I
I (1 -
)<
>
>

2 (3 μ +
I
eq
R

0)

eq
I = 1
eq
I = 4
-3 K 'F
Z
Z&
I
I (-
)

< > ~
I
I

2 i = 1

The expression of ~&
depends on the sign of the term (criterion of load-discharge)

~
 $I = 4$
: ~&
 3μ
 $- 3\mu K F'$
 I
 $I (Z_i) < \&$
 $Z >$

I
 $eq.$
 eq
 $I = 1$

~& is approximated
by:

~
 $I = 4$

~
 9μ
 3

3μ

$\&$
~
<
>
 $= 2\mu \&$
: ~&
~

-
 K
 $1 Z$
 Z
 1
 D

 2

2 (

-

-

<

>

I

I

-

$3\mu + R$

I

μ

R

0)

F' (

) & ~

2

3 +

eq

I

0

1

=

éq 7.2.2.1 - 1

with D = 1 if one plasticizes and if one is in load at the moment T and D = 0 in the contrary case.

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It is noticed that ~&

is a function closely connected of ~&. The plasticity of transformation, like thermal deformation, generate in the problem of speed a second member.

That introduced by the plasticity of transformation is form:

$i=4$

3

μ

3

$L_{pt} = 2\mu$

K'

F

Z

$Z\& \sim$

1

1 D

v D

II (-)

< >

I

-

()

2 1

μ

I =

3 + R0

To determine U_0 , it is necessary to solve after discretization spaces the following linear system of it:

K

$BT u_0$

L

0

$L_{th} L_{pt}$

0

+

+

+

B

0

0

=

0

ud

0

0

On simple cases tests for which there is an analytical solution, one noted that the fact of to neglect the second member $LP T$

() could lead, to converge, with a significant number

iterations. This is why this term is taken into account for the phase of prediction.

7.2.2.2 Iterations of Newton - Option FULL_MECA

In the method of NEWTON, knowing one, one determines one as well as possible +1 checking:

$F((u_{n+1}) =$

$((u_{n+1}), T) (v) D - L (T) 0$

$$F$$

$$F((un+1) F(one) +$$

$$((un+1) - (one) = 0$$

$$((one), T)$$

From where:

$$F$$

$$K$$

$$N$$

$$N =$$

$$= - F(U)$$

$$((one), T)$$

With each iteration one solves the linear system:

$$K$$

$$BT one$$

$$L$$

$$Fn$$

$$N$$

-

$$with$$

$$Fn$$

$$N$$

$$=$$

$$, T v D$$

$$N =$$

$$((U)) ()$$

$$B$$

$$0$$

$$0$$

0

During iterations of a step of time given, the method of NEWTON thus uses the calculation of the tangent operator K_N , which is given by the derivation of the implicit problem according to the increment of deformation. The tangent operator K_N can be recomputed or not with each iteration.

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One gives the expression of for the constitution of the consistent tangent matrix of the method iterative of Newton.

~ 1 (tr)

~ ~ ~

=

+

Id

with

=

3

~

~

=

~

Pt

vp

~

~ (2μ (

-

-

)

One a:

(~

) = *Id*

with

Id

=

~

(*ijkl*)

ik

jl

~

~ *Pt*

3

~ (

) = *F (Z, Z)*

2

~

~

~

~

3

vp

p

~ (

)

()

=

+

2 ~

p

~

eq

eq

with

$\sim e = 2\mu$

~

Id

$\sim eeq$

~

$= \mu$

~

3 *eq*

~

1

~

~

$2\mu Id\mu$

~

3

E

eq =

-

eq

eq

eq

(p)

~

The expression of ~ is obtained by deriving F = 0 compared to, which gives:

(p

)

3μ

~

~ =

1-n

p

N

3μ + R +

+

0

(1 3μF (Z,)) eq

Z

NT

T

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from where:

$3\mu p$

$2\mu Id1-$

$-(3\mu)$

2

E

eq

\sim

1

=

$\sim 1 + 3\mu F (Z, Z)$

1

$p \sim$

~

-

l-n
E

N

p
3μ + R +
eq
eq
eq
0

(1 + 3μF (Z, Z))

NTT

éq 7.2.2.2 - 1

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7.2.2.3 Operator tangent

That is to say = (,

, ,

2

,

2

,

2

)

11

22

33

12

23

the 13 virtual increase in constraint and

that is to say = (,
, , 2 , 2 , 2)

- 11
- 22
- 33
- 12
- 23

the 13 virtual increase in deformation,
the operator who binds to is given by the following expression:

$$2\mu$$

$$\mu$$

$$3p$$

$$C$$

~
p

$$\sim e \sim$$

$$=$$

$$1 - C$$

$$-$$

$$E \sim$$

ij

3rd ik jl
ij
kl
kl
has

has
eq

tr
 $() = 3K tr ()$

with:
1
die

with
drank
[é

(cf.

time

of

not

each

of

Q 7.2.2.1

-
,

(option

1])

,

TANG
RIGI_MECA_

)
 $= 1+3\mu F has (Z, Z)$

[é

(cf.

current

iterations

at the time

Q 7.2.2.2

-

(option

1])

,

,

FULL_MECA)

0

beginning

with

time

of

not

each

of

C =

3

1

current

iterations

at the time

(3 μ) 2

1

C
D

with the ébut
time

of

not

each

of

1
(eeq) 2 3μ + 0
R

and C =

p

2

(3μ)

1

p

C

-

2

2

at the time

iterations

-

1 N

E

E
(eq)
N

p

eq
 $3\mu + R +$
0

(+
 $1 3\mu ($
F Z, Z)

NT T

:
where

~ ~
I =
:
4
&
1

if elastoplastic and 3μ
-3MK 'F
I
 $I(1 - Z) < Z$
& >

$C =$
 0
 I
 eq
 1

eq
 $I = 1$
 0 if not
 1
(
 $charge$

in

is

one

if
 $\sim \sim$
 $0)$
 $plasticize$

one

if

and

$C =$
 2

0 if not

Let us note \mathbf{K} then the operator such as

$= \mathbf{K}$

and the vector of the diverter of the constraints is $S: S = (\sim$
 $, \sim$
 $, \sim$
 $,$
 \sim

2
,
~

2
,
~

2
)
11
22
33
12
23
13 .

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*D **
*d~**
=
+ K (I)
I
*d**
*d**
*d~**

d^*

1

=

$Id - 1 1$

d^*

d^*

3

d^*

2μ

μ

3

p

C

=

$1 - C$

Id

$p S S$

d^*

has

3

E

-

has

eq

from where:

D^*

2μ

μ

3

p

1

C

=

$1 - C$

Id

p

1 1

SS

*d**

has

3

E

-

3

-

has

eq

+ K (I)

1

*The operator **K** is written:*

2 2

μ

3μ p

C

K = K +

1 - C

p

- S S

1 1

3 A

3

E

1 1 has

eq

1 2

μ

$3\mu p$

C

K - 1 - C

p

- S S

3 A

3

E

1 has 2

eq

K

2 2 =

2 2

μ

$3\mu p$

C

K +

1 - C

p

- S S

3 A

3

E

has 2 2

eq

1 2
 μ
3 μ p

C

K - 1 - C
p
- S S

3 A
3
E

1 has 3

eq

1 2
 μ
3 μ p

C
K

33 = K -
1 - C

p
-
S S

3 A
3
E

has 2 3

eq

2 2
 μ
3 μ p

C

K + 1 - C
p
- S S

3 A
3
E

has 3 3

eq

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Author (S):

A. RAZAKANAIVO, A.M. DONORE, F. WAECKEL *Key*

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CP

-

SS

I has 4

C

P

-

SS

has 2 4

K = C

44

P

-

SS

has 3 4

2

μ

$3\mu p$

C

1 - C

P

S S

has

3

E -

has 4 4

eq

CP

-

S S

1 has 5

C

P

-

S S

has 2 5

C

P

K

-

S S

55 =

has 3 5

CP

-

S S

has 4 5

2

μ

$3\mu p$

C

$1 - C$

p

$S S$

has

3

$E -$

has 5 5

eq

CP

-

$S S$

1 has 6

C

p

-

$S S$

has 2 6

C

p

-

$S S$

has 3 6

$K = C$

66

p

-

$S S$

has 4 6

C

p

- $S S$

has 5 6

2

μ

$3\mu p$

$C S S$

$1 - C$

$p 6.6$

has

3

$E -$

has

eq

where “I” components of the K_{ii} vector correspond to “I” terms of the higher part of $i^{\text{ème}}$ column of the symmetrical matrix \mathbf{K}

It is noticed that the $\mathbf{K0}$ operator and operator \mathbf{kN} are different. The plasticity of transformation does not intervene in the same way in the calculation of the two operators.

Note:

These various terms were obtained by developing the case with isotropic work hardening but one obtains the same thing in the case of kinematic work hardening, R is then replaced

0

by the kinematic coefficient of work hardening H.

0

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Document: R4.04.03

*Law of behavior élasto (visco) plastic
in great deformations with transformations
metallurgical*

Summary

This document presents a model of behavior thermo-élasto- (visco) plastic at isotropic work hardening with effects of the metallurgical transformations writes in great deformations. This model can be used for three-dimensional, axisymmetric modelings and in plane deformations.

One presents the writing of this model and his digital processing.

To include/understand this document, it is practically essential to read the two notes [R5.03.21] and [R4.04.02]

devoted to the written models of behavior, respectively, in great deformations without effects metallurgical and in small deformations with metallurgical effects.

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This document presents a law of behavior thermo-élasto- (visco) plastic at isotropic work hardening in great deformations which takes into account the effects of the metallurgical transformations. It model can be used for three-dimensional, axisymmetric problems and in plane deformations.

This law represents a “assembly” of two models established in Code_Aster, namely one thermoelastoplastic model with isotropic work hardening written in great deformations (key word factor DEFORMATION: “SIMO_MIEHE”, cf [R5.03.21]) and a model small deformations thermo-élasto- (visco) plastic with effects of the metallurgical transformations (key word factor “META_P_ ** _ **” or “META_V_ ** _ **” of COMP_INCR of operator STAT_NON_LINE). The first model of great deformations was thus wide to take account of the consequences of metallurgical transformations on mechanics.

To include/understand this document, it is practically essential to read the reference documents [R5.03.21] and [R4.04.02] which concerns, respectively, the model great deformations without effects metallurgical and the model small deformations with metallurgical effects. Nevertheless, to facilitate the reading of this note, we make some recalls on these two models.

To justify the extension of the model written in great deformations to the model great deformations with metallurgical effects, we take again some theoretical aspects extracted from [bib1] related to

the writing of the model great deformations.

One presents then the relations of behavior of the complete model, his numerical integration and forms of the tangent matrix (options FULL_MECA and RIGI_MECA_TANG).

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2 Notations

One will note by:

Id

stamp identity

tr A

trace tensor A

AT

transposed of tensor A

det A

determinant of A

X

positive part of X

~

~

1

With

deviatoric part of tensor A defined by $A = A - (tr A) Id$

3

T

:

doubly contracted product: With: B = A B

ij ij = tr (AB)

I, J

tensorial product: (A B) ijkl = ij

With kl

B

3

With

~ ~

eq

equivalent value of von Mises defined by Aeq =

:

WITH A

2

With

TESTSTEMXÀ

gradient: TESTSTEMXÀ =

X

ij

With

divx A

divergence: (div

)

X Ad interim =

X

J

J

, μ

E

E

coefficients of Lamé: =

, $\mu =$

$(1 +) (1 -$

$2)$

$2(1 +)$

Young modulus

Poisson's ratio

E

modulate rigidity with compression: $3K = 3 + 2\mu =$

(1 - 2)

T

temperature

Tref

temperature of reference

Z

proportion of austenite

Zi

proportion of the four phases: ferrite, pearlite, bainite and martensite

In addition, within the framework of a discretization in time, all the quantities evaluated at the moment precedent are subscripted by -, the quantities evaluated at the moment $T + T$ are not subscripted and their increments are indicated par. One has as follows:

$Q = Q - Q$

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3

Recalls of the metallurgical model and the model large deformations

3.1

Model with metallurgical transformations

We present only here the consequences of the metallurgical transformations on mechanical behavior.

Determination of the mechanical evolution associated a process bringing into play metallurgical transformations requires a thermo-metallurgical calculation as a preliminary. This calculation

thermo-metallurgical is uncoupled and allows the determination of the thermal evolutions then metallurgical. For the metallurgical models of behavior of steels, one will be able to consult note [R4.04.01].

For the study of the metallurgical transformations of steel, there are five metallurgical phases: ferrite, pearlite, the bainite, martensite (phases) and austenite (phase).

The effects of the metallurgical transformations (at the solid state) are of four types:

- the mechanical characteristics of the material which undergoes the transformations are modified. Précisément, elastic characteristics (YOUNG modulus E and coefficient of Poisson) are not very affected whereas the plastic characteristics, such as the limit of elasticity, are it strongly,*
- the expansion or the voluminal contraction which accompanies the metallurgical transformations results in a deformation (spherical) of “transformation” which is superimposed on purely thermal deformation of origin. In general, one gathers this effect with that due to modification of the thermal dilation coefficient,*

· a transformation being held under constraints can give rise to a deformation irreversible and this, even for levels of constraints much lower than the elastic limit material. One calls “plasticity of transformation” this phenomenon. Total deflection is written then:

$$= E + HT + p + Pt$$

where E , HT , p and Pt are, respectively the elastic strain, thermal, plastics and of plasticity of transformation,

· one can have at the time of the metallurgical transformation a phenomenon of restoration of work hardening. The work hardening of the mother phase is not completely transmitted to the phases lately created. Those can then be born with a virgin state of work hardening or to inherit only one part, even totality, the work hardening of the mother phase. cumulated plastic deformation p is not then any more characteristic of the state of work hardening and it is necessary to define other variables of work hardening for each phase, noted rk which hold count restoration. The laws of evolution of these work hardenings differ from the laws usual so as to allow a “return towards zero” total, or partial, of these parameters at the time of the transformations.

One will be able to find in the document [R4.04.02] the expressions of the various relations of behavior.

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3.2

Model written in great deformations

3.2.1 Presentation

general

This model is a thermoelastoplastic law of behavior eulérienne written into large deformations which was proposed by Simo and Miehe ([bib2]) which tends under the assumption of small

deformations towards the model with isotropic work hardening and criterion of von Mises describes in [R5.03.02].

It makes it possible to treat not only the great deformations, but also, in an exact way, them great rotations.

The essential characteristics of this law are as follows:

- just like in small deformations, one supposes the existence of a slackened configuration, i.e. locally free of constraint, which makes it possible to break up the total deflection into a thermoelastic part and a plastic part,*
- the decomposition of this deformation in parts thermoelastic and plastic is not any more additive as in small deformations (or for the models great deformations written in rate of deformation with for example a derivative of Jaumann) but multiplicative,*
- as in small deformations, the constraints depend only on the deformations thermoelastic,*
- to write the law of behavior, one uses the tensor of the constraints of Kirchhoff, which is connected to the tensor of Cauchy by the relation $J = \dots$ where J represents the variation of volume between the configurations initial and current,*
- the plastic deformations are done with constant volume. The variation of volume is then only due to the thermoelastic deformations,*
- this model led during its numerical integration to a model incrémentalement objective what makes it possible to obtain the exact solution in the presence of great rotations.*

3.2.2 Kinematics

We make here some basic recalls of mechanics in great deformations and on the model of behavior.

Let us consider a solid subjected to great deformations. That is to say the θ_0 field occupied by the solid before deformation and (T) the field occupied at the moment T by the deformed solid. In the configuration

initial θ_0 , the position of any particle of the solid are indicated by X (Lagrangian description).

After deformation, the position at the moment T of the particle which occupied position X before deformation

is given by variable x (description eulérienne).

The total movement of the solid is defined, with U displacement, by:

$$x = x(X, T) = X + U$$

To define the change of metric in the vicinity of a point, one introduces the tensor gradient of transformation F :

$$\begin{aligned} x & \\ F & = \\ & = Id + U \end{aligned}$$

X
 X
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The transformations of the element of volume and the density are worth:

$$\begin{aligned} D & = J d_0 \text{ with } J \\ O & \\ & = \det F = \end{aligned}$$

where O and are respectively the density in the configurations initial and current.

To now write the model great deformations, the existence of a configuration is supposed slackened R , i.e. locally free of constraint, which then makes it possible to break up total deflection in parts thermoelastic and plastic, this decomposition being multiplicative.

One will note by F the tensor gradient which makes pass from the initial configuration 0 to the

configuration

current (T), by F_p the tensor gradient which makes pass from configuration 0 to the configuration slackened R, and F_e of the configuration R with (T). The index p refers to the plastic part, the index E with the thermoelastic part.

Initial configuration**Current configuration****F****(T)****0** **F_p** **F_e** **$T = T_{ref}$** **R****= 0****Slackened configuration**

Appear 3.2.2-a: Decomposition of the tensor gradient F in an elastic part F_e and plastic F_p

By composition of the movements, one obtains the following multiplicative decomposition:

$$F = F_e F_p$$

The thermoelastic deformations are measured in the current configuration with the tensor eulérien of left Cauchy-Green B_e and plastic deformations in the initial configuration by the tensor G_p (Lagrangian description). These two tensors are defined by:

 B_e **$F_e F_e T$** **=****, G_p** **$F_p T F_p$** **=****-****(****) 1 from where B_e** **$F G_p F T$** **=**

The model presented is written in such manner to distinguish the isochoric terms from the terms of change of volume. One introduces for that the two following tensors:

$$F = -$$

J I F

3 and Be = -2/B

3rd

J

with J = det F

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By definition, one a: det F = 1 and det Be = 1.

In this model, the plastic deformations are done with constant volume so that:

J p

p
= det F = 1 from where J I

E

=

= det F

One will find in the reference document ([R5.03.21]) the expressions of the relations of behavior.

4

Extension of the model great deformations

The objective of this paragraph is to justify the extension of the model written in great deformations for to take account of the metallurgical transformations. In particular, to take account of the plasticity of

transformation, we cannot add as in small deformations a term additional of deformation related to the plasticity of transformation. In fact, on the aspect decomposition kinematics, the taking into account of the plasticity of transformation does not change anything. One always has decomposition $F = F_e F_p$ where F_p thus contains all information on the “anelastic” deformation (including that related to the plasticity of transformation). It is only on the level behavior that fact, in particular, treatment of the plasticity of transformation.

Initially, we point out some theoretical elements which make it possible to write it model without metallurgical effects then we show the modifications to be made to hold account metallurgical effects and plasticity of transformation in particular.

4.1 Aspect thermodynamics

The writing of the law of behavior great deformations is from the thermodynamic framework with internal variables. The thermodynamic formalism rests on two assumptions. First is that the free energy depends only on the elastic strain B_e and of the variables intern related to the work hardening of the material (here cumulated plastic deformation associated the variable of work hardening isotropic R). This allows, thanks to the inequality of Clausius-Duhem, to obtain the laws of state. The second assumption is the principle of maximum dissipation, which corresponds to the data of a potential of dissipation, which then makes it possible to determine the laws of evolution of the internal variables.

The free energy is given by:

$$\psi = \psi(B_e, R) = E(B_e) + p(R)$$

One obtains by the first assumption, the laws of state, that is to say:

$$\begin{aligned} E &= \frac{\partial \psi}{\partial B_e} \\ p &= \frac{\partial \psi}{\partial R} \\ E &= \frac{\partial \psi}{\partial B_e} \\ 0 &= \frac{\partial \psi}{\partial R} \\ B \text{ and } R &= \frac{\partial \psi}{\partial \sigma} \\ E &= \frac{\partial \psi}{\partial B_e} \end{aligned}$$

B
0 p

It remains for dissipation:

1
:(-
& p T e-1
FG F B
) - &
RP 0

2
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With the help of the introduction of a function threshold such as F ($, R$) 0 , the principle of dissipation maximum (or in an equivalent way the data of a pseudopotential of dissipation [bib3]) allows to deduce some, by the property of normality, the laws of evolution, is:

F
- 1
p T e-1
F
FG
& F B
= & and &p = - &
2

R

It is here about a model of associated plasticity.

4.2 Extension

For the restoration of work hardening, there are no particular difficulties been dependent on large deformations. It is enough that the free energy depends, either to the cumulated plastic deformation, but

variables intern work hardening r_k associated with the variables with work hardenings Z, R

K

K of each one

metallurgical phases.

To take maintaining account of the deformations due to the plasticity of transformation, one proposes to add an additional term in the law with flow of the plastic deformation G_p which derives from a potential of dissipation.

One obtains thus for the laws of state:

E

p

=

2

E

0

B and Z. R =

E

B

K

K

0 r_k

and for the laws of evolution:

Pt

- 1

p T e-1

F

FG

& F B

= & +

2

123

plasticity of transformation

R

&

R = - &

F

K

-

(Z. R)

(Z. R)

K

K

K

K

1 2

4

3

4

restoration D work hardening

metallurgical and viscous

= Pt + R

()

One chooses the potentials Pt and R, respectively related to the plasticity of transformation and on restoration of work hardening, such manner to find, under the assumption of the small deformations, them

same laws of evolution as those of the model with metallurgical effects writes in small deformations.

4.3

Relations of behavior

A linear isotropic work hardening in the case of is placed.

The partition of the deformations implies:

Be

FG pFT

=

with $F = -$

J $1 F$

3, $J = \det F$ and $Be = -2/B$

3rd

J

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The relations of behavior are given by:

· **Thermoelastic Relation stress-strain:**

~

~

= μbe

3K

2

9K HT

1

tr =

(J -)

1 -

(J +)

$$\begin{aligned}
 &2 \\
 &2 \\
 &J \\
 &4 \\
 &HT \\
 &R \\
 &T_{ref} \\
 &R \\
 &T_{ref} \\
 &= \\
 &Z [(T - R_e \\
 &T F) - (1 - \\
 &Z) \\
 &F] + (iZ [) F (T - rT_{ef}) + Z \\
 &F] \\
 &i=1
 \end{aligned}$$

where: Z R characterizes the metallurgical phase of reference
 $Z_r = 1$ when the phase of reference is the austenitic phase,
 $Z_r = 0$ when the phase of reference is the ferritic phase.

T_{ref}
 HT
 HT
 F
 $= F (R_e$
 $T F) - (R_e$
 $T F)$ translated the difference in compactness between the ferritic phases
and austenitic at the temperature of T_{ref} reference,
 F is the dilation coefficient of the four ferritic phases and that of the phase
austenitic.

· *Seuil of plasticity:*

$$\begin{aligned}
 &F = - R \\
 &eq \\
 &- y
 \end{aligned}$$

R is the variable of work hardening of the multiphase material, which is written:

$$\begin{aligned}
 &F (Z) 4 \\
 &4 \\
 &R = (1 - F (Z))R + \\
 &Z. R, Z = Z \\
 &Z \\
 &I
 \end{aligned}$$

I
I
i=1
i=1

where R_k is the variable of work hardening of the phase K which can be linear or not linear by report/ratio with r_k and $F(Z)$ a function depending on Z such as $F(Z) [$

0,]
1 .

In the linear case, there are $R = R R$

K
 $0_k K$ where R_{0k} is the slope of work hardening of the phase K .

(I)
(I)
(I)
(I)

In the nonlinear case, one writes: $R = R$

+ R (R - R

K
K
K
K
)
0

where significances of R_k ,

R (I)

(I)

0_k and r_k

are represented on the figure below.

Rk

(3)

Rk (2)

(2)

Rk

R0 K

(1)

(1)

R

R

0 K

K

(0)

R0k

(0)

rk

Rk (0) (1)

(2)

(3)

R

R

K

rk

K

rk

Nonlinear curve of work hardening

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The elastic limit is worth there:

4

I

Z

y I

If Z 0

=

,

I I

y = (1 - F (Z)

) y + F (Z

) y,

y
=

Z
If Z = 0, y = y

where yi is the four limit elastic of the ferritic phases,
y that of the phase
autentic.

· Laws of evolution:

3
4
FG
& pFT = - &p
B
~ E - 3 B
~ E K F (1 - Z)
II
&i
Z

eq
i=1
4
- &Z (R - R)
I
II
&r
=1
= &p
I
+
- (Cr) m
Z
if
> 0
Z
moy

1 2

4

3

4

only in viscosity

$\&Z (R$

$- R)$

I

I

I

$\&r = \&p +$

$- (Cr) m$

Z

I

$if > 0$

Z

moy

I

I

1 2

4

3

4

only in viscosity

5

5

5

R

$= Z R$

moy

$K K, C = ZkCk, m = Zk K$

m

$K = 1$

$k = 1$

$k = 1$

where K_i, F_i, I

C and I

m are data of material associated with phase I, I it

coefficient of restoration of work hardening at the time of the transformation into $I (I [$

$0]$

I

),) and I

the coefficient of restoration of work hardening at the time of transformation I into (I [0] 1 ,).

*All the data material are indicated in operator DEFI_MATERIAU ([U4.43.01]) under various key words factors ELAS_META (_F0) and META_**.*

For a model of plasticity, the plastic multiplier is obtained by writing the condition of coherence $\dot{f} = 0$ and one a:

*&p,
0 F 0 and $\dot{p}f = 0$
In the viscous case, &p is written:
N
F
&p =*

or in an equivalent way:

*1/
4
F =
F
1
(- F (Z))
N
1 I
N
p&
+ Zii p&*

*Z i=1
where in and I
are the viscosity coefficients of material associated with phase I which depend possibly of the temperature.*

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The calculation of FG

& pFT gives:

~
~~
1

FG

& pFT = - (

E

eq

3 Aeq + &p) (tr B

+

)

3

2

eq

μ eq

4

where one posed A = K F &

Z

II

I.

i=1

Since ~

/

2

eq 1 and ~~

/eq 1, the second term of the expression above can be neglected

(in front of 1) for metallic materials insofar as:

eq R +y

-3

E

$\mu =$

μ

$10 \ll 1 \text{ tr } B$

tr Be 1 bus the tensor Be is symmetrical, definite positive and $\det Be = 1$.

It is this simplification of the law of evolution of Gp which makes it possible to integrate the law easily of

behavior i.e. to bring back it to the solution of a nonlinear scalar equation. One

will thus take thereafter:

tr Be

FG

& pFT - (&p +eq A)

~

éq

4.3-1

eq

4.4

Various relations

In operator STAT_NON_LINE, one reaches these various models by using the key words factors following:

/ COMP_INCR: (

RELATION

:

/

“META_P_IL”

/

“META_P_INL”

/

“META_P_IL_PT”

/

“META_P_INL_PT”

/

“META_P_IL_RE”

/

“META_P_INL_RE”

/

“META_P_IL_PT_RE”

/

“META_P_INL_PT_RE”

/

“META_V_IL”

/

“META_V_INL”

/

“META_V_IL_PT”

/

“META_V_INL_PT”

/

“META_V_IL_RE”

/

“META_V_INL_RE”

/

“META_V_IL_PT_RE”

/

“META_V_INL_PT_RE”

DEFORMATION

:

“SIMO_MIEHE”

)

We point out only here the significance of the letters for behaviors META:

- P_IL: plasticity with linear isotropic work hardening,***
- P_INL: plasticity with nonlinear isotropic work hardening,***
- V_IL: viscoplasticity with linear isotropic work hardening,***
- V_INL: viscoplasticity with nonlinear isotropic work hardening,***
- Pt: plasticity of transformation,***
- RE: restoration of metallurgical work hardening of origin.***

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Example: "META_V_INL_RE" = elastoviscoplastic law with nonlinear isotropic work hardening with

restoration of work hardening but without taking into account of the plasticity of transformation

The various characteristics of material are given in operator DEF1_MATERIAU. One return the reader to the note [R5.04.02] for the significance of the key words factors of this operator.

Caution:

If isotropic work hardening is linear, one informs under key word META_ECR0_LINE of DEF1_MATERIAU, the module of work hardening i.e. the slope in the plan forced deformation.

On the other hand, if isotropic work hardening is nonlinear, one gives directly under the key word META_TRACTION of DEF1_MATERIAU, the isotropic curve work hardening $R (R = - y)$ in

function of the cumulated plastic deformation $p (p = -$

).

E

Note:

The user must make sure well that the "experimental" traction diagram used for in to deduce the slope from work hardening is well given in the plan forced rational = F/S - deformation logarithmic curve $\ln (1+ L/L)$

0 where l_0 is the initial length of the useful part of

the test-tube, L variation length after deformation, F the force applied and S

$F L 1$

$F L$

current surface. It will be noticed that = $F/S =$

from where = $J =$

*. In
S L J
0 0
S L
0 0
F L
general, it is well the quantity
who is measured by the experimenters and this gives
S L
0 0
directly the constraint of Kirchhoff used in the model of Simo and Miehe.*

4.5 Internal constraints and variables

*The constraints of exit are the stresses of Cauchy, therefore measured on the configuration current.
For the whole of relations META_**, the internal variables produced in Code_Aster are:*

- V1: r1 variable of work hardening for ferrite,*
 - V2: r2 variable of work hardening for the pearlite,*
 - V3: r3 variable of work hardening for bainite,*
 - V4: r4 variable of work hardening for martensite,*
 - V5: r5 variable of work hardening for austenite,*
 - V6: indicator of plasticity (0 if the last calculated increment is elastic; 1 if not),*
 - V7: R the isotropic term of work hardening of the function threshold,*
- 1*
- V8: the trace divided by three of the tensor of elastic strain E*

B is

E

trb.

3

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5 Formulation

numerical

For the variational formulation, it is about same as that given in the note [R5.03.21] and which refers to the law of behavior great deformations. We point out only that it acts of a eulérienne formulation, with reactualization of the geometry to each increment and with each iteration, and which one takes account of the rigidity of behavior and geometrical rigidity. We now present the numerical integration of the law of behavior and give the form of the tangent matrix (options FULL_MECA and RIGI_MECA_TANG).

5.1

Integration of the various relations of behavior

In the case of an incremental behavior, key word factor COMP_INCR, knowing the tensor of constraints σ , the variables intern R -

K , the trace divided by three of the tensor of deformations

*1
rubber bands*

*E
trb, displacements U and U , the temperatures T - and T , and proportions of*

*3
1
various metallurgical phases Z*

*E
 K , Z_k , one seeks to determine (σ ,*

*R ,
b)*

*K
tr*

*.
3*

Displacements being known, gradients of the transformation of θ with σ , noted F , and of θ with (T) , noted F , are known.

One will pose thereafter:

4

- Z (R - R)

4

I

II

Z R

- R

(

)

WITH = K F Z

i=1

I

I

I

II

I, G =

and Gi =

(I = 1, 4)

i=1

Z

Zi

The implicit discretization of the law gives:

F = FF

J = det F

F = -

J 1 F

3

Be = -2/B

3rd

J

J =

~

~

= μbe

3K

9K

1

tr =

(2

J -)

1 -

HT

(J +)

2

2

J

4

HT

R

Tref

R

Tref

=

Z [(T - Re

TF) - (1 -

Z)

F] + (iZ [) F (T - rTef) + Z

F]

i=1

F 4

F = - (1 - F) R -

Z R

eq

-

Z

II

y

i=1

tr Be

Be = FG pFT = FG p-FT -

~ - tr Be

p

With

~

eq

If Z

-

m

-

> 0 then R = p + G -

T (Crmo)

, if not R = 0 and R

1 2

4

3

4

= 0

only in viscosity

If Z

-

m

-

I > 0, R = p + G -

T (Cr

I

I

moy)

, if not R = 0 and R

1 2

4

3

4

I

I = 0

only in viscosity

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In the resolution of this system, only the deviatoric constraint ~ is unknown because the trace of is function only of J (known).

One introduces Tr, the tensor of Kirchhoff which results from an elastic prediction (Tr: trial, in English test):

$$\begin{aligned}
 &\sim \\
 &\sim \\
 &Tr \\
 &eTr \\
 &= \mu b
 \end{aligned}$$

where

$$\begin{aligned}
 &beTr \\
 &FG p-FT \\
 &Fbe-
 \end{aligned}$$

$$\begin{aligned}
 &FT \\
 &= \\
 &= \\
 &, F \\
 &= (J) - 1 3F \text{ and } J = \det \\
 &(\\
 &) \\
 &F
 \end{aligned}$$

*One obtains **Be** starting from the constraints - by the thermoelastic relation stress-strain and starting from the trace of the tensor of the elastic strain.*

~-

E

l

E

B

=

+ *trb*

-

μ

3

One obtains for the tensor of Kirchhoff:

eTr

~

~

tr B

eTr

~

= μb

- μp

- μA

tr eTr~

B

eq

If $F < 0$, one has p then = 0 and:

~*Tr*

~

=

l + μ tr eTr

*With **B***

if not one obtains:

tr Be
tr beTr
=

eTr

~
tr B
l
+ μp
+ *A tr eTr*
~

B
= *Tr*

μ
eq

By calculating the equivalent constraint, one obtains the scalar equation out of p following:

eTr
eTr
Tr
eq + μp
tr B
+ μ
With eq tr B
= *eq*

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Expression of eq:

In plasticity:

-

$$eq = y + R p + D (R; T, Z)$$

with

$F = 4$

$$R = (1 - F) R_0 +$$

$Z R$

Z

$I_0 i$

$i = 1$

4

F

$$\text{and } D (R ; T, Z) = [1 - F] R (R - + G)$$

$+$

$$Z R (R + G)$$

Z

$I I I$

I

$i = 1$

4

$1 N$

F

In viscosity:

-

$$eq = y + R p + D (R; T, Z) + 1 (- F (Z))$$

$1 I$

N

(p/t)
+ Zii
(p/t)

Z i=1
with
D (R; T, Z) = [1 - F] R (R
+ G - T (Cr) m)
moy
4
F

+
Z R (R + G - T (Cr) m)
moy
Z
I I I
I
i=1

p checks:

Tr
eTr
4
1 N
F
- μ p
tr **B**
1
(- F (Z
)
(p
/T

1/
)
+ Z (p
/T N
eq
I

)
=
- $D(R; T, Z) - - R p$

Z
 II
 y
 $i=1$
 $1 + \mu A$
 eTr
 $'tr B$

The resolution is made in Code_Aster by a method of the secants with interval of research [bib4].

Note:

In the case of a nonlinear isotropic work hardening, slopes of R_0k work hardening and them work hardenings R

-
 K in the expressions of R and $D(R; T, Z)$ correspond to the variables

R
-
-
 m
 K taken at the moment T , i.e. $R = R + G + p - T (Cr$

K
 K
 K
moy). However, like one does not know a priori the value of these variables rk , one solves the equation out of p by taking them slopes R

-
-
 m
 $0k$ and Rk work hardenings for the quantities $R + G$

- $T (Cr$
 K
 K
moy). Once solved the equation out of p , one checks, for each phase, which one is well in the good

interval during the calculation of work hardening and the slope. In the contrary case, for phases concerned, the following interval is taken and the equation in p. again is solved. One continues this process until finding the good interval for all the phases.

One finds then for the diverter of the constraints:

$$\begin{aligned} & eTr \\ & \sim \\ & 1 \\ & p \text{ tr } \mathbf{B} \\ & \sim \\ & = \\ & 1 - \mu \\ & Tr \\ & 1 + \mu A \text{ tr } eTr \\ & Tr \\ & \mathbf{B} \end{aligned}$$

eq

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Once calculated cumulated plastic deformation, the tensor of the constraints and the tangent matrix, one carries out a correction on the trace of the tensor of the elastic strain E \mathbf{B} to hold account

plastic incompressibility, which is not preserved with the simplification made on the law of flow [éq 4.3.1]. This correction is carried out by using a relation between the invariants of \mathbf{E}

\mathbf{B}

~

and E

\mathbf{B} and by exploiting the plastic condition of incompressibility p

$J = 1$ (or in an equivalent way

$\det E$

$\mathbf{B} = 1$). This relation is written:

$$x^3 - J \text{ex} - (1 - J E) = 0$$

2

3

2

1 ~

()

~

~

E

E^2

eq

E

E

1

with $J^2 = (\mathbf{B}) \text{eq} =$

, $J = \det$

3

$\mathbf{B} = \det$ and

E

$X = \text{trb}$

2

2

(

2 μ)

μ

3

The solution of this cubic equation makes it possible to obtain

E

trb and consequently

thermoelastic deformation $\mathbf{B}e$ with the step of next time. If this equation admits

several solutions, one takes the solution nearest to the solution of the step of previous time. It is

1

moreover why one stores in an internal variable

E

trb.

3

Note:

If the plasticity of transformation is not taken into account, expressions obtained while taking $A = 0$ are the same ones.

If it is the restoration of work hardening which is neglected then one also has them same expressions but by taking all the equal ones to 1.

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5.2

Form of the tangent matrix

We give only here the forms of the tangent matrix (option FULL_MECA to the course iterations of Newton, option RIGI_MECA_TANG for the first iteration). For the assumptions concerning the metallurgical part, they are the same ones as those of the document [R4.04.02]. For part great deformations, one will find in appendix of [bib1], the detail of the linearization of the law of behavior.

One poses:

$J = \det \mathbf{F}$, $J -$

-
 $= \det \mathbf{F}$ and $J = \det \mathbf{F}$

· For option *FULL_MECA*, one a:

-

()/

J 1 3

1

J -

With =

=

H -

(HF) B -

B

F

J

3

JJ

J2

J -

3

HT

-2

+

KJ -

K

(1 - J) Id B

J

2

where **B** is worth:

B = F

F

- F

F

11

22

33

23 32

$B = F$

F

- F

F

22

11

33

13 31

$B = F$

F

- F

F

33

11

22

12 21

$B = F$

F

- F

F

12

31

23

33 21

$B = F$

F

- F

F

21

13

32

33 12

$B = F$

F

- F

F

13

21

32

22 31

$B = F$

F

- F

F

31

12

23

22 13

$B = F$

F

- F

F

23

31

12

11 32

$B = F$

F

- F

F

32

13

21

11 23

and where **H** and **HF** are given by:

In the elastic case ($F < 0$):

$$\begin{aligned} & \mu \\ & 2 \\ & H \\ & (B E - F \\ & + F \\ & B E - F \\ & B E - 2 A \\ & \sim F \\ & B E \\ & = \\ & -) \\ & ijkl \end{aligned}$$

$$\begin{aligned} & (1 + \mu A \\ & eTr \\ & ik LP \\ & jP \\ & IP pl \\ & jk \\ & tr \mathbf{b}) \\ & 3 ij \\ & kp LP \\ & ij \\ & kp pl \end{aligned}$$

$$\begin{aligned} & and \\ & 2\mu \\ & \sim \\ & HF = \\ & (beTr - A tr beTr \sim) \\ & (1 + \end{aligned}$$

$\mu A tr beTr)$
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if not in plastic or viscoplastic load, one a:

μ

H

$= (B E-F$

$+ F$

$B E)$

$ijkl$

$ik LP has$

jp

$IP pl$

jk

$R ($

With

$+ p$

$)\sim$

ij

eq

ij

$- 2\mu$

$+$

F

$B E$

has

3

(*R*
eTr
+ μ *tr B* (*1+ R A*
))
kp LP

eq

2
eTr
 μ
3
tr B
(*R p*
-)

eq
+
~ ~ *F*
B E
3 (R have
eTr
+ μ *tr B* (*1+ R A*
)) *ij kq qp LP*
eq

and

2μ
Id
R
(
With + p
)~

HF =
beTr - 2\mu tr beTr
eq

+

has
3a (R
eTr

eq
 + μ *tr B*
1
 (+ *R*
With))
3 2
 μ *tr beTr (R*
p - eq)
 +
 (~: *beTr*) ~

3
 (*R has*
eTr
eq
 + μ *tr B*
1
 (+ *R*
With))

with

4
F
1
 (- *N*)/*N*
F

1
 (-)/
R = 1
 (- *F*)
I
N
I
N
0
R +
Zi 0
R + 1
 (- *F (Z)*) (*p*)

\sqrt{T}
)
 \sqrt{NT}
+
 $Z(p$
 \sqrt{T}
)
 \sqrt{NT}

Z
 I
 II
 I
 $i=,$
 14

Z
 1
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 2
 II
 $=$
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4
 4

4
4
4
3

you
in viscosi

only

Tr

has

eq

=

eq

· For option *RIGI_MECA_TANG*

for the plastic model: they are the same expressions as those given for *FULL_MECA* but with $p = 0$ and $A = 0$, all variables and coefficients of material being taken at the moment T -. In particular, there will be $\mathbf{F} = \mathbf{Id}$.

for the viscous model: one takes only the expressions of *FULL_MECA* in the case rubber band with $A = 0$, all variables being taken at the moment T -.

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Organization (S): EDF/AMA

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***Models of metallurgical behavior
zircaloy in Code_Aster***

Summary:

This document presents the models of metallurgical behavior describing structure transformations, with the heating and the cooling, which the zircaloy (sheath of fuel pencil) between approximately 800°C undergoes and 1000°C.

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1 Introduction

The fuel sheaths of the nuclear engines with pressurized water consist of alloy of zirconium. These alloys undergo metallurgical transformations between 800°C and 1000°C, where they pass from a phase of compact hexagonal structure to a phase of cubic structure. In

certain cases of analysis such scenarios of accident of heart per primary education loss of cooling agent (APRP), the sheath reaches temperatures of the order 1000°C and undergoes metallurgical transformations then.

To analyze the mechanical behavior of the sheath in these situations, it is necessary of to take into account the influence of the metallurgy on mechanics: modification of the characteristics mechanics, change of volume at the time of the transformations, more possibly of others phenomena like the plasticity of transformation.

This document relates to the modeling of structure transformations of Zircaloy with heating and with cooling and this, a scale which, while remaining “reasonable” for metallurgist, is easily usable by the mechanic.

Metallurgy calculations in Code_Aster are done with the operator dedicated CALC_META, in “postprocessing” of a thermal calculation of evolution. With the model dedicated to the transformations

austenite-ferritic of steel, one has today two models of metallurgical evolution.

choice of the model is done with the key word RELATION; “ZIRC” or “STEEL”. This type of modeling is

realizable within Code_Aster for the whole of the elements (PLANE, AXIS, 3D) of the PHENOMENON

“THERMAL”. For the definition of the metallurgical behavior of Zircaloy the information of the word

key factor META_ZIRC under order DEFI_MATERIAU [U4.43.01] is necessary. Lastly, definition of the initial metallurgical state is realizable using order CREA_CHAMP, under key word factor ETAT_INIT of operator CALC_META.

The models presented (with the heating and cooling) are formulated within the framework of the relations

of behavior with internal variables (or mémoratrices). When one carries out a sequence of calculation thermo-metal-worker-mechanics of the zircaloy, one uses the same relations of behavior mechanics taking of account effects of the metallurgy, developed for steel:

(elasto) viscoplastic or elastoplastic, isotropic work hardening (linear or not linear) or kinematics, taken into account or not of the phenomena of plasticity of transformation and of restoration of metallurgical work hardening of origin.

Models established in Code_Aster to model the metallurgical transformations of Zircaloy are models developed by the ECA. These models were identified on the basis of test of dilatometry and calorimetry for alloys of sheaths (standard and new) within the framework of collaborations EDFCEA on the behavior of the sheath in accident by loss of réfrigèrent primary (program EDGAR).

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2

Proportion with balance.

Zircaloy has a compact hexagonal structure called phase, stable until temperatures about 800°C. Beyond approximately 800°C starts an allotropic transformation towards a phase cubic, and which is complete around 975°C.

That is to say T_{eq}

eq

D the initial temperature of transformation to balance and T_f

the temperature of end

of transformation. The proportion of phase Z eq

to balance is given by the equation, of type

Johnson-Mehl-Avrami, following:

eq

Z

$= 0$

for

$T < eq$

T

D

eq

N

eq

Z
= 1 - exp

- [KC T
(- T)
for
T
T
T
D
]
eq << eq
D
F

éq
2-1

eq
eq
Z
= 1
for
T < T

F

with:
T: temperature
K
N
C,
: parameters materials.

3
Equation of evolution to the heating

The transformation with the heating is the transformation:
The model of evolution of phase Z eq
to the heating is given by the differential equation (model of

Holt) following:

For

chauff

$T > T_d$

:

dZ

E

= Aexp-

$(T - T_d)m$

eq Z

éq 3-1

dt

RT

with:

T (Z)

eq

: T with balance corresponding to proportion Z

E

With,

, m: parameters materials

R

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4

Equation of evolution to cooling

The transformation with cooling is the transformation

The model of evolution of the phase eq

Z with cooling is given by the differential equation

following:

for

refr

T < Td:

$$dZ = K Z 1 (- Z)$$

R

éq 4-1

dt

with

$$K = - T - T (Z) \exp$$

R

eq

$$(Ar+BrT - T (Z))$$

eq

)

NB:

The model comprises equations giving the initial temperatures of transformation of phase according to the cooling and heating rates which today are not operational.

5 Formulation numerical

The integration of the differential equations is done with the explicit method of Runge-Kutta.

Note:

The discretization comprises in more one automatic cutting of the step of time when

T
> 5 C
°.

5.1**Feel metallurgical evolution**

One adopts the same principle as the models dedicated to steel. One considers that in a calculation of structure, certain zones can undergo a heating while others cool. One thus consider that there is one model of metallurgical transformations for Zircaloy which according to the temperature considered and signs it speed of evolution thermal is described either by model, is by the model.

T (T + T

) < eq

T

[eq eq

T T

;

eq

T

>

D

F

J
D
F
 $T & (T) > 0$

$T & (T) = 0$ if $Z Z$

if $Z < Z$

eq
eq

$T & (T) < 0$

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5.2

Proportion of phase and variables internal

The programme of study of the sheath in situation of APRP (EDGAR) comprises a mechanical part. In the mechanical model one considers that Zircaloy comprises 3 phases distinct from characteristics different mechanics; , -. The proportion of the three “mechanical” phases depends on initial proportion/given by the equations [éq 2-1], [éq 3-1] and [éq 4-1]. Characteristics mechanics of a multiphase point is obtained using a law of the mixtures on the characteristics of (pseudo) phases present. This requires from the metallurgical point of view to carry out a complement of development allowing the metallurgical exit it calculation to be able to recover 3 phases: one phase and a phase which met is in the form 1 (1: is the phase pure alpha having characteristic of) is in form 2. (2: alpha in mixture and which will have the characteristics mechanics of the phase -).

The models presented previously gives the proportion of the phase and such as $z=1-z$. cold phase is then distributed in 2 forms with the proportions $z1$ and $z2$ such as:
 $z=z1+z2$.

$$1 > z > 0.9 \quad 0.9 > z > 0$$

(Z -.

0.9)

$$z1 = 0$$

$$z1 =$$

Z

.

0.1

$$z2 = z$$

$$z2 = z - z1$$

distribution of the proportion of phase according to the proportion alpha

1

0,8

zalpha1

0,6

Z

zalpha2

0,4

zbeta

0,2

0

0.0,1.0,2 0,3.0,4.0,5 0,6.0,7.0,8 0,9 1

zalpha

5.2.1 Internal variables

The internal variables of the relation of behavior “ZIRC” are:

V1: z_1 proportion of the phase alpha1
V2: Z 2 proportion of the phase alpha2
V3: TPG, temperature at the points of Gauss

The use of the metallurgy model dedicated to Zircaloy requires to inform the parameters of model under the key word factor META_ZIRC of DEFI_MATERIAU.

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Document: R4.05.01

Seismic response by transitory analysis

Summary

The methods most frequently used for the seismic analysis of the structures are the methods spectral and transitory methods.

The transitory methods (direct linear or not, by modal synthesis) make it possible to calculate the answer of

structures under the effect of imposed seisms: single excitation (identical of each point of anchoring of

structure) or multiple and to take into account their possible nonlinear behavior.

With regard to the spectral methods, one calculates the maximum answer, for each mode of vibration, of each point of anchoring. The maximum response of the whole of the structure is then determined by

combination of the maximum answers of the modes. This type of analysis is clarified in the documentation of

reference [R4.05.03].

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1

Seismic behavior of a structure

1.1 Definitions

The analysis of the seismic behavior of a structure consists in studying its response to a movement imposed: an acceleration, in its various supports. Imposed acceleration is a temporal signal (T) called accélérogramme (cf [Figure 1.1-a]).

Appear 1.1-a: Accélérogramme LBNS

The seismic movement considered in calculation is a real accélérogramme known and read by operator LIRE_FONCTION [U4.21.08] is a synthetic accélérogramme calculated directly in the code, for example with the procedure FORMULATES [U4.21.11].

2

Seismic response of a system to a degree of freedom

That is to say a simple oscillator made up of a mass m connected to a fixed point by a spring K and one

shock absorber C which can move in only one direction X (cf [Figure 2-a]). This oscillator with one degree of freedom is subjected to a accélérogramme (T) horizontal in its support (not A).

locate galiléen

support

K

m

X_e

With

C

x_r

teststemxà

Appear 2-a: simple oscillator subjected to a seismic request

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Displacements of the oscillator are measured or calculated, that is to say in a relative reference mark related to point a:

relative displacement x_r , is in an absolute reference mark (R_a): absolute displacement teststemxà.

Displacement

absolute teststemxà breaks up into a uniform displacement of drive in translation X_e and one relative displacement x_r :

$$X = X + X$$

has

R

E

éq 2-1

One deduces from it by derivation the relation between accelerations:

!

$$X =$$

+

$$\text{with } (T) = X$$

has

!

X

(T)

R

! E!

éq 2-2

The mass is subjected to a horizontal force of recall which is proportional to relative displacement:

$$F = - K. X$$

R

R and with a horizontal force of damping presumedly proportional to the relative speed:

$$F = - C.X$$

v

! r.

The equation of the movement of the mass is written then: $- K. X - C.X' = m.x''$

R

R

! has

!.

Maybe, taking into account the equations [éq 2-1] and [éq 2-2]:

$$m.x'' + c.x' + k.x = - m. (T) = p (T)$$

R

R

R

)

éq 2-3

Note:

The study of the seismic response of an oscillator to a degree of freedom in the relative reference mark consists

thus in the study of the response of an oscillator to a force (

$p (T)$ of an unspecified form. The solution

equation of motion [éq 2-3] is then provided by the integral of Duhamel:

1

T

X =

$$p (T) . e^{-\zeta \omega_n T} . \sin(\omega_n \sqrt{1 - \zeta^2} T)$$

.

.

$$[D (T -)]D$$

R

m D 0

with:

$$p (T) = - m. (T)$$

K

C

=

, =

$$= 1 - 2$$

and

m

.

2 m

D

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3

Seismic response of a system to several degrees of freedom

3.1

Equations of the movement in the absolute reference mark

The balance of a mechanical system consists in writing, some is the moment of calculation considered, that

summon internal forces, inertias and of damping is equal to the imposed external forces on this known as system: F

+ F

+ F

= F

iner

amo

int

ext.

In the case of a linear behavior, known the system is represented by a model of finite elements or of discrete elements, one has (after discretization):

F

iner = $MR \cdot X!$ has

F

int = $K \text{ Teststemx}\grave{a}$

.

$\text{Teststemx}\grave{a}$ is the vector of nodal displacements of the discretized structure, in the reference mark absolute;

•
M is the matrix masses structure;

•
K is the matrix stiffness of the structure;

•
F
 = $F - F$

ext.

E

C is the vector of the forces imposed on the studied structure, *FC* that of possible forces of shock (cf [R5.06.03]).

To simplify the presentation, it is considered that the structure is only requested by displacements imposed on the level of its various supports. Thus, $F_e = 0$.

With an aim of simplifying the presentation, one generally separates the degrees of freedom into two, in

function of their type:

- degrees of freedom of structure not subjected to an imposed movement - also called active degrees of freedom - they are the unknown factors of the problem;

- degrees of freedom of structure subjected to an imposed movement - also called *ddl_impo* - they are the boundary conditions in displacement of the problem (limiting conditions of Dirichlet).

On the edges of the structure where *Xs* displacements are imposed, one $a: B X = X$ has

S.B is

stamp passage of all the degrees of freedom of the structure to the degrees of freedom of structure subjected to an imposed movement.

The balance of the system is written then, some is *v* pertaining to the space of displacements kinematically acceptable i.e., some such as $B v = 0$ are *v*:

$M \ddot{X} + F_{amo} + K X - F_{ext} = 0$

$B X$

= X_s has

That is to say:

M

\ddot{X}

T

+ $F_{amo} + K X - F_{ext} = - B$.

éq 3.1-1

$B X$

= X_s has

F

B

= - T has. is the vector of the forces of reactions exerted by the supports on the structure.

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By taking account of the partition of the degrees of freedom, the vector of displacements in the reference mark

X

has

m

m xs

absolute is written: Teststemxà =. The operators describing the structure become: M =

,

X

S

m

m

sx

ss

K

K xs

K =

with m

MT

=

and K

kT

=

and the vector of the external forces applied to

K
K

sx
xs
sx
xs
sx
ss

- FC

the structure is written: Fext =

.
0

The fundamental equation of dynamics in the absolute reference frame is written then, by taking account of

the partition of the degrees of freedom:

m

m X

!

K

K X

- F

xs

has

. + F

xs

has

C

+

. =

m

m

X

amo

!

K

K

X

F**sx****ss S****sx****ss S****has*****Maybe, by considering only the active degrees of freedom:*** **$m \ddot{X} + F$** **$+ K X = - F - m \dot{X}$** **has****amo****has****C****xs!** **$- K X$** **S****xs****S**

This approach requires the knowledge of displacements and absolute velocities associated the accélérogramme (T) but the recorders measure either of accelerations or speeds. One can go up with displacements by simple or double integration with order CALC_FONCTION [U4.62.04]. However, uncertainties of measurement give drifts which it is advisable to correct: displacements are thus well-known than speeds and accelerations. One will keep in memory orders of magnitude of the maximum amplitudes following:

- some tenth of “G” for accelerations;***
- a few tens of cm/s for speeds;***
- a few tens of cm for displacements.***

One will also make sure that at the end of the seism speed and displacement are realistic i.e. with more few tens of cm for displacement, null for speed.

3.2***Equations of the movement in the relative reference mark******3.2.1 Decomposition of the absolute movement***

The requests undergone by a structure at the time of a seism are classified in two types in the rules of construction (ASME, RCC-M):

- constraints induced by the relative movement of the structure compared to its deformation primary statics or constraints. These requests are due to the effects inertial of the seism;***
- constraints induced by differential displacements of anchorings or constraints secondaries.***

Generally, one thus breaks up the study of the structures into the study of the static deformation due to movements of the supports (it is the movement of drive) and in the study of the vibrations induced by accelerations of the supports around this deformation (it is the relative movement).

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X

Teststemxà

S displacement imposed of the supports

Xe

absolute movement Teststemxà

Xr

movement of drive Xe

relative movement Xr

(Ra)

The absolute displacement of any point M of the structure, not subjected to an imposed displacement, is equal

with the sum of relative displacement and displacement of drive of this point:

$$X(M) = X_r(M) + X_e(M)$$

has

R

E

)

éq 3.2.1-1

That is to say:

.

Teststemxà, the vector of displacements in the absolute reference frame;

.

Xr, the vector of definite relative displacements like the vector of displacements of structure compared to the deformation which it would have under the static action of displacements imposed on the level of the supports. **Xr** is thus null at the points of anchoring: **B Xr = 0**;

.

Xe, the vector of the displacements of drive defined as displacements of structure requested statically by imposed displacements of the supports

B

X = X

E

S

= R +

X. **X**.**K X = - B**

T

E

E =

S

E

. with

E

is the matrix of the static modes. The static modes represent, in the absence of external forces, the response of the structure to a unit displacement imposed on each degree of freedom of connection (others being blocked).

3.2.2 Simple or multiple excitation

To clarify more in detail the approach moving relative, and more particularly the calculation of components of drive, it is necessary to introduce concept of the simple or multiple excitation.

3.2.2.1 Excitation

simple

It is considered that the imposed seismic movement is a solid movement of body. One says generally that the **structure mono - is supported**.

Teststemxà (T)

Xr (T)

Xs (T)

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The absolute displacement of any point M of the structure, not subjected to an imposed displacement thus break up into a **relative displacement compared to a pointer related to the support** where is imposed the seismic movement and in a **rigid displacement of drive**.

In this case, the static modes correspond to the six modes of rigid body. Like the structure is linear rubber band, one separately studies the effects of the six components of the seismic movement. For each seismic direction, one writes simply the inertias induced by the seism under following form:

$$\mathbf{P}(T) = -\mathbf{M} \cdot \mathbf{X}$$

$$\mathbf{S} = -(\mathbf{T}) \cdot \mathbf{M}$$

(T) seismic movement in a direction is the accélérogramme;

is mode of the solid and unit body in this direction;

The seismographs measure only signals of translation. To consider that the structure studied mono is supported amounts supposing that all its points of supports undergo even translation. In this case, the components of [] are worth 1 for the degrees of freedom who correspond to displacement in the seismic direction considered and 0 for degrees of freedom which correspond to displacement in seismic directions perpendiculars with that considered or rotations.

However, considering the size of the models, the complete seismic analysis of equipment is carried out generally in several stages. Detailed seismic analysis of the equipment considered use then as excitations, the accelerations calculated at the time of the first stage. They compose of the six accélérogrammes of translation and rotation. The three are thus calculated modes corresponding to imposed displacements of translation and the three modes corresponding to imposed displacements of rotation. If the seismic movement is one

imposed rotation, in a point M, $M = OM$ for the degrees of freedom which correspond to displacement of translation and for the degrees of freedom which correspond to rotations.

3.2.2.2 Excitation

multiple

One cannot always only consider:

- the accelerations undergone by the whole of the points of anchoring of the studied structure are identical and in phase;
- the supports indeformable and are actuated by the same movement of rigid body.

In this case, one says that the **structure is multi - supported**. Static modes =

Id

correspond then to the 6.nb_supports static modes (or 3.nb_supports modes) where nb_supports is the number of accélérogrammes different undergone simultaneously by the structure. They are calculated by operator MODE_STATIQUE [U4.52.04] with option DDL_IMPO. They are solution of the equation following:

$$\mathbf{X} = \mathbf{X}$$

E

S

K**K** 0*xs*

that is to say

. =

éq 3.2.2.2 - 1

$$\mathbf{K} \mathbf{X} = - \mathbf{B}$$

T

E

. E

K**K****Id****F***sx**ss**has**E*

Maybe, by considering only the active degrees of freedom: $\mathbf{K} + \mathbf{K} \cdot \mathbf{Id} = 0$.

xs

The inertias induced by the seism are written then simply:

nb_supports

$$\mathbf{P}(T) = -$$

Mr. Xm! *S(T)*

m

m=1

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3.2.3 Modeling of damping

It is considered that the damping dissipated by the structure is of viscous type i.e. the force of damping is proportional to the relative speed of the structure:

F**= C X**

amo

! R where **C** is the matrix of damping of the structure.

That amounts neglecting the effect imposed speed. Indeed, one can more generally write:

F**= C X! = C X! + C. X**

amo

has

R

! S.

In the case of a uniform excitation at the base (case of the mono-support), damping only intervenes on relative displacements (the forces of damping are null for a rigid displacement).

In the case of a multiple excitation where the static solution is not any more one rigid displacement, to consider that the force of damping is proportional to the relative speed of the structure is one simplifying assumption.

3.2.4 Fundamental equation of dynamics

The fundamental equation of dynamics [éq 3.1-1], in the **relative reference mark**, is written then, taking into account

equations [éq 3.2.1-1] and [éq 3.2.2.2 - 1]:

MR. X**! + C X! + K X = - M. X! + F - BT.**

R

R

R

S
ext.

R

éq 3.2.4-1

Maybe, by partitionnant the degrees of freedom:

m

m X

! C

C X

! K

K X

- F

.

.

xs

R

xs

R

xs

R

C

(m + m I

xs

) dx! S

. +

. +

. =

-

m

m

0

C

C

0

K

K

0

F*SX**SS**SX**SS**SX**SS**has**R***(Mr. + m. I***SX**SS***) dx! S**with **C****= cT***SX**XS*

Maybe, by considering only the active degrees of freedom:

m.x! + c.x*R***! + k.x = - F***R**R***C - (Mr. + m. I***XS***) dx! S**

Principal advantages of the approach in relative displacement compared to that in displacement absolute are as follows:

- it is not necessary to integrate the accélérogramme (*T*);
- relative displacements obtained make it possible to determine the primary constraints directly induced by the seism.

3.3

Calculation of the seismic loading

The seismic loading (cf [§3.2]) - **Mr.** is - **(Mr. + m. I***XS***) dx! S** on the degrees of freedomcredits is built by operator **CALC_CHAR_SEISME** [U4.43.01]. It is usable directly at the time of a direct transitory analysis with **DYNA_LINE_TRAN** [U4.54.01] or of a transitory analysis by modal synthesis with **DYNA_TRAN_MODAL** [U4.54.03]. On the other hand, during a transitory analysisdirect nonlinear with **DYNA_NON_LINE** [U4.32.02], it should be transformed into a concept of the type

charge. This is carried out starting from operator AFFE_CHAR_MECA [U4.25.01] in the following way:

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```
char_sei = CALC_CHAR_SEISME (...);
```

```
charge = AFFE_CHAR_MECA (MODEL: ..., VECT_ASSE: char_sei);
```

```
dyna_nlin = DYNA_NON_LINE (
```

```
excit (LOAD: con_lim, LOAD: cham_no, FONC_MULT: acceler)
```

```
...);
```

In the case of a supported mono structure, it is enough to indicate the direction of the seism:

```
mono_x = CALC_CHAR_SEISME (MATR_ASSE: mass,
```

```
DIRECTORATE (...), MONO_APPUI: "YES");
```

In the case of a structure multi supported, should as a preliminary have been calculated the static modes.

One

calculate as many seismic loadings of supports which undergo a different acceleration.

```
multi_xi = CALC_CHAR_SEISME (MATR_ASSE: mass, DIRECTORATE (...),
```

```
NODE: NOI, MODE_STAT: mode_stat,);
```

3.4

Loading of incidental the wave type

It is also possible to impose a seismic loading by wave planes via

order AFFE_CHAR_MECA and the key word factor ONDE_PLANE. That corresponds to the loadings classically met during calculations of interaction ground-structure by the integral equations.

In harmonic, a wave planes elastic is characterized by its direction, its pulsation and its type

(wave P for the waves of compression, waves SV or HS for the waves of shearing). In

transient, the data of the pulsation, corresponding to a standing wave in time, must be

replaced by the data of a profile of displacement which one will take into account the propagation with run from time in the direction of the wave.

More precisely, one characterizes:

- a wave P by the function $U(X, T) = F(k.x - C T) K$

p

· a wave S by the function $U(X, T) = F(k \cdot x - C T) K$

S

With:

·

K , unit vector of direction

·

F then represents the profile of the wave given according to the direction K .

O

“Principal” face of wave

K

corresponding at the origin

profile

H

Function F

H_0 is the distance from the principal face of wave in the beginning O, carried by the directing vector of the wave with

the initial moment of calculation, H the distance from the principal face of wave in the beginning O, one unspecified moment.

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Note:

This type of load is available in a direct transitory calculation linear DYNA_LINE_TRAN or not DYNA_NON_LINE.

The use of this type of loading will be detailed in a specific note.

4

Transitory seismic response by modal synthesis

4.1

Description of the method

The method of modal recombination consists in breaking up the relative movement of the structure on the base of the clean modes. As this one is null on the level of the supports, one projects the equation of dynamics on the basis of blocked clean mode (clean modes obtained by blocking all them degrees of freedom of connection).

X
Q
 $R =$
 \cdot
 is the matrix of the blocked clean modes;
 \cdot

Q the vector of the unknown factors generalized on the basis of blocked clean mode.

The blocked clean modes are solution of:

$$\begin{pmatrix} 0 \\ \mathbf{K} - 2i.M \end{pmatrix} \mathbf{I} = \text{where } \mathbf{F}$$

F

I are the modal reactions at the points of supports.

I
 The equation of the movement projected on the basis of dynamic mode is written then:

$$\mathbf{MR} \cdot \mathbf{Q} \ddot{(\mathbf{Q} T)} + \mathbf{C} \dot{(\mathbf{Q} T)} + \mathbf{K} \mathbf{Q} (T) = - \mathbf{M} \cdot \mathbf{X} \ddot{(\mathbf{Q} T)} + \mathbf{S} \cdot \mathbf{F} \dot{(\mathbf{Q} T)} + \mathbf{B} T \mathbf{G}$$

G
 G
 G
 $\cdot r$
 where **MR**, G , **CG** and $\mathbf{K} G$ are the matrices of mass, of generalized damping and stiffness. For to simplify, one considers that they are diagonal. The matrix of damping generalized **CG** too because it is supposed that the assumption of Basile is checked (the matrix of damping is a combination linear of the matrices masses and stiffness).

Maybe, by considering only the active degrees of freedom:

$$\mathbf{Mr} \cdot \dot{(\mathbf{Q} T)} + \mathbf{C} \cdot \dot{(\mathbf{Q} T)} + \mathbf{K} \cdot (\mathbf{Q} T)$$

T

= - .f

T

C -. (**Mr. + Mr. I**

xs

) **D X**

G

G

G

! *S*

In the absence of shock, one is thus led to solve a whole of uncoupled equations (there is as much as clean modes).

Note:

It is possible to calculate a modal base with nondiagonal matrices. It is enough to specify during the construction of the classification generalized by the key word STORAGE: "FULL" with order NUME_DDL_GENE [U4.55.07].

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4.2

Choice of the modal base

For the seismic analysis of a linear structure, it would be necessary in theory to retain all the modes of which them

Eigen frequencies are lower than the cut-off frequency (generally about 33 Hz).

In practice, one is often satisfied to preserve in the modal base only the modes which contribute to a significant degree to the answer. One then preserves only the modes of which mass effective unit in a direction is higher than 1 and one also makes sure that, for the whole of these modes selected, the unit effective mass cumulated in each direction is little different from the total mass of the structure (higher than 90%). The criterion of office plurality of the

masses

modal effective is reached by connecting the following operators:

- Calcul of the total mass of the structure: POST_ELEM [U4.61.04]

masse_in = POST_ELEM (MASS_INER: (ALL: "YES"))

- Calcul of the blocked dynamic clean modes

: they are calculated in the operator

MODE_ITER_SIMULT [U4.52.02] or in MODE_ITER_INV [U4.52.01] according to the selected method.

mode = MODE_ITER_SIMULT (); or mode = MODE_ITER_INV ();

- Normalisation of the modes compared to the generalized mass: NORM_MODE [U4.64.02]

NORM_MODE (MODE: mode, STANDARD: "MASSE_GENE", MASSE_INER: masse_in);

- Extraction of the modal base of the modes whose unit effective mass exceeds a certain threshold (1 for example) and checking which the extracted modes represent at least 90% of the mass

total: EXTR_MODE [U4.64.03]

EXTR_MODE (

FILTRE_MODE (MODE: mode, CRIT_EXTRE: "MASSE_EFFE_UN", THRESHOLD: 1.e-3)

IMPRESSION (OFFICE PLURALITY: "YES");

Note:

Macro order MACRO_MODE_MECA [U4.52.05] makes it possible to connect the unit directly of the three last preceding orders.

Attention, certain local answers (in the particular case of nonlocalised linearities) can be strongly influenced by modes of a higher nature whose frequency is beyond the frequency of cut and whose effective modal mass is low (lower than 1). Key word VERI_CHOC of order DYNA_TRAN_MODAL [U4.54.03] allows to check a posteriori that the selected modal base is sufficient. If it is not the case, one highly advises to supplement it.

4.3 Calculation of the dynamic response of the structure studied by modal synthesis

After having calculated the base of the dynamic clean modes and having built a generalized classification by NUME_DDL_GENE [U4.55.07], one projects then the matrices of mass, damping and of stiffness, on this same basis with the operator PROJ_MATR_BASE [U4.55.01], vectors second member with PROJ_VECT_BASE [U4.55.02].

Note:

Macro order MACRO_PROJ_BASE [U4.55.11] makes it possible to connect the unit directly of the three operations.

The matrices and vectors thus projected, one calculates the generalized response of the mono system or multi-excited using operator DYNA_TRAN_MODAL [U4.54.03].

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4.4

Taking into account of the modes neglected by static correction

During the calculation of the generalized response of an excited mono structure, it is possible to take in count, a posteriori, the static effect of the neglected modes. In this case, once reconsidered the base physique one corrects the value of relative displacement calculated (respectively relative speed and relative acceleration) by the contribution of a pseudo-mode. The pseudo-mode is defined by difference between the static mode associated the unit loading of constant acceleration type imposed and projection on the calculated dynamic modes of displacement (respectively relative speed and relative acceleration).

One has then:

p

X

$= X +$

$F(T).$

I

I

-

.

$r_{\text{corrigé}}$

R

J

J

I

$j=1$

p

! X
F T

r_corrige =!

X +! ().

R

I

I -! .

J

J

I

j=1

p

! X
F T

r_corrige =! X +! ().

R

I

I -! .

J

J

I

j=1

Multiplicative functions of time *F (T*

I

) correspond to the accélérogramme imposed *I (T)* in each direction *I* considered.

The step to be followed is as follows:

· Calcul of the unit loading of type forces imposed (constant acceleration) in the direction of seism: AFFE_CHAR_MECA [U4.25.01]. One will pay attention to permute the sign of the direction since the seismic inertia is form (

$\mathbf{P} \mathbf{T}) = - \mathbf{M}.$

· $\mathbf{X}! \mathbf{S}$

cham_no = AFFE_CHAR_MECA (MODEL: model, GRAVITY: (VALE, DIRECTION)) ;

· Calcul of the linear static response of the structure to the preceding loading case:

MACRO_ELAS_MULT [U4.31.03].

mode_cor = MACRO_ELAS_MULT (CHAR_MECA_GLOBAL: con_lim,...

CAS_CHARGE: (NOM_CAS: "xx", CHAR_MECA: cham_no)) ;

It will be noted that there is as many loading case of direction of seism

· Calcul of the derived first and second of the accélérogramme: CALC_FONCTION [U4.62.04].

deri_pre and deri_sec = CALC_FONCTION (OPTION: DERIVES);

· Calcul of the answer generalized by taking of to account the modes neglected by correction statics:

dyna_mod = DYNA_TRAN_MODAL (MASS_GENE: , RIGI_GENE:

MODE_CORR: mode_cor

EXCIT (CORR_STAT: "YES"

D_FONC_DT: deri_pre, D_FONC_DT2: deri_sec.)

...);

Note:

In the case of an excited structure multi, the taking into account of the modes neglected by correction statics is not developed. One postraité absolute displacement in this case.

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4.5

Taking into account of the character multi - supported of a structure

It was seen previously (cf [§3.3]) that to calculate the seismic loading in the case of one

structure multi supported, should as a preliminary have been calculated the static modes. If one wants to be able

to restore the sizes calculated in the absolute reference mark or if one wants to be able to take into account

not located linearities, it also should be specified in DYNA_TRAN_MODAL that the studied structure is multi excited. Indeed, in this last case, one compares at every moment, the vector of absolute displacements of each point of shock considered, in order to determine if there is shock and of to calculate the corresponding forces of shock.

The step to be followed is as follows:

- Calcul of the static modes: MODE_STATIQUE [U4.52.04].

```
mode_stat = MODE_STATIQUE (DDL_IMPO: (...));
```

- Calcul of the answer generalized by taking of account the component of drive:

```
dyna_mod = DYNA_TRAN_MODAL (MASS_GENE: , RIGI_GENE:
```

```
MODE_STAT: mode_stat
```

```
EXCIT (MULT_APPUI: "YES"
```

```
ACCE: accelero, QUICKLY: speed, DEPL: move
```

```
DIRECTION: (...), NODE: NO1
```

```
...)
```

```
...);
```

4.6 Post

treatments

Operators REST_BASE_PHYS [U4.64.01] or RECU_FONCTION [U4.62.03] can then restore in physical space calculated evolutions:

- the operator

REST_BASE_PHYS restores overall (the complete field) displacements, speeds and accelerations;

- the operator

RECU_FONCTION restores locally (temporal evolution of a degree of freedom) displacements, speeds and accelerations.

One can restore the relative sizes by specifying (MULT_APPUI: "NOT") or sizes absolute by (MULT_APPUI: "YES").

One obtains then displacements of drive necessary to the calculation of the secondary sizes in withdrawing from absolute displacements relative displacements. This is carried out by the order CALC_FONCTION [U4.62.04] option COMB.

From the preceding evolutions, one can also extract the max. values and RMS and calculate it spectrum of response of associated oscillator. This is carried out by order CALC_FONCTION options MAX, RMS and SRO.

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5**Direct transitory seismic answer**

Direct integration is realizable is with assumptions of linear behavior: operator

DYNA_LINE_TRAN [U4.54.01] is with assumptions of nonlinear behavior: operator

DYNA_NON_LINE [U4.32.02]. Setting with share the way of taking into account the seismic loading (cf [§3.3]), syntaxes of DYNA_NON_LINE and DYNA_LINE_TRAN are identical.

5.1 Taking into account of a damping are equivalent to damping**modal**

Generally, the most precise information that one has on damping comes from the tests from vibration which makes it possible to determine, for a frequency of resonance given f_i , the width of corresponding resonance and thus damping reduces I to this resonance. **It is thus**

necessary to be able to take into account, in a direct transitory calculation, a damping equivalent with modal damping.

From the spectral development of the matrix identity:

n_mod be $\mathbf{X} \mathbf{X}^T \mathbf{K} n_modes \mathbf{X} \mathbf{X}^T \mathbf{K}$

Id = I I

=

 I I T

2

 I 1**X K X** I 1**M**

=

 I I

=

$G_{i,i}$

one shows:

· that one can develop the matrix of damping of the structure \mathbf{C} in series of modes

clean:

n_{mode}

T

$\mathbf{C} = \text{have. } (\mathbf{K}_i$

) $(\mathbf{K}_i$

)

$i=1$

· and that, account held of the definition of the critical percentage of damping:

T

I

I .

$\mathbf{C} I =$,

$2 \mathbf{M} G_{i,i} I .ai =$.

$2 \mathbf{K} G_{i,i}$

It is thus advised with the user to specify (syntaxes of DYNA_NON_LINE and DYNA_LINE_TRAN are identical), the values of modal depreciation for each Eigen frequency by

the intermediary of the key word factor AMOR_MODAL.

That amounts imposing a force of damping proportional to the relative speed of the structure:

n_{mode}

\mathbf{F}

$= \mathbf{C} \mathbf{X}$

I

T

amo

! R with $\mathbf{C} = 2$.

. $(\mathbf{K}_i$

) $(\mathbf{K}_i$

)

\mathbf{K}

$i=$

.

$G I$

1

—

I

5.2 Taking into account of a request multi supports with restitutions relative and absolute fields

By defect, the sizes are calculated in the relative reference mark. In DYNA_NON_LINE and

DYNA_LINE_TRAN, one uses a syntax identical to that of DYNA_TRAN_MODAL (presence of the words

keys MODE_STAT and MULT_APPUI: "YES") to calculate them in the absolute reference mark.

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6 Interaction

ground-structure

The seismic behavior of a building depends on the characteristics of the ground on which it is posed since it depends on the seismic movement imposed on the ground and the dynamic behavior of the building and

of its foundations. The interaction ground-structure most frequently contributes to decrease the answer of studied structure.

6.1

Impedance of a foundation

That is to say a surface rigid foundation without mass, subjected to a harmonic force of pulsation:

$P T$

it

$() = P .e$

0

It is thus actuated by a movement $X (T)$ of the same frequency. One calls

impedance of the foundation, the complex number (K), function of the frequency such as:

$($

$($

$P (T)$

$K) =$

$.$

$X (T)$

Several analytical or numerical methods make it possible to calculate the impedance of a foundation

according to the complexity of the foundation and ground on which it is posed or partially hidden.

Among

most frequently used, one quotes:

- analytical methods within the competences of WOLF or DELEUZE where it is supposed that it to erase circular, rigid and is posed on a homogeneous ground. The foundation must be surface;
- numerical method of code CLASSI where it is supposed that the foundation raft is of form unspecified, rigid and posed on a possibly laminated ground. The foundation must be surface;
- numerical method of the code MISS3D where the foundation raft can be of an unspecified form, possibly deformable and posed on a possibly laminated ground.

It is possible to treat the interaction ground-foundation by the **frequential method of coupling** (taken in count frequency response of the matrix of impedance) by carrying out a coupled calculation MISS3D/*Code_Aster*. This type of calculations is not detailed in this reference material. One present here only the case more the current where the interaction ground-foundation is treated by the **method**

springs of ground (it is considered that the terms of the matrix of impedance are independent of frequency).

In the case of a surface rigid foundation, the impedance is calculated in the centre of gravity of surface in contact in a reference mark related to the principal axes of inertia of this surface. For each frequency, it is expressed in the shape of a matrix of dimension (6, 6). One adjusts then the value of each term according to a particular clean mode of the building studied in blocked base:

- frequency of the first mode of swinging 0 for the horizontal stiffnesses $K_x(0)$, $K_y(0)$ and of rotation $K_{rx}(0)$, $K_{ry}(0)$;
- frequency of the first mode of pumping

1 for the vertical stiffness $K_z(1)$ and of torsion

$K_{rz}(1)$.

As the Eigen frequencies of the building depend on the stiffnesses of ground, the calculation of the values

total within the six competences of ground results from an illustrated iterative process appears [Figure 6.1-a].

first stiffnesses of ground $K_x(0)$, $K_y(0)$, $K_z(1)$, $K_{rx}(0)$, $K_{ry}(0)$ and $K_{rz}(1)$ are selected according to the first Eigen frequencies of swinging (0) and pumping (1) of structure in blocked base. The stiffnesses of grounds are then adjusted at the first frequencies clean significant of the structure on spring until correspondence of the frequencies to which functions of impedance are calculated with the values of the Eigen frequencies of the coupled system ground-building.

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Selection of the stiffnesses of ground

K_v (J) and K (I) in the matrix

of impedance

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Modal calculation with structure on
comes out from ground.

Not

Eigen frequencies

models on spring

correspond to

frequencies I and J

Yes

End

Appear 6.1-a: Process of adjustment of the stiffnesses of ground

6.2 Taking into account of a modal damping calculated according to the rule

RCC-G

One breaks up damping due on the ground into part of material origin and a part
geometrical: damping due to the reflexion of the elastic waves in the ground.

The rule of the RCC_G consists in summoning, for each mode, depreciation of each under
structure constitutive of the building considered and depreciation structural and geometrical of the
ground

balanced by their respective rate of potential energy compared to total potential energy:

$E_{k_i.k} + E_{s_i.s_i}$

K

S

$I =$

$E_{k_i.k} + E_{s_i.s_i}$

K

S

with:
·
 I , damping reduces average mode I ;
·
 K , the reduced damping of the kème element of the structure;
·
 if , the reduced damping within the competence of ground S for mode I ;
·
 E_{ki} , potential energy of the kème element of the structure for mode I ;
· and
 E_{si} , potential energy within the competence of ground S for mode I .
In the payment, modal damping is limited to a maximum value of 0,3.

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The part of material origin of the damping of the ground is calculated by balancing damping of each under structure by the report/ratio: rate of potential energy on total potential energy. As for the geometrical part of damping, it is calculated by distributing the values of damping for each direction (three translations and three rotations) balanced by the rate of potential energy in the ground of the direction. The directional values of damping are obtained while interpolating, for each calculated Eigen frequency, directional functions of damping exit of a code of interaction ground-structure (PARASOL, CLASSI or MISS3D). The report/ratio of the imaginary part on two

Im (K ())

time the real part of the matrix of impedance:

, provides the values of this damping

. R

2nd (K ())

radiative.

The step to be followed is as follows:

- Calcul of the potential energy dissipated in the studied structure: POST_ELEM [U4.61.04]

$E_k = \text{POST_ELEM (ENER_POT: (ALL: "YES"))}$;

- Calcul of modal damping by the rule of the RCC_G: CALC_AMOR_MODAL [U4.64.04]

$l_{\text{amor}} = \text{CALC_AMOR_MODAL (}$

ENER_SOL: (MODE_MECA: base_modale, GROUP_NO_RADIER: ,

$K_x ($

$K_y ($

$K_z (1)$

$)$

$)$

KX:

, KY:

, KZ:

,

$K_{rx} ($

$K_{ry} ($

$K_{rz} (1)$

$)$

$)$

KRX:

, KRY:

, KRZ:

));

AMOR_INTERNE: (GROUP_MA: , ENER_POT: $E K$, AMOR_REDUIT: K)

Im (K ())

AMOR_SOL: (FONC_AMOR_GEO:

. R

2nd (K ())

);

The calculation of the contribution of the ground to the potential energy E_s (key word factor

ENER_SOL) is calculated with

to leave the values of impedance of ground determined previously (cf [§6.1]). It can be calculated

according to two different methods according to whether one average modal efforts (key word

RIGI_PARASOL)

or modal displacements with the node of the foundation raft.

The reduced damping within the competence of ground $S S$ (key word factor AMOR_SOL) is calculated from

values of radiative damping.

6.3

Distribution of the stiffnesses and damping of ground

If one wants to study the effect of a seism on the possible separation of the foundation raft for example, one can be

brought to model the ground either by a single spring in the centre of gravity of the interface ground-building

but by a carpet of springs. This is possible thanks to order AFFE_CARA_ELEM [U4.24.01] option RIGI_PARASOL.

The step consists in calculating in each node of the grid of the foundation raft the elementary stiffnesses ($K_x, K_y, K_z, K_{rx}, K_{ry}, K_{rz}$

X

y

Z

X

y

Z) to apply starting from the total values within the three competences of translations:

k_x, k_y, k_z and within the three competences of rotations: k_{rx}, k_{ry}, k_{rz} resulting from a code of interaction ground-structure

(or calculated analytically).

It is supposed that the elementary stiffnesses of translation are proportional to surface S (P) represented by the node P and with a function of distribution F (R) depend on the distance R of the node P in the centre of gravity of the foundation raft O :

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$K = K$

.

X

$X(P) = K$

$S(P). F(P)$

X

O)

P

P

$$K = K$$

y

$$y(P) = K \cdot S(P) \cdot F(\text{COp})$$

y

)

P

P

$$K = K$$

Z

$$Z(P) = K \cdot S(P) \cdot F(\text{COp})$$

Z

)

P

P

One deducts *K X* then from them then *K (P*

X

) starting from calculation:

$$S(P) \cdot F(\text{COp})$$

$$kx(P) = K \cdot S(P) \cdot F(P)$$

$$O) = K$$

X

X.

.

$$S(P) \cdot F(\text{COp})$$

P

One deducts of the same *K (P from them*

y

) and *K (P*

Z

).

For the elementary stiffnesses of rotation, one distributes what remains after having removed the contributions

had with the translations in the same way that translations:

$$K = K$$

2

2

2

2

X-ray

X-ray (P) + [K y (P) .zO + K

P

Z (P). yOP] = K. S (P). F (COp

X-ray

) + [ky (P) .zO + K

P

Z (P). y P

O]

P

P

P

P

K = K

2

2

2

2

ry

ry (P) + [kx (P) .zO + K

P

Z (P). xOP] = K. S (P). F (COp

ry

) + [kx (P) .zO + K

P

Z (P). X P

O]

P

P

P

P

K = K

2

2

=

. (). (O) +

2

2

K
S P F
P
[*kx (P) .y PO +ky (P) .x*

rz
COp]
rz
rz (P) +
[*kx (P) .yO +k*

P
y (P). X P
O]

P
P
P
P
One deduces *krx* then from it then *K (P*

X-ray
) starting from calculation:
krx (P) = K. S (P). F (COp
X-ray
)

S (P). F (COp)
= K
. 2
. 2
X-ray - [K y (P) zO + K
P
Z (P) y P
O].

S (P). F (COp
P
)
P
One deducts of the same *K (P from them*

ry
) and *K (P*
rz
) .

Note:

By defect, one considers that the function of distribution is constant and unit i.e. each surface is affected same weight.

One can distribute in the same way six total values of damping, analytical or calculated by one code interaction ground-structure.

6.4

Taking into account of an absorbing border

If one wants to calculate the seismic response of a stopping, it is necessary, amongst other things, capacity to take into account

not reflexion of the waves in the valley. This is possible thanks to elements at absorbing border: option IMPE_ABSO in DYNA_NON_LINE and DYNA_LINE_TRAN. This functionality is not detailed in this document. It will be the subject of a specific note.

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R4.05 booklet: Seismic analysis

Document: R4.05.02

Stochastic approach for the seismic analysis

Summary:

This document presents a method of calculation probabilistic to determine the response of a subjected structure

with a random excitation of seismic type starting from the interspectres of the excitation at the points of support of

structure. The answer itself is expressed in the form of interspectres.

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1 Introduction

Classically the response of a structure subjected to a seismic excitation can be calculated by two approaches:

- transitory calculation of dynamics if the excitation is defined by a accélérogramme (cf [R4.05.01]).
- calculation by the traditional spectral method if the excitation is defined by a spectrum of answer of oscillator (SRO) (cf [R4.05.03]).

However a seismic excitation is by random nature. These two methods are not envisaged initially to hold account of it: in a case it is necessary to reiterate for various excitations the many ones temporal calculations then to make a statistical average of it (important cost calculation), in the other case one

carry out very conservative assumptions by considering averages (of quadratic type simple or supplements for example) for the maximum of the answers.

Also it was developed a method of calculation of the probabilistic type, also called "approach stochastic of the seismic calculation ", based on the calculation of the dynamic response expressed in interspectres of power starting from the spectral concentrations of power of the excitation. This method have in particular the advantage of better taking into account the correlations between the excitations to various supports of the structure.

The discussion of the various advantages of this method can be thorough in the reference [bib1].

We thus present the principle of the method and the notations retained starting from the steps traditional, then in third part probabilistic calculation itself.

Finally in fourth part the various methods will be presented to obtain the interspectre discharger.

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2

Principle of the step

2.1

Position of the problem considered and principle general

A multi-supported structure is in the case of placed, i.e. the structure has **m** ddl-supports, each one being subjected to its own excitation (not necessarily equal everywhere). One supposes that the structure is represented by a model finite elements comprising **N** ddl. The answer is sought in a number finished (and low) of **L** ddl.

It is supposed that the **size excitation is of imposed movement type** and results in a family of accélérogrammes $G(T)$

$J()$ for each one of the ddl-supports $J, j=1, \dots, m$.

The absolute movement of the structure is **broken up** classically moving of **drive and relative movement**.

The calculation of the response in interspectres of power is carried out by **modal recombination**. Following this modal calculation, a calculation of dynamic response random breaks up into three parts:

- definition of the interspectre of power discharger,
- calculation of the interspectre of power answer.

These the first two parts are the subject of order DYNA_ALEA_MODAL [U4.56.06].

The restitution of the interspectre of power response on physical basis is carried out with the order REST_SPEC_PHYS [U4.80.01].

- calculation of statistical parameters starting from the interspectre of power result.

This last stage is treated by order POST_DYNA_ALEA [R7.10.01] [U4.76.02].

2.2

Decomposition of the movement

The following decompositions and projections are detailed in the reference material relating to the resolution by transitory calculation of a seismic calculation [R4.05.01]. We retain any here only the broad outline.

That is to say **Teststemxà the** vector absolute displacement (of dimension **N**) of all ddl of the structure. The total answer known as **absolute Xa** of the structure is expressed as the sum of a contribution **relative Xr** and of the contribution of **drive Xe** due to displacements of anchoring (subjected to accelerations represented by a accélérogramme $G(T)$

J of each ddl-support $J, j=1, m$).

$$\mathbf{X}(T) = \mathbf{X}(T) + \mathbf{X}$$

has

R

$E(T)$

Are \mathbf{M} , \mathbf{K} and \mathbf{C} the matrices of mass, rigidity and damping of the problem, limited to ddl not supported.

The equation of the movement is written then in the reference mark related to the relative movement:

MR. X

$$\mathbf{M} \ddot{\mathbf{X}}(T) + \mathbf{C} \dot{\mathbf{X}}(T) + \mathbf{K} \mathbf{X}(T) = -\mathbf{M} \ddot{\mathbf{X}}(T) + \mathbf{F}$$

R

R

R

E

ext.

ext.

F vector of the external forces

In general the external forces are null during a calculation of seismic answers.

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2.3

Decomposition on the modal basis

The calculation of response in interspectres of power is carried out by **modal recombination** and is made

call, moving imposed, at a modal base which includes/understands at the same time dynamic modes and static modes.

That is to say $\{I I, = N,$

1} matrix (N, N) of the dynamic modes calculated for the conservative system associated, by maintaining the m blocked supports.

That is to say = $\{J, j= m,$

1} the matrix (N, m) of the static modes. The mode J corresponds to the deformation of the structure under a unit displacement imposed on the ddl-support J , other ddl-supports being blocked.

The imposed displacement of anchorings $\mathbf{X}_s(T)$ is connected to $\mathbf{X}_e(T)$ by the relation: $\mathbf{X}(T) = \mathbf{X}$

E

$S(T)$. Components of the acceleration of the points of anchoring! $\mathbf{X}_s(T)$ are the accélérogrammes $G(T$

$J),$

$j=1, Mr.$

m

One can thus write! $\mathbf{X}(T) = \mathbf{X}(T) = G(T)$

E

S

$J J$

$j=1$

One carries out the change of variable $\mathbf{X}(T) = ($

$\mathbf{Q}), ($

\mathbf{Q}

$R T$

T

$T)$ is the vector of the co-ordinates

generalized. By prémultipliant the equation of the movement per T , one obtains - in the absence of forces external others that the seismic excitation - the equation projected on the basis of dynamic mode:

T

T

T

T

$\mathbf{M}\dot{\mathbf{q}}(T) + \mathbf{C}\dot{\mathbf{q}}(T) + \mathbf{K}\mathbf{q}(T) = - \mathbf{M}\mathbf{X}(T) S(T)$

It is supposed that the matrix of damping is a linear combination of the matrices of mass and of rigidity (assumption of damping of constant Rayleigh on the structure or assumption of Basile allowing a diagonal damping). The base

, which orthogonalise the matrices \mathbf{M} and \mathbf{K} ,

orthogonalise thus also the matrix \mathbf{C} .

Taking into account this assumption, the preceding equation breaks up into N equations scalar uncoupled in the form:

m

!

$\mathbf{I}\ddot{\mathbf{q}} + 2$

2

\mathbf{I}

$$\mathbf{I} \dot{\mathbf{Q}} + \mathbf{Q} = - \mathbf{p} \mathbf{G} (T)$$

\mathbf{I}

\mathbf{I}

\mathbf{I}

ij J for i=1, N

$j=1$

Where one noted:

μ T

$\mathbf{I} =$

\mathbf{I}

M \mathbf{I}

modal mass

T

$k_i = i\mathbf{K} \mathbf{I}$

modal rigidity

\mathbf{K}

\mathbf{I}

$\mathbf{I} =$

μ

the modal pulsation

\mathbf{I}

T **C**

\mathbf{I}

\mathbf{I}

$\mathbf{I} = 2\mu$

reduced modal damping

$\mathbf{I} \mathbf{I}$

T iM_j the factor of modal participation of

ij

p =

μ_i

the support J on the dynamic mode I

Solution $\mathbf{IQ} (T)$ of this equation corresponds to the response of the dynamic mode I to the whole of the seismic excitation.

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One can still break up the problem by introducing the unknown factor **dij** (*T*) solution of the equation differential: $\mathbf{D} + 2$

$\mathbf{D} + 2$

$\mathbf{D} = G (T)$

ij

I I ij

I

ij

J

, this last equation corresponds to the answer of dynamic mode *I* with acceleration *G T*

J (). Relative displacement on the physical basis is expressed

then:

N m

$\mathbf{X} (T) = - \mathbf{p} \mathbf{D}$

R

ij ij (T) I

i=1 j=1

Information on the position of the point of support is contained in the factor of modal participation.

2.4 Answer

harmonic

One thus broke up the total response of the structure into a relative contribution and a contribution differential due to displacements of anchorings such as:

$\mathbf{X} (T) = \mathbf{X} (T) + \mathbf{X}$

has

R

E (T)

with

m

$\mathbf{X}! (T) = \mathbf{X}$

E

$$! S (T) = j g j (T)$$

$$j=1$$

N m

X (T) = - p D (T) where **D (T)** are

T solution of **D**

! +

$$\mathbf{D}! + 2$$

2

D

R

ij ij

I

ij

ij

I I ij

I

$$ij = G J (T)$$

$$i=1 J$$

=1

The solution of this last differential equation by the method of the transformation of made Fourier

1

to intervene modal transfer transfer functions *hi ()* such as: *hi () =*

.

$$(2 - 2 + I$$

I

$$2 II)$$

One thus obtains: **D () = H () . G ()** and **D**

2

ij

I

J

$$! ij () =$$

-

I

$$H () . G J ()$$

The total harmonic response of the structure results from the preceding formulas by recombination modal.

$$! \mathbf{X} () = ! \mathbf{X} () + ! \mathbf{X} ()$$

has

R

*E**N m**m*

$$\mathbf{X}(\omega) = 2\mathbf{p} \mathbf{H}(\omega) \mathbf{G}(\omega) + \mathbf{G}(\omega)$$

*has**ij J**J**I**J J**i=1 j=1**j=1*

One then makes apparaitre the complex matrix (N, m) , known as matrix of transfer $\mathbf{H}(\omega)$ following:

$$\mathbf{H}(\omega) = 2\mathbf{p}$$

H**()**

+

where \mathbf{p} is the matrix of the factors of participation, $\mathbf{H}(\omega)$ the vector of the modal transfer transfer functions

hi ().

The total response of the structure is worth! $\mathbf{X}(\omega) = \mathbf{H}(\omega) \mathbf{E}(\omega)$, where!

E**()***has*

$\mathbf{E}(\omega)$ is the vector of m lines

constituted of the transforms of Fourier of accelerations $\mathbf{G}(\omega)$

$\mathbf{J}(\omega)$ with the **m ddl**-supports.

It is seen that this expression determines the response in acceleration. This then forces to integrate twice the answer to obtain displacement, this problem is presented in [bib4]. One of additional interests of the method which we propose here is to abstract itself from this difficulty.

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3**The random dynamic response****3.1****Recall on the spectral concentrations of power [bib2]****3.1.1 Definitions**

That is to say a probabilistic signal defined by its density of probability $p_X(x_1, T_1; \dots; x_n, T_n)$. This density of

probability makes it possible to calculate the functions moments of the signal.

Moment of order 1 or hope of the signal:

$$+ \mu_X(T) = [E X(T)] = \int X p(X, T) dx$$

-

Moments of order 2 or intercorrelation of two signals:

$$+ \mu_{XY}(T, T) = [E X_1(T) Y_2(T)] = \int \int X_1 Y_2 p(X_1, T; Y_2, T) dx_1 dy_2$$

1

2

1

2

-

When the signal is stationary, the intercorrelation depends only on $\tau = T_2 - T_1$

2

1 .

It is written R

$$(\tau) = [$$

$E(T)$
 \mathbf{XY}
 \mathbf{X}
 \mathbf{Y} -)]

Spectral concentration of power and interspectre

One defines $\mathbf{SXY}()$ the interspectre of power or density interspectrale of power between two stationary probabilistic signals by the transform of Fourier of the function of intercorrelation, what one writes:

+
 1
 \mathbf{S}
 $() =$
 \mathbf{R}
 $- I$
 \mathbf{XY}

D
 2
 $\mathbf{XY}()$ E

-
 +
 The opposite formula is written: \mathbf{R}
 $() = \mathbf{S}$

I
 \mathbf{XY}
 $\mathbf{XY}()$ E
 D

-
 $\mathbf{SXY}()$ is generally complex and checks the relation of symmetry: $\mathbf{S}() = \mathbf{S}$
 \mathbf{YX}
 $\mathbf{XY}()$.

When $\mathbf{X} = \mathbf{Y}$, $\mathbf{SXX}()$ is called **autospectre power or spectral concentration of power (DSP)**. This function with the real and always positive property to be.

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3.1.2 Relations between the DSP and the other characteristics of the signal

Note:

Most of the time, the signal is defined over a limited time, its transform of Fourier does not exist not, one defines a transform of Fourier then estimated over one period length T by:

$T/2$

"

1

$X()$

$X(T)EI$

=

-

D

T

.

2

$-T/2$

One then has the following relationships to this estimated transform of Fourier:

2

S

=

" "

XY()

lim

E

T

T

$T+T$

[X() Y()]

2

S

=

" "

XX()

lim
 E
 T
 T
 T+ T

[X () X ()]

Bond between the autospectre of power and the power of the signal:

The power of a signal is equal to its variance. For a centered signal, the variance is worth:

$2X = X$

$R X (0)$.

+

One thus has: $2X = X$

$R X (0) = SXX$

() D.

-

3.2

Equations of motion

The total response of the structure is determined by the relation: $X () = H () E ()$

E

has

),

where $E ()$ is the vector of m lines made up of the excitations represented by the transforms of Fourier of the accélérogrammes $G (T$

J

) with the m ddl-supports,

(
H) is the matrix of transfer defined by (

$H = 2$

)

$p H () +$

where p is the matrix of the factors of participation,

(

H) the vector of the modal transfer transfer functions $h_i ()$

base dynamic modes

base static modes

it comprises N lines (= a number of ddl free of the structures) and m columns.

3.2.1 Stamp

“interspectrale-excitation”

NB:

This name “stamps interspectrale-excitation” is abusive: it means “matrix of density interspectrale of power of the excitation”.

It is supposed that the seismic excitation can be regarded as a stationary signal - taking into account

relationship between times representative - and centered. This makes it possible to use a certain number of result of the probabilistic analysis. One is interested then in the stationary response of the system to one stationary excitation.

S is noted

()

EE

!! the matrix of the interspectres of power corresponding to the excitation. Its data is clarified in the chapitre 4.

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For memory we recall here that it is calculated starting from transforms of Fourier of accelerations. It is a matrix (mxm). The ij term corresponds to the interspectre between the signals! **I.E. (internal excitation)** and

! **E J** is still between the transforms of Fourier of the accélérogrammes *G* and *G*

I

J.

3.2.2 Random dynamic response

It was seen that the interspectre power between two probabilistic signals is the transform of Fourier function of intercorrelation of the two signals. One applies it to the total response of the structure:

+

+

1

1

T

S

()

R

$\int E I$
 $- D$
 E
 $\int X(T) X(T) E I$
 $-$
 $=$
 $=$
 $-$
 D
 $!$

$X! X$
 2
 $! X! X$
 2

has
has
has has
has has

$-$
 $-$
 One works then in the temporal field to express the function of intercorrelation of the answer total R

$T T$
 $! has! (, ')$
 $X X$

\cdot
has
 One notes ($H T$) the impulse response of the system: $H(T) =$

$-$
 $TF1 [H ()]$
 and! $E(T)$ the transform of Fourier reverses exiting DSP: $! E(T) =$

$-$
 $TF1 [! E ()]$
 By transform of Fourier relation reverses: $! X () = H ()! E ()$
has

one has! $\mathbf{X}(T) = \mathbf{H} *$

has

! $\mathbf{E}(T) = \mathbf{H}(U)$

! $\mathbf{E}(T - U) D$

U

R

T

R

$(T, you) = \mathbf{E}! \mathbf{X}(T)! \mathbf{X}(T)$

! $\mathbf{X}! \mathbf{X}$

has

has

)

has has

T

R

$(T, you) = \mathbf{E} \mathbf{H}(U) \mathbf{E}$

-

H

E -

X X

! $(T U)$ of

(*v*)

! $(you v) FD$

! *has! has*

R

R

R

$(T, you) = \mathbf{E}$

**H (U) E - E -
H**

X X

! (T U T
)! (you v T
)
(v) FD
! has! has

R R

One supposes in this analysis the deterministic system, one can thus leave the impulse response calculation of the expectation. It comes

R
T T =
H U
E T - U T E T - v T H v
FD of
!

[

has!
(,)
) E! (
)! (')] ()
X Teststemxà
R R

The excitation is supposed a stationary process, the intercorrelation thus depends only on the variation on time = T - you:

R (T - you-U + v) = E [E -
E -
=

for = - - + = - +
EE

! (T U T
)! (you v)] R ()
T you U v
U v
!!

!!

EE

T

from where *R*

(T, you) =

H (U) R () H (v) FD

= R

!

()

X

what justifies a posteriori the approach.

has! Teststemxà

! E! E

! Teststemxà! Teststemxà

R R

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One now defers this expression in the expression of the spectral concentration of power of answer:

+

+

1

1

S

()

R

I

$\int \mathbf{H}(U) R(U, v) T$
 $\int \mathbf{H}(v)$
 I
 $\int E$
 $=$
 $=$
 $- +$
 $FD \text{ of } D$
 $!$

\mathbf{X}
 2
 2

has! $\mathbf{Teststemx\grave{a}}$
 $!$ $\mathbf{Teststemx\grave{a}}$ $!$ $\mathbf{Teststemx\grave{a}}$
 $!$ $\mathbf{E! E}$

$-$
 $- R R$

By répartissant the dummy variables of integration one reveals the transforms of Fourier

T
 respective of (
 $H(U), R(-U + v), \mathbf{H}(v)$
 $!!$

\mathbf{EE}
 , it comes finally:

\mathbf{S}
 $\mathbf{H}() \mathbf{S}$
 $() T$
 $=$

$\mathbf{H}()$
 \mathbf{X}
 $! \mathbf{X}$
has!
 \mathbf{E}
has
 $! \mathbf{E!}$

with $\mathbf{H}() = 2 \mathbf{p}$
 \mathbf{H}

()

+

Taking into account the relations between the transforms of Fourier of displacement, the speed and of acceleration, one has moreover:

-1

S

=

H () S () T**H ()****X! X!**

2

E**! E!***has has*

1

S

=

H () S () T**H****X X**

()

has has

4

EE**!!**

These relations make it possible to express the response of the structure by the DSP of displacement or of speed.

Note:

· According to the expression given to **H ()**, one respectively expresses the DSP of the displacement (of the speed or of acceleration) total, relative or differential:

absolute movement: **H () = 2 p**

H

()

+

relative movement: **H () = 2 p**

H

()

differential movement (IE of drive): **H () =**

· It is of use, during a calculation with Code_Aster, to restrict the matrix of the function of transfer to the lines of the **L ddl** of observation. This makes it possible to reduce of as much calculations as soon as **L**

is small in front of N.

3.3

Application in Code_Aster

The whole of the spectral approach for seismic calculation is treated in the order DYNA_ALEA_MODAL [U4.56.06]. The data are gathered under three key words factors and a word single-ended spanner.

The modal base is consisted of the dynamic modes calculated by the order MODE_ITER_SIMULT [U4.52.02] or MODE_ITER_INV [U4.52.01] stored in a concept of the type mode_meca recovered by the key word factor BASE_MODAL, on the one hand; calculated static modes

by order MODE_STATIQUE [U4.52.04] stored in a concept of the mode_stat type recovered by key word simple MODE_STAT, in addition. The key word factor BASE_MODAL also has the arguments which make it possible to determine the frequency band or the modes retained for calculation

and corresponding depreciation.

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The data corresponding to the excitation are gathered under the key word factor EXCIT (cf paragraph [§4]): one specifies there the type of excitation within the meaning of the SIZE: excitation in

displacement or in effort, the nodes NODE and component excited NOM_CMP, the name of interspectres or autospectres INTE_SPEC, complex functions read beforehand or calculated, respectively by operators LIRE_INTE_SPEC [U4.56.01] or CALC_INTE_SPEC [U4.56.03] and stored in a table of interspectre of concept tabl_intsp which applies in each ddl excited.

Under the key word factor ANSWER are the data related to the choice of the discretization.

Order DYNA_ALEA_MODAL provides the response in the form of spectral concentration of power on modal basis. To obtain the restitution of the DSP on physical basis, REST_SPEC_PHYS will be used [U4.80.01] which makes it possible to specify the type of size of the answer (displacement or effort),

with

“points of observation” (node-component) of the result. In the presence of a response of the type displacement, one will specify here also if the answer corresponds to absolute displacement, relative or differential.

REST_SPEC_PHYS provides a table of interspectres which contains according to the request of the user, stamp interspectrale in displacement SXX , of speed $S!$!

XX, or in acceleration $S!$!

XX for one

expression in the absolute reference mark (index has), the relative reference mark (index R) or of drive (index E).

Each preceding “combination” requires a call specific to order REST_SPEC_PHYS.

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4 Definition of the matrix interspectrale of power exciter

The seismic excitation is by nature, we said it, random. Also it can be known not by its temporal expression but in frequential form by a spectral concentration of power known as also interspectre.

When there are several supports, they can be excited by identical or different excitations, it last case is that of the multi-supports.

For m supports, one defines the matrix of density interspectrale of power of order m , or per abuse language the interspectre of order m , which is a matrix ($m \times m$) of complex functions depending on frequency.

The diagonal terms represent the “auto-” densities spectral of powers - or autospectres- to the points of excitation, the extra-diagonal terms correspond to the densities interspectrales enters the excitations in two points of support distinct (each line or column of the matrix represents in fact a point of support in physical grid or a mode in modal calculation). By definition of these terms, it results from it that the matrices of density interspectrales of power handled are square.

(See [bib2] or reference material associated with order POST_DYNA_ALEA [R7.10.01])
 We present hereafter the various orders of *Code_Aster* which make it possible to obtain one stamp density interspectrale of power.

4.1

Reading on a file

The most elementary way to define a matrix of density interspectrale of power is of to give, “with the hand”, the values with the various steps of frequency.

Operator LIRE_INTE_SPEC [U4.56.01] is used then.

LIRE_INTE_SPEC reads in a file “interspectre excitation”. The format of the file in which is consigned the matrix interspectrale is simple: one describes successively the function of each term matrix interspectrale; for each function, one gives a line by frequency while indicating frequency, parts real and imaginary of the complex number; or the frequency, the module and the phase complex number (key word FORMAT).

Example of file interspectre excitation (for a matrix reduced in the term):

INTERSPECTRE

DIM = 1

FONCTION_C

I = 1

J = 1

NB_POIN = 4

VALUE =

2.9999 0. 0.

3. 1. 0.

13. 1. 0.

13.0001 0. 0.

FINSF

END

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4.2**Obtaining a interspectre starting from functions of time**

One can deduce the matrix from density interspectrale of power starting from functions of time. One use then operator CALC_INTE_SPEC [U4.56.03] in *Code_Aster* [bib3].

Starting from a list of NR functions of time, this operator allows to calculate the interspectre NxN power which corresponds to them.

For each term of the matrix interspectrale (NxN) one uses the following step [bib3].

To calculate the interspectre of two signals one uses the relation of Wiener-Khichnine [bib7] which allows

to establish a formula of computation of the spectral concentration of power by the transform of Fourier finished samples of the signals (

$\mathbf{X}(T)$ and (

$\mathbf{y}(T)$).

It comes then:

$$S(F) = \lim_{T \rightarrow \infty} \frac{1}{T} E [\mathbf{Y}^*(F, T) \mathbf{Y}(F, T)]$$

\mathbf{xy}

\mathbf{Xk}

K

)]

$T T$

T

$$\mathbf{X}(F, T) = \text{TF} [\mathbf{X}](F) = \int_{-T}^T \mathbf{X}(t) e^{-i2\pi Ft} dt$$

) E

dt

K

K

K

where

0

T

$$\mathbf{Y}(F, T) = \int_{-T}^T \mathbf{y}(t) e^{-i2\pi Ft} dt$$

$$= \text{TF} [\mathbf{y}](F) = \int_{-T}^T \mathbf{y}(t) e^{-i2\pi Ft} dt$$

) E

dt

K

K

K

0
are the discrete transforms of Fourier

X
of and
y
of.

When one is interested in signals resulting from measurements, one has most of the time only known signals in a discrete way, in the same way a transitory computation result is a discrete signal. An approximation of the interspectre of the discrete signals $X [N]$ and $y [N]$ definite on L points spaced of T , cut out out of p blocks of Q points is obtained by the relation:

$$\begin{aligned}
 & \sum_{i=1}^p \sum_{k=0}^{Q-1} X [K] \sum_{n=0}^{Q-1} Y [K] \sum_{n=0}^{Q-1} y [N] e^{-2i kn/q} \\
 & = \sum_{k=0}^{Q-1} X [K] \sum_{n=0}^{Q-1} Y [K] \sum_{n=0}^{Q-1} y [N] e^{-2i kn/q} \\
 & = \sum_{k=0}^{Q-1} X [K] \sum_{n=0}^{Q-1} Y [K] \sum_{n=0}^{Q-1} y [N] e^{-2i kn/q}
 \end{aligned}$$

The various blocks can or not overlap. The values p and Q are with the choice of the user. This method is that of the periodogram of WELCH [bib8].

Calculation is done on a window which moves on the field of definition of the functions. The user specifies in the order the length of the window of analysis, the shift between two windows of calculation successive and the number of points per window.

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4.3 Excitations preset or reconstituted starting from functions complexes existing

One can wish to define a matrix of density interspectrale of power in various ways:

- by a white vibration: the values are constant
- according to the analytical formula of useful KANAI-TAJIMI in seismic calculation (filtered white vibration),
- or by taking again existing complex functions.

Operator DEFI_INTE_SPEC [U4.56.02] is used then.

4.3.1 Existing complex functions

It is enough under the key word factor PAR_FONCTION to give the name of the function for each pair of index NUME_ORDRE_I, NUME_ORDRE_J, corresponding to the higher triangular matrix (in reason of its hermiticity).

4.3.2 Noise

white

A white vibration is characterized by a constant value on all the field of definition considered. Under the key word CONSTANT factor, one gives this value (VALE_R or VALE_C) on the frequency band [FREQ_MIN, FREQ_MAX] for each pair of index INDI_I, INDI_J, corresponding to the matrix triangular higher (because of its hermiticity). To define the function perfectly, one specifies the interpolation and prolongations.

4.3.3 White vibration filtered by KANAI-TAJIMI [bib9]

For a structure pressed on the ground, it is common to take as excitation the spectral concentration of power of Kanai-Tajimi. This spectral concentration represents the filtering of a white vibration by the ground.

The parameters of the formula make it possible to exploit the centre frequency and the bandwidth of spectrum.

The spectrum $G()$ is expressed by the following relation:

$$G(f) = \frac{G_0}{\sqrt{1 + 4 \left(\frac{f - f_0}{\Delta f}\right)^2}}$$

G

G

G

G

$(f) =$

G

$(2$

$2 \Delta f$

2

2

2

0

$G_0) + 4 G$

G

$= 2 F$

G

G pulsation clean

G

modal damping

G_0

white sound level before filtering

The user must specify the Eigen frequency $F G$ of the filter, modal damping G and the level of white vibration $G_0 (= VALE_R)$ before filtering; like as for any function: the interpolation, them profiles external and the field of definition (frequency band).

By defect a ground running is well represented by the values $F G = 2.5$ Hz and $G = 0.6$.

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Example of use for a white vibration filtered by KANAI_TAJIMI:

Interex =
DEFI_INTE_SPEC (
DIMENSION: 1
KANAI_TAJIMI: (
NUME_ORDRE_I: 1
indices of the term of the matrix of density
NUME_ORDRE_J: 1
interspectrale of power
FREQ_MOY: 2.5
Eigen frequency
AMOR: 0.6
modal damping
VALE_R: 1
white sound level
Interpol: "FLAX"
linear interpolation
PROL_GAUCHE: "CONSTANT"
prolongation
PROL_DROIT: "CONSTANT"
FREQ_MIN: 0.
field of definition
FREQ_MAX: 200.
NOT: 1.
));

4.4

Other types of excitation

Calculations of the preceding paragraphs were carried out within the framework of the assumption of an excitation

moving **imposed** on a ddl. With the help of some modifications it is possible to use the same one approach for an excitation **in effort** [§4.4.1] or **by fluid sources** [§ 4.4.2], this one being expressed in a finite element [§4.4.3] or on a function of form of the structure [§4.4.4].

In the continuation of this paragraph, one supposes the random excitation known and provided by the user under

the form of a DSP, spectral concentration of power.

4.4.1 Case of the excitation in imposed forces

Under key word EXCIT there is SIZE = EFFO.

When the excitation with the supports is of type forces imposed, the general equation of the movement is:

m

!(

 $\mathbf{M} \mathbf{X} T) + ! ($ $\mathbf{C} \mathbf{X} T) + \mathbf{K} ($ $\mathbf{X} T) = \mathbf{F} \mathbf{j}$ $j=1$

The response of the structure is then calculated on a **basis of dynamic modes** = $\{I I, = N,$

 $1 \} ,$

these modes being calculated by supposing the **free exiting supports**. One does not distinguish, in it case of absolute, relative and differential movement and one does not use static modes.

 $T i F_j$

One defines the factor of modal participation in the form: $P_{ij} =$

 I μ

The transitory, harmonic and random answers have the **same expressions** as the answers of **relative movement** of the excitation multi-support in the case general [§3]. (What corresponds to the absence

static modes). The exiting force is represented in each ddl-support by its DSP in form of a term are equivalent to **S**

EE

!! () .

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4.4.2 Excitation by fluid sources

The fluid sources appear, for example, in the study of a network of pipings. They correspond to active bodies or connections of secondary pipings. They are generally sources of pressure or sources of flow. These various types of source are

presented hereafter according to their mathematical working and what *Code_Aster* makes in each configuration.

These fluid sources are not directly seismic excitations but can be induced by a seism. The resolution of the mechanical problem calls upon very the methods, because of their randomness, which justifies their presentation here.

The modeling of the network of piping is supposed to be realized using acoustic beam vibro of *Code_Aster*.

The response to fluid sources is calculated within the framework of the response to imposed forces (cf [§4.4.1]), within this framework one is interested in answers of size of the type “displacement” (SIZE = DEPL_R under the key word ANSWER).

The sources of pressure and force, for reasons of modeling of the fluid sources are represented by dipoles [bib5], it is thus necessary to give **two points of application**.

Source of flow-volume: SIZE = SOUR_DEBI_VOLU under key word EXCIT

A volume flow rate is expressed in m³/s, its spectral concentration of power in (m³/s)²/Hz.

A source of flow-volume is considered, in the formulation *P* - elements of piping with fluid, like an effort imposed on the ddl of the node of application of the source [R4.02.02].

The user provides the DSP of volume flow rate **S_{vv}** (), the DSP *vv* () applied in effort to the ddl is: () = () 2S ()

vv

vv

where is the density of the fluid.

Source of flow-mass: SIZE = SOUR_DEBI_MASS under key word EXCIT

A flow-mass is expressed in kg/s, its spectral concentration of power in (kg/s)²/Hz.

flow-mass is the product of flow-volume by the density of the fluid.

The user provides the DSP of **S_{mm} flow-mass** (), the DSP

()

mm applied in effort to

ddl is:

= 2S

mm

mm

Source of pressure: SIZE = SOUR_PRESS under key word EXCIT

A source of pressure is applied in *Aster* in a **dipole P1P2**.

For a source of pressure whose DSP is **S_{PP}** (), expressed in Pa²/Hz, *Aster* builds one stamp density interspectrale of power

()

PP which is applied in force imposed to

the ddl of the points P1 and P2.

*S*²

*S*²

dx

-

dx

$() = \mathbf{S} ()$

PP

PP

$S 2$

$S 2$

-

dx

dx

where S is the fluid section, dx the distance between the two P1 points and P2.

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Source of force: SIZE = SOUR_FORCE under key word EXCIT

The force corresponds simply to the product of the pressure by the fluid section of the tube: $F = PS$.

It thus is also applied to a **dipole P1P2**.

For a source of force whose DSP is **SFF** (), expressed in N^2/Hz , *Aster* applies in force imposed on the ddl of the points P1 and P2, (distant of dx), the matrix of density interspectrale of power

()

FF such as:

1 2

1 2

dx

-

dx

$() = S ()$

FF

FF

1 2

1 2

-

dx

dx

4.4.3 Excitation distributed on a function of form

If the spectral concentration of power of the excitation $E()$ corresponds to an effort imposed on one function of form f_i , $E()$ gives the frequential dependence of the level of the excitation.

The space weighting of the effort is represented in *Code_Aster* by a field with the nodes which does not depend on the frequency: key word CHAM_NO under the key word factor EXCIT. This field with the nodes

is a "assembled vector". From the theoretical point of view the formalism of calculation is the same one as

previously (excitation in imposed force [§4.4.1]), for a vector of force in second member equal to f_i .

4.5 Applications

These various types of excitation are included in the tests of validation, and are presented by examples in the report/ratio [bib6]. In particular the excitations of the fluid type are in the test: pipe subjected to random fluid excitations [V2.02.105] (SDLL105). Excitations on functions of form are tested in the case test: beam subjected to a random excitation distributed [V2.02.106] (SDLL106).

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Seismic response by spectral method

Summary:

The study of the response of a structure under the effect of imposed movements of seismic type, with one single imposed movement (mono support) or multiple (multi supports) is possible in transitory analysis (time history). One will refer to the note [R4.05.01].

For studies of dimensioning, one can be interested only in one estimate of the induced maximum efforts by the requests, to evaluate the safety margin with payments of construction, without resorting to a transitory analysis.

The spectral method is based on the concept of spectrum of oscillator of a accélérogramme of seism. One detail the method of development of this spectrum of answer available in operator CALC_FONCTION [U4.32.04].

It is shown how this spectrum of oscillator can be used to evaluate one raising of the answer in relative displacement of a simple oscillator. This approach is justified if one does not wish to know the history of displacements and of the efforts, while limiting themselves to the analysis of the inertial effects.

The spectral method uses general notions of the method of modal recombination [R5.06.01].

One describes the various rules of combination usable to obtain one raising realistic but conservative the maximum response of the structure. These methods are available in operator

COMB_SISM_MODAL

[U4.84.01].

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1

Concept of spectrum of oscillator

Spectral method for the study of the answer of a structure under the effect of imposed movements of seismic type is based on the concept of spectrum of oscillator of a accélérogramme of seism.

1.1

Imposed movement defined by a accélérogramme A (T)

For a movement imposed S of the seismic type, one can deal with the problem in absolute displacement

X or in relative displacement X such as: $X = X + S$. general equations of the movement of one simple oscillator are written then:

Absolute movement

Relative movement

X

S

X

X

m &+ X

c& + kx = - S

m

K

& + & +

= & +

X
m
X
C
kX
S
C
ks
&
m
C

One retains the formulation starting from the relative movement for two principal reasons:

*.
the seismic analysis of the structures uses the constraints induced by the inertial effects of seism, constraints calculated starting from the structural deformations which are expressed with to leave relative displacements;*

*.
the characterization of the signal of excitation can be reduced in this case to the accélérogramme seism s=*

*& (
With T), size provided directly by the seismographs. Signals of displacement S and speed &s are in general not available in the bases of data geotechnics.*

For the determination of the response of a simple oscillator to an imposed movement and the notations conventional one will refer to appendix 2 [R4.05.03 Annexe 2].

*The reduced equation is in this case, if the seism is defined by a accélérogramme (
With T), acceleration absolute applied to the base:*

*x+
& 2 X*

*éq
1.1-1*

0 &+
2 X = - S
0
& = - (
With T)

The solution of this problem is the integral of DUHAMEL presented at appendix A [éq A3.3-1]:

T
X (T) 1
=
(
With)
- T
0 (
)
E
sin (T
-) D F
= (,
With,

-

éq
1.1-2
0)
0
0
0
=
1-
0
(2
0
)

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1.2
Spectrum of oscillator of a accélérogramme

The concept of spectrum of oscillator was introduced initially to compare between them the effects of different accélérogrammes. The spectrum of FOURIER of a signal (With T) informs about its contents frequential. The response of a mechanical system to a movement imposed on the base largely depends dynamic characteristics of this system: Eigen frequencies and reduced damping (0) Appendix A details this aspect.

If one wishes to know the maximum value of the response of a simple oscillator to the parameters (, A, one must evaluate the integral of DUHAMEL which provides the response of the oscillator [éq 1.1-2] to 0) an excitation imposed on the base.

Accélérogramme

0.7
0.6
0.4
0.2
0.0
0.2

Absolute acceleration

0.4
0.6
0.7
0
3
6
9

12
15
18
Time

Appear 1.2-a: Accélérogramme

1.2.1 Spectrum of oscillator in relative displacement

From the integral of DUHAMEL, one can define the spectrum of oscillator of a accélérogramme

*(
With T) like the function of the maximum values of relative displacement $X(T) F$*

*= (
With, for each*

*0)
value of (ω , by recalling that '*

*=
1-*

*.
0*

*(2
0*

*)
0)*

*Srox (ω ,
With, $\omega = X T$*

*0)
(ω) max*

*T
X(T) 1*

*=
(
With) - T*

0 (ω

*)
E*

*sin(T
-) D F*

*= (ω ,
With,*

-

0)
0
0
0

One notes, on the figure [Figure 1.2.1-a] that beyond a certain frequency (35 Hz here), known as cut-off frequency of the spectrum, it does not have there significant dynamic amplification: displacement relative is null.

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Spectrum of oscillator (relative Displacement)

*1
0.1
10
Increasing depreciation
from 0.01 to 0.20*

*10 2
Depreciation
103
1%
5 %*

*Relative displacement
10%
20%
10 4
10 5*

1e05

10 0

10 1

Frequency

Appear 1.2.1-a: Spectrum of oscillator in relative displacement

1.2.2 Spectrum of oscillator in pseudo relative speed

For structures with weak reduced damping < 2

.

0 =

%

20, for which it is

acceptable to assimilate 0 and 0 one usually uses the spectrum pseudo speed defined by:

Srox& (,

With, = Srox,

With, = X T

0)

0

(

0)

0

() max

Pseudo speed is the value the speed which gives a value of the kinetic energy of the mass oscillator equal to that of the maximum deformation energy of the spring:

1

1

1

1

E = m x& T 2

() = m Srox& A

2

”

= m2

. X T 2

()

= K X T 2

()

= E

C
()
[
(
0)]
0
p
max
max

2
2
2
2
Spectrum of oscillator (Pseudo relative speed)
0.5
Increasing depreciation
from 0.01 to 0.20
I
ve
1
10
Depreciation
sse relat
E
1%
5 %
10%
20%
Pseudo saw 102
0.004
10 0
10 1
Frequency

Appear 1.2.2-a: Spectrum of oscillator in pseudo relative speed
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1.2.3 Spectrum of oscillator in pseudo absolute acceleration

In the same way for a weak reduced damping one can define the spectrum of pseudo acceleration definite

by:

***Srox (&
With, = Srox,
With, = X T***

0)

2

0

(

0)

2

0

() max

Spectrum of oscillator (Pseudo absolute acceleration)

4

Increasing depreciation

from 0.01 to 0.20

10 0

Depreciation

1%

5 %

10%

20%

Pseudo absolute acceleration

1

10

0.1

1

100
10
Frequency

Appear 1.2.3-a: Spectrum of oscillator in pseudo absolute acceleration

*The interest of this spectrum of pseudo acceleration lies in the fact that $S R OX ($
&*

*With, is good
0)*

approximation of the maximum of absolute acceleration $X& (T)$. Indeed, at the moment when relative displacement

*is maximum, relative speed is cancelled and the reduced equation is written $x& + 0 + 2$
 X*

*= - S what us
0*

*max
&
show that*

*2
2
 $X&$
 $= x& + s&$
 $= X$
 $= Srox,$*

*With, = S R ox&
With,*

*0
max
0
(
0)
(
0)*

*max
max*

For this reason, this spectrum of oscillator is called spectrum pseudo absolute acceleration.

The asymptote of this high frequency spectrum (acceleration at null period) corresponds to the answer

of a clean high frequency oscillator, i.e. very rigid. In this case, the mass tends to follow completely the imposed movement of the base. This asymptote thus corresponds to acceleration

maximum

(

With T)

imposed movement (ground or not of fixing of the oscillator). It is

max

attack in practice starting from the cut-off frequency of the spectrum. For this reason, one says that one

accélérogramme is fixed, for example, on 0.15 G, when its maximum amplitude and its spectrum of oscillator of pseudo absolute acceleration at null period are equal to 0.15 G.

1.3

Determination of the spectrum of oscillator

Determination of the spectrum of oscillator of a accélérogramme (

With T) is available in the operator

CALC_FONCTION [U6.62.04] with key word SPEC_OSCI: it is obtained by numerical integration of the equation of DUHAMEL by the method of NIGAM [R5.05.01]. This order provides the spectrum of pseudo absolute acceleration and, on request, the spectrum pseudo speed or the spectrum of relative displacement.

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1.4

Representation and use of the spectra of oscillators

1.4.1 Representation tri logarithmic curve

The spectra of response of oscillator are usually represented by tri graphs

logarithmic curves which make it possible to read on only one graph the three sizes: relative

*displacement,
pseudo relative speed, pseudo absolute acceleration.*

This representation is obtained by tracing the spectrum pseudo relative speed S_{rox} in co-ordinates log-log such as $\log S_{rox} = F(\log)$ on which one defers two graduations

complementary to $\pm 45^\circ$ if the scale of the graduations logarithmic curves is the same one on the two axes:

a graduation logarithmic curve with $+45^\circ$ to measure relative displacements

$$\log S_{rox} = \log (S_{rox} = \log S_{rox} + \log$$

0

)

0

.

a graduation logarithmic curve with to measure absolute accelerations

S_{rox}

$$\log S_{rox} = \log$$

&

&

$$= \log S_{rox} - \log 0$$

0

10

Speed m/s 10

103

1

Displacement m

$F = 3 \text{ Hz}$

\dot{Y}

$X f = 1$

$= 3 \text{ Hz}$

m/s

1

$x \& = 1 \text{ m/s}$

$X = \dot{Y}$

$X = 1 = 0.05305$

With

$x \&$

1

C 1

1

C

6

0 2

é

X =

=

= 05305

.

0

breadth

10

rat

6

io

Ý

N

X = Ý

X = 6 = 18.849

ms2

x& = X = 6 =

849

.

18

101

10

2

10

1

10

Frequency Hz

3

1

10

3 Hz

Appear 1.4.1-a: Representation tri logarithmic curve

1.4.2 Use of the spectra of oscillators

To evaluate the maximum response of a modal oscillator (

*,
with a accélérogramme (*

With T), one

I

I)

use the spectrum of pseudo absolute acceleration.

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It is represented in Code_Aster, by a tablecloth made up of several functions

Srox& = F (freq) with = cte

N

One uses a linear interpolation on the damping reduced for < < because amplification

N

I

N I

+

X

I

dynamics with resonance for = (that is to say = 1) is equal to m = [éq A2.2-3].

0

S

2

0

I

The variation of the module of the response in the vicinity of resonance also justifies an interpolation logarithmic curve for < <

. The spectrum of oscillator must be represented with one

m

I

m I

+

discretization in sufficiently fine frequency to limit the effects of the interpolation.

1.5

Spectra of oscillators used for studies

For the studies of industrial facilities, such as the nuclear thermal power stations, the seismic analysis conduit to establish several models:

*.
a model of the civil engineering of design of the buildings to determine:*

*-
accidental requests for the calculation of the frameworks of these buildings;*

*-
movements imposed on the points of fixing of the equipment (reactor vessel, supports of the networks of pipings, electrical equipment boxes.) at various levels of buildings;*

*.
models of study of checking of each equipment subjected to the imposed movements amplified by the dynamic behavior of the buildings.*

1.5.1 Spectrum of ground of design and checking of the buildings

This stage, the equipment is known only like inertial overloads and one can to admit that they do not bring any rigidity to the building. The structures in this case are subjected to one spectrum of ground.

The frequential contents of a spectrum of oscillator reflect that of the accélérogramme used and are thus “marked” by the properties of the ground instead of recording. To work out the spectrum of ground at the stage project, it is thus recommended to establish the spectra of oscillators for several accélérogrammes and to build a spectrum envelope which smoothes anti resonances.

Spectrum of oscillator of a accélérogramme and the spectrum of associated ground

10 0

1

Representation

tri logarithmic curve

10 1

Damping 1%

102

Pseudo_vitesse_relative

0.001

10 3

1

101

10 0

10

10 2

Frequency

Appear 1.5.1-a: Spectrum of ground for a project

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Note:

In many cases one does not know the rotational movement imposed by the seism, since the accélérogrammes of known seisms result from recordings of sismo-graphs, sensors with a degree of freedom of translation.

1.5.2 Spectrum of floor of checking of the equipment

The study of the dynamic behavior of the equipment subjected to the movements imposed by structure support at the points of supports is possible starting from the accélérogrammes of answer in these points, results of the transitory analysis of the behavior of the building: these accélérogrammes, known as of floor, make it possible to build spectra of floors.

For a checking of the equipment, one can limit oneself to a spectral analysis starting from the spectra of floor and the differential displacements imposed on the supports.

The spectra of floor are representative of the dynamic amplification brought by the structure support: a smoothing of the spectrum can be useful to take into account uncertainty on the position Eigen frequencies of the building, but one will take care to preserve realistic margins, since it spectrum of ground is already one raising of the seismic request. The spectrum of oscillator must be represented with a discretization in sufficiently fine frequency "to collect" resonances of structure.

Note:

Techniques of direct determination of the spectra of floors, starting from the spectrum of ground and modes of the structures were developed [bib1], but are not available currently in Code_Aster.

2

Seismic response by modal recombination

2.1

Recalls of the formulation

The spectral method of seismic analysis is based on the formulation of the dynamic response transient by modal recombination presented in the documents "Methods of RITZ in linear and nonlinear dynamics " [R5.06.01] and "Analyzes seismic by direct method or modal recombination " [R4.05.01].

Let us summarize the principles of the step detailed in the note [R4.05.01] for a structure represented in form discretized by the matric system:

$$MU\ddot{q} + CU\dot{q} + KUq = F(T)$$

éq

2.1-1

Notations moving absolute

U represents all the components of the movement (ddl of structure and ddl subjected to one

X

*imposed movement): one separates them in the form $U =$ describing **S. the operators***

K

K_{xs}

C

cxs

m

mxs

structure become: $K =$

C =
M =

K
K
C
C
m
m
sx
ss
sx
ss
sx
ss

*The problem moving relative of the structure compared to the supports with the decomposition Absolute movement = relative Movement + Mouvement of drive results in introducing it change of variable $U = U + E$.
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Notations moving relative

U represents all the components of the movement (ddl of structure and ddl subjected to one **X** movement imposed) which is written, with the preceding partition, $U = \mathbf{0}$. The partition of components of drive gives, by expressing the drive of the ddl of structure like one **be**

combination liéaire of the movements imposed on the supports, $\mathbf{E} = \mathbf{S}$

Assumption

It is supposed that no force of excitation is applied to the d.d.l structure what reduces it

$\mathbf{0}$

second member \mathbf{F} (T), with the same partition with $\mathbf{F} = \mathbf{R}$

The passage of the absolute movement to the relative movement can be written by introducing the operator of

passage:

\mathbf{X}

$\mathbf{X} \mathbf{b} \mathbf{e}$

\mathbf{X}

$\mathbf{I} \mathbf{E}$

$\mathbf{U} =$

$\mathbf{U.E.}$

with

$\mathbf{S} = + = \mathbf{0} + \mathbf{S} =$

\mathbf{S}

$= \mathbf{0} \mathbf{I}$

The system [éq. 2.1-1] takes the general form then:

$\mathbf{x} \&$

$\mathbf{x} \&$

\mathbf{X}

$\mathbf{0}$

\mathbf{T}

$+ \mathbf{T} \mathbf{C}$

$+ \mathbf{T} \mathbf{K}$

$= \mathbf{T}$

\mathbf{M}

éq

2.1-2**s&****s&****S****R****2.1.1 Single imposed movement: mono support**

The movement of drive corresponds then to a movement of solid body: the vector of drive in any point of the structure can express like a linear combination $R S$ components of displacement imposed on the centre of gravity of the foundation, where the R are stamp modes of bodies rigid of the structure reduced to the ddl of structure, which leads to:

X**X****R S** **$U = S = 0$** **+ S****I****=****R****0****I**

The properties of the modes of rigid body cf [R4.05.01] lead to:

K 0**C 0****m****m + m****R****T K =****T C =**

T M

xs

= T

0 0

0 0

m + m

m

R

sx

R

what makes it possible well to uncouple the system [éq 2.1-2]

m x&+c x&+k X = - m (

+ m

m

l

R

xs) S

& éq

2.1.1-1

In this case the transformation highlights well the inertial effect of the seismic loading.

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2.1.2 Multiple imposed movement: multi supports

This situation corresponds to a discrete number of points of connection of the structure in supports subjected to different imposed displacements. In this case, the diagonalisation of the term of rigidity is acquired while imposing:

 K **$K 0$** **x_s** **S** **$=$** **$of or K + K$** **$= 0$** **$that is to say = - K^{-1} K$** **$éq 2.1.2-1$** **$K$** **$K$** **$I$** **$R$** **$S$** **$x_s$** **$S$** **$x_s$** **$s_x$** **$s_s$**

The matrix S gathers 6. nappuis static modes for the models of structures and 3 times the number supports for the models of continuous mediums. Each static mode

 I **$S = -$** **$-$** **K** **K** **J** **x_s is a mode** **J**

of fastener, corresponding to a unit displacement imposed on a component of support, the others

null components, and being produced by operator $MODE_STATIQUE$ [U4.52.14].

The change of reference mark can then be expressed by:

X

X

X

S S

U = S = 0

+ S

X

I

S

= 0 I

S 1

S 2

m

m

S + m

T M =

xs

TS m+ msx

ms

Concerning the terms of damping, decoupling is acquired only if damping is proportional to rigidity, usually allowed assumption, but was not necessary with the modes rigid.

This makes it possible well to uncouple the system [éq 2.1-2]

$m\ddot{x} + c\dot{x} + kx = m$

- (

m m

I

-

+

S

xs) s&

éq

2.1.2-2

This formulation must be interpreted like the decomposition of the movement of the structure in one movement of drive corresponding to an instantaneous static deformation (displacement differential of the supports) and a relative movement corresponding to the inertial effects around this new static deformation.

This interpretation is in conformity with the classification of the requests defined by the rules of construction (ASME, RCC-M):

the constraints induced by the relative movement are, as for the static stresses, primary constraints (effects of inertia),

constraints induced by differential displacements of the supports which are they classified in secondary constraints.

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2.1.3 Summary

The equations [éq 2.1.1-1] and [éq 2.1.2-2] lead to the general form

$m\ddot{z} + c\dot{z} + kz = m\ddot{z}_1 + m\ddot{z}_2$

$= m\ddot{z}_1 + m\ddot{z}_2$

1

X $xs) s\& = -$ S O $m \&$

éq

2.1.3-1

The m_{xs} terms correspond under the terms of coupling of the matrix of mass with the degrees of freedom of support: this fraction of the total mass is very weak and it is justified to neglect it. Let us recall

that this term is indeed null for the models of structures whose matrix of mass is diagonal: models masses - arises, models with elements of the type "lumped mass".

In this case, one obtains the simplified formulas:

.

mono support: $O =$

where

 R

R are the six modes of solid body

.

multi supports: $O =$

where

 S

S are 6 N supports modes of fastener

The second member - $m O$ is built by operator $CALC_CHAR_SEISME [U4.63.01]$.

2.2

Response in modal base

2.2.1 Temporal response of a modal oscillator

If the studied structure is represented by its spectrum of low frequency real clean modes in embedded base, solution of (

2

 $K -$ M

) = 0 or of (

2

$K - m$) = 0 one can introduce one

new transformation $X = Q$ and the system of equations [éq 2.1.3-1] is written, by using the matrix modal factors of participation P

T C

2

T Mo

q& +

Q

éq

2.2.1-1

T

& + Q = -

S

T

& = - Ps&

m

m

Assumption:

For industrial studies concerned with the seismic analysis by spectral method, one limits oneself with the case of damping proportional, known as of RAYLEIGH, for which one can diagonaliser it

T C

term

=

2. Damping is then represented by a modal damping

T m

I,

possibly different for each clean mode [R4.05.01].

Each clean mode, characterized by the parameters (,

I

I) is compared to a simple oscillator

whose behavior is represented in the case general by

q& + q& + 2

2

Q = - PS éq

2.2.1-2

I

I

I

I

I

I

() &i

Let us recall that the \ddot{u}_g are accelerations of drive.

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2.2.2 Modal factor of participation in mono support

When the movement of drive is single, [éq 2.2.1-2] becomes

$$q_{\ddot{u}_g} + q_{\ddot{u}_g} + 2$$

2

$$Q = - p S$$

éq

2.2.2-1

I

I

I

I

I

I

I &

with

T

T

Mo m

I

I

P

R

=

=

éq

2.2.2-2

I

T

m

μ

I

I

I

where μ_i is the generalized modal mass, which depends on the standardization of the clean mode. Let us state

some properties of the factors of modal participation p_i in the case of rigid modes of translation, but extensible with the modes of rotation.

.

A mode X-ray, that we will note X, to recall that components in the direction X are unit, belongs to the space of dimension NR degrees of freedom of which NR clean modes constitute a base in which =

X

I I.

I

From the properties of orthogonality of the clean modes $T m = \mu$

I

I

I ij, one identifies them

coefficients I with the factors of modal participation p_{iX} in direction X and

= p

X

iX

I

éq

2.2.2-3

I

.

Moreover T m

X

X =

T

m masses total structure what leads to:

2

$p \mu$

T

T

2

2

m

=

$p p$

m

$p \mu$ and m

$p \mu$ or

éq 2.2.2-4

X

X

=

iX

jX

J

I

=

iX

I

T

$iX I iX I = I$

ij

ij

ij

ij

MT

Modal parameter p_i depends on the standard of the clean mode and is accessible, for each mode

X

clean in the concept result of the type mode_meca [U5.01.23] under name FACT_PARTICI_DX; of even $p_{2iX I}$

μ , independent of the standard, is accessible under the name of MASS_EFFE_UN_DX.

2.2.3 Modal factor of participation in multi supports

For a multiple imposed movement, [éq 2.2.1-2] becomes:

$$q_i + q_j + 2$$

2

$$Q = -$$

$p S$

$\dot{e}q$

2.2.3-1

I

I

I

I

I

I

$i_j \& j$

J

with

T

T

Mo m

I

I

$S J$

$p =$

$=$

$\dot{e}q$

2.2.3-2

i_j

T

m

μ

I

I

I

where μ_i is the generalized modal mass, which depends on standardization on the clean mode and the p_{ij} can be regarded as factors of participation relating to mode I and a direction J of imposed movement of a support.

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As previously, one can establish [bib4] the two properties

= p and T m = p2 μ

S J

ij

I

S J

S J

ij

I

éq

2.2.3-3

I

I

One makes, at this stage, no assumption of dependence between the various pij terms. Let us recall that the components &sj express the acceleration of drive applied to a direction of support J. The factors of pij participation are not built independently and only appear like intermediate variables in order COMB_SISM_MODAL [U4.84.01].

3

Seismic response by spectral method

The spectral method is an approximate technique of evaluation of the maximum of the answer of structure starting from the maxima of answer of each modal oscillator read on the spectrum of oscillator of the excitation.

3.1

Spectral response of a modal oscillator in mono support

The maximum response in relative displacement of a modal oscillator (,

I

I) for a direction X is

determined by reading on a spectrum of oscillator of pseudo absolute acceleration cf [§1.4.2] the value

pseudo absolute acceleration has

= Sro x& (A

2

iX

X

, I, I

) and while dividing by I from where:

Sro x&X (A, I, I

) has

Q

iX

iX max =

=

2

2 éq

3.1-1

I

I

The contribution of this oscillator to the relative displacement of the structure for the component xk depends on the factor on participation and component K

I in physical space:

has

xk

K

= p Q

K

= p

iX

iX max

I

iX

iX max

I

iX
I
2

éq
3.1-2

and the contribution to the pseudo absolute acceleration &xk is of the same &xk

K
max =
p has
iX
I
iX
iX.

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3.2
Spectral response of a modal oscillator in multi supports

One proceeds in the same manner to determine, starting from the value read &S jX on the spectrum of oscillator of pseudo absolute acceleration associated &sj, the contribution of the support J in direction X:

Sro s&j (A, I
, I
) S&
Q
jX

$iX \max J =$

=

2

2

éq

3.2-1

I

I

The expression of the contribution of this oscillator to the relative displacement of the structure for component x_k in physical space and for a movement imposed J becomes:

S&

x_k

K

= p

Q

K

= p

jX

$iX \max J$

I

ijX

$iX \max J$

I

ijX

I

2 éq

3.2-2

3.3

Generalization with other sizes

Note:

The method of spectral analysis is strictly limited to the sizes depending linearly on displacements in linear elasticity: generalized strains, stresses, efforts, nodal forces, reactions of supports.

In particular it cannot apply to equivalent sizes of deformation or of constraints (Von Mises).

For each R_k size, component of a field by elements it is possible to calculate modal component $R K$

I associated with the clean mode I what leads to

has

Rk

= rk p Q

= rk p

iX

iX max

I

iX

iX max

I

iX

I

2

éq

3.3-1

or

S&

Rk

= rk p

Q

= rk p

jX

iX max J

I

ijX

iX max J

I

ijX

I

2 éq

3.3-2

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4

Rules of combination of the modal answers

To evaluate one raising of the answer R of the structure, one must now combine the answers modal R_{kimax} defined previously. Several levels of combination are necessary:

- combination of the clean modes selected,***
- static correction by pseudo mode,***
- effect of the excitations different applied to groups from supports,***
- combination according to the directions of excitation seism.***

4.1

Direction of the seism and directional answer

Various considerations result in separately studying the seismic behavior according to each direction of space:

• for the study of a building on a ground, the accélérogramme of the vertically imposed movement is different from that describing the horizontal movement, him even different according to two orthogonal directions of space;

• for the study of equipment, the spectra of floor differ significantly according to three directions of space, since they integrate the participations of various modes of building (inflection of floors, inflection or torsion of the framework.).

This resulted in establishing a directional modal answer X-ray starting from spectra of oscillator different and from factors of modal participation established in each direction X representative one from directions of the TOTAL reference mark of definition of grid (X, Y or Z) or a particular direction defined explicitly by the user.

4.2

Choice of the modes suitable to combine

To correctly represent the modes of deformation likely to be excited by imposed movement, it would be necessary to know all the clean modes of frequency lower than cut-off frequency of the spectrum, beyond which there is no dynamic amplification significant. This condition can prove to be difficult to fill for the complex structures having one large numbers of clean modes.

The size of the modal base necessary must thus be evaluated to make sure that no mode having one important contribution in the internal efforts and the constraints was not omitted in each studied direction.

4.2.1 Expression of the modal deformation energy

1
The deformation energy associated with each clean mode U
T
 $I =$
 $\sum_{xi} K X$
 \max
 I
 2
max can be expressed
for a particular direction

2
 2
 1
has
 1
has
 $1 a^2$
 U
 $= p iX$
 T
 $K = p$
 iX
 2
 iX
 $\mu =$
 p^2

iX
I
μ
2
I
I
iX
2
I
I
2
iX
I
éq
4.2.1-1
2

2

2
I

I
I

This expression corresponds to an excitation mono support and can extend to the case from the multi supports.

The classification of the modes with decreasing deformation energies makes it possible not to retain systematically, for a general study of the structure, modes which do not produce significant deformations. On the other hand, for the study of the effect of the requests in a zone particular of the structure, it will be necessary to use the “local” modes which can be detected by an analysis of the distribution of the deformation energy on groups of mesh.

Let us note that one does not have an estimate of the total deformation energy to quantify the error made by being unaware of certain modes.

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4.2.2 Expression of the modal kinetic energy

l
The kinetic energy associated each clean mode is written V
T
I =
ximax m xi
who gives
2 &
& max

l
has 2
l a2
V
= p
iX
T
iX
m =
p2
iX
iX
I
I
μ
2
iX
I
éq
4.2.2-1
2

I
2 I

The expression [éq 4.2.2-1] utilized effective modal mass p_{2iX} I μ defined in [§2.2] what allows to state the criterion of office plurality of the unit effective modal masses [éq 2.2.2-4]

Criterion of office plurality of the effective modal masses

The quality of a modal base, the point of view of the representation of the inertial properties of structure, is evaluated by cumulating, for this direction, the unit effective modal masses modes available. A threshold of admissibility of 95% of the total mass is usually allowed. The same criterion can apply partially in the case of an excitation multi supports with N

T
N
modes while comparing

m
and

p^2

μ

S J

S J

ij

I.

I

Estimate of the error made with an incomplete modal base

The criterion of office plurality of the effective modal masses cannot always be satisfied. Indeed one limit in general at a modal base of N clean modes with N modes $\ll NR$ ddl. For rigid foundations, the spectrum of the Eigen frequencies necessary usually exceeds the frequency of cut of the spectrum of oscillator.

From the expression [éq 4.2.2-1] one can write the total kinetic energy in the form

N

NR

$V = V + V$

X

iX

iX

I

I

who allows to express the absolute error from [éq 3.1-1]

NR

NR

2

2

has

has

NR

(+

2

iX

2

N I) X

2

V = V =

p μ

p

X

iX

μ

2

iX

I

2

iX

I

NI

+

NI

+

I

I

NI

+

while noting has

= Sro X

(

)

I

& (A

N

X

X

, m

in,

+

N

+1) the value read on the spectrum of pseudo acceleration absolute for

N

n+1 and the modal damping weakest min likely to give

the raising amplitude. If the maximum frequency of the base f_n exceeds the cut-off frequency then has

= has

= (

With T)

($n+$)

$I X$

nX

max. This gives one raising of the absolute error

$1 a^2$

NR

2

(

$1)$

$1 A$

N

$n+ X$

$(n+1) X$

V

2

2

μ

μ

2

p

=

m -

2

p

X

iX

I

T

iX

I

éq

4.2.2-2

2

2

I

n+I

I

1

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4.2.3 Conclusion

The sizes allowing for choice of the modes necessary to each analysis are available in Code_Aster (operator POST_ELEM with options MASS_INER, ENER_POT and ENER_CIN and modal parameters FACT_PARTICI_DX and MASS_EFFE_UN_DX in the concept result of the type mode_meca [U5.01.23]).

No criterion of automatic admissibility is currently programmed and the sizes

N

T m and p2 μ

Sj

Sj

ij

I, necessary to the checking of the criterion for an excitation multi supports,

I

are not printed.

4.3

Static correction by pseudo-mode

4.3.1 Mono

support

The evaluation of one raising of the response to a seismic excitation requires, as suggests it preceding analysis, a correction by a term representing the static contribution of the modes clean neglected.

If one subjects the structure to a quasi-static constant acceleration in direction X, the answer aX is solution of $K = m I$

aX

X

, without dynamic amplification. The field of displacement aX of the nodes of the structure subjected to a constant acceleration in each direction is product by operator $MODE_STATIQUE$ [U4.52.14] with key word $PSEUDO_MODE$.

By breaking up this deformation on the basis of clean mode one obtains cf [§2.2.2]

NR

NR

NR p

K

= m p from where = k-1 m p

iX

=

aX

iX

I

aX

iX

I

2 I

I

I

I

I

This makes it possible to introduce a pseudo-mode cX , for each direction, while withdrawing from the mode quasi

statics has X the static contributions of the modes used I

N p

=

iX

-

cX

aX

éq

4.3.1-1

1 2

I
I
N

The expression [éq 4.3.1-1] is homologous with the term m -

p μ
2
T
iX

I of [éq 4.2.2-2] and the pseudo one

I

mode makes it possible to supplement the incomplete base of clean modes to introduce a correction of static effects of the neglected modes. The contribution of the pseudo mode is the value read on the spectrum

pseudo absolute acceleration has

= Sro X

(
)
& (A
N
X
X
, m
in, N

),
+
+
for
N

I
I
N
+I and
the modal damping weakest min.

Correction to be brought to relative displacements and the sizes which result (efforts from them generalized, forced, reactions of supports) in excitation mono support is then K

K
X

= *has*

cX

cX

(n+)

1 X

in accordance with the conditions of estimate of the error cf [§ 4.2.2].

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For the evaluation of the correction of absolute acceleration one obtains:

K

N

K

x& = -

cX

X

piX I has (n+) 1X

1

4.3.2 Multi

supports

In excitation multi supports, the formulation of the pseudo mode and its contribution take again the principle precede.

The field of displacement

K I

-

=

m

nodes of the structure subjected to an acceleration

ajX

SjX

unit of the support J in direction X is produced by operator MODE_STATIQUE [U4.52.14] with key word PSEUDO_MODE.

The correction to be brought to relative displacements and the sizes which result some writes then for the support J in direction X:

N P

X

ijX

I

cjX = cjX has (n+)

I jX with

=

-

cjX

ajX

2

1

I

For absolute acceleration, the correction is written:

N

&

=

-

cjX

X

SjX

I ijX

P

(n+) I has jX

I

4.4 ***General information on the rules of combination***

Rules of combination or office plurality of the various components, modal or directional, are multiple and more or less complexes to be implemented.

One presents the methods “natural” from the point of view of their aptitude required one raising realistic requests induced in a structure represented by a base of real clean modes resulting of a model in linear elasticity, raising estimated without transitory analysis for a size of G_k component, which one will name G_{kmax} . For the continuation the suffix max indicates the estimate of maximum value attack during the seismic excitation, by being unaware of the moment when it was reached) and the index R applies to clean modes, pseudo modes, directions of supports,...

Note:

Whatever the method of combination used, the value of a component obtained by combination cannot be used as data to calculate a new size: for example calculation of one raising of a differential displacement between two points must be calculated mode by mode, then compound.

4.4.1 Combination arithmetic

**G_k
= G_k
max
 R max
 R**

It is not usable since the spectral method disregards moment when values maximum are reached in two directions or for two different modes. No relation of phase, and thus of sign, does not exist between the contributions to combine. They is thus available only in the case of differential displacements in multi support.

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4.4.2 Combination in absolute value

G_k

= G_k

max

R_{max}

R

In an obvious way, it can provide an upper limit, since it supposes that all them contributions reach their maximum at the same moment with the same sign. Too much penalizing, it is available, but unusable industrially.

4.4.3 Simple quadratic combination

This method is also known under denomination SRSS (Public garden Root of Sum of Squares).

2

$G_{kmax} = \sqrt{(G_{krmax})^2}$

R

Assumption:

The assumption which justifies this method of combination can be stated: the probable maximum of the energy stored in the structure is the sum of the maxima probable of the energy stored on each mode and each component directional of the seism, i.e. with respect to energy, the clean modes and them components of the seism are uncoupled. They is similar to the rule of addition of the variables

random Gaussian and with null average.

Validity of this assumption, which will be discussed for each particular case of use of this method of combination, is not established and various proposals were presented to obtain a better approximation whenever it is put at fault cf [§ 3.4.1.2] following.

In addition, one will be able to refer to [bib3] for a criticism of this approach, in particular of sound aptitude to consider a maximum probable of the deformations and constraints, but the approach alternative which it evokes was the subject of any development in Code_Aster.

4.5

Establishment of the directional response in mono support

The directional answer, previously definite, is obtained by simple quadratic combination of two terms which we will discuss:

$$R = R_1 + R_2$$

X

m

C

R_m answer combined of the modal oscillators

R_c contribution of the static correction of the neglected modes (pseudo mode)

The assumptions justifying the method of quadratic combination simple, on this level, do not seem not to have to be called into question [bib1]. To simplify the notations one notes R_m instead of R_{mX},

...

4.5.1 Combined response of the modal oscillators

The response of the R_m structure, in a direction of seism, is obtained by one of the combinations possible of the contributions of each clean mode taken into account for this direction.

The number of possible methods proves simply the difficulty in releasing a justification sufficient to guarantee a conservative and realistic estimate. If simple quadratic combination (SRSS or CQS) is evoked by all, one will retain [bib1] that it is often put at fault and one he will prefer the complete quadratic combination (CQC). The other methods are available for possible comparisons.

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4.5.1.1 Summons absolute values

This combination corresponds to an assumption of complete dependence of the oscillators associated with each mode clean and led to a systematic overvaluation of the answer:

N
R =
R
m
I
I

Simple 4.5.1.2 quadratic Combination (CQS)

By considering that the contribution of each modal oscillator is a random variable independent, an estimate of the maximum answer, for the component of x_{kmax} displacement, can be obtained by simple quadratic combination of the contributions of each mode from where, for an excitation mono support:

N
2
N

2
xk
= xk
K

max
I max

=

(p Q
I
I I max)
éq
4.5.1.2-1
I
I

Generally, for any size IH associated with a modal oscillator (,) I
I.

N
R = R2
m
I
I

It constitutes a good approximation of reality when the spectrum of oscillator defining it seism is with broad frequency band and where the clean modes of the structure are quite separate them from/to each other and are located inside or in the vicinity of this band. It is in particular put in defect if clean modes are at close frequencies or for distant modes peak of excitation. [bib2]. The other methods of combination of the modal answers try to correct this point.

Quadratic 4.5.1.3 Combinaison supplements (CQC)

The quadratic combination supplements (established by DER KIUREGHIAN [bib5]) makes a correction to the preceding rule by introducing coefficients of correlation depending on depreciation and distances between close clean modes:

R = R R
m
I I
I
I
1 2
1
2
I
I
1

2
with the coefficient of correlation:

$$\begin{aligned}
& 8 \\
& (+) \\
& I J I \\
& J \\
& I I \\
& J \\
& J \\
& I \\
& J \\
& ij =
\end{aligned}$$

$$\begin{aligned}
& 2 \\
& (\\
& - 2 2 \\
&) + \\
& 4 2 \\
& (\\
& + 2) + 4 2 \\
& (\\
& + 2 \\
&)
\end{aligned}$$

$$\begin{aligned}
& 2 2 \\
& I \\
& J \\
& I J I \\
& J \\
& I \\
& J \\
& I \\
& J \\
& I \\
& J
\end{aligned}$$

or by introducing the report/ratio of pulsation or frequencies between two modes =/
J
I

8

(+)

I J

I

J

ij = 1

(- 2 2

) +

4 1

(+ 2) +

4 2 2

(

+ 2

I J

I

J)

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and for constant

8 2 1

(+)

ij =

1

(- 2 2

) + 4 2 1

(+ 2) + 8 2 2

4.5.1.4 Combination of ROSENBLUETH

This rule (proposed by E. ROSENBLUETH and J. ELORDY [bib6]) introduced a correlation enters modes, different from that of method CQC. The answers of the oscillators are combined by double nap (Double Sum Combination):

$$R_m = R R$$

I

I 2i li 2i

I

I

2

I

It requires an additional data, the duration S of the “strong” phase of the seism. The coefficient of correlation is then:

-1

' - '

2

2

I

J

= I +

where '

I 2 ' and '

ij

' + '

I

I

I

I

I

=

-
=
+ S
I
I
J
J
I

4.5.1.5 Combination with rule of the 10%

The close modes (of which the frequencies different from less than 10%) are initially combined by summation of the absolute values. The values resulting from this first combination are then combined quadratically (simple quadratic combination). This method was proposed by American payment U.S. Nuclear Regulatory Commission (Regulatory Guides 1.92 - February 1976) to attenuate the conservatism of the method of nap of the absolute values. It remains at fault for structures with an own frequency spectrum dense and should not be used any more.

4.5.2 Contribution of the static correction of the neglected modes

The contribution of the pseudo mode cf [§4.3.1] can be combined quadratically because independence with the contributions of the modes of vibration is not disputed.

4.6

Establishment of the directional response in multi supports

4.6.1 Calculation of the total answer

In this case, one retains like order of the combinations a step similar to that retained for the excitation mono support without that being completely justified.

One establishes the directional answers for each movement &sj applied: one will note RjX it result of this combination. To obtain this directional answer a news will thus be needed stage of combination by taking account of the dependence or the independence of the &sj.

The flow diagram for treatment becomes:

for each movement imposed &sj calculation of the directional answers

$$R = R_2 + R_2 + R_2$$

***jX
mj***

cj
ej

Rmj answer combined of the modal oscillators
Rcj contribution of the static correction of the neglected modes (pseudo mode)
Rej contribution of the movement of drive of the support J

.
combination of the answers R jX
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4.6.2 Separate calculation of the components primary education and secondary of the answer

Each component is the subject of a similar separate treatment. This step is adapted to the post treatment RCC-M in force for the seismic analysis of pipings [§ 4.9]:

primary component RIX (inertial answer):

.
for each movement imposed &sj calculation of the directional answers

R
= R2 + R2
I J X
mj
cj

Rmj answer combined of the modal oscillators
Rcj contribution of the static correction of the neglected modes (pseudo mode)

combination of answers IH J X

secondary component RII (quasi static answer):

combination of the Rej answers

4.6.3 Combined response of the modal oscillators

The response of the Rmj structure, in a direction of seism, is obtained by one of the combinations possible of the contributions of each clean mode taken into account for this direction. The selection criterion of the method of combination of the contributions of the modes is the same one as for an excitation mono support and one will use method CQC preferentially.

4.6.4 Contribution of the pseudo mode

The corrective term by pseudo mode cf [§4.3.2] can be combined quadratically.

4.6.5 Contribution of the movements of drive

The movement of drive of the structure not being uniform, one can add a term with calculation directional answer. This is not necessary if one chooses to consider this contribution statics like a specific loading case inducing of the secondary constraints. This term is defined starting from the maximum relative displacement which cannot be known starting from the only spectra of pseudo absolute acceleration of the supports.

*Rej = J
S
J max*

*Sj
static mode for the support J
jmax maximum relative displacement of the support J compared to a support of reference
(for which J max = 0)*

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4.6.6 Combination of the directional answers of supports

This stage is obligatory, but the choice of the method of combination of the directional answers remain very open. Indeed the assumption of independence of the &s J strongly depends on the clean modes

structure support of the studied equipment. An analysis of the studied system is necessary for to gather the supports by groups: for examples for a piping connecting two buildings, the group supports of the building 1, that of building 2 and finally that of the intermediate stanchions.

For each group one will be able to choose one of the three methods:

Combinaiso quadratiqu

N
E:
R =
2
X
RjX

Linear Combinaiso

N
:
R =
X
RjX

Combinaiso value

in

N

:

absolute

$R =$
 X
 RjX

The rule of combination can be the same one for all the supports or differentiated according to the supports or groups of supports defined by an occurrence of the key word factor COMB_MULT_APPUI. In this case total answer is obtained by:

2
 2
 $R =$
 $R^2 +$
 $R +$
 R

X
 $Q X$
 L
has
 Q
 L

has

where Q supports combined quadratically, L linearly combined supports, has supports combined in absolute value.

4.7
Combination of the directional answers

Two rules of combination of the directional answers are available.

4.7.1
Combination quadratic

This combination corresponds to the assumption of strict independence of the answers in each direction cf [§ 3.3.3]. Let us recall that this rule of combination does not have any geometrical

*significance,
although the three directions of analysis are orthogonal.*

$R = R_2 + R_2 + R_2$
X
Y
Z

The assumptions justifying the method of quadratic combination simple, on this level, do not seem not to have to be called into question [bib3], but this method is not used.

4.7.2 Combination of NEWMARK

This rule of empirical combination is most usually used and in general leads to estimates slightly stronger than the preceding one. It supposes that when one of the answers directional is maximum, the other are with most equal to the 4/10 their maximum contributions respective. For each direction I (X, Y, Z), one calculates the 8 values:

$R = \pm R \pm 0.4$
 $, R \pm 0.4$
 $, R$
L
X
Y
Z

What leads, by circular shift, with 24 values and $R = \max (R_l)$

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4.8

Warning on the combinations

Several remarks are essential to warn the user on the way of using the methods of combination and sizes combined in a note of study.

Notice 1:

If one wishes to use arithmetic combinations (direction) and combinations quadratic (modes) the quadratic office pluralities must be always carried out in the last.

Notice 2:

Any quadratic combination applies only to the sizes for which in values instantaneous the office plurality has the direction of a sum: combination of the components of displacement, or effort generalized or of constraint of each clean mode. The modal or directional quadratic combination cannot thus apply to intensities of constraint (constraint principal, of Von Mises, Tresca).

Notice 3:

The results of a combination, whatever the rule of office plurality, should not be used as data to calculate other sizes: for example a differential displacement between two points (or a deformation) can be calculated only starting from modal differential displacements that one combine then.

A fortiori the generalized efforts and the constraints can be calculated only mode by mode before any combination and not starting from inertias deduced from the fields from acceleration obtained by combination of modal accelerations.

4.9 Practical lawful

4.9.1 Partition of the primary and secondary components of the answer

The various supports of a line of piping can be animated different movements. One even section of piping can be left again on different buildings, levels or different equipment. It thus undergoes a multiple excitation. This results in two types of loading [§ 2.1.2]:

an excitation whose frequential contents vary from one support to another and who constitutes one primary education loading according to classification RCC-M,

seismic Differential Displacements (DDS) inducing a state of stress by displacements imposed on the supports and classified like secondary.

The generalized moments resulting from these 2 loadings intervene separately in inequations of dimensioning RCC-M and on several levels. Thus, for pipings of levels 2 and 3, the DDS not taken into account with the inertial seismic loading in inequation 10, are cumulated with the cases of displacements of thermal anchoring of origin in inequation 7. In the sight of a post deepened treatment RCC-M, It is thus necessary to have the components primary education and secondary of the seismic answer.

In a more general way, the method of combination of the answers of supports can differ according to whether one treats the case of the inertial or differential components. Moreover the number of support

concerned with these two summations can not be equal. One is caused to often to impose overall differential movements even for supports associated with spectra users different. In addition, of the DDS formulated in rotation are sometimes to consider. They cannot be associated an inertial loading (limited to the translations).

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Code_Aster thus proposes two treatments:

Determination of the total answer:

The contributions inertial and static of drive are cumulated during calculation of directional answers of support [§4.6].

• Partition of the primary and secondary components of the total answer:

The two contributions précédentes are not cumulated any more during the calculation of the answers

directional and are the subject of 2 independent treatments:

The inertial component is obtained by removing the term of drive $Re J$ in calculation of the total answer [§ 4.6].

*The static component is given by combining the terms of drive defined under key word **DEPL_MULT_APPUI**. Methods of combination of these loading cases **DDS** are indicated in key word **COMB_DEPL_APPUI**.*

4.9.2 Method of the spectrum envelope

*Even if pipings are subjected to a multiple seismic excitation, the current practice is of to be reduced to the calculation of a structure mono-supported while preserving the loading cases **DDS**.*

*This simplified step implies to define a single spectrum by direction for all the supports of piping. For each direction, one adopts a spectrum then “wraps” various spectra with the supports. The spectra retained for the horizontal directions **X** and **Y** are identical. In almost the whole of the cases, this method is generating of “margin of dimensioning”.*

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Transitory appendix 1 Response of a deadened simple oscillator

A1.1 Vibration forced of a system to a d.d.l in translation

For a simple oscillator of rigidity K, mass m and viscous damping C, the equation of the movement is form:

$$m\ddot{X} + c\dot{X} + kX = 0$$

for which the traditional notations are:

K
the own pulsation of the system not deadened:

$\omega_0 = \sqrt{k/m}$
critical damping:

$C_c = 2m\omega_0$
the amortissement réduit:

$\xi = \frac{C}{C_c}$
 $\xi = \frac{C}{2m\omega_0}$
(expressed as a percentage damping criticizes)
 C criticizes $2m\omega_0$

the own pulsation of the deadened system:

$$\omega =$$

2

0

0

(1 -)

F

the static deflection for a force F:

0

0

$$St = K$$

the reduced frequency:

$$= 0$$

reduced equation of the movement:

$$X'' + 2 X' + 2$$

0

$$0 X = 0$$

The total response to a harmonic excitation of the form $F(T) = F \cos(T)$

0

) is the sum:

.

of a free answer $X(T)$

L(0) deadened oscillatory general solution where $X(0)$ and

0

are given

by the initial conditions

$$X(T) = X$$

- T

E

$$0 \cos ($$

S T

L

10

$$0 + 0$$

)

.

of a forced answer X

T
F () permanent particular solution X (T) = X
cos (T
F
f0
-)

F

C

X

0

=

= arctg

f0

2

éq A1.1-1
K - m2 2 + (C) 2

K - m

(

)

who is written in reduced form:

X

K X

f0

f0

1

2

=

=

= arctg

éq A1.1-2

F
St
0
(
2
2
2
1 - 2) + (
2)
1-

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Harmonic response of a system to 1 ddl: modulate

30
Modulate
28
displacement
X relative/X static 24
function of
reduced frequency
20
16
Depreciation
20 %
12
10 %
05 %

Relative amplitude

02 %

8

01 %

4

0

0

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

1.6

Reduced frequency

Harmonic response of a system to 1 ddl: phase

180

Phase of

displacement

160

X relative/X static 140

120

100

Depreciation

20 %

Phase 80

10 %

05 %

60

02 %

01 %

40

20

0

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

1.6

Reduced frequency

Appear A1.1-a: Response of an oscillator in imposed force (module and phase)

The response to a harmonic excitation of the form $F \cos(\omega t)$

$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = F \cos(\omega t)$

()

ω

with a forced answer solution is written

$x(t) = X \cos(\omega t - \phi)$

particular permanent $X(\omega) = \frac{F}{\sqrt{(K - m\omega^2)^2 + (C\omega)^2}}$

F

ω

F

C

X

ω

$=$

$= \arctg \frac{C\omega}{K - m\omega^2}$

ω

2

éq A1.1-3

$K - m\omega^2 + (C\omega)^2$

$K - m$

(

)

who is written in reduced form:

$X(\omega) = \frac{F}{\sqrt{(K - m\omega^2)^2 + (C\omega)^2}}$

1

2

$=$

H

2

$(J) = \arctg$

éq A1.1-4

F

2

0

$1 - + J 2$

$1-$

where $H (J)$ is the harmonic answer complexes of a simple oscillator

1

$H (J) = ($

2

$1- 2) + ($

$2) 2$

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Appendix 2 Movement imposed of a system on a d.d.l in translation

A2.1 absolute Movement of a system to a d.d.l.

For a simple oscillator of rigidity K , mass m and viscous damping C , the equation of the movement absolute is form:

$$m\ddot{X} + (c\dot{X} - s\ddot{X}) + K(X - S) = 0$$

$$m\ddot{X} + c\dot{X} + kX = ks + cs\ddot{X}$$

$$0 \ddot{X} + X = S + 2S$$

$$0 \ddot{X} + F X = X$$

The response forced to a harmonic imposed movement of the form $S(T) = S \cos(T)$

is form $X(T) = X \cos(T)$

m
 $m0$

2) nap of two terms of answer, particular solutions permanent:

term induced by the excitation in displacement X

$\cos(T)$
 $d0$
 $- D$
)

$K S$

C

X

0

=

= arctg

d0

D

2

K - m2 2 + (C) 2

K - m

(

)

.

term induced by the excitation of speed X

cos (T

vo

- v

)

C S

C

X

0

=

= arctg

v0

v

2

(K - m2) 2 + (C) 2

K - m

what leads to a total forced answer

K 2 + C 2

X (T) = X cos (T - -)

()

S

[

cos T

m

m

--

1

2

0

1

2

(K - m²)² + (C)²]

(

)

from where the reduced form of the absolute amplitude:

2

X

1 + 2

2

1

m

(

)

=

= arctg

= arctg

S

2

1

2

2

0

[1 (- 2 2

) + (2)]

1-

2

If the movement imposed on the base is expressed in form S T complexes =

(S J T

() Re 0 E), the relative amplitude or transmissibility can be written starting from the harmonic answer complexes of an oscillator simple H (J)

$$X_m = \frac{2}{1 + (2) H (J)}$$

éq A2.1-1

*s0
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A2.2 Movement relating of a system to a d.d.l.

The problem of the response to an imposed movement can be dealt with in relative displacement of the mass by report/ratio at the base by posing $X = X - S$.

The equation of the relative movement for a harmonic imposed movement of the form $S (T) = S \cos (T - 0)$ is then of the form $m \ddot{x} + C \dot{x} + K X = - m \ddot{s}$ or in reduced form:

$$\ddot{X} + 2$$

$$x + 2X = -s = 2$$

$$\begin{pmatrix} S \\ \cos T \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ \acute{e}q A2.2-1}$$

The relative forced answer is then, for a permanent solution X ($T = m_0 \cos \omega t$),

$$m_2 S = C X = \frac{m_0}{2} = \arctg \frac{m_0}{2}$$

$$\text{\acute{e}q A2.2-2} \quad (K - m_2 \omega^2) + (C \omega)^2$$

$$K - m$$

who is written in reduced form:

$$\frac{X}{m_0} =$$

éq A2.2-3

s0

(- 2 2

) + () 2

1

2

Harmonic response of a system to 1 ddl: relative module

24

21

Modulate displacement

X relative/X static

18

function of

the reduced frequency

15

12

Depreciation

20 %

9

10 %

5 %

2 %

6

1 %

Relative amplitude

3

0

0.0

0.2

0.4

0.6

0.8

1.0

1.2

1.4

1.6

Reduced frequency

Appear A2.2-a: Response of an oscillator moving imposed (module of relative displacement)

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Appendix 3 Movement imposed not periodical of a system on one d.d.l.

The problem dealt with previously was limited to a periodic imposed movement. For an excitation not periodical, of variable amplitude with time, being exerted for one finished length of time.

A3.1 Impulse response

The simplest form is the unit impulse force, which applied to a rest mass front the application of the impulse ($X = X = \text{for } T < \text{ or } T = -$

& 0

0

0) can be written

~

t+ T

$F = \lim F dt = F \cdot dt = 1 = m \& X (T =)$

$0 - m \& X (T = -$

$0) = m \& X0$

T

T

0

1

The initial conditions are then noted $X (T =)$

$0 = X = 0 \text{ and } X \& (T =)$

$0 = X$
 0
 $\&0 =$
 m

The general equation of the response in free vibration of a system to a d.d.l.

$X\& + X$

$X T$
 $- T$
 0
 $() = E$
 $X cost$
 0
 $0 0$
 $+$
 $sin T$
 L
 0
 0
 0
 0

G (T) of a system then becomes the impulse response to a d.d.l.

$- T$
 0
 E
 $X (T) = G (T) =$
 $sin T$
 L

éq A3.1-1
 m
 0
 0

~

F

*For an impulse nonunit $F = F \cdot T$
initial speed is $\dot{X}_0 =$ and the answer becomes*

m

~

F - T***E******0***

~

*$X(T) =$
 $\sin T = F G(T$*

L***)******m******0***

éq A3.1-2

0

If the impulse force is applied to one unspecified moment the answer is

~

$X T = F G(T$

L ()***-)***

A3.2 Response in unspecified forced vibration

*The force of excitation $F(T)$ can be broken up into a series of impulses of variable amplitude $F()$
applied to the moment during a time. If the 0 response to one moment T is obtained by*

T

$X(T) =$

$F() G(T -$

) D***0***

*and while replacing by the expression of the impulse response [éq A.3-2] one obtains the equation of
convolution*

for a system at rest at moment 0 of the form

I
T
X (T) =
F () -0 (T -)
E

sin 0 (T -) D

éq A3.2-1
m 0 0
known under the name of Integral of DUHAMEL
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A3.3 Response moving unspecified imposed

For an analysis moving relative represented by [éq An2-2]

&
X + 2
2
2
0
&x + X = -
0
&s = s0
(
cos T)

the integral of DUHAMEL becomes

1 T
X (T) =
S (
&) -0 (T -)
E
sin 0 (T -) D

éq
A3.3-1
0 0

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R4.06 booklet: Under-structuring

Document: R4.06.02

Modal calculation by dynamic under-structuring
traditional and cyclic

Summary:

This report/ratio presents the theoretical bases of the methods of calculation modal per modal synthesis. We begin

by the description of the transformation of RITZ and methods of modal recombination which result from this.

Then, we present the modal synthesis which uses the traditional techniques of under-structuring and of modal recombination.

The modal computational tools by modal synthesis implemented in Code_Aster, then, passed in review. We present, first of all, the techniques of traditional dynamic under-structuring of CRAIG-BAMPTON and of MAC NEAL. Then, we approach the methods of dynamic under-structuring cyclic. Completely dedicated to the study of the structures with cyclic repetitivity, they benefit the best from geometrical characteristics of the structure. Methods of CRAIG-BAMPTON and MAC NEAL, developed within this framework, are exposed.

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1 Introduction

In front of the complexity of the structures mechanical, often made up of an assembly of several components, numerical or experimental methods traditional of vibratory mechanics reveal expensive, sometimes even unusable. In perfect coherence with the modular organization great projects, the methods of under-structuring seem the most effective means to make a vibratory study of the whole starting from the dynamic behavior of the components

[bib4].

In this report/ratio, we present, first of all, the theoretical bases of the methods of synthesis modal. They associate techniques of under-structuring and modal recombination. Each substructure is represented by a base of projection made up of dynamic clean modes and of static deformations to the interfaces. The study of the conditions of connection between substructures is

simplified by the consideration of compatible grids.

Then, we present the two techniques of modal calculation per traditional under-structuring, implemented in Code_Aster [bib6]: methods of Craig-Bampton and Mac Neal. They distinguish by the use of different bases for the substructures.

Lastly, we present the techniques of modal calculation per cyclic under-structuring. Methods implemented in Code_Aster [bib5], allowing the calculation of the modes of a structure repetitivity cyclic starting from the study of the one of its sectors are exposed.

General notations:

:

Maximum pulsation of a system (rad.s-1)

m

M

:

Stamp of mass resulting from modeling finite elements

K

:

Stamp rigidity resulting from modeling finite elements

Q

:

Vector of the degrees of freedom resulting from modeling finite elements

F

:

Vector of the forces external with the system

ext.

F

:

Vector of the bonding strengths applied to a substructure

L

:

Stamp containing the vectors of a base of projection organized in column

:

Vector of the generalized degrees of freedom

B

:

Stamp extraction of the degrees of freedom of interface

L
:
Stamp connection
T
:
Kinetic energy
U
:
Deformation energy
Id
:
Stamp identity
:
Stamp diagonal generalized rigidities
R
:
Stamp residual dynamic flexibility
E ()
Re ()
0
:
Stamp residual static flexibility

Note:
The exhibitor K characterizes the sizes relating to the substructure S K and the sizes generalized are surmounted by a bar: for example \bar{M} K is the matrix of mass generalized of the substructure S K .
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2 Synthesis

modal

The dynamic under-structuring consists in determining the behavior of a structure from vibratory characteristics of each one of its components ([bib3] and [bib4]). Methods implemented in Code_Aster, use simultaneously the traditional techniques of modal recombination and of dynamic under-structuring.

These methods, although different from that of the finite elements, adopt a step enough comparable. They reveal three essential stages:

Stage 1: numerical study of each component by the determination of their characteristics vibratory. Work consists in identifying clean modes and static deformations by traditional techniques of vibratory mechanics. If one compares each substructure to one super-element, this stage is similar to elementary calculation.

Stage 2: connection of the substructures. The given vibratory characteristics are used previously for each component, and one takes account of their liaisonnement. This work constitutes the stage of under-structuring itself. It is connected with an assembly.

Stage 3: the resolution and a phase of increase makes it possible to obtain the solution sought in locate physical total structure.

2.1

Transformation of RITZ

The transformation of RITZ is the subject of the reference material [R5.06.01]. We recall here its principle. For the problem of the numerical determination of the real clean modes of the system not deadened associated the structure, that we will indicate by clean modes, one is reduced to resolution of the problem of minimization according to:

Either virtual displacement, one seeks: Min

1 T (

2

K - M)

2

whose solution Q checks:

$$(K - 2M) Q = 0$$

éq 2.1-1

The method of RITZ consists in seeking the solution of the equation of minimization on a subspace space of the solutions. Let us consider the matrix containing the vectors of the base of under space in question, organized in columns. Restricted with this space of reduced size, the equation of minimization takes the form:

Min

1 T (

2

)

p

$p K$

$M p$

$=$

$-$

2

That is to say the required solution:

$Q =$

éq

2.1-2

it checks:

$(K - 2M) = 0$

éq 2.1-3

where: is the vector of generalized displacements,

$K = TK$

$M = T$

and

M

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After having solved the system [éq 2.1-3], obtaining the clean modes in the physical base is done with assistance of the relation [éq 2.1-2]. The transformation of RITZ thus makes it possible to replace the problem with

eigenvalues initial [éq 2.1-1] by a problem of comparable nature [éq 2.1-3], but of dimension reduced. The new matrices of rigidity and mass remain symmetrical.

However, this transformation must be used with prudence. Indeed, the new base being incomplete, an approximation is made on the level of projection. The precision of the final result depends then choice of the basic vectors and the relative error due to this reduction in the number of unknown factors must be estimated.

2.2 Recombination

modal

A traditional use of the transformation of RITZ, is the dynamic analysis by recombination modal. It is usually used for the calculation of the response of a structure to an excitation low frequency. We will limit ourselves here to the calculation of the response to an excitation of a structure

conservative. In this case, the finite element method enables us to be reduced to the equation following matric differential:

Mq**! + Kq = fext****éq 2.2-1**

If one applies the transformation of RITZ, with as incomplete projection, the first bases clean modes of the structure, the relation [éq 2.2-1] becomes:

M! + K = fext**éq 2.2-2**Where: **F****T****= F***ext.*

ext. is the vector of the generalized forces.

The clean modes are orthogonal relative with the matrices of mass and rigidity. The equation differential [éq 2.2-2] thus revealed diagonal matrices: the system is then made up uncoupled equations. Each one of them is the equation of an oscillator to a degree of freedom of type mass-arises which reveals the mass, generalized rigidity and force relating to the mode J (respectively: m_j, K_j, F_j).

If one considers the transformation of RITZ [éq 2.2-2], on the level of a degree of freedom, one a:

$$IQ = ij$$

J

J

Where:

IQ is the i ème co-ordinate of the vector Q ,

J is the coordinated j ème vector,

ij is the component of the i ème line and the j ème column of the matrix.

It thus appears that the response of the structure is expressed like the recombination balanced of answers of oscillators to a degree of freedom uncoupled. The transformation of RITZ allows, in it cases, to define a diagram are equivalent of the structure, which reveals the oscillators with a degree of freedom associated with the identified clean modes. Their stiffness and their mass are generalized rigidities

(K_j) and generalized masses (m_j) of the corresponding modes.

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F

F

F

1

2

Fm

m1

m_2
 m_m
 K
 K
 K
 (K, M)
 1
 2
 \dots
 m
 \Leftrightarrow

Held primarily being studied in low frequencies, the modal recombination consists with to use the properties of orthogonality of the clean modes of a structure to simplify the study of its vibratory answer. In addition to the interest to decrease the order of the numerical problem to solve, transformation of RITZ, in this case, also makes it possible to uncouple the differential equations and to release a physical interpretation of the result obtained. According to the frequency of excitation, one will use a more or less truncated modal base. It is however necessary to estimate the truncation error for to ensure itself of the validity of the result.

2.3 Synthesis modal

In a general way, the methods of modal synthesis consist in using simultaneously dynamic under-structuring (cutting in substructures) and modal recombination on the level of each substructure. Often confused, by abuse language, with the under-structuring dynamics, the modal synthesis is only one particular case of this one.

The dynamic under-structuring consists in considering the displacement of a substructure in overall movement, as its response to the bonding strengths which connect it to the others components.

The modal synthesis means that one calculates this movement, on the level of each substructure, by modal recombination. One thus uses a base of projection which characterizes each substructure. Indeed, if the total structure is too important to be subjected to a modal calculation, dimensions substructures make it possible to carry out this work. The modal synthesis forces to study initially separately each component, in order to determine their base of projection.

In the continuation of this chapter, we present the types of modes and deformations static used in methods of modal synthesis using the following simple example:

q_1
 q_2
 I
 I
 q_1
 2
 J
 QJ

1

Substructure 1

Substructure 2

2

The vector of the degrees of freedom of the substructure is characterized by an exhibitor who defines its number of the substructure, and an index which makes it possible to distinguish the degrees of freedom internal (index

I), of the degrees of freedom of border (index J).

q_k

q_k

I

=

q_{kj}

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One is brought, to study the substructure *K*, to define an impedance in the level of the degrees of freedom of connection. Within the framework of the developments carried out in *Code_Aster*, it is null, that is to say infinite.

The basic vectors used in the methods implemented in *Code_Aster* are:

- normal modes,
- constrained modes,
- modes of fastener.

2.3.1 Normal modes

The clean modes or normal modes are advantageously used as bases projection of

substructures for several reasons:

- they can be calculated ou/et measured,
- they offer interesting properties of orthogonality compared to the matrices of mass and of rigidity of the substructure.

They can be of two types according to the condition given to the interfaces of connection:

- modes specific to blocked interfaces,
- modes specific to free interfaces.

Substructure 1

Substructure 2

Modes specific to blocked interfaces

Substructure 1

Substructure 2

Modes specific to free interfaces

Let us note that in the case of a free substructure, modes of rigid body (or overall modes) existing belong to the base of transformation.

2.3.2 Static deformations

One defines a mode of interface in each degree of freedom of connection of each substructure.

According to

case, it can act of constrained modes or modes of fastener.

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The constrained modes are static deformations which one joint with the normal modes with interfaces blocked to correct the effects due to their boundary conditions. A constrained mode is defined by static deformation obtained by imposing a unit displacement on a degree of freedom of connection, them other degrees of freedom of connection being blocked.

q=1

q=1

Substructure 1

Substructure 2

Constrained modes

The modes of fastener are static deformations which one joint with the normal modes with interfaces free to decrease the effect of modal truncation. A mode of fastener is defined by the deformation statics obtained by imposing a unit force on a degree of freedom of connection, other degrees of freedom of connection being free.

f=1

Substructure 1

Mode of fastener

In the case of a substructure having of the modes of rigid body (here, substructure 2), its stamp rigidity is not invertible and it is not possible to calculate its modes of fastener. It is then necessary to block certain degrees of freedom to make the structure isostatic.

2.4

Conditions of connection between substructures

Let us consider the problem of two substructures bonded $S1$ and $S2$ in a rigid way. So that the movement of the structure supplements is continuous, it is necessary to impose the equality of displacements of

two components with the interface and the law of action-reaction:

Mr. S1 S2

$1 (M) = 2 (M)$

1

) and

$(M) = - 2$

U

U

F

F (M

L

L

)

éq 2.4-1

Where:

u1 (M) represents the field of displacements of substructure 1,

u2 (M) represents the field of displacements of substructure 2,

F 1L (M) represents the field of the bonding strengths applied to substructure 1,

F 2L (M) represents the field of the bonding strengths applied to substructure 2.

In *Code_Aster*, we limit ourselves to the compatible cases of grids. That means that they check following properties:

- the meshes of each substructure $S1$ and $S2$ rest strictly on the same ones

nodes in their intersection,

- the finite elements associated these meshes of connection are the same ones on both sides of

border.

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Consequently, the condition [éq 2.4-1] is strictly equivalent to the formulation below:

q1

2

1

2

1

2 = **Q** 1

2

and

F

= - **F**

éq 2.4-2

S S

S S

L

L

SIS 2

SIS2

Where:

qk

is the vector of the degrees of freedom to the nodes of *S1* interface

S2

S1 S2

substructure **K**,

F K

1

2

 L

is the vector of the bonding strengths to the nodes of interface S of S_1 and S_2

substructure K .

Indeed, grids of the two substructures S_1 and S_2 coinciding, functions of form associated the finite elements are the same ones with the interface. It is thus enough to impose the equality on the nodes

interfaces of connection of each substructure to impose the equality on all the field of connection.

Let us introduce the matrices of extraction of the degrees of freedom of interface \mathbf{B}_k

:

 S_1 S_2

$$\mathbf{q}_k = \mathbf{B}_k \mathbf{q}$$

éq 2.4-3

 S_1 S_2

By using the equation of projection [éq 2.1-2], the condition of continuity of displacements [éq 2.4-2] and

the formulation applied above to the two substructures, one obtains:

 \mathbf{B}_1

1

2

2

1

$$2 = \mathbf{B}$$

 S_1 S_2

That is to say:

 \mathbf{L}_1

1

2

2

1

$$2 = \mathbf{L}_1$$

2

with

$$\mathbf{L}_k = \mathbf{B}_k$$

 K

éq 2.4-4 $S S$ $S S$ $S S$ $SIS2$

where:

L1is the matrix of connection of $S1$ associated with the $S1$ interface $S2$

,

 $SIS2$ **L2**is the matrix of connection of $S 2$ associated the $S1$ interface $S2$

.

 $SIS2$

As we will see it in the next chapters, whatever the method chosen, the problem with the eigenvalues of the total structure, provided with its boundary conditions, can be written under form:

$$(\mathbf{K} - 2\mathbf{M}) + \mathbf{L}\mathbf{T} = 0$$

éq 2.4-5

$$\mathbf{L} = 0$$

Matrices of generalized mass and rigidity, the vector of the generalized degrees of freedom and stamp connection which appears here, are defined on the total structure. They take a form particular with each method (Craig-Bampton, Mac Neal, traditional, cyclic) which will be clarified more late. The vector of the multipliers of Lagrange makes it possible to translate the law of action-reaction to which

the interfaces are subjected.

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It is thus about a traditional problem of search for eigenvalues, to which is associated one linear equation of constraint. In *Code_Aster*, this type of problem is solved by double dualisation of the boundary conditions [R3.03.01].

Thus, one can show that this system is also solution of the problem of minimization of following functional calculus [bib6]:

*F**B*

=

*1 dt***F** (! ,).*has*

2

where **F** (,

!,

1

T

1

*T**T*

1

2

1,)

2 =

2

K + 2! **M**! + (1 +) **L**

2

-(

2

1 -)

2

The variables of this functional calculus are the generalized co-ordinates and the multipliers of Lagrange 1 and 2 (in a number equal to 2 times the number of equations of connection). The last term of functional calculus imposes the equality of the coefficients of Lagrange.

The extremum is reached for the values of the variables which cancel the derivative of *F*, whatever *has* and *B* realities:

- 1 + **L** + 2 = 0**LT** (

2

$$1 +) + (\mathbf{K} - \mathbf{M})$$

2

$$= 0$$

$$1 + \mathbf{L} - 2 = 0$$

$$- \mathbf{Id} \mathbf{L}$$

Id

0 0

0

1

1

0

*T**T*

2

L**K**

$$\mathbf{L} - 0 \mathbf{M}$$

$$0 =$$

0

$$\mathbf{Id} \mathbf{L} - \mathbf{I}$$

D 2

0 0

0 2

0

The double dualisation thus leads to a real symmetrical matrix problem. One shows [R3.03.01] that it makes it possible to make the algorithms of triangulation of matrix unconditionally stable. This method thus makes it possible to treat the connection of interfaces corresponding to basic types modal different without cost from management of an always delicate elimination. In addition, it is relatively simple. The major disadvantage of this formulation is to lead to systems assembled final of size more important than in the case of elimination. Indeed, it coupling of the matrix equations was made by introducing a number of degrees of freedom

additional equal to twice the number of equations of connection. This increase in dimensions matrices can thus be very important. Let us note that the degrees of freedom of Lagrange introduced are, in this case, the forces applied to the interfaces to ensure the connection between the two pennies structures.

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3

Modal calculation by traditional dynamic under-structuring

3.1 Introduction

After having separately studied the various stages of the under-structuring, and the techniques that they bring into play, it appears interesting to present the two principal methods of dynamic under-structuring: method of Craig-Bampton and that of Mac Neal.

Craig-Bampton uses, as bases projection of the substructures, modes constrained and normal modes with fixed interfaces [bib1].

In addition, Mac Neal uses, as bases projection of the substructures, modes of fastener and normal modes with free interfaces [bib2].

3.2

Method of Craig-Bampton

The following presentation utilizes only two substructures $S1$ and $S2$, but it is generalizable with an unspecified number of components. After having studied separately each substructure, their bases of projection (normal modes with fixed interfaces and constrained modes) are known. For each one of them (identified by the exhibitor K), one establishes a partition of the degrees of freedom, distinguishing the vector from the degrees of freedom intern \mathbf{q}_{ki} and the vector of the degrees of freedom

of connection \mathbf{q}_{kj} :

\mathbf{q}_k

\mathbf{q}_k

I
=
 q_k

Are:

K the matrix of the clean vectors of the S_k substructure,
 K the matrix of the constrained modes of the S_k substructure.
The base of projection of S_k is characterized by the matrix:
 $K [K K$

=
]
The transformation of RITZ (equation [eq 2.1-2]), enables us to write:

q_k
 K

q_k
 I
 K
 K
 I
 K
 K
= $K = [] =$
eq 3.2-1
 Q
 K
 J
 J

k_i is the vector of the generalized degrees of freedom associated the clean modes of S_k ,
 k_j is the vector of the generalized degrees of freedom associated the constrained modes of S_k .
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However the normal modes are given with fixed interfaces, and each constrained mode is obtained by imposing a unit displacement on a degree of freedom of connection, others being blocked. generalized co-ordinates relating to the static deformations are then the values of the degrees of freedom of connection:

qk

K

J = J

Let us interest in the contribution of the component *S K* from an energy point of view. Energies kinetics and of deformation are:

T

T K

1 K T

K K

1

1

2 !

Q M! Q

2 (K K

!)

K

K K

K T

K K

=

=

M! = 2! M!

T

The U.K.

1 K T

K K

1

1

2 Q

K Q

2 (K K

) K K K

K T

K K

=

=

K = 2 K

These expressions reveal projections of the matrices of mass and rigidity on the basis of substructure. These matrices, known as generalized, check a certain number of properties:

- because of orthogonality of the normal modes compared to the matrices of rigidity and mass, the left higher block of these matrices is diagonal. Moreover, we will consider that these modes are normalized compared to the matrix of mass,

- one can also show that the constrained modes are orthogonal with the modes normal compared to the matrix of rigidity [bib6].

The matrices of generalized rigidity and mass thus have the following form:

K

K T

K

K

0

K

K

Id**M****K =****M =**

K T

éq 3.2-2

0

K K K

K T

Mk K

K T

Mk K

where K is the matrix of the generalized rigidities associated the clean modes of $S K$.

The choice of the base of projection to blocked interfaces thus leads to a coupling of the modes normal and of the static deformations by the matrix of mass.

In the case of the calculation of the clean modes of the total structure, external forces applied to system are null. In addition, on the level of each connection, because of law of action-reaction and of continuity of displacements, the work of the bonding strengths is null. This is explained physically by the fact that the bonding strengths are internal forces with the total structure.

Thus, on the level of the complete structure, only the energies kinetic and of deformation are not null:

T

T

$$T = T1 + T2 = 1 1$$

1 1

$$! M! + 1 2$$

2 2

2

2 !

M!

T

T

$$U = U1 + U2 = 1 1$$

1 1

$$\mathbf{K} + 1 2$$

2 2

2

2

K

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The work of the bonding strengths being null, equations of Euler-Lagrange relating to the clean modes total structure are:

T

K1 0 L1

M1

1

0

0

0

T

0

K 2 L2

2

-

0

M2

2

0 = 0

éq 3.2-3

L1

L2

0

0

0

0

0

Consequently, the problem with the eigenvalues of the complete structure can be expressed by the system [  q 2.4-5] that we studied in chapter 2.4:

$$(\mathbf{K} - 2\mathbf{M}) + \mathbf{L}\mathbf{T} = 0$$

$$\mathbf{L} = 0$$

Thus, the calculation of the clean modes of the total structure by the method of Craig-Bampton consists with

to solve a problem with the eigenvalues matrix of reduced size. The matrices which it brings into play are symmetrical and are calculated starting from the bases of the substructures. This resolution gives us Eigen frequencies and generalized co-ordinates of the required modes. Theirs are obtained physical co-ordinates on the grids of the substructures by using the relation [  q 3.2-1]. This stage is called: stage of restitution on physical basis.

This method is interesting if one considers a numerical study of the substructures. Indeed, them normal modes with fixed interfaces and the constrained modes lend themselves well to calculation. On the other hand, them

experimental determination is delicate.

It is shown moreover that this method, is order 2 in m where m is the greatest pulsation clean identified.

3.3

Method of Mac Neal

It would be possible to present this method in a way similar to that of Craig-Bampton, the single one difference residing in the use of the normal modes at free interfaces and the modes of fastener.

However, it appears interesting to adopt a slightly different step, which makes it possible to lead to a criterion of truncation, and to reveal the modes of fastener like the static contribution of not identified clean modes.

As previously, one considers the problem of the calculation of the clean modes of a structure with 2 components. The method is generalizable with an unspecified number of substructures. In the continuation

we will identify any size associated with the substructure $S K$ by the exhibitor K .

We limit ourselves, here, with the modal calculation of the total structure, therefore the external forces are null.

Also, the displacement of $S K$ in the movement of the total structure, checks the equilibrium equation following dynamics:

$$(\mathbf{K} K - 2\mathbf{M}k) \mathbf{q}k$$

$$\mathbf{F} K$$

-

$$= L$$

  q 3.3-1

where $\mathbf{F} kL$ is the vector of the bonding strengths applied to $S K$.

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We will consider that the base of projection of $S K$ consists of **all** its modes suitable for free interfaces. Thus, the dimension of the projected problem is equal to the dimension of the problem resulting from modeling finite elements. We suppose, moreover, that a certain number of these modes was determined, others being unknown:

K

K

1

\mathbf{q}_k

[

K

1 2]

$K K$

=

=

K

2

Where:

k_1

is the matrix of the identified modal vectors of the substructure $S K$,

k_2

of the not identified modal vectors substructure $S K$ is the matrix,

K

is the vector of the generalized degrees of freedom associated the clean modes identified of

1

$S K$,

K

is the vector of the generalized degrees of freedom associated the not identified clean modes

2

of $S K$.

The equation [éq 3.3-1] becomes, with the previously definite generalized co-ordinates:

$$(kT K K - 2kT K K) K = kT K$$

K

M

F L

that is to say:

$$(\mathbf{K} K 2Mk) K$$

$K T$

F K

-

=

L

éq 3.3-2

The clean modes are orthogonal compared to the matrices of mass and rigidity and us let us choose to normalize them compared to the matrix of mass. One thus has:

K

0

0

1

Id

K K

M K

=

=

K

éq 3.3-3

0

0

Id

2

Now let us consider all two substructure. Each one of them checks them equations [éq 3.3-2] and [éq 3.3-3]. The whole of these dynamic equations constitutes the system according to:

T
1
1
1
1

Id 0
0
0

F
1
0
0
0
1
1
1

L

0

2
2
2
2
0

Id
0
0

F
1
0

0
2
1 1

1

L

-
=

éq 3.3-4

0
0
1
1
1
1

0
0
Id
0

2
0
2

2

F
2

L

0

0
0

2
2
2
2
0
0
0

Id

2

2

2

F

2

L

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By gathering the identified modes and the not identified modes:

1

1

1

2

co-ordinates of the identified modes

1

=

éq 3.3-5

1

2

co-ordinates of the not identified modes

2

2

2

The equation translating the transformation of RITZ becomes:

q1

1

Q = = [1

2

]

Q

2

éq 3.3-6

2

With these notations, the system of dynamic equations [éq 3.3-4] becomes:

F

1

0

Id 0*L*

2

1

*T**T*

1

=

éq 3.3-7

0

-

0

Id

[1 2]

F

2

2

2

L

This system of equations translates the dynamic behavior of the substructures separately. It do not represent the movement of the total structure. For that, it is necessary to associate the conditions to him of connection between the two components.

The equations between substructures which ensure their liaisonnement derive as of the equations [éq 2.4-2]

and [éq 2.4-4], as of the organization of the base which we chose [éq 3.3-5]:

1

L = [L L

1

2]

= 0

éq 3.3-8

2

F = - F

L

L

éq 3.3-9

1

2

The equations [éq 3.3-7] and [éq 3.3-9] enable us to express the generalized co-ordinates relating to the not identified modes:

2

1

T

2 = - 2 -

-

(

)

)

Id

2 F 1L

éq 3.3-10

From the equations [éq 3.3-8] and [éq 3.3-10], one thus obtains:

L - L

2

- Id 1

-

(

) Tf

1 1

2

2

2 L = 0

éq 3.3-11

1

However, according to the formula [éq 2.4-4], one knows that one can write the matrices of connection, in the form:

L = B

K

K

K

éq 3.3-12

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One thus has, according to [éq 3.3-11] and [éq 3.3-12]:

- +

(- 2

-

B

B

Id 1

) *Tf*

1 1 1

2 2

2

2 $L = 0$

éq 3.3-13

1

One sees appearing the matrix of residual dynamic flexibility associated the not identified modes:

R () = (

2

1

2 2 - **Id** -

T

E

) 2

éq 3.3-14

The bonding strength is nonnull with the degrees of freedom of connection of the substructures. We have

thus:

$$\mathbf{F} = \mathbf{B}\mathbf{T}\mathbf{f}$$

L

$1 L$

1

1

Consequently, the matrix problem [eq 3.3-7] can be reduced to the system are equivalent according to:

2

$$- \mathbf{I}d - \mathbf{T}\mathbf{B}\mathbf{T}$$

0

1

1

1

1

=

éq 3.3-15

- \mathbf{B}

$\mathbf{B} \mathbf{R} \mathbf{E}$

$() \mathbf{F} \mathbf{L}$

0

11

2

1

This problem has as unknown factors the generalized co-ordinates associated the identified clean modes and bonding strengths applied to the first substructure.

There are 2 cases according to whether one takes into account or not the matrix of residual flexibility.

3.3.1 First

case

One neglects the matrix of residual dynamic flexibility associated the not identified modes:

$$\mathbf{R}e () = 0.$$

The method of resulting dynamic under-structuring is thus very simple. It has the disadvantage of to be based on a method of modal recombination very sensitive to the effects of truncation.

3.3.2 Second

case

A limited development is used: $\mathbf{R} () = \mathbf{R}$

2

2

E

$E ()$

$0 + O (/m):$

Let us adopt the following notations:

Are:

N

the number of modes of the complete base,

m

the number of not identified modes,

the matrix of N normal modes to free interfaces,

the matrix of N modes of fastener (definite for all the degrees of freedom of system),

2

$I = I$

the diagonal matrix of N eigenvalues.

The complete matrix of the modes of fastener checks: $\mathbf{K} = \mathbf{Id} = \mathbf{K}^{-1}$

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The complete modal base of the normal modes with free interfaces constitutes an orthonormal base. stamp stiffness, expressed in this base is written:

$\mathbf{K} = T\mathbf{K} =$

Same manner:

\mathbf{K}^{-1}

$T\mathbf{K}^{-1}$

-

=

=

1

One thus deduces a new form from it of the complete matrix of the modes of fastener:

= - =

-

K 1

1T

As the clean modes are orthogonal two to two and stamps it eigenvalues is diagonal, the complete matrix of the modes of fastener takes the final form:

N

=

-

1 *T*

II I

i=I

Now let us consider the matrix of residual dynamic flexibility, resulting from the method of Mac Neal:

N

R () = (

2

1

2

1

2 2 - **Id** -

T

-

T

E

) 2 = *I*

(*I* -) *I*

i=m+I

To highlight the effect of modal truncation, we can approximate this matrix by sound development with order 1:

N

2

1

R

-

T

1

$$E() I$$

$$I$$

$$+$$

$$2 I$$

$$i=m+1$$

$$I$$

When the number of identified modes is sufficiently important, the dynamic contribution becomes negligible in front of the static contribution:

$$2$$

$$N$$

$$-$$

$$T$$

$$\ll$$

Re

$$() \mathbf{Re} () = II$$

$$1$$

$$1$$

$$0$$

$$2$$

$$I$$

$$I$$

$$i=m 1$$

$$+$$

where $\mathbf{R} E ()$

0 are the matrix of residual static flexibility.

This matrix can be calculated according to the matrix of the modes of fastener:

$$N$$

$$m$$

$$m$$

R

$$1$$

$$-$$

$$T$$

$$1$$

$$-$$

$$T$$

$$()$$

$$0 = -$$

R

$$1$$

$$-$$

T
E
II
I
II
I
E ()
 $0 = - I$
II

I 1
 =
I 1
 =
I 1
 =

The second term of this formulation is calculable in an exact way, since it utilizes only them modes (with free interfaces) identified.

Lastly, let us note that in the method of Mac Neal, only the contribution of the modes of fastener to the nodes of interface is necessary.

The resolution of the system [éq 3.3-4] enables us to determine the Eigen frequencies of the structure total and generalized co-ordinates of the clean modes. Increase with the expression of the modes clean in the physical bases of the substructures is done by the following relation:

qk
K K
 1
R K (0) Bk Tf K
 =
 1 +
E
 1
LI

The method of Mac Neal thus succeeds, in the case of the calculation of the modes of the total structure, with one

problem with the eigenvalues of reduced size. Matrices of mass and rigidity exits of under structuring are symmetrical. Two methods are actually proposed, according to whether one takes in count or not residual flexibility. The literature on the subject tends to show that the use of residual flexibility is essential to obtain reliable results [bib2].

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3.4**Implementation in Code_Aster****3.4.1 Study of the substructures separately**

The base of projection of each substructure is made up of dynamic clean modes and of static deformations.

The dynamic clean modes of the substructure are calculated with the traditional operators of *Code_Aster*: `MODE_ITER_SIMULT` [U4.52.02] and `MODE_ITER_INV` [U4.52.01]. In the case of under-structuring of Craig-Bampton, the interfaces of connection must be blocked. This is carried out with operator `AFFE_CHAR_MECA` [U4.25.01].

Operator `DEFI_INTERF_DYNA` [U4.55.03] allows to define the interfaces of connection of substructure. In particular, one specifies the type of the interface, which can be is “CRAIGB” (Craig-Bampton), either “MNEAL” (Mac Neal), or finally “NONE”.

Operator `DEFI_BASE_MODAL` [U4.55.04] allows to calculate the base of complete projection of substructure. Thus, the dynamic modes calculated previously are recopied. In addition, them static deformations are calculated according to the type defined in operator `DEFI_INTERF_DYNA` [U4.55.03]. If the type is “CRAIGB”, one calculates the constrained modes of the interfaces of substructure. If the type is “MNEAL”, one calculates the modes of fastener of the interfaces of substructure. If the type is “NO”, one does not calculate static deformation, which corresponds to a base of the type Mac Neal without static correction.

Operator `MACR_ELEM_DYNA` [U4.55.05] calculates the generalized matrices of rigidity and mass of the substructure, as well as the matrices of connection.

3.4.2 Assembly and resolution

The model of the complete structure is determined by operator `DEFI_MODELE_GENE` [U4.55.06]. In private individual, each substructure is defined by the macronutrient which corresponds to him (resulting from

`MACR_ELEM_DYNA`) and the swing angles which make it possible to direct it. The connections enters substructures are defined by the data of the names of the two implied substructures and those of the two interfaces in opposite.

The classification of the complete generalized problem is carried out by operator `NUME_DDL_GENE` [U4.55.07]. The matrices of generalized mass and stiffness of the structure supplements are

assemblies according to this classification with operator ASSE_MATR_GENE [U4.55.08]. The calculation of the clean modes of the complete structure is carried out by the operators MODE_ITER_SIMULT [U4.52.02] or MODE_ITER_INV [U4.52.01].

3.4.3 Restitution on physical basis

The restitution of the results on the initial grids of the substructures is carried out by the operator REST_BASE_PHYS [U4.64.01].

To decrease the duration of the graphic treatments during visualizations, it is possible to create one coarse grid by operator DEF1_SQUELETTE [U4.75.01]. This grid, ignored during calculation, is used as support with visualizations.

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4

Modal calculation by cyclic dynamic under-structuring

4.1 Introduction

In this chapter, we make the synthesis of the methods of cyclic dynamic under-structuring.

We give a definition of the cyclic repetitivity (or cyclic symmetry) and we present them principal incidences of this property on the dynamic behavior of the structure (circles and nodal diameters, double modes). Then, we expose, in a rather detailed way, both methods of cyclic dynamic under-structuring, implemented in *Code_Aster*.

improvements were made to the traditional methods, by the taking into account of the presence of nodes of the axis.

These methods suppose that the grid of the basic sector is such as its traces on the interfaces right-hand side and left are coinciding (compatible grids).

Notations specific to the cyclic under-structuring:

NR

=

a number of sectors

=
angle formed by the basic sector

=
dephasing AND element
OZ

=
cyclic axis of symmetry

=
rotation of angle and axis *OZ*
Re (*Z*)

=
real part of complex *Z*
Im (*Z*)

=
imaginary part of complex *Z*

=
stamp passage of the nodes of right-hand side to the nodes of left

=
stamp change of sector for the nodes of the axis
has

Note:
The index

D
is relative
with the degrees of freedom of right-hand side

"
G
"

with the degrees of freedom of left

"
has
"

with the degrees of freedom of the axis

"
I
"

with the identified clean modes

"
2
"
"

with the unknown clean modes

4.2 Repetitivity

cyclic

4.2.1 Definition

It is said that a structure is with cyclic repetitivity of axis *OZ*, if there is an angle $0 < \alpha < 2\pi$ such as structure is geometrically and mechanically invariant by rotation around *OZ* of this angle.

If α is the smallest angle checking this property, then any angular portion of angle of structure is called "basic sector" (or "irreducible sector").

The total structure is then made up of *NR* sectors:

2
NR =
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4.2.2 Propagation

of wave

One notes the rotation of axis *OZ* and angle defined in R3.

Let us consider a basic sector of a structure with repetitivity of axis *OZ*, and two similar points of two contiguous sectors G and D:

G
OZ

D

One with the relation between the points G and D:

$G = (D)$

It is noticed that the structure is left invariant by any rotation m (with m whole). One can note that all rotations leaving the invariant structure (geometrically and mechanically) are in a finished number:

$$m \in \{0, 1, \dots, NR - 1\}$$

Let us consider a scalar variable of state of the mechanical system studied U , and Z the associated complex:

$$U = \operatorname{Re}(Z) = \operatorname{Re}(U + jV)$$

It is possible to show, by the theory of the finished groups, the following relation for the points D and G [bib5]:

$$Z_G = E^{jm} Z_D$$

$$\{ , \dots, \}$$

$$()$$

$$Z(D)$$

$$2$$

such as

éq 4.2.2-1

Note:

- the quantities are expressed in the cylindrical reference mark (R, Z) ,
- for an axisymmetric structure (particular case of cyclic repetitivity), m is called index of FOURIER,
- in the case of a wave planes not deadened, E^{jm} is complex dephasing between two contiguous sectors; the equation means that this dephasing can take only one number finished known values,
- it is possible to limit the number of the values of m to the values ranging between 0 and $NR/2$; indeed, it is shown that the wave associated with dephasing $NR - m$ is identical to that associated dephasing m , but progresses in opposite direction [bib5].

If NR is even: $m = 0$ and $m = NR/2$ correspond to real modes:

$$m = 0$$

D

$$U((D)) = U(D)$$

$$m = NR/2$$

D

$$U((D)) = U$$

- (D)

All the other values of m correspond to modes appearing per orthogonal pairs with a given frequency (one speaks then about degenerated modes):

$$U = \text{Re}(Z) \text{ and } V = \text{Im}(Z)$$

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If NR is odd: $m = 0$ corresponds to a real mode not degenerated:

$$m = 0$$

D

$$U((D)) = U(D)$$

All the other values of m correspond to degenerated modes appearing per pairs orthogonal:

$$U = \text{Re}(Z) \text{ and } V = \text{Im}(Z)$$

4.2.3 Concept of diameters and nodal circles

The cyclic property of repetitivity, translated by the equation [éq 4.2.2-1] makes it possible to know a priori

pace of the clean modes of the structure, which strongly approaches what one can observe for axisymmetric structures. If one considers a clean mode of a structure with symmetry cyclic, all the sectors have the same deformation but with an amplitude function of their position angular, which one can translate by a dephasing between substructures. This mode can be classified starting from the number of nodal diameters and circles which characterize it. A nodal diameter (which is not

confused with a diameter that if the structure is axisymmetric) is a line of points of null movement passing by the axis of repetitivity; a nodal circle (which with the circular form only for the axisymmetric structures) is a line of points of null movement, it even with repetitivity cyclic. It is noted that it is the deformation of the mode of the substructure on which the mode is pressed structure supplements which determines the number of circle (S) nodal (with). On the other hand, the

number of
diameter (S) nodal (with) is defined by dephasing between two consecutive sectors.

Deformation

Phase enters

Deformation

Family

sector

sector

overall

NR sectors

0 circle

Inflection 1

in phase

0 diameter

N/2 sectors

0 circle

Inflection 1

in phase

1 diameter

NR sectors

1 circle

Inflection 2

in phase

0 diameter

N/secteurs

1 circle

Inflection 2

in phase

1 diameter

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4.2.4 Boundary conditions

Let us consider a structure with cyclic repetitivity, and two basic sectors successive of this one:

G

k+1

D

R_{k+1}

Interface

G

K

D

R_k

The connections between sectors being regarded as perfect, there are the conditions between the sectors:

$$\mathbf{q}_k = \mathbf{q}_{k+1}$$

G

D

continuity of displacements

éq 4.2.4-1

$$\mathbf{F}_K = -\mathbf{F}_{k+1}$$

L

L

principle of action - reaction

G

D

The exhibitor indicates the number of the sector considered. The preceding conditions of connection are expressed in the total reference mark.

By the formula [éq 4.2.2-1] relating to the propagation of wave in the structure and while posing: =

m,

one a:

$$\{\mathbf{q}_{k+}\}$$

1

J

$$1 = E$$

$$\{\mathbf{q}_k\}$$

K +

K

$$\{\mathbf{F}_{k+}\}$$

1

J

$$1 = E$$

$$\{\mathbf{F}_K\}$$

L
 $K +$
 $L K$

The index K means that the quantity is expressed in the reference mark related to the sector K : Rk . The equations of connection [éq 4.2.4-1], written in the reference mark related to the sector K thus utilize stamp passage of the sector K to the sector $K + 1$. This matrix is not other than the matrix of rotation of the degrees of freedom of right-hand side towards those of left, is the matrix of rotation of axis OZ and of angle, noted: .

We thus obtain the following system:

$$\begin{aligned} & \{ \mathbf{qk} \} \\ & J \\ & = E \{ \\ & \mathbf{qk} \} \\ & G K \\ & D K \\ \text{éq 4.2.4-2} \end{aligned}$$

$$\begin{aligned} & \{ \mathbf{F K} \} \\ & J \\ & = - E \{ \\ & \mathbf{F K} \} \end{aligned}$$

L
 K
 L
 K
 G
 D

The boundary conditions [éq 4.2.4-2] make it possible to calculate the clean modes of the whole of structure starting from one only basic sector.

This formalization can be generalized with the case of the nodes of the axis. One obtains then:

$$\begin{aligned} & \{ \mathbf{qk} \} \\ & J \\ & = E \{ \mathbf{qk} \} \\ & K \text{ has} \\ & \text{has} \\ & K \text{ has} \\ & \{ \mathbf{F K} \} \\ & J \\ & = - E \{ \mathbf{F K} \} \end{aligned}$$

L
 K
 has
 L

K

has

has

It is checked that if is nonnull, the displacement of the nodes of the axis is null (in fact, one notes then presence of one or several nodal diameters).

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4.3

Methods of cyclic under-structuring

4.3.1 Method of Craig-Bampton

One considers the problem with the eigenvalues of the total structure expressed on the basic sector. This last is thus subjected to the bonding strengths which are applied to him by the contiguous sectors.

By

elsewhere, the basic sector checks the equations of connection [éq 4.2.4-2]. We thus have:

$$(\mathbf{K} - 2\mathbf{M}) \mathbf{Q} = \mathbf{F} \mathbf{L}$$

\mathbf{Q}

\mathbf{J}

$\mathbf{G} =$

\mathbf{E}

\mathbf{Q}

\mathbf{D}

éq 4.3.1-1

\mathbf{F}

\mathbf{J}

$\mathbf{L} = -$

\mathbf{E}

\mathbf{F}

*L**G**D*

We suppose that the base is made up of the dynamic clean modes of the basic sector embedded with its interfaces, noted, and of the constrained modes relating to the degrees of freedom of interfaces

right-hand side and left, noted *D* and *G*.

Taking into account the fact that the only contribution to displacements of a degree of freedom of interface

comes from the constrained mode corresponding, the transformation of RITZ can be written:

Q*I**I*

$$\mathbf{Q} = \mathbf{Q}$$

$$D = [D \ G] \mathbf{Q}$$

D =**Q****Q***G**G*

Consequently, by using the transformation of RITZ, the system of equations [éq 4.3.1-1] becomes:

0

*I**T*

2

$$(\mathbf{K} - \mathbf{M}) \mathbf{Q}$$

$$D = [D \ G] \mathbf{fL}$$

*D***Q****F***G*

L_g \mathbf{Q} J $G =$ E \mathbf{Q} D

éq 4.3.1-2

 \mathbf{F} J $L = -$ E \mathbf{F} L G D

The surmounted matrices of a bar are projections of the matrices finite elements on the basis modal of the basic sector (generalized matrices).

One can show that the constrained modes are orthogonal with the normal modes with respect to stamp rigidity [bib5]. Thus, the corresponding products are null.

Let us adopt the following notations:

m: index relating to the clean modes of the sector,

D: index relating to the constrained modes of the right interface,

G: index relating to the constrained modes of the left interface.

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One can thus write these matrices in the form:

K

0

0

M

M

M

mm

mm

Mandeleivium

Mg

K = 0

K

K

dd

dg

M = M

M

M

DM

dd

dg

0

K

K

M

M

M

Gd

gg

gm

Gd

gg

Taking into account their definition, the constrained modes check:

di

di

gi
gi

$$D = dd = \mathbf{Id}$$

$$G = Gd = 0$$

Id
dg

gg

The second member of the matric equation [éq 4.3.1-2] becomes:

I
0
0
0
0

di **Id** 0 **F**
L = F
L
D
D

gi
0
Id F
L
F
L

G
G

By taking account of these notations, let us develop the matric equation checked by the basic sector:

K
2
mm I
- (**M + M Q + M Q**)
mm I
Mandeleivium D
Mg G) = 0
K Q + K Q
2
dd D
dg G - (M + M Q + M Q
DM I
dd D
dg G) = F Ld
K Q + K Q
2
Gd D
gg G - (M + M Q + M Q
gm I
Gd D
gg G) = F Lg
Q
J
= E Q
G
D
F
J

$= - E F$

L

L

G

D

Let us introduce the two last equations of this system into the three first:

$(K - 2M$

2

J

mm

$mm) I$

$- (M + E M$

Mandeleevium

$Mg) qd = 0$

$(K$

J

$+ E K$

2

J

dd

$dg) qd - (Mdm I$

$+ (M + E M$

dd

$dg) qd) = fLd$

$(K$

J

$+ E K$

2

J

J

Gd

$gg) qd - (Mgm I$

$+ (M + E M$

Gd

$gg) qd) = - E$

F

Ld

The association of the two last equations makes it possible to eliminate the terms from the bonding strengths. One

leads then to a problem with the eigenvalues final which one can put in the form:

$(\sim$

$2 \sim$

$() -$

()~

K

M

Q = 0

éq 4.3.1-3

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With:

~ I

Q =

Q

D

~

K

mm

0

K =

I

- I T

T

0

K

dd + E K dg + E

K Gd + K

gg

I

~

M

M

mm

Mandeleivium + E M

M =
 Mg

M
 IT
 I
 IT
 T
 $DM + -$
 E
M
M
 gm
 $dd + E Mdg + -$
 E
Mgd + M

gg

The matrices of mass and rigidity of the final problem are square. Eigenvalues solutions are thus real. In addition, the problem is of reduced size.

The resolution of the problem to the complex eigenvalues [éq 4.3.1-3] makes it possible to determine them

complex generalized co-ordinates of the clean modes of the total structure. Values complexes of displacements of the basic sector in the total mode are given, from generalized co-ordinates, by the following formula:

$$\mathbf{Q} = [$$

$$J$$

$$\sim$$

$$D + E D] \mathbf{Q}$$

éq 4.3.1-4

To determine the actual values of displacements, it is necessary to distinguish three cases according to values' from

dephasing AND element:

Case n° 1: = 0:

The displacements \mathbf{Q} given by the formula [éq 4.3.1-4] are then with actual values. All sectors deformed even and vibrate in phase. There is then only one real clean mode:

$$\mathbf{Q} = \text{Re} (\mathbf{Q}) = \mathbf{Q}$$

éq 4.3.1-5

Case n°2: $0 << (NR + 1)/2$:

The displacements provided by the formula [éq 4.3.1-4] are with complex values. With each one of these complex modes correspond two orthogonal degenerated real modes:

$$\mathbf{Q} = \text{Re} (\mathbf{Q})$$

$$\mathbf{Q} = \text{Im} (\mathbf{Q})$$

1
2
)

éq 4.3.1-6

Case n°3: = $NR/2$ ($\Rightarrow NR$ is even):

The displacements provided by [éq 4.3.1-4] are then with complex values. There are $NR/2$ diameters nodal, two contiguous sectors vibrate then in opposition of phase. Each complex mode is with the origin of only one real mode:

$$\mathbf{Q} = \text{Re}(\mathbf{Q}) = -\text{Im}(\mathbf{Q})$$

éq 4.3.1-7

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Taking into account of the nodes of the axis:

One supposes in this paragraph that the degrees of freedom carried by the nodes of the axis, with same titrate that the nodes of interfaces right-hand side and left, were blocked for the calculation of the modes dynamic of the basic sector and were the subject of calculations of constrained modes.

The base of projection is thus made up of the dynamic clean modes of the basic sector embedded with its interfaces, noted, and of the constrained modes relating to the degrees of freedom of interfaces

right-hand side, left and axis, noted D , G and A .

As we saw in chapter 4.2.4, if is nonnull, the displacement of the nodes of the axis is

no one (presence of at least a nodal diameter). The taking into account of the nodes of the axis thus does not have a direction

that if = 0. In this demonstration, we will limit ourselves to this case.

The problem with the eigenvalues of the total structure and equations of connection, expressed on this base are worth then:

I

0

Q**F***D**Ld* $(\mathbf{K} - 2\mathbf{M}) = D G \text{ has}$ **Q**

[

] **F***G**Lg*

éq 4.3.1-8

Q**F***has***HQ = Q**

and

Q*D* $= \mathbf{Q} \text{ has}$, **F***has has* $L = - \mathbf{F}$

and

F*L* $L = - \mathbf{F}$ *L has**G**D**has**has*

Let us adopt the following notations:

m: index relating to the clean modes of the sector,

D: index relating to the constrained modes of the right interface,

G: index relating to the constrained modes of the left interface,

a: index relating to the constrained modes of the interface centers.

One can thus write the matrices in the form:

K

mm

0

0

0

M

M

M

M

mm

Mandeleevium

Mg

my

0 **K**

K

K

M

M

M

M

K =

dd

dg

da

M DM

dd

dg

da

=

0

K

K

K

M

M

M

M

Gd

gg

ga

gm

Gd

gg

ga

0

K

K

K

M

M

M

M

AD

Ag

aa

amndt

AD

Ag

aa

Taking into account their definition, the constrained modes check:

di

di

gi

gi

have have

Id

dd

Gd

0

0

AD

$D =$

$=$

$G =$

$=$

$=$

$=$

0

has

dg

gg

Id

Ag 0

0

da

ga

0

aa

Id

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The second member of the matric equation [éq 4.3.1-8] becomes:

0 0

I

0

0

0

F

F

di

Id

0

0 L L

D

D

=

F

F

gi

0

Id

0

L

L

G G

F F

have

0

0

Id L

L

has has

By taking account of these notations, let us develop the matrix equation checked by the basic sector:

K

2

mm I

- (**M + M Q + M Q + M Q**)

mm I

Mandeleevium D

Mg G

my A) = 0

K Q + K Q + K Q

2

dd D

dg G

da has - (M + M Q + M Q + M Q

DM I

dd D

dg G

da has) = F Ld

K Q + K Q + K Q

2

Gd D

gg G

ga has - (M + M Q + M Q + M Q

*gm I**Gd D**gg G**ga has) = F Lg***K Q + K Q + K Q**

2

*AD D**Ag G**aa has - (M + M Q + M Q + M Q**amndt I**AD D**Ag G**aa has) = F***HQ = Q**

and

F*D**L = - F L**G**D***qa = Q**

and

F*has has**L = - F**L has**has**has*Let us replace, in the first four equations of the system **HQ** and **F L** by their expressions*G*respective, according to **qd** and **F L** and let us rewrite, in another form, the two last*D*

equations of the system, relating to the nodes of the axis.

(K -2M

2

*mm**mm) I**- ({M + M**Mandeleivium**Mg) Q + M Q**D**my A} = 0***(K +K**

2

*dd**dg*) $\mathbf{Q} + \mathbf{K} \mathbf{Q}$ *D**da has -*({ $\mathbf{Mdm} I + (\mathbf{M} + \mathbf{M}$ *dd**dg*) \mathbf{qd}) + $\mathbf{M} \mathbf{Q}$ *da has*} = \mathbf{fLd} ($\mathbf{K} + \mathbf{K}$

2

*Gd**gg*) $\mathbf{Q} + \mathbf{K} \mathbf{Q}$ *D**ga has -*({ $\mathbf{Mgm} I + (\mathbf{M} + \mathbf{M}$ *Gd**gg*) \mathbf{qd}) + $\mathbf{M} \mathbf{Q}$ *ga has*} = - \mathbf{F} *Ld*($\mathbf{K} + \mathbf{K}$

2

*AD**Ag*) $\mathbf{Q} + \mathbf{K} \mathbf{Q}$ *D**aa has -*({ $\mathbf{Mam} I + (\mathbf{M} + \mathbf{M}$ *AD**Ag*) \mathbf{qd}) + $\mathbf{M} \mathbf{Q}$ *aa has*} = \mathbf{fLa} \mathbf{Q} \mathbf{Q} *has**has* = $(1 + A) 2$ *(T* $1 + A) \mathbf{fL} = 0$ *has*

One replaces, in the first four equations of the system \mathbf{qa} by his given expression in before last equation. In addition, the association of the second and the third equation allows to eliminate the terms from the bonding strengths on the right. Lastly, the fourth equation is multiplied

by $(1 + T)$ *has*, which makes it possible to eliminate the terms from the bonding strengths to the axis. One leads

then to one

problem with the eigenvalues final which one can put in the form:

~

~

(- 2)~

K

M Q = 0

éq 4.3.1-9

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With:

I

~

Q = qd

Q

/

has 2

K

mm

0

0

~

K = 0

K

T
T
T

dd + **K** *dg* + **K** *Gd* + **K** *gg*
(Kda + Kga) (1+a)

T
T
0
1

1
1

(+ *has*) **(Kad + Kag)**
(+ *has*) **Kaa** (+ *has*)

M
M
mm
Mandeleivium + **M**

M
Mg
my (1+ A)

~
M = M
T
T
T
T
DM + **M**
M
gm
dd + **M***dg* + **M** *Gd* + **M** *gg*
(Mda + Mga) (1+a)

T
T
T
 1
 1

 1
 1

(+ *has*) **Mam**

(+ *has*) (**Mad** +Mag)

(+ *has*) **Maa** (+ *has*)

Displacements of the axis, divided by two, make it possible to preserve at the matrices of rigidity and of mass their square character. One restores modal complex displacements by the formula following:

Q = [
J
 ~
D + *E*
D 2a] **Q**

4.3.2 Method of Mac Neal

One considers the problem with the eigenvalues of the total structure expressed on the basic sector. This last is thus subjected to the bonding strengths which are applied to him by the contiguous sectors. By

elsewhere, the basic sector checks the equations of connection [éq 4.2.4-2]. We thus have:

(K - 2M) Q = F L

Q
J
G =

E
Q
D
éq 4.3.2-1

F
J
L = -

E
F
L
G
D

The modal base used to reduce dimensions of the problem to be solved, is a modal base with

free interfaces including/understanding of the dynamic modes and the modes of fastener relating to the degrees of freedom of the interfaces right and left. Let us suppose that the degrees of freedom of the basic sector are ordered in the following way:

Q
I
degrésde freedom internal
Q = Q
D degrésde freedom of the right interface
Q

degrésde freedom of the left interface
G

Are **data base** and **Bg**, the rectangular matrices of extraction such as:

Q = B Q and **Q = B Q**
D
D
G
G

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The boundary condition on displacements becomes with these notations:

B Q
J
= E B Q B Q = 0 with **B**
J
= E

B**- B***G**D**dg**dg**D**G***éq 4.3.2-2**

For the forces, the boundary condition becomes:

$$\mathbf{F} = \mathbf{B}\mathbf{T}\mathbf{f} + \mathbf{B}\mathbf{T}\mathbf{f}$$

F*T**- J T T**T**L**D L*

$$L = (\mathbf{B} - E$$

B*G**D) F*

$$= - \mathbf{B} \mathbf{F}$$

*L**G**L**dg L**G**D**G**G*

Let us regard as base, for the transformation of RITZ, the whole of the clean modes dynamic of the basic sector, by distinguishing the identified modes and the unknown modes:

1

$$\mathbf{Q} = [1 \ 2]$$

éq 4.3.2-3

2

where index 1 (resp. 2) refers to the known modes (resp. unknown). In the continuation, us will suppose that the clean modes are normalized with the unit modal mass.

By replacing **Q** by its expression according to the clean modes, and while multiplying on the left by transposed of the matrix of the modes, the matric equations [éq 4.3.2-1] and [éq 4.3.2-2] become:

(

2

$$\begin{aligned}
 & - \\
 &) \\
 & T \\
 & 1 \\
 & \mathbf{Id} \ 1 = 1 \ \mathbf{F} \ L
 \end{aligned}$$

$$\begin{aligned}
 & (\\
 & 2 \\
 & - \\
 &) \\
 & T \\
 & 2 \\
 & \mathbf{Id} \ 2 = 2 \ \mathbf{F} \ L
 \end{aligned}$$

éq 4.3.2-4

$$\mathbf{Bdg11} + \mathbf{Bdg22} = 0$$

where is the matrix of generalized rigidities (the generalized masses are unit).

One can thus draw a formulation from it from 2:

$$\begin{aligned}
 & 2 \\
 & 1 \ T \\
 & 2 = 2 - \\
 & - \\
 & (
 \end{aligned}$$

$$\begin{aligned}
 &) \\
 & \mathbf{Id} \\
 & 2 \ \mathbf{fL} \\
 & \text{éq 4.3.2-5}
 \end{aligned}$$

Consequently, one can eliminate 2 from the system of equations [éq 4.3.2-4]. One obtains the problem then with

eigenvalues according to:

$$\begin{aligned}
 & (\\
 & 2 \\
 & - \\
 &) \\
 & T \ T \\
 & 1 \\
 & \mathbf{Id} \ 1 + 1 \ \mathbf{B} \ \mathbf{F} \\
 & dg \ L = 0 \\
 & G \\
 & \mathbf{B} \\
 & T \ T \\
 & dg \\
 & 2
 \end{aligned}$$

1
-
11 - **Bdg** (
2 - **I**)
2
D
2 **B F**
dg L = 0
G

The final system to solve can be written:

~
~
(- 2)~

K
M Q = 0
éq 4.3.2-6

With:

1
~

Q = F

Lg
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The forms of the matrices of rigidity and mass are:

T T

~
 1
B
 1
dg
 ~
Id
 0
K =
M
T
 =
B

dg
 0
 0
 1
- B R () B

dg
E
dg

The matrix [**R ()**]
E
 is the matrix of residual dynamic flexibility of the not identified modes:

$$\mathbf{R} () = ($$

$$2$$

$$1$$

$$2 2 - \mathbf{Id} -$$

$$T$$

$$E$$

$$) 2$$

One approximates residual dynamic flexibility by his static contribution, by taking of account them modes of fastener. Then, the formula of restitution which makes it possible to calculate the complex values of displacements starting from the generalized co-ordinates of the solutions modes of [éq 4.3.2-6] is following:

$$\mathbf{Q} = [$$

$$T \sim$$

$$1$$

$$- \mathbf{R}(0) \mathbf{B}$$

$$E$$

$$dg] \mathbf{Q}$$

The actual values of displacements are determined, as for the method of Craig-Bampton, by the formulas [éq 4.3.1-5], [éq 4.3.1-6], [and éq 4.3.1-7].

Taking into account of the nodes of the axis:

We suppose, in this paragraph, that the degrees of freedom carried by the nodes of the axis, with even title that the nodes of interfaces right-hand side and left, were the subject of calculations of modes of fastener.

We limit ourselves to $= 0$ who is the only case modified by the taking into account of the nodes of the axis

(§ 4.2.4 and 4.3.1). We thus have:

$$(\mathbf{K} - 2\mathbf{M}) \mathbf{Q} = \mathbf{F} L$$

$$\mathbf{H} \mathbf{Q} = \mathbf{Q}$$

and \mathbf{F}

D

$$L = - \mathbf{F}$$

L

G

D

\mathbf{Q}

éq 4.3.2-7

$= \mathbf{Q}$ has

$$\text{has has } (1 - \text{has}) \mathbf{q} \mathbf{a} = 0$$

\mathbf{F}

\mathbf{F}

T

$$L = - \mathbf{F}$$

$$L \mathbf{F} L \text{ has } =$$

has

has

has

(1 - has) 2

The organization of the degrees of freedom of the basic sector is similar to that of the preceding chapter:

$\mathbf{I} \mathbf{Q}$

degrésde freedom internal

\mathbf{Q}

D
degrésde freedom of the right interface

Q =
Q
G degrésde freedom of the left interface

Q
has
degrésde freedom of the interface centers

Are **Ba** the rectangular matrix of extraction of the degrees of freedom of the axis:

$$\mathbf{Q} = \mathbf{B} \mathbf{Q}$$

has

has

The boundary condition on displacements of the axis becomes with these notations:

$$(1-a) \mathbf{B} \mathbf{a} \mathbf{q} = 0 \quad \mathbf{B} \mathbf{a} \mathbf{a} \mathbf{q} = 0 \quad \text{with } \mathbf{B} \mathbf{a} \mathbf{a} = (\textit{has -}) 1 \mathbf{B} \mathbf{a}$$

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For the forces, the boundary condition becomes:

F

$$\mathbf{F} = \mathbf{B} \mathbf{T} \mathbf{f}$$

$$+ \mathbf{B} \mathbf{T} \mathbf{f} + \mathbf{B} \mathbf{T} \mathbf{f}$$

$$\mathbf{F} = - \mathbf{B} \mathbf{T} \mathbf{F} + \mathbf{B} \mathbf{T}$$

T

L

G L

D L

L has

L

*dg L*1 *has -**G**D**has**G*

(A) 2

F

$$\mathbf{F} = -\mathbf{B}^T \mathbf{F} - \mathbf{B}^T$$

*L**dg L**aa**G*

2

The base of the transformation of RITZ is made up of the whole of the free clean modes of basic sector, by distinguishing the identified modes (index 1) and the unknown modes (index 2) definite by the equation [éq 4.3.2-3].

The equation [éq 4.3.2-7], written in this base takes the following form:

(

2

-

)

T

1

$$\mathbf{Id} 1 = 1 \mathbf{fL}$$

(

2

-

)

T

2

$$\mathbf{Id} 2 = 2 \mathbf{fL}$$

éq 4.3.2-8

$$\mathbf{Bdg11} + \mathbf{Bdg22} = 0$$

$$\mathbf{Baa11} + \mathbf{Baa22} = 0$$

The second equation makes it possible to determine 2 (cf [éq 4.3.2-5]), which can thus be eliminated from

system. One then obtains the problem with the eigenvalues according to:

F

(

2

-

)

TT
TT

1
Id 1 + 1 B F
dg L + 1 Baa
= 0

G
2
F
B
T
T

dg 1 - B
R (
dg
E) B
F

dg L - B
R (
dg
E)
1
Baa
= 0

G
2
F
B
T
T

aa 1 - B
R (
aa
E) B
F

dg L - B
R (
aa
E)
1
Baa

= 0

G

2

Thus, by defining the following unknown vector:

1

~

$$\mathbf{Q} = \mathbf{fL}$$

G

F

/

2

The following final system is obtained:

~

~

(- 2)~

K

$$\mathbf{M Q} = 0$$

éq 4.3.2-9

with:

T T

T T

1

B

1

dg

B

1

aa

Id 0

0

~

T

T

~

K = B
dg1 - B R () B

dg
E
dg
- B R () B

dg
E
aa
M = 0 0
0
B

T
T
aa

0
0 0
1
- B R () B
aa
E
dg
- B R () B

aa
E
aa

Division by two of the bonding strengths applied to the axis makes it possible to preserve at the matrices of rigidity and of mass their square character.

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One restores modal complex displacements by the following formula:

$\mathbf{Q} = [$

T

T

1

$- \mathbf{R}$

\mathbf{B}

$E(0) dg$

$- 2\mathbf{R} (\mathbf{B}$

$E 0)$

\sim

$aa] \mathbf{Q}$

4.4

Implementation in *Code_Aster*

The treatment of the basic sector is identical to that of the substructures in the under-structuring traditional. It utilizes the operators: `MODE_ITER_SIMULT` [U4.52.02] or `MODE_ITER_INV` [U4.52.01], `DEFI_INTERF_DYNA` [U4.55.03] and `DEFI_BASE_MODAL` [U4.55.04].

The clean modes of the structure with cyclic symmetry are calculated by operator `MODE_ITER_CYCL` [U4.52.03] according to the base of projection of the basic sector previously defined and of a number of sectors of the complete structure.

The restitution of the results on physical basis is identical to the traditional under-structuring. It makes to intervene the operator `REST_BASE_PHYS` [U4.64.01] and possibly operator `DEFI_SQUELETTE` [U4.75.01].

5 Conclusion

The principles of under structuring make it possible to expose the transformation of RITZ and modal recombination to lead to the modal synthesis which integrates these two techniques. Rules of liaisonnement between substructures are clarified.

Two methods were developed in *Code_Aster*: that of Craig-Bampton and that of Mac Neal. We present, here, their characteristics, as well in the definition of the initial modal base, that in its exploitation.

After having exposed the definition of a structure to cyclic symmetry and the properties which result

from this,

we presented the methods of cyclic under-structuring implemented in *Code_Aster*. They appear very interesting for the calculation of the clean modes of a structure with cyclic symmetry, such as the rotors of the revolving machines of which they benefit fully from geometrical and mechanical characteristics.

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**Harmonic response by under-structuring
traditional dynamics**

Summary:

After having made some recalls concerning the methods of modal synthesis and having introduced the base of

Harmonic Craig-Bampton, we present the theoretical bases of the methods of calculation of answer harmonic by under-structuring. Initially, we establish the dynamic equations checked by the substructures separately. Then, the taking into account of the conditions of assembly enters

substructures, enables us to determine the dynamic equations checked by the total structure. In private individual, we stick to well highlighting the treatment of the matrix of damping and of vector of the external efforts, which intervene in the harmonic calculation of response per under-structuring.

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1 Introduction

After having developed in *Code_Aster* of the modules of modal calculation by under-structuring of which

the theoretical bases are presented in reference [R4.06.02], the operators of calculation of answer harmonic by modal synthesis were implemented.

The methods of modal synthesis which consist in condensing the degrees of freedom resulting from modeling finite elements on fields of displacement particular to each substructure, translate by important profits into computing times and place memory.

For the harmonic problems, the studied system is subjected to a force spatially unspecified,

but sinusoidal in time. The form of the loading, the frequency of excitation and properties modal, play each one an essential role. It is also necessary to take account of dissipation in solid, which one can translate by the introduction of a matrix of damping. Method of calculation harmonic by under-structuring, programmed in *Code_Aster*, which makes it possible to replace it total problem by a simplified problem, proceeds in four times. First of all, of the clean modes and of the static deformations are calculated on each substructure composing the system. Then, the total problem is projected on these fields, and one takes account of the couplings between substructures, on the level of their interfaces. One can then solve the problem classically tiny room obtained. Finally, it any more but does not remain to deduce the overall solution by reconstitution from it.

General notations:

:

Pulsation (rad.s-1)

J

:

Imaginary pure unit ($j^2 = -$)

1

NS

:

A number of substructures

M

:

Stamp of mass resulting from modeling finite elements

K

:

Stamp rigidity resulting from modeling finite elements

C

:

Stamp damping exit of modeling finite elements

Q

:

Vector of the degrees of freedom resulting from modeling finite elements

F ext.: Vector of the forces external with the system**F L**

:

Vector of the bonding strengths applied to the system

:

:

Stamp vectors of the base of the substructures

:

:

Vector of the generalized degrees of freedom

B

:
Stamp extraction of the degrees of freedom of interface

L

:
Stamp connection

Note:

The exhibitor K characterizes the sizes relating to the substructure S K and the sizes generalized are surmounted by a bar: for example $\bar{\mathbf{M}} K$ is the matrix of generalized mass substructure S K .

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Harmonic response by under-structuring

2.1

Base of harmonic Craig-Bampton

Methods of modal synthesis (cf [R4.06.02]), whatever their applicability

(modal, harmonic and/or transitory) the techniques of under-structuring associate those of modal recombination. To carry out a calculation by under-structuring means that the structure is cut out in several elements and that its displacement is calculated like the response to the bonding strengths which

connect to the other components and the external forces which are applied to him. In addition, response of each substructure is calculated by modal recombination. Thus, all components are defined by a base of projection made up of clean modes and modes of interface. Two bases of projection introduced into *Code_Aster* were presented in the documentation of reference [R4.06.02]:

- bases of Craig-Bampton,
- bases of Mac Neal without/with static correction.

A third base of projection was introduced within the framework of the developments concerning

harmonic calculation of response per traditional dynamic under-structuring. It is about the base of Harmonic Craig-Bampton.

We present this new base of projection using the following simple example:

q^1

q^2

I

I

q^1

2

J

$Q J$

1

Substructure 1

Substructure 2

2

The vector of the degrees of freedom of the substructure is characterized by one

q_k

exhibitor who defines the number of the substructure, and an index which allows

q_k

I

$=$

q_k

to distinguish the degrees of freedom intern (index I), of the degrees of freedom of

J

border (index J).

The harmonic base of Craig-Bampton consists of modes specific to interfaces blocked and of harmonic constrained modes [bib6]. The latter are joined to the normal modes with interfaces blocked to correct the effects due to their boundary conditions. A harmonic constrained mode is defined by the response of the substructure not deadened to a harmonic displacement, of amplitude unit and of frequency given, imposed on a degree of freedom of connection, the other degrees of freedom of connection being blocked.

Substructure 1

Substructure 2

Modes specific to blocked interfaces

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 $Q = E J T Q = E J T$

Substructure 1

Substructure 2

Harmonic constrained modes

This modal base is more particularly appropriate to the problems of interactions fluid-structures, for which the static loadings applicable in *Code_Aster* (are not not taken in count effect of added mass). It can be used for any type of calculation (modal, harmonic and transient).

2.2**Dynamic equations checked by the substructures separately**

We will consider a structure S made up of NS noted substructures $S K$. Us

let us suppose that each substructure is modelled in finite elements. We saw that in one calculation by dynamic under-structuring, the vibratory behavior of the substructures results from external forces which are applied to him, and of the bonding strengths which on them the others exert under

structures. Thus, on the level of the substructure $S K$, we can write:

$$\mathbf{M} \mathbf{k} \mathbf{q} \mathbf{k} + \mathbf{C} \mathbf{k} \mathbf{q} \mathbf{k} + \mathbf{K} \mathbf{k} \mathbf{q} \mathbf{k} = \mathbf{F} \mathbf{K} + \mathbf{F} \mathbf{K}$$

!

!

ext. L **éq 2.2-1**

where:

 $\mathbf{M} \mathbf{k}$ is the matrix of mass resulting from modeling finite elements of $S K$ **$\mathbf{C} \mathbf{k}$** is the matrix of damping resulting from modeling finite elements of $S K$ **$\mathbf{K} \mathbf{K}$** is the matrix of rigidity resulting from modeling finite elements of $S K$ **$\mathbf{F} \mathbf{k} \mathbf{ext}$** is the vector of the external forces applied to $S K$ **$\mathbf{F} \mathbf{k} \mathbf{L}$**

is the vector of the bonding strengths applied to S K

\mathbf{qk} \mathbf{qk} and \mathbf{qk}

, !

!

are the vectors speed displacement and acceleration resulting from modeling finite elements.

In a harmonic problem, one imposes a loading dynamic, spatially unspecified, but sinusoidal in time. One is interested then in the stabilized answer of the system, without holding account transitory part.

The field of the external forces is written:

$\mathbf{F} K (T) = \mathbf{F} K$

$J T$

ext.

{*ext.*} E

The field of the bonding strengths is written:

$\mathbf{F} K (T) = \mathbf{F} K$

$J T$

L

{ $L E$

}

The field of displacements is written:

$\mathbf{qk} () = \mathbf{qk}$

$J T$

T

{

E

}

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The fields speed and acceleration are written:

$$\dot{\mathbf{q}}_k(T) = \mathbf{J} \{ \mathbf{q}_k \} \dot{J} T$$

E

!

$$\mathbf{q}_k(T) = -2 \{ \mathbf{q}_k \} J T$$

E

Finally the substructure S_k checks the following equation:

$$(\mathbf{K}_k + \mathbf{C}_k - 2\mathbf{M}_k) \{ \mathbf{q}_k \} = \{ \mathbf{F}_k \} + \{ \mathbf{F}_k \}$$

J

}

ext.

L

éq 2.2-2

The method of modal synthesis consists in seeking the field of unknown displacement, resulting from modeling finite elements, on an adapted space, of reduced size (transformation of Ritz).

We saw that for each substructure, this space is composed of clean modes

dynamic and of static deformations:

K

q_k[K_k

] I

K

K

=

K =

éq 2.2-3

J

K_k are the modal vectors associated the dynamic clean modes of S_k ,

K_s are the modal vectors associated the static deformations of S_k ,

k_i is the vector of the generalized co-ordinates associated the clean modes of S_k ,

k_j is the vector of the generalized co-ordinates associated the static deformations of S_k ,

K is the vector of the generalized co-ordinates of S_k .

The equation [éq 2.2-2] is projected on the basis of S_k by taking account of [éq 2.2-3]. This allows us to write:

$$(\mathbf{K}_k + \mathbf{C}_k - 2\mathbf{M}_k) \{ K_k \}$$

$$= \{ \mathbf{F}_k \} + \{ \mathbf{F}_k \}$$

J

}

ext.

L

éq 2.2-4

where:

\mathbf{Mk}

$K^T \mathbf{Mk} K$

=

is the matrix of generalized mass of $S K$,

\mathbf{Ck}

$K^T \mathbf{Ck} K$

=

is the matrix of generalized damping of $S K$,

\mathbf{Kk}

$K^T \mathbf{Kk} K$

=

is the matrix of generalized rigidity of $S K$,

$\{\mathbf{F} K\}$

$K^T \{\mathbf{F} K\}$

=

}

is the vector of the generalized harmonic external forces applied

ext.

ext.

with $S K$,

$\{\mathbf{F} K\}$

$K^T \{\mathbf{F} K\}$

=

}

is the vector of the generalized bonding strengths applied to $S K$,

L

L

$\{K\} O$

is the vector of generalized harmonic displacements.

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By supposing that the dynamic clean modes and the static deformations are organized like show the formula [éq 2.2-3] and by considering that the clean vectors associated the modes dynamic are normalized compared to the unit modal mass, the matrices of mass and rigidity generalized take the following form:

K T

K

K

K

K T

K

K

K

Id

M

K

K

M =

K =

K T

Mk K

K T

Mk K

K T

Kk K

K T

K K K

where:

Id is the Identity matrix,

K is the diagonal matrix of the squares of the own pulsations of the base.

It is shown, in the case of the method of Craig-Bampton, that the normal modes and the modes constrained are orthogonal with respect to the matrix of rigidity whose diagonal terms are, as of at the time, null [R4.06.02]. However, this property is not used in the algorithm programmed in *Code_Aster*.

We consider, like type of dissipation, only viscous damping (it is the only one which is supported by the tools for under-structuring in *Code_Aster*). Two methods are usable for to take into account this damping:

- the damping of Rayleigh applied at the elementary level which consists in supposing that stamp elementary damping **This** partner with each finite element of the model is one linear combination of the matrices of elementary mass and rigidity **K E** and **Me**:

$$\mathbf{C} = \mathbf{K} + \mathbf{M}$$

E

E

E

E

E

The matrix of damping then is assembled **Ck** then projected on the basis [éq 2.2-3]:

kT K K

K T

K

K

K

C

C

C =

K T

Ck K

K T

Ck K

- the damping proportional applied to the dynamic clean modes of each substructure. The resulting matrix is thus an incomplete diagonal (one does not know to associate damping proportional to the static deformations):

K

0

Ck =

0

0

2.3**Assembly of the substructures**

After having studied each substructure separately, one proposes to establish the equations which govern their assembly. Let us consider two substructures S_K and S_L connected between them to the level interface S_K

 S_L

. They are represented by their modeling finite elements and it is admitted that their respective grids are compatible. Thus, on the level of the interface, the nodes coincide and their meshes in opposite are identical. Consequently, the law of action-reaction and the continuity of displacements

with the interfaces, which represent the assembly of S_K and S_L , are written:

F K**= - F L** L L $S_K S_L$ $S_K S_L$ **qk K****L = ql** S S $S_K S_L$ *Handbook of Reference**R4.06 booklet: Under-structuring**HP-51/98/015/A***Code_Aster** ®*Version**4.0**Titrate:**Harmonic response by traditional dynamic under-structuring**Date:**08/12/98*

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where:

F kL

is the vector of the bonding strengths applied to the substructure $S K$, the level of $S K S L$

the interface $S K$

Sl

,

qk

is the vector of the degrees of freedom of the interface $S K$

Sl

resulting from modeling

$S K Sl$

finite elements of the substructure $S K$.

Let us introduce the matrices of extraction of the degrees of freedom of the interface $S K$

Sl

:

qk K

$L = \mathbf{B} K K$

L **qk**

$S S$

$S S$

ql K

$L = \mathbf{B} l K$

L **ql**

$S S$

$S S$

By using the equation of projection [éq 2.2-3] and the formulation applied above to both substructures subjected to a harmonic loading, one obtains:

B K

K

K

K

L

{

$$\} = \mathbf{B}l$$

$$L$$

$$L$$

$$\{ \}$$

$$S$$

$$S$$

$$S K Sl$$

That is to say:

$$\mathbf{L}k$$

$$K$$

$$K$$

$$L \{ \} = \mathbf{L}l$$

$$L$$

$$\{ \}$$

éq 2.3-1

$$S$$

$$S$$

$$S K Sl$$

where:

$$\mathbf{L}k$$

is the matrix of connection of the interface $S K$

$$Sl$$

Sk substructure,

$$S K Sl$$

$$\mathbf{L}l$$

is the matrix of connection of the interface $S K$

$$Sl$$

Sl substructure.

$$S K Sl$$

This treatment can be carried out on the level of all the interfaces of the total structure. In particular, it is noted that the work of the bonding strengths is null on the interface $S K$

$$Sl$$

; it is thus null on

total structure:

$$W$$

$$K$$

$$K$$

L
L
= F
Q
+ F
Q
= 0
S K Sl
L
S
S
L
S
S
S K Sl
K
L
S K Sl
K
L

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2.4

Dynamic equations checked by the total structure

The dynamic equations that checks the total structure are:

K1

C1

M1

...

...

..

K K

+ *J*

Ck

2

-

M K

...

...

...

K NS

C NS

Mr. NS

1

F 1 1

ext.

F

L

...

... ..

K

= **F K**

K

+

ext.

F L

... ..

NR

NR

NS F S

F S

ext. L

Which, it is necessary to add the equations of connection (according to [éq 2.3-1]):

K, L

K

K $\mathbf{L} K$ $L \{ \} =$ L L \mathbf{L} $\{ \}$ $S S$ $S K S l$

This system is solved by double dualisation of the boundary conditions [R3.03.01]. Its formulation finale thus utilizes the vector of the multipliers of Lagrange and can be written in the form condensed:

éq 2.4-1

$$(\mathbf{K} + J \mathbf{C} - 2\mathbf{M}) \{ \}$$

$$+ \mathbf{L} \mathbf{T} = \{ \mathbf{F} \}$$

ext.

$$\mathbf{L} \{ \}$$

$$= 0$$

The problem defined by the equation [éq 2.4-1] is symmetrical. In addition, its dimension is given by the number of modes taken into account (dynamic modes and static deformations). One is thus brought to solve a traditional harmonic problem, of reduced size, to which is associated one linear equation of constraint. Its resolution thus does not pose a problem.

2.5

Implementation in *Code_Aster*

2.5.1 Study of the substructures separately

The parameters E and E of the damping of Rayleigh are introduced, if necessary, by operator `DEFI_MATERIAU` [U4.23.01].

The treatments of the substructures are identical to the case of modal calculation [R4.06.02]. Modes clean dynamic are calculated with the operators: `MODE_ITER_SIMULT` [U4.52.02] or `MODE_ITER_INV` [U4.52.01]. The conditions with the interfaces of connection are applied with the operator

`AFFE_CHAR_MECA` [U4.25.01]. Operator `DEFI_INTERF_DYNA` [U4.55.03] allows to define them interfaces of connection of the substructure. Operator `DEFI_BASE_MODAL` [U4.55.04] allows to calculate the base of complete projection of the substructure.

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Operator MACR_ELEM_DYNA [U4.55.05] calculates the generalized matrices of rigidity, mass and possibly of damping of the substructure, as well as the matrices of connection.

The damping of Rayleigh is taken into account by supplementing operand MATR_AMOR.

Damping proportional is introduced by operand AMOR_REDUIT.

The harmonic loading is defined, on the level of the substructure, by the operators

AFFE_CHAR_MECA [U4.25.01] (application of the force on the grid), CALC_VECT_ELEM [U4.41.02]

(calculation of the associated elementary vectors) and ASSE_VECTEUR [U4.42.03] (assembly of the vector of

loading on the grid of the substructure).

2.5.2 Assembly and resolution

As in the case of modal calculation [R4.06.02], the model of the complete structure is defined by operator DEFI_MODELE_GENE [U4.55.06]. Its classification is carried out by the operator NUME_DDL_GENE [U4.55.07]. Matrices of mass, rigidity and possibly of damping generalized of the structure supplements are assembled according to this classification with operator ASSE_MATR_GENE [U4.55.08].

The loadings are projected on the basis of substructure to which they are applied, then assembled starting from classification resulting from NUME_DDL_GENE [U4.55.07] by the operator ASSE_VECT_GENE [U4.55.09].

The calculation of the harmonic response of the complete structure is carried out by the operator DYNA_LINE_HARM [U4.54.02].

2.5.3 Restitution on physical basis

The restitution of the results on physical basis is identical to the case of modal calculation [R4.06.02]. It makes

to intervene the operator REST_BASE_PHYS [U4.64.01] and possibly operator DEFI_SQUELETTE [U4.75.01] (creation of a grid "skeleton").

3 Conclusion

Method of calculation of response harmonic per modal synthesis available in *Code_Aster* rest on that of modal under-structuring, also programmed. It consists in expressing the whole of the equations in a space of reduced size, made up of modes of different substructures, by a method of Rayleigh-Ritz. The definition of these fields is that used for the modal under-structuring and includes/understands normal modes as well as other statics or harmonics. The procedure employed results in a projection of the matrices and the second member on restricted space.

In the document, we presented the bases of this method. We showed

how the projected equations were obtained starting from the problem arising in continuous space. base of harmonic Craig-Bampton which is used for primarily to solve the problems of interaction fluid-structures was presented. The formulation of the conditions of connection was also evoked. We thus obtained the reduced equations, according to the generalized co-ordinates, which allow to solve the modular problem of way, with less costs. In addition, the catch in account of the phenomena of damping was examined. It leads to the introduction into calculation by under-structuring of depreciation of Rayleigh or clean modal depreciation with each substructure.

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Transitory response by under-structuring

traditional dynamics

Summary:

This document presents the theoretical bases of the two methods of calculation of answer transitory by dynamic under-structuring implemented in *Code_Aster*.

The first method consists in carrying out a transitory calculation by under-structuring for which equations of

problem are projected on the bases associated with each substructure. The second method consists with to determine the clean modes of the complete structure by under-structuring and to project on this basis

them

equations of the transitory problem.

In both cases, only the case of an excitation per force imposed on the substructures is currently available.

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1 Introduction

The components of nuclear thermal power station are often of important size, complex geometry and sometimes composed of an assembly of several elements. To model the dynamics of these structures, the tools for vibratory analysis traditional then are badly adapted and it is necessary to have resort to methods of reduction the such techniques of modal synthesis which were developed in *Code_Aster*.

The methods of modal synthesis associate techniques resulting from the under-structuring and modal recombination [bib4]. Thus, the field of study is cut out in several substructures and it vibratory behavior of the complete structure is given according to the characteristics vibratory of each one of them. In addition, each substructure, is represented by a base of particular projection, made up of clean modes and static deformations of interface, on which are projected the equations of the problem ([R4.06.02], [R4.06.03] and [bib4]). From the purely data-processing point of view, these methods have two important advantages. Of one leaves, they allow to limit the size memory necessary to the storage of the sizes used at the time calculation and in addition, the computing times are generally very reduced. From the point of view of the organization of a draft study, the techniques of under-structuring are particularly interesting because they make it possible to validate, stage by stage, the models of the substructures. difficulties related to the modeling of a complex structure can thus be approached separately, it who makes easier from there the resolutions.

Several computational tools by dynamic under-structuring are currently available in *Code_Aster*. They make it possible to carry out modal calculations [R4.06.02] and calculations of answer harmonic [R4.06.03]. Work completed to establish the harmonic calculation of answer by dynamic under-structuring in *Code_Aster*, resulted in defining the treatment of the vector of forces external and of the matrix of viscous damping.

The object of this reference material is to present the theoretical bases of the two methods of transitory calculation of response per dynamic under-structuring available in *Code_Aster*.

first consists in carrying out a transitory calculation by under-structuring for which equations of problem are projected on the bases associated with each substructure. The difficulty lies in double dualisation of the boundary conditions which leads to a matrix of singular mass. For to use the diagram of integration explicit (which requires the inversion of the matrix of mass), it is necessary

thus to modify the treatment of the interfaces in the operator of calculation of the transitory answer. second method consists in determining the clean modes of the structure supplements by under-structuring and to project on this basis the equations of the transitory problem. The stage of restitution on the basis of physical final generalized result must thus take account of this double projection.

The operator of transitory calculation of answer which receives the under-structuring is the operator DYNA_TRAN_MODAL [U4.54.03]. Being based on methods of modal recombination, it was conceived to solve transitory problems in generalized co-ordinates and it is very effective for the problems of big size of which it makes it possible to reduce the number by degrees of freedom.

Of other leaves, it supports the taking into account of localised non-linearities (with the nodes) which one wishes to generalize with the case of the under-structuring.

In this report/ratio, we present the two methods of calculation transitory per under-structuring available in *Code_Aster*, like their implementation.

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General notations:

NR

:

A number of substructures

S

M

:

Stamp of mass resulting from modeling finite elements

K

:

Stamp rigidity resulting from modeling finite elements

C

:

Stamp damping exit of modeling finite elements

Q

:

Vector of the degrees of freedom resulting from modeling finite elements

F

:

Vector of the forces external with the system

ext.

F

:

Vector of the bonding strengths applied to the system

L

:

Stamp vectors of the base of the substructures

:

Vector of the generalized degrees of freedom

B

:

Stamp extraction of the degrees of freedom of interface

L

:

Stamp connection

Id

:

Stamp identity

:

Multipliers of Lagrange

Note:

The exhibitor K characterizes the sizes relating to the substructure S K and the sizes generalized are surmounted by a bar: for example $\bar{\mathbf{M}} K$ is the matrix of generalized mass substructure S K .

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2**Transitory calculation by projection on the basis of under structures****2.1****Dynamic equations checked by the substructures separately**

That is to say a structure S made up of NS noted substructures $S K$. We suppose that each substructure is modelled in finite elements. The vibratory behavior of the substructures results forces external which are applied to him and of the bonding strengths which on them the others exert substructures. Thus, for $S K$, we have:

$$\mathbf{M}_K \mathbf{q}_k + \mathbf{C}_k \dot{\mathbf{q}}_k + \mathbf{K}_K \mathbf{q}_k = \mathbf{F}_K + \mathbf{F}_K$$

!

!

ext. L **éq 2.1-1**

where:

 \mathbf{M}_k is the matrix of mass resulting from modeling finite elements of $S K$, **\mathbf{C}_k** is the matrix of damping resulting from modeling finite elements of $S K$, **\mathbf{K}_K** is the matrix of rigidity resulting from modeling finite elements of $S K$, **\mathbf{F}_{kext}** is the vector of the external forces applied to $S K$, **\mathbf{F}_{kL}** is the vector of the bonding strengths applied to $S K$, **\mathbf{q}_k and $\dot{\mathbf{q}}_k$**

, !

!

are the vectors displacement, speed and acceleration resulting from modeling

finite elements.

The field of unknown displacement, resulting from modeling finite elements, is required on a space adapted, of reduced size (transformation of Ritz) according to the formula:

$$\mathbf{q}_k$$

$$K K$$

$$=$$

éq 2.1-2

where:

K
is the vector of the generalized co-ordinates of $S K$,

K
is the matrix containing the modal vectors associated the dynamic clean modes and with the static deformations of interface of $S K$.

The transformation of Ritz [éq 2.1-2], applied to the transitory dynamic equation of the substructure [éq 2.1-1], allows to write:

$$\mathbf{M} K K$$

$$+ \mathbf{C} k K$$

$$+ \mathbf{K} K K$$

$$= \mathbf{F} K + \mathbf{F} K$$

$$!$$

$$!$$

ext.

$$L$$

éq 2.1-3

where:

$$\mathbf{M} k$$

$$K T \mathbf{M} k K$$

$$=$$

is the matrix of mass generalized of $S k$,

$$\mathbf{C} k$$

$$K T \mathbf{C} k K$$

$$=$$

is the matrix of generalized damping of $S K$,

$$\mathbf{K} K$$

$$K T \mathbf{K} k K$$

$$=$$

is the matrix of generalized rigidity of $S K$,

$$\mathbf{F} K$$

$$K T$$

$$= \mathbf{F} K$$

is the vector of the generalized external forces applied to $S K$,

ext.

ext.

F K

K T

= **F K**

is the vector of the generalized bonding strengths applied to *S K*,

L

L

K K and *K*

, !

!

are the vectors generalized displacement, speed and acceleration.

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The problem defined by the equation [éq 2.1-3] is symmetrical. In addition, its dimension is given by the number of modes taken into account (dynamic modes and static deformations). One is thus brought to solve a traditional transitory problem but of reduced size.

2.2

Assembly of the substructures

After having studied each substructure separately, one proposes to establish the equations which govern their assembly. Let us consider two substructures *S K* and *Sl* connected between them to the level interface *S K*

Sl

. It is admitted that their respective grids are compatible [R4.06.02]. Thus, with level of the interface, the nodes coincide and the meshes in opposite are identical. Consequently, the law of action-reaction and the continuity of displacements to the interfaces, which represent the assembly of *S K*

and *Sl*, are written:

F K
 = - **F** L
 K
 L
 L
 L
Q K
 $L = \mathbf{Q} K$
 L
 $S K S L$
 $S K S l$

$S S$
 $S S$
 where:

F kL
 is the vector of the bonding strengths applied to the substructure $S K$, the level of $S K S L$

the interface $S K$
 $S l$

·
qk
 K
 L
 resulting from modeling
 $S K S l$

is the vector of the degrees of freedom of the interface S
 S
 finite elements of the substructure $S K$.

Let us introduce the matrices of extraction of the degrees of freedom of the interface $S K$
 $S l$

:
qk K
 $L = \mathbf{B} K K$
 $L \mathbf{qk}$
 $S S$
 $S S$
ql K
 $L = \mathbf{B} l K$
 $L \mathbf{ql}$
 $S S$

S

By using the transformation of Ritz and the formulation applied above to the two substructures, one obtains:

\mathbf{Bk}

K

K

L

$= \mathbf{B1}$

L

S

S

$S K Sl$

That is to say:

\mathbf{Lk}

K

K

L

$= \mathbf{L1}$

L

S

S

$S K Sl$

where: \mathbf{Lk}

K

L

of Sk .

$S K Sl$

is the matrix of connection associated with the interface S

S

2.3

Dynamic equations checked by the total structure

The matric writing of the dynamic equation checked by the total structure, is simply written to leave dynamic equations checked by each substructure:

$\mathbf{M1}$

1

$! \mathbf{C1}$

1

$! \mathbf{K1}$

1 1

F

1

F

ext.

L

...

...

..

...

...

... ..

K

K

K K K

Mk

!+

Ck

!+

K K

= **F**

+ **F**

ext. L

...

...

...

...

...

... ..

MR. NR
NR

NR
NR
NR
S
S

!
C NR

!
K NR
S
S
S

NS F S
S
ext.
fL

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Which, it is necessary to add the equations of connection:

K, L

K

K

L

$= L$

L

K

L

K

L

$L K$

L

$F L$

$= - F$

$S S$

$S S$

L

$S K S l$

$S K S L$

This system can be written in the condensed form:

$M! + C! + K = fext + fL$

éq 2.3-1

$L = 0$

éq 2.3-2

K, L

K

L

$F L$

$= - fL$

éq 2.3-3

$S K S L$

$S K S l$

2.4

Double dualisation of the boundary conditions

The condensed problem, given above, arises in the form of a transitory system to which is associated a linear equation of constraint (in force and displacement). In *Code_Aster*, this type of problem is traditional and it is solved by double dualisation of the boundary conditions [R3.03.01], i.e. by the introduction of auxiliary variables still called multipliers of Lagrange for dualiser boundary conditions. After introduction of the multipliers of Lagrange, the system matrix puts itself in the form:

0

0

0!

0

0 0! - **Id** **L****Id** 0

1

1

1

*T**T*0 **M** 0! + 0 **C** 0! + **L****K****L**= **F***ext.*

éq 2.4-1

0

0

0

!

0

0 0

!

Id L**- Id**

0

2

2

2

where: 1 and 2 is the multipliers of Lagrange.

It is noted, the introduction of the multipliers of Lagrange makes singular the matrix of mass. As of at the time, the use of the diagram of integration explicit developed in operator DYNA_TRAN_MODAL [U4.54.03] of *Code_Aster* is impossible because they require the inversion of the matrix of mass. To make the matrix nonsingular, it is enough to dualiser with the same multipliers of Lagrange, the condition on the derivative second of the equations of connection.

Thus the condition of continuity of displacements [éq 2.3-2] is modified by the equivalent system [éq 2.4-2]:

L (+**!) = 0****T L = 0****éq 2.4-2****L °=****0 and L! °=**

0

where

° and! ° is initial generalized displacement and speed.

The matric system which results from this is form [bib7]:

- Id L**Id! 0 0 0!**

1

1

- Id L**Id 0**

1

T**T**

T
T

L
M
L! + 0 C
0! + L
K
L = fext éq 2.4-3

Id L - Id! 2 0 0 0! 2 Id L - Id2 0
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It is noted that the matrix of mass to the same form as the matrix of stiffness. It is thus invertible. This system is thus perfectly equivalent to the equation [éq 2.4-1] (it checks at any moment the conditions of connection) and it can be treated, in this form, by operator DYNA_TRAN_MODAL.

2.5

Treatment of the matrix of damping

It is noted that the condition of continuity of displacements, formulated in the equation [éq 2.4-2], translated by an equation of the second order not deadened. At the time of the resolution by step of time of one

transitory problem, any numerical error is likely car-to discuss, thus decreasing the stability of the algorithm. To optimize the damping of the numerical error, it is enough to dualiser the condition on derived first from the equations of connection with the same multipliers of Lagrange multiplied by 2

(so as to make this damping critical):

$$\mathbf{L} (\dot{\mathbf{x}} + 2\mathbf{x}) = \mathbf{0}$$

$$T \mathbf{L} = \mathbf{0}$$

éq 2.5-1

$$\mathbf{L} \dot{\mathbf{x}} = \mathbf{0}$$

0 and $\mathbf{L} \dot{\mathbf{x}} = \mathbf{0}$

0

The matrix system which results from this is form [bib7]:

$$-\mathbf{Id} \mathbf{L}$$

$$\mathbf{Id} \dot{\mathbf{x}}$$

$$1$$

$$\dots$$

$$2 \mathbf{Id}$$

$$\dots$$

$$2 \mathbf{L}$$

$$\dots$$

$$2 \mathbf{Id} \dot{\mathbf{x}}$$

$$1$$

$$-\mathbf{Id} \mathbf{L}$$

$$\mathbf{Id} \dot{\mathbf{x}}$$

$$1$$

$$T$$

$$T$$

$$T$$

$$T$$

$$T$$

$$T$$

$$\mathbf{L}$$

$$\mathbf{M}$$

$$\mathbf{L} \dot{\mathbf{x}} + .2\mathbf{L}$$

$$\mathbf{C}$$

$$\dots$$

$$2 \mathbf{L} \dot{\mathbf{x}} + \mathbf{L}$$

$$\mathbf{K}$$

$$\mathbf{L} = \mathbf{fext}$$

Id L - Id! 2 .2Id .2L - .2Id! 2 Id L - Id2 0

It is thus noted that the treatment of the numerical error on the equations of connection results in modifying

the matrix of damping of the transitory problem. This modification is completely comparable with that who is carried out on the matrix of mass.

On the other hand, we did not wish to generalize this treatment the case of the resolution of the problems not deadened transients. That would have led us to create a matrix of temporary damping. One could have feared to increase the computing times, without real benefit. Moreover, it is completely possible with the user to define a matrix of damping whose coefficients are null.

modification of this one being automatic, the transitory system not deadened will be actually solved, while optimizing the treatment of any numerical error intervening on the equations of connection.

2.6

Treatment of the initial conditions

Let us consider a substructure $S K$ characterized by its base of projection K made up of modes normal and of static deformations. It is supposed that initially the substructure $S K$ is subjected to one field of displacement or speed (that does not modify of anything the demonstration) noted: q_k^0 .

transformation of Ritz enables us to write:

q_k

$K K$

$O = O$

where: KB is the vector of displacements (or speeds) generalized (E) S of $S K$ to be determined.

The vector of displacements (or speeds) generalized (E) S initial (ales) is given as follows:

q_k

K

K

$K T$

=

$\Rightarrow q_k$

$K T$

K

K

O

O

$O = ($

$) . o$

éq 2.6-1

K

$K T$

K

$K T$

$\Rightarrow =$

-

0 1 qk

O

O

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3**Transitory calculation on a modal basis calculated by under structuring****3.1 Calculation of the clean modes of the structure supplements by under-structuring**

The second developed method consists in solving the transitory problem on the basis of modal complete structure calculated by under-structuring.

Each substructure S_K is represented by a base of projection, composed of clean modes dynamic and of static deformations, which we noted: K . The base of projection of structure supplements which results from it is noted: \cdot .

The modal base of the complete structure is calculated by under-structuring. Each mode obtained is thus linear combination of the vectors of the bases of projection of the substructures:

 NS K K $p = =$ **éq 3.1-1** $K = 1$

where:

is the matrix of the clean modes of the complete structure,

 p

is the matrix of the generalized modal co-ordinates of the structure.

Projection of the matrices and the vectors constitutive of the transitory problem, on the basis of mode clean of the complete structure calculated by modal synthesis allows to determine:

- the matrix of generalized mass:

 $\mathbf{M} = \mathbf{T}\mathbf{M}$

- the matrix of generalized rigidity:

$$\mathbf{K} = T \mathbf{C}$$

· possibly the matrix of generalized damping:

$$\mathbf{C} = T \mathbf{C}$$

· the vector of the generalized external forces:

\mathbf{F}

T

$= \mathbf{F}$

ext.

ext.

Because of orthogonality of the clean modes of the structure calculated by modal synthesis, report/ratio with the matrices \mathbf{M} and \mathbf{K} , the matrices of generalized mass and rigidity obtained above are diagonals:

$$\mathbf{M} = T \mathbf{M} = ($$

T

$$) \mathbf{M} = T \mathbf{M}$$

p

p

éq 3.1-2

$$\mathbf{K} = T \mathbf{K} = ($$

T

$$) \mathbf{K} = T \mathbf{K}$$

p

p

3.2

Dynamic equation checked by the total structure

The complete structure is subjected to the external forces which are applied to him. Thus, we can to write:

$\mathbf{M} \ddot{\mathbf{q}}$

$$+ \mathbf{C} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{f}_{ext}$$

éq 3.2-1

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The field of unknown displacement, resulting from modeling finite elements, is replaced by its projection on the basis of clean mode of the structure, according to the formula:

$$\mathbf{Q} = \mathbf{p}$$

éq 3.2-2

where: is the vector of the generalized co-ordinates of the structure.

The transformation of Ritz [3.2-2], applied to the transitory dynamic equation of the structure [3.2-1], allows to write:

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{C}\dot{\mathbf{Q}} + \mathbf{K}\mathbf{Q} = \mathbf{f}_{ext}$$

éq 3.2-3

The stage of restitution on physical basis requires to take account of the double projection (on the basis modal of the complete structure, then on the basis of projection of the substructures - cf éq 3.2-4).

$$\mathbf{Q} = \mathbf{p} =$$

éq 3.2-4

The problem defined by the equation [éq 3.2-3] is of completely traditional form. One is brought to solve a symmetrical transitory problem whose dimension is determined by the number of calculated modes by under-structuring and whose matrices of mass and rigidity are diagonal.

Let us note finally that the treatment of the initial conditions is identical to the case of transitory calculation by

projection on the basis of substructure (cf § 2.6).

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4

Comparative study of the two developed methods

Theoretical bases, associated the two methodologies implemented in *Code_Aster* for to carry out a transitory calculation of response by using the techniques of under-structuring, were presented in the preceding chapters. We specify, here, their essential characteristics.

The first methodology consists in making a calculation of transitory response per under-structuring. The equation checked by the complete structure is then projected on the basis of substructure. precision of this method thus is directly determined by the extent of these bases. These last can be enriched without leading to prohibitory computing times because the substructures are, in theory, relatively reduced sizes. At all events, it is difficult to estimate the effect of modal truncation with the only knowledge of the modes of the bases of the substructures. In addition, them bases of projection of the substructures are made up of modes which all are not orthogonal between them (clean modes and static deformations). Matrices of generalized mass and rigidity constitutive of the final transitory problem are thus not-diagonals. All in all, their width of band can be given starting from the number of static deformations of the bases of projection of substructures [R4.06.02]. Duration of integration in the operator of transitory calculation DYNA_TRAN_MODAL will be thus all the more long as there will be degrees of freedom of interface. Of other

leaves, the step of acceptable time of integration maximum by the diagram of integration explicit is determined starting from the maximum frequency of the base of projection. In the case of a calculation transient by under-structuring, this frequency results, in theory, of the static modes of which them diagonal terms are high in the matrix of rigidity generalized and weak in the matrix of mass generalized. Consequently, the step of time of integration cannot be a priori given. The experiment shows that it is very weak, taking into consideration Eigen frequency of the bases of under structures and that the use of the diagram of integration to step of adaptive time of DYNA_TRAN_MODAL is very advantageous.

The second methodology consists in making a transitory calculation on the basis of modal structure supplements obtained by under-structuring. It is known that the stage consisting in calculating the clean modes structure can be expensive in term of computing time. This is all the more true when one consider nonlinear forces because the base of projection must then be sufficiently extended for to represent the dynamics of the system well. In addition, the modal base on which is calculated transitory answer is of size lower than that determined by the clean vectors of under structures (clean modes and static deformations). It thus does not constitute a generating system. Double projection thus amounts introducing a cut-off frequency. One must thus expect it that this method is less precise than the preceding one. However, the calculation of the clean modes allows to estimate the effect of modal truncation. In addition, it can make it possible to validate the models of under-structures if one has experimental results. Finally, essential interest of this method is that the matrices of mass and rigidity used in transitory calculation are diagonals. Numerical integration is thus very fast.

To conclude, it is noted that transitory calculation by under-structuring is identified with the methods direct of transitory calculation. One does not have access to modal information and the matrices are not diagonals. In this case, one can say that the vectors of the bases of projection of the substructures play the same part that the functions of form of the finite elements. Transitory calculation on basis modal calculated by under-structuring is identified, as for him, with the methods of modal recombination traditional.

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5

Implementation in Code_Aster

5.1

Study of the substructures separately

If one wishes to introduce a damping of Rayleigh, the parameters E and E of this damping are defined, by operator `DEFI_MATERIAU` [U4.23.01].

The treatments of the substructures are identical to the case of modal calculation [R4.06.02] and harmonic

[R4.06.03]. The dynamic clean modes are calculated with the operators: `MODE_ITER_SIMULT` [U4.52.02] or `MODE_ITER_INV` [U4.52.01]. The conditions with the interfaces of connection are applied

with operator `AFFE_CHAR_MECA` [U4.25.01].

Operator `DEFI_INTERF_DYNA` [U4.55.03] allows to define the interfaces of connection of substructure. Operator `DEFI_BASE_MODAL` [U4.55.04] allows to calculate the base of projection substructure supplements (recopy of the clean modes and calculation of the static deformations).

Operator `MACR_ELEM_DYNA` [U4.55.05] calculates the generalized matrices of stiffness, mass and possibly of damping of the substructure, as well as the matrices of connection.

The damping of Rayleigh is taken into account by supplementing operand `MATR_AMOR`.

Damping proportional is introduced by operand `AMOR_REDUIT`.

The transitory loading is defined, on the level of the substructure, by the operators

AFFE_CHAR_MECA [U4.25.01] (application of the force on the grid), CALC_VECT_ELEM [U4.41.02]

(calculation of the associated elementary vectors) and ASSE_VECTEUR [U4.42.03] (assembly of the vector of

loading on the grid of the substructure).

The operator AFFE_CHAM_NO [U4.26.01] who allows to affect a field on the nodes of a model the field of initial displacement ou/et makes it possible to describe the initial field speed of the substructure.

5.2

Assembly of the generalized model

As in the case of modal calculation [R4.06.02] and harmonic [R4.06.03], the model of the structure supplements is defined by operator DEFI_MODELE_GENE [U4.55.06]. Its classification is carried out by

operator NUME_DDL_GENE [U4.55.07]. Matrices of mass, stiffness and possibly of damping generalized of the structure supplements are assembled according to this classification with operator ASSE_MATR_GENE [U4.55.08].

The loadings are projected on the basis of substructure to which they are applied, then assembled starting from classification resulting from NUME_DDL_GENE [U4.55.07] by the operator ASSE_VECT_GENE [U4.55.09].

Initial generalized displacements and initial speeds generalized for each substructure, are calculated by operator ASSE_VECT_GENE [U4.55.09]. This operator also realizes assembly of these vectors according to classification resulting from NUME_DDL_GENE [U4.55.07].

In the case of a transitory calculation projected on the “bases” of the substructures, the matrices and vectors

assembled generalized obtained with resulting from this stage are directly used for calculation transient. In the case of a calculation on the basis of complete modal structure calculated by under-structuring, it is necessary to carry out specific operations which are presented at the § 5.3.

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5.3**Calculation of the modal base of the complete structure and projection**

This chapter is specific to transitory calculation on modal basis calculated by under-structuring. The modal base of the complete structure is calculated with the traditional operators of *Code_Aster*: `MODE_ITER_SIMULT` [U4.52.02] or `MODE_ITER_INV` [U4.52.01]. One defines a classification of final problem generalized with operator `NUME_DDL_GENE` [U4.55.07]. Matrices of mass, of stiffness and possibly of damping generalized is projected on the basis of clean mode structure with operator `PROJ_MATR_BASE` [U4.55.01]. Generalized vectors corresponding with the external loadings are projected on the basis of clean mode of the structure with operator `PROJ_VECT_BASE` [U4.55.02].

5.4**Resolution and restitution about physical base**

The calculation of the transitory response of the complete structure is carried out by the operator `DYNA_TRAN_MODAL` [U4.54.03].

The restitution of the results on physical basis utilizes operator `REST_BASE_PHYS` [U4.64.01]; it is identical to the case of modal calculation [R4.06.02] and harmonic [R4.06.03]. One can use operator `DEFI_SQUELETTE` [U4.75.01] to create a grid “skeleton”. Coarser than it grid of calculation, it makes it possible to reduce the durations of the graphic treatments.

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6 Conclusion

We presented, in this report/ratio, work completed to introduce, in *Code_Aster*, it calculation of transitory response linear per dynamic under-structuring. The methods which were chosen consist, for the first of them, to project the transitory equations on the “bases” of each substructure, made up of dynamic clean modes and static deformations and for the second, to calculate the clean modes of the structure supplements by under-structuring and with y to project the transitory equations.

We begin with a talk from the theoretical bases on which are based the first method of transitory under-structuring to lead to the matrix formulation of the final problem. In private individual, an original treatment of the equation of continuity of displacements to return the matrix

of invertible mass and to ensure an optimal stability of the algorithm of integration, led us to modify the shape of the matrices of mass and damping of the transitory problem.

For the second method, the essential difficulty consists in restoring the results obtained in co-ordinates generalized on the physical basis. Indeed, it is necessary to take account of the double projection:

on the basis of modal structure supplements on the one hand, and on the basis of substructure of other leaves.

The developments carried out resulted in modifications of the operators

DYNA_TRAN_MODAL [U4.54.03] and REST_BASE_PHYS [U4.64.01]. Their syntax was modified very little,

so that their use is identical during a calculation by under-structuring and of a calculation direct by modal recombination.

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Titrate:
Modeling of the turbulent excitations

Date:
05/04/05
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Organization (S): EDF-R & D /MFTT, CS

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R4.07 booklet: Coupling fluid-structure
Document: R4.07.02

Modeling of the turbulent excitations

Summary:

One describes the modeling of the turbulent excitations available in Code_Aster and the way in which these last are taken into account in a calculation of dynamics. The turbulent excitations are characterized

*by a spectral concentration of efforts, specified using operator **DEFI_SPEC_TURB** [U4.44.31]. Their catch in count in a calculation of dynamics is done by projection of the spectrum on the basis of modal structure of which one wants to calculate the answer. The operations of projection are carried out using the operator **PROJ_SPEC_BASE** [U4. 63.14].*

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1

Principle of calculation

1.1 Determination of a modal base of the system under flow and projection of the excitation

The calculation of the dynamic response of a system to a turbulent excitation induced by a flow fluid is carried out by respecting the following stages:

1) initially, one calculates the modal base of the system except flow using operator `MODE_ITER_SIMULT` [U4.52.03],

2) one defines then the characteristics of the studied configuration, for taking into account of phenomenon of coupling fluid-structure, using operator `DEFI_FLUI_STRU` [U4.25.01].

This operator allows for example to inform the profiles speed associated with the zones of fluid excitation, for configurations of the type "beam of tubes under flow transverse". It produces a concept of the type `[type_flui_stru]` intended to be used by operators implemented downstream in the command file,

the 3) modal characteristics of the system under flow are then calculated using operator `CALC_FLUI_STRU` [U4.66.02]. One has at exit a modal base for each rate of flow,

4) the definition of the turbulent excitation is done then by a call to the operator `DEFI_SPEC_TURB` [U4.44.31]. Modelings available are as follows:

spectra of the type "length of correlation", specific of the configurations of the type "beam of tubes under transverse flow", for the application to the vibrations of tubes of Steam Generator. The key words corresponding factors are `SPEC_LONG_COR_1`, `SPEC_LONG_COR_2`, `SPEC_LONG_COR_3` and `SPEC_LONG_COR_4`. These spectra are

preset; however, the user can adjust the parameters of them. This part is developed with the paragraph [§2.2],

model of turbulent excitation distributed. The key word factor corresponding is SPEC_FONC_FORME. The spectrum of excitation is defined by its decomposition on one family of functions of form while providing, on the one hand a matrix interspectrale, and in addition a list of functions of form associated with this matrix. Concepts [interspectre] and [function] associated must be generated upstream. In the case component "control rod", the user can also use a spectrum of turbulence preset, identified on model GRAPPE1. This part is developed with paragraph [§2.3],

model of localised turbulent excitation. The key word factor corresponding is SPEC_EXCI_POINT. It is used in the case of a spectrum of excitation associated with one or several specific forces and moments. The definition of the excitation is done then in providing:

- a matrix interspectrale of excitations (the concept [interspectre] associated must be generated upstream),

- the list of the nodes of application of these excitations,

- the nature of the excitation applied of each one of these nodes (force or moment),

- directions of application of the excitations thus defined.

This part is developed with the paragraph [§2.4].

5) The projection of the spectrum of turbulent excitation previously definite, on the basis of modal structure under flow, is then carried out using operator PROJ_SPEC_BASE [U4.63.14].

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1.2 Calculation of the response to the turbulent excitation

: resolution

frequential

1.2.1 Introduction

The calculation of the frequential response of the structure or the system coupled fluid-structure is done in three stages:

- 1) calculation of the interspectres of modal excitations,*
- 2) calculation of the interspectres of modal answer,*
- 3) recombination on the physical basis.*

Initially, one introduces for each mode the transfer transfer function of the mechanical system (structure alone or system coupled fluid-structure). Each of the three stages above is then detailed.

1.2.2 Calculation of the interspectres of modal excitations

Interspectres of modal excitations S

($F U$

$Q_i Q_j$

,) are determined by projection of the spectrum

of turbulent excitation on the basis of modal system mechanical (structure alone or coupled system fluid-structure). This stage of projection is detailed in paragraph [§2] for the various models applicable to telegraphic structures.

1.2.3 Calculation of the interspectres of modal answer

Interspectres of modal displacements S

($F U$

$q_i q_j$

,) result then from the interspectres

modal excitations S

($F U$

$Q_i Q_j$

,) using the following relation:

S

Q Q

$$(F, U) = H_i(F, U) S Q_i Q_j(F, U) H_j(F, U) \quad \text{éq}$$

1.2.3-1

I J

where $H_i(F, U)$ indicates the combined complex of the transfer function $H(F, U)$

I
,) of the system

mechanics considered. Being given a frequency F and a rate of flow U , the function of transfer $H(F, U)$

I
,) of the mechanical system for mode I is defined by:

$$H(F, U)$$

I
) =

$$\text{éq}$$

1.2.3-2
F 2
F

$$M$$

$$2$$

$$- + 2 J$$

I I
+

I
F
I

F

I
I

where M_i indicates the modal mass of mode I , and F

I
 I indicate respectively, at the speed U ,

pulsation and the Eigen frequency of mode I, I indicates, at the speed U, the reduced damping of mode I, and J indicates the complex number such as $J^2 = -1$.

The calculation of the interspectres of modal displacements starting from the interspectres of modal excitations

and of the transfer transfer functions is carried out using operator DYNASPEC_MODAL [U4.53.23].

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One deduces in particular from [éq 1.2.3-2] the relation binding the autospectres modal displacements to

autospectres of modal excitations:

S

2

$q_i q_i (F, U) = H_i (F, U) S Q_i Q_i (F, U)$

éq

1.2.3-3

2

where $H (F U$

I

,) the square of the module of $H (F U$ indicates

I

,)

1.2.4 Recombination on physical basis

Being given a rate of flow U , the interspectre of physical displacement Known $U(X, X, F$

1

2

)

1 2

at the points of X -coordinates X and X

1

2, at the frequency F , is obtained by modal recombination. This

operation is written:

NR NR

S

$$(X, X, F) = (X) (X) S$$

J

2

Q Q (F, U

U U

I

)

éq

1.2.4-1

1 2

1

2

1

I J

I 1

= J 1

=

Where NR indicates the number of modes of the base; $I(xk)$ is the component at the point of discretization

xk of the deformation of the i ème mode following the direction of space considered.

The recombination on physical basis is carried out using operator REST_SPEC_PHYS

[U4.63.22]. The direction of space considered is specified at the time of the call to this operator.

1.2.5 Elements

statistics

The modal variance $2()$

$I U$, associated at the speed U , is expressed as follows:

2i U

() = 2 S

Q Q (F, U) df

éq

1.2.5-1

I J

0

At the rate of flow U, value RMS

()

RMS X of response in an item X of the structure is data by:

NR

(X)

2

= (X) 2

(U)

RMS

I

I

éq

1.2.5-2

I I

=

Where NR indicates the number of modes of the base and ()

I X is the component as in point X of

deformation of the ième mode following the direction of space considered.

This operation is carried out by operator POST_DYNA_ALEA [U4.84.04].

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1.3

Calculation of the response to the turbulent excitation: temporal resolution

The temporal resolution proceeds according to the sequence of the following operations:

1.3.1 Factorization of the density interspectrale

Operator FACT_INTE_SPEC [U4.36.04] carries out the factorization of the density interspectrale modal excitations S

($F U$

$Q_i Q_j$

), before application of the method of Monte Carlo.

1.3.2 Generation of the random modal excitations

Operator GENE_FONC_ALEA [U4.36.05] generates random modal excitations $Q(T)$

I

in

carrying out pullings by the method of Monte Carlo. Operator RECU_FONCTION [U4.32.03] allows to recover each evolution $Q(T)$

I

.

1.3.3 Modification of a modal base and projection

Operator MODI_BASE_MODAL [U4.66.21] modifies the modal base of the structure in substituent with

initial characteristics those obtained for a rate of flow considered.

Operator PROJ_MATR_BASE [U4.63.12] allows the projection of the matrices of mass and stiffness assembled on the new modal basis previously definite.

1.3.4 Definition of the obstacles

The definition of the geometry of the obstacles is carried out, if necessary, using the operator DEFI_OBSTACLE [U4.44.21].

1.3.5 Resolution

dynamics

Transitory dynamic calculation for mode I ($1 I NR$) is carried out using a diagram of numerical integration with operator DYNA_TRAN_MODAL [U4.53.21].

$$M \ddot{q}(T) + C \dot{q}(T) + K Q(T) = Q(T)$$

II I

II I

II I

I

) éq

1.3.5-1

Where M , C and K

II

II

II indicate respectively the generalized mass, damping and stiffness associated the $i^{\text{ème}}$ mode; $Q(T)$ and $Q(T)$

I

I

displacement and the excitation indicate respectively generalized associated the $i^{\text{ème}}$ mode.

1.3.6 Projection of Ritz

The restitution on physical basis is carried out using a projection of Ritz:

NR

$$U(X, T) = U(X) Q(T)$$

I

I

éq

1.3.6-1

$i=1$

$U(X, T)$ indicates the assembled vector of physical displacements; $U(X)$

I

is the assembled vector

defining the $i^{\text{ème}}$ modal form and $Q(T)$

I

generalized displacement following the $i^{\text{ème}}$ mode.

This last operation is carried out using operator `REST_BASE_PHYS [U4.63.21]`.

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2
Models of turbulent excitation applicable to the structures telegraphic

2.1 Principles
Generals

2.1.1 Assumptions

One supposes that the linear excitation induced on the telegraphic structure by turbulence of the flow can be modelled in the form of a stationary process random ergodic Gaussian of null average. This turbulent excitation thus is entirely characterized by its density interspectrale $S(X, X$

F

1

2), where X and X

1

2 are two unspecified points of the beam and indicates

the pulsation. The turbulent excitation applied to the structure is thus characterized by its density interspectrale $S F$.

Moreover, one supposes that the turbulent forces are independent of the movement of the structure. The turbulent excitation is identified in experiments on a model of reference. It is then applicable to any real component in geometrical similarity with the model of reference.

2.1.2 Calculation of the interspectres of modal excitations

One indicates by $F(X S$

T

,) linear density of turbulent excitation exerted on the beam; X is

the current X -coordinate of a point of the beam and S the complex pulsation (variable of Laplace).

They are done

additional assumptions following H1 and H2:

H1. The excited length is lower than the overall length L of the beam.

H2. The expression of F (X S

T

,) does not depend on the origin of the excited zone Xe; that is translated

by F (X, S) = F (X - X, S

T

T

E

).

In this case, one can express the linear density ft in the following form:

1

D D

F (X, S) =

U 2

D C,

,

, S

T

F

, Re

éq

2.1.2-1

2

D

L

R

H

E

X - X

sD

UD

with: =

E

S =

=
L
R
Re

U

E

Where the density of the fluid indicates, U is the mean velocity of flow of the fluid, D and Dh are respectively the diameter of the structure and the hydraulic diameter, C F represents it adimensional coefficient of turbulent force, X is the current X-coordinate of a point of the beam, Xe indicate the X-coordinate of the origin of the excited zone, represents It the excited length, is the variable of space reduced, S is the complex pulsation (variable of Laplace), Sr is the complex pulsation reduced, is the kinematic viscosity of the fluid, finally "Re" indicates the Reynolds number.

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By geometrical assumption of similarity of the real component with the model of reference, one obtains:

*I
F (X, S) =
U 2*

D C (, S

T
F
R, Re)
éq
2.1.2-2
2

Thus, the modal turbulent excitation Q (S)
I
can be written in the field of Laplace (assumption H2):

X + L
E
E
I
Q (X) =
F

(X, S) (X) dx = L F
(L, S) (L
+ X) D
I
T
I
E
T
E
I
E
E

éq
2.1.2-3
Xe
0

where ()
I X is the component of the ième modal deformation according to the direction of space in which
acts the turbulent excitation.

By means of the expression [éq 2.1.2-2], one deduces:

I
I

$$Q(S) = U^2 DL C$$

$$(\rho, S, Re) (L + X) D$$

I

E

F

R

I

E

E

éq

2.1.2-4

2

0

The densities interspectrales of modal turbulent excitations are expressed then in the form:

1

2

1 1

2

D

S

$$(F, U) = U DL$$

$$T(\rho, F$$

1

2

$$R, Re) (L$$

$$X) (L$$

$$X) D D$$

$$Q Q$$

E

2

+

+

U

I

1 E

E
J
2nd
E
I J
1
2
0 0
éq 2.1.2-4

with
1 I, J NR, where NR is the number of modes selected to determine the answer of structure;

T: interspectre of cf enters and
1
2 ;

fD
Fr =
: reduced frequency.
U

Note:

In what follows, one preserves the assumptions H1 and H2 and one notes I (F

ij
R, Re) the integral:

1 1
Iij (F, Re
R
) =
T (1,2, F, Re
R
) (

I
1L + X) (
E
E
J
2 L + X)
D
E
E

1
D 2 éq
2.1.2-5
0 0
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Using this notation, the interspectres of modal excitations are written:

1
2
2
D
S
(F, U) = U DL
I (F
Q Q
E
ij
R, Re)
éq
2.1.2-6
I J

2

U

The expression of the autospectres of modal excitations is similar:

*1
2
2
D
S
(F, U) = U DL*

*I (F
Q Q
E
II
R, Re)
éq
2.1.2-7
II*

2

U

*2.2
Spectra of the type “length of correlation”*

2.2.1 Key words

The key words factors SPEC_LONG_COR_i (I varying from 1 to 4) of operator DEFI_SPEC_TURB [U4.44.31] give access spectra of the type “length of correlation”. These spectra, specific of the configurations of the type “beam of tubes under transverse flow”, are preset but the user can adjust the parameters of them.

2.2.2 Definition of the model

2.2.2.1 Density interspectrale

In the case of spectra of the type “length of correlation”, the density interspectrale characterizing the turbulent excitation is supposed to be able to be put in a form at separable variables such that:

*S (X, X,) = S () (X, X
I
1
2*

0
0
1
2)
éq
2.2.2.1-1

*In this expression, $S()$
0 represent the autospectre turbulence and 0 (1
 $X, x2$) indicates one
function of space correlation defined by:
- $X - X$*

2
1

0 (1
 $X, x2) = \exp \text{éq}$
2.2.2.1-2
C

*where X and X
1
2 indicate the X -coordinates of two points of observation and C represents the length of
correlation.*

*Four analytical expressions are available in operator $DEFI_SPEC_TURB$ [U4.44.31]. These
expressions correspond each one to a particular representation of $S()$
0 .*

*The user defines a spectrum of turbulence by choosing one of these analytical forms, of which it
can adjust the parameters.*

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2.2.2.2 Modeling of the spectrum of turbulence by an expression with separate variables

.

Case general

The function tt introduced into the relation is modelled by a form with variables separated:

NS

(, F , Re) = (,

1

2

1

2)

T

R

N

$N(F R, Re)$

éq

2.2.2.2-1

$N 1$

=

Where $NR S$ indicates the degree of the base of the functions of form N and N is a function independent of the variable of space. These two functions are stored in the base of data and can be selected by the user.

The autospectres of modal excitations are given by [éq 2.1.2-7] while introducing:

NS

$I(F, Re) = L2(F$

II

R

nor

N

$R, Re)$ éq

2.2.2.2-2 **$n=1$** **1 1****with: $L_2 = (1$** **, 2) $(L + X$** **1****) $(L + X$** **2****) $D D$** **nor** **N** **I** **E** **E** **I** **E** **E** **1 2****éq****2.2.2.2-3****0 0**

The principle of calculation is as follows: one first of all calculates the values of L_{2ni} while realizing the calculation of the double integrals; one calculates then $N(Fr, Re)$ for all the values of N ; one obtains finally the expression of S

 $(F, U$ **$Q Q$** **) using the equation [éq 2.1.2-4].** **$I I$** **.**

Particular case: model used for the tubes of steam generator

The particular case of the study of the tubes of Steam Generator corresponds to a particular case of the case general

presented previously by posing $NS = 1$. The interspectre of turbulent excitation between two points of reduced X -coordinates and

1**2 is then given by:****-****(, F**

1
2

1
2

, $Re) = exp -$
 L

T
 R

$E (F R, Re) \acute{e}q$
2.2.2.2-4

C

where C represents the length of correlation of the turbulent forces and is the length excited. In general, one takes C of about 3 to 4 times the diameter external of the tube.

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Spectra of autocorrelation of modal excitations, in the case of profiles speed and of density constant, are given by:

2

D
SQiQi (F U)
1
,
2
=
U

E
DL
III (F, Re
R
) éq 2.2.2.2 - 5
2
U
with:

1 1
-

I (F, Re) = (F, Re) .exp- 2
1 L

E.
I
(L + X
1 E
E) I
(L + X
2nd
E) .d D
II
R
R

1
2
C

0 0

éq 2.2.2.2 - 6

***In the case general of profiles of density and unspecified rate of flow,
one a:***

2

SQ_iQ_j (F U)

1

D

***,
= D S (Fr)***

2 U

X +L X +L

E

E E

E

x² - 1

X

exp -

E (1

X) E (X)

2

2 U.E. (X) 2

1 U.E. (x²) I (1

X) I (x²) 1

dx dx²

X

X

C

E

E

éq 2.2.2.2 - 7

Where D is the diameter of the structure, L is the length of the excited zone, X_e is the X -coordinate of the origin of the excited zone, U is the mean velocity of the flow, S (Fr) is a spectral concentration of separate excitation the mean velocity of the flow U , X and X

X_1
 X_2 are the curvilinear X -coordinates of two points of observation on the tube, (X
 E
)
is the profile of density of the fluid along the tube, $U(X)$
 E
is the profile speed
transverse of the flow along the tube and C indicates the length of correlation.

Adimensional profiles of density and transverse speed of the flow external are in the following way defined:

()
 $E X$ indicating the evolution of the density of the external fluid along the zone immersed L_{imm} of the tube, one indicates by the density of the external fluid realised on the immersed part of the tube:

X
 $+ L$
 imm
 imm
 L
 $=$
 $(X) dx \text{ \acute{e}q 2.2.2.2 - 8}$
 L
 E
 imm
 $ximm$

One indicates by $R(X)$ the adimensional profile of density such as $() =$
 $E X$
 $R(X)$.

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U (X)

E

indicating the evolution rate of flow of the external fluid over the length excited of the tube, one indicates by U the rate of flow of the fluid realised on excited length of the tube:

X + L

E

E

l

U =

U (X) dx

éq 2.2.2.2 - 9

L

E

E

Xe

One indicates by (

U X) adimensional profile transverse speed of the external flow,

such as U

X = U U X

E ()

() .

By introducing the average sizes and the adimensional profiles into the expression [éq 2.2.2.2 - 7], one obtains:

$$\begin{aligned}
& 2 \\
& X + L X + L \\
& 1 \\
& 2 \\
& D \\
& E \\
& E E \\
& E \\
& X - X \\
& S
\end{aligned}$$

$$\begin{aligned}
& 2 \\
& 1 \\
& Q_i Q_j (F, U) = \\
& U D. \\
& S (Fr) \\
& exp
\end{aligned}$$

$$\begin{aligned}
& 2 \\
& - \\
& U
\end{aligned}$$

C

$$\begin{aligned}
& X \\
& X \\
& \acute{e}q \\
& 2.2.2.2-10
\end{aligned}$$

$$\begin{aligned}
& E \\
& E \\
& 2 \\
& 2 \\
& E
\end{aligned}$$

$$\begin{aligned}
& (x1) E \\
& (x2) Ue (x1) Ue (x2) I (x1) J (x2) dx dx
\end{aligned}$$

1
2

X - X
After having noted =
E, it comes:

1 1
1

-

2
3
3 2
x2

1
X
S

$(F, U) = U D L S (F) \times exp -$

R
 $Q_i Q_j$

E
R
 $(1 + X) R (2 + X)$

4

E
L
E
E
L
E
0 0

C

2

$$\begin{aligned}
 & U(L + X) \\
 & E \\
 & E) 2 \\
 & I \\
 & U(2L + X) \\
 & E \\
 & E) I(1L + X) \\
 & E \\
 & E) I(2L + X) \\
 & E \\
 & E)] d1d 2 \\
 & \text{éq 2.2.2.2 - 11}
 \end{aligned}$$

Where $S(Fr)$ represents the spectrum of turbulence, definite according to a reduced frequency Fr (a Strouhal number). For a tube in interaction with a transverse flow, $F R$ is written:

$$fD$$

$$Fr =$$

U

where F is the dimensioned frequency, D is the diameter of the tube and U is speed average of the flow.

The double integral of the expression [éq 2.2.2.2 - 11] is evaluated by the operator **PROJ_SPEC_BASE [U4.63.14]**.

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Case of multiple zones of excitation

If there are several zones of excitations, the notations are introduced additional following:

The zone of excitation K being located by its X-coordinate x_k beginning and its L_k length, one note $U(X)$

K

profile speed transverse of the fluid flow on the level of this zone.

average transverse speed on the zone of excitation K is then given by:

$X + L$

K

K

1

$U =$

$U(X) dx$

K

L

K

K

x_k

One deduces the adimensional profile from it transverse speed, standardized on the zone K:

$U(X)$
 $U(X)$
 K
 K
 $) =$

The U.K.

K indicating the total number of zones of excitation, average transverse speed on the whole of the zones of excitation is defined by:

K
 l
 $U =$
 U
 K
 K
 $K = l$

If V_{gap} is the speed intertube at the entry of the Steam Generator (the beach speeds retailers is defined in CALC_FLUI_STRU [U4.66.02] using key word VITE_FLUI), one proceeds to one the second standardization; transverse speed in an item X located in the zone of excitation K is given by:

$U(X)$
 U
 $V(X) = V$
 K
 $= V$
 $K U(X)$
 K
 gap
 $)$
 U

gap the U.K.

Thanks to this standardization, the arithmetic mean transverse speed on all them zones of excitation is equal at the speed intertube; one has indeed:

$+$
 $l K$
 $l X L$
 K

K

V(X) dx V

K

L

K

gap

=

K 1

=

K

X

K

The calculation of the interspectres of modal excitations, realized by operator PROJ_SPEC_BASE [U4.63.14], is done by adding the contributions with each zone of excitation according to the relation:

2 K

1

D

S

,

=

I J (F V

)

D

Lk

S

K

ij

(*F*

Q Q

gap

R)

2
 ×
 ×
 V

 $K = 1$
 K
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with:
 U
 fD
 $V = V$
 K
 K
 K
 $gap \times$
 $and F$
 $=$

U
 R
 Vk

$X + L X + L$
 K

K K

K

- X - X

2

1

and Lk =

exp

(X) (X V

) 2 (X V

) 2 (X) (X) (X) dx dx

ij

E 1 E 2 K 1 K

2

1

1

1

2

1

2

C

X

X

K

K

that is to say:

X + L X + L

K

K K

K

- X - X

Lk = V 4 ×

exp

2

1 (X) (X) u2 (X) u2 (X) (X) (X) dx dx

ij

K

E 1 E 2 K 1 K 2 I 1 I

2
1
2
C

X
X

K
K

One poses:

X + L X + L
K
K K
K
- X - X
L K =
exp
2
l (X) (X) u2 (X) u2 (X) (X) (X) dx dx
ij

E 1 E 2 K 1 K 2 I 1 I

2
1
2
C

X
X
K
K

The expression of the interspectres of modal excitations becomes then:

$2K$

l

D

S

,

=

4

$IJ(FV$

)

D

V

LK

S

K

K

ij

(F

$Q Q$

gap

R)

2

\times

\times

\times

V

$K = l$

K

from where:

K

l

S

,

=

$3 \times 3 \times \times$

$IJ(FV$

) D
(V lk S (F K
Q Q
gap
K
ij
R)
4
K = I

Analytical expressions of the spectra available for the user

Various analytical expressions of the spectra available in the operator
DEFI_SPEC_TURB [U4.44.31] are as follows:

SPEC_LONG_COR_I

Each U_i speed defined by the user by discretizing the beach speeds

[U - U

kN

min

max] explored is initially standardized in the U_i form while applying
the equation:

The U.K.

$U_{kN} = U$

I

I

U

where the U.K. and U respectively indicate the speed realised on the zone of excitation "K", and
mean velocity on the unit of the zones of excitation.

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A Reynolds number “local” R_{ik}

E, associated the zone “K” and the U_i speed is then

calculated starting from the local characteristics of the flow:

kN

U

D

Re_{ik}

I

=

The spectrum of turbulent excitation associated the zone “K” and the U_i speed is given under the shape of a vector S_{ik} , having as many components as of points used for to discretize the frequential interval $[F$

- F

min

max], support of the excitation. J -ième

component S_{ikj} of this vector is provided by the expression:

S_{ik}

0

$J =$

éq

2.2.2.2-12

2

F_{ik}^2

F_{ik}^2

r_j

r_j

$1 -$

2

$+ 4$

F

F

rc

rc

F_{ik}

r_j is provided by:

$F D$

F_{ik}

J

$r_j =$

$U \text{ kN}$

I

where:

F_j is the value of frequency associated with the j -ième component in the discretization with the frequential interval $[F$

$- F$

min

$max]$, F_{rc} is a cut-off frequency being worth 0.2; O ,

depend on the Reynolds number according to equations' provided in the table below:

R_{ik}

E

O

J- ; 1.5 104]

2.83504 10-4 3

0.7

] 1.5 104 ; 3.5 104]

Idem Idem

20 42

*.
- 14 10-4 Rik - 9 8*

. 110-8

2

Rik

1

+ 19

. 7 10 12

3

-

ik

R

13

. 10-4

E

E

E

-

ik

-

ik

- 35 95

. 10 17

4

R

+ 34 69

.

10 22

5

R

E

E

] 3.5.104; 5.104] Idem

4 0.3
] 5 104 ; 5.5 104] 50.18975
104 Idem
Idem
] 5.5 104 ; +]
Idem
4 0.6

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.
SPEC_LONG_COR_2

The spectrum of turbulent excitation is written:

$S(F$

0

$R) =$

$\acute{e}q$

2.2.2.2-13

F

R

$I +$

frc
The default values of the parameters are as follows:

-3
0 = 15
. 10
= 2 7
.

F = 01

.
rc

.
SPEC_LONG_COR_3

The spectrum of turbulent excitation is written:

S (F
0
R) =
éq
2.2.2.2-14

F
R
with:

=
0
0 (F
)
rc
=

(F)
rc

The default values of the parameters are as follows: F rc = 2

If F F
R
rc, one a:

3
-

$0 = 510$

$= 05$

.
if not

-5

$0 = 4 10$

$= 35$

.

.

SPEC_LONG_COR_4

The spectrum of turbulent excitation is written:

$S(F$

0

$R) =$

éq 2.2.2.2 - 15

F

$R v$

with:

1

$0 =$

10

-2

$6 8$

$. 10$

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The other parameters are defined by:

=
To 0 5. -
B 15. -
C 2 5. -
D 35.

v
v
v
v
= 2
= 4

v the rate of vacuum indicates; v
=
is the volume throughput defined by

v
mU; m is the flow
mass and U indicates the mean velocity of the flow. Values of the coefficients of
polynomial in v are as follows:

To = 24 042
.
B = -50 421
.
C = 63 483

.
 $D = 33\ 284$
 .

2.3

Model of turbulent excitation distributed

2.3.1 Key words

The key word factor SPEC_FONC_FORME of operator DEFI_SPEC_TURB [U4.44.31] makes it possible to define

a spectrum of excitation by its decomposition on a family of functions of form. The user with possibility of defining the spectrum by providing a matrix interspectrale and a list of functions of form associated. The concepts [interspectre] and [function] must then be generated in upstream. In the case of the component “control rod”, the user can also use one preset spectrum of turbulence, identified on model GRAPPE1.

2.3.2 Decomposition on a family of functions of form

The model of turbulent excitation distributed supposes that the instantaneous linear density of the forces

turbulent $F(X, T)$

T

,) can be broken up on a family of functions of form $()$

K, X of

dimension K in the following way:

K

$F(X, T) =$

T

$\sum_{k=1}^K K(X) K(T) \text{ éq}$

2.3.2-1

$k=1$

Coefficients $()$

K, T at every moment define the decomposition of the turbulent excitation on family of functions of form.

The density interspectrale of turbulent excitation between two points of the telegraphic structure of X -coordinates x_1

and x_2 is written then:

K, K

$S(X, X) = (X) (X) S$

F

K

L

kl ()

1

2

1

2

éq

2.3.2-2

K 1

= K 1

=

This formulation makes it possible to take into account an excitation whose space distribution is unspecified.

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2.3.3 Setting in equations

2.3.3.1 Application of a turbulent excitation distributed

The length of application L is characterized in an intrinsic way by the field of definition of functions of form associated with the excitation. The zone of application is determined by the data of name of the node around of which it is centered.

xn indicating the X-coordinate locating this node, the turbulent excitation is imposed on field $[X - L/2, X + L/2]$

N

N

2].

The turbulent excitation being able, in addition, to be developed in a way correlated in both directions Y and Z orthogonal with the axis of the telegraphic structure, the functions of form are a priori

vectors with two components.

One thus informs, by convention, these functions about the interval [0,2L], the field [0, L] being associated the direction Y and the field [L, 2L] being associated direction Z.

2.3.3.2 turbulent Excitation identified on model GRAPPE1

The functions of form K are the first 12 modal deformations of inflection of the structure identified in experiments, distributed according to two orthogonal directions' with the principal axis of beam. The general analytical expression of these deformations is as follows:

R

(X)

Yk

K (X) =

éq

2.3.3.2-1

Zk (X)

with:

N

N

N

N

Yk

Yk

Yk

Yk

Yk (X) =

Yk

With cos

X

Yk

B

sin

X

Yk

C

CH

X

Yk

D

HS

X

L + L + L +

L éq 2.3.3.2 - 2

N

N

N

N

Zk

Zk

Zk

Zk

Zk (X) =

Zk

With cos

X

BZk sin

X

CZk CH

X

Zk

D

HS

X

L +

L +

$L +$

L éq 2.3.3.2 - 3

where nY_k and nZ_k indicate numbers of waves, L is the length of application of the excitation and them coefficients $AY_k, BY_k, CY_k, DY_k, AZ_k, BZ_k, CZ_k, DZ_k$ are constant real coefficients characteristics of the function of form considered.

The first 6 functions of form are associated the direction **Y** and AZ_k, BZ_k, CZ_k, DZ_k are thus null, for $1 \leq k \leq 6$.

The 6 last functions of form are associated direction **Z** and AY_k, BY_k, CY_k, DY_k are thus null, for $7 \leq k \leq 12$.

This family of functions of form is thus characterized by $5 \times 12 = 60$ real coefficients.

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The turbulent excitation identified on model GRAPPE1 is homogeneous in the two directions orthogonal with the axis of the telegraphic structure, turbulence being *décorrélée* between these two directions.

The matrix interspectrale $[Skl]$ identified on model GRAPPE1 is thus a matrix of dimension 12×12 , consisted of two identical diagonal blocks of dimension 6:

[

S

0

S

O
 $K =$
 $L]$
 $[()] []$

$[]$

0
 $[S ()$
 O
 $]$

*By square property of symmetry, this matrix is entirely defined by the data of the part triangular higher (or lower) of $[S ()$
 $O]$, is 21 interspectres. For each one among them, their characteristic parameters are the level of plate, the cut-off frequency and the slope of the spectrum beyond this frequency.*

The matrix interspectrale of turbulent excitation identified on model GRAPPE1 is thus characterized by 63 real coefficients (3x21).

Note:

*Excitations GRAPPE1 are available to **two flows of reference**. The whole of data characterizing these excitations thus represents **246 real coefficients** ($[60+63] \times 2$).*

2.3.3.3 Projection of the excitation on modal basis

One notes:

$DY (X)$

I
 $I (X) =$

the modal deformed i -ème of the structure.

$DZi (X)$

Are ik the co-ordinates of the modal i -ème deformed of the structure on the basis of the functions of form $K (X)$:

K
 $(X) =$
 I
 ik

$K(X)$

$\acute{e}q$

2.3.3.3-1

$K=1$

The interspectres of modal excitations $SQ Q ()$ applied to the structure are written then:

$I J$

$K K$

S

$() =$

S

$Q Q$

ik

jl

$($

$)$

$\acute{e}q$

2.3.3.3-2

$I J$

$K L$

$K=1k=1$

For each mode I of the structure, the coefficients ik are given by integrating the equation [éq 2.3.3.3 - 1] prémultipliée by the functions J , on the applicability of the excitation. One obtains as follows:

$X + L$

0

2

K

$X + L$

0

2

$(X + L 2) (X) dx = (X + L 2)$

J

I

ik

J

$K (X + L 2) dx$

$X - L 2$

$K=1$

$X - L$

0

0

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$X + L$

0

2

K

L

$(X + L^2) (X) dx = (X)$

J

I

$ik J$

$K (X) dx$

(

I, J)

éq

2.3.3.3-3

$X - L^2$

$K = I$

0

0

For each I, the equation [éq 2.3.3.3 - 3] is written in matrix form:

[has] () = (B

jk

ik

ij)

éq

2.3.3.3-4

with:

L

has

=

$(X) (X) dx$

jk

J

K

0

that is to say:

L

has

$= ((X) (X) + (X) (X))dx$

jk

Yj

Yk

Zj

Zk

0

and

$X + L$

0

2

$B =$

$(X + L^2) (X) dx$

ij

J

I

$X - L$

0

2

that is to say:

$X + L$

0
 2
 $B =$
 $(DY(X)(X + L^2) + DZ(X)(X + L^2))dx$
 ij
 I
 Yj
 I
 Zj
 $X - L$
 0
 2

The resolution of each linear system of equations leads to the ik .
 The calculation of the scalar products is carried out in operator PROJ_SPEC_BASE [U4. 63.14].

Note:

1) functions $K(X)$ represent, in practice, the modal deformations raised on model. The system (\cdot) , with dominating diagonal, is thus well conditioned. In private individual, when the telegraphic structure model has a homogeneous linear density, them functions $K(X)$ are orthogonal and the matrix $[jk \text{ has}]$ is diagonal.

2) Tests comparing the applicability of the excitation with the field of definition of the structure are carried out.

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2.4

Model of localised turbulent excitation

2.4.1 Key words

The key word factor SPEC_EXCI_POINT of operator DEFI_SPEC_TURB [U4.44.31] is used in cases of a spectrum of excitation associated with one or more forces and moments specific. The user can define the spectrum while providing:

*.
a matrix interspectrale of excitations (the concept [interspectre] associated must be generated upstream),*

*.
the list of the nodes of application of these excitations,*

*.
the nature of the excitation applied of each one of these nodes (force or moment),*

*.
directions of application of the excitations thus defined.*

It can also use a preset spectrum of turbulence, identified on model GRAPPE2.

2.4.2 Bases

The model of localised turbulent excitation is a particular case of the model of turbulent excitation distributed. Thus, one supposes just as in paragraph [§2.3.2] that instantaneous linear density turbulent forces $F(X, T)$

T (,) can be broken up on a family of functions of form

$K(X)$ in the following way:

K

$F(X, T) = K(X) T$

T

K

K

) éq

2.4.2-1

$K = 1$

The coefficients $K(T)$ at every moment define the decomposition of the turbulent excitation on family of functions of form.

The density interspectrale of turbulent excitation between two points of the telegraphic structure of X -coordinates x_1

and x_2 is written then:

K

$S(X, X) = (X) (X) S$

F

K

L

(

)

1

2

1

2

$\acute{e}q$

2.4.2-2

K

L

$= L$

1

$=$

The characteristic of the model of localised turbulent excitation is due to the **specificity of the functions of form $K(X)$** :

$(X) =$

K

$(X - x_k)$ allows to represent a **specific force** applied to the point of X -coordinate

x_k

$(X) =$

K

$(X - x_k)$ allows to represent one **specific moment** applied to the point of X -coordinate

x_k

$(X - x_k)$ and $(X - x_k)$ the distribution of Dirac and the derivative indicate respectively of distribution of Dirac at the point of X -coordinate x_k .

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Taking into account the specificity of the functions of form, the projection of a localised turbulent excitation

on modal basis is much simpler than in the case general (excitation distributed), since one can analytically calculate the expression of the projected excitation.

2.4.3 Setting in equations

2.4.3.1 Application of a localised turbulent excitation

One considers a turbulent excitation applied to a structure telegraphic and made up of forces and of specific moments. This excitation is entirely characterized by the following data:

.

list nodes of application of the forces and moments specific,

.

nature of the excitation applied in each node (force or moment),

.

direction of the excitation applied in each node.

K

M

Thus $F(X, T) = F(S)(X - x_k) n_k - m_m(S)(X - x_m) N$

T

K

m

éq 2.4.3.1 - 1

K = 1

m = 1

is the expression of a located turbulent excitation, characterized by K forces and M moments specific, applied respectively to the nodes of X-coordinates x_k and x_m in the directions n_k and r_{nm} .

0

One a: nk =

cos (K) and definite Nm in a similar way.

sin (

K)

represent the azimuth giving the direction of application of the force (or the moment) in the plan P orthogonal with neutral fibre with the node of application, such as defined in the figure [2.4.3.1

Figure - has]

below:

Z

P

F

Nap

node of application

X

neutral fibre

Appear 2.4.3.1 - has: Definition of the direction of application

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*The generalized excitation associated the ième mode of the structure, $Q S$
 $I ()$, being defined by:*

L

$$Q(S) = \int_0^L F(X, T) dx$$

I

I

T

éq

2.4.3.1-2

0

where L represents the length of the beam and $I(X)$ the deformation of mode I , one obtains, taking into account

expression [éq 2.4.3.1 - 1]:

K

M

$$Q(S) = F(S)(X) N - M(S)(X)$$

I

K

I

K

K

m

I

K) Nm éq

2.4.3.1-3

$$K = 1$$

$$m = 1$$

The calculation of the interspectres of modal excitations leads then to:

$$K$$

$$K$$

$$S$$

$$(S) = S$$

$$Q Q$$

$$($$

$$).n$$

$$.($$

$$).n$$

$$I J$$

$$F F (S) ($$

$$I$$

$$I$$

$$2$$

$$2$$

$$K$$

$$K$$

$$I x k$$

$$K)$$

$$J x k$$

$$K$$

$$K = 1 K =$$

$$I$$

$$2$$

$$I$$

$$2 I$$

$$K$$

$$M$$

$$+ S$$

$$F M$$

$$($$

$$).n$$

$$.($$

$$).n$$

$$(S) ($$

1

1

2

2

K

m

I xk

K)

J xm

m

K = 1 m =

1

2

1

2 1

éq

2.4.3.1-4

M

K

+ S

,

MR. F

(

).n

.

).n

(S) (

X

I

ml

ml)

K

m

K

J xk 2

2

$m = I K =$

I
 2
 I
 $2 I$
 M
 M
 $+ S$

$MR. M$
 X

$S ('$
 $) .n$
 I
 mI
 $mI).$
 $($
 $) .n$
 $()$
 m

2
 m
 m
 m
 JX
 2
 $m I$
 $= m I$
 I
 2
 I
 $2 =$

Note:

*When the user defines the spectrum of turbulent excitation, it must inform the matrix interspectrale of the specific excitations whose terms intervene above. **This stamp has as a dimension K+M (a number of forces and specific moments applied).***

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2.4.3.2 turbulent Excitation identified on model GRAPPE2

The turbulent excitation identified on model GRAPPE2 is represented by a force and one moment resulting, applied in the same node following the two orthogonal directions to the axis of structure. The linear density of this excitation has as an expression:

0**1****F****2****2****T (X, S) =****U****H****D [LP tF (rs) (X - x0) - LP MT (rs) (X - x0)]****0****éq 2.4.3.2 - 1****2****1**

Where is the density of the fluid, U is the mean velocity of the flow, Dh is it hydraulic diameter, LP is the thickness of the plate of housing (corresponding to the length

$S D$

excited), x_0 is the X -coordinate of the point of application of the excitation, $S_r =$
is the complex frequency

U

reduced, $F(S)$

T

R) and $M(S)$

T

R) are the adimensional coefficients representing the force and the moment
resulting.

The sizes, U , D_h and LP make it possible to dimension the excitation.

In substituent the expression [éq 2.4.3.2 - 3] in the relation [éq 2.4.3.1 - 4] defining the modal excitation

$Q(S)$

$I()$, one obtains:

0

0

1

2

2

$Q(S)$

$U D L F(S)$

$(X$

$0)$

$0 + L M(S)'$

$=$

$(X$

I

H

P

T

R

I
p
T
R
I
0)
0
éq 2.4.3.2 - 2
2

1

1

The specific force and moment identified on model GRAPPE2 being décorrélés, the calculation of interspectres of modal excitations leads finally to:

0

0

1

2

2

D

S

=

U

D

L2

Q Q

H

p I

(x0)

I × J (x0)

1 SF F (Sr)

I J

2

U
TT

1

1

éq
2.4.3.2-3

0

0

4

,

,

+ *LP I (x0)*

I × J (x0)

I S MR. M (Sr)

T

T

1

1

In this expression, D is the diameter external of the structure, S

(S

FF

R) and S

(S

MR. M

R)

T T

T

T

the adimensional autospectres of force and moment represent respectively identified on model GRAPPE2. Operator PROJ_SPEC_BASE [U4.63.14] calculates the interspectres excitations modal according to the relation [éq 2.4.3.2 - 3] above.

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Note:

1) Autospectres adimensional GRAPPE2 are usable to simulate it behavior of any structure in similarity with the model; one then utilizes geometrical parameters structural feature to dimension the excitation. Model GRAPPE2 having been built in similarity with configuration engine, the following reports/ratios are fixed and characteristic of this geometry:

D

L

H

P

and

D

D

It is pointed out that D_h and D respectively indicate the hydraulic diameter and its diameter external of the structure; LP is the thickness of the plate of housing, corresponding to the excited length.

The data of, U and D are thus sufficient to dimension in manner univocal the turbulent excitation starting from the autospectres adimensional.

2) Adimensional autospectres S

(S
 $F F$
 R) and S
(S
 MR , M
 R) one and the other being defined by

$T T$
 T
 T

three real coefficients (level of plate, reduced frequency of cut and slope beyond of this frequency), only six constants make it possible to characterize the excitation turbulent adimensional identified on model GRAPPE2.

*Four configurations having been studied (ascending flow or going down, stem of order centered or offset), the whole of the data characterizing the excitations GRAPPE2 thus represents **24 real coefficients**.*

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R4.07 booklet: Fluid coupling structure

Document: R4.07.03

Calculation of matrix of added mass

on modal basis

Summary:

This document has an aspect of the fluid coupling/structure: when a vibrating structure is immersed in a fluid which one supposes at rest, incompressible and nonviscous, it feels forces of pressure whose resultant is proportional to the acceleration of the structure in the fluid: the coefficient of

proportionality is homogeneous with a mass: it is called added mass. One specifies here the means of estimating one

stamp of mass added for one (or of) structure (S) to several degrees of freedom on the modal basis of (of) structure (S) in the vacuum.

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1 Notations

p

:

fluctuating pressure in the fluid,

!
:
contour of the structure indexed by!
"
X
:
S
the field of displacements in the structure
!
!,
F, S: density of the fluid, the structure,
X
:
I!
clean mode of order I of the structure! in air
has, a:
generalized co-ordinates, speeds, accelerations
I
I
! " !
relating to mode I of the structure
has "
! in air
I
!"
:
the tensor of the constraints in the structure
:
the fluid vector of flow
H
:
the matrix of rigidity of the fluid
v
:
the field fluid speeds
N
:
the interior normal of the fluid.

2 Introduction

Many industrial components are in contact with fluid environments, which more is often in flow. These surrounding fluid environments disturb the vibratory characteristics of the structures,

in particular their modal characteristics. This action of the fluid on the structure results in effects of fluid coupling/structure.

One supposes the incompressible, perfect fluid environment here surrounding and at rest. One will show that then,

a structure which vibrates with a small amplitude in this fluid modifies the field of pressure in fluid at rest, and thus feels a compressive force, proportional to its acceleration.

proportionality factor is a mass. It describes the inertial effect of the fluid on the structure: it is why one names this mass masses added fluid on the structure.

When several structures are in contact of the same fluid, when one of the structures is put at to vibrate, not only it feels the inertia of the fluid, but it modifies the field of pressure around interfaces with the fluid of all the other structures. The efforts that each one feels are proportional to the acceleration of the vibrating structure: there still proportionality factors called added masses of coupling are masses.

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Recalls of the equations of the problem

3.1

Equations in the fluid

It is supposed that K vibrating structures are immersed in a true fluid (nonviscous), incompressible and at rest. One neglects the effect of gravity. One can thus write the equations of Euler associated with the fluid at rest:

N

y or X 2

!

Z

N

X or x1

· conservation of the mass:

$$F + \text{div} (v) =$$

F

0

éq 3.1-1

T

· conservation of the momentum:

v

1

+ (v ·)

v +

p

= 0

éq 3.1-2

T

F

Because of incompressibility of the fluid, the equation [éq 3.1-1] becomes:

$$\text{div } v = 0$$

éq 3.1-3

In the volume of the fluid, one neglects the convection induced by the movement of low amplitude of the structure. The equation [éq 3.1-2] thus becomes:

v

1

+

p

= 0

éq 3.1-4

T

F

v

While deriving [éq 3.1-3] compared to time and by deferring the expression of according to

T

pressure in this equation, one obtains:

$$\text{div } p = 0$$

that is to say:

$$p = 0 \text{ in}$$

who is the equation of Laplace in a fluid at rest.

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With the fluid interface/structure, one can write that the normal acceleration of the wall of the structure is

equalize with the normal acceleration of the fluid (continuity of the normal accelerations - condition of impermeability of the structure). One uses here following convention for the normal: it is about normal external with the structure, directed structure towards the fluid.

v

$$\cdot N = X'' \cdot N$$

T**S!**

With the equation [éq 3.1-4], one obtains:

v

$$p \cdot N = - F$$

$$\cdot N = - X'' \cdot N$$

T**F S!**

That is to say: p

(

$$) = - "X \cdot$$

N on**N****F S****!****!**

!, fluid interface/structure of the structure indexed by!.

In short, the fluid problem consists in solving an equation of Laplace with boundary condition of type von Neumann:

$$p = 0 \text{ in}$$

p

(
 $) = -F \cdot N \text{ on } 1, 1 =$

$N 1$

$\$!$

$! = 1, K$

éq 3.1-5

$\#$

p

(
 $) = 0 \text{ out of } 2, 2 = -$

$N 2$

1

3.2

Equations in the structures

Let us consider K structures elastic divings in a fluid environment. The equation of their movement in presence of fluid is written:

! index of structure! $\{, 0 \# , K\}$, $M \cdot \ddot{X} + K X = 0$ in, volume of the structure

$!$

$!$

$!$

$!$

S

$!$

$!$

$!, N = -$

pn on! , contour of the structure!

$M!$ is the matrix of mass of the structure, $K!$ its matrix of rigidity. The boundary condition on contour of the structures translates the continuity of the normal constraint to the fluid interface/ structure (it

tensor of the fluid constraints being tiny room to its nondeviatoric part, fluid being perfect). In integrating on the contour of each structure this normal constraint, a force F is obtained!

resultant of the structure/compressive forces of the fluid to the fluid interface. This force is the integral of

field of pressure on contour! %de each structure:

! index of structure! $\{, 0 \# , K\}$, $F = -$

!

pnd

!

The field of pressure checks the problem [éq 3.1-5].

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3.3 Equations of the coupled problem - Description of the matrix of mass added

Ultimately, the fluid coupled problem/structure is written:

$p = 0$ in

! {

p

0, K

},

= -

"

X

N on

S

N
F

.
!

!

!

! {0, K
#, M “X

+ K X
0 in

=
S
!!
!!

!

! {0, K
#, F = - pn
D

!

on!

!
éq 3.3-1

One will show from now on that the effort that feel the immersed structures is proportional to their acceleration. A good means of showing that is to place itself in the modal base of the structures in the vacuum. One can thus break up acceleration on this basis (which is in fact the meeting of modal bases of each structure). As follows:

xS (R, T) = has (T
I
) Xi (R)
!

!
!
 $i=1$

By deferring this expression in the second equation of the system [éq 3.3-1], one is brought to seek the field of pressure in the form:

$p =$
 $(T) p$ has
"I"
 $I(R)$

!
=,
 $1, K i=1,$
!#
#

By deferring in the problem [éq 3.3-1] these expressions, one has to solve in the fluid as much problems of Laplace whom one chose of modes for each structure. This results in:

p
 $I =$

 0 in
!

p
! {1, #, K}, I {1, #,}

,
!
 I

= -
 X

N on

N
 F
 $I \cdot$

!
!

!
[semi (A) + K (A) = (F) in
!] " !

[! I]!
I!
!

The "matrices" of mass and rigidity written in these bases are diagonal.
Each component of the effort of pressure resulting projected on modal basis is written:

K
I {1,
,

}
! {1, K
#}, (F

!) = - has "
p X ·

N NR D
I
jk
jk
I
J
!
K =1 j=1
!

One can then write the vector of the effort generalized of pressure on a structure immersed under
matrix form:

(F) = - [m] "with m = p X has · N
I
I jk
jk
I jk
jk I D

!
!
!
!
!

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Here! is fixed: the matrix [**semi** jk !] is called **matrix of mass added** of the fluid on the structure of

contour!. When one considers the modal base of the whole of K structures, one generalizes

notation of the **matrix of added mass** [**semi** jk

!] on modal basis in the vacuum! varying from 1 to K.

This matrix is in general not diagonal.

3.4 Some**definitions****3.4.1 Definition****1**When! = K (even structure) and I = J (even order of mode), the coefficient *semi* $I!!$ is the car-**mass****added** mode I of the structure!. It is about additional inertia due to the fluid moved by

mode of order I of the structure, taking into account the geometrical containments induced in the fluid by

presence of the other presumed fixed structures.

3.4.2 Definition**2**When! = K (even structure) and I J (different orders of mode), the coefficient *semi* $J!!$ is the **mass****added coupling** between the modes of order I and J of the structure!. In air, these terms of mass

extradiagonaux are null, because the modes are orthogonal between them. Taking into account the

expression

general of the coefficient *semi* jk

! , modes I and J can be coupled in mass, because the field of

pressure p $J!$ created by the mode J of the structure! is not necessarily orthogonal with the mode of order

I of this same structure. It is enough that this structure is immersed in an environment

not comprising geometrical symmetry so that this coefficient is nonnull. In an environment

symmetrical, on the other hand, the orthogonality of the field of pressure with the mode is observed.

3.4.3 Definition**3**When! K (different structures) and I J (different orders of mode), the coefficient *semi* jk

! is

mass added coupling between the modes of order I and J respectively of the structures I and K. It coefficient translates the inertial effort which makes undergo the structure K vibrating on its mode of order J to the structure I vibrating on its mode I.

3.5

Properties of the matrix of added mass

3.5.1 Theorem 1: the matrix of added mass is symmetrical

To simplify the demonstration, we will consider a single structure immersed in a fluid perfect, incompressible and nonviscous. We break up the movement of the structure on its basis modal (truncated with N modes), but the result can be just as easily shown in “physical” base (i.e the base of the nodal functions of interpolation). Lastly, the result spreads with the case of K structures immersed in the same fluid.

One must show that:

$$m_{ij} = p \mathbf{X}_i \cdot \mathbf{N}_j D = m_{ji} = p \mathbf{X}_j \cdot \mathbf{N}_i D$$

i, j

I, J

j, i

J, I

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.

p_i (respectively p_j) represents the field of pressure created in the fluid and to the interface with the structure by the mode of order I (respectively of order J) of the structure,

.

\mathbf{X}_j (respectively \mathbf{X}_i) respectively represents the modal deformation of the mode of order J (of order I).

However:

p
 0 in
 fluid volume

= 0 in fluid volume

$I =$

p_j

p
 and

p

I

J

= - $F \mathbf{X}_i \cdot \mathbf{N}$ on

= - $F \mathbf{X}_J \cdot \mathbf{N}$ on

N

N

From where, by using the formula of Green with a normal directed of the structure towards the fluid and harmonicity of p_i and p_j :

1

p
 $m = p \mathbf{X} \cdot \mathbf{N} D = -$

p
 $J D$

ij

$I J$

I

N

F

1

= -

$(p p$
 $D -$

p

$\cdot p$

D

$I J$

I

J

F

$\& '$

$($

$)$

$($

0

1

$= -$

$(p p$

$D -$

p

$\cdot p$

D

$J I$

$J I$

F

$\& '$

$($

$)$

$($

0

1

p

$= -$

p

$$I D = p \mathbf{X} \cdot \mathbf{N} D$$

J

N

J

I

F

$$= m j i$$

C.Q.F.D.

3.5.2 Theorem 2: the matrix of added mass is definite positive

One returns to the reference [bib1] for the complete demonstration.

3.5.3 Theorem

3

Let us suppose that one has K structures having properties of linear elasticity identical and who are immersed in the same fluid. Moreover, these structures admit two degrees of freedom of displacement in the plan Oxy (cf diagram). Each one of these structures admits the same spectrum

F

F

1,

, *N*,

*

* of Eigen frequencies in the vacuum.

For any Eigen frequency *fn*, there exists 2K Eigen frequencies {1,

2K} of the coupled system

fluid/structure checking *I* {1,

2

, *K*},

#

F

I

N

One returns to the reference [bib1] for the complete demonstration.

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3.5.4 Others

properties

· the coefficients of added car-mass are always positive

One always supposes that one has only one structure immersed in a true fluid, incompressible and with rest. The demonstration spreads without difficulty with K immersed structures.

One must show that:

I index of mode $\{1,$

$\# \}$

$$N, m = p \mathbf{X} \cdot \mathbf{N} D$$

II

I

I

0

However:

I

p

$$m = p \mathbf{X} \cdot \mathbf{N} D = -$$

p

ID

II

II

I

N

F

I

$= -$

$(p p$

$D -$

p

$\cdot p$

D

II

ID

F

$\& '$

$($

$)$

$($

0

1

=

((p
) 2D

I

F

0

· let us suppose that one has K structures immersed in the same fluid. It is supposed that they have one only degree of freedom of translation according to OX . Then the sum of all the coefficients of mass added this matrix gives the car-mass added on the whole of K structures moving very of the same sinusoidal rectilinear motion.

One returns to the reference [bib2] for the complete demonstration.

4

Implementation numerical

4.1

Resolution of the equation of Laplace by finite elements of volume

Let us take again the fluid problem of Laplace with boundary condition of the type von Neumann:

$$p = 0 \text{ in}$$

$$p$$

$$(\$$

$$) = -F \mathbf{x}_s \cdot \mathbf{N} \text{ on } \Gamma, \Gamma =$$

$$\Gamma_1$$

$$\Gamma_2$$

$$\Gamma_3, K$$

#

$$p$$

$$(\$$

$$) = 0 \text{ out of } \Gamma_2, \Gamma_2 =$$

$$\Gamma_2$$

1

Let us write a variational formulation of this problem:

$$v p$$

$$D =$$

0

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By using the formula of Green with a normal which one supposes directed of the structure towards the fluid

(thus interior with fluid volume) and by posing $\nu = 1$ 2:

 p

$$\nu \cdot p D + \nu D =$$

0

 N

That is to say:

$$\nu \cdot p D =$$

$$\nu X " D$$

 F **éq 4.1-1** N

1

One considers a partition of volume in a finished number of elements. On this discretization of field, one can write the approximate shape of the hydrodynamic field of pressure:

 NR

$$p = NR (\mathbf{R}) p$$

 I I $i=I$

N_i represents the nodal functions of interpolation definite on the elements: they are worth 1 with the node

$n^{\circ}i$, and 0 on all the others.

Then, by taking as function-tests v successively the nodal functions of interpolation, one obtains a system of NR equations while deferring in [éq 4.1-1]:

$$NR \cdot NR(\mathbf{R}) \cdot D = NR \cdot X \cdot D$$

$$J = 1, NR;$$

$$I$$

$$I$$

$$J$$

$$F$$

$J "N"$

$i=1$
1

what can be written in the form:

$$\mathbf{H} \mathbf{P} = \text{with vector of components } J = F \cdot NR \cdot jx " N "$$

$$D$$

éq 4.1-2

with

\mathbf{H} stamps coefficients ij

$$H = Nor \cdot$$

$NR \cdot J \cdot D$

In any rigour, this system is singular. It admits an infinity of solutions differing from a constant. It is thus necessary to impose a pressure (boundary condition of the Dirichlet type) in a point of the fluid for raising indetermination on the solution.

These precautions taken, by reversing the system [éq 4.1-2], one obtains the field of pressure in all the volume of fluid, including with the fluid interface/structure, where it interests us obviously.

4.2 Calculation of the coefficients of the matrix of mass added on basis

modal

It is necessary to estimate the value of the integral numerically:

$$m = \int p \cdot \mathbf{X} \cdot \mathbf{N} \cdot D$$

$$I \cdot jk$$

$$jk \cdot I$$

!

!

éq 4.2-1

!

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starting from a field with the nodes of pressure represented by a vector column noted P_{jk} and of a field with the nodes of displacement corresponding to a modal deformation of structure in air and represented by the vector column X_i . However, on the fluid interface/structure, the field of pressure approximate p_{jk} due to

the discretization of the interface in NR elements of edge can be written:

 NR

$$p = \sum_{jk} N_m(\mathbf{R}) p_{jk}$$

 jk jk $m=1$

while the field of “modal” displacement is written on this same discretization:

 NR I

$$\mathbf{X} = \sum_{n=1}^N N_n(\mathbf{R}) X_n$$

!

 \mathbf{X}_n $n=1$

Thus, by deferring these two expressions in the integral [éq 4.2-1], one obtains:

 NR NR NR

$$\text{semi } \sum_{jk} (N_m(\mathbf{R}) p_{jk}) [\sum_{n=1}^N N_n(\mathbf{R}) X_n \cdot \mathbf{N} + \sum_{n=1}^N N_n(\mathbf{R}) X_n \cdot \mathbf{N} D]$$

!

jk

!

X

I y

!

y]

m

N

N

m=1

n=1

n=1

!

NR

NR

NR

NR

m

p

$(N_m(\mathbf{R}) N_n(\mathbf{R}) N D) X +$

p

$(N_m(\mathbf{R}) N_n(\mathbf{R}) N D) X$

I jk

!

jk

X

I X

!

jk

y

I y

m

N

m

! *N*

m=1 n=1

m=1 n=1

!

!

One supposes in the demonstration that the problem is two-dimensional.

This can be put in the shape of a scalar product, utilizing a product stamps vector:

$$m$$

$$T$$

$$T$$

$$= \mathbf{P} \mathbf{A} \mathbf{X} + \mathbf{P} \mathbf{A} \mathbf{X}$$

with \mathbf{A} stamps coefficients NR NR $N D$

$$I j k$$

$$j k$$

$$X I X$$

$$j k$$

$$y I y$$

$$X$$

$$I J X$$

$$!$$

$$!$$

$$!$$

$$!$$

and \mathbf{A} stamps coefficients NR NR $N D$

$$y$$

$$I J y$$

$$!$$

$$5$$

Implementation in *Code_Aster*

5.1 Analogy

thermics

To solve the problem of Laplace in pressure, a thermal analogy is used: it is about to in hover solve the equation of heat with a thermal material of conductivity equalizes with the unit. As follows:

$$p = 0 \text{ in}$$

$$div$$

$$(gradT) = 0 \text{ in } T = 0 \text{ if}$$

=

1

p

T

$$($$

$$) = - F \text{ "xs} \cdot \mathbf{N} \text{ in}$$

(
) = N
in

N

N

T represents the temperature in the medium, it plays the part of the pressure in the fluid environment. N is the normal heat flow to the wall, it plays the part of the term $-F \mathbf{x}_s \cdot \mathbf{N}$ which is comparable to the variation in the course of the time of the flow of mass (fluid) to the wall of the structure. This quantity $-F \mathbf{x}_s \cdot \mathbf{N}$ is in homogeneous effect with a mass divided by a surface and a time squared.

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5.2

Implementation practical

A new operator CALC_MASS_AJOU was developed to take into account the inertial coupling (added mass) between structures bathed in the same true, incompressible fluid and with rest. The fluid is described by equivalent thermal characteristics (operator DEFI_MATERIAU [U4.23.01]) and the part of the grid representing are affected by elements thermics (operator AFFE_MODELE [U4.22.01]).

The operator uses five obligatory key words:

- the key word: MODELE_FLUIDE: it is on this model that one solves the problem of Laplace with boundary conditions of Von Neumann (or its thermal problem are equivalent),
- the key word: MODE_MECA (or CHAM_NO, or MODELE_GENE): this key word makes it possible to calculate them

boundary conditions of the flow type to the wall of the structure,

- the key word: **MODELE_INTERFACE**: it is on this model which includes/understands all the elements thermics of edge of the fluid interface/structure which one calculates the scalar product mentioned in the paragraph [§4.2],

- the key word: **CHAM_MATER**: it is about fluid material (described by characteristics thermics equivalent),

- the key word: **CHARGE**: it is a thermal load (temperature imposed in a node unspecified of the fluid grid) which corresponds to the boundary condition of Dirichlet for raising the singularity of the problem of Laplace (see [§4.1]).

One thus obtains a matrix of generalized added mass. This matrix having a profile line of full sky but (operator **NUME_DDL_GENE** [U4.55.07]) can be summoned with the matrix of mass generalized of the structure by using operator **COMB_MATR_ASSE** [U4.53.01]. This allows to calculate the coupled modes fluid/structure of the immersed structures ("wet" modes) (operator **MODE_ITER_SIMULT** or **MODE_ITER_INV** [U4.52.02], [U4.52.01]).

6 Bibliography

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***Coupling fluid-structure for the structures
tubular and coaxial hulls***

Summary:

This document describes the various models of coupling fluid-structure available starting from the operator CALC_FLUI_STRU. These models make it possible to simulate the forces of coupling fluid-rubber band in following configurations:

- beams of tubes under transverse flow (primarily, tubes of Steam generator),***
- passage stem of order/plate of housing (exclusively for the control rods),***
- coaxial cylindrical hulls under annular flow (for example, space ferments/envelope of heart),***
- beams of tubes under axial flow (for example, fuel assemblies).***

For each configuration, the model of forces fluid-rubber bands is initially presented. The resolution of modal problem is then described. The methods of resolution employed intégrent specificities of various models of forces fluid-rubber bands.

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1 Presentation

general

1.1 Recalls

The dynamic fluid forces being exerted on a structure moving can be classified in two categories:

- forces independent of the movement of the structure, at least in the range of small displacements; they are mainly random forces generated by turbulence or diphasic nature of the flow,***
- fluid forces dependent on the movement of the structure, known as “forces fluid-rubber bands”, persons in charge for the coupling fluid-structure.***

In this document, one is interested in the four models of forces fluid-rubber bands integrated in operator CALC_FLUI_STRU. The data-processing aspects related to the integration of these models made the object of notes of specifications [bib1], [bib2].

1.2 Modeling

The dependence of the forces fluid-rubber bands with respect to the movement of the structure is translated, in range of the low amplitudes, by a matrix of transfer enters the force fluid-rubber band and it vector displacement. The projection of the equation of the movement of the system coupled fluid-structure on the basis of modal structure alone is written, in the field of Laplace:

$$\{M\} \ddot{S} + \{C\} \dot{S} + \{K\} S - \{B_{ij}(U, S)\} (Q) = (Q_t) \quad \text{éq}$$

1.2-1

$[M_{ii}]$, $[C_{ii}]$ and $[K_{ii}]$

where

the diagonal matrices of mass indicate respectively,
of structural damping and stiffness in air;

(Q) the vector of the displacements generalized in air indicates;

(Q_t) the vector of the generalized random excitations indicates (forces independent of movement);

$[B_{ij}(U, S)]$

and

represent the matrix of transfer of the forces fluid-rubber bands, projected on the basis modal of the structure alone. This matrix depends in particular on U , speed characteristic flow, as well as frequency of the movement via the variable of Laplace S .

A priori, $[B_{ij}(U, S)]$ is an unspecified matrix whose extradiagonaux terms, if they are not null, introduce a coupling between modes. In addition, terms of $[B_{ij}(U, S)]$ evolve/move in manner not linear with the frequency S . complexes.

With each model of force fluid-rubber band is associated a specific matrix of transfer. In all the cases, the formulation of the modal problem under flow can be characterized by the relation [éq 1.2-1].

For the various types of configurations being able to be simulated using the operator CALC_FLUI_STRU, the representations of the matrices of transfer of the forces fluid-rubber bands are clarified in the continuation of this document.

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Excitation fluid-rubber band acting on the beams of tubes under transverse flow (primarily for tubes of Steam Generator)

The integration of this model of excitation fluid-rubber band in Code_Aster was approached in the note specifications [bib1]. The note of principle of software FLUSTRU [bib3] constitutes documentation theoretical of reference. The principal principal ones of modeling are pointed out hereafter.

2.1

Description of the studied configuration

One considers a beam of tubes excited by a transverse external flow. Physically, them transverse external flows tend to destabilize the mechanical system when rate of the flow increases.

An industrial case to treat in practice is that of the vibrations of the tubes of steam generators. On this component, the transverse flows are observed in the input area of the beam of tubes (monophasic flow liquidates), and in the curved part of the tube out of U (flow diphasic) [Figure 2.1-a].

Exit vapor

Separators

Excited zone

by

Food water

flow

diphasic

Water return

Excited zone

Beam of tubes

by

flow

Plate spacer

monophasic

Tubular plate

Primary fluid entry

Primary fluid exit

Appear 2.1-a: Diagram of steam generator

From the point of view of the coupling fluid-rubber band, the study of the dynamic behavior of different tubes of a beam subjected to a transverse flow is brought back to the study of an equivalent tube; the definition of the equivalent tube depends on the environment of the tube to treat.

When the tube considered has vibratory characteristics appreciably different from those of its neighbors, this tube can be compared to only one tube, vibrating in the middle of a beam of tubes rigid.

In the contrary case, the problem is more complex because one must consider a mechanical system with coupling between tubes of the beam and thus comprising a great number of degrees of freedom. To treat this kind of configuration, a model was developed at Department TTA, "the model total" [bib7]; this model allows the definition of a system equivalent to a degree of freedom, which represent the complete coupled system.

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The approach adopted to lead calculations can be summarized in the following way [Figure 2.1-b]:

- Taking into account the telegraphic nature of the structures studied, the calculation of the coupling fluid-rubber band in the beam of tubes is carried out by describing the tube by its X-coordinate curvilinear.*
- In calculation, the fluid environment of the tube is characterized, at the same time by the properties physiques of the fluid circulating inside the tube (fluid primary education), and by those of the fluid circulating outside the tube (fluid exiting secondary). These physical properties, such that the density, can vary along the tube, according to the curvilinear X-coordinate.*
- The rate of flow taken into account for the calculation of coupling fluid-rubber band is component, normal with the tube in the plan of the tube, the speed of the secondary fluid. This*

speed can vary along the tube.

· In order to be able to take into account the various possible types of excitation, several zones of excitation can be defined along the structure. In the case of the generator of vapor, for example, one may find it beneficial to distinguish, on the one hand the zones where the excitation is exerted

by a fluid in a monophasic state, which is located in foot of tube, and in addition, the zone where the excitation is diphasic at strong rate of vacuum, localised in the curved part of the tube.

· The calculation of coupling is carried out starting from the mechanical characteristics of the structure in

“fluid at rest”. The forces fluid-rubber bands of coupling are estimated from adimensional correlations which are obtained on analytical experiments in similarity. On each zone of excitation, one can thus apply the adequate correlations; the zones of excitation must be disjointed.

S U p p O r t s

Z O N E 2

Z O N E 3

Z O N E 1

0

has X E D E the fib Re N E U tre D U you B E

X

(B C M is S.E has. C U rv ilig N E)

Appear 2.1-b: Representation of the configuration studied

For this configuration of coupling fluid-rubber band, the following notations will be used:

L

Overall length of the tube

L_k

Length of the zone K

D

Diameter external of the tube

E

d_i

Internal diameter of the tube

I

Modal deformation of mode I

(X

E

)

Density of the external fluid to the curvilinear X-coordinate X

(X

I

)

Density of the fluid interns with the curvilinear X-coordinate X

T

Density of the tube (structure alone)

(*X*

eq

)

Density equivalent to the curvilinear X-coordinate X

U

Speed of the external fluid specified by the user in the operator

DEFI_FLUI_STRU

V (X)

Speed of the external fluid to the curvilinear X-coordinate X

Vk (X)

Speed of the external fluid to the curvilinear X-coordinate X (zone of excitation

K) challenge I

L

D it D U T of

wire D

it

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K) defined by the product of U and a profile speed specified by the user in operator DEFI_FLUI_STRU

U

V X

K

Mean velocity of the external fluid calculated starting from K () for zone of excitation

U

Average speeds the U.K. on all the zones of excitation

2.2

Stages of calculation

· The first stage of calculation consists in calculating the structural features in "fluid with rest ". One proceeds by considering an equivalent mass of the tube; this equivalent mass gather, on the one hand the mass of the tube alone, and on the other hand the masses added by the fluids intern and external.

An equivalent density is thus defined along the tube according to the X-coordinate curvilinear X by the expression:

1

(X)

2

2

2

2

=

+
-
+
eq
(
éq
2.2-1
2
2
I
I
T
E
I
E
eq

$D E - di [(X) .d$
 $(.d D) (X) .d]$
with

. C. 2
2
2 m D
D
E
eq =

éq 2.2-2

In the equation [éq 2.2-1], the term $(X) \cdot 2$ represents the mass added by the external fluid. This term depends, via the parameter C_m , of the arrangement of the beam of tubes (not square or triangular), and of the containment of the beam (not reduced). For calculations of coupling fluid-rubber band of the beams of tubes subjected to a transverse flow, one uses

usually, to estimate the coefficient C_m , of the given analytical expressions from experimental results. The whole of the data necessary to the estimate of the coefficient C_m is collected by operator `DEFI_FLUI_STRU`.

- Knowing the equivalent density of the tube, the elementary matrices of mass and of stiffness out of water at rest are then calculated by means of the profile of density equivalent, by operator `CALC_MATR_ELEM`; one uses options `MASS_FLUI_STRU` and `RIGI_FLUI_STRU`. Operator `MODE_ITER_SIMULT` allows, after assembly of the matrices elementary, to directly calculate the modes out of water at rest of the studied structure.
- The forces fluid-rubber bands of coupling are calculated by operator `CALC_FLUI_STRU` to leave adimensional correlations established on analytical models in similarity. These forces of coupling, $[B_{ij}(U, S)]$, dependent on the movement of the structure are then taken in account in the general equation of the movement [éq 1.2-1] to calculate the characteristics of system coupled flow-structure for a given speed of flow.

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2.3

Form of the matrix of transfer of the forces fluid-rubber bands

In the case of beams of tubes excited by a transverse flow, the forces fluid-rubber bands of coupling are forces distributed along the structure. They are characterized by linear adimensional coefficients of added damping and stiffness, named respectively C_d and C_k . The expression of the coefficients of the fluid matrix of transfer of the forces rubber bands projected on the basis of modal structure in “fluid at rest” is then the following one:

1

2

$$E(X) V(X) \text{ of } Cd(X, Sr) (X) dxs$$

B
2

$$ij(U, S)$$

L

I

=

ij

éq

2.3-1

1

2

2

+

$$E(X) V(X) Ck(X, Sr) I(X) dx$$

L

2

Dependence of the coefficients Cd and Ck with respect to the movement of the structure and the speed of the flow of the fluid is represented by their evolution according to the reduced frequency complexes Sr, defined by:

sD

Sr =

éq 2.3-2

U

The expression [éq 2.3-1] shows that one retains a diagonal matrix of transfer. That implies that:

- the various clean modes of the structure are rather distant from/to each other so that one can suppose that there is not coupling between modes.
- the modal deformations of the structure in "fluid at rest" are not disturbed by the setting in flow of the fluid.

These two assumptions could be checked in experiments on the beams of tubes subjected to one transverse flow.

In practice, taking into account the various zones of excitation taken into account along the structure, the diagonal coefficients of the matrix of efforts fluid-rubber bands projected on modal basis are written :

1
sd U

E
2

E (X) K
V (X) deCdk
I (X) dxs
L

K 2
UU
Bii (U, S) =

K

éq
2.3-3
K

1
2
sd U

E
 2

$+$
 $E(X) K$
 $V(X) C_{kk}$
 $I(X)$

dx
 L

K^2
 $U U_k$

where C_{dk} and C_{kk} indicate the adimensional coefficients of coupling respectively,
 $U U$

of damping and stiffness, retained for the zone of excitation K . Fluid speed
 K
 U

intervening in the reduced frequency complex in argument of the coefficients of coupling corresponds at the mean velocity on the zone of excitation K , after renormalisation of the profile $V_k(X)$, so that its average on all the zones of excitation is worth U .

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2.4**Resolution of the modal problem under flow**

In the configuration “Beam of tubes subjected to a transverse flow”, the problem is solved on the modal basis characterizing the structure in “fluid at rest”.

Generally, the characteristics of the system coupled flow-structure are obtained by seeking the solutions of the equation:

$$[M_{ii}] \dot{Q} + [C_{ii}] \dot{Q} + [K_{ii}] Q - [B_{ii}(U, S)] Q = 0$$

éq

2.4-1

[M_{ii}], [C_{ii}] and [K_{ii}]

where

the diagonal matrices of mass indicate respectively, of damping and stiffness structural features in “fluid at rest”;

(Q) the vector of the displacements generalized in “fluid at rest indicates”.

As the matrix of efforts fluid-rubber bands retained is diagonal, and that modal deformations are supposed not to be modified under flow, the problem of coupling fluid-rubber band bring back to the resolution of NR scalar problems, NR indicating the number of modes taken into account in the modal base.

For each mode I and each rate of flow U, the problem to be solved is written:

sd U

2

1

M_{ii}

+

C_{ii}

E (X) K

$V(X)$

E

2

$deCdk$

$I(X)$

$dx S$

$Lk 2$

UU

K

K

$éq$

$2.4-2$

1

$sd U$

$+$

$Kii -$

$E(X) 2k$

$V(X)$

E

2

Ckk

$I(X)$

dx

$= 0$

Lk 2

UU

K

K

It will be noted that the equation [éq 2.4-2] is non-linear in S; its solutions are obtained using one iterative method of Broyden type.

For each mode I, one obtains a solution if equation [éq 2.4-2]. One then deduces from it, for this mode, pulsation I and damping I of the system coupled flow-structure, in using the relation:

$$S^2 + 2\zeta \omega_n S + \omega_n^2 = 0$$

$$I = \omega_n \sqrt{1 - \zeta^2}$$

$$J = -\zeta \omega_n$$

with
1 éq

2.4-3

The coupled system dynamically becomes unstable when one of the damping coefficients I becomes negative or cancels themselves.

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3 Excitation fluid-rubber band acting on the stem of order on the level of the plate of housing (exclusively for the control rods)

The forces fluid-rubber bands acting on this type of configuration were identified on the model GRAPPE2 of department TTA. The theoretical aspects of the identification of these sources are developed in reference [bib4]. The integration of model GRAPPE2 in Code_Aster is approached in the note of specifications [bib2].

3.1

Description of the studied configuration

Model GRAPPE2 represents the stem of order, the higher part of the guide of bunch, and the thermal cuff of an engine of the type 900 or 1300 MWe [Figure 3.1-a].

Cuff

thermics

Tube envelope

Heart

power station

Plate

housing

Appear 3.1-a: General diagram of the model BUNCH 2

This model primarily consists of a hollow cylindrical tube low thickness, fixed on a full cylindrical central heart. The hollow tube is entirely immersed in water with ambient temperature. A plate, representing the plate of housing, makes it possible to reproduce it annular containment. The flow through the plate can be ascending or descendant. The stem of order can be centered or offset (50% of the average play) on the level of the plate of housing.

Four experimental configurations are thus possible, according to the direction of the flow and of centering or not of the stem of order. The coefficients of forces fluid-rubber bands were identified for each one of these configurations and are available in Code_Aster.

Model GRAPPE2 was dimensioned in geometrical, hydraulic similarity and of frequency reduced compared to the configuration engine. The only data of the diameter of the stem of order thus allows, in particular, to deduce the unit from the other geometrical magnitudes.

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3.2

Stages of calculation

*· The first stage of calculation consists in calculating the modal base of the water structure at rest, locally induced effects of mass added to the level of the containment of the plate of housing being neglected. This stage is carried out by operator **MODE_ITER_SIMULT**.*

With this intention, a homogeneous equivalent density is assigned to the whole of structure, in order to take into account the apparent mass added by the fluid, except for that induced by the effects of containment on the level of annular space. This mass voluminal equivalent is defined by:

R2

eq =

F + tube

éq

3.2-1

S

where:

indicate an adimensional coefficient of containment depend on the configuration studied; = 1 is the value used for calculations of control rods. It corresponds to a vibrating roller in an unlimited fluid field.

R
indicate the ray external of the tube,

S
indicate the surface of the cross-section of the tube,
tube indicates the density of material constituting the vibrating tube.

· The second stage is the taking into account of the coupling with the fluid flow. It is carried out with assistance of operator CALC_FLUI_STRU.

3.3 Representation of the excitation fluid-rubber band

That is to say X direction of neutral fibre of the tube. The excitation fluid-rubber band identified on the model

GRAPPE2 is represented by a resulting force and a moment, applied in the same point of X -coordinate $X O$, corresponding to the central zone of the passage of the stem of order through plate housing. The excitation is thus defined, in the physical base, by the relation:

$$f^c(X, S) = C$$

$$F(S)(X - x_0) - Mc(S)'(X - x_0)$$

éq
3.3-1

where $'$ the derivative compared to X of the distribution of Dirac indicates.

The resulting force, C
 F , acts thus under the effect of transverse displacements of the stem of order;
and the resulting moment, Mc , acts under the effect of the rotation of the latter.

One notes $X(S)$
 T
) the vector of transverse displacements and (
 S) the vector of associated rotations,
defined by:

$$0$$

$$XT(S)$$

$$= uy(X, S)$$

$$0$$

) éq 3.3-2

uz (*X*, *S*
0
)

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0

(

S) uy

=

(X, S)

0

éq 3.3-3 X $uz(X, S)$ 0 X

The following relations are used to calculate the forces and the moments fluid-rubber bands resulting starting from the added masses

 1

$Cm, cm2$, of added depreciation $Cd1 (Vr)$, $Cd2 (Vr)$ and

added stiffnesses $Ck1 (Vr)$, $Ck2 (Vr)$, adimensional coefficients identified on the model

GRAPPE2:

 1 2 2 1 1 2 C

$F(S) = - D L Cm S$

 1

$+ DUL Cd$

 F P F P

$1 (Vr) S +$

$U L Ck$

 F P

$1 (Vr) XT(S) \text{ éq}$

3.3-4 2 2 2 1

2
3
2
1
3
1
2
3

$$\mathbf{Mc}(S) = -D L C_m S$$

$$+ D U L C_d$$

F

p

F

p

$$2 (V_r) S +$$

$U L$

F

$$p C_k^2 (V_r) ($$

$S) \text{ \acute{e}q 3.3-5}$

2

2

2

In order to simplify the writing of the equations, one notes thereafter:

C

$$\mathbf{F}(S) = H_1(S) \mathbf{X} \mathbf{T}(S) \text{ and } \mathbf{Mc}(S) = H_2(S) ($$

$S)$

U

The fallback speed adimensionnelle V_r is defined here using the V_r relation =

, where S indicates

sD

variable of Laplace.

The expressions [éq 3.3-4] and [éq 3.3-5] utilize the thickness LP of the plate of housing. This thickness results from the value of the diameter of the stem of order, D , because of similarity geometrical with the configuration engine. The effort fluid-rubber band $f^c(X, S)$ is thus completely characterized by the data of the following sizes:

F

Density of the fluid,

U

Rate of the average flow in annular space between stem of order and plate housing,

D

Diameter of the stem of order,

1

C_m

Coefficient of added mass associated the translatory movement,

C_d

1 (V_r)

Added damping coefficient associated the translatory movement,

C_k

1 (V_r)

Coefficient of added stiffness associated the translatory movement,

C_{m2}

Coefficient of added mass associated the rotational movement,

C_d

2 (V_r)

Added damping coefficient associated the rotational movement,

C_k

2 (V_r)

Coefficient of added stiffness associated the rotational movement.

Adimensional coefficients of added mass,

1

C_m and

2

C_m, allow the taking into account of inertial effects induced by local containment of the stem of order on the level of the plate of housing. These effects are estimated as follows.

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That is to say H the thickness of the annular flow on the level of containment, deduced from D by similarity

geometrical compared to the configuration engine; indicate the adimensional coefficient of containment introduced by the relation [éq 3.2-1]. One obtains then [bib 4]:

1

3

2

2

2

D

D

D D

D L Cm =

-

L =

- L

2 F

p

1

F

8

F

H

4

p

F

4 2

p

H

3

1

2

D D

D D

L

2

3

p

F D LP Cm2 = F

-

(X - X)

2

2 dx

=

-

2

4 2

F

L

O

H

p

4 2H

3

One deduces the values from them from

1

Cm and cm2 by:

D

$Cm1 =$

- $\acute{e}q$

3.3-6

2 2H

$Cm1 D$

$Cm2 =$

=

-

$\acute{e}q$ 3.3-7

3

6 2H

The $Cd1$ coefficients, 1

Ck , $Cd2$ and $Ck2$ are directly deduced from measurement and are expressed in form adimensional correlations.

3.4 Projection on modal basis and expression of the terms of the matrix of transfer of effort fluid-rubber band

Decomposition of the movement on modal basis

One notes $J(X)$ the modal deformation of the j ème mode of the structure. Decomposition of the vector of displacements in the modal base is expressed in the form:

$DX(X)$

NR

NR

J

$(ux, S) = J(X) qj(S) = DY(X)$

J

$Q J(S) \acute{e}q$

3.4-1

$J=1$

$J=1$

$DZ J(X)$

Where $DX J$, $DY J$ and $DZ J$ correspond to the three components of translation characterizing them modal deformations calculated using Code_Aster.

Calculation of the generalized excitation associated mode I

The generalized excitation $Q(S$

I

) associated mode I is defined by the relation:

$I($

L

$S) = f^{\wedge}$

Q

$C(X, S)$

$\cdot I(X) dx$

éq 3.4-2

0

where L indicates the length of the structure on which one wants to impose excitations GRAPPE2.

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Transfer transfer functions $H1(S)$ and $H2(S)$ being defined starting from the relations [éq 3.3-4] and [éq 3.3-5],

one deduces some, taking into account the expressions [éq 3.3-1], [éq 3.3-4] and [éq 3.3-5]:

0

0

$IQ(S)$

NR

L

=

$H(S) \sum_{j=1}^J DY_j(x_0) q_j(S) (X - x_0) \cdot I$

$DY(X) dx$

0

1

$J=1$

$DZ J(x_0)$

$DZ_i(X)$

éq

3.4-3

0

0

NR

$-L$

$H(S) DY'$

$J(x_0) Q J(S) (X - x_0)$

.

$$I$$

$$DY(X) dx$$

$$0$$

$$2$$

$$J=1$$

$$,$$

$$DZ X$$

$$DZ X$$

$$J(O)$$

$$I()$$

From where, after integration:

$$IQ(S)$$

$$NR$$

$$= \{H1(S) [I$$

$$DY(xo) .DY J(xo) + DZi(xo) .DZ J(xo)]$$

$$J=1$$

$$+ H$$

$$,$$

$$,$$

$$,$$

$$,$$

$$2(S) [$$

$$I$$

$$DY(xo) .DY J(xo) + DZi(xo) .DZi(xo)]\} Q J(S) \acute{e}q$$

$$3.4-4$$

$$NR$$

$$= ij$$

$$B(S) Q J(S)$$

$$J=1$$

Note:

$$,$$

$$,$$

$$I$$

$$DY(xo) = DRZi(xo) \text{ and } DZi(xo) = -$$

$$I$$

$$DRY(xo)$$

3.5

Resolution of the modal problem under flow

The modal problem is solved by supposing, at first approximation, that the diagonal terms of the matrix of transfer of the efforts fluid-rubber bands [$\mathbf{B}(S)$] are dominating compared to the terms extradiagonaux.

The matrix [$\mathbf{B}(S)$] being thus reduced to its diagonal, the modal deformations are not disturbed by the taking into account of the coupling fluid-rubber band; the only modified parameters are them Eigen frequencies and modal reduced depreciation.

The modal problem under flow breaks up then into NR independent scalar problems, solved by a method of the Broyden type:

$$\begin{aligned}
 &(\mathbf{M} + a_j \\
 & \mathbf{II} \\
 & \mathbf{M} \mathbf{II})^2 \\
 & S + (\mathbf{C} + a_j \\
 & \mathbf{II} \\
 & \mathbf{C} \mathbf{II}(S))S + (\mathbf{K} + a_j \\
 & \mathbf{II} \\
 & \mathbf{K} \mathbf{II}(S)) = 0 \quad \text{é}q
 \end{aligned}$$

3.5-1

where
 a_j
 $\mathbf{M} \mathbf{II}$
 indicate the generalized mass added by the fluid,

a_j
 $\mathbf{C} \mathbf{II}(S)$ indicates the generalized damping added by the fluid,

a_j
 $\mathbf{K} \mathbf{II}(S)$ indicates the generalized stiffness added by the fluid.

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a_j

M

a_j

a_j

M , C II (S) and K II (S) are calculated using the relations:

a_j

1

2

2

2

2

2

2

,

,

$$M_{II} = + F D LP C_{m1} \{DYI(x_0) + DZi(x_0)\}$$

$$+ LP c_{m2} DYI(x_0) + DZ I(x_0)$$

éq 3.5-2

2

a_j

1

2

2

2
2
2
,

$C II (S)$

$$= - F DULp Cd1 (Vr) \{DYI (xo) + DZi (xo)\} + LP Cd2 (Vr) DYI (xo) + DZ I (xo)$$

2

éq 3.5-3

a_j
1
2
2
2
2
2
2
2
,

$K II (S)$

$$= - fU LP Ck1 (Vr) \{DYI (xo) + DZi (xo)\} + LP Ck2 (Vr) DYI (xo) + DZ I (xo)$$

2

éq 3.5-4

a_j
U
C
 a_j

II and $K II$ depend implicitly on S via the fallback speed $Vr =$

.

The three sizes necessary to dimension these terms are thus only D

F,
and
U, LP being deduced from D thanks to the geometrical property of similarity.

Like that was indicated previously, the adimensional coefficients
Cd1 (Vr), Ck1 (Vr), Cd2 (Vr) and Ck2 (Vr) result from the identified empirical correlations
in experiments on model GRAPPE2.

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4 Excitation fluid-rubber band acting on two hulls ***cylindrical coaxial under annular flow*** ***(example: space tank/envelope of heart)***

The integration of this model of excitation fluid-rubber band in Code_Aster was approached in the note specifications [bib2]. The note of principle of model MOCCA_COQUE [bib5] constitutes theoretical documentation of reference.

4.1

Description of the studied configuration

*The studied hardware configuration is made up of two coaxial, separate cylindrical hulls by an annular space in which runs out a viscous incompressible monophasic fluid [Figure 4.1-a]. The flow is done in the direction of the axis of revolution of the cylinders; to fix them notations, one supposes in the continuation of the document that it is about axis **X**.*

One notes:

L

the common length of the two cylindrical hulls,

R

$1 (, X T$

,)

the interior ray of annular space,

R

$2 (, X T$

,)

the ray external of annular space,

R, X,

1

T + R, X,

2

T

R (, X T,)

the average radius R (, X, T)

(

)

(

)

=

,

2

H (, X T,)

play annular (H (, X T,) = R2 (, X T,) - R1 (, X T,),

E

R, E, E

X

vectors of the base of cylindrical co-ordinates.

R1

R2

External hull

L

Internal hull

Appear 4.1-a: General diagram coaxial hulls

4.2

Stages of calculation

- *The first stage of calculation consists in determining the modal base in air of the structure. This operation is carried out by operator `MODE_ITER_SIMULT`. This calculation is necessary because decomposition of the matrix of transfer of the forces fluid-rubber bands [$\mathbf{B}(S)$] is expressed in this base.*
- *The second stage relates to the taking into account of the forces fluid-rubber bands. It intervenes in operator `CALC_FLUI_STRU`. This stage breaks up into eight sub-tasks:*

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4.2.1 Preprocessings

1°/Determination of the characteristic geometrical magnitudes, starting from the topology of grid: common length of the two hulls, average radius, average annular play.

2°/Characterization of the modal deformations in air: determination of the orders of hull, of principal plans, of the numbers of wave and the coefficients of associated deformations of beam with each mode of the structure, both for the hull interns the external hull.

4.2.2 Resolution of the modal water problem at rest

3°/Calculation of the matrix of mass added by the fluid [Maj] in the modal base of structure in air

4°/Calculation of the modal characteristics of the water structure at rest while solving:

$$[(\{Semi\} + [Maj]) 2s + [Ki]] (Q) = 0$$

One obtains the new structural features out of water at rest

E

E

E

Semi, Ki, fi

(generalized mass and stiffness, Eigen frequency of mode I) as well as the deformations modal

E

I, expressed in the base in air.

5°/Calculation of the water deformations at rest in the physical base, by basic change:

**[I.E.(internal excitation)] = [have] [E
.i]**

4.2.3 Resolution of the modal problem under flow

For each rate of flow:

6*/Calculation of [B (S)] in the modal base in air.

This calculation is carried out by solving the non stationary fluid problem according to the method specified in the paragraph § 4.3.1.

7*/Calculation of the forces fluid-rubber bands induced by the effects of damping and stiffness additions, in the modal water base at rest.

[B (S)] T

E

= [E

2

I] [

{B (S)}- [Maj] S} [E

I]

8*/Resolution of the modal problem by neglecting the extradiagonaux terms of the latter stamp, by the method of Broyden (buckles on the sub-tasks 6* and 7*).

E 2

M S

I

+ E

C S

I

+

E

Ki -

E

Bii (S) = 0

Modal characteristics of the structure:

EC.

EC.

EC.

Semi, fi

, I (generalized mass, Eigen frequency and damping of mode I, under flow) are given. modal deformations are supposed to be identical to those out of water at rest.

*End of loop on the rates of flow
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Note:

• The calculation of the terms of the matrix of transfer of the forces fluid-rubber bands requires resolution of the non stationary fluid problem (sub-task 6°). This resolution is not itself possible that if certain sizes beforehand were determined geometrical characteristics of the configuration, as well as the coefficients of the forms analytical of the modal deformations of the structures (preprocessings 1° and 2°).

• If the user chooses to carry out the first stage (calculation of the modal base by operator `MODE_ITER_SIMULT`) by taking directly into account the effects of mass added, those should not be taken any more into account by operator `CALC_FLUI_STRU`. For that, key word `MASS_AJOU` of order `DEFI_FLUI_STRU` must be informed by “NOT”. The sub-tasks 3° with 7° become then:

3° Calculation of the effects of mass added by the fluid, in the modal base of

/

water structure, in order to be able to cut off these effects of the effort fluid-rubber band total, since the terms of added mass are already taken into account.

4° removed Sub-task.

/

5° removed Sub-task.

/

For each rate of flow

6° Calculation of the matrix $[B(S)]$ in the modal water base.

/

7° Calculation of the forces fluid-rubber bands induced by the effects of damping and of

/

stiffness added in the modal water base:

$$[eB(S)] = [B(S)] - [Maj] 2s$$

The sub-tasks 1°, 2° and 8° are not modified.

4.3

Resolution of the non stationary fluid problem

4.3.1 Assumptions

simplifying

Some assumptions on the nature of the flow make it possible to simplify the equations of Navier-Stokes non stationary, at the base of the problem fluid-structure.

H1

It is supposed that the flow is the superposition of a stationary average flow, obtained when the structures are fixed, and of a non stationary flow induced by the movement of walls.

H2

It is supposed that the vibrations of structure are of low amplitude with respect to the thickness of the average annular flow.

H3

One supposes that the disturbances speed induced by the vibratory movements are, in average on a ray, primarily directed in the directions and X: one supposes thus that the vibratory movement induced a helicoid movement of fluid around the structures rather than a radial movement compared to these last. These disturbances speed define order 1.

H4

One supposes finally that the speed and pressure field is uniform, with order 1, in radial direction.

These simplifying assumptions make it possible to solve the fluid problem analytically. The matrix of transfer of the forces fluid-rubber bands [B (S)] is deduced from the non stationary flow resulting from this resolution.

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4.3.2 Analyze in disturbances

With the help of the assumptions stated previously, the analysis in disturbances of the fluid problem conduit to seek the non stationary flow in the form:

U = 0

R

+ 0 +

2

order

éq

4.3.2-1

U = + u~

0

(, X, T) +

2

order

éq

4.3.2-2

U

~

$$X = U(X) + U X(, X, T) +$$

2

order

éq

4.3.2-3

$$P = P(X) + p~(, X, T) +$$

2

order éq

4.3.2-4

with:

$$R = R + r~(, X, T$$

1

1

1

)

éq

4.3.2-5

$$R = R + r~(, X, T$$

2

2

2

)

éq

4.3.2-6

~

~

~

~

R

~

~

+

One defines the variables H and R like:

~

~

 $H = R$

2

 R $2 - R$ let $R =$

.

2

By limiting the development of the Navier-Stokes equations to the first order, one obtains two systems of equations characterizing the stationary part and the disturbed part of the flow, the second system being a linear system.

The resolution of the stationary fluid problem leads thus to:

 P

1

 $U(X) = U$ constant and

2

 $= - C U$ éq

4.3.2-7

 X H F

In the equation [éq 4.3.2-7], ρ indicates the density of the fluid and C_F the stationary part of coefficient of friction to the wall. The incompressible fluid being supposed, its density is not broken up partly stationary and fluctuating part. C_F is deduced from the law of Nikuradzé characterizing the flows in control:

 $2 U$ H C X $F = C_f \rho U^2$ $R, \rho(R, U)$ E R with E $R =$

éq

4.3.2-8

where m indicates the value of an exhibitor, indicates the kinematic viscosity of the fluid and the roughness of walls.

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It results from this:

C

*,
F*

= C fo (R

E

E,)

m (R

)

Re

~

~

~

2HU

Hu~

~

2

C

X

F

= C F (Re) - C F (Re)

with Re =

R =

and E

~

= (m +

U

2) *C*

X

F

+

2

order

U

The linear differential connection of a nature 1 characterizing the non stationary part of the flow induced

by the movements of walls is written in the field of Laplace:

~

~

u~

~

X

1 U

U

H

S ~ U R

S ~

+

= -

+

H -

+

R

X

R

H X U R X U

$u \sim$
 U
 I

$p \sim$
 U
 $+ S + C$
 \sim
 F
 $U +$
 $= 0$
 $\acute{e}q$
4.3.2-9

X

 H
 R

\sim
 \sim
 2
 U

U
 $I p$
 $U \sim$
 U
 $X + S + C (m$
 \sim
 F
 $+ 2)$
 $U X +$
 $= C H$

F

 X

 H
 X
 H

Three boundary conditions of input-output make it possible to solve this system. The first of these conditions is obtained by supposing that the flow is sufficiently regular upstream of space

annular, so that the tangential component the speed of entry can be neglected:

$U = 0$ in $X = 0$ éq

4.3.2-10

The two others are obtained by applying the conservation equation of the kinetic energy, under its quasi-stationary form, between the infinite upstream and the entry of annular space, then between the exit of

annular space and infinite downstream. One obtains then respectively, with the order disturbances:

R2

1

~

~

~

2

$p + U U X (1 + C$

0

0

D

in

E)

+

C U

$U r dr = X =$

D E

2

R

1

éq

4.3.2-11

R2

1

~

~

~

2

$p + U U X (1 - C$

0
 D
 in
 $S)$

$-$
 $C U$
 $U rdr = X = L$

$D S$

2

$R1$

In these expressions, C_d and C represent the stationary parts of the loss ratios

E

ds

of load singular of entry and exit. They take into account the dissipation of induced energy, when the walls are fixed, by possible abrupt evolutions of the geometry at the entry or exit of annular space. In the majority of the cases, these coefficients can be estimated simply using data of the literature (Idel' cik for example). When geometrical configuration of entry or exit is very particular, these coefficients can also be given with the assistance of a two-dimensional code of mechanics of the fluids adapted to the study of the problems with fixed walls, of type N3S.

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~
~

C_d and C are the non stationary parts of the singular loss ratios of load. These

*E
 ds*

coefficients take into account the disturbances of the lines of separation induced by movements of structure. They can be modelled thanks to a quasi-stationary approach of even natural that that introduced for the estimate of the coefficient of friction of wall. The system [éq 4.3.2-9] is solved analytically, using the limiting conditions [éq 4.3.2-10] and

~
~

[éq 4.3.2-11], by clarifying the functions H and R characterizing the second member.

The disturbances r_{\sim}

~

$l_1(X, S)$ and $r_2(X, S)$ defining the movement of the walls, the parts disturbed annular play and average radius are then defined, in the field of Laplace, by:

~

$H(X, S) = r_{\sim}$

,
 $2(X, S) - r_{\sim}$

,
 *$l_1(X, S)$ éq
4.3.2-12*

~

~

~

,,

,,

$R(X,$

+

$S) l_1(X, S) 2r_2(X, S)$

=

éq

4.3.2-13

2

4.3.3 Decomposition on modal basis

That is to say NR the number of oscillatory modes of the structure in the studied frequency band. decomposition on the basis of modal movement of the walls is expressed in the following way:

$$NR$$

$$R$$

$$\sim 1 (, X, S) = [$$

$$\cos K$$

$$*$$

$$I$$

$$I (- I$$

$$I)] R$$

$$\cdot i1 (X)$$

$$\cdot I (S) \acute{e}q$$

4.3.3-1

$$I I$$

$$=$$

$$NR$$

$$R$$

$$\sim 2 (, X, S) = [$$

$$\cos K$$

$$*$$

$$2i (- 2i)] R$$

$$\cdot 2i (X)$$

$$\cdot I (S)$$

4.3.3-2

$$I I$$

$$=$$

where

K i1 and k2i represent the orders of hull of the ième mode for the respective movements hulls internal and external,

i1 and 2i make it possible to characterize the principal plans of these modes,

$$R *$$

$$*$$

I

I (X) and r2i (X) is deduced from the deformations of beam of the structures internal and external associated the mode considered,

and

(S
I
) represents generalized displacement.

Note:

The functions R *

*

I

I (X) and r2i (X) is represented, within the framework of the analytical resolution, in the form of linear combinations of sine, cosine, hyperbolic sine and cosine hyperbolic:

*

R

I

I

I

I

I

I (X)

I

I

I

I

= Ad interim I cos

X + B il sin

X + C CH

I

I

X + D HS

I

I

X éq

4.3.3-3

L

L

L

L

*

R
2
2
2
2
2i (X)
I
I
I
I
= A2i cos
X + B2i

sin
X + C CH
2i

X + D HS
2i

X éq 4.3.3-4
L
L
L
L

with i_1 and $2i$ numbers of wave of the i ème mode for the movements of the hulls internal and external respectively.

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Solutions of the fluid problem ~, ~

~

p U

U

X and are required in the form of decompositions on modal basis deduced from those of ~

~

R and R

l

2 clarified by the relations [éq 4.3.3-1] and [éq 4.3.3-2]. One obtains thus, in the field of Laplace:

*NR **

,

p~ (, X, S)

p il (X S)

=

[

cos kl

-

+ 2

l

cos 2

-

I (

I)] p I (X S) [K I (2i)] I (S)

éq 4.3.3-5

2

2

I l

K

K

=

l

l

2i

NR

u~

*

*

X (, X, S) = (U I

I (X, S)

[
cos K i1 (- i1)]+ u2i (X, S)

[
cos k2i (-2i)] I (S)

éq

4.3.3-6

I =1

*NR **,

*

,

u~

,,

=

1

sin 1

-

+ 2

1

sin 2

-

(X S)

v I (X S)

[K I (I)] v I (X S) [K I (2i)] I (S)

éq

4.3.3-7

K

K

I =1

I

1

2i

4.3.4 Expression of the terms of the matrix of transfer of the forces fluid-rubber bands

The surface effort fluid-rubber band, F , are the resultant of the field of pressure and the constraints viscous and turbulent exerted by the flow on the walls of the structure moving.

$$F = - P N + T$$

+ T

X X

éq

4.3.4-1

The effort generalized fluid-rubber band associated the i ème oscillatory mode of the structure, $Q(S)$, is written as follows:

$$IQ(S) = \int F \cdot X_i ds$$

ds

éq

4.3.4-2

If

Where I is the surface of the walls of the structure indicates wet by the flow, and vector X_i represent the i ème vector deformed modal in this expression. The representation of the field of speeds and of pressure and the representation in the form of a law of wall of the viscous constraints and turbulent exerted on the structure moving allow to express the effort fluid-rubber band generalized $Q(S)$ in the following way:

IQ (

NR

$$S) = \sum_{j=1}^{NR} B_{ij}(S) J_j(S)$$

éq

4.3.4-3

J =1

with $B_{ij}(S) = B_{1ij}(S) + B_{2ij}(S)$

$B_{1ij}(S) = \int_{I_j} p(S) X_{ij} \cdot X_i ds$

(S

ij

I

B

) and B2 (S

ij

) the contributions of the hulls indicate respectively interior and external.

These contributions are defined by:

R1

1

B ij

l (S) =

-

cos [K il

,

,

.

2

(i1 - 1j) K ki11j LP i1 (X S) + CfUv i1 (X S)

r1j (X) dx

K

0

2

l

l

- R1

l

cos

,

,

.

2

$[k_{2i} (2i - 1j)] K k_{2i1j} L_{p2i} (X S) + C_{fUv2i} (X S)$

$r_{1j} (X) dx$

K

0

2

$2i$

éq 4.3.4-4

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and

R_2

*

l

*

*

$B_{2ij} (S) = -$

$[$

$\cos K_{il}$

,

,

.

2

$(i1 - 2j)] K_{ki12j} L_{P i1} (X S) + C_{fUv i1} (X S)$

$r^2 J(X) dx$

K

0

2

I

1

$- R^2$

$*$

1

$*$

$*$

cos

,

,

.

2

$[k_{2i} (2i - 2j)] K k_{2i2j} L_{p2i} (X S) + C_f U_{v2i} (X S)$

$r^2 J(X) dx$

K

0

2

$2i$

éq 4.3.4-5

4.4

Resolution of the modal problem under flow

As one explained in the paragraph [§ 4.2], one solves beforehand the modal problem out of water with rest, in order to take into account the inertial coupling between modes. One estimates the matrix thus of

mass added by the fluid, while calculating $[B(S)]$ for a mean velocity of the flow null.

modal characteristics of the system under flow are then obtained by disturbing them

water characteristics at rest. One does not hold any more account but of damping and the stiffness additions: the terms of mass added previously calculated are cut off from the matrix $[B(S)]$.

The coupling between modes is then neglected; consequently, the modal deformations remain unchanged compared to those out of water at rest. Only parameters disturbed by the setting in

flow of the fluid are the frequency and reduced modal damping. These parameters are calculated by solving NR nonlinear equations mode by mode, implementation of a method of the type Broyden.

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5

Axial flow (example: fuel assemblies)

The integration of this model of excitation fluid-rubber band in Code_Aster was approached in the note

specifications [bib2]. The note of principle of model MEFISTEAU [bib6] constitutes documentation theoretical of reference.

5.1

Description of the studied configuration

One considers a beam of K circular cylinders mobile in inflection and subjected to a flow incompressible of viscous fluid, limited by a cylindrical rigid enclosure of circular section or rectangular [Figure 5.1-a].

L

X

Circular enclosure

Z

Y

Rectangular enclosure

Appear 5.1-a: Beam under axial flow

The cylinders all parallel, are directed along the axis of the enclosure. They have a common length, noted L. To simplify the notations, it is supposed thereafter that X is the directing axis. The flow stationary axial and is supposed to be uniform in each section. Density of the which can fluid to be variable along axis X (heat gradients), the rate of the stationary flow depends also of variable X.

5.2

Stages of calculation

- The first stage relates to the determination of the modal base in air of the beam. This operation is carried out by operator `MODE_ITER_SIMULT`. This stage is essential because them forces fluid-rubber bands are projected on this basis.*
- The second stage relates to the taking into account of the forces fluid-rubber bands with the operator `CALC_FLUI_STRU`. This stage breaks up into 7 sub-tasks:*

5.2.1 Preprocessings

1°/By means of the topology of the grid, deduction of the co-ordinates of the centers of the cylinders beam then checking of the good provision of the cylinders ones compared to others (it is checked in particular that there is not overlapping between two cylinders) and by report/ratio with the rigid enclosure.

2°/Determination the length of excitation of the fluid, commune to all the cylinders, like of an associated discretization along the directing axis.

3°/Constitution of the tables giving the modal deformations in air of each cylinder of beam, for each mode taken into account for the coupling fluid-structure. One interpolate for that the deformations at the points of the discretization determined before.

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5.2.2 Resolution of the modal problem under flow

4°/Resolution of the disturbed fluid problem. Determination of the potential speeds disturbed the inversion of linear systems of high natures requires calling the setting in work of the method of Crout.

For each rate of flow

5°/Calculation of the matrices of mass, damping and stiffness added giving the matrix of transfer of the forces fluid-rubber bands in the modal base in air:

[B
2
 $ij(S)] = - [My] S - [Ca] S - [K has]$
[My] full symmetrical; [Ca] and [Ka] a priori full and nonsymmetrical.

6°/Resolution of the modal problem under flow; one solves the complete problem with vectors and with the clean ones

$\{[Mij] 2s + [Cij] s + [Kij] - [Bij(S)]\} \cdot (Q) = (0)$

One does not neglect the extradiagonaux terms of [B (S

ij
)]. After reformulation, the resolution is
carried out using algorithm QR: obtaining the masses, frequencies and depreciation
modal reduced under flow

EC.

EC.

EC.

Semi, fi

, I, modal deformations complex EC.

I expressed

in the base in air; of these last, one retains only the real part after minimization of
imaginary part (calculation of a criterion on the imaginary part).

7°/Restitution of the deformations under flow in the physical base.

[EC.

I] = [I] [EC.

I]

[I] is the matrix whose columns are the modal deformations in air, expressed in base
physics.

End of loop on the rates of flow

Note:

- ***The knowledge of the co-ordinates of the centers of the cylinders (preprocessing 1°) is necessary to the resolution of the disturbed fluid problem (sub-task 4°). This resolution conduit with the estimate of the terms of the matrix of transfer of the forces fluid-rubber bands (sub-task 5°), which utilizes the disturbances of pressure and speed.***
- ***Determination a common length of excitation and the creation of a discretization associated (preprocessing 2°) allow to define a field of integration on the structures for the projection of the forces fluid-rubber bands on the modal basis. The interpolation of modal deformations at the same points is thus necessary (preprocessing 3°).***
- ***The dynamic behavior of the beam under flow can also be studied with assistance of a simplified representation of the beam (with equivalent tubes). Stages of calculation for the taking into account of the coupling fluid-structure are then identical to those described previously, only differences appearing in the preprocessings. This second approach is described more precisely in the note [bib2]. In the stage 1° of preprocessings, the co-ordinates of the centers of the cylinders of the beam are then specified directly by the user, who also establishes the correspondence between the cylinders of beam and beams of the simplified representation given by the grid. In the stage 3° preprocessings, this correspondence makes it possible to assign to the cylinders of the beam, with points of discretization determined in the stage 2°, the modal deformations of the beams of simplified representation.***

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5.3

Resolution of the non stationary fluid problem

5.3.1 Assumption simplifying

H1

The field non stationary fluid speeds is analytically given while supposing

~

that the disturbed flow is potential in all the fluid field, and that the flow stationary is uniform transversely, but function of the axial position X:

$$\mathbf{U} = U + \mathbf{u} \sim = U(X) \mathbf{X} + ($$

)

éq

5.3.1-1

Such a field speeds admits a slip on the walls of the cylinders which will allow to calculate the viscous constraint by a law of friction.

H2

~

The movement of the cylinders does not generate disturbances speed ~

$$U = ($$

) that

radially and orthoradialement (assumption of the slim bodies): ~

~

~

$$U = U y$$

y

$$+ U Z$$

Z

H3

The field of pressure is broken up into parts stationary and disturbed according to $P = P + p \sim$

The stationary field of pressure depends only on X and its gradient is worth:

P

$$D(X)$$

U

D

C

$$= - U$$

(X) - fl

2

U U + g.x

éq

5.3.1-2

dx

dx

DH

where

DH indicates the hydraulic diameter of the beam,

C fl indicates the coefficient of local friction for stationary speed U. It depends on Reynolds number, calculated using stationary speed U, of the hydraulic diameter of beam and of the surface roughness. This coefficient is deduced from the law of Nikuradzé characterizing the flows in control;

G indicates the field of gravity. Its action on the stationary field of pressure depends on the slope of the beam (g.x).

5.3.2 Determination of the potential disturbed speeds

~

One seeks an analytical solution for (R, X, T) in the form of a superposition of elementary singularities which are written:

NR trunk

{C (X, T) .r-n.co (Sn

- N

K) + Dnk (X, T) .rk .si (

N N

nk

K

K)} éq

5.3.2-1

N =1

in the center of each cylinder K and:

Ntronc {Na (X, T) n.or.cos (No) + Nb (X, T) n.or.sin (No)}

éq

5.3.2-2

n=1

in the center of the rigid enclosure when this one is circular where:

NR

NR

3,
trunk
indicate the order of truncation of the series of Laurent (trunk =)
K
R
, K
the polar co-ordinates in a plan perpendicular to axis X indicate,
centered in the center of the cylinder K,
O
R
, O
the polar co-ordinates in a plan perpendicular to axis X indicate,
centered in the center of the circular rigid enclosure.

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The coefficients $C_{nk}(X, T)$, $D_{nk}(X, T)$, $Year(X, T)$ and $B_n(X, T)$ of the expressions [éq 5.3.2-1] and [éq 5.3.2-2]

are given by applying the boundary condition of nonpenetration:

· on the contour of each mobile cylinder K , this condition is written:

~

D_y

D_z

K ,

$(Kr = R)$

K

$K =$

$(X, T) \cos ()$

K

$K +$

$(X, T) \sin (K)$

K

R

Dt

Dt

where

$y_k(X, T)$ and $z_k(X, T)$ indicate the components of the displacement of neutral fibre of the cylinder K with X -coordinate X in the reference mark $(y,)$

$Z,$

rk and K indicate the polar co-ordinates in the reference mark $(y,)$

Z whose origin is taken with center cylinder $K,$

Rk indicates the ray of the cylinder $K,$

D

$+ U(X)$

\cdot
 Dt

T

X

\cdot on the contour of a circular rigid enclosure, it is written:

\sim

$O,$

$(gold = O$

$R) = 0$

where R_o indicates the ray of the enclosure.

O

R

In the case of a rectangular rigid enclosure, this condition is taken into account by a method derived from the method of the "images" [bib6]; the fluid problem confined by the rectangular enclosure

is made equivalent to the problem in infinite medium by creating images of the mobile cylinders of beam compared to the sides of the enclosure. This method results in introducing news singularities of the form [éq 5.3.2-1], placed at the center of the cylinders "images", in the expression of

\sim

\cdot It does not add however an unknown factor to the problem since the coefficients for this news singularities are derived from those of the mobile cylinders of the beam by the play of the images.

Finally, the potential disturbed speeds is written:

~
 $(R, X, T) K$
 $=$
 Dy
 Dz
 $F (R,)$
 K
 K
 $(X, T) + G (R,) K$
 K
 (X, T)
éq
5.3.2-3
 Dt
 Dt
 $K = 1$

Where K indicates the number of mobile cylinders of the beam. The functions $F (R,)$ and $G (R$
 K
 K
 $,)$ are
linear combinations of RN .

(
 $\cos N$), RN .

(
 $\sin N$), RN .

(
 $\cos N$) and RN .

(
 $\sin N$) of which them

coefficients are determined by the boundary conditions preceding. That requires the resolution of linear systems of high natures and with full matrices. The inversions are carried out while putting in work the method of Crout.

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5.3.3 Modeling of the fluid forces

One retains initially the forces due to the disturbances of pressure $\sim p$, connected to the potential speeds disturbed by:

~

~

 D $p = -$ $\text{é}q$ **5.3.3-1** Dt

The resultant of the field of pressure disturbed around each mobile cylinder is a linear force

2

 $D y$

2

 $D Z$ $F \sim$ K K

p acting according to y and Z . This force depends linearly on

*and**, thus generating*

2

 Dt

2

 Dt

terms of mass, damping and stiffness added.

One then takes into account the forces related to the viscosity of the fluid.

In a quasi-static approach, one considers the action of the fluid field speed ($U + u\sim$) around of a cylinder at the moment T : in the reference mark related to the cylinder, the flow, speed U to order 0, present an incidence compared to the cylinder which is a function of the disturbances speed and of movement of the cylinder itself. It results from it a force from trail and a force of bearing pressure. One

show that the components following y and Z of the resulting linear force f_v are written, for roll L :

(
 y
 Dy
 L
 F)

= -
 L
~
~

y
 $R U.a. fl$
- $U ly$ -
 L

$R U.a. p$
 $L - U ly$
 $éq$
5.3.3-2
 T

L
 Dt

(
 Z
 Dz
 L
 F)

~
~

= -

 Z $R U.a. fl$ $L - L$ $uy - R U CP$ $L - L$ uz L $éq$ **5.3.3-3** T L Dt *where**CP very indicates the slope with null incidence of the coefficient of bearing pressure around a cylinder slightly tilted ($CP = 0,08$).*

~

uy and ~uz indicate the averages of the disturbances speed along axes y and Z around Dy Dz *cylinders, which depend linearly on* K and K (cf [éq 5.3.2-3]). Dt Dt *These forces generate terms of added damping and stiffness.**One finally takes into account the action of the stationary field of pressure on the mobile structures deformations. One shows that the resulting linear force F_{pl} on the cylinder L has as components, with order 1:*

(

 L

2

L
F p)

y
= R
P

éq
5.3.3-4

y
L X X
(

L
2
L
F p)

Z
= R
P

éq
5.3.3-5

Z
L X X

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These forces generate only terms of added stiffness and any coupling between cylinders.

The expressions [éq 5.3.3-1], [éq 5.3.3-2] and [éq 5.3.3-3] highlight the need for solving the disturbed fluid problem before estimating the forces fluid-rubber bands.

5.3.4 Expression of the terms of the matrix of transfer of the forces fluid-rubber bands

Summary of the linear forces

For each cylinder L, the forces fluid-rubber bands are written according to y and Z:

$$F = F_{l-p} + F_L + F_L$$

L

p

éq

5.3.4-1

and are linear combinations of:

y

2

2

2

2

2

2

2

K

yk yk zk zk zk zk

,

,

,

,

,

,

,

(K =,

1 K)

T

t2

tx

x_2
 T
 t_2

$T X$
 x_2

Decomposition of the movement on modal basis

The movement of the beam of cylinders is broken up according to NR modes of vibration into air. One notes

k_j (1 K K and 1 J NR) deformations following y and Z of the cylinder K corresponding to the jème mode

beam. Components of the displacement of neutral fibre of the cylinder K to X-coordinate X can then be written:

$$y_k(T) = \sum_{NR} Q_j(T) J(X)_y$$

·
 \dot{e}_q
 5.3.4-2

$$z_k(T) = \sum_{NR} Q_j(T) J(X)_Z \quad \text{éq 5.3.4-3}$$

$J=1$
where $(Q) = (Q_j)$ is the vector of generalized displacements.

$J=$
 1 NR

Projection of the forces on modal basis

· *One notes (T*
 I

F) the projection of the forces fluid-rubber bands according to the ième mode of the beam.

I
 $F ($
 K
 L

$T) = fk (X, T) K$

. $I (X) dx \acute{e}q$

5.3.4-4

0

$K = 1$

(T

I

F) is a linear combination of ($Q J, q \& J, q \& J$)

$J =,$

1 NR

· One notes $F (T)$ the vector of the modal forces fluid-rubber bands: $F (T) = (I$
 $F (T) i =, 1N$ which is written:

$F (T) = - [My] (q \& (T) - [Ca] (q \& (T) - [Ka] (Q (T) \acute{e}q$

5.3.4-5

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Where

[My] the matrix of the terms of mass added by the fluid indicates,

[Ca] the matrix of the terms of damping added by the fluid indicates,

[Ka] the matrix of the terms of stiffness added by the fluid indicates.

These matrices are square real of order NR and their terms are independent of the movement structures. The matrix [My] is symmetrical; the matrices [Ca] and [Ka] are not it necessarily.

• The projection of the equations of the movement on modal basis provides:

$$[(M_{ii}] + [My]) (\ddot{q}(T) + [(C_{ii}] + [Ca]) (\dot{q}(T) + [(K_{ii}] + [Ka]) (Q(T) = (0) \text{ \acute{e}q 5.3.4-6}$$

[M_{ii}], [C_{ii}] and [K_{ii}]

where

the matrices of masses, depreciation indicate and stiffnesses of structure in air; these matrices are of order NR and diagonals.

In the field of Laplace, the relation [éq 5.3.4-6] becomes:

$$[(M_{ii}] + [My]) 2s + [(C_{ii}] + [Ca]) S + [(K_{ii}] + [Ka]) (Q(S) = (0) \text{ \acute{e}q 5.3.4-7}$$

• One introduces then the matrix of transfer of the forces fluid-rubber bands [B(S)] defined by:

$$[B(S)] = - [m_2] S - [Ca] S - [K_{has}] \text{ \acute{e}q 5.3.4-8}$$

And one finds the relation [éq 1.2-1] paragraph [§ 1.2]:

$$[(M_{ii}] 2s + [C_{ii}] S + [K_{ii}] - [B(S)]) (Q(S) = (0)$$

5.4

Resolution of the modal problem under flow

The modal problem under flow is formulated by the relation [éq 5.3.4-7] of the preceding paragraph.

This problem is solved after rewriting in the form of a standard problem to the vectors and with eigenvalues of type [A] (X) = (X).

The new formulation is as follows:

$$\begin{bmatrix} [] \\ 0 \\ [Id] \\ Q \\ Q \end{bmatrix}$$

= S
éq 5.4-1

-1
-1
sq
sq
- ([Mii] + [My]) [(Kii) + [Ka)] - ([Mii] + [My]) [(Cii) + [Ca)]

Note:

- 1) One doubles the dimension of the problem compared to that of the initial problem.*
- 2) The properties of the matrices [Mii] and [My] allow the inversion.*

The resolution of this problem is done by means of algorithm QR. Modules implemented by operator CALC_FLUI_STRU are the same ones as those used by MODE_ITER_SIMULT.

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The problem with the clean elements that one solves is a complex problem. One is thus obtained numbers even combined complex eigenvalues two to two. One preserves only those of which the imaginary part is positive or null.

The clean vectors complex, are defined except for constant a complex multiplicative. Like

one takes into account only real modes, it initially acts to determine, for each clean vector, the constant which minimizes the imaginary part of the vector compared to its real part, within the meaning of the euclidian norm. The clean vectors are then redefined compared to this standard.

Taking into account standardization used, it is then possible not to preserve in the concept mode_meca that the real part of the clean vectors. One restores however, in the file MESSAGE, indicators on the relationship between imaginary part and real part of the clean vectors thus normalized, so that the user can consider skew introduced by not taken into account of the part imaginary of the normalized vectors.

5.5

Taking into account of the presence of the grids of the beam of tubes

Modeling described previously, of the forces induced by an axial flow on a beam of cylinders, does not take into account the presence of the grids of the beam (for example, the grids of mix and of maintenance of the fuel assemblies). A comparison between this model and tests carried out on the model CHAIR (in the configuration of a beam of nine flexible tubes comprising a grid) is presented in a note of synthesis [bib8]: it is noted that the coupling fluid-rubber band between the grid and the axial flow is not negligible and that it generates one increase in the reduced modal damping of the tubes. The object of this paragraph is description additional effects due to the grids and of their taking into account in model MEFISTEAU.

5.5.1 Description of the configuration of the grids

One restricts here the study with two types of grids:

- the grids of maintenance which are located at the ends of the beam,*
- the grids of mixture which are distributed between the grids of maintenance.*

The grids all are positioned perpendicular to the beam of cylinders and are presented in the form of a prismatic network at square base on side $D G$ and height H_g (along axis X of cylinders). The grids of the same type are characterized by identical dimensions.

5.5.2 Additional stages of calculation

- The first additional stage relates to the specification of the type of configuration of the grids by operator $DEFI_FLUI_STRU$, then the checking of the good provision of the grids ones compared to the others, and the ends of the beam.*
- The second stage relates to the resolution of the modal problem under flow. In the loop on the rates of flow, the matrix of transfer of the forces fluid-rubber bands in the base modal in air is supplemented by the calculation of a matrix of added damping and a matrix of added stiffness, been dependent on the grids.*

5.5.3 Modeling of the fluid forces exerted on the grids

Calculation of the jump of pressure

First of all, the presence of grids disturbs the stationary field of pressure $P(X)$; one considers each grid like a singularity involving a jump of pressure, whose expression is put under form:

$$P(X) = (X) U^2 (X) K(X)$$

G
G
G
G
G
G
G

éq 5.5.3-1

2

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where

K_g indicates the loss ratio of load due to the grid,

U_g indicates the stationary speed of the flow on the level of the grid,

G indicates the density of the flow on the level of the grid,

xg indicates the axial position of the medium of the grid along the beam.

Density ()

G xg is calculated by linear interpolation of the profile of density (X) of the flow in the absence of grid. Stationary speed U ()

G xg is calculated in

application of the conservation of the mass throughput, which results in the following equation:

G (xg U) G (xg) Fg WITH =oUo F With

where

U O and O respectively indicate the profile of density and stationary speed of the flow in foot of beam,

F A indicates the fluid section of the beam in the absence of grid,

Fg A indicates the fluid section of the beam on the level of the grid: Fg WITH = F

With - G With with G With solid section of the grid.

One deduces the expression from it:

1 1 Ug (xg) = oUo

G With G (xg) 1

-

F
With

The loss ratio of load K G is calculated starting from the expression of the hydrodynamic force total which applies to the grid, and we obtain:

2
1
With
Kg =

G
WITH C (X)
G dg
G + 1 -
H PC (X)

éq 5.5.3-2
F
With

G m fl G
F
With

*The 1st term (in G
In Cdg) comes from the effort of trail; C ()
dg xg is the coefficient of drag of
roast. The 2nd term (in m
PC fl) is an effort corrector term of friction applied to
beam alone with the altitude of the grid (Pm is the wet perimeter of the beam in the absence of grid).
By introducing the expression [éq 5.5.3-2] into the relation [éq 5.5.3-1], one thus obtains the*

expression of

jump of pressure $P(X)$

G for each grid of altitude x_g . This jump of pressure is taken into account with level of the calculation of the stationary field of pressure $P(X)$, in the following way:

$P(X)$

)

$+I = P(X) - P$

(X)

I

I

G X

[

X, X]

G

I I

+

I

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Calculation of the specific fluid forces exerted on each grid

According to the same quasi-static approach as that carried out in the paragraph [§5.3.3], one shows that

the action of the fluid field speed ($U + u\sim$) around a grid implies a force of trail and one force bearing pressure, according to the incidence of the flow compared to the grid. The components y

and Z of the resulting specific force $F G$ are thus written, for each basic cell K of one roast:

(
With

I
 y
 Dy

\sim
 $fg)$

G
 K
 K

K
 K
 $= - G U$

\sim
 G
 C
 U
 C
 U

y
 2

 dg
 $- y +$

pg
 $- y$
 K
 T

Dt

(
With
 I
 Z
 Dz
 \sim

fg)

G

K

K

K

K

= - G U

~

G

C

U

C

U

Z

2

dg

- Z +

pg

- Z

K

T

Dt

where

C_{pg} very indicates the slope with null incidence of the coefficient of bearing pressure around a grid slightly tilted.

*A_g indicates the solid section of the basic cell K of the grid (which includes/understands K of them).
K*

These forces thus will generate additional terms of added damping and stiffness, that one obtains after modal decomposition of the movement and projection of these forces on the basis modal.

5.6

Catch in depreciation account in fluid at rest

Until now, the damping brought to a beam of tubes by the presence of a fluid at rest was not taken into account in modeling. One thus proposes here a model of damping in fluid at rest, whose appendix 1 of the note of synthesis of the tests CHAIR [bib8] constitutes reference material.

5.6.1 Modeling of the fluid force at rest exerted on a beam of tubes

The method of calculation of damping in fluid at rest which is implemented here, is one generalization of the method of CHEN [bib9].

It is a question of calculating the force resulting on each tube from the constraints due to shearing in boundary layer. It is a nonlinear problem because the fluid damping coefficient depends on frequency. One thus introduces following simplifications:

- the problem is written using the water frequencies at rest calculated without taking in count fluid damping,*
- one neglects the coupling between modes.*

R

The linear force F_K

I being exerted on the tube K subjected to a harmonic movement of the beam according to mode I at the frequency f_i is given by the following relation:

R

K

F = U the U.K.R C

I

I

I

K

D_{ki}

éq 5.6.1-1

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R

where U_{ki} indicates the speed of slip between the tube K and the fluid at rest, on both sides of the boundary layer, defined by:

R

R

The U.K. the U.K.

=

$q \& (T)$

I

in I

éq 5.6.1-2

with $Q (T) \sin ($

=

F T)

I

2 I and

R

U_{kim} depends on the averages $\sim u_y$ and $\sim u_z$ of the disturbances speed around the cylinders, calculated beforehand by the model.

CD_{ki} indicates the coefficient of drag of a cylinder of R_k ray, subjected to a flow

R_K

R

harmonic of amplitude ad infinitum U

= the U.K. 2 F

I

im

I, and is defined by:

max

3

F 2R 3

I

K

CDki = R

éq

5.6.1-3

K

2

U

2

F (2R)

I

I

K

max

where the kinematic viscosity of the fluid indicates.

The relation obtained while replacing [éq 5.6.1-2] and [éq 5.6.1-3] in the equation [5.6.1-1] is linearized

by a development in Fourier series (of the term $q&(T) Q(T)$)

I

& I

) one retains only the first

term; it comes:

R

R

F K

2 (2R) the U.K.F $q&(T)$

I

K

im

II

Projection on modal basis

By projection on modal basis and by neglecting the coupling between modes, one obtains the force generalized being exerted on the beam of tube following mode I:

$K L$

K

L

$R R$

R

R

I

$F (T) = K K$

$f_i . i (Z dz$

)

$2 (2 K$

$R) f_i$

K

K

$U_{im.i} (Z dz$

) $q \& i (T)$

$K = 1 0$

$K = 1$

0

$F (T)$

I

is thus proportional to $q \& (T)$

I

and the vector of modal force associated $F (T) = (I$

$F (T))_{i=, 1N}$

puts in the form:

$F (T) = - [Ca] (q \& (T))$

where $[Ca]$ the matrix of damping added by the fluid at rest indicates.

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**Homogenisation of a bathing network of beams
in a fluid**

Summary:

This note describes a model obtained by a method of homogenisation to characterize the behavior vibratory of a periodic network of tubes bathed by an incompressible fluid. Then the development of one finite element associated this homogenized model is presented.

The tubes are modelled by beams of Euler and the fluid by a model with potential.

This modeling is accessible in order AFFE_MODELE by choosing modeling 3D_FAISCEAU.

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1 Introduction

In nuclear industry, certain structures make up of networks quasi-periodicals of tubes bathed by fluids: “combustible” assemblies, steam generators,... For to determine the vibratory behavior such structures, the traditional approach (each tube is modelled, the volume occupied by the fluid is with a grid) is expensive and tiresome even impracticable (in private individual, development of a complicated grid containing a great number of nodes). Structures studied presenting a character quasi-periodical, it seems interesting to use methods of homogenisation.

Techniques of homogenisation applied to a network of tubes bathed by a fluid were with various already elaborate recoveries [bib1], [bib5], [bib4]. The models obtained differ by the assumptions carried out on the fluid (compressibility, initial speed of the flow, viscosity). According to allowed assumptions, the action of the fluid on the network of tubes corresponds to an added mass (drops frequencies of vibration compared to those given in absence of fluid), with one damping even added to an added rigidity [bib5].

At the beginning, finite elements associated two-dimensional models (network of runners bathed by a fluid) were elaborate [bib2]. To study the three-dimensional problems (network of tubes), a solution to consist in projecting the movement on the first mode of inflection of the beams [bib4]. Later on, of the three-dimensional finite elements were developed [bib3], [bib8].

2**Initial physical problem****2.1****Description of the problem**

One considers a whole of identical beams, axis Z, laid out periodically (either the period of space). These beams are located inside an enclosure filled with fluid (see [2.1-a]). One wish to characterize the vibratory behavior of such a medium, while considering for the moment only the effect of added mass of the fluid which is dominating [bib6].

Side surface L of beam $N \circ L$

External edge

L

Z

y

X

F

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2.2

Assumptions of modeling

It is considered that the fluid is a true fluid initially at rest, incompressible. Like the assumption of small displacements around the position of balance was carried out (fluid initially at rest), the field of displacement of the fluid particles is irrotational so that there is one potential of displacement of the noted fluid. There is no flow of fluid through external surface

It is considered that the beams are homogeneous and with constant section according to Z. To model them

beams, the model of Euler is used and the movements of inflection are only taken into account.

section of beam is rigid and the displacement of any point of the section is noted:

S

(sl Z (sl Z sl

=

,

Z

X

y

)

L the inflection of the beam n° L

()

$() () .$

The beams are embedded at their two ends.

The variational form of the problem fluid-structure (conservation of the mass, dynamic equation of each tube) is written:

=

$(sl \cdot N)$

V

éq 2.2-1

L

F

L

L

$2 L$

L

$2 L$

$2 L$

L

S

S

S

2

L

L

L

S

$S + I.E.(internal\ excitation)$

= -

NS

SV

éq 2.2-2

S

2

2
2

T
Z
F
Z
2
S
T

0
0
0

L

with:

$$V = (H^2$$

2

1

0]

(0, [*L*) and *V*

S

$$= H (F)$$

where:

.

N is the normal entering to the beam $n^\circ L$,

.

F is the constant density of the fluid in all the field,

.

S is the density of material constituting the beam,

.

S is the section of the beam,

.

E is the Young modulus,

.

I is the tensor of inertia of the section of the beam.

The second member of the equation [éq 2.2-2] represents the efforts exerted on the beam by the fluid.

2

pressure *p* of the fluid is related to the potential of displacement by: $p = -$

. In the same way, it

F
T
2

second member of the equation [éq 2.2-1] represents the flow of fluid induced by the movements of beams. At the border of each beam L one a: $sl \cdot N = \cdot N$.

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3 Problem

homogenized

3.1

Homogenized problem obtained

To take account of the periodic character of the studied medium, one uses a method of homogenisation based in this precise case on an asymptotic development of the variables intervening in physical starting problem. With regard to the operational step, one returns the reader to following references [bib2], [bib4], [bib5], [bib6]. One will be satisfied here to state the results obtained. In the homogenized medium (see [3.1-a]), the two following homogenized variables are

0

considered: S (displacement of the beams) and (potential speeds of the fluid). In form

0

0

variational, these variables are connected by the equations:

With

hom

0 = -

Ds

0
V

0
0

2

éq 3.1-1

S
2
S 2
S
2

M
0 S
0
0

hom
2
+ K
2
2
=
D
S
2
S V

T

Z

Z

F

T

S

0

0

0

where:

2

$V_{hom} = L^2$

2

2

S

$(0) H^0(\cdot)$

where

$0 = S \times]_0, 0 [$

L

H^2

2

$0(0) = \{v; (X, y) \in S$

Z

v

$\} (X, y, Z) \in H^0]$

$(\cdot, 0 [L])$

Higher edge

0, high

External edge

side 0, lat

L

Z

y

X

0

Lower edge

0, low

S

.

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1

YF

y

2

YS

N

y1

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The various tensors which intervene in [éq 3.1-1] are defined using two functions

=

(

1 2

),) in the following way:

1

1

0

y

y

Y

1

Y

2

F

F

1

2
2

Y

$B = (b) =$

0

With

= (*has*

F

=

-

éq 3.1-2

ij)

B

ij

Y

y

y

Y

ij

ij

Y

1

Y

2

F

F

0

0

0

Y
0
0
S
0
0
S
2
S
1

μ

D = (D = +
0
0
=
=
+
0
0
ij)
B

Y
M
S
(mij) B
S
Y
F
Y
S

0 0 0

0
0

0

éq 3.1-3*I**I*

0

E 2*X**xy* μ $K = (K =$

0

ij)*I**I**Y**xy**y*

0 0 0

where *Y* and *Y* respectively represent the surfaces of the fluid fields and structure of the cell

*F**S*

elementary of reference (cf [3.1-b]). *Y* represents the sum of the two preceding surfaces.

basic cell of reference is homothetic of μ report/ratio to the real cell of periodicity of heterogeneous medium.

Two functions

=

(

1 2

),) are solutions of a two-dimensional problem, called problem

cellular. On the basic cell of reference, the functions

=

(

1 2

),) are defined by:

 $v =$ *N v**v**F*

V
Y

éq 3.1-4

=
0 (to have a solution uniqu)

E
YF
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where:
 $V = \{v \text{ H}$
periodical in period 1
1 (Y), v (y)

y
F
}

Note:
It is shown that the two-dimensional part of B is symmetrical and definite positive [bib5].

Note:
In the matrix M, the term B plays the part of a matrix of mass added suitable for
F
each beam in its cell.

Note:
1
For the various tensors, one can put in factor the multiplicative term

. It was added
Y

in order to obtain the “good mass” of tubes in absence of fluid. One has then
 MFD

=
 0
mass tubes composing 0.

**3.2 Problem
 matric**

By discretizing the problem [éq 3.1-1] by finite elements, one leads (with obvious notations) to following problem:

“ $A = - “Ds$

0
 0

2
 2

éq 3.2-1

”
 $s0$
 M
 $+ “Ks = “DT$

0

$T 2$
 0
 F

$T 2$

what can be put in the form (one pre - multiplies the first equation by):

F
 $2s$
 2

0
 $s0$

~
T
*t*₂
~ *S*

0
"
M
- "*D*
2
"*K*
0
S
M
0

2
+ *K*
F
T
=
+

éq 3.2-2

2
0

-
-

0

0
"*D*
0

0
0
“A
F
F
0

t_2
 t_2

Note:

The problem obtained is symmetrical. If instead of choosing the potential of displacement for to represent the fluid, one had chosen the potential speed, one would have obtained a matrix problem nonsymmetrical also revealing a matrix of damping.

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Resolution of the cellular problem

4.1

Problem to be solved

On the two-dimensional basic cell (see [4.1-a]), one seeks to calculate the functions

=

(

1, 2) checking:

* $v =$

$N v$

v

V

Y

éq 4.1-1

0

to have a single solution

$* =$

(

)

Y

where:

$V = \{v H^*,$

periodical in period 1

$1 (Y) v (y)$

y

}

$y^2 Y^*$

Y

N

$y 1$

Appear 4.1-Error! Argument of unknown switch.

After having determined the functions

$=$

(

1, 2), the homogenized coefficients are calculated

B

$= 1, 2 ; =$

(

1, 2) defined by the formula:

B

$=$

éq 4.1-2

$Y y$

By using the formula of Green and the periodic character of, one shows that the coefficients B

can be written differently:

$$B =$$

$$N$$

éq 4.1-3

To estimate this quantity, it is necessary at the time of a discretization by finite elements, to determine for each element the outgoing normal, which can be tiresome. Another way then is operated; while taking in the equation [éq 4.1-1] $v =$, one obtains:

$$B =$$

éq 4.1-4

Y

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From the function potential energy defined by the traditional formula:

$$1$$

$$W(v) = -$$

$$v$$

éq 4.1-5

$$2$$

$$v$$

Y

one can rewrite the coefficients homogenized in the form:

$$B = - (W (+) - W () - W$$

()

éq 4.1-6

In the two-dimensional case general, one must calculate three coefficients of the homogenized problem $B, B = B, B$ (one knows that the matrix $B = (b)$ is symmetrical). One must solve both

11

12

21

22

following problems:

Ca

$$\text{lculer } V/* \nu = N \nu$$

1

1

Y

* 1

Y

Ca

$$\text{lculer } V/$$

éq 4.1-7

*

$$\nu =$$

$$N \nu$$

2

2

Y

* 2

Y

To calculate

$$* V/* = +$$

1

2

One has then:

B

$= -2W$

11

(1)

B

$= -2 W$

éq 4.1-8

22

(2)

B

$= B = -$

* -

-

12

21

$(W () W (1) W (2)$

Note:

If the basic cell has symmetries, that makes it possible to solve the problem on one part of the cell with boundary conditions adapted well and to only calculate certain coefficients of the homogenized problem. For example for the cell of the figure n°4.1 - has one a: $B = B$

$B = B = 0.$

11

22

12

21

4.2

Problem are equivalent to define

In the equation [éq 4.1-1], the calculation of the second member requires the determination of the normal with edge. To avoid a determination of the normal, one can write an equivalent problem, checked by functions.

1

0

1

$G = G$ are the vectors =

G

,

and
=, the functions are sought,

, V
1
2
1
2
0
1
1

such as:

v
G
v
v V
1
Y

= 1

Y

v
G
v
v V

éq 4.2-1

2
Y

= 2

Y
v
Y
G v
v V

=

Y

By using the formula of Green and the anti-periodicity of normal N , one shows that the problems [  q 4.1-1] and [  q 4.2-1] are equivalent.

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4.3

Practical application in Code_Aster

In *Code_Aster*, to solve the problem [  q 4.2-1], the thermal analogy by defining one material having a coefficient C equal to zero and one coefficient equal to is used. To impose it

p

calculation of the second member utilizing the term in G , key word GRAD_INIT in the order AFFE_CHAR_THER is selected. The thermal problem is solved by using the order THER_LINEAIRE. The calculation of the potential energy W is provided by order POST_ELEM with option ENER_POT. In the case general, three calculations are carried out to determine the values W (,

,

and then, the values of the coefficients of the homogenized problem are

1) W (2) W ()

deduced manually. To impose the periodic character of the space in which the solution is sought, key word LIAISON_GROUP in order AFFE_CHAR_THER is used.

5

Choice of the finite element for the homogenized problem

5.1

Choice of the finite elements

In the model presented previously, axis Z has a dominating role as a principal axis of beams. The developed finite elements check this characteristic. The meshes are of the cylindrical type: the quadrangular bases are contained in plans $Z = Cte$ and the axis of the cylinder is parallel to axis Z (see [5.1-a]).

Lz

Center Z

Appear 5.1-Error! Argument of unknown switch.

According to the equations [éq 3.1-1], derivative second following co-ordinate Z intervene in model, which requires finite elements CI in direction Z. Functions of form of the type Hermit to represent the variations of S following axis Z are thus used. At the points of S

S discretization, displacements S

X

y
 X , s_y but also the derivative,

who are related to the degrees of

Z

Z

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sy

S

freedom of rotation,

by the formulas

X

=

,

= -

must be known. In what

X

X

y

y

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

Z

relate to the variations according to X, y, one limits itself for the moment to functions of form Q.

1

For the degree of freedom of potential, functions of form Q or Q following the three directions

1

2

3

4

5

6

7

8

9

10

11

12

13

14

X, y, Z of space are used.

The finite element thus has as unknown factors the following degrees of freedom: S, S.

X

y

X

y

X

y

X

y

X

y

Note:

The order of the nodes of the meshes support is very important. Indeed, edges parallel with

axis Z are in the same way represented only the edges contained in the plans $Z = \text{Cte}$. The nodes of the meshes are thus arranged in a quite precise order: list nodes of the lower base, then list of with respect to the higher base (or vice versa). With regard to the geometry, the functions of form allowing to pass from the element of reference to the element running are Q . The finite element is thus under-parametric.

1

Two finite elements were developed:

- a associate with a mesh HEXA 8. In each node of the mesh, the unknown factors are S, S . The functions of form associated with the potential are Q .

 X y X y

1

- another associate with a mesh HEXA 20. In each node top of the mesh, unknown factors are S, S . In each node medium of the edges, the unknown factor is. Functions

 X y X y

of form associated with the potential are Q .

2

5.2

Finite elements of reference

5.2.1 Net HEXA 8

On the finite element of reference HEXA 8 (see [5.2-a]), the following functions of form are defined:

$$NR L$$
with L or D or R

éq 5.2-1

$$1, 1, 1 () = P 1 (1) P 1 (2) P L$$

$$\pm \pm \pm$$

$$\pm$$

$$\pm$$

$$\pm 1 (3)$$

=

The indices ± 1 represent the co-ordinates of the nodes of the mesh support of reference.

The functions which make it possible to define the functions of form write:

1-

1+

P

$$\begin{aligned}
& 1 \\
& = \\
& 1 \\
& = \\
& - () \\
& P () \\
& () \\
& () \\
& 2 \\
& 2 \\
& 1- \\
& 1+ \\
& P
\end{aligned}$$

$$\begin{aligned}
& 1 \\
& = \\
& 1 \\
& = \\
& - () \\
& P () \\
& () \\
& () \\
& 2 \\
& 2 \\
& 1 \\
& 3 \\
& 1 \\
& 1 \\
& 3 \\
& 1 \\
& [\\
& - ,]
\end{aligned}$$

11 éq 5.2-2
3

PD
3
1

1

1

=

-

+

- ()

PD1 ()

()

()

2

2

2

=

+

-

2

2

2

1

1

PR

2

3

2

3

1

1

1

=

- - +

1

=

- - + +

- ()

(

) $PR()$ (
)
()
()
4
4

The functions $P D P R$

,
are related to Hermit.

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The unknown factors of the problem homogenized, on a mesh, break up in the following way:

8

L 8

S

D

Z

R

=

+

$X()$

$DX NR$

I

$I()$

$DRX NR$

I

$I()$

2

$i=1$

I

=1

8

$L 8$

S

D

Z

R

=

-

= , ,

éq 5.2-3

1 2

$y()$

$DY NR$

I

$I()$

$DRY NR$

I

$I()$

(

3)

2

$i=1$

I

=1

8

(

) = NR

J

$J()$

J

=1

where DX , DY , DRX , DRY , are the values of displacement according to X , displacement according to

I
 I
 I
 I
 I

y , of rotation around axis X , rotation around the axis y and the potential of displacement with top I of the mesh. In *Code_Aster*, for each node, the degrees of freedom are arranged in the order quoted previously.

8
 7
 3
 6
 1
 5
 2
 4
 3
 1
 0
 0
 -1
 -1
 1
 2
 -1
 0
 1
 1

Appear 5.2-Error! Argument of unknown switch.

5.2.2 Net HEXA 20

On the finite element of reference HEXA 20 (see [5.2-b]), the following functions of form are defined:

$NR L$

with $L D$ or R

éq 5.2-4

$1, 1, 1 () = P 1 (1) P 1 (2) P L$

$\pm \pm \pm$

\pm

±
±1 (3)
=
NR () = Q (

éq 5.2-5

3)

J =

J

J

The indices ± 1 represent the co-ordinates of the nodes tops of the mesh support of reference.

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The functions *P, P L*

±

were already defined in the paragraph [§5.2.1]. The functions *Q* are defined

1

±1

I

by:

1

Q

I

I

I

I

I

I

I

= 1+

,....,

1

1

1

+ 2 1

2

+ 3 3 1 1 + 2 2 + 3 3 - 2

= 1

8

I ()

(

)(

)(

)(

)

I

8

1

2

Q

I

I

I

= 1-

, ,

,

1

1

1

+ 2 1

2

+ 3 3

= 9 11 17 19

I ()

4 (

())(

)(

)

I

1

éq 5.2-6

2

Q

I

I

I

= 1-

,

,

,

2

1

2

+ 1 1

1

+ 3 3

= 10 12 18 20

I()

4 (

())(

)(

)

I

1

2

Q

I

I

= 1-

(

I

I = 13, 14, 15, 16

2

2)

3

1

3

+ 1 1

1

+

I()

4 (

())(

)

where (*I*

I

I

, ,

are the co-ordinates of node *I* of the mesh.

1

2 3)

The unknown factors of the problem homogenized, on a mesh, break up in the following way:

8

L 8

S

D

Z

R

=

+

X ()

DX NR

I

I ()

DRX NR

I

I ()

2

i=1

I

=1

8

L 8

S

D

Z

R

=

-

= , ,

éq 5.2-7

1 2

y ()

DY NR

I

I ()

DRY NR

I

I ()

(

3)

2

I = 1

I

=1

20

(

) = *NR*

J

J ()

J

=1

where *DX, DY, DRX, DRY*, are the values of fluid displacement according to X, of displacement

I

I

I

I

I

according to y, rotation around axis X, of rotation around the axis y and the potential of displacement at top I of the mesh (*I = 1*,)

8 and fluid potential of displacement to the node medium of the edges

J

(*J = 9,2*) 0.

8

19

7

20

18

16

15

1

6

5
17
4
3
1

11
3
13
0
12
14 10 0 2
-1
-1
1
9
2
-1
0
1
1

Appear 5.2-Error! Argument of unknown switch.

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5.3

Choice of the points of Gauss

Each integral which intervenes in the forms of the elementary matrices, is transformed into an integral on the element of reference (a change of variable is carried out) who is then

calculated by using a formula of squaring of the GAUSS type.

The points of Gauss are selected in order to integrate exactly the integrals on the element of reference. Families of different points of integration are used to calculate the matrices of mass and the matrices of rigidity (the degrees of the polynomials to be integrated are different). But here, for

to calculate the various contributions of the matrix of mass, various families of points of Gauss can still be used.

The element of reference being a HEXA 8 or one HEXA 20, the integral on volume can be separate in a product of three integrals which correspond each one to a direction of the space of reference.

a number of points of integration necessary is determined by direction.

According to the mesh of reference, the number of points of integration by direction is as follows:

Net HEXA 8

Net HEXA 20

direction X or y

direction Z

direction X or y

direction Z

stamp “

K

2

2

2

2

stamp “

With

2

2

3

3

stamp “

D

2

3

2

3

stamp “

M

2

4

2

4

Four families of points of Gauss were defined. Each family corresponds to one of the matrices of problem to be solved.

On the segment $[-1, 1]$, the co-ordinates of the points of integration and their weights are as follows [bib7]:

A number of points of integration

Co-ordinates

Weight

2

 $\pm 1/3$

1

0

8/9

3

 $\pm 3/5$

5/9

 $3 - 2\sqrt{6/5}$

1

1

 \pm

+

4

7

 $2\sqrt{6/5}$

1

1

 $3 + 2\sqrt{6/5}$

-

 \pm $2\sqrt{6/5}$

7

The weight of a point of Gauss in the three-dimensional element of reference is obtained by multiplying them

three weights corresponding to each co-ordinate of the point of Gauss.

5.4

Addition of the problems of traction and torsion

To supplement the problem of inflection homogenized described previously, the problem of traction and it

problem of torsion are added in an uncoupled way.

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5.4.1 Problem of traction

The problem of traction homogenized is written in the following form:

$E S 2 U$

v

$2 S 2u$

Z

S

Z

$+$

$v = 0$

v

V

with $V = H^1 [(0, L)]$

Y

Z

Z

Y

T

2

T

T

The finite element of reference is a HEXA 8 having for unknown factor displacement DZ in each node.

The associated functions of form are Q1.

5.4.2 Problem of torsion

The problem of torsion homogenized is written in the following form:

$E 2$

$U v$

2

2u

Z

S

Z

with $V = HI [(0, L)]$

(

+

=

+)

J

J

v 0

v V

Y

Z Z Z

Y

Z

t2

2 1

T

T

where J is the constant of torsion.

Z

The finite element of reference is a HEXA 8 having for unknown factor displacement DRZ in each node. The associated functions of form are Q1.

5.5

Integration in *Code_Aster* of this finite element

The finite element is developed in *Code_Aster* in 3D. A modeling was added in catalogue modelings:

- “FAISCEAU_3D” for the 3D.

In the catalogue of the elements, the element can apply to the two following meshes:

Net

A number of nodes

A number of nodes

Name of the element

in displacement and

in fluid potential

in the catalogue

rotation

HEXA 8

8

8

meca_poho_hexa8

HEXA 20

8

20

meca_poho_hexa20

In the routines of initialization of this element, one defines:

- two families of functions of form respectively associated with displacements and rotation beams (linear in X , y and cubic function of form in Z) and under the terms of potential fluid (linear function in X , y , Z),
- four families of points of Gauss to calculate the matrix of rigidity and the various parts matrix of mass.

During the calculation of the elementary terms, the derivative first or seconds of the functions of form on the element running are calculated. In spite of the simplified geometry of the finite element (the axis of the mesh

cylindrical is parallel to axis Z and the sections lower and higher are in plans $Z = Cte$), a general subroutine to calculate the derivative second was written [bib7]. In addition, two news subroutines was developed starting from the subroutines existing for the elements isoparametric to take account of the under-parametric character of the element.

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6

Use in Code_Aster

6.1

Data necessary

The characteristics of the beams (section S , tensor of inertia I , constant of torsion J) are

Z

informed directly under the key word factor BEAM of order AFFE_CARA_ELEM.

The characteristics of the homogenized coefficients and the cell of reference are indicated under the key word factor POUTRE_FLUI of order AFFE_CARA_ELEM. For the single-ended spanner words,

correspondence is as follows:

B_T: b_{11}

B_N: b_{22}

B_TN: b_{12}

A_FLUI: YF

A_CELL: $Y = Y + Y$

F

S

COEF_ECHELLE: μ

The characteristics of materials are indicated in order DEFI_MATERIAU. For

tubes, the key word factor ELAS is used to indicate the Young modulus (E : E), the coefficient of Poisson (NAKED:) and density (RHO:). For the fluid, the key word factor

S

FLUID is used

to indicate the density of fluid (RHO:).

F

6.2

Orientation of the axes of the beams

The generators of the cylindrical meshes are obligatorily parallel to the axis of the beams and their bases of the meshes perpendicular to this same axis. During the development of the grid, it is necessary to be ensured

that the order of the nodes (local classification) of each cylindrical mesh is correct: nodes of base lower than the nodes of the higher base (or vice versa). Direction of the axis of the beams is well informed under the key word factor ORIENTATION of order AFFE_CARA_ELEM.

The following assumption was carried out: the reference mark of reference is the same one as the principal reference mark

of inertia of the characteristic tube representing the homogenized medium. That means that in equations [éq 3.1-3], term I is null.

xy

6.3 Calculation

modal

The developed finite element makes it possible to characterize the vibratory behavior of a network of beams

bathed by a fluid. It is interesting to determine the frequencies of vibration of such a network in air and out of water.

To carry out a modal calculation in air ($= 0$), it is necessary to block all the degrees of freedom corresponding

F

with the fluid potential of displacement, if not rigidity stamps it (and even the matrix shifted of modal problem) is noninvertible [R5.01.01].

To carry out a modal water calculation (0), it is necessary to use in the order

F

~

~

MODE_ITER_SIMULT, the option CENTER key word factor CALC_FREQ. The shifted matrix ($K - M$) is then invertible if it is not eigenvalue or if it is different from zero.

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7

Characterization of the spectrum of the homogenized model

7.1 Model

heterogeneous

That is to say a network with square step of N beams fixed in their low ends and of which ends higher move in the same way (uniform movement) (cf appears [7.1-a]). Only their movements of inflection are considered.

Uniform movement

Z

y

L

X

H

Embedding

H

Appear 7.1-Error! Argument of unknown switch.

The spectrum of vibration in air of this network to the following form. For each order of mode of

vibration of inflection, the modal structure consists of a frequency double correspondent with one mode in X and with a mode in where all the higher part moves (all the beams have *there* even deformed) and of a frequency of multiplicity $(2N - 2)$ correspondent with modes where all higher part of the beams is motionless and where beams move in opposition of phase.

In the presence of fluid, the spectrum is modified. For each order of mode of vibration in inflection, there are $2N$ frequencies of vibration are lower than the frequencies of vibration obtained in air. The effect of incompressible fluid is comparable with an added mass. There is always a double frequency correspondent with a mode in X and a mode in *there* where all the higher part moves (all the beams have the same deformation). On the other hand, one obtains $(N - 1)$

1 couples different of double frequency

(one in X and one in y) correspondent with modes where all the higher part of the beams is motionless and where beams move in opposition of phase.

For an order of mode of inflection

Frequency

Frequency of

double

multiplicity $2n-2$

in air

frequency

of vibration

out of water

Frequency

Spreading out of

double

spectrum

Appear 7.1-Error! Argument of unknown switch.

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7.2 Model

homogeneous

The heterogeneous medium was replaced by a homogeneous medium.

7.2.1 Problem

continuous

Recent work, concerning a problem of homogenisation planes of a network of runners retained by springs and bathed by a fluid, show that the spectrum of the homogeneous model continuous consists of a continuous part and two frequencies of infinite multiplicity [bib10]. spectrum of the Eigen frequencies of the water problem is also contained in an interval well defined limited supérieurement by the fréquence of vibration in air of a runner [bib5]. These results are transposable for each order of inflection of the network of tubes.

7.2.2 Problem

discretized

That is to say the homogeneous field with a grid by hexahedrons. That is to say p the number of generators parallel with axis Z of the network of beams.

Uniform movement

Z

y

L

X

H

Embedding

H

Appear 7.2.2-a

One finds results similar to those obtained for the heterogeneous model. It is enough to replace N by p . For an order of inflection of beam, the number of frequencies corresponding to modes where beams do not vibrate all in the same direction, depends on the discretization used in transverse directions with the axis of the beams.

According to the finite element used (mesh HEXA 8 or nets HEXA 20), the distribution of $(2p - 2)$ last frequencies is different. The first frequency doubles (that corresponding to the mode where the part higher moves) is the same one for the two finite elements.

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For an order of mode of inflection

Frequency

Frequency

Frequency of

double

of vibration

multiplicity 2p-2

HEXA 8

HEXA 20

Order of

frequencies

Appear 7.2.2-b

All in all, the homogeneous model makes it possible to obtain the frequencies of vibration easily correspondent with modes where all the beams vibrate in the same direction. Other modes obtained provide only one vision partial of the spectrum. In the discretized spectrum, one can turn over one or the two frequencies of infinite multiplicity presents in the spectrum of the continuous model.

8 Conclusion

The use of the finite elements developed associated the homogenized model of a beam of tubes periodical bathed by a fluid makes it possible to characterize the overall vibratory movements (all the structure moves in the same direction) of such a structure.

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R4.08 booklet: Influence thermics on mechanics

Document: R4.08.01

Calculation of the thermal deformation

Summary

This document is devoted to the presentation of the calculation of the thermal deformation. One indicates them to it information necessary to the calculation of the thermal deformation and the various possibilities of definition of these information by the user.

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1 Introduction

The values of the dilation coefficients are determined by tests of dilatometry which take place with to leave the ambient temperature (0°C or more generally 20°C). So one lays out in general values of the dilation coefficient defined compared to 20°C (temperature to which one supposes null thermal deformation).

Certain studies require to take a temperature of reference different from the temperature ambient (null thermal deformation for another temperature that the ambient temperature). It is then necessary to carry out a change of reference mark in the calculation of the thermal deformation (equation

[*éq 1-1*] and appears below).

HT

HT (T)

HT (T

HT (T)

m

ref.)

m

T

Tref F

T

of F

Temperature

$HT (T) = HT$

HT

$m (T) - m (ref.$

$T)$

éq 1-1

HT

where

m is the measured thermal deformation (definite compared to the ambient temperature)

HT is the calculated thermal deformation (definite compared to a temperature of reference)

In *Code_Aster*, the thermal deformation is calculated by the expression

$HT (T) = \alpha (T) (T - Tref.$

$T)$ where $\alpha (T)$ is the average dilation coefficient (with direction *RCC_M*) with

temperature *T* determined compared to the temperature *Tref* (*Tref* being the temperature to which one consider that *HT* (*ref.*

T) = 0.).

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Thermal dilation coefficient known compared to T_{ref}

The values of the thermal dilation coefficient were determined by tests of dilatometry carried out starting from the T_{ref} temperature.

In this case, key word TEMP_DEF_ALPHA should not be specified in the order DEFIN_MATERIAU [U4.23.01].

The equation [éq 1-1] is simplified, since HT
 m ($ref.$
 T) = 0.

From where:

$$HT(T) = \alpha(T) (T - T_{ref})$$

éq 2-1

and HT ($ref.$

$$T) = 0$$

where values of the dilation coefficient!

(T) are well informed under the key word ALPHA (or ALPHA_*) in DEFIN_MATERIAU.

3 Dilation coefficient known compared to one temperature T_{def}

def

$ref.$

The values of the thermal dilation coefficient were determined by tests of dilatometry who took place starting from a T_{def} temperature different from the temperature of T_{ref} reference.

Indeed, in general one has the values of the dilation coefficient defined compared to ambient temperature, 0°C or more generally 20°C, and certain studies require to take one temperature of reference different from the ambient temperature.

It is then necessary to carry out a change of reference mark [éq 1-1].

In this case, the user informs under key word TEMP_DEF_ALPHA of the order

DEFIN_MATERIAU, the value of the T_{def} temperature, and under the key word ALPHA (or ALPHA_*) them

values of the dilation coefficient (T) (definite compared to the T_{def} temperature). In

order AFFE_MATERIAU under key word TEMP_REF, it indicates the value of the temperature of T_{ref} reference.

The calculation of $HT(T)$ is done by using the formula:

$$HT(T) = \alpha(T) (T - T_{def}) - \alpha(T_{def}) (T_{ref} - T_{def})$$

T

$- D$

T_{def}

$$= \alpha(T) (T - T_{ref})$$

T

éq 3-1

and HT ($ref.$

$T) = 0$

The calculation of $HT (T)$ requires the preliminary calculation of the values of the function!

(T) .

The function!

(T) remains defined (or well informed) for the same values of T as (T) , $I =$,

1 NR and

keep the same attributes (even standard of interpolation (“FLAX”, “LOG”) and even type of prolongation

(“CONSTANT”, “LINEAR”, “EXCLUDED”).

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3.1 Calculation

of!

(T)

I in temperatures different of T_{ref} (to one precision near)

One obtains the expression of!

(T)

I by using the equation [éq 3-1].

I t.q. $T_i - T_{ref} Prec$

$(T) (T)$

I

$I - T_{def} - (T$

) $(T$

$ref.$

$ref. - T$

)

éq 3.1-1

!

 $(T$ def $I) =$ $T_i - T_{ref}$

The value of the precision is:

- is specified by the user under the key word PRECISION of the key word factor ELAS_FO (order DEFI_MATERIAU [U4.23.01]),
- is equal to 1. : default value.

3.2 Calculation

of!

 $(T$

I for temperatures close to T_{ref} (to one

precision near)

One cannot use the equation [éq 3-1] directly. One derives the equation [éq 3-1] compared to temperature and one take the value of derived at the T_{ref} temperature.

$$HT(T) = (T) (T - of$$

$$TF) - (R$$

$$T_{ef}) (ref.$$

$$T$$

$$- D$$

$$T_{ef}) = (! T) (T - ref.$$

$$T$$

$$)$$

$$HT$$

from where

$$= (T) ($$

$$T - of$$

$$TF) + (T) =! (T) ($$

$$T - R$$

$$T_{ef}) + (! T)$$

éq 3.2-1

$$T$$

and thus! $ref.$

$$(T$$

$$) = (R$$

$$T_{ef}) ref.$$

$$(T$$

$$- D$$

$$T_{ef}) + ref.$$

$$(T$$

)

The equation [éq 3.2-1] gives the expression of!

(*Tref*).

It is considered that!

(*T*) =! (*T*) I t.q. $T - T$

< *Prec*

I

ref.

I

ref.

The value of the precision is:

· is specified by the user under the key word PRECISION of the key word factor ELAS_FO

(order DEFI_MATERIAU [U4.23.01]),

· is equal to 1. : default value.

Also, to calculate!

(*T*)

I it is necessary as a preliminary to calculate (*T*)

)

ref.

.

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3.2.1 Calculation

of

(*T*)

ref.

1st case: t.q *I* -

I

T
 $T_{ref} < Prec$
 and I and
 $I = NR$

$T_{i+1} - T$
 T
 $ref.$
 $ref. -$

T
éq 3.2.1-1
 (
 $i-1$
 T

$ref.)$
 ()

() () ()
 $= 2$
 $+$
 $T_{i+1} - T$
 T
 $ref.$
 $ref. - T$

$i-1$

2nd case: $I = T$
 t.q $I - T_{ref} < Prec$ and
 if

$I = NR$
 $T_{ref} - T$
éq 3.2.1-2

(
 $I = 1$
 $T_{ref})$
 () (-)

$=$
 $T_{ref} - T_{i-1}$

3rd case: $I = 1$ t.q. $T_i - T_{ref} < prec$ and if $I = 1$
 $T_{i+1} - T$

éq 3.2.1-3

(

ref.

Tref)

() ()

=

T_{i+1} - T_{ref}

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R4.10 booklet: Estimator of error a posteriori

Document: R4.10.01

**Estimator of error of ZHU-ZIENKIEWICZ
in elasticity 2D**

Summary:

One exposes in this document the method of estimate of the error of discretization suggested by ZHU-ZIENKIEWICZ and applied to the system of linear elasticity 2D.

This estimator is based on a continuous smoothing of the calculated constraints allowing to obtain the best

precision on the nodal constraints compared to the methods standards.

Two successive versions of this estimator are described, corresponding each one to a different smoothing. These

two versions are available in *Aster*.

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1 Introduction

Research on estimators of error on the solutions obtained by calculations finite elements and their coupling with procedures of adaptive grid made these last years great strides considerable. The set aim is to mitigate the possible inadequacy of a modeling while adapting of one automatic way grid with the solution sought according to certain criteria (equal distribution of the error of discretization, minimization of the number of nodes to reach a given, less precision cost).

One introduces here an estimator of error of the type a posteriori within the framework of linear elasticity and

homogeneous 2D. Historically, this estimator, proposed by ZHU-ZIENKIEWICZ [bib1] in 1987, was largely used because of its facility of establishment in its existing weak and computer codes cost. Nevertheless, the bad reliability of this estimator for the elements of even degree was noted empirically (undervaluation of the error) and led the authors to a modification of their method in 1992 [bib2], [bib3] with numerical checking of the asymptotic convergence of the estimator on all the types of elements.

Nevertheless, the applicability of version 92 being for the moment more reduced (see [B3.2]), them two versions of this estimator were established in *Aster* and are the subject of this note.

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2

Principle of the method

2.1

Equations to be solved

One considers the solution (U ,) of a linear elastic problem checking:

· equilibrium equations:

Lu

= **Q** in

$ijnj =$

T on

I

T

with $\mathbf{L} = T \mathbf{BDB}$ operator of elasticity

· equations of compatibility:

$= \mathbf{Drunk}$

$\mathbf{U} =$

\mathbf{U} on \mathbf{U}

with $= \mathbf{U}$

T

!

· the law of behavior:

$= \mathbf{D}$

The problem discretized by finite elements consists in finding (\mathbf{uh}, H) solution of:

$\mathbf{U} = NR \mathbf{U}$

H

H

éq 2.1-1

checking $\mathbf{K} \mathbf{U}$

\mathbf{F}

$\mathbf{H} =$

with $\mathbf{K} = T (\mathbf{BN}) \mathbf{D} (\mathbf{BN}) D$

$\mathbf{F} =$

$NR \mathbf{Q} + T$

D

NR

T

TD

T

where:

\mathbf{uh} represents the nodal unknown factors of displacement

NR associated functions of form

The constraints are calculated starting from displacements by the relation:

$H = \mathbf{dB} \mathbf{uh}$

éq 2.1-2

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2.2

Estimator of error and index of effectivity

One notes

$$\mathbf{E} = \mathbf{U} - \mathbf{U}$$

the error on displacements

H

$$\mathbf{E} = -$$

the error on the constraints

H

The standard of the energy of the error **E** is written:

$$1/2$$

$$\mathbf{E} =$$

$$\mathbf{E} \mathbf{L} \mathbf{E}$$

$$\mathbf{T}$$

$$\mathbf{D}$$

in the case of elasticity

$$1/2$$

$$= \text{you } \mathbf{D} \mathbf{1} \mathbf{E} \mathbf{D}$$

-

éq

2.2-1

The total error above breaks up into a sum of elementary errors according to:

$$NR$$

$$\mathbf{E}^2 = \mathbf{E}^2_i$$

$$I = 1$$

where

NR is the total number of elements.

\mathbf{E}_I represents the local indicator of error on element I .

The goal is to consider the error exact starting from the equation [éq 2.2-1] formulated in constraints.

The idea of

base method is to build a new stress field noted \mathbf{e}^* from \mathbf{H} and such

that:

$$\mathbf{E} \mathbf{e}^*$$

*

$$= - \mathbf{H}$$

The estimator of error will be written then:

$$1/2$$

0

$$T^*$$

1

$$\mathbf{E} = \mathbf{E} \mathbf{D} \mathbf{e}^* \mathbf{D}$$

-

The quality of the estimator is measured by the quantity, called index of effectivity of the estimator:

$$0 \mathbf{E}$$

$$= \mathbf{E}$$

An estimator of error is known as asymptotically exact if 1 when $\mathbf{E} \rightarrow 0$ (or when $H \rightarrow 0$),

what wants to say that the estimated error will always converge towards the exact error when this one décroitra.

In an obvious way, the reliability of $0 \mathbf{E}$ depends on “quality” on \mathbf{e}^* .

The two versions of the estimator of ZHU-ZIENKIEWICZ are different on this level (see [§3]).

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2.3**Construction of an estimator asymptotically exact**

The characterization of such an estimator is provided by the following theorem (see [bib 2]).

TheoremIf \mathbf{e}^* \mathbf{U} \mathbf{u}^*

=

-

is the standard of error of the rebuilt solution, then the estimator of error

$\mathbf{0} \mathbf{E}$ previously defined is asymptotically exact

 \mathbf{e}^*

if

 $\mathbf{0}$ when $\mathbf{E} \mathbf{0}$ \mathbf{E}

This condition is carried out if the rate of convergence with H of \mathbf{e}^* is higher than that of \mathbf{E} .

Typically, if it is supposed that the exact error of the approximation finite element converges in

$\mathbf{E} = \mathbf{0} (HP)$

and the error of the solution rebuilt in

$\mathbf{e}^* = (+$

$\mathbf{0} H p)$ with $> \mathbf{0}$

then a simple calculation gives:

$\mathbf{1} - \mathbf{0} (H) \mathbf{1} + \mathbf{0} (H$

)

and thus $\mathbf{1}$ when $H \mathbf{0}$

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3

Construction of the stress field recomputed (*)

3.1 Version

1987

The solution **uh** resulting from the equation [éq 2.1-1] being *C0* on (because of the choice of functions from form *C0*), it follows that *H* calculated by [éq 2.1-2] is discontinuous with the interfaces of the elements. To obtain acceptable results on the nodal constraints, one generally resorts to one average with the nodes or a method of projection. It is this last method which is adopted here.

It is supposed that * is interpolated by the same functions of form that **uh**, is:

*

*

= *NR*

éq 3.1-1

and one carries out a total smoothing by least squares of *H*, which amounts minimizing the functional calculus

T

$J() = (-\mathbf{H}) (-\mathbf{H}) D$ in the space generated by *NR*.

By derivation, * must check $T NR (* -) D =$

H

0

by using the equation [éq 3.1-1], one obtains the linear system:

$\mathbf{M} \{*\} = \{\}$

B

with $\mathbf{M} =$
 $\mathbf{NR NR}$
 $T D$ and $\{ \}$
 $\mathbf{B} =$
 \mathbf{NR}
 $T H D$

This total system is to be solved on each component of the tensor of the constraints.
stamp \mathbf{M} is calculated and reversed only once.

3.2 Version 1992

The constraint of the field * differs compared to the version 1987 in the following way:
one supposes * polynômial of the same degree than displacements on the whole of the elements
having a node top interns S joint.

S is noted

=

K

K

! this unit called patch.

$S K$

For each component of *, one writes:

*

= \mathbf{Pa}

S

\mathbf{S}

éq 3.2-1

K

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where

P contains the suitable polynômiaux terms

have the unknown coefficients of the corresponding students'rag processions

Example: 2D

$P1 \mathbf{P} = [$

1, X, y] **have** = $T [has, has, has$

1

2

3]

$Q1 \mathbf{P} = [$

1, X, y, xy] **have** = $T [has, has, has, has$

1

2

3

4]

The determination of the coefficients of the polynomial **have** is done by minimizing the functional calculus:

NR

2

$F(A) =$

*

$\mathbf{H}(X, y$

I

$I) -$

(X, y)

S

I

I

I

K

=1

NR

= (

2

$\mathbf{H}(X, y$

I

$I) - \mathbf{P}(X, y$

I

$I) \mathbf{have})$

$I = 1$

(discrete local smoothing of \mathbf{H} by least squares)

where

$(X, y$

I

) are the co-ordinates of the points of GAUSS on SK .

NR is the total number of points of GAUSS on all the elements of the patch

The solution \mathbf{a} checks:

NR

NR

T

\mathbf{P} (

T

I

$X, iy) \mathbf{P}(ix, iy) \mathbf{h} = \mathbf{P}$

\mathbf{S}

$(ix, iy) \mathbf{H}(ix, iy)$

$I = 1$

$I = 1$

NR

from where \mathbf{h}

With

\mathbf{B}

$\mathbf{S} =$

-1 with $\mathbf{A} = tP(ix, iy) \mathbf{P}(ix, iy)$

$I = 1$

\mathbf{A} can very badly be conditioned (in particular on the elements of high degree) and consequently, impossible

to reverse in this form. To cure this problem, the authors [bib4] proposed one

standardization of the co-ordinates on each patch, which amounts carrying out the change of variables:

$X - X$

$X = -1 + 2$

min

$x_{max} - x_{min}$

$y - y$

$y = -1 + 2$

min

$y_{max} - y_{min}$

where X

, X

, y

, y

min

max

min

max represent the values minimum and maximum X and y on the patch.

This method notably improves conditioning of \mathbf{A} and removes the problem completely precedent.

Once determined, the nodal values are deduced according to the equation [éq 3.2-1] only on the nodes intern with the patch, except in the case of patches having nodes of edge.

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Patches internal:

QUAD4

QUAD8

QUAD9

TRIA3

TRIA6

points of GAUSS where are calculated the constraints \mathbf{H} according to the equation [éq 2.1-2]

nodes of calculation of **

internal top defining the patch

Patches edges:

The nodal values with the nodes mediums belonging to 2 patches are realised, in the same way for nodes intern the QUAD9 in the case of.

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Note:

*In the case of finite elements of different type, the choice of \mathbf{P} in the equation [éq 3.2 - 1] is delicate (problems of validity of **ace** if space is too rich, loss of super-convergence if it is not it enough). A thorough study seems essential.*

This is why estimator ZZ2 is limited for the moment to grids comprising only one only type of element. This restriction does not exist for ZZ1.

The authors showed numerically [bib3] that with this choice of \mathbf{e}^* , their estimator was asymptotically exact for elastic materials of which the characteristics are independent field and for all the types of elements and that the rates of convergence with H of \mathbf{e}^* were improved compared to the preceding version (especially for the elements of degree 2: to see case test Manual SSLV110 of Validation), from where a better estimate of the error.

One will find an illustration of these rates of convergence in the reference [bib 5].

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4 Establishment

in

Aster and current limits of use

4.1 Establishment

in

Aster

The two preceding estimators are established in *Aster* in the ordering of postprocessing CALC_ELEM [U4.61.02]. They are activated starting from options (ERRE_ELEM_NOZ1 for ZZ1 and ERRE_ELEM_NOZ2 for ZZ2) and enrich a structure of data RESULT.

Moreover, the calculation of the stress field smoothed by one or the other of the methods described with [B3]

can be started separately of the calculation of estimate of the error (option SIGM_NOZ1_ELGA for ZZ1 and SIGM_NOZ2_ELGA for ZZ2). It should be noted that this field is discretized directly with the nodes and

not by element with the nodes, which reduces the volume of exits.

The estimator of error provides:

- a field by element comprising 3 components:

-

- the estimate of the relative error on the element,

-

- the estimate of the absolute error on the element,

-

- the standard of the energy of the calculated solution h .

- of the exit-listing comprising same information at the total level (on all the structure)

All the fields obtained are displayable by IDEAS via order IMPR_RESU.

4.2 Limits

of use

Linear elasticity and homogeneous 2D (forced and plane deformations, axisymmetric),

Types of elements:

triangles with 3 and 6 nodes,

quadrangles with 4, 8 and 9 nodes.

For ZZ2, the grid must comprise one type of elements.

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Code_Aster ®

Version

2.6

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Estimator of error of ZHU-ZIENKIEWICZ in elasticity 2D

Date:

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Estimator of error in residue

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Organization (S): EDF-R & D /AMA

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Document: R4.10.02

Estimator of error in residue

Summary

The estimator of error in residue allows to estimate the error of discretization due to the finite element method

on the elements of a grid 2D or 3D. It is an explicit estimator of error utilizing the residues of equilibrium equations and jumps of the normal constraints to the interfaces, contrary to the estimator of

Zhu-Zienkiewicz, which uses a technique of smoothing of the constraints a posteriori [R4.10.01] and [bib5].

This estimator is established in Code_Aster in elastoplasticity 2D and 3D.

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1 Introduction

The estimator of error in residue was developed in 1993 by Bernardi-Métivet-Verfurth [bib1]. It is one explicit estimator of error utilizing residues of the equilibrium equations (from where its name). It apply to elliptic problems (Poisson, Stokes, or linear elasticity) in dimension 2 or 3. These problems are supposed to be discretized by finite elements associated a regular triangulation.

Historically, the first estimator of explicit error relating to the unbalances is due to Babuska and Rheinbolt [bib2] for the problems 1D with linear elements. Gago extended this estimator to the 2D and added to the formulas the jumps of traction to the interfaces of the elements [bib3] and [bib4]. Of new estimators were then proposed, in whom defects of surface traction with borders of the field were also taken into account as well as an improvement of the estimate of jumps inter-elements giving of the more reliable results.

One is interested here in the estimator in residue applied to the case of linear elasticity. The set aim is, with the exit of an elastic design, to if required determine the chart of error on the grid in sight to adapt this one (by refinement and/or déraffinement) or simply for information. The adaptation can be done by chaining with the software of Homard cutting.

2

Formulation of the estimator in residue

That is to say open from RN , $N = 2$ or 3 , of border, and T a regular triangulation of.

In linear elasticity, the continuous problem is written:

to find $(U,)$ such as:

div

+ $F = 0$ in

U

$= uD$
on D

$.n = G$

NR
on NR

D is the border of Dirichlet of the grid
 uD is the displacement imposed on this border
 NR is the border of Neumann
 N the unit normal with NR

gN is the loading applied to this border; it can continuous or be discretized.
 F is force of a voluminal type (gravity, rotation); it can continuous or be discretized.

H is the constraint obtained by the resolution of the discrete problem:

div
 $+ F$
 H
 $= 0$ in
 U

$= U$
 H
 D
on D

$.n = G$
 H
 NR
on NR

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with the relation $H =$

U

DB H where:

D is the matrix of Hooke

B is the linearized operator of the deformations

If K indicates a current element of the grid, the estimator of error (noted ()) is defined like being the quadratic average of the site indicators of error, noted (K):

$1/2$

$() = (K) 2$

KT

The indicator by local residue

The indicator is composed of three terms; the first represents the residue of the equilibrium equation on each mesh, the second term the jump of the normal constraints on the interfaces, the third term the difference between the normal constraints and the loading imposed on NR if the element intersects

NR.

V2

V1

K

S (K)

V3

K:

Element running where one wishes to calculate the error,

V1 with V3:

Elements having a common edge with the current element,

S (K):

Together edges of the element running having neighbors.

Appear 2-a: Elements intern with a grid

.

the first term of the estimator is the L2 standard of the residue of the equilibrium equation on net K, multiplied by HK which is, that is to say the diameter of the circle circumscribed for a finite element

triangular, that is to say the maximum diagonal for a quadrangle,

.

the second term is the integral, on S (K) definite [Figure 2-a], of the jumps of constraints normals integrated on each edge F of the element which has a neighbor, and multiplied by root of HF, which is the length of the edge F,

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NR: Border of Neumann

gN: Force applied to the border of Neumann

NR

gN

K
F

Appear 2-b: Elements located on the border of a grid

.

the third term is the integral, on the intersection of each edge F of the edges K of the element running K with the border of Neumann NR, the jumps between the constraints normals of the element and the force of Neumann G NR, multiplied by the root HF, length edge F.

There is thus the following formula for the estimator in residue:

$$\left(\frac{1}{K} \right) = H F + d_i$$

$$v$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$2$$

$$+$$

$$H [.n$$

.

éq 2-1

$$2$$

$$H$$

$$]$$

$$($$

$$+$$

$$H$$

$$G -$$

$$)$$

$$N$$

$$K$$

$$H L K$$

$$F$$

$$2$$

$$L F$$

$$F$$

$$NR$$

$$H$$

$$L2 F$$

$$F S$$

(K)
()
()
FKN

For the choice of the various terms of [éq 2-1], one returns to [bib1].

3 *Properties of the estimator in residue*

One notes
()
EX K the exact error U - uh
on the element K (unknown factor a priori)
H1 (K)

and
()
EX the total exact error U - uh
H1 ()

There are then the following properties ([bib1]):

.
some is the element K, the elementary error (K) is raised by the exact site error
(multiplied by a constant independent of the triangulation),

that is to say K
(K) C × (K)
1
EX

.
the exact total error is raised by the error considered total () (multiplied by one
constant independent of T)

that is to say
()
× ()
EX
C2

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Constant C and C

1

2 depend a priori on the type of finite element and the boundary conditions on problem. Kelly and Gago [bib3] proposed in 2D a C2 constant depending only on the degree p polynomial of interpolation used:

1 2

1

1

C =

that is to say

C =

for

1)

(degree

QUAD4

and

TRIA3

2
2

24 2
 p
2 p 6

1
 $C =$
for
2)

(degree
QUAD8

and
TRIA6

2
4 p 6

For the 3D, one does not have evaluation of the constant. One can nevertheless say that the error estimated total the total exact error in all the cases over-estimates. This result is not inevitably truth at the local level.

4 Establishment in Aster

The estimator in residue is established in 2D and 3D on all the types of elements (except the pyramids). It is calculated by order CALC_ELEM by activating option "ERRE_ELGA_NORE".

This option calculates on each element:

*.
the absolute error (K) (see [éq 2-1]),*

.
the standard of the tensor of the constraints H
who is used to normalize the absolute error,
 $L2(K)$
 (K)

.
the relative error (K)
rel
 $= 100 \times$

.
 $($
 2
 $K) 2 + H L2 (K)$

Foot-note:

This definition of the relative error implies that in the zones where the constraints are very weak, the relative error can be important and nonsignificant. It is then the absolute error that it is necessary to consider.

It also calculates at the total level:

$1/2$
 .
the absolute error () = (K) 2,

KT

$1/2$

2

.
the total standard of the tensor of the constraints

$=$
 $2 ($

H
 ,
 $L)$
 $H L2 (K)$

K T

the relative error ()

()

rel

= 100 ×

() 2 + 2

H L2 ()

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According to the expression [éq 2-1], one sees that to calculate the indicator of error on the mesh K, one must

to know:

1) possible loadings F on K and gN on K NR (or their discretization

F

and G

H

Nh),

2) them

quantities

H, H

K

F and N related to the geometry of the element,

3) the stress field H,

4) the list of the neighbors of K to recover the constraints on these elements, necessary to calculation of the 2nd term of [éq 2-1].

1 and 2 can be calculated or recovered easily.

3 must be calculated as a preliminary by one of options "SIGM_ELNO_DEPL", "SIEF_ELNO_ELGA" or "SIRE_ELNO_DEPL" (option without smoothing of the constraints).

In the contrary case, a fatal error message is transmitted.

4 requires the calculation of a particular connectivity mesh-meshes, in addition to standard connectivity mesh-nodes. This new object is stored in the structure of data of the grid type.

For the detail of the establishment in Aster, to see [bib6].

For the validation of the estimator, to see [bib7].

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Document: R4.10.03

Indicator of space error in residue
for transitory thermics

Summary

During digital simulations by finite elements, obtaining a rough result is not sufficient any more. The user is increasingly petitioning of space error analysis compared to his grid. He has need for support methodological and pointed tools “numériquo-data processing” to measure the quality of its studies and to improve them.

To this end, the indicators of space error a posteriori make it possible to locate, on each element, one cartography of error on which the tools of mending of meshes will be able to rest: the first calculation on one

coarse grid makes it possible to exhume the chart of error starting from the data and the solution discretized (from where it term “a posteriori”), refinement is carried out then locally by treating on a hierarchical basis this information.

The new indicator a posteriori (known as “in pure residue”) which has been just established post-to treat them

thermal solveurs of Code_Aster is based on their local residues extracted the semi-discretizations in time. Via option “ERTH_ELEM_TEMP” of CALC_ELEM, it uses the thermal fields (EVOL_THER) emanating

THER_LINEAIRE and THER_NON_LINE. It thus supplements the offer of the code in term of advanced tools

allowing to improve quality of the studies, their mutualisations and their comparisons.

The goal of this note is to detail theoretical, numerical work and data processing which governed sound establishment. With regard to the theoretical study we, initially, limited ourselves to linear thermics of a motionless structure discretized by the finite elements isoparametric standards. But, in practice, the perimeter of use of this option was partially extended to thermics not linear.

One gives to the reader the properties and the theoretical and practical limitations of the exhumed indicator, all in

connecting these considerations, which can sometimes appear a little “éthérées”, to a precise parameter setting of

operators accused and with the choices of modeling of the code. One tried constantly to bind the different ones

items approached, while detailing, has minimum, of the a little technical demonstrations seldom clarified in

specialized literature.

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1

Problems Description of the document

*During digital simulations by finite elements obtaining a rough result is not sufficient any more. **The user is increasingly petitioning of space error analysis compared to sound grid. It has need for support methodological and pointed tools “numériquo-data processing” to measure the quality of its studies and to improve them.***

For example, the precision of the results is often degraded by local singularities (corners, heterogeneities...). One then seeks the good strategy to identify these critical areas and for to refine/déraffiner in order to optimize the compromise site/total error. And this, with largest possible precision, in an automatic, reliable way (the error analysis must be itself less approximate possible!) robust and at lower cost.

*For each type of finite elements, one in general has **estimates a priori of the space error** [bib1], [bib3]. But those are checked only asymptotically (when the size H of the elements tends towards zero) and they require a certain level of regularity which is precisely not reached in zones with problem. Moreover, these increases underlie two types of strategies for to improve calculation:*

the “methods p” which consist in locally increasing the order of the finite elements,

“
methods *H*

” which locally refines in order to decrease the characteristics geometrical of the elements.

We are interested here in the second strategy, but through an other class of indicators:

indicators of errors a posteriori

. Since work founders of I. BABUSKA and

W. RHEINBOLDT [bib18], the importance of this type of indicator is well established and they arouse an interest

growing, as well in pure numerical analysis [bib5], [bib6], [bib7] as in the field of applications [bib4], [bib16]. They were in particular established and used in N3S, TRIFOU and it Code_Aster (for linear mechanics cf [R4.10.01], [R4.10.02]). For a “review” of the string indicators existing, one will be able to consult the reference work of R. VERFURTH [bib7] or, it report/ratio of X. DESROCHES [bib16], for a vision plus mecanician of these projections.

To take again a sales leaflet of M.FORTIN (cf [bib17] pp468-469), the development of the estimate a posteriori is justified mainly by three reasons:

.
the first is the need for establishing the precision of the results obtained by a calculation elements finished: which credit to grant to them? All phenomena and all the data which they intervene are well taken into account in modeling?

.
the second objective is to make it possible whoever to use a computer code without having with to intervene in the construction of the grid in order to obtain the necessary total precision,

.
finally, the third direction of study is more particularly directed towards the problems three-dimensional for which the size of the grids is limited by the place memory available and the cost of the resolution.

These specifications reveal two duaux problems: to estimate the precision of the solution obtained on the principal parameters of simulation and to propose means of calculating one new solution which respects a minimal precision. The first problem is truly that of the estimate of error whereas the second relates to the associated adaptive methods (refinement/déraffinement, mending of meshes, displacement of points, follow-up of border...). Thus these indicators make it possible to locate on each element a cartography of error on which the tools of mending of meshes will be able to rest.

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Note:

One prefers the denomination to him of

«

indicator” with the usual terminology

of “estimator” (translation literal of English “error estimator”). Taking into account the fact that it has the same theoretical limitations that those of the solvor finite elements (that it “post-draft”), which it is him even often sullied with numerical approximations and which it is exhumed via relations of equivalence utilizing the many ones constants dependent on the problem... the information which it underlies does not give truly “that an order of magnitude” of the required space error. In spite of these restrictions, these cartographies of error a posteriori do not remain less important about it, and in any case, they constitute the only type of accessible information in it field.

The first calculation on a coarse grid makes it possible to associate, with each element triangulation, one

indicator calculated starting from the discretized data and of the first discrete solution. Refinement be carried out then locally by treating on a hierarchical basis this information.

*In short, and in a nonexhaustive way, the **use of an indicator possibly coupled with one remailor:***

.

provides a certain estimate of the error of space discretization,

.

get a better frequency of errors due to the local singularities,

.

allows to improve modeling of the facts of the case (materials, loadings,

sources...),

allows to optimize (even precision at lower cost) and to make reliable the process of convergence of the grid,

to estimate and qualify a calculation for a class of grid given.

These considerations show clearly that the calculation of these estimators (which is finally only one postprocessing of the problem considered) must:

to be much less expensive than that of the solution,

to require only the discretized data and the calculated solution,

to be able to be located,

to be equivalent (in a particular form) to the exact error.

We will see, that with the indicators in residue, one can obtain only one total increase exact error joined to a local decrease of this same error. But these high delimiters and lower of the error are supplemented because, the first ensures us to have obtained a solution with a certain tolerance, while the second enables us to optimize the number of points locally to respect this precision and not to over-estimate it. They utilize constants which not depending on the discretizations space and temporal.

The goal of this note is to detail theoretical, numerical work and data processing which have governed the indicator installation of of error a posteriori allowing “post-to treat” them thermal solveurs of Code_Aster. It is about an indicator in pure residue initiated by the option “ERTH_ELEM_TEMP” of CALC_ELEM.

With regard to the theoretical study we, initially, limited ourselves to thermics linear of a motionless structure discretized by the finite elements isoparametric standards. But, in practice, the perimeter of use of this option was partially extended to thermics not linear. For more details on the perimeter of use and functional of the thermal indicator and one example of use, one will be able to refer to [§6].

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The indicator a posteriori that we propose is an indicator in pure residue based on local residues of the strong equation semi-discretized in time. For certain elements of the study theoretical (and in particular its groundwork) we took as a starting point the innovative work by C. BERNARDI and B. METIVET [bib6]. They have extended they-even, of elliptic with parabolic, them results of R. VERFURTH [bib7]. They in particular were interested in calculations of indicators on case models equation of heat with homogeneous condition of Dirichlet, semi-discretized in time by a diagram of implicit Euler. We extended these results to the problems really treaties by the linear operator of thermics of the code, THER_LINEAIRE. They are problems with limits mixed (Cauchy-Dirichlet-Neumann-Robin) inhomogenous, linear, with variable coefficients and discretized by one - method.

A basic work was thus undertaken for encircling the theoretical springs of the problem well subjacent thermics and to extrapolate the results of the problem models preceding. This so to try to approach modelings and the perimeter of the code while detailing subtleties often induced mathematics in the articles of art. A particular effort was brought to put in prospect choices led in Code_Aster compared to research, passed and current, like clarifying the general philosophy of these indicators.

One gives to the reader the properties and the theoretical and practical limitations of the released indicators

while connecting these considerations, which can sometimes appear a little “éthérées”, to a parameter setting

precis of operator CALC_ELEM accused in this postprocessing. One tried constantly to bind different the items approached, to limit the recourse to long mathematical digressions, all in retailer has minimum many “technical” demonstrations seldom clarified a little in specialized literature.

This document is articulated around the following parts:

Initially, one leads a theoretical study in order to underline holding them and outcomes of the subjacent thermal problem, and, their possible bonds with the choices of modeling of the code. First of all, the Abstracted Variational Framework is determined (CVA) minimum (cf [§2.1]) on which one will be able to rest to show the existence and unicity

of a field of temperature solution (cf [§2.2]). By recutting these pre-necessary theoretical one few “*éthérés*” with the practical constraints users, one deduce some from the limitations as for the types of geometry and the licit loadings. Then one studies the evolution of properties of stability of the problem (cf [§3]) during the process of semi-discretization in time and in space.

These results of controllability are very useful to create the standards, the techniques and them inequalities which intervene in the genesis of the indicator in residue. After having introduced them usual notations of this type of problems (cf [§4.1]), a **formulation is exhumed possible of the indicator** as well as the increase of the total error (cf [§4.2]) and the decrease site error associated (cf [§4.4]). Various types of space indicators (cf [§4.3]) are evoked and one details several used strategies of construction of indicators in parabolic (cf [§4.5]). In this same paragraph, the temporal aspect of the problem is too examined through the contingencies of management of the space error with respect to that of the step time.

In a third part (cf [§5]), principal contributions of these theoretical chapters and theirs bonds with the thermal solveurs of the code are summarized.

Finally, one concludes by approaching the practical difficulties from implementation (cf [§6.1]), environment necessary (cf [§6.2]), the parameter setting (cf [§6.3]) and the **perimeter of use (cf [§6.4]) of the indicator actually established in the operator of post-treatment CALC_ELEM**. An example of use extracted from a case official test (TPLL01J) is also detailed (cf [§6.5]).

Warning:

The reader in a hurry and/or not very interested by the theoretical springs genesis indicator of error and subjacent thermal problem can, from the start, to jump to [§5] which recapitulates them principal theoretical contributions of the preceding chapters.

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Code_Aster ®

Version

6.0

Titrate:

Indicator of error in residue for transitory thermics

Date:

03/06/02

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2 *The problem in extreme cases*

2.1 Context

One considers a limited open motionless body occupying related of Q

R ($Q = 2$ or 3) of

3

border

=

: =

: I

regular characterized by its voluminal heat with constant pressure

$i=1$

C

(X

p

(the vectorial variable X symbolizes here the couple (X, y) (resp. (X, y, Z)) for $Q = 2$ (resp. $Q = 3$)) and its coefficient of isotropic thermal conductivity (X).

Note:

One will thus not take account of a possible displacement of the structure (cf THER_NON_LINE_MO [R5.02.01]).

These data materials are supposed to be independent of time (modeling THER of Code_Aster) and constants by element (discretization 0 P).

Note:

With modeling THER_FO these characteristics can depend on time. As of first versions of the code and before the installation of THER_NON_LINE, it allowed to simulate “pseudo” non-linearities. Taking into account its use rather marginal, we will not be interested in this modeling.

One is interested in the changes of the temperature in any item X of opened and at any moment T [0 [> 0), when the body is subjected to limiting conditions and loadings independent of the temperature but being able to depend on time. It is about voluminal source $S(X, T)$, of boundary conditions of imposed the temperature type $F(X, T)$ (on the portion of surface external l), normal flow imposed $G(X, T)$ (on 2) and exchanges convectif $H(X, T)$ and $Text(, T, X)$ (out of 3).

One places oneself thus within the framework of application of operator *THER_LINEAIRE* [R5.02.01] of Code_Aster by retaining only the conductive aspects of this linear thermal problem.

Note:

Non-linearities pose serious theoretical problems [bib2] to show the existence, the unicity and the stability of the possible solution. Some are still completely open... But in practice, that by no means prevents from “stretching” it perimeter of use of the estimator of error which will be exhumed rigorously for linear thermics, with nonlinear thermics (operator *THER_NON_LINE* [R5.02.02]).

This problem in extreme cases **interfered** (type Cauchy-Dirichlet-Neumann-**Robin** (also called condition of Fourier) inhomogenous, linear and with variable coefficients) is formulated

$$\begin{aligned} T \\ C \\ - \\ p \\ \text{div}(T) = S \\ \times], \\ 0 [\end{aligned}$$

$$\begin{aligned} T \\ T = F \\ \times \\ l \\], 0[\end{aligned}$$

$$\begin{aligned} T \\ (P) \end{aligned}$$

0

= G

×

2

], 0 [éq

2.1-1

N

T

+ $HT = HT$

×

ext.

3

], 0[

N

$T(,$

$\mathbf{X})$

$0 = T(\mathbf{X})$

0

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Note:

.

In this theoretical study of the problem mixed (0

P), one supposes that the border dissociates

in portions on which acts inevitably a condition limits nonhomogeneous. This

assumption is not in fact not of primary importance and one can suppose the existence of a portion 4

, such

3

that = -

4 :

, on which a condition of homogeneous Neumann intervenes

=

I

I I

(thus, when one builds the variational formulation associated with the strong formulation (0

P), them

terms of edges related to this zone disappear. The problem remains well posed then since it

is thermically unconstrained in this zone. By means of computer, it is well it besides

who does, since the terms of edges are initialized to zero). In practice, it is besides

often the case.

.

It will be supposed that the coefficient of exchange $H(T, X)$ is positive what is the case in

Code_Aster (cf [U4.44.02 §4.7.3]). And that will facilitate a little the things to us in

demonstrations to come (cf for example property 5).

.

The condition of Robin modelling the convectif exchange (key word EXCHANGE) on a portion of
edge of the field, can be duplicated to take account of exchanges between two under-parts

border in opposite (key word ECHANGE_PAROI). This limiting condition models one thermal resistance of interface

$$T + HT = HT$$

*With =
, T =
N
3
12
21
T
I
has*

one

éq 2.1-2
ij

$$T + HT = HT$$

Not to weigh down the writing of the problem and insofar as this option is similar to condition of Robin with the external medium, we will not mention it specifically in calculations which will follow.

The condition of Dirichlet can spread in the form of linear relations between the dds (key word LIAISON_) to simulate, in particular, of geometrical symmetries of the structure.*
With =

I
I
21 T = T
I
(
has

one
)
OUP
LIAISON_GR
ij
I
J
li 1
T(X, T) + 2 T
J
(X, T) = (X, T) on ×
2
1
], 0 [éq
2.1-3
I
J
simply

more

or
T
II(,
X T) = (,
X T) on 1
×],
0 [(
)
L
LIAISON_DD
I

In the same way functionalities LIAISON_UNIF and LAISON_CHAMNO make it possible to impose one even temperature (unknown) with a whole of nodes. They constitute a surcouche preceding conditions by imposing couples (,) particular. Not to weigh down the writing of the problem and insofar as these options are only particular cases of generic condition of Dirichlet, we will not specifically mention them in

calculations which will follow.

.
When the material is **anisotropic** conductivity is modelled by a diagonal matrix expressed in the reference mark of orthotropism of material. That does not change basically following calculations which hold account only isotropic case. It is just necessary to take guard of not to commutate more, under the conditions limit of Neumann and Robin, the scalar product with the normal and the multiplication by conductivity.

.
For a **transitory calculation**, the initial temperature can be selected in three manners different: by carrying out a stationary calculation over the first moment, by fixing it at one uniform or unspecified value created by a AFFE_CHAM_NO and by carrying out a recovery with to start from a preceding transitory calculation. This choice of the condition of Cauchy does not have any incidence on the theoretical study which will follow.

.
We will not treat the case where (almost) all the loadings are multiplied by one even function dependent on time (option FONC_MULT, this well adapted functionality Handbook of Reference

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for certain mechanical problems is disadvised in thermics, because it can return in conflict with the temporal dependence of the loadings and, in addition, it applies selectively with each one of them. It was not included besides in THER_NON_LINE).

It is shown that the functional framework the more most convenient general and for “the catch in hand” of it parabolic problem is as follows.

For the geometry: opened locally limited the only one with dimensions one of its border,
(H1)

variety of dimension $Q - 1$, lipschitzienne or 1
C per piece
(H2)

For the data:

2

SL (

1

,

0 ; -

H ()

2

0

TL ()

1

2

-

-

2

FL,

0; H

(1)

1

2

2

,

GL,

0; H

(2

)

1

2

2

,
 TL
ext.
,
 $0; H$
 $(3$
 $)$
 $(H$

$3)$
 $, C,$
 L
 p
 $()$
 2
 $HL (,$
 $0;$
 $L(3$
 $))$

who allows us to obtain a solution in the following intersection

$T 2$
 $L ($
 1
,
 $0; H ()) 0$
 $C ($
 2
,
 $0; L ()) \acute{e}q$

2.1-4
Note:

*That is to say $(X,$
 $)$ Banach, one notes $LP (, 0; X)$ the space of the functions $T v (T)$
 X
strongly measurable for dt measurement such as
 1*

v

=

 $v(T) dt$

0,

. It is Banach, therefore a space of Hilbert for

 $; p, X$ p $p < +$

0

 X

the associated standard.

The introduction as of these spaces of Hilbert particular “space times” comes from the **need from to separate variables** X and T . Any function $U: (X, T) Q =$

 $: \times$

], 0 [U (, \mathbf{xt}) can in fact

to be identified (by using the theorem of Fubini) with another function

 $u\sim: T] [\{u\sim$

,

0

 $(T) X u\sim$

:

$(T) (X) = U (X, T)$. The transformation $U u\sim$

constituting one

isomorphism, one will simplify the expressions thereafter by noting U what should have been meant $u\sim$.

Note:

.

The fact of separating, in first, the time of the variable of space makes it possible to be strongly inspired conceptual tools developed for the elliptic problems. It is besides completely coherent with the sequence “

semi-discretization in time/total discretization in

space” which usually chairs the determination of a formulation usable in

practical.

.

The assumptions on the geometry ensure us of the property of 1-prolongation of the open one

. Thus one will be able to confuse the space of Hilbert

1

 $H ()$

=
 Q
: $U L2 ()/U (L2 ())$

on which it is convenient to work, with space

$H^1 () =$
: $\{U \text{ Of } ()/U H^1 (Q) \text{ with } U$
 $= U\}$

for which standard theoretical results on the traces, the densities of space and them equivalent standards are licit.

Taking into account the character lipschitzien of the border the theoretical results which will follow will be able to apply to the **structures comprising of the corners** (outgoing or returning). By

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against the treatment of **points** or **points of graining** leaves this theoretical framework general. In the same way, the fact that the open one must locally be located the same with dimensions one of its border,

prevent (theoretically) the treatment of **crack**. To treat this type rigorously of problem, an approach consists in correcting the basic functions of the finite elements by one suitable function centered on the internal end of the crack (cf P. GRISVARD. School from Numerical Analysis CEA-EDF-INRIA on the breaking process, pp183-192, 1982).

The indicator in residue using the solution of the problem in temperature, its limitations theoretical are thus, at best, identical to those of the aforesaid problem.

Taking into account the formulation [éq 2.1-1] one thus will be interested in a solution belonging to space functional according to:

Note:

This space comprises also the possible conditions of Dirichlet “generalized” of linear relations type between U and V .

$$TW: = \{ \\ 1 \\ UH(U)/U = \\ : U \\ = F \\ 01 \\ ,$$

} éq
2-1-5

Moreover, thanks to the geometrical assumptions (H1) and (H2), there is an operator of raising (compound of the operator of usual raising and the operator of prolongation by zero apart from

$$1 \\ 1 \\ 2 \\ 1 \\) R: H(1) H() \text{ linear, continuous and surjective such as:}$$

$$1 \\ 2 \\ 01, RF = F \\ FH(1$$

) éq
2-1-6

One thus will not be able to make the problem initial homogeneous in Dirichlet while being interested any more but in the solution

$$UV = \\ : \{U 1 \\ H() / \\ U: = U = \\ 01 \\ ,$$

} 0 éq

2-1-7

1
resulting from the decomposition

$$T = U + RF$$

éq
2-1-8

Note:

That is to say $(X, \|\cdot\|_p)$ Banach, one notes $L^p(\Omega; X)$ the space of the functions $T v(T)$ strongly measurable for dt measurement such as

$$\int_0^T v(t) dt \in X$$

. It is Banach, therefore a space of Hilbert for $p=2$; X

p

$$p < +\infty$$

the associated standard.

This change of variable produces the problem simplified out of U

$$U = C^1(\Omega) \times \mathbb{R}^s$$

T

$U = 0$

×

1

], 0[

U

(1

P)

= \hat{g}

×

2

], 0 [éq

2-1-9

N

U

+ $hu = \hat{h}$

×

3

], 0[

N

$U ()$

$0 = U$

0

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with the new second member

RF
 $\hat{s} =$
 $: S - C$
 $+ div$
 p
 $(RF)^2$
 $L ($
 -1
 $,$
 $0; H ()), \acute{e}q$
2-1-10
 T
new loadings

RF
 2
 1
 $\hat{g} =$
 $: G -$
 $- 2$
 L
 $,$
 $0, H$
 (2)

$\acute{e}q$
2-1-11
 N

1
 $\hat{h} =$
RF
 $: H (T$
 $- RF$
ext.
 $)$

2
 $-$

- 2

L

,

0, H

(3)

éq

2-1-12

N

and the new initial condition

u0 () =

: T () - RF (0

,) 2

0

L ()

éq

2-1-13

Note:

.

*This **theoretical raising**, which can appear a little “**éthéré**”, has an anchoring completely concrete in the digital techniques implemented to solve this type of problem.*

*It corresponds to a **substitution (this technique is not used in Code_Aster, one prefers to him the technique of double dualisation via dds of Lagrange [R3.03.01]) conditions limit of Dirichlet**. By renumbering the unknown factors so that these conditions appear in the last, the comparison can be schematized in the matrix form following*

With

0

T

s^ =

: S -

F has

ji J

=

$j > J$

0 Id

T

$=$

$: RF$

1

1

F

The assumptions of regularity on the border also ensure us of the good following properties for the workspaces. One then will be able to place itself within the usual abstracted variational framework.

Lemma 1

Under the assumptions (H1) and (H2) the workspaces W and V of Hilberts are provided with the standard induced by

1

$H ()$.

Proof:

The result comes simply owing to the fact that the application traces

1

$01: H ()$

2

$L (1$

$)$ is

1

composed of the application traces usual

1

$0: H () H () 2$

2

$L ()$ linear, continuous and

surjective (taking into account the assumptions selected) and of the operator of restriction on 1 he too

linear, continuous and surjective. From share their definition, one deduces from it that W and V are closed sev

1

H (). It of Hilberts is thus provided with the standard

.
,
1
!

Lemma 2

Under the assumptions (H

1
H

1) and (H2), the standard and the pseudo norm induced by

() are equivalent on

functional space V. One will note $P () > the 0$ constant of Poincaré relaying this equivalence

v V

v

P () v

,
1

,
1

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Note:

U thereafter will be noted

=
: *supess U (T) and*
,

pp. T
(
2
2
2
U,)
v (m
H ()
) (U,)
v

2
2

m =
, :
(
U,) *vL2 (), U*

=
:
U
and U

=
:
U
.
m

,
2
m
,

L ()
L2 ()
m

m

=

*m****Proof:***

This result is a corollary of the inequality of Poincaré checked by the open ones called of “Nikodym” of which fact part taking into account the assumptions selected. There are however two cases of figures:

.

that is to say the problem is really mixed and comprises conditions limit others that those of Dirichlet, my (-) 0 (see the demonstration [bib1] §III.7.2 pp922-925),

1

.

either one takes into account only conditions of imposed the temperature type, my (-) = 0, V =

*1**H*

and one finds the standard result of equivalence of the standard

*0 ()**1*

and of the pseudo norm on this space (see for example the demonstration [bib3] pp18-19).

!

The compilation of the preceding results makes it possible to encircle the Variational Framework Abstracted (CVA) on which will rest the weak formulation:

.

*1**H**V**H*

,

*0 ()**1()*

.

*V H = 2**: L () = H “V”**-1*

H () by identifying H and its dual,

.

there is a continuous linear canonical injection of V in H,

•
V is dense in H and the injection is compact (it inherits in that the properties 1 H () with respect to H),

•
V is provided with the pseudo norm induced by 1 H () and H of its usual standard.

Note:

According to a formulation of the theorem of compactness of Rellich adapted to spaces of Sobolev on open (for example, theorem 1.5.2 [bib3] pp29-30).

2.2

Strong formulation with weak

By multiplying the principal equation of the problem in extreme cases [éq 2.1-1] by a function test $v \in V$ and in using the theorems of Green and Reynolds (to commutate the integral in space and derivation in time, with fixed and from the characteristics materials independent of time), one obtains:

D C U

•
 \wedge

éq

2.2-1

$$p(T) v dx + U(T) v dx = S(T) U(T)$$

vdx +

v

D

dt

N

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By introducing the conditions limit in [éq 2.2-1], it occurs the weak formulation (within the meaning of distributions (within this framework general, the temporal derivative is thus to take with the weak direction)

temporal of]

(, 0 []) following:

The solution is sought

$U \in L^2(\Omega; V)$

$C^0(\Omega; H)$

éq

2.2-2

checking the problem

To find $U: T \rightarrow V$ such that

$(U, v) = (f, v) \quad \forall v \in V$

that

such

such

D

(P)

$v \in V$

$C^0(\Omega; T, v)$

T has; $U \in T, v$

$B \in T, v$

2

$(p(\cdot)) + ((\cdot)) = ((\cdot)) \quad \text{éq}$

2.2-3

0,
dt
U ()
0 = 0
U

with
(T has; U (T), v): = U (T) .v dx + H (T)

U T vd
0,3 () 0,3

3
éq
2.2-4
(B (T), v): = ^s (T), v
+ ^g (T)
^

, v 1 1
+ H T, v
0,2
() 0,3 1 1

1
- ×,
1
- ×, 2
- ×, 3
2 2
2 2

while noting,
the hook of duality enters spaces

p
H () and Q
H ().
p×q,

Note:

.
The unknown field and the function test belong to the same functional space, which is more comfortable from a numerical and theoretical point of view.
.

The hooks of duality will not be able to be transformed into integrals with the traditional direction (like for the surface term of A (T;. .)) that if the space of membership of the news is restricted source and of the new loadings with

2

$\wedge_s L ($

2

,

$0; L ()$

2

$, \wedge_g L ($

2

,

$0 L ()$

2

\wedge

;

and H L

,

0; L éq

2.2-5

2

(

2 (3)

According to [éq 2-1-10] [éq 2-1-12] this restriction can be translated on the initial loadings in the form

3

2

2

F L

,

$0; H ()$

2

$, S L$

,

$0; L, G L$

,

$0; L$

and T

L,

$0; L$

1
(
2 (
2 (
2 (
2
2
ext.
(
2 (3)

éq 2.2-6

.

The formulation (P has a direction well, because it is shown that

2)
T has (T; U (T), v)
2
L] (,
0 [] Of]
(,0[)
T C U

p (T)
2
L (,
0; V) and v V
T
(C up (T), v)
2
L]
(, 0 [] Of] (, 0 [)
0,
T ^s (T)
2
L (,
0 ;
1
H - ()

1
and v H ()

1
H - ()
T ^s (T),

2
v
L] (,
0 [] Of]
(,0[

1
- ×,
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T ^g (T)

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L
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H (
and
v H H

2)
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(2)
2 (2)

T ^g (T),
2

v 1 1 L

D

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] (, 0 [

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(, 0 [

- × ,

2 2 2

and one finds obviously the same thing for the term of exchange on 3.

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.

In the surface integrals one will henceforth note $U(T)$ and v what should be noted (in any rigour) U

and

.

0, I (T)

v

0, I

.

Membership of the solution with L^2 (

0; V) rises from the assumptions on the data and of

properties of the differential operators and trace. The fact that it must also belong to

C^0 (

0; H) comes just from the necessary justification of the condition of Cauchy.

One can then be interested in the existence and unicity of the solution of the initial problem (P in 0) showing its equivalence with (P and by applying to this last a parabolic alternative of 2) theorem of Lax-Milgram.

Theorem 3

Within the abstracted variational framework (CVA) definite previously and by supposing that assumptions

(H1), (H2) and (H3) are checked, then the problem (P admits a solution and only one 2)

U L2 (, 0; V) C 0 (, 0; H).

Proof:

This result comes from theorems 1 & 2 of the “Dautray-Lions” (cf [bib3], §XVIII pp615-627). For to use it is necessary nevertheless to check

Mesurability of the bilinear form (

U (T), v)

2

V

T has (T; U (T), v) on],

0 [

Its continuity on V × V

pp T],

0 [have (T; U (T), v)

U (T)

v

+ H (T)

U (T)

v

,

,

1

,

1

1

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1

,

,

3

2 3

2 3

(

$U(T, v)$

V

max (

$H(T)$

2

2

CP

UT

v

3

()

,

,

) (), 1, 1

3

with C the constant of continuity of the operator of trace on and P () the constant of

3

3

Poincaré.

.

Its V - ellipticity compared to H

$pp T]$,

$0 [have (T; v, v)$

2

-2

+

v

C

$- H T$

$C v$

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) 20,
2
3

v V has (

T; v, v)
2
-2
+ v
+ C

- H T
C
v
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> 0
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4

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4
4
4
4
3
> 0
with C the constant of continuity of the canonical injection of
1
H () in 2
L ().
0
.
The continuity of the linear form B (T) on V
pp T],
0 [(B (T), v) ^s (T)
v
+ G (T)
^
^
1
v 1
+ H (T)
v
-,
1
,
1
-,
1
1
,
,
3
2
2
-,

2
2
3
2 3
2

$v V$
 $P ()$

$\max \wedge s (T)$
 $, \wedge g (T)$
 \wedge
 1
 $C, H T$
 C
 v
 2
 $()$

$- ,$
 1
 $- ,$
 1
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 $,$

$,$
 1
 $-$

2
 $2 3$

with C the constant of continuity of the operator of trace on.

2
 2

$!$

Theorem 4

*The problems (P and (P are equivalent and thus the initial problem admits a solution and one
2)
0)
only
U L2 (,
0; V) C 0 (,
0; H).*

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Proof:

The existence and the unicity of the solution of the problem (P result of course from the preceding theorem, one

0)

time that the equivalence of the two problems was shown. It thus remains to prove the implication opposite (P P which is very hard to exhume “not formally”. In particular conditions

2)

(0)

limits of Neumann, Robin and the condition of Cauchy are difficult to obtain rigorously.

“Dautray-Lions” proposes a very technical demonstration ([bib1] §XVIII pp637-641). While adapting his results one shows that in our case of figure, the limiting conditions on I in fact are checked,

-1

-1

not on L2

,
0, H2 (
, but on space (B) 'H2
(by noting I: = ×
)
I
]

,
0 [
00 (I

I
)
I)

defined as being the dual topological one of

I
B =
: W H (

) L2
2
(I/
2
,
0 ;
with
0 and

I
) v L (
V)

v {
v
v
W

×
0} =
{
×
}
=

I
!

Note:

.
Because of low regularity imposed on thermal conductivity, L (), one cannot not to claim with the “standard” regularity U 2 H (). Indeed in the case, for example, of a Bi-material (with =) from which the characteristics are distinct from share and 1 2 of other of the border, [éq 2-1-9] and the theorem of the divergence imposes 1 2 1 2

Appear 2.2-a: Example of Bi-material

U (T)
U (T)
1
-
=
2
in H
pp T
1
2

00 ()
],0[

N
1
N
2

However, therefore the condition of transmission cannot be carried out on the internal border
1
2

U (T)
U (T)

pp pp T],
0 [

N
1
N
2

Thus U (T)
2

H ()
2
H

do not involve U (T)
2

H (). This restriction us
1
(2)

will not allow to exhume, as in [bib6], of increases of the “strong” type of the error space total and of the site indicator of error. Within our framework of work plus general one will have to be satisfied with estimates of the “weak” type.

This type of problem also meets when one treats the open polyhedric ones not convex (for example comprising a returning corner). Open polyhedric (known as polygonal in two-dimensional) is a finished meeting of polyhedrons. A polyhedron is an intersection finished of closed half spaces.

To obtain estimates of the “strong” type, it is necessary to concede more regularity on geometry and on the loadings

variety of dimension $Q - 1$, C per piece (property of 2-prolongation)

(H4)

2

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2

S L (

2

,

0; L ()

1

T H

0

()

3

1

1

2

2

FL

,
0; H (, GL

,
0; H
, HT

,
0; H

(H5)

1)

2

2 (2)

2

2

ext.

(3)

, *C,*
L

p

()

2

HL (,

0;

L (3)

What allows obtaining a solution in the following intersection

U 2

L (

2

*,
0; H () 0*

C (

1

*,
0; H () éq*

2.2-7

Now that we made sure of the existence and the unicity of the solution within the framework functional required by the operators of Code_Aster, we semi-will discretize in time (P0) then to spatially discretize the whole by a method finite elements. In parallel, us will study its properties of stability. They we will be very useful to create the standards, them techniques and the inequalities which will intervene in the genesis of the indicator of error in residue.

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3

Discretization and controllability

3.1

Controllability of the continuous problem

By not making any concession on the assumptions of regularity seen in the preceding paragraph, one

with increase known as “weak” (to take again a terminology in force in the article which served as base with our study [bib6]) following.

Property 5

Within the abstracted variational framework (CVA) definite previously and by supposing that assumptions

(H1), (H2) and (H3) are checked, one with the “weak” controllability of the continuous problem (with

K

my

P >)

1 (

,

(I),

0, I

()

) 0

,

T

2

2

2

pp T

C U T

U

D

C U

P

() +

()

+

0,

p

0 0,

0

0,

éq 3.1-1

T

2

K

^
^
^

$$I
S () 2
+ G () 2I + H ()$$

D
-,
I
-,
I

2
-,
0
2
2 3

Proof:

One here will detail this a little technical demonstration because, on the one hand, the specialized literature returns

seldom in this level of details and, in addition, one will re-use same methodology for to exhume all increases which will follow one another in this theoretical part of the document. All initially, by multiplying the equation of [éq 2.1-1] by U (T), while integrating spatially on, then temporally on [,

0 T] with T [, 0 [one obtain, like the characteristics materials are presumed independent of time,

$$T
T
T
I (C U, U D div U, U D s^, U
D éq 3.1-2$$

$$p ()
()
- ((() ()
=
0,$$

$0,$
 $()()$

$- \times$
 $1,$
 1
 $2 T$
 0
 0
 0

By using the formula of Green and the conditions limit [éq 2.1-1] one obtains

1
 T
 T
 2
 2
 $C U T$
 $C U$
 U, U
 D
 $H U 2D$
 p
 $() -$
 p
 0

$+ (()())$
 $+$
 $0,$
 $()()$
 $0,$
 $0,$

$=$
 2
 0
 0
éq 3.1-3
 $T s^{\wedge} (), U ()$
 $+ g^{\wedge} () U ()$

^
,
1 1
+ H (), U ()

D
- ×
1,
1
- ×,
1 1

2
,
0
- ×
2 2
2 2 3

One can évincer the term of exchange of [éq 3.1-3] because it is supposed that H (T) 0 pp T. By using one argument of duality, the inequality of Cauchy-Schwartz, lemma 2 and the relation

*2
has
2ab + (
B) 2 (> 0), one obtains*

T
T
2
T
2

1 1
s^ (), U ()

D
s^ () 2
P ()
D +
2

U ()

D
- ×
1,
1
2 2
-,
1
0,

éq
3.1-4
0

0
,
0

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One carries out same work on the loadings, thus defining the parameters and in taking again the notations of theorem 3 (for $C_i...$), then one inserts these inequalities in [éq 3.1-3]

2

2

T

P

2

C U T

2

2

C 2 2

C 2 2

U

D

P

()

()

+

-

(+

+

2

3

)

()
0,

0,
,

0

2
2
^

éq
3.1-5

2
^

H
T

2

$s^{\wedge}()$
 $G() 1 1$
()

- × ,
-1×1,

C U

+

1,

1

2 2 2

2 2 3

D

p

0 0,

- ×

+

+
2
2
2

0

It now remains to seek a triplet of strictly positive realities (α, β, γ), not privileging any particular term, in order to reveal a constant independent of the solution and parameter setting in front of the term in gradient. One arbitrarily chooses to pose

$$2 \\ P(\alpha) \\ 2 - \\ (2 \alpha^2 + 2 \beta^2 + 2 \gamma^2) \\ + C + C =$$

éq
3.1-6

2
3
) 1

,
Maybe, for example,

((
my 1) +)

1
2
=

2
 $P(\alpha)$ (
(

my) +)
3

((
my 2) +)

1
2
=
,

éq
3.1-7

2
2
CP

2
)((
my) +)

3

((
my 3) +)

1
2
=
,

2
2
CP

3
)((
my) +)

3

From where increase [éq 3.1-1] while taking

2
P () (
(
my) +)

3

1
2

2

C

C

K =

2

3

max

éq

3.1-8

1

,

,

my

1

my

1

my

1

,

(

(1)+) ((2)+) ((3)+)

!

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Note:

The recourse to measurements of the external borders is an easy way allowing the inequality to support the passage to limit (0) when one or more limiting conditions come to

miss in this mixed problem.

While placing itself within the particular framework of a homogeneous problem of Cauchy-Dirichlet with

characteristics materials constants equal to the unit $C = 1$, $\nu = \nu$ and $s^{\wedge} = S$

éq

3.1-9

p

2

3

and by introducing particular standards on $V =$

1

H

and its dual

$O()$

$s^{\wedge}(T) v$

my

$s^{\wedge}(T)$

,

*

- \times

1,

1

*

$()+$

= sup

with

v

=

1

v

éq

3.1-10

- ,

1

*

,

1

2

,

1

 $v \in V, v_0$ v $(my() + 3) P()$

,

1

one finds well the inequality (2) pp427 of [bib6].

.

If one allows more regularity on the geometry (H4) and the data (H5), one can to exhume **during, known as “extremely”**, of the preceding property. The control of the solutions that it operate is of course more precise than with [éq 3.1-1] because it is carried out via stronger standards. Contrary to “weak” increase, it also utilizes **directly the infinite standard** coefficient of convectif exchange. One will not detail his obtaining here because this family of increase is not essential for the calculation of the required indicator.

3.2

Semi-discretization in time

One fixes a step of time T

such as

either an entirety NR and such as the temporal discretization or

 T

regular: $T =$,

 $0 T = T$ $, T = 2 T$ $T = N T$

.

0

1

2

 LN

Note:

This assumption of regularity does not have really importance, it allows just

to simplify the writing of the semi-discretized problem. To model a transient unspecified at the moment T

=
-
 N , it is just enough to replace
 T
by T
 T
 T .
 N
 $n+1$
 N

Semi-discretization in times of [éq 2.1-1] by **- method** leads to the following problem:
The continuation is sought

(one)
 V
éq
3.2-1
 $0nN$
such as

$n+1$
 U
- one
 C
- div

p
 $(n+1$
 U
 $) - (1 -) \text{div} (one) = n+1$
 \wedge_s
 $+ (1 -) \wedge_{sn} 0 N NR -1$

T
 $n+$
 1
 U
 $= 0$

0 N NR -1

1

(

N 1

n+1

U

P

^ N

1

) +

=

+1

G

0 N NR -1

2

N

n+1

U

n+1

n+1

n+1

^

+ H U

= H

0 N NR -1

3

N

0

U (.) = U

0

éq 3.2-2

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while posing

N

= ,

X N

with {U, S h^

*,
^, H, g^} and 0 N NR*

T

While multiplying [éq 3.2-2] by a function test v and while integrating on, one finds (via the formula of Green) of course the variational formulation [éq 2.2-3] semi-discretized in time

N

N

n+1

N

n+1

N

n+1

N

n+

Being donn

és U, s^, s^

, g^{\wedge} , g^{\wedge}
 h^{\wedge}

,
 h^{\wedge}

,
, H , $H 1$

($n+$
P 1 To calculate
that

such

2

)

$n+$

$U 1 V$

$\acute{e}q$

3.2-3

(

$n+$

$C U 1, v$

p

) + *T has* (N

$n+$

T; $U 1, v$

) = ($C one, v$

p

) + *T* (bn, v

) (vV)

0,

0,

with

$N 1$

+

$N 1$

:

+

=

+

(1 -) N where $\{U, hu, B, S, \wedge \wedge g\} \wedge$

, H

has (

$N I$

NT ; +

U

, v

)

$N I$

:

+

= $U . v dx$ +

(hu) $N I$ +

$v D$

éq

3.2-4

3

($N I$ +

B, v

)

$N I$

+

$N I$

+

$N I$

^

:= $\wedge s, v$

+ $\wedge g,$

+

v

+ H, v

0,2

1 1

0,3

1 1

1

- × ,

1

- ×, 2

- ×, 3

2 2

2 2

This semi-discretization in time made it possible to transform our parabolic problem into one elliptic problem to which one can apply the theorem of standard Lax-Milgram. Assumptions of this theorem are checked easily thanks to the results of continuity and ellipticity of demonstration of theorem 3. From where the existence and the unicity of the continuation (U N) V required.

0nN

Note:

· While posing $RF = 0$ one finds the variational formulation semi-discretized well of Code_Aster (cf [R5.02.01 §5.1.3]). (Or them) the condition (S) of Dirichlet (generalized or not) are checked in the workspace W to which the solution must belong. Moreover, in implicitant it completely - method (Euler retrogresses) one finds the formulation of the code SYRTHES [bib9].

· To be able semi-to discretize by - method one needs to restrict the membership of the new source with $\wedge s 0$

C (

-1

,

0; H () (to be able to take a value in one moment

given). In addition, the initialization of the iterative process [éq 3.2-3] necessarily involves

U 1

H

·

0

()

· To simplify the expressions one will not mention any more the temporal dependence of the form bilinear A (T; .) (for the implicitation of the term of exchange), it will remain implied by that of the solution.

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As for the continuous problem, by not making any concession on the assumptions of regularity, one with “weak” increase following:

Property 6

By supposing that the assumptions of property 5 are checked, that it - diagram is

1
unconditionally stable (

), that $\Delta t \leq 0$

C (

-1

,

0; H () and U 1

H

, one with

0

()

2

controllability “

weak

” of the problem semi-discretized in time (with

K

my

P >)

1 (

,

(I),

0, I

()

) 0

,

n+

2

N

1

N

4

3

1

+ 2

1

+ 2

1

-

2

C U

+ *T*

U

C U

+

N

C U

p

p

p

0,

0,

0,

0,

2

2

T

K T

n+

2

1

2
2
2

0 N NR -1

+
U
+ 1
n+1
n+1
n+1
^
^s
^g

1
H
0,

+
+
-,
1
-, 2
-1,

2
2

2
2 3
éq 3.2-5

Proof:

This inequality is obtained easily by taking again the stages described in the demonstration of property 5. It is necessary, on the other hand, to multiply [éq 3.2-2] by the particular function test

U n+1 =
: U n+1 +

(1 -) a V éq

3.2-6

and évincer the term of exchange by the argument

0 < min (N N 1

H, +

H

)

2

N 1

+

U

(hu) N 1+ N 1+

U dx

max (N N 1

H, +

H

)

2

N 1

+

U

éq 3.2-7

0,

0,

3

3

3

In addition there are not this time the source term and the loadings which require the easy way

[éq 3.1-4], it should also be set up on the cross term (2 -) C U N 1

1

+ undx

. From where one

P

fourth parameter checking a system of the type [éq 3.1-6]

2

P ()

2 -
(2 2 2 2 2
+ C + C =
2
3
) 1
,

éq
3.2-8
2
2
- 1 - 2 = 1
!

Note:

- If a **conditionally stable diagram** in the case of is not placed, in addition to numerical problems which are likely to occur at the time of implementation the effective of the operator, one will not be able to determine the parameters (,) governing the equation [éq 3.2-8].
- While placing themselves within the particular framework [éq 3.1-9] of the article [bib6] and by taking again the standards

4 - 3
1
equivalent [éq 3.1-10], like
<, one finds well the inequality (5) pp428.

2
2
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While stating [éq 3.2-5] for the values of $m \{1$

,
0

,
 $K\}$

N and by summoning these increases to N ,

one obtains the “weak” increase following which takes account of the history of the solutions and of data.

Corollary 7

Under the assumptions of property 6, one with increase

$N 1$

$N 1$

2

2

2

2

N

$C U$

+ T

U

$C U$

$C U$

P

-

$m+$

1

+ 4

(1-)-

m

P

(4 - 3)

P

0,

0

0,

0,

0,

m=

0

m=

0

N 1

2

2

2

0 N NR

+ K

m

m

^

T

^s

^

m

G

H

1

-

+1

+1

+1

+

1

+

-,

1

-,

1

2

,

$m =$

-

0

2

2 3

éq 3.2-9

or more simply

$N 1$

2

2

2

N

$C U$

+ T

U

$C U$

p

-

$m +$

1

p

0,

0

0,

0,

$m =$

0

éq

3.2-10

$N 1$

2

2

2

$0 N N R$

+ K

m

m

^

T
 \wedge_s
 \wedge
 m
 G
 H
 I
 $-$

$+I$
 $+I$
 $+I$

$+$

I
 $+$
 $-$,
 I
 $-$,

I
 2
 $,$

$m=$
 $-$
 0
 2
 $2\ 3$

Proof:

Obtaining [éq 3.2-9] being already explained, it remains to be shown [éq 3.2-10]. This inequality more “coarse” comes simply owing to the fact that

(
 N
 $4\ 1\ -) 1$
 2
 $C\ um\ 0$
 p
 $0,$

$m =$

0

éq

3.2-11

(4 -)

2

2

3

C U

C U

p

0

p

0

0,

0,

!

Note:

.

One can obviously make the same remark as [bib6] by noting that the last term of [éq 3.2-9] is a sum of Riemann which tends towards the last term of [éq 3.1-1] when it no time tends towards zero. In addition, if one introduces the function (with [

$N T$

, $(n+1) T$

]

temporal function characteristic of interval $[N T$

, $(N +)$

$1 T$

]) $U (T) un+$

=

1

[

$N T$

, $(n+1) T$

](T)

closely connected per pieces in [éq 3.1-1], one finds exactly [éq 3.2-9].

.

As for [éq 3.1-1], by adopting the less restrictive approaches (H4) and (H5), one finds a “strong” version of properties 6 and 7.

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3.3

Error of temporal discretization

The preceding results on the continuous problem and its form semi-discretized in time are re-used jointly to study the controllability of the error of temporal discretization

0

$N \ N R$

$in: = U \ N - U \ (NT)$

éq

3.3-1

0

$E = 0$

One starts by revealing this error by withdrawing from the equation [éq 3.2-2] the relations

$(n+) \ T$

1

1

U

$()$

$U \ (N +)$

1 T

$) - U \ (N \ T$

)
 $D =$
 T
 T
 T
 NT
 U
 $(N +)$
 IT
 $)$
 C
 $= div$
 $+I + ^$
 $+I$
éq
3.3-2
 P
 $(U(N) T)$
 $S(N$
 $) T)$
 T

$(1-)$
 U
 $(NT$
 $)$
 C
 $= 1 - div$
 $+ 1- ^$
 P
 $($
 $) (U(N T) ($
 $) S(N T)$
 T

that is to say
 $n+1$
 N

$n+1 T$

E

$- E$

1

C

$-$

1

div

$\acute{e}q$

3.3-3

p

$($

$n+$

U

E

$=$

$D +$

U

$)$

$($

$)$

$($

$)$

$C p$

T

T

T

T

NT

while noting

$en+1 =$

$: en+1 +$

$(1 -) in$

U

U

$\acute{e}q$

3.3-4

=

:

(

 $N +$) $1) + (1 -) U$ T $(N T$

)

 T T T

From this expression one can describe, via the recourse to the formula of Taylor, controllability “weak” of the error of temporal discretization. But to be able to use the derivative temporal of the solution continues one needs a minimum of regularity in T , for example by conceding that

 $U 1$ $H (,$ $0; V) 2$ $H ($ -1

,

 $0; H ()$ *éq***3.3-5**

Property 8

By supposing that the solution checks the additional assumption of temporal regularity [éq 3.3-5], one with the “weak” controllability of the error of temporal discretization

 $N 1$ 2 2 $0 N$ N NR CE $+ T$ E P

-
 $m+$
 l
 $0,$
 $m=$
 $0,$
 0
 $K(T) 3$
 N
 C
 l
 $(p) 2^{-1} ($
 U
 U
 $l) 2$
 2
 $-$
 $($
 $m T)$
 $-$
 $(m +) l t)$

2
 2

4
 $m=0$
 T
 T

éq 3.3-6
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Proof:

While evaluating [éq 3.3-3] by a formula of Taylor to order 2, one utilizes the derivative second temporal of the solution and one shows that the continuation of error (in)

V checks a similar problem

OnN

with [éq 3.2-2] (by supposing that the temporal discretization of the conditions limit are exact)

n+1

E

- in

C

- div

p

(n+1

E) =

T

C

T

p

2u

2u

(1-)

(NT) -

(N +) It)

0 N NR -

2
2

1
2

T
T

(n+1
P
in
N
NR

3
) +1

= 0
0
-1
1
n+1
E

= 0

0 N NR -1
2

nn+1
E

+ n+1 n+1
H
E
= 0

0 N NR -1

3
N

0

 $E() = 0$ **éq 3.3-7**

One can then apply the second result of the corollary 7 to him from where [éq 3.3-6] (one could, of course, just as easily to apply the rough result of this corollary or that of the property 6 from which it rises).

!

Note:

.

While placing itself within the particular framework [éq 3.1-9] of the article [bib6] with an implicit scheme

(=1) and by taking again the equivalent standards [éq 3.1-10] one finds well the inequality (8) pp429. It is enough to make tighten T_0 and to approximate the integral by the sum of Riemann what constitutes the second member of [éq 3.3-6].

.

The existence and the unicity of the continuation (in) rise of course from that of (one) but one also can redémontrer by applying the theorem of Lax-Milgram to the weak formulation rising from [éq 3.3-7].

3.4**Total discretization in time and space**

*It is supposed that the field is **polyhedric or not** and that it is discretized spatially by one **regular family** (H) H of triangulations. Because of this regularity finite element method applied to (N)*

+

 P

converge when the largest diameter of the elements K of $($

2

)

H) H tends towards zero

$H := \max H_0$

éq**3.4-1** K K Th **Note:**

The finite elements $(K, km No, K)$ are closely connected equivalents with same elements of reference, they relations of compatibility on their common borders and the constraints check geometrical [éq 3.4-1] and [éq 3.4-2].

It is pointed out that the diameter of K is real $H: = \max X - Y.$

K
 $(X y)$
 2
 $, K$

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By noting K the roundness (one recalls that the roundness of K is reality
=
: $\max \{diameter$
 sph

eras K) associated K , finite elements of (
 K
}
 H) H also satisfy
constraint

>
 HK
 $0/$

éq
3.4-2
 K

In the usual triplet $(K, km No, K)$ one defines polynomial space as being that of the polynomials of total degree lower or equal to K on K

$P =$
 $: P$

éq
3.4-3
 K

$K(K)$
and approximation spaces it (with the “weak” direction) associated

$V =$
 $:$
 $/$
 P

éq
3.4-4
 H

$\{v \in V \mid T v$
 H
 H
 $H K$
 $K(K)\}$
 V

To conclude, one will note H , the operator of projection which associates the solution continues its V_h interpolated

$: V V$
 H
 H
éq
3.4-5
 $v v_h$

Note:

With a regular family of triangulations, this operator of interpolation is continuous and it can

to be written $v =$

H

$v(\mathbf{x}_i)$ Neither by noting the \mathbf{x}_i tops of the grid and N_i their function of

I

form associated.

It will be of a very particular importance when it is necessary to describe the increase which will exhume

the indicator of error.

Note:

In practice the grids are often polygonal, the approximation H of becomes then more rudimentary than in the polyhedric case. To preserve the convergence of the method it is then necessary to resort to isoparametric elements (cf [bib3] pp113-123 or P. GRISVARD. Behavior of the solutions of year elliptic boundary problem in A polygonal gold polyhedral domain. Numerical solution of PDE, ED. Academic Press, 1976).

The indicator in residue was established in Code_Aster only for the elements isoparametric (triangle, quadrangle, face, tetrahedron, pentahedron and hexahedron). Moreover, as they are **simplexes** or **parallélotopes**, the **associated triangulation is regular** (cf [bib3] pp108-112).

For the simplexes the relation [éq 3.4-2] results in the existence of a lower limit on angles and, for the parallélotopes, by the existence of an upper limit controlling them relationship between the height, the width and the length.

In the definition [éq 3.4-4] of V_h , they are the intrinsic relations of compatibility with family of elements which assures us

H

, $K v$

l

P

$l =$

:

éq

3.4-6

$H K$

$K (K)$

$H (K)$

v

H

H

(

K)

In the literature one often prefers the more regular definition to him

*

V =

: V 0

C

éq

3.4-7

H

H

()

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By regaining semi-discretized shape (N 1

+

P

with functions tests in V

2

)

H one obtains the problem

completely discretized in time and space (for a H fixed) according to:

The continuation is sought

(naked

V

$\acute{e}q$

3.4-8

$H) 0nN$

H

initialized by

0

$U: = U$

$\acute{e}q$

3.4-9

H

$H 0$

checking the following problem

N

N

$n+1$

N

$n+1$

N

$n+1$

N

$n+$

Being donn

$\acute{e}s U, s^{\wedge}, s^{\wedge}$

$, g^{\wedge}, g^{\wedge}$

h^{\wedge}

,

h^{\wedge}

,

$, H, H 1$

(

H

$H, n+$

P

1

To calculate

that

such

2

)

n+

U I V

(

H

H

n+

C U I, v

T has U I, v

C U, v

T B I, v

v

V

p H

H)

+ (*n+*

, *H*

H) = (

N

p H

H)

+ (*n+*

H)

(

H

H)

0,

0,

éq 3.4-10

*Just as one supposed in the preceding paragraph as the **temporal discretization of loadings was exact***

in

N

=

∴ -

(N T

) = 0 with $\{S H, \wedge$

,

H, g^{\wedge} and $0 N NR$

(H6)

, one supposes here moreover than their **space discretization is too**

H

N

N

N

=

: = with

,

\wedge

,

\wedge

, \wedge and 0

(H7)

H

H

$\{S H H G\}$

N

NR

In Code_Aster, these assumptions can not be checked and it will be seen that they impact the quality of the indicator in residue and its relations between equivalence and the exact error (cf [§4.3]). In

practical, even if one is obliged to compose with this approximation, it is not truly problems as long as the loadings “are not chahutés too much” in time and space.

By applying the theorem of standard Lax-Milgram following the groundwork developed in demonstration of theorem 3, one shows the existence and the unicity of continuation (N

U

in the closed sev

$H) N$

(it is thus Hilbert, pre-necessary essential for the use of the famous theorem) V_h of Hilbert V .

*Moreover, by applying the second result of corollary 7 (one could, of course, just as easily to apply the rough result of this corollary or that of the property 6 from which it rises), **controllability “weak” of the completely discretized problem** takes the following form:*

Property 9

While being based on the triangulation defined previously and by supposing that the assumptions (H6)

and

(H7) are checked, one with increase

N

2

2

2

N

C

$+ T$

U

C

U

p

H

-

$m+$

1

H

p

H

$0,$

,

0

$0,$

$0,$

$m=$

0

$\acute{e}q$

3.4-11

N

2

2

2

$0 N NR$

$+ K$

m

m

\wedge

T

\hat{s}
 $\hat{}$
 m
 G
 H
 1
 $-$

$+1$
 $+1$
 $+1$

$+$

1
 $+$
 $-$,
 1
 $-$,

1
 2
 $,$

$m=$
 $-$
 0
 2
 $2\ 3$

by noting $m+1$

m
 U
 $=$
 $: U + 1 + 1 - U$

\cdot
 $, H$
 H
 $($
 $) mh$

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Note:

.

While placing itself within the particular framework [éq 3.1-9] of the article [bib6] with an implicit scheme

(=1) and by taking again the equivalent standards [éq 3.1-10] one finds well the inequality (14) pp430.

.

By adopting the less restrictive approaches (H4) and (H5), one finds a version “strong” of this increase utilizing the H1 standard of the field result.

Now that we determined the functional framework ensuring us of the existence and the unicity of continuation discrete solution and to study the evolution of the controllability of the problem during discretizations, we go mutualiser these results a little “éthérés” to release increase where the indicator will intervene.

4

Indicator in pure residue

4.1 Notations

To build the site indicator of error one will require the following notations:

.

The whole of the faces (resp. nodes) of the element K is indicated by $S(K)$ (resp. $NR(K)$).

The whole of the nodes associated with one with its faces F (pertaining to $S(K)$) $NR(F)$ is noted.

Note:

To make simple, one will indicate under the term “face”, the with dimensions one of a finite element in 2D or one of its faces in 3D.

The diameter of the element K (resp. of one of its faces F) HK (resp is noted. HF).

The whole of the triangulation (H) breaks up in the form

$T: = T$

$T T T$

H

$H,$

$H 1$

,

$H, 2$

$H, 3$

by noting (H, I) the whole of the finite elements having a face contained in border I .

With same logic, the whole of the faces of the triangulation (H) breaks up in the form

$S: = S$

$S S S$

H

$H,$

$H 1$

,

$H, 2$

$H, 3$

with

I

{,

1

}

3

,

2

$S: = K$

$/K T$

K
= S K
H, I
{
H
I}
()
K Th, I
.

In the same way, the whole of the nodes of the triangulation (H) breaks up in the form

NR: = NR
NR NR NR
H
H,

H 1
,
H, 2
H, 3
.

The function “bubble” associated with K (resp. F) is noted K (resp. F).

Note:

It is related to D () (together of the indefinitely derivable functions and with support compact) resulting from the theorem of truncation on compact: its support is limited to compact in question (here K or F) and it is worth between 0 and 1 on its interior (with the direction topological of the term). It is thus null on the border of compact and outside this one.

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.

One notes P the operator of raising on K of traces on F , built starting from an operator of F raising fixed on the element of reference.

.

The union of the finite elements of the triangulation dividing at least a face with K is noted

=

:

K'

K

$S(K) S(K')$

.

The union of the finite elements of the triangulation containing F in their border is noted

:=

K'

F

$F S(K')$

.

Union of the finite elements of the triangulation which share at least a node with K (resp. with F) is noted

=

:

K'

(resp.

=

:
K'
).
K
F
NR (K) NR (K')
NR (F) NR (K')

Th
H

F
K
F
K

F
K

Appear 4.1-a: Designation of the types of vicinities for K and F.

4.2
Increase of the total space error

We thus will see how to obtain a local indicator of calculable error from data and of the discrete solution (N

U
. As the discretized workspace is included in
H) N
continuous space V V, one can re-use [éq 3.2-3] with v
H
h. While withdrawing to him [éq 3.4-10] it occurs
(with N and H fixed and while supposing (H6) and (H7))

(C

éq
4.2-1
p (n+1
N
U
- U +1, v
+ T

$U + I - U + I, v \text{ has } =$

$C U - U, v$

v

V

H

$) H)$

$((N$

N

$, H) H)$

$(p (N nh) H) (H H)$

$0,$

$0,$

Note:

.

This relation states the orthogonal character of the space error with respect to the elements of V_h .

.

It supposes in addition which the discretization is “consistent” i.e. there is not additional errors introduced by the numerical integration of the integrals. In practical it is of course not the case!

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Let us consider the following linear form

(
To v) =
 : (*C*
éq
4.2-2
p (U n+1 - U n+1,
 +
 +1 - +1,

H
) *v*)
T has (one
U N v

, *H*
) (*v V*)
 0,

who will be used to us as discussion thread during this demonstration. By packing it via [éq 4.2-1], one obtains

(
To v) = (*CP (N*
N
U - U, v
 + *C U - U, v - v*
 +
H)
)

(*p (N nh) (H)*
 0,
 0,

(
éq
4.2-3

v
V)
(CP (n+1 N
U
 - *U +1, v - v*
 + *T*
U +1 - U +1, v - v has
H

) (*H*)
 ((*N*

N

, H)

H)

0,

While taking [éq 3.2-3] after having replaced v

-

H by v

v

V, one can build

H

(CP (n+1 n+

U

-

1

U

- N

U + N

U

v v

T has U

U

v v

H

H),

- H) + (n+1

n+

-

1, -

, H

H)

=

0,

0,

(

éq 4.2-4

v V)

T (n+1

B, v - v

T has U

v v

C U

U

v v

H)

-

N

,

N

N,

0,

(+1 - -

1 -

-

, H

H)

(p (+h H)

H) 0,

Then A (v) becomes

(

To v) = (CP (N

N

U - U, v

+ T

B +1, v - v -

H)

)

(N

H)

0,

(

éq

4.2-5

v

V)

(CP (n+1 N

U

- U, v - v

- T

U +1, v - v has

H

H)

H)

(N, H)
 $H)$

0,

Then one breaks up the last three terms on each element K of the triangulation and one applies, with the last, the formula of Green

$n+$

(

$To v)$

I

N

$= (CP ($

U

U

N

$U - N$

U

v

T

S

C

U

$v v dx$

$H),)$

+

$N I$

H

H

$N I$

\wedge

div

0,

+

-

-

+

p

$(+, H) (- H)$

T

K

K

Th

n+1

T

U

-

H,

(v - v D

H)

2

N

F

F

S

H,

n+1

U

v - v V

+ T

G

v v D

H

n+1

^

-

H,

(- H)

N

F
F
S

H, 2
n+1
U
+ T

n+1
^h -
H,

-

(hu
v v D
H)

n+1
(- H)

N
F
F
S

H, 3
éq 4.2-6

Note:

.

One allowed oneself to replace the hooks of duality of [éq 3.2-4] by integrals and one the formula of Green bus to compact K the assumptions (H4 can apply) and (H5) are checked (while replacing by K and

I by
K

). *One thus has*

I

1

v - v H

and

éq 4.2-7

H

(K)

2

, U H

H

(K)

2

, ^S L (K)

2

, G L (K)

2

^

^

H L K

2

(

3)

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Let us point out some properties of the operator H of L2-local projection introduced by

P. CLEMENT [bib8]

V L2

:

V

H

()

H

éq

4.2-8

v vh

It checks in particular increases of errors of projection

v

V

v - v

: = v - v

C H v

H

H

4

K

0, K

0, K

,

1 K

éq

4.2-9

K

T,

F

S K

v - v

: = v - v

C

H v

H

()

H

H

5

F

0, F

$0, F$

,
 $1 F$

where the constants $C4$ and $C5$ depend on the smallest angles of the triangulation. While taking this operator of space projection and by applying the inequality of Cauchy-Schwartz to [éq 4.2-6] it occurs thus:

$n+$
 $($
 $To v)$
 1
 N
 $-(CP ($
 U
 U
 N
 $U - N$
 U
 v
 Tc
 HS
 C
 U
 v
 $H),)$
 $N 1$
 $-$
 $^$
 $-$
 H
 $H + div N 1$
 0
 4
 $,$
 $+$
 K
 p
 $($
 $+$
 $, H)$

,
I
T
K
K
O, K
Th
n+

I
T
U
+
C

H

v
F
H,
5

2

,
I
N
F
F
S

H,
O, F

n+I
U
v V
+ Tc

H
^N
H
G

v
5

+1 -
,

F

,

1
N
F
FS
0, F
H, 2
n+1

U
+
^n
H
N
Tc
HH

hu
v
5

+1 -
,

-
F

(H) +1

,

I
N
F
F S
O, F
H, 3

éq 4.2-10

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This inequality clearly lets show through a possible formulation of the indicator in pure residue:

Definition 10

K

In the framework of the operator of transitory thermics linear of Code_Aster, continuation (N

(K)

H

T

OnN

theoretical local indicators can be written in the form

n+1

N

n+

U

- U

I
 $U I$
 $n+1$
 (K)
 $n+$
 $=$
 $: H S I$
 \wedge
 $-$
 H
 H
 C
 $+ \text{div } U I$
 H
 $,$
 K
 p
 $($
 $n+$
 $H,)$
 $+$
 H
 F
 $+$
 T
 $2 FS$
 N
 (K)
 $0, K$
 $0, F$
 $\acute{e}q 4.2-11$
 $n+1$
 $n+1$
 U
 U
 $n+$

H G 1 -

H,

+

H H 1

,

^

^

hu

1

F

n+ -

H

-

F

(H) n+

FS2 (K)

N

FS3 (K)

N

0, F

0, F

It is initialized by

1

0

(

0

K)

U

=

: H S 0

^ + div U 0

H

K

(H) +

0, K

H
F

+

2 FS
N
(K)
0, F
éq 4.2-12
0
0

U
U
H G 0 -
H

+

H h0
^
^

h0u 0
F

-
H -
F
H
FS2 (K)
N
FS3 (K)
N

0, F
0, F
Continuation (N
()

theoretical total indicators is defined as being
0nN
1

0

N
N
NR () = N
(K) 2
2
:

éq 4.2-13
K HT

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Note:

.

While placing itself within the particular framework [éq 3.1-9] of the article [bib6] with an implicit scheme (=1) one finds well the definition (24) pp432.

.

Whatever the initialization retained for thermal calculation, one starts the temporal continuation of cartography of indicators of error as if one were in hover: no the term in temporal finished difference, $n+1=0$ (in Code_Aster a transitory field of temperature is initialized with index 0) and =1.

.

It should be stressed that this indicator is composed of four terms: the term principal, named voluminal term of error, controlling the good checking of the equation of heat, to which three secondary terms are added checking the good behaviour of the conditions limits (terms of jump, flow and exchange). In 2d-PLAN or 3D (resp. in 2D-AXI), if the unit of the geometry is the meter, the unit of the first is W.m (resp.

1

W.m.

-

rad) and that

1

1

other terms is it

2

W.m (resp.

1

2

W.m.

-

rad). Attention thus with the units taken in

count for the geometry when one is interested in the gross amount of the indicator and not in its relative value!

.

While taking as a starting point the the increases developed by R. VERFURTH (cf [bib7] pp84-94) for the Poisson's equation one could have taken as indicator the root of the sum of the squares terms quoted above.

1

2

2

N 1

+

N

n+

U

- U

U

n+

H

H

n+

1

2
1
H ^s - **C**
+ div U
H

K

p
(
1
H

+
H
F

+
,
)
2
1
,

T
2 FS K
N

K

~

N 1
+

(K)
()

0,
0, F

:=

2

2

N 1

+

N 1

U

+

U

N 1

+

H,

N 1

+

H,

^

H ^

G

-

+

H H

hu

F

-

-

F

(H) N 1+

FS K

N

FS K

N

2 (
)
3 (
)

0, F
0, F

éq 4.2-14

This definition leads to an increase of the total error which is more optimal than that which will be released thereafter. But we preferred, to remain homogeneous with the writings of B. METIVET [bib6] and with the estimator in linear mechanics already installation in code, us to hold some with the version of definition 10.

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While resting on [éq 4.2-10] and definition 10 one can then exhume the increase of the total error following:

Property 11

Under the assumptions of properties 6, of (H6) and by using definition 10, one has, at the total level, the “weak” increase of the error (with K

PC C >) via the history of

2 (
, (),
,

4

5

) 0

,
indicators

1

1

2

2

2

C U

U

4 1

C U

U

T

U 1 U 1

p (

N

N

N - nh)

+ (-)-

p (m -

m

H)

+ -

(m+

m+

-

, H)

0,

0,

0,

m=

0

m=

0

N

2

0 N NR

(4 -)

3

C U

u0

K T

2

p (

-

0

) +

2

(m

H

()

0,

m=0

éq 4.2-15

or more simply

1

2

2

C U

U

T

U 1 U 1

p (

N

N - nh)

+ -

(m+

m+

-

, H)

0,

0,

m=

0

N

2

0 N NR**C U****u0****K T****2****éq****4.2-16****p (****-****0****) +****2****(m****H****()****0,****m=0****Proof:**

The estimates [éq 4.2-15] [éq 4.2-16] are obtained by reiterating the same process as for properties 5, 6 and 7. One takes in [éq 4.2-10] the particular function test

N 1**+****N 1****v:****+****= U****- U****éq****4.2-17****, H**

One évince the term of exchange by the usual argument

(H (U - U**éq****4.2-18****H)****n+1**

$(n+1) n+$
 U
 $-$
 1
 U
 dx

$, H)$
 > 0
 3

It is necessary to apply the easy way [éq 3.1-4] to the cross term (2 -) C

$+1$
 1
 $- +$
 $-$

1
and on
 $p (U N$
 $U nh) (a unh) dx$

the product utilizing the indicator. One has then to find the parameters and checking a system type [éq 3.2-8]

2
 $P ()$
 2
 2
 $-$
 $= 1$
 $,$

éq
4.2-19

2
 2
 $- 1 - 2 = 1$
 1

who admits solution only if the diagram is unconditionally stable (). From where increase

2
[éq 4.2-15] [éq 4.2-16] while taking

2
 $P ()$

**$K =$
 $\max C, C$**

**éq
4.2-20**

**2
(2 2
4
5)**

**,
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**The inequality [éq 4.2-16] more “coarse” results from the same sales leaflet as for corollary 7.
!**

**Note:
.
While placing itself within the particular framework [éq 3.1-9] of the article [bib6] with an implicit
scheme
(=1) one finds well the inequality (25) pp432 (with $c = \max(1, K2)$).**

**.
By adopting the less restrictive approaches (H4) and (H5), one finds a version “strong” of
this property.**

**.
This property can be shown more quickly while noticing than the inequation [éq 4.2-10]**

is similar to the equation of the problem semi-discretized in time [éq 3.2-3]: except for the inequality, in

changing U by $u-uh$ and while taking as term $(bn, v$

) the second member of [éq 4.2-10].

One can then directly apply the corollary 7 to him which is during estimate sought!

Of [éq 4.2-15] [éq 4.2-16] it appears that, at one moment given, the error on the approximation condition of Cauchy and the history of the total indicators intervenes on total quality of the solution obtained. One will be able to thus minimize the error overall of approximation due to the finite elements in the course of time while re-meshing “with good knowledge”, via the continuation of indicators, the structure. Because, in practice, one realizes that it refinement of the meshes makes it possible to decrease their error and thus cause to drop temporal sum of the indicators. The total error will butt (and it is moral) against the value floor of the error of approximation of the initial condition (which will have tendency it also to drop of course!). The indicator “over-estimates” the space error overall.

With the other alternative of indicator [éq 4.2-14] one finds the same type of increase. However the K_2 constant changes. It is multiplied by checking the C_6 constant (cf [bib7] pp90)

2
 2
 2
 v
 $+$

éq
4.2-21

K
 v
 $C v$
 6
 $,$
 1
 $,$
 1
 $,$
 1
 KT
 F
 F

H

HS

~

K: = C K

éq

4.2-22

2

6

2

According to the definitions [éq 2-1-8], [éq 2-1-10] with [éq 2-1-13] if the taking into account of the limiting conditions

of Dirichlet (generalized or not), via the ddls of Lagrange, is exact (what is the case in

Code_Aster)

H

RF N =

: RF N = RF N = RF

0

(H8)

H

H

(N T)

N

NR

the preceding property produces the following corollary then:

Corollary 11bis

Under the assumptions of property 11 while supposing (H8), one with the increase of the space error total expressed in temperature

1

2

2

C T

T

T

T1 T1

p (

N

N - nh)

+ -

(m+

m+

-

, H)

0,

0,

m=

0

N

2

0 N NR

C T

T 0

K T

2

éq

4.2-23

p (

-

0

) +

2

(m

H

()

0,

m=0

by using definition 10 of the indicator also expressed in temperature

U T, s^ S, G G and h^

^

HT

éq 4.2-24

ext.

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4.3
Various types of possible indicators

By extrapolating a remark of [bib5] (pp194-195) it appears that increases of property 11 can exhume itself while taking as indicator

n+1
N
n+
U
- U
1
U 1
n+1

K: H S 1
^
C
div
U 1
H
,

p, T (
)
R
n+
=
-
H
H +
K

p
(
n+
H,)
+
sF
H
+
T
p
2 F S K
N
L (K)
()

Tl (K)
éq
4.3-1
n+1
n+1

U
U
S
n+
H G 1 -
H,

+
H H 1
,
^
^

hu
1
F

S n+ -
H

-

F

(H) n+
FS2 (K)

N

N

T

F S K

L (K)

3 ()

T

L (K)

where constant the R and S are worth

6

T,

1

p > 1 (
2D Q = 2) or

p

(

3D Q =)

3

5

R (Q, p)

Q

Q

:= Q + 1 - -

éq

4.3-2

2

p

1

-1

-1

S (Q, T)

Q

Q

:= Q - -

-

2

2

T

Note:

Just to introduce this generic shape of indicators, one passes from the notation hilbertienne standards of spaces to the notation of Lebesgue

It is parameterized by the types of standards voluminal and surface which intervene for its obtaining. Contrary to the indicator which we chose (N 1

+

who corresponds to $p=t=2$), some

2,2 (K)

use the voluminal standard L1 ($p=t=2$) or on the contrary the infinite standard.

This last formulation, just like its simplified form of definition 10 (or [éq 4.2-14]), constitute an indicator of error well a posteriori because its calculation requires only knowledge geometrical materials, loadings, data, of and approximate field solution u_h accused thermal problem. However the exact estimate of the indicator is not always possible when one has complicated loadings. Two approaches are then possible:

.

Either one approximates the integrals which return in the composition of definition 10 by one formulate squaring.

.

Either one approximates the loadings by a linear combination of simpler functions who will be able to allow an exact integration. Generally same architecture is used that that which was installation for the finite elements modelling the field of temperature.

Note:

.

In both cases the loadings are “prisoners of the selected vision finite elements” to model the field solution.

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These two strategies are equivalent and in Code_Aster it is the first which was reserve: the voluminal integral is calculated by a formula of Gauss, those surface by one formulate Newton-Dimensions.

Both introduce a skew into the calculation of the estimator who can be represented in introducing the approximate versions of the loadings and source (into the initial problem in T and in the problem transformed into U)

N 1

+

N 1

+

N 1

+

N 1

S

, G

,

+

T

H

and

éq

4.3-3

, H

, H

ext., H

, H

N 1

+

N 1

+

N 1

+
N I
^
^s, ^g,
+
H
H

and

éq
4.3-4
, H
, H
, H
, H
in spaces of voluminal approximation (for the source) and surface (for the loadings)

X =
: v L2/K
T
v
P K
H (
) {H
(
H
H K
L (
)}
1

éq
4.3-5
X =
: v L2/F
S
v
,
P F
H (I)
{H (I)
H I

H
(
F
I)}
I
it

In fact, one introduces two types of numerical errors during the calculation of the indicator: that inherent with the formulas of squaring (for polynomial loadings of a high nature) and that due to voluminal term. Indeed, this last requires a double derivation which one carries out in three stages because in Code_Aster one does not recommend the use of the derived seconds of the functions of forms.

Note:

They were recently introduced to treat the derivation of the rate of refund of energy (cf [R7.02.01 § Annexe 1]).

On the one hand, one calculates (in the thermal operator) the heat flux at the points of gauss, then one extrapolate the values with the nodes corresponding by smoothing local (cf [R3.06.03] CALC_ELEM with OPTION=' FLUX_ELNO_TEMP') before calculating the divergence of the vectors flow at the points of Gauss.

With finite elements quadratic the intermediate operation is only approximate (one affects like value with the median nodes the half the sum of their values to the extreme nodes). However numerical tests (limited) showed that, even in P2, this approach does not provide results very different from those obtained by a direct calculation via good the derivative second.

Note:

· The indices l1, l2, l3 of these polynomial spaces can be unspecified and different from that of the approximate solution: K. However, to prevent that these terms do not become prevalent (it is a question of rather estimating the error on the solution than that on modeling of the loadings) one will tend to take $L K - 2$

·
I
(I =, 1)
3
,
2

Definition 10 and the weak estimate 11 associated are rewritten then in the following form. This

new definition, N 1

+

, by a R (one is subscripted takes again in that the usual notations of [bib6]

R

(K)

and [bib7]) (for “reality”) in order to notify well that it corresponds better to the values which are calculated

indeed in the code.

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Definition 12

K

In the framework of the operator of transitory thermics linear of Code_Aster, continuation (N

K

R (

) HT

$0nN$

real local indicators can be written in the form

$n+1$

N

$n+$

U

- U

1

U 1

n+1

K: H S 1

^

C

div

U 1

H

,

R

()

n+

=

-

H

H +

K

, H

p

(

n+

H,)

+

H

F

+

T

2 FS

N

(K)

0, K

0, F

n+1

n+1

U

U

n+
H G 1 -
H,

+
H H 1

,
^
^

H U
1
F
, H

n+ -
H
-
F
, H
(H H) n+
FS2 (K)

N
FS3 (K)
N
0, F
0, F
éq 4.3-6

It is initialized by

1
U 0
0
K: H s0
^

div
U 0
H

R (
) =
+
K
H

**(H) +
0, K**

**H
F**

**+
2 FS**

**N
(K)
0, F
éq 4.3-7
0
0**

**U
U
H G 0 -
H**

**+
H h0
^
^**

**h0u 0
F
H**

**-
H -
F
H
H
H
FS2 (K)**

**N
FS3 (K)
N
0, F
0, F
Continuation (N**

()
real total indicators is defined as being
0nN
1

0
N
N
NR
K

éq 4.3-8
R () =
NR () 2
2
:

K HT

Note:

• One can make the same remarks as for its alter “theoretical” ego. They is declined too according to the formulations [éq 4.2-14] N

~
and [éq 4.3-1], [éq 4.3-2] N

•
R, p, T (K)
R (K)

While being based on the results of property 11, definition 12 and the triangular inequality one can then
to exhume the increase of the following real total error (one began again that the simplified version):
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Property 13

Under the assumptions of properties 6, of (H6) and by using definition 12, one has, at the total level, “weak” increase of the error (with K

*PC C >) via the **history of***

2 (
, (),
,
4
5

) 0
,
real indicators

1
2
2
C U
U
T
U 1 U 1
p (
N
N - nh)
+ -
(m+
m+

-

, *H*)

0,

0,

m=

0

N 1

2

0

2

2

2

N *N* *R*

C *U*

*u*0

K *T*

0

K

1

K

*H*2*S* 1

^

S 1

^

p (

-

0

H)

+

2

(*R* ()) + - (*m*+*R* ())

m+

m+

+

-

K

, *H*

}+

0,

O, K

KT

m=

H

0

N 1

2

2

K T

^

^

^

^

2

-

m+1

m+

H G

- G 1

+

H H 1. 1

H U

1

hu

1

F

, H

0, F

m+

m+

-

-

F

, H

(H H) m+ + (

H) m+

0, F

$KT m=0 F$

H

$S2 (K)$

$FS3 (K)$

éq 4.3-9

*Under (H8), one with the same expression in **temperature***

1

2

2

CT

T

T

$T1 T1$

$p ($

N

$N - nh)$

+ -

$(m+$

$m+$

-

, $H)$

0,

0,

$m=$

0

$N 1$

2

0

2

2

2

NNR

$CTT 0$

KT

0

K

1

K

H2S 1 S 1

p (

-

0

H)

+

2

(R () + - ({m+R ()

m+

m+

+

-

K

, H

}+

0,

0, K

KT

m=

H

0

N 1

2

2

K T

2

-

m+1

m+

H G

- G 1

+

H

HT

T

1

HT

T

1

F

, H

0, F
F (H (
-
ext., H
H) m+ - ((
-
ext.
H) m+

0, F
KT m=0 F
H
S2 (K)
FS3 (K)

éq 4.3-10
by using definition 12 of the indicator also expressed in temperature

U T, s^ S, G G and h^
^
HT

éq 4.3-11
ext.

Note:

· As for the theoretical value there is a morals with the history because, when one will refine, the error total will butt against the value floor due to the approximations of the initial condition, of limiting conditions and of the source. One cannot obtain results of better quality how data input of the problem!

4.4 **Decrease of the local space error**

Before exhuming the decrease of the space error, one will have to introduce some results complementary:

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Lemma 14

It is shown that there are constants strictly positive C_i ($i=6... 11$) checking

I

v

P

$sup K, L, L, L (K)$

$\{ 1 2 3 \}$

C

v

v

C

$2 v$

6

K

7

K

$0, K$

$0, K$

$0, K$

v

$C h-1 v$

K

$8 K$

K
0, K
0, K
-1
1
v

2
P

éq 4.4-1
sup K, L, L, L (F)
{ 1 2 3}
CH
Statement

v
C
2 v
9 F
K
F
10
F
0, F
0, F
0, F
v
Ch-1 v
K
11 F
K
0,
0,
F
F

Proof:

One passes to the element of reference then one uses the fact that the standards are equivalent on polynomial spaces considered, because they are of finished size (cf [bib5] pp196-98, [bib7] [§1]).

!

These preliminary results are crucial to determine a decrease of the site error by the real indicator. But one will see that one will be able to obtain only one opposite room of [éq 4.3-9], [éq 4.3-10].

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Property 15

Under the assumptions of property 6, of (H6) and while being based on definition 12 and the lemma 14, one

has, at the local level, the “weak” decrease of the error (with $K C I =$

>) via

3 (

,

6

I

L)

11

0

the real indicator

N 1

+

N 1

U

- +

U

- *one* - *N*

U

H

C

H

H

+

K

p

(

N I

+

N I

U

- +

U

+

, *H*) 0,

T

K

0,

K

l

n+

n+

n+

n+

n+

l

R (K) K

H ^ l

S

- ^ 1

S

+ H ^ 1

2 G

- ^ 1

G

+

3

K

, H

F

, H

0,

0,

K

K

2

1

^ n +

n +

n +

n +

1

2

^ 1

H H

- H

- hu

+ H U

F

, H

(H) 1 (

H H) 1

0,
K
3

0
N NR -1

éq 4.4-2
Under (H8), one with the same expression in temperature

N 1
+
N 1
T
-
+
T
- T N - N

T
H
C
H
H
+
K
P
(

N 1
+
N 1
T
-
+
T
+

, H) 0,

T
K

0,

K

1

n+

n+

n+

n+

n+

1

R (K)

1

1

1

1

2

K

HS

- S

+ HG

- G

+

3

K

, H

F

, H

0,

0,

K

K

2

1

N 1

2

+

H H

F

(N 1+ N 1

T

- +

T

- n+

n+

H

T

- n+

T

ext.,

)

1

, H (

1

1

ext., H

, H)

0,

K

3

0

N NR -1

éq 4.4-3

by using definition 12 of the indicator also expressed in temperature

U T, s^ S, G G and h^

^

HT

éq 4.4-4
ext.

Proof:

This a little technical demonstration comprises three stages which will consist in raising successively each term of the indicator [éq 4.3-6] (by using the inequalities of the property 14) and to gather increases obtained:

Firstly, one will replace in the equation [éq 4.2-6] the term in $v - v$ by the product W

H
 K making
to intervene the function “bubble” of K

$n+1$
 N
 $+1$
 U
 $- U$
 N
 H
 H
 K
 T
 $v =$
 $: s^{\wedge}$
 $- C$
 $+ \text{div } U +1$

H
 K
, H
 p
(
 N
, H)
 T

éq
4.4-5

$W =$
 $: v$
 K
 $K K$

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From where succession of increases, via [éq 4.4-1] and the inequality of Cauchy-Schwartz,

2

2

$n+$

$U 1$

2

$- n+$

$U 1 - N$

$U - N$

U

v

$C W v dx C$

, W

$U 1$ has $U 1, W$

$S 1$

\wedge

$S 1$

\wedge, W

$K 0, K$

$7 K K$

7

H

H

+

K

(n+ - n+

, **H**

K) - (n+ -

n+

, **H**

K)

T

K

0,

n+1

n+1

N

N

C 2max,

1

^

^

7

(C8) U - U - U - U

H

H

+ -

H 1

U 1 U 1

S 1 S 1

W

K

(n+ - n+

, *H*)

n+

n+

+

-

, *H*

0, K

T

0,

0,

0, K

K

K

K

2

n+1

n+1

N

N

v

C

U

U

U

U

7 max,

1 C

U 1 U 1

S 1

^

S 1

^

K

(*8*)

-

-

-

H

H

+

(n+ - n+

, H)

n+

n+

+

-

, H

0, K

C6

0, K

0, K

T

0, K

éq 4.4-6

Then, one reiterates the same process for the surface terms WF, I

N I

+

F

S (K)

uh,

S

v

:=

H,

F I

,

N

éq

4.4-7

W

: = statement

F 1

,

K

F K 1

,

N 1

+

F

S (K)

U

N 1

+

H,

S

v

: = ^g

-

H, 2

F, 2

H,

N

éq

4.4-8

W

: = statement

F, 2

F

F F, 2

N 1

+

F

S (K)

U

N 1

+

H,

^

S

v

:

N

= H

-

- H U +

H, 3

F, 3

H,

(H H) 1

N

éq

4.4-9

W

: = statement

F, 3

F

F F, 3

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Maybe, for example, for $i=1$ succession of increases, via [éq 4.4-1] and the inequality of Cauchy-Schwartz,

$n+$

$U 1 - n+$

$U 1 - N$

$U - N$

U

H

H, W

$+ U 1 U 1$ has, W

2

$F 1$

,

$(n+ - n+$

, H

$F 1$

,)

2

2

v

C

$W v D$

T

C

$F 1$

,

$0, F$

$10 F 1, F 1,$

10

$0,$

F

- (*n*+
S 1
^
- *n*+
S 1
^, *W*
v, *W*
, *H*
F 1
,)
- (*K F* 1,)

0,
0,

n+
U 1 - *n*+
U 1 - *N*
U - *N*
U
H
H
+ -
H 1 *U* 1 *U* 1
F
(*n*+ - *n*+
, *H*)

C 2 *max*,
1
10
(*C*11)

0,
T
F

0,
W

F

F 1,

0, F

n+1

n+1

+ s^

- s^

+ v

, H 0,

K 0, F

F

1 n+

U 1 - n+

U 1 - N

U - N

U

-1

2

2

H

H

H

+ H

U 1

2

U 1

F

F

(n+ - n+

, H)

0,

C

T

v
 $10 \max,$
 $1 C$
 $0,$
 $F 1$
,

(11)
 F
 F
 $0, F$
 $C9$
 1
 1

$+$
 $n+$
 $H S 1$
 $2 ^$
 $- n+$
 $S 1$
 $^$
 $+ H2 v$
 F

$, H$
 F
 K

$0,$
 $0, F$
 F

éq 4.4-10
Finally it is enough to carry out the linear combination implying [éq 4.4-9] and [éq 4.4-10] to conclude (bus H H and

v
 v

v
with F S
).
F
K
 (K)
 0,
 0,
F
K
 !

Note:

.
This local decrease of the error is also declined according to the formulations [éq 4.2-14]
N
 ~
and [éq 4.3-1], [éq 4.3-2] N

.
R, p, T (K)
R (K)
 .

While placing itself within the particular framework [éq 3.1-9] of the article [bib6] with an implicit scheme (=1) one finds well the inequality (29) pp432.

.
By adopting the less restrictive approaches (H4) and (H5), one finds a version “strong” of this property.

.
This result provides only one opposite room of total increase [éq 4.3-9], [éq 4.3-10] but within the framework of this type of indicator one will not be able to obtain better compromise. These estimates are optimal within the meaning of [bib5]. They show the equivalence of summon hilbertienne indicators with the space part of the total exact error. constants of equivalence are independent of the parameters of discretizations in space and in time, they depend only on the smallest angle of the triangulation.

.
This increase of the real indicator of error shows, which if one very locally refines (around K) in order to decrease N

, *one is not ensured of a reduction in the error*
R (K)

in an immediate vicinity of the zone concerned (in K). The indicator “under consider locally” the error space and only a more macroscopic refinement realizes theoretically a reduction in the error (cf property 13).

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4.5 Complements

The K3 constant just like its alter preceding ego, K2, depends intrinsically on the type on limiting conditions enriching the equation by initial heat as well as type by temporal and space discretization. To try to free itself from this last constraint, SR. GAGO [bib10] proposes (on a problem 2D models) a dependence of the K2 constant in function of the type of finite elements used. It is written

~

K2

K: =

éq

4.5-1

2

2

24 p

where p is the degree of the polynomial of interpolation used (p=1 for the TRIA3 and QUAD4, p=2 for TRIA6 and QUAD8/9). From where the idea, once the indicator of total error calculated, to multiply it by this

1

“corrective” constant

. This strategy was implicitly retained for the calculation of

2

24 p

the indicator of error in mechanics (option “ERRE_ELGA_NORE” of CALC_ELEM, cf [R4.10.02 §3]).

We however did not adopt it for thermics because this constant was not given that empirically on the equation of Laplace 2D. We do not want to thus skew the values of indicators.

It was question, until now, only of charts of indicators of space errors calculated with one moment given of the transient of calculation. But, in fact, there are several ways to build one indicators of error on a parabolic problem:

.

one can very well, first of all, semi-to discretize the strong formulation space some and to control sound

space error by indicator of error adapted a posteriori to the stationary case (in our elliptic case). Then one applies a solvor, of step and order variables, treating the equations ordinary differentials (for example [bib10] [bib11] [bib12]),

.

a second strategy consists in semi-discretizing in time then in space and determining the indicator of one moment space error given (for example [bib4] [bib6] [bib13]) from local residues of the semi-discretized form. One applies a linear solvor to the form variational allowing to repeatedly build the solution at one moment given from the solution of the previous moment,

.

another possibility consists in discretizing simultaneously in time and space on suitable finite elements and to control their “space-time” errors in manner coupled (for example [bib14] [bib15]).

This last scenario is most tempting from a theoretical point of view because it proposes a control complete of the error and it allows to avoid unfortunate decouplings as for the possible ones refinements/déraffinements controlled by a criterion with respect to the other (cf following paragraph). It is

however very heavy to set up in a large industrial code such as Code_Aster. It supposes indeed, to be optimal, nothing less than one separate management step of time by finite elements. It who from the point of view of architecture supporting the finite elements of the code is a true challenge!

One thus prefers the second scenario to him which has the large advantage of being able to be established directly

in a code D finite elements because this it is based above all on the resolution of the system completely discretized. It is this type of indicators which was set up in N3S, TRIFOU and Code_Aster.

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Within the framework of a true “space-time” discretization of the problem (scenario 3), one obtains, in any rigour, a “space-time” indicator for each element of discretization

$K \times [T, T$

who is the balanced sum of three terms:

N

$N 1$

$+]$

1) the residue of the calculated solution and the data discretized compared to the strong formulation problem (P

$K \times$

0) evaluated on

$[T, T,$

N

$N 1$

$+]$

2) the jump space through $K \times [T, T$

of the operator traces associated (who connects naturally

N

$N 1$

$+]$

formulations weak and strong via the formula of Green),

3) the temporal jump through $K \times [T, T$

calculated solution.

N

N 1
+]

The solution which was installation does not make it possible obviously to reveal explicitly it term of temporal jump. It re-appears however implicitly, because of method of semi particular temporal discretization, in all the terms in definitions 10 and 12.

On the other hand, the fact of being interested mainly only in the space discretization and sound possible refinement/déraffinement should not occult certain contingences with respect to management of the step of time. Indeed, during transitory calculations comprising of abrupt variations of loadings and/or sources in the course of time, for example of the shocks thermics, the fields of calculated temperatures T_N ($0 < N \leq N_{max}$) can oscillate spatially and temporally. Moreover, they can violate the "principle of the maximum" by taking values in outside terminals imposed by the condition of Cauchy and the conditions limiting. To surmount it numerical phenomenon parasitizes one shows, on a canonical case without condition of exchange (cf [R3.06.07 §2]), that the step of time must remain between two terminals:

T

H < T
< T

éq
4.5-2
min ()
(
max
)

In practice, it is difficult to have an order of magnitude of these terminals, one thus has evil, if one detects oscillations, to modify the step of time in order to respect [éq 4.5-2]. In addition, this type of operation is not always possible sometimes because it is necessary to take into account the abrupt ones precisely variations of loadings (in particular when T is too small).

When T is too large one can function as implicit Euler ($=1$) what will cause of to gum the upper limit.

On the other hand when it is too weak, two palliative strategies are offered to the user:

- diagonaliser the matrix of mass via the lumpés elements (cf [R3.06.07 §4] [§5]) proposed in the code (that requires installation to treat the P2 elements or modeling 2D_AXI),*
- to decrease the size of the meshes (that increases complexities necessary calculation and memory).*

It is from this point of view that the refinements/déraffinements practised on the faith for our indicator can have an incidence. The fact of refining will not pose any problem on the other hand in déraffinant one can deteriorate very well the decrease of [éq 4.5-2]. It is necessary thus to be very circumspect if

one uses the option déraffinement software LOBSTER (encapsulated for Code_Aster in MACR_ADAP_MAIL option "DERAFFINEMENT" [U7.03.01]) on case test comprising a thermal shock.

We now will summarize the principal contributions of the preceding theoretical chapters and theirs holding and bordering with respect to the thermal calculation set up in Code_Aster.

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5

Summary of the theoretical study

That is to say (P0) the problem in extreme cases interfered (inhomogenous Cauchy-Dirichlet-Neumann-Robin type

linear and with variable coefficients) solved by operator THER_LINEAIRE

T

C

- div

p

(T) = S

×],

$0 [$ T $T = F$ \times 1 $], 0 [$ T (P) G 0 $=$ \times 2 $], 0 [\acute{e}q$ $5-1$ N T $+ HT = HT$ \times $ext.$ 3 $], 0 [$ N $T (X)$ 0 $, = 0$ $T (X)$

Taking into account the choices of modelings operated in Code_Aster (by AFFE_MATERIAU, AFFE_CHAR_THER...) one determines the Abstracted Variational Framework (CVA cf [§2]) minimal on which one will be able to rest to show the existence and the unicity of a field of temperature solution (cf [§2]). By recutting these pre-necessary theoretical a little “éthérés” with the practical constraints users, one deduces some from the limitations as for the types of geometry and the licit loadings. Then, while semi-discretizing in time and space by the usual methods of the code (of which one ensures oneself of course of the cogency and owing to the fact that they preserve the existence and the unicity of the solution),

one studies the evolution of the properties of stability of the problem (cf [§3]). These results of controllability are very useful for us to create the standards, the techniques and the inequalities which intervene in the genesis of the indicator in residue. In these stages of discretization us also briefly let us approach the influence of such or such theoretical assumption on the perimeter functional of the operators of the code.

Before summarizing the principal theoretical results concerning the indicator of error, we go préciser some notations:

one fixes a step of time T such as either an entirety NR and that temporal discretization or T

*regular: $T =$,
 $0 T = T$
 $, T = 2 T$*

*$T = N T$
,
 0
 1
 2
 $L N$*

Note:

This assumption of regularity does not have really importance, it allows just to simplify the writing of the semi-discretized problem. To model a transient unspecified at the moment tn , it is just enough to replace T by $tn=tn+1-tn$.

that is to say the parameter of - method semi-discretizing temporally (P0),

*are N
 T and N
 T fields of temperatures at the moment $T 0$
, exact solutions*

*$N ($
 N
 $NR)$*

*H
initial problem (P0), respectively semi-discretized in time and completely discretized in*

time and in space.

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Taking into account the modelings installation in the code, we can suppose that temporal discretization of the loadings and the source is exact and that the taking into account, via Lagranges, conditions limit (generalized or not) of Dirichlet is too. By against, one of the approaches to model the numerical approximations carried out during calculations

integral of the indicator of error, consists in supposing inaccurate the space discretization of loadings and of the source. Their approximate values are noted

N 1

+

N 1

+

N 1

+

N 1

S, G

,

+

T

H

and

éq
5-2
, H
, H
ext., H
, H
while posing

N I
+

=

,
X (N +)
I
+ (I-),
X N
with {T, S, T, G,
and

éq
5-3
ext.
}
H
0 N NR -1

T

T

Note:

The establishment of this type of indicator (in mechanics as in thermics) is also sullied of another type of numerical approximations related to calculations of the derived seconds of the term voluminal (cf [§4.3]). Its effect can possibly feel when one is interested in intrinsic value of the voluminal error for sources very chahutées on a grid coarse.

They exist then two constants K2 and K3 independent of the parameters of discretization in time

and spaces some, depending only on the smallest angle on the triangulation and the type of problem, which allow to build:

*.
An increase of the total space error (the history of the total real indicator “over-estimates” the total space error)*

*1
2
2
C T
T
T
T1 T1
p (
N
N - nh)
+ -
(m+
m+
-*

*, H)
0,
0,
m=*

*0
N 1
2
0
2
2
2
N NR
C (T - H
T
K T
0
K
1

K
H2S 1 S 1*

p
0
0)
 +
 2
 (*R*) + - (*m*+*R*)
m+
m+
 +
 -
K
 , *H*
 }+
0,
0, *K*
KT
m=
H
0

N 1
2
2
K T
2
 -

m+1
m+
H G
 - *G 1*
 +
H
H T
T
1
H T
T
1
F
 , *H*

0, F
F (H (
-
ext., H
H) m+ - ((
-
ext.
H) m+

0, F
KT m=0 F
H
S2 (K)
FS3 (K)

éq 5-4
.

A decrease of the local space error (it “underestimates” the local space error)

N 1
+
N 1
T
-
+
T
- T N - N

T
H
C
H
H
+
K
p
(

N 1
+
N 1
T

-
+
T
+

, **H**) 0,

T
K

0,

K

1

n+

n+

n+

n+

n+

1

R (K)

1

1

1

1

2

K

H S

- **S**

+ **H G**

- **G**

+

3

K

, **H**

F

, **H**

0,
0,
K
K
2

1

N 1
2

+
H H
F

(N 1+ N 1

T
- +
T
- n+
n+
H
T
- n+
T
ext.,

)
1
, H (
1
1
ext., H
, H)

0,
K
3

0
N NR -1

éq 5-5
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K
.
With continuation (N
K of local real indicators (by using the notations of
R (
) HT
0nN
[\$4.1])
n+1

K
1
:
K
1

K
1

K
1

K
R
()
n+
= R, flight ()
n+
+ R, jump ()
n+
+ R, flow ()
n+
+ R, éch ()
n+
T1 - N
T
I
N
T1
éq 5-6
n+
=
: H S 1 -
H
H
C
+ div T1
H
,

K
, H
p
(nh,)
+
+
+

H
F

+
T
2 FS

N
(K)
0, K

0, F
n+1
n+1

T
T
n+
H G 1 -
H,

+
H
H T
T
1
,

F
, H

F ((
-
ext.
) n+ -
H

, H
FS2 (K)

N
FS3 (K)

N
0, F
0, F

who is initialized by

1
T 0
0

K: H s0 div T 0

H

R (

) =

+

K

H

(H) +

0, K

H

F

+

2 FS

N

(K)

0, F

0

0

T

0

0

T

H G -

H

+

H

HT

T

F

H

F ((

-

ext.

)

-

H

H

FS2 (K)

N

FS3 (K)

N

0, F

0, F

éq 5-7

This local continuation makes it possible to build continuation (N

()

real indicators

0nN

total

1

0

N

N

NR

K

éq 5-8

R () =

NR () 2

2

:

K HT

Of [éq 5-4] (cf [§4.2]) it appears that, at one moment given, the error on the approximation of the condition of

Cauchy and the history of the total indicators intervenes on the total quality of the solution obtained.

One will be able to thus minimize overall the error of approximation due to the finite elements with the course

time while re-meshing “advisedly”, via the continuation of indicators, the structure. Because, in practice,

one realizes that the refinement of the meshes makes it possible to decrease their error and thus cause

to drop

temporal sum of the indicators. The total error will butt (and it is moral) against the value floor had with the approximations of the initial condition, the limiting conditions and the source (which will have

tendency it-also to drop of course!). One cannot obtain results of better quality how data input of the problem!

The result [éq 5-5] (cf [§4.4]) provides only one opposite room of total increase [éq 5-4] (it “must” would have been to reveal also an increase at the local level) but, within the framework of it type of indicator, one will not be able to obtain better compromise. These estimates are optimal with feel [bib5]. They illustrate the equivalence of the sum hilbertienne indicators with the part space of the total exact error. The constants of equivalence are independent of the parameters discretizations in space and time, they depend only on the smallest angle of the triangulation and of the type of dealt with problem.

According to this increase of the indicator [éq 5-6], if one refines very locally (around the element K) in order to decrease N

, one is not ensured of a reduction in the error in an immediate vicinity

$R(K)$

zone concerned (in K). The indicator “locally underestimates” the error space and only a more macroscopic refinement carries out a reduction in the error theoretically.

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*Only in pure residue, all a “zoology” of indicators of space error are permissible (cf [§4.3]), we retained a type similar of it to that already set up for the mechanics of Code_Aster. Being based on the solutions and the discrete loadings of the moment running and the moment precedent (except with the first step of time), **its theoretical limitations are thus, at best, those inherent in the resolution of the problem in temperature**: no zones comprising of points of graining or of point, not of crack, problem to the multi-material interfaces, - diagram unconditionally stable, regular family of triangulation, polygonal grid discretized by isoparametric finite elements, oscillations and violation of the principle of the maximum (cf [§4.5]). Of course, in practice, one very often passes in addition to, and this without encumbers, this perimeter of “theoretical” use.*

But it is necessary well to keep in mind, that as a “simple postprocessing” of (P0), the indicator cannot unfortunately provide more reliable diagnosis in the zones where resolution of the initial problem stumbles (close to crack, shock...).** Its denomination prudently reserved of indicator (instead of the usual terminology of estimator) is in these particular cases more that never of setting! But if, in these extreme cases, its gross amount perhaps prone to guarantee, **its utility as an effective and convenient supplier of charts of error for one mending of meshes or a refinement/déraffinement remains completely justified.

In the same vein, even if the formulation [éq 5-6] were established only in the transitory linear case, isotropic or not, defines by (P0), one could also stretch his perimeter of use to the non-linear one (operator THER_NON_LINE), to conditions limit different (ECHANGE_PAROI for example) or to other types of finite elements (lumpés isoparametric elements, elements of structure...) (cf. [§2.1]). For more information on the “data-processing” perimeter corresponding to its establishment effective in the code, one can refer to [§6.2] or the user's documentation of CALC_ELEM [U4.81.01].

It was question, until now, only of charts of indicators of space errors calculated with one moment given of the transient of calculation. But, in fact, **there are several ways to build one indicators of error on a parabolic problem** (cf [§4.5]). That which we retained does not allow a complete control of the error and it always requires a **certain vigilance when one draft of the problems of the type shocks** (the same one as for the problem post-treaty!). It does not make to appear that implicitly the term of jump temporal in all the terms in [éq 5-6].

To finish, it should be stressed that this indicator is thus composed of four terms:

.
the principal term, called term of voluminal error, controlling the good checking of the equation of heat,

.
to which **three secondary terms** are added checking the good behaviour of the **space jumps** and limiting conditions: **terms of flow and exchange.**

In 2d-PLAN or 3D (resp. in 2D-AXI), if the unit of the geometry is the meter, the unit of first is

1
1
W.m (resp.

1
W.m.

-
rad) and that of the other terms are it

2
W.m (resp.
1

2
W.m.

-
rad). **Caution**

thus with the units taken into account for the geometry when one is interested in the gross amount of the indicator and not with its relative value!

We now will approach, after the practical difficulties of implementation in the code, environment necessary and its perimeter of use. One will conclude for a drawn example of use of a case official test.

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6 **Implementation in Code_Aster**

6.1 Difficulties **particular**

To calculate this type of indicator it is necessary to compose with the vision “elementary calculation + assembly”

*generally deployed in all the codes finite elements. However the estimate, at the local level, of (K) requires, not only the knowledge of its local fields, but also that of its meshes neighbors. **One thus needs to carry out a “total calculation” on a K scale, in calculation room!** A copied strategy on what had been set up for the estimator in mechanics consist in transmitting this type of information in the **components of wide charts** which they will be transmitted in argument of entry of CALCULATION. It is this type of contingency which explains the heterogeneity of treatment at the time of the overloads of loadings between the thermal solveurs and it calculation of our indicator (cf [§6.2]).*

***Another type of difficulty, numerical this time, relates to the calculation of the voluminal term.** Indeed, it requires a double derivation which one carries out in three stages, because in Code_Aster one do not recommend the use of the derived seconds of the functions of forms.*

Note:

They were recently introduced to treat the derivation of the rate of refund of energy (cf [R7.02.01 § Annexe 1]).

*On the one hand, one calculates (in the thermal operator) the vector flow at the points of gauss, then one extrapolate the corresponding values with the nodes by local smoothing (cf [R3.06.03] CALC_ELEM with **OPTION=' FLUX_ELNO_TEMP'** and [§6.2]) in order to calculate its divergence at the points of Gauss.*

With

quadratic finite elements the intermediate operation is only approximate (one affects like value with the median nodes the half the sum of their values to the extreme nodes). However numerical tests (limited) showed that this approach does not provide results very different from those obtained by a direct calculation via good the derivative second.

Lastly, it was necessary to determine various geometrical characteristics (diameters, normals, jacobiens...), connector industries of the elements in opposite and to reach the data which they underlie in all cases of figures envisaged by the code (started from grid symmetrized and/or heterogeneous, loading function or reality, non-linear material, all isoparametric elements 2D/3D and all them thermal loadings).

*Beyond these fastidious developments, a **large effort of validation “géométrico-data processing”** was deployed to try to track possible bugs in this entrelac of small formulas. These hard tests on small cases model tests (TPLLO1A/H for 2D_PLAN/3D and TPNA01A for the 2D_AXI) appeared profitable (including for the indicator in mechanics and lumpés elements!) and essential. Because one does not lay out, to my knowledge, theoretical values allowing to validate in certain situations these indicators: **“nothing resemble more one value of indicator... than another value of indicator!”**. In the absence of other thing and, although in a process of validation that is not the panacea, it are thus necessary to try to release a maximum of confidence in all these components.*

6.2 Environment necessary/parameter setting

*The calculation of this indicator is carried out, via option **“ERTH_ELEM_TEMP”** of the operator of post-treatment **CALC_ELEM**, on a **EVOL_THER** (provides to the key word **RESULT**) coming from a calculation*

*former thermics (linear or not, transient or stationary, isotropic or orthotropic, via **THER_LINEAIRE** or **THER_NON_LINE**, cf more precise perimeter [§6.4]).*

*As one already underlined, it **requires as a preliminary the recourse to option “FLUX_ELNO_TEMP”** of **CALC_ELEM** which determines the values of the vector heat flux to the nodes (cf example of use [§6.5]).*

*This indicator consists of fifteen components by elements and for a given moment. In order to to be able post-to treat them via **POST_RELEVE** or **GIBI** one needs to extrapolate these fields by element in*

*fields with the nodes by element. The addition of option **“ERTH_ELNO_ELEM”** (after the call to **“ERTH_ELEM_TEMP”**) makes it possible to carry out this purely data-processing transformation. For one moment*

and a given finite element, it does nothing but duplicate the fifteen components of the indicator on each nodes of the element.

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For carrying out the integral postprocessing of desired thermal calculation well, it is necessary:

To carry out on all the geometry, TOUT=' OUI' (default value, if not calculation stops in ERREUR_FATALLE). This provisional choice was led by data-processing contingencies and functional calculuses, bus thus all the finite elements are seen affecting a homogeneous indicator calculated with the same number of terms (if not quid of the concept of term of jump and of term of CL at the edge of the field considered ?). In addition the tool of refinement/déraffinement of the code (the software LOBSTER encapsulated in MACR_ADAP_MAIL), natural outlet of our cartographies of error, does not make it possible to treat only parts of grids.

Note:

That poses problems of propagation of subdivisions to preserve conformity triangulation. In fact, to divert this type of contingency, it would be necessary, that is to say to define a buffer zone making the junction enters the “dead” zone of the grid and the zone “activates” to treat, that is to say in a way more optimal but also much more difficult from a point of view structures, to reduce it to a layer of joined elements.

To provide the same temporal parameter setting: value of (default value equalizes to 0.57) provided to key word PARM_THETA; if necessary if transitory problem is dealt with, it is necessary to inform usual fields TOUT/NUME/LIST_ORDRE with licit values opposite

thermal calculation. The calculation of the history of the indicator can then be carried out from any moment of a transient, knowing that with the first increment one carries out calculation as in hover ($=1$, $n+1=0$ and not of term of finished difference cf [éq 5-7]).

Moreover, in hover, if the user provides a value of different from 1, one imposes to him this last value after having informed some.

In a related way, one detects the request for supply of charts of errors between noncontiguous sequence numbers (there is an ALARM) or the data of a EVOL_THER not comprising a field of temperature and vector flow to the nodes (calculation stops in ERREUR_FATALE). The value of and numbers it sequence number taken into account are layouts in the file message [§6.3]. The sequence number and the corresponding moment accompany also each occurrence by indicator of error in the file result ([§6.3]).

To use the same loadings and by complying with the rules of particular overloads with the options of error analyses of this operator. Thus, in the thermal solveurs (and mechanics) one incorporates the limiting conditions of the same type, whereas in calculations errors of CALC_ELEM (and thus also with our indicator) one cannot take into account, for a limiting type of condition given, that the last provided to key word EXCIT. The order of these loadings revêt thus a crucial importance!

Note:

This restriction finds its base in the first remark of the paragraph precedent. For making well it would be necessary, that is to say concaténer on the elements of skin concerned all the limiting conditions, is to provide to elementary calculations charts variable sizes containing all the loadings exhaustively. The first solution seems by far most optimal but also hardest to put in work. It would then be necessary also to make the same thing for the indicator in residue of mechanics (OPTION=' ERRE_ELGA_NORE').

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However, in the event of conflict between loadings of the same type, one often can and easily find a solution palliative via the adequate AFFE_CHAR_THER. The user is informed presence of several occurrence of the same type of loading by one message of ALARM and the list of the loadings actually taken into account is traced in the file message ([§6.3]).

The code stops on the other hand in ERREUR_FATALE if the provided loadings pose some problems (interpolation of loadings function, access to the components, presence of CHAMPGD of the coefficient of exchange and absence of the CHAMPGD of the outside temperature or vice versa.....),

***Within the same framework general:** value of the model (parameter MODELS), of necessary materials (CHAM_MATER), of the structure EVOL_THER given (RESULT) and result (assignment of Réentrant CALC_ELEM with possibly “reuse”). They are traced in the file message ([§6.3]).*

If the user does not respect this necessary homogeneity of parameter setting (to the rules of overload near) between the thermal solver and the tool for postprocessing, it is exposed to skewed results even completely false (without inevitably a message of ALARM or one ERREUR_FATALE stops it, one cannot all control and/or prohibit!). There remains only judge then relevance of its results.

Let us recapitulate all this parameter setting of operator CALC_ELEM impacting the calculation directly of the indicator of space error in thermics.

Key word factor

Key word

Default value

Value obligatoire (O)

or advised (C)

MODEL

Idem thermal calculation

(O)

CHAM_MATER

Idem thermal calculation

(O)

ALL “YES”

“YES” (O)

TOUT/NUME/LIST_ORDRE “YES”

“YES” (C)

PARM_THETA

0.57

Idem thermal calculation

(O)

RESULT

EVOL_THER of calculation

thermics (O)

reuse

EVOL_THER of calculation

thermics (C)

EXCIT CHARGES

Idem thermal calculation +

regulate of overload (O)

OPTION

“FLUX_ELNO_TEMP”

“ERTH_ELEM_TEMP”

“ERTH_ELNO_ELEM”

INFORMATION

1 1

(C)

Table 6.2-1: Summary of the parameter setting of CALC_ELEM impacting the calculation of the indicator

Note:

.
In transient, it (strongly) is advised to calculate the history of the indicator on moments of calculations contiguous. If not, the postprocessing of the temporal semi-discretization will be distorted, and according to the devoted formula... the user will become only judge of relevance of its results.

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6.3***Presentation/analysis of the results of the error analysis***

*Option “ERTH_ELEM_TEMP” provides in fact, not one, but **fifteen components by elements** stop K and by step of time t_{n+1} . Indeed, **for each one of the four terms** of [éq 5-6], $_$ the term the voluminal main thing and the three surface secondary terms $_$, one calculates not only the **error absolute**, but also a **term of standardization** (the theoretical value of the discretized loadings that one would have had to find) and the **associated relative error**. By summoning these three families of four contributions, one establishes also the total absolute error, the total term of standardization and the relative error total. What makes the account well!*

The fact of dissociating the contributions of each component of this indicator allows to compare their relative importances and to target strategies of refinement/déraffinement on a certain type of error. Even if the voluminal term (representing the good checking equation of heat) and the term of jump (related to modeling finite elements) remain them dominating terms, it can prove to be useful to measure the errors due to certain type of loading in order to refine their modeling or to re-mesh the accused frontier zones.

Moreover this type of strategy can be easily diverted of its goal first in order to make refinement/déraffinement by zone: it is enough to impose, only in this zone, a type of fictitious limiting condition (with very bad value in order to cause a large error).

Mode of calculation of these components and the name of their component “of reception” in the field symbolic system “ERTH_ELEM_TEMP” of the EVOL_THER are recapitulated in the table below (in being based on the nomenclature of [éq 5-6]).

Absolute error

Relative error

Term of standardization

(in %)

Term

N 1

+

N 1

+

K

N

NR 1

+

K: = H S 1

+

R, flight (

)

N

R, flight (

)

R, flight (K)

voluminal

×100

K

, H 0, K

TERMVO

N 1

+

NR

K

R, flight (

)

.

TERMV1

TERMV2

Term of

N 1

+

N 1

+

K

l

R, jump (

)

R, jump (K)

jump

×

N

+

l

+

H2

T1

N

F

, H

TERMSA

n+

NR

NR

K: =

R, jump (

)

R jump (K)

.

100

l

,

2

N

TERMS2

O, F
TERMS1
Term of flow
N 1
+

N 1
+

K
l
R, flow (
)
R, flow (K)
×
N
NR 1

+

K: = H2 G l
+

R, flow (
)
N
TERMFL
n+
NR
F

, *H 0, F*
R flow (K)
.
100
l
,
TERMFI
TERMFI2
Term
N 1
+

N 1

+

K

l

R, éch (

)

R, éch (K)

of exchange

×100

N

NR l

+

K: = H2 H T - T

l

+

R, éch (

)

F ((ext.

) N

TERMEC

N l

+

NR

K

, H 0, F

R, éch (

)

.

TERME2

TERME1

Total

n+l

K

l

:

K

N l

+

n+

NR l K:

NR I K

R

() = n+R, I ()

R (K)

R (

) = n+R, I ()

×

I

n+

NR

I

R

(K)

.

100

I

ERTABS

TERMNO

ERTREL

Table 6.3-1: Components of the indicator of error.

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For the absolute error and the term of standardization, in 2d-PLAN or 3D (resp. in 2D-AXI), if the unit geometry is the meter, the unit of the first term is W.m (resp.

I

W.m.

-
rad) and that of

1
1
other terms is it

2
W.m (resp.

1
2
W.m.

-
rad).

Attention thus with the units taken into account for the geometry when one is interested in gross amount of the indicator and not with its relative value!

This information is accessible in three forms:

.
For each moment of the transient, these fifteen values are summoned on all the grid (one fact the same thing as in the table [Table 6.3-1] by replacing K by) and traced in a table of the file result (.RESU).

THERMICS: ESTIMATOR Of ERROR IN RESIDUE

IMPRESSION OF THE TOTAL STANDARDS:

SD EVOL_THER RESU_1
SEQUENCE NUMBER 3
MOMENT 5.0000E+00
ABSOLUTE ERROR/RELATIVE/STANDARDIZATION
TOTAL 0.5863E-05 0.2005E- 04% 0.2923E+02
VOLUMINAL TERM 0.3539E-05 0.0000E+ 00% 0.0000E+00
TERM JUMP 0.2217E-05 0.1006E- 04% 0.2205E+02
TERM FLOW 0.4384E-06 0.3886E- 05% 0.1128E+02
TERM EXCHANGE 0.4591E-06 0.5755E- 05% 0.7977E+01

Example 6.3-1: Layout of option “ERTH_ELEM_TEMP” in the file result

.

It is stored by means of computer in the fifteen components of the field symbolic system “ERTH_ELEM_TEMP” of the thermal SD_RESULTAT. The variables of access of this field are , for each mesh (in our M1 example), the sequence number (NUME_ORDRE) and the moment (INST). With option “ERTH_ELNO_ELEM” one with the same thing for each node of the element considered.

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FIELD BY ELEMENT AT THE POINTS OF GAUSS OF REFERENCE SYMBOL

ERTH_ELEM_TEMP

SEQUENCE NUMBER: 3 INST: 5.00000E+00

M1 ERTABS ERTREL TERMNO

TERMVO TERMV2 TERMV1

TERMSA TERMS2 TERMS1

TERMFL TERMF2 TERMF1

TERMEC TERME2 TERME1

1 0.5863E-05 0.2005E-04 0.2923E+02

0.3539E-05 0.0000E+00 0.0000E+00

0.2217E-05 0.1006E-04 0.2205E+02

0.4384E-06 0.3886E-05 0.1128E+02

0.4591E-06 0.5755E-05 0.7977E+01

.....

FIELD BY ELEMENT AT THE POINTS OF GAUSS OF REFERENCE SYMBOL ERTH_ELNO_ELEM

SEQUENCE NUMBER: 3 INST: 5.00000E+00

M1 ERTABS ERTREL TERMNO

TERMVO TERMV2 TERMV1

TERMSA TERMS2 TERMS1
TERMFL TERMF2 TERMF1
TERMEC TERME2 TERME1
N1 0.5863E-05 0.2005E-04 0.2923E+02
0.3539E-05 0.0000E+00 0.0000E+00
0.2217E-05 0.1006E-04 0.2205E+02
0.4384E-06 0.3886E-05 0.1128E+02
0.4591E-06 0.5755E-05 0.7977E+01
N3 0.5863E-05 0.2005E-04 0.2923E+02

Example 6.3-2: Layouts, via IMPR_RESU, of the components of the field symbolic system “ERTH_ELEM_TEMP”/“ERTH_ELNO_ELEM” in the file result Handbook of Reference R4.10 booklet: Estimator of error a posteriori HI-23/02/014/A

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.
One can also trace the values of each one of these components in the file message (.MESS) by initializing the key word INFORMATION to 2. However this functionality rather reserved for developers (for maintenance or of the pointed diagnoses) also revealed complementary impressions (documented but too exhaustive) on the elements constituting the indicator and the characteristics of the finite elements and their vicinities.

TE0003 *****

NOMTE/L2D THPLTR3/T

RHOCP 2.000000000000

- null limiting condition “it does not have there limiting conditions on this edges,

.
Tests of not-regression “numérico-data processing” showed that the manner of to model the loadings and the source, as constants or functions, could to especially influence notably the values of very small terms of error (in relative error well sure) and to worry the user unnecessarily. This phenomenon is explained by differences of codings of the discretized loadings [éq 5-2]. This type of behavior is found too as soon as one changes linear solvor, preconditionnor, method of renumerotation, of platform...

.
In hover, when one uses a nonnull source with linear finite elements, it term principal is very badly estimated since it requires a double derivation of the field of temperature. An ALARM thus warns the user and the enjoint to pass into quadratic.

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6.4 Perimeter

of use

*This indicator was developed, for the moment, only on the isoparametric elements (TRIA3/6, QUAD4/8/9, TETRA4/10, PENTA6/13/15 and HEXA8/20/27) and for modelings PLAN, PLAN_DIAG, AXIS, AXIS_DIAG, 3D and 3D_DIAG. It thus does not calculate the contributions of elements of structure of the type hull (COQUE_PLAN, COQUE_AXIS, HULL), of the pyramids (PYRAM5 and PYRAM13) and of the modeling of Fourier (AXIS_FOURIER). It does not allow either to calculate the terms of jumps of these elements with the authorized elements. **However, if a grid comprise licit and illicit elements, calculation does not stop and, via OPTION 2 in***

*the suitable catalogues of elements, **one warns the user of not taken into of the aforesaid account elements.***

*Indeed to carry out this postprocessing, it is necessary as a preliminary to call, explicitly, the option “FLUX_ELNO_TEMP” (calculation of the vector heat flux to the nodes) and, implicitly, “INIT_MAIL_VOIS” (determination of the characteristics of the vicinity K of an element K). **One is thus tributary of their respective perimeters of use.***

*It is also necessary to keep in mind **some more minor rules** but which can cover one very particular importance for very precise studies:*

1) The calculation of the indicator treats only the elements of the grid pertaining to the model indicated by the MODEL key word of order CALC_ELEM. One can thus work with grids (not cleaned) comprising “meshs of outline” to which one allots one different model.

2) In one

***grid in dimension Q**, one calculates the terms of jump and loading, only on **elements of skin of dimension q-1**. Therefore, the relations of the TRIA/QUAD are treated with the SEG and relations TETRA/PENTA/HEXA with the FACE. For example, in the event of presence of segments in a three-dimensional grid, the option will not stop but it will not take into account their (possible) contributions.*

*3) Preliminary option “ERTH_ELEM_TEMP” and its options do not know the **PYRAM**. Their contributions will be ignored. This gap comes from their introduction into Code_Aster more recent than those of the already quoted preliminary options.*

Note:

In any event these elements are minority in a grid 3D and are not generated that by the voluminal free maillor of GIBI, which creates some locally for to supplement portions of grids hexahedral.

4) In 2D,

***one should not accidentally intercalate a segment between two triangles or quadrangles**, if not the term of jump of these elements will not be calculated and one will enquérira oneself with*

wrong of the existence of a possible limiting condition. Calculation will not stop but with this interface, the value of the indicator will be incomplete. However, for needs private individuals (charging density internal and localised in a structure, fissures...), one can of course allow this kind of situation. In 3D, the problem arises of course too when one intercalates quadrangles or triangles between two FACE contiguous.

*5) the same type of imbroglio occurs when **two points of the grid are superimposed** geometrically. There still, calculation should not stop, but the value of the indicator will be incomplete on the level of this zone of covering,*

6) **If one works with a grid which results from operations of symmetrization**, it is necessary to test not to be in the two preceding cases of figures. Moreover, on both sides of the axis of symmetry, the **close meshes do not have inevitably (with in particular the maillor GIBI)** of the orientations which meet the standard of Code_Aster (they should be reversed). The calculation of the indicator, for which this information is crucial (to define them external normals with each mesh and the connector industries in opposite), detects the problem in calculating the jacobien each mesh. In 2D, an algorithm of substitution allows to circumvent the problem and to rebuild the tables of connector industry “nodes of the element running nodes of its neighbors”. In 3D, the problem is much more difficult and private individual with each element, the code thus stops in **ERREUR_FATALE** in the event of problem.

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7) **If one wants to refine or déraffiner his grid with MACR_ADAP_MAIL [U7.03.01]**, it grid should comprise only triangles or tetrahedrons. Concerning the loadings surface or voluminal, the “good practice” consists in using only groups of meshes. If groups of nodes are used, one must expect distorted calculations, because after some refinements, other points will have probably fit geometrically in the zone concerned with the **GROUP_NO** without seeing itself affecting one unspecified loading (one cannot modify the composition of a **GROUP_NO** in the course of session!).

For specific loadings or points of statement (on which go, for example, to rest **POST_RELEVE_T**) the **GROUP_NO** is licit. On the other hand, it is not advised to directly use meshes (**MA**) or nodes (**NO**) (apart from group), because in it case, to the liking of the renumérotations, **LOBSTER** probably will lose their trace. It cannot to preserve the memory of the meshes or the nodes that through a name of **GROUP_MA** or of **GROUP_NO**. Thanks to this mechanism, it can adopt a Lagrangian vision of becoming of these meshes or of these points!

The calculation of the indicator takes place indifferently on a EVOL_THER coming from THER_LINEAIRE or of THER_NON_LINE, stationary or transitory, isotropic or orthotropic, and, on a motionless structure with a grid by elements answering the preceding criteria.

Into non-linear one takes into account non-linearities of materials and the modification of the problem in

enthalpy. However one does not treat the possible contributions of non-linear loadings (FLUX_NL and RADIATION). The user is informed by it by an ALARM, just like it is informed not taken into account of a limiting condition of type ECHANGE_PAROI. Indeed, into linear one recognizes, for the moment, that the contributions of the loadings SOURCE, FLUX_REP and EXCHANGE. For

the taking into account of these loadings, of the particular rules of overload are applied (cf [§6.2]).

6.5 Example of use

To familiarize itself with the use of this indicator in thermics and its possible coupling with the encapsulation of HOMARD® (for more information, one will be able to consult the site <http://www.code-aster.com/outils/homard>) via MACR_ADAP_MAIL [U7.03.01] one can to take as a starting point this expurgée version of the case test TPLL01J [V4.02.01]. It is however only about one

case data-processing test of not-regression putting forward the use of certain functionalities of new process control language PYTHON (buckles, test...).

MATERI=DEFI_MATERIAU (THER=_F (LAMBDA = 0.75, RHO_CP = 2.0))

*M= [None] *5*

MAIL=LIRE_MAILLAGE ()

initial Grid

M [1] =DEFI_GROUP (reuse=MAIL, MAILLAGE=MAIL,

CREA_GROUP_NO=_F (TOUT_GROUP_MA = "YES"))

Vecteurs results has each iteration

*MODE= [None] *4*

*MATE= [None] *4*

*CHAI= [None] *4*

*RESU= [None] *4*

Buckles indicating calculation/mending of meshes; PYTHON makes 3 iterations for K in arranges (1,4):

Assignment of materials/model/loading

SUBDUE [K] =AFFE_MATERIAU (MAILLAGE=M [K],

AFFE=_F (ALL = "YES", MATER = MATERI))
MODE [K] =AFFE_MODELE (MAILLAGE=M [K],
AFFE=_F (ALL = "YES", MODELING = "3D",
PHENOMENON = "THERMAL"))
CHAI [K] =AFFE_CHAR_THER (MODELE=MODE [K],

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*TEMP_IMPO=_F (GROUP_NO = "FIINF", TEMP = 100.),
FLUX_REP=_F (GROUP_MA = "FLOW", FLUN = -1200.),
ECHANGE=_F (GROUP_MA = "ECHA", COEF_H = 30. ,
TEMP_EXT = 140.)*

Calculation thermal

*RESU [K] =THER_LINEAIRE (MODELE=MODE [K],
CHAM_MATER=MATE [K],
EXCIT=_F (LOAD = CHA1 [K]))*

Calculation of the indicator of error

*RESU [K] =CALC_ELEM (reuse=RESU [K], MODELE=MODE [K],
TOUT=' OUI',
CHAM_MATER=MATE [K], RESULTAT=RESU [K],
EXCIT= _F (CHARGE= CHA1 [K]),
PARAM_THETA=0.57,
OPTION= ("FLUX_ELNO_TEMP", "ERTH_ELEM_TEMP", "ERTH_ELNO_ELEM"))*

Subtlety PYTHON to define the new grid

M [k+1] =CO ("M_%d' % (k+1))

Adaptation of the grid while basing itself on component ERTABS of

ERRE_ELEM_TEMP of RESU [K].

Old grid: M [K]. Grid refines: M [k+1]

MACR_ADAP_MAIL (ADAPTATION=_F (

MAILLAGE_N = M [K],

MAILLAGE_NP1 = M [k+1],

RESULTAT_N = RESU [K],

INDICATOR = "ERTH_ELEM_TEMP",

NOM_CMP_INDICA = “ERTABS”))

Example 6.5-1: Expurgé of the command file of case-test TPLL01J

In this other example extracted Internet site of HOMARD®, coupling EARTH_ELEM_TEMP/MACR_ADAP_MAIL [U7.03.01] simulates the circulation of a “hot” fluid of share and

of other of a metal part bent (in top and bottom, via a condition of EXCHANGE depend on time in AFFE_CHAR_THER_F). The circulation of the fluid is carried out left towards the line.

The precision is especially necessary at the ends of the structure, on the level of the propagation of the fluid:

thanks to the indicating coupling of error/tool of refinement-déraffinement, the grid thus remains fine in edge of part, in the zone where concentrates the “hot” fluid. Finally it is déraffiné with the back, one time that the fluid passed.

It is also noted that, as envisaged by the theory (cf remarks [§2.2]), the resolution of the problem thermics “is blunted” in the returning corners and that the indicator of error (although it is him also penalized in these zones) this established fact announces (even when the part cooled).

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6.0

Titrate:

Indicator of error in residue for transitory thermics

Date:

03/06/02

Author (S):

O. BOITEAU *Key*

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Example 6.5-2: Use of option “EARTH_ELEM_TEMP” coupled with LOBSTER

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Outline conclusion

During digital simulations by finite elements, obtaining a rough result is not sufficient any more.

The user is increasingly petitioning of space error analysis compared to the grid.

He has need for support methodological and pointed tools “numérique-data processing” to measure the quality of its studies and to improve them.

To this end, the indicators of space error a posteriori make it possible to locate, on each element, a cartography of error on which the tools of mending of meshes will be able to rest: a first calculation on a coarse grid makes it possible to exhume the chart of error starting from the data and of solution discretized (from where the term “a posteriori”), refinement is carried out then locally in treating on a hierarchical basis this information.

The new indicator a posteriori which has been just established post-to treat the thermal problems of Code_Aster is based on their local residues extracted the semi-discretizations in time. Via the option “ERTH_ELEM_TEMP” of CALC_ELEM, it uses the thermal fields (EVOL_THER) emanating from THER_LINEAIRE and of THER_NON_LINE.

This new indicator supplements the offer of the code in term of advanced tools making it possible to improve quality of the studies, their mutualisations and their comparisons. Indeed, of the indicators of error in mechanics and macro of refinement/déraffinement MACR_ADAP_MAIL [U7.03.02] is already available. It remains to supplement the perimeter of use of these tools and, to pack them, in particular for to better manage non-linearities and the interactions space error/temporal error.

Note:

Estimator by smoothing of constraints of Zhu & Zienkiewicz (CALC_ELEM + OPTION

“ERRE_ELEM_NOZ1/2” [R4.10.01]) and indicator in pure residue (“ERRE_ELGA_NORE” [R4.10.02]).

Thereafter, the prospects for this work are several orders:

.
From a functional point of view, the complétude of this indicator could also improve in taking into account possible nonlinear limiting conditions (FLUX_NL and RADIATION) and of the exchanges between walls (ECHANGE_PAROI). In the long term, it would also be necessary to be able to rest

on finite elements of structure (hull...), of the pyramids and capacity to deal with problems of convection-diffusion (operator THER_NON_LINE_MO [R5.02.04]).

.
From a theoretical point of view, when new limiting conditions are used and/or when one be based on new modelings (hull, beam...), a study “numérique-fonctionnel calculus” similar to that of this document, should be carried out to judge theoretical and practical limitations (with respect to Code_Aster) of such an indicator and to exhume its adhoc formulation.

.
 Let us recall finally that a **string of indicators of error a posteriori are available**, and, that enough little was tested and validated on industrial cases. In order to refine diagnoses, to establish comparisons and to set up strategies of mending of meshes per class of problem, it would be interesting to pack the list of the indicators available. Different indicators in residue plus local problem thus appeared more effective (but also more expensive) during numerical tests (into elliptic) in N3S [bib5].

Note:

The indicator is the standard of the solution of a local, of the same problem standard than it problem initial, but discretized on spaces of higher degree and of which the second member is the residue. According to the limiting conditions affixed with this local problem, one in distinguishes from various types. They thus mix the vision “bases hierarchical” and them aspects “residue” of the indicators of error a posteriori.

.
 The ideal consists in discretizing simultaneously in time and space on finite elements adapted and to control their “space-time” errors in a coupled way. **This “space-time” indicator** gives access to a complete control of the error and it allows to avoid unfortunate decouplings as for possible refinements/déraffinements controlled by a criterion with respect to the other (cf discussion [§4.5]). It is however very heavy with to set up in a large industrial code such as Code_Aster. It supposes indeed, for to be optimal, nothing less than one separate management step of time by finite elements. What from the point of view of architecture supporting the finite elements of the code is true challenge!

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Handbook of Reference

R4.20 booklet: Methods of analysis

Document: R4.20.01

Indicators of discharge and loss of proportionality of the loading in elastoplasticity

Summary

One presents a whole of scalar parameters called indicators, allowing to appreciate a loss of proportionality of a loading during its history. Two types of indicators are proposed: indicators being appeared as scalar fields allowing to detect the zones of the structure undergoing nonradial discharges or loadings, and total indicators integrated on a zone of the structure chosen by the user. The latter are more especially intended for the evaluation of the validity of rate of refund of energy in nonlinear breaking process.

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1 Introduction

1.1

Definition of a loading proportional

Considering a structure subjected to a thermomechanical loading in the interval of time $[0, T]$

it will be said that this loading is proportional (or radial) with the material point P if the field of constraint represented in this point by the tensor is proportional to a tensor independent of the moment considered, the proportionality factor being a monotonous function of time.

Formally, that will be expressed by:

$[0, T], (P,) =$

(
)

(P),

() > 0

0

monotonous function in [0, T].

This definition implies, in particular, that the principal directions of the constraints remain constant, at the point considered, throughout the way of loading (these directions can be of course variable of a point with another).

1.2

Importance of the loading proportional and utility of indicators

For plastic materials, the mechanical fields depend on all the history run out during the way of loading. The laws of flow thus have an incremental character and their integration depends on each case of loading. A notable exception precisely relates to the loading proportional for which the law of flow can be integrated once and for all. For example, the law of plasticity of Prandtl-Reuss based on the criterion of Von Mises can be replaced by a law nonlinear rubber band (called law of Hencky-Settings). Cases of loading strictly proportional are rather rare. Indeed, it is necessary to fill a great number of conditions to carry out such a case [bib1]

and these last are seldom checked for the industrial structures. One can even say that for structures presenting of the geometrical defects such as cracks, these conditions are not never strictly checked.

When the loadings are multiaxial, cyclic, or thermomechanical transients, some sections of the way of loading can be strongly nonproportional. It is then useful to locate these sections and to evaluate the importance of the loss of proportionality, so for example adjusting discretization in time of the elastoplastic problem for the section considered, or to measure the validity certain postprocessings (in breaking process for example).

1.3

Various types of indicators of loss of proportionality

It seems difficult to define a single and simple size which could detect at the same time zones space of loss of proportionality and the sections of way of loading (temporal zones) in a material point. This is why we propose four scalar sizes having each one them specificity: two, defined by fields measuring in each point the discharge and the deviation of constraints between two steps of time (indicating buildings), two others of more total nature, characterizing in a given zone of the structure a history of loading nonproportional.

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2 Indicators

buildings

The goal of these indicators is to determine the zones of the structure where, at one particular moment, occurs

that is to say a discharge or a loss of radiality of the stress field. They are produced in postprocessing of a static or dynamic calculation, 2D or 3D, using a law of behavior rubber band or not. They are appeared as fields of scalars whose examination can to be carried out by tracing their isovaleurs by a graphic post-processor.

2.1

Indicator of discharge

This indicator measures at the point M and between the moment T and $T + T$, relative variation of the standard of

$(M, T + T$

) - (M, T)

constraints within the meaning of Von Misès. It is written formally: $I1 =$

. This

$(M, T + T$

)

quantity is negative in the event of local discharge at the point M . the standard (M, T) can be written four ways different according to the choice from the modelisator:

3

1)

(M, T)

$D.D$

=

, where D is the deviatoric part of the tensor of the constraints (this

2

standard is useful in plasticity with isotropic work hardening).

3

2)

 $(M, T) =$

., where one considers the totality of the tensor of the constraints in order to detect by

2

example pressure decreases hydrostatic.

3

3)

 (M, T) $(D X). (D$

=

-

- X), with X the tensor of the constraint of recall in

2

case of an elastoplastic law with a kinematic work hardening.

3

4)

 $(M, T) =$ $(- X). (- X)$

2

2.2

Indicator of loss of radiality

This indicator measures at the point M and between the moment T and T + T

, variation of the direction of

constraints. It is written:

 $(M, T).$ $I_2 = 1$

, where the scalar product “.” is associated the one of the four standards

 (M, T)

the preceding ones. This quantity is null when the radiality is preserved during the increment of time.

Note:

These indicators are closely related to the discretization in time of the problem. In particular, if this discretization is too coarse, one can not detect the discharge or the loss very well of radiality occurring during the increment of time.

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3 Indicators

total

These indicators are intended to detect if, during the history of the structure and until the moment current T , and for a zone of the structure chosen by the modelisator, there was loss of proportionality loading (these indicators thus leave a trace of the history contrary to the indicators buildings which are instantaneous). They are only usable within the framework of a behavior elastoplastic with isotropic work hardening (in 2D or 3D).

3.1

Indicator on the parameters of plasticity

This quantity allows, in the case of the plasticity of Von Mises with isotropic work hardening, on the one hand

of knowing (on average on a zone S of the field) if constraints and deformations

plastics have the same directions and if the plastic threshold is reached at the current moment, and in addition if

during the history the plastic deformation changed direction. This quantity is written:

p

1

\cdot

$I =$

1

$(-$

$) D$

3

S

$($

$,$

$+ R(p) p$

S

Y

where Y is the initial plastic threshold, R the extension of the surface of load related to work hardening and p

cumulated plastic deformation. The scalar product is associated the standard within the meaning of Von Misès. This

indicator is standardized and has a value ranging between zero and one. It is null if the loading has preserved its character proportional in each point of S throughout the past history.

Notice 1:

The indicator is not affected if during the history, there were discharges then refills rubber bands without change of direction of the constraints when one reconsiders the threshold (cf [Figure 3.1-a]).

$(Pt) = (Pt$

1

$2)$

$(pt1)$

$(pt2)$

Y

$Y + R (p)$

$I3 = 0$

$I3 0$

Appear 3.1-Error! Argument of unknown switch. : Way of loading between T1 and t2 in the plan deviatoric of the constraints

Notice 2:

In the formulation of this indicator three ingredients related to plasticity intervene:

· the variation enters the direction of the constraints and the current plastic deformations

$(. p$

p

$),$

· the position of the constraints compared to the current threshold (

$($

$+ ()$

Y

$R p),$

· the variation enters the standard of the current plastic deformation and the cumulated plastic deformation

$(p p).$

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A loss of proportionality could occur during the history without the indicator not detecting it via the first two ingredients (i.e. one can have at the end of the loading coincidence of the directions of the constraints and the plastic deformations and being on the plastic threshold).

On the other hand, there will be p_p , and the indicator will be obligatorily higher than zero, consequently the user will be informed loss of proportionality.

Notice 3:

If the indicator detects obligatorily a loss of proportionality in a zone, in practical it is necessary that the latter not contains sufficient material points with loading proportional. Indeed, if one chooses a very vast zone with few points concerned, standardization of the indicator carried out with division by the volume of the zone implies some "crushing" towards zero of the value of the quantity. Typically, for a structure containing a defect source of nonproportionality, one may find it beneficial to choose a zone of integration S surrounding the defect with a weak vicinity in order to obtain a significant value of the indicator.

3.2 Indicator energetics

This indicator with the same function that the precedent, but is founded on the density of energy. It is written:

1

$I =$

1

(-

) D

4

$S S$

,

T

where is the density of deformation energy defined by: $(T) =$

.

D , and is the density

0

of elastic energy associated the traction diagram if nonlinear elastic material were considered.

More precisely, this quantity is written:

1
 2
 2
 μ
 $((T)) = Ktr () +$
 $2, if < (+ R$
 Y
 $(p),$
 2
 3
 1
 2
 2
 R
 (P)

P
 $((T)) = Ktr () +$
 $+$
 $R Q$
 2
 6
 $() dq, if = (+ R$
 Y
 $(p),$
 μ
 0

with K the module of compressibility, μ the coefficient of shearing of Lamé, R the threshold of traction diagram associated with the standard with the plastic deformation P

P = (this one can be different true plastic threshold, because P p if the loading is nonproportional). This indicator is also standardized between 0 and 1. It is null for a loading having always kept its character proportional (=).

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4

Environment of use of the various indicators

The indicators presented here are usable in postprocessing of a mechanical calculation and are available for the finite elements of the continuous mediums in 2D (mode of plane deformations, constraints plane or axisymmetric, triangular or quadrangular meshes) or 3D (meshes hexahedral, tetrahedral, pentaedric or pyramids). Telegraphic elements, beams, plates and hulls are excluded from this application.

4.1 Indicators

buildings

These indicators are accessible after a static or dynamic calculation whatever the law from behavior of material. Operator CALC_ELEM presents options "DCHA_ELGA_SIGM" and "DCHA_ELNO_SIGM" for the indicator of I1 discharge evaluated with the nodes or the points of Gauss of the element, and options "RADI_ELGA_SIGM" and "RADI_ELNO_SIGM" for the indicator of loss of

radiality I2 evaluated with the nodes or the points of Gauss of the element [bib2]. The key word STANDARD allows

to use one of the four standards described in paragraph 2.1, two last being used only if one carried out as a preliminary an elastoplastic calculation with kinematic work hardening.

4.2 Indicators

total

These indicators are accessible only after one elastoplastic calculation with isotropic work hardening. Operator POST_ELEM presents options "INDIC_SEUIL" and corresponding "INDIC_ENER" respectively with the total indicators I3 and I4. Those are evaluated on a group of mesh previously defined by the user (for example by order DEFI_GROUP).

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R5.01 booklet: Modal analysis

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Algorithm of resolution for the problem

generalized

Summary

In this document, we present the algorithms of resolution for the generalized modal problems which are established in *Code_Aster* via operators `MODE_ITER_INV` and `MODE_ITER_SIMULT`:

- method of the iterations opposite,
- method of Lanczos,
- the method WILL GO (known as of “Sorensen”),
- method of Bathe & Wilson.

One gives to the reader the properties and the limitations, theoretical and practical, of the modal methods approached

while connecting these considerations, which can sometimes appear a little “éthérées”, to a precise paramétrage of

operators. For each method, one recapitulates in the form of tables the aforementioned paramétrage with his values by

defect and of the references to the paragraphs of the document.

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1

Introduction - Description of the document

A majority of study concerning the **dynamic behavior of structures** are carried out in carrying out a **transitory analysis on modal basis**. To exhumate these modes of vibrations, one string of algorithms were developed since about fifty years. In order to face continual increase in the size of the problems and with the degradation of conditionings of the discretized operators, only most effective and most robust, in practice, were built-in in the two modal operators of *Code_Aster*.

The optimal perimeters of use of these operators can be dissociated. When it is about **to determine some eigenvalues** (typically a half-dozen) or **to refine some** estimates, operator **MODE_ITER_INV** is completely indicated. He gathers algorithms heuristic and those of type powers (cf [§3]).

On the other hand, **to capture a significant part of the spectrum**, one A resorts to **MODE_ITER_SIMULT**.

This last federates the methods known as of “subspace” (Lanczos [§4], [§5], IRAM [§6], Bathe & Wilson

[§7]) which projects the operator of work in order to obtain an approximated spectrum of more reduced size (treated

then by a total method of type **QR** or Jacobi).

Until now, these algorithms stumbled regularly on the same shelves: detection correct of multiple modes, modes of rigid body and a general way, treatment of packed spectrum. All this led to the appearance of “**phantom**” modes sometimes badly easy to detect (modes corresponding to multiplicities missed and being able to generate correct residues with the direction

of Aster, and this, more especially as the criteria of the post-modal checks were sometimes permissive (residue in 102 instead of 106 current), decontaminated even insufficient (test of Sturm limited to the values

clean positive) which causes distortions of results downstream from calculation, during projections on base modal.

To be been free from the recurring problems to this type of approach, one thus proposed to enrich **MODE_ITER_SIMULT** (starting from V5) of the **algorithm WILL GO** (“Implicit Restarted Arnoldi” [§6]). This

alternative of Arnoldi, initiated by D.C. Sorensen in 1992, makes real great strides for the resolution of large

modal systems on parallel supercomputers.

It tries to bring an elegant remedy for the recurring numerical problems raised by the others approaches. In short, IRAM gets a better total robustness In V5, it made it possible to balance all of software anomalies related to the modal problems generalized (in V5, it allowed to balance all software anomalies related to the modal problems generalized) while improving complexities calculation and memory for a fixed precision, and, it allows a real control on quality of the modes via a suitable paramètrage. **Its use (method by defect in MODE_ITER_SIMULT) is thus with advising in all the cases of figures.**

This document is articulated around the following parts:

- initially, one recalls the context of modal calculation in *Code_Aster* to through matrices and limiting conditions used, particular properties of modes clean exhumed and of the difficulties of estimate of the spectrum. One recapitulates also what it is necessary to know with minima about the spectral transformations, the families of modal algorithms and on the broad outline of the course of a modal calculation in the code,
- in the second time, one describes the method of the powers opposite and its phases preliminaries of localizations of eigenvalues (method of bisection and the secant) installations in **MODE_ITER_INV**,
- the third part treats the framework general of the methods of subspace installation in **MODE_ITER_SIMULT**. One details in particular the implications of the choice of the shift and the type of

subspace of projection in the paramétrage of the operator,

· the three following chapters show, in the order, the three methods of this operator:

Lanczos, IRAM and Bathe & Wilson,

· in the appendices one approaches more in details of the processes “of second level” to which make call three preceding methods. It is about the algorithms **QR** and Jacobi which are known as general practitioners because they allow to capture all the spectrum of an operator. The first, algorithm **QR**, is fundamental because it intervenes in the majority of the methods. It is the algorithm of reference which offers a very great robustness but calculation complexities and memory prohibitory. One approaches also the various algorithms of orthonormalisation put in

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place in the code. Indeed, one will cease hammering, throughout this document, only their quality and their robustness are crucial for the good unfolding of the algorithms.

This last version of the document was almost entirely rewritten while taking as a starting point the the indices

written precedents, respectively by D. Seligmann [R5.01.01] index A and B. Quinnez [R5.01.01]

index B, this in order to try to approach the code while clarifying more in details them

characteristics of the methods and the subjacent numerical phenomena. A particular effort was

brought to put in prospect the choices led in *Code_Aster* compared to research,

passed and current, like clarifying the general philosophy of a modal calculation.

One gives to the reader the properties and the limitations, theoretical and practical, of the modal methods approached while connecting these considerations, which can sometimes appear a little “éthérées”, to one

precise paramétrage of the operators. For each method, one recapitulates in the form of tables the aforementioned

paramétrage with its default values and of the references to the paragraphs of the document.

At the time of the first passages, one strongly engages the user has to modify only the parameters principal noted in fat in these tables and with reading the traces of the file message. Others, concerning more the mysteries of the algorithms, were initialized empirically with values standards.

One tried constantly to bind different the items approached and to limit to the bare minimum the recourse with long mathematical demonstrations. In any event, the many references which enamel the text must make it possible to seek precise information.

The object of this document is not to detail all the aspects approached, of the complete works having already fulfilled this mission. One will quote in particular F. Chatelin [bib3], G.H. Golub [bib6], P. Lascaux

[bib11], B.N. Parlett [bib18] and Y. Saad [bib32], and one recommend the synthesis particularly brought up to date and exhaustive made by J.L. Vaudescal [bib23].

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2 Context

2.1 Problems

We consider the problem generalized with the eigenvalues:

To find (λ, \mathbf{U}) such as $\mathbf{A} \mathbf{U} = \lambda \mathbf{B} \mathbf{U}$, $\mathbf{U} \neq 0$, where \mathbf{A} and \mathbf{B} are symmetrical matrices with coefficients realities. This type of problem corresponds, in mechanics, in particular with:

- The **study of the free vibrations of a** not deadened and nonrevolving **structure**. For this structure, one seeks the smallest eigenvalues or those which are in one interval given to know if an exiting force can create a resonance. In this case, stamp \mathbf{A} is the matrix of rigidity, noted \mathbf{K} , (possibly increased matrix of geometrical rigidity noted \mathbf{kg} , if the structure is prestressed) and \mathbf{B} is the matrix of mass or of noted inertia \mathbf{M} . **the** eigenvalues obtained are the squares of the associated pulsations at the sought frequencies.

The system to be solved can be written: $(\mathbf{K} + \mathbf{kg}) \mathbf{U} = \lambda \mathbf{M} \mathbf{U}$ where $\lambda = \omega^2$ is the pulsation,

ω the Eigen frequency and \mathbf{U} the vector of associated clean displacement.

- The **search for linear mode of buckling**. Within the framework of the linearized theory, in supposing a priori that the phenomena of stability are suitably described by system of equations obtained by supposing the linear dependence of displacement by report/ratio at the level of critical load, the research of the mode of buckling \mathbf{U} associated on this level with

critical load, is brought back to a problem generalized to the eigenvalues of the form:

$(\mathbf{K} + \mathbf{kg}) \mathbf{U} = \mathbf{0}$ with \mathbf{K} stamps rigidity and \mathbf{kg} stamps geometrical rigidity.

Note:

- *this type of generalized modal problem is treated in Code_Aster by two operators: `MODE_ITER_INV` and `MODE_ITER_SIMULT`. Each one having its perimeter of application, its functionalities and its limitations,*
- *in V5, the user can specify the class of membership of its calculation by initializing it key word `TYPE_RESU` with “DYNAMICS” (default value) or with “MODE_FLAMB”. The posting of results will then be formatted by taking account of this specificity. In the first case one will speak about frequencies whereas in the second, one will speak about critical load,*
- *in the presence of depreciation and of gyroscopic effects, the **study of dynamic stability** of a structure leads to the resolution of a modal problem of a nature higher, known as quadratic: $(\mathbf{K} + i \mathbf{C} - 2 \mathbf{M}) \mathbf{U} = \mathbf{0}$. It is solved by the two modal operators and is the object of a specific note [R05.01.02].*

Now that bonds between the mechanics of the structures and the resolution of modal problems generalized were recalled, we will be interested in the treatments of the conditions limit in code and with their incidences on the matrices of mass and rigidity.

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2.2

Taking into account of the limiting conditions

There are two ways, during the construction of the matrices of rigidity and mass, to take into account the boundary conditions (this description in dynamic term of problem is extrapolated easily with buckling):

· **double dualisation**, by using ddls of Lagrange [R3.03.01], makes it possible to check

$\mathbf{C} \mathbf{U} = \mathbf{0}$ (CLL for Linear Limiting Condition),

with \mathbf{C} stamps real of size $p \times N$ (\mathbf{K} and \mathbf{M} are of order N). Matrices of rigidity and of mass dualized have the form then

K
CT
CT
M 0 0
 ~

~

K = C - Id
Id
M = 0 0 0

C Id - Id

0 0 0

with and strictly positive realities which will be initialized to 1 without that involving a loss of general information.

The dimension of the problem was increased by $2p$, because to N ddls known as “physics”, one has added ddls of Lagrange. There are two ddls of Lagrange by linear relation assigned to p limiting conditions.

· The **setting has zero of p lines and columns of the matrices of rigidity and mass**. This is not valid that for blockings of ddls. One cannot take into linear account of relation and one will speak about kinematic blocking (CLB for Limiting Condition of Blocking). Matrices of rigidity and mass become:

~
K 0

~
M 0
K =

M =

0 Id
0 0

The dimension of the problem remains unchanged but it is however necessary to withdraw the participations of

ddls blocked with the components of the initial matrices (**K** is obtained starting from **K** in eliminating the lines and the columns from the ddls which are blocked; idem for **M**).

When limiting conditions are imposed, the number of eigenvalues (with all their multiplicities) really implied in physics (with the flat of modeling close (grid, frequency of rupture...)) phenomenon is thus lower than size N of the transformed problem:

$3p'$

·
 N
 $= N -$
with p'= 2 p
ddl - active
 (double dualisation),
 2
 ·
 N
 $= N - p$
ddl-credits
 (kinematic blocking).

Framed below the posting dedicated to these parameters in the file message shows.

 THE NUMBER OF DDL
 TOTAL EAST: 220 N
 LAGRANGE EAST: 58 $p'= 2p$
THE NUMBER OF ACTIVE DDL EAST: 133 *nddl_actifs*

Example 1: Management of the ddls
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*In addition, in the modal calculation algorithms, one must ensure oneself of the membership of the solutions with acceptable space. One brings back oneself there via auxiliary treatments. Thus when blockings are used kinematics (CLB), it is necessary in the various algorithms and for each iteration, to use a vector "of positioning " **ubloq**, defined by:*
 · *if*

*ième ddl is not blocked **ubloq** (I) = 1,*

· if not

***ubloq** (I) = 0,*

u1** (I) = **u0** (I). **U

(I) (I = ... N

bloq

*) **u1***

1

*If one uses the method of double dualisation, one needs a vector of positioning of the ddls for lagrange **ulagr** definite like **ubloq**. It is only used at the time of the choice of the random initial vector. So that this vector **u0** checks the conditions limit (CLL) one operates in the following way:*

u1** (I) = **u0** (I). **U

(I) (I = 1... N

lagr

*) **u2***

~

K u2** = **u1

Thereafter, to simplify the notations, we will not make the distinction between the initial matrices and theirs

*hanging dualized (noted with a tilde) that if necessary. Very often, they will be indicated by **A** and **B** in order to approach the usual modal notation without being attached to such or such class of problems.*

2.3

Properties of the matrices

*As we wrote previously the matrices considered are **symmetrical** and with **real coefficients**. According to the cases of figure indexed in the table below, they can be defined positive (noted > 0), semi-definite positive (0), indefinite (0 or 0) even singular (S).*

Structure

Lagranges *

Buckling

Fluid

free

structure

K

0 and S

< 0 or > 0

> 0

0 and S

M

> 0

0 and S

0 or 0

> 0

(resp. **Kg**)

Table 2.3-a: Properties of the matrices of the generalized problem

* This column relates to of course the properties of the matrices dualized made up to leave initial matrices [R3.03.01]

Columns of this mutually being excluded table, in practice, a problem of using buckling Lagranges doubles to model some of its limiting conditions, sees its dualized matrices

~
~

(**K** and **K G**) to become potentially indefinite.

Note:

This range of properties must be taken into account at the time of the choice of the couple operator of work -

(pseudo) produced scalar. This framework can thus reinforce, with effectiveness and transparency, robustness and the perimeter of the modal calculation algorithm in all the encountered cases of figure by Code_Aster.

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The following paragraphs go enable us to measure the incidence of these properties on the spectrum of generalized problem.

2.4

Properties of the clean modes

Let us recall first of all that if the matrix of modal problem standard $A U = U$ is real symmetrical, then its own elements are real; The clean elements of a matrix are its clean values and its vectors. In addition, **A** being normal, its own vectors are orthogonal. In the case of the problem generalized $A U = B U$, this condition is not sufficient. Thus, let us consider the following generalized problem:

1
1 U

1
0
U
1
1
=
1
0 U
0

- 1

U
2
2
1
1
- ±

its own modes are $\pm = (1 \pm I 3)$ and $u_{\pm} =$

2
1 + 2 1 .
±

If one adds the assumption “one of matrices **A** or **B** is **definite positive**”, then the problem generalized has its **real solutions**. There is even the characterization (sufficient condition) more precise following.

Theorem 1

Are **A** and **B** two symmetrical matrices real. If there exists and such as **A + B** that is to say definite positive, then the generalized problem has its real own elements.

Proof:

This result is obtained immediately by multiplying the problem generalized by and by carrying out one spectral shift. The problem then is obtained $(A +)$

$B U = (+) B U$. Like $(A +)$

B is definite positive, it admits a single decomposition of Cholesky in the form CCT with C stamps regular.

1
The problem is written **C B** then

1

C-T Z = μ **Z** with **Z** = **CT U** and $\mu = +$, which allows to conclude, because matrix **C-1B C-T** is symmetrical.

Notice

This characterization is not necessary, thus the generalized problem associated the matrices

With = *diag* (1, - 2, -)

1 and **B** = *diag* (- 2 1

, ,)

1 admits a spectrum real all while not answering condition of definite positivity.

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Proposal 2

If the matrices **A** and **B** are real and symmetrical, the clean vectors of the generalized problem are **With** and **B** - orthogonal, which means that they check the relations

C

B U

I

J = I

J has **J**

C A U

I

J = J I

J has **J**

where **J** has is a scalar depend on the standard of the *jième* clean vector, *ij* is the symbol of Kronecker and **U J** is the clean vector associated the eigenvalue **J**.

Proof:

Immediate for distinct eigenvalues, by writing \mathbf{A} and \mathbf{B} - product scalar between two couples (I, J) and (J, I) , then by using the symmetry of the matrices (cf [bib9]).

Note:

- *one shows that \mathbf{A} and \mathbf{B} orthogonalities of the clean vectors are a consequence of hermiticity of the matrices. They are clearly a generalization of the properties of the problem square standard (even normal),*
- *orthogonality compared to the matrices does not mean especially that the clean vectors are orthogonal for the **traditional euclidian norm**. This one can be only the fruit of particular symmetries (cf TP n°1 [bib25]),*
- *this property simplifies **calculations of modal recombinations** (DYNA_TRAN_MODAL [R5.06.01]), when one handles matrices of generalized rigidity and mass which are diagonals. The k_j quantities = $J a_j$ and $m_j = a_j$ are called, respectively, modal rigidity and modal j ème mode masses.*

Knowing that the modes are real, we now will worry we about their estimate.

2.5

Estimate of the real spectrum

*Because of membership of the spectrum to the real axis, the problems of **counting of eigenvalues** go to be largely **simplified**. One does not have to treat réionnement complex plan and one can to be based on the corollary of the law of inertia of following Sylvester.*

Corollary 3

Are \mathbf{A} and \mathbf{B} two real matrices symmetrical, \mathbf{B} being of more definite positive. The number of eigenvalues, strictly lower than, of the problem generalized $\mathbf{A} \mathbf{U} = \mathbf{B} \mathbf{U}$ is then equal to a number of strictly negative diagonal coefficients of the matrix \mathbf{D} such as $(\mathbf{A} - \mathbf{B}) = \mathbf{LDLT}$.

Proof:

Cf paragraph n°1 of the article of Y. Haugazeau [bib7].

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Remarks

- *this corollary extends to the square matrices and results from the properties of the series Sturm*

*(N)**and of the principle of inclusion (one notes p* *)**(N)**(N)**= die (your - **B**) the characteristic polynomial**(N) (N)**(N) (N) (N)*problem generalized **A****U***= **B** **U** obtained by removing N last**()**lines and columns of matrices **A** and **B**. the continuation of polynomials (p N) constitutes a continuation of* *N* *Sturm because the roots of the i ème polynomial frame those of $(i+1)$ the ème. This property**of interlacing of the spectrum of symmetrical real submatrices “principle is called**of inclusion”) [bib7], [bib9],*

- *the possible multiple eigenvalues are taken into account their multiplicity,*

- *thereafter, one will call modal position of and one will note it p_m (), this number of*

*strictly negative diagonal coefficients.**This corollary thus makes it possible to easily determine the number of eigenvalues contained in one interval $[\mu, \mu]$ and the modal position of these eigenvalues in the spectrum. It is enough to carry out two**decompositions **LDLT**, that of $(\mathbf{A} - \mathbf{B})$ and that of $(\mathbf{A} - \mu \mathbf{B})$ and to enter the difference**number of strictly negative terms between the two diagonal matrices. In the jargon of the code,**one indicates (improperly besides!) this test under the term of “**test of Sturm**”.**It must however be extended to the quite particular shapes of the matrices met in**Code_Aster, and in particular, it is necessary to be able to take into account buckling with or without**Lagrange.*

With this intention, one **widened the criterion with a matrix B unspecified** and one **generalized** it while taking in count the **dualized matrices**.

Let us recall that one defines usually the **signature** of a matrix (and that of the quadratic form associated) like the triplet of natural entirities (R, S, T) where R indicates the number of eigenvalues > 0 , S it a number of null eigenvalues and T concealment be < 0 . The latter entirety is thus what one notes $pm()$ in our case of figure.

Property 4

Are **A** and **B** the two real matrices symmetrical (of order N) related to the generalized modal problem

~

~

(S): **With $U = B U$** . Let us note **A** and **B**, their matrices associated resulting from the double dualisation with p

Lagranges making it possible to check $C U = 0$ (CLL), with **C** stamps real of size $p \times N$.

~

~

Then, the signature of $(A - B)$ is written, by noting card $[$,

$B]$ the number of values *has*

clean of the generalized problem included in the interval $[has, B]$:

if B is indefinite and A is definite positive

then card $\{ \}$

$0 = 0$ and

R

$= \text{card}] - , [+ \text{card} [, 0 + [$

if < 0 : $S = p + \text{card} \{ \}$

éq 2.5-1

T

$= ($

$pm) =$

card],]

$0 + p$

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R

= card] -, + [

if = 0: $S = p$

éq 2.5-2

T

= (

pm) = p

R

= card] -,]

0 + card], + [

if > 0: $S = \text{card } \{ \} + p$

éq 2.5-3

T

= (

pm) =

card [,

0 [+ p

if B is definite positive and if A is semi-definite positive

then card]

,

- [

0 = 0 and

R

= card], 0+ [

if = 0: $S = p + \text{card } \{ \}$

0

éq 2.5-4

T

= (

pm) = p

R

= card], + [

if > 0 : $S = p + \text{card } \{ \}$

éq 2.5-5

T

= (

pm) =

card [,

0 [+ p

Proof:

To impose boundary conditions linear, a technique of double dualisation is used

[R3.03.01] which leads to the transformed generalized system

WITH CT

CT

B 0 0 U

(~

~

-) ~ = **C - Id**

Id - 0 0 0 v = 0

(~

With

B U

S)

C

Id

- **Id**

0 0 0 W

J

One will note thereafter the **VI** null vector column of size p ($i= 1..p; j= 2,3$) except with index I , for which it is worth 1, and **0n**, the null vector column of size N . Now let us consider the three families of vectors following:

u1

I

~
.
 $N - p$ **V1** vectors = **v1**

I
 I which is the clean vectors of the system (S). Clearly,

v1i ~ ~
according to proposal 2, they are **A** and **B** - orthogonal (according to the properties of **A** and **B**) and one notes their eigenvalues I .

0
 N

.
 p independent vectors **V2** = **v2**

I
 I ,

v2

I
0
 N

.
 p independent vectors **V3** = **v3**

I
 I .

- **v3i**

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U

These $N + p$ vectors form a base B of space $E = \sim u = \mathbf{v}/\mathbf{C}\mathbf{u} = \mathbf{0}$ of the solutions

W

acceptable of the dualized problem (as a free family of a space of finished size). \sim

\sim

Let us consider, for a given real number, the quadratic form associated matrix $(\mathbf{A} - \mathbf{B})$

(\sim)

\sim

$\sim \sim$

$= (-), \sim, \sim$

U

With

B U U

U.E.

While breaking up on the basis B generated by the preceding vectors

\sim

U =

$aI \mathbf{V}$

1

2

2

3

3

I

I

+

V has

$II + has \mathbf{V}$

$II,$

$i=,$

1 $N p$

$i=,$

$1 p$

$i=,$

$1 p$

one obtains

\sim

T

$1 2$

1

1

$3 2$

$(U) = (A) (-) (U B U) + (- 4) (has$

I

I

I

I

I

I

$i=,$
 $1 N p$

$i=,$

$1 p$

J

(one used in particular the properties of orthogonality of the families of vectors \mathbf{VI} and the relation

$(\mathbf{u1}, \mathbf{CTv2}$

1

2

$I) = (\mathbf{C U}, \mathbf{v}$

I

I

$I) = 0$). From where, by noting L_i the linear form which associates \sim

\mathbf{U} its *ième*

coordinated in the base \mathbf{B}

\sim

\sim

T

2

1

1

2

\sim

2

\sim

$(U) = L_i (U) (I -) (U B U$

$$I$$

$$J) + 0 \text{ Ln-p+i (U) + (- 4) Ln+i (U)}$$

$$i=,$$

$$1 \text{ N p}$$

$$i=,$$

$$1 \text{ p}$$

$$i=,$$

$$1 \text{ p}$$

Clearly them (Li)

are linearly independent from where an immediate reading of the terms of

$i=,$
 1 n+ p
 the signature according to the sign of the factors. Moreover, it is noticed that this decomposition comprises

obligatorily p zeros and p minus signs. The relations of the property result all then naturally, by using the principle of inertia of Sylvester (who ensures us that this decomposition is invariant), the relations of **A** and **B** - orthogonality of property 2, the properties of the table [Table 2.3-a] and by noticing that:

$$\sim T \sim \sim$$

U With U

$$C A U$$

$$\sim T \sim \sim$$

$$I$$

$$J$$

$$I$$

$$J$$

$$U B U = C B U$$

$$I$$

$$J$$

$$I$$

$$J =$$

$$=$$

for J

$$0 .$$

$$J$$

$$J$$

Restriction 0 of the case of figure (2) comes owing to the fact that if **B** is definite positive then the spectrum problem (S) is positive and thus the shift with a strictly negative value does not have any interest (one find within the framework of application of corollary 3).

According to the table [Table 2.3-a] this property applies to the matrices handled by

Code_Aster and one can build the following corollary.

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Corollary 5 (theoretical wide Sturm)

In the matrix configurations of *Code_Aster*, the number of clean modes of the problem generalized (S) whose clean vector checks the linear conditions limiting (CLL) and of which the value clean is contained in the interval], μ [is

If $\mu > 0$

card

], [$= p ($

m

$) - p ($

m

μ

μ

$)$,

if not

card], [$= p ($

m

$) + p ($

m

μ

μ

$) - 2p$.

If no dualisation of Lagranges is required, $p = 0$ is posed.

Proof:

It is immediate by combining the results of the table with those of property 4, and, while noticing that the generalized modal configurations of *Code_Aster* can be only of two types:

- buckling

A is definite positive and **B** is indefinite, the spectrum can be negative and one uses relations of the type (1),

· dynamic

A is semi-definite positive and **B** is definite positive, the spectrum is positive and one can use the relations of the type indifferently (1) or (2).

Of course, if no dualisation of Lagranges is required, the same reasoning applies in

~

~

posing $p = 0$,

= ,

~

With

WITH $B = B$ and $U = U$.

To finish, the principle of inertia of Sylvester ensures us that the signature of factorization

(~

~

-) ~ ~ ~

With

B = LDLT is identical to that of the shifted matrix. In spite of this transformation, them

modal positions thus remain many perennial information.

However this corollary is tributary of arithmetic exact, in practice it is necessary to adapt it and control its application.

2.6

Establishment of the test of Sturm

This test is confronted into arithmetic finished with two concomitant problems:

· the factorization of the shifted matrix when is very close to an eigenvalue,

~

· the calculation of the strictly negative terms of **D** to evaluate the modal position

(

pm).

The first point requires, au préalable, the determination of a criterion of membership of the shift with spectrum of the problem. In *Code_Aster* that C_i is founded on the loss of decimals at the time of

~

~

~ ~ ~

factorization of matrix $(A - B) = LDLT$. More precisely, is regarded as being one eigenvalue, if during factorization one loses more decimal $NPREC_SOLVEUR$ (in any rigor it is only one test of bad numerical conditioning). It is then necessary to modify the value of in shifting this shift of $PREC_SHIFT$ % following the algorithm:

For $I = 1, NMAX_ITER_SHIFT$

If loss of more than decimal $NPREC_SOLVEUR$

then (1+ *PREC_SHIFT*),

If not

exit;

Fine buckles.

Algorithm 1: Shift of the shift

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If at the end of tentative *NMAX_ITER_SHIFT*, the matrix is still not numerically invertible, one continues all the same calculation with the last shifted value. Framed below shows trace of such a shift in the file message.

ONE MODIFIES THE VALUE OF SHIFT OF: 5.00000E+00POURCENT

THE VALUE OF SHIFT BECOMES: 5.96840E+04

ONE MODIFIES THE VALUE OF SHIFT OF: 5.00000E+00POURCENT

THE VALUE OF SHIFT BECOMES: 5.66998E+04

VALEUR_MIN IN FREQUENCY EAST: 2.00000E-01

VALEUR_MAX IN FREQUENCY EAST: 5.78813E+01

THE VALUE OF SHIFT IN FREQUENCY EAST: **3.78975E+01**

Example 2: Shift of the shift

In algorithm 1 if *F*

then = - *F*

corig

corig. This parameter *F corig* corresponds to a value

threshold in lower part of which it is considered that one has a numerically null eigenvalue (in traditional mechanics, that corresponds to a mode of rigid body, cf [§5.5.4]). The imposition

= - *fcorig* thus makes it possible to dissociate these modes of the remainder of the spectrum and to avoid instabilities

numerical during the test of Sturm. This threshold is paramètrable with key word *SEUIL_FREQ*.

Note:

· the preceding key words are accessible starting from the key word factor *CALC_FREQ* from both

modal operators,

· Y. Haugazeau [bib7] proposes more generally to deal with these problems of pivots numerically "small" by building a matrix, unitairement similar (i.e. similar via a unit matrix of passage (orthogonal in reality) to the shiftée matrix, of which factorization would have less instability.

By taking into account these elements and knowing that, numerically, the calculation of the pivots strictly negative of the diagonal matrix includes in fact also the elements (theoretically) null of the signature, one can rewrite the preceding corollary.

Corollary 5bis (numerical wide Sturm)

According to the assumptions of corollary 5, there is the numerical criterion of accountancy of the modes according to:

If $\mu > 0$

card $[,] = p ($

m

) - p (

m

μ

μ

),

if $\mu < 0$

card $[,] = p ($

m

) + p (

m

μ

μ

) - 4p

Proof (heuristic):

The fact is applied that numerically the operator of factorization provides the "modal position numerical "

(

pm) = S + T with property 4 and corollary 5. In addition, establishment of the criterion

neither the nullity of the product allows, nor the estimate of card { }.

Note:

This criterion is used in preprocessings in MODE_ITER_INV (options "ADJUSTS" or "SEPARATE") and

in MODE_ITER_SIMULT (option "BANDAGES"), in postprocessings to check the validity of the number

modes (MODE_ITER_SIMULT) and in auxiliary controls (IMPR_STURM).

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Now that we are able to enter the spectrum of the generalized problem, it remains with to determine! The generic algorithms being intended for the standard problems, it is necessary to transform our initial problem.

2.7 Transformation spectral

These techniques make it possible to answer objective triple:

- to exhume a standard modal problem,
- to direct the research of the spectrum,
- to separate the eigenvalues.

The spectral calculation algorithms converging of as much better than the spectrum (of work) which they treat

is separated, these techniques can be regarded as prepacking of the problem of departure. They make it possible to return the separation of certain modes much more important than those of other modes, and to improve their convergence thus.

Most widespread of these transformations is the technique known as of “**shift and invert**” which consists with

to work with operator **A** such as:

μ

-
With U = B U (A -)

B 1
1
B U =

U
!# "

\$
##

-
With

!"\$

μ

Appear 2.7-a: Effect of the “shift and invert” on the separation of the eigenvalues.

The figure [Figure 2.7-a] shows that this separation and this orientation of the spectrum of work in μ are had with the particular properties of the hyperbolic function. In addition, it is observed that only them eigenvalues are affected by the transformation. At the end of the modal process it is thus enough to to pass by again in the plan of by a suitable change of variable.

Note:

- the variable is usually indicated by the term of “shift” or spectral shift,
- the matrix of work **A** must of course be invertible, that can become one besides of motivations of this shift (cf [§5.5.1]).

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For memory, let us note that with a shift complexes several scenarios are possible:

- to work completely into arithmetic complex,
- into arithmetic real,
- mixer two steps by isolating the real and imaginary contributions from **A**

~

~

1 1

1

With =

(

Re With

) μ =

+
,

--

~

~

~

~

1 1

1

With =

(

Im A) $\mu =$

-

.
2i

--

Each one of its approaches has its advantages and its disadvantages. For the quadratic problems, it is for the moment the second solution which was adopted in the code.

Remarks

· *this choice of the operator of work is indissociable among that of (pseudo) - scalar product. It allows to direct itself towards such or such algorithm and can thus influence the robustness of calculation,*

· *another class of spectral transformation, with double shifts, makes it possible to select them eigenvalues located on the right vertical axis. It is about the rational transformation of*

-1

-

Cayley:

(A - B

2

1

) (A - B

2

) U =**U**

.

!### "

\$

#

- 1

With

!"\$

μ

We now will enumerate the various families of methods allowing to solve it standard modal problem.

2.8 Calculation

modal

The methods of calculation modal can gather in (at least) **four families**:

- The algorithms of the type **QR** (cf [Appendix 1]) which were presented per H. Rutishauser (1958) and formalized jointly by J.C. Francis and V.N. Kublanovskaya (1961). It is one fundamental algorithm often implied in the other methods.

Perimeter of use: Calculation of all the spectrum.

Advantages: Good convergence, robustness, calculation of the form of Schur (any matrix complex **A** admits the decomposition of Schur $\mathbf{Q}^* \mathbf{A} \mathbf{Q} = \mathbf{T}$, with **Q** and **T** of the matrices respectively, unit and triangular higher. In the real case, **T** is only diagonal by block 1x1 or 2x2 (real form of Schur). Columns of the matrix unit, known as of Schur, vectors of Schur are called).

Disadvantages: Prohibitory memory complexities and calculation, sensitivities to the “balancing (them techniques of balancing (English balancing) consist in transforming reversibly the operator of work so that its matrix standard decreases. Thus its handling will be less sensitive to the effects of round (cf [§10.3]))”.

Alternatives: With implicit shift or clarifies, simple or double...

- The algorithms of the **powers** type which were historically developed the first for to solve generic modal problems. These are basic algorithms of which them others are an improvement. They are implied in operator `MODE_ITER_INV` (cf [§3]).

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Perimeter of use: Calculation of the extreme values of the spectrum.

Advantages: Simplicity, very good estimate of the clean vector in some iterations.

Disadvantages: Convergence can become problematic, bad capture of multiplicities, of the clusters... (together of eigenvalues “very close”).

Alternatives: Powers opposite, (Bi) iteration of the quotient of Rayleigh (cf [§3.3.2])...
· Them

methods of subspace which consist in projecting the operator of work on a space H such as the spectrum of the projected operator is a good approximation of the part of initial spectrum which one seeks. These algorithms are the hard core of the operator MODE_ITER_SIMULT (cf [§4]).

Perimeter of use: Calculation of part of the spectrum.

Advantages: Reduction of the size of the problem and memory complexities and calculation, require only the calculation of a product matrix-vector and not the knowledge of all the matrix.

Disadvantages: Use the many pre one and postprocessings, convergence can become problems, more or less easily capture the multiplicities and the clusters according to alternatives.

Alternatives: Iterations of subspace, Bathe and Wilson (1971), Lanczos (1950), Arnoldi (1951), Davidson (1975), Sorensen (1992)...

· Them

other approaches more or less empirical and are specialized. They are often connected to other problems: search for roots of polynomials, functions unspecified... One can thus quote the method of bisection used in preprocessings in MODE_ITER_INV, but also that of Jacobi, Laguerre etc...

Note:

*Many parallels can be led between these families (method **QR** is not thus that a method of iterations of subspaces applied to entire space), but they also lead to processes similar to those developed for other problems:*

· *optimization; The method of the quotient of Rayleigh is with the method of the powers opposite, what the method of Newton is for a method of descent traditional,*

· *resolution of linear systems; The method of the combined gradient is a method of subspace for the positive definite symmetrical systems,*

· *search for roots of polynomials: the method of the powers is a method of Bernoulli applied to the matrix “companion” of the polynomial (cf [bib11] pp502/503).*

The following paragraph will synthesize what precedes in the total flow chart by resolution of one generalized modal problem of *Code_Aster*.

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2.9**Establishment in Code_Aster**

This one can break up into four phases:

1)

The first operation consists in **determining the shift** like certain parameters of the problem.

That is carried out in a more or less transparent way according to the option of calculation chosen by the user:

- MODE_ITER_INV + option: "SEPARATE" or the shift "ADJUSTS" is determined by the first phase of the algorithm and the number of clean modes sought by frequential bands (provided by the criterion of Sturm) is limited by NMAX_FREQ,
- MODE_ITER_INV + option: "NEAR" *the* shift is fixed by the user and the number of modes clean is equal to the number of shifts,
- MODE_ITER_SIMULT + option: "PLUS_PETITE" the shift is null and the number of modes is paramètré by NMAX_FREQ,
- MODE_ITER_SIMULT + option: "BAND" the shift is equal in the middle of the band fixed by the user and the number of modes are determined by the criterion of Sturm,
- MODE_ITER_SIMULT + option: "CENTER" *the* shift is fixed by the user and the number of clean modes is paramètré by NMAX_FREQ.

2)

In one second phase of preprocessings, one **factorizes partially the matrix of work****With** $= (\mathbf{A} - \mathbf{B}) - 1\mathbf{B}$ while being interested only in its reversed part. The action of this operator on an unspecified vector \mathbf{U} , noted $\mathbf{u2}$, will be built thus more effectively $\mathbf{u1} = \mathbf{B} \mathbf{U}$

(

 $\mathbf{u2}$ **With - B) $\mathbf{u2} = \mathbf{u1}$**

This factorization undergoes the same risks besides as the criterion of Sturm when the shift is near to an eigenvalue. One carries out the same shifts then according to algorithm 1.

Notice

In V5 when NMAX_ITER_SHIFT is reached calculation stops in fatal error (only for this preprocessing), this problem of factorization which can give place to numerical instabilities.

3)

Modal calculation itself is carried out: the standard problem is solved, then one returns to initial problem. In MODE_ITER_INV two alternatives are available: method of the iterations opposite (option: “**DIRECT**”) and its acceleration by quotient of Rayleigh (“**RAYLEIGH**”). For it who is MODE_ITER_SIMULT, it allows the use of three distinct methods: method of Bathe and Wilson (“**JACOBI**”), that of Lanczos (“**TRI_DIAG**”) and finally, that of Sorensen (“**SORENSEN**”).

Each one of these methods have internal tests of stops. Without counting that methods of projection employ auxiliary modal methods: Jacobi (cf [Appendix 3]) for the first and **QR/QL** (cf [Appendix 1]) for the others. They require also tests of stops.

The user often has access to these parameters, although it is warmly recommended, at least initially, to preserve their default values.

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4)

This last part gathers the **tests of total stops** which check the good unfolding of calculation. They are of two types:

1

2 2

· **Norme** (one recalls that **U**

U

= max **I**, **A**

max

With

=

ij, **U** =

U

2

I and

I

I

J

I

1

With = max *I*

(**AA**)

* 2

= max *I*

(**A**) if **A** square

2

residue of the initial problem

I

I

U

U U

If >

SEUIL_

FREQ then

With $U - B U^2 < THRESHOLD$,

With U^2

If not

With $U - B U$

? < *THRESHOLD*,

2

End if.

Algorithm 2: Test of the standard of the residue

This sequence is paramètrée by key words THRESHOLD and SEUIL_FREQ, pertaining respectively with the key word factor VERI_MODE and CALC_FREQ. In addition, it

postprocessing is activated by initialization with “YES” (default value) of STOP_ERREUR in the key word factor VERI_MODE. When this rule is activated and non-observance, calculation arrête, if not the error is just announced by an alarm. One would know of course only too to recommend not to decontaminate this parameter preferential treatment!

· **Comptage of the eigenvalues**

This postprocessing is set up in MODE_ITER_SIMULT and it makes it possible to check that it a many eigenvalues contained in a band test [1,2] are equal to the number detected by the algorithm. The inclusion of the initial band (in option “BANDAGES” it acts of values indicated by the user, if not they are the extreme values of the calculated spectrum) [I, F] in this band test is led in order to detect possible problems of clusters or of multiplicities at the initial boundaries (cf TP n°1 [bib25]).

+
1

I

2

-

I

F

F

Initial band

Bandage tested

Appear 2.9-a: Counting of the eigenvalues

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While noting +

-

I and F , respectively, largest and the smallest eigenvalue not requested by the user and including the initial band (cf [Figure 2.9-a]), one has the algorithm of construction of bandage following test:

+

-

$I - I$

$F - F$

If + exists

-

I

(resp.f) and if

$< PREC_SHIFT$ resp.

I

F

then

+

-

+

+

= **I**

I
 resp.
 1
 = F
 F,
 2

 2
 2

If not
 =
 1
 I (1 - S (
 ign I) *PREC_SHIFT*) (resp. =
 2
 F (1+ S (
 ign F) *PREC_SHIFT*),
 End if.

Algorithm 3: Construction of the band test

This sequence is paramètrée by key word *PREC_SHIFT* of the key word factor *VERI_MODE*.
 Framed below the trace of the error messages shows when these post-checkings
 start.

CHECKING A POSTERIORI OF THE MODES

<E> <MODE_ITER_SIMULT> <VERIFICATION OF THE MODES> FOR THE CONCEPT
 >MOD_4< IT

MODE NUMBER 2 OF CRITICAL LOAD 8.7243E+07 A a STANDARD Of ERROR
 OF 6.3919E-01

<E> <MODE_ITER_SIMULT> <VERIFICATION OF THE MODES> FOR THE CONCEPT
 >MOD_4< IN

The INTERVAL (- 1.0608E+08,1.8130E+08) IT THERE A THEORETICALLY 3 LOAD (S)
 CRITICAL (S) AND ONE OF A CALCULEE (S) 4.

Example 3: Post-checkings

The activation of the second postprocessing is subordinated to the initialization of STURM to "YES" in
 key word factor *VERI_MODE*. When this rule is activated and not respected, the continuation of the
 events

is controlled by *STOP_ERREUR* (so "YES" calculation stops, if not the error is just announced by one
 alarm). It would not be known of course that too much to recommend not to decontaminate these
 parameters 'passes

droit'!

Now that the context of the generalized modal problems of *Code_Aster* was brushed, us
 let us be interested more particularly in the methods of the power type and their establishment

in operator MODE_ITER_INV.

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3

Method of the powers opposite (MODE_ITER_INV)

3.1 Introduction

To calculate several eigenvalues of the generalized problem, the **general method is not used iterations opposite just as it is.**

One can, for example, to combine it with a **technique deflation** (technique of consistent filtering with to transform the operator of work so that it has the same eigenvalues except some preset modes which see themselves affecting a zero value) so as to filter automatically spectral information updated more to find it with the following iteration. With deflation by restriction (of other types of deflation exist, such as the method of Ducan-Collar which uses for to filter spectral information the first line of the matrix and the clean vector. These techniques vectorial spread of course per blocks) of Wielandt one builds the operator repeatedly of work (in the symmetrical case), by noting (μ_k , the **U.K.**) the mode to be filtered,

With

With

1

- μ

U C

$K + =$

K

K

K

K .

This strategy, which does not apprehend the multiplicities, must be supplemented by a criterion of Sturm.

In addition, the fact of working into arithmetic finished and of not building the operator indeed

With K with each iteration, constrained with *mâliner* this step of process of orthogonalisation powerful.

All these complications resulted in choosing another way, which breaks up into two parts:

- 1) localization of the eigenvalues (determination of an approximate value of each value clean contained in an interval given by a **technique of bisection**, refined or not, by a **method of the secant**),
- 2) improvement of these estimates and the calculation of their own vectors associated by one method of iterations opposite.

The search for a value approached for each eigenvalue considered is selected in key word factor CALC_FREQ via OPTION:

- if
OPTION = "SEPARATE", in each interval of frequencies defined by key word FREQ, an approximate value of each eigenvalue contained in this interval is calculated in using the method of dichotomy (cf [§3.2.1]),
- if
OPTION = "ADJUSTS", one carries out the same operations first of all as previously and then, on the basis of these approximations, one refines the result by the method of the secant (cf [§3.2.2]).

For these two options, one calculates at the same time the modal position of each value clean what makes it possible to detect the multiple modes. Either only the NMAX_FREQ are retained the lowest frequencies contained in the maximum interval specified by the user, is one calculates all the values of this interval (if NMAX_FREQ = 0).

- if
OPTION = "NEAR", the frequencies given by key word FREQ, are considered like the approximate values of the sought eigenvalues.

Note:

- *of course, as one already specified (cf [§2.8]), this operator is to be used only for to determine or refine some eigenvalues. For a wider research it is necessary to use operator MODE_ITER_SIMULT,*
- *with the "CLOSE" option one cannot calculate multiple modes.*
- *it is an expensive algorithm because it calls much upon the test of Sturm and thus with its associated factorizations.*
- *the terminals of the intervals of research are provided by FREQ or following CHAR_CRIT the initialization of TYPE_RESU.*

We now will detail the various algorithms (and their paramètrages) which are put in place in the first part of the process.

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3.2**Localization and separation of the eigenvalues****3.2.1 Method of bisection**

As one already saw previously, the corollary 5bis of the law of Inertia of Sylvester (cf [§2.5]) allows to determine the number of eigenvalues contained in an interval given while carrying out two decompositions **LDLT**. This criterion can thus result in refining the interval until not including that an eigenvalue. This piloting being set up, one passes from one iteration to the other by using it **principle of the dichotomy**.

On a starting interval $[has, B]$, one thus operates in the following way, knowing

(
pm has) and

(
pm b):

1
Calculation of $*$ = (+).

2
 has

B
Calculation of

(
pm $*$).

If
(
pm $*$) =

(
pm has) (resp. p (
m b) then

to start again on $[*, B]$ (resp. $[, a^*]$),

If not
to start again on $[, a^*]$

and on $[*, B]$,

End if.

Algorithm 4: Method of Bisection

The process is stopped if one cut out more NMAX_ITER_SEPARE time the interval of departure, or if

- *B has*

for a given interval, one has
SEPARATE PREC

*

—
(in this case one does not refine any more
seek in this interval).

One obtains finally a list of frequencies. In each interval defined by the arguments of this list, is an eigenvalue having a certain multiplicity. Like approximation of this value clean, the medium of the interval is taken.

Note:

· *one could have used as criterion the change of sign of the characteristic polynomial, but in addition to the fact that it is very expensive to evaluate, it does not make it possible, just as it is, to detect them*

multiplicities,

· *the initialization of the process can be carried out in an empirical way according to the needs for the user. To include part of the spectrum one can also use the régionnements plan complexes theorems of Gerschgorin-Hadamard (on \mathbf{A} , $\mathbf{A} T \dots$). Accordingly, the method of bisection can prove more effective than a **QR** in the presence of cluster. Its convergence, although linear, is indeed raised by $1/2$ whereas that of **QR** can tend towards 1 [bib11].*

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has

*

B Interval of will tra

the 1st division

p ()

has

has

*

B

*

B

the 2nd division

p ()

has

B

B

has

B

has

*

*

*

the 3rd division

p ()

has

B

has

B

has

B

has

*

*

*

*

B

the 4th division

Appear 3.2.1-a: Method of bisection

These approximated eigenvalues can be improved by a unidimensional minimization seeking to cancel the characteristic polynomial (p) = $\det(\mathbf{A} - \mathbf{B})$. But only the calculation of one value of this determinant requires $NR!$ operations.

3.2.2 Method of the secant

The method of the secant is a **simplification of the method of Newton-Raphson**. With the stage unspecified K , knowing a value K and by approximating the nonlinear function (p) by its

tangent in this point, one determines the next value $k+1$ as being the intersection of this line with the axis of, and so on, according to the iterative diagram

 $p()$ $K -$ $K - 1$ $K + 1 = K - ($ $p(K) ($ p $K) - ($ $p^{k-1})$ $k-1$ $k+1$ K **Appear 3.2.2-a: Method of the secant**

The tangent being approximated by a difference finished in order not to have to calculate of derived from (p), only the estimate of the polynomial is necessary.

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$k+1 - K$

It is considered that one reached convergence when

<

PREC_ADJUSTS and, in addition, one

K

limits itself to *NMAX_ITER_AJUSTE* iterations if this criterion is not reached.

Note:

This method has a quadratic convergence when it is close to the solution, in the case opposite, it can diverge. From where interest to combine the method of bisection with this approach.

We now will detail the algorithm of the powers opposite (coupled to an acceleration of Rayleigh) constituting the second part of the process.

3.3

Method of the powers opposite

3.3.1 Principle

To determine the eigenvalue of the problem generalized $\mathbf{A} \mathbf{U} = \mathbf{B} \mathbf{U}$ nearest in module to μ , one applies the method of the powers to operator $(\mathbf{A} - \mu \mathbf{B})^{-1} \mathbf{B}$.

$\mathbf{B}) \mathbf{B}$. In fact, one only builds

the factorized matrix (this notation should not be confused that symbolizing the “shift and invert”

WITH $= (\mathbf{A} - \mu \mathbf{B})^{-1} \mathbf{B}$

)

$\mathbf{B}^{-1} \mathbf{B}$

) $\mathbf{A} = (\mathbf{A} - \mu \mathbf{B})^{-1} \mathbf{B}$

\mathbf{B} and that amount dealing with the generalized problem

(-1)

1

With) Drunk =

\mathbf{U}

-

. Method of the powers opposite converging proritainment towards

μ

!"\$

eigenvalues of stronger module (in μ), one will thus capture the closest to the shift.

The principle is as follows, knowing an estimate of the sought eigenvalue and on the basis of one

initial vector standardized \mathbf{y}_0 , one builds a clean vector series approximate (\mathbf{y}_k) by

K

recurring formula

For F

$K = 1, \dots$ anger

~

With $\mathbf{y}_k = \mathbf{B} \mathbf{y}_{k-1}$,

~

$\mu_k = \mathbf{y}_k$,

~

\mathbf{y}_k

$\mathbf{y}_k =$

,

μ_k

Fine buckles.

Algorithm 5: Method of the powers opposite

With 1, the 2... first eigenvalues (of clean vector \mathbf{u}_i) closest in module to,

it is shown that one has a linear convergence of \mathbf{y}

\mathbf{U}

K

1 and one quadratic convergence of

1

~

$\mathbf{y}_k (I)$

μ

1

K

and

$I = 1. \dots n$ (the factor of convergence of these continuations is of

1 -

$\mathbf{y}_{k-1} (I)$

(

)

1 -

1 -

the order of

).

min $I -$

iI

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Note:

- *this result is not acquired (with the factor of convergence near) only when the eigenvalue dominant (here that nearest in module to the shift) is single. In the contrary case, results remain however possible, even in the complex framework, but the analysis rigorous of the convergence of this algorithm is still incomplete [bib6], [bib23],*
- *in theory, these results require that the initial vector is not orthogonal with clean subspace on the left required. In practice, the rounding errors avoid it problem (cf [bib11] pp500-509),*
- *even if the estimate of the eigenvalue is coarse, the algorithm provides one quickly very good estimate of the clean vector,*
- *the major disadvantage of this method is that it is necessary to carry out a factorization of **A** for each eigenvalue to calculate.*

In general one works in euclidian norm or infinite standard, but to facilitate calculations post-modal one seeks here **B-to standardize the** clean vectors (when **B** is indefinite, one works with the associated pseudo-standard). The basic algorithm can be rewritten while posing $\mathbf{Z} = \mathbf{B}\mathbf{y}$

0

0

For $K = 1 \dots$ to make**With $\mathbf{y}_k = \mathbf{z}_{k-1}$,** **$\mathbf{z}_{\sim k} = \mathbf{B} \mathbf{y}_k, T.$**

(

y**Z****y****K****K 1****K) =**

-

y^T. z_~,**K****K**

T

~

 $K =$

(

 $sign \mathbf{Y} \cdot \mathbf{Z}$ K

),

 $\mathbf{z} \sim$ \mathbf{Z} K $K = K$

,

1

 $\mathbf{y}^T \cdot \mathbf{z} \sim 2$ K K

Fine buckles.

Algorithm 6: Method of the powers opposite with B-pseudo-standard

1

Let us note that (\mathbf{y}^k) . This presentation avoids the products matrix-vector by the matrix \mathbf{B}

1 -

during the calculation of the scalar products and already the coefficient of Rayleigh of the following paragraph precedes.

It is observed that the factor of convergence is all the more small as the spectral shift is close

sought eigenvalue and thus which $\mathbf{A} - \mathbf{B}$ is close to the **singularity**. That is not in fact **not****prejudicial** with the process because the error made by solving the system is “mainly” in

direction generated by the clean vector which is the sought direction. That means that at the time of

resolution of $\mathbf{A} \mathbf{y} = \mathbf{Z}$ K $K - 1$, one does not find the solution exact \mathbf{y}^K but only the rounding errors

lead to solution close to the form ~

 $\mathbf{y} = \mathbf{y} + \mathbf{W}$ K K

. This one is proportional to the solution

exact, but as standardization is arbitrary, all happens correctly [bib18].

Note:*This bad conditioning, far from having an unfavourable effect, improves even convergence of the algorithm.*

This algorithm is thus used to improve the clean vector associated with the value approximated with phase 1. To refine this estimate of the eigenvalue, a quotient of Rayleigh is introduced.

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3.3.2 Method of iteration of the quotient of Rayleigh

Let us recall that the **quotient of Rayleigh** applied to the generalized problem is defined by the number $R(\mathbf{X})$, with \mathbf{X} a vector not no one of N , such as:

$\mathbf{x}^T \mathbf{A} \mathbf{X}$

$R(\mathbf{X}) = \frac{\mathbf{x}^T \mathbf{A} \mathbf{X}}{\mathbf{x}^T \mathbf{B} \mathbf{X}}$

This quotient has the remarkable property of stationnarity in the vicinity of any clean vector and to reach a **extremum** (local) which is the corresponding **eigenvalue**: for each \mathbf{X} fixed, $R(\mathbf{X})$ minimize $(\mathbf{A} - \lambda \mathbf{B}) \mathbf{X}$.

2

What we can translate by "if \mathbf{X} is an approximation of a clean vector of the system

With $\mathbf{X} = \mathbf{B} \mathbf{X}$, then $R(\mathbf{X})$ is an approximation of the eigenvalue associated vector \mathbf{X} ", and

reciprocally, we saw that if one had a good estimate of an eigenvalue,

method of the iterations opposite made it possible to obtain a good estimate of the clean vector corresponding.

From where the idea to combine these two properties by considering the algorithm of iteration reverses with

spectral shift for which one revalues, with each iteration, the eigenvalue via the quotient of Rayleigh. One then obtains the algorithm known as of iterations of the quotient of Rayleigh (in B-pseudo-standard)

For $K = 1 \dots$ to make

$(\mathbf{A} - R(\mathbf{y}_{k-1})) \mathbf{B} \mathbf{y}_k = \mathbf{z}_k$, 1

$\mathbf{z}_k = \mathbf{B} \mathbf{y}_k$,

$R(\mathbf{y}_k) = \frac{\mathbf{z}_k^T \mathbf{y}_k}{\mathbf{y}_k^T \mathbf{B} \mathbf{y}_k}$

$R(\mathbf{y}_k)$

\mathbf{y}_k

\mathbf{z}_k

\mathbf{y}_k

K
 K 1
 $K) =$
 $- +$
 T
 \sim
 $R(\mathbf{y}_k - \mathbf{1}),$
 $\mathbf{y} \cdot \mathbf{Z}$
 K
 K
 T
 $K =$
 $($
 $\text{sign } \mathbf{Y} \cdot \mathbf{z} \sim$
 K
 $K),$
 $\mathbf{z} \sim$
 \mathbf{Z}
 K
 $K =$
 $,$
 K
 1
 $\mathbf{y}^T \cdot \mathbf{z} \sim 2$
 K
 K

Fine buckles.

Algorithm 7: Method of iterations of the quotient of Rayleigh with B-pseudo-standard

For the standard modal problem, one can show [bib18] that the convergence of this algorithm is

cubic if the operator of work is normal (a matrix \mathbf{A} is known as normal if $\mathbf{A}\mathbf{A}^* = \mathbf{A}^*\mathbf{A}$).

It is thus the case of the operators square, antihermitiens or unit) (a fortiori in the case

square) and at best quadratic in the other cases.

If one used this method without modifying it, it would be necessary for each iteration of the process of improvement

of each eigenvalue, to carry out a factorization **LDLT**, which would be very expensive. From where the idea of

not to even carry out (this strategy of the coupling of methods to the complementary characteristics

antagonists is often used in numerical analysis. For example, in optimization, the method of

Levenberg-Marquadt couples one steepest-descent and a Newton) this shift of Rayleigh, that if one

is in a vicinity (arbitrary concept to define) solution.

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Notice*Within a framework plus general, certain authors introduced an algorithm known as “of Bi-iterations of T***y With X***quotient of Rayleigh”. Based on the stationnarity of the Bi-quotient $R(\mathbf{X},$* *B**y) = T**in the vicinity***y B X***clean vectors on the right and on the left, it provides (in nonsquare) the two types of vectors**clean. Its cost is however crippling because it requires twice more factorizations**(cf B.N. Parlett, 1969 [bib18]).***3.3.3 Establishment in Code_Aster**

This spectral shift is activated only if the key word OPTION of the key word factor CALC_MODE is initialized

in “RAYLEIGH”. By defect, one has “DIRECT” and the shift is then traditional (total correction rather than progressive of the eigenvalue). The algorithm set up in the code cuts out as follows

(in B-pseudo standard):

- Initialisation of the eigenvalue starting from the estimate of the first phase: .
- Calcul of an initial vector \mathbf{y}_0 random checking the limiting conditions.
- **B** - orthonormalisation of \mathbf{y}_0 compared to the modes previously calculated (if it is a mode multiple according to the first phase) by Gram-Schmidt Modified (since the development of this operator, more effective and more robust methods of orthogonalisation are spread, such as Gram-Schmidt Modified Iterative (IGSM) used in MODE_ITER_SIMULT (cf [Appendix 3] and B.N.Parlett [bib18])) (GSM).

·

T

Calculation of $\theta = \text{sign}(\mathbf{y}_0^T \mathbf{B} \mathbf{y}_0)$.

- For $k=1,$
NMAX_ITER to make

To solve (A -

B) $y_k =$

B y

$K - 1$

$K - 1$.

B-orthonormalisation (possible) of y_k .

T

$y^T B y$

Calculation of the correction of the eigenvalue C

K

K

=

-1 .

T

$y_k^T B y_k$

If $y^T B y$

K

K

$1 \text{ } PREC$

-1 -

then

=

+ C ,

exit;

If not

If $OPTION = \text{"RAYLEIGH"}$ and if C

01

. then

=

+ C ;

End if.

End if.

Fine buckles.

Algorithm 8: MODE_ITER_INV

The standard of maximum error acceptable $PREC$ and the maximum number of authorized iterations

$NMAX_ITER$

are arguments of the key word factor $CALC_MODE$.

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Note:

- the clean vector **B-being normalized**, one considers to have reached convergence when the value absolute of the unsuitable scalar product is close to the unit,
- to avoid taking an **B-orthogonal** initial vector with the sought eigenvalue one uses a method of pulling random for the components of this vector,
- in addition to be able to determine multiple or close modes, one uses one **B-orthogonalisation** with the already calculated modes,
- the option “of acceleration” of the algorithm by quotient of Rayleigh being expensive, it is not used with each iteration that if one is in the vicinity of the required eigenvalue.

3.3.4 Posting in the file message

In the file message, the results are posted in the form of table

Position

Frequency Deadened

Dichotomy

Method

Secant

Method

Opposite

modal

sow

Numbers

Numbers

Precision Numbers

Precision

iterations

iterations

iterations

*

*

0.

*
*
*
*
***Table 3.3.4-a: Trace MODE_ITER_INV in the file message**-----
THE NUMBER OF DDL

TOTAL EAST: 192

LAGRANGE EAST: 84

THE NUMBER OF ACTIVE DDL EAST: 66
-----**CHECKING OF THE FREQUENCY SPECTRUM (CONTINUATION OF STURM)**The NUMBER OF FREQUENCIES IN the BAND (1.00000E-02, 6.00000E-02) IS 4
-----**MODAL CALCULATION: METHOD OF ITERATION OPPOSITE****OPPOSITE SECANT DICHOTOMY**

NUMBER FREQUENCY (HZ) AMORT NB_ITER/NB_ITER/PRECISION/NB_ITER/PRECISION

4 1.97346E-02 0.00000E+00 4 6 2.97494E-07 4 1.22676E-07

5 2.40228E-02 0.00000E+00 4 5 4.21560E-05 3 4.49567E-09

6 4.40920E-02 0.00000E+00 3 5 2.19970E-05 3 2.62910E-09

7 5.23415E-02 0.00000E+00 3 5 2.34907E-07 5 1.32212E-07
-----**CHECKING A POSTERIORI OF THE MODES**
-----**Example 4: MODE_ITER_INV**

With the “CLOSE” option, the columns “Dichotomy” and “Secant” do not appear, while with the option “SEPARATED”, only the “Secant” column disappears. The last column precision gathers

intermediate data and does not represent, as for the other modal operator

MODE_ITER_SIMULT, the standard of error of the residue. It is an artifact which will be brought to disappear.

Let us recapitulate now the paramètrage of operator MODE_ITER_INV.

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3.3.5 Summary of the paramétrage

Key word factor

Key word

Default value

References

“DYNAMIC” TYPE_RESU

“DYNAMIC”

[\$2.1]

“MODE_FLAMB”

[\$2.1]

CALC_FREQ

FREQ

[\$3.1]

CHAR_CRIT

[\$3.1]

OPTION “SEPARATE”

“ADJUSTS”

[\$2.1]

“ADJUSTS”

[\$3.1]

“NEAR”

[\$3.1]

NMAX_FREQ

0

[\$3.1]

NMAX_ITER_SEPARE

30

[\$3.2.1]

PREC_SEPARE

1.E-04

[\$3.2.1]

NMAX_ITER_AJUSTE

15

[\$3.2.2]

PREC_AJUSTE

1.E-04

[\$3.2.2]

NPREC_SOLVEUR

8

[§2.6]

NMAX_ITER_SHIFT

5

[§2.6]

PREC_SHIFT

0.05

[§2.6]

SEUIL_FREQ

1.E-02

[§2.9]

CALC_MODE

OPTION “DIRECT”

“DIRECT”

[§3.3.3]

“RAYLEIGH”

[§3.3.3]

PREC

1.E-05

[§3.3.3]

NMAX_ITER

30

[§3.3.3]

VERI_MODE

STOP_ERREUR “YES”

“YES”

[§2.9]

“NOT”

THRESHOLD

1.E-02

[§2.9]

Table 3.3.5-a: Summary of the paramètrage of MODE_ITER_INV

Note:

- *in V5, the user can specify the class of membership of its calculation by initializing it key word TYPE_RESU. According to this value, one informs vector FREQ or CHAR_CRIT,*
- *one finds all the “tripaille” of parameters related to the preprocessings of the test of Sturm (NPREC_SOLVEUR, NMAX_ITER_SHIFT, PREC_SHIFT) and with postprocessings of checking (SEUIL_FREQ, VERI_MODE),*
- *at the time of the first passages, it is strongly advised to modify only them principal parameters noted in fat. The others relate to more the mysteries of the algorithm and they were initialized empirically with values standards.*

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4

Method of subspace (MODE_ITER_SIMULT)

4.1 Introduction

If one **only** wishes to calculate the **p** clean elements of a generalized problem of order N where $N \ll p$ (for example, p smaller eigenvalues or all eigenvalues included/understood in a given interval), one saw that it was preferable to have resorts to methods of subspace. They are based on the analysis of Rayleigh-Ritz which consists in effectively projecting the problem considered

on under space of dimension m ($p < m < N$) and to seek certain clean elements of it problem projected (much easier to treat) using robust algorithms (**QR** or **QL** for Lanczos and IRAM, Jacobi for the method of Bathe and Wilson).

The criteria of effectiveness of the aforesaid projection are:

- small size of the space of projection (directly related to calculation complexities and memory),
- facility of its construction,
- the robustness of orthogonal projection (in the nonsquare case, of oblique projections were developed by F. Chatelin [bib2],
- the setting in the canonical shape of the projected matrix,
- the good approximation of the part of the initial spectrum sought by that of the projected operator,
- and, of course, the minimization of calculation complexities and memory and that of the effects of round-offs

(those are especially related to the problems of orthogonality raised by the 3rd point).

One will first of all present the analysis of Rayleigh-Ritz before detailing three methods resulting from this

analyze: method of Lanczos, that known as of Sorensen (WILL GO) and that of Bathe and Wilson.

4.2

Analyze of Rayleigh-Ritz

Let us consider the standard modal problem of order N , $\mathbf{A} \mathbf{U} = \mathbf{U}$, and the subspace H of N generated by a orthonormée base ($\mathbf{Q}_1, \mathbf{Q}_2, \dots, \mathbf{Q}_m$).

The latter constitutes the matrix

orthogonal Q

T

allowing to define the operator of projection P

$$= Q Q^T$$

m

m

Mr. method of

Galerkin used by this analysis consists in solving the following problem

m

$$= m$$

\times

(

\sim

\sim

\sim

$$U Q X,$$

,

X

, $\sim u$)

With

to find (

)

such as

To find

$\times H$ such as

*

\sim

Q WITH

Q X

m

m

$$= X$$

P

$m (\sim \sim \sim$

With - U) = 0

! "
\$

B

m
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~
The research of the elements of Ritz (\sim

U) which thus interest cost to us to find the modes
clean of the matrix of Rayleigh **B = QT AQ**

m
m

Mr. the eigenvalues remain unchanged in
two formulations, on the other hand knowing a clean vector **X** of **Bm** one must go up with space all
entirety via the transformation \sim

$$\mathbf{U} = \mathbf{Q} \mathbf{X}$$

m

- . The step can thus be summarized in the form:
- Choix of a space H and a base (**h1, H2, ..., hm**) represented per **H**.
- Orthonormalisation of the base

R
0

· Matrice of Rayleigh

N
X
H
=

Qm

m

~

· Clean Elements of **Bm**: (\cdot , **X**)

~

· Elements of Ritz: (\cdot , ~**U = Q X**

m),

~

· Test of error with the residue: **R = Au~ - u~**

.

Algorithm 9: Procedure of Rayleigh-Ritz**Note:**

- the elements of Ritz are in fact the clean modes of the matrix (of order NR) of the approximation of Galerkin **Cm = Pm A Pm**,
- calculation complexity of this process, the order at worst of $O(m^2(4n+m))$ (much less with a good space, cf Lanczos and IRAM), are without common measurement with that of a good **QR** ($O(N^2)$) or that of an iteration of the quotient of Rayleigh encapsulated in a process heuristics (the cost of factorizations is prevalent on that of the products matrix-vector algorithm itself) such as that developed in *MODE_ITER_INV* ($\gg pn3$). But it is quite clear that these methods are more complementary than concurrent.

To consider the error made by using the clean elements of the **Bm** matrix there is the result according to.

Theorem 6

That is to say (\cdot , **U**) a clean element of the matrix diagonalisable (it is the case of the normal matrices and of

nondefective matrices (matrices of which all the eigenvalues have even arithmetic multiplicity and

~

geometrical) **A** and, **Bm** the matrix of Rayleigh associated, then this one admits an eigenvalue checking

, **U**

(-

~

IP**(I - P**m) **U**m) **U**

-

2

P U 2**Pmu**

m

2

H

~ ~

where is a number dependent on, **A** and **P**

, **U**

Mr.

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In the square case the numerator of this increase is squared. The associated clean vector, ~

U,

check as for him

(I - P) U

(

sin, ~

U U)

m

2 .

P U

m

2

Proof:

Cf [bib11] pp531-537.

It is shown that for *m* rather large, can be raised by a number which does not depend that on **A** and on min *I* -. The estimate of the second member **(I - Pm) U** depends on the choice of the subspace.

J

2

I J

Note:

The number min I - (and its alternatives) is omnipresent in the analyses of errors, of

*J**I J*

convergence or of conditioning spectral. One sees, once more, the importance of separation of spectrum of work on the good numerical behaviour of the algorithms.

Of course, to take account of spectral information already obtained, even to improve it or filter it, one reiterates this procedure of Rayleigh-Ritz by modifying the subspace of projection. One will connect then the construction of this new space (recurring of the precedent) and this process of projection. Two types of spaces (each one attached to distinct methods) are most often used.

4.3

Choice of the space of projection

Two choices of space of projection H are most often set up:

· The first, that of the **method of Bathe and Wilson** (cf [§7] METHOD = "JACOBI"), consist in choosing under space H of dimension m then to build successively:

$$H = A H$$

1

$$H = A H = A^2 H$$

2

1

...

H*I*

$$I = A H I - = A H$$

1

This method, which holds at the same time of generalization in form block of the method of powers and of the truncation of the algorithm **QR**, conduit with an impoverishment of space of work: $\dim H \dim H$

I

i-1. Not to always find the same clean mode

dominating it is necessary to insert in the process a reorthogonalisation (with all the problems of calculation complexity and round-offs that that implies).

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· The second, that of the **method of Lanczos** (cf [§5], METHOD = "TRI_DIAG") and of **IRAM** (cf [§6], METHOD = "SORENSEN") starting from an initial vector \mathbf{y} consists, to build continuation of spaces of Krylov (H_i) $i = 1, m$ where H_i is the space generated by the family of vectors $(\mathbf{y}, \mathbf{A}\mathbf{y}, \dots, \mathbf{A}^{i-1}\mathbf{y})$. This last is called the space of Krylov of \mathbf{A} and order i initiate by \mathbf{y} :

$H = K(\mathbf{A}, \mathbf{y})$ Vect

$i-1$

i

i

$(\mathbf{y}, \mathbf{A}\mathbf{y}, \dots, \mathbf{A}^i \mathbf{y})$.

They check the following property: $\dim H_i \leq \dim H_{i+1}$

i

$i+1$. Contrary to the preceding choice, one

thus a certain enrichment of the workspace has. In addition, it will be seen that they allow to project in an optimal way within the meaning of the criteria defined previously.

Historically, the first type of space was very much used in mechanics of the structures. But for to decrease calculation complexity related on the size of the subspaces and the orthogonalizations, one prefers to them

from now on methods of the Lanczos/Arnoldi type.

Note:

*One meets an impressive variety of algorithms using a space of the first type. They are called "iterations of subspace", "iterations orthogonal" or "iterations simultaneous". Beyond the various terms, it should especially be retained that these adaptations the strategies of restartings (technique concern consisting in starting again an algorithm modal with an initial vector comprising spectral information already. Typically, one chooses a linear combination of the clean vectors or already exhumed vectors of Schur (cf [§5.4.2]), techniques of acceleration (technical consisting in replacing the matrix of work **With** by a matrix polynomial $\mathbf{P}(\mathbf{A})$ which with the characteristic to be of great amplitude in the areas spectral of interest (cf [§6.4]) and of factorization implemented to enrich the workspace. There are even versions based on the powers opposite making it possible to calculate p more small modes (cf [bib11] pp538-45, [bib23] pp49-54).*

For these methods of projection, by supposing that initial space H is not too poor according to the p dominant directions, the factor of convergence of the i ème mode When they are arranged classically, i.e. by order descending of module (when they are classically arranged, i.e. by order descending of module) at the end of K iterations is written:

K

O

$m+$

1

.

I

It expresses two phenomena clearly:

- the priority convergence of the dominant modes,
- improvement of these convergences (and thus of their standards of error for a number iterations fixed) when the size of the subspace increases.

To transform the modal problem generalized into a standard problem one A resorts to spectral transformations. They also make it possible to direct the research of the spectrum and to improve convergence (cf [§2.7]).

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4.4

Choice of the spectral shift

To calculate the smallest eigenvalues close to a given frequency or all the values clean included/understood in a given frequency band, one carries out a spectral shift of generalized problem. That is to say the value of shift, one transforms initial problem $\mathbf{A} \mathbf{U} = \mathbf{B} \mathbf{U}$ into a shifted standard problem.

With $\mathbf{B} \mathbf{U}$

$1 = \mu \mathbf{U}$ with $\mathbf{A} = \mathbf{A} - \mathbf{B}$ and $\mu = -$

This transformation spectral, known as of “simple shift”, is used in the method of Bathe and Wilson. As the process detects the smallest eigenvalues in μ , those gradually are captured who are closest to (in module).

One carrying out the reverse transformation (“shift and invert”) it occurs

1

With $U = U$

μ with $A = (A -$

-1

B) B

and $\mu = -$

It is the problem dealt with with Lanczos and IRAM which makes it possible to calculate the greatest eigenvalues

μ thus the eigenvalues closest to the shift (in module).

In order MODE_ITER_SIMULT, there are three ways of choosing this shift (for buckling, transposition at the levels of critical loads is immediate):

.

= 0, one seek the smallest eigenvalues of the starting problem. This corresponds to

OPTION = “PLUS_PETITE” under the key word factor CALC_FREQ.

2

.

= 0 with $0 = ($

$2 f_0)$, one seeks the frequencies close to the frequency $FREQ = f_0$.

(OPTION = “CENTER”).

$0 +$

2

2

.

=

1 with $0 = ($

$2 f_0)$ and $1 = ($

$2 f_1)$, one seeks all the frequencies

2

included/understood in the band $[F, F$

0

$1]$ (OPTION = “BAND” with $FREQ = \{f_0, f_1\}$).

The number of frequencies to be calculated is given in general by the user using NMAX_FREQ under the key word factor CALC_FREQ. But for the option “BANDAGES”, it is automatically given in using the corollary 5bis (cf [§2.6]).

Note:

It is pointed out that the factorization of the matrix of work, just like the tests of Sturm of the option “BAND” are dependent on numerical instabilities when the shift is close to an eigenvalue.

Palliative treatments, paramétrables by the user, were established (cf [§2.6], [§2.9]).

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5**Method of Lanczos (METHOD = "TRI_DIAG")****5.1 Introduction**

This algorithm, developed at the point by **Lanczos** [bib10] **in 1950**, was hardly used until the medium of

the Seventies. First of all very simple and effective, it is the seat nevertheless **of great instabilities numerical** being able to cause the capture of phantom modes! Those are dependent mainly on **problems of maintenance of orthogonality** enters the vectors generating the space of Krylov. comprehension of this type of behavior vis-a-vis arithmetic finished computers was long with to exhume.

Since, many palliative solutions were born and an abundant literature covers the subject.

Let us quote for example the work of J.K. Cullum [bib3] and AI which is entirely devoted to him, that of B.N. Parlett [bib18] and the brought up to date and exhaustive synthesis of J.L. Vaudescal [bib23] (pp55-78).

In the continuation of this paragraph, we first of all will delay we on the theoretical framework of operation of the algorithm. Then, before detailing the alternative installation in *Code_Aster*, we will stick to the realistic framework of arithmetic finished.

5.2**Theoretical algorithm of Lanczos****5.2.1 Principle**

Its perimeter of application covers the couples operator with work (pseudo) produced scalar ensuring **the hermiticity** of **A**. It makes it possible to repeatedly build a matrix of Rayleigh **Bm** of **size flexible, tridiagonale and square** (with a true scalar product, if not it loses this last property). This particular form largely facilitates the calculation of the modes of Ritz: with **QR** one loses an order busy magnitude then, when one seeks p clean modes while projecting on one subspace of size m , $O(pm^2)$ with $O(20pm)$ [bib6].

The algorithm consists in gradually building a family of vectors of Lanczos **q1, q2, ..., qm** while projecting orthogonally, with the iteration K , vector **A qk** on the two preceding vectors **qk** and **qk-1**. The new vector becomes **qk+1** and thus, gradually, one ensures structurally

the orthogonality of this family of vectors. The iterative process is summarized thus.

$$\mathbf{Q} = \mathbf{0}$$

0

$$, 0 = 0.$$

Calculation of \mathbf{Q}/\mathbf{Q}

1

$$1 = 1.$$

For $K = 1, m$ to make

$$\mathbf{Z} = \mathbf{A} \mathbf{Q}$$

-

K

\mathbf{Q}

,

k-1

K -1

=

K

$(\mathbf{Z}, \mathbf{q}_k)$,

$$\mathbf{v} = \mathbf{Z} - \mathbf{Q}$$

K

K

$$= \mathbf{v},$$

K

If $K = 0$

then

\mathbf{v}

\mathbf{Q}

=

,

K +1

K

If not

Déflatio;

N

End if.

Fine buckles.

Algorithm 10: Theoretical Lanczos

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While noting, **EM** the *mième* vector of the canonical base, the **vector residue of the factorization of Lanczos** is written **R**

= **Q**

and

m

m

m+1 *Mr.*

B

m

m

N

With

Q_m

=

Q_m

+

0

m

N

m

R

T

m=m **q_{m+1}e_m**

Appear 5.2.1-a: Factorization of Lanczos

The matrix of Rayleigh takes the form then (in complex with a square scalar product)

1

0

0

1

1

...
0
B
2

 $m =$
.
0
...
...

m
-1

0
0
 m
-1 m

By construction, this algorithm ensures us of the following results (into arithmetic exact):

- them
- $(\mathbf{q}_k)_{k=1, m}$ constitute an orthonormal family,
- they generate the space of Krylov of order m initiated by \mathbf{q}_1 ,

$K(\mathbf{A}, \mathbf{Q}$
-1
 $1) = Vect$
 m
 m
 $(\mathbf{Q}, \mathbf{A} \mathbf{Q}, \dots, \mathbf{A} \mathbf{Q}$

1
1

- they make it possible to build a matrix \mathbf{B}_m tridiagonale, square and of flexible size,
- the spectrum of \mathbf{B}_m does not admit that the m dominant simple modes on which \mathbf{q}_1 has one component.

Note:
· the algorithm thus does not allow, in theory, the capture of multiple modes. That can be explained by noticing that any matrix of irreducible Hessenberg (a matrix \mathbf{A} is said of Hessenberg higher (resp. lower) if \mathbf{Have} , $j= 0$ for $i > j+1$ (resp. $j > i+1$). It is of more irreducible if \mathbf{Have} 1, 1

01

+

) (thus a fortiori a matrix tridiagonale) does not admit that simple modes,

- the cost of an iteration is weak, about that of a product matrix-vector $\mathbf{A} \mathbf{qk}$, that is to say for the m first iterations a calculation complexity of $O(Nm(c+5))$ (with C the number means of nonnull terms on the lines of the matrix of work). In addition, complexity necessary memory is weak but one does not need to build \mathbf{A} in entirety, one has right need to know its action on a vector. This characteristic is very interesting for to solve problems of big sizes [bib23],

- in general the sphere of activity directs the choice of a vector of initial Lanczos, this one must by example to belong to a space of acceptable solutions (checking constraints, conditions limit...) like with the image unit of the operator. This last point is important because it makes it possible more quickly to enrich modal research while not being limited to the core,
- the algorithm of Lanczos is only one means of approaching the subspaces of Krylov. Its generalization with the nonsymmetrical configurations is called algorithm of Arnoldi (cf [§6.2]).

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5.2.2 Estimates of errors and convergences

Because of particular form of \mathbf{Bm} , the extraction of the required elements of Ritz is simplified, and in more, the following result allows us quickly (via a product of two realities!) to estimate their qualities.

Property 7

~

The euclidian norm of the residue of the element of Ritz (\sim

$\mathbf{U} = \mathbf{Q} \mathbf{X}$

m) is equal to

~

\mathbf{R}

= (A

~

T

- I) U =

E X

2

2

m

m

Proof:

Commonplace by taking the euclidian norm of the factorization of Lanczos

WITH Q X

Q B X

Q

and

=

+

X

m

m

m

m m+1 m where **qm+1** is normalized with the unit.

Note:

A result plus general, independent of any method of resolution, us ensures that for

~

each clean mode (, X) of Bm, there exists a clean mode (, U) of A such as:

R 2

- ~

2

with = min

T

I - ~

~

~

U With U

(

R

sin U, ~u)

I

2

Therefore, more than minimization of the residue, the criterion of stop of the methods of the Lanczos/Arnoldi type

m < makes it possible to approach the original spectrum as well as possible. These increases are not accessible

that in the square case.

In the case general (and in particular nonnormal) they are more difficult even impossible with to build without additional information (level of nonnormality, "defectivity").

The estimate of the residue is not then any more the good criterion to estimate the quality of the approximated modes.

Let us interest being maintained in the quality of convergence of the algorithm. Let us particularize theorem 6

for this algorithm. By limiting the standard of error of $(\mathbf{I} - \mathbf{Pm}) \mathbf{U}$ one obtains increases

2
following:

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Theorem 8

That is to say $(\mathbf{I}, \mathbf{u}_i)$ the *ième* clean mode dominating of \mathbf{A} diagonalisable, there exists a mode of Ritz

then

$(\sim, \sim$
 $\mathbf{I} \mathbf{u}_i)$ such as:

2

~

2

J -

N

-2
 $I - I$

m
 $T - I(I)$
 $j < i J - I$
(

$J - N$
sin
, ~u

-1
ui I)

m
 $T - I(I)$
 $j < i J - I$

-
I
i+1
with $I = 1 + 2$
, T
-

the semi (X) polynomial of Tchebycheff of *semi* degree, the constant of
1+i
N

theorem 6 and =
your (
N Q, U
1
 I).
P U
 $m I 2$

Proof:
Cf [bib11] pp554-557.

These increases indicate (cf [bib23] pp59-63) that:

- if the initial vector does not have any contribution along the clean vectors, one cannot capture the aforementioned modes (+),

- one

has

firstly convergence of the extreme modes and as much better than the spectrum

is **separate** (these methods are less sensitive to the clusters than the methods of the types reiterated powers), without compressing of eigenvalues (famous “the clusters”),

- the **error decrease when m increases** (like the reverse of the polynomial $T_m-I(X)$, therefore like the reverse of exponential),

- the estimate on the eigenvalues is better than that their associated own vectors.

Note:

The convergence of the method was studied by P. Kaniel then improved (and corrected) by S.C. Paige; One will find a synthesis of these studies in the paper of Y. Saad [bib20].

Let us look at now how behaves the algorithm into arithmetic finished. We will see that it is the seat of surprising phenomena.

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5.3

Practical algorithm of Lanczos

5.3.1 Problem

of orthogonality

At the time of implementation the effective of this algorithm, one realizes that very quickly the **vectors of**

Lanczos lose their orthogonalities. The matrix of Rayleigh is not then any more the matrix projected of the initial operator and the captured spectrum are sullied more and more with errors. A long time this phenomenon

inescapable was allotted wrongly to the effects of rounded the arithmetic one finished. In 1980, D.C. Paige showed

that the loss of orthogonality was especially **ascribable with the convergence of a clean mode**. Thus, as of

that one captures a mode dominating, one disturbs the fitting of the preceding vectors of Lanczos. following result expresses this paradox clearly.

Property 9 (Analysis of Paige)

By taking again the notations of algorithm 10 and by noting the precision machine, with the iteration $k+1$ the orthogonality of the new vector of Lanczos is expressed in the form:

$$\mathbf{v}^T \mathbf{q}_{k+1} + \mathbf{A}$$

$$\mathbf{q}_{k+1}^T \mathbf{Q}$$

2

 $K \ll 1$ $(K$

+

=

)

 K **Proof:**

Cf [bib26].

The problem occurs in fact during the standardization of the new vector of lanczos \mathbf{q}_{k+1} . When it mode converges, according to property 7, that comes from the smallness of the coefficient K and thus in spite of

theoretical orthogonality ($\mathbf{v}^T \mathbf{I} \mathbf{Q} = 0$ (K)), effective orthogonality is far from being acquired

$$(\mathbf{q}_{k+1}^T \mathbf{Q}$$

$$K \ll 1 \gg (K)).$$

The digital processing of these problems has been the research object many for thirty years and of many palliative strategies were developed. The choice of such or such method depends type of required spectral information and average data processing available, the synthesis of Besides JL.Vaudescal proposes a very good overflight of these alternatives (cf [bib23] pp66-78):

- **Algorithme of Lanczos without reorthogonalisation**, developed by J.K. Cullum and R.A. Willoughby [bib3] which consists with expurger the calculated spectrum of its phantom modes of studying interlacings of the eigenvalues of the matrix of work and one of its submatrices. This alternative admits a weak overcost calculation and memory to determine them eigenvalues, but it complexes (sometimes largely) the research of the vectors clean.

- **Algorithmes of Lanczos with total reorthogonalisation** (B.N. Parlett) or **selective** (J. Scott) which consists with each step with réorthogonaliser the last vector of Lanczos obtained compared to all the already calculated vectors or simply compared to the vectors of Ritz converged (this alternative thus makes it possible to control the loss of orthogonality dynamically acceptable). These alternatives are much more expensive in complexity calculation (them automatic réorthogonalisations) and memories (storage of the preceding vectors) but

they prove more effective and more robust.

In the alternative of Newmann-Pipano [bib14] (METHOD = "TRI_DIAG") of *Code_Aster*, it is **strategy of reorthogonalisation supplements** which was selected: the vector **qk+1** is thus not completely calculated as in the theoretical algorithm.

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Note:

- *these problems of loss of orthogonality occur with all the more of acuity that the size of subspace increases. It is an additional argument for the installation of one iterative process limiting this size (cf [§5.4.2]),*
- *these strategies are declined vectoriellement or per blocks, they are established in simple or iterative algorithms. The explicit latter being able to profit from restarts or implicit, controlled or automatic. Alternative IRAM profits thus from an implicit restart calculated automatically,*
- *the central point of these algorithms is consisted the method of orthogonalisation put in place. All the process is dependent on its success and its robustness. With orthogonal of type Housholder or Givens, very expensive transformations but very robust, one prefers from now on the algorithms of the type Gram-Schmidt Itératifs (IGSM) which are a better compromise between effectiveness and complexity calculation (cf [Appendix 2]). It is it besides choice which was made for the alternatives of Lanczos/Arnoldi installation in the code,*
- *the alternative of selective reorthogonalisation compared to the converged modes returns to to carry out an implicit deflation by restriction on the operator of work not to have with to recompute (cf [§3.1]).*

5.3.2 Capture multiplicities

It was seen that in theory the algorithm of Lanczos did not allow, some is his strategy of reorthogonalisation, the theoretical capture of multiple modes. **In practice**, for once, **effects from round-off come to the rescue** and “powder” with small components along almost all clean vectors. One can thus **capture multiplicities**, however they can be erroneous and require a complementary postprocessing of checking.

Note:

In fact, only a version per blocks (G. Golub & R. Underwood, 1979) can allow us one correct detection of the multiplicities, at least as long as the size of the blocks is sufficient. This version was wide (Mr. Sadkane [bib22], 1993) with the algorithm of Arnoldi but it would be a pity to deprive itself of the alternative of Sorensen (IRAM) who is more effective and more robust.

5.3.3 Phenomenon of Lanczos

The success of this algorithm rested at the beginning on what is called the “**phenomenon of Lanczos**”. This conjecture predicts that for a sufficiently large size of subspace ($m \gg N$) one is

able to detect all the spectrum (with load to thereafter distinguish the “good grain from the ryegrass”) of the operator of work. Taking into account weak the pre-necessary report of “basic” Lanczos (grosso-modo a matrix tridiagonale and some vectors), this is particularly interesting for to treat hollow systems of very big sizes (other algorithms of reiterated the powers type or **QR** require them it knowledge of all the matrix of work).

We now will detail (a little) some elements whose interest largely exceeds the framework of this algorithm.

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5.4 Treatments

complementary

5.4.1 Detection of spaces invariants

In algorithm 10, the nullity of the coefficient $l-1$ prevents the standardization of the new vector and an additional treatment called of **deflation** requires. The subspace of calculated Krylov is then under **space invariant** and its spectrum is thus exact. In other words, the $l-1$ first eigenvalues exhumed by the algorithm of support (**QR** or **QL**) are those of **A**.

To continue, it is then necessary to choose a new random vector observing the limiting conditions, the orthogonaliser with all the already calculated vectors of Lanczos and to build a second family of vectors of Lanczos that one orthogonalise compared to the first family constituting space:

$H = K(A, Q$

1

1) = Vect

L -

L

L

(Q, A Q, ..., A Q

1

1

1) .
 The matrix tridiagonale obtained with the following form:

1

1

...

0

1

2

...

...

0

Bm =

0 *L*

L

L L

+1

...

0

...

...

M-1

M-1 m

The intermediate factorization of Lanczos (with the order *L*) is written:

WITH Q = Q B

L

L

L

The values $(K, K)_{k=1, L}$ are obtained starting from the first random vector and the values $(K, K)_{k=1+1, m}$ are obtained starting from a second random vector. To detect the nullity of the term extra-diagonal the numerical criterion is used

*L**PREC_LANCZOS*

-1

L then $l-1 = 0$,

where *PREC_LANCZOS* is initialized under the key word factor *CALC_FREQ*.

Note:

· a more robust criterion retained by large mathematical bookshops *EISPACK* [bib26], *LAPACK* [bib26] and *ARPACK* [bib27] use the preceding diagonal term, a parameter flexible *C* and the precision machine,

 $L \ 1 < C$

-

 $(l-1 + L)$

it is this criterion which at summer appointed for IRAM (but the user cannot modify the parameter C, this one is fixed at a standard value by the code), various messages preventing besides the user of the detection of a space invariant.

· *Obtaining such a space is completely sympathetic nerve since it ensures us of very good quality of the first modes. Moreover, one reduces the size of the problem by dividing it into two parts: one solved, the other to solve. Many techniques seek with to reproduce artificially this phenomenon (cf [§3.1], [§5.3.1]).*

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5.4.2 Strategies of restartings

To limit the endemic problems of orthogonalisation, to limit to them pre-necessary report and to take dynamically into account spectral information already obtained, of the strategies of

restartings were coupled with the algorithm of Lanczos. It loses its “simple” character and becomes “iterative”. One reiterates the “restarts” until satisfying the desired criteria of convergence. The algorithm does not undergo the convergence of the modes imposed by theorem 8 and it can support interactivement that of certain modes.

In theory however, plus the size of the subspace is large, better is convergence. One compromise is thus to find between the size of the subspace and the frequency of the restartings. Various vectors of restartings can be used, being generally written like a sum balanced p required clean modes (Y. Saad 1980)

$$\mathbf{Q} = \mathbf{Q} \mathbf{X}$$

$$\begin{matrix} p \\ 1 \\ I \\ m I \\ \cdot \\ I=1 \end{matrix}$$

Indeed, if one starts again with an initial vector pertaining to the subspace invariant generated by sought modes, one is then sure to obtain (with a good algorithm of orthogonalisation) a standard residual about the precision machine and of the almost exact clean modes. In addition to the decision to start the restarting, all the problems lie in the search for these weights I . Us will see that with IRAM, the restarts set up via implicit polynomial filters solve elegantly and automatically this question.

Note:

- *this philosophy of the restarts was initiated by W. Karush (1951),*
- *restartings based on the vectors of Schur associated with the spectral decomposition sought seem more stable (J. Scott 1993),*
- *of the criteria of effectiveness was required to decide restart appropriateness (B. Vital 1990),*
- *instead of a simple linear combination of clean vectors, one can determine a vector of restarting via polynomials (Tchebycheff in reality and Faber in complex) allowing to center modal research in such or such zone. One speaks then about polynomial accelerations explicit [bib22].*

The following paragraph will show some of the concepts presented hitherto to clarify alternative installation in the code.

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5.5**Establishment in Code_Aster****5.5.1 Alternative of Newmann & Pipano**

This alternative developed by Mr. NEWMAN & A. PIPANO [bib3], [bib14], in 1977, is a method of Simple Lanczos, into arithmetic real, with total reorthogonalisation (via a GSM). It uses it crossbred "shift and invert" traditional of the scalar pseudo-product introduced by the shifted matrix (**A-B**).

(A)**B -**

1

1

B U =**U**

!# "

#

\$

##

μ -

With

!"\$

(X, y) = yt (A)**B X,**

with =

(

sign **X, X**), 1**X = (X, X) 2.**

This choice returns the couple operator of work - symmetrical scalar product and it are adapted to the matrices

particular of the mechanics of the structures. One can thus deal with problems:

- of free structure and fluid structure (**A** can be singular),
- of buckling (**B** is indefinite).

The scalar pseudo-product introduced by the shifted matrix is thus regular (answers this prerogative cf [§2.6], [§2.9]) and it makes it possible to seek vectors of Lanczos (**A - B**) - orthogonal. The total gonalisation is carried out only if it proves to be necessary and this criterion is paramètrable.

Note:

· **B-orthogonality** and a fortiori that with the Euclidean direction cannot be but the fruit any more of particular configurations. This does not modify the properties of orthogonalities of the modes clean (cf proposal 2),

· as one already announced [§1.7] it exists a whole zoology of couples operator - product scalar, this one being only one possibility among others. Thus, the libraries [bib29] traditional propose to deal with the problems of buckling in “buckling mode” via the same one “shift and invert” and the scalar pseudo-product introduced by **A**. But because of introduction quasi-systematic of Lagranges, this matrix becomes indefinite even singular, which disturb the process largely. The same causes produce the same effects when for a calculation of dynamics, one uses the scalar **B-product**.

The price to be paid for this profit in robustness is the loss of possible symmetry of the matrix of Rayleigh

1 1
0
0

1
...
0
=
B
2
I
I I

m =
with

·
0
...
...
m 1

-
I = sign (**Q**, **Q**
I + 1
I + 1)

0
0
M - 1 *m*
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After balancebeing balanced and putbeing put in the form of Hessenberg higher, **Bm** is diagonalisée by one

implicit method **QL** if it remains in spite of very symmetrical or by a method **QR** if not (cf [Appendix 1]). The difference in cost calculation between these solveurs remains negligible vis-a-vis the costs of

réorthogonalisations. The diagram of established Lanczos becomes then:

Random pulling of **qinit**.

q~

-1

1 = (A -

)

B

(qinit (I) .ulagr (I)) .i

q% = A

1

(q~1 (I) .ubloq (I)) .i

1 =

(

sign qT

% 1 (A -)

B q%1)

1

-

Q = Q

2

1

1% qT

1% 1 (A -

)
B q%1
Q = 0, =,
 0 = .
 0
 0
 0
 0
 For $K = 1$, m to make
Z = A Q -

Q
 ,
 K
 k-1
 k-1
 k-1
 = T
 K
qk (A -)
B Z,
v = Z - Q,

K
 K
 K
 Réothonalisation from
v
 report/ratio with **Q**

(I)
 (IGSM),
 $I = 1, K$
 T
 $K = v (A -)$
B v,
 T
 $K =$
 (
sign v (A -)
B v),
 If K
 then
 $0 Kv$

qk+1 =

,

1

K 2

If not

Deflation;

End if.

Fine buckles.

Algorithm 11: Alternative of Newman-Pipano

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The reorthogonalisation is carried out thanks to an alternative “house” of the Method of Gram-Schmidt Iterative (IGSM) according to the following process:

For I = 1, K

has

T

= **qk+1 (A - B) Q,**

I

If (**qT 1 (A)**

B Q

K

I

PREC_ORTHO

+

-

) then

For J = 1, *NMAX_ITER_ORTHO*

X = Q*T* $K + 1 - (\mathbf{qk} + 1 (\mathbf{A} -))$ **B IQ) IQ,** $B = \mathbf{xT} (\mathbf{A} -)$ **B IQ**If (*B PREC_ORTHO*) then**Q****X** $K + 1 =$

,

 $I + 1 I$

, exit;

If not

If (*B has*) then**Q****X** $K + 1 =$

,

 $= B has;$

If not

Failure, emission of a message of alarm,

 $I + 1 I$

, exit;

End if.

End if.

Fine buckles in J.

End if.

Fine buckles out of I.

Algorithm 12: Procedure of reorthogonalisation of “TRI_DIAG”**Note:**

After some tests, it seems that this alternative specific to the code is less effective than the IGSM of the type Kahan-Parlett [bib18] selected for the IRAM (cf [Appendix 2]).

5.5.2 Paramétrage

To be able to activate this method, it is necessary to initialize the key word METHOD with “TRI_DIAG”. In addition,

the size of the subspace of projection is determined, either by the user, or empirically to leave formula:

$$m = \min (\text{my} ($$

$$X \ 4 \ p, \ p + 7), \text{nddl-credits})$$

where

p

is the number of eigenvalues to calculate,

nddl-credits

is the number of degrees of active freedom of the system (cf [§2.2])

The user can always impose to him even dimension by indicating it with the key word

DIM_SOUS_ESPACE of the key word factor CALC_FREQ.

Parameters of the total reorthogonalisation, PREC_ORTHO and NMAX_ITER_ORTHO, *the* number maximum of iterations **QR**, NMAX_ITER_QR, and the criterion of deflation (cf [§5.4.1]),

PREC_LANCZOS, are

accessible by the user under the key word factor CALC_FREQ. When this phenomenon of deflation product, a specific message specifies its row L.

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5.5.3 Warning on the quality of the modes

Let us recall that this alternative is not iterative. From the number of desired frequencies, one estimate the dimension of the subspace of calculation and one hopes that the clean modes of the problem projected standard will be a good approximation of those of the generalized problem. One checks has posteriori the validity of the results (cf [§2.9]) but one does not have any active control on this precision. If they are not satisfactory, one has as only only resorts to carry out a new test in increasing the dimension of the subspace.

Note:

- *the method WILL GO that we will approach proposes a flexible and dynamic control of precision of the results, it is an iterative method,*
- *in V5, in parallel of the modal position and the eigenvalue (in frequency if one is in dynamics), one posts from now on the standard of the residue of the initial problem calculated in postprocessing (after having standardized the clean vectors ad infinitum (in order to exhume clean vectors “neutral” with respect to the string of standardizations which they can undergo at the time calculations of the modal parameters of the structure (factors of participation...)).*

5.5.4 Perimeter

of use

Attention, this method is usable one to the only deal with quadratic problem. In the event of error at the time of the initialization of the key word METHOD, calculation stops and a trace is left in the files of exit.

In V5, an **algebraic method calculating the null eigenvalues** of the modal problem generalized was introduced. Physically those correspond to the movements with energy of null deformation of a free structure (cf [bib25] TP n°2). Except the numerical difficulties of handling of quasi-null quantities, because of their multiplicities, their capture correct was until often problematic present for Lanczos established in *Code_Aster*: phantom modes appeared corresponding to missed multiplicities!

This algorithm of detection of the modes of rigid body, which one activates by initializing OPTION with "MODE_RIGIDE" (default value `WITHOUT `), intervenes in preprocessing of modal calculation properly

known as. It is based on the analysis of the matrix of rigidity and breaks up into three phases:

- detection of the null pivots of this matrix,
- blocking of these pivots,
- resolution of a linear system whose are solutions the associated clean vectors.

During the process of Lanczos it is enough consequently to orthogonaliser, progressively with their determination, basic vectors with the latter.

However, the introduction of the method WILL GO reduced the interest (if it is not as comparison) of one

such option (which is not free in calculation complexity since it requires inversions of systems).

What good is it to deprive itself of such an algorithm which is by no means affected by the presence of these modes

a little particular multiples!

This method remains nevertheless useful as solution of help in the event of failure of IRAM (that can occur in the presence of clusters in the vicinity of the terminals or for operators pathological (badly conditioned, nonnormal, defective...) who should however be rather rare (that often reveals a badly posed problem). One can also plan to encapsulate it in one auxiliary operator (like IMPR_STURM).

Note:

The quadratic treatment of problem is incompatible with options "MODE_FLAMB" and "MODE_RIGIDE".

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5.5.5 Posting in the file message

The example below resulting from the list of case tests of the code (sdll112a) recapitulates the whole of the traces

managed by the algorithm. The iteration count effective **QR** (or **QL**) can be only identical for all eigenvalues. It is an artifact of information which will be brought to disappear.

THE NUMBER OF DDL

TOTAL EAST: 86

LAGRANGE EAST: 20

THE NUMBER OF ACTIVE DDL EAST: 56

The SELECTED OPTION EAST: **PLUS_PETITE**

THE VALUE OF SHIFT IN FREQUENCY EAST: 0.00000E+00

INFORMATION ON CALCULATION REQUIRES:

A NUMBER OF REQUESTS MODES: 10

THE DIMENSION OF REDUCED SPACE EAST: 0

IT IS LOWER THAN THE NUMBER OF MODES, ONE TAKES IT EQUALIZES A 40

FREQUENCIES CALCULEES INF. AND SUP. ARE:

FREQ_INF: 1.54569E+01

FREQ_SUP: 1.01614E+02

THE FIRST HIGHER FREQUENCY NOT SELECTED EAST: 1.29375E+02

MODAL CALCULATION: METHOD OF SIMULTANEOUS ITERATION

METHOD OF LANCZOS

NUMBER FREQUENCY (HZ) STANDARD OF ERROR ITER_QR

1 1.54569E+01 1.29798E-12 4

2 1.54569E+01 7.15318E-13 4

3 3.35823E+01 3.98618E-13 4

4 3.35823E+01 4.25490E-12 4

5 4.73076E+01 4.26463E-12 4

6 4.73076E+01 1.43391E-12 4

7 5.45850E+01 9.52006E-12 4

8 8.80156E+01 3.45489E-13 4

9 1.01614E+02 6.13949E-12 4

10 1.01614E+02 3.18663E-12 4

CHECKING A POSTERIORI OF THE MODES

IN the INTERVAL (1.54182E+01, 1.16326E+02)
IT THERE A WELL 10 FREQUENCY (S)

Example 5: MODE_ITER_SIMULT with “TRI_DIAG”

Now let us recapitulate the paramètrage available of operator MODE_ITER_SIMULT with this option METHOD = “TRI_DIAG”.

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5.5.6 Summary of the paramètrage

Key word factor

Key word

Default value

References

“DYNAMIC” TYPE_RESU

“DYNAMIC”

[§2.1]

“MODE_FLAMB”

[§2.1]

METHOD “TRI_DIAG”

“SORENSEN”

[§5.5.3]

OPTION “MODE_RIGIDE”

“WITHOUT”

[§5.5.4]

“WITHOUT”

[§5.5.4]

CALC_FREQ

FREQ

[§4.4]

CHAR_CRIT

[§4. 4]

OPTION “PLUS_PETITE”

`PLUS_PETITE "

[§4.4]

“BAND”

[§4.4]

“CENTER”

[§4.4]

NMAX_FREQ

10

[§4.4]

DIM_SOUS_ESPACE

Calculated

[§5.5.2]

PREC_ORTHO

1.E-12

[§5.5.1], [§5.5.2]

NMAX_ITER_ORTHO

1.E-04

[§5.5.1], [§5.5.2]

PREC_LANCZOS

1.E-04

[§5.5.1], [§5.5.2]

NMAX_ITER_QR

15

[§5.5.1], [§5.5.2]

NPREC_SOLVEUR

8

[§2.6]

NMAX_ITER_SHIFT

5

[§2.6]

PREC_SHIFT

0.05

[§2.6]

SEUIL_FREQ

1.E-02

[§2.9]

VERI_MODE

STOP_ERREUR “YES”

“YES”

[§2.9]

“NOT”

[§2.9]

PREC_SHIFT

5.E-03

[§2.9]

THRESHOLD

1.E-06

[§2.9]

STURM “YES”

“YES”

[§2.9]

“NOT”

[§2.9]

Table 5.5.6-a: Summary of the paramètrage of MODE_ITER_SIMULT with “TRI_DIAG”

Note:

- *in V5, the user can specify the class of membership of its calculation by initializing it key word TYPE_RESU. According to this value, one informs vector *FREQ* or *CHAR_CRIT*,*
- *one finds all the “tripaille” of parameters related to the preprocessings of the test of Sturm (*NPREC_SOLVEUR*, *NMAX_ITER_SHIFT*, *PREC_SHIFT*) and with postprocessings of checking (*SEUIL_FREQ*, *VERI_MODE*),*
- *at the time of the first passages, it is strongly advised to modify only them principal parameters noted in fat. The others relate to more the mysteries of the algorithm and they were initialized empirically with values standards,*
- *in particular, to improve quality of a mode, the only flexible parameter is dimension of the subspace, *DIM_SOUS_ESPACE*.*

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Algorithm WILL GO (*METHOD* = "SORENSEN")

6.1 Introduction

We saw that one of the crucial problems of the method of Lanczos is the loss of orthogonality inescapable of its basic vectors (cf [§5.3.1]). A **generalization** of this algorithm **to the case not square** imagined by **W.E. Arnoldi** [bib31] in **1951** makes it possible to solve partially this problems. It was given to the style of the day by Y. Saad [bib32] in 1980 and one abundant literature covers the subject. Put aside the article founder and papers of Y.Saad and Mr. Sadkane [bib22], one recommends the brought up to date and exhaustive synthesis of J.L. Vaudescal [bib23] (pp79-112). This method being at the base of **algorithm IRAM known as "of Sorensen" (IRAM for `Implicit Restarted Arnoldi Method')**, we first of all will detail his operation, its behaviors and its limitations. Thereafter, we will determine the stakes to which IRAM must answer (and it does it in the majority of the standard cases!) and we will consider its theoretical mysteries and numerical. We will conclude by the summary from effective sound paramètrage in *Code_Aster* and for an example of file message.

6.2 Algorithm of Arnoldi

6.2.1 Principle

Its perimeter of application covers **all the couples operator with work (pseudo) produced scalar**. During this opening is the filling of the matrix of Rayleigh which becomes form **Hessenberg higher**. It is not very prejudicial, because one can thus apply to him directly algorithm **QR** (the first stage of a good **QR**, except balancing, consists in reducing the matrix of work in the form of Hessenberg. That makes it possible to gain an order magnitude at the time of the resolution itself (cf [Appendix 1]), from where a profit about $O(10m^3/3)$.

The algorithm is very similar to that of Lanczos, it consists in building a family gradually of vector of Arnoldi **q1, q2, ..., qm** while projecting orthogonally, with the iteration K , vector **A qk** on K vectors precedent. The new vector becomes **qk+1** and thus, gradually, one ensures the orthogonality of this family of vectors.

With the difference of Lanczos, the orthogonality of the new vector compared to all the precedents is thus explicitly and not assured implicitly. This one is managed by the algorithm of Gram-Schmidt Modified (GSM) (cf appendix 2) which proves sufficiently robust in the majority of the cases.

While noting, **EM the mième** vector of the canonical base, the **vector residue of the factorization of Arnoldi**

R is written

= B

Q

and

m

$m+1, m$

$m+1$ Mr.

B

m

m

N

With

Q_m

$=$

Q_m

$+$

0

m

R

T

N

m

$m = bm + 1, m \text{ } qm + 1em$

Appear 6.2.1-a: Factorization of Arnoldi

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The iterative process is summarized as follows:

Calculation of **Q/Q**

1

1 = 1.

For K = 1, m to make

Z = A Q,

K

For L = 1, K to make (**GSM**)

B =

lk

(**Z, ql**),

Z = Z - blk ql,

Fine buckles;

B +

= **Z,**

K 1, K

If B +

0 then

K 1, K

v

Q

=

K +1

,

bk+1, K

If not

Deflation;

End if.

Fine buckles.

Algorithm 13: Theoretical Arnoldi

The matrix of Rayleigh is written then:

B

B

...

B

11

12

m

1

B

B

...

B

B

21

22

2m

$m =$

.

0

...

...

$bm-1, m$

0

0

B

m, m

B

-1

mm

Except the shape of this matrix and a less acuity with the problems of orthogonality, **this algorithm us ensures the same theoretical and numerical properties that Lanczos**. However if one let grow indefinitely the size of the subspace until convergence of the eigenvalues wished, the effects of round-off despite everything will take again the top and one goes to the front of large troubles.

From where need, as for Lanczos, to make this process iterative via restartings.

Note:

- *this method can be seen like an implicit alternative of the algorithm of Lanczos with total reorthogonalisation (cf [§5.3.1]),*
- *the implicit orthogonalisation of the algorithm can be led by more expensive algorithms but more robust such as **QR** or IGSM. This is strongly necessary when the operator of work presents a too strong defect of normality,*
- *because of structure of **Bm**, memory complexity is requested than with Lanczos, by against complexity calculation remains of the same order of magnitude $O(Nm(c+3+m))$ (with C it numbers average nonnull terms on the lines of the matrix of work),*
- *to improve this first point, Y. Saad [bib32] showed that the structure of Hessenberg higher can see to cancel its extreme on-diagonals if one carries out only partially reorthogonalisation,*
- *the vectorial algorithms having a tendency natural to miss by the multiplicities, one prefers often to use a version blocks (Mr. Sadkane [bib22], 1993). But the size of those influence on the quality of the results, for this reason one prefers the vectorial versions to them or blocks of IRAM,*
- *the choice of the vector of initial Arnoldi is carried out same manner as for Lanczos.*

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6.2.2 Estimates of errors and convergence

With regard to the evaluation of the quality of approximation of the clean modes obtained, there is one as simple and effective criterion as for Lanczos.

Property 10

~

The euclidian norm of the residue of the element of Ritz (\sim

$\mathbf{U} = \mathbf{Q} \mathbf{X}$

m) is equal to

$\mathbf{R} = (\mathbf{A}$

~

T

$$- \sim \mathbf{I} \mathbf{U} = \mathbf{B}$$

E X

2

$m +$

2

1, m

m

Proof:

Commonplace by taking the euclidian norm of the factorization of Arnoldi

$$\mathbf{Q} \mathbf{X} = \mathbf{Q} \mathbf{B} \mathbf{X} +$$

Q

and

\mathbf{B}

X

m

m

m

$m + 1, m$

$m + 1$ m and as \mathbf{q}_{m+1} is normalized with the unit.

Always by focusing us on the standard of the additional projector $(\mathbf{I} - \mathbf{P}_m) \mathbf{U}$ one can

2

to generalize the theorem of convergence 8 with the nonsquare case.

Theorem 11

That is to say $(1, \mathbf{u}_1)$ the first (traditional arrangement, by order descending of module) clean mode

N

dominating of \mathbf{A} diagonalisable and is $\mathbf{Q} =$

U

1

K K the initial vector of Arnoldi broken up on

$K = 1$

\sim

base clean vectors, it exists a mode of Ritz then $(, \sim$

1 $\mathbf{u}_1)$ such as:

\sim

2

m

1 - 1

1

1

N

with =
, the constant of theorem 6, =

K
and
P U
1

$m I 2$
 $K = 1, K 1$

1
1
 $m +$
-

$m + 1$

m
 $K -$

1

1 =
-

. This result is declined in the same way on the other modes.

$J = K =, K J$
 K

2
2
 J

Proof:

By taking again to the demonstration of Y. Saad ([bib33] pp209-210) and the result of theorem 6.

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These increases very different from those obtained with Lanczos guide the same ones however phenomena:

- if the initial vector does not have any contribution along the sought clean vectors, one cannot to capture ($I +$),

- one

has

firstly convergence of the peripheral modes of the spectrum, and this, of as much better than it is separate (property of m

I),

- the decrease of the error is proportional to the increase in m (property of m

I).

Note:

- *when an eigenvalue is badly conditioned $I +$ then it should be increased m so that m*

I decreases,

- *of the similar results was exhumed in the case of a defective operator (cf Zia 94 [bib23]).*

Extremely from this lesson, we now will recapitulate the stakes to which must answer IRAM.

6.3

stakes

The algorithm tent will bring an elegant remedy for the recurring problems raised by the others approaches:

- **minimization of the space of projection:** it proposes with minima $m > p+1$ instead of the $m = 4p$ of Lanczos,

- optimal management of the overcosts of orthogonalisation establishing a **compromise** enters the **size of subspace** and the **frequency of the restartings**,

- transparent, dynamic and effective management of these restarts,

- taken into automatic account of **spectral information**,

- fixing

pre-necessary report and of the **quality** of the **results**.

There is thus more question to be posed concerning the strategy of reorthogonalisation, the frequency of restarts, their establishments, criteria of detection of possible phantom modes... "super-IRAM"

charge of all!

In short, it gets:

- one

better

total **robustness**,

- of

complexities calculations $O(4nm(m-p))$ and **memories** $O(2np+p^2)$ **improved** (especially by report/ratio in simple Lanczos such that of Newmann & Pipano) for a **fixed precision**,

- a more rigorous capture of the multiplicities, clusters and rigid modes of body

(thus **less parasitic modes!**).

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Note:

- *on this last point, only a version per blocks of IRAM or a version incorporating of the “purging and lock” (techniques of capture and filtering, cf D. Sorensen & R.B. Lehoucq, 1997) can us to guarantee a correct detection of the spectrum of a standard operator (i.e not too badly conditioned),*

- *it seems, that in practice in Code_Aster, the report/ratio in calculation complexity between IRAM and Lanczos/Bathe & Wilson are with minima of order 2 in favour of the first. With sizes of reasonable problems (a few tens of thousands of ddl and $p = O(100)$) this one can to go up up to 10 (without the encapsulation of MACRO_MODE_MECA). In certain cases semi-industrial, it made it possible to unroll a search for spectrum which had failed with Jacobi and who were inaccessible with Lanczos (taking into account the time limits),*

- *a class of algorithm known as of “Jacobi-Davidson” (cf R.B. Morgan 1990) seems even more promising to treat pathogenic cases. It uses an algorithm of the Lanczos type that it packaged via a method of Rayleigh.*

In the following paragraph we will clarify the operation of IRAM.

6.4

Algorithm “Implicit Restarted Arnoldi” (WILL GO)

This algorithm was initiated by D.C. Sorensen [bib30] in 1992 and makes real great strides for the resolution

great modal systems on parallel supercomputers. Its framework of application is very with fact general. It deals with as well the real problems as complex, square or not. It is summarized in a succession of factorizations of Arnoldi whose results control automatically static and implicit restartings, via polynomial filters modelled by implicit **QR**.

First of all it carries out a **factorization of Arnoldi of order $m = p + q$** (in theory $Q = 2$ is enough, in practical $q = p$ is preferable. It is this last default value besides which was retained (cf [§6.5.3]) of the matrix of work. Then once this preprocessing carried out it **reiterates a process of filtering of the part of the undesirable spectrum** (numerically and méthodologiquement, it is easier to exclude to incorporate).

It starts by determining the spectrum of the matrix of Rayleigh (via indétronable **QR**) and it in dissociate the nondesired part (while referring to the criteria fixed by the user) which it uses then like shift to set up a series of Q **QR** implicit with simple shift (cf [Appendix 1]).

factorization is written then:

$$\mathbf{In} \mathbf{Q}^+ \\ \mathbf{Q}^+ \mathbf{B}^+ \\ = \\ + \mathbf{R} \mathbf{Q}$$

m

m

m

m

where $\mathbf{Q}^+ = \mathbf{Q} \mathbf{Q}$

+

T

m

m

, $\mathbf{B} = \mathbf{Q} \mathbf{B} \mathbf{Q}$

m

m

and $\mathbf{Q} = \mathbf{Q} \mathbf{Q}$

\mathbf{Q}

1 2 ..

Q the unit matrix associated the **QR**. Afterwards

to have updated the matrices, one truncates them until the order p

$\mathbf{In} \mathbf{Q}^+ = \mathbf{Q}^+ \mathbf{B}^+ + \mathbf{R}^+$

p

p

p

p

and thus, at the price of Q news iterations of Arnoldi, one can find a factorization of Arnoldi of order m which is viable. All the subtlety of the process rests on this last sequence. Let us note:

Q

(

~

WITH) = (A -

Id

$p+i$),

$i=1$

the matrix polynomial of order Q generated by the operator of work. In fact, the implicit **QR** have acts on the p first lines of the matrix of Arnoldi, Rayleigh and residue, **so that factorization complementary to order Q produces the same effect as a factorization of order m initiated by the vector**

~

Q = (

With) Q

1

1 .

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The initial vector was implicitly modified (it does not have construction and effective application there of polynomial) so that it generates the desired modes preferentially and this, by withdrawing them components considered to be “unsuitable”. As one already pointed out (cf [§5.4.2]) this type of restart allows to decrease the residue by reducing the components of the initial vector following the modes undesirable. Into arithmetic exact, there would be immediately **Rm = 0** and the process would stop there! **Iteration after iteration, one thus improves quality of the modes sought while resting on auxiliary modes.** Of course this one is estimated at each stage and conditions the opportunity of to simulate another restart or not. The process can be summarized (very macroscopically!) like

follows:

Factorization of Arnoldi of order m : $\mathbf{A}\mathbf{Q} = \mathbf{Q}\mathbf{B} + \mathbf{R}$.

m

m

m

For $K = 1$, $NMAX_ITER_SOREN$ to make

~ ~

~

~

~

To calculate

1

, 2

... p

, $p+1$... $p+q$,

!"

#

\$

#

!"

\$

#

Preserved for Used like

améliorat ion

shifts

QR with implicit shifts,

Update **Q**, **B** and **R**

m

m

m ,

Truncation of these matrices has the order p ,

$\mathbf{A}\mathbf{Q} = \mathbf{Q}\mathbf{B} + \mathbf{R}$ $\mathbf{A}\mathbf{Q} = \mathbf{Q}\mathbf{B} + \mathbf{R}$,

p

p

p

m

m

m

Quality estimate of the p modes.

Fine buckles.

Algorithm 14: Method WILL GO (known as of Sorensen)

The maximum number of iterations is controlled by key word $NMAX_ITER_SOREN$ of the key word factor

CALC_FREQ.

It should be noticed that the Arnoldi algorithm is prudently supplemented by a **reorthogonalisation total** (started that if that proves to be necessary). This overcost is all the more acceptable as it is established via algorithm IGSM of Kahan-Parlett (cf [Appendix 2]) which is particularly effective. All this makes it possible to ensure us of the good behaviour of projection with respect to the initial spectrum.

In addition, the evaluation of the **quality of the modes** is not carried out simply by building p **residues R**

= (A

~

- ~ I) U

I

I

I

via property 10. One already mentioned that in the case not

2

2

square, they could not suffice with this task, in particular in the event of strong defect of normality. For of to discharge, without having recourse to other information (to obtain a criterion exact it would be necessary to be able

to consider conditionings spectral of spaces invariants and the angles which they make between them. It who is sometimes difficult to obtain, even a posteriori!) a priori, the preceding property is used supplemented by a criterion due to Z.Bai et al. [bib34]

~

B

T

+1,

E X < my (

X B, *PREC_SOREN*

m

m

m

m

)

where is the precision machine and *PREC_SOREN* is a key word initialized under *CALC_FREQ*. The user has

thus a quality control (partial) of the modes, that of which it did not lay out with the other methods established in the code. Taking into account the various standards used, this error is different from that resulting from the total postprocessing (cf [§2.9]) which is posted in the columns results.

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Note:

- *the technique of polynomial acceleration used is more effective than that of Tchebycheff, since the latter is explicit and requires m produced matrix-vector,*
- *to avoid the deterioration of the vectors of Ritz (and thus of the approximate clean vectors) by eigenvalues of very large modules (partners with the core of \mathbf{B} in “shift and invert”) a filtering of the type Ericsson & Ruhe [bib35] was established. More robust techniques exist but they require information a priori concerning in particular the blocks of Jordan associated with the core with the operator (cf Meerbergen & Spence, 1996),*
- *this algorithm can be seen like the truncated shape of implicit algorithm **QR** of J.C.F. Francis (cf [Appendix 1]).*

The following paragraph will clarify the choices which led to the alternative installation in the code.

6.5

Establishment in Code_Aster

6.5.1 ARPACK

The package of origin [bib29] (**ARPACK** for ARnoldi Package) coding the method is available in freeware on Internet. These originators, D.Sorensen, R.Lehoucq and C.Yang of Rice University of Houston, wanted it at the same time:

- simple of access (FORTRAN77, “reverse communication”),
- flexible (it is based on libraries LINPACK, LAPACK [27] and BLAS [bib36] (BLAS is the acronym for BASIC Linear Algebra Subprograms)),
- and rich in functionalities (decomposition of Schur, shifts flexible, many spectral transformations).

Its effectiveness is multiplied by ten by the use of **very optimized BLAS** from level 2 and 3 and by the setting in

place “**reverse communication**”. The user is thus a Master of his structures of data and of its procedures of treatment concerning the operator and the scalar product of work. It is him which provides to routines ARPACK this type of information. That thus makes it possible to manage as well as possible, with

tools and the procedures ASTER, the products matrix-vector, factorizations, resolutions of system...

6.5.2 Adaptations of the algorithm of Sorensen

To deal with generalized modal problems, this package proposes a whole series of spectral transformations, in reality or complex, square or not. Into square, the algorithm of Sorensen is based on the couple Lanczos-QL (IRLM for Implicit Restarted Lanczos Method). two approaches (square or not) are not designed besides to treat pseudo-products scalars related to indefinite matrices.

Precisely, for at the same time **circumscribing numerical problems involved in their properties enough**

heterogeneous in the code and, in addition, to ensure itself of a **better total robustness**, we have chosen **to work in nonsymmetrical (IRAM with Arnoldi and QR)**, on the **couple operator of following scalar work-product:**

(A -

)

B -

1

1

B U =

U

!# "

#

\$

##

μ -

With

!"\$

(X, y) = yTx

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One could have dealt with the problems of buckling in “buckling mode” via the same “shift and invert”

and

the scalar pseudo-product introduced by **A**. But because of quasi-systematic introduction of Lagranges, this matrix becomes indefinite even singular, which disturbs the process largely. The same causes produce the same effects when, for a calculation of dynamics, one **A** resorts to scalar **B-product**. Rather than to modify all the package while introducing a scalar pseudo-product, we thus chose a simple scalar product “Euclidean” more robust and much less expensive.

It was thus necessary to modify the procedures of “reverse-communication” of the package, because it did not envisage

this option (with matrices standards one classically prefers to enrich the components with one matrix scalar product, even in nonsymmetrical). Contrary to the alternative of Newmann & Pipano (cf [§5.5.1]) introduced for Lanczos, we deliberately placed ourselves in one nonsymmetrical configuration. But in order to avoid as much as doing it can the problems of orthogonality

recurring, even into symmetrical, we would have chosen the version of IRAM using Arnoldi.

The disadvantage of this step is that it is necessary, in preliminary postprocessing of IRAM, **B-orthonormaliser** approximate clean vectors to find property 2 numerically exploited by the modal recombinations. This stage does not disturb the base of clean modes exhumed and it is very effectively carried out via the IGSM of Kahan-Parlett.

The taking into account of the limiting conditions and, in particular of the double dualisations, was led as for Lanczos according to the procedure describes in [§2.2]. In particular, once the vector initial in acceptable space one is applies the operator of work to him. This traditional process allows to purge the vectors of Lanczos (and thus the vectors of Ritz) of the components of the core. In addition, in certain configurations for which the **number of frequencies requested** p is **equal to the number of ddls active** (cf [§2.2]), one owed bluffer the algorithm which stopped in error fatal! Indeed, it detected generally well space invariant awaited (from size p), but because of particular structure of the clean vectors associated Lagranges (cf proof of property 4, [§2.5]) it had much evil to generate an initial vector which is proportional for them.

One would have needed particular treatments taking account of the classification of these Lagranges, which

would have been all the more expensive as they are not fundamentally necessary to solve it problem requested! All the spectral information being already present at deepest of the algorithm, it is not thus the sorrow to complete the two remaining iterations (when the user asks $p = \text{nddl-credits}$, one imposes $m = p + 2$ automatically). It is enough to short-circuit the natural wire of the algorithm, to withdraw the interesting modes of Ritz and post-to treat them to return in space initial.

Note:

This type of case of figure in which one seeks a number of clean modes very near to ddls numbers leaves the “ideal” perimeter of use of this type of algorithm (cf [§2.8]). A good QR would be without any doubt more effective, but it is a good means of testing the algorithm.

6.5.3 Paramétrage

To be able to activate this method, it is necessary to initialize the key word METHOD with “SORENSEN”. Size of

subspace of projection is determined, either by the user, or empirically from formulate:

$$m = \min (my (X 2p, p + 2), nddl-credits)$$

where

p
is the number of eigenvalues to calculate,

$nddl-credits$

is the number of degrees of active freedom of the system (cf [§2.2])

The user can always impose to him even dimension by indicating it with the key word DIM_SOUS_ESPACE of the key word factor CALC_FREQ.

The parameter of the IGSM of Kahan-Parlett (cf [Appendix 2]) PARA_ORTHO_SOEN, *the* maximum number

iterations of the total process, NMAX_ITER_SOEN, and the criterion of quality control of the modes, PREC_SOEN, are accessible by the user under the key word factor CALC_FREQ. When this last key word is null, the algorithm initializes it with the precision machine (= 2.22.E-16 on SGI).

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6.5.4 Posting in the file message

The example below resulting from the list of case tests of the code (ssll103b) recapitulates the whole of the traces

managed by the algorithm. One finds in particular, for each critical load (or frequency), the estimate of its quality via the standard of error.

Here, IRAM reiterated only once and used 30 IGSM (in its first phase). The resolution total consumed 91 products (makes some, less than that because of “implicit” introduction of the product Euclidean scalar) matrix-vector and 31 inversions of system (just increase because the operator of work is already factorized).

THE NUMBER OF DDL

TOTAL EAST: 68
LAGRANGE EAST: 14
THE NUMBER OF ACTIVE DDL EAST: 47

The SELECTED OPTION EAST: **PLUS_PETITE**
THE VALUE OF SHIFT CRITICAL LOAD EAST: 0.00000E+00

INFORMATION ON CALCULATION REQUIRES:

A NUMBER OF REQUESTS MODES: 10
THE DIMENSION OF REDUCED SPACE EAST: 30

=====
= METHOD OF SORENSEN (CODE ARPACK) =
= VERSION: 2.4 =
= DATE: 07/31/96 =
=====

A NUMBER OF RESTARTINGS = 1
A NUMBER OF PRODUCTS OP*X = 31
A NUMBER OF PRODUCTS B*X = 91
A NUMBER OF REORTHOGONALISATIONS (STAGE 1) = 30
A NUMBER OF REORTHOGONALISATIONS (STAGE 2) = 0
A NUMBER OF RESTARTINGS OF A NULL V0 = 0

CRITICAL LOADS CALCULEES INF. AND SUP. ARE:
CHARGE_CRITIQUE_INF: -9.96796E+06
CHARGE_CRITIQUE_SUP: -6.80007E+05

MODAL CALCULATION: METHOD OF SIMULTANEOUS ITERATION

METHOD OF SORENSEN

NUMBER CRITICAL LOAD NORMALIZES ERROR

1 -6.80007E+05 5.88791E-12
2 -7.04572E+05 1.53647E-12
3 -7.09004E+05 1.16735E-12
4 -7.10527E+05 1.72306E-12
5 -7.11205E+05 2.41783E-12
6 -7.11542E+05 7.88981E-13
7 -7.11703E+05 5.71621E-13
8 -1.50492E+06 1.17776E-11
9 -6.02258E+06 2.42221E-11
10 -9.96796E+06 3.55014E-12

CHECKING A POSTERIORI OF THE MODES

IN the INTERVAL (- 1.00178E+07, - 6.76607E+05)
IT THERE A WELL 10 LOAD (S) CRITICAL (S)

Example 6: MODE_ITER_SIMULT with “SORENSEN”

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Note:

The introduction of this method made it possible to balance many cards of anomalies related to multiplicities, of the clusters or research of eigenvalues of orders of magnitude very different on which Lanczos and Bathe & Wilson stumbled.

Now let us recapitulate the paramètrage available of operator MODE_ITER_SIMULT with this option METHOD = “SORENSEN”.

6.5.5 Summary of the paramètrage

Key word factor

Key word

Default value

References

“DYNAMIC” *TYPE_RESU*

“DYNAMIC”

[§2.1]

“MODE_FLAMB”

[§2.1]

METHOD “SORENSEN”

“SORENSEN”

[§6.4]

CALC_FREQ

FREQ

[§4.4]

CHAR_CRIT

[§4. 4]

OPTION “PLUS_PETITE”

`PLUS_PETITE "

[§4.4]

“BAND”

[§4.4]

“CENTER”

[§4.4]

NMAX_FREQ

10

[§4.4]

DIM_SOUS_ESPACE

Calculated

[§6.5.3]

PREC_SOREN

0.

[§6.4], [§6.5.3]

NMAX_ITER_SOREN

20

[§6.4], [§6.5.3]

PARA_ORTHO_SOREN

0.717

[§6.5.3], [Appendix 2]

NPREC_SOLVEUR

8

[§2.6]

NMAX_ITER_SHIFT

5

[§2.6]

PREC_SHIFT

0.05

[§2.6]

SEUIL_FREQ

1.E-02

[§2.9]

VERI_MODE

STOP_ERREUR “YES”

“YES”

[§2.9]

“NOT”

[§2.9]

PREC_SHIFT

5.E-03

[§2.9]

THRESHOLD

1.E-06

[§2.9]

STURM “YES”

“YES”

[§2.9]

“NOT”

[§2.9]

Table 6.5.5-a: Summary of the paramètrage of MODE_ITER_SIMULT with “SORENSEN”

Note:

- *in V5, the user can specify the class of membership of its calculation by initializing it key word TYPE_RESU. According to this value, one informs vector *FREQ* or *CHAR_CRIT*,*
- *one finds all the “tripaille” of parameters related to the preprocessings of the test of Sturm (NPREC_SOLVEUR, NMAX_ITER_SHIFT, PREC_SHIFT) and with postprocessings of checking (SEUIL_FREQ, VERI_MODE),*
- *at the time of the first passages, it is strongly advised to modify only them principal parameters noted in fat. The others relate to more the mysteries of the algorithm and they were initialized empirically with values standards,*
- *in particular, to improve quality of a mode, the fundamental parameter is dimension of subspace DIM_SOUS_ESPACE.*

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7

Method of Bathe and Wilson (METHOD = `JACOBI)

7.1 Principle

The method of Bathe and Wilson is a **method of simultaneous iterations** which consists in **extending the algorithm of the iterations opposite**. One works starting from the problem shifted $\mathbf{A} \mathbf{X} = (-) \mathbf{B} \mathbf{X}$.

One distinguishes four parts in the algorithm [bib1], [bib4]:

· to choose

p initial vectors independent $\mathbf{X}, \mathbf{X}, \dots, \mathbf{X}$

1

2

p and to build matrix \mathbf{X} that they generate,

- to calculate the clean elements in the subspace of Ritz while solving

$$(\mathbf{A} - \mathbf{B}$$

$$\mathbf{T}$$

$$\mathbf{T}$$

$$\mathbf{I}$$

) $\mathbf{u}_i = 0$ where $\mathbf{A} = \mathbf{Q} \mathbf{A} \mathbf{Q}$ and $\mathbf{B} = \mathbf{Q} \mathbf{B} \mathbf{Q}$. One returns then to space initial (for the clean vectors) via the transformation $\mathbf{X} = \mathbf{q} \mathbf{u}$ where $\mathbf{U} = \{ [\mathbf{u}_i] \}$,

- to test the convergence of the clean modes I.

7.2

Tests of convergence

The method of Bathe and Wilson converges towards p smaller eigenvalues provided that the p initial vectors are not **B-orthogonal** with the one of the clean vectors. In addition, matrices **With** and **B** tend towards diagonal matrices. For this reason and as the matrices are full, one uses the method of Jacobi (cf [Appendix 3]) to find the elements clean of the subspace of Ritz.

To test the convergence of the eigenvalues, one classifies them after each iteration by order growing in absolute value and one looks at if, for each eigenvalue, the following test is checked

$$k+1 \quad K$$

$$K$$

$$+$$

$$-$$

1

—

where the exhibitor K indicates the iteration count. If after NMAX_ITER_BATHE iterations, one does not have

converged for all the eigenvalues, a message of alarm is transmitted in the file message.

7.3

Establishment in *Code_Aster*

7.3.1 Dimension of the subspace

If one wishes to calculate p eigenvalues, it is recommended to use under space of dimension Q higher. One will check convergence only for the R smaller eigenvalues where $p \ll R \ll Q$. It seem that $r = p$ is not sufficient: one can find the good eigenvalues but the vectors clean are not correct (convergence is slower for the clean vectors than for eigenvalues). $R = (p + Q)/2$ seems a good choice. For Q one usually takes [bib1]

$$Q =$$

$$($$

$\min p + 8, 2 p)$.

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7.3.2 Choice of the initial vectors

To choose Q initial vectors, one operates in the following way:

With

· first vector such as X

H ,

I

$1 = \mathbf{Bii}$

· for the other vectors of 2 with ($Q -$)

1

0

0

1

-- line 2

I

$\mathbf{X} = 1$

-- line I

\mathbf{X}

2

$1 Q - 1 = 0$

,...

0

0

With
where I are the indices the corresponding to smallest successive values of II ,

Bii
· last
vector

xq, random vector.

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7.3.3 Parameters in Code_Aster

To be able to use the method of Bathe and Wilson, order `MODE_ITER_SIMULT` should be chosen and to select `METHOD = "JACOBI"`. Two parameters relating to convergence directly method are accessible under the key word factor `CALC_FREQ` using the key words `PREC_BATHE` and `NMAX_ITER_BATHE`. One finds there also those managing the internal method of resolution modal, `PREC_JACOBI` and `NMAX_ITER_JACOBI`.

Key word factor

Key word

Default value

References

“DYNAMIC” `TYPE_RESU`

“DYNAMIC”

[§2.1]

“`MODE_FLAMB`”

[§2.1]

METHOD "JACOBI"

"SORENSEN"

[Appendix 3]

CALC_FREQ

FREQ

[§4.4]

CHAR_CRIT

[§4. 4]

OPTION "PLUS_PETITE"

`PLUS_PETITE "

[§4.4]

"BAND"

[§4.4]

"CENTER"

[§4.4]

NMAX_FREQ

10

[§4.4]

DIM_SOUS_ESPACE

Calculated

[§7.3.1]

PREC_BATHE

1.E-10

[§7.2]

NMAX_ITER_BATHE

40

[§7.2]

PREC_JACOBI

1.E-12

[Appendix 3]

NMAX_ITER_JACOBI

12

[Appendix 3]

NPREC_SOLVEUR

8

[§2.6]

NMAX_ITER_SHIFT

5

[§2.6]

PREC_SHIFT

0.05

[§2.6]

SEUIL_FREQ

1.E-02

[§2.9]

VERI_MODE

STOP_ERREUR “YES”

“YES”

[§2.9]

“NOT”

[§2.9]

PREC_SHIFT

5.E-03

[§2.9]

THRESHOLD

1.E-06

[§2.9]

STURM “YES”

“YES”

[§2.9]

“NOT”

[§2.9]

Table 7.3.3-a: Summary of the paramétrage of MODE_ITER_SIMULT with “JACOBI”

Note:

- *in V5, the user can specify the class of membership of its calculation by initializing it key word TYPE_RESU. According to this value, one informs vector *FREQ* or *CHAR_CRIT*,*
- *one finds all the “tripaille” of parameters related to the preprocessings of the test of Sturm (NPREC_SOLVEUR, NMAX_ITER_SHIFT, PREC_SHIFT) and with postprocessings of checking (SEUIL_FREQ, VERI_MODE),*
- *at the time of the first passages, it is strongly advised to modify only them principal parameters noted in fat. The others relate to more the mysteries of the algorithm and they were initialized empirically with values standards.*

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8

Conclusion - Synthesis

The optimal perimeters of use of the modal operators of *Code_Aster* can be dissociated. When it is a question of **determining some eigenvalues** (typically a half-dozen) or **to refine some estimates**, operator **MODE_ITER_INV** is completely indicated. He gathers heuristic algorithms and those of type powers (cf [§3]), which were historically developed first to solve generic modal problems. He couples a phase of localization of eigenvalues (via a technique of bisection and a method of the secant) with an improvement of these estimates and a calculation of the clean vectors associated by a method of iterations opposite (crossbred, or not, of quotient of Rayleigh).

On the other hand, **to capture a significant part of the spectrum**, one A resorts to **MODE_ITER_SIMULT**.

This last federates the methods known as of “subspace” (Lanczos [§4], [§5], IRAM [§6], Bathe & Wilson

[§7]) which projects the operator of work in order to obtain an approximated spectrum of more reduced size (treated

then by a total method of type **QR** or Jacobi, cf [Appendix 1] and [Appendix 3]). In addition to theirs numerical qualities (reduced calculation complexities and memory, facilitated coupling of techniques of deflation, of restarting and acceleration...) and mathematics (good convergence, controls quality of the spectrum of the projected operator...), it should be noted that they necessarily do not require

knowledge of the operator of work but that of its action on a vector (this characteristic is very useful to treat large systems but it is not used in *Code_Aster* where one assembles all matrices before treating them).

Until now, these algorithms stumbled regularly on the same shelves: detection correct of multiple modes, modes of rigid body and a general way, treatment of packed spectrum. All this led to the appearance of “**phantom**” modes sometimes badly easy to detect (modes corresponding to multiplicities missed and being able to generate correct residues with the direction

of Aster, and this, more especially as the criteria of the post-modal checks were sometimes permissive (residue in 10^{-2} instead of the 10^{-6} current), decontaminated even insufficient (test of Sturm limited to the values

clean positive)) who cause distortions of results downstream from calculation, during projections on base modal. To be been free from the recurring problems to this type of approach, one thus proposed to enrich **MODE_ITER_SIMULT** (starting from V5) by the **algorithm WILL GO** (“Implicit Restarted Arnoldi”

[§6]).

This alternative of Arnoldi, initiated by D.C. Sorensen in 1992, makes real great strides for the resolution of

great modal systems on parallel supercomputers. Its framework of application is completely general, it deals with as well the real problems as complex, square or not. It is summarized in one

succession of factorizations of Arnoldi whose results control restartings automatically
statics and implicit, via polynomial filters modelled by implicit iterations **QR**.

The algorithm tent will bring an elegant remedy for the recurring problems raised by the others
approaches. There is not any more a question to be posed concerning the strategy of reorthogonalisation,
frequency of the restarts, their establishments, the criteria of detection of possible phantom modes...
In short, **numerically**, IRAM gets a better total robustness while improving them
calculation complexities and memory (especially compared to simple Lanczos such as that of Newman &
Pipano) for a fixed precision.

From a **functional point of view**, the establishment of this method allowed a profit in calculation
complexity
(observed) with minima of order 2. From now on, the user has a real control on the quality of the modes
via one
suitable parameter setting. One, moreover, did not reinforce the severity of the post-modal checks and
extended them
applicability.

The use of IRAM (by defect in MODE_ITER_SIMULT) is thus with advising in all them
case of figures, including for the search for some eigenvalues (one can even draw part
excellent properties of the method of the powers opposite concerning the calculation of vectors
clean and, to refine the result, by starting again MODE_ITER_INV with for estimate values
clean exhumed by MODE_ITER_SIMULT). Beyond their numerical specificities and
functional calculuses which are included in this document, one can synthesize the modal methods of
Code_Aster in the shape of table below (the **default values are materialized in
fat**).

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Operator

Algorithm

Key word

Advantages

Disadvantages

Perimeter

of application

MODE_ITER_INV

1st phase

(heuristics)

Calculation of some

Bisection

“SEPARATE”

modes

Calculation of some

Bisection +

“ADJUSTS”

Better precision

Cost calculation

modes

Secant

Improvement of

Initialization by

“NEAR”

Resumption of values

No the capture of

some estimates

the user

clean estimated

multiplicity

by another

process.

Cost calculation of this

phase quasi-no one

2nd phase

(method of

powers properly

said)

Basic method

Powers

“DIRECT”

Very good

Not very robust

opposite

construction of

clean vectors

Option of acceleration

Quotient of
"RAYLEIGH"
Improve
Cost calculation
Rayleigh
convergence
MODE_ITER_SIMULT
Calculation of part of
Bathe & Wilson
"JACOBI"
Not very robust
spectrum
Lanczos
"TRI_DIAG"
Not very robust
(Newman- Pipano)
IRAM (Sorensen)
"SORENSEN"
Increased robustness.
Better
calculation complexities and
memory.
Control
quality of the modes.

Table 8-a: Summary of the modal methods of Code_Aster

IRAM made it possible to balance all the software anomalies related to the generalized modal problems, but it seems that alone its variation per blocks or a version incorporating of the "purging and lock" [bib23] can guarantee to us "an quasi-infallible" detection of the spectrum of a standard operator (i.e too badly not conditioned).

A class of algorithm known as of "Jacobi-Davidson" [bib37] seems even more promising to treat pathogenic cases. It uses an algorithm of the Lanczos type that it packaged via a method of Rayleigh.

Let us conclude by noting that the algorithm WILL GO was not extended yet to the quadratic case in modal operators of the code [R5.01.02].

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Appendix 1 General information on algorithm QR

A1.1 Principle

The algorithms of the type **QR** (cf [§2.8]) were had a presentiment of per H. Rutishauser (1958) and were formalized

jointly by J.C. Francis and V.N. Kublanovskaya (1961). This fundamental method is often implied in the other approaches adapted better to deal with the problems of large sizes (in particular methods of projection).

For $K = 1, \dots$ to make

H = Q R (factorization **QR**),

K

K

K

H

= $\mathbf{R Q}$;

$K + 1$

K

K

Fine buckles.

Algorithm 1.1: Theoretical QR

The process leads repeatedly towards a matrix **triangular HK higher** (or triangular by blocks) whose diagonal terms are the eigenvalues of the initial operator $\mathbf{H} = \mathbf{H1}$. The notation \mathbf{H} is not in fact not innocent, because there is any interest with transforming beforehand orthogonally (in a manner to modify only the shape of the shifted operator and not his spectrum) the operator of work \mathbf{A} in the form of higher Hessenberg, that is to say into arithmetic real

$$\mathbf{H} = \mathbf{Q}^T \mathbf{A} \mathbf{Q}$$

1

0

0

This can be carried out via various orthogonal transformations (Householder, Givens, Gram-Schmidt...) and their cost (about $O(10n^3/3)$) negligible is compared with the profit that they allow to realize with each iteration of the total process: $O(N^2)$ (with Householder or Fast-Givens one more precisely has $O(2n^2)$ against $O(4n^2)$ with simple Givens) against $O(n^3)$. Its profit of an order magnitude can be even improved when the operator is tridiagonal symmetrical (it is the case of Lanczos with a true scalar product): $O(20n)$.

Convergence towards a simple triangular matrix is carried out only if all the eigenvalues are distinct modules and that if the initial matrix is not "pathologically" too poor in clean directions. The convergence of the *ième* mode (arranged classically by order descending of modulate) is carried out then in:

J

max

,

$J I$

I

what can prove very slow if no complementary process is implemented.

Note:

- *determination of the spectrum of the matrix of Rayleigh with the alternative of Newmann & Pipano (cf [§5.5.1]) is carried out via a QR (or a QL into symmetrical) simple of this type (with, with precondition, balancing). The only parameter accessible by the user is the maximum number acceptable iterations NMAX_ITER_QR,*

- *for IRAM (cf [§6.4]), this calculation is carried out via a method QR with double explicit shift whereas the polynomial filters managing the restarts use a QR with double implicit shift.*

By prudence, no parameter is accessible for the user in Code_Aster!

- *one should not confuse the method, algorithm QR, and one of its conceptual tools, factorization QR,*

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- *this class of algorithm is very much used to determine the complete spectrum of an operator, because it is very robust (it is the reference in this field). However it is very greedy memory from there what places makes its use crippling on great systems,*
- *the perimeter of application of algorithm **QR** is much more general than that of Jacobi (it is the second standard algorithm providing all the spectrum of an operator for can that one is ready to entirely store it) who is limited to the square matrices.*

To accelerate the convergence of the simple algorithm which can be very slow (in the presence of clusters

for example) a multitude of alternatives, based on the choice of shifts answering certain criteria, were born.

A1.2 strategy of the shift

This strategy consists in **causing artificially a phenomenon of deflation** (cf [§3.1], [§5.4.1]) within the matrix of work. That offers triple favours:

- of to be able **to isolate a real eigenvalue** even two combined complex eigenvalues,
- all in **reducing the size** of the problem to be treated,
- and in **accelerating convergence**.

In its version with simple explicit shift, the method is rewritten then in the following form:

For $K = 1, \dots$ to make

To choose the shift μ ,

$$\mathbf{S}_k = \mathbf{H}_k - \mu \mathbf{I},$$

$$\mathbf{S} = \mathbf{Q} \mathbf{R} \text{ (factorization } \mathbf{QR}),$$

K

K

K
 H
 $= R Q + \mu;$
 $K + 1$
 K
 K
 I

Fine buckles.

Algorithm 1.2: QR with simple explicit shift

Note:

This process spreads intuitively with several shifts. One builds then, for each total iteration K , as many auxiliary matrices S_{ik} of shift μ_i .

The convergence of the process is largely improved in the direction where under-diagonal terms cancel themselves asymptotically in:

$J - \mu$
max

.
 $J I$
 I

- μ

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In theory, if this shift μ is eigenvalue of the problem, then deflation is exact. In practice, the effects of round-off disturb this phenomenon, it is what is called the **property of direct instability of the algorithm**. The principal difficulty lies in the choice of (or of) the shifts. In addition, one does not keep

not the same shift for all the iterations. One must change some when it is associated a value clean converged. Indeed, it will have numerically caused its ousting of the spectrum of work in causing a deflation with the preceding iteration.

Since the Sixties a whole **zoology of shifts developed**. While simplest use the last diagonal term **H_{n, N}**, that of **J.H. Wilkinson** [bib24] consists in determining analytically the eigenvalue μ of the diagonal block

H**H** $n-1, n-1$ $n-1, N$ **H****H** $N, n-1$ N, N

nearest to this term. This technique makes it possible to even obtain a quadratic convergence cubic (in the symmetrical case) and it proves particularly effective to capture modes double or of the distinct eigenvalues of the same module. However this strategy can appear ineffective in the presence of combined complex clean modes.

The same author then proposed an **alternative including a double shift** corresponding to both complex eigenvalues μ_1 and μ_2 (beforehand given) of the accused block. But in addition to numerical difficulties with réfreiner appearance of invading complex components (which in theory do not take place to be), one has in addition much evil to preserve the character of “Hessenberg higher” of the matrix of work. As well as possible, that slowed down the convergence of algorithm **QR**, with

worse, that distorts its operation completely.

In order to stage with these underhand numerical disadvantages, J.C. Francis developed a version with **double implicit shift**. It is very well clarified in [bib6] (pp377-81) and one will be satisfied just here to summarize philosophy of it.

To minimize the effects of round-offs, it would be necessary to constitute the auxiliary matrix resulting **S_k** directly

simultaneous application of the two shifts μ_1 and μ_2

$$\mathbf{S} = \mathbf{H}^2 - (\mu_1 + \mu_2) \mathbf{H} + (\mu_1 \mu_2) \mathbf{I}$$

$$\mathbf{S} = \mathbf{H}^2 - (\mu_1 + \mu_2) \mathbf{H} + (\mu_1 \mu_2) \mathbf{I}$$
 K K K

before factorizing it in form **QR** and building the new one reiterated **H_{k+1}**

$$\mathbf{S} = \mathbf{Q} \mathbf{R}$$
 K K K ,
$$\mathbf{H}_{k+1} = \mathbf{Q} \mathbf{T} \mathbf{H} \mathbf{Q}^T$$
 K K K

K.

1

But only the cost of the initial assembly (about $O(n^3)$) the tactics make inoperative. This alternative, based on the **Q-implicit** theorem, consists in applying to the matrix of work of particular transformations of Householder allowing to find a matrix of Hessenberg “primarily” equalizes in **Hk+1**, i.e. of the type:

~

H

1

 $1 = \mathbf{E H}$ **E****E**

K +

K +1

with = dia (

G ± 1. ±

.)

1 .

The matrix of work thus preserves, at lower cost, its particular structure and its spectrum.

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All these alternatives are in fact very sensitive to the techniques of balancing established for préconditionner the initial operator. The following paragraph will summarize this technique of “setting with

the scale” (“balancing” for the Anglo-Saxons) of the terms of the matrix of work which is very employee in modal calculation.

A1.3 Balancing

It is a question of **mitigating the effects of round** by attaching the **perimeter of expansion of the terms of**

the operator of work, i.e. to prevent that they do not become too small or, on the contrary, too large. On this subject, E.E. Osborne [bib17] (1960) noticed that generally the error on the calculation of clean elements of \mathbf{A} is about \mathbf{A}^2 where is the precision machine. It proposed then to transform the initial matrix into a matrix

~
With $= \mathbf{D} \mathbf{A}$
 \mathbf{D} (with \mathbf{D} a diagonal matrix),
 such as:

~
With $\ll \mathbf{A}$.

2
 2
 In fact one calculates a succession of matrices repeatedly

With $= \mathbf{D}^{-1} \mathbf{A}$

\mathbf{D}
 K
 $K - 1$

$K - 1$

$K - 1$

such as:

With

With

K

K

F

checking **have**

$=$ **have** with **have** and **has**

2

2

I , respectively, the $i^{\text{ème}}$ column and line of \mathbf{A}^k .

Note:

· *this technique was generalized to any matrix standard induced by the discrete standard of l_q*

Q

with l_q the whole of complex continuations (one) N such as one \ll ,

N

· *its employment is very widespread in scientific computation and in particular among the direct solveurs of*

system of linear equations.

The base of calculation of the computer, noted, intervenes in the determination of the terms of matrices

\mathbf{D}^k .

In order to minimize the rounding errors, the elements of \mathbf{D}^k are chosen so that they are powers

of this base.

Are **Rk** and **Ck** the p standards (in practice $p= 2$ is often taken), respectively line and column I of the **Ak-1** matrix (I is selected in such way that $I - 1 K - 1 [M]$). By supposing that

R C

K

$K 0$, one shows whereas there is a single entirety signed such as:

$2 -1 \mathbf{R}$

$< K 2 +1$.

Ck

Either $F =$, one defines matrix **Dk** such as:

Id + (-)

1 E and

p

p

p

p

F

II

if $(\mathbf{Ck} F) + (\mathbf{R} K/F) < \mathbf{C} + \mathbf{R}$

D

$(K K)$

$K = \mathbf{Id}$

if not

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where $0 < 1$ is a constant and **I.E.(internal excitation)** the *ième* canonical vector. One builds then repeatedly:

D

= **D D**

K

K

K -1,

,

WITH = D-1A

D

K

K

K -1 K.

by initializing **D0= Id.** the process stops as soon as **D**

Id

K =

.

Note:

· before carrying out balancing, a research of the isolated eigenvalues is carried out in detecting the presence of lines and quasi-null columns (all the terms are null, except that placed on the diagonal). When there are some one can, by carrying out permutations suitable, to put the matrix of work in the following form blocks:

*

*

*

Y
Z

0
*

0
X
T

*
*

0
0
*

0
*

*It is then necessary to carry out balancing only on the central block **X** because the terms diagonal of the two higher triangular matrices are eigenvalues of the matrix of work.*

.
 $p = 1$
 $= 0.9$
.5

*in Code_Aster, one chose
and
(cf [bib19]),
· before applying method **QR**, one starts by balancing the matrix of work and then,
one transforms it in the form of Hessenberg higher. Once calculation **QR** carried out, one*

go up vectors of Ritz to the clean vectors via the reverse of the transformations orthogonal due to the setting in Hessenberg form and balancing. It is this process which is set up as well in Lanczos in IRAM. However in addition to the fact that it require the storage of these orthogonal matrices, it is also and especially extremely sensitive to the effects of round-offs. Thus, it would be largely preferable to estimate the vectors clean via a method of the powers opposite initialized by the eigenvalues exhumed.

A1.4 method QL

Factorization **QL** of a matrix **A** consists with orthonormaliser its vectors columns of starting with the last (contrary to algorithm **QR** which begin with the first) thus giving a matrix **L** triangular lower regular. It is shown besides that algorithm **QL** applied to invertible operator **A** is equivalent to algorithm **QR** applied to **A**-* (transposed matrix combined reverse of **A**).

The method installation in *Code_Aster* is identical to the method of simple shift of J.H. Wilkinson seen previously by adapting the search for this shift to the minor 2x2 highest.

Note:

*Contrary to **QR** which captures the eigenvalues by order ascending of module, one obtains here preferentially dominant modes, then others, by order descending of module.*

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Appendix 2 Orthogonalisation of Gram-Schmidt

A2.1 Introduction

One saw on several occasions in this document that the **quality of orthogonalisation** of a **family of vectors** is **crucial** for the good **unfolding of the algorithms** and the **quality of the modes** obtained.

This task is besides to permanently realize from where the importance of a fast algorithm.

The simple algorithms of orthonormalisation are deduced from the traditional process of Gram-Schmidt but they are often “conditionally stable” (the quality of their work depends on matric conditioning of the family with orthonormaliser). This defect of robustness can prove problems to treat situations particularly badly conditioned. One prefers to them then

more expensive but robust algorithms, containing projections or of rotations: transformations spectral of Householder and Givens.

In practice, for the establishment of the method WILL GO in *Code_Aster*, we retained one **iterative version of the process of Gram-Schmidt** (the IGSM of Kahan-Parlett [bib18]) It carries out one

good compromise between the robustness and calculation complexity since it is unconditionally stable and orthogonaliser allows except for the precision machine, in, to the maximum, twice more time that traditional Gram-Schmidt (GS).

In the following paragraphs we will detail the operation of the basic algorithm, thus that that of these two principal alternatives. First was installation in MODE_ITER_INV and second is used in MODE_ITER_SIMULT. Comparative very percussion of these methods is declined in [bib23] (pp33-36) starting from a very simple example.

A2.2 Algorithm of Gram-Schmidt (GS)

Being given K vectors independent of N , (\mathbf{x}_i)

one wishes to obtain K orthonormal vectors

$I=1, K$

(compared to an unspecified scalar product) (\mathbf{y}_i)

space which they generate. In others

$I=1, K$

terms, one wishes to obtain another orthonormal family generating same space. The process of traditional orthonormalisation of Gram-Schmidt is as follows:

For $I = 1, K$ to make

For $J = 1, I - 1$ to make

Calculation of $r_{ji} = (\mathbf{y}_j, \mathbf{x}_i)$

J

I);

Fine buckles.

Calculation of $\mathbf{y}^0_i = \mathbf{x}_i - R \mathbf{y}$

j_i

$J,$

$J =,$

$1 \ i-I$

Calculation of II

$R = \mathbf{y}^0_i,$

\mathbf{y}^0

Calculation of \mathbf{y}

I

$I =$

;

II

R

Fine buckles.

Algorithm 2.1: Algorithm of Gram-Schmidt (GS)

This process is simple but very unstable because of the rounding errors, which causes to produce nonorthogonal vectors. In particular when the initial vectors are almost dependent that creates important variations magnitude in the second stage of the process

$\%y = \mathbf{X} - R y$

I

I

ij

J

$J =,$

$1 i-1$

From a numerical point of view, the management of these variations is very difficult.

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Note:

· the problem is completely similar to that met by the inversions of systems put in place in the code (cf [§2.6]),

· by noting **Q** the matrix generated by (\mathbf{y}_i)

, one explicitly was thus built

$I=1, K$

factorization **QR** of the initial matrix **X** related to (\mathbf{x}_i)

. It is in fact the goal of very proceeded

$I=1, K$

of orthogonalisation.

A2.3 Algorithm of Modified Gram-Schmidt (GSM)

In order to évincer these instabilities numerical one reorganizes the preceding algorithm. Mathematically equivalent with the preceding process, this one is then much more robust because it avoids the variations of

magnitude important between the vectors handled in the algorithm.

In the initial process, the orthogonal vectors \mathbf{y}_i are obtained without taking account of the $i-1$

preceding orthogonalisations. With Modified Gram-Schmidt, one **orthogonalise more gradually by taking account of preceding deteriorations** according to the process below.

For $I = 1, K$ to make

$\%y_i = \mathbf{X}, I$

For $J = 1, I-1$ to make

Calculation of $r_{ji} = (\mathbf{y}, J \%y_i)$,

Calculation of $\%y_i = \%y_i - R_{ji} \mathbf{y}_j$;

j_i

J

Fine buckles.

Calculation of II

$R = \%y$,

I

$\%y$

Calculation of \mathbf{y}

I

$I =$

;

II

R

Fine buckles.

Algorithm 2.2: Algorithm of Modified Gram-Schmidt (GSM)

The orthonormality of the basic vectors is much better with this process and it can be even obtained with the precision machine and except for a constant (depend on the conditioning of \mathbf{X}). However, for to treat situations particularly badly conditioned, this “conditional” stability can prove quickly problematic, from where the recourse to the following iterative algorithm.

Note:

GSM is twice more effective than a method of Householder to obtain a factorization

QR of the initial matrix \mathbf{X} . It requires only $O(2nk^2)$ operations (with N the number of lines matrix).

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A2.4 Algorithm of Iterative Gram-Schmidt (IGSM)

To ensure itself nevertheless of **orthogonality except for the precision machine**, one recommends to realize

one **second orthogonalisation**. And if, at the end of the latter, orthogonality is not assured it is not any more the sorrow to start again, the handled quantities are then certainly very close and their variations oscillate around zero. This approach is based on a theoretical analysis due to W. Kahan and taken again by B.N. Parlett [bib18] (cf pp105-110).

For I = 1, K to make

%yi = \mathbf{X} , I

For J = 1, I -1 to make

Calculation of $r_{ji} = (\mathbf{y}, J \% \mathbf{y}_i)$,

Calculation of $\sim \mathbf{y}_i = \% \mathbf{y}_i - R \mathbf{y}$,

ji

J

If

$\sim \mathbf{y}_i \% \mathbf{y}$ then

I

%yi $\sim \mathbf{y}$, I

Exit buckles in J;

If not

Calculation of $r_{\sim ji} = (\mathbf{y}, \sim \mathbf{y}$

J

I),

Calculation of $\sim \sim \mathbf{y}_i = \sim \mathbf{y}_i - \sim r \mathbf{y}$,

ji

J

If

~

~

$\mathbf{y}_i \sim$

\mathbf{y} then

I

%yi $\sim \sim \mathbf{y}$, I

Exit buckles in J;

If not

%y $\mathbf{0}$,

I

Passage to I following;

End if.

Fine buckles.

Calculation of II

$R = \%y,$

I

y

Calculation of y

I

$I = \%;$

II

R

Fine buckles.

Algorithm 2.3: Algorithm of Iterative Gram-Schmidt of type Kahan-Parlett (IGSM)

During the use of IRAM in *Code_Aster*, the criterion is paramètré by the key word PARA_ORTHO_SOREN (cf [§6.5]). It is shown that its interval of validity is $[1.2e, 0.83-]$ with precision machine and one generally allot value 0.717 to him (by defect).

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Note:

- *more the value of the parameter is large, less the reorthogonalisation starts, but that affects the quality of the process,*
- *contrary to the version “house” developed for Lanczos (cf [§5.5.1]) here criteria stops concentrate on the standards of the vectors orthogonalized rather than on the products scalars inter-vectors which tend more to reflect the effects of round-off. That, joined to suppression of the iterations higher than two, therefore useless, can explain the addition of effectiveness of the version of Kahan-Parlett,*
- *according to D.C.Sorensen the paternity of this method is rather to allot to J. Daniel and Al (paper of 1976 subjected to Mathematics of Computation, vol.30, pp772-795).*

Appendix 3 Method of Jacobi

A3.1 Principle

Method of Jacobi [bib4], [bib6], [bib11] allows to calculate all the eigenvalues of one

generalized problem whose matrices are definite positive and symmetrical (the matrices obtained with each iteration by the method of Bathe & Wilson check these properties; cf [§7]). It consists with to transform matrices **A** and **B** of problem $\mathbf{A} \mathbf{U} = \mathbf{B} \mathbf{U}$ into diagonal matrices, while using successively orthogonal similar transformations (matrices of rotation of Givens). process can be schematized in the following way:

$$\mathbf{A}_1 = \mathbf{A}$$

$$\mathbf{B}_1 = \mathbf{B}$$

$$\mathbf{A}_2 = \mathbf{Q}_1^T \mathbf{A} \mathbf{Q}_1$$

1

$$\mathbf{B}_2 = \mathbf{Q}_1^T \mathbf{B} \mathbf{Q}_1$$

1

1

1

1

1

...

$$\mathbf{A}_k = \mathbf{Q}_{k-1}^T \mathbf{A}_{k-1} \mathbf{Q}_{k-1}$$

1

$$\mathbf{B}_k$$

*T**K* -*K* -*K* -

$$= \mathbf{Q}$$

$$\mathbf{B}$$

$$\mathbf{Q}$$

1

1

1

K -1*K* -1

stamp

stamp

$$\mathbf{A}_k$$

$$\mathbf{A}_D$$

$$\mathbf{B}_k$$

Data base*K*

diagonal

K

diagonal

Algorithm 3.1: Process of Jacobi

-1

AD

The eigenvalues are given by = **AD (data base)** that is to say = H and stamps it clean vectors

Bdii

check:

1 **AD**

11

D

1 2

K

1/ A

$X = Q Q \dots Q \dots$

22

D

1/**Ann**

Each Q_k matrix is selected so that a term (I, J) nondiagonal and not no one of A_k or of B_k is null after the transformation (for the choice of this term, cf [§0]).

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A3.2 Some choices

In this algorithm, one realizes that the important points are as follows:

- Comment to choose the terms to be cancelled?
- Comment to measure the diagonal character of the matrices when K tends towards the infinite one?
- Comment to measure convergence?

A3.2.1 nondiagonal Terms to cancel

For the choice of the terms to be cancelled, there are several methods:

- the first consists with each stage K , to choose the largest element modulates some not diagonal of the \mathbf{A}_k matrix or \mathbf{B}_k and to cancel it by carrying out a rotation (cf [§0]). This choice ensure the convergence of the method of Jacobi but is relatively expensive (search for the maximum element),
- the second solution consists in successively cancelling all the nondiagonal elements of these matrices while following the natural order $\mathbf{A}_k, \dots, \mathbf{A}_k, \mathbf{A}_k$

K

13

1n

23, When one arrives at **Year-1**, N , one

start again the cycle. This method converges slowly.

An alternative of this method, consists very of following the natural order of the terms, to cancel only those which are higher than a given precision K . At the end of a cycle, one decreases value of this criterion and one starts again. It is this strategy which is used in *Code_Aster*.

A3.2.2 Test of convergence

To test the convergence and the diagonal character of the matrices, one operates thus. It is checked that all

coupling coefficients defined by:

\mathbf{A}_{ij}

\mathbf{B}_{ij}

F

=

F

=

$I J$

\mathbf{A}_{ij}

\mathbf{B}_{ij}

$\mathbf{A}_{ii} \mathbf{A}_{jj}$

$\mathbf{B}_{ii} \mathbf{B}_{jj}$

are lower than a given precision (diagonal character of the matrices). One also controls convergence of the eigenvalues via the indicator

K

$K 1$

I

--

I

$F = \max$

I

$K - 1$

I

so that there remains lower than a given precision *jaco*.

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A3.2.3 Algorithm in Code_Aster

The algorithm implemented in *Code_Aster* is summarized with:

Initialization of the matrix of the vectors specific to the matrix identity.

Initialization of the eigenvalues.

To define the precision of necessary dynamic convergence.

To define the precision total *glob*.

For each cycle $k=1, n_{\max} jaco$

K

To define the dynamic tolerance: $K = (dyn)$,

$L = 0$,

For each line $i= 1, N$

For each column $j= 1, N$

K, L

K, L

A_{ij}

B_{ij}

Calculation of the coupling coefficients $F K L, =$

$F K L, =$

$I J,$

A_{ij}

K, L

K, L

B_{ij}

K, L K, L

A_{ii} A_{jj}

B_{ii} B_{jj}

If *F K L*

With, or *F K L*

K

ij

B, then

ij

Calculation of the coefficients of the rotation of Givens,

Transformation of the **A_k** matrices, *L* and **B_k**, *L*,

Transformation of the clean vectors,

L = L + 1

End if.

Fine buckles.

Fine buckles.

K, L

With

Calculation of the eigenvalues *K*

II

I =

K, L.

B_{ii}

K

K 1

I

- -

I

Calculation of *F = max*

-

I

K 1

.

I

K, L

K, L

A_{ij}

B_{ij}

Calculation of the total coupling coefficients $F = \max$

$F = \max$

With

B

I, J

K, L

K, L

I, J

$K, L K, L$

WITH A

B B

$I J$

II

jj

$I J$

II

jj

If F

and

F

and

F

With

$glob$

B

$glob$

$glob$ then

Correction of the clean modes (let us divide by **Bkii**),

Exit;

End if.

Fine buckles.

Algorithm 3.2: Method of Jacobi established in Code_Aster

One notes $n_{\max jaco}$ the maximum number of allowed iterations.

Note:

*The matrices **A** and **B** being symmetrical, only the half of the terms is stored in form of a vector.*

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A3.2.4 Stamps rotation

One seeks with each stage to cancel the terms being in position I and J ($I J$) of the matrices

A k , L

,

and **B k L** by multiplying them by a matrix of rotation which with the following form:

G = 1 L = 1, N; **G** = has; **G**

ll

ij

$ji = B$

+1

+

other terms being null. One wishes to have **A k , L**

= **B k , L 1**

ij

ij

= 0 what leads to:

has K, L

K, L

K, L

With

$II + (1 + b) \mathbf{A}$ has

+ B

ij

With $jj = 0$

has K, L

K, L

K, L

Bii +

$(1 + A B) \mathbf{B} + B$

ij

B $jj = 0$

because $\mathbf{A}k, L + 1$

WP $\mathbf{A}k, L \mathbf{G}$

B $k, l+$

=

1

and

= **GTB** $k, L \mathbf{G}$. If the two equations are proportional one a:

K, L

With

$has = 0$ and B

ij

= - K, L

With jj

if not:

C

C

= 2 $B has = - 1$

D

D

with:

C

K, L

K, L

K, L

K, L

= **A B - B A**

C

K, L

K, L

K, L

K, L

1

II

ij

II

ij

2 = **A** jj **B** $ij - B$ jj **A** ij

2

K, L

K, L

K, L

K, L

C

C

C = A

B

- **B A**

D

3

3

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jj

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jj

=

+

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sign c3)

C C

II

1 2

2

2 +

Note:

· if $D = 0$ then one is in the case proportional,

C 2

· if the matrix **B** is definite positive then

3

C C

1 2

2 +

is positive [bib11] what gives one well

feel with parameter *D*.

Summary A3.2.5 of the paramètrage

To reach the parameters which intervene in the method of Jacobi, the user must select following key words:

.

dyn key word *PREC_JACOBI* under the key word factor *CALC_FREQ*,

.

n_max_jaco key word *NMAX_ITER_JACOBI* under the key word factor *CALC_FREQ*.

The total precision *glob* is equal to the precision of convergence required in the method of Bathe and Wilson. There is thus $glob = bath$ (cf [§7]).

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Document: R5.01.02

Algorithms of resolution for the problem

quadratic with the eigenvalues

Summary:

In this document, we fix the theoretical framework of the methods of search for eigenvalues of the problem

quadratic which is developed in the code of *Aster* mechanics.

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1 Introduction

1.1

Position of the problem

Dynamic analysis or the study of the stability of the balance of a mechanical structure led, in tally of the linearized theory, to solve the matrix differential equation of the second order where

M

is the matrix of mass and inertia of the structure,

G

is the matrix induced by the gyroscopic effect (case of the revolving machines),

is a significant real parameter number of revolutions,

C

is the matrix of damping induced by dissipative forces.

K

is the matrix of rigidity of the structure,

E

is the matrix of viscous damping interns structure,

F

is the external force (which is null in the case of the search for balance).

1.2

Properties of the matrices

The matrices considered are with real coefficients.

Classically, one considers:

M

is symmetrical (semi) definite positive,

G

is antisymmetric,

C

is symmetrical,

K

is symmetrical not necessarily definite positive,

E

is antisymmetric.

Dice at the time, one realizes that the simultaneous presence of the matrix of damping and the matrix of gyroscopic effect destroys the symmetry or the antisymmetry of the term speed; pareillement internal damping producing an antisymmetric matrix, destroyed the property of symmetry of stamp rigidity.

In addition, the introduction of linear relations modifies the character of positivity of the matrices: stamp K is indefinite (with positive or negative eigenvalues).

1.3

Problem with the eigenvalues associated

The sought solutions are form (separation of the variables of space and time).

$$U(T) = E t.x \text{ with IC and X ICN}$$

What leads us to the quadratic problem with the eigenvalues according to:

the solution U (T) can be rewritten in the form:

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that one can put in matric form:

$$U(T) = [X] [E T] [K]$$

where

[X]

is the modal matrix

[N X 2n]

[E T] is a diagonal matrix

[2nx2n]

[K]

is a matrix unicolonne

[2nx1]

Proposal:

The modal matrix [X] perhaps used like stamps transformation to uncouple N equations of the quadratic problem of origin, its 2n columns are not linearly independent.

Note:

One determines [X], using the two following identities:

$$U(0) = [X] [K]$$

1.4

Some traditional particular cases.

These particular cases were lengthily studied (cf for example [MEI.67], [ROS.84])

M

G

C

K

E

values

vectors

clean

clean
rights
lefts
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2

Reductions with a linear form

One is interested in the possibilities of reduction of the quadratic problem in a generalized problem equivalent.

The principles of reduction are applicable to unspecified matrices, and if we consider it problem in (M, C, K) is only to simplify the talk.

2.1

Reductions with a linear form

There are several traditional methods to transform the quadratic problem into a problem generalized with the eigenvalues.

We will develop the method which consists in introducing speed like auxiliary variable:

For that we introduce an additional equality:

$$Z(T) - Z(T) = 0$$

where Z is a matrix not identically null.

Our initial system can then be rewritten in the matrix form in a space of dimension double of initial space

And the problem with the eigenvalues associated is in its opposite form

By supposing K and Z regular, the problem with the eigenvalues generalized thus obtained can formally to put itself in the standard form:

Notation: one poses $y = X$

The standard form associated the quadratic problem is independent of the matrix régulière Z chosen.

Definition

One will call linear reduction of the quadratic problem, any generalized problem of which all them clean elements ((X, y)) check:

,

(X) is clean solution of the quadratic problem,

.

,

and $My = MX$ (condition of coupling)

.

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Proposal:
If ((X, y) T) is a solution of the linearized equation and if Z is regular then (, X) is solution of Pr, obblème quadratic.

This result is immediate.
2.1.1 A particular choice for matrix Z: ±M

one takes for matrix Z the matrix - M of the initial system, dice at the time the linearized system (A.z =

B.z)

.
is written:

μ
If the matrices M, C and K are symmetrical and Si the matrix M is regular, this linear reduction allows to build a symmetrical generalized problem.

one takes for matrix Z the matrix M of the initial system, dice at the time the linearized system (A.z = B.z)

.
is written:

μ
If M and K are definite positive then B is also, but matrix A is not symmetrical.

Proposal:

If one chooses $Z = \pm M$ (matrix not identically null) and if $(, (X, y) T)$, with X not no one, is solution clean of the equation generalized then $(, X)$ is solution propr E of the quadratic problem.

2.1.2 Particular case of the positive matrix M definite

If the matrix of mass is definite positive then it a decomposition of Choleski admits $M = QT.Q$

If one introduces the linear transformation $U (T) = Q^{-1}.q$ into the basic equation of the quadratic problem after pre-multiplyhaving multiplied it by $Q^{-T} = (Q^{-1})^T$, one obtains:

and while defining $v =$
the preceding equation, leads us to the standard problem with the values clean $A.v = .v$ with

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2.1.3 Case of the symmetrical reduction for a singular matrix M

In this paragraph we suppose the matrices M, C and K symmetrical.

If M is singular and if the eigenvalue is semi simple then it Q exists an orthonormal matrix such as.

0

Indeed, the eigenvalue being semi-simple, the core of M admits a base made up of vectors clean. This base is C 0

omplétée and orthonormalized. The matrix Q admits for vectors columns them vectors of this base.

Let us note that M11 is symmetrical (bus M is) and regular.

One introduces then a regularization of the matrix M represented by the matrix MR. and defined by: where M22 is a regular symmetrical matrix (for example the matrix identity).

Property:

The generalized problem associated the matrices

is a symmetrical linear reduction of the quadratic problem.

Demonstration:

symmetry is ensured by construction.

· the “tildées” matrices check

·

thus

, that is to say still

However if

associated the eigenvalue of the generalized problem one is a clean vector has

MR. $y = MX$, therefore $My = M$

MX . The condition of coupling is established.

Let us suppose that under-vector X of the clean vector that is to say no one. Then the equation

·

MR. $y = MX$ led to $y = 0$ bus MR. is regular, which is absurd.

The condition $X \neq 0$ is thus established.

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Note: Interpretation of the regularized matrix MR.:

This symmetrical linear reduction is distinguished from that presented previously by substitution of M per MR . in matrix A .

If M is singulière, this substitution forces the vector to have a component there null in the core Mr . Ainsi the symmetrical linear reduction led to a generalized problem posed in under space of IR^{2n} such as $y \in \ker M - \{0\}$.

If K is regular, the preceding generalized problem admits a form reverses standard equivalent.

2.2

Property of orthogonality of the clean vectors

It is immediate to show that the complex clean modes check the properties of orthogonality exits of the following generalized problem if the matrices A and B are symmetrical:

If one develops the preceding expressions, by taking account of the linear reductions used one obtains the expressions:

Note:

The first equality is independent of the regularity of M ,

· the second equality is obtained directly if M is regular and is established if not in

·

using the basic change of the regularization of Mr .

The modes of the quadratic problem are thus not M , C or K orthogonal.

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3

Method of determinant

3.1 General.

The search for zero of the characteristic polynomial, a quadratic problem to the eigenvalues, pose the problems inherent in the polynomials of the complex variable with complex values. Not having relation of order in IC, usual methods, at two points, of dichotomy type or secant are inapplicable.

We present here the “popular” method more for research of zero of polynomials of variable complexes with complex values, the method of Muller.

3.2

Method of Muller.

The method suggested by Muller [MUL.56] is an iterative method using as curve of interpolation a parabola with horizontal axis.

This method is relatively easy to implement but it lends itself badly to research zeros of real functions with real roots, because it plunges the interpolation in the complex plan and this even on the basis of actual values.

Its interest is related to the class of this method “the methods by curves of interpolation”, namely:

sureté of the method by dichotomy, since research is carried out in a ball “

reducing " gradually,

that only the calculation of the function is necessary (calculation of not derived as in

method of Newton),

convergence is connected with a quadratic convergence.

The most widespread method of the methods by curves of interpolation is the method at two points known as secant.

3.2.1 Development of the method

Let us note $F(Z) = a_0z^n + a_1z^{n-1} + \dots + a_n$ the algebraic equation which one seeks the zeros, have are them

complexes and we suppose a_0 not no one.

The quadratic formula of interpolation of Lagrange gives us:

$$L_i(F(Z)) = b_0z^2 + b_1z + b_2$$

and we consider the curve which passes by the last three points (reiterated):

$$(z_i, F(z_i)), (z_{i-1}, F(z_{i-1})), (z_{i-2}, F(z_{i-2}))$$

and thus the coefficients b_0, b_1 and b_2 check:

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one can rewrite the formula of interpolation of Lagrange
the new point is: $z_{i+1} = Z$
One can solve the quadratic equation in, we obtain then:

by taking the reverse of the traditional solution of a quadratic equation:
with

From $i+1$, one obtains:
 $h_{i+1} = i+1 h_i$,

$z_{i+1} = z_i + h_{i+1}$ which is one zero of the equation.

The sign of the denominator is then taken of such kind that it is of larger possible module and thus of such kind that $i+1$ is of larger possible module, and finally z_{i+1} will be the root nearest to

z_i

Note:
Muller [MUL.56] proposes a process of initialization while using:
“arbitrary” values $z_0 = -1.$, $z_1 = 1.$, $z_2 = 0.$

and values
year + year-1+ year-2
for F (z_0)
year - year-1+ year-2

for F (z1)

year

for F (z2)

This choice of values then results in considering:

$$L_2(F(Z)) = \text{year} + \text{year} \cdot z + a_n \cdot z^2$$

who is an approximation of F (X) in the vicinity of the origin.

The advantage of this process of starting is that it does not require any evaluation of the polynomial F (X) and

that it is thus fast.

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3.2.2 Convergence of the method

Proposal: To consider that convergence is assured as soon as for given is one acceptable criterion.

Let us show initially that during the unfolding of the algorithm three the reiterated successive ones are distinct.

If it were not the case, by supposing that $z_i = z_{i-1}$ and that x_i 1st are reiterated for which one has convergence and thus

If it is supposed that Z

$I = z_{i-2}$, one would then have I and $i+1$ identically null and thus one would have $z_{i+1} = z_i$

from where contradiction.

Then into constant that the difference between two reiterated can only decrease, one obtains the result announced.

3.2.3 Application of the method for research of eigenvalues

3.2.3.1 Development

That is to say to determine the eigenvalues of the system $(2M + K + C) X = 0$,

We thus seek the zeros of the polynomial Ca

ractér

istic that we note

$$F(Z) = \det(z^{2M} + z^K + C)$$

To calculate in sequence the zeros of the polynomial, we use a technique of deflation.

Consequently the polynomial considered is:

where

are K zeros already calculated.

The use of the algorithm is immediate, and we benefit from this adaptation to formulate it way légèrement different.

Let us note

the value of the function to be interpolated at the point z_i at the time of the research from the k th zeros.

It is then practical, for the implementation, to reveal a few intermediate quantities:

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reiterated the l_{i+1} sought being solution of:

with

One deduces some:

the sign of the denominator being taken so that l_{i+1} (and thus h_{i+1}) are of smaller module

possible

consequently

To converge towards the eigenvalues with positive imaginary part (and thus at positive frequency), one takes

Notice on deflation:

When the eigenvalues appear per combined pairs, it is also necessary to eliminate combined found eigenvalues and which physically corresponds to a negative frequency

3.2.3.2 Cost of the method in term of factorization

With each iteration of the method one makes:

Matrix algebra part (evaluation of the characteristic polynomial)

a linear combination of three matrices

O (N²)

· a LDLt factorization of the combination die

O (nb²), B width of

.

bandage

the product of the diagonal terms divided by deflation

O (N)

.

Method part:

operations of the method with properly spoken

O (1)

.

Moreover it is advisable to add, as an assumption of responsibility (since it is a method at three points) twice the evaluation part of the characteristic polynomial.

Broadly this method costs (i+2) factorizations, for a solution calculated out of I iterations.

The cost of this method is such, that it must be to hold for small systems (case of development) or when it is important to have a very high degree of accuracy on the sought frequency.

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4

Methods of opposite iteration

The method of iteration opposite extends immediately to the quadratic problem by using its form “linearized”.

The linearization of the quadratic problem not being single, there exist several alternatives.

4.1

Method of iteration opposite suggested by Wilkinson

Wilkinson [WIL.65] proposes to bring back the quadratic problem to the following standard problem

By supposing M invertible, and by posing $y = X$

Being given an approximation of the sought eigenvalue, one can define the iterative process according to:

Matric equation, we deduce a system from equations to two unknown (y_{s+1} , x_{s+1}) and by combination of this system we deduce the expression from y_{s+1} and x_{s+1} .

4.2

Alternative developed by Jennings

When M and/or K are singular, one obtains a stable and equivalent quadratic equation in introducing an auxiliary parameter for a spectral shift of.

While replacing by
in the quadratic equation, one obtains the quadratic problem in:

μ

While noting
the dynamic matrix which is regular by construction, one
bring back to the standard problem such as it is proposed by Wilkinson while posing $y = X$

μ

This equation is stable in the direction where for strictly positive.

This process thus makes it possible “to regularize $\mu <$

“the 1

orders of magnitude of the matrices M , C , and K through
stamp dynamic $D()$.

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4.3

The algorithm of iteration reverses *Aster*

4.3.1 Implementation

This algorithm is available in *Aster* by operator `MODE_ITER_INV`.

That is to say a 0 approximate value of the sought eigenvalue, one builds the dynamic matrix $D(\lambda)$ that

one factorizes in LDLt form.

One initializes the iterative process by the vectors according to:

$$x_0 = \{(1, 0)\}$$

$$y_0 = 0 \cdot x_0$$

the iterative process to obtain the nth one reiterated:

Standardization of x_{n-1} and y_{n-1} to avoid the overflows of capacity:

Resolution of:

Calculation of y_n starting from x_n :

Evaluation

of

N:

This diagram can be put in the matrix form:

4.3.2 Criterion

of stop

We use a simple result [GOH.&al.86] relating to the polynomials of matrices of the form:

$$D(\lambda) = m^2 + C + K$$

That is to say λ an eigenvalue of the operator $D(\lambda)$ and x_0 an associated clean vector not no one checking $D(\lambda) x_0$

$= 0$ then we have the equality:

The criterion of stop is done on the relative variation of λ

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5

Lanczos method applied to the quadratic problem

In this chapter we suppose the matrices M , C and K real symmetrical, so that the problem quadratic associated to a linear form symmetrical $Az = Bz$ can be tiny room

where

MR is a regular matrix deduced from M which coincides with M if the latter is regular. One seeks to develop the arithmetic method preserving real one overall in order to obtain a problem reduces real.

5.1

Choice of a problem to be approached

5.1.1 Form reverses standard

One seeks an approximation of the couples (λ, X) of clean parts of the quadratic problem which correspond to the close eigenvalues of a shift $= +i$ complexes given.

The approximation of Galerkin of Pr
oblème spectral of an operator

S in

an S

ous-space of Krylov k_m

$= \text{span}(r_0, Sr_0, \dots, S^{m-1}r_0)$ makes it possible to approach the clean couples of elements of the operator S corresponding to the eigenvalues of larger modules.

The passage to the standard form reverses after spectral shift of generalized problem preceding, provides the spectral problem:

$$(A - B) - 1 B Z = Z$$

This problem admits the same clean vectors as the problem generalized and of the eigenvalues bound to those of the problem generalized by the relation:

Thus, an approximation of Galerkin of the spectral problem of the operator:

$$S = (A - B) - 1 B =$$

in under space of Krylov $k_m = \text{span}(r_0, Sr_0, \dots, S^{m-1}r_0)$ provides the approximation of the couples clean parts of the quadratic problem sought.

5.1.2 Strategies of shift

The operator S preceding being complex, it calls in a natural way the use of the arithmetic one complex. It is nevertheless possible to use the arithmetic real one to approach the couples clean elements in which one is interested. It is enough to use a real operator of the same vectors clean and whose eigenvalues of larger module correspond to those of the problem quadratic closest to.

Technique of double déc

alage spectral by and proposed by Francis within the framework of the method QR makes it possible to build such an operator

noted to him also S:

$$S = [(A - B) (A B)] - 1 B = [AB-1A - 2 A + I I2 B] - 1 B$$

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The major disadvantage of this technique is the filling of the matrix $AB - 1A - 2A + I$ if B is not diagonal.

Parlett and Saad [PAR.SAA.87] propose an alternative which uses the real part or the imaginary part of operator $(A - B) - 1B$:

$$S_+ = \text{Re} [(A - B) - 1B]$$

$$S_- = \text{Im} [(A - B) - 1B]$$

whose respectively noted eigenvalues μ_+ and μ_- are related to the eigenvalues of the problem

 μ_+ μ_-

quadratic by the relations:

μ_+ and μ_- carry out the maximum of their module in the vicinity of μ_0 .

 μ_+ μ_-

This approach preserves the arithmetic real one overall and avoids the disadvantage of the technique of Francis, if the calculation of a vector $S_{\pm} v$, where v are real, is carried out into arithmetic complex.

By noting that:

$$\text{Re} [(A - B) - 1B] = [(A - B) - 1 + (A B) - 1] B$$

$$\text{Im} [(A - B) - 1B] =$$

$$[(A - B) - 1 - (A B) - 1] B$$

This approach can be interpreted like a technique of double shift summons, by opposition with the approach of the double produced shift suggested by J-C.F. Francis.

5.2 Method of approximation

From now on S will indicate one of the real operators S^+ or S , one of its eigenvalues and P
 μ
 stamp of a scalar pseudo-product (i.e. a symmetrical bilinear form not necessarily
 defined positive).

5.2.1 Approximate problem and algorithm of Lanczos

When S is car-assistant for the pseudo scalar product induced by P , i.e.
 $(U, AV) P = (With, v) P U, v \in \mathbb{R}^{2n}$,

the method of Lanczos is used to generate a base of under space of Krylov k_m .

The spectral problem is then approached by projection P -orthogonal on k_m and the problem reduces thus
 obtained is represented in the base of the vectors of Lanczos by a real matrix tridiagonale of order
 m .

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The method of Lanczos extended to the scalar pseudo-products is defined by the formulas of recurrence following cf [R5.01.01]:

ro = arbitrary

.

O =, qo =

.

0

0

1 = Sign ((ro, ro) P)

1 =

0

for J = 1, 2, ..., m

.

$$J = (q_j, S q_j) P$$

$$r_j = S q_j - J J q_{j-1} J q_{j-1}$$

$$j+1 = \text{sign}((r_j, r_j) P)$$

$q_{j+1} =$
 If one notes Q_m the matrix $2n \times m$ vectors of Lanczos q_j these formulas write in the form matrix real following:

$$Q^T P Q_m = J_m = \text{diag}(1, \dots, m)$$

$$S Q_m - Q_m J_m T_m = m+1 \quad m+1 \quad r_m \quad E M^T \text{ with } E M^T = (\dots, 1)$$

0
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T_m the real matrix tridiagonale symmetrical $m \times m$:
 The product $J_m T_m$ is a nonsymetric matrix tridiagonale dice that J_m is not proportional to identity.

The two preceding matric relations make it possible to write:

$$Q^T P [S Q_m - Q_m J_m T_m] = 0$$

The application of this relation to a couple $((m), S(m))$ IC X IC_m of clean elements of the operator represented by the matrix J

μ

MTM gives:

$$Q^T P [S Q_m S(m) - Q_m S(m)] =$$

μ
0

who characterizes the couple $((m), Z(m) = Q_m S(m))$ IC X IC_{2n} like approximation of Galerkin by

μ

projection P-orthogonal on km of a clean couple of element of the operator S.

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5.2.2 Choice of a pseudo scalar product

The symmetry of matrices A and B ensures that bilinear forms associated the matrices $\operatorname{Re} [(A - B) - 1] - 1$, $\operatorname{Im} [(A - B) - 1] - 1$ and B is scalar pseudo-product,

that the operator left real S_+ (respectively imaginary part S) is autoadjoint for scalar pseudo-product induced by $\operatorname{Re} [(A - B) - 1] - 1$ or by B (respectively by $\operatorname{Im} [(A - B) - 1] - 1$ or by B).

The pseudo-products scalar induced by $\operatorname{Re} [(A - B) - 1] - 1$ and $\operatorname{Im} [(A - B) - 1] - 1$ is extensions of scalar product used in the alternative of Pipano-Neuman of the algor

ithme Lanczos.

If the matrix M is singular, the pseudo scalar product induced by B makes vectors where $y \in \operatorname{Ker} M$, of the quasi-null vectors. This disadvantage exists them also in the case of pseudo-Pr

oduits scalar of Pipano-Neuman type (in particular if the matrix of the scalar pseudo-product admits eigenvalues of null sum), but the occurrence of such an event is rare in practical.

5.3

Application to the quadratic Problem

The use of this approach requires, with the method of Lanczos, the calculation of the real vectors S_z and P_z for $Z \in \mathbb{R}^{2n}$.

5.3.1 Operator

spectral

That is to say the dynamic matrix $D()$ associated the spectral shift $= + I$ defined by:

2

$$D() = M + C + K$$

If $D()$ is regular then the complex operator $(A - B) - 1B$ can be written in the form

The calculation of Sz for $S = \text{Re} [(A - B) - 1B]$ and $S = \text{Im} [(A - B) - 1B]$ can be carried out without destroying structure dig matrices if the rear one arithmetic complex be T partially used in the algorithm:

Preparation into arithmetic complex

.
to form
 $D () = 2 M + C + K$
- to factorize
 $D ($
) under
form LDLT

-
Calculation of Sz

.
 $u1 = Cx \ u2 = MX \ u3 = My$
in IR
- $u4 = D () - 1 \ u1 + u2 + u3$
in IC
-

according to the choice of the operator one obtains:

-
 $S+ Z = \text{Re} [(A - B) - 1B] Z =$
 $S Z = \text{Im} [(A - B) - 1B] Z =$

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5.3.2 Operator of pseudo scalar product

Choice P = B

.

This choice is valid for the operators obtained in real left approach or part imaginary.

Calculations can be carried out without assembly of B and into arithmetic real.

Choice P = Re [(A - B) - 1] - 1

.

This choice corresponds to the partly real approach of the operator S.

If the dynamic matrix D (), where is the real part of is regular then the real operator (A - B) - 1B is defined by:

,

and the scalar pseudo-product is written:

$$\text{Re} [(A - B) - 1] - 1 = (A - B) + 2 B (A - B) - 1B$$

Consequently the calculation of Pz can be carried out like that of Sz while using exclusively the arithmetic real one.

This approach requires the use of an auxiliary real matrix to store factorized the dynamic matrix D ().

Choice $P = \text{Im} [(A - B) - 1]^{-1}$

This choice corresponds to the partly imaginary approach of the operator S .

Formally we have:

$$\text{Im} [(A - B) - 1]^{-1} = [(A - B) B^{-1} (A B) + 2 B B^{-1} B]$$

The matrix B is regular under the condition necessary and sufficient that M is it and

One obtains then:

$$\text{Im} [(A - B) - 1]^{-1}$$

If M is singular, one can establish this equality by defining pseudo-opposite

of B

by

The calculation of Pz can then be carried out without assembling the matrix P explicitly and while using exclusively the arithmetic real one.

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5.3.3 Cost of the Lanczos phase

The cost report corresponds to the allowance of additional vectors (3 realities and 1 complex or real) and with the allowance of the dynamic matrices used for the calculation of the operators S and P .

The following table summarize these allowances

= 0

= \mathbb{R}^*

$$= I I IR$$

$$= I IC$$

+

Approach

S

$K \in \mathbb{R}^{n \times n}$

$D \in \mathbb{R}^{n \times n}$

$D \in \mathbb{C}^{n \times n}$

$D \in \mathbb{C}^{n \times n}$

part

$P = B$

-

-

-

-

real

$P = \text{Re}(A - 1) - 1$

-

-

$K \in \mathbb{R}^{n \times n}$

$D \in \mathbb{R}^{n \times n}$

Approach

S

$D \in \mathbb{C}^{n \times n}$

$D \in \mathbb{C}^{n \times n}$

part

$P = B$

-

-

Imaginary

$P = \text{Im}(A - 1) - 1$

The cost in operation is divided into a fixed blow and a blow depend on the number of vector of Lanczos to calculate.

The fixed cost corresponds to factorization LDLT of the additional matrices (carried out in

the arithmetic associated one) and is worth $O(b^2n)$ if B is the bandwidth common to the matrices M , C and K .

The calculation of a vector of Lanczos requires:

2 scalar products of vector of \mathbb{R}^{2n} :

2

$O(2n)$

· 1 linear combination of 3 vectors of \mathbb{R}^{2n} :

3

$O(2n)$

· the calculation of Sz :

·

3 products matrix-vector in \mathbb{R}^n :

3

$O(2bn)$

1 linear combination of 3 vectors in \mathbb{C}^n :

3

$O(2n)$

1 descent-increase in \mathbb{C}^n :

$O(2bn)$

1 scalar product - vector in \mathbb{C}^n :

$O(N)$.

The calculation of Pz

·

For $P = b$:

3 products matrix-vector in \mathbb{R}^n :

$O(2bn)$

For $P = \operatorname{Re}(A - 1) - 1$:

5 products matrix-vector in \mathbb{R}^n :

5

$O(2bn)$

2 combinations linear of 3 vectors of \mathbb{R}^n :

6

$O(N)$

1 descent gone up in \mathbb{R}^n :

$O(2bn)$

1 linear combinations of 5 vectors of \mathbb{R}^n :

5

$O(N)$

For $P = \operatorname{Im}(A - 1) - 1$:

6 products matrix-vector in \mathbb{R}^n :

6

O (2bn)

2 linear combinations of 4 vectors of IRn:

8

O (N)

Broadly the cost of the Lanczos phase is out of O (b2n) + 10 m O (2bn).

5.4

Implementation in Aster

The matrices M and C are symmetrical semi-definite positive and the matrix K is symmetrical regular indefinite. The pseudo scalar product retained corresponds to the extension of that proposed by Neuman and

Pipano [R5.01.01].

This algorithm is available in *Aster* by operator MODE_ITER_SIMULT.

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5.4.1 Parameters of the implementation

The spectral problem with real operator is parameterized by:

value of the spectral shift IC,

· the choice of $\text{Re}(A - 1B)$

or of $\text{Im}(A - 1B)$

.

The pseudo scalar product is then $\text{Re}(A - 1) - 1$ or $\text{Im}(A - 1) - 1$ and allowances and factorizations of

additional matrices are carried out has minimum in type and a number, this in agreement with the table paragraph [§5.3.3].

5.4.2 Under space of approximation

The vector $r_0 \in \mathbb{R}^{2n}$ generating under space of approximation breaks up into where

R_N ; the choice selected consists in posing $\epsilon = 0$ and drawing by chance the components from,

while imposing to him null components in $\ker M_r$.

If the dimension of under space is not specified, it is calculated by the empirical formula:

$$m = 2 \text{ Min}(\max(p+7), 2p, N)$$

where p is the number of couples of elements suitable to approach.

the dimension of the subspace of approximation are doubled because the couples of clean elements complexes arise per combined pairs.

5.4.3 Strategy of reorthogonalisation

The use of arithmetic with finished precision, deteriorates the properties of orthogonality of the vectors of

Lanczos and with them the rate of convergence of the approximate couples.

The strategy of complete reorthogonalisation ensures the orthogonality of all the vectors of Lanczos, and the algorithm then has a behavior close to that into arithmetic exact.

This strategy of reorthogonalisation forces to preserve the vectors $P_{.j} \quad j=1, \dots, M_r$.

The reorthogonalisation of the vectors is carried out by the process of Gram-Schmid modified cf. [R5.01.01].

5.4.4 Implementation of the Lanczos phase

The selected implementation is that described in [R5.01.01] within the framework of the generalized problème.

El E is summarized by:

Entries

:

.

the matrices P and S: i.e. the matrices M, C and K and factorized the LDLT of the matrices

-
dynamic additional.

m the number of vectors to be generated,

- r0 the vector generating under space of Krylov,

- precision of orthogonalisation and the number maximum of authorized reorthogonalisation.

-
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Exits

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vectors of Lanczos (q_1, q_2, \dots, q_m),

- the diagonal $(1, 2, \dots, m)$ and on-diagonal $(1, 2, \dots, m)$ of the matrix tridiagonale

-

T_m

the vector $(1, 2, \dots, m)$ of the pseudo scalar products of the vectors of Lanczos.

-

Algorithm:

.

Generation of the first vector q_1 and coefficients $1, 1$ and 1

-

Buckle generation of q_j, J, J and J for $J = 2, \dots, m$

-

For $J = 2, 3, \dots, m$ to make

Calculation of the direction of q_j

Standardization of q_j , calculation of J and storage of P_{qj}

Réorthogonalisation so necessary compared to q_j for $I = 1, \dots, j-1$

Reactualization of q_j, J, J and J in the event of reorthogonalisation,

Calculation of J and J

5.4.5 Restoration of the approximations for the quadratic problem

Approximations

clean couples of the quadratic problem result from

clean couples

$J_m T_m$ matrix by:

extraction of the “high” part of the vector.

.
choice
of, root of the flexible quadratic equation
with
who checks the condition of

.
coupling $M [O N] Q_m$
=
 M

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I
T

C
2
C
I
K
K
2
Re
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Appendix 1 Interpretation of the complex eigenvalues

In the case of a symmetrical damping and in internal absence of damping, relations orthogonalities and owing to the fact that the clean elements appear per combined pairs, one has the relations

following:

If one notes

, one can then define:

and one can write the eigenvalue complexes in the following form:

for which one can give a physical interpretation of the eigenvalue

The imaginary part represents the oscillatory part of the solution

is the pulsation of the i-ème mode

the real term represents the dissipative character of the system

is the damping of the i-ème mode,

is the reduced damping of the i-ème mode.

Physical interpretation of the clean vectors:

The physical significance of the existence of a clean vector complexes, lies in the fact that if

structure vibrates on a clean mode, its various degrees of freedom do not vibrate with the same one phase ones compared to the others.

The modal bellies and nodes do not correspond of the stationary points, but

move during the movement.

Note:

one finds the traditional formulation of the deadened systems with 1 degree of freedom

are real and are many quantities intrinsic with a mode (modal quantities) and

dependent on the standardization of the mode.

We point out that the modes of the quadratic problem do not diagonalise the matrices M, K and C.

Notice on the real term of the eigenvalue:

If the real part of the eigenvalue is negative, then the clean mode is a movement

deadened periodical of pulsation

If the part R

éelle of the eigenvalue is positive, then the clean mode is a movement

periodical of amplitude increasing and thus unstable.

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2
M
0

1
C
C

X
M
0
0
N
M
1

K
K

X
1

0

M
C
0

N

.

- K

+
K

X

C

K
2n
- K
-1
M

-
+

0

-1
C
0
=

X
=
K

0
=
0

0

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Appendix 2 linear Reductions

form

problem

quadratic

generalized

(1)

generalized

(2)

standard

(1)

standard

(2)

Note: to obtain the forms standards it is necessary to suppose:

M and K régulières for the form (1)

· M regular for the form (2)

.

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Organization (S): EDF/IMA/MMN

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R5.01.03 document:

Modal parameters and standard of the clean vectors

Summary:

In this document, one describes:

- various possibilities in *Code_Aster* to normalize the clean modes,
- important modal parameters associated the clean modes.

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1

Definition of the problem to the eigenvalues

1.1 General

That is to say the problem with the eigenvalues according to:

To find

$$(\cdot) C \times Cn / (2 \mathbf{B} + \mathbf{C} + \mathbf{A}) = 0$$

éq 1.1-1

where **A**, **C**, **B** are positive symmetrical real matrices of order N .

Two cases are distinguished:

- quadratic problem: $\mathbf{C} = 0$,
- generalized problem: $\mathbf{C} \neq 0$.

is called eigenvalue and clean vector. In the continuation, one will speak about clean mode for and one will introduce the concept of Eigen frequency.

To solve this problem, several methods are available in *Code_Aster* and one returns it reader with the documents [R5.01.01] and [R5.01.02].

1.2 Problem**generalized**

The generalized problem can be written in the form:

To find

$$(\cdot) \times N / (-2\mathbf{B} + \mathbf{A}) = \mathbf{0}$$

éq 1.2-1

One introduces two other sizes which make it possible to characterize the clean mode:

$$= = ($$

$$2 F)$$

éq 1.2-2

where

: own pulsation associated the clean mode,

F : Eigen frequency associated the clean mode.

One also shows that the clean modes are **A** and **B** orthogonal, i.e.:

$$i^T \mathbf{A} \mathbf{J}$$

$$i^T$$

$$I$$

$$=$$

$$ij$$

With

éq 1.2-3

$$i^T \mathbf{B} \mathbf{J}$$

$$i^T$$

$$I$$

$$= ij$$

B

where $(I \mathbf{J}$

,
) are two clean modes.

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1.3 Problem**quadratic**

The quadratic problem [éq 1.1-1] can be put in another form of size doubles (one speaks about linear reduction [R5.01.02]):

To find

(

0 B - B 0,) $C \times Cn/$

+

= 0**éq 1.3-1****B****C****0****With****! 0 B!****- B 0**One poses in the continuation: **B =****With****B****C****= 0 A.**

Like matrices **A**, **C**, **B** are real, the values and clean modes imaginary are combined two to two.

One introduces three other sizes which make it possible to characterize the clean mode:

$$\begin{pmatrix} 2F \\ +I \\ +I \end{pmatrix} = +IB \text{ has } = -$$

éq 1.3-2

$$\begin{pmatrix} 1-2 \\ 1-2 \end{pmatrix}$$

where
 : own pulsation associated the clean mode,
F: Eigen frequency associated the clean mode,
 : reduced damping.

$$\mathbf{0} \mathbf{B} \\ - \mathbf{B} \mathbf{0}$$

It is also shown that the clean modes are orthogonal, i.e.:

$$\mathbf{B}$$

C and

$$\mathbf{0} \\ \text{With}$$

$$\begin{pmatrix} iT \\ J \\ iT \\ J \\ iT \\ I \\ iT \\ I \end{pmatrix} I + J) \mathbf{B} + \mathbf{C} = ij (2 I \mathbf{B} + \mathbf{C})$$

éq 1.3-3

$$- \\ iT \\ J$$

iT

J

2

iT

I

iT

I

$I J \mathbf{B} +$

With = ij

$(- I \mathbf{B} + \mathbf{A})$

where (

I

J

I, J) are the eigenvalues respectively associated with the clean modes (\cdot).

Note:

the clean modes are thus not \mathbf{A}, \mathbf{B} or \mathbf{C} orthogonal.

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2

Normalizes clean modes of the generalized problem

One supposes to have calculated a couple (\cdot) solution of the problem [éq 1.2-1]: λ is the eigenvalue associated the clean mode. One considers for the moment only the case of the generalized problem.

In *Code_Aster*, order NORM_MODE [U4.06.02] makes it possible to impose a type of standardization for the whole of the modes.

2.1

Components of a clean mode

That is to say a clean mode of components (J)

·
 $j = N$
 1,

Among these components, one distinguishes:

- the components or degrees of freedom called “physics” (they are for example the degrees of freedom of displacement (DX, DY, DZ), degrees of freedom of rotation (DRX, DRY, DRZ), it potential characterizing an irrotational fluid (PHI),...),
- the components of Lagrange (the parameters of Lagrange are unknown factors additional which is added with the “physical” problem initial so that the conditions with limits are checked [R3.03.01]).

In *Code_Aster*, one has three families of standards:

- standard Euclidean,
- standard: “larger component with 1” among a group of degrees of freedom defined,
- standard masses or unit generalized rigidity.

They successively are described.

Previously, one defines L a family of indices which contains m terms:

$$L = \{L, K, 1m \text{ with } 1 L\}$$

N and $1 m N$

K

K

·

2.2 Normalizes

Euclidean

$1/2$

m

2

The following standard is defined:

$= ($

2

$lk)$

$K 1$

$=$

1

1

1

One then obtains the normalized vector “

: ” =

“ $J =$

$J J =, 1$

$N.$

2

2

In *Code_Aster*, two standards of this family are available:

- NORME=' EUCL': L corresponds to the whole of the indices which characterize a degree of physical freedom,
- NORME=' EUCL_TRAN': L corresponds to the whole of the indices which characterize a degree of physical freedom of displacement in translation (DX, DY, DZ).

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2.3

“Larger component with 1 normalizes”

The following standard is defined:

= max

L

$K =, m$

K

1

1

1

One then obtains the normalized vector “

: ” =

“ $J =$

$J J =, 1$

N .

In *Code_Aster*, five standards of this family are available:

- $NORME='SANS_CMP=LAGR'$: L corresponds to the whole of the indices which characterize one physical degree of freedom,
 - $NORME='TRAN'$: L corresponds to the whole of the indices which characterize a degree of physical freedom of displacement in translation (DX, DY, DZ),
 - $NORME='TRAN_ROTA'$: L corresponds to the whole of the indices which characterize a degree of physical freedom of displacement in translation and rotation (DX, DY, DZ, DRX, DRY, DRZ),
 - $NORME='AVEC_CMP'$ or “SANS_CMP”: L is built either by taking all the indices which correspond to types of components stipulated by the user (for example the standard displacement following axis X: “DX”) ($NORME='AVEC_CMP'$), is by taking the complementary one of all the indices which correspond to types of components stipulated by the user ($NORME='SANS_CMP'$),
 - $NORME='NOEUD_CMP'$: L corresponds to only one index which characterizes a component of one node of the grid. The name of the node and the component are specified by the user (key words NOM_CMP and $NODE$ of order $NORM_MODE$ [U4.64.02]).
- By defect the modes are normalized with standard “SANS_CMP=LAGR”.

2.4

Mass or unit generalized rigidity normalizes

1/2

That is to say a positive definite matrix of order N . The following standard is defined:

$= (T$

E

E)

1

1

One then obtains the normalized vector “

: ” =

“ $J =$

$J J =, 1$

N

.

E

E

In *Code_Aster*, two standards of this family are available:

· NORME=' MASSE_GENE': $\mathbf{E} = \mathbf{B}$. In a traditional problem of vibration, B is the matrix of mass.

· NORME=' RIGI_GENE': $\mathbf{E} = \mathbf{A}$. In a traditional problem of vibration, A is the matrix of rigidity.

Note:

For a rigid mode of body, one a:

=

E

To = 0

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3

Normalizes clean modes of the quadratic problem

3.1

Euclidian norms and “larger component with 1”

For the quadratic problem, one has the same standards as for the generalized problem. clean modes being complex, one works with the square product. Various standards “traditional” become:

1/2

m

· square standard:

= $(L$

2

L

where is combined of,

K

K)

lk

lk

K 1

=

/

1 2

· standard “larger component with 1”:

= max *L* =

max

L

L

(the value

$K,$
 $1 m$
 K
 $K,$
 $1 ($

 K
 $K)$
 $=$
 $= m$

absolute in the real field becomes the module in the complex field).

3.2
Mass or unit generalized rigidity normalizes

With regard to the standard “masses or generalized rigidity”, denomination by analogy with generalized problem, one uses as matrix associated with the standard, that which intervenes in the writing quadratic problem put in the reduced form [éq 1.3-1].

One has then:

· standard masses generalized:

T
 T
 $!$
 T
 $T \mathbf{0}$
 \mathbf{B}
 $! =$
 \mathbf{B}
 $=$

 $= 2 TB + T$
 $,$
 $,$
 $\mathbf{C},$
 \mathbf{B}
 $($
 $($

B C

^
= 1
! #B,
· standard generalized rigidity:

T
T
!
T
T - B

0
!
=
With
=

= -2 *TB*
+ *T*
,

,
With
,
With
()
()

0
With

"
1
=

.
! With
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4

Modal parameters associated for the generalized problem

One in the case of places a traditional generalized problem of vibration. One a:

.

With = **K** is the matrix of rigidity,

.

B = **M** is the matrix of mass.

That is to say a couple (,) solution of the problem:

$$(- 2 \mathbf{M} + \mathbf{K}) = \mathbf{0}$$

éq 4-1

In the continuation, one defines successively the following sizes:

- sizes generalized,
- effective modal mass and unit effective modal mass,
- factor of participation.

To know the names of the parameters associated with the clean modes and how y to reach in structure of data RESULT mode_meca, one returns the reader to the document [U5.01.23].

4.1 Sizes

generalized

4.1.1 Definition

Two generalized sizes are defined:

- Masse generalized of the mode: m

T

= **M**,

- Rigidité generalized of the mode: K

T

= **K**.

These quantities depend on standardization on. These sizes are accessible in the concept RESULT of the type mode_meca [U5.01.23] under names MASS_GENE, RIGI_GENE.

Notice 1:

One with the following relation between the pulsation (or the frequency) of the mode and the mass and

rigidity

generalized of the mode:

T

\mathbf{K}

K

$= = ($

$2 F) =$

$= .$

T

$\mathbf{M} m$

Notice 2:

From the physical point of view, the generalized mass (which is a positive value) can be interpreted like the mass moving:

m

T

2

$= \mathbf{M} =$ where is the density of the structure.

The kinetic energy of the structure vibrating according to the mode is equal then to:

1

1

$E =$

2

m

2

T

C

$=$

Mr.

2

2

The potential energy of deformation associated with the mode is equal to:

1

1

$E =$

K

T

p

$=$

K.

2

2

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4.1.2 Use

During a calculation by modal recombination [R5.06.01], one seeks a solution of the equation of dynamics:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(T),$$

in the form $\mathbf{X} =$

$$I$$

$I(T)$ where I is the clean mode real associate with the eigenvalue I ,

$$i = m$$

1,

solution of the generalized problem (in general one has m N (N is the number of degree of freedom) because one

does not take into account that part of the modal base):

(- \mathbf{M}^2

$$I$$

$$I + \mathbf{K}) = 0$$

The generalized vector = (I)

is solution of:

$$i = m$$

1,

~

~

~

$$\mathbf{M} \ddot{\mathbf{x}} +$$

$$\mathbf{C} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{F}$$
 (problem of order m) with:

~

$$\mathbf{M} = (\sim \mathbf{M}$$

$$iT$$

$$J$$

iT J $ij) = ($ $\mathbf{M})$ \sim $\mathbf{C} = (\sim C_{ij}) = (\mathbf{C})$ \sim \cdot $\mathbf{K} = (\sim \mathbf{K}$ iT J iT $ij) = ($ $\mathbf{K})$ \sim $F = (\sim f_i) = (F)$

The modes of vibration of the generalized problem are \mathbf{K} and \mathbf{M} orthogonal [R5.01.01]. Matrices

 \sim \sim

\mathbf{M} and \mathbf{K} are then diagonal and are consisted of the rigidities and masses generalized of each

 \sim

mode. The matrix \mathbf{C} is usually full if one does not make additional assumptions on \mathbf{C} [R5.05.04].

4.2

Effective modal masses and unit effective modal masses

4.2.1 Effective modal masses

That is to say \mathbf{U}_D a unit vector in direction D . In each node of the \mathbf{U}_D vector having them components of displacement (DX, DY, DZ) one a:

$(DX = X, DY = y, DZ = Z$

 D D

$D)$ where $(X, y, Z$

 D D

$D)$ are the cosine Directors of the direction D (one has

thus: $x^2 + y^2 + z^2$

 D D

$D = 1$).

For example, if D is direction X , the vector \mathbf{U}_D has all its components DX equal to 1 and its other components equal to 0.

One defines the effective modal masses in the direction D by:

(T

2

Mr. Ud)

m D, = (

.

T

M)

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4.2.2 Property

Statement:

The sum of the effective modal masses in a direction D is equal to the total mass m_{totale} structure. That is written:

(iT

2

Mr. Ud)

m

=

where N is the total number of modes associated with

,

=1, (

=

iT

I

ID

I

N

M)

m

total

i=1 N

,

problem [éq 4-1]

4.2.3 Unit effective modal masses

By using the preceding property, one defines the unit effective modal masses:

2

T

MR. U

~

1

(

D)

m, D =

,

m

T

total

(M)

and one a:

~

m

= 1

I

.

, *D*

i=,

1 *N*

Modal masses ~

m, D and *m D*, are independent of the standardization of the mode of vibration.

4.2.4 Use

“Empirical” relation:

At the time of a study “seismic request of a structure in a direction *D*” by a method of modal recombination, one must preserve the modes of vibration which have a unit effective mass important and it is considered that one has a good modal representation if for the unit of the modes preserved one a:

~

m 0 9

I

, .

, D

$i =$,

1 N

This empirical relation is stated in the RCC_G (Rules of design and construction applicable to the Civil Engineering).

4.2.5 Directions privileged in *Code_Aster*

In *Code_Aster*, one has three directions which are those of the reference mark of definition of the grid:

$D =$ direction X,

$D =$ direction Y,

$D =$ direction Z.

The effective modal masses and the unit effective modal masses are accessible in concept RESULT of the type mode_meca [U5.01.23] under names MASS_EFFE_DX, MASS_EFFE_DY, MASS_EFFE_DZ, MASS_EFFE_UN_DX, MASS_EFFE_UN_DY, MASS_EFFE_UN_DZ.

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4.3

Factors of participation

4.3.1 Definition

One defines other parameters called factor of participation:

(T

Mr. Ud)

$p D, = ($

T

M)

This parameter depends on the standardization of the mode of vibration.

As for the effective masses, one has three directions D which are those of the reference mark of definition of the grid.

The factors of participation are accessible in the concept RESULT of the mode_meca type [U5.01.23] under names FACT_PARTICI_DX, FACT_PARTICI_DY, FACT_PARTICI_DZ.

4.3.2 Property

Statement:

The factors of participation associated with a direction D check the following relation:

 $(iT$ 2 iT **MR. U** 2 $D)$ **MR. U** 2 m $=$ **M**

,

where N is

 $=1, ($ D iT I $=$ iT I iT I I $=$ D I N **M)** $($ $)$ **M**

$(p) m$

total

$i=1 N$

,

$i=1 N$

,

the total number of modes associated with the problem [éq 4-1].

This result is obtained easily by expressing the factor of participation according to the modal mass effective and by using the result stated with [§ 4.2.3].

4.3.3 Use

These parameters are used in particular to calculate the response of a structure subjected to one seism by spectral method. One returns the reader to the document [R4.05.03].

4.4

Unit vector displacement

In what precedes, a unit vector of displacement **Ud** was considered which relates to only them degrees of freedom of translation (DX, DY, DZ). This concept can be extended to rotations in considering the following definition. One defines a matrix **U** of dimension $(N \times)$

6. If all nodes

grid support 3 degrees of freedom of translation and 3 others of rotation, the matrix **U** is formed of the stacking of the matrices **uktr** $(6 \times)$

6 following (the index K corresponds to the node of

D

number K):

1 0 0

0

(Z

$K - zc)$

- $(yk - yc)$

0 1 0 - $(zk - zc)$

0

$(xk - xc)$

K

0 0 1

$(yk - yc) - (xk - xc)$

0

utr =

0 0 0

1
0
0

0 0 0
0
1
0

0 0 0
0
0
1

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where $(X, y, Z$

K

K

K) are the co-ordinates of the node and $(X, y, Z$

C

C

c) are the co-ordinates of the center

instantaneous of rotation.

One can thus define effective modal masses, factors of participation associated with

degrees of freedom of rotation.

For the moment, the calculation of these parameters is not available in Code_Aster.

5

Modal parameters associated for the quadratic problem

One writes the quadratic problem in the form: $(2\mathbf{M} + \mathbf{C} + \mathbf{K}) = 0$.

For the quadratic problem, one calculates only three parameters which correspond to the sizes generalized following:

· generalized mass (real quantity):

m

T

$= \mathbf{M}$,

· generalized rigidity (real quantity):

K

T

$= \mathbf{K}$,

· generalized damping (real quantity):

C

T

$= \mathbf{C}$.

Attention, if one normalizes the clean mode with the standard “masses generalized”, one does not have in the case

quadratic: $m = 1$. One can make the same remark concerning generalized rigidity.

By using the relations of orthogonality and the fact that the clean elements appear per pairs combined, one can write the following relations:

TC

C

2

$2 (2 F)$

$=$

$= 2$

$($

$\text{Re}) = -$

$= -$

$,$

$TM m$

$1 - 2$

$1 - 2$

T

2

\mathbf{K}

K

2

2

(2 F)

=

= =

=

.

TM m

2

2

1-

1 -

6 Bibliography

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*Organization (S): EDF/IMA/MMN***Handbook of Reference****R5.02 booklet: Thermics****Document: R5.02.01****Algorithm of linear thermics transitory****Summary:**

One presents the algorithm of transitory thermics linear established within order THER_LINEAIRE [U4.33.01]. The various options of calculation necessary were presented in the elements of structure plans, axisymmetric and three-dimensional [U1.22.01], [U1.23.01] and [U1.24.01].

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Expression of the equation of heat in linear thermics

1.1

Equation of heat

One places oneself in open of \mathbb{R}^3 of regular border.

In any point of, the equation of heat can be written:

$T(R, T)$

- div ((

$\mathbf{Q}(R, T)) + S(R, T) = C$

$p T$

with:

\mathbf{Q}

vector heat flow (directed according to the decreasing temperatures),

S

heat per unit of volume dissipated by the internal sources,

C

voluminal heat with constant pressure,

p
 T

temperature,

R

variable of space,

T

variable time.

This equation translates the phenomenon of change of the temperature (only through phenomenon of diffusion, convection having been neglected) in any point of opened and at any moment.

It

in theory an infinity of solutions admits, but the data of the initial conditions and variation of boundary conditions in the course of time determines the evolution of the phenomenon perfectly.

1.2

Fourier analysis

In thermal conduction, the Fourier analysis provides an equation connecting the heat flow to the gradient temperature (normal vector on the isothermal surface). This law reveals, in its form more general, a tensor of conductivity. In the case of an isotropic material, this tensor is reduced to one simple coefficient, the thermal coefficient of conductivity.

$$\mathbf{Q}(R, T) = -T(R, T)$$

For the elements of anisotropic thermics one will refer to Implantation of the elements 2D and 2D Axisymmetric in mechanics and thermics [R3.06.02].

1.3

Equation of heat in the case of the linear model of thermics

By combining the two equations above, one obtains:

$$T(R, T) - \text{div}(-T(R, T)) + S(R, T) = CP T$$

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2

Boundary conditions, loading and initial condition

One describes here only the boundary conditions thermal leading to linear equations in temperature, which excludes the conditions of the radiation type.

2.1 Temperatures**imposed**

The conditions of the Dirichlet type, are usually treated by dualisation in *Code_Aster* (cf [R3.03.01]), but they can also be eliminated in certain cases (loads kinematics).

$$T(R, T) = T(R, T)$$

1

on

1

where $T R T$

1 (,) is a function of the variable of space and/or time.

2.2 Relations**linear**

It is of the conditions of the Dirichlet type, making it possible to define a linear relation between the values of

the temperature:

· between two or several nodes: with an equation of the form

 N

$$i T(R, T) = (T)$$

 $i=1$

· between couples of nodes: with an equation of the form

1

 N

2

 N

$$1i i T(R, T) + 2i i T(R, T) = (T)$$

/

/

 $I 1$

12

=

 $I 1$

21

=

where 12 and 21 are two under-parts of the border which one binds two to two the values of temperature. This type of boundary condition makes it possible to define conditions of periodicity.

2.3**Imposed normal flow**

It is of the conditions of the Neumann type, defining flow entering the field.

$$- \mathbf{Q} (R, T) \cdot \mathbf{n} = F (R, T)$$

on $\partial \Omega$

where $F (R, T)$ is a function of the variable of space and/or time and \mathbf{N} the normal indicates with border $\partial \Omega$.

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2.4 Exchange

It is of the conditions of the Neumann type modelling the convectifs transfers on the edges of field.

$$- \mathbf{Q} (R, T) .n = H (R, T) (T (R, T)$$

ext.

$$- T (R, T))$$

on 3

where T

$R T$

ext. (\cdot) is a function of the variable of space and/or time representing the temperature of external medium, and $H (R, T)$ is a function of the variable of space and/or time representing it coefficient of convectif exchange on border 3.

2.5 Exchange

wall

It is of the conditions of the Neumann type bringing into play two pennies left the border in opposite. This type of boundary condition models a thermal resistance of interface.

T

1

$$= H (R, T) (T (R, T) - T (R, T)) \text{ on}$$

N

N

2

1

12

1 normal external to 12

1

T

2

$= H(R, T) (T(R, T) - T(R, T))$ on

N

N

1

2

21

2 normal external to 21

2

$(N = -N$

1

2 in general)

2.6 Source

voluminal

It is the term $S(R, T)$ function of the variable of space and/or time.

2.7 Condition

initial

It is the expression of the field of temperature at the initial moment $T = 0$:

$T(R,)$

$0 = T(R)$

0

where $T R$

0 () are a function of the variable of space.

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Variational formulation of the problem

We will restrict ourselves here to present the problem with only the boundary conditions of

imposed temperature [§2.1], of imposed normal flow [§2.3] or of exchange [§2.4]. Conditions with limits of exchange wall [§2.5] are treated with [the §4] and those with linear relations [§2.2] are brought back

without difficulties with that of [§2.1].

That is to say open of \mathbb{R}^3 , border = 1 2 3.

The weak formulation of the equation of heat is:

$$\begin{aligned}
 &T \\
 &T \\
 &C \\
 &\int_V D + T \\
 &\int_V \\
 &D - \\
 &\cdot \\
 &= \\
 &\cdot
 \end{aligned}$$

$$\begin{aligned}
 &\nu D \\
 &S \nu D \\
 &P \\
 &T \\
 &N
 \end{aligned}$$

where ν are a sufficiently regular function cancelling itself uniformly on 1. With the conditions with following limits:

$$\begin{aligned}
 &T \\
 &= T(R, T) \\
 &\text{on} \\
 &1 \\
 &1 \\
 &T \\
 &= Q(R, T) \\
 &\text{on} \\
 &N \\
 &2 \\
 &T
 \end{aligned}$$

$$= H(R, T) (T(R, T) - T)$$

on

N

ext.

3

The variational formulation of the problem is:

T

C

$\int_V D +$

T

\int_V

$$D + hT \int_V D = S \int_V D + q \int_V D + HT \int_V D$$

P

T

ext.

3

2

3

4

Variational formulation of the problem with condition

of exchange between two walls

One considers the “simplified” problem where does not appear any more source term and where boundary conditions

are only of imposed the temperature type and exchanges wall.

That is to say open of \mathbb{R}^3 , border = $\{1, 2, 3\}$.

The boundary conditions are in this case:

T

$$= T(R, T)$$

1

)
on 1

 T

1

 $= H(R, T) (T(R, T$
2
 $) - T(R, T))$
on
 N
1
12

1
 T

2

 $= H(R, T) (T(R, T$
1
 $) - T(R, T$

))
on
 N

2
21
2
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In substituent in the weak formulation of the equation of heat, one obtains:

T

C

$\cdot \nu D$

p

+

T

$\cdot \nu$

D

T

+

$H(T$

$- T) \cdot \nu D$

/

$12 + H(T - T) \cdot \nu D$

/

0

12

21

21

21 =

12

12

21

where ν are cancelled uniformly on 1.

This type of boundary conditions reveals new terms connecting degrees of freedom located on the two borders in relation.

5

Discretization in time of the differential equation

A traditional way to discretize a first order differential equation consists in using one - method. Let us consider the following differential equation:

$$y(T) =$$

$$(T, y(T))$$

$$T y() = y$$

0

0

- Method consists in discretizing the equation by a diagram with the finished differences

$$1 (y - y) = (T, y) + 1$$

N 1

+

N

N 1

+

N 1

$$(-) (T, y)$$

T

+

N

N

where y_{n+1} is an approximation of $y(t_n)$

+1, y_n being supposed known

and is the parameter of the method, [

0]

1

, .

Note:

if = the 0 diagram is known as explicit,

if the 0 diagram is known as implicit.

5.1.1 Precision of the method

Let us suppose *there* sufficiently regular (at least 3 times differentiable), by a development of Taylor at the point tn one obtains:

t^2

$$y(T)$$

$$) y(T)$$

T

$$y'(T)$$

$$y''(T) O(t^2)$$

N 1

N

N
 N
 $)$
 $+ -$
 $=$
 $+$
 $+$
 2
and
 $(T, y(T))$
 1
 1
 $+$ (
 $n+$
 $n+$
 $1 -) (T, y(T))$
 N
 N
 $= y'(T)$
 $1 + ($
 $n+$
 $1) y'(T)$
 N
 $=$
 $y'(T$
 $)$
 $n+1 + (y'(T$
 $)$
 $1 - y'(T))$
 $n+$
 N
 $=$
 $y'(T)$
 2
 $N +$
 T
 $y''(T) + O(T$
 $)$
 N

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The solution thus checks roughly:

1 (

2

 $y(t_{n+1}) - y(T$

+

 $N))$

1

 $= (T, y(T)) + 1$ $N 1$

+

 $N 1$ $(-) (T, y(T$

+

 N $N)) + ($ $-) T y''(T) + O$ N (T) T

2

1

1

The diagram is of order 1 in time if, and of order 2 if = (diagram of Crank-Nicolson).

2

2

5.1.2 Stability of the method

Let us consider the following differential equation:

$$y' = -y$$

T O R

$$y(0) = y_0$$

0

By using it - method in this differential equation one obtains:

$$1 - 1$$

$$(-) T$$

y

y

1

$$0 N NR$$

$$n+ =$$

$$-1$$

$$1+ T$$

$$N$$

That is to say still:

$$1 - 1$$

$$(-) X$$

y

$$RN (T$$

) y

$$\text{with } R (X$$

$$n+ l =$$

$$0$$

$$) =$$

$$1+ X$$

The approximate solution y_n must be limited (the exact solution of the initial problem being it), which imposes

the following condition:

$$R (T$$

$$) 1$$

By studying the variations of the function $R (X)$, one notes easily that:

· if

12 the condition is checked whatever the T , the diagram is unconditionally stable;

2

· if

< 12 the condition is checked that if T

, the diagram is conditionally

1

$$(- 2)$$

stable.

In order THER_LINEAIRE [U4.33.01], the parameter is a data being able to be provided by the user, the default value is fixed at 0.57. This value with the reputation at Department MMN to be preferable with the value of Crank-Nicolson (0,5) and “optimal” for the quadratic interpolations, but we did not find trace of the justifications.

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5.1.3 Application to the equation of heat

Let us use it - method in the variational formulation of the equation of heat, where one posed:

$$T_+ = T(R, T_+ T)$$

$$) T = T(R, T) h_+ = H(R, T_+ T)$$

)

$$H = H(R, T)$$

$$F_+ = F(R, T_+ T)$$

$$) F_- = F(R, T) T_+ = T(R, T_+ T)$$

$$) T = T(R, T)$$

ext.

ext.

ext.

ext.

)

$$s_+ = S(R, T_+ T)$$

)

$$S = S(R, T)$$

T_+

$$1 = T(R, T)$$

1

$+ T$

)

$T -$

$$1 = T(R, T)$$

1

Let us introduce following spaces of functions:

$$V =$$

$$1()$$

$$/ =$$

+

(,)

$t+$

{ $v H$

v

$T R T$

1

1

}

$$V =$$

$$1()$$

$$/ =$$

-

(,)

T

{ $v H$

v

$T R T$

1

1

}

V

1

$$0 = \{v H () v = 0$$

1

}

The field $T -$

V

+

T being supposed known, one seeks T

$Vt+$:

$T+ - T -$

C

$v D$

+

-

p

+

(*T*

. *v* + (1) *T*

. *v*

) *D*

T

-

(*F* + + (1) *F* -) *v d2*

-

(*h* + *T* +

-

-

+

+

-

-

ext. + (1 -) *H Text - H T*

- (1) *H T*) *v d3*

2

3

=

(*s* + + (1) *S*) *v D*

v V

0

While posing:

(*HT*) = *h* + *T* + + (1) *HT* -

ext.

ext.

ext.

F = *F* + + (1) *F* -

one obtains finally:

$$CP T + v D + T +$$

. v

$$D + h + T + v D$$

T

3

3

CP

=

$$T - v D - (1)$$

-

$$T. v D + F v D$$

T

2

2

$$+ ((HT$$

- -

+

-

$$ext.) - (1) H T) v d^3 + (S + (1) S) v D$$

3

v V

0

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6 Discretization

space

That is to say pH a division space, let us indicate by NR the number of nodes of the grid, pi function of form associated with node I . One indicates by J the whole of the nodes belonging to border 1.

Are:

$VH =$

=

()

;

=

+

(,)

1

$t+$

{v

$v p X$

v

$T X T$

$J J$

I

I

J

J

}

$i=,$

1 NR

$VH =$

=

()

;

=
 -
 (,)
 1

T
 {*v*
v p X
v
T X T
J J
I
I
J
J
 }
i=,
 1 *NR*
V H
 0 = {*v = v p (X)*
 ;
v = 0
J J
I
I
J
 }
i=,
 1 *NR*

Let us pose:

C

K T
p
 =
T p p D
 + *T p. p*

D + h+ T p D
ij I

T I I J

H

I

I

J

H

I

I

H

3

H

H

H

3

C

L

P

=

T - p D - (1) T

- · P

D +

F p D

J

T

J

H

J

H

J

H

2

H

H

H

2

+ ((HT

) - (1) *HT* -) *p D* 3 +

(*s* + + (1) *S*) *p D*

ext.

J

H

J

H

H

3

H

By dualisant the boundary conditions in imposed temperature ([R3.03.01]), one reveals the operator *B* defined by:

0 if *J J*

(*Bv*) *J = v*

if

J J

J

One obtains finally the following system:

NR

K T

T

ij I

+ (*B*) = *L*

J

J

J

i=1

(*B T*) = *T (X, T)*

J

1

J

J

J

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7

Implementation in Code_Aster

7.1 Equations

introduced

Order THER_LINEAIRE [U4.33.01] makes it possible to treat the equation in the transitory case such that it is described above, but it also makes it possible to solve the stationary problem which is reduced with the following equation:

$$- \operatorname{div} (T) = S$$

in

and boundary conditions following:

T

$$= T (R, T$$

1

S)

on

1

T

$$= Q (R, T)$$

on

N

S

2

T

$$= H (R, T) (T (R, T) - T)$$

on

*N**ext.**S*

3

ts being the moment taken to evaluate the boundary conditions of the equation.

In the transitory case, it is necessary to provide an initial state, this initial state (field of temperature) can be selected among the following:

- a field which can uniform or unspecified be created by order AFFE_CHAM_NO,
- a field result of a stationary problem describes by the equations above, the moment of calculation is taken at the first moment defined in the list of realities describing the discretization temporal defined by the user,
- a field extracted the result of a transitory problem.

The discretization in time (value of T) must be provided in the shape of one or more lists moments. These lists are created by the user by order DEFI_LIST_REEL [U4.21.04].

A thermal transient can be calculated by carrying out several calls to the order THER_LINEAIRE [U4.33.01] by enriching the same concept of the evol_ther type while providing to leave the second call the initial moment of resumption of calculation (to obtain T -) and possibly the final moment.

The fields of temperatures resulting from a calculation contain at the same time the value with the nodes of the grid and

with the nodes of Lagrange. During a resumption of calculation, it is possible to vary the type of boundary conditions, the field used to initiate new in-house calculation is then tiny room to only nodes of the grid. The concept result of the evol_ther type will contain fields then with nodes being based on different classifications. The operators of *Code_Aster* interpolate then only with the nodes of the grid when classification differs.

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7.2

Principal thermal options calculated in *Code_Aster*

7.2.1 Boundary conditions and loadings

TEMP_IMPO

DDLI_R

$T + * D$

1 +

$v D$

DDLI_F

1

1

1

DDLI_R

$*T D$

1

DDLI_F

1

1

FLUX_REP

CHAR_THER_FLUN_R

$qv D$

CHAR_THER_FLUN_F

2

2

EXCHANGE

CHAR_THER_COEF_R

$h+ T+ v D$

CHAR_THER_COEF_F

3

3

CHAR_THER_TEXT_R

$((HT) - (-) H T -) v D$

1

CHAR_THER_TEXT_F

ext.

3

3

ECHANGE_PAROI
RIGI_THER_PARO_R

$h+ T+$

$T+$

$/ -$

$v D$

$($

$) 1$

RIGI_THER_PARO_F

$/$

12

21

12

12

CHAR_THER_PARO_R

$(1-) - (- - -)$

HT

T

$v D$

CHAR_THER_PARO_F

$/$

$/$

21

12

1

12

12

SOURCE

CHAR_THER_SOUR_R

$(s+ ($

$) S$

$+ -$

$) v D$

1

CHAR_THER_SOUR_F

7.2.2 Calculation of the elementary matrices and transitory term

RIGI_THER

$+ .$

T
v D

MASS_THER
CP T v D

+

CHAR_THER_EVOL
CP T v D - (1) T
. *v*
D

T

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Nonlinear thermics

Summary

Operator THER_NON_LINE [U4.54.02] allows to solve the problems of transitory thermics in

solids in the presence of non-linearities of the properties of the materials (heat-storage capacity and conductivity), or of

boundary conditions (heat exchange of radiation type). One presents here the formulation and the algorithm

employee, this last being close to that related to operator STAT_NON_LINE [R5.03.01]. Various options of calculation necessary were presented in the plane, axisymmetric elements of structure and three-dimensional [U3.22.01], [U3.23.01] and [U3.24.01].

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1**Expression of the equation of heat in thermics not linear****1.1****Equation of heat for a motionless solid**

In all this document, one treats only the thermal of the solid bodies, even if the change of liquid/solid phase is taken into account. There is thus no heat transfer by convection but only by conduction.

The first principle of thermodynamics connects the temporal variation of total energy dE_{totale} of one system included/understood in a volume of control to the work of the external efforts W and heat Q received by this same system:

of

 $= D (E$ $+ E$ $) = W$ $+ Q$

total

intern

kinetics

éq 1.1-1

By injecting the theorem of the kinetic energy in this equation, one reveals thus power of the interior efforts, function of the field speed [bib1]:

$$! E_{interne} = ! Q - P (U)$$
 I **éq 1.1-2**

For the resolution of the problem of thermics, the system is supposed without movement. Power interior efforts $pi (U)$ is thus null. Indeed, in the majority of the applications concerned, thermal phenomena and mechanics are uncoupled; density power density dissipated by plastic deformations, $pi = C. ! plastic$, is neglected in front of the heat exchanged on the surface or

other voluminal heat sources.

The equation [éq 1.1-2] which expresses the variation of heat in volume is written then:

D

S

$$E D = Q! = (rvol - div \mathbf{Q} D$$

éq 1.1-3

dt

)

S

S

where one noted:

E

internal energy,

density,

$rvol$

the voluminal rate of contribution external of heat,

\mathbf{Q}

the vector heat flow.

Moreover, since the solid is motionless, for any volume of control (

T) =, one obtains then

the local equation of conservation of heat:

of

$$= R - div \mathbf{Q}$$

éq 1.1-4

dt

flight

If all the system is actuated by a rigid movement of body, an additional term

appears in the member of left, utilizing the speed of the solid and the gradient of energy.

This situation is treated by order THER_NON_LINE_MO [R5.02.04].

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In the case of a reversible transformation, the equation [éq 1.1-4] becomes, with the assistance of the second

principle of the thermodynamics which makes it possible to write in our case of

$= TdS:$

intern

$T S! = R - divq$

éq 1.1-5

flight

and finally the equation of heat in its traditional form:

$C T! = R - div Q$

éq 1.1-6

P

flight

S

with the heat-storage capacity with constant pressure defined by: $C = T$

P

T

P

As it is explained in chapter 1.4, it can be advantageous to write the term of left of the equation [éq 1.1-6] with the enthalpy which does not depend whereas temperature:

!

$= R - div Q$

flight

éq 1.1-7

T

where $(T) =$

$C dT$

P

T0

1.2

Fourier analysis

In thermal conduction, the Fourier analysis provides an equation connecting the heat flow to the gradient temperature (normal vector on the isothermal surface). This law reveals, in its form

more general, a tensor of conductivity. In the case of an isotropic material, this tensor is reduced to one coefficient (being able to depend on the temperature), thermal conductivity:

$\mathbf{Q}(X, T) = - (T) \nabla T$

(X, T)

éq 1.2-1

1.3

Equation of heat in the case of the model of thermics non-linear transient

By combining the equations [éq 1.1-5] and [éq 1.2-1], one obtains:

$$D$$

$$R$$

$$\text{div} ((T) T$$

$$\text{flight} -$$

$$-$$

$$) =$$

éq 1.3-1

dt

or, if the heat-storage capacity does not depend on the temperature:

$$dT$$

$$R$$

$$- \text{div} (- (T) T$$

$$) = C$$

$$\text{flight}$$

p

éq 1.3-2

dt

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1.4

Numerical advantage of the formulation in enthalpy for problems with phase shift.

The relation between enthalpy and heat-storage capacity is:

T
 $(T) =$
 $C(U)$ of

p
 T_0

When this function enthalpy presents abrupt variations, it is more precise to handle (T) that its derivative. Indeed, paces characteristic of these functions in the vicinity of the melting point are as follows:

Enthalpy

Liquid

C
 $H(T)$

C

H

2

p

has

L

have

R

late

nte

$C(T)$

p

1

$C(T)$

p

2

Solid

Solid

$H(T)$

Liquid

1

Temperature

Temperature

During an iteration, either because the thermal transient is violent, or because the beach of phase shift very small (pure substance), two the is reiterated successive ones of the temperature can to be located discontinuity on both sides. The evaluation of the slope of the function enthalpy with vicinity of the melting point will be very false if one considers $C(T)$

p

$1), C(T)$

p

2) or an average

balanced of both. On the other hand, the slope of the right-hand side in dotted lines is always an approximation

correct of $D dT$ at the melting point.

2

Boundary conditions, loading and initial condition

One will refer to [R5.02.01] for the boundary conditions thermal and the loadings leading with linear equations in temperature like for the initial condition.

2.1

Non-linear normal flow

It is of the conditions of the Neumann type, defining flow entering the field.

- $\mathbf{Q} \cdot \mathbf{n} = G(X, T)$ on the border

éq 2.1-1

where $G(X, T)$ is a function of the temperature and possibly of the variable of space X and/or of time T and \mathbf{N} indicates the normal external with the border, \mathbf{Q} is the vector heat flow (directed according to the decreasing temperatures).

This expression makes it possible to introduce for example conditions of the type exchanges with a coefficient

of convectif exchange depend on the temperature:

- $\mathbf{Q} \cdot \mathbf{n} = G(X, T) = H(X, T) (T(X, T) - T)$

ext.- T)**éq 2.1-2***Handbook of Reference**R5.02 booklet: Thermics**HI-75/99/013 - Ind A***Code_Aster** ®*Version*

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2.2

Non-linear normal flow - condition of the radiation type ad infinitum

A particular case of the boundary conditions preceding is the radiation ad infinitum of gray body which results in a particular case of function $G(X, T)$:

$$- \mathbf{Q}(X, T) \cdot \mathbf{n} = [(T(X) +$$

273)

15 4 - (

4

$T +$

.

273)

15]

éq 2.2-1

The characteristics to be defined at the time of the definition of this loading are emissivity, the constant of

Stefan-Boltzmann $= 5,73.108$ $\text{W/m}^2 \cdot \text{K}^4$ and the temperature ad infinitum.

$T(R)$ and T are then expressed in degrees Celsius. 273.15°C is the temperature of the absolute zero.

3

Variational formulation of the problem

We will restrict ourselves here to present the problem with only the boundary conditions of imposed temperature [R5.02.01 §2.1], of imposed normal flow [R5.02.01 §2.3], of exchange [R5.02.01 §2.4], of nonlinear flow [§2.1] and radiation [§2.2].

That is to say open of R^3 , border = 1 2 3 4 5.

One must solve the equation [éq 1.1-4] in T on $\times] 0, T$ [with the boundary conditions:

T

$$= Td(R, T)$$

on

1

T

(T)

$$= F(R, T)$$

on

N

2

T

(T)

$$= H(R, T) (T(R, T) - T$$

)

on

N

ext.

3

éq 3-1

$$T$$

$$(T)$$

$$= G(R, T)$$

on

 N

4

$$T$$

$$(T)$$

=

4

4

$$[(T +$$

$$273)$$

$$15 - (T +$$

.

$$273)$$

15] on

 N

5

and with, possibly, of the initial conditions $T(T =)$

0. If these last are not specified, one

solves as a preliminary the stationary problem, i.e. the equation [éq 1.3-1] without the term of evolution temporal.

That is to say a v sufficiently regular function cancelling itself on 1, while noticing:

 D

$$(T) \cdot v D$$

$$! (T) \cdot v D$$

=

 dt T **éq 3-2**

$$(T) T$$

$$\cdot v D = - \operatorname{div} ((T) T$$

$$) \cdot v D + (T)$$

$$\cdot v D$$

N

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the weak formulation of the equation of heat can then be written:

D

T

(T) .v D

(T) T

. v

D - (T)

.v D = R

.v D

+

éq 3-3

dt

N

flight

One deduces the variational formulation from it from the problem:

$$D(T) \cdot v D + (T) T \cdot v d + hT \cdot v d$$

$$3 = \frac{d}{dt}$$

$$3 R \cdot v D + F \cdot v D$$

éq 3-4

$$2 + HT \cdot v D$$

3 + *flight ext.*

2
3
g.v D
4
4
27315
27315
4 +

$$[. (T + .) - (T + .)] \cdot v d 5$$

4
5
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4**Discretization in time of the differential equation****4.1****Introduction of - method**

A traditional way to discretize a first order differential equation is it - method.

Let us consider the following differential equation:

$$! y (T) = (T, y (T))$$

éq 4.1-1

$$y ()$$

$$0 = y0$$

- Method consists in discretizing the equation [éq 4.1-1] by a diagram with the finished differences

$$1 (y - y) = (T, y) + 1$$

$$N 1$$

+

$$N$$

$$N 1$$

+

$$N 1$$

$$(-) (T, y)$$
éq 4.1-2

$$T$$

+

$$N$$

$$N$$

where y_{n+1} is an approximation of $y (tn)$

+1, y_n being supposed known and is the parameter of method, [0]

1

, .

Note:

if = the 0 diagram is known as *explicit*,

if the 0 diagram is known as *implicit*.

4.2

Application to the equation of heat

Let us use it - method in the variational formulation of the equation of heat, where one posed:

$$T_+ = T(R, T_+ T)$$

)

$$T_- = T(R, T)$$

$$h_+ = H(R, T_+ T)$$

)

$$H = H(R, T)$$

$$F_+ = F(R, T_+ T)$$

)

$$F_- = F(R, T)$$

$$T_+ = T(R, T_+ T)$$

$$) T_- = T(R, T)$$

ext.

ext.

ext.

ext.

$$R_+ = R(R, T_+ T)$$

$$) R_- = R(R, T) \quad T d_+ = T D(R, T_+ T)$$

$$) T D = T D(R, T)$$

flight

flight

flight

flight

)

$$G_+ = G(R, T_+)$$

$$G_- = G(R, T_-)$$

where $T D(R, T)$ represents the temperature imposed on the border of the field, according to time and space.

Let us introduce following spaces of functions:

V

1

D

$=$

$()$

$/ =$

$+$

t_+

$\{v H$

v

T
1
}
V
1
D
=

()
/=

T
{v H

v
T

1
}

V
1

$$0 = \{v H () v = 0$$

1
}

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The field T -

V

+

v V

T being supposed known, one seeks *T*

Vt+ such as 0:

$$(T +) - (T -) \nu D$$

+

$$((T +) T +$$

. *v*

$$+ (1 -) (T -) T -$$

. *v*

$$) D$$

T

$$- (F + + (1 -) F -) \nu d2$$

$$(h+ T + + ($$

ext.

$$1 -) H Text - H T - (1 -) H T) \nu d3$$

2

3

$$(G + + (1 -) G -) \nu D$$

4

=

$$(R + + ($$

$$flight 1) rvol) \nu D$$

4

éq 4.2-1

Not to weigh down the writing excessively and insofar as the process is identical to the different one terms, one did not make appear the term of radiation in these equations (integral on 5).

While posing:

$$(HT) = h + T + + (1 -) HT -$$

ext.

ext.

ext.

$$F = F + + (1 -) F -$$

$$R = R + + (1 -) R +$$

flight

flight

,

one obtains finally:

$$(T +) v D + (T +) T +$$

. v

$$D + h + T + v D$$

T

3

3

$$- G (T +) v d4 = L (v, T -)$$

1

éq 4.2-2

4

v V

0

where one posed:

(T -)

$$L (v, T -) =$$

$$v D - 1$$

$$(-) (T -) T -$$

$$. v D + F v D$$

1

T
2

2

éq 4.2-3

$$+ ((HT) - 1$$

$$(-) HT -) \nu D + R \nu D + 1$$

$$(-) G (T -) \nu D$$

ext.

3

4

3

4

At one moment of calculation given, this term is known. Indeed, only the temperature at the previous moment,

T_{-} , as well as the *known* values at the moment running of function of time, intervene.

If the distribution of temperature in the system at the initial moment is not provided, one solves the stationary problem. The terms of evolution disappear, $= 1$; the field of temperature to the initial moment is given by:

$$(T_{t=0}) T_{t=}$$

0. ν

$$D + ht=0 T_{t=0} \nu D$$

0

3

-

$$G (T_{t=}) \nu D$$

4

3

4

$$\begin{aligned}
&= \\
&F t=0 v D \\
&0 \\
&0 \\
&0 \\
&2 \\
&+ ht= T t= v D 3 \\
&+ rt= v D
\end{aligned}$$

ext.

éq 4.2-4

2

3

v V
0
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The problem is written finally in the condensed form:

Oneself
T - - known tr
ouver

,

+
T
V
T
V
 + such as

T
T

éq 4.2-5

v V

$$has (v, T +) = L (v, T$$

-
 0
 1
)
5

Space discretization and adaptation of the algorithm of Newton with the problem

The principle of the method of Newton is very detailed in [R5.03.01], one will expose only them here adaptations specific to the nonlinear algorithm of thermics.

5.1 Discretization

space

That is to say *pH* a division space, let us indicate by *NR* the number of nodes of the grid, *pi* function of form associated with node *I*. One indicates by *J* the whole of the nodes belonging to border 1.

Are:

VH

D

=

=

()

;

=

+

(,)

t+

$\{v$
 $v p X$
 v
 T
 $X T$
 J
 J
 I
 I
 J
 J
 $\}$
 $i=,$
 $1 NR$
 $V H$
 D
 $=$
 $=$
 $()$
 $;$
 $=$
 $-$
 $(,)$

éq 5.1-1

T
 $\{v$
 $v p X$
 v
 T
 $X T$
 J
 J
 I
 I
 J
 J
 $\}$
 $i=,$
 $1 NR$
 $V H$
 $0 = \{v = v p (X)$
 $;$

$v = 0$

JJ

I

I

J

}

$i=,$

1 NR

The problem [éq 4.2-5] can be replaced by the problem discretized with a finished number of unknown factors

according to:

Oneself

T -

H

- known tr

ouver

,

+

T

V

T

VH

+ such as

T

T

éq 5.1-2

v

VH

has (v, T +

0

) = L (v, T

-

H

H

1

H

)

that one can as write, with the same formalism as STAT_NON_LINE [R5.03.01], in form vectorial:

$$\mathbf{v}^T \mathbf{R}(\mathbf{T}+, t+) = \mathbf{v}^T \mathbf{L}(\mathbf{T}, t+) \mathbf{v}$$

such as \mathbf{B}

$$\mathbf{v} = 0$$

éq 5.1-3

$\mathbf{B} \mathbf{T}+$

$$= \mathbf{T}d(T +)$$

where the operator \mathbf{B} expresses the boundary condition of imposed temperature $T +$
 VH

$t+$. It is defined by:

$$0 \text{ if } J J$$

$$(\mathbf{B}\mathbf{v}) J = v$$

éq 5.1-4

J

if

$J J$

The case where the application \mathbf{R} is linear is treated by order THER_LINEAIRE [R5.02.01].

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The dualisation of the boundary conditions, detailed in [R3.03.01], led to the nonlinear problem in $\mathbf{T}+$:

R

$$(\mathbf{T}+, t+) + \mathbf{B}\mathbf{T} + \\ = \mathbf{L} (\mathbf{T}, t+)$$

éq 5.1-5**B T+****Td**

+

=

(T)

The unknown factors are the couple $(\mathbf{T}+, +)$, where $+$ represents the “multipliers of Lagrange” of boundary conditions of Dirichlet.

To solve the system [éq 5.1-5] amounts cancelling in $(\mathbf{T}+, +)$

+

+

*I**I*

the vector $\mathbf{F} (\mathbf{T},)$, called residue, defined by:

$$\mathbf{L} (\mathbf{T}, + \\ T) - \mathbf{R} (\mathbf{T}+, + \\ T) - \mathbf{B}\mathbf{T} +$$

$$\mathbf{F} (\mathbf{T}+, + \\) =$$

Td**éq 5.1-6***(+**T) - B T+*

The method of Newton consists in building a vector series $\{x_n\}$ converging towards the solution N

of $\mathbf{F} (X) = \mathbf{0}$ using the tangent linear application of \mathbf{F} .

5.2 Calculation**stationary**

The variational problem is that of the equation [éq 4.2-4]. To note: in stationary calculation, enthalpy does not intervene in application **R**.

One introduces the matrix of the tangent linear application of the function $\mathbf{R}(\mathbf{T}_n)$:

\mathbf{R}

$\mathbf{K} N = \mathbf{T} \mathbf{T}_n$

That of the function $\mathbf{F}(\mathbf{T}_n, N$

) is then:

$\mathbf{K} N$

$\mathbf{B} \mathbf{T}$

\mathbf{B}

0

In the case of stationary calculation, one must reiterate starting from a uniform value of initialization of the field

of temperature; in fact $T_0 = 0$ in any node. The first iteration of calculation, known as iteration of prediction, consists in solving the following system:

\mathbf{K}

$(\mathbf{T}) \mathbf{B} \mathbf{T} \mathbf{T}$

$-\mathbf{H} \mathbf{T} - \mathbf{R}(\mathbf{T}) - \mathbf{B} \mathbf{T}$

0

1

0

0

0

=

éq 5.2-1

\mathbf{B}

0

D

1 - 0

\mathbf{T}

$-\mathbf{B} \mathbf{T}$

0

As one can see it in the equation of the stationary problem [éq 4.2-4], the temperature does not appear not with the second member: are written \mathbf{L} and not $\mathbf{L}(\mathbf{T})$

0 .

If the problem is linear, $\mathbf{R}(\mathbf{T}) = \mathbf{K}(\mathbf{T}) \mathbf{T} = \mathbf{K} \cdot \mathbf{T}$

0

0

0

0. All the terms in \mathbf{T}_0 disappear by

simplification. The solution is obtained in an iteration by inversion of a system identical to that described in [R5.02.01 §6].

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The following iterations are iterations of Newton, with reactualization or not of the matrix tangent \mathbf{K} .

\mathbf{K}

$(\mathbf{T}) \mathbf{B} \mathbf{T} \mathbf{T}$

+ - \mathbf{T}

\mathbf{T}

$I \ 1$

I

$\mathbf{L} - \mathbf{R}(\mathbf{T}) - \mathbf{B}$

(I)

I

I

=

éq 5.2-2**B**

0 I

1

+ - I

0

For the iteration of prediction, the writing of the lower subsystem of the equation [éq 5.2-1], afterwards simplification, ensures us that **B T**

T

1 =

D. The reiterated first and all the following thus check them conditions of Dirichlet.

Brackets around the index of iteration in the expression **K (T)**

(*I*) mean that one can

to reactualize or not the tangent matrix with the wire of the iterations.

Note:

The temperature of T0 initialization has of influence only for one nonlinear stationary calculation. In being of about size of the awaited temperatures, it would make it possible “to leave” less far from the solution that a null field everywhere; and thus the iteration count would decrease. Today, it is not possible to enter a value of T0. The vector temperature is initialized, into hard, to zero.

5.3 Calculation**transient**

For the first iteration of the step of time, known as iteration of prediction, one “makes like if” it problem describes in [éq 5.1-5] was linear. This formulation must make it possible to obtain directly solution to a linear problem of thermics. But here, the situation is a little different from calculation stationary because of the formulation in enthalpy. The linearization of [éq 5.1-5] gives:

R

$$(\mathbf{T}, t+) + \mathbf{K} (\mathbf{T}, t+) (\mathbf{T}+ - \mathbf{T}) + \mathbf{B}\mathbf{T} +$$

$$= \mathbf{L} (\mathbf{T}, t+)$$
éq 5.3-1**B T+****Td +**

=

(T)

What amounts solving, for the problem presented in matric form:

K

(T) BT T+

1

L

(T, +

T) + K (T) T - R (T)

=

D

B

+

0

éq 5.3-2

T (+

T)

1

The function enthalpy is known with a constant of integration close which appears in the relation flexible

R (T)

-

-

-

+

with **K (T) T**. This same constant is found in the expression of **L (T, T)**. One can

then to eliminate it while leading to the system from equations according to:

K

-

T

+

~

(T) B T1

L

(T, +

T)

= **D**

B

+

0

éq 5.3-3

T (+
 T)

1

~

where $L(T, T +)$ is the second member calculated with the heat-storage capacity and not the enthalpy (option

CHAR_THER_EVOLNI [§6.2]).

Lastly, as for the stationary case seen in the preceding chapter, the following iterations are iterations of Newton:

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K (T, T +) BT T+

1 - T+ L (T, t+) - R (T, T +

) - BT

(I)

i+

I

I

I

=

éq 5.3-4

B

+

+

0 i+1 - I

0

This time, on the other hand, **L (T, T +)** is calculated with the enthalpy and not the heat-storage capacity to be

coherent with **R (T+)**

I

.

5.4 Convergence

Since time intervenes in the form of the tangent matrix, and also the step of time, one prefer systematically to bring up to date this one at the beginning of each step not not to degrade too

much

speed of convergence. On the other hand, freedom is left with the user control his frequency of calculation during a step of time.

With each iteration, one can carry out the search for an optimum step of progression towards the solution by

some iterations (2 or 3) of linear research. This method is described in detail in [R5.03.01].

Calculation famous is converged when the vector residue is null [éq 5.1-6]:

$$\mathbf{L}(\mathbf{T}, T+) - \mathbf{R}(\mathbf{T}+ + T+)$$

$$I, T)$$

$$\mathbf{F}(\mathbf{T}+ + +$$

$$I, T)$$

$$\text{éq 5.4-1}$$

$$\mathbf{Td}(T+) - \mathbf{B T}+$$

$$I$$

The lower part of the vector is always null (conditions of Dirichlet). One thus checks:

$$\mathbf{L}(\mathbf{T}, + T) - \mathbf{R}(\mathbf{T}+ T)$$

$$I, + T) - \mathbf{B}$$

$$+ I 2$$

$$\text{éq 5.4-2}$$

$$\mathbf{L}(\mathbf{T}, + T) - \mathbf{B T} + I 2$$

The user also has the possibility of stopping the iterations on an absolute criterion:

$$\mathbf{L}(\mathbf{T}, + T) - \mathbf{R}(\mathbf{T}+ T)$$

$$I, +$$

T) - B

+i

éq 5.4-3

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6

Principal calculated options of non-linear thermics

in Code_Aster

6.1

Boundary conditions and loadings

One will refer to [R5.02.01] for the boundary conditions and the loadings linear.

(1 -) (-)

Flow not

G T v D

CHAR_THER_FLUNL

4

linear

4

[(T +.

273)

15 4 - (

-

4

1 -) ($T +$

.

273)

15]

Radiation

$v D$

CHAR_THER_RAYO_R

4

CHAR_THER_RAYO_F

4

6.2

Calculation of the elementary matrices and transitory term

Inertia

CP

thermics,

MTAN_RIGI_MASS

$v.v D +$

(T) $v.v$

D

+

conductivity

T

4 +

Radiation

($T + 27315$ 3

.) $v.v D$

MTAN_THER_RAYO_R

4

MTAN_THER_RAYO_F

4

$hv.v D$

Coefficient

MTAN_THER_COEF_R

4

of exchange

MTAN_THER_COEF_F

4

dg

Flow not

MTAN_THER_FLUXNL

-

+

$(T) v.v d4$

linear

dT

4

Term

$(T -)$

transient

CHAR_THER_EVOLNI

$v D - (1 -) (T -) T -. v D$

T

$CpT - v D - (1) (T) T$

. v

D

T

6.3

Calculation of the residue

1

I

I

I

RESI_RIGI_MASS

$(T) v$

$D + ((T)) T. v$

D

T

I

Radiation

(*T* +
.) *v D*

27315 4

RESI_THER_RAYO_R

4

RESI_THER_RAYO_F

4

(*h* + *Ti*) *v D*

Coefficient

RESI_THER_COEF_R

3

of exchange

RESI_THER_COEF_F

3

Flow not

RESI_THER_FLUXNL

-

I

(

)

linear

G T v D 3

3

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Nonlinear thermics in pointer

Summary

One presents the formulation and the algorithm of the problem of convection-diffusion in nonlinear thermics

stationary introduced within order THER_NON_LINE_MO [U4.33.04].

The goal is to solve the equation of heat in a mobile reference frame related to a loading and moving in a given direction and at a speed.

Nonthe linearities of the problems come as well from the characteristics of material which depend on temperature, that boundary conditions of the radiation type.

The problems of this type can be dealt with with models using of the finite elements of structure plans, axisymmetric and three-dimensional.

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Presentation of the problem

The equation of heat has strong not linearities under certain conditions. It is the case when the material undergoes phase shifts: those are accompanied by abrupt variation of the characteristic sizes (heat-storage capacity, enthalpy). This nonlinearity is all the more accentuated when the problem of convection-diffusion is dealt with, where the term appears of

transport depend on the function enthalpy. The goal of this modeling is to treat this last problem in steady operation (stationary case).

In all the cases, one supposes that the field speed is known a priori. The case of a mobile solid is rather frequent in practice. It relates to in particular the applications of welding or the treatment of surface which brings into play a heat source moving in a direction and at a speed data. The problem of thermics is then studied in a reference frame related to the source.

The problem with the derivative partial results from the equation of the total heat balance on any field who is written:

D

$D =$

$Qd -$

$q.n D$

dt **éq 1-1**

accumulation

creation + input-output

In this equation, represents a related, interior field with the studied system, which one follows in its movement, the specific enthalpy of material represents and indicates its mass voluminal. Q is a voluminal heat source, \mathbf{Q} is the heat flow through the border (\mathbf{N} being the external normal), and D/dt is the **particulate derivative**.

The first term of [éq 1-1] is written (see for example [bib1]):

 D $()$ $D =$ $+ div$ $(\mathbf{V}) D$ **éq 1-2** dt T

or, taking into account the conservation of mass

 $+ div (\mathbf{V}) =$

0 [bib1]:

 T D $D =$ $+ \mathbf{V} \cdot \text{grad } D$

éq 1-3*dt**T*

where \mathbf{V} is the Flight Path Vector of displacement of the field. \mathbf{V} is indicated under the single-ended spanner word

CONVECTION of orders AFFE_CHAR_THER and AFFE_CHAR_THER_F.

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The second term of the second member of [éq 1-1] is written, taking into account the theorem of the divergence

and of the Fourier analysis ($\mathbf{Q} = -K(T) \mathbf{gradT}$):

$\mathbf{q.n} D = D \mathbf{Q}$

iv D = - div

$(K(T) \mathbf{gradT}) D$

éq 1-4

where T is the temperature and $K(T)$ is the thermal conductivity of material, function of the temperature. The equation [éq 1-1] having to be satisfied for any field, it comes then:

+ $\mathbf{V} \cdot \text{grad} - \text{div} (K (T) \text{grad} T)$

= Q , in,
éq 1-5

T

Note:

Let us note that the traditional case with, $K (T) = K$ (constant) and $\mathbf{V} = 0$, and where the specific enthalpy is

a linear function of the temperature, $(T) = CT$ gives again the traditional equation good known:

T

C

- =, in

$K T$

Q

,

where Δ is the Laplacian and C (constant) represents the specific heat.

The problem with the derivative partial treaty by order THER_NON_LINE_MO [U4.33.04], consists to solve the equation [éq 1-5] in the stationary case (directly at the permanent state) with boundary conditions on the border.

This problem is formally written in the following form:

$\mathbf{V} \cdot \text{grad} ($

$U T) - \text{div} (K (T) \text{grad} T) = Q$, in,

éq 1-6

+ boundary conditions

on

where we adopted the notation, valid for all the continuation, ($U T) = (T)$ where is constant,

defining the voluminal enthalpy.

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2**Boundary conditions. Problem of reference to be solved**

One will refer, for example, with [R5.02.01] for more information on the boundary conditions thermics of the type Dirichlet, Neumann and linear Fourier, and with [R5.02.02] for the conditions with limits of the nonlinear normal flow type (nonlinear Fourier).

Of enthalpic formulation, the **stationary** problem of thermics thus consists in solving in one field of border on.

V.grad ($U T) - \text{div} (K (T) \mathbf{gradT}) = Q, \text{ in,}$ **éq 2-1** T with $K (T)$ $= (T - T$ $ext.$

) on

 \mathbf{N}

1,

éq 2-2 T $K (T)$ $= q^0$

on

,

éq 2-3 \mathbf{N}

2

 T $K (T)$ $= (T)$

on

 \mathbf{N}

3,

éq 2-4 $T = T_0$

on

4

,
éq 2-5

where:

.
 T_0 : is the temperature imposed on 4;

.
 q_0 : is the normal flow imposed on 2;

.
: is the coefficient of heat exchange;

.
 T_{ext} : is the outside temperature;

.
 (T) : is the normal flow of nonlinear Fourier type (radiation).

The equations [éq 2-2], [éq 2-5] the boundary conditions of the types represent, respectively:
Linear Fourier, Neumann, nonlinear Fourier and Dirichlet.

The problem of reference [éq 2-1], [éq 2-5] is strongly nonlinear because of nonthe linearities on
 $K(T)$, (

$U(T)$ (phase shift) and (T) (radiation).

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Variational formulation of the problem

That is to say open of \mathbf{R}^3 , border = 1 2 3 4 such as,

for I, J and $I, J = 1, \dots, 4$, one $a_{IJ} =$.

That is to say still a sufficiently regular function which is cancelled on 4:

$V = \{\text{regular and } = 0\}$

·
4
}

Let us multiply by the two members of the equation [éq 2-1], then integrate on. An integration by parts gives then:

$$Q D = \mathbf{V} \cdot \mathbf{grad} (U T) D - \text{div}$$

$$(K(T) \mathbf{grad} \mathbf{T}) D$$

$$= \mathbf{V} \cdot \mathbf{grad} (U T) D + K(T) \mathbf{grad} \mathbf{T} \cdot \mathbf{grad} D - K(T) D$$

N

-4

éq 3-1

since is null on 4.

From where, by taking account of the boundary conditions [éq 2-2], [éq 2-3] and [éq 2-4], the formulation

variational of the problem of reference which is given by the following equation:

$$V K(T) \mathbf{grad} \mathbf{T} \cdot \mathbf{grad} D + \mathbf{V} \cdot \mathbf{gradu} (T) D + D - (T) D$$

1

3

éq 3-2

$= Q D +$

$T D$

$+ q_0 D$

ext.

,

1

2

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4**Treatments of nonthe linearities**

In the sight of the numerical resolution of the nonlinear problem that we consider, it is necessary of to treat all nonlinearities.

In our case, let us quote the strong not linearity related to the function enthalpy (

$U T$) which takes into account it

solid-liquid phase shift, as well as nonthe linearity related to the possible presence of one boundary condition of nonlinear normal flow (radiation).

Let us recall that in the traditional case of the problems of transitory thermics nonlinear without convection, i.e. $\mathbf{V} = 0$, several methods of resolution is proposed in the literature. There exists

as well methods using of the enthalpic formulations as methods using of

formulations in temperature, all having for goal as well as possible to treat nonthe linearity related to the enthalpy

(phase shift).

We return the reader to the reference [bib5] for a summary of the principal methods met

in the literature. However, let us note that because of the difficulty related to the presence of the term of transport " $\mathbf{V} \cdot \text{grad}$ (

$U T$)" in the problem, none of these methods will be employed in

continuation.

As in any iterative process, the goal of the numerical diagram in sight is to find a field of

temperature T_{n+1} with the iteration $N + 1$, the field of temperature T_N , solution of the iteration

the preceding one.

4.1**Treatment of nonthe linearity related to the enthalpy**

In order to treat this nonlinearity, the strategy employed in this study was inspired by a technique

of resolution of the free problems of borders [bib3], which, in the beginning was proposed in [bib4].

Let us consider the function enthalpy (

$U T$) as being given in a reciprocal form:

Temperature function of the enthalpy (opposite of the function (

$U T$)). In other terms one will have to treat

relation following Temperature-enthalpy:

$$T = (U)$$

éq 4.1-1

The reason of this choice will be clearer in what follows. Indeed we will have to deal with problem with two fields: a field of temperature and a field enthalpy. Discretization of the function opposite [éq 4.1-1] allows to increment the field enthalpy according to the current field of temperature (and not the reverse) as follows:

The development with the first order of the function (U) is as follows,

$$T_{n+1}$$

$$(one) (one) (un+$$

=

+

1

,

- one),

éq 4.1-2

where ' is the derivative of the function defined by [éq 4.1-1] compared to its argument.

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In order to take into account this nonlinearity, and from [éq 4.1-2], one replaces $un+1$ by one approximation according to the unknown field of temperature T_{n+1} in the following way:

$$un+1 \text{ one}$$

$$(Tn+$$

-

=

1

- (one),

éq 4.1-3

where is a parameter of relieving, constant on all the field and during all the iterative process,

1
representing the term

.

'(one)

Because of the nonconvexity of the function (U), this parameter of relieving necessarily must to check the following condition [bib2], [bib3]:

1

éq 4.1-4

max '(N

U)

N

1

In practice one takes =

.

max '(N

U)

N

By taking of account the approximation [éq 4.1-3], discretization of the second term of the equation [éq 3-2] is expressed in the following way:

V.gradun+1 D = V.gradun D +

V.gradT n+1

D

-

V.grad

(one) D,

éq 4.1-5

4.2 Treatment of nonthe linearities related to the condition of Fourier not linear and with thermal conductivity

Nonthe linearity related to the condition of normal flow nonlinear is treated by considering it development with the first order of the function (supposed sufficiently regular) (T) which is given by:

$$(T_{n+1}) (T_n) (T_n) (T_n +$$

=

+

1

,

- T N),

éq 4.2-1

where (.)'the derivative of the function indicates (.) compared to its argument.

It appeared necessary to decide of a strategy of discretization of the term “ $K(T) \text{ grad } T$ ” in the equation [éq 3-2] in order to be able to treat this nonlinearity for the stationary problem that us let us consider. For that, we adopted the following approximation:

$$K(T_{n+1})$$

$$T_{n+1}$$

$$K(T_n)$$

$$T_{n+1}$$

$$[K(T_n) K(T_{n-1})]$$

=

-

-

1

grad**grad**)] **grad**Tn**éq 4.2-2***Handbook of Reference**R5.02 booklet: Thermics**HI-74/98/007/A***Code_Aster** ®*Version*

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This discretization is in fact a simplification of the development to the first order of the term $K(T) \text{ grad } T$. It is effective being in particular because of the low not linearity of the function

$K(T)$ in practice.

Note:

Also let us note that the following purely explicit approximation:

$$K(T_{n+1})$$

$$T_{n+1}$$

$$K(T_n)$$

$$T_n$$

1

grad

grad

,

also give satisfactory results. This observation was checked from several numerical experiments.

5

Algorithm established in Code_Aster

The numerical diagram employed for the resolution of the problem of reference [éq 2-1], [éq 2-5] is deduced from the variational formulation [éq 3-2] and from the treatment of the various not linearities, [éq 4.1-5],

[éq 4.2-1], [éq 4.2-2], discussed in the preceding section.

The algorithm of resolution is consisted the sequence of two successive operations with each iteration of calculation.

Knowing the fields solutions with iteration N : T_N with the nodes and *one* at the points of Gauss, one seek the solutions T_{n+1}

$$u_{n+1}$$

and

with the iteration $N + 1$ as follows:

$$V,$$

$$K$$

$$(T_n) \text{ grad } T_{n+1} \cdot \text{grad } D + \mathbf{V} \cdot \text{grad}$$

$$T_{n+1} D$$

+

$$T_{n+1} D$$

$$- '(T_n) T_{n+1} D$$

1

3

$$= Q D$$

+
 $T D +$
 $Q D$

ext.
 0

éq 5-1

1
 2
 $+ ((T_n) - (T_n) T_n) D + [K (T_n) - K (T_{n-1})] \text{grad} T_n \cdot \text{grad} D$

3
 +
V.
grad

(one) $D - \mathbf{V} \cdot \text{grad} u_n D$

u_{n+1}
one
 (T_{n+1})
 =
 +
 1

- *(one)*

éq 5-2

With each iteration, a linear problem of convection-diffusion is solved to obtain the field with nodes T_{n+1} [éq 5-1], and then a simple on-the-spot correction is carried out to obtain the field with points of Gauss u_{n+1} [éq 5-2].

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The criterion of stop adopted in *Code_Aster* utilizes at the same time the two fields solutions: the field of temperature, and the field enthalpy.

The algorithm continues the iterations as long as at least one of the relative variations of reiterated is higher than the corresponding tolerance given by the user:

1/2

1

2

$(T_{n+1} - T_N$

I

$)$

$i=1. , nddl$

$> \text{tole } 1$

1/2

1 2

$(T_{n+1}$

I

$)$

$i=1. , nddl$

1/2

1

2

$(u_{n+1} - T_N$

I

D)

 $i=1. , npg$

> tole 2

1/2

1 2

(un+

I

)

 $i=1. , npg$

where *nddl* is the total number of the degrees of freedom to the nodes, and *npg* is the total number of the points of

Gauss.

tole 1 is indicated under the key word *crit_temp_rela* key word factor convergence of the operator *ther_non_line_mo*.

tole 2 is indicated under the key word *crit_enth_rela* key word factor convergence of the operator *ther_non_line_mo*.

6

Principal options of calculation in *Code_Aster*

One presents below the principal options of *Code_Aster* specific to the unfolding of the algorithm [éq 5-1], [éq 5-2] above. On the other hand, we will not mention the options not specific of *Code_Aster* and which is used in calculation:

- Boundary conditions:

Linear Fourier

RIGI_THER_COET_R

 $n+$

RIGI_THER_COET_F

TD

1

1

Nonlinear Fourier

,

 $(Tn) n+1$

RIGI_THER_FLUTNL

T

D

3 ((*Tn*) (*Tn*) *Tn*

-

,

)

CHAR_THER_FLUTNL

D

3

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· Elementary Matrices and second member:

N

n+1

RIGI_THER_TRANS

K

(*T*) **gradT** .grad *D*

n+

RIGI_THER_CONV_T

V. gradT

D

1

-

CHAR_THER_TNL

$[K(T_n) - K(T_{n-1})] \text{grad} T_n \cdot \text{grad} D$

+ $\mathbf{V} \cdot$

$\text{grad}(\text{one}) D - \mathbf{V} \cdot \text{grad} u_n D$

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Document: R5.03.01

Quasi-static nonlinear algorithm

(operator STAT_NON_LINE)

Summary:

Operator STAT_NON_LINE [U4.51.03] of Code_Aster allows a quasi-static request in the case of to integrate various types of non-linearities coming from the behavior of material or of large geometrical transformations. One describes the total algorithm of resolution here employed. The integration of the relations of behavior themselves is described in other documents, like [R5.03.02] for the élasto-plasticity, to which one will be able to refer for examples. For calculations in great transformations geometrical, one will be able to consult for example the document [R5.03.20] on nonlinear elasticity into large displacements, or the document [R5.03.21] on the thermoelastoplasticity with isotropic work hardening.

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matters**

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1 Presentation

1.1

General information on the relations of behavior

STAT_NON_LINE is the operator of Code_Aster making it possible to carry out mechanical calculations not linear when the effects of inertia are neglected.

Calculation relates a priori only to the variables mechanical (displacements, constraints, variables interns) by excluding any coupling with other physical phenomena (thermics,...). By consequent, associated fields influencing the mechanical behavior (thermal fields, metallurgical) are calculated as a preliminary by other operators (THER_LINEAIRE [U4.33.01], THER_NON_LINE [U4.33.02]), even by other codes (SYRTHES, SATURN,...).

This assumption suffers an exception with regard to modeling thermo-hydro-mechanics (modeling known as "THM") for which milked STAT_NON_LINE the whole of the coupled problem of the equations of diffusion of thermics, the pressure of fluid and of mechanical balance [R7.01.10].

In STAT_NON_LINE, two families of behaviors are available:

that which corresponds to the key word factor COMP_ELAS (Elastic Behavior) led to through equilibrium equation to a nonlinear system depending explicitly on the field displacements \mathbf{U} compared to the configuration of reference, and parameterized by the moment of calculation (through inter alia the thermal evolution). For more details, one will be able to defer, for example, with the document [R5.03.20] concerning elasticity into large transformations (hyper-elasticity), or the document [R5.03.21] on the thermoelastoplasticity with isotropic work hardening,

the other family, which corresponds to the key word factor COMP_INCR (Incremental Behavior), is associated relations of behavior expressed by a differential equation implicit (for example elastoplasticity, viscoplasticity, hypo-elasticity). In this case, relation of behavior is integrated as presented for example in [R5.03.02]: in connecting an increment of displacement \mathbf{U} calculated starting from a mechanical state given (the state mechanics being represented by a field of displacements \mathbf{U} , a stress field and a field of variables intern) with the stress field at the moment T of calculation. The equilibrium equation thus leads to a nonlinear system out of \mathbf{U} , but which is also parameterized by the moment of calculation through the facts of the case (variation of the loading mechanics and thermal evolution for example).

In both cases, one calculates the solution gradually. It is theoretically not essential in the nonlinear elastic case, but it may be that nonthe linearity of the solution sought either too strong for the algorithm of resolution used, and that it is essential, for numerical reasons, to operate step by step.

It is necessary nevertheless to have for the spirit the fundamental difference between the two approaches. The elastic case suppose the existence of a state of reference per report/ratio to which the elastic strain is written: this state corresponds in a state without forced deformation nor. It is the “absolute” value of the loading which create the deformation. The incremental case is based on the state previously calculated and “forgets” all reference to the former states except that given by the internal variables. In this case, it is variation of the loading which modifies the state of the system: in particular, one needs a variation of the field of temperature to create thermal deformations.

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1.2

Position of the nonlinear quasi-static problem

Consequently [§1.1], it is seen that it is legitimate to consider that the nonlinear problem that one draft has like unknown factor a displacement and is parameterized by time. That is to say thus the problem not

linear quasi-static according to:

$$\mathbf{v}^T \mathbf{R}(\mathbf{U}, T) = \mathbf{v}^T \mathbf{L}(T)$$

\mathbf{v} such as $\mathbf{B}\mathbf{v} = 0$

$$\mathbf{Drunk} = \mathbf{u}d(T)$$

where:

.

T represents the variable of moment,

.

\mathbf{U} the field of displacement taken starting from a configuration of reference,

.

\mathbf{L} the mechanical loading to which the structure is subjected (pressure, imposed force,...),

.

the relation $\mathbf{Drunk} = \mathbf{u}d(T)$ corresponds to the boundary conditions imposed in displacements (imposed displacements, connections between degrees of freedom,...) : \mathbf{B} is a linear operator of the space of the fields of displacements on a space of functions defined on part of edge of the structure, $\mathbf{u}d$ is a function given on this part,

.

\mathbf{R} represents the internal forces of the problem of quasi-static mechanics nonlinear (in the linear case, there are $\mathbf{R}(\mathbf{U}, T) = \mathbf{K}\mathbf{u}$, where \mathbf{K} is the matrix of rigidity of the structure). The notation $\mathbf{R}(\mathbf{U}, T)$ is a short cut which one will use in the continuation.

In fact, more precisely, $\mathbf{R}(\mathbf{U}, T)$ is connected to the stress field by the operator of work of virtual deformations \mathbf{QT} [§ 2.2.1] according to the relation:

$$\mathbf{R}(\mathbf{U}, T) \mathbf{QT}$$

=

(\mathbf{U}):

éq

1.2-1

In small displacements, \mathbf{QT} is independent of displacements; for the large ones displacements, to refer to [R5.03.20]. Subsequently, one will place oneself on the assumption small displacements and small deformations.

The stress field \mathbf{I} at the moment T_i is written ($\mathbf{U}, T, T, \mathbf{H}$)

\mathbf{I}

$\mathbf{I} \mathbf{I}$

$i-1$, if one notes T_i the field of temperatures and H_{i-1} last history of the structure. For the elastic behaviors, the history does not intervene: **the H_{i-1} unit is thus empty.** For the incrémentaux behaviors, the history is the whole of the states (fields of displacements, constraints and variables intern) at the moment precedent: **H**

=
 I
 $\{U$
 I
 $I\}, I\}, I$
 $, T$
 $-$
 $-$
 $-$
 $-I\}$
 $I.$

In the case general, the dependence of the operator \mathbf{R} is, as we saw in [§1.1], implicit compared to time: it results from the integration of the relation of behavior in time (for the problems of elastoplasticity for example). The dependence clarifies compared to time is very rare: it in particular in the case of appears relations of fascinating behavior in count a phenomenon of work hardening by time known as time-hardening.

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The dualisation of the boundary conditions of **Drunk** Dirichlet = $\mathbf{ud}(T)$ led to the problem according to [R3.03.01]:

R

$$(\mathbf{U}, T) + \mathbf{BT} = \mathbf{L}(T)$$

éq

1.2-2

$$\mathbf{Drunk} = \mathbf{ud}(T)$$

The unknown factors are now at any moment T the couple (\mathbf{U}, T) , where represents the "multipliers of Lagrange" of the boundary conditions of Dirichlet [R3.03.01]. Vector \mathbf{BT} is interpreted like opposite of the reactions of support to the corresponding nodes.

The formulation of the quasi-static problem consists in expressing the balance of the structure (forces external interns = forces) for a succession of moments of calculation $\{T_i\}$ who parameterize it

 $i \in I$

loading:

R

$$(\mathbf{U}, T) + \mathbf{BT}$$

$$= \mathbf{L}(T)$$

 $i \in I$ $i \in I$

*I**éq**1.2-3**Drunk**= ud (T)**I**I**what amounts cancelling in (U, T)**I**I I the vector **F** (U, T) defined by:****R** (U, T) + **BT** - **L** (T)****F** (U, T) =****Drunk** - **ud** (T)*

The state of the structure in t_0 is supposed to be known. One carries out I increments (or not) of load definite as follows:

*not**charge n°* *1**2**I**moment* *t_0 T_1 t_2* *...* *T_{i-1} T_i*

The unknown factors are calculated in an incremental way by the total algorithm of resolution (even for elastic behaviors). From $(\mathbf{u}_{i-1}, i-1)$, solution satisfying T_{i-1} balance, one determine \mathbf{u}_i and I , which will make it possible to obtain the T_i solution:

U*= **U****+ **U******I***

I

-I

I

I = I +

-I

I

*The increments **ui** and I are initially estimated by linearizing the problem compared to time around (**ui**, T*

-I

*i-1 i-1) (phase known as of prediction or Euler [bib 3]). Then a method is used of NEWTON or one of its alternatives to solve the equation [éq 1.2-3] in an iterative way (one calculate a continuation (**one**, N*

I

I)).

For the incremental relations of behavior, one needs to know out of T_{i-1} it stress field $i-1$ and the field of variables intern $i-1$ (cf [R5.03.02] for one example).

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2

Method of resolution

2.1

Principle of the method of NEWTON

The method of NEWTON is a traditional method of resolution of the equations of the type:

$$\mathbf{F}(\mathbf{X}) = 0$$

where \mathbf{F} is a vector function (nonlinear) vector \mathbf{X} .

It consists in building a vector series $\{\mathbf{x}_n\}$ converging towards solution \mathbf{X} . To find

N

the new one reiterated \mathbf{x}_{n+1} , one approaches $\mathbf{F}(\mathbf{x}_{n+1})$ by a development limited to order 1 around \mathbf{x}_n ,

and

one expresses that $\mathbf{F}(\mathbf{x}_{n+1})$ must be null:

0

N

+

N

' N

N

= (

) () +

+

$\mathbf{F}(\mathbf{X})$

$\mathbf{F}(\mathbf{X})$

$\mathbf{F}(\mathbf{X}) (\mathbf{X}$

- \mathbf{x}_n),

that is to say:

-1

$\mathbf{F}'(\mathbf{x}_n) \mathbf{x}_{n+1} - \mathbf{x}_n$

= - $\mathbf{F}(\mathbf{x}_n)$

or $\mathbf{x}_{n+1} = \mathbf{x}_n - [\mathbf{F}'$

() (

)

()

(\mathbf{x}_n)] $\mathbf{F}(\mathbf{x}_n)$

Recall:

$\mathbf{F}'(\mathbf{x}_n)$ is the tangent linear application associated the function \mathbf{F} . The derivative as in point \mathbf{X}

in the direction \mathbf{H} is defined like:

$$\mathbf{F}(\mathbf{X} + \mathbf{H}) - \mathbf{F}(\mathbf{X})$$

$$\mathbf{F}'(\mathbf{X}) \mathbf{H} = \lim_{\mathbf{H} \rightarrow 0} \frac{\mathbf{F}(\mathbf{X} + \mathbf{H}) - \mathbf{F}(\mathbf{X})}{\mathbf{H}}$$

The matrix of $\mathbf{F}'(\mathbf{X})$ in the bases chosen for the vector spaces concerned be called the matrix jacobienne \mathbf{F} as in point \mathbf{X} .

When \mathbf{F} is related to an Euclidean vector space with actual values, $\mathbf{F}'(\mathbf{X})$ is one linear form, and one can show that there is a vector (single), noted $\mathbf{F}(\mathbf{X})$ and called it gradient of \mathbf{F} , such as:

$$\mathbf{F}'(\mathbf{X}) \mathbf{H} = \mathbf{H} \mathbf{T} \mathbf{F}(\mathbf{X})$$

(\mathbf{X}) (produced scalar of \mathbf{H} and the gradient).

When one is close to the solution, the convergence of the method of NEWTON is quadratic (it numbers of 0 after the comma in the double error with each iteration: 0.19 - 0.036 - 0.0013 - 0.0000017 for example). But this method (using the true tangent) has several disadvantages:

it requires the calculation of the tangent to each iteration, which is expensive (especially in case where a direct solver is used),

if the increment is large, the tangent (known as coherent or consistent) can lead to divergences of the algorithm,

it can not be symmetrical, which obliges to use particular solveurs.

For this reason one can use other matrices in the place of the tangent matrix: stamp elastic, a tangent matrix obtained before, the symmetrized tangent matrix,...

[§2.2.1].

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2.2

Adaptation of the method of NEWTON to the problem arising

2.2.1 Resolution without boundary conditions dualized

If one initially forgets the boundary conditions of Dirichlet, one must solve one system of the form:

$$\mathbf{R}(\mathbf{U}, \mathbf{T}) = \mathbf{Lm\acute{e}ca}$$

\mathbf{I}

\mathbf{I}

,

where $\mathbf{Lm\acute{e}ca}$

\mathbf{I}

indicate the mechanical loading at the moment \mathbf{T}_i .

Using the notations of [§ 2.1], that reverts cancelling the vector function \mathbf{F} defined by:

$$\mathbf{F}(\mathbf{U}, \mathbf{T}) = \mathbf{R}(\mathbf{U}, \mathbf{T}) - \mathbf{Lm\acute{e}ca}$$

\mathbf{I}

\mathbf{I}

\mathbf{I}

The nodal forces \mathbf{R} can symbolically be noted \mathbf{QT} , where \mathbf{QT} is the matrix associated with the operator divergence (expression of the agricultural work of virtual deformations), with $(\mathbf{U}, \mathbf{T}, \mathbf{H})$. One a:

\mathbf{I}

\mathbf{I}

\mathbf{I}

$i-1$

$$(\mathbf{QT}) =$$

(U): (W) D

K
K

(L) =
F W D +
T W D

I K
I
K
I
K

where

•
w_k indicates the basic function associated with the kth degree of freedom with the structure,

•
F indicates the voluminal forces applying at the moment T to,

I
I
•

T indicates the surface forces applying at the moment T to the border of.

I
I

The matrix Q depends on displacements U in great displacements [R5.03.20].

The application of the method of NEWTON results in solving a linear succession of problems of the type

(N is the number of the iteration of NEWTON, I that of the variable of moment):

KN un+1 = Lméca - RN

I
I
I
I,

where u_{n+1} is noted

u_{n+1}

=

$1 - one$

I

I

I the increment of displacement between two iterations of NEWTON

successive. The matrix K_N

N

N

I is the matrix of tangent rigidity in u_i and vector I_H represents them

forces intern with the n th iteration of NEWTON of I ème not of load. The $L_méca$ quantity - R_N

I

I

represent the not balanced forces, which one can also call the “residue”.

The matrix K nor is the matrix of the tangent linear application of the function F , it is thus worth:

F

R

$L_méca$

KN

I

$I =$

=

-

.

U

N

U

N

U

(U, T)

(U, T)

N

II

I, I

(U, T)

I, I

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In the absence of following forces [§2.2.3], the second term is null. The matrix K nor is thus the derivative

at the plain point of the nodal forces (or interns) compared to displacements:

 R **$K_{ni} =$** **U (one T** **I, I)**

A small error in the evaluation of the residue can have serious consequences, because it is its calculation

exact which guarantees, if one converges, that it will be towards the sought solution. On the other hand, it is not

always necessary to use the true tangent matrix, whose calculation and factorization are expensive.

For example, an alternative of the method uses the elastic matrix K_0 .

Method using the true tangent matrix K nor (known as also coherent or consistent matrix)

be called the method of NEWTON; methods using of other matrices (such as for example

stamp elastic K_0) are called modified methods of NEWTON. The choice enters a matrix

tangent (the last obtained or a preceding matrix) is carried out in Code_Aster by

the intermediary of the key word STAMPS: "TANGENT" or "RUBBER BAND" of the key word

factor NEWTON. Moreover,

it is possible to use a matrix of discharge, i.e. of a matrix with internal variables

constants (the evolution of nonthe linearities is thus not taken into account in this matrix), in

below of a certain step of time, for certain laws of behavior. One will refer to

documentation [U4.51.03] for the use of this functionality.

The method of NEWTON with consistent tangent matrix can be illustrated simply using diagram of the figure [Figure 2.2.1-a].

R
Li
L-R2
2
R L-R1
1
R
R0
0
1
U
U
2
U
U
I
I
I
I
U
1
U
2
U
I
I

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2.2.2 Resolution with boundary conditions dualized

When one takes into account the conditions of imposed displacements, the system to be solved is written:

R

(U, T) + BT

= Lméca

I I

I

I

Drunk

= ud

I

I

One will use the symbol to note the increments since the preceding balance (out of Ti-1) of displacements, of the parameters of Lagrange, the loading and imposed displacements:

U = U - U

I

I

i-1

= -

I

I

i-1

Lméca = Lméca - Lméca

I
I
i-1
ud = ud - ud

I
I
i-1

Knowing (U

,)
i-1
I
I
, the couple (U,)

I
I will be determined by the resolution of the system:

R

(U
+ U
, T) + BT (
+) = Lméca + Lméca

i-1
I
I
i-1
I
i-1
I

éq
2.2.2-1
B (U
+ U
) = ud + ud

i-1
I
i-1
I

One will use a method of NEWTON to solve this system. However, the experiment shows that the convergence of the method of NEWTON is strongly dependent on a judicious choice on the initial estimate: “more the initial estimate is close to the solution, plus the algorithm converges quickly”.

To start the iterative process of the method, it is thus useful to determine “a good” increment initial $(u_0, 0)$

I

I. For that, one linearizes compared to time the continuous problem: it is what one call the phase of prediction (or of initialization). One connects with the loop of the iterations of NEWTON which makes it possible, with convergence, to obtain the values of $(U,$

I

I, and thus those of

(U

)

I, I.

2.2.2.1 Phase of prediction

One linearizes the system [éq 2.2.2-1] compared to time around $(U$

,)

i-1

I

I

; by taking account of

the balance obtained at the moment T_{i-1} , one obtains the equations making it possible to calculate values

predictive $(u_0, 0)$

I

I:

R

R

$u_0 BT_0$

+

= $L_{méca}$ -

T

U

I

I

I

I

T

U

I

T

-1

i-1

Drunk = ud

I

I

R

= (): (T T (T))

indicate the total differential of R

Q U

***,
compared to T. This notation***

T Ti-1

T

particular be to draw the attention to the fact that

=

+

as one sees it in

T

T

T T

continuation.

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Therefore, while using [éq 1.2-1] and dependence from report/ratio with time and compared to temperature:

T

T

Q

0

0

Q:

+

: *U + BT*

= *Lméca - QT:*

T

- *QT:*

T

U

U

I

I

I

I

I

T

T

ui-1

U

T

T

I

I

I

-

-

-

l

l

l

B

U

= *ud*

I

I

If the constraints depend on other variables, such as for example of proportions Z of various metallurgy i.e. phases (T, T (T), Z (T)) , one sees appearing to the second member

the term corresponding: QT:

Z

. It is supposed thereafter that the effects

I

Z Ti-1

metallurgical [R4.04.02] are integrated into the second member representative of the effects thermics. Moreover, one currently does not hold account for the phase of prediction of dependence clarifies constraints compared to time. Lastly, dependence of stamp QT compared to displacements is neglected on the assumption of small

QT

displacements: the term

, known as term of geometrical rigidity, thus disappears from

U ui-1

the preceding equation.

With obvious notations, the system of equations obtained is also written:

K

u0

+ BT

0

= $Lméca + Lther$

$i-1$

I

I

I

I

$éq$

2.2.2.1-1

$B u_0$

=

ud

I

I

Let us note that one can replace the K_{i-1} matrix formally, derived from R compared to U in u_{i-1} by the elastic matrix K_0 .

For the developers, let us specify that the calculation of the tangent matrix at the time of the phase of prediction

is carried out via the option of calculation $RIGI_MECA_TANG$ [§ 2.2.2.3].

The effective increment of loading L_i appearing to the second member includes/understands two terms:

.

the mechanical increment of loading $Lméca$

I

, composed of the dead loads

(independent of the geometry, like gravity) and of the following loads (dependent geometry, like the following pressure [R3.03.04]). Actually, there is cases (it first increment of load, for example) where $méca$

L

is unknown: this $Lméca$ fact is

$I I$

-

I

rather calculated by the relation

méca

L

= méca

L

- T

Q

- T

B, which returns exactly to

I

I

I I

-

I I

-

even, in the case running, by taking account of balance with the increment (i-1). One will notice whereas the expression utilized the multipliers of Lagrange to the increment (i-1), which is sometimes unknown (with the first increment of load, for example). But linearity boundary conditions in imposed displacements, which results in the fact that the matrix B is constant, allows in this case an abuse language: one poses = 0 and

I

0

truth

0 truth

=

+

I

I I

-

(the true exhibitor corresponds to the variables appearing indeed

I

in the system of equations above) and one solves:

0

T

0

K U + B = méca

L

+

ther

L

- T

Q
I I
-
I
I
I
I
I I

- ,
0

Drunk = D
U - U
B
I
I
I I

-
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.
the increment of loading says “thermal” (and metallurgical [R4.04.02]) Lther
I
, resulting from

derivation of the nodal forces compared to the temperature and which is an estimate of the effect of a variation in temperature. Indeed, if one notes K the module of compression hydrostatic and the thermal dilation coefficient, the increment of loading “thermal” is written (since $R = QT$ and $ther = - 3K T$ I where I is the matrix

*$I D$
 D
identity):*

*$L_{ther} = - QT:$
 $T = QT: 3 K T$
 $I,$
 I
 I
 I
 T
 D
 $Ti-1$*

In the elastic case, they are the nodal forces associated a thermal dilation (it is not strictly speaking a loading, that is assimilated rather to the effect of a deformation initial). This estimate is used in the phase of prediction and the criterion of stop. If thermal dilations make leave the structure of the elastic range (plasticity by example), this estimate will be corrected at the time of the iterations of NEWTON.

Note:

A particular case relates to the use of an excitation of the type TYPE_CHARGE: Meaning “DIDI” Differential Dirichlet, i.e. compared to the initial state. That consists, for the conditions with D limits of the blockings type, to impose, not $B U = U$, but $B (U - U) = ud$ 0 . In this case, it system to be solved in the phase of prediction for the new increment of load is:

*$K u0 + T$
 $B 0 = méca$
 L
 $+$
 $ther$
 L*

- *T*
Q
i-1
I
I
I
I
i-1

Bu0 = D
U
I
I

2.2.2.2 Buckles on the iterations of NEWTON

One must find the values (U,)

I
I of the increments of displacements and parameters of Lagrange
since the values (U

,)
i-1

I
I
obtained with preceding balance (urgent Ti-1). One takes as
initial values (u0, 0)

I
I obtained at the end of the phase of prediction, before beginning them
iterations of the method of NEWTON.

With each iteration, one must solve a system allowing to determine (un+1 n+1
,
), increments

I
I
displacements and parameters of Lagrange since the result (one

N
,)
I
I of the iteration
the preceding one:

KN

un+1 + BT

N

+1 = Lméca - R (one) - BT N

I

I

I

I

I

I

éq

2.2.2.2-1

B un+

I

I

= 0

with R (one) = QT N

N

N

I

I, constraints I being calculated starting from ui displacements by the intermediary of the relation of behavior of the material [§1.1]. In fact, in the case of them incrémentaux behaviors, nor is calculated starting from (i-1, i-1) and of the increment of deformation (un+1) induced by the increment of un+1 displacement = one + un+1 - U

I

I

I

I

i-1.

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Stamp “tangent”

As in the phase of prediction, one is not obliged to use the true tangent matrix K nor. In private individual, operator STAT_NON_LINE authorizes the use of the elastic matrix K_0 , or reactualization of the tangent matrix all i_0 not of time (key word REAC_INCR) or all n_0 iterations of NEWTON (key word

N

REAC_ITER). Thus, the matrix K_I perhaps replaced by one stamp K

m

J, with J_I , or a K_i matrix, with $m N$.

Caution: a “stiff” matrix too does not pose problems of stability but can produce a very slow convergence; a “flexible” matrix too can lead to divergence, it is advised in this case to make linear research [§ 2.3].

It is difficult to give a rule making it possible to know when one must reactualize the tangent matrix: that strongly depends on the degree of nonlinearity of the problem and the size of the increments of load.

In discharge, it is recommended either to use the elastic matrix for the phase of prediction and of resolution, the elastic matrix for the phase of prediction then the matrix is to use tangent for the resolution.

The figures hereafter illustrate the various possibilities of reactualization of the tangent matrix: stamp elastic K_0 used with each iteration [2.2.2.2 Figure - has], tangent matrix reactualized with each iteration and with each step of time [2.2.2.2 Figure - B], reactualized tangent matrix all them i_0 not of time ($i_0 = 1$ here) [2.2.2.2 Figure - C], and stamps tangent reactualized all the n_0 iterations of NEWTON ($n_0 = 2$ here) [2.2.2.2 Figure - D].

R

Li

U

***Appear 2.2.2.2 - has: use of the elastic matrix
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R

Li

U

***Appear 2.2.2.2 - B: use of the true revalued tangent matrix
with each iteration***

R

Li+1

LI

U

***Appear 2.2.2.2 - C: use of the tangent matrix
revalued with each step of time***

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R

L I

U

**Appear 2.2.2.2 - D: use of the revalued tangent matrix
all 2 iterations of NEWTON**

Method of NEWTON modified (using another matrix than the consistent tangent matrix) converge less quickly than the method of NEWTON traditional, but is less expensive. It is all the more economic as the number of degrees of freedom of the system is high. This is why one can advise, when the system is of big size, to keep the same tangent matrix during some iterations. Lastly, let us not forget to announce that in certain cases, it is calculation with stamp elastic which is fastest in terms of computing time, even if the iteration count carried out is much more important (they are cheap iterations since the matrix is not calculated and factorized that only once); moreover, it is the elastic matrix which it is recommended to use during the discharges.

As the equation [éq 2.2.2.2 - 1] shows it, it is necessary to carry out with each iteration of NEWTON calculation

possible of the new tangent matrix K_N

N^T

and of the “nodal forces” $R + B$

I

I

I: for

developers, let us specify that these operations are carried out by the option of calculation

FULL_MECA

(RAPH_MECA if the tangent matrix is not recomputed).

The tangent matrix obtained by option RIGI_MECA_TANG and the tangent matrix obtained by option FULL_MECA are in general calculated differently [§ 2.2.2.3].

Actualization of the unknown factors

The actualization of displacements and their increments is done as follows, that of the parameters of Lagrange is carried out in an identical way:

un+

$$I$$
$$= one + un+I$$

I

I

I

un+I

$$= one + un+$$

I

I

I

I

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Criteria of convergence

There are three criteria of convergence.

Criterion RESI_GLOB_MAXI consists in checking that the infinite standard of the residue

T

N

Q + T N

B - méca

L

, is lower than the value specified by the user. He is not advised

I

I

I

to use this criterion alone, because one cannot easily have an idea of the absolute orders of magnitude acceptable.

The criterion of convergence chosen by defect amounts checking that the residue is sufficiently small, as previously, and this relative with a quantity representative of the loading (it is it criterion RESI_GLOB_RELA):

QT N

+ BT N

- Lméca

I

I

I
 ,
éq
 2.2.2.2-2
Lméca + Lther - BT N

I
I
I

being desired relative precision given by the user (or the default value of 10-6) and

the standard of the maximum.

One can notice that, in the case of use of RESI_GLOB_RELA, the criterion can become singular if the external loading

Lméca
Lther
BT N
 +
 -
becomes null. This can arrive in

I
I
I

case of total discharge of the structure. If such a case of figure arises (i.e loading 10-6 times smaller than the smallest loading observed until now the increment), the code use then criterion RESI_GLOB_MAXI with as value that observed with convergence preceding increment. When the loading becomes again not no one, one returns to the criterion initial.

The third criterion is criterion RESI_REFE_RELA: the idea of this criterion is to build a force nodal of reference, which will be used to estimate term in the long term, the nullity (approximate) of the residue:

J
 {
 }
 ddls
 (T N T N méca
Q + B - L
F
I

I
I
)
ref.
J
J

More precisely, the nodal force of reference is built starting from the data of an amplitude of constraint of reference (in mechanics; in the case of the THM, it is necessary to give one reference to each physical phenomenon entering the generalized constraint)

ref.

. From this amplitude of constraint of reference, one defines the tensor test : it is null for all these

J
components, except j-ième which is worth ref.

. One defines then, for each node of each element following nodal force (the goal being to give an idea of the importance of a component in a point of Gauss of the constraint on the nodal force):

NR
M
~
1
E
R =
B

I
test
I, J
J
NR = 1 j=1

with NR the number of points of Gauss of the element, M the component count of the tensor of constraint; the exhibitor being used to note the evaluation of quantity at the point of Gauss, are them weight of the points of Gauss.

The nodal force of reference is then defined by:

ref.
~e

F
= min R
I
I
E
I
where *I* is the whole of the elements connected to node *I*.
I
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Convergence is issued carried out when all the criteria specified by the user are checked at the same time. By defect, one makes a test on the relative total residue (RESI_GLOB_RELA) and numbers it maximum of iterations of NEWTON (ITER_GLOB_MAXI).

2.2.2.3 RIGI_MECA_TANG and FULL_MECA

It is important to stress that the tangent matrix resulting from option RIGI_MECA_TANG and the matrix tangent resulting from option FULL_MECA are in general not identical.

Let us suppose that one reached convergence for the moment T_{i-1} and that one seeks now with to obtain balance for the moment following T_i . The matrix resulting from IH GI_MECA_TANG comes from one linearization of the equilibrium equations compared to time around (U ,)

*i-1**1**I**i.e around*

balance at the moment T_{i-1} . It is thus the tangent matrix of the system with convergence at the moment T_{i-1} .

On the other hand, the matrix resulting from FULL_MECA comes from a linearization from the equilibrium equations by

report/ratio with displacement around (one

*N
,) i.e around balance at the moment T .*

*I**I**I*

Moreover, one can interpret the differences between RIGI_MECA_TANG and FULL_MECA in others

terms. One can thus show that the matrix resulting from RIGI_MECA_TANG corresponds to the operator

tangent of the problem continuous in time, known as also problem of speed (and connects the speed of constraint

at the speed of deformation), whereas the matrix resulting from FULL_MECA corresponds to the operator

tangent of the problem discretized in time [bib1]. The document [R5.03.02] gives the expression in each of the two cases for the relation of elastoplasticity of Von Mises with isotropic work hardening.

It is pointed out that the treatment of a relation of behavior [R5.03.02 § 5] consists with:

*.
to calculate constraints N*

*N
I and the variables intern I starting from the initial state (*

*,)
i-1
i-1
and of the increment of linked displacement,*

*.
to calculate the nodal forces $R_N = Q^T N$*

*N
I
I and reactions of BT_i support,*

*.
to calculate (possibly) the tangent matrix (option RIGI_MECA_TANG for the phase of*

prediction, option FULL_MECA for the iterations of NEWTON).

2.2.3 Case of the following loadings

A following loading (in mechanics) is a loading which depends on the geometry of the structure, as for example the pressure which is exerted in the direction opposed to the normal (or the forces of inertia in a reference mark not galiléen). Thus, when the structure becomes deformed with the evolution of charge, the loading, expressed in an absolute reference mark, is transformed. The loads which do not depend

no the geometry of the structure are called dead or fixed loads (for example, gravity). To indicate that a load must be treated like a following load in STAT_NON_LINE, one indicates TYPE_CHARGE: "SUIV" under key word EXCIT.

A mechanical loading $L_{méca}(T)$ comprising following loads is written $L_{fixe}() + L_{suiv}$

(U, T)

(the fixed exhibitor indicates here the died loads, and suiv the following loads). The system of equations

to solve becomes then:

R

$(U, T) + BT$

$= L_{fixe}(T) + L_{suiv}(U, T)$

II

I

I

II

Drunk

$= ud$

I

I

Operations of derivation allowing to write the phase of prediction and the iterations of the method of NEWTON thus utilize the derivative of L_{suiv} compared to displacements U .

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The phase of prediction becomes:

Lsuiv

(

K

) **u0**

BT

0

Lfixe Lsuiv Lther

I -

+

=

+

+

I

U

I

I

I

I

I

U

I

-I

B

u0

=

ud

I

I

and the iterations of NEWTON consist in solving the system:

Lsuiv

(

KN -

) un+1 + BT

n+

l = Lfixe + Lsuiv (one) - R (one) - BT N

I

U

I

I

I

I

I

I

one

I

B

un+

I

I

= 0

Thus, with the beginning of each step of load (prediction) and with each iteration of NEWTON, one must

to calculate a matrix of “rigidity” ($-L_{suiv}/U$) and a L_{suiv} vector **one**

(
) related to the loadings

one

I

I

follower.

The only loads which can be treated like following loads in the current state of operator `STAT_NON_LINE` are:

·
pressure for modelings `3D`, `3D_SI`, `D_PLAN`, `D_PLAN_SI`, `AXIS`, `AXIS_SI`,
`C_PLAN`, `C_PLAN_SI` [R3.03.04],

·
the loading of gravity for elements `CABLE_POULIE` [R3.08.05], elements with three nodes comprising a pulley and two bits of cables: the force of gravity being exerted on the element depends on the respective lengths of the two bits,

·
the centrifugal force in great displacements, which for a number of revolutions is given by:

[OM]

D

=

[(OM +

)

;

0

D

U]

·
the loading of gravity for all modelings `THM` of the porous environments not saturated [R7.01.10]: indeed, the density depends on the nodal variables U , p and T to take account of the relations of behavior of the géomatériaux one.

2.3 Seek

linear

Linear research here exposed relates to linear research in the absence of piloting. For description of linear research in the presence of piloting, one will refer to documentation [R5.03.80].

2.3.1 Principle

The introduction of linear research into operator STAT_NON_LINE results from a report: method of NEWTON with consistent matrix does not converge in all the cases of figure, in particular when one leaves too much far from the solution. In addition, the use of matrices other than stamp tangent consistent can, when they are too “flexible”, lead to divergence. linear research makes it possible to be guarded against such divergences.

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It consists in considering ($un+1$, $n+1$

I

I

), either like the increment of displacements and of parameters of Lagrange, but as a direction of research in which one will seek with to minimize a functional calculus (the energy of the structure). One will find a step of advance in this direction, and the actualization of the unknown factors will consist in making:

$un+1 = one + un+1$

I***I******I***

$$n+I = N + n+$$

I***I******I******I***

In the absence of linear research (by defect) the scalar is of course equal to 1.

2.3.2 Minimization of a functional calculus

In order to be better convinced of the founded good of linear research, one can interpret the method of NEWTON like a method of minimization of a functional calculus (if matrices tangents are symmetrical). We insist on the fact that the equations obtained are rigorously those of the method of NEWTON exposed in [§2.2] and that only the way of y to arrive is different.

“The talk the dualisation of the boundary conditions of Dirichlet and we place Forget” to simplify on the assumption of the small deformations. The functional calculus is considered:

J: V***IR***

$$U J (U) = W ((U)) D F .u D t.u D$$

where the density of free energy W makes it possible to connect the tensor of the constraints to the tensor of

W

deformations linearized by the relation =.

The functional calculus J being convex, to find the minimum of J is equivalent to cancel its gradient, that is to say:

$$J (U) .v = 0 \quad \forall v \in V,$$

what is exactly the Principle of Virtual Work since:

$$J (U) .v = (U): (v) D - F .v D t.v D$$

Thus, to solve the equations resulting from the Principle of Virtual Work (bases problem formulated in [§1.2]) is equivalent to minimize the functional calculus J which represents the energy of the structure (energy decreased intern of the work of the external forces F and T).

2.3.3 Method of minimization

Minimization is made in an iterative way, classically in two times with each iteration:

· calculation of a direction of research along which one will seek reiterated according to,

· calculation of the “best” step of advance in this direction: $u_{n+1} = u_n + \Delta u$.

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In a problem of minimization, the natural idea is to advance in the direction opposed to the gradient functional calculus, which is locally the best direction of descent since this direction is normal with the lines of isovalues and directed in the direction of the decreasing values [Figure 2.3.3-a].

U N

- $J(U, N)$
 $J(U) = \text{cste}$

Appear 2.3.3-a

However, it is possible to improve the choice of the direction of descent by using this method of gradient in a metric news. It is what will enable us to find the equations traditional of the method of NEWTON [§2.3.4].

Let us take the simple example of a problem with two variables X and there for which the functional calculus with the form of an ellipse whose minimum is in $(/a, /b)$:

$$J(X, y) = ax^2 + by^2 - X - y$$

While choosing like direction of descent the reverse of the gradient of J , one passes from one reiterated to the following (only let us reason on X) by:

$$x_{n+1} = x_n - (ax_n -),$$

who does not point towards the solution since the normal in a point of an ellipse does not pass in general not by the center of the ellipse [Figure 2.3.3-b].

y

B

X
has

Appear 2.3.3-b

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On the other hand, if a change of variables is carried out so that the isovaleurs of J become circles [Figure 2.3.3-c]:

X = has X

y = B y

1

X y =
X 2 + y2 -
X -
y

J (,)
(
)

2
has
B

y

B

X
has

Appear 2.3.3-c

The use of the opposite direction of the gradient of J then makes it possible to obtain the solution in one iteration:

$$X_{n+1}$$

$$X_N$$

$$X_N$$

$$x_{n+1}$$

$$=$$

$$-$$

$$-$$

$$=$$

$$1$$

$$($$

$$)$$

$$=$$

has

has

has

Thus, the use of the method of gradient in the metric news allows a convergence immediate. In a more complicated case (functional calculus convex but different from an ellipse), convergence is not instantaneous but the change of variables makes it possible to reduce appreciably the iteration count necessary.

2.3.4 Application to the minimization of energy

To simplify, one will place oneself in the discretized linear case where the functional calculus J is worth:

$$1$$

$$J(U) =$$

$$uTK U - uTL$$

$$2$$

O one notes K the matrix of rigidity of the structure, and L the vector of the imposed loadings.

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To minimize J , we will use the same method of descent as previously while making with precondition a completely similar change of variables. The matrix K being symmetrical defined positive, its eigenvalues are real positive: one can thus define the “square root” of K that K will be noted (also symmetrical). One poses $U = K U$, the minimization of J is then equivalent to that of:

$$\begin{aligned}
 &1 \\
 &J(U) = \\
 &U U - U \\
 &K - 1 \\
 &T \\
 &T \\
 &L \\
 &2
 \end{aligned}$$

By taking as direction of descent the opposite direction of the gradient of J , one obtains:

$$\begin{aligned}
 &u_{n+1} \\
 &one \\
 &one \\
 &K - \\
 &= \\
 &- \\
 &-
 \end{aligned}$$

I
(
L)

Maybe, while returning to the initial variables:

$$\begin{aligned} & \mathbf{u}_{n+1} \\ & \mathbf{one} \\ & \mathbf{K} - \\ & = \\ & - \\ & \mathbf{1} (\mathbf{K} \mathbf{one} - \mathbf{L}) \end{aligned}$$

Or:

$$\begin{aligned} & \mathbf{K} (\mathbf{u}_{n+1} - \mathbf{one}) = \\ & \mathbf{L} - \mathbf{K} \mathbf{one} \end{aligned}$$

One finds the equations of the method of NEWTON: the matrix K is Hessienne of functional calculus J (matrix of the derived second) and the second member is the difference of the loading and of the nodal forces, also called residue.

Thus method of NEWTON perhaps interpreted like resulting from the minimization of energy structure via a method of gradient applied after a change of metric.

2.3.5 Determination of the step of advance

Let us return to the real problem, that of the resolution of $\mathbf{R}(\mathbf{U}, \mathbf{T}) = \mathbf{L}$
II

I. This problem can be interpreted like the minimization of:

$$\begin{aligned} & \mathbf{W} \\ & \mathbf{T} \\ & (\mathbf{u}_i) - \mathbf{u}_i \mathbf{L}_i, \end{aligned}$$

where W (u_i) corresponds to the discretization, on the basis of function of form, of energy interns structure

$$\begin{aligned} & ((\mathbf{U})) \\ & \mathbf{W} \mathbf{D}. \end{aligned}$$

One calculated by the method of NEWTON an increment of \mathbf{u}_{n+1} displacement
I

who, in the problem

of minimization, is interpreted like a direction of research. One will calculate the step of advance in this direction allowing to minimize the value of the functional calculus:

Min {W N

(U +

n+

U 1) -

N

L (U +

n+1

I

I

I

I

ui)}

R

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To find the minimum of this scalar function of that one will note $F()$, one seeks the point where its derivative is cancelled (that amounts making orthogonal the final residue and the direction of research):

$F'() = [N T$

+1

$$\begin{aligned}
 &T \\
 &N \\
 &n+ \\
 &U \\
 &I \\
 &I \\
 &] [Q (ui + ui) - Li] = 0
 \end{aligned}$$

(F '() is the projection of the residue on the direction of research)

With the notations of [§2.2.2] and by taking of account reactions of support, the scalar equation with to solve to determine the step of advance, is written:

$$\begin{aligned}
 &[T \\
 &un+1] [QT (a un+1) BT (N n+ \\
 &+ \\
 &+ \\
 &+ \\
 &1) - Lméca \\
 &I \\
 &I \\
 &I \\
 &I \\
 &I \\
 &I \\
 &I \\
 &] = 0
 \end{aligned}$$

So that the determination of is not too expensive, one uses a method of secant of which it numbers maximum iterations is fixed by the user. The method of secant can be interpreted like a method of NEWTON where the derivative at the point running is approached by the direction joining it not running and the preceding point:

$$\begin{aligned}
 &p - p-1 \\
 &p-1 p \\
 &G - p p \\
 &G -1 \\
 &p+1 = p \\
 &p \\
 &- \\
 &G \\
 &= \\
 &p \\
 &p-1 \\
 &p
 \end{aligned}$$

p
 $G - G$
 $G - G - 1$
,

where $G p$ was noted

F
 p
 $= ' () .$

One leaves $0 = 0$ and $1 = 1$. The method of secant has an order of convergence of about 1.6 [bib2]. It is represented schematically on the figure [Figure 2.3.5-a].

1
 $G = F ' ()$
 4
 0
 1
 3
 2
 0

Appear 2.3.5-a

At the end of linear research, one brings up to date displacements and parameters of Lagrange by:

$un+1 = one + un+1$
 I
 I
 I
 $n+1 = N + n+$

I
 I
 I
 I

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The test of convergence carries:

.
on the maximum number of iterations of linear research indicated by the user under key word ITER_LINE_MAXI of the key word factor NEWTON (the default value 0 inhibits linear research, and is worth 1 then),

.
on criterion RESI_LINE_RELA given by $F()$. $F()$ 0, where is worth by defect 0.1.

Linear research is to some extent a “policy insurance” allowing of to secure against serious divergences of the method of NEWTON. When direction of seek ($\mathbf{un}+1$, $n+1$ I I) is “bad” (if the tangent matrix is too flexible, by example), the linear algorithm of research leads to a low value of, which avoids of going “in the decoration”. It is not necessary to do many iterations in the method of secant (2 or 3 are enough to avoid the catastrophes) because each one is rather expensive (it is necessary to recompute the internal forces) and there is not the ambition to find with each iteration of NEWTON the really optimal one.

2.4 Algorithm

of

STAT_NON_LINE

One will use like previously index I (like “moment”) to note the number of an increment of charge and exposing it N (like “newton”) to note the number of the iteration of NEWTON in progress. The algorithm used in operator STAT_NON_LINE can then be written way schematically

following:

$(\mathbf{u}_0, \mathbf{0})$ and 0 known

Buckle over moments T_i (or increments of load): loading $\mathbf{L} = \mathbf{L}$

I

(T_i)

.

$(\mathbf{u}_{i-1}, i-1)$ known

.

Prediction: calculation of \mathbf{u}_0

0

I and I

.

Buckle on iterations of NEWTON: calculation of a continuation (\mathbf{one} , N

I

I)

-

$(N$

N

N

I, I) and $(\mathbf{U}$,

I

I) known

-

Calculation of the matrices and vectors associated with the following loads

-

Expression of the relation of behavior

-

calculation of constraints N

N

I and of the variables intern I starting from the values

$i-1$ and $i-1$ with preceding balance (T_{i-1}) and of the increment of displacement

$\mathbf{one} = \mathbf{one} - \mathbf{U}$

I

I

$i-1$ since this balance

-

calculation of the "nodal forces": $\mathbf{Q}_n + \mathbf{BT} N$

I

I

-

possible calculation of the matrix of tangent stiffness: $\mathbf{K} N = \mathbf{K} (\mathbf{one}$

I

I)

-

*Calculation of the direction of research (**un+1**, n+1*

I

I

) by resolution of a system

linear

-

Iterations of linear research:

-

Actualization of the variables and their increments:

un+

$$I = one + un+1$$

un+1

$$= one + un+1$$

I

I

I

I

I

I

and

n+1

N

n+1

n+1

N

n+

I

I

$$= I + I$$

I

$$= I + I$$

-

Test of convergence
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.

Filing of the results at the moment T_i

U

$= U$

$+ U$

I

$i-1$

I

$I = I +$

$-I$

I

I

I

It is noticed that there are three overlapping levels of iterations: a loop external on the steps of time,

a loop of iterations (known as total) of NEWTON and subloops possible for linear research (if it is asked by the user) and certain relations of behavior requiring iterations (known as interns), for example for elastoplasticity in plane constraints.

3 Piloting

One will refer to documentation [R5.03.80].

4 Large deformations

4.1 Objective

Until now, we made the assumption that displacements and deformations remained moderate so as to respect the assumption of the small disturbances. This assumption becomes null and void in many cases and the method of resolution of the problem must then integrate the evolution of geometry of the problem, to handle a particular kinematics and to use an adequate formulation of the law of behavior.

In practice, the assumption of the small deformations can be applied as long as the square of modulus of deformation remains lower than the precision of calculations considered. In the same way, the assumption of small rotations can be applied as long as the product enters the square of the swing angle and the modulus of deformation remain lower than the precision of calculations considered.

Various alternatives exist within Code_Aster; our objective is not here to do one of them detailed presentation and we return to the various documents treating specifically each problems:

*.
Relation of nonlinear elastic behavior in great displacements: [R5.03.20],*

*.
Beams in great displacements: [R5.03.40],*

*.
Voluminal elements of hulls into nonlinear geometrical: [R5.07.05],*

*.
Elasto (visco) plasticity, metallurgy and great deformations: [R4.04.03],*

*.
Elastoplastic modeling with isotropic work hardening in great deformations: [R5.03.21].*

The objective is here to present a general functionality of the code which allows a simple treatment of problems great deformations: argument `PETIT_REAC` of the key word `DEFORMATION` under

key word factor COMP_INCR.

In the continuation, we will limit ourselves to the case of the great plastic deformations, which allows of good

to include/understand problems PETIT_REAC. We base ourselves for that on [bib4] and on documentation [R5.03.21].

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4.2

Great plastic deformations

Initially let us point out the requirements of a modeling great plastic deformations in mechanics of the continuous mediums in term of equilibrium equations, of kinematic description and of relation of behavior so as to determine the limitations of modeling PETIT_REAC well.

Equilibrium equations

If one makes the choice write the equilibrium equations on the current configuration and use it tensor of constraints of Cauchy, they are summarized with:

: \mathbf{v}

$$D = \mathbf{F} \cdot \mathbf{v}$$

$D +$

.

$\mathbf{T} \mathbf{v}$

D

\mathbf{v}

Vad

éq

4.2-1

The preceding equation can be written on the configuration of reference; it is besides what is made for nonlinear elasticity in great deformations [R5.03.20]. In the case of laws of behavior of incremental nature i.e whose evolution is controlled by the current state, this writing loses of sound interest. One prefers then the writing [éq 4.2-1], although the current configuration is an unknown factor of problem.

Kinematic description

For the laws of plastic behavior, one uses commonly like measures deformation it rate of deformation \mathbf{D} :

$$\mathbf{D} = \frac{1}{2} (\mathbf{U} + \mathbf{U}^T)$$

Moreover, one defines also the rate of rotation \mathbf{W} :

$$\mathbf{W} = \frac{1}{2} (\mathbf{U} - \mathbf{U}^T)$$

Let us stress that in the preceding expressions, the operator gradient is defined on the configuration current.

Elastoplastic relation of behavior

Before writing the relations of behavior, we will make three simplifying assumptions:

the elastic strain are small in front of the plastic deformations (what is checked generally well in the case of metals). This makes it possible to break up the rate of deformation in an additive way in a plastic part and an elastic part:

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p$$

the plastic deformation is isochoric. This makes it possible to write that the tensors of constraints of Kirchhoff (stress measurement adequate to the great deformations) and of Cauchy

(intervening in the equilibrium equations) are identical to the first order.

the behavior is isotropic.

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Under these assumptions, the laws of behavior in great deformations can be written under one form near to the small transformations. Thus for the plasticity of Von-Settings to isotropic work hardening one obtains:

$\mathbf{J} =$

E

p

$\mathbf{A} : (\mathbf{D} - \mathbf{D})$

$F(, p) = - R(p)$

éq 4.2-2

eq

3

~

$$Dp = p$$

&
2

eq
where

.

is the diverter of,

.

A is the tensor of elasticity,

.

J indicates it derived from Jaumann of, which takes account of terms of transport defined in assistance of the rate of rotation \mathbf{W} and which is given by:

$$J = \& - \mathbf{W} \cdot + \cdot \mathbf{W}$$

4.3 Functionality

PETIT_REAC

The principle of modeling PETIT_REAC simply consists in reactualizing the geometry of problem during iterations of Newton (and not at the end of each step of time). This means that all the quantities intervening in the equations of the problem are evaluated on the configuration current. Anything else is not modified compared to the case small disturbances.

We have just seen which ingredients are necessary to a “clean” integration of a law of elastoplastic behavior in great deformations, with the help of three simplifying assumptions. Now let us detail the differences which the resolution by PETIT_REAC implies.

Equilibrium equations

Taking into account the geometrical reactualization, the interior efforts, first term of [éq 4.2-1], are estimated in an exact way, with the help of the approximation by finite elements. The calculation of the efforts

outsides east him independent of the resolution by PETIT_REAC: the dead loads are calculated on the configuration of reference and the loads following on the current configuration. Equations with balance are thus dealt in an exact way.

Kinematic description

Kinematic description is the same one as that of the small disturbances. This means that one increment of deformation is calculated by:

$$1 \\ = ((\mathbf{U}) + T(\mathbf{U}))$$

2

The total deflection is then the sum of each one of these increments of linearized deformation, calculated on different configurations. It is thus delicate to give him a physical direction and better is worth to use it like an indicator of the level of deformation reached.

Elastoplastic relation of behavior

The difference between the formulation [eq 4.2-2] and the formulation small disturbances lies in replacement of derived from Jaumann by the simple derivative in time. The disadvantage of this last is due to the fact that it is not incrémentalement objective i.e not invariant by rigid rotation structure (for more details, to see documentation [R5.03.21]).

The use of PETIT_REAC is thus not appropriate to great rotations but it is it with large deformations.

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Integration within Code_Aster

In term of finite elements, the resolution by PETIT_REAC implies with each step of load resolution of the same nonlinear system as previously:

R

$$(U, T) + BT$$

$$= L(T)$$

I I

I

I

Drunk

$$= ud(T)$$

I

I

With the difference close the forces intern are formally calculated by:

$$\mathbf{R}(U, T)$$

T

$$= \mathbf{Q}(U):$$

I

I

I

where the operator Q depends on displacements.

Within this framework, the calculation of the tangent matrix carries out formally to:

R

$$\mathbf{Q}(U)$$

$$\mathbf{KN} =$$

$$= \mathbf{Q}(U):$$

+

:

I

U

N

U

N

U

$$(U, T)$$

(U, T)
(one, T
I
I
I
I
I
I)

The first term is the contribution of the behavior, similar to what was presented into small transformations, with the difference which this contribution is evaluated here in current configuration. second term is the contribution of the geometry which is not present in small transformations. Within the framework of resolution PETIT_REAC, this term is not present in the calculation of the matrix tangent. One thus has:

$\mathbf{K} \mathbf{N} = \mathbf{Q} (\mathbf{U})$:
I
 \mathbf{U} (one, tii)

One can announce that the absence of the geometrical contribution in the tangent matrix can sometimes to make convergence difficult. Moreover, the presence of important plastic deformations can to make the problem quasi-incompressible, because of isochoric nature of these deformations. problems of numerical blocking can then appear which can be circumvented by employment elements under integrated

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Organization (S): EDF/MTI/MMN, RNE/MTC

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R5.03.02 document

Integration of the relations of behavior

elastoplastic of Von Mises

Summary:

This document describes the quantities calculated by operator STAT_NON_LINE necessary to the implementation of the quasi static nonlinear algorithm describes in [R5.03.01] in the case of the elastoplastic behaviors. These quantities are calculated by the same subroutines in operator DYNA_NON_LINE in the case

of a dynamic stress [R5.05.05].

This description is presented according to the various key words which make it possible the user to choose the relation

of behavior wished. The relations of behavior treated here are:

- the behavior of Von Mises with isotropic work hardening (linear or not linear)*
- the behavior of Von Mises with linear kinematic work hardening (model of Prager)*

The method of integration used is based on a direct implicit formulation. From the initial state, or to leave

moment of preceding calculation, one calculates the stress field resulting from an increment of deformation. One

also calculate the tangent operator.

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1 Introduction

1.1

Relations of behaviors described in this document

In operator STAT_NON_LINE [U4.51.03] (or DYNA_NON_LINE [U4.53.01]), two types of behaviors can be treated:

- **the incremental behavior: key word factor COMP_INCR,**
- **the behavior in nonlinear elasticity: key word factor COMP_ELAS.**

For each behavior one can choose:

- **the relation of behavior: key word RELATION,**
- **mode of calculation of the deformations: key word DEFORMATION.**

For more details, to consult the document [U4.51.03] user's manual, the behaviors described

here raising only of the key word factor COMP_INCR.

The relations treated in this document are:

VMIS_ISOT_LINE:

Von Mises with linear isotropic work hardening,

VMIS_ISOT_TRAC:

Von Mises with isotropic work hardening given by a traction diagram,

VMIS_CINE_LINE:

Von Mises with linear kinematic work hardening.

1.2

Integration drank

To solve the nonlinear total problem posed on the structure, the document [R5.03.01] described the algorithm used in Aster for nonlinear statics (operator STAT_NON_LINE) and it document [R5.05.05] described the method used for nonlinear dynamics (operator DYNA_NON_LINE).

These two algorithms are based on the calculation of local quantities (in each point of integration of each finite element) which results from the integration of the relations of behavior.

With each iteration N of the method of Newton [R5.03.01 § 2.2.2.2] one must calculate the nodal forces

R one

()

N

N

I

= QT I (options RAPH_MECA and FULL_MECA) constraints I being calculated in each point of integration of each element starting from displacements linked via relation of behavior. One must also build the tangent operator to calculate Kni (option FULL_MECA).

Before the first iteration, for the phase of prediction, one calculates Ki -1 (option RIGI_MECA_TANG).

The calculation of Ki -1, which is necessary to the phase of initialization [R5.03.01 § 2.2.2.2] corresponds to

calculation of the tangent operator deduced from the problem of speed.

This operator is not identical to that which is used to calculate Kni by option FULL_MECA, to run of the iterations of Newton. Indeed, this last operator is tangent with the problem discretized of implicit way.

One describes here for the relations of behavior VMIS_ISOT_LINE, VMIS_ISOT_TRAC and VMIS_CINE_LINE, the calculation of the tangent matrix of the phase of prediction, Ki - 1, then the calculation of

stress field starting from an increment of deformation, the calculation of the nodal forces R and stamp tangent Kni.

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Code_Aster ®**Version****5.0****Titrate:****Integration of the elastoplastic relations****Date:****20/03/01****Author (S):****J.M. PROIX, E. LORENTZ, P. MIALON****Key:****R5.03.02-C Page:****4/26****2****General notations and assumptions on the deformations****-****All the quantities evaluated at the previous moment are subscripted par.****The quantities evaluated at the moment $T + T$ are not subscripted.****The increments are indicated par. One has as follows:**

$$Q = Q T$$

$$(+ T) = Q T () + Q = Q + Q.$$

For the calculation of the derivative, one will note: ! **Q derived from Q compared to time****tensor of the constraints.****operator déviatoire: ~**

$$ij = ij - 1$$

$$3 kk ij.$$

$$() eq$$

$$3 \sim \sim$$

equivalent value of Von Mises:

$$=$$

$$eq$$

$$ij ij$$

$$2$$

increment of deformation.**With****tensor of elasticity.****, μ , E , ν , K** **moduli of the isotropic elasticity, respectively: coefficients of Lamé,****Young modulus, Poisson's ratio and module of compressibility.**

$$3K = 3 + 2\mu$$

modulate compressibility

thermal dilation coefficient average.

T

time.

T

temperature.

()

positive part.

+

To calculate the tangent operators, one will adopt the convention of writing of the symmetrical tensors of order 2 in the form of vectors with 6 components. Thus, for a tensor a:

“T

a= [axx ayy azz

2axy

2axz

2ayz]

”

One introduces the hydrostatic vector 1 and stamps it deviatoric projection P:

“1=t [1 1 1 0 0] 0

1 ” ”

P = Id - 1 1

3

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2.1

Partition of the deformations (small deformations)

One writes for any moment:

T () = E T () + HT T () + p T (),

with

$E(T) =$

-

To 1 (T (T)) (T)

with

$HT(T) = (T(T)) (T(T) - T_{Id}$

ref.)

or in a more general way:

$HT(T)$

$= (T) (T - T - T$

$T - T$

def) (ref.) (ref.

def)

$= \#(T) (T - T_{ref})$

and

$HT(T$

$= 0$

ref.)

A depends on the moment T via the temperature. The thermal dilation coefficient

(T (T)) is an average dilation coefficient which can depend on the temperature T.

temperature T is the temperature of reference, i.e. that for which thermal dilation

ref.

is supposed to be null if the average dilation coefficient is not known compared to T, one can

ref.

to use a temperature of definition of the dilation coefficient average T

(defined by the key word

def

TEMP_DEF_ALPHA of DEFI_MATERIAU) different from the temperature of reference [R4.08.01].

.

$\$ \& \%$

$\&$

,

$\&\&$

What leads to: ! (T)

-

To 1 (T (T)) (T)! HT (T)! p

=

+

+

(T)

This choice is made by preoccupation with a coherence with elasticity: it is necessary to be able to

find the same solution in

elasticity (operator MECA_STATIQUE) and in elastoplasticity (operator STAT_NON_LINE) when

them

characteristics of material remain elastic. This choice leads to the discretization:

$= p + A^{-1}$

$() + HT$

with:

A^{-1}

$() = A^{-1} T + T$

$($

$) -$

$(+$

$) - A^{-1} T$

$() -$

and

$HT = T$

$(+ T$

$($

$) T (- T)$

$($

$)$

ref.

$- T -$

$() T - Tref Id$

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2.2 Reactualization

In STAT_NON_LINE, under the key word factor COMP_INCR, three modes of calculation of the deformations

are possible:

• “SMALL”

- “SIMO_MIEHE” [R5.03.21] (which carries out calculation in great deformations for one isotropic work hardening)
- “PETIT_REAC” (which is a substitute with calculation in great deformations, valid for the small ones increments of load, and for small rotations [bib2]).

This last possibility consists in reactualizing the geometry before calculating:

X is written

$= X$

N

N

$O + u_i - 1 + u_i$, the calculation of the gradients of u_i is thus made with geometry X instead of the initial geometry x_0 .

2.3 Conditions

initial

They are taken into account via $-, p, U$.

In the event of continuation or resumption of a preceding calculation, there is directly the initial state $-, p, U$ in

on the basis of p, U of preceding calculation at the specified moment.

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3

Relation of Von Mises with isotropic work hardening

3.1

Expression of the relations of behavior

These relations are obtained by key words VMIS_ISOT_LINE and VMIS_ISOT_TRAC.

For these two relations, the mode of calculation of the deformations is DEFORMATION: "SMALL":

~

.

\$%'

p

3

!

=

- 1

! *p*

=! - A -! HT

2

eq

eq - *R* (*p*) 0

! *p* = 0 if *eq* - *R* (*p*) <

0

! *p* 0 if *eq* - *R* (*p*) = 0

! *p*: speed of plastic deformation,

p: cumulated plastic deformation,

HT : thermal deformation of origin: $HT = (T - ref.$

$T)$.

Id

The function of work hardening $R(p)$ is deduced from a simple test tensile monotonous and isothermal
In this case:

0

$eq = L$

0

L

= 0 0

0

$p = P$

L

L

= $L -$

.

E

0 0 0

$L - R(p)$ 0

The user can choose a linear work hardening (relation VMIS_ISOT_LINE) or a traction diagram data by points (relation VMIS_ISOT_TRAC).

3.1.1 Relation

VMIS_ISOT_LINE

The data of the material characteristics are those provided under the key word factor ECRO_LINE or ECRO_LINE_FO of operator DEFI_MATERIAU [U4.43.01].

/ECRO_LINE: (D_SIGM_EPSI: AND

SY: y)

/ECRO_LINE_FO: (D_SIGM_EPSI: AND

SY: y)

ECRO_LINE_FO corresponds if AND and y depend on the temperature and are then calculated for the temperature of the point of current Gauss.

The Young modulus E and the Poisson's ratio are those provided under the key words factors ELAS or ELAS_FO.

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In this case the traction diagram is as follows:

L

AND

y

E

L

I.e.:

y

$$L = E L$$

if $L < E$

.

y

y

$$L = y + \text{AND } L -$$

E if L E

Note:

y is the elastic limit (the choice of falls on it on the user: it can correspond to the end of linearity of the real traction diagram, either lawful elastic limit or conventional. At all events, one uses here the single value defined under ECRO_LINE).

When the criterion is reached one a:

()

$$L - R p$$

$$= 0 ,$$

thus

L

$L - R L -$

$E = 0,$

from where

E

$R(p) =$

$T E p +$

$E - E$

$y.$

T

3.1.2 Relation

VMIS_ISOT_TRAC

The data of material are those provided under the key word factor TRACTION: (SIGM: F), of operator DEFI_MATERIAU.

F is a function with one or two variables representing the simple traction diagrams. The first variable is obligatorily the deformation, the second if it exists is the temperature (parameter of a tablecloth). For each temperature, the traction diagram must be such as:

- the X-coordinates (deformations) are strictly increasing,
- the slope between 2 successive points is lower than the elastic slope between 0 and the first point curve.

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The interpolation compared to the temperature is carried out in the following way:

That is to say the temperature considered, if there is K such as

[

]

$K, K + 1$ where K indicates the index of

traction diagrams contained in the tablecloth, one point by point builds the traction diagram with temperature while interpolating compared to the X-coordinates and the ordinates of the points of both extreme traction diagrams.

K

$K + 1$

If is apart from the intervals of definition of the traction diagrams, one extrapolates conformément with the prolongations specified by the user in DEF1_NAPPE [U4.31.03] and according to the principle precedent.

Note:

It is disadvised and dangerous to extrapolate the traction diagrams for values of temperature very far away from the extreme temperatures to which the curves are defined. It is always preferable to provide traction diagrams for values of temperature framing temperatures of calculation.

If the numbers of points of discretization of the traction diagram to K and $K + 1$ are different, one interpolate between the last point of the poorest curve with all the remaining points of the curve richer. Consequently, it is preferable enough to have a number of points of discretization homogeneous for the various temperatures.

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In all the cases, the traction diagram considered is a linear function per pieces:

=

$I + 1 - I ($

)

[

]
 $I +$
 - for
 for $I + 1$ N .

I
 $I, I + 1$
 $I + 1 - I$

N being the number of points of interpolation with a linear extrapolation, constant or excluded according to the choice carried out in DEF1_FONCTION by the user (cf [U4.31.02] for more precise details on extrapolation considered).

2
 $y = 1$
 E

1
 2

The first point makes it possible to define:

$y = 1$
 $E = 1$.

1
 It is this Young modulus who is used in the integration of the relation of behavior.
 One thus has for any I :

p
 I
 = -
 .
 I
 I
 E

The function of work hardening is then:

$I + 1 - I$
 $R(p) =$

(
)
 [
]
 $I +$
 $p - p$ for $p p$

.
 p
 I
 $I, pi + 1$
 $I + 1 - pi$

The user must also give the Poisson's ratio, and a fictitious Young modulus y_g (which is only useful to calculate the elastic matrix of rigidity if the key word NEWTON: (MATRIX: "ELASTIC") is present in STAT_NON_LINE) by the key words:

/ELAS: (NAKED: E: E)
/ELAS_FO: (NAKED: E: E)

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3.2

Tangent operator. Option RIGI_MECA_TANG

The goal of this paragraph is to calculate the tangent operator K_{i-1} (option of calculation RIGI_MECA_TANG

called with the first iteration of a new increment of load) starting from the results known with the moment previous T_{i-1} .

For that, if the tensor of the constraints with T_{i-1} is on the border of the field of elasticity, one writes condition:

$$! F = 0$$

who must be checked (for the continuous problem in time) jointly in the condition:

$$F$$

$$= 0$$

with:

$$F$$

$$(\cdot, p) =$$

$$()$$

$$eq - R p.$$

So on the other hand the tensor of the constraints with T_{i-1} is inside the field, $F < 0$, then the tangent operator is the operator of elasticity.

The quantities intervening in this expression are calculated at the moment previous T_{i-1} , which are them only known at the moment of the phase of prediction. One thus obtains:

$$\begin{aligned}
 & \dot{F} \\
 & \dot{F} \\
 & \dot{F} \\
 & \dot{F} \\
 & \dot{F} \\
 & \dot{F} \\
 & \dot{F} \\
 & = \\
 & \dot{F} + \\
 & \dot{F} \\
 & \sim \\
 & p = \\
 & \dot{F} + \\
 & \dot{p} = \\
 & (\sim \\
 & 2 \mu \dot{F} - 2 \mu \dot{P} \\
 &) + \\
 & \dot{p} \\
 & p \\
 & p \\
 & p \\
 & F \\
 & = \\
 & (\\
 & F \\
 & 2 \mu \dot{F} - 2 \mu \dot{P} \\
 &) + \\
 & \dot{p},
 \end{aligned}$$

because is deviative.

With

-

=
 = (T
 p
 p -
 ,
 -
 =
 = (T, = = p (T and p
 = p = (
 p Ti-1)
 i-1)
 i-1)
 i-1)

Note:

One does not hold account in this expression of the variation of the elastic coefficients with temperature. It is an approximation, without important consequence, since this operator is useful that to initialize the iterations of Newton. On the other hand, dependence of the tangent operator by report/ratio with the thermal deformations is well taken into account on the level of the total algorithm [R5.03.01].

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 3 ~
 3 ~

One has then:

2μ
 ! - 2μ!
 ,

p
 R
 $-(p) p = 0$

2
 2
 $!$
 eq
 eq
 $(3 \mu) (\sim \sim$
 $. !)$

what leads to: $! p =$
thus
,

$3 \mu + R (p$
 eq
)

9μ
 $(\sim \sim$
 $. !)$
 \sim

if

2
 $=$
 $-$
 $=$
 p
,

$F(, p)$
 $R p$
 0
 $!$
,

$= 2 3 \mu + R (p)$
 eq
 $()$
 eq

0 ,
 if
 - $R p < 0$
 eq

()

~
 !
 p
 $= K$
 $+ 2 \mu$
 -
 ij
 ! $kk ij$

(! ij ! ij)

Note:

Information -

-
eq - $R(p) = 0$ is preserved in the form of an internal variable which is worth 1 in this case and 0 if -

-
 $eq < R(p)$.

The tangent operator binds the vector of virtual deformations * to a vector of virtual constraints *.

The matrix of tangent rigidity is written for an elastic behavior:

" "
 $(K \mathbf{1} \mathbf{1} 2\mu \mathbf{P})$

=
 +
 and for a plastic behavior:

" "
 $(K \mathbf{1} \mathbf{1} 2\mu \mathbf{P} \mathbf{C} \mathbf{S} \mathbf{S})$

p
)
 =
 +
 -

with **S** the vector of the deviatoric constraints associated - defined by:

St

$$= (\begin{matrix} \sim & & & \\ - & & & \\ - & & & \\ - & & & \\ , & \sim & , & \sim & , \\ \sim & & & & \\ 2 & & & & \\ , & & & & \\ \sim & & & & \\ 2 & & & & \\ , & & & & \\ \sim & & & & \\ 2 & & & & \\ 11 & 22 & 33 & & \\ 12 & & & & \\ 23 & & & & \\ 31) & & & & \end{matrix}$$

and:

$$(3 \mu)^2$$

$$1$$

$$CP = (2 \ 3 +$$

$$\mu$$

$$'$$

eq)
R

if -
eq - R (-
1
p)
= 0
= 0 if not

In the case of the first increment of loading, therefore if the state at the previous moment corresponds to one nonconstrained initial state, the tangent operator is identical to the operator of elasticity.

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3.3

Calculation of the constraints and the internal variables

The decomposition of the deformations makes it possible to write:

$$= p + (-) + HT$$

To 1

Maybe, by taking the spherical and deviatoric parts:

~

p

$$\sim = +$$

HT

$$= 0$$

because

~

.

2μ

tr

$$tr =$$

tr

HT

because $tr p$

= 0.

K

3 +

By direct implicit discretization of the relations of behaviour for isotropic work hardening, one obtains then:

3

-

-

~

~

~

~

~

+

2μ

2μ

(

)

2μ

-

-

p

-

~

-

+

= 2

(- +

)

2μ

eq

$3 K$

tr =

tr -

HT

-

+ $3 K$ tr

- $3 K$ tr

$3 K$

(- +

) - R (-

$p + p$

) 0

eq

p

= 0 if (- +

) < *R p*

p

eq

(- +)

p

0

if (- +

)

= *R p*

p

eq

(- +)

One defines, to simplify the notations, the tensor *E* such as:

~

μ

E

2

~ -

~

=

+ 2 μ

E

-

and tr = tr.

2 μ

Two cases arise:

.

(-

)

R (-

+

<

p + *p*

eq

)

in this case

p
0 are ~
~
= - ~
~ E

=
+ =
thus
(~e) < $R p$

eq
()
.
(-
)
 $R (-$
+
=
 $p + p$

eq
)
in this case

$p 0$
thus
(~e) $R p$

eq
()
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One deduces the algorithm from it from resolution:

.
~
if
 $E R p$ then p

0
~
~ -
~
~

that is to say

=

E

=

+ =

eq

()

· if

~

$eeq > R p$

()

then it is necessary to solve:

~

~

3

- + ~

$E = \sim - + \sim + 2 \mu p$

2

(+

) eq

thus by factorizing ~ -

~

+ and by taking the equivalent value of Von Mises

3

2

p

E

μ

= +

-

1

eq

2 (+

) (

) eq

eq

+

-

that is to say:

$$E = R p + p$$

+ 3

p

eq

(

) μ

because:

-

eq

= (+

)

$$= R (p + p$$

)

eq

It is a scalar equation out of p , linear or not according to $R (p)$. p is obtained analytically, because R is a linear function per pieces.

· If work hardening is linear (relation VMIS_ISOT_LINE), one obtains directly:

$E -$

$p =$

eq

$$y - R' p$$

$$R' + 3 \mu$$

with:

E E

R'

=

T.

E - AND

· If work hardening is given by a traction diagram, one benefits from the linearity by pieces to determine *p* exactly to see [§An1].

Once *p* determined, one calculates by:

E

~

eq - 3 μ p

-

~

+

=

. ~

E

eeq

and

tr (- +

) = tr *E*.

Options RAPH_MECA and FULL_MECA carry out both the preceding calculation, which clarifies it calculation of **R one**

()

N

()

N

N

I. It is noticed that actually, **R ui**

= **QT I** where *I* is calculated not in function

N

of **linked**, but of *I* -1 and **ui**.

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Note:

Particular case of the plane constraints.

The model of Von Mises with isotropic work hardening (VMIS_ISOT_LINE or VMIS_ISOT_TRAC) is also available in plane constraints, i.e. for modelings C_PLAN, DKT, COQUE_3D, COQUE_AXIS, COQUE_D_PLAN, COQUE_C_PLAN, PIPE, TUYAU_6M.

In this case, the system to be solved comprises an additional equation. This calculation is detailed in appendix 2.

3.4

Tangent operator. Option FULL_MECA

The option

N

FULL_MECA makes it possible to calculate the tangent matrix \mathbf{K}_i with each iteration. The operator tangent which is used for building it is calculated directly on the preceding discretized system (one notes to simplify: ~

= ~

- + ~

, $p = p + p$) and one writes the expressions only in isothermal case.

· If the tensor of the constraints is on the border of the field, F

= 0 then one have, in

differentiating the expression of the law of normality in ~

~ -

~

= + :

3

~

~

~

: ~

p

~

~

3

$$2 \mu = 2 \mu$$

$$2 \mu p$$

p

p

\sim

-

=

+

-

2

eq

2

3

eq

eq

where $\mathbf{p} \sim$

\sim represents infinitesimal increases around the solution in incremental elastoplastic problem obtained previously.

Like:

3 \sim

: \sim

,

$$= R(p) p$$

2 eq

by carrying out the tensorial product of the first equation by \sim

one a:

$$2 \mu \sim: \sim \sim$$

$$-: \sim = 2 \mu.$$

$p,$

eq

by eliminating p from the two last equations:

~

~

$2 \mu: \sim$

: ~

=

$3 \mu.$

$1 + '$

$R(p)$

· Si on the other hand if the tensor of the constraints is inside the field, F

< 0 , then

the tangent operator is the operator of elasticity.

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By expressing p and \sim : ~

in the first equation, one obtains:

$3 \mu p$

$2 \mu \sim -$

$\sim =$

$\sim + C .p (\sim: \sim) \sim,$

+

eq

with:

9 2

μ

$R'(p) p$

1
C
=
1
2
-

p

R'
+ 3 μ
eq
eq
(*p*)
~

The positive part (: ~
) allows to gather in only one equation the two conditions:

+
· is
F
< 0, which implies $p = 0$
· is *F*
= 0

One obtains then:

2 μ
C
~
~
p (~: ~) ~
=
-

+
has
has
while posing:
3 μp

= 1 + *R* (*p has* + *p*
)

The tangent operator binds the vector of virtual deformations $\delta \mathbf{u}$ to a vector of virtual constraints $\delta \mathbf{c}$.

The matrix of tangent rigidity is written for an elastic behavior:

$$\mathbf{K} = \mathbf{K}_e + 2\mu \mathbf{P}$$

and for a plastic behavior:

$$\mathbf{K} = \mathbf{K}_e + \mathbf{C} + \mathbf{P}$$

$$\mathbf{S}$$

has
has

with \mathbf{S} the vector of the deviatoric constraints associated - defined by:

$$\mathbf{S} = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \\ \delta_{10} \\ \delta_{11} \\ \delta_{12} \end{pmatrix}$$

11 22 33

12

23

31)

and:

1

~

if led to a plasticization and. ~

0

= 0 if not

It is noted that the tangent operator with the system resulting from the implicit discretization differs from the operator

tangent with the problem of speed (RIGI_MECA_TANG). One finds it while making: $p = 0$ in expressions of C and A .

p

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3.5

Produced internal variables

The relations of behavior VMIS_ISOT_LINE and VMIS_ISOT_TRAC produce two variables interns: p and (useful for the calculation of the tangent operator).

4

Relation of Von Mises with linear kinematic work hardening

4.1

Expression of the relation of behavior

This relation is obtained by key word VMIS_CINE_LINE of the key word factor COMP_INCR. It is written:

~
~
~
- **X**
3
-
.
3
X
\$%'
P

!
=
! p
- 1

With

2
(

=

 p

=

-

-

- X)

!

 HT

2

- X eq

(

)

!

!

 eq $\mathbf{X} = C p$ $HT = (T - T \mathbf{Id}$ $ref.)$

(-

éq 4.1-1 $\mathbf{X}) -$

0

 eq y ! $p = 0$ if $(- \mathbf{X}) -$

0

 eq y ! $p = 0$ if $(- \mathbf{X}) -$ $= 0$ eq y

y is the elastic limit (the choice of falls *on it* on the user: it can correspond to the end of

linearity of the real traction diagram, either lawful elastic limit or conventional... At all events, one uses here the single value defined under ECRO_LINE). C is the coefficient of work hardening deduced from the data by a simple tensile test. In this case (tensor of constraints uniaxial, tensor of plastic deformations isochoric and

orthotropic):

X
 0
 0
 L
 0

0
 L

X
 $= 0 0$
 0
 $\mathbf{X} = 0$
 $- L$
 0

2

$0 0 0$

X
 0
 0
 $- L$

2

$($

3

$- \mathbf{X})$

$=$

X

$L -$

eq

L

2

P
L
X
C
C
L
=
L =
L -

E
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L

AND

y

E

L

The data materials are those provided under the key word factor ECRO_LINE or ECRO_LINE_FO of operator DEFI_MATERIAU:

/ECRO_LINE (D_SIGM_EPSI: AND SY: y)

/ECRO_LINE_FO (D_SIGM_EPSI: AND SY: y)

ECRO_LINE_FO corresponds if AND and y depend on the temperature and are then calculated for the temperature of the point of current Gauss.

The Young modulus E and the Poisson's ratio are those provided under the key words factors ELAS or ELAS_FO.

y

y

For

$L >$

E

L

$= y + \text{AND } L -$

$E,$

but one also has:

3

$L -$

$XL = y$

2

L

$XL = CL -$

E

from where, by eliminating XL and while identifying:

$E E$

$C = 2$

$T.$

3rd - AND

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4.2

Tangent operator. Option RIGI_MECA_TANG

The goal of this paragraph is to calculate the tangent operator K_{i-1} (option of calculation RIGI_MECA_TANG

called with the first iteration of a new increment of load) starting from the results known with the moment previous T_{i-1} .

For that, if the tensor of the constraints with T_{i-1} is on the border of the field of elasticity, one writes condition:

$$! F = 0$$

who must be checked (for the continuous problem in time) jointly in the condition:

$$F$$

$$= 0$$

with

$$F = F (- -$$

$$\mathbf{X}) = (-$$

-

, ,

-

$$- \mathbf{X}) -$$

eq

y

So on the other hand the tensor of the constraints with T_{i-1} is inside the field, F

< 0 , then

the tangent operator is the operator of elasticity.

One poses:

-

-

1

0

Dev.

~ -

-

if $(- \mathbf{X}) -$

= (information given by the variable interns)

\mathbf{X} and

eq

y
=
-
= 0 if not

The problem of speed is written in this case:

$$2$$
$$1 \ 3 \ 2 \ \mu \ (\sim - \mathbf{X}) \sim$$
$$. \ !) (\sim - \mathbf{X})$$

if
- \mathbf{X} -
= 0
 p

()
!
 y
= $2 \ \mu \ 2$
 $C + 2 \ \mu$
 y

0
if $(- \mathbf{X}) - < 0$
 y

eq

 \sim
!
 p

= $K + 2 \ \mu -$
 ij
! $kk \ ij$

(! $ij!$ ij)

The tangent operator binds the vector of virtual deformations * to a vector of virtual constraints

*

The matrix of tangent rigidity is written for an elastic behavior:

" "

$(K \mathbf{1} \mathbf{1} 2\mu \mathbf{P})$

=

+

and for a plastic behavior:

" "

$(K \mathbf{1} \mathbf{1} 2\mu \mathbf{P} \mathbf{C} \mathbf{S} \mathbf{S})$

p

)

=

+

-

with **S** the vector of the deviatoric constraints associated *Dev.* defined by:

St =

Dev. Dev. Dev.

Dev.

Dev.

Dev.

(

)

11 , 22 , 33 ,

2 12 , 2 23 , 2 31 .

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and:

2

2
μ
1
C

p
= 3

·
2 y 2 μ + C

In the case of the first increment of loading, therefore if the state at the previous moment corresponds to one nonconstrained initial state, the tangent operator is identical to the operator of elasticity.

**4.3
Calculation of the constraints and variables internal**

The direct implicit discretization of the continuous relations results in solving:

~ -
~
~
~
- **X**
p

3

2
μ
= 2 μ +
-

-
=

2 μ p

2 μ
2 μ
2
y

C
X =

-
X
-
+ Cp
(
C
-
X)
 eq
 y
 p
= 0 if $(- \mathbf{X}) <$
 eq
 y
 p
0 if not

tr (
K
- +
)
3
=
tr -
tr
tr
-
+ 3 K
- 3 $K HT$
3 K

One still poses:
~
2 μ
C
E
~ -
~
=

-
2 μ
X
-
+

-
.
2 μ
C

The first equation is also written:

2
~
~
μ

3
~
X

-
~
-
2 μ +

p
-
= +
2 μ

2 μ
2
y
C

by cutting off **X**
=
X + *C*
p

-
has each term, one obtains:
C
2
~

~
μ
C
3
~
X
-
-
~
-
2 μ
+
X
X
p
C
P

--
-
= - +
2 μ
+
2 μ
C
2
y

or, by using the law of flow:

~
3
p
E
= (~

- **X**) 1
+
(2 μ + C)

2
y

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One still obtains a scalar equation out of p by taking the equivalent values of Von Mises:

3

$eeq = y +$

$(2 \mu + C) p$

2

what gives directly:

E

-

p

eq

y

$= 3 (2\mu+C)$

2

2μ

And is obtained by: ~

~ -

~

=

2μ

2μ

p

-

+

-

2μ

By noticing that:

3

~

~

X

3

E

p

-

= *p*

= *p*

2

2

E

y

eq

because

~

~

- **X**

E

=

E

y

eq

one thus has:

E

2 μ

2 μ

(*eq* - *y*)

~

~ -

~

=

+ 2 μ

-

+ .~

E

-

 2μ $2 \mu + C$ *eeq*

The variables intern X are calculated by:

 $\sim \mathbf{E}$ C C

3

 $\mathbf{X} =$ \mathbf{X} p

-

 C $C p$

-

+

=

 \mathbf{X}

-

+

 C C E

2

*eq***Note:**

Particular case of the plane constraints.

The direct taking into account of the assumption of the plane constraints in the integration of the model of Von

Settings with linear kinematic work hardening was not made in *Code_Aster*.

On the other hand, to take into account this assumption, i.e. to use VMIS_CINE_LINE with modelings C_PLAN, DKT, COQUE_3D, COQUE_AXIS, COQUE_D_PLAN, COQUE_C_PLAN, PIPE,

TUYAU_6M, one can use the method of condensation static (due to R. of Borst [R5.03.03]) which allows to obtain a plane state of stresses with convergence of the total iterations of the algorithm of Newton.

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4.4**Tangent operator. Option FULL_MECA***N*

Option FULL_MECA makes it possible to calculate the tangent matrix \mathbf{K}_i with each iteration. The operator

tangent which is used for building it is calculated directly on the preceding discretized system (one notes to simplify: ~

= ~

- + ~

, $p = p + p$) and one writes the expressions only in isothermal case.

1

~

if $p > 0$ and $(-\mathbf{X})$. ~

0

Dev. is posed

= ~

- \mathbf{X} and = 0sinon

The tangent operator binds the vector of virtual deformations * to a vector of virtual constraints

*.

Then the matrix of tangent rigidity is written:

" "

 $(K \mathbf{1} \mathbf{1} 2\mu \mathbf{P} \mathbf{C} \mathbf{S} \mathbf{S})$

2

p

)

=

+

with **S** the vector of constraints associated to *Dev.* by:

St =
Dev. Dev. Dev.
Dev.
Dev.
Dev.
 (
)
 11 , 22 , 33 ,
 2 12 , 2 23 , 2 31 .

and:

2
 2
 μ
 1
 C

p
 = 3
 .a
 2
 1
 $y^2 \mu + C$
 $a1 =$
 1
 2
 $(\mu + C) p$
 $1 + 32$
 y
 $3 p$
has

2
 = $a1 1 + C$

2

y

4.5

Produced internal variables

The variables intern are 7:

- it
- tensor
- X** stored on 6 components,
- the scalar variable.

5 Bibliography

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Appendix 1 Relation VMIS_ISOT_TRAC: complements on integration

Implicit discretization of the relation of behavior led to solve an equation out of p [§5].

$eeq - 3 \mu p - R p + p$

(

) = 0.

One solves the equation exactly while drawing left the linearity per pieces.

One examines initially if the solution could be apart from the terminals of the points of discretization of the curve

$R(p)$, i.e., if p is a possible solution.

N

For that:

- if

E +

p

3

- *p* -

0

eq

μ (

N)

N

then one is in the following situation:

eq

R (*p + p*)

eeq + 3μp -

(

) - 3μ (*p - + p*) = 0

p

p

N

+ *p*

-

if the prolongation on the right is linear then:

that is to say

-

N

N - 1

1

=

H 1 =

1 +

p

1

- *p*

N -

N -

N -

N -

N - (

N - 1)

p - p

N

N - 1

then:

E
 $- H$
 p
 eq
 N
 $=$
 $- 1$

3
 1
 μ
 $-$
 $+$
 N
 $-$

if the prolongation is constant:

$E -$
 $p =$
 eq
 N
 3μ
 $-$

if not an error message is transmitted,

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· if not, the solution p is to be sought in the interval $[p, p$
such as:

I
 $I + 1]$

E
3
1 >
+
p - p
i+
eq
 μ (
i+I)
and

E
+
p
3
- *p*
I
eq
 μ (
I)
E
-
eq + 3 \mu (p - I
p) = 0
E
-
eq + \mu
3 (p - p) = 0
E
-
eq + 3 \mu (p - I
p +) = 0
1
p
p
p
I
i+I
+1 -
I

I
=
H
=
+
p - p
for *I*
= 1 with *N - 1*

I
I
I I (
I)
p + 1 - p
I
I

then, *p* is such as:

E
- *H*
p
eq
I
=
and
p + p
,
3
[*p p*
I
I + 1]
+ μ
I

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Isotropic appendix 2 Work hardening in plane constraints

In this case, the system to be solved comprises an equation moreover:

$= 0$. It then is obtained

33

following system:

~ -

~

3

+

2μ

~ - ~ =

$2 \mu p$

2

(- +

) eq

tr = 3 K tr

(- +

) - R (-

p + p) 0

eq

p = 0 if (- +

) < R

eq

(- p + p)

p 0

if (- +

)

= R

eq

(- p + p)

= 0

33

With this assumption, is not entirely known:

N

33 cannot be calculated only starting from **ui**.

Note:

In the case of modelings other than C_PLAN, therefore for example for modelings of hulls (DKT, COQUE_3D), the assumptions on the transverse terms of shearing and are defined

13

23

by these modelings (in general, the behavior related to transverse shearing is linear, elastic and uncoupled from the equations above). These terms thus do not take into consideration here.

One poses =

$Q +$

y with Q entirely known starting from **linked** and of elasticity, therefore

0 0 0

Q

Q

Q

y

33

= -

11 +

and

=

- (

22

0 0 0 are unknown.

1

)

0 0 y

Compared to the preceding system, there are an additional unknown factor, Y .

· If

(~ ~

)

$R (-$

+

<

$p + p$

) then p

= 0 thus $2 \mu \sim = \sim$,

eq

i.e. $y = 0$.

· If not, the technique of resolution consists in expressing y according to p . One obtains one then nonlinear scalar equation in p .

2 μ

One poses: '~ E

~ -

~

2 μ

-

Q

=

+

. In the same way that for integration except plane constraints, one

2 μ

obtains:

y

~

~

~-

~

3 μ p

2 μ

1

E +

= +

+

.

R (p + p

)

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But this expression utilizes an additional unknown factor y : In particular:

y

~

~

~

-

~

$3 \mu p$

+ 2

33

=

+

33

1

E

μ

33

33

+

$R(p + p$

)

however y

~

$33 = 2 y$

3
 3 K+
 and tr -
 (+
) = 3 K tr Q + 3 K+ y + 3 K tr - - 3 K+ HT.

Like:

tr + -

(
)

tr + -

(
)

~

-

-

33 + ~

33 = 33 + 33 -

= 0 -

.

3

3

One obtains an equation binding y and p:

~

3 μ p

-

tr E - 3 K y

E

+ 2 μ 2 y =

1 +

33

3

R (p + p)

3

with

3 K

tr E = 3 K tr - + 3 K tr Q - 3 K HT.

That is to say:

4 μ

$3 \mu p$

~

tr

3μ

E

E

p

y

+ KI +

1

3

-

-

$R(p + p)$

33

= -

-

+

3

$(RP + p)$

by noticing that:

~

tr

tr

E

E

E

E

33

= 33 -

= 0 -

3

3
 and by clarifying μ , K , one obtains:

$$(31 - 2) p$$

y
 E

=

~

$$E p + (2 1 -) R (p + p) 33$$

to defer in the equation out of p (identical to the preceding cases)

~

$$E + 2 \mu \sim y$$

(

$$) - 3\mu p R p + p$$

(

$$) = 0.$$

eq

1

y

-

y

where expresses itself *there* according to p since:

~

=

- 1

3

2

The scalar equation out of p thus obtained is always nonlinear. This equation is solved by a method of search for zeros of functions, based on an algorithm of secant (cf [R6.03.02]). Once the solution p known one calculates *there* then.

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Organization (S): EDF/MTI/MMN

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Document: R5.03.03

Taking into account of the assumption of the constraints

plane in the nonlinear behaviors

Summary:

This document describes a general method of integration of the nonlinear models of behaviors (elastoplastic, viscoplastic, damaging,...) in plane constraints.

This is carried out by a method of static condensation due to R. of Borst.

This method makes it possible to use modeling C_PLAN, or modelings COQUE_3D, DKT and PIPE for all the models of incrémentaux behaviors of STAT_NON_LINE available into axisymmetric or in plane deformations. It is not operational for the moment in DYNA_NON_LINE.

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1 Introduction

One presents here a general method of integration of the nonlinear models of behaviors (plasticity, viscoplasticity, damage) in plane constraints. It is activated by the key word ALGO_C_PLAN: "DEBORST" of the operand of the incrémentaux nonlinear behaviors COMP_INCR of STAT_NON_LINE, for modeling C_PLAN, DKT, COQUE3D and PIPE.

2

Difficulty of integration of the nonlinear behaviors in plane constraints

Modeling C_PLAN, (as well as modelings COQUE_3D, DKT, PIPE) supposes that the state of constraints room is plane, i.e. that $\theta = 0$, Z representing the direction of the normal with

zz

surface. The tensors of constraints and deformations thus take the following form (in C_PLAN):

0

$Dn. Dn$

xx

xy

$K N = B T Dn - 12 21 B$

!

!

11

0

Dn

$= xy$

yy

22

0

0

zz

Note:

For the hulls, it is necessary to add terms due to transverse shearing (γ_{xz} , γ_{yz}), but

xz yz

those are treated elastically and do not intervene in the resolution of the behavior room.

This assumption implies that the corresponding deformation is a priori unspecified (contrary with other two-dimensional modelings where one makes an assumption directly on). It

zz

can be given that using the relation of behavior. However the condition $= 0$ is not

zz

alleviating for the integration of the behavior where one calculates an increase in constraint in function of the increase in deformation provided by the algorithm of Newton. In the case of linear elasticity, the taking into account of this condition is simple and makes it possible to find:

= -

+

zz

(xx yy)

1 -

But if the behavior is nonlinear, cannot be calculated only starting from **U** and

zz

does not result simply from the other components of the tensor of the deformations. The catch in count EC assumption must then be made (when it is realizable) in a way specific to each behavior, and very often brings to additional difficulties of resolution: it is the case in private individual for the behavior of Von Mises to isotropic work hardening [R5.03.02]. So many models of behavior are not available in plane constraints.

The method presented here has the large advantage of not requiring any particular development in the integration of the behavior to satisfy the assumption of the plane constraints. It is usable as soon as the model of behavior is available into axisymmetric or in plane deformations.

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3

Principle of the treatment of the plane constraints by method De Borst

The idea of the method due to R. of Borst [bib1] consists in treating the condition of plane constraints not on the level of the law of behavior but on the level of balance. One obtains thus during iterations of the algorithm of total resolution of STAT_NON_LINE of the stress fields which tend towards a plane stress field progressively with the iterations:

N

zz

0

where N indicates the number of iteration of Newton.

One thus obtains the condition of constraint planes not exactly, but in an approached way, with convergence of the iterations of Newton, for each calculated increment. One checks, as specified by the continuation, that the component above is lower than a given tolerance.

The method consists in breaking up the fields (strains or stresses) into a part purely planes (specified by a "hat") and a component according to Z . One then reveals explicitly the component zz in the expression of the tangent operator in plasticity:

!

!

=

, =

zz

zz

! 11

D

D12

The tangent operator $D =$

$D D D$ becoming $D =$

.d

=

.

=

D

D

zz **21**

22

D

11

D**D12**

where D and D indicate infinitesimal increases, and where by definition

=

is

 D **D** D **21**

22

the coherent tangent matrix with the **behavior without the assumption of plane constraints**, is in axisymetry, is in plane deformation to see for example [R5.03.02] for the models of Von Mises).

4

Implementation of the method

The method consists of each point of integration of each element with:

- 1) to use the axisymmetric relation of plane behavior or deformation (they are identical) to calculate the constraints starting from the deformations,
- 2) to carry out a static condensation on the relation stress-strain
- 3) to write the infinitesimal increases D and D which are connected above by the operator tangent in the form of increase between two iterations in Newton N and $n+1$:

$$D = n+1 N$$

-

$$n+1 (-$$

$$N) n+1 N$$

-

=

+

-

+

=

-

and the same for D . A convergence, this variation must tend towards zero.

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By writing $n+1$

N

=

+

=

zz

zz dzz 0 one obtains, for the iteration $n+1$:

D

N

N

n+1

!! -

-

+

+

11

N 1

! N

! N 1! N

D

D 12

D!

+1

.

D

= N

$N =$

N

=
N
N

-
-
zz
zz
zz
zz

D
n+1
21
D 22
D

zz
what, by using the last equation of this system, enables us to be reduced to:

$! n+1 = ! N$

N
+ **D D**
1
N 1
D

D
N 1
0
11 !

+
N
-
-
.

12 (
N
D22) (*N*
N
+

D

21

=

zz

! +)

N

N

-1

N

N

D = - D

D

N 1

22

+

D

zz

()

(

21 !

+

zz

)

with the stress field which is written:

N

D D

D

1

. *N*

N

12

21

! *n*+

N

N 1

N

12

N

= **D**

11 -

D

-

N

!++!

N

zz

D22

D

22

N

N

N

D

D

n+l

N

zz

=

-

+ 21

N

- 21

N 1

zz

zz

N

N

!

N

!+

D

D

D

22

22

22

By using the preceding expression of the stress field, one finds then:

LT $n+$

. FD

1

LT $n+$

=! . ! FD

1

= **L**

=

N

N

N

!

D. D

D

+

BT

Dn

12

21

N

12

N

11 -

D! N 1

+ !

$zz FD$

-

22

D

22

D

N

N

N

= !

D. D

D

BT Dn

12

21

-

! B.dun+1dv +! BT N

12

N

11

!

zz FD

-

22

D

22

D

Dn

$$= \mathbf{K} \mathbf{D} \mathbf{N} + \mathbf{1} \mathbf{B} \mathbf{T}$$

$\mathbf{1} \mathbf{N}$

12

\mathbf{N}

-

$\mathbf{z} \mathbf{z} \mathbf{F} \mathbf{D}$

22

\mathbf{D}

It is thus noted that the taking into account of the plane constraints intervenes on two levels:
 · in the matrix of tangent rigidity, by a corrective term (second term of the expression below) compared to the expression 2D of the tangent matrix:

$\mathbf{D} \mathbf{n} \mathbf{D} \mathbf{n}$

\mathbf{N}

12

21

$\mathbf{K} \mathbf{N}$

$\mathbf{B} \mathbf{T}$

=

\mathbf{D}

$\mathbf{F} \mathbf{D}$

11 -

\mathbf{N}

\mathbf{B}

!

·

!

$\mathbf{D} \mathbf{2} \mathbf{2}$

· in the writing of the second member by a corrective term (second term of expression Ci below) compared to the expression 2D of the tensor of the constraints:

$\mathbf{D} \mathbf{n} \mathbf{1} \mathbf{2} \mathbf{N}$

$\mathbf{R} (\mathbf{one} \mathbf{1}) = \mathbf{B} \mathbf{T}$

+

-

$\mathbf{F} \mathbf{D}$

N
ZZ
!!N

D22

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To implement the method De Borst for the unit of the incrémentaux behaviors, it is enough thus to calculate these corrective terms and to add them to the constraints and tangent matrices obtained by integration 2D (in fact axisymmetric or deformation planes) these behaviors. With this intention, it is necessary to store some additional information during iterations of Newton. One thus adds (in a transparent way for the user) 4 variables intern with the behavior used.

The data-processing realization is as follows:

1) during the iteration n+1 of the algorithm of Newton, one has in entry of the routine calculating it behavior: **un+1**, -

-

, and 4 following additional variables internal resulting

N

N

D

Dn

preceding iteration: 1 scalar variable -

ZZ

21

N

! and 3 variables - 21,

N

+ N

D

D

N

D

22

22

22

2) before carrying out the integration of the nonlinear behaviors (which will be made in

N

N

N

D

D

n+1

N

axisymmetric), one calculates

zz

=

-

+ 21

N

- 21

N 1

!

! + ,

zz

zz

N

N

N

D

D

D

22

22

22

3) One lets the routines of integration of the behavior calculate the constraints as well as

! n+1

tangent behavior *D* from

as if modeling were axisymmetric or

$n+1$

zz

of plane deformation,

4) one modifies at exit the second member and the tangent matrix (if the reactualization of

Dn Dn

N

12

21

stamp tangent was asked) so that: **K N**

BT

=

D11 -

B

!

.

! FD and

N

22

D

Dn

R (one 1) = BT

+

- 12 N

!! N

FD ,

N

zz

$D22$

N

N

D

Dn

5) one stores the new internal variables -

zz + 21

N

! and - 21.

N

N

D

D

N

D

22

22

22

To check convergence, one checks, always on the level of each point of integration of each finite element if $\mathbf{n}+1 <$, where

N 1

+

=

with provided by the user under the key word

zz

RESI_INTE_REL. The default value is 10-6.

At the time of testing the convergence of the total iterations of Newton (defined by RESI_GLOB_REL and

N +1

RESI_GLOB_MAXI) one examines whether all the points of integration check the condition

$<$. If

zz

it is not the case, one carries out additional iterations of Newton until complete checking of this condition.

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5

Aspects practise use

To use this method, it is necessary to specify under the key word factor COMP_INCR the key word ALGO_C_PLAN: "DEBORST". It is necessary also that modeling (specified in AFFE_MODELE) elements concerned with this behavior is "C_PLAN" or a model of the hull type to plasticity local: COQUE_3D, DKT, PIPE.

In practice, that increases (automatically) by 4 the number of internal variables of the behavior. For converging well, it is advised to reactualize the tangent matrix if possible (, with all them iterations: REAC_ITER: 1, or all N iterations, with N small).

This method thus allows a great flexibility in use compared to the behaviors: it is enough that a behavior is available in axisymetry or plane deformation so that it is too usable in plane constraints.

As for all integrations of models of behaviors nonlinear, it is highly advised to give a small criterion of convergence (to leave the default value to 106.).

6 Bibliography

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R of Borst "the zero normal stress condition in plane stress and Shell elastoplasticity"

Communications in applied numerical methods, Flight 7, 29-33 (1991)

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Document: R5.03.04***

***Relations of behavior élasto-visco-plastic
of Chaboche***

Summary:

This document describes the integration of the model of behavior élasto-visco-plastic of J.L. Chaboche with nonlinear and isotropic kinematic work hardening. Two versions of the model are available in Code_Aster:

· a version with one or two variables kinematics, introduced recently, takes into account all the variations of the coefficients with the temperature, and has an effect of work hardening on the variables tensorial of recall. This version also makes it possible to model (in an optional way) the character viscous of the material (viscosity of Norton). It is integrated by the solution of only one equation nonlinear scalar.

This model is available in 3D, plane deformation, axisymetry. Modeling in plane constraint use a method of condensation static (of Borst).

· a version with two variables kinematics which exists in Code_Aster since the version 2, which does not take into account all the variations of the coefficients compared to the temperature, but which was used for several studies, and for which one has sets of identified parameters. This version is integrated in environment PLASTI. It does not make it possible to model viscosity.

It does not take into account of effect of work hardening on the tensorial variable of recall. This model is available in 3D, plane strain, plane stress and axisymetry. One gives also elements to identify the coefficients of the relation of behavior.

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1 Models élasto-visco-plastics of J.L. Chaboche available in Code_Aster

For the structural analysis subjected to cyclic loadings, work hardenings isotropic (linear or not) and linear kinematics traditional [R5.03.02] and [R5.03.16] are not sufficient any more. In private individual, one cannot correctly describe the stabilized cycles obtained in experiments on a tensile specimen subjected to an alternated imposed deformation or a traction and compression.

If one seeks to precisely describe the effects of a cyclic loading, it is desirable to adopt modelings more sophisticated (but easy to use) such as the model of Said Taheri, by example, cf [R5.03.05], or if the number of cycles is limited the model of Jean-Louis Chaboche who is introduced here.

Actually, the model of J.L. Chaboche can be more or less sophisticated. Developed models in Code_Aster either a kinematic variable (VISC_CIN1_CHAB comprise) or two (VISC_CIN2_CHAB and CHABOCHE), and of isotropic work hardening.

The choice to use two variables kinematics complicates certainly the model, but makes it possible to identify correctly uniaxial tests in a broader range of deformations [bib2], [bib7]. One certain number of indentifications of the parameters of this model were carried out mainly for

the stainless steels A316 and A304 ([bib7], [bib8]).

The models comprise 8 parameters (a kinematic variable) or 10 (two variables kinematics), introduced into order DEFI_MATERIAU:

CINI_CHAB (CINI_CHAB_F0)

= _F (

R_0 = R_0,

R_I

=

R_I, (useless

if

B=0)

B

=

B,

(defect: 0.)

$C_I = C_I,$

K

=

$K,$

(defect: 1.)

W

=

$W,$

(defect: 0.)

G_0

=
G_0,

A_I
=
A_I, (defect
:
0.)

)
CIN2_CHAB (CIN2_CHAB_FO)

= ***_F (***

R_0 = R_0,

R_I
=
R_I, (useless
if
B=0)

B
=
B,
(defect: 0.)

C1_I = C1_I,

C2_I = C2_I,

K
=

K, (defect: 1.)

W
=

W, (defect: 0.)

G1_0 = G1_0,

G2_0 = G2_0,

A_I

=
A_I
,
(*defect*
:
0.)

)

The 8 or 10 parameters are real constants. All these parameters can depend on temperature (key words CIN1_CHAB_FO or CIN2_CHAB_FO) and the awaited values are of type function.

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If one wants to introduce in addition to viscosity (models VISC_CIN1_CHAB and VISC_CIN2_CHAB), it is also necessary to provide in order DEFI_MATERIAU, under the key word LEMAITRE (or LEMAITRE_FO) parameters NR and UN_SUR_K, which can depend on temperature.

Parameter UN_SUR_M of key word LEMAITRE (respectively LEMAITRE_FO) must obligatorily

to be put at zero (respectively with the identically null function). In the absence of one of the key words

LEMAITRE or LEMATIRE_FO, the behavior are supposed plastic.

Model CHABOCHE is a model with two variables kinematics with isotropic work hardening, but without the effect of work hardening on the term of recall and without taking into account of the variation of C1 and C2

with the temperature. The characteristics of work hardening are given by 9 constants, introduced in order DEFI_MATERIAU:

CHABOCHE = _F

(

R_0

=

R_0,

R_I

=

R_I,

B = B,

K = K,

W = W,

A1
=
A1,

A2
=
A2,

C1
=
C1,

C2
=
C2,

)

In this case the characteristics do not depend any more a temperature since version 5, because these variations were badly taken into account by this model.

The use of these laws of behavior is accessible in orders STAT_NON_LINE or DYNA_NON_LINE by key words VISC_CIN1_CHAB, VISC_CIN2_CHAB or CHABOCHE of COMP_INCR.

In the continuation of this document, one describes models VISC_CIN1_CHAB precisely and VISC_CIN2_CHAB. One presents then the detail of his numerical integration in bond with construction of the coherent tangent matrix. One also briefly describes the integration of the model with two variables kinematics CHABOCHE. Lastly, one also gives some elements for identification of the characteristics of material.

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2 Models

VISC_CIN1_CHAB and VISC_CIN2_CHAB

2.1

Description of the models

At any moment, the state of material is described by the deformation, the temperature T , the deformation

plastic p , cumulated plastic deformation p and the tensor of recall X . Equations of state then define according to these variables of state the constraint = $H Id +$

~ (broken up into

parts hydrostatic and deviatoric), the isotropic share of work hardening R and the kinematic share X :

H

=

() =

(- HT) with $HT = (réf$

tr

$K tr$

$T-T$

) Id éq 2.1-1

3

~ = - $HId = \mu$ (~

- p

2

) éq 2.1-2

$R = R(p)$

éq 2.1-3

$X = ($

$X p p$

$) = X$

éq

2.1-4

$1 (p$

p

$) + X2 (p p$

,

,

,)

where K , μ , and coefficients of (

$X p$) and $R (p)$ are characteristics of the material which can

to depend on the temperature. More precisely, they are respectively the modules of compressibility and of shearing, the thermal dilation coefficient, functions of isotropic work hardening and kinematics. As for $T \text{ réf}$, it is about the temperature of reference, for which one considers thermal deformation as being null.

Note:

For model *VISC_CIN1_CHAB* one thus considers only the only tensorial variable $X p$

$1 ($

$X p = 0$. This remains valid for all the continuation: one will describe the two models formally of

$2 ($

the same way, model *VISC_CIN1_CHAB* resulting from *VISC_CIN2_CHAB* while supposing

$X p = 0$.

$2 ($

The evolution of the plastic deformation is controlled by a normal law of flow to a criterion of plasticity of von Mises:

(

$3 \sim \sim$

$F, R, X) = (\sim - X1 - X2) - R (p)$

with

With

=

With: With

éq

2.1-5

eq

eq

2

~

3

- X1 - X

p

& = & F

= &

2

éq 2.1-6

2

(~ - X1 - X2) eq

&p = &

2

=

&p &p

:

éq 2.1-7

3

***As for the plastic multiplier &
, it is obtained by the condition of coherence:***

if $F < 0$ or $\dot{F} < 0$ & $\dot{F} = 0$

éq 2.1-8

if

F

= 0 and $\dot{F} = 0$ & $\dot{F} = 0$

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Note:

The evolution of variables X and X is given by:

1

2

2

$X = C p,$

1

1()

3

1

2

$X = C p,$

2

2 ()

3

2

éq 2.1-9

p

&

$1 =$
& -
 1

$p p$
 $1 ()$
&
 p
&

2
=
& -
 2

$p p$
 $2 ()$
&

The functions $C(p)$ (p) and $R(p)$ are defined, in accordance with [bib2] by:

$R(p) = R$
-
+ (R
 $0 - R) ELP$
 C
 1
 1

$1(p) = C$
 $1 (+ (K -) e-wp)$
 C
 1
 1
 $2(p) = C$
 $2 (+ (K -) e-wp)$

0
 $p =$
has
-
+ $1 - has$
ELP
 $1 ()$

1 (
())

0
p =
has

-
+ 1 - has
E LP

2 (
2(
())

The presence of viscosity can model in a simple way (cf Lemaitre and Chaboche [bib2]) in replacing the condition of coherence [éq 2.1-8] by:

NR
F
& =

éq 2.1-10
K

*F left positive F (hooks of Maccauley)
K, NR characteristic of viscosity (Norton) of material*

Unchanged all the other equations of the model are left. It will be seen that such an introduction of viscosity involves only minor modifications of the implicit algorithm of integration of the law of behavior.

Note:

The definition of X and X in the form [éq 2.1-9]:

1

· makes it possible to keep a formulation which takes into account the variations of the parameters with temperature without introducing term in & T as in [bib.4], in the same way that it viscoplastic model of Chaboche. These terms necessary because their are not taken in account would lead to inaccurate results [bib4].

· makes it possible to have a coherent writing with the thermodynamic expression of the potential plastic [bib2] (p.221).

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Note:

Significance of the functions C (p), (p), C (p), (p), R (p:

1

1

2

2

)

it is noted that the functions C (p), (p), C (p), (p), R (p intervening in the equations

1

1

2

2

)

the preceding ones allow all the three to model various nonlinear effects of work hardening. The introduction of work hardening, either on the level of the kinematic part, by $C(p)$, or on the level of recall, by the function (p) , does not have the same effect on the classification tests [bib2]. The use of a model with (p) makes it possible in particular to identify more easily of strong cyclic work hardenings. Several work of identification of the coefficients of the models of Chaboche were carried out besides on the basis of model with a work hardening represented by (p) ([bib5], [bib6]), in particular for stainless steel A316L.

2.2

Integration of relations VMIS_CIN1_CHAB and VMIS_CIN2_CHAB

To numerically carry out the integration of the law of behavior, one carries out a discretization in time and one adopts a diagram of implicit, famous Euler adapted for relations of behavior elastoplastic. Henceforth, the following notations will be employed: H_n , A_n and Δt_n represent respectively values of a quantity at the beginning and the end of the step of time considered thus that Δt_n increment during the step. The problem is then the following: knowing the state at time T_n - like increments of deformation (resulting from the phase of prediction (cf Doc. R STAT_NON_LINE [R5.03.01])) and of temperature T_n , to determine the state of the variables intern at time T_{n+1} as well as constraints.

One takes into account the variations of the characteristics compared to the temperature while noticing that:

$$H_n = H_0 + K \text{tr}(\epsilon_p)$$

$$K = K(T)$$

$$\epsilon_p = \epsilon_p(T)$$

$$K = K(T)$$

$$\sim$$

$$\mu \sim$$

$$=$$

$$2\mu$$

$$p$$

$$E$$

$$2\mu p$$

$$-$$

$$+$$

(~
) ~

- = -

μ

éq
2.2-2

with

~

μ

E

~-

~

= - + 2

μ

μ

Within sight of the equation [éq 2.2-1], one notes that the hydrostatic behavior is purely elastic if K is constant. Only the treatment of the deviatoric component is delicate.

In the absence of viscous term, the relation of discretized coherence is:

Elastic mode: $F = 0$ and $p = 0$

Plastic mode: $F = 0$ and $p = 0$

On the other hand, in the presence of viscosity, the condition of coherence is replaced by the equation [éq 2.1-10] which, discretized, is written:

NR

$1 NR$

p

F

p

=

$F = K$

T

K

T

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In other words, while posing:

1 NR

~

P

F = F - K

T

the viscoplastic increment of cumulated deformation is determined by:

~

:

rubber band

Mode

F 0 and p = 0

~

éq 2.2-3*viscoplast**Mode*

:

*ic**F = 0 and p 0*

Finally, by adopting an implicit discretization, the only difference between the laws of behavior plastic and viscoplastic lies in the form of the function of load F: a term there is observed complementary in the event of viscosity. In fact, incremental plasticity seems the borderline case incremental viscoplasticity when K tends towards zero. This convergence was already described by J.L. Chaboche and G. Cailletaud in [bib3].

In the continuation of this paragraph, one will thus detail the integration of the viscoplastic law. To find the case of the plastic behavior, it is enough to take $K = 0$ in the equations below (one recall that the user to place itself in this case must obligatorily remove key word LEMAITRE or LEMAITRE_FO of order DEFI_MATERIAU).

~

~

2

2

2

-

-

*= E - C - - C -**X**X**- 2μ p**C**C*

1

2
 1
 1
 2
 2
 - (1 1 + 2 2)
 3
 3
 3

Equations of flow [éq 2.1-6] and [éq 2.1-7], once discretized, and the condition of coherence [éq 2.2-3] are written (by noticing that $p =$):

2
 2
 2
 2
 ~e - C -
 -
p
 C
 μ
 C
 C
 1
 -
 2
 - 2
 -
 1
 -
 1
 2

 1
 2 2
 3
p =
 3
 3
 3
 3
 p

éq 2.2-4

2

2

2

2

2

$\sim e - C -$

-

p

C

μ

C

C

1

-

2

- 2

-

1

-

1

2

1

2 2

3

3

3

3

eq

\sim

\sim

$F_0 p_0 F p = 0$

éq

2.2-5

The treatment of the condition of coherence (preceding equation) is traditional. One starts with one test elastic ($p = 0$) which is well the solution if the criterion of plasticity is not exceeded, it be-with

to say if:

~
2
2
E -
-
-
C
-

l(-
p) 1 - C2 (-

p)
R
0

éq
2.2-6

3
3

2 - (p) <

eq

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In the contrary case, the solution is plastic ($p > 0$) and the condition of coherence is reduced to

~

*F = 0. To solve it, it is shown that one can bring back oneself to a scalar problem by expressing **p** and 1,2 according to p. By gathering the equations of the problem resulting from the discretization implicit, the system of equations is obtained:*

1 NR

2

E

-

2

-

p

2

2

~

- C 1 - C 2 - 2

μ - C 1 - C 2 = R

1

2

1

2

(p)

p

+ K

éq 2.2-7

3

3

3

3

eq

T

2

E

-

2

-

p

$$\begin{aligned}
& 2 \\
& 2 \\
& \sim \\
& - C 1 - C 2 - 2
\end{aligned}$$

$$\mu - C 1 - C$$

- 1
- 2
- 1
- 2
- p**
- 3
- 2
- =
- p
- 3
- 3
- 3
- 3

$$\begin{aligned}
& \mathbf{éq} \\
& \mathbf{2.2-8} \\
& 1 NR \\
& 2
\end{aligned}$$

$$\begin{aligned}
& R(p) \\
& p \\
& + K
\end{aligned}$$

$$\begin{aligned}
& T \\
& = \\
& \mathbf{p} - \\
& 1
\end{aligned}$$

$$\begin{aligned}
& p \\
& 1 1
\end{aligned}$$

$$\begin{aligned}
& \mathbf{éq} \\
& \mathbf{2.2-9} \\
& = \\
& \mathbf{p} - \\
& 2
\end{aligned}$$

p
 $2\ 2$

*In this writing, it should well be noted that $p = p + p$
and $= - +$ and that C , are*

I
 I
 I
 I
 I

functions of p . By considering the three last equations, this linear system out of p and can

I
to be solved to express these quantities according to p . Indeed, it is equivalent to:

$I\ NR$

p
 p
 $3 \sim e$

-
-

$R(p) + 3$
 $\mu p + K$

$= p - 1$
 $C\ 1 - C22 - 1$
 $C\ 1 - C22$

$\acute{e}q$
2.2-10
 T

2

$l + p = p -$
-

p
 $l($

1)

1 1

éq

2.2-11

+

p =

p -

-

2 (1

p

2)

2 2

*By calculating C11 and C2 2 and by replacing them in the expression of **p** one obtains one expression of **p** according to p only:*

C

C

p

- p

1

C

1

1

1

p

-

1 =

-

M

1

=

1 (p)

- M1 (p)

p

$I +$
 p

$I +$
 p
 1
 1
 1
 1

C

C
 p
 $-p$
 2
 C
 2
 2
 2

p
 $-$

éq 2.2-12

$2 =$

$-$

M
 2

$=$

$2 (p)$

$- M2 (p)$

p

$I +$
 p

$I +$
 p
 2

2
2
2

C p
with M

I
I (p)
()
= I+i (p) p

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By deferring this expression in the expression of p one finds:

p
1

=
3p~e - p ((C1 - M11p) -
1 + (C2 - m2 p) -
2)

1 NR
2
2

$$R(p) + (3\mu + M1 + m2)$$

p

$$p + K$$

T

what is simplified in:

1

3

p

$p \sim e$

$=$

$-$

éq

2.2-13

$()$

$(pM -$

$-$

$1 + m2)$

2

1

2

$D p$

with:

$1 NR$

$$D(p) = R(p) + (3\mu + M1(p) + m2(p)$$

p

p

$+ K$

T

*It now only remains to replace **p** in the expressions of C*

*1
1 and C2
2 for
to express this term according to p by:*

*M3
~ **E***

*C
1
=
p
-
-
-
1
1
-*

*(pM + M
1 1
2 2)
M
p*

*D
1 1
2
1
-*

*M3
~ **E***

*C
2
=*

p
-
-
-
2
2
-

$(pM + M$
 $1 1$
 $2 2)$
 M
 p

D
 $2 2$
 2
 2

-

~

*then to substitute the expression obtained thus that **p** according to p in the equation $F = 0$, and one obtains a scalar equation out of p to be solved, namely:*

~

$1 NR$
 $F(p) E 2$

-

2

-

p

2

2

~

p
 $= - C 1 - C 2 - 2$

1

2

$\mu -$

1

$$C 1 - C22 - R (p) - K$$

$$= 0$$

3

3

3

3

eq

T

what is simplified in:

1 NR

R (p)

p

+ K

~

1 NR

F (p)

T

2

-

2

~

p

=

-

E

éq 2.2-14

D (p)

$$- M1 1 - m2 2 - R (p) - K$$

$$= 0$$

3

3

eq

T

This scalar equation out of p is solved numerically, by a method of search for zero of function (method of secants which one briefly describes in appendix 2).

Once determined p , one can calculate \mathbf{p} using the equation [éq 2.2-13] then 1 and 2 with assistance of the equations [éq 2.2-11]. It any more but does not remain to calculate the tensor of the constraints, by equations [éq 2.2-1] and [éq 2.2-2], and to bring up to date the variables intern 1 and 2.

Note:

*· an interesting borderline case (for the validation of this model) is presented by posing $\epsilon = 0$. One finds itself then exactly in the situation of linear kinematic work hardening (if $R(p) = y$, [R5.03.02]) or of mixed work hardening for $R(p)$ unspecified (cf [R5.03.16]),
· these models are also available in plane constraints, by a total method (static condensation due to R. of Borst) [R5.03.03].*

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2.3

Calculation of tangent rigidity

In order to allow a resolution of the total problem (equilibrium equations) by a method of Newton [R5.03.01], it is necessary to determine the coherent tangent matrix of the problem incremental.

This matrix is composed classically of an elastic contribution and a plastic contribution:

E

p

=
- 2

μ

éq

2.3-1

μ

with $E = + 2\mu$

p, which gives again in particular $\sim e$

$\sim -$

\sim

=

+ 2μ

-

μ

One immediately deduces from it that in elastic mode (traditional or pseudo-discharge), the matrix tangent is reduced to the elastic matrix:

E

=

éq 2.3-2

*For that, one once more adopts the convention of writing of the symmetrical tensors of order 2 pennies form vectors with 6 components. Thus, for a tensor **a**:*

$$\begin{aligned}
 &T \\
 &= [axx \ ayy \ azz \ \text{has} \\
 &2axy \\
 &2axz \\
 &2ayz] \\
 &\text{éq} \\
 &2.3-3
 \end{aligned}$$

If one introduces moreover the hydrostatic vector $\mathbf{1}$ and stamps it deviatoric projection \mathbf{P} :

$$\begin{aligned}
 \mathbf{1} &= t [1 \ 1 \ 1 \ 0 \ 0] \\
 &0
 \end{aligned}$$

$$\begin{aligned}
 &\text{éq 2.3-4} \\
 &1 \\
 &\mathbf{P} = \mathbf{Id} - \mathbf{1} \mathbf{1}
 \end{aligned}$$

$$\begin{aligned}
 &\text{éq 2.3-5} \\
 &3 \\
 &\text{where is the tensorial product}
 \end{aligned}$$

Then the matrix of coherent tangent rigidity is written for an elastic behavior:

$$\begin{aligned}
 &\mathbf{E} \\
 &= K \mathbf{1} \mathbf{1} + 2\mu \mathbf{P}
 \end{aligned}$$

éq 2.3-6

On the other hand, in plastic mode, the variation of the plastic deformation is not null any more.

~
*One derives compared to **E**, knowing that one a:*

$$\begin{aligned} & \mathbf{p} \\ & \mathbf{p} \sim e \\ & \mathbf{p} \\ & = \\ & = 2 \mu \end{aligned}$$

P éq 2.3-7

~ .
E

~ .
E

S spaces symmetrical tensors
P projector on the diverters
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p
To calculate ~, one uses the expression of p according to ~

E
E and p:

1
3
p =
p ~

E -

()
(pM -
-
1 + m2)
2
1
2
D p

what is written in the form:

p = (
With p) ~e + B1 (p) -
-
1 + B2 (p) 2

Thus:

p
~ E
With p
B p
B p
= A p Id
1
2

~
+
+
+

E

()
() ()

-
()

-
~ **E**
~ **E**
1

~e
2

(
With p)
(
With p) (
With p) p
Quantities of the type
=

~e is calculated using: ~e
p ~

E

p
Finally, it any more but does not remain to calculate the variation of p:

~e

~

One uses for that: F (~

p, E

)= 0

1 NR

R (p)

p

+ K

~

1 NR

F (~

T

E

-

p

p, E)

2

2

~

=

-

D (p)

- M1 1 - m2 2 - R (p) - K

= 0

3

3

eq

T

~

~

~

p

p

F, p (

F
 \sim
 $,$
 $\sim e$
 $p,$
 \sim
 \sim
 \sim
 $,$
 p
 $= - F \sim e p,$

$= -$
 $\acute{e}q$
2.3-8
 $,$
 \mathbf{E}
 $) \mathbf{E}$
 $\mathbf{E} ($
 $\mathbf{E})$

$\sim e$
 \sim

$F, p (\sim e$
 $p,)$

The detail of calculations is given in appendix 1.

The initial tangent matrix, used by option RIGI_MECA_TANG is obtained by adopting it behavior of the preceding step (elastic or plastic, meant by internal variable being worth 0 or 1) and while making tighten p towards zero in the preceding equations.

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2.4

Significance of the internal variables

The internal variables of the two models at the points of Gauss (VELGA) are:

- V1 = p: cumulated plastic deformation (positive or null)*
- V2 =: being worth 1 if the point of Gauss plasticized during the increment or 0 if not.*

The following internal variables are, for modeling 3D:

· For model VISC_CINI_CHAB

-

$$V3 = I_{xx}$$

-

$$V4 = I_{yy}$$

-

$$V5 = I_{zz}$$

-

$$V6 = I_{xy}$$

-

$$V7 = I_{xz}$$

-

$$V8 = I_{yz}$$

· For model VISC_CIN2_CHAB

-

$$V3 = I_{xx}$$

-

$$V4 = I_{yy}$$

-

$$V5 = I_{zz}$$

-

$$V6 = I_{xy}$$

-

$$V7 = 1xz$$

-

$$V8 = 1yz$$

-

$$V9 =$$

$$2 \text{ xx}$$

-

$$V10 =$$

$$2 \text{ yy}$$

-

$$V11 =$$

$$2zz$$

-

$$V12 =$$

$$2 \text{ xy}$$

-

$$V13 =$$

$$2 \text{ xz}$$

-

$$V14 =$$

$$2 \text{ yz}$$

For modelings C_PLAN, D_PLAN, and AXIS:

-

$$V7 = 0$$

-

$$V8 = 0$$

-

$$V13 = 0$$

-

$$V14 = 0$$

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3

Model with two variables kinematics: CHABOCHE

3.1

Description of the model

This model comprises two tensorial variables which describe the kinematic share of work hardening:
 $\mathbf{X} = \mathbf{X1} + \mathbf{X2}$. *The equations of the behavior are then:*

$\dot{\mathbf{H}} =$
 $(\dot{\mathbf{H}}) =$
 $(\dot{\mathbf{H}}) = \mu (\dot{\mathbf{H}})$
 tr
 $K \text{tr}$
 $T-T$
 $) \text{Id éq 3.1-1}$
3
 $\dot{\mathbf{H}} = - \mathbf{H} \dot{\mathbf{H}} = \mu (\dot{\mathbf{H}})$

$\dot{\mathbf{p}}$
2
 $) \text{éq 3.1-2}$
 $R = R(p)$

éq 3.1-3

$$\mathbf{X} = \mathbf{X}$$

 p p

$$1(p, \cdot) + \mathbf{X}2(p, \cdot)$$

éq 3.1.4

where K , μ , and the coefficients of the functions $R(p)$, $\mathbf{X}1(p)$ and $\mathbf{X}2(p)$ are characteristics of material which can depend on the temperature. More precisely, they are respectively the modules of compressibility and shearing, the thermal dilation coefficient, functions of isotropic and kinematic work hardening. As for $T_{\text{réf}}$, it is about the temperature of reference, for which one regards the thermal deformation as being null.

The evolution of the internal variables is controlled by a normal law of flow to a criterion of plasticity:

$$F, R, \mathbf{X}, \mathbf{X} = (\sim - \mathbf{X} - \mathbf{X}) - R(p)$$

with

With

=

With: With

1

2

1

2

éq**3.1-5**

eq

eq

2

~

p

3

$$-X1 - X$$

$$\& = \& F$$

$$=$$

$$\&$$

$$2$$

éq 3.1-6

$$2$$

$$(\sim - X1 - X2) eq$$

$$\&p = \&$$

$$2$$

$$=$$

$$\&p: \&p$$

éq 3.1-7

3
 As for the plastic multiplier $\&$
 , it is obtained by the condition of coherence:

$$if F < 0 or \&F < 0 \& = 0$$

$$if F$$

$$= 0 and \&F = 0 \&$$

éq 3.1-8
0

The evolution of variables X_i is given in model CHABOCHE by:

&
2

$X = C$
has (p) & p
- X
 I
 I
 I
 I & p, I
,

1 2
3

=
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The functions (p) and $R(p)$ are defined by:

$$(p) ((K) E w p$$

$$= +$$

$$-$$

$$-$$

$$I$$

$$I$$

$$)$$

$$R(p) = R$$

$$-$$

$$+ (R0 - R) E LP$$

3.2

Integration of the relation of behavior CHABOCHE

As for the relations of behavior $VISC_CIN1_CHAB$ and $VISC_CIN2_CHAB$, one adopts one diagram of implicit Euler. One takes into account the variations of the elastic characteristics of even way that previously [éq.2.1-1], [éq.2.1-2].

One starts with an elastic test, while taking for stress field:

$$\sim E$$

$$\mu \sim -$$

$$\sim$$

$$=$$

$$+ 2\mu$$

$$-$$

μ
who is well the solution of the problem if:

$$\sim$$

$$2$$

$$2$$

$$E - C1 (-$$

$$p) - 1 - C2 (-$$

$$p) - 2 - R(p) < 0$$

$$3$$

$$3$$

In the contrary case, the solution is elastoplastic. It is then necessary to solve the system of equations not linear according to:

$$\begin{aligned}
 &\mu \\
 &\sim \\
 &\sim \\
 &- \\
 &- \\
 &1 - - 2\mu \\
 \mathbf{p} = & \\
 &- \\
 &(-) 0 \\
 &\mu \\
 &2 \\
 \mathbf{X1} - 1 & \\
 C 1 & \\
 (p) \mathbf{p} \text{ has} & \\
 + C \mathbf{X} & \\
 1 & \\
 1 p = 0 & \\
 3 & \\
 &2 \\
 \mathbf{X2} - 2 & \\
 A.c. 2 (p) \mathbf{p} & \\
 + C \mathbf{X} & \\
 2 & \\
 2 p = 0 & \\
 3 & \\
 (\sim - \mathbf{X1} - \mathbf{X2}) - R (p) = 0 & \\
 eq &
 \end{aligned}$$

that one can write in a way more contracted in the following form:

$$\begin{aligned}
 &G (y \\
 &)
 \end{aligned}$$

H

y

X

$F L (y$

)

()

$= 0 =$

with

y

I

$=$

$I (y$

)

X

2

J

(y

)

p

One solves this system by the method of Newton proposed in environment PLASTI, (described in detail in [R5.03.10]), is:

$F l$

$D (y$

L

$K) = - F (y$

$K)$

y

K

y

$+I = y$

+ D

K

K

(y

K)

While reiterating in K until convergence. This resolution takes place for each point of integration.

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The resolution requires the calculation of Jacobien of the local system F L. His general expression is given hereafter; analytical calculations are not detailed in this document.

G

G

G

G

X1 X2 p

H

H

H

H

X1 X2 p

J =

I

I

I

I

X1 X2 p

J

J

J

J

X1 X2 p

3.3

Operator of tangent behavior

After resolution of the preceding discretized system, the solution obtained is such as the equation (F L (y

) =)

0 are checked at the end of the increment. One seeks to evaluate the tangent operator in

this point, i.e.

.

T +t

For a small variation of F L, by regarding this time as variable and not parameter, the system remains with balance and one checks $dF L = 0$, i.e.:

F L

F L

F L

F L

L

+

$$\begin{aligned} &+ \\ &X1 + \\ &F \\ &X2 + \\ &= 0 \end{aligned}$$

$$\begin{aligned} &X1 \\ &X2 \\ &P \\ &P \end{aligned}$$

This system can be still written:

$$\begin{aligned} &H \\ &0 \\ &F L \\ &(y \\ &), \text{ with} \\ &0 \\ &= X \\ &X = \\ &y \\ &0 \\ &0 \end{aligned}$$

This system of equations can be put in the form:

$$K = H$$

from where the required tangent operator:

$-I$

$K H$

$=$

$t+t$

One is led to re-use the same matrix jacobienne J as previously to evaluate the operator tangent. The calculation of K^{-1} is carried out numerically by a method of decomposition of Gauss. Its expression is detailed in appendix 1.

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3.4

Significance of the internal variables

The internal variables of the model at the points of Gauss (VARI_ELGA) are, for modeling 3D:

$\cdot VI = XI_{xx}$

- $V2 = X1yy$
- $V3 = X2zz$
- 1
- $V4 = X1xy$
- $V5 = X1xz$
- $V6 = X1yz$
- $V7 = X2xx$
- $V8 = X2yy$
- $V9 = X2zz$
- $V10 = X2xy$
- $V11 = X2xz$
- $V12 = X2yz$
- $V13 = p$: cumulated plastic deformation (positive or null)

For modelings *D_PLAN*, and *AXIS*,

- $V5 = 0$
- $V6 = 0$
- $V11 = 0$
- $V12 = 0$

4 **Comparison of models *VMIS_CIN2_CHAB* and *CHABOCHE***

The principal difference between models *VISC_CIN2_CHAB* and *CHABOCHE* relates to the evolution of variables kinematics.

In case *VISC_CIN2_CHAB*, one a:

&
2
2
2

$$\begin{aligned}
 \mathbf{X} &= (\mathbf{C} + \mathbf{C}) = \\
 &\mathbf{C} + \mathbf{C}(p) \mathbf{C} p \\
 &- I(p) \mathbf{X} \\
 I \\
 I \\
 I \\
 I \\
 I \\
 I
 \end{aligned}$$

I
I
I
I & p
3

3
3

In case CHABOCHE, one a:

&
2

X = C
has (p) & p
- X

I
I
I
I & p, I

,

1 2
3

=

These two models are not equivalent: in particular, variation of the coefficients with temperature is not well taken into account in model CHABOCHE.

In the particular case where the coefficients C are constant, the two models are equivalent. For

I
that, it is necessary to choose:

.

0
I(p) =
I
C what implies: I = I
C, has = 1

2

.

C(p) = C = a.c.

I

I

II and $W = 0$, because of the additional term:

&C

3 I which allows

to take into account the variation of C_i with the temperature and the plastic deformation cumulated p .

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5

Principle of the identification of the parameters of the model.

In case the simplest (only one kinematic variable, $l = cste$, l

$C = cste$, $R(p) = y$) them

coefficients of model 1, 1

C can be identified on a simple tensile test uniaxial, or

on a cyclic curve of work hardening.

Indeed in the uniaxial case, the model is reduced in 1D to [bib2]:

$$dX = C D p$$

$$- X D p$$

1

1

1 1

,

$$= \pm I$$
$$- XI = y$$

that one can integrate (in monotonous loading) in the following way:

C

C

X

I

=

+ X 0

I

I

I -

- -

= ±

(

exp

I (p

p

0),

I

I

I

= + X

y

I

C

whose asymptote of the traction diagram makes it possible to obtain I by:

I

C

C

p X

I

I

I

thus $y +$

1

1

and whose slope in the beginning provides C (If X_0

1

1 =)

0 :

$p_0 X$

0

0

&1 $C_1 - y X$

1 1

$X_1 = C_1 - y X$

1

1

For a model has two variables kinematics, without isotropic work hardening, a traction diagram still allows to find these relations:

C

C

p

1

2

$y +$

and the slope in the beginning is worth $C + C$

+

1

2

1

2

But apart from these simple cases a numerical identification is necessary to obtain them parameters. One will be able to make this identification for example on tensile tests compression with imposed deformation.

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Appendix 1 Stamps behavior tangent for the models

VMIS_CIN1_CHAB and VMIS_CIN2_CHAB

p

To obtain the tangent behavior in the elastoplastic case, \sim should be calculated [éq 2.3-7].

E

One uses for that the expression of \mathbf{p} according to \sim and p , which is written in the form:

E

p

$3 p \sim$

*

-
=
*

$$2D(p)E + B1(p)I + B2(p) - 2$$

with
 $B^*i(p)$
 $MI(p)$
 $= -p$

$D(p)$
 Cp
 M
 I
 $I(p)$
 $()$
 $=$

$$I + I(p)p$$

INR

$$D(p) = R(p) + (3\mu + MI(p) + m2(p))p + K$$

T

The following definitions are pointed out:

$$R(p) = R + (R0 - R)ELP$$

$$I(p) = Ci(I + (K -)E)$$

- *LP*

$$I(p) = I(\textit{has} + (1 - \textit{has}) E)$$

thus:

3 p

p

3 p

2D (p)

*

*

~e

1

B (p)

-

B2 (p)

-

=

Id

~e

2D (p) +

+

1 +

2

~e

~e

~e

(

With p)

(

With p) (

With p) p

Quantities of the type

=

~

E are calculated using: $\sim e$

$p \sim$

E
These various terms are expressed by:

$3 p$

$2 ($
 $D p)$

3

1

$D (p)$

\cdot

$= I (p)$

with $I (p) =$

-

p

p

2

$($

$D p) D^2 (p)$

B^*

$I (p)$

$M I (p)$

\cdot

$= -$

-

\cdot

$=$

p

$D (p) p \text{ Semi } (p) I (p) \text{ Hi } (p)$

p

It remains to calculate:

~e

~

~

F ~e p,

p

, (

E

)

One thus uses, following [éq 2.3-8]:

= -

~

E

~

F, p (~e

p,)

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~

E

E

E

$F ($
 $1/$
 $1/$
 $p \sim$
 $,)$
 p
 p
 $= S$
 \sim
 \sim
 $eq (p,)$
 NR
 NR
 $- R (p) - K$
 $= G (p,) - R (p) - K$
 T

 T

 $1 NR$
 $1 NR$

$R (p)$
 p
 M
 $+ K$
 $I (p) R (p)$
 p
 $+ K$

T

E

-

-
T
2
~

with $S = A + B11 + B$

With
2 2
=
= -

D ()
B
p
I
3
D (p)
I NR
p

Then, by posing v
 $R (p) = R (p) + K$

:
T

3 v
R p S
v
R
~ E

-
-
~e
WITH + B + B
G ~e p,
p

, (
)
()
(

1 1

2 2)

2D (p) S

DS

= -

= -

eq

= -

eq

~

E

G

E

S

S

p (~

p,)

3

,

3

- R p

2

3

,

v ()

,

: S, p - v

R (p)

,

: S, p - v

R (p)

2 S

2

eq

Seq

~

3

E

-

-

1

L (p). + L21 (p) 1 + L22 (p) 2

= - 2

3

$L(p)$

2

v

Rp

With p

with

1

$L(p)$

()

2 ()

=

=

2

$D(p) * S$

S

eq

eq

v

Rp

1

v

Rp

1

21

$L(p)$

()

= $D(p) 1B(p)$

$L22(p)$

()

=

Bp

Seq

$D(p) 2 () Seq$

1 1

3

-

S

~ E

-

-

$K p NR$

3

$L(p) =$

$: (A(p)$

,

$+ I$

$B(p)$

,

$I + B^2(p) - R(p) -$

2 Seq

NT T

P

Finally, \sim is put in the form:

E

P

3 p

3

=

Id +

I

p

S

$\sim e + I$

p

has

-

-

$\sim \mathbf{E}$

\sim

$+ I$

p

has

E

1

2

2 (

$D p)$

(()

()

())

2

1

2

+ (1

$Hs \sim e + 1$

$Ha -$

1

$1 1 + Ha2 -$

2) -1

+ (2

$Hs \sim e + 2$

$Ha -$

2

$1 1 + Ha2 -$

2) -2

with:

3

Lp

1

$3 Ip L$

p

I

21

$S(p) = -$

$I(p)$

()

.

I

p

1

= -

2

$L3(p)$

has ()

() ()

2

$L3(p)$

$3 Ip L$

p

I

22

$a_2(p)$
 $() ()$
 $= - 2 L_3(p)$
 $3 H p. L p$
 $3 H p L$

p
 $3 H p L$

p
 $H 1$

1

1

1

1

21

1

1

22

$S(p)$

$() ()$

$= -$

H

p

1

$= -$

H

p

2

$= -$

2

$L_3(p)$

$has ()$

$() ()$

2

$L_3(p)$

$has ()$

$() ()$

2

$L_3(p)$

$3 H p. L p$

$3 H p L$

p

$3 H p L$

p

H2

2

1

2

2

21

2

2

22

S(p)

() ()

= -

H

p

1

= -

H

p

2

= -

2

L

L(p

3

)

3(p)

has ()

() ()

2

L3(p)

has ()

() ()

2

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*Behavior élasto-visco-plastic of J.L.Chaboche**Date:**08/12/03**Author (S):**P. of BONNIERES, J.M. PROIX, E.LORENTZ Key**:**R5.03.04-B Page**: 21/22****Appendix 2 Resolution of the equation $F(p) = 0$***

It is a question of solving a nonlinear scalar equation by seeking the solution in a confidence interval. For that, one proposes to couple a method of secant with a control of the interval of research. That is to say

the following equation to solve:

$$F(X) = 0$$

$$X \text{ [has, B]} \quad F(\text{has}) < 0 \quad F(b) > 0$$

éq A2-1

The method of the secant consists in building a succession of points x_n which converges towards the solution. It is

defined by recurrence (linear approximation of the function by its cord):

 x_n ***1******1******1******1******- x_{n-}*** ***x_{n+}*** ***= x_{n-}*** ***- $F(x_{n-})$*** ***éq A2-2***

$$F(x_n) - F(x_{n-1})$$

In addition, if x_{n+1} were to leave the interval, then one replaces it by the terminal of the interval in question:

if

$x_{n+1} < has$

then

$x_{n+1} := has$

éq A2-3

if

x_{n+1}

$1 > B$

then

$x_{n+1} := B$

On the other hand, if X_{n+1} is in the interval running, then the interval is reactualized:

if

$x_{n+1} [has, B] \text{ and } F(x_{n+1}) < 0$

then

$= x_{n+1} has$

éq A2-4

if

$x_{n+1} [has, B] \text{ and}$

1

1

$F(x_{n+1}) > 0$

then $B = x_{n+1}$

One considers to have converged when F is sufficiently close to 0 (tolerance to be informed). As for both first leader characters, one can choose the terminals of the interval, or, if one has an estimate of the solution, one can adopt this estimate and one of the terminals of the interval.

Note:

This method functions well if there is only one solution in the interval $[a, B]$. Without that being formally shown, one can note that $F(a) > 0$.

$0 > 0$.

One seeks then B such as $F(b) < 0$.

$\sim e^2$

-

2

-

C

1 1 -

C2

R -

3

3

2

-(p)

eq

One leaves for that $B =$

μ

3

If $F(b)$ is > 0 , one multiplies B by 10 and one tests if $F(b) > 0$, and so on, until finding a value B such as $F(b) < 0$.

One is sure that there is then at least a solution on $[a, B]$.

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Viscoplastic relation of behavior of Taheri

Summary

One presents in this document the establishment of the relation of viscoplastic behavior of Taheri, available for the whole of the isoparametric elements (continuous medium 2D and 3D) except for plane constraints. After a presentation of the equations of evolution of this law, one describes the system obtained

by implicit discretization; it is shown in particular that it always admits a solution.

This model is well adapted to describe the response of the austenitic steels under cyclic requests, and in particular the phenomenon of progressive deformation. On the other hand, because of its complexity (two surfaces of

charge, semi-discrete internal variable), it does not appear desirable to employ it for applications different (monotonous way of loading, for example).

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1

Description of the model

The relation of behavior proposed by Taheri [bib5] makes it possible to describe the response of steels austenitic under cyclic requests: it is indeed well adapted to represent it phenomenon of progressive deformation. Before stating the equations themselves, one can

to specify that this model differs from traditional plasticity (criterion of von Mises with work hardening kinematics and isotropic) by two characteristics, sources of difficulties in the numerical formulation. On the one hand, the evolution of the dissipative variables rests on two criteria of load instead of one: first, traditional, conditions the appearance of plastic deformation, the second makes it possible to keep one trace "maximum" work hardening reached by material to account for the phenomenon of ratchet. In addition, satisfactorily to represent the progressive deformation, an internal variable semi-discrete was introduced. Constant when the behavior is dissipative, it evolves/moves only in the elastic mode of material. Of original appearance, this model does not rest any less on physical bases, always exposed in Taheri [bib5]. It is accessible, in a wide version viscoplastic (necessary to describe the behavior under high temperatures), by order STAT_NON_LINE under the key word RELATION: VISC_TAHERI.

1.1 Behavior

plastic

A detailed description of the law of behavior is given in Taheri and Al [bib6]. Briefly, the state of material is described by its state of deformation, its temperature like four internal variables:

tensor of total deflection

T

temperature

P

cumulated plastic deformation

p

tensor of plastic deformation

P

constraint of peak, memory of maximum work hardening

$p N$

plastic tensor deformation due to the last discharge (variable semi-discrete).

The equations of state which express the thermodynamic forces associated according to the variables of state are written:

$$= K \text{Tr}(-HT) + 2 \mu (\sim - p)$$

HT

Id

$$= (T - \text{Tr}é) Id$$

éq 1.1-1

P

has

has

- B p 1

R

R0 (2

pp

l

éq 1.1-2

3) With (

N)

m E

S

=

+

-

D

D

eq

= -

-

p

B p l -

X

[p p p

=

S -

S

C

N]

C =

C + C E

l

éq 1.1-3

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~

has

deviatoric part of a tensor **has**

R

isotropic variable of work hardening

X

kinematic variable of work hardening

K, μ

modules of compressibility and shearing

thermal dilation coefficient

T_{réf}

temperature of reference

S

constraint of ratchet

B, R°, A,

has **m, C, C**

other characteristics of work hardening of material

1

Let us note that the moduli of elasticity and the thermal dilation coefficient are indicated by the user by the order **DEFI_MATERIAU**, key word **ELAS**, while characteristics of work hardening are fixed by key word **TAHERI**. These characteristics can depend on temperature, by employing key words **ELAS_FO** and **TAHERI_FO**. Also let us specify that one example of identification of the characteristics of work hardening on uniaxial situations is given in Geyer [bib2].

The evolution of the internal variables is defined by two criteria. The first controls plasticity traditional with work hardenings kinematics and isotropic compounds:

~

0

X

$$F = (\sim - \mathbf{X}) - R \ 0$$

and

S

=

-

éq 1.1-4

eq

$$(\sim - \mathbf{X}) \ eq$$

()

1

eq

equivalent standard: **has**

$$(\mathbf{a} \sim : \mathbf{a} \sim 2$$

$$\ eq = 32$$

)

F

criterion of plasticity

s0

normal external with the criterion F

This criterion is matched traditional condition of load/discharge:

if $F < 0$ or **s0** 0

$$p = 0$$

(elasticity

! :

!

)

éq 1.1-5

if

F

= 0 and

s0 0

$p \ 0$ such as

>

!

$$! \ F = 0$$

(plasticity

! :

)

And the law of flow associated with the criterion F is:

3

2

! **p** =

! **0**

p **S**

and thus

! *p* =

! *p*

éq 1.1-6

2

3 eq

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The second criterion controls the evolution of the constraint of peak. Geometrically in the space of diverters of the constraints, it translates the fact that the first surface of load ($F = 0$), represented by a sphere of center \mathbf{X} and ray R , remains inside a sphere of centre the origin and

p. It is written simply:

$$G = X + R$$

p

eq

-

0

éq 1.1-7

G criterion of maximum work hardening

According to the preceding geometrical considerations, the evolution of the constraint of peak is:

if

$G < 0$ or

! X

p
 $eq +!$
 $R 0$
 $! = 0$

éq 1.1-8

if
 $G = 0$ and
 $! X$
 p
 $eq +!$
 $R > 0! 0$ such as! $G = 0$

It should be noticed that in the natural state of material, the constraint of peak is not null but is worth initial elastic limit, namely:

p (init)
 ial
 $= (1 - m) R0$

Until now, we did not evoke the evolution of the semi-discrete internal variable $\mathbf{p N}$. In fact, it evolves/moves only in elastic mode. More exactly, this variable takes account of the state of plastic deformation during the last discharge; in other words, at the beginning of each discharge, this variable should take the value of the current plastic deformation instantaneously. However, for to preserve a continuous behavior, one regularizes the evolution of $\mathbf{p N}$ in the following way:

In elastic mode:

\mathbf{p}
 \mathbf{p}

 \mathbf{p}
 \mathbf{p}
 \mathbf{p}
 if
 elasticity classi
)
 $\mathbf{N} =$
 $! = 0$
 (
 that
 $! = !$
 \mathbf{N}
 $(\mathbf{N} -)$
 éq 1.1-9
 if \mathbf{p}
 \mathbf{p}

(pseudo - déchar)

N

! 0 tq! F = 0

Ge

In plastic mode:

! pn = 0

The behavior is thus completely given. Before passing to the introduction of viscosity, the observation of two surfaces of load calls an important remark. One could think that surface $G = 0$ is actually activated only in plastic mode. In practice, it is nothing. One can for example quote the case of a thermal loading: a cooling involves (generally) one dilation of the surface of load $F = 0$, so that the constraint of peak is brought to evolve/move for to preserve $G = 0$, and this same in elastic mode.

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1.2

Taking into account of viscosity

To model the behavior of the stainless steels under cyclic loading when temperature is about 550°C , it is not more possible to neglect the terms of creep. To return count these effects of viscosity while preserving the properties of the preceding model, a method simple consists in making viscous the evolution of the plastic deformation. In other words, viscosity intervenes only in plastic mode: no direct influence on the semi-discrete internal variable nor on the surface of load $G = 0$. For that, while following Lemaitre and Chaboche [bib3], one replaces condition of coherence [éq 1.1-5] by:

NR

F

! p =

éq 1.2-1 $K p1 M$

F

positive part of F (hooks of Macauley)

 K, NR, M

characteristics of viscosity of material

The characteristics of viscosity of material are indicated in order `DEFI_MATERIAU`, either by key word `LEMAITRE` if they do not depend on the temperature, or by the key word `LEMAITRE_FO` in the contrary case. In the absence of one of these key words, the behavior is supposed plastic.

Unchanged all the other equations of the model are left. It will be seen that such an introduction of viscosity involves only minor modifications of the implicit algorithm of integration of the law of behavior.

1.3**Description of the internal variables calculated by *Code_Aster***

The internal variables calculated by *Code_Aster* are 9. They are arranged in the following order:

1
 p
cumulated plastic deformation

2
 p
constraint of peak

3 to 8

p N
plastic tensor of deformation due to the last discharge
(arranged in order xx, yy, zz, xy, xz, yz)

9
loadmeter/discharge (cf [§2.3])

0 elastic discharge
1 traditional plastic load
2 plastic load on two surfaces
3 pseudo-discharge

As for the tensor of the viscoplastic deformations, it is not arranged among the internal variables but can be calculated in postprocessing via the order `CALC_ELEM`, options “`EPSP_ELGA`” or “`EPSP_ELNO`”, (cf [U4.61.02]).

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2**Numerical formulation of the relation of behavior**

In order to be able to treat within the same framework plasticity and viscoplasticity, one chooses to proceed to one implicit discretization of the relations of behavior, (cf [R5-03-02]). Let us note moreover that one explicit procedure of integration is delicate for two reasons: on the one hand, treatment of the variable semi-discrete is necessarily implicit and can lead to numerical oscillations (one pseudo-discharge, therefore one solves $F = 0$, and of the blow, F can be (very weak but) higher than zero, from where load with the step following instead of continuing the discharge), and in addition, the equation [éq 1.1-2] is not not derivable when $\mathbf{p} = \mathbf{p} \mathbf{N}$.

2.1**Implicit discretization of the equations of behavior**

Henceforth, one adopts the convention of following notation. If U indicates a quantity, then:

 U quantity U at the beginning of the step of time U increment of the quantity U during the step of time U quantity U at the end of the step of time (not of exhibitor +)

Let us start by introducing the elastic constraint, i.e. the constraint in the absence of increment of plastic deformation. One can notice besides that only the term deviatoric cheek a role in the nonlinear part of the behavior:

 $\mathbf{E} =$ $(-\mathbf{HT}) + \mu (\sim\mathbf{p}-$ $K tr$

2

 $)$ and $\sim\mathbf{e} = \sim\mathbf{e} - 2\mu \mathbf{p}$ **Id****éq 2.1-1**

"\$#

\$ \$%

\$

~ **E**

By taking account of the equations of state [éq 1.1-1] and [éq 1.1-3] and of the law of flow [éq 1.1-6], one a:

déf

~

~

-

3

S

X

E

C (p

p p

0

=

-

=

-

S

- **N**) - (2μ + CS) *p*

S

éq 2.1-2

2

By noting that s° is not other than **S** normalized, one deduces some immediately:

3

~

p -

S + (2 μ + C S) p

s0

E

C

-

p

éq 2.1-3

2

=
- (S
p
eq
N)
"\$\$\$ #
\$
%
\$\$\$\$

Consequently, S is entirely determined by:

E
0
0
S
3
S = S S
S =
S
=
with
and
-
2 μ

éq 2.1-4

E
eq
eq
(+ C S) p
eq
S
2
eq

Finally, the functions of load are:

has

has
0
p -
p
3

$$F = S - D R + (2$$

0

S

3) *With*

- **N** +

p

eq

éq 2.1-5

2

eq

has

p

3

has

p

0

0

2

p -

3

G C S

P

Sp

S

D

N

R

(

3) *With*

p

p **0**

p

=

-

+

N

S

-

éq 2.1-6

2

+

+

-

+

2

eq

eq

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2.2

Taking into account of the viscous terms

In the absence of viscous terms, the relation of discretized coherence is:

Elastic mode:

$F = 0$ and

$$p = 0$$

éq 2.2-1

Plastic mode:

$F = 0$ and

$$p = 0$$

On the other hand, in the presence of viscosity, the condition of coherence is replaced by the equation [éq 1.2-1] which, discretized, is written:

1

NR

p

F

$$1 M p$$

F

$$= K p$$

NR

$=$

éq 2.2-2

T

$$K p 1 M$$

T

In other words, while posing:

1

~

1 M

p

$$F = F -$$

$K p$
 NR

éq 2.2-3

T

the viscoplastic increment of cumulated deformation is determined by:

~

Elastic mode:

$F = 0$ and

p

$= 0$

~

éq 2.2-4

Viscoplastic mode:

$F = 0$ and

p

0

Finally, by adopting an implicit discretization, the only difference between the laws of behavior plastic and viscoplastic lies in the form of the function of load F : a term there is observed complementary in the event of viscosity. In fact, incremental plasticity seems the borderline case (without associated numerical difficulty) of incremental viscoplasticity when viscosity K tends towards zero. Let us note that this remark was already mentioned by Chaboche and Al [bib1].

2.3

Discretization of the conditions of coherence

Before discretizing the conditions of coherence and describing the various modes of behavior possible, a remark is essential as for the treatment of the semi-discrete variable. Like only intervenes “to control” \mathbf{p}_n , one can always bring back itself during a step of time to:

\mathbf{p}

$\mathbf{p} -$

$\mathbf{p} -$

\mathbf{N}

$= \mathbf{N} + (1 -)$

$0 \ 1$

éq 2.3-1

The value of is then fixed by the conditions of coherence, which translates the equation of evolution [éq 1.1-9] on the continuous level. Such a parameter setting with each step of time makes it possible to be freed from

storage of, in condition well-sure of preserving the values of \mathbf{p}_n .

After this opening remark, one can be interested in the conditions of coherence. For the criterion G who controls the evolution of the constraint of peak, the discretized form of the condition of coherence is:

$G(p, p) = 0$

$p = 0$

$G(p, p)$

$= 0$

éq 2.3-2

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The condition of coherence relating to F is more delicate insofar as it controls the evolution plastic deformation in plastic mode of load and evolution of in mode of discharge.

Once discretized, she is written:

In plastic mode of load ($=$)

1 :

$F(p, p) = 1 = 0$

$p = 0$

$F(p, p)$

$= 1 = 0$

éq 2.3-3

In mode of discharge ($p = 0$):

$F(p, 0)$

$= 0$

$0 = 1$

$F(p, 0)$

$=$

$=$

=

,) = 0

éq 2.3-4

To be able to select the mode of behavior of material, and thus equations to be solved, first question is:

Are we in plastic situation or rubber band?

In fact, there is a solution in elastic mode (pseudo-discharge > 0 or traditional elasticity $= 0$) if one can find an increment of constraint of peak such as:

Incremental condition of discharge (scalar equation out of p):

$$F(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$
éq 2.3-5

$$G(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$
 p

0

 p

$$G(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$

In the event of plastic load, i.e. when there is not p satisfactory [éq 2.3-5], one then has with to solve the nonlinear system out of p and \dot{p} following:

Plastic mode (nonlinear system out of p and \dot{p}):

$$F(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$
 p

0

éq 2.3-6

$$G(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$
 p

0

 p

$$G(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$

On the other hand, in elastic situation, two choices are still possible: pseudo-discharge (> 0) or traditional elasticity ($= 0$). The second case being more favorable, one starts by examining whether it is not

not realizable, i.e. if there is an increment of constraint of peak such as:

Incremental condition of traditional elastic mode (scalar equation out of p):

$$F(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$
éq 2.3-7

$$G(p, \dot{p}) = 0$$

$$, \dot{p} = 0$$

p

0

p

$G(p =, 0 p$

, = 0) = 0

Lastly, if it were to be a question of a discharge pseudo-rubber band, it remains to solve the nonlinear system in

and p following:

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Discharge pseudo-rubber band (nonlinear system in and p):

$F(p =, 0 p$

,) = 0

0 1

éq 2.3-8

$G(p =, 0 p$

,) 0

p

0

p

$G(p =, 0 p$

,) = 0

Let us note as of now that nonlinear systems [éq 2.3-6] and [éq 2.3-8] can be reduced to solution of a simple scalar equation if $p = 0$ makes it possible to obtain a solution.

One can summarize the algorithm of choice of the equations to be solved by the decision tree below.

Charge/discharge

(2.3-5)

Plastic resolution

Elasticity

(2.3-6)

traditional/pseudo

(2.3-7)

Resolution

Resolution

traditional elasticity

pseudo-discharge

(2.3-8)

Appear 2.3-Error! Argument of unknown switch. : **Decision tree to choose the mode of behavior**

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2.4

Framing of the solutions

With the reading of the preceding paragraph, one could note the need for solving (numerically) one certain number of scalar equations or nonlinear systems. For that, it is always interesting to have an interval on which to seek the solution. On the one hand, a framing of solution shows its existence (what strongly reinforces the chances of success of an algorithm of resolution!), and in addition, it allows a suitable digital processing, therefore surer.

Concerning p , one undervaluing is of course 0. In addition, the constraint of ratchet S represents a limit beyond which the model does not have any more a smell. In fact, with the examination of constant materials

obtained by identification, cf Taheri and Al [bib6], G becomes indeed negative when $p = S$ if the difference between the plastic deformation and the plastic deformation due to the last discharge too is not

important (a few %):

$G(p)$

= ,
S) = (*C*
P
P
+ *CI*) (- **N**)
S
eq
" #
\$ %
\$
13
"\$ #
\$ \$ %
\$
3%

éq 2.4-1

0
+ (
R
A has
has
1 - *m*)
+ (2
P P
1
0
3)
(- **N**) -
"#%
S
S
eq

0,75
&
"#% " #
\$
%
\$

20%

0,5

0,7

One can also seek one raising for p . By examining the expression of F:

$F(p, p)$

$) S$

$- D R 0$

eq

3

0

$eq -$

$(2 \mu + CS) p - D R$

éq 2.4-2

2

3

$- (2 \mu + C S$

$-$

0

$) p -$

$(D$

\max

$p) R$

2

One deduces one from them raising for p , such as $F(p, p)$

$) 0 :$

0

$\max (p$

$)- ($

$D p$

p

$) R$

p

3

éq 2.4-3

$(2 \mu + C S$

$)$

2

$\sim e -$

-

p -

p

(CP *p*

,)

p

-

N

E

p

(*S*

)

smax ()

eq

= max

éq 2.4-4

~

E -

p -

C

p

(*S*

p

-

N) *eq*

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In particular, one can give one raising (coarse) p independent of p :

max

0

max - (1 - m) R $p_{max} = 3$ **éq 2.4-5** $(2 \mu + C S$

)

2

p - p

-

- p**N** E max E [S

]

 S eq max = $eq + (C1 + C)$ max [**éq 2.4-6****p -** S **- S pn**] eq

One can then notice that the systems [éq 2.3-6] and [éq 2.3-8] always admit a solution. In effect, if for each system, one writes p respectively ()

 p p and

() solutions of

 $G = 0$, then one a:

· Nonlinear Système of plastic load:

$F(p, p)$
 $0(p=0), (=)$
 10 and $F(p, p)$
 $\max(p_{max}), (=)$
 10 **éq 2.4-7**

· Nonlinear Système of pseudo-discharge:

$F(p,$
 p
 $= 0 (=)$
 $1, (=)$
 10
 and
 $F(p, p)$

$0 (= 0), = 0) 0$ **éq 2.4-8**

3 Methods of numerical resolution

The resolution of the incremental equations confronts us either with a nonlinear scalar equation, or with a nonlinear system with two unknown factors. The numerical methods below are exposed employees. One also examines the calculation of the tangent matrix, possibly used by the total algorithm of STAT_NON_LINE, (cf [R5.03.01]).

3.1 Scalar equation: method of secants

It is a question of solving a nonlinear scalar equation by seeking the solution in an interval of confidence. For that, one proposes to couple a method of secant with a control of the interval of research. That is to say the following equation to solve:

$F(X) = 0$
 $X [$
 $,$
 $B] F \text{ has } (has) < 0 F(b) > 0$

éq 3.1-1
The method of the secant consists in building a succession of points X_N which converges towards the solution.

It is defined by recurrence (linear approximation of the function by its cord):

N
 $n-1$
 $n+1$
 $n-1$
 $N 1$
 $X - X$
 X
 $= X$

- F (X -)

éq 3.1-2

F (xn) - F (xn-1)

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In addition, if X_{n+1} were to leave the interval, then one replaces it by the terminal of the interval in question:

if

$X_{n+1} < has$

then

$X_{n+1} := has$

éq 3.1-3

if

X_{n+1}

$1 > B$

then

$X_{n+1} := B$

On the other hand, if X_{n+1} is in the interval running, then the interval is reactualized:

if

X_{n+1}

1

[
 $has, B]$ and $F(x_{n+1}) < 0$

then

$a := X_{n+1}$

éq 3.1-4

if
 $X_{n+1} \in [has, B]$ and

1
 1

$F(x_{n+1}) > 0$

then

$b := X_{n+1}$

One considers to have converged when F is sufficiently close to 0 (tolerance to be informed). As at the first two leader characters, one can choose the terminals of the interval, or, if one lays out of an estimate of the solution, one can adopt this estimate and one of the terminals of the interval.

3.2

Nonlinear systems: method of Newton and linear research

One presents here a method of Newton which one associated a linear technique of research and a control of the direction of descent not to leave the field of research (terminals on unknown factors).

That is to say the following nonlinear system:

$F(X, y) = 0$

$x_{min} \leq X \leq x_{max}$

with

éq 3.2-1

G

$(X, y) = 0$

y

$\min y \leq y_{max}$

If (X, y) is a point of the field of research, then one builds a succession of points (x_n, y_n) ,

,

) which

converge towards a solution (or at least, it is hoped for) by the following process.

· Détermination of the direction of descent

A direction of descent (X, y) is given by the resolution of the linear system 2×2 :

$F_n \quad F_n \quad X$

N

, X

, y

F

éq 3.2-2

Gn

Gn y = -

N

, X

, y

G

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· Correction of the direction of descent

The direction of descent (X, y) is corrected so that the points considered are in the field of research (with max the maximum length which one is authorized to describe it length of the direction of descent):

$xmin - X$

$xmax - X$

if $X +$

X

$max < X$

X

min

:=

if $X +$

X

$max > X$

X

max

:=

max

max

y

éq 3.2-3

min - y

$y_{max} - y$

if

y +

y
 $\max < y$
 y
 \min
 $:=$
 so $y +$
 y
 $\max > y$
 y
 \max
 $:=$

\max
 \max
 · Recherche
 linear

It any more but does not remain to minimize the quantity $E = (F^2 + G^2)/2$ in the direction of descent. Let us note that the standard E that one minimizes thus is a measurement of the error made in resolution of the system: it is null when (X, y) is solution of the system [éq 3.2-1]. For to minimize E , one simply will seek to cancel his derivative, i.e. to solve the scalar equation:

$$[E(X + X, y + y)] = 0 \text{ and } 0$$

éq 3.2-4

\max
 "\$\$\$ #
 \$ \$\$\$ %
 \$
 (FF +GG
 , X
 , X) X + (F F +G G
 , y
 , y) y

· Critère of convergence

One considers to have converged when the error E is lower than a prescribed size. By elsewhere, if the standard of the direction of descent becomes too weak (another size with to inform), one can think that the algorithm does not manage to converge.

3.3 Criteria of stop

Until now, values of stop and iteration counts maximum of the methods of resolution the preceding ones were not specified. Two cases should be distinguished.

· When one seeks to check the conditions of coherence (scalar equation or system not linear according to the situation), one awaits precise results, whose relative tolerance is

fixed by the user in order STAT_NON_LINE under key word RESI_INTE_RELTA,
(cf [U4.32.01]). According to whether one seeks to solve $F =$,

$0 G = 0$ or simultaneously

$F = G = 0$, the criterion of stop is expressed respectively:

F

G

F2 G2

1

or

+

or

R0

R0

R0

2

R0 limit elastic initial, provided by the user, cf [§ 1.1].

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In addition, the user always specifies a maximum iteration count in the order
STAT_NON_LINE under key word ITER_INTE_MAXI, (cf [U4.32.01]).

· When iterations of linear research are carried out, one seeks to obtain a convergence
faster (or at least sourer). One should not therefore devoting an excessive time to it.
This is why one fixed once for a a whole iteration count maximum equal to 3, one
limit maximum max equal to 2 and one criterion of relative stop of 1%:

[E (X + X, y + y)]

$$10 \ 2$$

$$[E (X + X, y + y)]$$

=0

3.4 Stamp tangent

In the optics of a resolution of the equilibrium equations (total) by a method of Newton, it is essential to determine the consistent matrix of the tangent behavior, (cf Simo and Al [bib4]). This matrix is composed classically of an elastic contribution and a plastic contribution:

E**p**

=

- 2 μ **éq 3.4-1**

One immediately deduces from it that in elastic mode (traditional or pseudo-discharge), the matrix tangent is reduced to the elastic matrix:

Elastic mode:

E

=

éq 3.4-2

On the other hand, in plastic mode, the variation of the plastic deformation is not null any more. Rules of made up derivation allow to obtain:

p**0**

3

p

3

*p**p*

3

0**S**

0

0

0

=

S

+ *p*

S

Id

S

S éq 3.4-3

=

+

-

~e

2

~e

~e

2

~e

2

eq

tensorial product

~

One can note that one preferred to derive compared to **E**, knowing that one a:

p

p ~e

p

S S

=

= 2 μ **P**

with

P
éq 3.4-4

~ .
E

~ .
:
E
' ~

S spaces symmetrical tensors

P projector on the diverters

Finally, it does not remain any more that to calculate the variation of *p*. For that, it is necessary to distinguish if it is about one traditional mode of plasticity (*p* = 0) or of plasticity on two surfaces. As follows:

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Traditional plasticity: $F(, \sim$

$p \mathbf{E}) = 0$

$F \mathbf{E}, \sim \mathbf{E}$

E

E

E

p

p

F

, ~

, ~

, $p(, \sim$

$p) p$

$$= - F E p$$

= -

éq 3.4-5

, ~
(, ~) ~
()

~

E

F

E

, p (, ~

p)

Plasticity on two surfaces: F (, p, ~e) 0 and G (, p

, ~

p

p

E

=

) = 0

F p

F

,

, ~

E

F

F p p

F

G

E

p

G

, p

E

,

, ~

E

p
,
,~

=
G
G

p =
-
éq 3.4-6

E
p
G
. ~
~
, *p*

E
F
F

,

,~

, *p*

, *p*

G
G
, *p*
, *p*

An attentive examination of the expressions [éq 2.1-5] and [éq 2.1-6] makes it possible to note that the variations of

F

~

and \mathbf{G} compared to \mathbf{E} are not necessarily colinéaires with \mathbf{S}° . By taking account of [éq 3.4-3], one from of deduces whereas the tangent matrix is in general not symmetrical in mode plastic. Rather than to impose the use of a nonsymmetrical solver, much more expensive in time calculation, one prefers to symmetrize this matrix.

3.5 Constraints

plane

The treatment of the plane constraints adds a nonlinear equation to solve, coupled with systems [éq 2.3-6] and [éq 2.3-8], (cf [R5-03-02]). In front of this considerable difficulty and the absence of expressed need, one preferred not to make it possible to force a state of plane stresses with level of the law of behavior. In other words, modeling C_PLAN is not available for law of behavior VISC_TAHERI.

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Model of Rousselier in great deformations

Summary

One presents here the model of Rousselier which makes it possible to describe the first stages of the plastic growth cavities in a steel. The relation of behavior is elastoplastic with isotropic work hardening, allows the changes of plastic volume and is written in great deformations. To describe the large ones deformations, one uses the theory suggested by Simo and Miehe. The original formulation of Simo and Miehe is modified so, on the one hand, facilitating the numerical integration of the law of behavior and, on the other hand, of to replace the theory of Simo and Mihe within the variational framework of generalized standard materials.

This model is available in order STAT_NON_LINE via the key word RELATION: "ROUSSELIER" or "ROUSSELIER_FO" under the key word factor COMP_INCR and with the key word

DEFORMATION: "SIMO_MIEHE".

This model is established for three-dimensional modelings (3D), axisymmetric (Axis) and in plane (D_PLAN).

One presents the writing and the digital processing of this model.

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1 Introduction

The mechanisms at the origin of the ductile rupture of steels are associated the development of cavities within material. Three phases are generally distinguished:

- germination*
: it is about the initiation of the cavities, in sites which correspond preferentially with the defects of material,
- growth: it is the phase which corresponds to the development itself of the cavities, controlled primarily by the plastic flow of the metal matrix which surrounds these cavities,*
- coalescence: it is the phase which corresponds to the interaction of the cavities between them to create macroscopic cracks.*

In what follows, we treat only the phases of growth and coalescence.

The model of Rousselier [bib1] presented here is based on microstructural assumptions which introduce a microstructure made up of a cavity and a plastic rigid matrix thus isochoric.

In this case, porosity F , definite like the relationship between the volume of the cavity C and volume

total V of representative elementary volume, is directly connected to the macroscopic deformation by:

$$1 - F$$

$$V C$$

$$J = \det \mathbf{F}$$

$$0$$

$$=$$

$$\text{with } F =$$

$$f \& = (1 - F) \operatorname{tr} \mathbf{D}$$

$$\dot{e}_q$$

$$1-1$$

$$1 - F$$

$$V$$

where f_0 indicates initial porosity, \mathbf{F} the tensor gradient of the transformation, J the variation of volume and \mathbf{D} the rate of deformation.

To build the law of growth of the cavities, Rousselier takes as a starting point a phenomenologic analysis

who leads it to the following ingredients:

- *great deformations figure,*
- *irreversible changes of volume,*
- *isotropic work hardening.*

These considerations leads it to write the criterion of plasticity F in the following form:

(

$$F, R)$$

$$H$$

$$= e_q + D F \exp$$

$$1$$

$$- R(p) -$$

y

$$\dot{e}_q$$

$$1-2$$

$$1$$

where is the constraint of Kirchhoff, R isotropic work hardening function of the plastic deformation cumulated p and 1 , D and y of the parameters of material. The presence in the criterion of plasticity of

hydrostatic constraint H authorizes the changes of plastic volume. One also notices that this model does not comprise a specific variable of damage because only information microstructural reserve is porosity, directly related to the macroscopic deformation by the equation [éq 1-1].

As for the treatment of the great deformations, one adopts the theory of Simo and Miehe but in one slightly modified formulation. The approximations brought make it possible to make easier the numerical integration of the law of behavior but also to replace the theory of Simo and Miehe within the variational framework of generalized standard materials.

Thereafter, one briefly gives some concepts of mechanics in great deformations, then one point out the theory of Simo and Miehe as well as the made modifications. One presents finally them relations of behavior of the model of Rousselier and its numerical integration.

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2 Notations

One will note by:

Id

stamp identity

tr A

trace tensor A

AT
transposed of tensor A

det A
determinant of A

~
With
~
1
deviatoric part of tensor A defined by $A = A - (tr A) Id$
3

H
With
tr A
hydrostatic part of tensor A defined by H
With =

3
:
doubly contracted product: $A : B = A B = tr ($
T
AB)
ij ij
I, J

tensorial product: $(A B) ijkl = ij$
With kl
B

3
With
~ ~
eq
equivalent value of Von Mises defined by $Aeq =$
:
WITH A

2

With

With

With =

X

gradient: X

X

, μ , E, K moduli of the isotropic elasticity

*y
elastic limit*

thermal dilation coefficient

*T
temperature*

*Tref
temperature of reference*

In addition, within the framework of a discretization in time, all the quantities evaluated at the moment

precedent are subscripted by -, the quantities evaluated at the moment $T + T$ are not subscripted and them

increments are indicated par. One has as follows:

$$Q = Q - Q$$

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3

Theory of Simo and Miehe

3.1 Introduction

We point out here specificities of the formulation suggested by SIMO J.C and MIEHE C. [bib2] for to treat the great deformations. This formulation was already used for models of thermoelastoplastic behaviour with isotropic work hardening and criterion of Von Mises, [R5.03.21] for a model without effect of the metallurgical transformations and [R4.04.03] for a model with effect metallurgical transformations.

The kinematics choices make it possible to treat great displacements and great deformations but also of great rotations in an exact way.

Specificities of these models are as follows:

- just like in small deformations, one supposes the existence of a slackened configuration, i.e. locally free of constraint, which makes it possible to break up the total deflection into a thermoelastic part and a plastic part,**
- the decomposition of this deformation in parts thermoelastic and plastic is not any more additive as in small deformations (or for the models great deformations written in rate of deformation with for example a derivative of Jaumann) but multiplicative,**
- the elastic strain are measured in the current configuration (deformed) tandis that the plastic deformations are measured in the initial configuration,**
- as in small deformations, the constraints depend only on the deformations thermoelastic,**
- if the criterion of plasticity depends only on the deviatoric constraint, then deformations plastics are done with constant volume. The variation of volume is then only due to thermoelastic deformations,**
- this model led during its numerical integration to a model incrémentalement objective**

(cf [§3.2.3]) what makes it possible to obtain the exact solution in the presence of great rotations.

Thereafter, one briefly points out some concepts of mechanics in great deformations.

3.2 General information on the great deformations

3.2.1 Kinematics

Let us consider a solid subjected to great deformations. That is to say the θ field occupied by the solid before deformation and (T) the field occupied at the moment T by the deformed solid.

Initial configuration

Current configuration deformation

F

θ

(T)

Appear 3.2.1-a: Representation of the initial and deformed configuration

In the initial configuration θ , the position of any particle of the solid is indicated by X (Lagrangian description). After deformation, the position at the moment T of the particle which occupied position X before deformation is given by variable x (description eulérienne).

The total movement of the solid is defined, with U displacement, by:

$$x = x(X, T) = X + U$$

éq

3.2.1-1

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To define the change of metric in the vicinity of a point, one introduces the tensor gradient of transformation F :

$$\begin{aligned} x^i & \\ F &= \\ &= Id + U \end{aligned}$$

ϵq
 3.2.1-2
 X
 X

The transformations of the element of volume and the density are worth:

$$\begin{aligned} D &= J d_0 \text{ with } J \\ O & \\ &= \det F = \epsilon q \end{aligned}$$

3.2.1-3
where O and ϵq are respectively the density in the configurations initial and current.

Various tensors of deformations can be obtained by eliminating rotation in local transformation. For example, by directly calculating the variations length and angle (variation of the scalar product), one obtains:

$$\begin{aligned} 1 \\ E &= (C - Id) \text{ with } C = FTF^T \epsilon q \\ 3.2.1-4 \end{aligned}$$

2
 1
 With =
 Id - b-1

(
) with $B = FFT$

ϵq
 3.2.1-5
 2

E and A are respectively the tensors of deformation of Green-Lagrange and Euler-Almansi and C and B , tensors of right and left Cauchy-Green respectively.

In Lagrangian description, one will describe the deformation by the tensors C or E because it are

quantities defined on θ , and of description eulérienne by tensors B or A (definite on).

3.2.2 Constraints

The tensor of the constraints used in the theory of Simo and Miehe is the tensor of definite Kirchhoff by:

$J =$

éq 3.2.2-1

where is the tensor eulérien of Cauchy. The tensor thus results from a “scaling” by variation of volume of the tensor of Cauchy.

3.2.3 Objectivity

When a law of behavior in great deformations is written, one must check that this law is objectify, i.e. invariant by any change of space reference frame of the form:

$$x^* = C(T) + Q(T) X \quad \text{éq}$$

3.2.3-1

where Q is an orthogonal tensor which represent the rotation of the reference frame and C a vector which translates translation.

More concretely, if one carries out a tensile test in the direction e_1 , for example, followed of one rotation of 90° around e_3 , which amounts carrying out a tensile test according to e_2 , then the danger with a nonobjective law of behavior is not to find a tensor of the constraints uniaxial in the direction e_2 (what is in particular the case with kinematics PETIT_REAC).

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3.3

Formulation of Simo and Miehe

Thereafter, one will note by **F** the tensor gradient which makes pass from the initial configuration 0 to current configuration (T), by **F_p** the tensor gradient which makes pass from configuration 0 to slackened configuration R, and **F_e** of the configuration R with (T). The index p refers to the part plastic, the index E with the elastic part.

Initial configuration

Current configuration

F

(T)

0

F_p

F_e

T = T_{ref}

R

= 0

Slackened configuration

Appear 3.3-a: Decomposition of the tensor gradient F in an elastic part F_e and plastic F_p

By composition of the movements, one obtains the following multiplicative decomposition:

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_p$$

éq 3.3-1

The elastic strain are measured in the current configuration with the tensor eulérien of Left Cauchy-Green \mathbf{B}_e and plastic deformations in the initial configuration by the tensor \mathbf{G}_p (Lagrangian description). These two tensors are defined by:

 \mathbf{B}_e $\mathbf{F}_e \mathbf{F}_e^T$

=

 \mathbf{G}_p $\mathbf{F}_p^T \mathbf{F}_p$

=

-

(

) 1 from where \mathbf{B}_e $\mathbf{F}_p^T \mathbf{G}_p \mathbf{F}_p$

=

éq**3.3-2**

However, one will employ alternatively another measurement of the elastic strain \mathbf{E} , which coincides with the opposite of the linearized deformations when the elastic strain are small:

1

 $\mathbf{E} = ($ \mathbf{E} **Id - b) éq 3.3-3**

2

In the case of an isotropic material, one can show that the potential free energy depends only on left tensor of Cauchy-Green \mathbf{B}_e (where in our case of the tensor \mathbf{E}) and in plasticity of the variable p dependent on isotropic work hardening. Moreover, one supposes that the voluminal free energy breaks up, all

as in small deformations, in a hyperelastic part which depends only on the deformation rubber band and another related to the mechanism of work hardening:

(,

 \mathbf{E}_p)

el

= (\mathbf{E})

bl

+ (p) éq**3.3-4**

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If instead of using the constraint of Cauchy, one uses the constraint of Kirchhoff, the inequality of Clausius-Duhem is written (one forgets the thermal part):

: $\mathbf{D} - \dot{\theta} \geq 0$

éq 3.3-5

expression in which \mathbf{D} represents the rate of deformation eulérien.

Under the preceding assumptions, dissipation is still written:

E

l

$p T$

+

$\mathbf{b} : \mathbf{D} +$

$(\dot{G}$

\mathbf{F}

$\mathbf{F})$

:

-

p & 0 éq
3.3-6

E

2nd

p

The second principle of thermodynamics then requires the following expression for the relation stress-strain:

E

= -

B

éq 3.3-7

E

One defines finally the thermodynamic forces associated with the elastic strain and the deformation figure cumulated in accordance with the framework of generalized standard materials:

E

S = -

that is to say

= S B

éq

3.3-8

E

= -

With

éq 3.3-9

p

where thermodynamic force A is the opposite of the isotropic variable of work hardening R .

It remains then for dissipation:

$$\begin{aligned}
 & \dot{p} - T \dot{E} \\
 & : (- \\
 & - \\
 & \mathbf{F} \& \\
 & \mathbf{G} \mathbf{F} \mathbf{B} \\
 &) + A \dot{p} = S : (- \\
 & \dot{p} \mathbf{T} \\
 & \mathbf{F} \& \\
 & \mathbf{G} \mathbf{F}) + A \dot{p} = 0
 \end{aligned}$$

éq 3.3-10

2
2

3.3.1 Formulation

original

The principle of maximum dissipation applied starting from the threshold of elasticity F , function of the constraint of

Kirchhoff and of thermodynamic force A makes it possible to deduce the laws of evolution from them from

plastic deformation and of the cumulated plastic deformation, is:

$$\begin{aligned}
 & \dot{p} - T \dot{E} \\
 & - \\
 & - \\
 & \mathbf{F} \\
 & \mathbf{G} \mathbf{F} \& \mathbf{F} \mathbf{B} \\
 & = \&
 \end{aligned}$$

éq 3.3.1-1

2

F

$\&p = \&$

éq 3.3.1-2

With

$\&$

$0 F$

$0 F \& = 0 \text{ éq}$

3.3.1-3

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Note:

One can show easily that the derivative compared to the time of the variation of volume

plastic p

J is written:

p

J & =

F

p

& J tr

éq

3.3.1-4

so that if the surface of load F depends only on the deviatoric part of the tensor of forced of Kirchhoff, then the plastic deformations are done with constant volume is:

J p

p

det F

1 from where J

J E

E

=

=

=

= det F = det F éq

3.3.1-5

3.3.2 Formulation

modified

The approximation introduced here on the original formulation of Simo and Miehe relates to the expression of

law of flow, all the more reduced approximation as the elastic strain are small,

since

E

= S B. Indeed, one henceforth expresses the threshold of elasticity like a function of the forces thermodynamic and either of the constraints F (S, A) 0, and it is compared to these variables that one apply the principle of maximum dissipation, which leads to the following laws of flow:

1

p T

-

F

FG& F = &

$\dot{\epsilon}q$

3.3.2-1

2

S

F

 $\dot{\epsilon}p = \dot{\epsilon}$ $\dot{\epsilon}q$

3.3.2-2

With

 $\dot{\epsilon}$

0 F

 $0 F \dot{\epsilon} = 0 \dot{\epsilon}q$

3.3.2-3

3.3.3 Consequences of the approximation

By replacing the constraint by the thermodynamic force S associated with the elastic strain into the expression of the criterion of plasticity, one introduces in fact a disturbance of the border of field of reversibility of a factor 2nd. Compared to the initial formulation, it results from it obviously an influence on the elastic limit observed but also on the direction of flow: in particular, the derivative compared to the time of the plastic variation of volume is written then:

P

P

E 1

-

 $J \dot{\epsilon} =$

F

 $\dot{\epsilon} J B$

:

 $\dot{\epsilon}q$

3.3.3-1

S

so that if the criterion F depends only on the diverter of the tensor of the constraints S, one do not find p

J = 1: the isochoric character of the plastic deformation is not perfectly any more preserved.

Insofar as the elastic strain remain small, results obtained with this model modified do not deviate significantly from those obtained with the old formulation (cf [bib3]), while numerical integration will be simplified by it. Indeed, it will be seen thereafter that this model follows the same diagram of integration as that of the models written in small deformations.

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Note:

This new formulation of the great deformations makes it possible to replace the theory of Simo and Miehe within the framework of generalized standard materials. From a numerical point of view, this results in to express the resolution of the law of behavior like a problem of minimization compared to the internal increments of variables.

Indeed, one recalls that within the framework of generalized standard materials, the data of two potentials free energy ψ (, has) and potential of dissipation D ($\dot{\alpha}$), function of the tensor of deformation and a certain number of internal variables has, allows to define completely the law of behavior (one places materials in the case of independent of time).

=

, $A = -$

$D(\dot{\alpha})$

éq
3.3.3-2

has

where $D(a\&)$ is under differential of the potential of dissipation D .
The laws of generalized behavior of the standard type which do not depend on time are characterized by a potential of dissipation positively homogeneous of degree 1, which translated by the following property:

$\&$
has
 $>$
 $D ($
 0
 $a\&) = D (a\&$
 $)$

D
 $(a\&) = D (a\&)$ **éq 3.3.3-3**

Now if one writes the problem [éq 3.3.3-2] in form discretized in time and if one uses the property of under differentials [éq 3.3.3-3], one obtains the following discretized problem:

=

, $A = -$
 $D (has$
 $)$ **éq**
3.3.3-4

has

One can show that the equation [éq 3.3.3-4] is equivalent (cf [bib4]) to solve the problem of minimization compared to the internal increments of variables *has* according to:

-
D (*has*
) *has*
 = *ArgMin* [(*A* + *)
 + *D* *has* (*)
has]

éq
3.3.3-5

has

*

has

The application of the equation [éq 3.3.3-5] to the model of Rousselier in great deformations modified is written:

(,
E *p*
D (
and

)
p
D, *p* &
)

=>
 (*Tr*
E +,
E *p* + *p*

D (
and

)
 ,
E *p*
) **éq**

3.3.3-6

tion
discretized
continuous

energy

E
discretized

energy

S = -

E
With = -
=
D (,
E p
)
has

- R = -

éq.
3.3.3-7

p

Min [(Tr
E +,
E p + p
) + D (,
E p
)]
E, p

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One will find in the paragraph [§4], the relation which binds the rate of plastic deformation
 \dot{p}
D
once discretized and the increment of elastic strain \mathbf{E}
, as well as the definition of Tr
E.

One sees well here whom if one takes the initial formulation of Simo and Miehe, one cannot write any more
the problem of minimization [éq 3.3.3-7] with the constraint of Kirchhoff because of the term
in \mathbf{E}
B *in the expression:*

\mathbf{E}
= -
B
éq
3.3.3-8
E

4 *Model of Rousselier*

We now describe the application of the great deformations to the model of Rousselier presented in introduction.

4.1

Equations of the model

To describe a thermoelastoplastic model with isotropic work hardening (the equivalent into small deformations with the model with isotropic work hardening and criterion of Von Mises), Simo and Miehe propose one elastic potential polyconvexe. By reason of simplicity, one chooses here the potential of Coming Saint who is written:

$$\begin{aligned} & (\\ & 1 \\ & , \\ & \mathbf{E} p) = [K (tr \mathbf{E})^2 \\ & \sim \sim \\ & + 2\mu \mathbf{E} : \mathbf{E} + 6K T \\ & tr \mathbf{E}] p \\ & + R (U) \text{ of the } \acute{e}q \end{aligned}$$

4.1-1

2

0

In accordance with the equations [éq 3.3-8] and [éq 3.3-9], the laws of state which derive from the elastic potential above are written then:

$$\begin{aligned} \mathbf{S} = [\\ - K tr \mathbf{E} \mathbf{Id} + \mu \mathbf{e} \sim \\ 2 \\ + 3K T \\ \mathbf{Id}] \acute{e}q \end{aligned}$$

4.1-2

With = - R (p)

éq 4.1-3

The threshold of elasticity is given by:

$$\begin{aligned} HS \\ F (\mathbf{S}, R) = S \\ + Df exp \\ - R \end{aligned}$$

eq

1

y
éq
4.1-4

1
According to the equations [éq 3.3.2-1] and [éq 3.3.2-2], the laws of flow are defined by:

1
p T
s~
3
Df
S
-
G
F & F = &
+
exp H Id

éq
4.1-5
2

2s
3
eq
1

&p = &

éq 4.1-6

&
0 F
0 F & = 0 éq
4.1-7
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4.2

Treatment of the singular points

In fact, the equation of flow [éq 4.1-5] translated the membership of the direction of flow to the cone normal on the surface of the field of elasticity. It is valid only at the regular points, characterized by: $\text{seq } 0$

éq 4.2-1

It thus remains to characterize the normal cone at the singular points, i.e. checking:

~

S

H

$$S = 0$$

and

$D F \exp$

1

$$- R =$$

y

éq

4.2-2

1

The normal cone with convex of elasticity in such a point is the whole of the directions of flow which carry out the problem of maximization according to:

*

$$(\mathbf{S}, \mathbf{R}) = \sup [$$

p

$$\mathbf{S}: \mathbf{D} - R p \& - (p$$

$$\mathbf{D}, p \&)] \text{ éq}$$

4.2-3

p

$$\mathbf{D}, p \&$$

where *

is the indicating function of convex F and $(p$

$\mathbf{D}, p \&)$ potential of dissipation obtained by

transform of Legendre-Fenchel of the indicating function of F :

p

$$(\mathbf{D}, p \&) = \text{Sup} [$$

p

$$\mathbf{S}: \mathbf{D} - R p \&] \text{ éq}$$

4.2-4

S, R

$$F(S, R) 0$$

After some calculations, one obtains:

D

p

p

$tr p$

2

$$(D, p\&) = p\& + tr D \ln$$

$$-1 + I + (tr p$$

$$D) + I + ($$

$$p$$

$$p\& - D)$$

$$y$$

$$1$$

éq
4.2-5
IR
IR
eq
p
Df

&
 3

with
 0
if
X 0
I + (X) =

éq
4.2-6
IR
 + *if not*

For ~
*S = 0, **
is worth:

*

p
tr D

p
p

(, Sr) = Sup
HS tr D - tr D ln

1
-

1 - R p& -
&
éq
4.2-7

p

p
y
D, p&
D F p&

p
tr D 0 1

4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
3
p

p& 2D p
G (tr D)
0

-

eq
3
By noticing that for tr
p
D 0, the function G (tr p
D) is concave, the suprémum compared to
trace rate of plastic deformation
p
D is obtained for:

p
p
S
G (tr D) = 0 of
where
tr D = D F p
H
&

exp

éq
4.2-8
1

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Note:

One finds well then for the indicating function of the threshold of elasticity F .

$$0$$

$$\text{if}$$

$$F 0$$

$$* (S, R) = \text{Sup} [F \&p] =$$

éq
4.2-9

p
+
&
if not
2 p
&p Deq 0
3

In a singular point, the normal cone, together of the acceptable directions of flow, thus characterize by:

p
S
tr D = D F p
H
&

exp

éq
4.2-10
1

p& 2 peq
D 0

éq
4.2-11
3

4.3

Expression of porosity

One saw in introduction that the microscopic inspiration of the model of Rousselier is based on one microstructure made up of a cavity and a plastic rigid matrix, therefore isochoric. In this case, porosity is directly connected to the macroscopic deformation by:

$$1 - F$$

$$J = \det F$$

$$0$$

$$=$$

$$f\& = (1 - F) \operatorname{tr} D$$

$$\acute{e}q$$

$$4.3-1$$

$$1 - F$$

However, on a macroscopic scale, one supposes that the material can also become deformed reversible elastic manner. The expression above is not thus exact any more, even if it represents still a good approximation as long as the elastic strain are small. Unfortunately, it prohibits even reasonable elastic compressions, because very quickly, porosity is cancelled and impose an isochoric behavior again ($J = \text{constant}$ bus $F = 0$).

Rousselier proposes as for him to express porosity while basing himself on the rate of plastic deformation

$$p$$

D. The relation is written in incremental form:

$$f\& = (1 -$$

$$p$$

$$F \operatorname{tr} D \acute{e}q$$

$$4.3-2$$

That means that the variable porosity employed to parameterize the criterion of plasticity F does not depend

that plastic deformation. In fact, the rate of plastic deformation is an evaluated quantity in the slackened configuration. Its transport in the current configuration (as D) is expressed still:

$$T$$

$$E$$

$$p E$$

$$1$$

$$p T$$

$$F D F$$

$$= -$$

$$G$$

$$F \& F \acute{e}q$$

$$4.3-3$$

$$2$$

Finally, one adopts like law of evolution of porosity:

1
 $f = (1 - F)$
 $p = T$
tr-
G
F & F
éq
4.3-4
2

Again, this law of evolution of porosity remains close to that employed by Rousselier when the elastic strain are small.

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4.4 Relation

`ROUSSELIER`

This relation of behavior is available via the argument `ROUSSELIER` of key word COMP_INCR under operator STAT_NON_LINE, with the argument `SIMO_MIEHE` of the key word factor DEFORMATION.

The whole of the parameters of the model is provided under the key words factors `ROUSSELIER` or `ROUSSELIER_FO` and `TRACTION` (to define the traction diagram) order DEFI_MATERIAU ([U4.43.01]).

Note:

The user must make sure well that the “experimental” traction diagram used, is directly, that is to say to deduce the slope from it from work hardening is well given in the plan rational constraint = F/S - deformation logarithmic curve $\ln(1 + L/l_0)$ where l_0 is initial length of the useful part of the test-tube, L variation length afterwards deformation, F the force applied and S current surface.

4.5

Internal constraints and variables

The constraints are the constraints of Cauchy, thus calculated on the current configuration (six components in 3D, four in 2D).

The internal variables produced in Code_Aster are:

- *V1, cumulated plastic deformation p ,*
- *V2, porosity F ,*
- *V3 in V8, the tensor of elastic strain E ,*
- *V9, the indicator of plasticity (0 if the last calculated increment is elastic, 1 if solution figure regular, 2 if singular plastic solution).*

Note:

If the user wants to possibly recover deformations in postprocessing of sound calculation, it is necessary to trace the deformations of Green-Lagrange E , which represents a measurement of deformations in great deformations. The traditional linearized deformations measure deformations under the assumption of the small deformations and do not have a direction into large deformations.

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5 Formulation numerical

For the variational formulation, it is about same as that given in the note [R5.03.21] and which refers to the law of behavior with isotropic work hardening and criterion of Von Mises into large deformations. We recall only that it is about a eulérienne formulation, with reactualization of the geometry to each increment and each iteration, and which one takes account of rigidity of behavior and geometrical rigidity.

We now present the numerical integration of the law of behavior and give the form of the tangent matrix (options FULL_MECA and RIGI_MECA_TANG).

5.1

Expression of the discretized model

Knowing the constraint -

, cumulated plastic deformation p , elastic strain -

E ,

displacements U and U , one seeks to determine (\cdot , p , E).

Displacements being known, gradients of the transformation of θ with \cdot , noted F , and of \cdot with, noted F , are known.

To integrate this model of behavior, one does not choose a purely implicit algorithm because, on the one hand, that led to the resolution of a rather complex nonlinear system, and on the other hand,

allows more to express the problem like the minimization of a functional calculus. This is why one prefer to treat in an explicit way the variation according to porosity in the threshold of elasticity. For other terms, one employs a diagram of implicit Euler.

Once discretized, the following system then is obtained:

-

$$F = FF$$

éq 5.1-1
 $J = \det F$

éq 5.1-2
 $J =$

éq 5.1-3

E

$= S B$

éq 5.1-4

$Be = Id - E$

2

éq 5.1-5

· Equations of state:

S

[-

$= \mu e \sim$

2

**+ $K \operatorname{tr} E \operatorname{Id} + 3K T$
 Id]**

éq. 5.1-6
With -
= $R(p)$

éq 5.1-7
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Thereafter, one expresses the laws of flow and the criterion of plasticity directly according to tensor of the elastic strain E.

· *Lois of flow*

***p
1
p T
1
p T
- p T
-
T
D - FG& F = -
FG F - FF G F F***

***2
2 T
4
1 4
23
4
1
4
2 3***

***E
E***

***-
B
B***

= - 1

***[Id-2nd-F {Id- -
2nd} T
F]***

éq

5.1-8

2 T

=

E - 1

(

[Id-F {Id- -

2nd} T

F])/T

Tr

= (E - E)/T

2

1

4

4

4

4

2

4

4

4

4

3

Tr

E

By taking the parts traces and deviatoric of the equation [éq 4.1-5], one obtains:

Tr

-

3K T

K tr E

tr E - tr E

p

= Df exp (-

) exp (-

)

éq 5.1-9

1

1

~

Tr - 3

~

E

E

p

regular

solution

if

~

2

E

E =

eq

éq 5.1-10

2 ~ ~Tr

0

and

p (E - E)

singular

solution

if

eq

3

· Conditions of coherence

3

-

KT

K tr E

2μ

+ D F exp (-

) exp (-

) - R

regular

solution

if

eq

E

1

y

1

1

F =

3

-

KT

K tr

E

éq

5.1-11

D F

exp (-

) exp (-

) - R

singular

solution

if

l

y

l

l

with F

0 p

F

0

p = 0

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5.2

Resolution of the nonlinear system

The integration of the law of behavior is thus summarized to solve the system [éq 5.1-9], [éq 5.1-10] and [éq 5.1-11]. We will see that this resolution is brought back to that of only one scalar equation, of which

unknown factor X is the increment of the trace of the elastic strain:

Tr

$$X = tr \mathbf{E} - tr \mathbf{E}$$

éq 5.2-1

Thanks to this choice, that the solution is elastic or plastic, attack in a singular point or not, the equation [éq 5.1-9] bearing on the trace of the elastic increment is always valid and allows to express the increment of cumulated plastic deformation:

$K tr Tr$

Tr

\mathbf{E}

$3K$

-

T

*K (tr E - tr Tr
E)*

tr E - tr E = p

D F exp

-

exp (-

) exp

-

1

1

1

1

4

4

4

4

4

4

2

4

4

4

4

4

4

3

G

éq

5.2-2

K X

(X) 1

p

$= X \exp$

G

1

This equation shows us that one can seek $X = 0$ to guarantee a plastic deformation cumulated positive and that the elastic solution is obtained for $X = 0$. It is also noticed that the increment of cumulated plastic deformation is a continuous and strictly increasing function X . With the help of these remarks, if one notes by S the term [éq 5.2-3] in the criterion of plasticity, it acts then, there too, of a continuous and strictly increasing function of X :

Kx

$F = 2 \text{ eq}$

$\mu e - ($

$S X) \text{ with}$

$S ($

$X) = - G \exp$

1

-

+

$R (p (X)) + y \text{ éq}$

5.2-3

1

This stage, the step of resolution breaks up into two times.

5.2.1 Examination of the singular points

Such a singular point is characterized by [éq 5.1-10] (low) and [éq 5.1-11] (low), therefore in particular by

$(Sx) = 0$. Because of the properties of S , this equation admits with more the one positive solution, say S

who exists if and only if (

$S = 0$). The knowledge of S

X makes it possible to deduce the tensor from it from

elastic strain E , cumulated plastic deformation p as well as the thermodynamic forces

S and R .

Finally, this singular point will be solution if the inequality in [éq 5.1-11] (low) is checked, i.e.

if:

S

2 ~s ~Tr

p
(E - E) eq éq

5.2.1-1

3
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5.2.2 Solution
regular

The equation of flow [éq 5.1-10] (high) which determines the deviatoric part of the tensor of elastic strain makes it possible to deduce a scalar equation from it function from the increment from cumulated plastic deformation:

Tr
3
~
eq
E = eq
E - p
~ ~
3
2
Tr
E

E - E

-

=

p

E

éq

5.2.2-1

2

eq

E

e~ = eq ~Tr

E

Tr

eq

E

***One notes that because of the positivity of eq
E, the value sells by auction p
is limited:***

2 Tr

p

eq

E

éq 5.2.2-2

3

The condition of coherence determines X now:

F = 2µeTr

eq - S (X) - 3

µ p 0

*éq***5.2.2-3**

Being given this expression, the increase of the licit value of p is reduced to the only condition

(Sx) 0 or, in an equivalent way, with

*S**X X.*

The elastic solution is obtained for $X = 0$. It is the solution of the problem if and only if:

F () *$0 = 2\mu eTr$* *$eq - S (0 <$* *) 0 éq***5.2.2-4**

In the contrary case, one must then solve:

 3μ *Kx* *$X > xs$* *Tr* *xs* *$F (X) = 2\mu eeq - S (X) -$* *$X exp ($* *) =**if**exist**0 with**éq***5.2.2-5***G**l* *$X > 0$* *if not*

This function is continuous and strictly decreasing and tends towards - with X . It thus admits with more one solution. The demonstration of the existence of this solution is immediate. Indeed, it is enough to

to prove that F is positive on the lower limit of the interval of research.

When S

X does not exist, (

F)

0 > 0 since the solution is not elastic.

When S

X exists, the function is worth:

S

Tr

S

S

2 Tr

F (X) = 2μ eq

E - 3 p

μ

> 0 p

<

eq

E éq

5.2.2-6

3

This condition is checked since the singular solution was rejected.

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5.3

Course of calculation

The step to solve the whole of the equations of the model is as follows:

1) One seeks if the solution is elastic

· calculation of $F()$

0

· if $F()$

$0 < 0$, the solution of the problem are the elastic solution Sol

X

$= 0$

· if not one passes into 2)

2) If

$S()$

$0 > 0$, the solution are plastic and regular

· one passes into 4)

3) If

$S()$

$0 < 0$, one seek if the solution is singular

· one solves $S(S$

$X) = 0$

2

· if S

X checks the inequality

S

$\sim s \sim Tr$

p

$(E - E)$

Ground

eq, then the solution is singular

S

X

$= X$

3

· if not, S

X is a lower limit to solve $F(X) = 0$, one passes into 4)

4) The solution is plastic and regular

· one solves $F(X) = 0$

5.4 Resolution

To solve the two equations $S(X) = 0$ and $F(X) = 0$, one employs a method of Newton with controlled terminals coupled to dichotomy when Newton gives a solution apart from the interval of the two terminals. One now presents the determination of the terminals for each case precedents (items 2) 3) and 4) of the preceding paragraph).

5.4.1 High delimiters and lower if the function S is strictly positive at the origin

One solves:

μ
 Kx
 $2 Tr$
 eq
 $\mu e - S (X = 3$
)
 $X exp ($
)
 $F (X) = 0$

4
42
1
4
43
G

1

F
1 4
4 2 4
4 3
éq
5.4.1-1

F ()
 $0 > 0$
1

$3\mu p$

F ()
0
1
 > 0

where the function $p(X)$ is continuous, strictly increasing and null at the origin and the function $F(X)$ is continuous, strictly decreasing and strictly positive at the origin (see [Figure 5.4.1-a]).

One poses:

T_r

K_x

$$F = 2\mu e^{\epsilon} - R(X) - y + G \exp(-)$$

) then F

$$(X) < F(X)$$

X_0

1

1

2

1

éq 5.4.1-2

1 4

4

4 2

4

4

4 3

1

f2

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where the function $F(\cdot)$

and X are continuous, strictly decreasing. In this case, the resolution of the equations:

$$\inf$$

$$\inf$$

$$\inf$$

$$Kx$$

$$F(p$$

$$\inf$$

$$2$$

$$) = 3\mu p$$

and

$$\inf$$

$$X$$

$$\exp(\cdot)$$

$$) = G p$$

éq

5.4.1-3

1

to deduce p successively from it

then X gives a lower limit \inf

$$X$$

who corresponds to

solution of the model with isotropic work hardening and criterion of Von Mises. If $F(0)$

$$0$$

$$2$$

$<$, the lower limit

is taken equalizes to zero: \inf

$$X$$

$$= 0.$$

The upper limit \sup

$$X$$

is such as:

$$Kx\sup$$

$$\sup$$

$$G$$

$$X$$

$$\exp(\cdot)$$

$$) =$$

F (xInf)

éq

5.4.1-4

3 1

1

μ

Kx

The equation of type X exp (

) = constant is solved by a method of Newton.

1

3μ p (X)

f1

3μ p (sup

X

) = F

inf

1 (X

)

F 2

3μ p (inf

X

) = F

inf

2 (X

)

X

Ground

inf

Sup

X

X

X

Appear 5.4.1-a: chart of the hight delimiters and lower

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5.4.2 Hight delimiters and lower if the function S is negative or null at the origin

The system to be solved is as follows:

X

Kx

-

Kx

S (X) = 0

R p + exp (

) + y = G exp (

1

-

)

G

1

1

éq

5.4.2-1

S (0) < 0

-

R (p) + y < G

1

The part of left is a continuous, strictly increasing X and strictly positive function with the origin, the part of right-hand side is a continuous, strictly decreasing function X and strictly positive at the origin. Using the properties of these two functions, a chart (cf [Figure 5.4.2-a]) of these functions shows that the upper limit Sup

X
is such as:

$$\text{Sup} \left(\frac{1}{1 - Kx} \right)$$

$$) = R(-p)$$

$$\text{Sup} \frac{1}{1 - Kx}$$

+

$$\frac{1}{y}$$

$$X$$

$$=$$

$$\log$$

éq 5.4.2-2

$$K$$

$$-$$

$$1$$

$$R(p)$$

$$+ y$$

The lower limit Inf

$$X$$

is such as:

$$\text{Inf}$$

$$Kx$$

Sup

Sup

-

X

Kx

G exp (

1

-

) = R p +

exp (

) +

y

1

G

1

+

éq

5.4.2-3

Inf

G

X

=

1 log

1

K

Sup

Sup

-

X

Kx

*R p +
exp (
) +*

y

*G
I*

*R (X)
I G
+ y
Kx
I G exp (-
)*

*R (p -)
+
I*

*y
X*

*Inf
Ground
Sup*

*X
X
X*

*Appear 5.4.2-a: chart of the hight delimiters and lower
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5.4.3 Hight delimiters and lower if the function S is strictly negative at the origin and xs not solution

The following system is solved:

$$\begin{aligned}
 &Tr \\
 &3\mu \\
 &Kx \\
 &2\mu ee q - S(X) = \\
 &X \exp (\\
 &) \\
 &4 \\
 &42 \\
 &1 \\
 &4 \\
 &43 \\
 &G \\
 &1
 \end{aligned}$$

$$\begin{aligned}
 &14 \\
 &424 \\
 &43 \\
 &F(X) = 0
 \end{aligned}$$

$$\begin{aligned}
 &f1 \\
 &3\mu \\
 &p \\
 &S(0) < 0
 \end{aligned}$$

$$\begin{aligned}
 &F(0) \\
 &1 \\
 &> 0
 \end{aligned}$$

éq

5.4.3-1

S

$$S(X) = 0$$

S

Tr

μS

Kx

$$2\mu eq = 3$$

$X \exp ($

)

G

1

The solution Ground

X

is such as S (Ground

X

) > 0 .

For the lower limit, one takes Inf

S

X

= X . Being given properties of the functions If

(strictly decreasing) and $3\mu p(X)$ (strictly increasing), the upper limit Sup

X

is such

that (cf [Figure 5.4.3-a]):

Sup

Sup

Kx

$2G Tr$

X

$\exp ($

) =

eq

E

$\acute{e}q$

5.4.3-2

3

1

This equation is solved by a method of Newton.

$S(X) < 0$ $S(X) > 0$

$2\mu eTr$

$eq - S(0)$

$S(X) = 0$

3μ

Kx

Tr

$X \exp ($

)

$2\mu eq$

G

1

X

Inf

S

Ground Sup

X

= X X X

Appear 5.4.3-a: chart of the hight delimiters and lower

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5.5

Integration of porosity

This stage, it any more but does not remain to integrate the law of evolution of porosity. According to the equations [éq 4.3-4] and [éq 5.1-8], it is still expressed using unknown factor X:

$$\frac{f}{X} =$$

éq 5.5-1

$$\frac{1 - F}{T}$$

maybe while integrating:

$$\frac{F df}{X T}$$

$$=$$

$$\frac{F = 1 - 1}{(- F) \exp(-)}$$

éq

5.5-2

$$\frac{1 - F}{T}$$

$$\frac{F}{0}$$

where one carried out an exact temporal integration by supposing X constant lasting the step of time. It

choice makes it possible to ensure that F is increasing and remains lower than 1, whatever the step of time.

5.6 Form of the tangent matrix of the behavior

One gives the form of the tangent matrix here (option `FULL_MECA` during iterations of Newton, option `RIGI_MECA_TANG` for the first iteration).

For the option `FULL_MECA`, this one is obtained by linearizing the system of equations which governs the law of

behavior. We give hereafter the broad outline of this linearization.

For option `RIGI_MECA_TANG`, they are the same expressions as those given for `FULL_MECA` but with $p = 0$. In particular, there will be $\mathbf{F} = \mathbf{Id}$.

The law of behavior can be put in the following general form:

$\mathbf{e} = (\mathbf{F}$
)

éq 5.6-1

$\mathbf{E} = (\mathbf{E})$

éq 5.6-2

$\mathbf{E} = \mathbf{E} (\text{Tr}$

$\mathbf{E})$ éq 5.6-3

$\mathbf{eTr} = \mathbf{eTr} (\mathbf{F}$

)

éq 5.6-4

The linearization of this system gives:

E

E

Tr

=

:

:

:

+

: F

= H: F

éq

5.6-5

E

E

Tr

F

F

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where H is the tangent matrix. Thereafter, the five terms of the equation are separately determined the preceding one.

In the linearization of the system, one will often use the tensor C defined below and both following equations:

1
has
ij = (ik jl + jkil) has

kl
éq
5.6-6

2
app = K has
L
kl

éq 5.5-7
1
ijkl
C
= (ik jl + jkil) éq

5.6-8
2

· *Calcul of
and of*

F

*Linearization of the relation which binds the constraint of Cauchy and the constraint of Kirchhoff
give:*

1

J

J

=

= -

: F

éq

5.6-9

J

J

F

By using the relation [éq 5.6-6], one obtains for

:

= C éq 5.6-10

and for

:

F

= - J

éq

5.6-11

F

J

F

with

J

= 22

F 33

F - 23

F 32

F

11

F

J

= 11

F 33

F - 13

F 31

F

22

F

J

= 11

F 22

F - 12

F 21

F

33

F

éq

5.6-12

J

J

= 31
F 23
F - 33
F 21
F
= 13
F 32
F - 33
F 12
F
12
F
21
F
J

J

= 21
F 32
F - 22
F 31
F
= 12
F 23
F - 22
F 13
F
13
F
31
F
J

J

= 31
F 12
F - 11
F 32
F
= 13
F 21

F - 11

F 23

F

23

F

32

F

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· Calcul of

E

The relation which binds the constraint of Kirchhoff and the tensor of elastic strain E is given by:

= $S B$

$E = -2\mu E - Tr E Id$

+ $4\mu E E$

+ $2 (tr E E$

) éq

5.6-13

-

$3K T$
 $Id + 6K T$
 E

One obtains after linearization:

$$= (2 \operatorname{tr} E - \mu + 3KT) E + (E - Id) \operatorname{Tr} E + 4\mu (E E + E E)$$

éq
5.6-14
from where

$$ij = (2 \operatorname{tr} E - \mu + 3KT) Cijkl + (e2ij - ij) kl + 2\mu (ikelj + ilekj + eilkj + eikjl)$$

kl
 E
éq 5.6-15

E
 Tr
 · *Calcul of*

F

The relation between the tensor of elastic strain $\operatorname{Tr} E$ and the increment of the gradient of transformation F is written:

$$\operatorname{Tr} E = (I - Id - F B F)$$

éq
5.6-16

Its linearization gives:

$$\operatorname{Tr} E = - I (ij E$$

E

E

ik F

jp bpl + IP

F bpl jk) éq

5.6-17

kl

F

2

E

· Calcul of

Tr

E

Elastic case

E

*In the elastic case, the calculation of
is immediate since*

Tr

E

= E

from where

Tr

E

E

= C éq 5.6-18

E

Tr

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Plastic case regular Solution

E

To determine
, one operates in two stages. By the law of flow discretized, one calculates

Tr
E

in first E
according to Tr
E

and
p

. Then the condition of coherence makes it possible to deduce some
p

according to Tr
E

. These two stages are thereafter detailed.
The deviatoric part of the law of flow discretized is written:

~
~ ~Tr

3
E
E - E = -

p

éq

5.6-19

2

eq

E

One obtains after linearization:

3 p

~

~Tr

3e~

9

e~

1

(+

)

E = E -

p + p

(e~ e~

:

) éq

5.6-20

2nd

2nd

4

3

eq

eq

eq

E

4

1

4

2 3

1/

To determine E

~ e~

:

, one contracts the equation [éq 5.6-20] with E

~ what gives:

~

~ ~

~Tr

E: E = E: E - E

p

eq

éq

5.6-21

from where

~

9 p

~

~ ~

~Tr

E =

E E + C

E

: E - 3

p

éq

5.6-22

E

4 3

2nd

eq

eq

1

4

4

4 2

4

4

4 3

3

2

1A

With

2

1

For the part law of flow traces discretized, one a:

Tr
3KT
K

$$Tr E - Tr E = Df p$$

exp

exp -
Tr E éq
5.6-23

1

1

what gives:

1
Tr E =
Tr
Tr
E
DfK p
3KT
K

1+
exp
exp -
Tr

E
1

1

1

1

4

4

4

4

4

4

4

2

4

4

4

4

4

4

4

3

1

K

3KT

éq

5.6-24

Df exp -

Tr E exp

1

l

+

p

DfK p

3KT

K

l+

exp

exp -

Tr

E

l

l

l

l

4

4

4

4

4

4

4

2

4

4

4

4

4

4

4
3
2

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In the plastic case, the condition of coherence is worth:

$3KT$

K

2 eq

$\mu e + Df exp -$

$exp -$

$Tr E - R -$

0

1

y =

éq

5.6-25

1

1

from where

3μ (~ ~

E: E)

$3KT$

K

- $DfK \exp$ -

\exp -

$Tr E Tr E - HP = 0$

éq

5.6-26

eeq

1

1

By injecting the relation [*éq 5.6-21*] in the equation above, one obtains then:

3μ

1

~ ~ Tr

p

=

E: E

eq

E

$3K T$

K

$3\mu + H + DfK \exp$ -

\exp (-

Tr)

E

2

1

1

1

4

4

4

4

4

4

4

4

4

2

4

4

4

4

4

4

4

4

4

3

3

3K T

K

éq 5.6-27

DfK exp -

exp (-

Tr)

E

1

1

1

Tr

-

Tr **E**

3K T

K

$3\mu + H + DfK \exp -$

$\exp (-$

Tr)

E

2

1

1

1

4

4

4

4

4

4

4
4
4
2
4
4
4
4
4
4
4
4
4
3
4

*While replacing
p*

by its value obtained above in the equations [éq 5.622] and [éq 5.6-24], one obtains:

$$\begin{aligned}
& 1 \\
& \mu \\
& 3 \\
& 3 \sim \\
& Tr \\
& 1 \\
& \sim \\
& 1 \\
& Tr \\
& \mathbf{E} \\
& = \mathbf{A1} + \mathbf{A2} + \mathbf{Id} \\
& 3 \\
& \mathbf{E}: \mathbf{E} + \\
& \mathbf{Id} \\
& 1 \\
& + 4\mathbf{A2} + \mathbf{Id}
\end{aligned}$$

2

Tr E

3

eq

E

3

3

1

4

4

4

4

4

4

2

4

4

4

4

4

3

1

4

4

4

4

4

4

4

2

4

4

4

4

4

4
3
D *etr*
ddvetr
tr
éq 5.6-28
from where
E

1

= **ddvetr** + **dtretr** - **ddvetr**: **Id Id éq**
5.6-29
E
Tr

3

Plastic case singular Solution

*The step is identical to that used previously.
One obtains for the law of flow discretized:*

~
~
E = 0

E = 0
éq
5.6-30

for the deviatoric part and the part traces, the relation is identical to that found for regular solution.

Tr
Tr = **1Tr** +

p
2

éq
5.6-31
where 1

and 2 have the same definitions as in the preceding paragraph.

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The condition of coherence then makes it possible to find

p

according to Tr

E

.

3KT

K

Df exp

exp -

Tr E - R -

0

1

y =

éq

5.6-32

l

l

maybe after linearization:

3K T

K

DfK exp

exp (-

Tr)

E

l

l

l

Tr

p

= -

Tr E

éq

5.6-33

3K T

K

H + DfK exp

exp (-

Tr)

E

2

1

1

1

4

4

4

4

4

4

4

4

2

4

4

4

4

4

4

4

4

3

4

that is to say finally:

1

$$\mathbf{E} = [1 +$$

$$4 2]$$

Tr

Id Tr E éq

5.6-34

31 4

4 2 4

4 3

dtretr

from where

E

= dtretrId

éq

5.6-35

E

Tr

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Organization (S): EDF-R & D /MMC

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Model of Rousselier for the ductile rupture

Summary

The model of Rousselier describes the damage due to the plastic growth of cavities in a metal. It allows to model cracking and the ductile rupture. The relation of behavior is elastoplastic or viscoplastic with isotropic work hardening. It allows the changes of plastic volume and is written in small deformations. The writing in great deformations with a formulation of Simo and Miehe modified, in

the elastoplastic case only, is described in [R5.03.06].

This model is available in order STAT_NON_LINE via the key word RELATION =

“ROUSS_PR” or “ROUSS_VISC” under the key word factor COMP_INCR and with the key word DEFORMATION =

“PETIT_REAC”.

This model is established for modelings three-dimensional (3D), axisymmetric (AXIS), in constraints plane and in plane deformations (C_PLAN, D_PLAN).

One presents the writing and the digital processing of this model.

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1 Introduction

The mechanisms at the origin of the ductile rupture of metals are associated the development of cavities within material. Three phases are generally distinguished:

- germination: it is about starting or nucleation of the cavities, in sites which correspond preferentially with the particles of second phase present in material,***
- growth: it is the phase which corresponds to the development itself of the cavities, controlled primarily by the plastic flow of the metal matrix which surrounds these cavities,***
- coalescence: it is the phase which corresponds to the localization of the deformation between the cavities***
to create macroscopic cracks.

The model of Rousselier [bib1], [bib2], [bib3] presented here is based on assumptions microstructural which introduces a microstructure made up of cavities and of a matrix of which them elastic strain negligible are compared with the plastic deformations. In this case, and in the absence of nucleation of new cavities, porosity F , definite like the relationship between volume cavity C

V and total volume V of representative elementary volume, is directly connected to macroscopic plastic deformation by:

C

$1 - F$

0

0
V
=

with $F =$

$$f_0 = (1 - F)$$

p
 tr &
 $\dot{\epsilon}q$
1-1

1 - F
V

where f_0 indicates initial porosity, and are respectively the density in

O
configurations initial and current (one takes in the continuation = 1) and p

O
& the rate of deformation
plastic of total volume V .

The construction of the model rests on a thermodynamic and phenomenologic analysis which brings to write the plastic potential F in the following form:

$$F(p, \dot{\epsilon}) = + D F \exp m - R(p) \dot{\epsilon}q$$

1-2
eq
1
1

1
where σ is the constraint of Kirchhoff, τ is the constraint of Cauchy, R isotropic work hardening function of the cumulated plastic deformation p , and D of the parameters of material. The presence

1
1
in the plastic potential of the hydrostatic constraint authorizes the changes of volume
 m
plastic.

In the event of nucleation of new cavities, one considers that the voluminal fraction created is proportional to the cumulated plastic deformation. It is thus enough to replace F by $F + A p$ in

N

equations of the model. A is a parameter of material. The equation [éq 1-1] is not modified.

N

In the viscoplastic case, one writes the viscoplastic potential v_p

F like a function of the potential

plastic F:

v_p

$F = ($

$F p, F)$

éq 1-3

One will consider only the particular case such as:

m

F

$p \& =$

$= \&$

éq 1-4

0 HS

F

0

who is reduced to a function power (law of the Norton type) when two parameters of material & and are very large.

0

0

Thereafter, one presents the relations of behavior of the model of Rousselier and his integration numerical.

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2 Notations

One will note by:

Id

tensor second-order identity

II

tensor identity of the fourth order

tr A

trace second-order tensor A

~

~

I

With

With = A - (tr A)

deviatoric part of tensor A defined by

Id

3

tr A

With

With =

hydrostatic part of tensor A defined by

m

m

3

3

With

~ ~

eq

equivalent value of von Mises defined by Aeq =

:

WITH A

2

T

:

doubly contracted product: A: B = A B = tr (AB)

ij ij

I, J

tensorial product: (A B)

=

ijkl

ij

With kl

B

, μ, E, K

moduli of the isotropic elasticity

2

p&

~

~

speed of equivalent plastic deformation

p

p
p =
&
&
: &
3

In addition, within the framework of a discretization in time, all the quantities Q evaluated at the moment precedent are subscripted by -, quantities evaluated at the moment $T = T - + T$ are not subscripted and the increments are indicated par. One has as follows:

$$Q = Q + Q$$

The numerical resolution is carried out by one - method, with 0 1. For all the quantities, one defines:

$$Q = Q + Q$$

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3 **Model of Rousselier**

We now describe the derivation of the equations of the model of Rousselier presented in introduction.

3.1 **Derivation of the equations of the model**

It is supposed that the specific free energy breaks up into three parts: a hyperelastic part who depends only on the elastic strain, a part related to the mechanism of work hardening and one part related to the damage:

$$(E, p, F) = E(E) + p(p) + (F)$$

éq 3.1-1

The inequality of Clausius-Duhem is written (one does not consider the thermal part):
 $\dot{\psi} + \theta \dot{\eta} \leq 0$

éq 3.1-2 *expression in which*

$$E = E_0 + \frac{1}{2} E_1 \epsilon^2 + \dots$$

$\dot{\epsilon}$ = $\dot{\epsilon}$ + $\dot{\epsilon}$ the rate of deformation represents.

Dissipation is still written:

E

p

-
: & +: & -

p & -

f & 0 *éq*

3.1-3

E

p

F

The second principle of thermodynamics then requires the following expression for the relation elastic stress-strain:

=

éq 3.1-4

E

One defines the thermodynamic forces associated with the elastic strain, with the deformation figure cumulated and with porosity in accordance with the framework of generalized standard materials:

(*E*

) =

éq 3.1-5

E

(

p) =
With

éq 3.1-6

p

(

F) =
B

éq 3.1-7

F

It remains then for dissipation:

: p
& - A p& -
B f& 0

éq
3.1-8

The principle of maximum dissipation applied starting from the viscoplastic potential $F_{vp}(\cdot, A, B)$ allows to deduce the laws of evolution from them from the plastic deformation, the plastic deformation cumulated and of porosity, is:

P
 νP
 F
& =

éq 3.1-9

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F
 νP
 $p\& = -$

éq 3.1-10

With

F

v_p

$f \& = -$

éq 3.1-11

B

It is supposed that $F_{vp}(\cdot, A, B)$ is a function of the plastic potential $F(\cdot, A, B)$ and that this last break up into two terms depending respectively on the second invariant on coupled to A and on first invariant of coupled to b :

$$F_{vp} = (F) = (F$$

+

éq

3.1-12

$VM(\cdot,$

A

eq

$) F_m(\cdot, B$

m

)

By assumption, the first term breaks up in an additive way like the potential of von Mises:

F

$= -$

$-$

$= -$

éq

3.1-13

$VM(\cdot,$

A

eq

)

With

eq

(p) R

R (p)

0

eq

Not to obtain a commonplace result, the decomposition of the second term must be multiplicative:

F

=

éq

3.1-14

m (

, B

m

) G (m) H (B)

Taking into account the equation [éq 1-1], the laws of evolution for

p

tr & and f& lead to the equality:

g' (

-1 B

m)

H ((F)

=

éq

3.1-15

G (

1

- B

m)

F H ((F)

The two members of this equation are functions of the two independent variables and F,

m

thus they is equal to a constant of dimension the reverse of a constraint, it is the parameter of material 1. The parameter without dimension D appears in the integration of G/G:

1

1

G ()

= $D \exp m$

éq

3.1-16

m

1

1

1

The function (B and the function reverses $F = H$ (B are unknown. The simplest choice and more

naturalness is to take H H , which gives:

1

)

naturalness is to take H H , which gives:

1

$H(B) H$

éq 3.1-17

$1(B) = F$

(B)

F

D

1

H

=

= -

$F(1 - F)$

éq

3.1-18

dB

1

The plastic potential is written finally:

$F = + D F \exp m - R(p)$ **éq**

3.1-19

eq

1

1

1

The law of evolution for $p\&$ gives:

$D(F)$

$p\& =$

$= V(F)$

éq 3.1-20

dF

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The function $V(F)$ defines the viscosity of material. One will consider only the particular case such

as:

m

F

$$V(F) = \&$$

éq

3.1-21

0 HS

0

who is reduced to a function power (law of the Norton type) when two parameters of material & and are very large. Conversely one a:

0

0

$$F - S(p\&) = 0$$

éq 3.1-22

l

l

m

p&

$$S(p\&) =$$

-

HS

éq

3.1-23

0

&0

In the case of plasticity independent of time, the preceding equation becomes $F = 0$ (criterion or threshold of plasticity) and \dot{p} is given by the equation of F & consistency = 0 if $F = 0$ and $\dot{p} = 0$ if $F < 0$.

The equations of the model are now completely defined, in the case without nucleation of new cavities. In the event of nucleation of new cavities, one considers that the voluminal fraction created is proportional to the cumulated plastic deformation. It is thus enough to replace F by $F + A p$ in the equations of the model. A is a parameter of material. The equation [éq 1-1] is not modified.

3.2

Equations of the model

One summarizes the equations of the model deduced from the thermodynamic and phenomenologic analysis which precede:

1

m

p

m

-

&

$= + D F + A p$

$- R p - HS$

=

éq 3.2-1

v_p

eq

1

1 (

N

) exp

()

1

0

0

1
&0

=
= [(**Id Id**) + 2μII] E

:
éq
3.2-2

1 - F - A p
N
=

éq 3.2-3

1 - f0
~
~
~ p
3
3
& = p&
= p

éq 3.2-4

&
2

2
eq
eq
p

$$tr \& = p \& D F A p \exp m$$

**éq
3.2-5**

1 (
+ N)

$$1
f \& = A (1$$

éq 3.2-6

1
) p

F tr &
with A = 1, this parameter being introduced only for numerical reasons.

1
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4 Formulation numerical

4.1

Key words, given internal material and variables

For the foreseeable applications, the model was established under two distinct key words: `ROUSSELIER_PR` for the plastic model with nucleation of cavities or `ROUSSELIER_VISC` for the viscoplastic model without nucleation. That makes it possible to avoid useless numerical calculations.

corresponding simplified equations are obtained starting from the general equations while posing respectively $\nu = 0$ or $A = 0$.

0

N

*The whole of the parameters of the model is provided under the key words factors `ROUSSELIER` or `ROUSSELIER_FO` and `TRACTION` (to define the traction diagram) order *DEFI_MATERIAU* ([U4.43.01]). Parameters of the viscoplastic model (,*

0

& and m) are provided

0

by the key word `ROUSSELIER_VISC`.

The internal variables produced in Code_Aster are:

V1, cumulated plastic deformation p ,

V2, porosity F ,

V3 with V8, the tensor of elastic strain E

V9, the indicator of plasticity (0 if the last calculated increment is elastic, 1 if solution figure regular, 2 if singular plastic solution).

We now present the numerical integration of the law of behavior and give the form of the tangent matrix (options *FULL_MECA* and *RIGI_MECA_TANG*).

4.2

Expression of the discretized model

The numerical resolution is carried out by one - method, with 0 1. For all the quantities Q , one defines:

$$Q = Q + Q$$

$$Q = Q + Q$$

The system of equations discretized is:

$$\begin{aligned} &\sim \\ &\sim e \\ &= 2\mu = 2\mu (\sim \end{aligned}$$

$$\begin{aligned} &\sim p \\ &-) \acute{e}q \end{aligned}$$

4.2-1

$$\begin{aligned} &E \\ &= Ktr \\ &= K tr \\ &- tr \end{aligned}$$

$$\begin{aligned} &\acute{e}q \\ &4.2-2 \end{aligned}$$

$$\begin{aligned} &m \\ &(\\ &p) \end{aligned}$$

$$\begin{aligned} &\sim \\ &p \\ &3 \\ &\sim \end{aligned}$$

$$= p$$

éq 4.2-3

$$2 \text{ eq}$$
$$p$$

$$tr = Pd F$$

With $p \text{ exp } m$

éq
4.2-4
 $1 ($
 $+ N)$

$$1$$
$$F$$
$$= A$$

éq 4.2-5

$$1 (1 -$$

 $)$

p
F
tr

l

m

p
m
-

=
+ *D F + A p*

- *R p - HS*
= **éq 4.2-6**

vp
eq
l
l (
N
) exp
(
)
l
0
0

&

T

1
0

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This system is reduced to the solution of only one equation scalar for the unknown factor F , knowing

, T

and quantities -

Q . It is noted that does not intervene in the algorithm, on the other hand it

will intervene in the calculation of the coherent tangent matrix. One calculates successively:

F

=

-

+ $K tr$

ϵq

4.2-7

m

m

-

$A1 (1 -$

$F)$

p

is the positive root of the quadratic equation:

With

F

p

F

With p

p

éq

4.2-8

N

$()^2 + (+$

-

N

)

1

-

=

With -

F

D

1 (1

) *exp*

1

$(m - 1) 0$

/

3

μ

p

$\sim = 1$

-

- [

~

~

+ 2

μ

éq

4.2-9

~- +

$$\mu \sim \frac{1}{2} J(\dots)$$

$$eq = \dots +$$

$$\mu \sim \frac{1}{2}$$

$$- 3 \mu$$

éq 4.2-10

$$eq [\dots] p$$

The scalar equation for F is the equation [éq 4.2-6] = 0.

Notice 1:

Like F is very weak in most of the structure, it would be preferable to use p like principal unknown factor. But in this case it is not possible to bring back to a scalar equation, which makes more difficult the use of a method of the type Newton. It is also one of the reasons why the equations [éq 1-1], [éq 3.2-6] and [éq 4.2-5] were not modified by the introduction of the nucleation of the cavities.

Notice 2:

The equation [éq 3.2-6] can be integrated exactly:

$$1 \dots p$$

- F

$tr =$

ln

0

$A1 1 - F$

from where:

1

p

$1 - -$

F

$tr =$

ln

With

1

$1 - F$

As numerical parameter A can be modified in a discontinuous way, the derived form

1

[*éq 4.2-5*] was preserved, including in the calculation of the coherent tangent matrix. If

the use of parameter A was to be abandoned in a later version, it would be necessary

1

to consider the use of the integrated form.

Notice 3:

The integrated form $= 0$ is used, including in plasticity instead of the relation of

vp

F & consistency $= 0$ which gives p &. The coherent tangent matrix is calculated with this

form integrated.

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Date:

10/09/04

Author (S):

G. ROUSSELIER, R. MASSON, G. Key BARBER

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4.3

Resolution of the nonlinear scalar equation

The resolution of the equation

(F

be carried out by an algorithm of Newton on controlled terminals

vp

) = 0

in routine LCROUS. (F

and its derivative compared to F

are calculated in the routine

vp

)

RSLPHI called by LCROUS. The initial values of the terminals are:

· limits lower: F = 0 since

(it was checked as a preliminary that the branch

vp (0) < 0

1

rubber band (negative threshold) is not solution),

· limits higher: F

such as

sought by dichotomy between 0 and

-

1 - F

vp (0) > 0

2

1

-

- F

(first value for this research:

).

2

The algorithm of Newton begins with the value F = 0. Whatever the value found for F one

thus note for the continuation that the function (F) and its derivative compared to F are at least

νp
 $)$
 1
 $-$
 $- F$

calculated for $F = 0$ and

$.$
 2

The developments carried out to improve convergence and the robustness of the algorithm are described in [bib5].

4.4 **Form of the tangent matrix of the behavior**

One gives the form of the tangent matrix here (option *FULL_MECA* during iterations of Newton, option *RIGI_MECA_TANG* for the first iteration).

For option *RIGI_MECA_TANG*, the tangent operator is the same one as that which connects \mathbf{E} with in

[*éq 3.2-2*].

For option *FULL_MECA*, the tangent matrix is obtained by linearizing the system of equations which governs the law of behavior: [*éq 4.2-1*] with [*éq 4.2-6*]. It is thus about a coherent tangent matrix.

To simplify the expressions, one notes in this paragraph [*§4.5*]: Q for

Q , quantities being

all expressed at the moment $T = T - + T$

. The coherent tangent matrix is:

has - has

has

has

y

1

3

~ ~

~

5

1

= $\mathbf{II} + \mathbf{Id}$ has

has

has

y

3

Id +

2

+ +

Id

4

+ +

5 ~

4

-

1 Id +

3

3

3K

K

éq 4.4-1

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This operator is calculated in routine RSLJPL. The coefficients are calculated as follows:

$$= 3K \text{ has} + y K (Z + Z p$$

)
éq 4.4-2

$$1$$
$$1$$
$$eq$$
$$7$$
$$2$$
$$= \mu \text{ has} (y + y)$$

éq 4.4-3

$$2$$
$$1$$
$$3$$
$$1$$
$$2\mu$$
$$\text{has}$$
$$eq$$
$$=$$

éq 4.4-4

3

z5

= $3\mu y$ X has

éq 4.4-5

4

2 2

= $3\mu y$ has

éq 4.4-6

5

1

1

= $3\mu K$ p has

- has

éq 4.4-7

6

2 eq

1

$$3K_z Z (F + A p)$$

$$6 l$$

$$N$$

$$y = -$$

$$\text{éq 4.4-8}$$

$$l$$

$$X$$

$$l \text{ eq}$$

$$3\mu$$

$$y = -$$

$$\text{éq 4.4-9}$$

$$2$$

$$2$$

$$X Z$$

$$15 \text{ eq}$$

$$3K_z Z A p$$

$$6 l$$

$$N$$

$$y = -$$

éq 4.4-10

3
X
1 eq
With Z
Z
y = 1 8 +
9
1

éq 4.4-11

4
Z
Z + Z p
1
7
2
A to Z
Z has
1 2 8
9 6
y =
-

**éq
4.4-12**

5
Z
(Z + Z p
)
1
eq
7
2

$$Z = I + A$$

$$Pd (F + A p) \exp m$$

éq

4.4-13

1

1

1

N

1

$$Z = \mu$$

$$3 + R$$

éq 4.4-14

2

vp

$$Z = K (F + A p) Z - A I$$

(- F) éq

4.4-15

3

N

1

1

1

$$Z = R p$$

-

éq 4.4-16

4

νp

eq

$Z = + \mu$

3

p

éq 4.4-17

5

eq

$Z =$

$exp m$

D

éq 4.4-18

6

1

1

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$Z = Z(F + A p)$

éq 4.4-19

7

6

1

N

1 - F

Z =

éq 4.4-20

8

1 - F - A p

N

With

Z

N

=

éq 4.4-21

9

1 - F - A p

N

X = Z Z (Z + Z p

) + Z Z - X

éq

4.4-22

1

3 6

7

2

1 2

1

3

X = - Z Z p

(Z + Z) - Z Z + X p

éq

4.4-23

2

3 6

4

7

1 4

1

3

2

$$X = A Z Z$$

éq 4.4-24

$$\begin{aligned} & 3 \\ & N 1 6 \\ & 1 \\ & Dr. (p) \\ & 1 dS (p \\ & /T \\ &) \\ & R = \\ & + \end{aligned}$$

**éq
4.4-25**

$$\begin{aligned} & vp \\ & dp \\ & T \\ & p \\ & d\& \end{aligned}$$

For the plastic model with nucleation of cavities `ROUSSELIER_PR` and for the model viscoplastic without nucleation `ROUSSELIER_VISC`, the corresponding simplified equations are obtained starting from the equations above by posing R respectively $= Dr. (p)/dp$ and $A = 0$.

$$\begin{aligned} & vp \\ & N \end{aligned}$$

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Organization (S): EDF/MTI/MMN

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Document: R5.03.08

Integration of the relations of behavior

viscoelastic in operator STAT_NON_LINE

Summary

This document describes the viscoelastic behaviors in the case of the ingredients necessary to the setting in

work of non-linear algorithm STAT_NON_LINE described in [R5.03.01]. Data input of all them viscoelastic relations of behavior integrated in Aster have in a general way the same form. Only the way of introducing the principal data (the function speed of viscous deformation) varies: it is presented

according to the various key words which make it possible the user to choose the relation of behavior wished.

These quantities are calculated by a method of integration semi-implicit. From the initial state, or from

the moment of preceding calculation, one calculates the stress field resulting from an increment of deformation.

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1 Introduction

The tubes of sheath in Zircaloy of the fuel pin of the power stations REFERENCE MARK present a behavior strongly viscous mechanics.

Within the framework of the chaining enters Code_Aster and the code of the fuel pin CYRANO3, two non-linear viscoelastic models specific to Zircaloy were introduced into Code_Aster.

One of them is the model used in Cyrano2 and Cyrano3. Other was developed by the EPRI.

In addition, a model much more general and correspondent with other materials that Zircaloy has also introduced summer. It is about the non-linear viscoelasticity of Lemaître, which can be brought back for

certain particular values of the parameters to a relation of viscoelastic behavior of Norton.

For these three models, one supposes that the material is isotropic. They can be used in 3D, in plane deformations (D_PLAN) and into axisymmetric (AXIS).

One presents in this note the equations constitutive of the models and their establishment in Code_Aster.

2 Relation

continuous

One places oneself on the assumption of the small disturbances and one divides the tensor of the deformations into

an elastic part, a thermal part, a anelastic part (known) and a viscous part.

equations are then:

$$\sigma = E \epsilon + H \dot{\epsilon} + \eta \int \dot{\epsilon} dt + v$$

$$= A(T) E$$

3 ~

$$\dot{\epsilon} = (\dots)$$

$$G \text{ eq, } T) 2 \text{ eq}$$

with:

2

$$\epsilon : \text{cumulated viscous deformation} =$$

! :

v!

3

v

~

1

$$\sigma : \text{diverter of the constraints } \sim = - \text{Tr} (\dots) I$$

3

3

~ ~

$$\text{eq: equivalent constraint eq} =$$

:

2

WITH (T): tensor of elasticity

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3

Nature of the function G for each relation of behavior

3.1 Relation

LEMAITRE

In this case, G is expressed explicitly (is a scalar here):

N

(

1

1

1

G, T) =

with

,

0

,

0 N 0

/

K 1 m

>

K

m

The data of the material characteristics are those provided under the key words factors LEMAITRE or LEMAITRE_FO of operator DEFI_MATERIAU.

1

1

/LEMAITRE:

NR:

N UN_SUR_K: UN_SUR_M:

K

m

The Young modulus E and the Poisson's ratio are those provided under the key words factors ELAS or ELAS_FO.

3.2 Relations

ZIRC_CYRA2 and ZIRC_EPRI

For these relations, G is not expressed explicitly. The behavior is represented by one unidimensional creep test, with constant constraint, which utilizes time passed since the moment when the constraint is applied. The relation of behavior is defined here by the data of

four functions F, G, F, G

1

1 2

2 describing the evolution of the viscous deformation in the course of time:

$$v = f_1(T) g_1(\cdot, T) + f_2(T) g_2(\cdot, T)$$

éq 3.2-1

The function G is calculated then numerically by eliminating time T in the following way:

1) for a given triplet (\cdot, T) , one solves in T the equation [éq 3.2-1] by the method of

Newton (see [bib2]). An approximation of the solution T is found (\cdot, T) ,

2) one obtains the value of the function G in (\cdot, T) by deriving compared to time the equation [éq 3.2-1] (see [bib1]):

! = !

$$v = (G, T) = F "1(T) g_1(\cdot, T) + F"$$

2 (T) g2 (, T)

and in substituent in this new equation the value of $T(\cdot, T)$ found previously.

One finds the formulation uniaxial following:

! = !

$$v = (G, T) = F "1(T(\cdot, T) g_1(\cdot, T) + F"$$

2 (T (, T) g2 (, T)

For each of two relations ZIRC_CYRA2 and ZIRC_EPRI, the form of the four functions

F, G, F, G

1

1 2

2 is preset and the user introduces only some parameters into the file of order.

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Thus, for ZIRC_CYRA2, one a:

F

- T

1 (T) = [Cth fab (1 - E

) +t] Frec

(0,00266 -0,413)

2

2nd

g1 (, T) =

With ex

HT

(p K/(T +,

273)

15

3

3

F

- K T

irr

2 (T) = [Cirr fab (1 - E

) +t] Frec

4

g2 (, T) = A

3 irr

with: C = 4450

HT

= 4,

-3

1

5 10 H

To = 9,

17

529 10

HT

$K = 39000 \cdot K$

F

=,

-4

1.816 10 ex

rec

(p6400/(T +,

273

rec

)

15

C

= 4000

irr

-3

1

K

= 3 10 H

irr

Airr =

-

1 2 10 22

,

Positive parameters fab, R

T EC. and are those provided under the key word factor ZIRC_CYRA2 of operator DEFI_MATERIAU:

/ZIRC_CYRA2: (EPSI_FAB: TEMP_REHEATED: T

FLUX_PHI:

fab

rec

)

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In the same way, for ZIRC_EPRI, one a:

F

5

With

1 (T) = T

With

G

A3

4

A6 - A7 T+273 15

1 (, T) = A (

1 S (

H2

With)

/(

,)

R E

p

F

B2

2 (T) = T

B

G

B3 B4 - B5 T+273 15

B6

7

2 (, T)

/(

,)

= 1

B E

RP (cos max

)

with:

1
With =,
8
1.603 10
2
To = 4,
5
567 10-
3
To = 2,28
4
With =,
0 997
5
With =,
0 77
6
With =,
0 956
7
To = 2,
4
3 10
1
B =,
-21
3.296 10
B2 =,
0 811
B3 =,
0 595
B4 = 1 352
,
5
B = 22 9
, 1
B6 = 1 5
, 8
B7 = 2 228
,

Positive parameters, RP and max 0 max

2 are those provided under the key word factor

ZIRC_EPRI of operator DEFI_MATERIAU:

/ZIRC_EPRI: (FLUX_PHI:

R_P: R THETA_MAX:

p

max

)

It will be noted that, for the two relations of behavior and all the functions:

T is expressed in hours

T is expressed in °C

express yourself in MPa

The document of Young E and the Poisson's ratio are those provided under the key words factors

ELAS or ELAS_FO.

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4**Integration of the relation of behavior****4.1 Establishment of the scalar equation for the implicit scheme and with constant elastic coefficients**

One indicates by *early* the total deflection at the moment $T + T$

and by *early* the variation of deformation

total during the step of current time. One calls *O* the deformation imposed at the moment $T + T$ and

O variation of deformation imposed during the step of current time.

This imposed deformation results from thermal dilation and the anelastic deformations. One has thus:

$$O = (T + T)$$

$$[((T+t) - rTef) - (T) ((Tt) - rTef)] \mathbf{I}_3 + a (t+t) - has (T)$$

where \mathbf{I}_3 is the tensor identity of order 2 in dimension 3.

One poses = *early* - *O*

As it is supposed here that μ is constant, one with the following relation between the diverters of and:

$$\sim = 2\mu (\sim$$

- v)

éq 4.1-1

However, the law of flow is written, an implicit way:

~

v

3

-

$$= G eq, +$$

,

éq 4.1-2

2

(v) T

eq

T

eq

One thus has, by eliminating ν between [éq 4.1-1] and [éq 4.1-2]:

~

-

$$2\mu\sim = \sim + 3\mu t G,$$

eq +

(v), T

eq

eq

-

G

éq 4.1-3

eq +

(v)

(

, T

~

eq

$$- + 2\mu\sim) = 1+ 3\mu$$

~

T

eq

By posing $\sim e$

$\sim -$

\sim

$= + 2\mu$, one thus has:

E

-

$eq = 3\mu T$

$G eq, +$

$(v), T$

éq 4.1-4

eq

eq

+

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However, one has according to [éq 4.1-2]:

(

-

v)

$= T G, +$

$, T$

eq
eq

(*v*) *eq*

From where:

$$eeq = 3\mu (v) +$$

eq

eq

(

1

E

v)

=

-

eq

$$3 (eq eq)$$

μ

In substituent this last expression in [éq 4.1-4], one a:

1

E

-

E

$$eq = 3\mu T$$

G *eq*, +

,

$$3 (eq - eq) T + eq$$

μ

If one poses, *eeq*, -, *T* and *T*
being known:

1

F (*X*)

μT

G *X*, -

3

$3(E - X), T + X$

E

$=$

$+$

eq

$- eq$

μ

one can then calculate the quantity

$-$

$eq = (+)$ as being the solution of the scalar equation:

eq

$F(X) = 0$ where $X = eq$, convention adopted for the following paragraphs.

4.2

Resolution of the scalar equation: principle of routine ZEROF2

One easily shows that, if the requirements in the paragraph [§3] on the characteristics of materials are checked, the function F is strictly increasing and the equation $F(X) = 0$ admits one single solution.

If E

E

$eq = 0$, then the solution is $X = 0$. If not, one a: $F()$

$0 = - eq < 0$

The problem thus consists in finding for a function F unspecified the solution of the equation

$F(X) = 0$ knowing that this solution exists, that $F()$

$0 < 0$ and that F is strictly increasing.

The algorithm adopted in ZEROF2 is as follows:

· one leaves $A = 0$ and $B = X$

where X

0

0

ap

ap is an approximation of the solution. If it is

necessary (i.e. if $F(b0) < 0$), one brings back oneself by the method of the secants

year $F(bn) - bn F(\text{year})$

$(Zn =$

then $has 1 = B$ and $B 1 = Z)$ in one or more iterations with the case

$F(B$

$n+$

N

$n+$

N

$N) - F(\text{year})$

where $F(has) < 0$ and $F(b) > 0$:

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F (b1)

= B has

1

0

0

b1

F (b0)

F ()

0

(In the case of the figure above, this first sentence was done in an iteration:

has = B and F (B

1

0

1) > 0).

· one

calculate

$Nd = \text{left whole } (Nmax)$ or $Nmax$ is the maximum number of iterations that

one was given. One then solves the equation by the method of the secants while using however

method of dichotomy with each time N is multiple of Nd :

1)

If Nd divides N

+ B has

Z

N

N

$N =$
 2
if not
 $year F (bn) - bn F (year)$
 $Zn =$
 $F (bn) - F (year)$
finsi
 $N = N + 1$
if $F (Z) >$
if $F (Z) < 0$
 $1 = Z has$
 $B 1 = B$
 $n+$
 N
 $n+$
 N
if not
 $has 1 = has$
 $B 1 = Z$
 $n+$
 N
 $n+$
 N
finsi
to go into 1)
if not
The solution is: $X = Z FINE$
 N

finsi
This second part of the algorithm makes it possible to treat in a reasonable iteration count the cases where F is very strongly non-linear, whereas the method of the secants would have converged too much slowly. These cases of strong non-linearity meet in particular with the law of LEMAITRE, for
 N
values of
large.

m
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4.3

Calculation of the constraint at the end of the step of current time

According to [éq 4.1-3], if X is the solution of the scalar equation, while posing:

$$\begin{aligned} & (\\ & 1 \\ & X \\ & B X, E \\ & eq) = \\ & = E \end{aligned}$$

1

-

$$\begin{aligned} & G X, + 3 (E \\ & - X \\ & eq \\ & eq \\ &), T \end{aligned}$$

μ

$$1 + 3\mu T$$

X

one a:

$$\begin{aligned} & \sim = B (X, E E \\ & eq) \sim \end{aligned}$$

éq 4.3-1

If $eeq = 0$, which is equivalent according to the scalar equation to $X = 0$, one prolongs B by

1
continuity. For that, one poses (

y X)

-

(E

=

+

- X

G X = G (X, there X, T)

eq

)

μ

and ()

()). The derivative of G

3

express yourself according to the derivative partial of G at the point (X (

, there X), T):

G

1 G

G' (X) =

(X (

, there X), T) -

(X (

, there X), T)

X

μ

3 y

The prolongation of B by continuity gives then:

(

1

B,

0)

0 = 1 + 3μtG' () 0

and one has, always if E

~

eq = 0, = 0.

Once one calculated ~

, one obtains by the relation (K is supposed to be constant here):

= - + = ~

1

+ Tr (-

) + KTr ()

I

éq 4.3-2

3

3

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4.4 Diagram

semi-implicit

With an implicit numerical diagram [éq 4.1-2], in the case, for example, where G does not depend on, only intervenes by the calculation of the ν value of the constraint at the end of the step of time. It can in to result from the important numerical errors if the constraint strongly varies in the course of time (see [bib2]).

To cure that and to improve the resolution, one discretizes the law of flow in way semi-implicit:

~
~-

+
 ν
3

-
(ν) eq

T

$$2$$

$$= G +$$

$$-$$

$$,$$

$$, T -$$

éq 4.4-1

$$T$$

$$2$$

$$2$$

$$+$$

$$+$$

$$2$$

$$2 -$$

$$eq$$

$$+$$

2 eq

To transform in the most economic way what was programmed previously (while following implicit formulation [éq 4.1-2]), it is enough to divide each member of the equation [éq 4.4-1] by 2:

$$-$$

$$(v) eq$$

$$-$$

$$-$$

$$T$$

$$G +$$

$$,$$

$$, T$$

$$- \sim$$

$$\sim$$

$$($$

2
+
+
2
2
+
v/2)

eq
3

2
=
T
2
2
-
+

2 eq
and to make the same thing with the relation [éq 4.1-1]:

~
~

= 2μ
-
v
2

2
2

It is noted that this system is same form as that consisted the equations [éq 4.1-1] and

[éq 4.1-2], the data being
instead of, unknown factors being respectively
and
v with

2
2
2
G
2
2
2

place of and ν and the function replacing the function G.

One can thus use the resolution of the paragraphs [§4.1] with [§4.3] as well as the corresponding algorithm

while introducing
and by dividing the function G by 2. It then remains to multiply the results

and
 ν
by 2 to obtain the increments of calculated constraint and viscous deformation

2
by the semi-implicit diagram (it and the ν of the equation [éq 4.4-1]).

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It will be noticed that the calculation of the tangent operator is not affected by this modification of the diagram

numerical. Indeed, one has obviously:

()

2

=

2

4.5 Taking into account of the variation of the elastic coefficients in function of the temperature

One has, if **A** is the tensor of elasticity:

= + (-1

v

With)

with:

(A-1) A-1 (-

T

T) (-) A-1

(- T) -

=

+

+

-

This is translated in the equations of [§4.4] by:

-

(v)

-

eq

-

T

G +

,

,

T

-

~

~

~

2

+

+

2

2

+

eq

-

-

2

$2\mu + 2\mu$

2μ

~

3μ

~-

T

-

2 -

+

2 =

2

-

-

4μ

+

2 eq

While posing:

-

~

~

2μ

2μ

E

+

~

-

=

+

2μ

-

4μ

2

and

3K- + 3K

-

Tr (*E*

) =

Tr

-

()+

3K *Tr*

6K

2

one is reduced exactly to the preceding case [§4.4].

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5

Calculation of the tangent operator

If $eeq = 0$ and $X = 0$, one take the tensor of elasticity as tangent operator.

If not, one obtains this operator by deriving the equation [éq 4.3-1] compared to:

~

~

(

$B X, E$

E

eq)

~

~e

E

=

=

+ (

$B X, eq$)

then while also deriving [éq 4.3-2] compared to:

~

$Tr () \sim$

=
 + K^3
 =
 + $K T$
I
II

3 3

It will be noted that, in these equations, the tensors of order 2 and order 4 are respectively compared to vectors and with matrices. **I3** is here a tensor of a nature 2, compared to a vector:

$T \mathbf{I3} = (11,1,0, 0,)$

0

One has moreover:

(bx, E

E

eq) **B**

X

=

$X E$

+

$X E$

eq

$X ($

B

, eq)

$E (, eq)$

eq

X

It is thus necessary to calculate. For that, one derives the scalar equation implicitly compared to. To simplify, one will omit thereafter in the writing of G and his derivative the parameter T .

One has then:

$$\begin{aligned}
 &E \\
 &E \\
 &[\\
 &X \\
 &G \\
 &eq \\
 &3\mu G (X) +] \\
 &1 \\
 &+ T \\
 &(X, y) \\
 &eq \\
 &= \\
 &y
 \end{aligned}$$

From where:

$$\begin{aligned}
 &G \\
 &1 - T \\
 &(X, y) \\
 &X \\
 &y \\
 &E \\
 &eq \\
 &= \\
 &1 + \mu \\
 &3 \\
 &T \\
 &G (X) \\
 &G \\
 &1 - T \\
 &(X, y) \\
 &X \\
 &y \\
 &\mu \\
 &3 T \sim e \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 &1 + \mu \\
 &3
 \end{aligned}$$

T

G (X) E

eq

with the expression of G' (X) obtained with [§4.3].

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One obtains finally the following expression of the tangent operator:

$= K T$

I I

~

~

3

$3 + 2\mu E T E + B X, E$

With

[

(eq)]

with:

~

1

With =

= J6 -

T

I I where **J** is the matrix identity of row 6.

3

3

6

3

G

1 -

,
3

T
(X y)

E

=
y
(

3

eq
- X

2nd

1 3

G

eq)

+ μt (X)

Note:

In the case of laws ZIRC_CYRA2 and ZIRC_EPRI, it is checked easily that:

1

2

2

G (X) =

,

G G F'

1 1 1 -

,

''

,

,

''

,

FF
1 1 + G G F
2 2
2 -

FF
2 2 + G G
1 2 (F F
1 2 - F F
1 2)
F G
1 1 + F G
2 2
,

''
,
1
,

+ G G
2 1 (F F
1 2 - F F
1 2) -
(F g11+f g22)
3μ

G (
F " G
F 'G
X, y, T)
1 1 +
=
2 2

F 'G
1 1 + F '2 g2
where F, F "F " F, F" F '

1

,

1

,

1 2 2, 2 indicate the values of F and F

1

2 and of their derivative at the point $T(X, y, T)$ and

where G, g', G, g'

1

1

2

2 indicate the values of G and G

1

2 and of their derivative compared to at the point

(X, T) (see [bib1]).

6 Bibliography

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Nonlinear relations of behavior 1D

Summary:

This document describes the quantities calculated by operator STAT_NON_LINE necessary to the implementation of the quasi static nonlinear algorithm describes in [R5.03.01] in the case of the elastoplastic behaviors or viscoplastic monodimensional. These behaviors are applicable to the elements of BAR, with elements of beam and beams multifibre (direction axial only) and to the elements of concrete reinforcement (modeling ROASTS).

The behaviors described in this document are:

- the behavior of Von Mises with linear isotropic work hardening: VMIS_ISOT_LINE, and unspecified VMIS_ISOT_TRAC,***
- the behavior of Von Mises with linear kinematic work hardening: VMIS_CINE_LINE,***
- the behavior of Von Mises with linear, nonsymmetrical work hardening in traction and compression:
with restoration of the center of the elastic range: VMIS_ASYM_LINE. This last was developed to model the action of the ground on the Cables with Gas Insulation,***

- *the behavior of PINTO-MENEGOTTO which makes it possible to represent the elastoplastic behavior uniaxial of the reinforcements of the reinforced concrete. This model translates nonthe linearity of the work hardening of the bars under cyclic loading and takes into account the Bauschinger effect. It makes it possible of more than simulate it buckling of the reinforcements in compression. This relation is available in Code_Aster for elements of bar and elements of grid,*
- viscoplastic behaviors of the LMA-RC and J.Lemaître, usable by ASSE_COMBU in axial direction of the elements of beam.

The resolution is made in all the cases by a method of integration implicit as from the moment of calculation precedent, one calculates the stress field resulting from an increment of deformation, and the behavior tangent which makes it possible to build the tangent matrices.

One describes finally a method, similar to the method due of Borst [R5.0303] allowing to use all them behaviors available in 3D in the elements 1D.

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1

Use of the relations of behavior 1D

1.1

Relations of behavior 1D in Code_Aster

The relations treated in this document are:

VMIS_ISOT_LINE:

Von Mises with symmetrical linear isotropic work hardening

VMIS_ISOT_TRAC:

Von Mises with unspecified isotropic work hardening

GRILLE_ISOT_LINE:

Von Mises with symmetrical linear isotropic work hardening

VMIS_CINE_LINE:

Von Mises with symmetrical linear kinematic work hardening

GRILLE_CINE_LINE

Von Mises with symmetrical linear kinematic work hardening

VMIS_ASYM_LINE:

Von Mises with asymmetrical linear work hardening and restoration

PINTO_MENEGOTTO:

Behavior of the reinforced concrete reinforcements

GRILLE_PINTO_MEN:

Behavior of the reinforced concrete reinforcements

ASSE_COMBU

Viscoplastic behaviour of the fuel assemblies:

Model of LEMAITRE or the LMA-RC

These relations of behavior (incremental) are given in operator STAT_NON_LINE [U4.51.03] under the key word factor COMP_INCR, by the key word RELATION [U4.51.03]. They are not valid that in small deformations.

*One describes for each relation of behavior the calculation of the stress field from one increment of deformation given (cf algorithm of Newton [R5.03.01]), the calculation of the **nodal** forces R and of the tangent matrix K nor.*

1.2 Notations

general

All the quantities evaluated at the previous moment are subscripted by -.

Quantities evaluated at the moment $T + T$ are not subscripted.

The increments are indicated par. One has as follows:

$$\begin{aligned} \mathbf{Q} &= \\ (& \\ \mathbf{Q} &+) = \mathbf{Q} () + \mathbf{Q} = \mathbf{Q} \\ T & \\ T & \\ T & \\ + \mathbf{Q} & \\ . & \end{aligned}$$

tensor of the constraints (in 1D, one is interested only in the single component nonnull uniaxial).

\sim
1

deviative operator: \sim

$ij = ij - kk ij.$

3
()

equivalent value of Von Mises, equalizes in 1D with the absolute value
eq

increment of deformation.

With

tensor of elasticity, equal in 1D to the Young modulus E

, μ , E , K

moduli of the isotropic elasticity.

thermal dilation coefficient α .

T

temperature.

()

positive part.

+

*p
cumulated plastic deformation*

p

plastic deformation

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1.3

Change of variables

Whatever the type D finite element referring to a law of behavior $1D$, it is necessary to carry out one change of variables to pass from the elementary quantities (efforts, displacements) to constraints and deformations.

1.3.1 Calculation of the deformations (small deformations)

For each finite element of Code_Aster, in $STAT_NON_LINE$, the total algorithm (Newton) provides to the elementary routine, which integrates the behavior, an increase in field in displacement.

For the elements of bar, one calculates the deformation (only one axial component) by:

$$\begin{pmatrix} U \\ L \end{pmatrix} - \begin{pmatrix} U \\ U \end{pmatrix} = \begin{pmatrix} 0 \\ L \end{pmatrix}$$

and the increase in deformation by:

$$\begin{pmatrix} U \\ L \end{pmatrix} - \begin{pmatrix} U \\ U \end{pmatrix} = \begin{pmatrix} 0 \\ L \end{pmatrix}$$

For the elements of grid (modelings $ROASTS$ and $GRILLE_MEMBRANE$), one calculates the deformation membrane as for the elements of hulls DKT . Simply, only one direction corresponds physically with the directions of reinforcements. One thus finds oneself in the presence of a behavior $1D$.

In addition, in small deformations, for all the models described in this document, one writes for any moment the partition of the deformations in the form of an elastic contribution, dilation thermics, and of plastic deformation:

$$\begin{pmatrix} T \end{pmatrix} = E \begin{pmatrix} T \end{pmatrix} + HT \begin{pmatrix} T \end{pmatrix} + p \begin{pmatrix} T \end{pmatrix}, \text{ with}$$

$$E \begin{pmatrix} l \end{pmatrix} =$$

-

To 1 (T (T) (T) =

T

E (T) ()

HT (T) = (T (T) (T (T) - Tref) Id

1.3.2 Calculation of the generalized efforts (forced integrated)

For integration of the behavior 1D, it is necessary to integrate the component of constraints obtained, for to provide to the total algorithm (Newton) a vector containing the generalized efforts.

For the elements of bar, one calculates the effort (uniform in the element, by supposing that the section

is constant) by:

NR = S.,

and the vector forces nodal equivalent (as for the elements of beam, [R3.08.01]) by:

- NR

F = NR

For the elements of GRID, one calculates the efforts as for the elements of hulls DKT (efforts membrane) by integration of the constraints in the thickness (only one layer and only one point of integration).

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2
Relation of behavior of Von Mises with work hardening isotropic linear

: VMIS_ISOT_LINE or unspecified

:
VMIS_ISOT_TRAC

2.1
Equations of model VMIS_ISOT_LINE

They are the restriction of the behavior 3D [R5.03.02] on the uniaxial case:

~
p
3

& =
p&
= p&

2
eq

= - p
- HT

E

eq - R (p) = - R (p)

0
p& = 0 if eq - R (p) < 0

p& 0 if eq - R (p)

= 0
with:

.

&p:
speed of plastic deformation,

p:
cumulated plastic deformation,

**HT = (T -
thermal deformation,
Re
T F):
E.E**

**R (p)
T
=
. p +:
E - E
y
function of linear work hardening isotropic, or R (p) refines by
T
pieces, deduced from the traction diagram.**

In case VMIS_ISOT_LINE, the data of the material characteristics are those provided under the key word factor ECRO_LINE or ECRO_LINE_FO of operator DEF1_MATERIAU [U4.43.01].

**/ECRO_LINE = (D_SIGM_EPSI = AND, SY = y)
/ECRO_LINE_FO = (D_SIGM_EPSI = AND, SY = y)**

In case VMIS_ISOT_TRAC, the data of the characteristics of materials are provided under key word factor TRACTION of operator DEF1_MATERIAU [U4.43.01].

TRACTION = _F (SIGM = courbe_traction)

courbe_traction represents the traction diagram, point by point. The first point allows to define the elastic limit y and the Young modulus E [R5.03.02].

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ECRO_LINE_FO corresponds if AND and y depend on the temperature and are then calculated for the temperature of the point of current Gauss. The Young modulus E and the Poisson's ratio are those provided under the key words factors ELAS or ELAS_FO. In this case the traction diagram is the following one:

L

E

T

y

E

L

y
L = E L
if L <

E

•
y

$$L = y + LTE - \text{if } L$$

E
E

When the criterion is reached one a:

$$L - R(p) = 0, \text{ therefore } L - R L -$$

E = 0, from where:

$$E E R(p) T = p + = H. p + E - E y y T$$

In the case of a traction diagram, the step is identical to [R5.03.01].

**2.2
Integration of relation VMIS_ISOT_LINE**

By direct implicit discretization of the relations of behavior, a way similar to integration 3D [R5.03.02] one obtains:

$$- + - R (- p +p) 0$$

-

HT

E

+

E (-) - (-

+)+

-

= Ep

-

-

E

+

p

-

0 if + = R (-

p +p)

p =

-

0 if + < R (-

p +p)

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**:
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Two cases arise:

HT

**.
-**

**R (-
+
<
p + p) in this case p = 0 is = (
-)
-
+
-**

**thus
-**

**+ (
- HT
) < R (-
p
-
)**,

**.
-**

**R (-
+
=
p + p) in this case p 0
-**

thus
+ (
- *HT*
) *R* (-
p
-
).

One deduces the algorithm from it from resolution:

-
let us pose E
=
+

-
(
HT
-
)

if E R (p) then
p

= 0 and = (- HT)

if E > R (p) then it is necessary to solve:

-
E
-
+

= +
+ *E p*
-

+

E
E p

= ***I*** +

-

-

(+)

+

thus by taking the absolute value:

E

E p

= +

-

I

+

+

-

maybe, while using -

R (-

+

=

p + p).

E

= ***R (p***

+ p

) + E p

One thus deduces some:

E

- ***(+ HP***

y
)
· *in the case of a linear work hardening: p*
=

$E + H$
· *and in the case of an unspecified work hardening, the curve $R(p)$ being refined per pieces, one solves the equation out of p directly*

:
 p
+ $R(p)$
 E
+)
 E
 $p =$ *in the same way that in*
3D [R5.03.02].

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Let us notice in the passing that:
 E

=
 E
 $R(p)$

then

E

E

= (- +

) =

R (p) =

E

E p

1 + R (p)

Moreover, the option

N

FULL_MECA makes it possible to calculate the tangent matrix K_i with each iteration.

The tangent operator who is used for building it is calculated directly on the preceding discretized system.

One obtains directly:

E

-

if $> R (p)$

= AND

if not

= E

Note:

The option

0

RIGI_MECA_TANG which makes it possible to calculate the tangent matrix K_I used in phase of prediction of the algorithm of Newton, takes account of the indicator of plasticity with the previous moment:

· if = 1

= T

E

· if = 0

= E

2.3 Variables

interns

The relation of behavior VMIS_ISOT_LINE produces two internal variables: p and.

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3

Relation of behavior of Von Mises, work hardening

linear kinematics 1D: VMIS_CINE_LINE

3.1

Equations of model VMIS_CINE_LINE

They are the restriction of the behavior 3D ([R5.03.02] and [R5.03.16]) on the uniaxial case.

behavior 3D is written:

= K (- p - HT) with K operator of elasticity

X

p

= C

(

3 ~ ~

F, R, X) = (~ - X) -

with

With

=

WITH A

eq

y

eq

2

F

~

p

3

- X

& = &p

= &p

2

(~ - X) eq

if

F < 0

&p = 0

if

F

= 0

&p 0

In the uniaxial case, the tensors are written:

2 3

~

3

= D

X =

D

p

X

= p D

with

D =

-

1 3

2

- 1

3

As long as the loading is monotonous, the following relations immediately are obtained:

3

3

E.E

p

p

=

X = C p

= C p

+

T

y = F () = y +

p

2

2

E - AND

2 EET

EE

3

C is determined by: C =

. One poses: H

T

=

= C

3rd - AND

E - AND

2

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The relation of behavior 1D is written then:

p

- X

& = $p\&$

- X

= (- HT

- p

)

E

3

$X =$

p

$C =$

p

H

2

- $X - y \ 0$

$p\& = 0$ if - $X - y < 0$

$p\& 0$ if - $X - y = 0$

The data of the material characteristics are those provided under the key word factor ECRO_LINE or ECRO_LINE_FO of operator DEFI_MATERIAU [U4.43.01]:

/ECRO_LINE = (D_SIGM_EPSI = AND, SY = y)

/ECRO_LINE_FO = (D_SIGM_EPSI = AND, SY = y)

3.2

Integration of relation VMIS_CINE_LINE

By direct implicit discretization of the relations of behavior, a way similar to integration 3D ([R5.03.02] and [R5.03.16]) one obtains:

-
+ - -
 $X - X - y \theta$

$E p$
 $= E (- HT$
 $) - (-$
 $+) E -$

+

-

E

-

+ - -
 $X - X$

p
 $= p$

-

+ - -
 $X - X$

-
X
X

-
 $= p$

-
H
H

-
-
 $p = 0 \text{ if } + - X - X = y$

-
-
 $p = 0 \text{ if } + - X - X < y$

with
HT
 $= (T -$

-
-
ref.
T
 $) - (T - \text{ref.}$
T
 $)$

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Two cases arise:

.

-

-

+ - $X - X < y$ in this case $p = 0$ is

= (- HT

) E - H -

+

-

X

-

-

E

H

E

H

thus

-

-

- X

+ - HT

< R p

-

(

) (-

-

),

E

H

· if not $p = 0$.

E
-
H
To simplify the writings one will pose: E

=
-
X - + E

.
-
-
(
HT

-

)
E
H

One deduces the algorithm from it from resolution:

E
H
HT
E
· *if*
y then p =,
0

-
X = X
, = E -
+

-
(
)
- .
-
H
E

· *if not it is necessary to solve:*

p

HT

E

-

- H

$$E = E (-) - - - (+) + X$$

-

H

-

+ - -

X - X

p

- X

= p

= p

-

+ - -

X - X

- X

H

-

p

X -

X =

H

-

H

-

+ - -

$$X - X - y = 0$$

H

Let us notice that:

-

p

$$X = X - H$$

. One deduces then from the first equation:

$$H -$$

$$E$$

$$p$$

$$= - X + (E + H)$$

One thus obtains, while eliminating - X from the second equation:

$$p$$

$$E$$

$$p$$

$$= ($$

$$E + H) p + y$$

By replacing p in the relation between E and - X, one obtains:

$$-$$

$$y$$

$$X = E$$

$$(E + H) p$$

$$+ y$$

By taking the absolute value of the two members of the preceding equation, one finds p:

$$(E H) p$$

$$E$$

$$+$$

$$+ y =$$

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Once p determined, one can calculate:

E

p

$= p$

E

-

E

-

HX

$X = X + X$

=

+ $H p$

-

E

H

- X

E

and while using: =

, one obtains directly:

E

y

E

= y

+ X

E

Moreover, the option

N

FULL_MECA makes it possible to calculate the tangent matrix $K I$ with each iteration.

The tangent operator who is used for building it is calculated directly on the preceding discretized system.

One obtains directly:

E

-

if $> R(p)$

= AND

if not

= E

The option

0

RIGI_MECA_TANG which makes it possible to calculate the tangent matrix $K I$ used in the phase of prediction of the algorithm of Newton is obtained using the indicator of plasticity - moment

precedent:

· if -

= 1 then

= T

E

· if -

= 0 then

= E

3.3 Variables

interns

The relation of behavior VMIS_CINE_LINE produces two internal variables: X and.

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4

***Relation of behavior of Von Mises with work hardening
linear asymmetrical: VMIS_ASYM_LINE***

4.1

Equations of model VMIS_ASYM_LINE

4.1.1 Behavior

asymmetrical

in traction and compression

It is a behavior uncoupled in traction and compression, built starting from VMIS_ASYM_LINE, but with elastic limits and different modules of work hardening in traction and in compression. We adopt an index T for traction and C for compression. The behavior rubber band in traction and compression identical and is characterized by the same Young modulus. There is two fields of isotropic work hardening defined by RT and RC. The two fields are independent one other.

YT

elastic limit in traction. In absolute value.

YC

elastic limit in compression. In absolute value.

Pt

Variable interns in traction. Algebraic value.

PC

Variable interns in compression. Algebraic value.

ETT

Slope of work hardening in traction.

Etc

Slope D`work hardening in compression.

The equations of the model of behavior are:

$$\dot{\epsilon} = \dot{\epsilon}_t + \dot{\epsilon}_c$$

$$\dot{\epsilon}_t = \dot{\epsilon}_t$$

$$\dot{\epsilon}_c = \dot{\epsilon}_c$$

$$\dot{\epsilon}_t = \dot{\epsilon}_t$$

$$\dot{\epsilon}_c = \dot{\epsilon}_c$$

$$\dot{\epsilon}_t = \dot{\epsilon}_t$$

$$\dot{\epsilon}_c = \dot{\epsilon}_c$$

with

$$\dot{\epsilon}_c = 0 \text{ if } \dot{\epsilon}_c < 0$$

$$\dot{\epsilon}_c = 0 \text{ if } \dot{\epsilon}_c = 0$$

$$\dot{\epsilon}_t = 0 \text{ if } \dot{\epsilon}_t = 0$$

R
0

(Pt) <

&pT 0 if = RT (Pt)

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& p: speed of plastic deformation in compression, p

C

&:

T speed of plastic deformation in traction,

HT: thermal deformation of origin: $HT = (T - Re$

T F).

It is noticed that one cannot have simultaneously plasticization in traction and compression: that is to say

&pC = 0, either &pT = 0, or both is null.

The data of the material characteristics are those provided under the key word factor ECRO_ASYM_LINE of operator DEFI_MATERIAU [U4.43.01].

$ECRO_ASYM_LINE = _F (DT_SIGM_EPSI = HT, ETT,$
 $SY_T = yT,$
 $DC_SIGM_EPSI = HC, ETC,$
 $SY_C = yC,)$

The Young modulus E is provided under the key words factors $ELAS$ or $ELAS_FO$.

One calculates the functions of work hardening by:

E
 E
 $R(p)$
 TT
 $=$
 $p +$
 $= H. p$
 T
 $+$
 $E - E$
 T
 yT
 T
 T
 yT
 TT
 E
 E
 $R(p)$
 TC
 $=$
 $p +$
 $= H. p$
 C
 $+$
 $E - E$
 C
 yC
 C
 C
 yC
 TC

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4.2

Integration of behavior VMIS_ASYM_LINE

By direct implicit discretization of the asymmetrical relation of behavior, a way similar to the preceding one, one obtains:

$$p = p$$
$$p$$
$$T + C$$

$$p$$
$$HT$$

$$= - - E$$

$$- +$$

$$Pt = Pt$$

$$- +$$

$$(- +$$

$$)-$$

-
-
T
R (Pt + Pt) 0
p 0
if
-
-
-

T
(+) - TR (Pt + Pt) = 0

p = 0
if

-
-
-

T
(+) - TR (Pt + Pt) < 0

- +
PC = PC

- +
- (- +) -
-

-
C
R (PC + PC) 0

P
0
if - (- +
) - RC (-
PC +
-
PC)
C
= 0

$$\begin{aligned}
 PC &= 0 \\
 \text{if } (- &+ \\
) - RC &(- \\
 PC &+ \\
 - & \\
 PC) &< 0
 \end{aligned}$$

Integration is similar to that of VMIS_ISOT_LINE for each direction of traction and of compression. It should well be seen that the centers of the fields of elasticity are data (calculated explicitly with the preceding step) for the incremental problem to solve.

Four cases arise:

$$\cdot - HT > 0:$$

E is posed

-

HT

T =

+ E (

-

)

-

E

E

T <

-

T

R (Pt) in this case Pt = 0 thus = T and = E

- if not

:

E

-

T - (+ H p

yT

TT)

p =

, p

T

= 0

E + H

C

T

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E

E

T

=

= T R p

E p

E

T (T)

T

I +

T

T

R (Pt)

= ETT

· - HT < 0

E is posed

-

HT

C =

+ E (

-

)

-

- E <

E

C

R (-

C

PC) in this case PC = 0 thus = C and = E

- if not

:

E

-

C - (+ H p

yC

C C)

p =

, p

C

= 0

E + H

T

C

E

E

C

=

= C R p

E p

E

C (C)

C

I +

C
C
R (PC)

= etc

Note:

*The initial tangent matrix (option **RIGI_MECA_TANG**) is taken equal to the elastic matrix.*

4.3 Variables

interns

The relation of behavior VMIS_ASYM_LINE produces 2 internal variables: PC Pt.

It is not usable for the elements of grid

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5 Model

of

PINTO_MENEGOTTO

The model presented in this chapter describes the behavior 1D reinforcing steels of the concrete armed [bib1]. The law constitutive of these steels is made up of two distinct parts: the loading monotonous composed of three successive zones (linear elasticity, plastic stage and work hardening)

and

the cyclic loading whose analytical formulation was proposed by A. Giuffré and P. Pinto in 1973 [bib2] and was then developed by Mr. Menegotto [bib3].

During cycles, the way of loading between two points of inversion (semi-cycle) is described by an analytical curve of expression of the type $= F()$. The interest of this formulation is that the same one equation controls the discharge and load diagrams (see for example the figures [Figure 5.1.1-a] and [Figure 5.1.1-b]). The parameters attached to the function F are reactualized after each inversion of loading. The reactualization of these parameters depends on the way carried out in the plastic zone during the preceding semi-cycle.

In addition, this model can treat the inelastic buckling of the bars (G. Monti and C. Nuti [bib4]). The introduction of new parameters into the equation of the curves then makes it possible to simulate the softening of the answer stress-strain in compression.

5.1

Formulation of the model

5.1.1 Loading

monotonous

This chapter describes the first loading which the bar undergoes, i.e. the part preceding activation curve of Giuffré [Figure 5.1.1-a].

The monotonous traction diagram of steel is typically described by the three successive zones following:

- The linear elasticity, defined by the Young modulus elastic $Y.E$ and limit.

=

E (zone 1, [Figure 5.1.1-a])

- The plastic stage, ranging between the elastic strain limits $0y$ and the deformation of work hardening H , higher plate in deformation limits. During the stage constraint remains constant.

= $0y$ (zone 2, [Figure 5.1.1-a])

- Work hardening, describing the traction diagram up to the ultimate point of constraint and of deformation, (U, U) . This part is represented by a polynomial of the fourth degree:

4

0

$U -$

= - (-)

U

U

y

(zone 2, [Figure 5.1.1-a])

U - H

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The slope of work hardening (used thereafter, for the cyclic behavior) is defined here by:

0

U - y

Eh =

0. It is the average slope of zones 2 and 3 of the following figure.

U - y

U

0y

zone 3

zone 2

zone 1

0

y
H
U

Appear 5.1.1-a

5.1.2 Loading cyclic

One places oneself now if the bar undergoes a consecutive discharge with the first loading. Two cases arise then:

- the starting position is located in the elastic zone. The discharge remains in this case rubber band,*
- the starting position is located in the plastic zone (0y). The answer is first of all rubber band, then, for a certain value of the deformation, the discharge becomes nonlinear [Figure 5.1.2-a] (this is true for a discharge starting from zone 2 or of zone 3).*

The relation which the deformation must satisfy so that the curve of Giuffré is activated is as follows:

0
max - > y, with

·
3 0
max maximum deformation reached in load.

As soon as one crossed this limit with the first discharge, it is the cyclic behavior (curve of Giuffré [Figure 5.1.2-a]) which is activated.

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discharge elastic

0y
3

*max
nonlinear discharge:
activation of the curve of
Giuffré*

Appear 5.1.2-a

5.1.2.1 Presentation of the *n*th semi-cycle

*The shape of the curve of the *n*th semi-cycle depends on the plastic excursion carried out during preceding semi-cycle. The following quantities are defined [5.1.2.1 Figure - has]:*

***y_n** : Elastic limit of the *n*th semi-cycle. (Calculation clarified with [§ 5.1.2.2])*

R_{n-1}

R

*: Constraint at the last point of inversion (forced maximum attack with the *n*-Iième semi-cycle).*

R_{n-1}

R

*: Deformation at the last point of inversion (maximum deformation attack with the *n*-Iième semi-cycle).*

$N - n - 1$

N

N

N

N

y

R

-1

***y** : Deformation corresponding to y : $y = R$*

$+$

E

***$F(T)$** : Plastic excursion of the *n*th cycle*

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$n-1$

p

$n-1$

$n-1$

($N, 1 n-1$

(,

R

)

R

$n-1$

y)

y

$n-1$

Eh

$n+1$

N

Eh

(N, N

y

)

y

*(N, N
R)
R
Np*

Appear 5.1.2.1 - has

*5.1.2.2 Law
of work hardening*

The model is based on a kinematic law of work hardening. The branches of the semi-cycles are included/understood between two asymptotes of slope Eh (asymptotic slope of work hardening).

N is thus determined

N

n-1

n-1

n-1

y in the following way: $y = y$

*.
(
sign - p) +*

where the function

*(
sign X) = 1 if $x < 0$ and 1 if $x > 0$ and where n-1 is the plastic increment of constraint of the semi-cycle precedent [5.1.2.1 Figure - has] which is defined by: n-1*

N

E

-

=

1

H p

.

For each semi-cycle one thus determines N

n-1

n-1

N

y according to y

and p, one deduces y from them, then

the following semi-cycle is calculated (by the law of behavior below). Maximum deformation (in absolute value) attack before changing direction will make it possible to calculate the plastic excursion

N

N

N
p = R - Y.
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Analytical 5.1.2.3 Description of the curves = F ()

The expression chosen in the model to follow the curves of loading is as follows:

1 -

B

**** = B * +***

1 R

(

1 + ()

/

**** R)***

E

With B

H

=

report/ratio of the slope of work hardening on the slope of elasticity.

E

- n-1

* =

R

N

N 1

y - -

R

- n-1

* =

R

N

N 1

y - -

R

n-1

N

p

1

p

= N N 1

y - -

R

The size R makes it possible to describe the pace of the curve of the branches. It is a function of the way

plastic carried out during the preceding semi-cycle:

A1.

R () = R0 - G () where G () =

A2 +

Parameters R, A and A

0

1

2 are constants without unit depending on the mechanical properties

steel. Their values are obtained in experiments and Menegotto [bib3] proposes:

$R = 20\ 0$

$To = 18\ 5$

With

0

1

$2 = 0\ 1$

. 5

5.1.3 Case of inelastic buckling

Monti and Nuti [bib4] show that for a relationship between the length L and the diameter D of the bar lower than 5, the curve of compression is identical to that of traction. On the other hand, when $L/D > 5$

a buckling of the bar is observed. In this case the curve of compression in the plastic zone has a lenitive behavior. The model available in Code_Aster makes it possible to describe also this phenomenon.

The following variables are defined [Figure 5.1.3-a]:

$E0$: Initial elastic Young modulus (correspondent with E without buckling).

bc : Report/ratio of the slope of work hardening on the elastic slope in compression.

LT : Report/ratio of the slope of work hardening on the elastic slope in traction (refill after compression with buckling).

Er : Modulus Young reduced in traction (slope of the curve of refill after compression with buckling).

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S
E

B × E

C

5

ny

B × E

E

R

S

Appear 5.1.3-a

5.1.3.1 Compression

One introduces a negative slope B

E

C ×

, where bc is defined by:

E

B

'0

B = has (5 0

. -

-

L/D) E

y

C

0y
With
N
= 4 0
.
and '= max
the greatest plastic way carried out during
L/D
(p)
loading.

*It is necessary then, as in the model without buckling, to determine n_y . The method is identical, but one adds a complementary constraint *s in order to position the curve correctly compared to the asymptote [Figure 5.1.3-a].*

-
B

C
B
.
110 - L/D
S = S B E
where S is given by: S =

Cl D
1 - C
B
(
10th
- .1) 0

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And one thus has: N

N

*

$y = (y)$

+

without buckling

S

N - n-1

R

This modifies also the value of N

N

y

-1

$y = R$

+

E

5.1.3.2 Traction

At the time of the semi-cycle in traction according to one adopts a reduced Young modulus defines by:

- has 2

E = E has

6

0

**5 + (10
. - a5) (
p)
E
with a5 10
. (50. L/D
R
)**

**=
+
-
/7 5
.**

Note:

The parameters has, C and a6 is constants (without unit) depend on the properties mechanics of steel and is in experiments given. Values adopted by Monti and Nuti [bib4] are:

has = 0 006

C = 0 500

**= 620 0 have
6**

5.2

Establishment in Code_Aster

This model is accessible in Code_Aster starting from key word COMP_INCR (RELATION = "PINTO_MENEGOTTO") or (RELATION = "GRILLE_PINTO_MEN") of the order STAT_NON_LINE [U4.51.03]. The whole of the parameters of the model are given via the order DEF1_MATERIAU (key word factor PINTO_MENEGOTTO) [U4.43.01]. The parameters here are indexed

intervening in the model:

Parameter of the model

Intervenes in

value adopted by defect in Aster

0

First loading

—
y

First loading

—
U

First loading

—
U

First loading

—
H

E

Cycles

If no value entered one takes

B

H

=

computed value with the first loading

E

R

Cycles 20

0

has

Cycles 18.5

1

has

Cycles 0.15

2

L/D

Cycles with buckling

4 (to be by defect except buckling)

(if L/D >5)

has

Buckling 620

6

C Buckling

0.5

flambage has

0.006

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The parameters R , has , has , has , C and has

0

1

2

6

depend on the mechanical properties of steel and are determined in experiments. The values adopted by defect in Code_Aster are those proposed in the literature [bib1].

E

One gives in [Figure 5.2-a] a comparison of the model following the value of B

H

=

for two

E

values: $B = 0 0$

. 1 and $B = 0 001$

.

.

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Mechanical department and Digital Models

Electricity

TEST LOADING THERMAL AND CYCLIC ON A BAR (ELEMENT MECA_BARRE)

from France

ELASTOPLASTIC BEHAVIOR

X

4

10

10

MODEL OF PINTO-MENEGOTTO

WITHOUT BUCKLING.

COMPARISON BETWEEN TWO VALUES OF B

5

(

NR)

BADLY

NOR

T

0

B=0.001

FFOR

B=0.01

E

-5

0

5

10

15

20

25

30

35

40x10⁻⁴

THERMAL DEFORMATION

agraf 29/06/98 (c) EDF/DER 1992-1998

Appear 5.2-a

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One gives in [Figure 5.2-b] a comparison of the model without buckling and the model and buckling.

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from France

X

4

10

10

MODEL PINTO-MENEGOTTO

COMPARISON OF THE MODEL WITHOUT BUCKLING

AND OF THE MODEL WITH BUCKLING.

5

)

NR

(

BADLY

NOR

T

0

TOKEN ENTRY WITH BUCKLING

FFOR

TOKEN ENTRY WITHOUT BUCKLING

E

-5

-2

-1

0

1

2

3

4x10⁻³

THERMAL DEFORMATION (- ALPHA*DT)

agraf 29/06/98 (c) EDF/DER 1992-1998

Appear 5.2-b

5.3 Variables

interns

They 8, and are defined by:

V1 =

n-1

R

V 2 =

N

R

V 3 =

N

R

V 4 =

-

+ - (T - -

T)

V 5 = - (T - -

T)

V 6 = cycl

= 0

comporteme

if

cyclic

NT

activ

not

is

é
= 1
opposite

case

in
V 7 =
= 0
flax

evolution

one

with

corresponds

time

of

not

if
éaire
= 1
(indicateu

opposite

case

in
R
plasticity

of

)

***V 8 = indicating
buckling***

of

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6

Relation of behavior of LEMAITRE (ASSE_COMBU)

The model presented in this chapter describes the nonlinear viscoelastic behavior 1D of J. Lemaître developed for the modeling of the fuel assemblies, and applicable to elements of beams, in the axial direction, with behavior ASSE_COMBU [bib6].

6.1

Formulation of the model

The equations are as follows:

vp

& = p&

N

Q

-

&

RT

p& =

.E

.

+L

11

Q

1

K

, N >

0

,

,

0

0

0

m

p

K m

R

}.

vp

G

HT

= & - & - & -

&

E

The coefficients are provided under key word LEMAITRE of DEFI_MATERIAU and & is flow neutronics (derivative of the fluence compared to time).

G

(T) = (At + b). ((X, y, Z) S

).

Note:

20

2

• The neutron flux & (X, y, Z) is expressed obligatorily into 10 n/cm /s. This implies that the units of the other sizes are fixed:

-

E, K, are in MPa,

-

times are in seconds,

-

co-ordinates in mm

-

Q

T,

in Kelvin

R

Two types of integrations are available according to the value of key word PARM_THETA:

• purely implicit integration, if PARM_THETA =1.0 (default value)

• implicit semi integration, if PARM_THETA =0.5

Only these two values are authorized.

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6.2 Integration

implicit

By direct implicit discretization of the relations of behavior, one obtains:

-

νp

+

$=p$

-

+

N

-

Q

+

RT

-

-

p=

. E

(-)

+ L

.

1

T

K0t

(-

p + p) m

-

-

= - vp

- G

- HT

-

E

E

with

HT

-

-

= T

$(T - T_{ref}) - T$

$(T - T_{ref})$

S
 G
 $= (+$
 $At + b) () S$
 $. - (-) . (-$
 $At + b)$

One can still bring back oneself there to only one nonlinear scalar equation out of p, while posing:

E
 E
 $=$
 $- + E$
 $-$
 $(- G$
 $- HT$
 $)$
 E

then the system is reduced to:

N

Q

I

$-$

RT

$p =$
 $. E$

+ ***L***.

l

T

K

-

K0t

m

(p + p)

E

vp

E p

= +

E = +

E p

= ***l***+

and by taking the absolute value of the two members of the last equation, one obtains:

E = + E p

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what results in solving the equation:

N

E

$- E p$

Q

1

$-$

p

$=$

$. E R T$

$+ L. T$

K

l

0

(

-

p + p

)

K

T

m

Once this solved equation (by a method of search for zero of function scalar), one
E

E p

obtains the constraints by: =
= E l

l

E

E p

+

E - E p

6.3 integration
semi-implicit

In fact, in elastoplasticity, one uses the implicit integration of the models of behavior, because
convergence towards the solution of the problem continuous in time, excellent, and is led moreover to

unconditionally stable diagrams.

For viscoelastic or viscoplastic behaviors, utilizing explicitly time physics, the implicit discretization always leads to unconditionally stable diagrams, but convergence towards the solution is not also any more fast. It is preferable to use an integration then semi-implicit. It is the choice which we made here, following in that the integration of the model of Lemaître in Aster and Cyrano3 [bib5]. The method implemented here is not one general theta-method: it functions only for theta=0.5. It makes it possible however to obtain correct results. For more general information, it would be necessary to use more sophisticated method, by example method of RUNGE KUTTA of order 2 or 4.

Here, one writes simply:

-

+

$$\frac{vp}{2}$$

-

+

2

N

Q

-

-

T

+

I

2

RT +

2

p=

E

.

+ ***L***.

l

T

K

p

0

m

K T

-

p +

2

1

-
vp
G
HT
-
=
-
-
-
-
-
2nd E
2
2
2
2
HT = T
() (T - Tref) -
-
T
(
)(-
T - Tref)
= (At + b) () S
G
. - () S
.. (-
At + b)

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-

One seeks to calculate +

. One can write:

2

-

E -

E

E

vp

G

HT

=

+

=

+

-

- E

- E

2

2

2

-

E

2

2

2

2

2

thus

-

-

E

G HT

v

-

E

+

=

+

E

-

+

-

-

-

2

2

E

2

2

2

2

2

As previously, one solves while posing:

-

-

G

HT

E
E

=
+
+ ***E***
-
-

E - 2
2
2
2
2

then the system is reduced to:

N

Q

-
-

T

+

p

1
2
RT

+

T

=

·

2

E

+ *L*.

2

1

K

K T

2

0

-

p

p

m

+

2

-

p

p
+
E
2

-
-

E

= +
+

E
1
2

2
2

2

-
= +

+

-

+

+

2

2

from where:

p
E
-

= +
+ *E*

2
2

The equation to be solved is exactly same form as the implicit equation:

N

Q

-
E
p

T

p

1 - E
RT +
2

T

=
.
2
E
+ *L.*

2

1

K

K T

2

-

0

p

p

m

+

2

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Once this solved equation, one obtains the internal variables while multiplying by 2 the value obtained and constraints by:

E p
-

E
+
= 1-

2

E
-
-
+
= 2 +
-

2 -

One can thus use the same routines of resolution as in the implicit case, while calculating G HT simply E in

,

,

.

2

2

2

On an elementary test of creep (test SSNL109A), one obtains by the semi-implicit method one

correct result (to 0.02% of the analytical solution) if one uses 2 steps of time (instead of 100 steps of times required to have a correct solution with implicit integration).

6.4 Variables

interns

Two variables intern are calculated in this model: p and neutron fluence calculated with the step current time.

6.5

Identification of the parameters of the model

It is done starting from creep tests (uniaxial test with constant constraint imposed under flow neutronics constant). By integration of the equations of the model, one obtains then:

m

Q

$n+m$

+ &

-

νp

$(T) N m$

N

=

RT

+ LE

$m K$

0

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7

Relation of behavior of LMA-RC (ASSE_COMBU)

The model presented in this chapter describes the viscoplastic behavior 1D LMA-RC (Laboratory of Mechanics Applied R.Chaléat of Besancon) developed for the modeling of fuel assemblies, and applicable to the elements of beams, in the axial direction, with behavior ASSE_COMBU [bib6].

7.1

Formulation of the model

The élasto-viscoplastic model developed with the LMA-RC to describe the orthotropic behavior of tubes of sheaths of the fuel pin [R5.03.10] is written in 1D isotropic:

}

= & - v p

&

- G

& - HT

&

E

v p

&

= p&.

(- X)

=

- X

N

- X

$$p\& = \&0 \sin H$$

K

m

X

$$X\& = Q (Y (p) \nu p$$

()

$\&$

$$- (X - X 1) p\&)$$

-

X

$$R \sinh$$

m

$X 0$

X

()
 $X \& I = q1 (Y (p) \nu p$
 () 1
 (2)
 &
 - (X - X) p&
 (2)
 X&
 = Q Y
 2
 (p) \nu p
 (2)
 (& - X p&
 with: Y () Y (Y
 Y
 0
) ebp
 \nu =
 +
 -

The coefficients, as in 3D, are provided by key word LMARC (one does not use the coefficients here dependent on the anisotropy) (Q, Q, Q 1 2 correspond respectively to the parameters p, p, p 1 2 of the key word LMARC).

The law of growth is identical to that used for the model of Lemaitre:
G
 $(T) = (At + b) ((X, y, Z) S$

The neutron flux is the product of a function of X (clevis pin, having to be confused with one of the axes of the total reference mark) and a function of y and Z.

Note:

- *The fluence is worth (X, y, Z).*
- *Only one diagram of integration is available: a purely implicit diagram.*

20

2

- *The neutron flux (X, y, Z) is expressed obligatorily into 10 n/cm /s. This implies that the units of the other sizes are fixed:*

-

E, K, are in MPa,

-

times are in seconds,

-

co-ordinates in Meters.

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7.2 Integration

implicit

To integrate these relations of behavior, while bringing back itself if possible to only one equation to (- X)

to solve, it is necessary to make an assumption on =

. Indeed, it can take only two values:

- X

(- - -

X)

*+1 or -1. One thus supposes this known sign (initialized by =
) . If one cannot solve*

- - -

X

*the equation obtained with this assumption, one takes the opposite sign. The remainder of the
equations can*

to be integrated in a purely implicit way. The system is written:

-

= - +

= E

+

G HT - p

E

p

E

-

- -

= -

E

-

N

X

p =

&0 tsi

= F

nh T

,

v (X)

K

l

m

X X

X =

Q p (Y (p)

()

- (X - X)

()

- R sinh

,,

l

m

= F (p X X)

X0

X

()

X l = Q

l

2

() l

(2)

(, X, X)

l (

p Y (p)

()

()

- (X - X) = F p

l

$$\begin{aligned}
 &(2) \\
 &X \\
 &= Q p Y \\
 &2 \\
 &2
 \end{aligned}$$

$$\begin{aligned}
 &- \\
 &= \\
 &2
 \end{aligned}$$

$$\begin{aligned}
 &(p) \\
 &() \\
 &(\\
 &X) \\
 &() \\
 &f^2 (, \\
 &p X)
 \end{aligned}$$

There is thus a system of 5 equations to 5 unknown factors:

$$\begin{aligned}
 &() 1 \\
 &(2) \\
 &, p, X, X, X
 \end{aligned}$$

The second equation is also written:

$$\begin{aligned}
 &1 \\
 &2 \\
 &- X \\
 &p N \\
 &p N \\
 &= \log \\
 &+ 1+
 \end{aligned}$$

K

$$\begin{aligned}
 &\& T \\
 &\&0 T \\
 &0
 \end{aligned}$$

By using the first equation, one can express X according to p:

- X =

p

E -

E

X = - X

1

2

p N

p N

X = F (

log

1

1 p) =

p

E -

-

E

X - K

+

+

0 & T

0 & T

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In addition, by integration successive of the functions f2 and f1 one can also bring back oneself to one equation utilizing only X and p:

$Q Y (p)$

(2)

-

-

p

2

2

(

X

)

()

$X =$

$1+ Q p$

2

-

Q

1

2 -

2

p

-

-

-

1
 1
 $(Y(p)())$
 $()$
 $(X X X)$
 $()$
 $X =$

$1 + Q p$
 1

$()$
 $()$

like $X = F(p, X, X 1)$, and $X 1 = G(p)$ according to the preceding expressions one can write: $X = F2(p) = F1(p)$. The equation to solve to find p is thus:

$F(p) = F$
 0
 $2(p) - F1(p) =$

Once calculated p , one obtains the constraints by: =

$E p$
 $E -$

7.3 Variables

interns

They are 5:

$V1 = p$
 $V2 = X(1)$
 $V3 = X(2)$
 $V4 = X$

In Code_Aster, one adds a last internal variable: $V5 =$ neutron fluence calculated with no current time.

7.4

Identification of the parameters of the model

The identification of the parameters is carried out in the reference [bib7]. It relates to the ZIRCALOY 4 with 350°C .

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Method to use in 1D all the behaviors 3D

As for the treatment of the plane constraints [R5.03.03], it is possible to profit for modelings 1D of the behaviors available in 3D. One extends for that the method due to R. of Borst with the case 1D, by treating this condition (unidimensional stress field) not with the level law of behavior but on the level of balance. One obtains thus during iterations of the algorithm of STAT_NON_LINE of the stress fields which tend towards a field one-way. It is checked, with convergence of the total iterations of Newton, that the fields of constraints are indeed one-way, except for a precision, if not the iterations are continued. The method consists in breaking up the fields of strains and stresses into a part purely one-way (direction X) and a part relating to the other directions, and to carry out one static condensation by writing that components of the constraints relating to the other directions are null. One does not consider in the tensors (order 2) only the diagonal terms, written under form vectors with 3 components. Direction X corresponds to the direction of the element (bars, multifibre beam) or with the direction of the reinforcements of grid. At one unspecified moment of the resolution of incremental behavior, the tangent operator D connects the increase in constraints to

the increase in deformation by:

$D =$

$D =$

Dd

that one rewrites:

D

X

D

D

D

11

12

$13 D$

X

$D y =$

D

D

D

21

22

$23 D Y$. By writing these increases like the difference between

D

D

D

D

Z

31

32

$33 D Z$

iterations N and $N + 1$ of Newton, one obtains:

$n+1$

N

$n+1$
 N
 $n+1$
 N
 $D =$
 $- =$
 $-$
 $-$
 $, D =$
 $-$

With convergence, this variation must tend towards zero.

By introducing the conditions

$N 1$
 $+$
 $N 1$
 $: y = 0$
 $+$

*and 2 (one-way behavior), one obtains,
for the iteration $N + 1$:*

1
 1
 D
 $n+$
 $X - N$
 $n+$
 X
 $X - N$
 X

$X D$
 D
 D
 11
 12
 $13 D$

X
 $n+1$
 N

N

$$Dy = y - y =$$
$$-y$$
$$=$$

D
 D
 D
 21
 22
 $23 Dy$

D
 $n+1$
 N

N
 D
 D
 D

Z
 31
 32
 33
 D

Z
 $-Z$
 $-$

Z

Z

The two last equations make it possible to express $D y$ and $D Z$ according to $D X$:

$$\begin{aligned}
 &1 \\
 D y = & \\
 (N & \\
 - - D & \\
 y & \\
 21d & \\
 - D & \\
 X & \\
 23d Z) &
 \end{aligned}$$

D22

$$\begin{aligned}
 &1 \\
 D Z & \\
 = & \\
 (N & \\
 - Z - 31 & \\
 D D X - 32 & \\
 D D y) &
 \end{aligned}$$

D33

$$\begin{aligned}
 &1 \\
 \text{that is to say} & \\
 D y = (& \\
 N & \\
 N & \\
 - 33 & \\
 D + D & \\
 y & \\
 23 & \\
 + D D & \\
 Z &
 \end{aligned}$$

y X)

l

D Z = (

N

N

- 32

D - D

y

22

+ D D

Z

Z X)

with

= 33

D D22 - D23 32

D, D = D

y

23 31

D - D21 33

D, Dz = 32

D D21 - 31

D D22

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by deferring these expressions in the first equation, one obtains:

$$\begin{aligned}
 & D D \\
 & 1 \\
 & 12 \\
 & + D D \\
 & 13 \\
 & \\
 & n+ \\
 & N \\
 & y \\
 & Z \\
 & D D \\
 & 12 \\
 & 23 - D \\
 & D \\
 & 22 13 \\
 & N \\
 & D D \\
 & 12 32 - D D \\
 & 12 33 \\
 & N \\
 & X = X + D 11 + \\
 & D X + \\
 & Z + \\
 & y
 \end{aligned}$$

Balance with the iteration $N + 1$ is written:

$$\begin{aligned}
 & D D \\
 & T \\
 & N 1 \\
 & T \\
 & N 1
 \end{aligned}$$

T
12
y + D D

+
B
FD =
+
B
FD
13
X
=
Z
B D11 +

D X

+ T N D D
12
23 - D
D
22 13
N
D D
12 32 - D D
12 33
N
B X +
Z +
y FD

= N n+
K of 1 + T N D D
12
23 - D
D
22 13
N

D D
12 32 - D D
12 33
N
B X +
Z +
y FD

It is thus noted that the taking into account of the unidimensional behavior intervenes to two levels:

· in the tangent matrix, by the corrective term:

D D
12
+ D D
BT
y
Z B FD

13

· in the writing of the second member, by the corrective term:

BT

((D D - D D
12
23
22 13) N
Z + (D D - D D
12 32
12 33) N
y) FD

To implement this method, it is enough to calculate these corrective terms and to add them to constraints and tangent matrix obtained of the resolution 3D of the behavior. For that it is necessary to store information of an iteration of Newton to the other, by the means of 4 variables additional interns. The stages of the resolution are:

1) with

the iteration $N + 1$, the data is:

$n+1$

-

-

U

, and the 4 variables intern (calculated with

I

D

$V1 = N$

$y + ($

N

$D23 Z -$

N

$D33 y - Dy N$

$X), V 2 = y,$

iteration N):

,

I

$V 3 = N$

D

$Z + ($

N

$D32 y -$

N

$D22 Z - Dz N$

$X), V 4 = Z$

2) before carrying out the integration of the behavior (carried out into axisymmetric) one calculates

$n+1$

N

I

y

$=$

$y + ($

N

N

$- D$

$33 y + D$
 $23 Z + Dy D X)$

,

$n+1$
 N

1
 Z
 $=$
 $Z + ($
 N
 N
 $- D$
 $32 y - D$
 $22 Z + Dz D X)$

3) the integration of the behavior provides constraints $N 1$
+

and the tangent operator D ,

4) one modifies the second member and the tangent matrix as indicated above,

5) one stores the new variables intern and one checks if

$n+$
 1
 Z
 $<$
 $n+$
and 1
 y
 $<$, with =

$n+$
 1 ,
 X
 $=$
 $RELA$
 $RESI_INTE_$

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Élasto-viscoplastic relation of behavior

LMARC for the tubes of sheath of the pencil

fuel

Summary:

A model was developed with the LMA-RC of the university of Besancon (Laboratory of Mechanics Applied R.

Chaléat) to describe the élasto-viscoplastic behavior of the tubes of sheath of the fuel pin. It the taking into account of the strong mechanical anisotropy of the tubes in Zircaloy 4 allows.

It is established in Code_Aster under the name of LMARC; the equations of speed are integrated numerically by an implicit scheme of Euler in environment PLASTI.

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1 Introduction

The tubes of sheath in Zircaloy of the fuel pin of the power stations REFERENCE MARK present a

behavior

anisotropic and strongly viscous mechanics. A phenomenologic model was developed with LMA-RC of Besancon [bib1] to obtain a fine description of the behavior of material in sight to evaluate realistic states of stresses in situation of Pellet-sheath Interaction (IPG).

Being given the crystallographic texture of the tubes, one makes the assumption of orthotropism. The model was

confronted with the experimental results on tubes of sheath in Zircaloy 4 in various states metallurgical and subjected to loadings plain and multiaxés [bib1], [bib2], [bib3]. The model is initially intended to be used within the framework of the coupling between Code_Aster and the code of fuel pin CYRANO3. It could however be used for other metallic materials presenting an orthotropic viscoplastic mechanical behavior.

The model is introduced into Code_Aster in 3D, plane deformations (D_PLAN), and axisymetry (AXIS) under the name of LMARC. It is about a unified viscoplastic model with internal variables: cumulated viscoplastic deformation and three variables of kinematic work hardening.

The taking into account of the anisotropy is carried out by four tensors of a nature 4 affecting the sizes equivalent mechanics but also laws of evolution of the internal variables.

One presents in this note the equations constitutive of the model and his establishment in Code_Aster.

2

Formulation of the model

2.1 Tally

theoretical

The model of behavior developed to the LMA-RC lies within the scope of the thermodynamics of irreversible processes and of the mechanics of the continuous mediums. It is about a model elastoviscoplastic unified, i.e. dependent inelastic deformations or independent of time are gathered in only one term. By considering the assumption of small disturbances, one divides the tensor of the deformations into an elastic part, a thermal part and a viscoplastic part:

$$= E + HT + vp$$

The elastic part is given by the law of Hooke, the anisotropy of behavior which can be neglected in this case. The concept of surface of load used in plasticity is replaced by a family of equipotential surfaces: they are surfaces of the space of the constraints in each point of which the module the speed of deformation is the same one (dissipation is the same one) [bib4]. Being given texture tubes of sheath of the fuel pin, one can make the assumption of orthotropism of mechanical behavior and one use a formulation of the Hill type to describe surfaces equipotential:

3

$$F =$$

ij

ij

ijkl kl

kl

R

R
2 (~ - X) M
(~ - X) - 0 = - X - 0
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where

~

1

= - tr () I D

is the deviatoric part of the tensor of the constraints.

3

X

a kinematic variable of work hardening (tensorial).

M

tensor of order 4 for the description of the anisotropy

(with the formulation of Hill, only 6 coefficients are independent).

0

R

initial elastic limit.

The direction of evolution of the viscoplastic tensor of deformation is given by the rule of normality on equipotential surfaces:

vp

F

! ij =! v ij

where v represent the cumulated viscoplastic deformation, obtained starting from the equation of state of which

formulation was established in experiments with the LMA-RC [bib1]:

N

F

v = sinh

0

K

with S

X = 0 I X 0 and X = X if X 0

To entirely define the model, it remains to give the equations of evolution of the variables of work hardening representing the state of internal stress of the material which is opposed to the deformation

(constraints induced by the interactions on various scales between mobile dislocations and substructure). The kinematic work hardening of nature is described in the model via three nonlinear variables.

m

2

vp

1

X

X

! X

mn

ij = p

Y (v)

()

Nijkl! - Q

kl

ijkl

3

(X - X

kl

kl)! v

R sinh

NR R

m

ijkl

klmn

-

X0

X

()

2

! 1

1

2

X

vp

ij = p

Y

1

(v)

()

()

Nijkl! - Q

kl

ijkl

3

(X - X

kl

kl)! v

(2)

2

!

2

X

vp

ij

= *p*

Y

2

(*v*)

()

Nijkl! - *Q*

X

kl

ijkl

kl

! *v*

3

3

with

X =

X R

X

2

ij

ijkl

kl

Xij ()

()

O = *X 1*

ij ()

(2)

O = *Xij* ()

O = *O*

(

$Y) = Y$

- v

v

+ $(Y_0 - Y) E B$

(

$Y v)$ makes it possible to describe hardening or softening under cyclic loading.

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These equations comprise for each variable of kinematic work hardening a first term of linear kinematic work hardening compared to the deformation and a second term of restoration dynamics. Lastly, the first equation comprises a third static term of restoration for to take into account the effects dependent on time.

For the description of the anisotropy, one distinguishes an intrinsic anisotropy with the structure of material

(form of equipotential) and an additional anisotropy induced by the viscoplastic flow.

The anisotropy is introduced into the model of the LMA-RC by the means of tensors of order 4 in relations between the various tensorial variables. Anisotropy induced by the flow

viscoplastic is translated in the laws of evolution of the variables of work hardening. Terms of these equations are connected to different mechanisms of deformation in material: work hardening linear kinematics, dynamic and static restorations. The taking into account of the induced anisotropy by the viscoplastic deformation is thus made by the introduction of three distinct tensors NR, Q and R.

Bene foot-note:

The model suggested by the LMA-RC [bib2] is without threshold. The initial elastic limit R0 was added during integration in Code_Aster to widen the possibilities of the model. It is enough to consider a zero value to work with a model without threshold.

2.2

Description of the tensors of anisotropy

To simplify the writings, thereafter a matric notation, image of the notation are used

tensorial intrinsic in a orthonormé reference mark. Moreover, one uses notations of a lower order (tensor 2 = vector, tensor 4 = matrix), in order to be identified with what is in the code. Because of symmetry of the handled tensors of order 2, one reduces them to vectors (1 X 6) by multiplying them components of shearing by root of 2.

$$T = [1 = 11, 2 = 22, 3 = 33, 4 = 2\ 12, 5 = 2\ 13, 6 = 2\ 23]$$

$$T = [1 = 11, 2 = 22, 3 = 33, 4 = 2\ 12, 5 = 2\ 13, 6 = 2\ 23]$$

According to cylindrical co-ordinates' related to the tube, one considers thereafter $11 = rr$, $22 = \theta\theta$, $33 = zz$.

As we mentioned in the first chapter, being given the crystallographic texture of tubes, one can make the assumption of orthotropism and confuse the axes of anisotropy with the axes materials. The conditions of symmetry which result from it lead to nine independent components for each tensor of anisotropy. The incompressibility of the viscoplastic flow is translated by three additional relations and the component count independent is tiny room to six [bib1].

With the matric notation, each of the four tensors takes a form identical to that of M is:

M

M

M

11

12

13

0

0

0

M

M

M

12

22

23

0

0

0

$$M_{11} + M_{12} + M_{13} = 0$$

$$M_{13} = 0$$

M

M

M

13

23

33

0
0
0

$M =$
with $M12 + M22 + M23 = 0$

0
0
0
 M

44
0
0

$M13 + M23 + M$

$33 = 0$
0
0
0
0
 M
0

55

0
0
0
0
0
 M

6
6

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To find the isotropic version of the model, it is necessary to take the following values for the four tensors:

2

1

M

$= M = M =, M = M = M = - \text{ and } M = M = M =.$

11

22

33

12

13

23

44

55

66

1

3

3

In experiments, one cannot reach in our study the components 44, 55 and 66 which correspond to shear tests. Within the framework of the work of thesis realized to the LMA-RC, only component 66 was determined by tests of traction-torsion, the two last (R and rz.) not being able to be reached because the low thickness of the tubes.

Being given the preceding considerations, only components 11, 22, 33 and 66 are put in reading of the command file, the other components being is given starting from the equations had with the plastic incompressibility, is taken equal to the isotropic values for the components of shearing.

Bene foot-note:

The matric notations used in the references [bib 1, 2 and 3] are those of Voight.

Only the terms of shearing are influenced; conversion to work with notations of Code_Aster is obtained using the following formulas (I = 4,5,6):

1

1

$M =$
 $MVoight, Q = QVoight, NR = 2 NVoight \text{ and } R =$
 $RVoight$

II
 II
 II
 II
 II
 II
 II
 II

2
 2

2.3

Equations of the model

$$= E + HT + vp$$

$$= A(T) E$$

3

$$F =$$

$$(\sim - X) T M (\sim - X) - R0 = \sim - X - R$$

2

0

\sim

vp

F

3

$$M(-X)$$

$$! = v!$$

$=$

v

$!$

2

$\sim - X$

if F,

0

$()$

1

(2)

$v! =,$

$$0! X =! X$$

$$=! X$$

$$= 0$$

if $F >$,

0

N

2

$vp T$

$-1 vp$

F

$v! =! 0$

$(!) M! =$

$! 0 \sinh$

3

K

$X m$

!

2

X

$X =$

p

$Y (v)$

vp

$()$

$NR! - Q$

1

$rm \sinh$

$NR R$

3

$(X-X) v!$

-

$X0$

X
 $()$
 $!$
 2
 2
 $X 1 =$

p
 Y
 1
 2
 2
 2
 1
 (v)
 vp
 $()$
 $()$
 $NR! - Q$

3
 $(X - X) ()$
 v
 p
 Y
 $!$
 $! X$
 2
 (v)
 vp
 $()$
 $NR!$
 $Q X$
 $v!$

$=$
 $-$

3

with

(

3

$Y v) =$

$- b v$

T

$Y + (Y0 -$

$Y) E$

$X =$

$X R X$

2

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Bene foot-note:

In Code_Aster, the whole of the parameters of the model

R! , K, N, Y, B X, R, m, p, p, p

0

0

0

1

2, M, NR, Q, R (*I*

m

II

II

II

II

= 1, 2, 3,)

6 can

to be a function of the temperature.

2.4 Relation

LMARC

The model is accessible in *Code_Aster* in 3D, plane deformations (D_PLAN), and axisymetry (AXIS) starting from key word COMP_INCR of order STAT_NON_LINE. The whole of parameters of the model is provided under the key word factor LMARC or LMARC_FO of the order DEFI_MATERIAU [U4.23.01].

/

LMARC: (

R_0

:

R0

DE_O

:

0
NR
:
N
K
:
K
y_0
:
yo
y_I
:
y
B
:
B
A_0
:
X0
RM
:
rm
M
:
m
P
:
P
P1
:
p1
P2
:
p2
M11
:
M11
N11
:
N11
M22
:
M22

N22
:
N22
M33
:
M33
N33
:
N33
M66
:
M66
N66
:
N66
Q11
:
Q11
R11
:
R11
Q22
:
Q22
R22
:
R22
Q33
:
Q33
R33
:
R33
Q66
:
Q66
R66
:
R66
)

3
Establishment of the model in *Code_Aster*
3.1

Algorithm of resolution of the quasi-static problem

One seeks to check the balance of the structure at every moment. In incremental form, it is about one nonlinear problem whose variational formulation in the case of the small deformations can be put in the form:

To find U such as:

kinematically

$$((U + U), T) D = L(T)$$

acceptable and T

$$\text{Drunk} = ud$$

(T) where U indicates the field of displacement, Bu

ud

=

(T) corresponds to the boundary conditions in displacement and $L(T)$ are the loading at the moment T .

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One is thus led to solve, for each increment of time T :

$$F_{t+t}(U + U)$$

T

) = 0 on the basis of a state with balance F

= 0

0

U

being the increment of the solution U on T

, C being known

The diagram general adopted by *Aster* to solve this discretized total system is a method of Newton [bib5] which is written, K being an indication of iteration:

$F D$

$$(U$$

 $K) = - F (U$

$K)$

U

K

U

$$+1 = U$$

$+ D$

K

K

$(U$

$K)$

This diagram requires, starting from the estimate of displacements to the iteration K , to calculate in each not Gauss:

$T T$

+ which checks the law of behavior

$$M C t + T =$$

the operator of tangent behavior

$t + t$

F

T

$$= K = K \text{ with } K =$$

B

Data base

U

E

E

E

E

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3.2 Environment

PLASTI

It is thus necessary, with each total iteration and in each point of Gauss, to integrate the equations model described in [§2.3] for the calculation of $t+t$ and to calculate the operator of behavior tangent.

An environment was created in *Code_Aster* with an aim of parameterizing the establishment of models elastoviscoplastic presenting a function threshold (field of elasticity).

This algorithm:

- manages the choices of integration elastic or (visco) plastic,
- proposes various routines to contribute to the resolution of the nonlinear system (local) formed by the equations of the model,
- updates the variables at the end of the increment,
- calls the routines user for the calculation of the operator of tangent behavior.

The step to establish a new model can be schematized in the following way:

Writing of the equations of the model of speed

$\dot{y} = F(y, T)$

Choice of a diagram of integration

Writing of the system discretized R

$(y) = 0$

Writing of the routines specific to the model:

- recovery of the data materials,
 - evaluation of the function threshold,
 - evaluation of the operator of tangent behavior
 - routine for the resolution of the system R
- $(y) = 0$
(the algorithm proposes a method of Newton for one implicit nonlinear system).
+ Modification of the routines of shunting of the algorithm

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3.3

Discretization of the equations of the model

If the increment of time corresponds to a loading elastoviscoplastic, a diagram is used implicit of Euler whom one solves by a method of Newton.

In discretized form, the system of equations is written:

(

F

G) - H

-

v

$= 0$

$t+t$

$t+t$

(

2

F

L) X

p

$Y(v) NR$

(

Q X X1)

v

3

-

-

$T +t$

m

X

X

+ R

sinh

NR R

T

m
 $= 0$

X0

X
T

+t
(
2
F

D) X1-

p
 Y
1
(v) NR
Q (X X
1
2)
 v
=

0
3

-
-

$t+ T$

(
2
F

J) X2-

p

Y

2

(v) NR

Q X

v

2

0

3

-

=

$T + T$

N

(

F

$K) v$

-

0! sinh

T

= 0

K

with $Y(v) = Y$

-

+ ($Y0 - Y$) $E bv$

3

T
 $X =$
 $X R X$
 2
 In more contracted way, one poses:

$G(y)$
 $)$

L
 (y)
 $)$

X
 $F L(y)$
 $) = 0 = I(y)$
 $)$
 with
 y
 $= X$

1
 J
 (y)
 $)$

$X2$

K
 (y)
 $)$
 v

One solves this system by the method of Newton proposed in environment PLASTI, that is to say:
 Fl

$D(y)$
 L
 $K) = - F(y)$
 $K)$
 y
 K
 y
 $+1 = y$
 $+ D$
 K
 K
 $(y$
 $K)$

While reiterating in K until convergence.

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The resolution requires the calculation of Jacobien of the local system $F L$. His general expression is given

hereafter; analytical calculations block per block are given in appendix (cf [§An1]).

G

G

G

G

G

X X

1
X2 v

L
L
L
L
L

X X1 X2 v

I
I
I
I
I
J=

X X1 X2 v

J
J
J
J
J

X X

1
X2 v

kT
kT
kT
kT
K

$X X$

1
 $X_2 v$

3.4

Operator of tangent behavior

The formed system of the equations of the model written in discretized form ($F_l(y)$) =)

0 are checked in

end of increment. For a small variation of $F L$, by regarding this time as variable and not like parameter, the system remains with balance and one checks $dF L = 0$, i.e.:

$F l$

$F l$

$F l$

$F l$

$F l$

$F l$

+

+

X_+

X_{1+}

X_{2+}

v

= 0

X

X_1

X_2

v

This system can be still written:

H

X
0
FL

(y
) = X, with y
= X and
1
X = 0
y

X2
0
v

0
By successive substitution and elimination (cf [§An2]), one deduces from it that:
K = H
from where the required tangent operator:

C
-1

M
KH

=

=

$t+t$

The preceding equations show that one is led to re-use the same matrix jacobienne J that previously to evaluate the tangent operator. This operator is known as coherent (insinuation with the system of integration) and still noted MC.

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In the case of the elastoplastic models, one can also calculate the tangent operator said of speed (MV) starting from the equations of the model of speed [bib7].

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P. GEYER

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Appendix

1

Expression of Jacobien of the equations

elastoviscoplastic integrated

That is to say thus to evaluate the terms of the hypermatrice jacobienne J at the moment $T + T$

G

G

G

G

G

X X

1

X2 v

L

L

L

L
L

X X1 X2 ν

I
I
I
I
I
J =

X X1 X2 ν

J
J
J
J
J

X X

1
X2 ν

kT
kT
kT
kT
K

X X

1
X2 ν

G

2 F

G

2 F

= I + H

v

= H

v

2

X

X

t+t

t+ v

G

G

G

F

=

= 0

=

H

X1 X2

v

t+ T

L

2

2 F

= - p Y (v)

NR

v

3

2 $t+ T$

L

m

2

2 F

X

$NR R$

$= I p Y(v)$

NR

v

$+ (pQ)$

v

$+ R$

\sinh

T

X

3

X

$t+t$

m

$X0$

X

$t+t$

$t+ T$

$t+$

T

m

T
X 1
m
m

R X
m 1
X
3
-
(
)
+ - *R sinh*

+ *R*
cosh

m
X

NR R X
T

X
m
2

0
X
m

X
X
2
0
0

X

$T + t$
 $t + T$

L

= -(
L
p Q)
v

= 0
X
 $t + T$
1

X2
L
2
F

Y

= - p NR
Y

$T + t (v)$
 $v + p Q (X X1)$
v

3

+

v
 $t + T$
 $t + T$
 $T + T$

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I

2

2 F

= - p Y

1

(v)

NR

v

3

2 t+ T

I

2

2 F

= - p Y

1

(v)

NR

X

3

X
v
t+ T
I = I + (pIQ)

v
X
t+t
1
I = - (pIQ) v

X2
I
2
F

Y
= - p NR
Y

1 Q (1
X - X2)
1
t+ T (v)

v
+ p
v
3

+
t+ T
+

v
T T
t+t

2

2

J

2

F

J

2

F

$$= -p Y v$$

v

$$p Y v$$

2

() NR

= -

() NR

v

2

2

3

X

3

X

t+ T

t+ T

J

J

= 0

= I + (2

p Q)

v

1

X

X²

t+t

J