2
F

Y

= -

+ (*p2* Q X2)
2
p NR
T
Y T (*v*)
v

v

3

+
+ *v*
t+
t+*t*

T + *T*

T
K

F *n-1* 1
F
F
K
= -

N 0!

sinh

cosh

T
K

K

K
= - X
T +t
K
K
K
=
= 0
= 1
X1 X2
v

with *I the* matrix of identity.
It appears in the preceding expressions the derivative first and seconds of the expression of equipotential surfaces F compared to and X. Their evaluations hereafter are given:
F
3
(~ - X)
= M
2
~ - X
2 F
1

F
F *T3*
=

M
X
~ - X

-

2

2 F
1
3
F
F *T*
=
2
M D -

~ - X 2

---

with D tensor unit in the space of the diverters:
2
1
1
-
-

0 0

0
3
3
3
1
2
1

-
-
0 0

0
3
3
3

D = 1
1
2
-
-

0 0 0
3
3
3

0
0
0
1 0 0

0
0
0
0 1 0

0
0
0
0 0 1

Note:
For detailed examples of calculations of these expressions, one will refer to the reference [bib7].

Appendix 2 Evaluation of the coherent tangent operator MC

While regarding as variable in the discretized system, one can write:

G
G
G

0
0

X
v

L
L
L

L
D

H *D*

0

X X1
v

D X
0
I
I
I
I
I

X 0

X X
1 =
1
X2
D
v

J
J
J
J
D X2

0

0

X
X

0
2
v

D v

K T
kT
K

0
0

X
v

While operating by successive eliminations and substitutions, the fourth block of the system of equation give:

J
1
-

J
J
J

D X = -

D

2
+
D X+
D v

X2
X
v

While posing:
I J -

1
B =

X
2 *X*
2
and by replacing *D X*
2 in the second and third block of the system of equations, one obtains:
L
L
L
L
D

D X
D X1+
D v

0

+

+ X1

v

=
X
éq An2-1
I
J
I
J
I
- *B*
D

B
D X
D

X

+
-

X
X
+ X
1
1
éq An2-2
I
J
+
- *B*
D v

0

v

v

=
the equation [éq An2-2] gives:

I
1
-

I
J
I
J
I
J

D X = -

- *B*
D
1

B
D X

B
D v

X

+
-

+

-

1

X
X
v

D v

While posing:

L I -

1
C =

X X
1
1
and by replacing *D X*
1 in the equation [éq An2-1], one obtains:
L
I
J
- *C*
- *B*
D

L
I
J

+
- *C*
- *B*
D X
éq An2-3

X

X
X
L
I
J
+
- *C*
- *B*
D v

0

v

v

v

=
The fifth block of the system of equation gives:
K -1 *K T*
K -

1 *kT*
D v
= -
D
D

X
v

-

v
X
While posing:
L
I
J K
F =
- C
- B

v

v

v
v

and by replacing *D v*
in the equation [éq An2-3], one obtains:
L
I
J
kT

- C
- B
F
D

-

éq An2-4
L
I
J
kT
+
- *C*
- *B*
F
D X = 0
X

X
X -

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P. GEYER
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While posing:
L
J
I
J

kT
D =
- *A*
- *C*
- *B*
F

-

and
L
J
I
J
kT
E =
- *A*
- *C*
- *B*
F

X
X

X
X - X
the equation [éq An2-4] is written:
D D + *E D* X = 0
from where by replacing *D X*
and *D* in the first block of the system of equation, one obtains:

G
G K -1 *kT G*
G K -1 *kT*

-1

-

-
-

E D D

= H *D*

v

v

X *v*

v

X

Finally, the coherent tangent operator is written:
-1
T
-1
T

-1
G
G K
K
G
G K
K

-1

M =
-

E D H
C

v

v

-
-

X *v*

v
X

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R5.03.11 document

Mono behaviors elastoviscoplastic and polycrystalline

Summary:

The goal of this document is to describe the integration of the mono and polycrystalline behaviors, while specifying
independent way the criterion, the flow, work hardening etc

One treats here integration of these laws of behavior associated with systems with slip corresponding to the usual crystalline families. This integration can be made (method explicitly of Runge_Kutta with control of the precision and local recutting of the step of time) or an implicit way
(method of Newton with local recutting of the step of time).

These behaviors can be employed for the calculation of microstructures (grid of an aggregate, with geometrical representation of each physical grain) or for calculation of polycrystals, mediums "homogenized" having in any material point (or not of integration or calculation) several phases

simultaneous, in variable proportions.

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1 Introduction

The objective general of the development of the "microphone-macro" functionalities in Code_Aster is of
to be able to integrate in a modular way of the models into several scales (with a possibility of choice laws of behaviors, rules of localization, types of microstructures, bond between
not integration in the element and the "module law of behavior"). What can carry out to
types of calculation different (polycrystalline calculations, use of a law of the Berveiller-Zaoui type or one
standard law "regulates in", calculations of aggregates multi-crystalline lenses with grid of a microstructure,…).

The step presented here consists in allowing decoupling, by modularity, of the various elements *who constitute a law of behavior. This flexibility is accessible directly to the user. Of more, for the developer, it is possible to add a law of behavior (macroscopic or microscopic) by simply defining the derivative partial of the problem, in terms of calculation of constraints and of internal variables. This is sufficient if one is satisfied with an explicit integration; for an implicit integration, it is necessary to define in more the tangent operator.*

More precisely, for the aspect behavior of monocrystal, the modularity is total on the level of calculation "not material": the material, represented by some homogeneous equations in the case of them
macroscopic phenomenologic models, is now more complex: for a finite element given, it consists of a monocrystal having an orientation given, and having some a number of systems of slip. Each family of systems of slip has her own law of behavior local.

In the case of a polycrystalline model, one supposes that in a material point (not of integration of one finite element), several metallurgical phases are present simultaneously, each phase being able to consist of grains with orientations given, each grain having a certain number systems of slip (not inevitably the same ones for each phase). The representation of material can also include the shape of the grains and the type of phases involved, inducing such or such type of rule of transition from scale. Each family of systems of slip has her characteristic local law of behavior. One finds a separation between the crystallographic structure, the law of crystalline viscoplasticity and rules of transition from scales. This mode of separation is also wide to the law of viscoplasticity itself, with a separation enters elasticity, the criterion and the law of flow. The representation of material can also include the shape of the grains and the type of involved, inducing such or such type phases of rule of transition from scale.

2
Formulation of the mono and polycrystalline behaviors

2.1

Relations of behavior of the monocrystal

The behavior related to each system of slip of a monocrystal is (in the whole of behaviors considered) of élasto-visco-plastic type. Owing to the fact that one is interested each time in only one direction of slip, the behavior is mono dimensional. It can break up into three types of equations:

· Relation of flow:
& = G (, p), with, p& = and
S
S
S
S
S
S
& S
for an elastoplastic behavior a criterion of the type: F (, p) and F.p&

S = 0
S
S
S
S
0
for a élasto-viscoplastic behavior, p& = F, p
S
(S S S S)

· Evolution of kinematic work hardening: & = H (, p)
S
S
S
S
S

· Evolution of the isotropic work hardening defined by a function: R (p)
S
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These relations become, after discretization in time:

· Relation of flow:

= G (, p), with, p
=

S
S
S
S
S
S
S
for an elastoplastic behavior a criterion
type
: F (, p)
and
S
S
S
S
0
F.p

S = 0
and for a élasto-viscoplastic behavior, p
= F,

S
(

p
S
S
S
S) T

· Evolution of kinematic work hardening:
= H (, p)
S
S
S
S
S

· Evolution of isotropic work hardening: R (p)
S

The quantities (, p) are evaluated at the moment running for an implicit discretization and with
S
S
S
S
the previous moment for an explicit discretization.

To fix the ideas, here examples of relations of viscoplastic or elastoplastic flow, and of work hardening. The names of these relations correspond to their name in the order DEFI_MATERIAU [U4.43.01].

2.1.1 Examples of relations of flow

ECOU_VISC1
- C
S
S
= G (, p) = p

S
S
S
S
S
S - C
S

S
N

- C - R (p)
S
S
S
S
p
= T
.
S
K
The parameters are: C, K, N.

ECOU_VISC2
- C - has
S
S
S

= G (, p) = p

S
S
S
S
S
S - C - has
S
S
S
N
D
- C - - R (p) has +
(C
2
)

S
S
S
S
S

S
C
p
2
=
S
K
The parameters are: C, K,
N has, D.

ECOU_VISC3

-

-

*

G

V *

0

= G, p
S
=
S

μ
& exp
exp
.
,
S
(S S S S) 0 kT

kT
S

The parameters are: K,
*

,
μ &, G
, V

0
0

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ECOU_PLAS1
F (, p)
C
R p
,
S
S
S
S
= S - S - ()
S
S
0
F.p
,
S = 0
F

= p

S
S

S
The associated parameter is: C.

2.1.2 Examples of relations of kinematic work hardening

ECRO_CINE1
= H
(, p) = - D p

S
S
S
S
S
S
S
S
The parameter is: D.

ECRO_CINE2
C

S
m
S

= H
(, p) =
- D p
- (
)

S
S
S
S

S
S
S
S
M
S
Parameters being then: D, M, m, C.

2.1.3 Examples of relations of isotropic work hardening

One taking the very simple shape of the matrix H translating the interaction enters the systems of slip credits, isotropic work hardening can be form:

ECRO_ISOT1
NR
R
R (p) R
Q (
H 1
(
E
))
S
S
= 0 +
-
- LP
Sr
r=1
with:
H = 1
(-) +
Sr
Sr
Sr
The parameters are: R, Q, B, h.
0

ECRO_ISOT2
1s
2s
R (p) = R + Q (H Q) + Q Q
S
0

1
rs
2
sg
with:
dqis = B 1
(- qis) dp
I
The parameters are: R, Q, B,
H Q.
0
1
1
2

2.2
Systems of slip and total behavior of the monocrystal

A monocrystal is composed of one or more families of systems of slip, (cubic, octahedral, basal, prismatic,…), each family including/understanding a certain number of systems (12 for the octahedral family for example).

To each family of system of slip are associated a law with flow, a type of work hardening kinematics and isotropic, and of the values of the parameters for these laws. In other words, one does not envisage
not to vary the relations of behavior or the coefficients within the same family of systems of slip. On the other hand, from one family to another, the laws of behavior can change, as well as the values of the parameters.
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A system of slip is determined by the tensor of orientation
S
m, built from
crystallographic definitions of:

· direction of slip (defined by the unit vector L)
· and of the normal in the plan which slips (definite by unit vector N).

S
1
m = (N L + L N)
ij
2
I
J
J
I

From the point of view of the behavior at the material point, this tensor intervenes for the calculation of the scission
reduced S
S
= m
:
and that the speed of total viscoplastic deformation vp
E, defined to leave
ij
ij
knowledge speeds of slip S
& for all the systems of
slip: vp
E&
S
m

ij
ij
=
S
&

S G
Moreover, the monocrystal can be directed compared to the total axes of definition of the co-ordinates.
This orientation is defined for each mesh or groups meshs (typically for each grain)
by the data of 3 nautical angles. Components of the tensor of orientation
S
m, defined in
reference mark related to the monocrystal, are then expressed in the total reference mark by using these nautical angles.

2.3 Behavior of the homogenized polycrystal

In the case of homogenized polycrystal, it is necessary to define each single-crystal phase by sound orientation, its proportion (voluminal fraction) and the associated behavior. It is necessary moreover define one
regulate localization.
The single-crystal behavior is built like previously starting from the behavior
preceding elasto-visco-plastic and of the data of families of systems of slip.

2.3.1 Recall of what exists

Code_Aster has, since version 4, one only polycrystalline law of behavior
(POLY_CFC), specific to steels C.F.C, (thus having 12 systems obligatorily of
slip), and limited to 40 grains (40 definite phases each one by a voluminal fraction and one orientation). The law of behavior is fixed (élasto-visco-plasticity, with kinematic work hardening nonlinear), and the 2 methods of localization and homogenisation are that of Berveiller-Zaoui, and that of Pilvin-Cailletaud. The introduction of the orientations of the phases, the voluminal fractions and of
orientations of the systems of slip is done using operator DEFI_TEXTURE. This operator
create a table, which is provided to DEFI_MATERIAU, in complements of the parameters of the law of
behavior [R5.03.13]. This is validated in test SSNV125.

2.3.2 Behavior of the POLYCRYSTAL type

In addition to the single-crystal behavior describes previously, one adds a scale of modeling,
who represents that of the phases.

On the level of a point of Gauss, there are always the relations of elasticity on
total tensors (homogeneous):

· total deflection macroscopic E

· viscoplastic deformation macroscopic VP
E
· macroscopic constraint:

= D (
HT
vp
E - E - E)
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· Moreover, knowing the whole of the internal variables relating to the systems of slip of each phase, parameters of behavior of each phase, orientations and voluminal fractions of each phase, and the type of method of localization,

for each single-crystal phase (or "grain"), defined by an orientation and one proportion fg, a relation of localization of the constraints, general form (to be expressed in the local reference mark of each phase)

= L, E,
G
(vp vpg G)

and for each system of slip of each phase, of the relations of behavior relating to each system of slip, similar to the case of the monocrystal:

Relation of flow:
& = G (, p), with, p& = and p& = F, p

S
(S S S S)
S
S
S
S
S
S
& S
Evolution of kinematic work hardening: & = H (, p)
S
S
S
S
S
Evolution of the isotropic work hardening defined by a function: R (p)
S
Viscoplastic deformations of the phase: vpg
&
S
m

ij
ij
=
S
&

S G

There remain the equations of homogenisation then: vp
vp
E& = F
G &g
G

2.3.2.1 Relation of scaling

Two relations of localization of the type = L, E, are available in the version
G
(vp vpg G)
current:

· The relation of Berveiller-Zaoui [bib5] established on the concept of autocoherence. This relation is validated under certain conditions, namely: isotropy of material, elastic behavior homogeneous and monotonous loading:

vp
E
1
3
G
vp
vp
ij
G
ij = ij + μ

Eij - ij
= 1+ μ

2
J2 (ij)

· The second relation, developed more particularly for cyclic loadings [bib4] allows to give a good description to schematize the interactions between grains:

G
G
G
-
ij = ij + μ (Bij - ij)
Bij = F gij
G

G
vp
G
vp
vp
-
& =
G
&
- (

D -
G

) ||
G
&
||
ij
ij
ij
ij
ij

where D and are parameters characteristic of material and temperature.

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3
Local integration and implementation numerical

3.1
System of equations to be solved

3.1.1
Behavior of the MONOCRYSTAL type

The local behavior of the monocrystal is defined, at one moment given of the discretization in time

and to
level of a point of integration of a finite element, by the data:

· of the tensor of macroscopic constraints at the previous moment
-
(T)
,
I 1
=
-
· of the variables intern at the previous moment, for each system of slip:
T
T
p T
,
S (
,
,
I 1
-)
S (I 1
-)
S (I 1
-)
· and of the tensor of increase in total deflection provided by iteration N of the algorithm
total of resolution
N
E
= E
(with the notations of [R5.03.01]).
I

Integration consists in finding:

· the macroscopic tensor of constraints = (T)
I
· and the variables intern = (T), = (T), p = p (T)
S
S
I
S
S
I
S

S
I

checking the equations of behavior in each system of slip (which are relations mono dimensional), and relations of passage between the tensors macroscopic and the unit directions of slip. Notation: one writes the equations in the form discretized of way:

· clarifies, if the noted quantities +
With
are evaluated at the moment T: +
-
With
= A = (
With T

I 1
-)
I 1
-
· implicit, if they are evaluated at the moment T: A+/- = A+ = (
With T
I)
I

The equations to be integrated can be put in the following general form:

Being given, in a point of Gauss, the tensors:
E
: variation of deformation at the moment T,
I
-
E T
(
)
: deformation at the moment T,
I 1
=
-
E
I 1
-
-
(T)
: macroscopic constraint at the moment T,

I 1
=
-
I 1
-
T
T
p T
: variables intern for each system of slip to T,
S (
,
,
I 1
-)
S (I 1
-)
S (I 1
-)
I 1
-

It is necessary to find:

= (T): macroscopic constraint at the moment T, in the reference mark corresponding to the orientation
I
I
total
= (T)
S
S
I
= (T)
S
S
I
p = p (T)
S
S
I
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checking:

1
D = (1
D)
-
- + (
HT
vp
E
- E

- E

), where D can depend on the temperature, and can correspond to one
orthotropic elasticity.
vp
E = m
S S
S
for each system of slip (of the whole of the families of systems):
N equations:
+ /-
=: m
S
S
S
N relations of flow: maybe in viscoplasticity

G

p

S =
(+/- , +/- , +/-, +/-)
S
S
S
S
S
with p
F

p

S =
(+/-, +/-, +/- , +/-)
S
S
S
S
maybe in plasticity F (+
, +/-
, +/-
, +/-
p
)
,
F p
,
S = 0
S
S
S
S
0
with p
=

S

S
N equations of evolutions of kinematic work hardening:
H

p

S =
(+/- , +/- , +/-, +/-)
S
S
S
S
S
N equations of evolution of isotropic work hardening: R (+
p
)
S
S
S

This is solved either explicitly (Runge_Kutta), or implicit (Newton).

3.1.2
Behavior of the POLYCRYSTAL type

The discretized relations of behavior are:

Being given (in a point of Gauss) total tensors:

· increase in total deflection E
,
· total deflection at the moment previous E (T
,
I 1)
-
=
-
E
· forced at the previous moment: (T

I
=
-)
- ,

1
· the whole of the internal variables - -
-
, p relating to the systems of slip of
S
S
S
each phase,
· parameters of behavior of each phase, orientations and fractions voluminal of
each phase, and the type of method of localization.

It is necessary to find = (T, = T, = T, p = p T checking:
I)
S
S (I)
S
S (I)
S
S (I)

· on the level of the point of Gauss: = D (D-1) - + D (
HT
vp
E
- E

- E

), in the total reference mark,
for each phase (or "grain"), defined by an orientation and a proportion fg, one
relation of localization of the constraints, the general form (to be expressed in the reference mark
room of each phase)
= L, E,
G
(vp vpg G)

and for each system of slip of each phase:
-
vp

= m

G

S
S
S
N

S equations:
=: m
S
S
N

=
S relations of flow:
G
, p, with p
=

S
(S S S S)
S
S
N

=
S evolutions of work hardening:
H
, p
S
(S S S S)
-
F (, p
F p
, (in plasticity independent of time)
S
S
S
S)
,
0. S = 0

· It remains the equations of homogenisation then:
vp
vp
E

=
F

G
G
G
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The viscoplastic behaviors relating to each system of slip are identical to the case of the microstructure.

In the current version of Code_Aster, these relations of behavior are only integrated explicit way.

3.2 Resolution implicit

It is thus necessary to solve a system of the following general form:

vp
+
+
+
1
-
1
-
-
HT
vp
S (, E

, p)
D - (D) - (E
- E

- E

)
S
S
S
-

vp
vp
+
+
+

vp
R Y
() = R (, E

, p
) = E (, E

, p)

=
E

- m
= 0
S
S
S
S
S
S
S
S

S

vp
has (,
E

, +
, +
, p+)
S
S
S

- H (+
, +
, +
, p+)
S
S
S
S
S
vp
N G (, E

, +
, +
, p+)

S
S
S
S

N - G (+
, +
, +
, p+)

S
S
S
S
S
S

vp

p (,
E

, +
, +
, p+)

S
S
S

p
- F (+
, +
, +
, p+)
S
S
S
S
S

+
+
=: m
S
S

In more contracted way, one poses:

S (y)

(ey)

vp
E

R (y) = 0 = (
y has)

avecy =

S
G (y)

S

p (y)

PS

To solve this system of 6+6+3ns nonlinear equations (in 3D), one uses a method of Newton: one builds a vector series in the following way solution:

Dr.
Y
= Y - (
) 1
- R (Y)
K 1
+
K
K

dYk
Dr.
It is thus necessary to define the initial values Y, and to calculate the matrix jacobienne system: (this one
0
dYk
is detailed in appendix for the viscoplastic behaviors described previously). It with following form:

S
S
S
S
S

vp

E

p
S
S
S
E
E

E
E
E
vp
E

p
S
S

S
has
has
has
has
has
J =

vp
E

p
S
S
S
G
G
G
G
G

vp

E

p
S
S
S
p
p

p
p
p
vp
E

p
S
S

S

R (Y)
The criterion of stop of the iterations relates to the nullity of the residue:
K
<. If convergence is not
R (Y)
0
reached after the maximum number of iterations, the stationnarity of the solution is also tested:

Y
Y

K +1 - K <

The method used allows a local recutting of the step of time, either systematic, or in the event of not convergence.

3.2.1 Operator of tangent behavior

The formed system of the equations of the model written in discretized form (R (Y) = 0) is checked in end of increment. For a small variation of R, by regarding this time as variable and not like parameter, the system remains with balance and one checks dF L = 0, i.e.:

R

+ R

+
R
E

R
R
R
vp
E +

p

vp
S +
S +
S = 0

E

E

p
S
S
S

This system can be still written:

E

vp

E
0
R
(Y) = X, avecY = etX

S
= 0
Y

S
0
p
0
S

By successive substitution and elimination (cf [§Annexe2]), one deduces from it that the matrix jacobienne
calculated for implicit integration allows to calculate the tangent operator without intervention additional in the code.
This one is written directly (see [§Annexe2]):

=

= (
1
-
-
Y - Y Y Y

0
1 3
) 1
2
E

T

T

+

E t+t
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By writing the matrix jacobienne in the form:

Y
1
0
0
[ ] [ ]

J.Y
= [ ]
0
[ ]

vp
1
Y

1

E

Y
0
2
[

] Y
3
Z

With:

Y = D-1
0
S

Z
= ×n
S
S
p
S

The submatrices have as dimensions:

dim (Y = D 1
-
0
)=[6,
6]
dimY1 = [3
,
*6 * NS]*

dimY2 = [3 NS 6*
,]
dimY1 = [3 NS 3*
*, * NS]*

3.3 Resolution explicit

Another method of resolution, very simple to implement to solve the equations differentials of the single-crystal behavior is the explicit resolution. So that it is effective numerically, it is essential to associate an automatic control of step to him. As in [R5.03.14], one uses the method of Runge and Kutta. The calculation of the variables intern at the moment T + H

Dy
is a function only values of their derivatives
= F (Y, T):
dt

H (-
, -
, -
, p)
S
S
S
S
S

G (-

, -
, -
, p)

S
S
S
S

S
Y

=
=

= F (-
, -
, -
, p)

p
S
S
S
S
S
S

vp
E
m

S
S
S
with =: m = - + D (
HT
vp
E
- E

- E

)
S
S

One integrates according to the following diagram:

Yt+h = Y (2) if the criterion of precision is satisfied
H
Y (2) = Y + [F (Y, T) + F (Y (1), T + H)] with Y (1) = Y + H F (Y, T)
2
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( )
( )
The difference between Y 2 (diagram of order 2) and Y 1 (diagram of order 1, Euler) provides an estimate
error of integration and makes it possible to control the size of the step of time H which is initialized with Ti for
the first attempt. The strategy of the control of the step is defined on the standard basis of the variation enters
two methods of integration: || Y (2) - Y (1) || and of the precision required by the user (key word: RESI_INTE_RELA). The criterion selected is as follows, where Y is noted = (y1, y2,…, yN):

| y (2) - y (1) |
Y (T) = sup
J
J

<
J =1, NR
max [

, | yj (T) |]

The parameter is fixed at 0,001. The precision of desired integration must be coherent with level of precision necessary for the total stage.

If the criterion is not checked, the step of time Re-is cut out according to a discovery method (a number of under-not defined by the user via key word ITER_INTER_PAS). When the step of time becomes too much weak (H < 1.1020), calculation is stopped with an error message.

4 Variables

interns

4.1
Case of the monocrystal

The internal variables in Code_Aster are called ***V1***, ***V2,… Vp***.
The six first are the 6 components of the viscoplastic deformation.
V7, ***V8***, ***V9*** *are the values of p for the system of slip S = 1*
1
1
1
V10, ***V11***, ***V12*** *correspond to the system S = 2, and so on.*
The last internal variable, Vp, (p=6+3n+1, N being the total number of systems of slip) are an indicator of plasticity (threshold exceeded in at least a system of slip to the step of time running). If it is null, there no was increase in internal variables at the current moment. If not, it the iteration count of Newton contains (for an implicit resolution) which was necessary for to obtain convergence.

4.2
Case of the polycrystal

The internal variables in Code_Aster are called ***V1***, ***V2,… Vp***.
The six first are the 6 components of the viscoplastic deformation. The seventh is viscoplastic deformation are equivalent cumulated (macroscopic).
Then, for each phase, one finds:
Viscoplastic deformations or the tensor Beta
values of p for each system of slip
S
S
S
The last internal variable, Vp, (p=6+1+m (6+3n) +1), p = 7 + (6 + 3n
, m being the number of
S) + 2
G =,
1 m
phases and N being the number of systems of slip of the phase G).
S
is an indicator of plasticity (threshold exceeded in at least a system of slip to the step of time running). If it is null, there no was increase in internal variables at the current moment. If not, it the iteration count of Newton contains (for an implicit resolution) which was necessary for to obtain convergence.
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5 Numerical establishment in Code_Aster

Generally, the single-crystal behaviors are integrated into the methods of Runge-Kutta for explicit integration [R5.03.14], and with the environment "plasti" for implicit integration [R5.03.10]. The tensors of orientation of the systems of slip are as for them all defined in a routine, providing the tensor in total reference mark for the nth system of the provided family of which it name is provided by the appealing routine.

To add a new behavior of monocrystal, or simply a new law of collapse or work hardening, it is advisable to define its parameters in DEFI_MATERIAU. According to the case (flow, isotropic work hardening or kinematics), it is necessary to add the reading of these parameters in routines LCMAFL, LCMAEI, LCMAEC. For integration, it is enough to write the definition of increases in variables intern in routines LCMMFL (flow), LCMMEC (work hardening kinematics) and LCMMEI (isotropic work hardening), so that explicit integration functions.

Implicit integration also uses routines LCMMFL, LCMMEC and LCMMEI. It asks moreover to define the derivative of the equations compared to the various variables. The derivative are to be written in routines LCMMJF (derivative of L `equation of flow), LCMMJI (derivative of the relation of isotropic work hardening) and LCMMJC (derivative of the kinematic relation of work hardening).

6 Use

These models are accessible in Code_Aster in 3D, plane deformations (D_PLAN), forced plane (C_PLAN) and axisymetry (AXIS).

6.1
Case of the monocrystal

In the case of microstructures with a grid, various grains of a monocrystal being represented by groups of meshs, it is necessary to affect the parameters of materials and the behaviors of monocrystals like their orientations with the various grains.

The values of the parameters of the relations of behavior are provided using the order *DEFI_MATERIAU. Currently, this is defined starting from key words ECOU_VISC1, ECOU_VISC2, ECOU_VISC3 for the flow, ECRO_ISOT1, ECRO_ISOT2 for isotropic work hardening and ECRO_CINE1, ECRO_CINE2 for kinematic work hardening [U4.43.01]. For example [V6.04.172]:*

```
MATER1=DEFI_MATERIAU (
ELAS_ORTH=_F (E_L=192500,
E_T=128900,
NU_LT=0.23,
G_LT=74520,),

# RELATIONS Of FLOW
ECOU_VISC1=_F (N=10, K=40, C=6333),
ECOU_VISC2=_F (N=10, K=40, C=6333, D=37, A=121),
ECOU_VISC3=_F (K=40, V=, GAMMA0=),

# WORK HARDENING ISOTROPIC
```

ECRO_ISOT1=_F (R_0=75.5, Q=9.77, B=19.34, H=2.54),
ECRO_ISOT2=_F (R_0=75.5, Q1=9.77, B1=19.34, H=2.54, Q2=-33.27, B2=5.345,),

 # WORK HARDENING KINEMATIC

ECRO_CINE1=_F (D=36.68),
ECRO_CINE2=_F (D=36.68, GM=, PM=,),
);

One can thus dissociate, on the level of the data, the flow of the isotropic work hardening and of kinematic work hardening.

It is now necessary to define it (or them) standard of studied monocrystal. For that, one defines the behavior
in an external way with STAT_NON_LINE, via operator DEFI_COMPOR, for example:

MONO1=DEFI_COMPOR (MONOCRYSTAL = (_F (MATER=MATER1,
ECOULEMENT=ECOU_VISC1,
ECRO_ISOT=ECRO_ISOT1,
ECRO_CINE=ECRO_CINE1,
FAMI_SYST_GLIS= (“CUBIQUE1”,),

_F (MATER=MATER1,
ECOULEMENT=ECOU_PLAS1,
ECRO_ISOT=ECRO_ISOT2,
ECRO_CINE=ECRO_CINE2,
FAMI_SYST_GLIS=' CUBIQUE2',),
),

_F (MATER=MATER2,
ECOULEMENT=ECOU_PLAS1,
ECRO_ISOT=ECRO_ISOT2,
ECRO_CINE=ECRO_CINE2,
FAMI_SYST_GLIS=' PRISMATIQUE',),
),
)
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The structure of produced data contains names of systems of slip, associated names material parameters, for each behavior of monocrystal.

FAMI_SYST_GLIS MATE_SYST
TYPE_LOI
FLOW
ECRO_ISOT
ECRO_CINE
“CUBIC”
MATER1

VISC ECOU_VISC1
ECRO_ISOT1
ECRO_CIN1
“BASAL” MATER1

VISC ECOU_VISC1
ECRO_ISOT1
ECRO_CIN1
“PRISMATIC”
MATER1
PLAS ECOU_PLAS1
ECRO_ISOT2
ECRO_CIN2
... ...

...

Operator DEFI_COMPOR calculates the total number of variables intern associated with the

monocrystal.

Lastly, to carry out a calculation of microstructure, it is necessary to give, grain by grain, or group of meshs (representing sets of grains) an orientation, using the MASSIVE key word of AFFE_CARA_ELEM. For example:

```
ORIELEM = AFFE_CARA_ELEM (MODEL = MO_MECA,
SOLID MASS = (
_F (GROUP_MA=' GRAIN1',
ANGL_REP= (348.0, 24.0, 172.0),
),
_F (GROUP_MA=' GRAIN2',
ANGL_REP= (327.0, 126.0, 335.0),
),
_F (GROUP_MA=' GRAIN3',
ANGL_REP= (235.0, 7.0, 184.0),
),
_F (GROUP_MA=' GRAIN4',
ANGL_REP= (72.0, 338.0, 73.0),
),
...)
```

Note:

· Contrairement to current operator DEFI_TEXTURE, one gives only the name of crystallographic structure, knowing that directions of slip of each family systems of slip will be defined once and for all in the source.
· For the same monocrystal, the values of the parameters can be different from one family of systems of slip to the other. This is why one can define a material different by occurrence from the key word factor MONOCRYSTAL. But in this case, how to provide to transmit to STAT_NON_LINE information stipulating that in a point of gauss (all those of the group of meshs concerned), are there several materials present? This is possible thanks to an evolution of AFFE_MATERIAU [U4.43.03] and structure of data material [D4.06.18]):

```
MAT=AFFE_MATERIAU (MAILLAGE=MAIL,
AFFE =_F (GROUP_MA=' GRAIN1',
MATER= (MATER1, MATER2),),
);
```

The other data of calculation are identical to a usual structural analysis.

Lastly, in STAT_NON_LINE, the behavior resulting from DEFI_COMPOR is provided, under the key word

COMP_INCR via key COMPOR, obligatory word with key word RELATION=' MONOCRISTAL'.
COMP_INCR = _F (RELATION = ' MONOCRISTAL',

COMPOR
=
COMP1

Specified that for explicit integration, (RESO_INTE=' RUNGE_KUTTA'), it is useless to ask the reactualization of the tangent matrix since this one is not calculated. To begin from iterations of Newton of the total algorithm, it can be useful to specify PREDICTION=' EXTRAPOL' [U4.51.03].

One will be able to find an example of use in the tests: SSNV171 and SSNV172.
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6.2
Case of the polycrystal

In the case of multiphase polycrystals, each phase corresponds to a monocrystal. One will use thus parameters preset previously in DEFI_MATERIAU for the monocrystal. Here, it is about to lay down, for each phase, the orientation, the voluminal fraction, the monocrystal used, and the type of law of
localization. This is carried out under the key word factor POLYCRYSTAL of DEFI_COMPOR.

MONO1=DEFI_COMPOR (MONOCRISTAL=_F (MATER=MATPOLY,
ECOULEMENT=' ECOU_VISC2',

ECRO_ISOT=' ECRO_ISOT2',
ECRO_CINE=' ECRO_CINE1',
ELAS=' ELAS',
FAMI_SYST_GLIS=' OCTAEDRIQUE',),);

POLY1=DEFI_COMPOR (POLYCRISTAL= (_F (MONOCRISTAL=MONO1,
FRAC_VOL=0.025,
ANGL_REP= (- 149.676, 15.61819, 154.676,),),
_F (MONOCRISTAL=MONO1,
FRAC_VOL=0.025,
ANGL_REP= (- 150.646, 33.864, 55.646,),),
_F (MONOCRISTAL=MONO1,
FRAC_VOL=0.025,
ANGL_REP= (- 137.138, 41.5917, 142.138,),),
.....
_F (MONOCRISTAL=MONO1,
FRAC_VOL=0.025,
ANGL_REP= (- 481.729, 35.46958, 188.729,),),),),
LOCALISATION=' BETA',
DL=321.5,
DA=0.216,);

The key word POLYCRYSTAL makes it possible to define each phase by the data of an orientation, of one
voluminal fraction, of a monocrystal (i.e. a model of behavior and systems of
slip).
The key word LOCALIZATION makes it possible to choose the method of localization for the whole of the phases
polycrystal.

Lastly, in STAT_NON_LINE, the behavior resulting from DEFI_COMPOR is provided, under the key word
COMP_INCR via key COMPOR, obligatory word with key word RELATION=' POLYCRISTAL'.
COMP_INCR = _F (RELATION = ' POLYCRISTAL',

COMPOR
=
COMP1
)
This behavior is tested for example in SSNV125A (where one can check that the results are identical to those obtained with POLY_CFC).

6.3 Example

As example of implemented, one presents here briefly a calculation of aggregate, of form cubic (elementary volume) including/understanding 100 single-crystal grains, definite each one by a group of
meshs. The total number of elements is 86751. With meshs of order 1 (TETRA4) it comprises 15940 nodes. With meshs of order 2 (TETRA10), it comprises 121534 of them.
The loading consists of a homogeneous deformation, applied via one
normal displacement imposed on a face of the cube (direction Z). One reaches a deformation of 4% in 1s and 50 increments.
Calculation (tetra4) lasts 140000 seconds is 39 hours of CPU Alphaserveur.

```
ACIER=DEFI_MATERIAU (ELAS=_F (E =145200.0, NU=0.3,),
ECOU_VISC1=_F (N=10., K=40., C=10.,),
ECRO_ISOT2=_F (R_0=75.5,
B1 =19.34,
B2 =5.345,
Q1 =9.77,
Q2 =33.27,
H=0.5),
ECRO_CINE1=_F (D=36.68,),
);
```

```
COEF=DEFI_FONCTION (NOM_PARA = ' INST', VALE = (0.0, 0.0, 1.0, 1.0,),);

MAT=AFFE_MATERIAU (MAILLAGE=MAIL, AFFE=_F (ALL = " YES ", MATER=
(STEEL),),);
```

```
COMPORT=DEFI_COMPOR (MONOCRISTAL= (
_F (MATER =ACIER,
ECOULEMENT= " ECOU_VISC1 ",
ECRO_ISOT = " ECRO_ISOT2 ",
ECRO_CINE = " ECRO_CINE1 ",
ELAS= " ELAS ",
FAMI_SYST_GLIS=' OCTAEDRIQUE',),
),);

ORIELEM = AFFE_CARA_ELEM (MODEL = MO_MECA,
SOLID MASS = (
_F (GROUP_MA=' GRAIN1', ANGL_REP= (348.0, 24.0, 172.0),),
_F (GROUP_MA=' GRAIN2', ANGL_REP= (327.0, 126.0, 335.0),),
_F (GROUP_MA=' GRAIN3', ANGL_REP= (235.0, 7.0, 184.0),),
.................
_F (GROUP_MA=' GRAIN99', ANGL_REP= (201.0, 198.0, 247.0),),
_F (GROUP_MA=' GRAIN100', ANGL_REP= (84.0, 349.0, 233.0),),
))

FO_UZ = DEFI_FONCTION (NOM_PARA = “INST”,
VALE = (0.0, 0.0, 1.0, 0.04,),)

CHME4=AFFE_CHAR_MECA_F (MODELE=MO_MECA,
DDL_IMPO=_F (GROUP_NO=' HAUT', DZ=FO_UZ,),)

LINST = DEFI_LIST_REEL (DEBUT= 0. ,
INTERVAL = (_F (JUSQU_A = 1. , NOMBRE= 50),))

SIG=STAT_NON_LINE (MODEL =MO_MECA,
CARA_ELEM=ORIELEM,
CHAM_MATER =MAT,
EXCIT= (_F (CHARGE=CHME1),
_F (CHARGE=CHME2),
_F (CHARGE=CHME3),
_F (CHARGE=CHME4),),
COMP_INCR= (_F (RELATION
= ' MONOCRISTAL',
COMPOR =COMPORT,
ALL = ' OUI',),),
INCREMENT= (_F (LIST_INST=LINST,
SUBD_PAS =4,
SUBD_PAS_MINI=0.000001,
),),
```

NEWTON =_F (REAC_ITER =5,),),
);

The following figures represent isovaleurs of the deformations the constraints according to Z. One notes
nonhomogeneity of the values, and one can even distinguish the contour of the grains.
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To be able to exploit this type of results, one can for example calculate average fields by grains. On the following figure, one represented the equivalent constraints according to equivalent plastic deformations for the whole of the grains.

---

7 Bibliography

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Appendix

1
Expression of Jacobien of the equations elastoviscoplastic integrated

The system to be solved is form:

vp
+
+
+
1
-
1
-
-
HT
vp
S (,
E

, p)
D - (D) - (E
- E

- E

)
S
S
S
-

vp
vp
+
+
+

vp
R Y
() = R (, E

, p
) = E (,
E

, p)

=
E

- m
= 0
S
S
S
S
S
S
S
S

S

vp
has (, E

, +
, +
, p+)
S
S
S

- H (+
, +
, +
, p+)
S
S
S

S
S
vp
N G (,
E

, +
, +
, p+)

S
S
S
S

N - G (+
, +
, +
, p+)

S
S
S
S
S
S

vp

p (, E

, +
, +
, p+)

S
S
S

p
- F (+
, +
, +

, p+)
S
S
S
S
S

+
+
=: m
S
S

That is to say thus to evaluate the terms of the hypermatrice jacobienne J at the moment T + T

S
S
S
S
S

vp

E

p
S
S
S
E
E
E
E
E
vp
E

p
S

S

S
has
has
has
has
has
J =

vp
E

p
S
S
S
G
G
G
G
G

vp

E

p
S
S
S
p
p
p
p
p
vp
E

p
S

S

S

With regard to the first line of the matrix, independently of the equations of work hardening and of flow, one a:

S

1
= -
S
D
=
S
Id
=
S = S = 0

vp
E

p
S
S
S

The second line can be written also independently of the flow and work hardenings:
E = E
E
E
E
0
= Id
= 0
= - m
vp
S

= 0

E

p
S
S
S

The first column of the lines corresponding to the equations (A), (G) and (p) is written:

has

has

=
S

S
G

G

=
S

S
p

p

=
S

S
with

S = (m) Ts

The second column is identically null (because of equation (E): relations of flow and of work hardening can express itself only according to
and not of
vp
E

.
S

The last block of equations, depends as for him on the selected behaviors:

has
has
has

p

S
S
S
G
G

G

p

S
S
S
p
p
p

p

S
S
S

Example

Let us choose the viscoplastic relation of flow ECOU_VISC1

C
S -
(G)
p
S -
S
S
= 0

C
S -
S

C
R p
S -
S -
() N
(p) p

T
S -.
S
S
= 0
K

NR
with isotropic work hardening
- LP
ECRO_ISOT1:
R
R (p) R
Q (
H 1
(
E
)) , H = 1
(-) +
S
S
= 0 + Sr -
Sr
Sr
Sr
r=1
and a kinematic work hardening defined by ***ECRO_CINE1***
(A)

D p

S - S +
S S = 0

then:
has
= 0

S
G
= 0

S
p

- N T

-1 - C
=
- C - R p
N
S
S
S (S) N
S

K
- C
S
S
S
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has =
1+ D p
S

S
G
=
0

S
p
nc T

-1 - C
=
- C - R p
N
S
S
S (S) N
S
S

K
- C
S
S
S
has

= -1

S
G

= 1

S
p

= 0

S
has

=
ds
p
S
G

- C
S
S
=
p

- C
S
S
S
p

N T

-1 Dr. p
= 1+
- C - R p
N
S
S
S (S) N
S (S)
p

K
D p

S
S
Dr. p
S (S)
- bps
= Qbh E
ss
D p
S

and, concerning the interaction between systems of slip, it there only one term not no one:

p

N T

-1 Dr. p
=1+
- C - R p
N
S
S
S (S) N
S (S)
p

K
D p

R
R

Dr. p
S (S)
- bpr
= Qbh E
Sr
D p
R
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Appendix 2 Evaluation of the coherent tangent operator

It is a question of finding the operator tangent coherent, i.e. calculated starting from the solution of (R (Y) = 0) in end
of increment. For a small variation of R, by regarding this time as variable and not
parameter, one obtains:

R

+ R

+
R
E

R
R
R
vp
E +

p

vp
S +
S +
S = 0

E

E

p
S
S
S

This system can be written:

E

vp

E
0
R
(Y) = X, avecY = etX

S
= 0
Y

S
0
p
0
S

By writing the matrix jacobienne in the form:

Y
1
0
0
[ ] [ ]

J.Y
= [ ]
0
[ ]

vp

1
Y

1 E

Y
0
2
[] Y
3
Z

With:
Y = D-1
0
S

Z
= ×n
S
S
p
S

While operating by successive eliminations and substitutions, the third block of the system of equation gives:

Z
= - (Y3) - 1Y2
Evp

= Y
-
Z
1
= Y1 (Y3) - 1Y2
(Y0 +Y1 (Y3) - 1Y2) = E

the required tangent operator can thus be written directly:

=

= (
1
-
-
Y - Y Y Y

0
1 3
) 1
2
E

T
T

+

E t+t

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Élasto-viscoplastic relation of behavior
for cubic polycrystalline materials with faces
centered
Summary:
A polycrystalline model was developed with the Center of Materials of the School of the Mines of Paris to describe it
élasto-viscoplastic cubic polycrystalline material behavior with centered faces. It is about a model with great number of variables intern which allows, thanks to a simplified description of the microstructure (texture
crystallographic, systems of slip), to describe many phenomena observed on these materials (examples: Bauschinger effect, anisotropy of surface of plasticity).
It is introduced into Code_Aster under the name of POLY_CFC on the level of order DEFI_MATERIAU;
behavior is integrated by an explicit diagram of Runge-Kutta of order 2 into adaptive step.
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1 Introduction

Many research related during the twenty last years to a modeling phenomenologic of the behavior of materials. The model suggested is, on the other hand, founded on a polycrystalline approach which integrates microstructural information and offers capacities of modeling extended compared to the phenomenologic approaches. Moreover, by comparison with the phenomenologic models using a criterion of Von Mises, the micromechanical approach allows to have a criterion of plasticity which can present angular parts.

The basic idea of crystallographic modelings of plasticity is as follows: the introduction of the variables attached to the physical mechanisms is beneficial for the capacities of modeling of the corresponding relations of behavior. Indeed, if the physical aspect of mechanisms of deformation is well represented in the model, the predictive character of the equations constitutive is improved, including for complex loadings located out of the field of identification initial of the studied model, since the mechanical behavior of material is associated its microstructure.

The model is designed to account for the mechanical polycrystalline material behavior of cubic structure with centered faces.
The model is introduced into Code_Aster in 3D, plane deformations (D_PLAN), forced plane (C_PLAN) and axisymetry (AXIS) under the name of POLY_CFC in DEFI_MATERIAU. It is about one viscoplastic polycrystalline model with great number of internal variables, namely 1688 per point of integration. The taking into account of the microstructure is carried out by the introduction of texture. texture includes/understands the directions of orientation and the voluminal fraction associated each orientation
grains which make the microstructure.
One presents in this note the equations constitutive of the model and his introduction into Code_Aster.
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2

Formulation of the model

It is about a élasto-viscoplastic model which lies within the scope of the micromechanical approaches with taking into account of microstructural information to model the elementary mechanisms of inelastic deformation. The general step is to build a law of behavior
macroscopic starting from a simplified description of the microstructure. The method consists then with to describe the microstructure (i.e. on a local scale, to identify the phases and to characterize them behavior) and to express the total sizes like the averages of the local sizes.
The elementary ladders and mechanisms to consider are in the following way selected [Figure 2-a]:
· an element of elementary volume representative of material, for the scale of arrival to which the process of modeling must succeed considered, which makes it possible to describe the behavior with the macroscopic scale,
· the grain, for the starting element of the scaling, for which the relation of behavior developed is associated the mechanism of deformation retained in this

modeling, i.e. crystallographic slip, without consideration concerning morphology and the space distribution of the grains of elementary volume (the grain is defined by an orientation).

Orientation G

Scale of the élém of volume

,

Scale of the grain

,

“Grain” 1

“Grain” G

“Grain” N

Appear 2-a: Scales considered for polycrystalline modeling

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The model is based on a self-coherent approach. One replaces the detailed analysis of the interactions there
mechanics between the various phases by an approximate evaluation which consists in considering successively interactions between each phase (grains having even orientation of network crystalline lens are indistinguishable and constitute a phase), the whole of these phases being gathered in an inclusion, and a fictitious homogeneous medium, which constitutes the matrix. The character of autocoherence is
ensured by the fact that the mechanical behavior of this fictitious medium is precisely that of the medium homogeneous equivalent in the studied heterogeneous medium. The model was defined and developed in work [bib1], [bib2], [bib3] and [bib4].

The modeling of the behavior of the phases is done via internal variables which go
to describe the work hardening of material in the grains, where plasticity is given by the law of Schmidt to
level of the systems of slip.

The sizes present on a grain scale are the inelastic deformation, the generated constraint
in the grain and a certain number of variables intern associated with each grain. With the scale macroscopic of the element of volume, one has of the total deflection, the deformation
inelastic, of the constraint (the variables intern are associated the grains which constitute the element of volume).

The presence of two scales of modeling consequently requires to define a stage of
scaling, which is the essential element of this type of modeling and by which the different ones polycrystalline models are distinguished.

Notations:

E, ***Ee***, ***Eth***, ***Evp*** Deflections total, elastic, thermal and viscoplastic.
ij
ij
ij
ij
vpg
Viscoplastic deformation in the grain G.

ij
F G
Voluminal fraction of matter constituting the grain G.
! *S*,
Speed of slip of the system S.
! *PS*,
Intensity the speed of slip of the system S.
ms, ***N***, ***L***
ij
I
I
Tensor of orientation and unit vectors corresponding respectively to
normal in the plan which slips and the direction of slip.
Fs
Function threshold.
Xs
Kinematic variable of work hardening.
R *S*
Isotropic variable of work hardening.
S
G
,
Cission solved on the system S and forced in the grain G.
ij
S
S
1
2s
, *Q*, *Q*
Variables associated with the systems with slip S for the description of
work hardening in the grains. *Q S*
1 and *q2s* represents the interaction between
systems of slip (dislocation).
hrs
Components of the matrix of interaction enters dislocations (*hrs* = 1 on
diagonal to take into account a car-work hardening when systems
are identical).
ij
Macroscopic constraint.
B
G
ij, *ij*

Variables of localization, sizes comparable to deformations (on the scale
macroscopic and on a grain scale).
J
3
2
2 (ij)
Dev. D

or
D is the deviatoric part of
2
ij (
ij)
ij
ij.
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2.1
Behavior on a grain scale

2.1.1 Definitions on the level of the systems of slip

The mechanism of deformation considered is the crystallographic slip. It is thus advisable to define the mechanical behavior related to this mechanism where it is supposed that a system of slip ***S*** is credit when its solved cission S reaches a breaking value. For a cubic material with faces centered, there are 12 systems of slip which are defined by their tensor of orientation ***ms*** to leave crystallographic definitions of the direction of slip (defined by the unit vector ***L***) and of the normal in the plan which slips (definite by unit vector ***N***).
S
G

=
S
S
1
ij: ***mij m***
= ***N L*** + ***L N***
ij
2 (*I*
J
J
I)
The speed of deformation viscoplastic! *vpg* is defined starting from the knowledge speeds of slip! *S* for all the systems of slip:
vp
! *G*
S
S
ij
= ***mij***!
S G

2.1.2 Law of behavior on a grain scale

The internal variables of the local behavior noted *S*, *S* and *PS* make it possible to define the evolution work hardening and to calculate the speed of deformation! *vpg* in all the grains. *S* is associated kinematic work hardening and *PS* are the cumulated viscoplastic slip.
The viscoplastic formulation retained in this modeling proposes a function power (by analogy with the law of Norton in creep) to define the intensity! *PS the* speed of slip and allows to have access to derived from the internal variables starting from the knowledge of the constraint *G* and of the internal values of variables:
Fs *N*
! *PS* =|! *S*
|=
with <x>=0 if x<0 and <x>=x if x>0
K
where *K* and *N* are parameters characteristic of material and temperature.
Fs depends on the initial threshold *0* of flow on the system of slip **S** of the solved cission *S* and of two variables of Xs work hardening and R *S. the* kinematic variable of work hardening Xs allows to take account of local heterogeneities in the grains due to the development of dislocations. isotropic variable of work hardening R *S* which accounts for the interaction between dislocations (by the intermediary of a matrix of interaction of components *hrs*) can present a value of saturation work hardening.
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The criterion of flow corresponding to the equipotential ones in viscoplasticity is written:

Fs =| *S*
- Xs|-
1
2
0 - R *S*
C
+
(*S*
X)
2D

where *0*, *C* and *D* are parameters characteristic of material and temperature.

The equations of state of the variables are as follows:

Xs
S
S
S
= *C*
+ = *X* +
S

has
has

R *S*
S
1
2s

= Q

1 rs
H Q
+ Q Q
2
R S

hrs = rs + H (1 - rs) with *rs*= 0 if *R S* and *rs* = 1 if *R = S*
where *C*, *has*, *Q1*, *Q2* and *H* are parameters characteristic of material and temperature.
It remains to define the laws of evolution of the variables of work hardening:
! *S* =! S - *S*! *PS*

D
! *qis B* (1 - *qis S*
= I
)! *p* (I = 1,2)
where *D*, *b1* and *b2* are parameters characteristic of material and temperature.

2.2

Relation of scaling

It is a question of modelling the internal stresses and heterogeneities of deformation of a phase to one other to have access to the total sizes. It is noted that the total deflection in each grain
is never calculated. One is interested in the viscoplastic deformation of the phases, which makes it possible to define
viscoplastic deformation of elementary volume representative of material from
voluminal weighting of the fractions of the phases *F G*:
vp
Evp
ij
F
G
= G ij
G
Two relations of localization are programmed (according to the presence or not coefficients *D* and in [éq 2.2-2]). The difficulty to justify, on macroscopic data of behavior,
relevance of the rule of scaling is avoided while having a relation [bib5] which can
to be used as reference because it validates the character of autocoherence. This validation is effective under
certain conditions, namely: isotropy of material, homogeneous elastic behavior and
monotonous loading:
vp
E
1

3
G
vp
vp
ij

G
ij = *ij* + µ **Eij** - *ij*

= 1+ µ

éq 2.2-1
2
J2 (*ij*)
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The second relation, developed more particularly for cyclic loadings [bib6] allows to give a good description to schematize the interactions between the grains:
G
G
G
ij = *ij* + µ (**Bij** - *ij*)
Bij = *F gij*
éq 2.2-2
G

!
G
vp
G
vp
vp
G
G
G
ij =!
ij - *D* (*ij* - *ij*

) ||! *ij* ||
where *D* and are parameters characteristic of material and temperature.

2.3

Equations of the model

Total behavior:

vp
E
Ee
Eth Evp
Evp
ij
ij
ij
ij
ij
F
G
=
+
+
= *gij*
G

Intragranular behavior:

vp
1
! *G*
S
S
S
ij
= ***mij***!

ij
m = 2 (*in* ***lj*** + ***lj*** *in*)
S G

Fs *N*
! *PS* =|! *S*
|=
with <x>=0 if x<0 and <x>=x if x>0
K
Criterion:
Fs | *S* - Xs|-
Rs 1 *C*
2
0
+
(xs
=
-
)
2D
S
G
=
S
ij: ***mij***
Xs
S
S
S
S
= *C* + *has* = *X* + *has*
!
S =! *S*
- *D S*! *S*
p

R *S*
S
1
2s
= *Q*

1 rs

H Q
+ Q Q
2
R S

! *qis B* (1 - *qis S*
= I
)! *p* (I = 1,2)
hrs = 1
H (- rs) + *rs* with *rs* = 1 if *R* = *S* and *rs* = 0 if *R S*
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Relations of scaling (accessible according to the presence from the parameters *D* and):
G
vp
vpg
ij = ij + µ (*ij*
E
- *ij*) (*D* and miss command file [bib5])
vp
1
||E ||
= 1 3
+ µ
ij

2 J2 (*ij*)
G

G
ij = *ij* + μ (**Bij** - *ij*) (*D* and are defined [bib6])
vp
vp
vp
B
G
G
G
G
G
G
ij =
F
G ij! *ij* =! *ij* - *D* (*ij* - *ij*) ||! *ij* ||
G
In Code_Aster, the whole of the parameters of the model
D, *N*, *K*, *Q*, B, *H*, *Q*, B, *C*, *D, have*
1
1
2
2
can be a function of the temperature.

3

Implementation numerical in *Code_Aster*

The diagram general adopted with the polycrystalline behavior to integrate the relation of behavior is a method of Runge Kutta to adaptive step [bib7]. The programmed diagram is of order 2.

From the state of deformation **E** (*T*), knowledge of the variables intern *y T* = (*S S*
PS
( )
,
,
)
and of the variables of localization *G* (*T*) at the moment T, one seeks to find, at the moment t+t the constraint
macroscopic to represent balance. From the increment of total deflection proposed with the total stage **E = E** (*T* + *T*) - **E** (*T*) and of an assumption of evolution of this linear deformation in time, one explicitly integrates the differential equations of the behavior by controlling the precision by a method with variable step.

One thus knows **Evp** (*T* + *T*
), which makes it possible to calculate (*T* + *T*
) as specified below.

3.1
Calculation of the macroscopic constraint
One calculates the constraint starting from the viscoplastic deformation **Evp**. This one corresponds to first size arranged in the vector which constitutes the whole of the variables of model *Y*.

HT
vp
ij = C (**Eij - Eij - Eij**)
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3.2
Evolution of the internal variables and the variables of localization
Knowing the cission *S* (calculated starting from the constraint *G* and of the tensors of orientation for all the systems of slip) and the variables intern *y* in the grains, one can calculate them
derived from the variables! *y* starting from the writing clarifies equations of the model and to have access to
speeds of viscoplastic deformation in the grains! *vpg*.
Variables characteristic of the relation of localization used to describe a behavior under
cyclic requests are accessible for all the grains starting from information from deformation and
of speed [éq 2.2-1].
4 Synthesis
The model is accessible in *Code_Aster* in 3D, plane deformations (D_PLAN), forced
plane (C_PLAN) and axisymetry (AXIS) starting from key word COMP_INCR of the order
STAT_NON_LINE. The whole of the parameters of the model is provided under the key word factor
POLY_CFC
or POLY_CFC_FO of order DEFI_MATERIAU [U4.23.01].
/
POCY_CFC:

(
DL
:
D
DA
:

NR
:
N
K
:
kMPa
TAU_0
:
MPa
0
Q1
:
Q1 MPa
B1
:
b1
HL
:
H
Q2
:
Q2 MPa
B2
:
b2
C1
:
C MPa
D1
:
D (MPa) 3
C2
:
MPa has)
The parameters D and are optional. When they are present, the rule of change of scale used [bib6] is that of the equation [éq 2.2-2]. If not, one uses the rule [bib5] of

the equation [éq 2.2-1] (valid only under monotonous loading); cf [§2.2].
The selection of the diagram of integration of the relation of behavior is done by the option RUNGE_KUTTA_2 of operand RESO_INTE starting from the key word NEWTON of the order STAT_NON_LINE.
The method does not provide a tangent matrix; one thus uses the elastic matrix for total balance what implies that the user must force himself to define a cutting of no relatively fine time, so that balance at the total level is facilitated.
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The significant number of internal variables used in this modeling implies to store them that at the last moment of calculation to allow the recoveries. One uses option CHAM_EXCLU then of key word factor FILING of order STAT_NON_LINE with the only possible argument "VARI". The variables intern are then safeguarded only with the last step of time.

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Appendix 1 Introduction of information textures

The texture of material intervenes in this model like a data of description provided once and for all in DEFI_MATERIAU, as well as the parameters material. This one makes it possible to calculate the tensors
of orientation ***ms*** by carrying out a change of reference mark the reference mark of material enters (reference mark of the laboratory) and
the crystallographic reference mark (the local reference mark) and also to have access to the voluminal fractions *fg* of
material which presents such or such crystallographic orientation.
This texture is thus defined in the following way:
1, 2, *fg* where 1, 2 is the angles of Euler.
One lays out by defect, of a description of texture distributed in an isotropic way on 40 different orientations
for $F\ G = 1/40$.
One can also introduce the results of texture obtained starting from an experimental study (diffraction of x-rays of the crystallographic plans of the cubic structure with centered faces). One then presents them under

form of a succession of 3 angles (angles of Euler which correspond to the setting in coincidence of the reference mark related to
the sample and of that related to the crystal) associated a voluminal fraction. If one follows 40 orientations for this
modeling, the file of texture comprises 40 lines then corresponding to the 40 orientations more represented in the matter sample analyzed.
For the modeling introduced into *Code_Aster*, one does not take account of the possible evolution of texture
with work hardening (assumptions of the small disturbances), which makes it possible to regard as fixed parameters
voluminal fractions *F G* and components of tensors ***ms*** (6 components), definite for each grain
(each orientation) and for the 12 principal systems of slip of structure CFC (40*12*6=2880 components).
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Handbook of Reference
R5.03 booklet: Nonlinear mechanics
Document: R5.03.14
Integration clarifies relations of behavior
nonlinear
Summary
This document not describes a method of integration explicit for the resolution of problems with behavior
linear used in elastoviscoplasticity (in operator STAT_NON_LINE [R5.03.01]).
The numerical method presented is that of Runge_Kutta of order 2 with adaptive step.
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Code_Aster ®
Version
4.0
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1 Introduction

Into operator STAT_NON_LINE [U4.32.01] is introduced a method of integration explicit of type Runge_Kutta of order 2 (option “RUNGE_KUTTA_2” of operand RESO_INTE of the key word factor CONVERGENCE) for the incrémentaux problems of nonlinear behavior (operand RELATION key word factor COMP_INCR).
For more details to consult the document [U4.32.01] user's manual. Relations concerned currently are:
VISCOCHAB:
Élasto-viscoplastic behavior of Chaboche,
V_ENDO_CHAB:
Viscoplastic behavior with damage of Chaboche
POLY_CFC:
Polycrystalline élasto-viscoplastic behavior for materials of cubic structure with centered faces.
Relation VISCOCHAB can also be integrated by the implicit method (cf [R5.03.11] while the 2 other relations mentioned are accessible only by the explicit method presented hereafter.
This type of integration makes it possible to very easily establish a new model of behavior [bib2].
One describes the calculation of the stress field starting from an increment of deformation given while following
evolution of the internal variables.

2 Notations

general

-
All the quantities evaluated at the previous moment are subscripted par.
The quantities evaluated at the moment T + *T* are not subscripted.
The increments are indicated par. One has as follows:
Q = **Q** *T*
(+ *T*) = **Q** *T*
() + **Q** = **Q** + **Q**.

tensor of the constraints.

increment of deformation.
With
tensor of elasticity.
Y
internal variables
µ, *E*, *v*
moduli of the isotropic elasticity.
T
temperature.

total deflection.
ij
p
plastic deformation.
ij
HT
thermal deformation.
ij
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3 Integration clarifies of a relation of behavior incremental

3.1 Integration drank

In a structural analysis, the writing of the equilibrium equations is translated in integral form and require the calculation of the interior efforts. It is thus necessary to calculate the stress field in each point of integration; in the case of linear elasticity, this calculation is elementary but, in

situations where the representation of the behavior of material requires the use of a model not linear, this one is more complex and it is necessary, moreover, to follow the evolution of the internal variables
([cf R5.03.01]).
The nonlinear behavior of material also modifies the strategy of calculation since one adopts, in the majority of the cases, an incremental step in time. This one consists in discretizing the interval of study in increments for better following nonthe linearity of the response of the structure.

3.2 Step general

One places oneself at one moment *T* where the structure is in balance and where one connait, at each point of Gauss,
the values of the variables intern *Y* -. For the models élasto (visco) plastic, these internal variables allow to calculate the constraint
p
ij with the law of Hooke (the plastic deformation *ij* is often present among the internal variables):
=
p
HT
ij
With (*T*) [*ij* (*U*) - *ij* - *ij* (*T*)]
To pass to the following increment (urgent *T* + *T*), it is necessary to estimate the constraint (*T* + *T*)
ij
, associated
by the relation of behavior, with the increment of displacement the *U.K.* suggested by the iteration *K* of the total stage. It is thus enough to follow the evolution of the variables intern *Y* solving the system differential equations: Dy/dt = F (*Y*, *K*, *T*) with the initial conditions *Y* (*T*) = *Y* -.
Classically, one adopts a linear interpolation of displacement and temperature (known for analyses uncoupled) with the course the interval from time [*T*, *T* = *T* - + *T*], which leads to:
- *T* -
T, *T*
[] *T* () = *T* + [*T* - *T*]
T
- *T*
K
-
K
-
K
-
ij () = *ij U*
(
) +

[*ij U*
(
+ U

) *- ij U*
(
)]
T

3.3
Use of an explicit method
One of the techniques simplest to implement to solve these differential equations
is the use of explicit methods. So that they numerically remain effective, it is
essential to associate an automatic control of step to them to preserve a good compromise cost
precision. Methods of explicit and encased Runge and Kutta [bib1], [bib2] are undoubtedly them
simplest diagrams of integration respecting these criteria. Their principle is to associate two
diagrams of integration of a different nature to control the step of time according to a precision
required. According to the order of integration chosen, several algorithms are available and the diagram
simplest is a method of order two. The calculation of the internal variables at the moment T + *H* is not
Dy
function that values of their derivatives
= *F* (*Y*, *T*) according to the following diagram:
dt
Yt+h = Y (2) if the criterion of precision is satisfied (cf [§3.4])
H
Y (2) = *Y* + [*F* (*Y*, *T*) + *F* (*Y* (1), *T* + *H*)] with *Y* (1) = *Y* + *H F* (*Y*, *T*)
2
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( )
( )
The difference between *Y* 2 (diagram of order 2) and *Y* 1 (diagram of order 1, Euler) provides an estimate error of integration and makes it possible to control the size of the step of time *H* which is initialized with *Ti* for
first attempt. Thus, the method remains effective if the behavior remains elastic during
the increment and one naturally have under local steps independent of the total increment for
to integrate, with a better precision, the evolution of the variables intern at the points of Gauss where not linearity of the behavior is most significant.

3.4

Control step of time

The strategy of the control of the step is defined on the standard basis of the difference between the two methods
of integration: || *Y* (2) - *Y* (1) || and of the precision required by the user (key word: RESI_INTE_RELA).
The criterion selected is as follows, where *Y* is noted = (*y1*, *y2*,…, *yN*):
(2)

- (1)

| *y*
y |
Y (*T*) = sup
J
J

<
J =1, *NR*
max [

, |yj (*T*) |]
The parameter is fixed at 0,001. The precision of desired integration must be coherent with
level of precision necessary for the total stage. For the models elastoviscoplastic, a value
traditional this parameter is 0,001. For materials with a low viscosity and for
elastoplastic models, the value of this parameter is a little lower (0,0001 or 0,00001).
If the criterion is not checked, the step of time is redécoupé according to a discovery method. When it no time becomes too weak (H < 1.1020), calculation is stopped with an error message.

3.5

Influence on the total stage

In its current version, the method does not provide a tangent matrix; its evaluation would be possible
by a technique of disturbances. One uses for the moment the elastic matrix for total balance,
what implies a more significant number of increments so that the total resolution is facilitated.
To mitigate these disadvantages, one can suggest the possibility of developing at the total level one diagram of the type BFGS [bib3]. Another simpler solution of development consists, instead of
to consider the increment of displacement for the first iteration at the moment running (suggested with

stamp elastic), to initialize the increment of displacement sought for the first *U1* iteration with
-
to leave the value *One*, solution with the preceding increment (cf appears following page).
The key word PREDICTION: “EXTRAPOL” (under NEWTON) makes it possible to initialize with the increment
converged of the preceding step (balanced by the report/ratio of the steps of time). This estimate is projected
on the field of displacements kinematically acceptable so that the final solution checks well boundary conditions of Dirichlet.

-
Loading
U
U
1
1
U *1
U
U -
N
One

3.6 Establishment of a new model of behavior by explicit method

To establish a model of behavior, it is enough to modify the 2 routines of shunting NMCOMP (routine general of environment PLASTI) and RDIF01 (routine suitable for explicit integration) and to write 2 additional routines:

· XXXMAT: recovery of the data material,

· RKDXXX: writing of the equations constitutive of the model (expression of the derivatives temporal internal variables; this requires the preliminary calculation of the plastic multiplier using the equation of consistency for the elastoplastic models).

The update of the catalogues material relates to only DEFI_MATERIAU and STAT_NON_LINE.

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Viscoplastic behavior with damage of CHABOCHE

Summary:

The viscoplastic model coupled with the isotropic damage of Chaboche (developed at the origin to predict
successfully the lifespan and the cracking of the paddles of the modern turbojets) is used for calculations of prediction of the time of ruin of structures at high temperatures.

It is established in Code_Aster under the name of VENDOCHAB; the equations of speed are integrated numerically by a diagram clarifies of Runge-Kutta of order 2 with automatic cutting in under-not buildings according to an estimate of the error of integration (method of Runge-Kutta encased, cf [R5.03.14]),
or by a method of integration implicit [R5.03.01].

Test SSNV126 validates the explicit integration of this model. The document of validation [V6.04.126] provides
analytical solution for an isothermal uniaxial creep test.
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1 Introduction

Calculations by finite elements carried out within the framework of the studies on the serious accidents of
nuclear engines highlighted the need to use models of damage in order to
to envisage the ruin of a structure such as the tank subjected to the severe thermal conditions and complexes (high temperatures going until fusion, high thermal gradients in space or
time, etc) which the corium [bib1] would impose to him.

The major interest this choice lies in the fact that the value of the variable of damage at rupture (or with cracking) can be regarded as an intrinsic parameter of the material which is accessible, although that is difficult and delicate, by measurements physical (ultrasounds, X-rays diffraction,
etc). The criterion of rupture with the theory of the damage is then more "physical" that them criteria in maximum deformation used sometimes in viscoplastic calculations without damage or criteria of damage not coupled (rule of addition of time
actually passed under certain conditions (, T) divided by the time of ruin for these same conditions).

The model implemented in Code Aster is a viscoplastic model of behaviour to work hardening viscosity-multiplicative coupled with the isotropic damage (model due to Chaboche [bib2]).

Bene foot-note:

One will find in the reference [bib3] a detailed description of the capacities of the model, one methodology for the identification of the parameters and the values of these parameters for steel 22 MoNiCr 3 7.

2
Formulation of the model

2.1 Tally
theoretical

In this sub-chapter, one insists on the specificity of law VENDOCHAB (i.e.
the damage) compared to the usual viscoplastic models. For more details, one
will refer to [bib2].

The theory of the damage describes the evolution of the phenomena between the virgin state and the starting of
the macroscopic crack in a material by means of a variable continues (scalar or tensorial)
describing the progressive deterioration of this material. This idea, due to Kachanov which was the first with
to use to model the creep rupture of metals in uniaxial request, was taken again in
France in the Seventies by Lemaitre and Chaboche. Evolution of material of its virgin state with
its damaged state is not always easy to distinguish from the phenomenon of deformation
generally accompanying and is due to several different mechanisms of which creep forms part.
The viscoplastic damage of creep corresponds to intergranular decoherences
accompanying the viscoplastic deformations for metals at average temperatures and
raised.

To define what is this variable of damage, let us consider the surface S of one of the faces of one element of volume located by its normal directed towards outside N. On this section, them
~
microscopic cracks and the cavities which constitute the damage leave traces of various forms. That is to say S
the effective resistant surface and SD the total surface of the whole of the traces.

One a:

$$D = \frac{S - \tilde{S}}{S}$$

and one defines the variable of damage by:

S
D
D
N = S

Dn is the measurement of the local damage relative to direction N. From a point of view physics, the variable of Dn damage is thus the relative surface of the cracks and cut cavities by the normal plan with the direction R
N. From a mathematical point of view, while making tighten S towards 0,
variable Dr. N is the surface density of discontinuities of the matter in the normal plan with N. Dn = 0 corresponds at the virgin state not damaged. Dn = 1 corresponds to the element of broken volume
in two parts according to a normal plan with N.

The assumption of isotropy implies that the cracks and cavities are uniformly distributed in orientation in a point of material. In this case, the variable of damage becomes a scalar who does not depend any more orientation and is noted D. One a:

D = Dn N

We will consider here only the isotropic variable of damage.

Total mechanical measurements (modification of the characteristics of elasticity, plasticity or of viscoplasticity) are easier to interpret in term of variable of damage thanks to the concept of effective constraint introduced by Rabotnov. The effective constraint represents the constraint paid to the section which resists the efforts indeed. In the case of the isotropic damage, it is written:

~

= (1 - D)

And one a:

.
~
= for a virgin material
.
~
+ at the instant of the failure
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The principle of equivalence in deformation implies that any behavior with the deformation, unidimensional or three-dimensional of a damaged material is translated by the laws of behavior of the virgin material in which one replaces the usual constraint by the constraint effective.

One distinguishes 2 types of variables to characterize the medium:

Observable variables (measurable):

· the temperature T
· the total deflection which breaks up as indicated below:

E
vp
HT
= + +

Internal variables:
· viscoplastic deformation vp

· the isotropic variable of work hardening R
· the isotropic variable of damage D

That is to say = (
vp
, T
, R, D), the potential of state, the laws of state describing this potential are:

=

***R* =**
-

R

***S* =**
-
T

***Y* =**
-

D

According to the law of normality, one has, with, the potential of dissipation:

vp
***&* =**

R
***&* =**

R

***D&* =**

Y

The modeling of the work hardening and the damage of material is done via internal variables (or hidden). In the case of the model VENDOCHAB, variables internal introduced into Code_Aster are:

.
vp
: tensor of the inelastic deformations
.
p: cumulated plastic deformation
.
R: variable of work hardening viscosity
.
D: scalar variable of isotropic damage

Bene foot-note:

In the made programming, the variable vp is stored in the shape of a vector with 6 components, vp , with the result that the variables intern are stored in a table of dimension 9 (cf [§5]).
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2.2 Equations of the model

The equations of the models are written then:

= E + HT + vp

= 1
(- D) E

'
vp

& = 3 p

&
2

eq

r&
p& =

1
(- D)

NR

D
eq -
1
(
y
-)
r& =
1

1
(- D)
M
K R

R

( )
- K (())
D& =

1
(- D)

With

with:

() = J () + J () + 1
(- -) J ()
0
1
2

:
where J ()
maximum

principal

constraint

is

0
J () = Tr ()

1
3
J
'
'
( ) = =

2
eq
ij
ij
2
X:

of

positive

part
X

where:

E
HT
vp

, ,

and

are respectively the deflections total, elastic, thermal and plastic,
= (

ijkl)
is the elastic tensor of rigidity,

'= 1
- Tr () Id is the deviatoric part of the tensor of the constraints,
3
p
is the cumulated plastic deformation,
R
is the variable of isotropic work hardening viscoplastic
D
is the scalar variable of isotropic damage

Bene foot-note:

The whole of the parameters of the model, NR, M, K,
WITH R and K can be related to
the temperature (in °C). K can be constant, depend on the temperature or on () (in
MPa) and of the temperature.
In addition, it is seen that this model considers that it can exist a viscoplastic threshold there which depends on the temperature.

It is seen that this model is reduced to the viscoplastic model of Lemaitre if it is considered that D = 0 and
if one neglects the equation of evolution of D.M, NR, and K are coefficients characteristic of purely viscoplastic behavior of material.
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The evolution of the damage is governed by a law with three parameters: With, R, and K. The constraint equivalent () allows to take account of a possible effect of the spherical part of the tensor of forced on the damage (a little as in the laws growth cavities at the base models of Gurson and Rousselier). The fact that the maximum principal constraint can play one role in () is difficult to imagine for materials as steel but returns the model more general.

3 *Calculation of the parameters material*

The parameters of the law of behavior can be calculated starting from creep tests carried out for various levels of constraints and temperature. For that one uses a law of behavior unidimensional because the request of a cylindrical test-tube in traction can be modelled in dimension 1. The tensor of the constraints is reduced to its axial component.

2

0
0

0 0
0

3

'

1

= 0 0 0
and
= 0 0 -
0
3

0
0 0

1
0
0
-

3

One thus has: J = J = J =
0 ( )
1 ( )
2 ( )
0
( )=
(
,
0
)

The system of equations to be solved is then:

1 0
0

vp
1
&
= p&0 -
0

2

1
0
0
-

2
R

p =
&
&
1
(- D)
NR
- 1
(- D)
0
R &
y
=
1
1
(- D)
M
K R
R
0
- K ()
0
D& =
1
(- D)
With

This system of equations is integrable, which makes it possible to have only one equation for the rate of cumulated viscoplastic deformation (which one can compare to the total deflection by neglecting them elastic strain).

One can then correlate this expression with the experimental data to adjust the coefficients, but the number of parameters and non-linearities make that difficult (moreover there is not unicity).

It is thus necessary to use a method of correlation calling upon "physical" assumptions on phenomenon of creep whose curve is represented hereafter.
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Deformation (%)
Deformation of end of
*primary education creep **
Time (H)
*T **
tr
Time of creep
Time of
primary education
Creep
rupture
primary education
Creep
Creep
secondary
tertiary sector

Appear 3-a: Various phases of creep on a curve of creep

The curve of deformation according to time obtained after a creep test breaks up into three parts:
· a part known as of primary education creep where the damage is negligible.
· a part known as of secondary creep where the speed of deformation is appreciably constant.

· a part known as of tertiary creep where work hardening is saturated and where phenomena of damage are dominating.

A method of calculation of the parameters using the experimental data (, T) (one uses also (&, T) which results some by a numerical procedure) for various levels from constraints and various temperatures was elaborate at the ECA. It uses the expressions found higher in the case of a homogeneous and unidimensional constraint by making assumptions according to the part of
the curve where the data are taken. For example, in the primary education phase of creep, one makes the assumption D = 0 and in the secondary phase of creep, one uses the fact that &
is constant.

One will find in the references [bib3] and [bib4] the complete description and of the examples of calculations
carried out on the German steel of tank 22 MoNiCr 3 7.

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4
Establishment in Code_Aster

4.1
Algorithm of resolution

The algorithm used is of the total-room type.

The total iterations use the elastic matrix of rigidity calculated starting from the matrix of Hooke damaged:

0
= 1
(- D)

On the level of the local iterations (i.e. in each point of GAUSS), the numerical integration of equations of speed can be carried out either by an explicit diagram of Runge-Kutta of order 2 with automatic cutting in under-not buildings according to an estimate of the error of integration (method of Runge-Kutta encased), that is to say by an implicit scheme of Euler solved by a method of Newton. One will refer to the references [bib3] for all the details concerning the methods numerical, and with [R5.03.14] for the explicit algorithms employed and their programming data processing.

4.2
Implicit integration of the relation of behavior

With each total iteration of resolution of the variational problem of balance and for each point of elementary integration, it is necessary to integrate the equations of the model described into [§3] to obtain it
tensor of the constraints and if required to calculate the operator of tangent behavior.
The problem written in a generic form at the moment T is consisted of the four systems of equations nonlinear following:
T-1

Rb *(, p, state) = 0*

,
éq
4.2-1

T-1
RP *(, p, state) = 0*

with

T-1

(, p, state) = 0

éq
4.2-2

T-1
(
, p,

state
state) = 0

Rb *is a system of six equations (six unknown factors) describing the unknown factors associated with constraints. One notes the vector 6 components of these unknown factors. The connection enters and is realized by means of the system of equations and the vector p contains the variables R and D.*
RP *is a system of equations describing the internal unknown factors. One chooses a system of 2 equations with D& and r& like internal unknown factors. The evolution of the variables of state is described by*
the system of equations.

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The implicit scheme of Euler is used and the algorithm is presented in the following form:

Initialization of the unknown factors of the discretized problem and recovery of the values of the variables of state
obtained with the preceding step
iterations of the method of Newton (a maximum number of pre iterations defined by the user):

· Récupération of the values of the parameters intervening in the material law (the operator of elasticity)
· Calcul of the criteria of constraint and their derivative compared to the constraints
· Récupération of the values of the parameter K intervening in the evolution of the damage and its derivative
· Calcul of the current price of the variables of state, the equations describing the unknown factors interns and of the equations describing the constraints

· Calcul of derived from the equations compared to the unknown factors
· Résolution of the linear system

N
Rb

N
Rb

N

p
D
Rb

N
N
= -

éq
4.2-3
RP
RP

N

dp
RP

p

· Test of convergence
Evaluation of the tangent operator

4.2.1 Discretization implicit of the equations of the model

Considering that an increment of time characterizes a new state of the system [éq 4.2-1] and [éq 4.2-2] solved by an algorithm of Newton, one chooses to identify the state of one quantity at the previous moment
by exhibitor T-1 whereas its current state is noted without exhibitor. Thus variation of a quantity for the increment of time considered is presented by U = U T-1 + U
= U T-1 + T
U& (T
)
For =0, one obtains an explicit diagram and for =1, one obtains a purely implicit diagram.

With these notations, the discretized form of the vectorial system is written:

T
T

***Rb** - (*

1- (1-
D
+ T D&)
1
-
3
( )

-
T
R
HT - vp +

&
=

2
T -

(1 (1
D
+ T D&)
0
(
eq)

éq 4.2.1-1

or more simply ***Rb*** *- (1 - (T 1*
D
+ T D&)

el = 0

=
where is the vector 6 components from the tensor of the constraints.
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NR
(

D
T D&
eq) - y (1 - (T-1 +
)
r& -
= 0

(1 (T-1
D
+ T D&) K (-
R
+ T r&) 1
1
M
T
RP

éq
4.2.1-2

() R
D& -
(1 (T1
D
+ T D&) -
-
K (()) = 0

With

The evolution of the variables of state is described by the system of equations:

T
D = D -1 + T
D&

-1
3

T
r&

-1

R

= + T

= + T
&

éq 4.2.1-3
vp
vp

2 (1 - (-1
-1
&
T
D
+ T
D&)
T
vp

1 - D + T
D&
eq
((T
) eq

T
R = R -1 + T
r&

where D, and R are the variables of state whose history is preserved.
vp

The deformations and the variables of states are not unknown factors of the problem. These sizes will be filed with each increment of time converged to be re-used with the following increment.

4.2.2 Resolution
numerical
***Rb** = 0*
The resolution of the nonlinear system
use the method of Newton-Raphson associated with
***RP** = 0*

a technique of tangential approximation in order to seek the solutions in a field where functions are correctly conditioned.
According to the algorithm of Newton-Raphson, one solves this system in an iterative way on the sequence
following:

· Initialisation of the unknown factors
· Recherche of a direction of descent by the resolution of the system [éq 4.2-3]
X
· Test of convergence err =
X

4.2.3 Operator of tangent behavior

The tangent operator is obtained by deriving the constraints compared to the total deflections according to
made up rules of derivation:

D

p

=
+
+
éq
4.2.3-1
D
p

where the function of constraints (, p, T-1
, v
. The derivative of the unknown factors compared to
state) =
total deflections are obtained by deriving the system [éq 4.2-1] that is to say:
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Rb Rb
Rb

p

= -
éq
4.2.3-2
***RP RP** p*
RP

p

4.2.4 Particular case of the plane constraints

The elements 2D in plane constraints having to be usable for this model of behavior, one carry out an additional treatment in on-layer of the general treatment carried out in 3D. A positive test on the case of the plane constraints means:

· With the resolution of the system [éq 4.2-1], one adds the additional equations

0
33 =
13 =
23 =
· One modifies the tangent operator to ensure energy balance

4.3 Syntax of use

The key word retained for this model is:

“Viscoplasticity with Damage of CHABoche” - > “VENDOCHAB”

This model is accessible in Code_Aster starting from key word COMP_INCR (RELATION: “VENDOCHAB”) of order STAT_NON_LINE [U4.32.01]. The whole of parameters of the model is given via order DEFI_MATERIAU (key words factors VENDOCHAB or VENDOCHAB_FO if the coefficients depend on the temperature and/or the equivalent constraint ()) [U4.23.01].

4.3.1 Operator : DEFI_MATERIAU

The following table gives the exact correspondences between the symbols of the equations and the words keys of Code_Aster:

Parameter material
Symbol in
Key word in Aster
equations
Threshold of viscoplasticity
y
“S_VP”
Coefficient 1 of the equivalent constraint of creep

“SEDVP1”
Coefficient 2 of the equivalent constraint of creep

“SEDVP2”
First exhibitor of the viscoplastic law
NR
“N_VP”
Second exhibitor of the viscoplastic law

M
“M_VP”
Coefficient of the viscoplastic law
K
“K_VP”
Coefficient of the law of damage
With
“A_D”
First exhibitor of the law of damage
R
“R_D”

K [[] T
,]
Second exhibitor of the law of damage

“K_D”

Note:

“_VP” *=> coefficient intervening in an equation of the viscoplastic behavior.*
“_D” *=> coefficient intervening in an equation of the behavior of damage.*
“SEDVP” *=> (Sigma) Equivalent in Viscoplastic Damage.*
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In short, that thus gives following syntax in Code_Aster.

my [to subdue] = DEFI_MATERIAU (

 # Behavior
Rubber band

/
ELAS = _F (

)
/
ELAS_FO
=
_F
(

)

#
Nonlinear Mechanical behavior

/
VENDOCHAB
=
_F (

S_VP
=
[R]

SEDVP1
= [R]

SEDVP2
= [R]

N_VP
=
[R]

M_VP
=
[R]

K_VP
=
[R]

A_D
=
[R]

R_D
=
[R]

K_D
=
[R]

)

/
VENDOCHAB_FO
=_F
(

S_VP
=
[function]

SEDVP1
= [function]

SEDVP2
= [function]

N_VP
=
[function]

M_VP
=
[function]

K_VP
=
[function]

A_D
=
[function]

R_D
=
[function]

K_D
=
[function],
[tablecloth]

)

)

4.3.2 Operator
:
STAT_NON_LINE

One does not detail here all the options of operator STAT_NON_LINE, but only those which it is possible to use for the law of behavior "VENDOCHAB". One will pay attention to the fact that it is necessary
to use:
· is
NEWTON = _F (MATRIX = "ELASTIC")
with
CONVERGENCE
=
_F
(
RESO_INTE

=
“RUNGE_KUTTA_2”)
(An error message is transmitted if not)

· is
CONVERGENCE
=
_F
(
RESO_INTE
=
“IMPLICIT”)
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4.3.3 Use of DEFI_NAPPE when K_D depends on the tensor of the constraints.

In the command files where one wishes to make depend parameter K_D on the constraint, it is necessary to use operator DEFI_NAPPE. It is necessary for that to use DEFI_NAPPE in the following way (it key word opposite NOM_PARA must be “TEMP” imperatively, and NOM_PARA_FONC is then “X”):

An example below is shown:

```
kd_t = DEFI_NAPPE (NOM_PARA = ' TEMP',
PROL_DROITE = ' LINEAIRE',
```

```
PROL_GAUCHE = ' LINEAIRE',
PARA = (900, 1000. , 1025. , 1050),
NOM_PARA_FONC = ' X',
DEFI_FONCTION = _F (PROL_DROITE = ' LINEAIRE',
PROL_GAUCHE = ' LINEAIRE',
VALE= (0.1, 14.355,
100. , 14.855,
200. , 14.355 ),
),
_F (PROL_DROITE = ' LINEAIRE',
PROL_GAUCHE = ' LINEAIRE',
VALE= (0.1. , 14.5,
100. , 15.,
200. , 15.5 ),
),
_F (PROL_DROITE = ' LINEAIRE',
PROL_GAUCHE = ' LINEAIRE',
VALE= (900. , 14.5363,
1000. , 15.0363,
1050. , 15.5363 ),
_F (PROL_DROITE = ' LINEAIRE',
PROL_GAUCHE = ' LINEAIRE',
VALE= (0.1, 14.5725,
100. , 15.0725,
200. , 15.5725 ),
) )
```

If there is only one dependence in constraint, it is imperatively necessary to use a DEFI_NAPPE with thus a “virtual” dependence in temperature.

For example:

```
kd_t = DEFI_NAPPE (NOM_PARA = ' TEMP',
PROL_DROITE = ' LINEAIRE',
PROL_GAUCHE = ' LINEAIRE',
PARA = (0. , 10000.),
NOM_PARA_FONC = ' X',
DEFI_FONCTION = (_F (PROL_DROITE = ' LINEAIRE',
PROL_GAUCHE = ' LINEAIRE',
VALE= (0.1. , 14.5,
100. , 15.,
200. , 15.5 ),
),
_F (PROL_DROITE = ' LINEAIRE',
```

PROL_GAUCHE = ' LINEAIRE',
VALE= (0.1. , 14.5,
100. , 15.,
200. , 15.5),
)),
)

It should be noted that the use of a tablecloth for K_D slows down calculations because the value of K_D in function
tensor of the constraints is reactualized with each local iteration.
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5
Significance of the internal variables

The internal variables of the model at the points of Gauss (key word VARI_ELGA) are accessible by:

· V1 = 11

vp
· V2 = 22

vp
· V3 = 33

vp
· V4 = 12

vp
· V5 = 13
vp
· V6 = 23

vp
· V7 = p, cumulated plastic deformation
· V8 = R, the variable of isotropic work hardening viscoplastic
· V9 = D, the variable of damage

6 Bibliography

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:

Organization (S): EDF/AMA

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Document: R5.03.16

Elastoplastic relation of behavior with linear and isotropic kinematic work hardening nonlinear. Plane modeling 3D and constraints

Summary:

This document describes an elastoplastic law of behaviour to mixed, kinematic work hardening linear and isotropic nonlinear. Equations to solve to integrate this relation of behavior numerically are specified, as well as the coherent tangent matrix.

This behavior is usable for modelings of continuous mediums 3D, 2D (AXIS, C_PLAN, D_PLAN), and for modelings DKT, COQUE_3D and PIPE.
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1 Introduction

When the way of loading is not monotonous any more, work hardenings isotropic and kinematic are not more equivalent. In particular, one can expect to have simultaneously a kinematic share and one isotropic share. If one seeks to precisely describe the effects of a cyclic loading, it is desirable to adopt modelings sophisticated (but easy to use) such as the model of Taheri, for example, cf [R5.03.05]. On the other hand, for less complex ways of loading, one can wish to include only one linear kinematic work hardening, all nonthe linearities of work hardening being carried by the isotropic term. That makes it possible to follow a curve precisely of traction, while representing nevertheless phenomena such as the Bauschinger effect [bib1] (see for example it [Figure 5-a]).

The characteristics of work hardening are then given by a traction diagram and a constant, said of Prager, for the term of kinematic work hardening linear. They are introduced into order DEFI_MATERIAU:

Linear isotropic work hardening
Nonlinear isotropic work hardening
DEFI_MATERIAU (
DEFI_MATERIAU
(
ECRO_LINE: (

TRACTION: (SIGM: curve of

SY: elastic limit
traction)
D_SIGM_EPSI
:
slope of the curve of PRAGER:
(C:
constant of Prager)
traction)
)
;

PRAGER: (C: constant of Prager)
) ;

These characteristics can also depend on the temperature, with the proviso of employing the words then
keys factors ECMI_LINE_FO and ECMI_TRAC_FO in the place of ECRO_LINE and TRACTION. The employment of
these laws of behavior is available in orders STAT_NON_LINE or
DYNA_NON_LINE:

Linear isotropic work hardening
Nonlinear isotropic work hardening
STAT_NON_LINE
STAT_NON_LINE
(
(
COMP_INCR
:
COMP_INCR
:

(

(
RELATION
: “VMIS_ECMI_LINE”
RELATION: “VMIS_ECMI_TRAC”

)

)
)
;
)
;

In the continuation of this document, one precisely describes the model of combined work hardening. One presents
then the detail of its numerical integration in bond with the construction of the tangent matrix coherent. Lastly, a tensile test uniaxial pressing illustrates the identification of the characteristics material.

2
Description of the model

At any moment, the state of material is described by the deformation, the temperature T, the deformation
plastic p and the cumulated plastic deformation p. the equations of state defines then in function of these variables of state the constraint = H Id +
~ (broken up into parts hydrostatic and
deviatoric), the isotropic share of work hardening R and the kinematic X, so called share forced of recall:

H 1
=
() =
(- HT) with HT = (réf
tr
K tr
T-T
) Id éq 2-1
3
~
1
= - H Id = μ (~ - p
2
) ~

where = - tr () Id
éq
2-2
3
R = R (p)

éq 2-3
X

p
= C

éq 2-4

where K, μ, R
C
and are characteristics of material which can depend on the temperature.
More precisely, they are respectively the modules of compressibility and shearing, it
thermal dilation coefficient average (see [R4.08.01]), the isotropic function of work hardening and
constant of Prager. As for T réf, it is about the temperature of reference, for which
thermal deformation is null.

K, μ are connected to the Young modulus E and the Poisson's ratio by:

E
K = 3 + 2μ = 1 - 2
E
2μ = 1+

Note:

Concerning the kinematic share of work hardening [éq 2-4], one notes that it is linear in it
model. In addition, it is necessary to take guard with the fact that in certain references, one calls
constant of Prager 2C/3 and not C. In the same way, for the isotropic function of work hardening,
elastic limit is included there by R ()
0 = y, certain references treating it separately.
The evolution of the variables intern p and p is controlled by a normal law of flow associated
a criterion of plasticity F:

(
3 ~ ~
F, R, X) = (~ - X) - R
with
With

=
WITH A
eq
eq

éq
2-5
2
~
p

3
- X
& = & F

= &

éq 2-6
2 (~
- X) eq
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&p = &
2
=
&p &p

éq 2-7
3

As for the plastic multiplier &
, it is obtained by the condition of following coherence:

if F<0 or &F < 0 & = 0

éq 2-8
if
F

= 0 and &F = 0 & 0

3
Integration of the relation of behavior

To numerically carry out the integration of the law of behavior, one carries out a discretization in time and one adopts a diagram of implicit, famous Euler adapted for relations of behavior elastoplastic. Henceforth, the following notations will be employed: Has, A E T A
represent
respectively values of a quantity A at the beginning and the end of the step of time considered thus that

its increment during the step. The problem is then the following: knowing the state at time T - thus that increments of deformation and temperature T, to determine the state at time T like constraints.

Initially, one takes into account the variations of the characteristics compared to temperature by noticing that:

H
K
=
H + K tr (-
-
HT) éq
3-1
K
~
μ ~
=
- +
-
2μ (~

-
p) éq 3-2
μ
C
X
X
p
=
+
-
C

éq 3-3
C

Within sight of the equation [éq 3-1], one notes that the hydrostatic behavior is purely elastic. Only the treatment of the deviatoric component is delicate. To reduce the writings to come, one introduced ~se the difference ~
- X in the absence of increment of plastic deformations, so that:

~
μ ~ - C -
~
- X =
-
X + 2μ -
p
-
(2μ +
-
C)

éq 3-4
μ
C
1444 2
4
3
4444
~se
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The equations of flow [éq 2-6] and [éq 2-7] and the condition of coherence [éq 2-8] are written one times discretized and by noticing that p =:

~
p
3
- X
= p

éq 3-5
2
(~ - X) eq
F 0 p 0 F p = 0

éq 3-6

The treatment of the condition of coherence [éq 3-6] is traditional. One starts with a test rubber band (p = 0) which is well the solution if the criterion of plasticity is not exceeded, i.e. if:

F = E - R (-
S
p
eq
) 0 éq 3-7

In the contrary case, the solution is plastic (p > 0) and the condition of coherence [éq 3-6] is reduced with F = 0. To solve it, one starts by showing that one can bring back oneself to a scalar problem by eliminating p. Indeed, by taking account of [éq 3-4] and [éq 3-5], one notes that p is colinéaire with ~se bus:

p
3
p

~e
p
=
2µ C

éq
3-8
2 (
S
~
-
+

- X) [
(
)]
eq

In addition, according to [éq 3-5], the standard of p is worth:

(
3
p) = p éq 3-9
eq
2

One thus deduces immediately the expression from it from p according to p:

~e
3
S
p = p

éq 3-10
2
seeq

It now only remains to replace p by its expression [éq 3-10] in the equation [éq 3-4]

3

(2μ + C) p

2
one obtains: ~
~
- X = S.E. 1

seeq

by deferring ~
- X in the equation F = 0, one is brought back to a scalar equation out of p to solve,
with knowknowing:

3
- (2μ + C) p
- R (p + p
eq
) = 0 éq
3-11
2
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Insofar as the function R is positive, which one will admit henceforth, there exists a solution p with this equation, characterized by:

3
E
(
2
S
2μ +) + R (-
C
p
p + p
) =
where 0
eq
eq
< p
<
éq
3-12
2
3 2μ + C

Let us note that in the interval specified in [éq 3-12], the solution is single. For details as for solution of this equation, one will refer to [R5.03.02].

The particular case of the plane constraints is studied with [§6].

4
Calculation of tangent rigidity

In order to allow a resolution of the total problem (equilibrium equations) by a method of Newton, it is necessary to determine the coherent tangent matrix of the incremental problem. For that, one once more adopts the convention of writing of the symmetrical tensors of order 2 pennies form vectors with 6 components. Thus, for a tensor a:

T
= [axx ayy azz has
2axy
2axz
2ayz]

éq
4-1

If one introduces moreover the hydrostatic vector 1 and stamps it deviatoric projection P:

1 =t [1 1 1 0 0]
0

éq 4-2
1
P = Id - 1 1

éq 4-3
3

Then the matrix of coherent tangent rigidity is written for an elastic behavior:

= K 1 1 + 2μ P

éq 4-4

and for a plastic behavior:

μ
3

p
~
~
2
E
E
p
1
S
S
= K 1 1 + 2μ1-
P 9μ

éq 4-5

+
-
E
3
E
E
seq
R
S
S

eq
eq
eq

(p) + (2μ + C)
2

The initial tangent matrix, used by option RIGI_MECA_TANG is obtained by adopting it behavior of the preceding step (elastic or plastic, meant by internal variable being worth 0 or 1) and by taking p = 0 in the equation [éq 4-5].
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Note:

RIGI_MECA_TANG is the operator linearized compared to time (cf [R5.03.01], [R5.03.05]) and to the problem of speed corresponds what is called; in this case, the linearization by report/ratio with U, out of U = 0, provides the same expression.

One now proposes to show the expression [éq 4-5]. By differentiating them [éq 2-1] and [éq 2-2] with fixed temperature, one obtains immediately:

= [K 11+ μ P] - μ p
2
2

éq 4-6

If the mode of behavior is plastic, the incremental law of flow [éq 3-10] provides then:

~e
~e

3
S
3
S
p = p
+
p

éq 4-7
2

2

eq
eq

As for p, it is obtained by differentiating the implicit equation [éq 3-12]:

3 (

2µ + C) + (p) p
R

éq 4-8

2

= eq

Lastly, it any more but does not remain to provide the variations of ~se:

~e
~e

~e
~e
~
1
= 2μ ~ seeq = μ S
3
~ S

2μμ S
S
E

3
~
S
éq
4-9

E
E
E
E
eq

seq =
-

seq
S
S
eq
eq

While replacing then [éq 4-7], [éq 4-8] and [éq 4-9] in [éq 4-6], one obtains well the expression [éq 4-5].

This expression is formally identical to that defined in R5.03.02: [éq 4-3] and is written:

μ
3

~e
~e
p
1
p
1
S
S
K1 1 2μ

1
Id
1 1
9μ2
=
+
-

-

3

+
-
E

3
E
E
seq
R+ (2μ + C) S
S
eq
eq
eq

2

with = 1 if led to a plasticization, and = 0 if not.
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While using [éq 3-12], one finds:

2
Dev.
Dev.
** R*
R
***
9μ
R. p
1

= 1 1 + 2µ Id -

1-

H (p)
R (p)
3
R+ (2µ + C) R (p) R (p)
2
2µ G (p)
G (p)
*with * = K -*
2
2
3 H (
** =*
p)
µ
µ H (p)

for the option

FULL_ MECA
Dev. = ~
-
: X
for option RIGI_ MECA_ TANG Dev. =
~ - - -
X

:
3 (2µ + C) p
with H (p) = 1+ 2
(
R p)
3

p
and
G (p) = 1+ C
2
(
R p)

5
Identification of the characteristics of material

Let us consider a tensile test uniaxial pressing, [Figure 5-a]. One proposes to show how it makes it possible to identify the constant of Prager and the isotropic function of work hardening. In such test, the various tensors are with fixed directions, i.e.:

2 3

~
p
3 p

= D
X = X D
= D
with D =
-

1 3
5-1
2

- 1
3

As long as the loading is monotonous, therefore in phase of traction, one obtains them immediately following relations:

3
3
p
p

X
C p
T

C p

R (p
=
=
=
+) 5-2
2
2

The constant of Prager is determined by the first plasticization in compression, since one a:

T
3
p
p
WITH = CA + R
2
(A)
T
C
WITH + A

C =
3
p

éq
5-3
C
p
p

3
With =

A.C. - R

2
(A)
With
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T (p)
your
p
3C 2 A
p

= - E
C
C (p)
With

Appear 5-a: Tensile test uniaxial pressing

The curve of work hardening T = (p
F
) from the traction diagram T = F is deduced () provided by
the user under key words ECRO_LINE ((SY and D_SIGM_EPSI (linear work hardening)) or TRACTION (unspecified work hardening). It finally makes it possible to obtain the isotropic function of work hardening
by [éq 5-2]:

3

R (p) = T (p) -
p
C
.
2

For the effective calculation of R (p), according to the R5.03.02 document, one titrates party of linearity (ECMI_LINE)
or of the linearity per pieces of the traction diagram (ECMI_TRAC):

ECMI_LINE:

T
E.E
= F (p) =
T
y +
p
E - AND
E.E
3
R (p) =
T
y +
- C p = y + R. p
éq
5-4
E - AND 2

The equation [éq 3-12] becomes then:

3 (2μ +C) p +
E

.

éq
5-5
y + R (p +
p) = S
2
eq

ECMI_TRAC:

T
p
i+1 -
= F () =
I
I +
(p - IP), for IP p ip+1
I
p -1 - I
p
éq 5-6
+1 -
3
+1 -
R (p)
I
I
I

=
I
I +
(p - IP) - CP = I -
I
p + R. p
I
p -1 - I
p
2
I
p -1 - I
p
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Note:

For the use: the correspondence enters the model of behavior VMIS_CINE_LINE and it behavior VMIS_ECMI_LINE is as follows:
For VMIS_CINE_LINE, it is necessary to introduce into DEFI_MATERIAU a linear work hardening of slope
And by:
D_SIGM_EPSI: And
With VMIS_ECMI_LINE, to reproduce same behaviour with kinematic work hardening linear, it is necessary to give in DEFI_MATERIAU.

· a linear work hardening of slope And: D_SIGM_EPSI: And
· The constant of Prager C: PRAGER: C

2 EE
C is determined by: C
T
=

3rd - AND
It should well be noticed that the identification of C and R (p
) have directions only in one field
deformations limited (small deformations). In particular, if T (p) presents an asymptote tmax for p sufficiently large, then the kinematic contribution of work hardening does not have any more
significance. It is thus advised to restrict itself with the field where work hardening is strictly positive.

6
Particular case of the plane constraints: calculation of p

It is necessary to add to the equations [éq 3-1] with [éq 3-4] the condition of plane constraints = 0, which
33
add an unknown factor (corresponding deformation):

H
K
=
H + K tr (-
-
HT) éq 6-1
K
~
μ ~
=
- +
-
2μ (~

-
p) éq 6-2
μ
C
X
X
p
=
+
-
C

éq 6-3
C
= 0

éq 6-4
33

Then, the equation [éq 3-4] becomes:

~
μ ~
C
-
-
~
- X =
C
p
y
E
p
y

-
- - X + 2
- (2 + C)
~
~
+ 2
= S - (2 + C)
~

μ
μ
μ

μ
μ
+ 2μ éq 6-5
C

where ~
C is entirely determined by the elastic behavior:

-
C

~ =
(~c + ~c), ~c = ~, ~c
33
11
22
11
11
22 = ~

-

22
1
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0 0 0

and ~
y = 0 0 0 is unknown. It is also supposed that =
=
=
= 0 .
13
23 13 23

0 0 Y

One always has:

~
p
3
- X
= p

éq 6-6
2

(~ - ***X)*** *eq*
F =
R p
0
p

0
p

0
éq
6-7
eq -
( )

F
=

Elastic test:

· If

F = E - R (-
S
p
0

éq 6-8
eq
)

then

~
~
=*, p = 0****, Y = 0* ***éq 6-9***

K

H =
-
tr
HT
- H + K
(C -
*) **éq***
6-10
K

· If not, the solution is plastic: p > 0, Y 0. One can still bring back oneself to a problem scalar in p.

By taking account of [éq 6-5] and [éq 6-6], one notes that ~
- X is colinéaire with ~
~
***S.E.** + 2µ*
y bus:

3

(2µ +C) p

*(~ - **X**) + 2*
*= (~ - **X**) H (p*
) = [~
~

S.E.
1
*+ 2µ y] **éq 6-11***
R (p)

Thus:

(~
E
4
-

X
H p
=

éq
6-12
33 +
Y

33
33)
() ~s
µ

3

We will only express the equation [éq 6-12] according to p. According to [éq 6-4]:

K
= 0 ~

~
=
+
=
+ K Y
-
, *with H*
H
C
=
tr
HT
-
+ K
-
éq
6-13
E

(
)
E
+ .
33
33
33
K
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While using [éq 6-5], [éq 6-6] and the incompressibility of the plastic deformations, one can show that:

~
C
S.E. +
= -
X -

éq 6-14

33
E
C
33

Then:

~
~

C
= S.E.

éq 6-15

33 - K.
Y +
-
33
X
-
C
33

As according to [éq 6-3]:
C
C
3
~
-
-
- X
X =
X
.

.

-
+ C
p =
X
-
+ C
p 33
33
éq
6-16
33
C
33
33
C
33
2
R (p)
C
3
~

3
p

X. G
-
33
=
G p
= 1+ C
-
+

, with ()

éq 6-17
33
(p)
X 33
C p
C
2
R (p)

2
R (p)

From [éq 6-12], [éq 6-15], [éq 6-17], one obtains an equation flexible p and Y:

4
H (p)
H

~
(p)
Y. µ + K
*= **S.E.***
*-1 **éq***
6-18
3
G (p)
33

G

(p)

The equation [éq 6-11] makes it possible to obtain the scalar equation out of p to be solved, namely:

(~ -) H (p
) = R (-
X
p + p
) H (p
) = [~
~

S.E.
y
2
µ
éq
6-19
eq
+
] eq

Equation in which Y is related to p by the equation [éq 6-18].

As in the case of isotropic work hardening, one obtains a scalar equation out of p, always not linear, which is solved by a method of secant.

Once p known, the calculation of ~
and X is carried out starting from the expression of Y, therefore of
entirely known, by a step similar to that of the equation [éq 3-10].

~e
~ y
~
p
3
S *+ 2*
μ
3
- ***X***
= p
p

éq
6-20
2
(
=
~e
S *+ 2*
μ ~ y)
2
H (p) ~
~ y
μ
eq
*(****be*** *+2) eq*
~
μ ~
=
- *+*
-
2μ (~

-
p) éq 6-21
μ

One obtains by eliminating ***p*** *from [éq 6-6], [éq 6-3] and [éq 6-2]:*

~
μ ~
G p

-
~
() 3
p
C
=
-
2μ
2μ
X
- +

+

éq
6-22
μ
H (p) 2
R (p) H (p) -

C
3
p

µ

3
p
~
~

C
X = *C*

2
µ
1
C

éq 6-23
2
R (p) H (p
)
-
-
-
+
+ -

µ

2
R (p) H (p
)
X
-
C

7
Significance of the internal variables

The internal variables of the model at the points of Gauss (VARI_ELGA) are for all them modelings:

· *VARI_1 = p: cumulated plastic deformation (positive or null)*
· *VARI_2 =: being worth 1 if the point of Gauss plasticized during the increment or 0 if not.*

X: tensor of recall:

For modeling 3D:

· *VARI_3 = X*
11
· *VARI_4 = X*
22
· *VARI_5 = X*
33
· *VARI_6 = X*
12
· *VARI_7 = X*
13
· *VARI_8 = X*
23

For modelings D_PLAN, C_PLAN, AXIS

· *VARI_3 = X*
11
· *VARI_4 = X*
22
· *VARI_5 = X*
33
· *VARI_6 = X*
12

For modelings of hulls (DKT, COQUE_3D), in local reference mark and each point of integration of each layer:

· *VARI_3 = X*
11
· *VARI_4 = X*
22
· *VARI_5 = X*
33
· *VARI_6 = X*
12
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Relations of behavior of the discrete elements

Summary:

This document describes the nonlinear behaviors of the discrete elements which are called by the operators
of resolution of nonlinear problems STAT_NON_LINE [R5.03.01] .ou DYNA_NON_LINE [R5.05.05].

More precisely, the behaviors described in this document are:

.
the behavior of the Von Mises type to isotropic work hardening used for the modeling of threaded assemblies, implemented in MACR_GOUJ2E_CALC, accessible by the key words DIS_GOUJ2E_PLAS and DIS_GOUJ2E_ELAS of key word COMP_INCR [U4.51.11],
.
the behavior of unilateral the contact type with friction of Coulomb (in translation), and it behavior of the Von Mises type to isotropic or kinematic work hardening linear (in rotation), used to model the behavior within the competences of connection - pencil of the fuel assemblies roasts, accessible by key word DIS_CONTACT from key word COMP_INCR [U4.51.11],
.
the behavior of the shock type with friction of Coulomb, accessible by key word DIS_CHOC from

key word COMP_INCR [U4.51.11].

The integration of the models of behavior mentioned above is implicit.

Other behaviors relating to the discrete elements are available, but nonhere detailed:

.
Armament of the lines (Relation ARMS) [R5.03.31],
.
Nonlinear assembly of angles of pylons (Relation ASSE_CORN) [R5.03.32],

---

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matters

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1

General principles of the relations of behavior of discrete elements

1.1 Nonlinear relations of behavior (of the discrete elements) currently available

The relations available in Code_Aster for the discrete elements are relations of behavior incremental data under the key word factor COMP_INCR by the key word RELATION in nonlinear operators STAT_NON_LINE and DYNA_NON_LINE. One distinguishes:

- *the behavior of the Von Mises type to isotropic work hardening used for the modeling of assemblies threaded, implemented in MACR_GOUJ2E_CALC and accessible by the words keys DIS_GOUJ2E_PLAS and DIS_GOUJ2E_ELAS,*
- *the behavior of unilateral the contact type with friction of Coulomb, used for to model the behavior in translation within the competences of connection roasts - pencil of fuel assemblies, accessible by key word DIS_CONTACT,*
- *the behavior of the Von Mises type to isotropic or kinematic work hardening linear used to model behaviour in rotation within the competences of connection - pencil roasts of fuel assemblies, also accessible by key word DIS_CONTACT, of STAT_NON_LINE.*

And the following behaviors, which are not here detailed:

- *Armament of the lines (Relation ARMS) [R5.03.31],*
- *Nonlinear assembly of angles of pylons (Relation ASSE_CORN) [R5.03.32],*
- *Contact with shocks (Relation DIS_CHOC).*

The parameters necessary to these relations are provided in operator DEFI_MATERIAU by key words:

Behavior in Type of element (modeling) key Words in
AFFE_CARA_ELEM
STAT_NON_LINE
in AFFE_MODELE
DEFI_MATERIAU
key words under DISCRETE
DYNA_NON_LINE

DIS_GOUJ2E_ELAS
2D_DIS_T: discrete element 2D TRACTION
CARA: “K_T_D_L”
DIS_GOUJ2E_PLAS
with two nodes in translation
DIS_CONTACT
DIS_T or DIS_TR: element DIS_CONTACT: (
CARA: “K_T_D_L”
friction of discrete Coulomb 3D with two nodes in COULOMB:
or
translation or translation/
RIGI_N_FO:
CARA: “K_TR_D_L”
rotation
EFFO_N_INIT: if rotation

)
DIS_CONTACT
DIS_TR discrete element 3D with DIS_CONTACT:
CARA: “K_TR_D_L”
rotation
two nodes in translation/RELA_MZ: curve
rotation
DIS_CHOC contact and DIS_T: discrete element 3D with DIS_CONTACT: (
CARA: “K_T_D_L”
shock with friction two nodes in translation
COULOMB:
To be able to use
of Coulomb
RIGI_NOR:

stamp rigidity
RIGI_TAN:
rubber band, option
AMOR_NOR:
RIGI_MECA
AMOR_TAN:
DIST_1
DIST_2
PLAY
)

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These behaviors were developed and are used for the following applications:

DIS_GOUJ2E_PLAS: Relation effort - displacement of the Von Mises type with work hardening isotropic for the modeling of the threaded assemblies
DIS_GOUJ2E_ELAS: Relation effort - linear displacement deduced from the curve effort - displacement characterizing L `assembly
DIS_CONTACT
Elastoplastic behaviour in rotation,
unilateral contact with friction of Coulomb in translation
DIS_CHOC
To take into account the shocks as well as friction enters one
structure and its supports or between the structures.

Contrary to the models of behavior 1D [bib3], these relations bind the efforts directly and displacements, instead of being formulated between constraints and deformations. They are not valid that in small deformations.

One describes for each relation of behavior the calculation of the field of efforts starting from an increment of
displacement given (cf algorithm of Newton [R5.03.01]), the calculation of the nodal forces R and of stamp tangent.

1.2
Calculation of the deformations (small deformations)

For each finite element of Code_Aster, in STAT_NON_LINE, the total algorithm (Newton)

provides to the elementary routine, which integrates the behavior, an increase in field in displacement [R5.03.01]

For the discrete elements, one deduces from it the increase in elongation (in translation) or rotation, between nodes 1 and 2 of the element, which is equivalent to the calculation of the increase in deformation
in the case of continuous mediums or of the behaviors 1D.

= U - U
2
1 ,

1.3
Calculation of the efforts and the nodal forces

For integration of the behavior, it is necessary to provide to the total algorithm (Newton) a vector containing
generalized efforts, on the one hand, and on the other hand a vector containing the nodal forces R [R5.03.01] in total reference mark (X, Y, Z).

For the discrete elements, the resolution of the nonlinear local problem provides the efforts directly in the element (uniforms in the element), in local reference mark (X, y, Z), which is form:

F (
1
node)
F = 1

F
with
2 (node 2)
Fx

in 2D: F = F = F
1
2
y

X

y
Fx

N2

F
F
y

X

Fz
in 3D: F = F = F
=
=
1
2
there of translation alone, F
F
1
2

in

M
F
X

N1
Z
My
M
Z
translation and rotation.
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The vector R of the equivalent nodal forces (which is expressed in the total reference mark) is deduced from
F by change of reference mark:

- F (node 1)
R = PT R P
R
with
=
1
loc
loc
F

(node
2
2)

where P is the matrix of change of reference mark, allowing the passage of the total reference mark towards the reference mark
room, as for the elements of beam [R3.08.01].

2
Relation of behaviour of the threaded assemblies

2.1 Notations
general

All the quantities evaluated at the previous moment are subscripted by -.
Quantities evaluated at the moment T + T
are not subscripted.
The increments are indicated par. One has as follows:

Q = (
Q +) = Q () + (
Q) = Q
T
T

T
T
+ Q

2.2
Equations of model DIS_GOUJ2E_PLAS

They are deduced from the behavior 3D VMIS_ISOT_TRAC [R5.03.02]: a relation there is represented
of behavior of the elastoplastic type to isotropic work hardening, binding the efforts in the element discrete with the difference in displacement of the two nodes in the local direction y.

In local direction X, the behavior is elastic, linear, and the coefficient connecting the Fx effort to Dx displacement is the Kx stiffness provided via AFFE_CARA_ELEM.

The nonlinear behavior relates to only the local direction y.

While noting = u1 - u2 and = F 1 = F 2
y
y
y
y

The relations are written in the same form as the relations of Von Mises 1D [R5.03.09]:

p

& = &p
= E (- p)
- R p =
- R p 0
eq
( )
( )
- R p < 0 p = 0
eq
( )
&
- R p = 0 p 0
eq
( )
&

In these expressions, p represents a “cumulated plastic displacement”, and the function of work hardening

isotropic R (p) is closely connected per pieces, data in the form of a curve effort - displacement definite
point by point, provided under the key word factor TRACTION of operator DEFI_MATERIAU [U4.43.01].

The first point corresponds at the end of the linear field, and is thus used to define at the same time the limit of
linearity (similar to the elastic limit), and E which is the slope of this linear part (E are independent of the temperature). The function R (p) is deduced from a curve characteristic of the assembly (modeling of some nets) expressing the effort on the pin according to difference in average displacement between the pin and the support [bib1]: F = F (U - v).
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2.3
Integration of relation DIS_GOUJ2E_PLAS

By direct implicit discretization of the relations of behavior, a way similar to integration 1D [R5.03.09] one obtains:

- +
E - = E p - +
- + - (
R p + p) 0

- + - (
R p + p) < 0 p = 0

- + - (
R p + p) = 0 p 0

Two cases arise:

.
-

R (-
+
<
p + p) in this case p = 0 is =
E thus
- +
< (-

R p),
.
-

R (-
+
=
p + p) in this case p 0 thus -

R (-
+

p).

One deduces the algorithm from it from resolution:
let us pose E
= - +

if E R (p) then
p

= 0 and =
if E > R (p) then it is necessary to solve:
- +

E = - +

+ E p
-

+

E p
E
= +
(-
1

)

+

+
-

thus by taking the absolute value:

E p

E = +
-
1

+

+
-

maybe, while using
-

R (-
+
=

p + p).

E = R (p + p
) + E p
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By taking account of the linearity per pieces of R (p), one can solve explicitly this equation to find p. Once calculated p one deduces some in the following way:
E

=

E
R (p)
then:
E
E
= (- +
) =
R (p) =

E
E p
1 + R (p)
Moreover, the option
N
FULL_MECA makes it possible to calculate the tangent matrix ***K*** *I with each iteration.*
The tangent operator who is used for building it is calculated directly on the preceding discretized system.
One obtains directly:

E.R p

if E > R (-

p)
( )

= E =

T
E + R (p)

if not
=

E

2.4 Variables interns

The relation of behavior DIS_GOUJ2E_PLAS produces two internal variables: “displacement plastic cumulated” p, and an indicator being worth 1 if the increase in plastic deformation is nonnull and 0 in the contrary case.

3 Relation of behavior within the competences of connection roasts fuel pin

Behavior DIS_CONTACT, used to model the behavior in translation of the springs of connection roasts - pencil of the fuel assemblies covers in fact two behaviors distinct, relating to different degrees of freedom:

.
the behavior of unilateral the contact type with friction of Coulomb, relates to them directions X and y local,
.
the behavior of the Von Mises type to isotropic or kinematic work hardening linear used to model behaviour in rotation relates to rotation around Z local and that around X local.

In the other directions (translations according to Z local, rotation around there local), the behavior is rubber band, defined by the stiffnesses provided in CARA: “K_T_D_L” or CARA: “K_TR_D_L” of the key word
DISCRETE factor of order AFFE_CARA_ELEM.

3.1
Model of contact with friction of Coulomb

3.1.1 Presentation of the model of contact - friction

The connection roasts - pencil is ensured by a flexible blade and two bosses. This system allows one slip with friction of the pencil in the axial direction. In addition, the neutron irradiation has for effect to soften this connection: the gripping force decreases according to time. To model this connection, one introduced a behavior which applies for discrete elements to two nodes MECA_DIS_TR_L and MECA_DIS_T_L

That is to say an element with two nodes. That is to say NR direction carried by the element (X local) and T a direction
perpendicular (in this case, it corresponds to the axial direction of the pencil): it is the direction y local. Are the efforts in the directions NR and T (cf [Figure 3.1.1-a]).
The relation of behavior of elastic type perfectly plastic and is characterized by [Figure 3.1.1-b]:

.
KTe an "elastic" slope,
.
Elastic kN rigidity in the direction NR,
.
a threshold of friction defined by R

= μ R
T
NR where μ is the coefficient of friction of
Coulomb,
.
a module "of work hardening" fictitious KTl to avoid a slip not control,
.
RN0 an initial tension in the direction NR,
.
a function of time F (T) or fluence G (), standardized ("of decrease") of
rigidity of the connection in the direction NR.
T
Center pencil
UT
Node 1
Node 2
Roast
NR
UN1
UN2

Appear 3.1.1-a

RT
R
K
S
Tl
KTe
&
UT

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These data are provided in DEFI_MATERIAU:

DIS_CONTACT: (
COULOMB: μ

(coefficient of friction)
EFFO_N_INIT

:
RN0
(initial tension of the spring)

/
RIGI_N_FO

:
F (T)

(standardized function of time)
/
RIGI_N_IRRA
:
G () (standardized function of the fluence)

.
KT_ULTM

:
Ktt (slope of work hardening)

)

Elastic characteristics of the springs K
and K
NR
Te are provided under the key word factor
DISCRETE of order AFFE_CARA_ELEM. Modeling supposes that the direction T of
slip is the there local one and that X local is the normal direction NR with the contact (to direct the element
discrete, one uses the key word factor ORIENTATION of AFFE_CARA_ELEM). The contact with friction
is written in directions X and Y. For third direction Z and the degrees of freedom of rotation,
the law of behavior is purely elastic and the characteristics of rigidity do not vary with time.

The slope of work hardening is a slope of regularization which simulates a nonperfect slip, but which allows to obtain a solution if the pencil is subjected to no imposed displacement and enters one mode of pure slip.

3.1.2 Equations of the model

The model of contact - friction is similar to a behavior of Von Mises in perfect plasticity:

R 0
NR
R = F T R
+ K U
- U
NR
() (N0
NR (
N2
N1)
R = K U - U
U

with

= U - U
T
Te (
p
T
T)
T
T 2
T1
R R + K.
T
S
Tl
R = - μ R
S
NR

R
p
T
U& = .sgn (R) =

T
T
RT

with >

if

0
R = R
T
S

=

if

0
R < R
T

S

K K
K is defined by:
Te
Tt
K =
.

Tl
Tl
K - K
Te
Tt

3.1.3 Integration of the relation

It is made on the basis of purely implicit integration.
One supposes known the solution at the moment previous T -: R -, R -
T
NR
and displacements and increases in displacements of iteration N of the algorithm of Newton of STAT_NON_LINE, noted:

U = U - U and U = U (T) - U (T)
T
T 2
T1
NR
N2
NR 1
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:

The problem is written then:
R = F (T)
+
0
NR
(R
K U
NR 0
NR
NR)

R
R

= K U
T
-

T
Te
T

R
T
R R +. K

T
S
Tl

with > 0 if R = R
T
S
= 0 if R < R
T
S

Phase of prediction:
One poses: R
= R + K. U

Te
T
Te
T
One can find R directly =
{
min,
0 F (T) (R + K U
NR
NR 0
NR
NR)}
Resolution:

.
If there are contact, then R 0 and:
NR
- If
R < R
Te
S then it does not have there slip:
= 0 and R = R
T
Te
-
If not, there is slip:
To solve, one writes like usually:
R
R
R = R + R

= R + K
U

- K
T
.
= R - K
T

.
T
T
T
T
Te
T
Te
R
Te
Te
R
T
T
R = R + K
-
. +
T
S
Tl (
)

thus by gathering the terms:

1
R 1 + K

.
R
T
Te

R = Te
T

R = R + K
-
. +
T
S
Tl (
)

that is to say still

RT (R + K. =
T
Te
) R
R
Te
T
R = R + K
-
. +

T
S
Tl (
)
thus R
= (R + K

Te
T
Te.), which makes it possible to find (
) :
R
= R + K (-
+
) + K (
) = R + K (-
.
.
.) + (K + K.

Te
S
Tl
Te
S
Tl
Te
Tl) (
)

then to find R by:
T

R
R
-
Te
=
+
+

.

T
(R K
S
Tl (
)) RTe
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.
If there is no more contact (R =
=
NR
0) then: RT 0

For the calculation of the tangent matrix, the option
N
FULL_MECA makes it possible to calculate the tangent matrix ***K*** *I*

with each iteration. The tangent operator who is used for building it is calculated directly on the system discretized preceding. One obtains directly:

- if R 0 and > 0 then
K and
F (T). K

NR

=

Tl

=
NR
T
NR

- if not
0 and
F (T)

=

=
. K
NR
T
NR

Note:

If there is separation (R
=
NR
0), it are necessary to take guard with the fact that the part which is normally “held” by the discrete element is not it any more. For example, in the case of one fuel pin, if none the springs is more in contact, this one can “fall”. In practical, to avoid these situations, it is necessary that the spring modelled by the discrete element is always in compression. That can be specified by the user using the initial effort, of coefficient of rigidity and the function F (T).

3.2
Elastoplastic model of kneecap

3.2.1 Equations of the model

They are deduced from the behavior 3D VMIS_ECMI_TRAC [R5.03.16]: indeed, it is here about to represent a relation of behavior of the elastoplastic type to unspecified isotropic work hardening, superimposed on a linear kinematic work hardening, binding the Mz moment (or MX) in the discrete element
with the difference in rotation of the two nodes around Z local (or X local). One is not thus interested here
that with rotation around Z (or X) local.

While noting = - and = M = M, (resp. = - and = M
= M)
z2
z1
z1
z2
x2
x1
x1
x2

The relations are written here still in the same form as the relations of Von Mises 1D [R5.03.09]. They can be deduced from behavior VMIS_ECMI_TRAC [R5.03.16], by noticing that in
the uniaxial case, the constant of Prager C must be multiplied by 3/2. In any rigour it would be necessary to write:
3
X
C p
=
, but here, one writes directly: X
C p
=. It is provided via the key word
2
PRAGER of the key word factor DIS_CONTACT of operator DEFI_MATERIAU.

p
- X
& = p& -

X
= E (- p
)

- X - R (p) 0

p& = 0 if (- X) - R (p) <
eq
0
p& 0 if (- X) - R (p)

=
eq
0

X =
p
C
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p represents here a "plastic rotation cumulated" around each local directions Z and X.
isotropic function of work hardening R (p) is closely connected per pieces, data in the form of a function
defined point by point under the key word factor RELA_MZ of the key word factor DIS_CONTACT of

operator DEFI_MATERIAU. It is supposed implicitly that the relation used for rotation around of Z is identical to that used for rotation around X local.

The function R (p) is deduced from a curve characteristic of the spring during a deflection test of a pencil in a grid, curves which expresses the moment applied with the pencil according to rotation of the pencil F = F () which is translated with our notations into: = F (). R (p) is deduced from F,
as in [bib2] by taking account of the linearity per pieces of F. The first point corresponds at the end of linearity, and thus defines at the same time the limit of linearity similar to the elastic limit and E
who is the slope of this linear part (independent of the temperature for this development).

3.2.2 Integration of the relation

By direct implicit discretization of the relations of behavior, a way similar to integration 1D [R5.03.09] one obtains:

- X - R (p) = - +
- -
X - X
- R (-
p + p) 0

- +
- -
X -

X

E
-

=

E p

- +
- -
X - X

p
0
if - +

- -
X - X = R (-
p + p)
p

= 0 if - +
- -
X - X <
-

R (p + p)

Two cases arise:

.
-
< (-
X
R p + p
) in this case p = 0 are =
thus - +
< (-

R p),
.
-
=
(-
X
R p + p
) in this case p 0
thus -

R (-
+

p).

One deduces the algorithm from it from resolution:

let us pose E
= - +

- X -

if E R (p) then p = 0 and =
if E > R (p) then it is necessary to solve:
- X
- X
E = - + - X -

- X + X + E p
= - X + (E + C)

p
-

X
- X
- X
because X =
C p -
X
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one thus obtains:

E + C
p

E
(
)

= 1 +
(X)

-

X

-
and by taking the absolute value:

(E + C) p

E = 1 +
-

X
-

X

maybe, while using
-
=
(-
X
R p + p
).

E = R (p + p
) + (E + C) p

By taking account of the linearity per pieces of R (p), one can solve explicitly this equation to find p. Once calculated p one deduces some in the following way: one with the relation of proportionality:

E
- X
=

E
R (p)
where X is calculated using:

- X
E
E

X =
C p
=
C p
=
C p
-

X
E

R (p) + (E + C) p
from where
E

=X +
R (p)
E

Moreover, the option
N
FULL_MECA makes it possible to calculate the tangent matrix K I with each iteration.
The tangent operator who is used for building it is calculated directly on the preceding discretized system.
One obtains directly:

E.R p + C
if E > R (-

p)
(( ))

=
=

AND
E + R (p) + C

if not
=

E

3.3 Variables
interns

The relation of behavior DIS_CONTACT produces 3 internal variables:

.
The first relates to the contact - friction: it is worth:
-
1 if there is slip,
-
0 so not slip,
-
1 if separation.
.
The two following ones relate to elastoplastic behaviour in rotation:
-
the second internal variable is equal to p around local direction Z,
-
the third internal variable is equal to p around the direction X local.
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4
Modeling of the shocks and friction: DIS_CHOC

Behavior DIS_CHOC translates the contact with shock and friction between two structures, via two types of relations:

.
the relation of unilateral contact which expresses the non-interpenetrability between the solid bodies,
.
the relation of friction which governs the variation of the tangential stresses in the contact. One will retain for these developments a simple relation: the law of friction of Coulomb.

4.1
Relation of unilateral contact

Are two structures
1/2
1/2
1 and 2. D NR is noted
the normal distance enters the structures, FN

force normal reaction of 1 out of 2.

The law of the action and the reaction imposes:

F 2 1
/= - F 1/2
NR
NR

éq 4.1-1

The conditions of unilateral contact, still called conditions of Signorini [bib5], are expressed following way:

D 1/2 0, F 1/2 0, D 1/2 F 1/2 = 0 and F 2 1
/= - F 1/2
NR
NR
NR
NR
NR
NR

éq
4.1-2
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F 1/2
NR
D 1/2
NR

Appear 4.1-a: Graph of the relation of unilateral contact

This graph translates a relation force-displacement which is not differentiable. It is thus not usable in a simple way in a dynamic calculation algorithm.

4.2
Law of friction of Coulomb

The law of Coulomb expresses a tangential limitation of effort
1 2
F
of tangential reaction of
T
1 on
. That is to say 1/2
U
compared to in a point of contact and is μ it
2
&
the relative speed of
T
1
2
coefficient of friction of Coulomb, one has [bib5]:

1 2
1 2
S = F
- μ F
,
0
1 2
1 2
u&
= F
, ,
0 .s = 0
T
NR

T
T

éq
4.2-1

and the law of the action and the reaction:

2 /1
1 2
F
= - F

éq 4.2-2
T
T

F1/2
T
Ý
U 1 2
1 2
u&T

Appear 4.2-a: Graph of the law of friction of Coulomb

The graph of the law of Coulomb is him also nondifferentiable and is thus not simple to use in a dynamic algorithm.
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4.3 Approximate modeling of the relations of contact by penalization

4.3.1 Model of normal force of contact

The principle of the penalization applied to the graph of the figure [Figure 4.3.1-a] consists in introducing one
1 2
F
= F
NR
(
1 2
D NR
)
univocal relation
by means of a parameter. The graph of F must tend towards
the graph of Signorini when tends towards zero [bib6].

One of the possibilities consists in proposing a linear relation enters
1 2
D
and
1 2
F
:
NR
NR

1 2
1
1 2
1 2
F
= - D
if D
;

0
1 2
F
= 0 if not
éq
4.3.1-1
NR
NR
NR
NR

1
If K is noted
= called commonly “stiffness of shock”, one finds the traditional relation,
NR

modelling an elastic shock:

1 2
1 2
F
= - K D

éq
4.3.1-2
NR
NR
NR

The approximate graph of the law of contact with penalization is as follows:
F 1/2
NR
D 1/2
NR

Appear 4.3.1-a: Graph of the relation of unilateral contact approached by penalization

To take account of a possible loss of energy in the shock, one introduces a “damping of shock " CN the expression of the normal force of contact is expressed then by:

1 2
1 2
1 2
F

= - K D
- C U

éq
4.3.1-3
NR
NR
NR
NR
&N
where
1 2
u&
is normal speed relative from report/ratio to. To respect the relation of
NR
1
2
Signorini (not of blocking), one must on the other hand check a posteriori that F 1/2
NR
is positive
+
or null. Only the positive part will thus be taken
expression [éq 4.3.1-3]:

+
X
= X if X 0
+
X
= 0 if X < 0
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The complete relation giving the normal force of contact which is retained for the algorithm is following:

1 2
1 2
1 2
1/ 2 +
2 /1
1 2
if D
0F
= - K D
- C u&
,
NR
NR
NR
NR
NR
NR
NR
NR
F
= - NR
F

if not

F 2 1
/= F 1/2
NR
NR
= 0.
éq
4.3.1-4

4.3.2 Model of tangential force of contact

The graph describing the tangential force with law of Coulomb is not-differentiable for the phase of adherence (1/2
u&
= 0. One thus introduces a univocal relation binding relative tangential displacement
T
)
1 2
D
1 2
F
= F
T
(
1 2
dT)
and the tangential force
by means of a parameter. The graph of F must
T

to tend towards the graph of Coulomb when tends towards zero [bib6].
One of the possibilities consists in writing a linear relation between D 1/2
1/2
T
and FT:
by noting 0
with the value of a quantity has at the beginning of the step of time:

0
1 2
1 2
1
F
- F
= - D

- D

éq
4.3.2-1
T
T
(
0
1 2
1 2
T
T
)

1
If one introduces a "tangential stiffness" KT =, one obtains the relation:

0
1 2
1 2
F
= F
- K D
- D

éq
4.3.2-2
T
T
T
(
0
1 2
1 2
T
T
)

For numerical reasons, related to the dissipation of parasitic vibrations [bib7] in phase of adherence, one is brought to add a "tangential damping" CT in the expression of the force tangential. Its final expression is:

0

1 2
1 2
F
= F
- K D
- D
- C U
éq
4.3.2-3
T
T
&
F
= - F
T
T
(
0
1 2
1 2
T
T
)
1 2
2 /1
1 2
,
T
T
T

It is necessary moreover than this force checks the criterion of Coulomb, that is to say:

1 2
1 2
1 2
1 2
1 2
u&
2 /1
1 2
F
µ F

apply
one
if not
T
F
= - μ F

, F
= - F
éq
4.3.2-4
T
NR
T
NR
1 2
T
T
u&T
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The approximate graph of the law of friction of Coulomb modelled by penalization is as follows:

F 1/2

T
KT
Ý
U 1 2
T

Appear 4.3.2-a: Graph of the law of friction approached by penalization

4.4
Definition of the parameters of contact

One specifies the key words here allowing to define the parameters of contact, damping and friction [U4.43.01]

Operand RIGI_NOR is obligatory, it makes it possible to give the value of normal stiffness of shock kN.
The other operands are optional.

Operand AMOR_NOR makes it possible to give the value of normal damping of shock CN.
Operand RIGI_TAN makes it possible to give the value of tangential stiffness KT.
Operand AMOR_TAN makes it possible to give the tangential value of damping of shock CT.
The COULOMB operand makes it possible to give the value of the coefficient of Coulomb.
Operand DIST_1 makes it possible to define the distance characteristic of matter surrounding the first
node of shock
Operand DIST_2 makes it possible to define the distance characteristic of matter surrounding the second
node of shock (shock between two mobile structures)
The operand PLAY defines the distance between the node of shock and an obstacle not modelled (case of a shock
between a mobile structure and an indeformable and motionless obstacle).

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Document: R5.03.18

Law of damage of an elastic material fragile

Summary:

This document describes the elastic model of behavior fragile ENDO_FRAGILE available in statics and in
dynamics. The damage is modelled in a scalar way; loadings in compression and in
traction are not distinguished. In addition to the local model, the nonlocal formulations with gradient of damage and with regularized deformation are also supported to control the phenomena of localization.
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1 Field of application

Law ENDO_FRAGILE aims at modelling in the manner a simplest possible behavior fragile rubber band. The material is elastic isotropic. Its rigidity can decrease in an irreversible way when the deformation energy becomes important, without distinguishing traction from compression. This loss of rigidity is measured by a variable interns scalar called damage which evolve/move of 0 (healthy material) to 1 (completely damaged material, i.e. without rigidity). Moreover, the constraint cannot exceed a threshold which also decrease him with the level of damage for to reach 0 when the material is completely damaged. One will refer to [bib1] for one description of this type of phenomenology.
*The property of decrease of the threshold in constraint with the level of damage is called softening and generally involves a loss of ellipticity of the equations of the problem. It results from it a localization of the deformations and damage in bands of which the thickness is directly controlled by the size of the finite elements. To mitigate this deficiency of the model, two nonlocal formulations are proposed, one founded on the introduction of the gradient of the damage and activated by modeling *_GRAD_VARI [R5.04.01], the other resting on one regularization of the deformations and activated by modeling *_GRAD_EPSI [R5.04.02]. In a case as in the other, the width of the bands of localization is henceforth controlled by a parameter material, well informed in operator DEFI_MATERIAU under key word LONG_CARA of the key word factor NON_LOCAL [U4.43.01]. However, obtaining a physical problem posed again well is not obtained that at the price of an important overcost in time calculation. In addition, it should well be noticed that only the relations of behavior are deteriorated and not the equilibrium equations. Consequently,*

the constraints preserve their usual direction.
Lastly, that one activates or not these nonlocal formulations, softening character of the behavior also involve the appearance of instabilities, physics or parasites, which result in snap-backs on the total answer and returns the piloting of the essential loading in statics. piloting of the type PRED_ELAS [R5.03.80] then seems the mode of control of the level of the most suitable loading.

2
Local law of behavior

2.1
Relations of behavior

The state of material is characterized by the deformation and the damage D ranging between 0 and 1. The forced relation deformation is elastic, rigidity is affected in a linear way by the damage:

*= (1 - D) **E***
éq
2.1-1
*with **E the** tensor of Hooke. In addition, the evolution of the damage, always increasing, is controlled by the following function threshold:*

2
y
+
F (D) 1
,
*= **E** - K (D)*
where
(D)
1
K
= W
éq
2.1-2
2

1

+ - D

The coefficients y
W and, both positive, is parameters of the model. The condition of
coherence then determines completely the rate of damage d&:

F (, D) 0
d& 0
d& F (, D) = 0 ***éq***
2.1-3
The equations [éq 2.1-1] with [éq 2.1-3] are enough to entirely describe the law of behavior
ENDO_FRAGILE, indeed very simple. One can also notice that it fits in
formalism suggested by Marigo [bib2].

2.2
Identification of the parameters of the model

The parameters of this law of behavior are four. On the one hand, the module of
Young E and the Poisson's ratio who determine the tensor of Hooke by:

-
+

1
1

E *=*
-
(tr) ***Id***
éq
2.2-1
E
E
In addition, y
W and which defines the lenitive behavior. They are determined by a test
of simple traction, cf [Figure 2.2-a]. To simplify the entry of the data of the model, one informs not not
y
W and but directly the tangent module T
E and the constraint with the peak y
under the key word
factor ECRO_LINE or ECRO_LINE_FO of operator DEFI_MATERIAU. As for E and, they are given classically under the key word factor ELAS or ELAS_FO.
For whatever purpose it may serve, here also expressions of the deformation with rupture R in this test of
simple traction, as well as voluminal energy 0
K consumed to damage a point completely
material, this last expression being valid whatever the history of loading:

R
1
1 y
1 1
1
2
0
y
1 R y
y 1 +
= -

K =
-
= = W

éq 2.2-2

T
E
E

2
T
E
E
2

2

y
y

wy =

2nd

T

E < 0

E
AND

= -

E
R

Appear 2.2-a: Simulation of a simple tensile test
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2.3
Integration of the law of behavior

Temporal discretization of the equations [éq 2.1-1] with [éq 2.1-3] on a step of time [T - T] is realized by a diagram of implicit Euler. For any function of time Q, one notes -
Q = Q (-
T) and
Q = Q (T). To integrate in time the law of behavior then means to determine the state of stress and of damage solution of the following nonlinear system, where deformation and the state of material at the beginning of the step of time (-
-
, D) are given:

*= (1 - D) **E***

éq 2.3-1
F (, D) 0
D - -
D 0
(D - - D) F (, D) = 0
éq
2.3-2

A method of resolution was proposed by [bib3]. It starts by examining the solution without evolution of the damage (also called elastic test) then, if necessary, proceeds to one correction to check the condition of coherence. In this case, the existence and the unicity of solution guarantee the correct operation of the method. Let us consider the elastic test:

D = -
D
solution if

F el () = F (, -
D) 0
éq
2.3-3

In the contrary case, the damage is obtained by solving F (, D) = 0:

y
= (
W
D
1 +)

-
where
W = 1
1
E

éq
2.3-4
W

2
As for the constraint, it is given by [éq 2.3-1] in all the cases.
It still remains to be made sure that the damage does not exceed value 1. In fact, when D = 1,
the rigidity of the material point considered is cancelled. Insofar as no technique of suppression
finite elements "broken" is not implemented (technical possibly delicate when them
finite elements have several points of Gauss), of the null pivots can appear in
stamp rigidity. This is why one introduces a numerical threshold beyond D which one is considered
C
elastic residual rigidity for the tangent matrix, the equations of remaining behavior
unchanged.

To preserve a reasonable conditioning of the matrix of rigidity, one chooses
5
D = 1 10
-

C
. One
indicator, arranged in the second internal variable, then specifies the behavior during the step time running:

.
= 0 elastic behavior (deformation energy lower than the threshold)
.
=1 evolution of the damage
.
= 2 (saturated damage) (D = 1).
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2.4
Description of the internal variables

The variables intern are two:

.
VI (1) damage D
.
VI (2) indicating

3

Formulation with gradient of damage

3.1
Tally standard generalized

To mitigate the pathological localization of the deformations and the damage inherent in softening character of material, a first alternative consists in extending the law of behavior by taking of account the gradient of the damage on a material point scale [bib4]. This approach is restricted with generalized standard materials. Model ENDO_FRAGILE enters indeed within this framework; microscopic free energy μ and potential of dissipation microscopic μ is written:

0el () = 1 ***E***

0
2
μ (, D) = (1 - D) el () + bl (D)

+ I [0] 1 (D)
where

éq
3.1-1
0 D 1 -
bl (D)
(D)
= - K

1+ - D

= 0 + I
μ (d&)
K d&

éq
3.1-2
0 + (d&)
[
[
where I indicates the indicatrix of the convex unit K, null in K and being worth + elsewhere. One K
the convexity of these potentials compared to the damage, strict convexity will notice for free energy: that confirms the existence and the unicity of the solution for the integration of the law.

To preserve a simple model, one will be satisfied to introduce a quadratic term in gradient of damage in the free energy. The potential of dissipation remains unchanged. Consequently, them macroscopic potentials are written:

(
0
1
, D, ***D*** *= 1 - D + D + I*
D + C ***D D***
.
)
(
·)
éq
3.1-3
el ()
bl (·)
[0]1(·)

2
(d&, &d = k0 d& + I
d&
.
)
.

éq
3.1-4
0 + (·)
[
[
where the factor C depends on the length characteristic L of the material (distance characteristic
B
of interaction enters the microscopic cracks given under key word LONG_CARA of DEFI_MATERIAU)
of
following manner:

2
2

C =
µ (

L
, 0) 4
8
B
0
2
=
K L
éq
3.1-5
2
D

13
13 (1+) 2
B
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The macroscopic potentials thus built then make it possible to define the total potentials in the origin of the variational formulation of the law of behavior. In this relatively simple case, one can interpret this variational formulation in term of laws of local state and evolution of which them expressions are close to those for the local law:

= (1 - D) ***E***
éq

3.1-6
F
E

éq
3.1-7
G (
1
, D) =
- K (D) + C D

2
F D
d&
d&
D =
éq
3.1-8
G (,
) 0
0
fG (,) 0

The essential difference with the local model lies in the definition of the function threshold which depends
henceforth Laplacian of the damage. This last confers on the condition coherence one differential character. The interpretation of the variational formulation also provides the conditions with the limits for the field of damage as well as the conditions of interface (or jump):

*D **N** = 0*
on
N

normal

of

edge

éq
3.1-9
*[C D **N**] = 0*
crosses

with
lies of

N

normal

of

interface

one

éq
3.1-10

3.2
Integration of the law of behavior

The integration of the laws of behavior to gradients of internal variables is described in the booklet [R5.04.01]. It is based inter alia on a local stage which depends explicitly on the relation on behavior treated. It is this one which one proposes to describe here.
First of all, it is necessary to define the standard used to build the term of penalization of
R
R
Lagrangian increased, i.e. the matrix NR. As recommended, one adopts an estimate diagonal of the matrix hessienne of energy compared to the variables of damage, which conduit with:

NR ·
·

2
0

NR =
·
·
K
2
R
R

NR

(1+)

NR =
with

éq
3.2-1
NR

2

2
4
NR
=

0
B
L
K
NR

2

(1+)
13

In accordance with the general theory, it is now necessary to solve the following nonlinear system, in which the state at the beginning of the step of time as well as the deformation, the multipliers of Lagrange µr,
R R
the coefficient of penalization R and the nodal damage evaluated at the point of current Gauss = ***B D*** *are given:*

R
R
R

R R R
R R
- R (, D) + μ + R NR (- D) R (
-
D - D) ***éq***
3.2-2
D
D
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R
where, once more, the notation compacts D = (D, ***D***
.
) is used. Being given decoupling enters
the value and the gradient of the damage in the free energy and the potential of dissipation, it
system [éq 3.2-2] makes it possible to separately treat the gradient of the damage and its value:

-
*(****D***
µ
R NR

D
) +
+

(
-

)
= 0 ***éq***
3.2-3
D

-

(, D +μ + R NR - D

D - D
·)
·
·· (·
·)
μ (
-
·
·)
éq
3.2-4
D ·
On the one hand, the equation [éq 3.2-3] makes it possible to determine the gradient of damage immediately:

***μ** + R NR*

D
=

éq
3.2-5
C + R NR

As for the equation [éq 3.2-4], it is interpreted like an equation of coherence:

(FAr ·) 0 D - D 0
·
·
(D - D
·
·) (
F Ar ·) = 0
with
(Ar ·) R 0
F
= A - K
·
éq
3.2-6

in which the thermodynamic force R
With · has as an expression:

Ar = -
, D + μ + R NR - D
·
(·) ·
·· (·
·)
éq
3.2-7
D ·
The solution of this equation of coherence is similar to that for the local model [éq 2.3-1] - [éq 2.3-4]. Initially, an elastic test then a correction so necessary. Being given strict convexity of the potentials, the solution is single. When the elastic test is not solution, resolution of the equation (
F R
With ·) = 0 conduit with:

W + μ + R NR = R NR D +
1
·
·
·
·
·
·
·
K (D ·)
where
W =
E
éq
3.2-8
2
After some handling, this equation is reduced to research roots of a polynomial of degree 3 in D
D
·, which one knows that only one (that which one seeks) is higher than
-
· .

3.3
Estimate of the Laplacian of the damage

Like the local law, the law with gradients can lead to structural instabilities. Piloting by elastic prediction PRED_ELAS can then appear essential. But this last requires given of a threshold of elasticity point by point, whose existence is not necessarily acquired in presence of nonlocal models. In fact, in this case, such a threshold exists thanks to the interpretation [éq 3.1-6] - [éq 3.1-8] of the variational formulation of the behavior: it is the function F.

However, the evaluation of this threshold requires to know the Laplacian of the field of damage who is not calculated by the algorithm of integration of the nonlocal laws. To know the nodal field to damage is not enough besides because the functions to form, polynomials of degree 1, do not allow to estimate a derivative directly second. Fortunately, the recourse to an artifice allows despite everything to estimate the Laplacian with a precision sufficient for the algorithm of piloting.
Indeed, at the end of a step of time, the equation of coherence [éq 3.1-8] is checked, and this of as much better than the precision requested by the user (key word RESI_DUAL_ABSO under the key word factor LAGR_NON_LOCAL) is large. In each point of Gauss, one can thus be confronted with two cases of appear. That is to say the threshold is not reached and, in this case, the damage in this point does not vary. One
suppose whereas it is the same for the Laplacian:

-
-
D = D
D = D
éq
3.3-1
In the contrary case, the damage evolves/moves and the equation F D = is checked, so that one
G (,
) 0
in deduced:

1
D =
K (D) 1

- E éq
3.3-2
C
2

In any rigour, this does not make it possible to determine the Laplacian when the damage is saturated. However, that does not have importance since these points are not taken into account to control it loading (one seeks points which dissipates).

3.4
Description of the internal variables

In the case of the formulation with gradient of damage, the variables intern are now with numbers of six:

.
VI (1) damage D
.
VI (2) gradient of the damage D, component X (R into axisymmetric)
.
VI (3) gradient of the damage D, component y (Z into axisymmetric)
.
VI (4) gradient of the damage D, component Z (0 in 2D)
.
VI (5) estimate of the Laplacian of the damage D
(as long as D <1)
.
VI (6) indicating

4
Formulation with regularized deformation

4.1
Formulation continues in time

The approach with regularized deformation [R5.04.02] also makes it possible it to control the phenomena of
localization and for this reason seems an alternative to the formulation with gradient of damage. But at the difference in the latter, this formulation has the advantage of to resort to the standard algorithms for the nonlinear problems. Indeed, the only difference by report/ratio with the local law of behavior lies in the data of two deformations instead of one, local deformation which intervenes in the forced relation deformation and the deformation regularized which controls the evolution of the damage. This one results from the local deformation by resolution of the system of partial derivative equations according to:

- 2 = ***0***
structure

in

B
L

éq
4.1-1

***N** = 0*
on
normal

of

edge

N

where the characteristic length L is again indicated under the key word
B
LONG_CARA of
DEFI_MATERIAU. Finally, the relation of behavior is written in the following way, where function threshold F was already defined in [éq 2.1-2]:

*= (1 - D) **E***

éq 4.1-2
F (, D) 0
d& 0
d& F (, D) = 0 ***éq***
4.1-3

4.2
Integration of the law of behavior

One of the advanced advantages for the nonlocal formulation with regularized deformation is the little of

modifications which it involves in the construction of the law of behavior. Indeed, the integration of internal variables is completely controlled by the regularized deformation. They thus are found expressions of the local law:

el
-
-
if F () = F (, D) 0
D = D

1

with
W =
E

y

éq 4.2-1

W
if F el () = F (
-
, D) > 0
D = (1+)

1-
2

W

The constraint is then obtained directly by the relation [éq 4.1-2]. Moreover, one preserves well-sure the introduction of a damage criticizes [éq 2.3-5] to preserve a residual rigidity.

4.3 Variables
interns

They are the same internal variables as for the local law:

.
VI (1) damage D
.

5
Piloting by elastic prediction

The piloting of the type PRED_ELAS controls the intensity of the loading to satisfy some equation related to the value of the function threshold el F during the elastic test [bib5]. Consequently, only the points where the damage is not saturated will be taken into account. The algorithm which takes in charge this mode of piloting, cf [R5.03.80], requires the resolution of each one of these points of Gauss of the following scalar equation in which is a data and the unknown factor:

~el
F () =

éq 5-1

~
The function el
F provides the value of the function threshold during an elastic test when the field of displacement breaks up in the following way according to the scalar parameter:

U** = **U** + **U

éq 5-2
0
1

*where **U** and **U** are given. Thanks to the linearity in small deformations of the operators deformation*
0
1
(calculation of the deformations starting from displacements) and regularized deformation, one also obtains
following decompositions:

= 0 + 1
and
= 0 + 1
éq
5-3
Consequently, whatever the adopted, local, nonlocal modeling with gradient
~
of damage or not local with regularized deformation, the function el
F can be put under
following form:

~el () = 1
F
*(**E** +*
0
1
***E**) **E** (**E** +*
0
1
***E**) - S*

2
local

law
***E** =*
***E** =*
S = K
0
0
1
1
(- D)

éq
5-4
gradient of
endommagem ent
***E** =*
***E** =*
S = K
0
0
1
1
(- D) - C - D
déformatio regularized

N
E
***E** =*
***E** =*
S = K
0
0
1
1
(- D)

The equation [éq 5-1] is reduced thus to research roots of the polynomial of degree 2 following:

1
*P = **E** **E** **E** -*

0
0
0

2
P ()
2

= P + 2 P +
1
0
1
2
P
where
*P = **E E***
1
0
1
E

éq
5-5

2

1
*P = **E E E***
0
1
1
2
One provides to the algorithm piloting the linear approximation of P in the vicinity of the roots (real) when they exist or if not the value of for which P reaches its minimum.
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Document: R5.03.20
Elastic relation of behavior
nonlinear in great displacements
Summary:
One proposes to describe here a relation of nonlinear elastic behavior which coincides with the law elastoplastic of Hencky-Von Put (isotropic work hardening) in the case of a loading which induces one radial and monotonous evolution of the constraints in any point of the structure. This model is selected in order STAT_NON_LINE via the key word RELATION: "ELAS_VMIS_LINE" or "ELAS_VMIS_TRAC" under the key word factor COMP_ELAS.
One extends then this relation of behavior to great displacements and great rotations, in measure where it derives from a potential (hyperelastic law); this functionality is selected via key word DEFORMATION: "GREEN". It is available for all the isoparametric elements 2D and 3D.
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1 Relation of nonlinear elastic behavior: ELAS_VMIS_LINE and ELAS_VMIS_TRAC

1.1 Objective

Within the framework of the global solution in breaking process, one cannot give a direction to the rate of restitution of energy that for hyperelastic relations of behavior, i.e. which derive of a potential, free energy. In order to be able nevertheless to deal with elastoplastic problems, one propose a relation of nonlinear elastic behavior which leads to results identical to those obtained by the plastic relation of behavior of Hencky-Von Put (isotropic work hardening) in the case of an evolution of loading radial and monotonous in any point. The definition of characteristics of the material (key word DEFI_MATERIAU) is identical to that of the behavior isotropic plastic. For further information on the model, one will be able to refer to [bib1]. For to illustrate the common points and the differences between the models plastic and rubber band, one presents below a traction diagram then compression obtained for a unidimensional bar.

Final State
Final State
F max

F max

Elastoplasticity
Nonlinear elasticity
1.2
Relation of behavior
After integration in time of the relation of behavior of Hencky-Von Put, formulated in speeds of strains and stresses in [R5.03.02] which one adopts the notations, the expression constraints according to the deformations is:
*= K (tr) **Id**+*
~

G (eq
)
éq 1.2 1

y

- if
eq

2µ

G = 2µ and
p = 0

y

- if
eq
>

2µ

R (p)
R (p)

2
G =
and
p such as: p +
=
eq
éq 1.2 2

eq

μ
3
3
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In a way similar to plasticity, the function of work hardening R (p) is deduced from the provided data by a simple tensile test (work hardening linear with key word ELAS_VMIS_LINE or well defined by points with the key word ELAS_VMIS_TRAC, cf [R5.03.02]).
As for the variable p, it deserves a few moments of attention. In the plastic model, its significance is clear. It is about the cumulated plastic deformation, always increasing; it is one variable interns model. On the other hand, in the elastic case, it does not have any more the internal statute of variable,
since there is no dissipation. Moreover, it decrease during discharges. In fact, its value coincides with that obtained in plasticity as long as the evolution of the loading is radial and monotonous.
In addition to the relation of behavior itself, it is necessary to know the value of energy free for a state given for calculations of rate of refund of energy. Without demonstration, it potential whose the relation derives from behavior is worth:
y
1
2
2μ
- if
2
eq
() = K (tr) +

2μ
eq
2
3
2
éq 1.2-3
y
1
R p
2
((eq)
p
-
eq
if eq >
() = K (tr)
( )
+
+
R (S)
2
2
6

μ
μ
ds
0
1.3
***Resolution of the equation out of** p*
One could note in the preceding paragraph that the expression of the constraints requires solution of an equation relating to the variable p. Insofar as the function of work hardening R is increasing, this equation can be written by gathering the terms where appear p in the first member (who is then increasing with p):
( )
p + R p
3μ
2 eq
2
( )
eq
p + R p =

X
3μ
3
3
p
More precisely, the first member is linear per pieces in p. to solve the equation, it is then enough sequentially to traverse each interval until finding that in which locate the solution. An equation closely connected provides the value of p. then.
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1.4

Calculation of the relation of behavior and tangent rigidity

The calculation of the constraints and tangent rigidity, i.e. variation of the constraints by report/ratio with the deformations, is carried out according to the algorithm presented below. By adopting the convention of
Code_Aster, the constraints and the deformations are arranged in a vector with six components, while tangent rigidity is a matrix 6x6.

xx

xx

1

yy

yy

1
zz

zz

{

1
} =

{ }
=

{ }
1 =
2 *xy*
2 *xy*

0
2

xz
2

0

xz

2 *yz*

2 *yz*

0
Relation of behavior:
{ }
= *K* (tr) {} + *G* {}
~
1
Tangent rigidity:
D {}
= **K** = **K1** + **K**
D {}
[ ]
[**2**]
3K - *G*
.
[**K**]
{ }
1 { }
1
+ *G* [**I**]

D
1 =
3

y
[

0]
if *eq*

2µ
·
[**K2**] = 3 2µ R' (*p*)
y

G ~ ~
if
2
'
eq >

2
R
2
eq
(*p*)
-
{ } { }
+ µ
3

µ
1.5
Taking into account of deformations of thermal origin
In a way identical to plasticity, one divides the total deflection into a mechanical part which checks the preceding relation of behavior [éq 1.2-1], [éq 1.2-2] and a thermal part, function of temperature. Let us note moreover that the various characteristics of material can also depend temperature.
= *m* + *HT*

= *K* (tr *m*)
+ G (

~
Id
eq)
éq 1.5 1
with *HT* =

(*T* - *Tréf*) **Id**
: thermal dilation coefficient
Tréf: temperature of reference
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It remains to supplement the potential free energy [éq 1.2-3] to include the temperature there. Several choices
are possible, depend on the way in which one wishes to define the entropy (derivative of the free energy compared to the temperature). In our case, the adopted potential is:
y
1
HT 2
2µ
if
2
eq
(, *T*) = *K* Tr
2
2
((-) +
µ
eq

3
2
y
1
R p
HT 2
(*eq*) p *eq*
if *eq* >
(, *T*) = *K* Tr
R
2
2
((-)
( )
( )
+
+
(*S*)
µ
6µ
ds
0

1.6

Particular treatment of the plane constraints

Usually, one seeks to determine the constraints knowing the deformations and the temperature. However, it is not completely any more the case under the assumption of the plane constraints insofar as three of the components of the tensor of the deformations are henceforth unknown, the dual sizes being fixed:

xz, *yz* and *zz* unknown

xz = *yz* = *zz* = 0

It is thus necessary to start by determining these unknown components. The adopted method is exposed in [bib1] and [R5.03.02]. One can however recall here that the components *xz* and *yz* do not pose of problem, being given the form of the relation of behavior [éq 1.2-1]:

xz = *yz* = 0

On the other hand the determination of the component *zz* requires the solution (numerical) of an equation nonlinear scalar.

Lastly, a last warning is essential. With the difference in the plane deformations, solutions that one obtains under the assumption of the plane constraints are generally not exact in measurement where they do not check the conditions of geometrical compatibility (integrability of the field of deformations). They are only approximate solutions.

2

Elasticity in great transformations

2.1 Objective

Henceforth, one proposes to take into account great displacements and great rotations, functionality accessible by the key word DEFORMATION: “GREEN” in the order STAT_NON_LINE. Let us specify as of now that one restricts oneself with finite elements isoparametric (D_PLAN, C_PLAN, AXIS and 3D) for which discretization of the continuous problem do not raise particular difficulties, cf [R3.01.00].

To this end, it is admitted that the second tensor of the constraints of Piola-Kirchhof, **S**, drift of the potential
of Hencky-Von Put expressed using the deformation of Green-Lagrange **E**:

S =
(**E**)
E

Also let us point out the definitions of **E** and **S.** One can also find information complementary in [bib1].

1
F = **Id** + **Gra** (
D U) **E** = (*TFF* - **I**)
D
2
S = Det (**F**) **F-1**
T
F-1

Such a relation of behavior, known as hyperelastic, makes it possible in any rigour to take into account great deformations and great rotations. However, we limit ourselves to the small ones deformations, and this for two reasons. First of all, the relation of behavior adopted does not present not the good properties (polyconvexity) to ensure the existence of solutions and does not control not more important compressions. Then, the plastic behavior differs notably from one

behavior hyperelastic as soon as the deformations become appreciable. It is for these reasons which we chose to preserve the assumption of small deformations, thus escaping polemic of the great deformations.

2.2 Virtual work of the external efforts: assumption of the loads died

To deal with the problem of hyperelastic structural analysis, one seeks to write balance under variational form on the initial configuration. In particular, it is necessary to express the virtual work of external efforts on this same initial configuration what requires the additional assumption of dead loads: it is supposed that the loading does not depend on the geometrical transformation. Typically, an imposed force is a dead load while the pressure is a loading follower since it depends on the orientation of the face of application, therefore of the transformation. Under this assumption, the virtual work of the external efforts is written like a linear form:

$$W_{ext}(\mathbf{v}) = \int_{O} F \cdot v \, dO + \int_{\partial O} Td \cdot v \, dS$$

F: voluminal loading

Td: surface loading being exerted on the edge ∂O

2.3

Virtual work of the interior efforts

We will not give here a demonstration of the expressions presented. For that, one will be able to defer to [bib1] and [R7.02.03]. There still, we choose the initial configuration like configuration of reference, to express the work of the interior efforts:

SW. **v** = - *F S*
v

D
int
ik kl I L, O
O
v
with: *v*
I
I L, = Xl
In the optics of a resolution by a method of Newton, it is important to also express
variation second of the virtual work of the interior efforts, namely:
2Wint. **U**. **v** = - *U.S.*
I, K kl v D
I L,
O
geometrical rigidity
O
2
- *U F*
F
I Q, IP
jk

v
D
elastic rigidity
!
E
J L,
O
pq Ekl
O

2.4 Formulation
variational

We now have at our disposal all the ingredients to write the variational formulation
problem:
W
v + *SW*
int.
ext. **v** = 0
v Kinematically acceptable

3 Bibliography

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Modeling élasto (visco) plastic with

isotropic work hardening in great deformations

Summary

***One describes here a thermoelastoplastic relation between behaviour and isotropic work hardening written into large
deformations and proposed by Simo and Miehe. This model is available in order STAT_NON_LINE via the key word RELATION: "VMIS_ISOT_TRAC" or "VMIS_ISOT_LINE" under the key word factor COMP_INCR and with the key word DEFORMATION: "SIMO_MIEHE". A viscous version of this model
is also proposed: "VISC_ISOT_TRAC" and "VISC_ISOT_LINE".
This model is established for three-dimensional modelings (3D), axisymmetric (Axis) and in deformations
plane (D_PLAN).***

***One presents the writing and the digital processing of this law, as well as the associated variational formulation. It
of a variational formulation eulérienne acts, with reactualization of the geometry and which takes account of
rigidity of behavior and geometrical rigidity.***

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Count

matters

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1 Introduction

We present here a thermoelastoplastic law of behavior written in great deformations proposed by SIMO J.C and MIEHE C. [bib1] which tends in small deformations towards the model of elastoplastic behaviour with isotropic work hardening and criterion of Von Mises, described in [R5.03.02].
The kinematics choices allow, as with the simple reactualization available via the key word PETIT_REAC, to treat great displacements and great deformations but also of great rotations in an exact way.
Specificities of this model are as follows:

.
just like in small deformations, one supposes the existence of a slackened configuration, *i.e. locally free of constraint, which makes it possible to break up the total deflection into a thermoelastic part and a plastic part,*
.
the decomposition of this deformation in parts thermoelastic and plastic is not any more additive as in small deformations (or for the models great deformations written in rate of deformation with for example a derivative of Jaumann) but multiplicative,
.
the elastic strain are measured in the current configuration (deformed) tandis that the plastic deformations are measured in the initial configuration,
.
as in small deformations, the constraints depend only on the deformations thermoelastic,
.
the plastic deformations are done with constant volume. The variation of volume is then only due to the thermoelastic deformations,
.
this model led during its numerical integration to a model incrémentalement objective (cf [§3.3]) what makes it possible to obtain the exact solution in the presence of great rotations.

A viscous version of this model is also available (law in hyperbolic sine as in the case of the model of Rousselier ROUSS_VISC, cf [R5.03.07]).

Thereafter, one briefly points out some concepts of mechanics in great deformations, then one present the relations of behavior of the model and its numerical integration to treat them equilibrium equations.
One proposes a variational formulation eulérienne, with reactualization of the geometry. For this reason,
one expresses the work of the interior efforts and his variation (with an aim of a resolution by the method
of Newton) for the continuous problem, which provides respectively after discretization by elements

stop the vector of the interior forces and the tangent matrix.

***Bene foot-note**:*

One will find in [bib2] or [bib3] a presentation deepened on the great deformations.
This document is extracted from [bib4] where one makes a more detailed presentation of the model elastoplastic, of its numerical integration and where one gives some examples of validation.

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2 Notations

One will note by:

Id
stamp identity

tr A
trace tensor A

AT
transposed of tensor A

det A
determinant of A

X
positive part of X

~
With
~
1
deviatoric part of tensor A defined by A = A - (
With
tr) Id
3

:
doubly contracted product: With: B = A B = tr (ABT)
ij ij

I, J

tensorial product: (A B)
=
ijkl
ij
With kl
B

3
With
~ ~
eq
equivalent value of von Mises defined by Aeq =
:
WITH A
2

With
TESTSTEMXÀ
gradient:

=
TESTSTEMXÀ

X

ij
With
div
=
X With
divergence: (divxA) I

X
J
J

, µ, E, K
moduli of the isotropic elasticity

y
elastic limit

coefficent of thermal dilation

T
temperature

Tref
temperature of reference

In addition, within the framework of a discretization in time, all the quantities evaluated at the moment
precedent are subscripted by -, the quantities evaluated at the moment T + T
are not subscripted and them
increments are indicated par. One has as follows:

Q = Q - Q
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3
Recalls on the great deformations

3.1 Kinematics

Let us consider a solid subjected to great deformations. That is to say the 0 field occupied by the solid before deformation and (T) the field occupied at the moment T by the deformed solid.

Initial configuration
Current configuration deformation
F
0
(T)

Appear 3.1-a: Representation of the initial and deformed configuration

In the initial configuration 0, the position of any particle of the solid is indicated by X (Lagrangian description). After deformation, the position at the moment T of the particle which occupied
position X before deformation is given by variable X (description eulérienne).

The total movement of the solid is defined, with U displacement, by:

X = x$ (X, T) = X + U

To define the change of metric in the vicinity of a point, one introduces the tensor gradient of transformation F:

x$
F =
= Id + U

X
X

The transformations of the element of volume and the density are worth:

D = J D O with J
O
= det F =

where O and are respectively the density in the configurations initial and current.

Various tensors of deformations can be obtained by eliminating rotation in
local transformation. For example, by directly calculating the variations length and angle
(variation of the scalar product), one obtains:

1
E = (C - Id) with C = FTF
2

1
With =
Id - b-1
(
) with B = FFT
2
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E and A are respectively the tensors of deformation of Green-Lagrange and Euler-Almansi and C and B, tensors of right and left Cauchy-Green respectively.

In Lagrangian description, one will describe the deformation by the tensors C or E because it are quantities defined on 0, and of description eulérienne by tensors B or A (definite on).

Note:

That is to say a solid undergoing two successive transformations, for example the first transformation makes pass the solid of the initial configuration 0 to a configuration 1 (tensor F1/0 gradient and vector u1/0 displacement), then the second transformation of configuration 1 to 2 (tensor gradient F2 1 and vector displacement u2 1).
F
F2/1
1/0
0
1
2
F2/0

The passage of configuration 0 to 2 is given by the tensor F2/0 gradient (displacement U
= U
+ U
2/0
2 1
/
1/0) such as:

F
= F F
2/0
2 1
/
1/0

One obtains then, for example, for the tensor of deformation of Green-Lagrange E

E

= FT E F + E
2/0
1/0 2 1
/
1/0
1/0

where E2/0, E1/0 and E2/1 are the deformations of Green-lagrange of configurations 2 by report/ratio with 0 associated F2/0, 1 compared to 0 associated F1/0 and 2 per report/ratio to 1 associated F2 1, respectively.
This constitutes one of the difficulties encountered at the time of the writing of a law of behavior in great deformations because one cannot write any more one formula similar to that written in small deformations, namely 2/0 = 2/1 + 1/0 where is the tensor of total deflection linearized.
To find 2/0 = 2/1 + 1/0 in small deformations starting from the expression of E2/0, it is necessary to neglect all the terms of order 2 of u2/0, u1/0 and u2 1. In this case, one has
E
~
E
~
T
F E
F
~
2 0 - 2 0, 1 0 - 1 0 and 1 0 2/1 1 0 - 2/1.

3.2 Constraints

For the model describes here, the tensor of the constraints used is the tensor eulérien of definite Kirchhoff
by:

J =

where is the tensor eulérien of Cauchy. The tensor thus results from a "scaling" by variation of volume of the tensor of Cauchy; this is not the case of other tensors of constraints used (first and second tensor of Piola-Kirchhoff).
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In description eulérienne, the equilibrium equations are given by:

*div + **F** = on*

0

X

D
F
N
*. = **T** on*

*where **F** is the voluminal force applied to the field, **N the** normal external with the border F and F the part of the border of the field where are applied the surface forces **td**.*

3.3 Objectivity

When a law of behavior in great deformations is written, one must check that this law is objectify, i.e. invariant by any change of space reference frame of the form:

x* = C (T) + Q (T) X

where Q is an orthogonal tensor which represent the rotation of the reference frame and C a vector which translates
translation.
More concretely, if one carries out a tensile test in the direction e1, for example, followed of one

rotation of 90° around e3, which amounts carrying out a tensile test according to e2, then the danger with a nonobjective law of behavior is not to find a tensor of the constraints uniaxial in the direction e2 (what is in particular the case with kinematics PETIT_REAC).

4 Presentation of the model of behavior

4.1 Aspect kinematics

This model supposes, just like in small deformations, the existence of a slackened configuration R, i.e. locally free of constraint, which then makes it possible to break up the total deflection in parts rubber band and plastic, this decomposition being multiplicative.
Thereafter, one will note by F the tensor gradient which makes pass from the initial configuration 0 to current configuration (T), by F p the tensor gradient which makes pass from configuration 0 to slackened configuration R, and Fe of the configuration R with (T). The index p refers to the part plastic, the index E with the elastic part.

Appear 4.1-a: Decomposition of the tensor gradient F in an elastic part Fe and plastic F p

By composition of the movements, one obtains the following multiplicative decomposition:

F = FeF p

The elastic strain are measured in the current configuration with the tensor eulérien of Left Cauchy-Green Be and plastic deformations in the initial configuration by the tensor G p (Lagrangian description). These two tensors are defined by:

Be
FeFeT
=
, G p
F pTF p
=
-
(
) 1 from where Be
FG pFT
=

The model presented is written in way to distinguish the isochoric terms from the terms of change from
volume. One introduces for that the two following tensors:

F = -
J 1 F
3 and Be = -2/B
3rd
J
with J = det F

By definition, one a: det F = 1 and det Be = 1.

4.2
Relations of behavior

The law presented is a model thermoélasto (visco) plastic with isotropic work hardening which tends under
the assumption of the small deformations towards the model [R5.03.02] with criterion of Von Mises (it acts of
plastic model). The plastic deformations are done with constant volume so that:

J p
p
= det F = 1 from where J J E
E
=
= det F

The relations of behavior are given by:

.
thermoelastic relation stress-strain:

~
~
E
= μb
3K
2
9K
1
tr =
(J -)
1 -
(T - T) (J
)
2
2
ref.
+ J

.
threshold of plasticity (it is admitted that it is expressed with the constraints of Kirchhoff):

F = eq - R (p) - y

where R is the isotropic variable of work hardening, function of the cumulated plastic deformation p.

.

laws of flow:

~
~
p T
3
E
1
~
E

FG& F = - &
B

= -

3 &
tr B +

eq
3
µ eq
&p = &

For the model of plasticity, the plastic multiplier is obtained by writing the condition of coherence f& = 0 and one a:
p&,
0 F 0 and p& F = 0
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In the viscous case, one takes &p equalizes with:

m
F
p& = &0 HS

0

where &, and m are the viscosity coefficients. Let us announce that this law is reduced to a law of the type
0
0
Norton when the 2 parameters materials & and are very large.
0
0

It is pointed out that:

Be = -2/B
3rd
J

F = -
J 1 F
3

and that the partition of the deformations is written:

Be
FG pFT
=

For metallic materials where the report/ratio
μ
eq/
is small in front of 1, the expression of the law
flow can be approached by:

~
p T
E
eq
FG& F = - & tr B
+O

éq.4.2-1
eq
μ
eq
where O
is negligible in front of the first term.
μ
It is this last expression which is established in Code_Aster.

Note:

If the deformations are small, one a:

J 1+ tr
E
E
B Id +
2

p
p
G Id -
2

where is the total deflection, E

elastic strain and p
plastic deformation in
small deformations.
By replacing these three expressions in the equations of the law of behavior presented here, one finds well the thermoelastoplastic traditional model with isotropic work hardening and criterion of Von Mises.

4.3
Choice of the function of work hardening

This relation of behavior is available in operator STAT_NON_LINE, under the key word factor COMP_INCR and argument "SIMO_MIEHE" of the key word factor DEFORMATION. One can choose
for the function of work hardening, a linear work hardening or to provide a diagram traction. Five relations can be used.

RELATION =
/"ELAS"
/
"VMIS_ISOT_TRAC"
[DEFECT]
/
"VMIS_ISOT_LINE"
/
"VISC_ISOT_TRAC"

/
“VISC_ISOT_LINE”

For a purely thermoelastic behavior, the user chooses argument “ELAS” (it behavior is then hyperelastic); for an isotropic work hardening given by a curve of traction, the user chooses argument “VMIS_ISOT_TRAC” in the plastic case or “VISC_ISOT_TRAC” in the viscous case and for a linear isotropic work hardening, the argument “VMIS_ISOT_LINE” in the plastic case or “VISC_ISOT_LINE” in the viscous case. The various characteristics of material are indicated in operator DEFI_MATERIAU ([U4.23.01]) under the key words:
· ELAS some is the law (one gives the Young modulus, the Poisson's ratio and possibly the thermal dilation coefficient),
· TRACTION for “VMIS_ISOT_TRAC” and “VISC_ISOT_TRAC” (one gives the curve of traction),
· ECRO_LINE for “VMIS_ISOT_LINE” and “VISC_ISOT_LINE” (one gives the limit of elasticity and the slope of work hardening),
· VISC_SINH for “VISC_ISOT_TRAC” and “VISC_ISOT_LINE” (one gives the three viscosity coefficients).

Note:

The user must make sure well that the “experimental” traction diagram used, is directly, that is to say to deduce the slope from it from work hardening is well given in the plan rational constraint = F/S - deformation logarithmic curve ln (1+ L/L)
0 where l0 is
initial length of the useful part of the test-tube, L variation length afterwards deformation, F the force applied and S current surface. It will be noticed that

F L 1
F L
F L
= F/S =
0
from where =
J =
0. In general, it is well the quantity
0 which is
S L J
S L
S L
0
0
0
measured by the experimenters and this the constraint of Kirchhoff gives directly

used in the model of Simo and Miehe.

4.4
Internal constraints and variables

The constraints are the constraints of Cauchy, thus calculated on the current configuration (six components in 3D, four in 2D).
The internal variables produced in Code_Aster are:

.
V1, cumulated plastic deformation p,
.
V2, the indicator of plasticity (0 if the last calculated increment is elastic, 1 if not),
1
.
V3, the trace divided by three of the tensor of elastic strain E
B is
E
trb.
3
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Note:

If the user wants to possibly recover deformations in postprocessing of sound calculation, it is necessary to trace the deformations of Green-Lagrange E, which represents a measurement of

deformations in great deformations. The traditional linearized deformations measure deformations under the assumption of the small deformations and do not have a direction into large deformations.

4.5 Field of use

The choice of a kinematics DEFORMATION: “PETIT_REAC” also makes it possible to treat a law of thermoelastoplastic behaviour with isotropic work hardening and criterion of von Mises into large deformations. The law is written in small deformations and the taking into account of the great deformations is done by reactualizing the geometry.
Between the law presented here (SIMO_MIEHE) and PETIT_REAC,

.
there is no difference if the deformations are small
.
there is no difference if the deformations are large but small rotations
.
there are differences if rotations are important.

In particular, the solution obtained with kinematics PETIT_REAC can deviate notably from exact solution in the presence of great rotations and this whatever the size of the steps of time chosen by the user, contrary to kinematics SIMO_MIEHE.

4.6 Integration of the law of behavior

In the case of an incremental behavior, key word factor COMP_INCR, knowing the constraint -
, cumulated plastic deformation p, the trace divided by three of the tensor of deformations 1
rubber bands E
tr B, displacements U and U and the temperatures T - and T, one seeks with 3
1
to determine (, tr E p
b).
3
Displacements being known, gradients of the transformation of 0 with -, noted F, and of - with, noted F, are known.

The implicit discretization of the law gives:
F = FF

J = det F
F = -
J 1 F
3
Be
FG pFT
=

J =
~
~
E
= μb
1
1
2
3
1
tr = K (J -)
1 - K (T - T
) (J
)
3
2
2
ref.
+ J
F =
-
eq - R (p
+ p
) - y ~
~
p
p
T
E
E
p

T

F (G -
-
G
) F = - tr B
p
from where B =
-
FG
F - tr E
B
p

eq
eq
In the plastic case: p,
0 F
0 and F
p = 0
1

-
- p m
In the viscous case:
1
- R (p + p) - - HS
0
eq
y
0
=

&0

T

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Note:

This formulation is incrémentalement objective because the only tensorial quantity incremental which intervenes in the discretization is &
G p. Like G p and G p are
measured on the same configuration, i.e. initial configuration, discretization of &G p, is G p G p G p
=
-
-, is incrémentalement objective.

One introduces Tr
, the tensor of Kirchhoff which results from an elastic prediction (Tr: trial, in English test):

~
~Tr
eTr

= µb

where

beTr
FG p-FT
Fbe-

FT

=
=
, F

= (J) - 1 3F and J = det
(
)
F

One obtains Be starting from the constraints -
by the thermoelastic relation stress-strain and
trace of the tensor of the elastic strain.

~-
E

1
E
B
=
+ tr B
-
μ
3

Note:

The interest of this formulation is that it is not necessary to calculate the deformation plastic G p which would oblige us to reverse the gradient of the transformation F. One needs only to know FG p-FT.

If Tr

-
eq < R (p) + y, one remains elastic. In this case, one a:
1
1
1
p = p, =~Tr + T R Id

and
E
eTr
tr B = tr B

3
3
3
if not, one obtains:

E
eTr
tr B = tr B

This last relation is not possible that if one makes simplification on the law of flow.

eTr
~Tr
μ tr
p

=
B
~(1+
)
eq

While calculating the equivalent constraint, one brings back oneself to a nonlinear scalar equation out of p:

Tr - - μ
eTr
tr B
p = 0
eq
eq

In the plastic case:
= + R (p + p
)
eq
y
, which leads to p solution of the equation:

Tr - -
-
R (p + p) - μ

eTr
tr B
p = 0
eq
y

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1
-
-1 p m
In the viscoplastic case:
= + R (p + p) +
eq
y
0 HS

, which leads to
& T
0

p solution of the equation:
1

Tr
-
1
- p m
- - R (p + p) - HS
- μ
eTr
tr B
p 0
eq
y
0
=

&0

T

The solution of this last equation is made in Code_Aster by a method of the secants with interval of research (cf [R5.03.05]). Integration can be controlled by the parameters RESI_INTE_RELA and ITER_INTE_MAXI under STAT_NON_LINE key word CONVERGENCE.

Once p known, one can then deduce the tensor from it from Kirchhoff, that is to say:

eq ~Tr K 2
3K
1
=

+
(J -)
1 -
(T - T
) (J +) Id
Tr

ref.

2

2
J
eq

Once calculated cumulated plastic deformation, the tensor of the constraints and the tangent matrix, one carries out a correction on the trace of the tensor of the elastic strain E
B to hold account
plastic incompressibility, which is not preserved with the simplification made on the law
of flow [éq 4.2.1]. This correction is carried out by using a relation between the invariants of E
B
~
and E
B and by exploiting the plastic condition of incompressibility p
J = 1 (or in an equivalent way
det E
B = 1). This relation is written:

x3- J ex- (1 - J E) = 0
2
3

2
~
~
E
1
2

E
eq
E
E

1
with J 2 = B
=
eq
, J = det B = det
= tr

2
2
3
and

E
X
B
2μ
μ
3

The solution of this cubic equation makes it possible to obtain
E
tr B and consequently
thermoelastic deformation Be with the step of next time. If this equation admits
several solutions, one takes the solution nearest to the solution of the step of previous time. It is
1
moreover why one stores in an internal variable
E
tr B.
3
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5 Formulation
variational

Insofar as the constraints provided by the law of behavior are eulériennes, one is chosen
variational formulation written on the current configuration (eulérienne) and not on the
configuration
initial, that is to say:

D =
v fv D +
D
X
T v dS v Kinematically acceptable

F

14 2
4
3
44
1444 2
4
3
4444
F
. v
int
F
.v
ext.

We are interested only here in work of the interior forces and its variation in optics of a resolution by the method of Newton. One will find in [bib 4] the demonstration of the expressions presented.

5.1
Case of the continuous medium

One rewrites here the work of the interior efforts in indicielle form, that is to say:

v
F .v
I
int
= ij
D

X J

We need also to express the variation of the interior efforts in the configuration current is:

U
U
p
J

v
F U v

I
int.
.
=
-

ij
ik
D rigidity géométriq
ue

X

p
xk

X J

U
ij
statement
+

I

D rigidity of behavior
pq
F
-
X

X
Q

J

where X are the punctual coordinates on the configuration -.

5.2
Discretization by finite elements

One discretizes virtual displacements U and displacements v by finite elements. The notations are the following ones, by adopting the convention of summation of the repeated indices:

U
U
U (X) = NR N (X) U N
I = Dn (X) U N
I = D-n (X) U N
I
I

X
J
I
-
J
I
J
X J
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where:

NR N (X) is related to form associated with node N
U nor, component I of the nodal displacement of node N
Dn (X
J
), components of the gradient of the functions of form on the configuration
D-n (X
J
), components of the gradient of the functions of form on the configuration -

One obtains for the vector of the interior forces:

(F
N
int) = DND
I
ij
J

and for the tangent matrix, which is not a priori symmetrical:

KN m
m
N
m
N
I p = [D D - D D
p ij
J
K
ik
p] D

+ D-n
ij
Dn D
Q

F
J

pq

In the case of a two-dimensional modeling (deformation planes), expressions of the vector of interior forces and of the tangent matrix are identical to this ready that the corresponding indices with the components only vary from 1 to 2.

In the case of an axisymmetric modeling, by numbering the axes in order (R, Z,), the vector interior forces is written:

NR N

(F axi N = Dn +
int)
33
1d, {1}
2
, , {1 }
2
,

R

and the tangent matrix:

[Kaxi] [K] [Kcorr
=
+

]
with:
[
N m
N
N
NR
NR
Kcorr
=

m
m
(1)
]
D D
-

D D
1

+
R
R 33
F

[
N m
m
m

NR

N
NR
Kcorr
N
= D
-
33
(2)
]

D
D
D

33

1

+
R

F
R

[
N m
N
m
NR
NR
Kcorr
33
(3)
] =

11
R 33
F
R

From an algorithmic point of view, we symmetrized the tangent elementary matrix K which is not it not a priori.
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5.3
Form of the tangent matrix of the behavior

One gives the form of the tangent matrix here (option FULL_MECA during iterations of Newton, option RIGI_MECA_TANG for the first iteration). This one is obtained by linearizing it system of equations which governs the law of behavior. We give here the final result of this linearization. One will find in [bib4] the detail of this calculation.

One poses:

J = det F, J -
-
= det F and J = det F

.
For option FULL_MECA, one a:

(-
J)/
1 3
-
1
J
With =
=
H -
(HF) B -
B
F

J

3
2
J J
J

-
J
3
-2
+

KJ - K (T - T
) (1 - J
) Id B

ref.

J
2

where B is worth:

B = F

F

- F

F
11
22
33
23 32
B = F

F

- F F
22
11
33
13
31
B = F

F - F F
33
11
22
12
21
B = F

F

- F F
12
31
23
33
21
B = F

F - F F
21
13
32
33
12
B = F

F

- F F
13
21
32
22
31
B = F

F

- F F
31
12
23
22
13

B = F

F

- F F
23
31
12
11
32
B = F

F

- F F
32
13
21
11
23

H and HF are given by:

In the elastic case (F < 0):

2μ
H
= μ (B E-F + F B E) -
F B E
ijkl
ik LP
jp
IP pl
jk
ij
kp LP
3
and
~
HF = 2μbeTr
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In the plastic case (F = 0) or viscoplastic:

μ
E
E
H
=
(B F + F b)
ijkl
ik
LP
jp
IP
pl
jk
has

R ~
p

ij
ij
E
-
2μ
+
F B

eTr

kp
LP
3a R
(+ μ tr B
)
eq

2
eTr
3μ tr B
R
(p)
eq
~ ~
E
+

F B
3
has
R
(+
eTr
ij kq
qp
LP
μ tr b)
eq
and

μ
2
2
p

μ
3
p
-
eTr
eTr ***Id***

~
R
tr ***B*** *eTr R*
(
)
HF *=*
B
- μ
2 tr ***B***
eq

+
+
~
*(: **B** eTr ~*
)
has
3a
R
*(+ μ tr **B** eTr)*
a3 R
*(+ μ tr **B** eTr)*

eq

eq

Tr

where has
eq
=
eq
1
-
2

2

p
1
1 -
and R = R' (p) +

0 × 1+
×
× (p)
1

m
, R (p) being the derivative of
&

1
0

T
m (&0t)
1
4
4
4
4
4
4
4
4
2
m
4
4
4
4
4
4
4
4
4
3
only
viscous

case

isotropic work hardening compared to the cumulated plastic deformation p.

.
For option RIGI_MECA_TANG, they are the same expressions as those given for FULL_MECA but with p = 0 and all the variables and coefficients of material taken with the moment T - (in theory, it would be necessary in the viscous case, to take the expressions of FULL_MECA in the elastic case, all the variables being taken at the moment T -). In particular, one will have ***F*** *=* ***Id****.*

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Law of behaviour in great rotations and small deformations

Summary:

***One describes here the formulation adopted to treat great rotations and small deformations. This formulation is valid for all the laws of behavior defined under COMP_INCR of the order STAT_NON_LINE and provided with modelings three-dimensional (3D), axisymmetric (AXIS), in deformations
plane (D_PLAN) and in plane constraints (C_PLAN).
This functionality is selected via the key word DEFORMATION = "GREEN" under COMP_INCR.***

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1 Some definitions

One points out here some definitions of tensors related to the great deformations.
One calls tensor gradient of the transformation F, the tensor which makes pass from the initial configuration
0 with the deformed current configuration (T).

x$
F =
= Id + U

with X = x$ (X, T) = X + U
éq
1-1
X
X

where X is the position of a point in 0, X the position of this same point after deformation in (T) and U displacement.
Various tensors of deformations can be obtained by eliminating rotation in local transformation. This can be done in two manners, that is to say by using the theorem of polar decomposition, is by directly calculating the variations length and angle (variation of scalar product).
Of Lagrangian description is obtained (i.e. on the initial configuration):

· By the polar decomposition:

F = RU

éq 1-2

where R is the tensor of rotation (orthogonal) and U the tensor of pure deformations right (symmetrical and definite positive).

· By a direct calculation of the deformations:

1
E = (C - Id) with C = FTF

éq 1-3
2

where E is the tensor of deformation of Green-Lagrange and C the tensor of right Cauchy-Green. The tensors U and C are connected by the following relation:

2
C = U

éq 1-4

2
Assumption of the small deformations and great rotations

When the deformations are small, there are no fundamental difficulties to write the laws of behavior: the various models "great deformations" lead to the same model

“small deformations”, and this as well for isotropic behaviors anisotropic. Only difficulty of a geometrical nature related to finished rotation remains.

*To write the model in great rotations and small deformations, one leaves the polar decomposition of F = RU is F. As the tensor U is a tensor of deformation pure and in addition small, one can calculate, by a law of behavior small deformations, the tensor of the constraints **

*associated the this history in deformation U. It is then enough to subject to this tensor ** *, rotation R*
to obtain the tensor of the constraints associated with the history in deformation F, as follows:

***T*
*= R R***

éq 2-1
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One can summarize this diagram as follows:

*ldc HPP **

**** T***
F U
= Id +

= R
R

éq 2-2

The disadvantage of this computation channel is that it requires the polar decomposition of F. Two assumptions are made then to avoid it.
On the one hand, to avoid the calculation of U, one can approach deformation HPP, by the deformation of
Green E, by benefitting owing to the fact that the deformations are small:

1
1
1
E =
FTF - Id =
U - Id U + Id = +
2
(
)
(
)(
)

éq
2-3
2
2
2

One then deduces from them by the law from behavior "small deformations".
In addition, in the same manner to avoid the calculation of R, one can approach the tensor of constraints HPP by the second tensor of Piola-Kirchhoff S:

S = F-1 F = Det (U) U-1 -1

=
+

(
O)
J
U

éq
2-4

One deduces some then by:

= 1
T
FSF

éq 2-5
J

Finally, in the presence of great rotations and to small deformations, it is enough to write the law of behavior "small deformations" with, in entry, history of the deformations of Green E, and in exit, the history of the constraints of Piola-Kirchhoff S. This approach is valid as well for isotropic laws of behavior that anisotropic.
As for the adapted variational formulation, it is about that adopted in hyper-elasticity (behavior ELAS, ELAS_VMIS_XXX under COMP_ELAS with the deformations of the type GREEN). For
more details, one will refer to the associated reference document [R5.03.20]. It is however necessary to be
sure that the problem studied induced many small deformations bus if not one cannot do them any more
simplifications [éq 2-3] and [éq 2-4]. Without this assumption, the variation with a plastic behavior increases quickly with the intensity of the deformations.

3 Bibliography

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Plasticity in the beams
Summary
This document describes the elastoplastic model of behavior of the elements of beam POU_D_E, POU_D_T and
POU_D_TG.
One describes the formulation of the model, his integration in the algorithm of STAT_NON_LINE or DYNA_NON_LINE
and method of local resolution (Runge-Kutta of order 4 with variable step).
One gives finally an example of application.
The revision B integrates a modification of the criterion of plasticity to treat the alternate loadings. One presents
also a method of integration implicit, usable in STAT_NON_LINE and DYNA_NON_LINE.
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1
Choice of modeling of plasticity in the beams

1.1 Field of application

The development of plasticity in the elements of beam of Code_Aster was carried out in drank to calculate the behavior of the pylons until the ruin (elastoplastic buckling). It was realized initially for elements of beam with warping (modeling POU_D_TG).
It follows a certain number of choices which were carried out to this end. This explicit document these choices.
The extension to the elements of beam without warping (modelings POU_D_E and POU_D_T) was made for other applications, such as for example the calculation of lines of pipings.

1.2 Choice of the type of element

Since it was mainly a question of calculating formed structures of beams with open mean profile (angles), for which warping is important, plasticity was introduced into the element of beam of Timoshenko with warping: POU_D_TG. Since version 4.3, this functionality is available also for the elements of right beam of Euler (POU_D_E) and Timoshenko (POU_D_T), but not for the elements of beam (POU_C_T curves) nor for the elements of bar.

1.3 Choice of the model of plasticity

Following a rather complete bibliographical study, studying the various types of modeling of plasticity in the beams [bib1], [bib2], [bib3], [bib4], it was selected a model of progressive plasticity [§2.2], while keeping a total criterion (i.e. expressing themselves on the efforts and the deformations of beams). This criterion is a generalization of the criterion says "circular" which utilizes way identical 3 moments and normal effort. It is simple and can be considered according to [bib1] wrap criteria more specific to various sections.

1.4 Numerical choice of resolution

The integration of this model in Code_Aster was made in the diagram of the usual resolution of plasticity [R5.03.01], for operators STAT_NON_LINE and DYNA_NON_LINE.
Two methods of integration of the behavior are available:
· a method based on a direct implicit discretization, as in the local integration of elastoplastic behavior in the elements of continuous mediums 2D and 3D [R5.03.02],
· a method of resolution numerical of the differential equations by a method of Runge-Kutta of order 4 with variable step. One solves the system of local equations (discretization semi-implicit), which is of order 10, with a control of the error. The step of time is Re automatically cut out in order to obtain the precision requested [bib5].
These two methods can be used indifferently. For certain applications, method implicit is faster, without it being a general property.
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2 Notations
Notations of this document
Significance
Notation of Doc. U
C
constant of torsion
JX
E E
eccentricity of the center of torsion/shearing
EY, EZ
y, Z
E
Young modulus
E

Poisson's ratio
NAKED
G
E
modulate of Coulomb = (
G
2 1+)
I, I
geometrical moments of inflection per report/ratio
IY, IZ
y
Z
with the axes y, Z
I
constant of warping
JG
K, K
coefficients of shearing
y
Z
1
1
AY
AZ
M, M, M
moments around axes X, y, Z
MT, MFY, MFZ
X
y
Z
NR
normal effort with the section
NR
S
surface of the section
With
U, v, W
translations on axes X, y, Z
DX DY DZ
V, V
sharp efforts along axes y, Z
VY, VZ
y

Z

rotations around axes X, y, Z
DRX DRY DRZ
X, y, Z
Note:
DX, DY, DZ and DRX, DRY, DRZ are in fact the names of the degrees of freedom associated with components of displacement U, v, W, X, y, Z
.
As in elasticity [R3.08.01], one makes a kinematic assumption of beams, i.e. that one admits that the cross-section of the beam is indeformable (remains right). One can thus express them displacements of an unspecified point of the section, according to displacements of the point corresponding located on the average line, and according to an increase in displacement due to the rotation of the section around the transverse axes.
The discretization is carried out on a linear element with two nodes and six degrees of freedom by nodes (or 7 ddl for modeling POU_D_TG). These degrees of freedom are the three translations U, v, W and three rotations X, y, Z (and warping for modeling POU_D_TG).
Finally when we arrange sizes related to the degrees of freedom of an element in one vector or an elementary matrix (thus of dimension 12 or 122), one arranges initially the variables for top 1 then those of top 2. For each node, one stores initially the sizes dependent on the three translations, then those related to three rotations. For example, a vector displacement will be structured in the following way:
U, v, W
,
,
,

, U, v,
W
,
,
,
1
1
1
X
y
Z
2
2
2
X
y

Z

1
1
1
2
2
2
!### "
# #
$
###
!##
"
##
$
####
top 1
top 2
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3 Relations of behavior

3.1 Variables generalized

The variables necessary to the description of plasticity are generalized variables, i.e. relating to a section of beam, as in elasticity [R3.08.01].
In plasticity, one utilizes only the normal effort, the two bending moments and the moment of torsion. One will note:

$$X = \begin{pmatrix} U \\ p \end{pmatrix}$$

X NR X

y p M

$y =$ $\mathbf{T}\, y$, $\mathbf{Q} =$ X, $\mathbf{qp} =$ $= YP$ M

Z
Z

Z =
Z

M
X

p
X

X
X
X = *X*
The relation connecting the constraints generalized with the elastic part of the generalized deformations is:
NR = *ES* (
p
X - *X*)
M
p
y = *I.E.(internal excitation) there y* -
T H. (**Q qp**
=
-
)
(
y)
that is to say **T** = *M*
p
Z = *I.E.(internal excitation) Z* (*Z* - *Z*)
M
p
X = *GC* (*X* - *X*)
One makes the assumption that the other variables generalized present in the element of beam (efforts edges and warping for POU_D_TG) do not intervene in plasticity. Constraints and
the corresponding generalized deformations are always bound by elasticity:
v
shearing action *V* = *K SG*
y
y

-

X
Z
W
shearing action *V = K SG*
Z
Z
+

X
y

2
Bi - moment and warping *M = I.E.(internal excitation)*
X

X
2
E
The Young modulus *E* and the Poisson's ratio (what makes it possible to obtain *G* = (
) are
2 1 +)
provided under the key word factor ELAS of order DEFI_MATERIAU.
Characteristics of the section of beam *I I I C K, K*
y, Z, y
Z are provided via
order AFFE_CARA_ELEM.
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Expression of the relation of behavior

The criterion of plasticity *P* (defining the limit elastic surface) depends on the generalized constraints **T**, and of three internal variables:

·

y, *Z* which are the cumulated plastic curves

· and the variable *p*.

It is a model with isotropic work hardening.

There is an important difference between such a model of plasticity (known as “total”) and the models of continuous mediums 2D and 3D: those express a relation of behavior in a material point, enters tensors of strains and stresses, whereas the total criteria are not intrinsic with material but they also depends on the form of the section of the beam (by the expression of the criterion on plasticity).

Plasticity is of type “associated”. *P* = 0 defines at the same time the surface of load and surface allowing to define the plastic flow. It is supposed to be convex. For the flow, one apply, by analogy with the elastoplastic models of the continuous mediums, the principle of Hill, it who implies the law of normality: the vector speeds of generalized plastic deformations %qp is normal on the surface of load. This is debatable insofar as the quantities intervening in criterion of plasticity are not of the same order (efforts and moments). However, the criterion is expressed here in standardized form. The parameter %p (multiplying plastic) is well the dimension a speed of plastic deformation: that easily in the case of checks a pure traction or of an inflection pure. By analogy with the continuous mediums, one will call *p the* “equivalent plastic deformation cumulated”.

The relation of behavior is expressed by:

P (**T**, *p*
y
Z
) = *F* (**T**, *y*, *Z*) - *R* (*p*) 0

T = **H** (**Q** - **qp**)

p
x%
P

p
% = *O*
if *P*
p
,
,
< 0
éq 3.2-1
p
(**T Q** *p*)
p
%
q% = *p*
p
y
%
with
with
q% =
T
p% O if *P*

(**T**, **qp**, *p*)
p
= 0
%zp
%x
p
p
%y = %y
%z = %z
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The criterion of plasticity is specific to the beams: it allows, in the case of an inflection, a passage progressive, in a section, the first plasticization until total plasticization. The function *F* is written:

N2
F (**T**,
2
2
2
y, *Z*) = *NR*
+ *A*
p
2
y (*y*). *M* + *A*
y
Z (*Z*). *M*
+ *A.M X*
NR
Z
X
p
(*y*
Z
y)
y
(
éq 3.2-2
Z)
Z
+
+
M2
M2
M2
M2
1
With
py
ey

pz
ez
y (*y*) = (
,
Z Z =
,
X =
y
Z
2

+

+
y)
With ()
y
(*Z*)
With
M
Z
px
On the other hand, for a section in pure traction or pure torsion, there is no plasticization progressive: the section either entirely elastic, or is completely plasticized.
Parameters *NR p*, *M px*, *Mey*, *M py*, *y*, *y*, *Mez*, *M pz*, *Z*, *Z*, are specific to section and with material. It are defined under key word VMIS_POUTRE of the order DEFI_MATERIAU. The value of the elastic limit is given *there* at the time of the definition of the function of work hardening [§2.3]. If *S* indicates the sectional surface, these parameters are expressed by:
I
I
y y
Z y
NR = S, M
= C, M =
, M
p
y
px
y
ey
=
.

Z
ez
y
max
max
M
= M

, M
= M
py
ey
pz
ez is the plastic moments limit section. The parameter
depends that form of the section (is worth for example 1.5 for a full rectangular section)
[bib2].
Parameters *y*, *y*, *Z* and *Z* do not have an analytical expression. It are to be adjusted as well as possible, by example on a calculation of pure inflection [§4.1]. (For a rectangular section, one finds in this case: *y* = .084 and *Z* = 0 0013
.
). Let us notice that for *y* = 0 (resp. *Z* = 0), one finds a criterion not
progressive in inflection: when the criterion is reached for one bending moment, all the section is then plasticized: *M = M*
y
py (resp. *M*
= M
Z
pz). (*M ey* (resp. *M ez*) does not intervene).
Note:
For the nonmonotonous loadings (alternating bendings) it is important to write the dependence of Ay (*y*) *and Az* (*Z*
) *compared to the cumulated plastic curves y, Z, and not by report/ratio with the plastic curves p*
p
y and Z. Indeed, those have an unspecified sign, can
p
p
to grow or decrease, therefore the monotony of Ay

With
is not ensured, which leads
y

and Z (*Z*)
with positive or negative work hardening according to the cases. On the other hand Ay (*y*) *and Az* (*Z*
) *are*
monotonous since the cumulated plastic curves y, Z are positive and increasing.
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Titrate:
Plasticity in the beams
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3.3 Work hardening

The function of work hardening *R* (*p*) can be given in two ways in the order DEFI_MATERIAU:
Maybe in the form of a linear work hardening via the key word factor ECRO_LINE: one provides then values of the elastic limit *y* and the slope of linear work hardening *AND*, relating to one curve of traction total stress-strain.
In this case the traction diagram is as follows:

y
L
L = EL
if *L E*

y

y
L = y + AND L -

if *L*

E
E

AND
y
thus
(*RP*) = *S* (*y* + *HP*)
E. *AND*
E.H
E
H =
that is to say
AND =
E - *AND*
E + *H*
L foot-note: to respect *E*
T < *E*
Maybe in the form of a nonlinear work hardening via the key word factor ECRO_FLEJOU: one then provides the elastic limit *y* and 3 realities: *E p*, *U*. *U* indicates the maximum constraint or ultimate in traction, *E p* is a parameter similar to a module of work hardening, is the power intervening in the expression below. These parameters must be readjusted on a curve of work hardening (deduced from the traction diagram), since in this case, work hardening is defined by:
400
350
300
250
L 200
150
100
50
0
0.00
0.03
0.09
0.15
0.21
0.33
0.45
0.57
L
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y
L = E L
if
L E

H. p

R (*p*)

= S
y
L = S
y +
if

1
L E

p

$1 +$

$U - y$

H

with
$E.E\ p$
$E.H$
$H =$
that is to say $E\ p =$
$E - E\ p$
$E + H$
foot-note: to respect E
$p < E$
Note:
· *This law has the advantage of having a horizontal asymptote: L cannot exceed U.*
· *It is often necessary to use this model to resort to a numerical identification of parameters E p, U.*
These two types of work hardening can be indicated in DEFI_MATERIAU. The behavior to introduce into STAT_NON_LINE or DYNA_NON_LINE under the key word factor COMP_INCR is then: VMIS_POU_LINE or VMIS_POU_FLEJOU [bib6].
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4

Integration of the behavior

One describes the calculation of the constraints generalized starting from an increment of generalized deformations,
and the calculation of the nodal forces and the tangent matrix.
To solve the nonlinear problem posed on the structure, the document [R5.03.01] described the algorithm of Newton used in *Aster*.
With each iteration *N* of the method of Newton, one must as in the case of the plasticity of the point material to solve a linear problem of the type:

K *N* **one**
+1 = **Lméca - R** *N*
I
I
I
I

where I is the sequence number corresponding to the variable of moment [R5.03.01], page 7, **K** *nor* is the matrix
of tangent rigidity in iteration *N* of the algorithm of Newton, **Lméca**
I
is the vector corresponding to
mechanical loading. This system provides an increase in **un+1** displacements
I
(expressed in
ddl of beams) which is provided to the behavior for the calculation of the generalized constraints. The vector
R *N*
N
T
N
I represents the nodal forces. It can symbolically be noted **R**

= **Q.**
I
I, where **QT** is
stamp associated with the operator divergence (**QT**) = (**U**)**. E (W)**

K where **W**
K
K indicates the function of
W
base associated with the *kth* degree of freedom with the structure.
The vector of the generalized constraints **nor** is composed of the vector **T** (which intervenes in the relation
of elastoplastic behavior) and other generalized efforts (sharp efforts and possibly
Bi-moment) which is related to the variables of deformation generalized by a relation of behavior rubber band.
T
NR

Vy
M y
One thus has: =
where **T** =

Vz
Mz

M

M X

Q

U
X =
v
X
- Z

X

y
y =
In the same way for the vector of the generalized deformations: **E** = *W*

, where **Q**
X
+
=

y

X

Z
Z =
2
X
X

X
x2
X = *X*
Integration on L `element is carried out using 3 points of Gauss.
The generalized constraints **Tn**
N
I are calculated starting from displacements of the beam **ui** (thus of deformations generalized) via the relation of behavior.
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One must also build the tangent operator to calculate **K N**
0

I or **K I** (options RIGI_MECA or
FULL_MECA). Two methods of integration are possible:
· method of RUNGE-KUTTA with step variable and recutting of the step of time, accessible
by key word RESO_INTE: “RUNGE_KUTTA_4” under the key word factor CONVERGENCE.
· a method based on the implicit of the relations of behavior, accessible discretization
by key word RESO_INTE: “IMPLICIT” (default value).

4.1
Local integration by the method of RUNGE-KUTTA

4.1.1 Discretization clarifies behavior

The explicit formulation from of speed of the relations of behavior [éq 2.2-1] is obtained by the writing of
condition of coherence [R5.03.02]:

P
P

P

P

If *P* (**T**, *p*
y
Z
) =, 0 *P%* (**T**, *p*
y
Z
) = 0 **T%+** % + % + *p*
y
Z
% = 0
T

p
y
Z

If not *P* (**T**, *p*
y
Z
)< ,0 %
%

T = H.q, qp

% = 0

In the first case, the condition of coherence makes it possible to express the plastic multiplier and speed of plastic deformation:

P

p

P

P

P

H (q% - q%) +

%y +

Z

% +

= 0

T

%

y

Z

p

p

P

P

P

P

P

P

R

thus

-

+

+

-

=

H (q% *p%*

)

0

T

T

p%

%

%

y

p
p
M y
Z Mz p
In the second case, %p = 0.
One obtains finally:
P

%
Hq
T
%p =
éq 4.1.1-1
P

P

P

P

P
P

R
H
-
-
+
T T
M

M

p
y
y
Z
Z

This expression utilizes the plastic condition of load [R5.03.02]. Once %p was calculated, one can find the plastic deformations generalized while using:

P

P

%p = *O* if
%
Hq 0

%qp = %p
,
T

éq 4.1.1-2
T
P
%p *O* if
%
Hq 0

T

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then constraints generalized by:
%T **H** (%q %qp
=
-
)

One can then express the constraints generalized according to the deformations generalized by one tangent operator:

P P

H

%
T
T
T = H1-
%q
éq 4.1.1-3

P P
P P
P P R
H

-
-
+
T

T

M
M
p
y
y
Z
Z

To carry out the local resolution, an elastic prediction is made:
knowing the state (**T**
-
-

-
, ,
y
,
Z
p) at the moment Ti
one seeks the state (**T+ T**
+
-
+
-
+
-
=
+ T, *y* = *y* +,
y Z = *Z* +,
Z p
= *p* + *p*) at the moment ti+1
· if
P (**T** *H*.**Q** - -
, *p*) 0 then (**T+** = **T** + *H*.**Q** +
-
+
-
, =, =, *p*+ = *p*
+
y
Z
y
y
Z
Z
)
· if not, it is then necessary to solve a differential equation of the type:
%Y = *F* (**Y**, %)
Q
with

P, *p*
Q p
(**T** *y Z*)
p%

(4 equations)

T

P **Hq%** +

T
p

P

P
P
P

P P
R

H

-
-
+

Y =
F (**Y**, *q%*)
T T
=
M
M
p
y
y
Z
Z

R
éq 4.1.1-4
R (*p*)

p%

p
p
T

H (Q
% - **q%)** (4 equations)

p
y
%y

Z
p

%z
and
(**Y** - *T*) = **y**
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One directly solves this system of differential equations (of dimension 12) by a method of

resolution of Runge-Kutta of order 4 [bib9]. To simplify the numerical establishment and to use one method of general resolution, the system was not simplified. One could however decrease it a number of equations to obtain a system of order 4.

4.1.2 Stamp

tangent

Should be calculated initially the tangent operator. Once the system [éq 4.1.1-4] solved, the operator tangent connecting a virtual increase in deformation **q*** to a virtual increase in constraint **T*** is given by:

P, *p*
*
ep
*

(**T** *y Z*)
T = **H Q** = **H** 1 -

p
q*

T

The formulation of this tangent operator is identical for two options RIGI_MECA_TANG or FULL_MECA.

The only difference comes from the estimate of the increase in deformation **Q**:

· In the case of option RIGI_MECA_TANG, corresponding to the phase of prediction, **Q** is worth either 0 for the first increment, or the increase with the preceding increment.

· In the case of the option FULL_MECA, **Q** changes with each iteration of Newton.

To calculate **K N**
0
I or **K I**, one assembles simply the elementary tangent matrix calculated by integration over the length of the beam (3 points of Gauss) of:

L
K
BTHep
=
B
elem
0

where **B** represents the matrix connecting displacements of the nodes of the element to the deformations generalized in each point of Gauss:

Q = **Drunk**

4.1.3 Method of resolution

Method used to solve **Y** = *F* (**Y**,)
Q is a method of Runge-Kutta of order 4,
described for example in [bib7].
The increase in deformation generalized **Q** is calculated starting from the increment in displacement **un+1**
I
provided by the total algorithm of Newton [R5.03.01]. One calculates in fact **Q** of each of the three points of Gauss of the element:
n+
U 1

I
X =
X
n+
1
N
U +1
y
I

I
n+1
y =

y
Q =
B U =
X
with **un+1**
N
n+1
I
I
= **U** + **U**

=
n+
1
I
I
n+1

zi
zi
Z =
X
n+1
xi
n+
1

xi
X =
X
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The variables **Q** are variables of piloting of the algorithm of resolution.
The method consists in calculating 4 estimates of *F*:
Y
-
1 = *F* (**Y**,)
Q

Y
-

Y = *F* **Y**
1
2

+
, **Q**

2

Y
-

Y = *F* **Y**
2
3
+
, **Q**

2

Y
-
4 = *F* (**Y** + **Y3**,)
Q
The result is obtained by:

1
Y = **Y** + 2Y + 2
1
2
Y3 + **Y**
6
4
Y+ = **y** + **Y**
One thus solves it by the method above, with a routine inspection of the precision. For that, two resolutions are carried out:
· one
with
Q, which provides **YI** solution of **YI** =
Y
(
+ **YI**
F

, **Q**), by the method
above,
Q
· the other with two steps of
, [bib8] page 72, which provides **Y** *II* solution of
2
1
2
YII = **Y** 2 + **Y** 2
1
1
Q
Y 2 = *F* (**y** + **Y2**,
)
2
2
1
2
Q
Y 2 = *F* (**y** + **Y2** + **Y2**,
).
2
One compares the solutions then **THERE** *I*
YII
and
obtained, and if the greatest difference is higher than
precision required by the user (key word RESI_INTE_RELA of STAT_NON_LINE), one decreases the step.
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II
I
1
Y - Y
More precisely, if *D* is noted
I
I
= max

, then:
i=1 1
, 2
max
I
Y
NR p
E.S
M py

E.I y
Mpz

E.Iz
M
px
G.C

1
Q

if *D* 100 one chooses **Q** =

NR
p

10

Q
max
E.S
S
I 1< *D* 100 one chooses 1
Q =
front
EC. **Y**
=

(*D* + 0 442
.
) 04.98
y

1
NR
S
I *D*
1 then
p
Q =

Q
Mpy

M pz

M px
M
py
E.Iy

M pz

E.I

Z
One then starts again the resolution with the new step **q1**. One thus divides the step by 10 if D > 100 and by 1.2 if D is close to 1, and by a factor varying continuously between these two values. If d>1, one decreases the increment **q1**, which gives **q2**, and so on by recurrence, until to satisfy the criterion D < 1. One then obtains a solution for a **qk** part of the initial increment **Q**. It the solution for the remainder of the increment **Q** - **qk** is then necessary to calculate. This new increment is him too likely to be decreased if d>1, and so on.
The total number of iterations necessary to convergence can be rather high (several tens), but it must be lower than the maximum iteration count fixed by user (ITER_INTE_MAXI). It is necessary however to take care that the resolution is sufficiently precise (= -10 6 per defect) to avoid propagation of error during step of time.
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4.2
Purely implicit local integration
4.2.1 Implicit discretization of the behavior
The direct implicit discretization of the relations of behavior [éq 3.2-1] gives it *in extenso* following system (P1):
NR = E.S. (
p
X - X)
M = E.I
p
y
y (*y - y*)

M = E.I
p
Z
Z (*Z - Z*)
M = E.I
p
X
X (*X - X*)
NR -
p
+ NR
X = p F (- **T** + -
-
T, *y + y, Z + Z*)
N2. With
-
+
-
.
+
p
p
y (*y*
y) (*My My*)
y = p F (- **T** + -
-
T, *y + y, Z + Z*)
N2. With
-
-
p
Z (*Z + Z*). (*M + M*
Z
Z)
p
Z = p F (- **T** +T, -
-
y + y, Z + Z)
N2. A.
- +
p
p
X (*M*

M
X
X)

= *p*
X
F (- **T** +T, -
-
y + *y*, *Z* + *Z*)
p
y = *y*
p
Z = *Z*
F (-
T + **T**, -
-
-
y + *y*, *Z* + *Z*) - *R* (*p* + *p*
) 0
p = 0
if *F* (**T** +
-
-
-
T, *y* + *y*, *Z* + *Z*) < *R* (*p* + *p*)
with *p* 0 siF (**T+** -
-
-
T, *y* + *y*, *Z* + *Z*) = *R* (*p* + *p*)
One has thus to solve a system of 11 nonlinear equations.
As previously, a purely elastic test is carried out:
· If
P (**T H.Q** - -
, *p*
+
) <, 0 then **T**
= H.Q
, qp

= ,
0 *p*
y
Z

= ,
0 *y* =,
0 *Z* = 0
· If not, it is necessary *a priori* to solve the nonlinear system of order 11.
The 11 unknown factors are: *NR*, *M*, *M*, *M*, *p*
, p
p
p
y, *Z*, *X*, *p*
y
Z
X
X
, y, *Z*
all them
other quantities being known. Affected variables of the sign `- `while exposing was given with
the previous moment.
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This level, several choices are possible:
· One can seek to directly solve this system by a method of the NEWTON type, in
using for example environment PLASTI [R5.03.10]. That presents the disadvantage of having
to reverse the matrix jacobienne with each iteration, stamps which can be badly conditioned. Of
more the solution in the vicinity of 0 is difficult to obtain.
· One can seek to reduce to the maximum the size of the system, as for the plasticity of Von
Settings with isotropic work hardening in plane constraints [R5.03.02]. 3 equations then are obtained
nonlinear. The method of selected resolution is triple seeks of zero of function.
It is this method which we describe and who proved to be most effective. It thus was

established in *Code_Aster*.
In the system to be solved (P1), one can easily eliminate the variables *p*
p
p
p
X, *y*, *Z*, *X*.
One obtains then the system of order 7 following (P2):
NR - + *NR*

NR - + *NR*
= *NR* - + *E.S.* - *E.S. p*
X
F (-
-
-
T + **T**, *y* + *y*, *Z* + *Z*)
N2. With
-
-
p
y
+ .
+
-
-
(*y*
y) (*M*
M
y
y)
M + *M*

= *M* + *E.I.* - *E.I. p*
y
y
y
y
y
y F (-
-
-
T + **T**, *y* + *y*, *Z* + *Z*)
N2 With

-
-
Z (*Z* + *Z*). (*M* + *M*
Z
Z)
p.
M - + *M*

= *M* - + *E.I.* - *E.I. p*
Z
Z
Z
Z
Z
y F (-
T + **T**, -
-
y + *y*, *Z* + *Z*)
N2. With
-
p
X.
+
-
-
(*M*
M
X
X)
M + *M*

= *M* + *G.J.* - *G.J. p*
X
X
X
X
X
X F (-
T + **T**, -
-
y + *y*, *Z* + *Z*)
F (-
T + **T**, -

-
-
y + *y*, *Z* + *Z*) - *R* (*p* + *p*
) 0
N2. With
-
-
p
y (*y* + *y*). *M* + *M*

=
y
y
p
y
F (- **T** +T, - +, -
y
y Z + *Z*)
N2. With
-
-
p
Z (*Z* + *Z*). *M* + *M*
Z
Z
= *p*
Z
F (- **T** +T, - +, -
y
y Z + *Z*)
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By expressing the efforts according to the internal variables, and by taking account of the criterion of plasticity,
the following system then is obtained (P3):
R p + p

NR - + NR
= (NR - + E.S.x)
(
)
R (p + p
) + E.S. p

R p + p

M - + M
-
y
y = (M + E.I
y
y y)
(
)
R (p + p
) + E.I. p
. N2. With
-
y
p
y (y + y)
R p + p

M - + M
-
Z
Z = (M + E.I
Z
Z Z)
(
)
R (p + p

) + *E.I. p*
. N2. With
-
Z
p
Z (*Z* + *Z*)
R p + *p*

M - + *M*

= (*M* - *G.Jxx*)
(
)
X
X
X +
R (*p* + *p*
) + *G.J. p*
. N2. With
X
p
X
F (-
T + **T**, -
-
-
y + *y*, *Z* + *Z*) = *R* (*p* + *p*
)
N2. With
-
-
p
y (*y* + *y*). *M* + *M*
y
y
= *p*
y

R (*p* + *p*
)
N2. With
-
-
p

Z (*Z* + *Z*). *M* + *M*
Z
Z
= *p*
Z

R (*p* + *p*
)
This system can be reduced by elimination of the efforts, by raising squared the first 4 equations,
1
by multiplying them by
, *Ay* ()
… , *Az* (.)
. , *A*, respectively, and by summoning them. One obtains then
2
NR
X
p
2
F (**T**
T
-
-
,
,
, *p*
+
+
+
+ *p*
y
y
Z
Z
)
with the member of left:
who is equal to
NR 2p
2
R (*p* + *p*
)
N2
, which leads to the following final system:

p
2
2
-
-
1
(*NR* + *E.S.x*)
(*M* + *E.I*
y
y y)
=
+
+
2
NR
2
p
NR
-
-
-
p [*R* (*p* + *p*
) + *E.S. p*
] 2 [*R* (*p* + *p*
)
2
2
+ *E.I. p*
. *NR. With*
y
p
y (*y* + *y*)]
(*M* + *E.I*
-
Z
Z Z) 2
(*M* + *E.I*
X
X X) 2
+ [
+
R (*p* + *p*
)

2
+ *E.I. p*
. NR. With
-
-
Z
p
Z (*Z* + *Z*)]2 [*R* (*p* + *p*
)
2
+ *G.J. p*
. NR. With
X
p
X] 2
N2
-
-
p. Ay (*y* + *y*). *M y* + *E.I y*

y
y = *p R* (*p* + *p*) + *E.I*
2
-
y. p. NR p. Ay (*y* + *y*)
N2
-
-
p. Z
With (*Z* + *Z*). *Mz* + *E.Iz*

Z
Z = *p R* (*p* + *p*) + *E.I*
2
-
Z. p. NR p. Z
With (*Z* + *Z*)
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On these three equations, the first is an equation out of *p* only, provided that one can to express *y* and *Z* according to *p*. These expressions are not analytical in the case general. On the other hand they are it if *Ay* or *Az* is constant (it is the case for example when *y* = 0 or *Z* = 0).

If one can know (numerically) *Ay* and *Az* for *p* given, the first equation is scalar in *p*. It is enough to solve it by a method of research to zero of function.

The three equations has to solve are then: *F* (*p*
) = 0, *G* (*y*) = 0 and (*H* = 0
y)
, with:
(
2
2
NR - + *E.S.*
-
X)
.

2
(*M* + *E I*
y
y y)
F (*p*
) = 1- [
- *NR*
2
p
+
2
R (*p* + *p*
) + *E.S. p*
]
[*R* (*p* + *p*) +E.I. p.N2.A -

y
p
y (*y* + *y*)]
2
2
.

.

2
(*M* + *E I*
-
Z
Z Z)
2
(*M* + *E I*
X
X X)
- *NR p* [
- *NR p*
R (*p* + *p*
) + *E.I. p*
. *N2*
2
. *With*
-
-
[
N2
2
. *With*
p
X]
Z
p
Z (*Z* + *Z*)]
R (*p* + *p*
) + *G.J. p*
X.
N2. With - +. M - + E.I
p
y
y

y
y
y y
G (*y*)
(
)
= - *p*
y
R (*p* + *p*) + *E.I.* p.N2.A -
y
p
y (*y* + *y*)
N2. With
-
-
p
Z (*Z*
+ *Z*)
(
. *M* + *E.I*
Z
Z Z
H Z
) = - *p*
Z
R (*p* + *p*) + *E.I.* p.N2.A -
Z
p
Z (*Z*
+ *Z*)
In practice, one uses a method of secants [R5.03.05] for the resolution of the first equation
(*F* (*p*
) = 0). This method is very stable because it ensures that the solution remains well in the terminals defined at the beginning. With each evaluation of the function *F* (*p*
I), one evaluates *y* (*IP*) and *Z* (*IP*) with
to leave the value given of *pi* by another method of search for zero [R5.03.09], while solving
G (*y*) = 0 and (
H y) = 0.
The interval of search for the solution (positive) of *F* (*p*
) = 0 are [0, 1
p], with:
F (+
-

-
-
T H Q, *y*, *Z*
) - *R* (*p*)
p1 =
E.S
Intervals of search for the solutions (also positive) of *G* (= 0
= 0
y)
and (
H
y)
are:
[0 1
, *y*] and [0 1
, *Z*], obtained by resolution of the equations *G* (*y*) = 0 and (
H y) = 0 while fixing
1
1
the functions *Ay* and *Az* with their initial value (respectively
and
).
2
M
2
ey
Mez
Once *p* is calculated, one obtains the efforts by the first 4 equations of the system (P3).
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4.2.2 Stamp

tangent

For option RIGI_MECA_TANG, (i.e. for the initial estimate of the tangent matrix)
the tangent operator is selected equal to the operator of elasticity **H.**
For option FULL_MECA, it is necessary to seek, after having obtained the solution of the system (P1), the operator
tangent connecting a virtual increase in deformation **q*** to a virtual increase in constraint:
T*
Hep q*
=

The calculation of the elementary tangent operator is done by considering a small variation of the equations of
system (for example P2) around the calculated solution, but by regarding this time **Q** as variable.
By differentiation of (P2), one obtains, when (*M*, *M*
y
Z) (,
0)
0

E.S. p
E.S. NR
NR 1 +
p
. B
E.S.

R

+
=
R
X

E.I. p
. N2. With
E.I. N2. A. Mr. B
p

. E.I. N2. A.M
M
y
p
y
y
p
y
y
y
p
y
y

y 1 +
p

= E.I
y
y.

R
+
+
R
R
y

E.I. p
. N2. With
E.I. N2. A. Mr. B
p
. E.I. N2. A.M
M
Z
p
Z
Z
p

Z

Z
Z
p
Z
Z
Z 1 +
p

+
= *E.I.*
R
Z
Z
Z

+
R
R

G.J. p
. N2. With
G.J. N2. A. Mr. B
M
X
p
X

1 +

p
X
p
X
X

G.J
X
.

R
+
=

R
X
X

F
F

F

F
F

F
NR

+ M

+ M

+ M

+
+
= R. p

NR
y

M
Z

M
X M
y
Z
y
Z
X
y
Z
M

. p. N2. A.M + p
N2. A.M2.B +.

2
2
p
y
y
p
y
y
y (.
p NR. A.M - R M
p
y
y
y)
y
= 0
M

. .
p N2. A.M + p
*N2. A.*M2.B +.
2
2
Z
p
Z
Z
p
Z
Z
Z (.
p NR. A.M - R M
p
Z
Z
Z) = 0
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where, one posed, to simplify the notations:
$R = R (p + p$
$)$
R
$M = M - + M$

$M = M - + M$

$R =$
y
y
y
Z
Z
Z
p
$WITH = A$
$-$
$-$
y
$y (y + y) A = A$
Z
$Z (Z$
$+ Z$
$)$
$R. p$

$H = 1$

With

R
Ay
Z

With
With =
y =
Z

NR = *NR* - + *NR*

y
Z
One thus obtains a linear system of order 7. To obtain the tangent operator, it is necessary to express.
NR, *M*, *M*, *M*
p
p
p
p
y
Z
X according to *X*, *y*, *Z*, *X* by thus eliminating *p*, *y*, *Z*. Like
it is a linear system, one could solve it matriciellement. But it is preferable to eliminate them
internal variables on the equations, because certain terms can be null according to the cases. The system
is written then in the following way (in the case general where *M* 0, *M*
y
Z 0):
N.a1 + *p.e1* = *H* .1x
M .b
y
2 + *p.e2* +. *F* 2 = *H*
y
.
2 *y*
M .c
Z
3 + *p.e3* +. *g3* = *H*
Z
.
3 *Z*
M .d
X
4 + *p.e4* = *h.*
4 *X*
N.a5 + *M* .b
y
5 + *M* .c

Z
5 + *M* .d
X
5 + *p.e5* +. *F*
y
5 + *G*
Z 5 = 0
M .b
y
6 + *p.e6* +. *F*
y
6 = 0
M .c
Z
7 + *p.e7* +. *g7*
Z
= 0
M y .b6 + *p.e6*
When the complete system is differentiable: *y* = -
.
F 6
Mz.c7 + *p.e7*
Z = -
.
g7
Then in substituent in the fifth equation, one obtains:
p = *N.k5* + *M* .l
y 5 + *M* .m
Z
5 + *M* .n
X
5
a5
1
F.
5 *b6*
1
G.
5 *c7*
d5
k5 =
, *l5* =
B

5 -
, *m5*
c5
, *n5*
r5
r5
F

=
-
6
r5
G

=
7
R

5
F.
5 *e6*
G.
5 *e7*
with *R*
5 =
+
- *e5*
F 6
g7
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Note:
· *if M*
0, *F*
2
'
2
y
6 < 0*: indeed, F* 6 = *NR A M - R* (*M*
p
p
y
y
y) *can be null only if*
M
'
y = 0 *bus A* <,
0 *R*
y
> 0,
· *in the same way, if M*
0, *G*
Z
7 < 0 ,
F 5.e6
g5.e7
· the *term*
resp.
M = 0 resp. *M* = 0,
F 6
g7 is nonnull except so y
(
Z
)
· *if r5* = 0*, one takes k5*, *l5*, *m5* and *n5 null.*
This makes it possible to also express *y*, *Z* according to *NR*, *M y*, *Mz*, *M X*:
y = N.k6 + M y.l6 + Mz.m6 + Mx.n6
e6
b6 E.

6 *l5*
e6
e6
k6 = -
K,
5 *l6* = -
+
, *m6*
m,
5 *n6*
n5
F 6
F 6
F

= -
= -
6
F 6
F

6
Z = N.k7 + M y.l7 + Mz.m7 + Mx.n7
e7
e7
c7 E.
7 *m5*
e7
k7 = -
K,
5 *l7* = -
L,
5 *m7* = -
+
, *n7*
n5
g7
g7
g7
G

= -

7
g7
Finally, in substituent in the first four equations, one obtains the expression of the operator
~
H:
NR. (*a1*+ *K*.
5th)
1 + *M* .e.
1 *L*
y
5 + *M* .e.
1 *m*
Z
5 + *M* .e.
1 *n5* = *H*
X
.
1 *X*
NR. (*E* .2k5 + *F* .2k)
6 + *M Y*. (*b2* + *E*.
2 *l5* + *F*.
2 *L*)
6 + *Mz*. (*E*.
2 *m5* + *F*.
2 *m*)
6 + *M X*. (*E*.
2 *n5* + *F*.
2 *N*)
6 = *h*.
2 *y*
NR. (*E* .3k5 + *G* .3k7) + *M*
(*e3*+ *e3.m5*+ *g3.m7*) + *M*
X. (*e3.n5* + *g3.n7*)
y. (*E*.
3 *l5* + *G*.
3 *l7*) + *Mz*.
= *h3.z*
NR
.e4.k5 + *M*

.e4.l5 + *M*

.e4.m5 + *M*

y
Z
X. (*d4* + *e4.n*)
5 = *h4.x*
~
~ -1
I.e. **H T**
= **H Q**
thus the tangent operator is **Hep** = (**H**). **H**
Particular cases:
· if
M = 0
y
, then the *e2* terms, *F* 2, *b6*, *e6*, *F* 6, *b5*, *K* 6, *l6*, *m6*, *n6*, *l5* and *l7* are null. That
~
amounts eliminating the terms in *M y* in **H**,
· if
M = 0
3, 3, 7, 7, 7, 5, 7, 7, 7, 7, 5, 6
Z
, the terms *E G C E G C K L m N m m* are null. That returns to
~
to write *m5* = 0 in **H**,
· if
M and *M* are null, all the terms above are null. However there remains one
y
Z
problem if moreover *e5* = *R* = 0 bus the tangent operator is then singular. It is the case of perfect plasticity in traction and compression or torsion. In this case one cannot to solve. One thus uses the operator of elasticity as tangent operator.
To avoid having problems of numerical instability or nonconvergences, one symmetrizes **Hep** and one use the symmetrical solvor.
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5 Variables

interns

The variables intern of the two behaviors: VMIS_POU_LINE and VMIS_POU_FLEJOU are:

V
p
1 = *X*
V
p
2 = *y*
V
p
3 = *Z*
V
p
4 = *X*
V 5 = *p*
R (*p*) - *S.*
V
y
6 =
0
S. y
F (*p p*
T, *p*
y
Z
)

siV 6 = 0

S.
V
y
7 =
R

(*p*) *siV6*> 0

S. y

V
p
8 = *y*
V
p
9 = *Z*
They are extrapolated with the nodes (option VARI_ELNO_ELGA of CALC_ELEM) using the functions of
form element.
Note: the variables V6 and V7 are not used in the calculation of integration of the behavior.
It are necessary to the postprocessings used for the calculation of the pylons.
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6 Example of application

6.1 Pure inflection on a beam

This example results from test SSNL106 of the documentation of validation of *Code_Aster*.
A right beam length L is subjected to a pure inflection, i.e. a bending moment
constant over all its length. It is embedded at an end and is subjected to one bending moment with the other end.
Mz
The section of the beam is rectangular or circular:
2v
R
B

Numerically, one can deal with this problem either with imposed moment, or with imposed curve, in

noticing that M
Z
Z and thus do not depend on X. The curve is then equal to:
X
Z Z (*L*) - *Z* ()
0
Z (*L*)
Z =
=
=
X
L
L

6.2 Solution
analytical

One places oneself on the assumption of a linear work hardening.
If one calls the v distance to the neutral axis of external fibre of the section (v = ymax = v for the section rectangular, and v = R for the circular section), and U the value of y from which the beam is plasticized, one can write:
xx = *Z*. *y*
xx = *Exx* for 0 *y U*

y
xx = *y* + *AND xx* -
for *U y v*

E
y
y
v

v
xx
U
xx
X
X
- v
- U
- v

The solution giving the moment according to the curve is obtained then simply by the integral:

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Mz ()
U = *xx* (*y*) .y.ds
S
what gives for the rectangular section:
M

E
2
3
1
E

Iz. y
M
Z
T
ez
T
Z
= 1-

with *M*
ez
and

M

E

-
2 2 +
=
=
E
ez
ez

v
I.E.(internal excitation)
ez

Z

ez
Z
and for the circular section:
R
3

3

y
E
4
E
2
- *E*
M
T
T

2
2

(
2)
Z =
(*E AND*)

+

-
(1-µ) +
arcsin (µ) - µ (1 - 2µ) 1 - µ
E

4 µ
3
2µ

U
y
with µ =
=
R
ERz

6.3 Data

The numerical values are:
B =.
0, v
1
= .
0 ,
1 *R* =.
0 ,
1 *E* =.
2 1011, *E*
9
6
T =.
2 10, *y* =
.
150 10
One deduces some:
I
I
section
S
NR = *S*
y y
Z y
p
y M

=
=
ey =
M =
M
M
M
M
Z
ez
py
ey
pz
ez
max
ymax
rectangle 0.02
3. 106
5. 104
1. 105
7.5 104
1.5 105
ring
0.01 4.7124 106
1.1781 105
1.1781 105
2. 105
2. 105
The coefficients and were adjusted by successive tests. One finds:
section

rectangle
0.084
0.0013
ring
0.084
0.0012
The beam is modelled by two elements.
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6.4 Results
obtained

In perfect plasticity, after precisely from the values of and, one obtains results which differ from the analytical solution of less than 0.5%.
The curve momentcourbure for the rectangular section is:

EDF
Mechanical department and Digital Models
Electricity
ssnl106a, rectangle, perfect plasticity, moment = F (curve)
from France
200000
3
X 10
180
150
120
90
analytical solution
total moment
calculation alpha=0.84 beta=0.0013
60
30
0
0
0
4
8
12
16
1
20

2
24
2
28
2
CURVE/courbure_el
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and for the circular section:
EDF
Mechanical department and Digital Models
Electricity
ssnl106a, circle, perfect plasticity, moment = F (curve)
from France
200000
3
X 10
180
150
120
90
analytical solution
total moment
calculation alpha=0.84 beta=0.0013
60
30
0
0
0
4
8
12
16
20
24
28
CURVE/courbure_el
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7 Conclusion

The total model of progressive plasticity used here makes it possible to find the behavior elastoplastic of a beam of section a priori unspecified subjected a normal effort, moments has of inflection and torsion. One in particular correctly simulates in inflection the progressive passage of the first plasticization with complete plasticization.

It is a model with purely isotropic work hardening.

The criterion of plasticity (voluntarily simplified compared to more realistic criteria for various profiles of section) is famous wraps for unspecified combinations of loadings. This is to check with the case not case, in particular for certain forms of sections (open mean profiles).

This modeling of plasticity presents however an additional difficulty of use by report/ratio with that used for the material point, in 2D or 3D: parameters intervening in the criterion depend on the form of the section. Some of them must be adjusted numerically.

Nevertheless, this approach makes it possible to model at lower cost the elastoplastic behavior of beams. It must be seen as complementary to approaches finer than are modelings with local criteria of plasticity (for example for modelings of hulls [R5.03.70] or them modelings of pipes [R3.08.06]).

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Law of behaviour of assembly ASSE_CORN

Summary:

This document describes the nonlinear behaviour of the nonlinear assemblies of angles of pylons modelled by discrete elements DIS_TR. This law of behavior is affected on the discrete elements by means of relation ASSE_CORN called by the operators of resolution of nonlinear problems STAT_NON_LINE [R5.03.01] or DYNA_NON_LINE [R5.05.05].

The law represents at the same time behaviour in traction of the assembly and the relation moment-rotation around
the axis of the bolts perpendicular to the assembly. The other directions of loading present one linear elastic behavior describes by traditional characteristics of rigidity.

One distinguishes in the law from behavior two phases associated with two mechanisms: the first

representing the friction and the slip of the bolts until the thrust, and the second representing plasticization of the assembly until the ruin. The laws of the plastic type describing each one of these phases have
even pace and have a concavity at their connection which makes convergence problematic and require a particular digital processing in the options of calculation to which the method appeals iterative of Newton.

Count

matters

---

1 Notations

SLF
Surface Limite of Friction
M y
Moment in the assembly around the axis y
N1
Effort limits slip of the assembly on axis X
M 1
Moment limits slip of the assembly on the axis y
SLU
Surface Ultimate Limite
N2
Ultimate limiting effort of the assembly on axis X
M2

Ultimate limiting moment of the assembly on the axis y
NR
Limiting effort
M
Limiting moment
U 1
Displacement limits mechanism 1 on axis X
1
Rotation limits mechanism 1 on the axis y
U 2
Displacement limits mechanism 2 on axis X
2
Rotation limits mechanism 2 on the axis y
U
Displacement of the assembly on axis X

Rotation of the assembly on the axis y
N
Effort reduces N = Nx/NR
m
Moment reduces m = My/M
U R
Displacement reduces Ur = U /U
R
Reduced rotation R =/
U
Displacement limits on axis X

Rotation limits on the axis y
H (X)
Scalar function
has
Parameter of nonlinearity
D
Constant scalar
D
Vector reduced generalized displacement
F
Vector reduced generalized effort
p
Variable interns scalar
feq
Effort generalized equivalent reduces scalar

F
Surface loading
R (X)
Scalar function R (X) H 1
-
=
(X)
D
Vector generalized displacement
F
Vector generalized effort
[D]
Stamp displacement generalized limit
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[F]
Stamp effort generalized limit
+
X
Value of X at the moment T + dt
-
X
Value of X at the moment T
E
Eccentricity of loading E = M y/NR X
er

Reduced eccentricity of loading E = m N
R
/

Sign N
[ ]
Stamp
{ }
Vector column
< >
Vector line
KB
Tangent operator at the moment T
KN
Tangent operator at the moment T + dt
Kor, Knr reduced tangent Operators

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2 Physical model of the one-way behavior of assembly

Assembly of an angle on the wing of another or a plate (bracket or splice plate) by bolts is schematized by [Figure 2-a].

Appear 2-a: locate local connection; axis X is confused with the axis of the bar and centers it is confused there with the axis of the bolts

The one-way behaviour of the assembly is modelled for the loading in traction or in inflection.

The modeling selected of the one-way behavior in loading of the assembly subjected to one normal effort or a moment around is represented there by [Figure 2-b].

Normal effort
Moment/y
SLU
SLU
mechanism 1
mechanism 2
mechanism 1
mechanism 2
SLF
SLF
U
butted
butted
1
Displacement

Rotation
1
0
0
0
U
0
2
2
Appear 2-b: mechanisms of assembly in normal effort and moment

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One distinguishes two phases of the behavior associated with two mechanisms:

· mechanism 1: friction and slip until the thrust (beginning of the shearing of bolts).
· mechanism 2: plasticization of the assembly until the ruin by shearing of the bolts or tearing of the grips.

The limiting surface of friction (SLF) is the curve corresponding to the appearance of the slip in space NR X M Y. friction is described by the law of Coulomb.

Ultimate limiting surface (SLU) is the curve corresponding to the ruin of the assembly in space NR X M Y. the ruin can be due, according to the design of the assembly, with the shearing of the bolts or with the tearing of the grips.

Tests on the same geometry but with tightening torques of the different bolts show that the tangent stiffness of mechanism 2 at the point of thrust decreases when the SLF bring closer the SLU.

This justifies the physical modeling retained for the assembly of the two mechanisms [Figure 2-b].

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3
Relation of behavior of the mechanisms

The behavior of mechanisms 1 and 2 is similar. It is nonlinear between a behavior
initial tangent rigid and an asymptotic limiting behavior.

It is described by two essential parameters: the parameter of nonlinearity and the parameter surface limit.

The thrust (mechanism 1) or ruins it (mechanism 2) are described by an associated kinematic criterion.

3.1 Behavior
one-way

We said to [§2] that one-way behaviors in normal effort and moment around
are similar there [Figure 2-b].

They can be described consequently relation if the adimensional sizes are used:

NR
M
.
X
y
reduced forces: *N* =
and
m =

NR
M
U

· reduced displacements: *U R* =
and *R* =
U

[Figure 3.1-a] represents in adimensional form the one-way behavior.
Analytically, it can be written (it is a choice):

U

R = *H* (*N*) or
R = *H* (*m*)
+
1
1
with *H* (*X*)
has
X
=
has

D 1 - *X*
a+1
N
D =
has
1 - *N*

A is the scalar parameter of nonlinearity. *N* and *A* are identified on the one-way tests. *N* who takes into account the variability of the tests generally takes value 0.95.

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Titrate:

N
m
N = Nx/NR
1
m = My/M
N m
Ur = U/U
R =/
criterion
kinematics
Ur
1

R

Appear 3.1-a: relation of behaviour of assembly

It is noticed that *H* (*N*) = 1 or *H* (*m*) = 1, i.e.: *U R* =1 or *R* = 1, or: *U* = *U* or = .

The one-way kinematic criterion is thus checked for *N* = *N* or *m* = *Mr*.

3.2
Incremental two-dimensional behavior

The coupling in extreme cases is defined by limiting surface:

2
2
NR X
M y

+
= 1

NR
M

The one-way behavior in reduced variables is described by the relation of [§3.1]:

D = H (*F*)

U R
where
D is the vector reduced displacements

R

N
F is the vector reduced forces
m

Into two-dimensional behavior, the isotropy is translated by a model with a variable interns **scalar** *p* such as:

p = H (*feq*)
loading

in

where *feq* is the equivalent reduced force (scalar).

feq is defined such as:
F = feq F *

where
NR X
F is the current point of loading *M*
y

*
NR X
F * is the limiting loading associated *F*

*
M y

The expression of *feq* results from the expression of limiting surface. Membership of *F* * with surface limit is written:

2
2
*
*

NR X
M y
+
= 1

NR
M

By the definition of *feq*, one can write:

2
2
NR
M
X
y
+
= 1

feqN
feqM

i.e. according to reduced forces *N* and *m*:

2
2
N
m
+
= 1

feq
feq

from where
2
2
feq = N + m

One defines then the surface of loading *F*, homothetic on limiting surface, by:

F: *feq - R* (*p*) = 0
R (*p*) *H* 1
where
-
=
(*p*)
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For a formalism similar to that of plasticity with isotropic work hardening [bib2], one obtains the relation
of behavior continues expressed in reduced sizes:

.
.
.
F
D = p F = p
F

feq
· *p* = 0
if *feq* - *R* (*p*) < 0
·
·
p = *h'* (*feq*) *F eq* if *feq* - *R* (*p*) = 0

The relation of behavior of the rigid type - plastic without elasticity is written finally:

·
· = *p*
1
D
[*D*] [*F*] - *F*
feq
U
Nx
where *D* =
and *F* =

M

y
[
0
0
D] *U*

=
NR

and [*F*]

=

0

0 *M*

The relation of incremental behavior in reduced sizes is obtained by integration of the relation continue between T (variable -) and $T + dt$ (variable +).

In loading, p
check $F = 0$ with $T + dt$:

$feq+ = R (p + p$
)
éq
2.2-1

By introducing the relation of behavior,

+
F
$D = p$

éq
2.2-2
+
feq
one deduces the value from p
,

2
2
p
$= D.D =$
U
$R +$
R

and one calculates the value of
+
feq by [éq 2.2-1]. The relation of behavior [éq 2.2-2] gives them
reduced efforts:

U
+

N
R
=
$R (p + p$
)

p

+

m
R
=
R (p + p
)
p

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In unloading, p
= 0 and one have by [éq 2.2-2]:

D = 0

4
Establishment in *Code_Aster*

The relation of behavior ASSE_CORN is assigned to discrete elements of modeling DIS_TR with 2 confused nodes. This relation is called by the operators of resolution of

nonlinear problems STAT_NON_LINE [R5.03.01] or DYNA_NON_LINE [R5.05.05].

The local axes of these elements X, y, Z are defined as on [Figure 2-a].

The integration of this relation of behaviour of the assemblies in operator STAT_NON_LINE of *Code_Aster the* formulation of the tangent operators requires *KB* and *kN* [bib3].

· *KB* is tangent rigidity at the beginning of the step of time, urgent *T*.
· *KN* is tangent rigidity at the end of the step of time, urgent *T* + *dt*.

The illustration of the operators *KB* and *kN* is given by [Figure 4-a].

Appear 4-a: definition of the operators KB and kN

4.1
Formulation in sizes reduced in loading

4.1.1 Operator
Knr

We saw with [§3.2] that the relation of behavior is written:

+
D

F
=
R (*p* + *p*
)
p

2
2
with
p
= *D*.*D* =
U
R +
R

The Knr operator is defined by:

F

K
= *I*
NR
1 *I*, *J* 2

D J

It is written:

p
[*Id*] - {}
p
D · <
>+
D

K
J
=
R +
NR
2

(*p*) +
p

{

+

'
D}
p
R
+
(*p*)
· <
>
D

p
J

Calculation gives then:

p

+
Ur
<
> =
R

;

D J
p
p
U 2r
Ur

R
{*D*}

p + *p*
p

· <
> =

D

2
J
Ur R

R

p
p

1 *x2*
and with *A* = 1
has

one: *H* (*X*) =

D 1 - *X*

1
2
1
2

R (*p*) = *H* (*p*) =
- *D p* + *D p* + 4 *D p*

2

1

1 -
2

R' (*p*)
D [*R* (*p*)]
=
=
h' [*R* (*p*)] *R* (*p*) [2 - *R* (*p*)]
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4.1.2 Operator Kor

For the elastoplastic behaviors, the operator *KB* with $T = 0$ is equal to the rigidity of the structure rubber band. In our case, the tangent initial behavior is rigid. *The Kor* operator is defined then by the passage in extreme cases when *p* tends towards 0 of the *Knr* operator. One obtains:

R' (*p*)
R (*p*)
=
p0
p

R (*p*)
from where *Kor* =

[*Id*]
p0
p

However *R* (*p*) < 1
p
and if one supposes that the user gives, for the first step of loading, of
values such as
4
p 10
>
, one can retain in practice:

4
10
0
K
=

T
however =0

4
0
10

These remarks are illustrated by [Figure 4.1.2-a].

Appear 4.1.2-a: operator KB with T = 0

At the moment *T* running, the *Kor* operator is equal to the *Knr* operator of the preceding step defined by [§4.1.1].
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4.2
Formulation in sizes reduced in unloading

To avoid numerical problems, one describes the behavior (rigid) in unloading by:

Kor = Knr = K

T
however =0

4.3
Tangent operators kN and KB

· Tangent operator *kN* is written:

F

K =
I
N
1 *I*, *J* 6

D J

with

F

NR
1
X
N

NR
=

=
×
D

U
1

U
R U
F

NR
1
X
N

NR
=
=
×
D
5

R
F

M

5
y
m

M
=
=
×
D

U
1

U
R U
F

M

5
y
m

M
=
=
×
D
5

R
F
2 = *Ky*
D
2
F
3 = *Kz*
D
3
F
4 = *KRx*
D
4
F
6 = *KRz*

D
6

The other values are null.
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· The tangent operator *KB*, with $T = 0$, is written:

4 *NR*

10
O

U

K y

K

KB =
Z

K X-ray

4 *M*

O
10

K Rz

4.4 Digital processing of connection enters the mechanisms of

law of assembly

During the resolution of each step of loading by the iterative method of Newton, one must calculate with each iteration the tangent with the curve of balance force-displacement of the law of behavior. The problem is that connection enters the mechanisms of the law of assembly, on the law of behavior, has a concavity (cf [Figure 2-b]) which returns convergence problems when, during a step of loading, one passes from one mechanism to the other.

In the subroutine TE0041 which calculates, for each increment of load, the elementary matrix of tangential rigidity of a discrete finite element with 2 nodes having of the degrees of freedom in translation and in rotation, it proved to be necessary to converge, to calculate a directed secant stiffness of the state initial of null effort and displacement towards the state, at the end of the step of loading, consisted the effort imposed and displacement corresponding on the curve of balance of the law of behavior. It was necessary for that, which was unusual on the level of this option, to know the number of iteration interns numerical process calculating the step of loading, then to consider the effort imposed on the element with the end of this step.

Indeed, if one notes
+
F effort imposed on the level of an element (a priori unknown since one does not know that assembled efforts),
+
U displacement corresponding on the curve of balance, and for iteration *I*, values respective *U* (*I*), *F* (*I*), *Ks* (*I*) of displacement, the effort and the matrix secant acting as tangent matrix calculated at the end of the iteration, one knows only in entry of the above mentioned subroutine
I
U, and values at the beginning of the step of load *F* (0) and *U* (0), because one the values with the preceding iteration *I* did not store -1. In the expression of the residue calculated in end
+
of iteration *I* -1: *F* - *F* (*I* -)
1 = *Ks* (*I* -)
1. (*U* (*I*) - *U* (*I* -)
1, one thus knows nothing any more but *U* (*I*) with iteration *I*, except in the particular case *I* = 1 where one a:

F + - *F* (0) = *K*
-

S (0). (*U* ()
1 *U* (0)
+
F there is the only unknown value at the beginning and results from the others. One also deduces it displacement
+
U at the end of the step according to the relation of balance:
.
p. [*N*, *m*]
.
.
= *R* (*p*)

. *U*
+
R, *R*, from where secant stiffness *K S* ()
+
1 = *F* /U.

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The problem is that in this first iteration, displacement *U* ()
1 imposed is different from
final displacement to calculate

+
U balances some with +
F from now on known (with the test of balance close to
no the preceding loading). Effort calculated at the end of this iteration *F* ()
1 must thus be also
different from
+
F and such as *F* ()
1 = *Ks* ()
1. *U* ()
1 so that starting from the couple *U* ()
1 and *F* ()
1, one points
with the secant *K* ()
1
S
on the couple
+
U and +
F. One thus obtains at the beginning of iteration 2 one
displacement *U* (2) very near to +
U and one can then calculate by the relation of balance *F* (2) very
near also to
+
F as well as the secant stiffness *Ks* (2) = *F* (2) /U (2).

If one converged exactly with the preceding step of load, 2 internal iterations are enough to converge exactly, if not one needs some additional iterations to satisfy the test of balance on residue.

The method known as of “directed secant” is schematized on [Figure 4.3-a] where one has them following correspondences:

U I = *U* (*I*)

K (*U I*
T
) = *Ks* (*I*)

for a law of behavior *LC* (*U I*) = *F* (*I*).

F (U1)
Elementary loop of Newton:
Entries: U0 Ui+1 Sig- VAr Mater->LC iter = i+1

If iter = 1:
One estimates F+ = LC (U0) + Kt (U0) (U1 U0)
One estimates U+ = LC-1 (F+)
K (U1) = F+/U+, F (U1) = K (U1) * U1
F+
Exits: K (U1) F (U1) Sig+ Var+
If iter > 1:

Exits: Kt (Ui+1) LC (Ui+1) Sig+ Var+
F
0
0
U0
U+
U1

Appear 4.3-a: method of directed secant

One thus sees now why it was necessary in the option calculated by the above mentioned subroutine to know it
number of iteration interns *I* in order to distinguish the particular case *I* = 1.
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5
Variables and parameters of the law of behavior

5.1
Variables of the law

The law of behavior comprises 4 internal variables per point of calculation of which 3 only are active:

· V1 is displacement reduces equivalent *p* maximum reached out of mechanism 1,
· V2 is displacement reduces equivalent *p* maximum reached out of mechanism 2,
· V3 is an indicator which is worth 1 or 2 according to whether one is respectively on surface limits mechanism 1 or 2, and 0 if one is under this limiting surface (after discharge for example),
· V4 is inactive for the moment (thus remains to 0).

5.2
parameters of the law

The parameters of the law of behavior entered like data under key word ASSE_CORN of order DEFI_MATERIAU [U4.43.01]:

· NU_: one enters behind this key word the value of the *N1* parameter of mechanism 1,
· MU_1: one enters behind this key word the value of the parameter *M* 1 of mechanism 1,
· DXU_1: one enters behind this key word the value of the parameter *U* 1 of mechanism 1,
· DRYU_1: one enters behind this key word the value of parameter 1 of mechanism 1,
· C_1: one enters behind this key word the value common to parameters *N* and *m* of mechanism 1,
· NU_2: one enters behind this key word the value of the parameter *N2* of mechanism 2,
· MU_2: one enters behind this key word the value of the parameter *m2* of mechanism 2,

· DXU_2: one enters behind this key word the value of the parameter *U* 2 of mechanism 2,
· DRYU_2: one enters behind this key word the value of parameter 2 of mechanism 2,
· C_2: one enters behind this key word the value common to parameters *N* and *m* of mechanism 2,
· KY, KZ, KRX, KRZ take the values of the characteristics of linear behavior in
local directions *y*, *Z*, *X-ray*, *Rz* respectively.

---

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Static and dynamic modeling of the beams
in great rotations

Summary:
This note gives a mechanical formulation of the beams in great displacements and great rotations but with an elastic behavior. The essential difficulty of the analysis of rotations holds so that they are not not commutable and a vector space, but a variety does not constitute.
At any moment, the configuration of a cross-section of beam is defined by the vector-displacement of sound
centre of gravity and the vector-rotation of the system of the principal axes of inertia compared to a position of
reference. As in classical theory of the beams, the interior efforts are reduced to their resultant and them moment on the locus of centres of the sections. The associated deformations are defined.
The linearization of the interior efforts compared to displacements leads to the matrix of usual rigidity, which
is symmetrical, and, because of great displacements and rotations, with the geometrical matrix of rigidity, which is
unspecified.
The linearization of the inertias carries out, for the translatory movement, with the matrix of usual mass who is symmetrical and, for the rotational movement, with a matrix much more complicated and without any
symmetry.
The diagram of temporal integration is that of Newmark.
This modeling was tested on five problems of reference: three of statics and two of dynamics.
This work was undertaken within the framework of the PPRD MEKELEC (M7-90-01) whose objective was to develop
tools of modeling for the components of the lines and the stations. The goal of the modeling presented in

this note is the dynamic study of the drivers provided with **spacers** (for the lines) or with **descents on equipment** (for the stations) and subjected to the forces of Laplace resulting from currents of short-circuit.
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symbol of the vector product.

operator of multiplication by the vector on the left.
Q
derived from *Q* compared to the curvilinear X-coordinate.
! *Q*

derived from *Q* compared to time.
S
curvilinear X-coordinate on the locus of centres of the sections.
“**U**
antisymmetric matrix of order 3 associated of axial vector **U**.
1
stamp unit of order 3.
Df. X

directional derivative of F in direction *X*.
D
D
D
D
1
,
,

ds
stamp diagonal **DIAG** *ds ds ds*.
WITH, I
,
,
1 2
surface and moments of inertia compared to the principal axes 1 2 or 3 of the section
or 3
right-hand side.
B
stamp deformation.
C
stamp behavior.
E, G,
Young modulus and rigidity to shearing, density.
E
general axes of co-ordinates.
I i=1,3
E
principal axes of inertia of the section of X-coordinate *S* in position of reference.
I (*S*) *i=1,3*
linear external force exerted on the beam.
F (*S, T*)
(
force in the beam with the X-coordinate *S* and.

F *S*, *T*)
F (*S*, *T*)
RT (*S*, *T*) (
F *S*, *T*).
F
forces external data with the nodes.
ext.
F
, **F**
inertias and efforts interior to the nodes.
iner
int
I
tensor of inertia a length unit of beam in deformed position, expressed
in the general axes.
J
tensor of inertia a length unit of beam in position of reference, expressed
in the general axes.
linear external moment exerted on the beam.
m (*S*, *T*)
moment in the beam with the X-coordinate *S* and.
m (*S*, *T*)
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M (*S*, *T*)
RT (*S*, *T*) **m** (*S*, *T*).
Ni
function of form relating to node *I*.

R (*S*, *T*)
operator or matrix, in general axes, of rotation of the cross-section
of X-coordinate *S*, configuration of reference to that at the moment T.
R
rotation making pass from the general axes to the principal axes of inertia of
O (*S*)
section of X-coordinate *S* in configuration of reference.
R early (*S*, *T*)
(
R *S*, *T*) **R** (*S*
O
) .
(
SO)
3
group operators of rotation in space with 3 dimensions.
T
principal axes of inertia of the section of X-coordinate *S* at the moment T.
I (*S*, *T*) *i*=, 13
X
position, at the moment T, of the center of the cross-section of X-coordinate *S.*
O (*S*, *T*)
'

X - T
O
1 .
E
RT.

R
0
early

.
0
Rtot

X

O (*S*, *T*)

(*S*, *T*)
: position of the section of X-coordinate *S* at the moment T, defined by

(*S*, *T*) vector position **xo** of the center and the vector rotation

.
(

xo (*S*)
S)

: virtual displacement with the X-coordinate *S.*
(*S*)
(
xo (*S*)
S)

: correction of displacement to the X-coordinate *S.*
(*S*)
(*S*, *T*)
defining vector, with the X-coordinate *S* and, variation of curve by
report/ratio with the configuration of reference.
X
RT.
(*S*, *T*)
vector rotation, at the moment T, of the section of X-coordinate *S* compared to its
position of reference.
nor-1, *I*
vector rotation enters moment *I* - 1 and iteration *N* of moment *I.*
nor
angular velocity of a section of beam calculated with iteration *N* of moment *I.*
Q, **Q-1**
operator of passage of a vector rotation to the associated quaternion and its reverse.
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2 Introduction

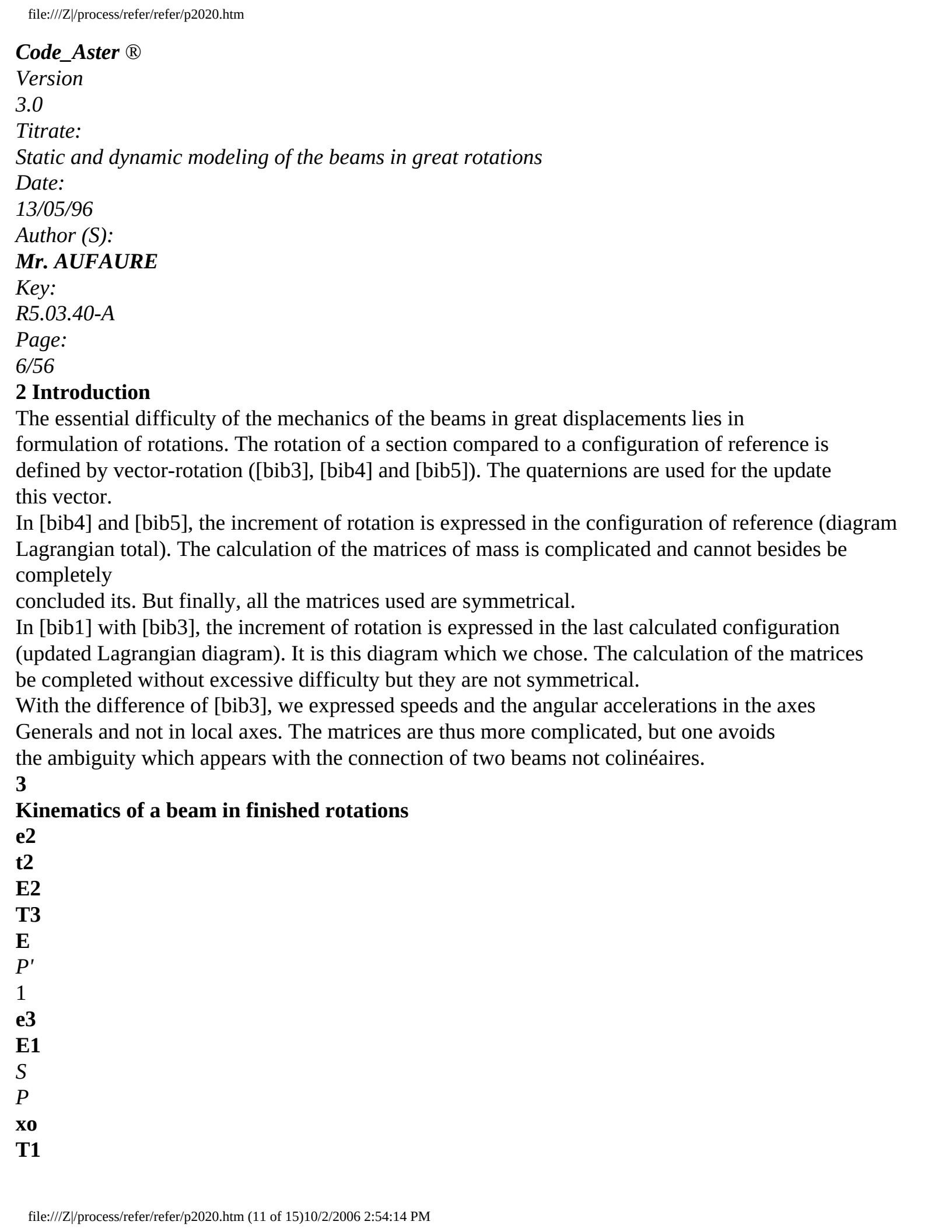
The essential difficulty of the mechanics of the beams in great displacements lies in formulation of rotations. The rotation of a section compared to a configuration of reference is defined by vector-rotation ([bib3], [bib4] and [bib5]). The quaternions are used for the update this vector.

In [bib4] and [bib5], the increment of rotation is expressed in the configuration of reference (diagram Lagrangian total). The calculation of the matrices of mass is complicated and cannot besides be completely
concluded its. But finally, all the matrices used are symmetrical.

In [bib1] with [bib3], the increment of rotation is expressed in the last calculated configuration (updated Lagrangian diagram). It is this diagram which we chose. The calculation of the matrices be completed without excessive difficulty but they are not symmetrical.

With the difference of [bib3], we expressed speeds and the angular accelerations in the axes Generals and not in local axes. The matrices are thus more complicated, but one avoids the ambiguity which appears with the connection of two beams not colinéaires.

3 Kinematics of a beam in finished rotations



S
E3
(A)
Position of reference
(b)
Position at the moment T
Appear 3-a: Evolution of a section of beam
Let us follow the evolution of a section of beam of its initial position - or **of reference** - [3-a] (A) with its position deformed at the moment T [3-a] (b).
The cross-section of the center *P* of the beam in position of reference is located by the X-coordinate curvilinear *S* of *P* on the locus of centres (or neutral fibre). One attaches to this section the trihedron orthonormé **E E E**: **E**
1
2
3
1 is the unit tangent of the locus of centres out of *P*; **E**
and **E**
2
3 are
directed along the principal axes of inertia of the section.
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As in [bib1] with [bib5], one makes the assumption that during the movement the sections initially right-hand sides remain plane and do not change a form.
Moment 0 at the moment T:
·
P comes out of *P* and the position of *P* is defined by the vector **xo** (*S*, *T*);
· the trihedron orthonormé **E E E**

1
2
3 becomes the trihedron orthonormé **T T**
T. **T** and **T**
1 2 3
2
3 are
always directed along the principal axes of inertia of the section and **T1** is always normal unit with this section. But **T1** is not inevitably tangent with the locus of centres out of *P*: in other words, there can be, in deformed position, a slip due to shearing, like in the model of Timoshenko.
The state of the section at the moment T is thus defined by:
· it
vector
xo (*S*, *T*), which gives the position of the centre of gravity;
· the vector-rotation which makes pass from the trihedron **E E E**
1
2
3 with the trihedron **T T**
T
1 2
3, and which is defined in
[§4.1].
The whole of these two vectors constitutes the vector (*S*, *T*).

4

Vector and operator of rotation

Appendix 1 gives preliminary results concerning the antisymmetric matrices of order 3.

4.1 Vector-rotation

Let us suppose that, in the system of general axes *P* **E E E**
1 2
3 [4.1-a], the point *M* results from
M by the rotation of angle around the axis passing by *P* and of unit vector **U**. Let us pose:
= **U**,
vector-rotation is called making pass from *M* to *Mr*.
According to the formula of Euler-Rodrigues [bib6] p. 186 and [bib7]:

PM = *PM* + sin **U** *PM*
(1 cos) **U U** *PM*

+ -

.

Me

M
U
e3
e2
E
P
1
Appear 4.1-a: Representation of a finished rotation
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In general, the vector-rotation of the product of two rotations **is not the** geometrical sum of vector-rotation components. The case 2D is a particularly simple exception:
vector-rotation, perpendicular to the plan, are added algebraically.

4.2
Operator of rotation

Taking into account [éq An1-3], the preceding equation is written:

PM = [**1** + sin **U** " + (1 - cos) **U** " 2] *PM*.
The expression between hooks defines the **operator of rotation R** making pass from *PM* to *PM*:
R = **1** + sin **U** " + (1 - cos) **U** " 2.
éq 4.2-1
One calls “parameters of Euler” of rotation the four following numbers:

E
= cos
E
= sin *U*
0
1
1
2
2

éq 4.2-2
E
= sin *U*
E
= sin *U*
2
2
3
3
2
2
One has obviously:
e2 + e2 + e2 + e2
0
1
2
3
= 1.
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Let us pose:

E
1

E = *e2* = sin **U**.

2
e3

By using the relation [éq A1-4], one easily puts the expression [éq 4.2-1] of **R** in the form:

R = (2 2
E -)
1 **1** + (
2
T
0
E E + 0
E **E** ").

éq 4.2-3

In addition, substitute sin and cos, with the second member of [éq 4.2-1], by their developments in whole series, it comes:

3
5
2 *p* -

1
p 1

-
R = **1** + -

+
+ # + (-)
1

"

3!
5!
(
+
2 *p* -)
U
1 !

2 4 6
2 *p*
p 1

+
-
+
+ # + (-) -
1
“2

2!
4 !
6!
(2 *p*) **U**
!
maybe, while using [éq A1-5] and [éq A1-6], the **exponential shape** of the operator of rotation:
(2
p
U ")
(**U** ")
R = **1** + **U** " +
+ # +
+
2 !
p!
#
éq 4.2-4

=
(
exp **U** ") =
(
exp “).
It appears on [éq 4.2-4] that when 0,
R 1 +.
éq 4.2-5
R =
(
exp
U
~
) is obviously not calculated by the development [éq 4.2-4], but by
the expression [éq 4.2-1].
Since “**C** = - “**U**, the transposition of all the terms of the second member of [éq 4.2-4] gives:
[(*T*
exp “)]
=
(
exp - “), that is to say:
éq 4.2-6
RT = **R** 1
and:
R RT
= **1** .
éq 4.2-7
The operators of rotation, orthogonal according to the equation [éq 4.2-7], form a group compared to the operation of multiplication - noncommutative in 3D - called **group of Dregs** and indicated by
(
SO)
3
(Special Orthogonal group).
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5

Passage of the local axes to the general axes

The components of the vectors are expressed in the general axes **E E E**
1 2
3 [3-a].
matrices of the operators who connect them are thus valid only in these axes. But mechanics
beams is formulated much more simply in the principal axes of inertia buildings **T T T**
1 2
3
in current configuration. One is thus brought to make the change of axes of the trihedron general **E E E**
1 2 3
with the local trihedron **T T T**
1 2 3 by the product **R** early of two rotations:
·
rotation
Ro, invariable, which brings the general axes (**E E E**
1 2
3) on the local axes in
position of reference (**E E E**
1
2
3) ;
·
rotation
R, depend on the time, which brings the trihedron (**E E E**
1
2
3) on the local trihedron in
current deformed position (**T T T**
1 2 3), is:
R
= **R R**
early
O.

éq 5-1

Being given a vector **v**, components known in the trihedron general, her components in

local trihedron are the components in the trihedron general of the vector:
V = RTtot v.
éq 5-2
One can thus replace calculations relating to vectors expressed in local axes in
current configuration, by same calculations relating to the same turned vectors of **RTtot** and
expressed in general axes. In other words, this **RTtot** rotation makes it possible to replace calculations in
local axes of the current configuration, by same calculations in general axes.
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6
Interior efforts, deformations and law of behavior
One places oneself within the framework of the theory of the beams. The efforts and the deformations are defined by
their end cells on the line of the centres of gravity of the sections. Thus, virtual work in
the beam is calculated by a simple curvilinear integral along this line.
6.1 Efforts
interiors
One calls interior efforts on the section of center *P* and normal **T1 the** efforts which exerts on
this section the part of the beam located in the direction of **T1** [6.1-a]. These efforts form one
torque whose end cells out of *P* are: the resultant **F**, resulting moment **Mr.**
P "
m
T1
F
P'
Appear 6.1-a: Section of beam reduced to the locus of centres
If **F** and **m** are respectively the force and the moment outsides given per unit of length not
deformation - **F** and **m** are supposed to be independent of the configuration, i.e. “conservative” or

“not-follower” -, the static balance of the section of beam *P P* length *ds* is written:

F
+ **F** = **0**
S

éq 6.1-1

m
X
+
O +
=
.

0
S

F
m
S

6.2

Variation of curve in a point of the locus of centres

The local axes of the section of X-coordinate *S* result from the general axes by the relation [éq 5-1]:

T () =
(
R). **R** () **E**
I S
S
S I.
O

éq 6.2-1

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While deriving compared to *S*:
you
= (**R Ro** + **R R'**
'
O) **E**
I
I,
maybe, by reversing the relation [éq 6.2-1]:
you
= (**R RT** + **R R'**
'
RT RT
O
O
) **T**
I
I.
Let us pose:
R' RT
= "
éq 6.2-2
The matrix "is antisymmetric because derivation compared to *S* of [éq 4.2-7] gives:
"+ "*T* = **0**.
It is checked that the matrix "O defined by:
"
'

T
T
O
= R Ro Ro R
is also antisymmetric.
Therefore, while utilizing axial vectors and O:
you
= (+ O) **T**
I
I.
.
ds O is the vector rotation which makes pass from **Ti** (*S*) to **Ti** (*S* + *ds*) when the beam undergoes one **uniform** rotation (**R'** =)

0 of its position of reference to its current position. O (*S*)
characterize the **curve of the configuration of reference** to the X-coordinate *S.*
.
ds is the increase in **Ti** rotation (*S*) with **Ti** (*S* + *ds*) due to the variation of **R** along
the beam. It is this vector which characterizes the **variation of curve** between the configuration
of reference and current deformed configuration.

6.3

Virtual work in the beam and vector of the deformations

This paragraph is the extension to the three-dimensional case of a step made in [bib8] on the beams plane.
The configuration of the beam at the moment T is defined [§3] by the field (*S*, *T*):
X *S*, *T*

(
O
S, *T*)
( )
=
.

(*S*, *T*)

Let us calculate work *W* of the linear external forces **F** and **m** in following virtual displacement:

xo (*S*)

(
S) =

.
(*S*)
One has obviously:
S
W =
2 (.xo +
F
Mr.) *ds*.
s1
The equilibrium equations [éq 6.1-1] make it possible to replace the external forces **F** and **m** by interior efforts **F** and **m**:
s2
F
m
xo

W =
-
.xo -
. -
F. *ds*

,
s1
S
S

S

maybe, by integration by parts of the first two terms:
2
S
W = - [

2
'
'
f.x + **Mr.**] + 1 [**f.x** + **Mr.** '- (**X**
O
O
O

F).
1
] *ds*.
S
If one indicates by *Wext the* total work of the external forces on the beam - length and in ends - the preceding equation is written:
s2
2
Wext = (**f.xo** + **Mr.**) *ds* + [**f.xo** + **Mr.**]
S
1
1
S
éq 6.3-1
=
2
'
'
F. **X - X**
+

Mr. '
.
s1 [
(O
O)
] *ds*
According to the theorem of virtual work for the continuous mediums, the second member is work virtual of the interior efforts, which one notes *Wint*. According to the idea of [bib8], let us seek to put them
coefficients of **F** and **m** in the form of the **virtual increase in two vectors.**
Let us make the assumption that the vector **x'** O tangent with the locus of centres but not inevitably length
unit, does not differ from unit vector **T1**, normal with the section, that by infinitely small of the 1st order (of

the order of). That implies that the beam lengthens little and undergoes a weak slip. Then:
S
W
2
'
int
=

.
1 [
.
F (**X - T**
O
1) + **Mr.**] *ds*
S
Indeed, in virtual displacement:
.
x'
'
O increases **xo**;
· unit vector **T1** turns of;
· it
vector
, defining the variation of curve [§6.2], increases '.
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One thus has:
S

W
2
int
= [.f + **Mr.**] *ds*,
éq 6.3-2
s1
with:

x' O - **T**
1
=
.
éq 6.3-3

F
In the equation [éq 6.3-2], and
m
thus seem respectively a **torque**

efforts or **generalized constraints** and the **torque of the associated deformations**.
defines lengthening and the slip; the variation of curve [§6.2] defines.
It is observed that, if must be small, on the other hand there is no limitation for. Majority of the beams enter within this framework.
The relation [éq 6.3.1] is written:
s2
F
Wint (;
) =
.

B *ds*
éq 6.3-4
s1
m
with:
D
'

1 xo

B = *ds*

éq 6.3-5

D
0
1

ds
B is called **matrix of deformation**.
6.4
Law of behavior
According to [§5], the components in local axes of the generalized constraint and deformation are components in general axes of the vectors:
F

E

and such as:
M
X

F

T **F**

=
éq 6.4-1
M

m
E

T

=
X

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One supposes that the law of behavior is elastic and that, in local axes, it with same expression that for a beam of Timoshenko:

F

E

= **DIAG** [*EA*, *GA*, *GA*, *GI*, *I.E.(INTERNAL EXCITATION)*, *I.E.(INTERNAL EXCITATION)*
2
3
1
2
3]
éq 6.4-2

M
X

With and *A*
2

3 being two surfaces depending on the size and the form of the section.
One poses:
C = **DIAG** [*EA, GA, GA, GI, I.E.(INTERNAL EXCITATION), I.E.(INTERNAL EXCITATION)*].
2
3
1
2
3
C is called **matrix of behavior**.
7
Elementary inertias
The inertias applied to an element *ds* form a torque which admits, in the centre of gravity:
· a general resultant, - *A ds*! **xo** (*S*, *T*);
· one equal moment resulting contrary to the absolute velocity of the elementary kinetic moment
H.
To express the angular velocity, let us proceed as to [§6.2] and derive the relation:
T
= **R**
I
I.E.(internal excitation),
compared to *T*, by holding account that **I.E.(internal excitation)** does not depend on time. One obtains:
!
T
=! **R RT T**.
I
I
éq 7-1
Let us pose:
! **R RT** = “.
éq 7-2
By deriving the relation [éq 4.2-7] compared to *T*, one sees that the matrix “is antisymmetric. If one indicate by the axial vector of this matrix, the relation [éq 7.1] is written:
!
T (*S*, *T*) = **T**
I
I (*S*, *T*).
is thus the **angular Flight Path Vector** of the section of beam of X-coordinate *S* at the moment T.
The elementary kinetic moment has as an expression:
H = *ds*
= *ds*
T
I
R J R

éq 7-3
where **J** is the tensor of inertia in the configuration of reference:
J
= **R**
T
O
DIAG [*I, I, I*
1
2
3] **R**

O.
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While deriving compared to time:
! **H** = *ds*
! + *ds* (!
T +
! *T*
I
R J R
R J R

).
But:
! **R J RT**
=! **R RT R J RT**

= “**I** = **I**

,
,
and:
R J R! *T* = **R J RT R RT**

! = **0**
because, according to the equation [éq A1-2]:
R R! *T* = **-** “= **0**.
From where moment of the inertias of the element:
-! **H** = - *ds* **I**! - *ds* **I**

.
The virtual work of the inertias thus has as an expression:
s2
With! **xo**
X
W
O
= **-**
.
ds
iner
.
s1 **I**! +

I
éq 7-4

8
Equation of the movement and course of a calculation
8.1
Equation of the movement not deadened
If one adds the inertias to the external forces, the weak form of the equations of the movement, in other words the virtual work of the forces not balanced in the beam is written:

W (! ! ;) = *W* (;) - *Winer* (! ! ;) - *Wext* (;).
int
éq 8.1-1
With balance *W* is null, for all.
F
Wint (;) is given by the equation [éq 6.3-4] where the torque of efforts, has as an expression,
m

according to [éq 6.4-2]:
F

= **C** *T*
éq 8.1-2
m

The torque of deformations generalized of the second member of [éq 8.1-2] results from the position current: is given by [éq 6.3-3] and rises from [éq 6.2-2].
W
! !
iner (;) is given by the equation [éq 7-4].
S
X
2 **F**

W
O
ext. (;) =
. *ds*+ work of the concentrated forces
éq 8.1-3
s1 **m**

If the external forces given are conservative, i.e. independent of the configuration,
Wext does not depend on.

8.2

Course of a calculation

· In the dynamic case, one seeks the fields of displacements, speeds and accelerations
in a discrete succession of moments: *T*, *T*
T
, *T*
T
1 2
I - 1 *I*
N.
#
#

· In the static case, one splits the total load in **increments** of load which one adds
successively from zero to reconstitute the truck load. With each stage of
loading, named by "urgent" abuse, one calculates the field of displacements.
Knowing the state of the structure at the moment *Ti* - 1, one deduces from it his state at the moment *Ti*
by **prediction**
correction:

· In the static case (STAT_NON_LINE), the prediction consists in calculating the answer of
structure with *I* - ème increment of load, by preserving its behavior at the moment *Ti* - 1.

· In the dynamic case, one must initially **initialize the** fields at the moment *Ti*, by formulas
rising from the algorithm of temporal integration used: in operator DYNA_NON_LINE,
it is the algorithm of Newmark [§A3]. Then one applies the increase in load enters
T
T
I - 1 and *I* with the behavior in initialized situation.
In the predicted state, the equilibrium equation is generally not satisfied and one must **correct** them
displacements by **iterations** resting on **linearized equations.**

9

Linearization of the equations of the movement

Let us suppose calculated the state of the structure to iteration *N* of moment *I*. *N* = 1 corresponds to the
phase of

prediction. The weak form of the equilibrium equations is, with this iteration [éq 8.1-1]:
W (*N N N*
N
N
N
N
N
! !

=
-
! !
I, *I*, *I*;)
W (*I*;) *Winer* (*I*, *I*, *I*;) - *Wext* (*I*;
int
)**.**
éq 9-1
· If this quantity is rather small, within the meaning of the criterion of stop [bib10], one considers that this *N* -
ième iteration gives the state of the structure to moment *I*.
·
N + 1

If not, one calculates **corrections** of displacement *I*
such as:

+ 1
+ 1 .
+ 1 .

L W
(*N*
N
N
N
N
N
I
+ *I*
), (*I* + *I*), (*I* + *I*);

=
éq 9-2

W (*N N N*
N
N
N
N + 1
I
! *I*! *I*! ;) + *DW* (*I*! *I*! *I*! ;). *I*
= 0.
DW (*N N N*
N
N
N
N
I
! *I*! *I*;). +1
I

is the differential of Fréchet of *W* (*I*! *I*! *I*;) in the direction
N + 1
I
[An4].
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9.1
Matrices of rigidity

They result from the differentiation of Fréchet de *Wint* (;
) in the direction. According to
equations [éq 6.3-4] and [éq 6.4-1]:

s2
F
Wint (;) = {**B**}
. *ds*.
s1
M
That is to say:
s2
F
D Wint. =
B.

S [{
D
}]
ds
1
M
éq 9.1-1
s2

F

2
F

+ {**B**}
. (
S
D.) *ds* +
M
{**B**}
. *D*. *ds*.
s1
S

1

M

However, according to the equation [éq 6.3-5]:
{

X
'
'
O + **xo**

B}

=
.

'

Thus [éq A4-2]:

X
'O
D {**B**}
. =
.

0

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In addition [éq A4-5]:

^

R
0
D. =
early
.

^

0

Rtot

Lastly, according to [§6.4]:

F

D = **C** *D*
T
.
M

^

T
T

=

C B -
0
C
^ ,

0

because [éq 6.3-2] and [éq 6.3-5]:

D
.
B,

=

and [éq A4.6]:
^
D T
T
R
= -

early.
Rtot
.
It is shown [A5] that:
· the sum of the first two integrals of the second member of [éq 9.1-1] can be put in the form:
s2 T T

E *ds*;
s1
· the third integral can be written:
s2 T T
T
S
B C B *ds* + 2 *T T*
B Z *ds*

,
S
S
1
1
with:
D

1
0
ds

D
= **0**

1 ;

ds
0
1

0
0 - F

E = **0**
0 - m;
éq 9.1-2

'
F

0
“**X F**”
O

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v
0

T
- 1
T

R C R
early
1
early (**R C**
R
early
1
early **F**)
Z =

v
éq 9.1-3
0

T
- 1
T

R C R
early
2
early (**R C**

R
early
2
early **m**)
C and **C**
1
2 being two submatrices of **C**:
C1 = **DIAG** [*EA*, 2
GA,
3
GA],
C2 = **DIAG** [1
GI, *EI2*, *EI3*].
T **E**
matrix is called of

rigiditegeometric;
éq 9.1-4
BT
C *T*
B
matrix of material rigidity is called.
éq 9.1-5
Lastly, during the differentiation a matrix appears which does not appear in [bib2]:
BT Z
that we call complementary matrix.
éq 9.1-6
9.2 Matrices
of inertia
They result from the differentiation of Fréchet de *Winer* [éq 7-4] in the direction. More precisely, one places oneself in the configuration of *N* - ième iteration of the moment *I* and one differentiates
N + 1
in direction *I*
.
9.2.1 Differentiation of the inertia of translation *A*! **xo**
One has immediately, according to the equations [éq An4-4] on the one hand and [éq An3.1-4] on the other hand:
(
N + 1
N + 1
With
D

With N
N
! **X**
+ 1
, *I*). **X**,
= *A*
I
! **X**, *I*
=
X
O
O
O
2
O, *I*
éq 9.2.1-1
T

9.2.2 Differentiation of the inertia of rotation I! + I

According to the equation [éq 7-3]:
I
= **R J RT**

,
J being a constant tensor.
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9.2.2.1 Terms coming from the differentiation of I

According to the equations [éq A4-3] and [éq A4-4], these terms are:

N
- (
N
!) + !" - "( ) +

+
I
I
I
“**I** 1,

éq 9.2.2.1 - 1

"
I
I

v
v
N
- (
N
!) + !" - "( ) +

+
I
I
I
“**I** 1 all sizes appearing in the hook

"
I
I

being taken with iteration *N* of moment *I*.

9.2.2.2 Terms coming from the differentiation from and!

According to the expressions [éq A3.2-3] and [éq A3.2-4], speed and angular acceleration with iteration *N*
moment *I* are:

N
N
N - 1, *T*
N - 1
N
T
N
N - 1
I
= **IH IH**
I
+
IH IH - 1 - 1, -

T
(*I I i-1*, *I*)
1
N
N
N - 1, *T*
N - 1
N
T
N
N - 1
! *I* = **IH IH**
! *I*
+
R

.
2
I **IH** - 1 (*I* - 1, *I* - *I* - 1, *I*)
T
A variation of vector-rotation can affect only the sizes relating to this iteration *N*
moment *I*, since, in the two preceding relations, the other sizes are fixed.
In other words, only are to be differentiated **R** *N*
N
I and *I* - 1, *I*, increment of vector-rotation of moment *I* - 1
with iteration *N* of moment *I*.
However [éq A4-5]:

N
^
+ 1
D N
N + 1
N
IH .i
=
R.
I
I
And, according to [bib3]:
D N
N + 1
N
N

+ 1
I - 1, *I*. *I*
=
(**Ti-1**, *I*) *I*,
with:

(

1
T
2
1
T
"
T) =
+
1 -
- .
2

2
2
tan

Terms coming from differentiation from and! are thus:

- **In**
N
N,
1 *T N* 1
N N
N
N
N
N,
1 *T N* 1
, *I* (
-
-
R R
I
I
! *I*) - “**I**
I
, *I* - (**I**, *I I*) (
-
-
R R

I
I
I)

1

+
N
N N
N
N

2 **I**, *I* +

T “**I**
I
, *I* - (**I**, *I I*)
éq 9.2.2.2 - 1
T

[

- *N T*
N
N

R R
1
N
T
(*N I*)
I
i-1 (*I*, 1i - *I*, 1i)
,

] +R **Rii- T1** *I*, 1
N + 1
combination of the three preceding matrices being to multiply by *I*

.

10 Implementation by finite elements

One gives below the representation in finite elements of the matrices of [§9]. These matrices appear in expressions to be integrated along the beam. One thus calculates them at the points of Gauss.

NR, *NR*

I

J… are the values taken, at the point of Gauss considered, by the relative functions of form with nodes *I*, *J*….

The matrices of rigidity connect the increase in displacement of the node *J* to the increase in force intern with node *I* for the element *E*.

10.1 Stamp interior deformation and efforts

The matrix of deformation has as a continuous expression [éq 6.3-5]:

D

'

1 xo

B = *ds*

.

D

0

1

ds

In finite elements, the contribution of the displacement of node *I* to the deformation to the point of Gauss considered is obtained by multiplying the 6 components of this displacement by the matrix:

1 *NR* **X**”

H

I

I “O (*G*)

Bi =

'

.

éq 10.1-1

0

NR **1**

I

The superscript *H* indicates than it is about the discretized shape of the matrix **B**.

According to [éq 6.3-4]:

Fe
=
BhT
int *I*

F
ds
éq 10.1-2
E
I **m**
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is the interior effort applied to node *I* of the element *E* and which had with the stress field generalized **F** in the element. This constraint is calculated according to the equation [éq 8.1-2]. But it is necessary to notice **m**

that:
· on the one hand [éq 6.3-3]:
RT
RT
=
X! **- E**
early
early
O
;

1
· in addition, the vector **is updated** at each iteration, as it is indicated to [§10.8].

10.2 Matrices of rigidity

The expression continues matrix of material rigidity [éq 9.1-5] is:
BT
C *T*
B.
One deduces some, for the finite element *E*, the matrix connecting the increase in displacement of the node *J* to
the increase in force interns with node *I*:

=
BhT C *T*
Bh
chechmate *I J*
I
J ds.
éq 10.2-1
E
One calculates numerically **C** *T* at the point of current Gauss.
The expression continues geometrical matrix of rigidity [éq 9.1-4] is:
T **E**,
where:
D

1
0
ds

D
= **0**
1,

ds
0
1

and **E** is given by the equation [éq 9.1-2].
One deduces some, as for the matrix of material rigidity:

HT
H

géom *I J*
= I **E** *J ds*,
éq 10.2-2
E
where:
1
0
I

'

hi =
0
NR

I **1**,
0

1
I
and where **E** is calculated numerically at the point of current Gauss.
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The expression continues complementary matrix of rigidity [éq 9.1-5] is:
BT Z,
where **Z** is given by the equation [éq 9.1-3].
One deduces some:

=

BhT
compl *I J*
I NR J **Z** *ds,*
E
where **Z** is calculated numerically at the point of current Gauss.

10.3 Forces of inertia

According to [éq 7.3-1]:

NR With **X**

! O

Fe
I
iner *I*
= -
ds
éq 10.3-1
E Ni

(**I**! + **I**)
is the inertia of the element *E* to node *I*.

10.4 Stamp of inertia

Let us pose:
To **1 0**
I

Me
I J =
NR NR **I**
O iner =
ds
0
J
and
O iner

O iner
.

I

J

E
1
It is seen, according to [éq 7-4] and [éq 9.2.1-1], that
E
M
2
T
O iner is well the matrix of inertia of the element *E*
for the translatory movement (diagonal submatrix *A* **1** of **Io** iner). But according to
[éq 9.2.2.1 - 1] and [éq 9.2.2.2 - 1], **it is not the** matrix of inertia of rotation.
Nevertheless, the matrix **M**
E
O iner, assembly of **Mo** iner, is used to calculate the **initial acceleration** of
beam when it leaves its position of reference with a **null initial speed** (O = **0**). Indeed,
one has then, according to [éq 7.3-1]:
X!
F
O
ext. (*T* =)
0
= **- Finer** (*T* =)
0
= **Mo** iner,
! O
since, in position of reference:
I
= **J**

.
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Let us call **J the** sum of the two matrices [éq 9.2.2.1 - 1] and [éq 9.2.2.2 - 1] and pose:
0 0
I
=
and
Me *I J = NR NR* **I**
belch

dS
0
'

J
belch
I
J belch

E
Also let us pose:
0 0
I
=

and
Me
I J =
NR NR **I**
O belch
ds
0 J
O belch
I J orot

E
The matrix of complete inertia of an element of beam is obviously:
1
1

Me
=
Me
-
Me
+ **Me**
iner
2
O iner
2
O belch
belch.
T
T

10.5 External forces data

According to [éq 8.1-3]:
NR

Fe
I
ext. *I*
=
F

ds
éq 10.5-1
E NR
I **m**
is the force applied to node *I* of the element *E* which is equivalent to the external forces distributed.

10.6 Linear system of iteration

By discretization in finite elements, the equation [éq 9-1] gives:
W H = (**F - F**
- F
int
).
iner
ext.
.
In addition, by supposing that the external forces are conservative, one has, according to [§10.2] and [§10.4] :

1
1

D W h. = **S**
+ **S**
+ **S**
+ **M** -
M
+
M
chechmate
geom
compl
belch
.
2
O belch
2
O iner.

T
T

The relation [éq 9-2], having to be checked by all, thus leads, with iteration *N* of moment *I*, with *N* + 1 linear system following out of *I*
:
[
1
1
xn +1
,

Sn
+ **Sn**
+ **Sn**
+ **mn**, -
M
+
M
O *I*
chechmate, *I*
geom, *I*
compl, *I*
belch *I*

2
O *belch*
2
O iner
N + 1
T
T

I **éq 10.6-1**
= **F**
- Fnint, + **Fn**
ext., *I*
I
iner, *I.*
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In *Code_Aster*, the elementary matrices **Meo** iner, which are independent of displacement, are assemblies only once to constitute the total matrix **Mo** iner, which is in particular used to calculate initial acceleration [§10.4].
Three matrices of elementary rigidity,
and
E
chechmate
geom
compl, the matrix of inertia of **Mrot** rotation,
who depend all the four on displacement, and inertia of corrective rotation stamps it
1
-

E
M
2
T
O belch, which is invariable, with each iteration, combined then are assembled for
~
to constitute a pseudo matrix of total rigidity **kN**.
I
The linear system [éq 10.6-1] thus becomes:
N

+ 1
~
1
X

N
O,

K

+
M
I
2
O iner
= **F**
- Fn + **Fn**
.
I
éq 10.6-2
N + 1
ext., *I*
int, *I*
iner, *I*

T

I
In the case of a static problem, the preceding system is simplified in:
[**xn+1**
KN]
O, *I*

= **F**
- Fn
I
,
éq 10.6-3
N 1
ext., *I*
int, *I*

+
I

where **Kni** is the assembly of the only matrices of rigidity to iteration *N* of the "moment" *I* [§8.2]:
Sn
+ **Sn**
+ **Sn**
chechmate,
geom, *I*
compl.
I
I

10.7 Update of displacement, speed and acceleration

The treatment of the translatory movement is traditional; that of the rotational movement is done with assistance of the quaternions [A8].

10.7.1 Translatory movement

One applies the formulas [éq A3.1-3] and [éq A3.1-4].

10.7.2 Rotational movement

The sizes to be updated are:
· on the one hand, vector-rotation, speed and angular acceleration;
· in addition, for later calculations, the matrix of rotation and the increment of vector-rotation moment *I* - 1 with the current iteration of moment *I* [§A6].

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The update of vector-rotations rests on the following property of the quaternions [§A8.4]: “it quaternion of the product of two rotations is equal to the product of the quaternions of rotations components ".
Thus while posing [§An8.6]:
(*X*
+ 1
O, **X**) =
Q (*nor*)
(*xo*, **X**) = **Q** (*N*
I),
it comes:
N + 1
- 1
I
= **Q** ([*X*
O, **X**) (*xo*, **X**
$
)].
éq 10.7.2-1
In addition, according to the equation [éq An6-2], if:
(*there* **y**) = **Q** (*N*
O,
I -1 *I*),
,
then:
N + 1
- 1
I 1, *I*
= **Q** ([*X*
,
O **X**) O (*y*
-
, **y**
O
)].
éq 10.7.2-2

The update of the matrix of rotation is immediate [§4.2]:
RN + 1
N + 1
I
=
(
exp “*I*
),
who is calculated according to [éq 4.2-1].
Finally speed and the angular acceleration are updated by the relations [éq A3.2-3] and [éq A3.2-4].

10.8 Update of the vector variation of curve

The vector, which defines the deformation of rotation [§6.2], should be calculated only at the points of Gauss.
In *Code_Aster*, it is treated by means of computer like a “internal variable”.
According to [éq 6.2-2]:
D
“*N*
N
N T
I
=
(**IH**). **R.**
I
ds
And, according to [éq 4.2-6]:

1
1
1
^ *N* +

^ *N* +
+
D

“*N*
=
exp
N
N T

R R exp-
.

I
I
I
I
I
ds

That is to say:

1
1
1
^ *N* +

^ *N* +
+
D

“*N*
=
exp

exp

I
-
I
I
ds

^ *N* + 1

^ *N* + 1
+ exp
“*N* exp-
.
I
I
I

In the preceding equation, the matrix of the first member is antisymmetric by construction; second matrix of the second member is obviously antisymmetric; thus the first matrix of second member is too.
One shows in [bib2] Appendix B, that the axial vector of this last matrix is:
N + 1

N + 1
N + 1
N +
'
1
N +
sin
1
I
'
sin

.
I

N
I
I
1

=
+

I
1
N + 1
(*I*)

(
)
+ -

N + 1

N + 1
N +

1
I

I

I
I

2
1
N + 1
1 sin
I

'
+
2

N +
1
N 1
I

+

.
2
1
N + 1
(*I*)

I
2

Thus:
N
^
+ 1
N
^
+

1
1

N +
N
AXIAL exp
"
exp

I
=
+

-
.
I

I
I

10.9 Initialization before the iterations

In the dynamic case, if the loading is constant in time, the iterations cannot
to start at the moment *I* which if one initializes some of the fields of displacement, speed and acceleration with
values different from those of moment *I* - 1. These initializations are done as follows.

10.9.1 Translatory movement

xo
= **X**
O,
O,
.
I
I - 1

Then, according to the equation [éq An3.1-1]:

1
2 - 1
!
xoo *I* = -
! **xo** *I* - 1 +
!
X
.
,
,
O, *I*
T
-

2
1

According to the equation [éq An3.1-2]:

! **xo**
=! **X**

O
I
O *I* - 1 + *T*
([1 -)! **xo** *I* -1 +! **X**
O
O *I*].
,
,
,
,
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10.9.2 Rotational movement

One takes expressions similar to the preceding ones:

oi = *I* -1**,**

éq 10.9.2-1

1
2 - 1
! oi = -
I -1 +
! *I***,**

éq 10.9.2-2

T
-

2
1

O
O
I
= *I* -1 + *T*
([1 -)! *I* -1 +! *I*].

éq 10.9.2-3

The second members of [éq 10.9.2-2] and [éq 10.9.2-3] have a direction because all the vectors which appear in it
are in the tangent vector space with
(
SO)
3 in **IH** - 1.
Like consequences of the equation [éq 10.9.2-1]:
Ro = **R**

I
I - 1
and:
oi-1, *I* = .0
It is seen that at the first moment (*I* =)
1, initializations require the knowledge of acceleration
!
xo
initial, whose calculation is indicated to [§10.4].
! O
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11 diagrammatic Organization of a calculation

This paragraph shows how the concepts presented to [§10] in unfolding are articulated of one calculation.

11.1 Calculation

statics

Calculation of the external forces [§10.5]
ITER = 0

ITER = ITER + 1
A max. number of iterations
yes
calculation is stopped
reached
?
not
Calculation of the interior forces [§10.1]

Test of stop: the forces are they
yes
balanced, except for a tolerance?
end
not
Calculation of the matrices of rigidity [§10.2]
Resolution of the linear system with
corrections of displacements [éq 10.6-3]
Update of displacements [§10.7]
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11.2 Calculation
dynamics

Calculation of the matrix of inertia **M** O iner [§10.4]

Calculation of initial acceleration

Buckle on the steps of time

Calculation of the forces external at the moment running [§10.5]
Prediction of displacements, speeds and accelerations [§10.9]
ITER = 0

ITER = ITER + 1
A max. number of iterations
yes
calculation is stopped
reached
?
not
Calculation of the interior forces [§10.1]
Calculation of the inertias [§10.3]
Test of stop: the forces are they
yes
no next time
balanced, except for a tolerance?
not
~ *N*
Calculation of the pseudo matrix of rigidity **K** [§10.6]
I

1
Combination of and of
~
K *nor*

t2 **Mo** iner
Resolution of the linear system with
corrections of displacements [éq 10.6-2]

Update of displacements, speeds
and accelerations [§10.7]
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12 Use by *Code_Aster*

This paragraph indicates how intervene the beams in great displacements in orders of *Code_Aster*.

Order
Key word factor
Key word
Argument
AFFE_MODELE
AFFE
PHENOMENON
“MECHANICAL”
MODELING
“POU_D_T_GD”
AFFE_CARA_ELEM
BEAM
SECTION
“GENERAL”
'RIGHT-ANGLED
“CIRCLE”
STAT_NON_LINE
COMP_ELAS
RELATION
“ELAS_POUTRE_GD”
and
DEFORMATION
“GREEN”
DYNA_NON_LINE

13 Simulations numerical

One gives five digital simulations below resting on the formulation presented in this note. The three first relate to static problems, the two last on problems dynamic.

13.1 Embedded right beam subjected to one moment concentrated in end (case-test SSNL103)

That is to say **M** this moment. The beam is the seat only one constant moment and the equation [éq 6.4-2] shows
that the variation of curve **X** is also constant. The beam thus becomes deformed in circle of ray:

$$R = \frac{E I}{}$$

3 ,
M
in a plan perpendicular to the vector moment.
The figure [13.1-a] shows the deformations of a beam length unit, of which *E I3* = 2 and subjected at the moments,
2 and
4 .
The beam is cut out in 10 finite elements of the 1st order. One applies from the start the final moment and
convergence is reached in 3 iterations.
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M
=

M
= 4

M
= 2

Appear 13.1-a: Beam subjected to one moment in end
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13.2 Arc embedded-rotulé charged at the top

The figure [13.2-a] shows the deformations of an arc of 215° of opening, embedded on the right, rotulé with
left and subjected to an increasing force concentrated at the top. The solution of this problem is
data in [bib11] for an initial ray of 100 and following characteristics of beam:
EA = *GA* = 5 × 107
I.E.(internal excitation) = 106
and
.
The arc is modelled by 40 elements of the 1st order. One made grow the force up to 890 per eight increments of 100, an increment of 50 and four increments of 10. Beyond **buckling** appears,
i.e. displacement continues to grow under a force which decrease brutally. The algorithm
described here does not allow to take into account such a phenomenon, and diverges for a force from 900. Da
Deppo, in [bib11], locates the force criticizes to 897.
There is the table of comparative results according to:

Force	**Vertical displacement**	**Displacement**
point	**horizontal of the point**	
of application	**of application**	
Our calculations		
890	110.5	60.2
Da Deppo		
897	113.7	61.2
F = 400		
F = 700		

F = 890

Appear 13.2-a: Arc embedded-rotulé charged at the top (Da Deppo)

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13.3 Arc circular of 45° embedded and subjected in end to a force perpendicular in its plan

The problem is three-dimensional. It was proposed in [bib12]. The figure [13.3-a] shows three successive configurations of the beam of initial ray 100, square section and characteristics:

EA = 107
GA
6
5
,
1 = *GA2* = 5 × 10,
GI1 = EI2 = EI3 = 8 333
.
× 10 .
Z
50
P = 600 lb
40
With (15.9 47.2 53.4)
30
P = 300 lb
CONFIGURATION
FINALE
To (22.5 59.2 39.5
20

10
0
Y
10
20
30
40
50
60
70
10
CONFIGURATION
20
INITIAL
30
To (29.3 70.7 0
X

Appear 13.3-a: Three configurations of the beam (extracted from [bib12])

We modelled the beam by 8 elements of the 1st order. The force grows by increments of 20. One has following comparative results, for the co-ordinates of the point of application of the force, in configurations of the figure [13.3-a].

FORCE
X
Y
Z
300
Our calculations
22.3
58.9
40.1
BATHE-BOLOURCHI
22.5
59.2
39.5
600
Our calculations
15.7
47.3
53.4
BATHE-BOLOURCHI
15.9
47.2
53.4

13.4 Movement of an bracket

It is about a three-dimensional dynamic problem dealt with in [bib3].
An bracket consists of a post and a cross-piece length 10 [13.4-a] (A). The foot of
post is embedded and one applies to the connection a not-following force, perpendicular to the plan of bracket at rest [13.4-a] (b).

10
F
50
F
(A)
(b)
10
Z
T
0
1
2
y
X

Appear 13.4-a: Bracket subjected to a dynamic force perpendicular in its plan

The characteristics of the elements provided by [bib3] are as follows:
EA = GA
6
1
= *GA2* = 10,

GI
3
1
= EI2 = EI3 = 10,
To = 1,
I2 = I3 = 10; *I1* = 20.
It is noticed that these data do not make it possible to identify a material and a section of beam, because one has at the same time:
I.E.(internal excitation)
E
EA
E
2
2
6
= 10
=
and
= 10
.

=
I

With
2

One can thus deal with this problem only by imposing, by program, a characteristic: one chose to impose product *EA.*
The post and the cross-piece are modelled each one by 4 elements of the 1st order and the duration of the analysis
comprise 120 equal steps of 0.25.
The figures [13.4-b] and [13.4-c] give the evolution of the component according to X of displacement respectively for the connection and the end of the cross-piece. In cartridge, one reproduced them corresponding curves data in [bib3] for two modelings.
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100
80
displacement
Displacement
60
40
6.0
20
00
5.0
20
40
With
60
B
4.0
80
C
100
D
0.0 30 60 90.120.150 180.210.240 270 300
3.0
Time
2.0
1.0
0.0
1.0
2.0
3.0
4.0
2.0
4.0.6.0.8.0 10.0 12.0 14.0 16.0 18.0 20.0 22.0 24.0 26.0 28.0 30.0 32.0 times

Appear 13.4-b: Bracket of SIMO - Displacement of the connection perpendicular to the initial plan

100

80
Displacement
60
40
20
displacement
00
20
10.00
40
With
60
8.75
B
80
C
7.50
100
D
0.0 30 60 90.120.150 180.210.240 270 300
6.25
Time
5.00
3.75
2.50
1.25
0.00
-1.25
-2.50
-3.75
-5.00
-6.25
-7.50
-8.75
-10.00
2.0.4.0.6.0 8.0 10.0 12.0 14.0 16.0 18.0 20.0 22.0 24.0 26.0 28.0 30.0 32.0 times

Appear 13.4-c: Bracket of SIMO - Displacement of the end perpendicular to the initial plan

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13.5 Setting in rotation of an arm of robot

An arm of robot *OA* is put moving in the plan **E E**
1 2 by a rotation (*T*) imposed on sound
center 0 [13.5-a]. One wants to calculate the displacement of end *A* in a system of axes **e' e'**
1 2
involved in rotation (*T*).

E 2
O
With
E 1
(A)
(*T*)
10
10 elements of beam of the 1st order
135.0
108.0
81.0
(radians)
(b)
)
(
T
54.0

27.0
angle
0.00.0 6.0 12.0 18.0 24.0 30.0
Time

Appear 13.5-a: Arm of robot subjected to an imposed rotation

Characteristics of material:
EA = 2 8

, × 107,
GA
7
2
= 1 × 10 ,
I.E.(internal excitation) = 1 4
, × 104,
To = 1 2
,
I = 6 ×
-
10 4
The step of time evolves/moves of 0,05 at the beginning of the analysis with 0,001 with the end.
The figures [13.5-b] and [13.5-c] give the evolution of following displacement e'1 and according to perpendicular direction. One reproduced, in cartridge, the corresponding curve of [bib3]. When number of revolutions becomes constant, the arm undergoes a permanent lengthening due to the centrifugal force
and it is subjected to an oscillation of inflection of low amplitude.
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0.00125
0.00000
0.00125
0.00250
0.00375
0.00500
0.00625
0.0040
0.00750
0.0000
0.00875
-0.0008
5.14 X 10
-4
0.01000
(L, I)
~
2
-0.0056
0.01125
0.01250
-0.0104
0.01375
-0.0152
0.01500
-0.0200
0.01625
Tip Displacement U
0.0
6.0

12.0
18.0
24.0
30.0
0.01750
Time
0.01875
0.02000
1.0.2.0.3.0 4.0.5.0.6.0 7.0.8.0.9.0 10.011.012.013 .014.015.016.017.018.0
Appear 13.5-b: Following displacement e'1
0.04
0.00
- 0.04
- 0.08
- 0.12
- 0.16
- 0.20
0.10
- 0.24
0.00
- 0.28
0.06
(L, I) 2
- 0.32
~

-0.22
- 0.36
- 0.40
-0.38
- 0.44
-0.54
- 0.48
Tip Displacement U
- 0.52
-0.70
0.0
6.0
12.0
18.0
24.0
30.0
- 0.56

Time
- 0.60
1.0.2.0.3.0 4.0.5.0.6.0 7.0.8.0.9.0 10.011.012.013 .014.015.016.017.018.0
Appear 13.5-c: Following displacement e'2
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Appendix

1 Some definitions and results concerning antisymmetric matrices of order 3

Any vector **U** of order 3 and components *U, U, U*

X
y
Z one can associate the antisymmetric matrix “
U of order
3 following:
0
- U
U

Z
y
“**U** = *U*
0
- U
Z
*X***.**

éq A1-1

- U
U
0
y
X

Conversely, any antisymmetric matrix of order 3 can be written in the form [A1-1] and him can thus to associate a vector **U**. This vector is called the **axial vector** of the matrix.
One sees without difficulty that:
“**U U** = **0,**

éq A1-2
in other words the axial vector is clean vector of antisymmetric associated for the eigenvalue 0.
· Whatever the vector **v**:
“**U v** = **U v**;
éq A1-3
“**U** “**v** = **v C** - (**U. v**) **1**.
éq A1-4
· If
U is unit:
“**u2** = **U C - 1** (matrix symétri)
that;
“**u3** = **-** “**U**
(antisymmetric matrix).

From where:
p - 1
“**u2p** = (-)
“**u2**
1
(matrix symétri)
that;
éq A1-5
p - 1
“**u2p** - 1 = (-)
1
“**U** (antisymmetric matrix).
éq A1-6
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Appendix 2 Treatment of the forces of damping

These forces are not taken into account currently in *Code_Aster*.

A2.1 Assumptions and end cells in the center of the section

The assumption is made that the element of volume *FD* bordering a point *M* interior with the beam is subjected to one
force damping being composed of two parts:
· for the translatory movement of the section of X-coordinate *S to* which **M** belongs:
D **F**
= **- X**,
;
1
1! O (*S T*) *FD*
· for the rotational movement velocity angular around the center *P* of the section:

D **F**
= - ,
.
2
2
(*S T*) *P M FD*
Integrated on the volume of beam length *ds*, these forces admit for end cells out of *P*:
· the force:
D **F** = - *A*
1
! **xo** (*S, T*) *ds*;
· moment:

D
2
m
= - *ds*
P
2
M

(*S, T*) *P M D*
I (*S, T*) *ds*.
section

= -
A2.2 elementary Forces of damping
The virtual work of the forces of damping is:
With
1
! **X**
S
O
X
2

W

O
= - **2**
.
ds
amor
.
S
I
1

If damping in account is taken, *Wamor* must be cut off with the second member from the equation [éq 9-1].
It results from this, as with [§10.3] for the inertias, that the contribution of the element *E* to the force of damping to node *I* is:
NR With **X**
I 1
! O

Fe

amor *I*
= -
2
ds.
E NR
I
I

If damping in account is taken, these forces must be added to the second member of the equation [éq 10.6-1].
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A2.3 Matrices of damping

According to the relations [éq A4-3] and [éq A4-7] giving the differentials of Fréchet of! **xo** and of in the direction
, and given that ([éq A4-5] and [éq A4-6]):
(

DI.) = - (**I**) + **I** “,

To **1**
0
1

0
0

S
S
D W
.
2
2
amor
= -
.

2
! *ds*

2

1 **0**
I
.

.

- *S*
0

-
1

(**I**) *ds*
S

One deduces the matrices from them from damping of the element *E*:
· except for the sign, relation between the increase speed to the node *J* and the increase in force of damping to node *I*:
To **1**
1
0
It

amor *I J*
=
NR
I NR J
2
ds;
E
0

I

· except for the sign, relation between the increase in displacement to the node *J* and the increase in force damping with node *I*:
0
0

2

amor *I J*
=
NR
I NR J
ds.
E
0 -

(**I**)

In the hook of the first member of the equation [éq 10.6.1], it is necessary to add:

.
stamp

C
T amor;
·
stamp
Samor.
The matrix **It**
E
amor is symmetrical, but the **Samor** matrix is antisymmetric.
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Appendix 3 Algorithm of Newmark in great rotations

Traditional A3.1 Diagram of Newmark in translation

The state of the locus of centres (displacement, speed and acceleration) is supposed to be known at moment *I* - 1.

The algorithm of Newmark [bib10] and [bib13], rests on the following developments of the displacement and of

speed, with the iteration *N* + 1 of moment *I*:

2
+ 1
T

xn
= **X**
1
1 +
T
X
N +
, *I*
, *I* -
! , *I* - 1 +
(1 2
2
2 [-
) **X**! 1 + **X**
O
O
O
O, *I* -
! O, *I*]

éq A3.1-1

! **xn** + 1

$N + 1$
, I
=! **X**, I 1 + T
-
([1 -)! **X**, I -1 +! **X**
O
O
O
O, I]
éq A3.1-2
1
1
and are the parameters of Newmark which, in the case of the “rule of the trapezoid”, are worth and:
hooks
4
2
$N + 1$
in the equations [éq A3.1-1] and [éq A3.1-2] are then the arithmetic mean of! **xo** I - 1 and of! **X**
.
,
O, I
Récrivant each one of these relations to iteration N of moment I and cutting off member with member from the relations
the preceding ones, it comes:
1

!
xn + 1
N
$N + 1$
N
$N + 1$
N
$N + 1$
N
I
=! **X** I +
2 (**X**
- **X**
I
I)
and
! **X** I

=! **X** *I* +

(**X**
- **X**
O
O
O
O
O
O
O *I*
O *I*).
,
,
,
,
,
,
,
,
T
T
If one poses:
xn + 1 = **xn** + **xn** + 1
O,
O, *I*
O,
,
I
I
éq A3.1-3
one thus has:
1

!
xn + 1
N
N + 1
N + 1
N
N + 1
, *I*
=! **X**, +

X
I
and

2
, *I*
! **X**, *I*
=! **X**, +
X
O
O
O
O
O *I*
O, *I*
éq A3.1-4
T

T

A3.2 Diagram of Newmark adapted to great rotations
To unify calculations, one would like to be able to write, in rotation, the following similar relations:

1
N + 1
N
N + 1
N
N + 1
N
N + 1
N
I
= *I* +
1, -
1,
and
!
= ! +
1, -
,
T
(*I* - *I I* - *I*)
I

I
2 (I - I
I -1, I)
T

N + 1
where *nor* 1, I and I - 1, I are respectively the vectors increment of rotation of the section considered, enters
moment I - 1 and iterations N and N + 1 of moment I [A6].
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But these relations are not exact because one cannot combine a speed (or an acceleration) angular with the iteration (N + 1) of moment I with the homologous size with iteration N and of vector-rotation
counted starting from the configuration at moment I - 1 [A7]. One can combine only after having brought by
rotation (one says “transported”) all sizes in the same configuration. One chooses to simplify configuration of reference.
Thus let us pose:
N + 1
N + 1, T
N + 1
N
N, T
N
I
= **IH**
I

I = **IH** *I*
éq A3.2-1
Year + 1
N + 1, *T*
N +
=
1
R
!
Year
N, *T*
N
I
I
I
I
= **IH**
! *I*
éq A3.2-2
N + 1
T
N + 1
N
T
N
I - 1, *I*
= **IH** -1 *I* -1, *I*
I -1, *I* = **IH** -1 *I* -1, *I*
The algorithm of Newmark in rotation results in the following relations:
· for the operator of rotation, [éq A6-1]:
N
^
+ 1
1

RN +
= exp
RN
I
;
I
I

· for the increment of vector-rotation since moment *I* - 1, [éq A6-2]:
(
1
1
^ *N* +
exp “*N* +
N

I - 1 *I*) = exp
E
I
(xp “*I*, 1i);
,

· for the angular velocity:

N + 1
N
N + 1
N
I
= *I* +
1, -
;

T (*I* - *I*
I - 1, *I*)
· for the angular acceleration:
1
Year + 1 = **Year**
N + 1
N
I
I +
,

.
2 (*I* - 1 *I* - *I* - 1, *I*)
T

N +
1
The two preceding relations give, by opposite transport on the configuration

and taking into account
I
relations [éq A3.2-1] and [éq A3.2-2]:

N + 1
N + 1
N, *T*
N
N + 1
T
N + 1
N
I
= **IH**
IH
I +
IH
IH 1
1, -

éq A3.2-3
T
-

(*I I i-1*, *I*)
1
! *N* + 1
N + 1
N,

T
N
N + 1
T
N + 1
N
I
= **IH**

IH
! *I* +
R
R
2
I
I - 1 (*I* -, 1 *I* - *I* -, 1 *I*).
éq A3.2-4
T

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Appendix 4 Calculation of the differentials of Fréchet

One will have to use thereafter the identity following, known as *hook of Dregs*, whose demonstration is immediate. If
"**A** and "**B** are two antisymmetric matrices of the 3rd order, of axial vectors **A** and **B**, then one has, for all vector **v**:
("**A** "**B** - "**B** "**A**) **v** = (**A**) **B v.**

éq A4-1
In other words: is the axial vector of the antisymmetric matrix "
With "**B** - "**B** "**A.**
One calls differential of Fréchet of the function **F** in direction **X**, the quantity:
D
D **F** (**X**). **X**

= grad **F**. **X**
= lim
F (**X** + **X**

).
0 *D*
It is the principal part of the increase in **F** corresponding to increase **X** in variable **X**.
Here the catalogue of the differentials of Fréchet intervening in this note.
D
D **x' X**
'
.
O
= lim
(**X** + **X**
O
O) '
= (**xo**) '= **X**
O
.

éq A4-2
0 *D*
O

D! **X**. **X**
=! **X**
O
O
O**.**

éq A4-3
D! **X**. **X**
=! **X**
O
O
O**.**

éq A4-4
D
^
^

Dr. =
lim
exp

R = **R**
éq A4-5
0 *D*

D
^
^

DRT =
RT

-
RT
.
lim
exp
= -

éq A4-6
0 *D*

because, according to the equation [éq 4.2-6]:
^
T
^

exp

exp

=
- .

“=! **R RT**.
However:
D

.
^
^

D! .
R =
lim
exp

R =! **R** +! **R.**
0 *D*

Therefore, according to [éq A4-6]:
^
^
^
D = (*DR.*) *T*
R + (
R *D T*
" .
! .
!

R.) =! + "-".
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Maybe, by using the **hook of Dregs** [éq A4-1]:
D. =! **-.**

éq A4-7
Appendix 5 Complements on the calculation of the matrices of rigidity
This appendix develops calculations of [§9.1].
A5.1 Stamps geometrical rigidity
[
F

F
{

D **B**}
.]. + {**B**}
. [*D.*] =
M
M

(
^
^
X
'
'
'

O). **F** + (**xo** + **xo**). **F** + '. **Mr.**
By rearranging the terms and by using the vectorial identity:
a. (**B c)** = **C.** (**b) has**,
the second preceding member is written:
- x'. **-** “. +. **X**”
F
m
F
O +. (**x'**
O
" "
O **F**) =

. **E**.
A5.2 Matrices of material and complementary rigidity
As it is shown with [§9.1]:
F

^

0

D
T
T

. = **C B -**

C
^ .

M

0

However:

-
T
F.
= **C** 1

m
Consequently:

{
F

B}
. *D.*
{**B**}. **C** *T* **B**

=
éq A5.2-1

M

^

T
0
1
T **F**
- {B}

C
^ **C**
.
.
0

m
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The term:
{**B**}. **CT B**
conduit immediately with the **matrix of material rigidity**.
In addition:
^

-

1
T

0
F
R
C
R

-
F
1
T
(early 1 early)
^
C = -
.

m
-

1
0

(
T

R C R
early
2
m
early
)
The second term of the equation [éq A5.2-1] is thus written:

0 R
C RT
1
T
early
1
early

(**R C R**
early
1
early **F**)
B.

.
0 R
C RT
1
T

early
2
early

(**R C R**
early
2
early **m**)
By indicating by **Z the** matrix between hooks of the expression which precedes, **BT Z is the matrix of rigidity**
complementary.
Appendix 6 Principle of the iterative calculation of rotations
This appendix visualizes the operation of exponentiation, defines the vectors increment of rotation which intervene
in appendix 3 and the relation between them gives.
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---

1 + ^
N
(*i-1*, *I*) **IH-1**
IH -1
Rni
n+1
(
) **RN**
RN +1
1 + ^
I
I
I
1 + ^
n+1
(*i-1*, *I*) **IH-1**
SO3
( )

Appear A6-a: Representation of the operators intervening in the rotation of the sections.

: symbol of exponential projection

The curved surface of the figure [. A6-a] represents the unit
(
SO)
3 of the operators of rotation **R.** One has
appeared tangent spaces in
(
SO)
3 in **R**
N

I - 1, rotation calculated at moment *I* - 1, and in **IH**, *N* - ième iteration
in the calculation of rotation at moment *I*.
That is to say
N
I - 1 the vector rotation corresponding to **IH** - 1. There is a vector increment of rotation *I* - 1, *I* such that:
RN = exp (*N*
"*I* -1 *I*) **R**
I
I -.
,
1
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If the equilibrium equation of the beam is not satisfied, at moment *I*, by **Rni**, one seeks a correction
N + 1
N
N + 1
I
of *I* - 1, *I*. *I*
is obtained by **linearization**, while replacing in the equilibrium equation, according to equivalence [éq 4.2-5]:
^ *N* + 1

^ *N* + 1
exp
N
N

R
by
1

I
+
R.
I
I
I

The second of the preceding expressions is not an operator of rotation, but is in space
+1
N + 1
tangent with
(
SO)
N
3 in **R**
N
I [A6-a]. Having calculated *I*
, **IH**
results some while projecting by exp on
(
SO)
3 :
N
^
+ 1
1

RN +
= exp
RN
I
.
éq A6-1
I
I

N + 1

N + 1
The increment of swing angle *I* - 1, *I* making pass from **R**
R
I - 1 with
I
is such as:
RN + 1 = exp (*N* +1
"*I* -1 *I*) **R**
I
I - 1.
,
Vector-rotation not being additive, **one does not have**:
N + 1
N + 1
N
I 1, *I*
= *I*
+ *I* -1, *I*,
-
but one a:
(
1
1
^ *N* +
exp "*N* +
N

I -,
1 *I*) = exp
.

éq A6-2
I
(
exp "*I* -, 1 *I*)

N + 1
With [§10.7.2], one solves the preceding equation compared to *I* - 1, *I* by using the properties of the quaternions.
N + 1
Increments of swing angle *nor* 1, *I* and *I* - 1, *I* are used to calculate the corrections speed and
N N +

1
of acceleration of with

.
I I
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Appendix 7 Need for transport in a space of reference
for the vectorial operations relative to
rotational movement

The sizes kinematics, speeds and accelerations, in a given configuration are in
tangent vector space with
(
SO)
3 at the point defined by the rotation of the configuration compared to the position of
reference. One can combine the sizes relating to two distinct configurations only after having them transported in the same vector space taken for reference. The example which follows will make include/ understand this
need.

Let us examine the angular velocity of a gyroscope at three moments *T*, *T* and *T*
1 2
3 [A7-a]. Let us suppose that one passes from
configuration 1 with configuration 2 by the rotation of angle - around **e1**, and configuration 1 with configuration 3 by the rotation of angle -/2 around the same axis. In 1, the angular velocity is carried by the axis of the gyro and has as components (0, 0,) in the general axes **E E E**
1 2
3. In 2, the angular velocity is
also carried by the axis and has as general components (0, 0, 3). One wants to determine the components

general speed in 3, knowing that this speed is the average speeds into 1 and 2.
That the angular velocity in 3 is the average angular velocities into 1 and 2 does not mean that it is average in the general axes, but in the axes related to the gyro. In the example, since the gyro turns around its axis, in 1 with speed, 2 with speed 3, then in 3 it turns around this axis with speed 2. Therefore, in 3, the general components angular velocity are (0, 2, 0).
Taking into account [§5], one obtains the preceding result in “transporting” the angular Flight Path Vector of
configuration 2 on configuration 1, taken for reference, i.e. while making turn this vector of the angle around **e1**. Its general components are then (0, 0, 3). One makes the average of this vector and the vector angular velocity in 1, to obtain the vector of components (0, 0, 2). One “transports” finally this last vector on configuration 3 by making it turn of -/2 around **e1**, and one leads well to the vector angular velocity of general components (0, 2, 0).
0

1 0

1
e3
3

O
0
E
3 2
2
0

e1
2

0
2 0

-

3
Appear A7-a: Evolution angular velocity of a gyroscope
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Appendix

8 Use of the quaternions in modeling of great rotations [bib14] [bib15]

A8.1 Definitions

A quaternion, noted (***Q***) or (*xo*, **X**), is defined by the whole of a scalar *xo* and a vector in three dimensions **X** (*X X X*
1
2
3) :
(*X*, **X**) = *X* + *x1* + *x2* + *X*
O
O
3 , ,
, and being three numbers satisfying the relations:
2
2
=
= 2 = - 1;
= - = ; = - = ; = - = .

Combined quaternion

Combined, noted *, from a quaternion is obtained by changing the sign of the vectorial part:
(X, **X**) * = (X
O
O, - **X**).

Purely vectorial quaternion

A quaternion is known as purely vectorial when its scalar part is null and that it is thus of the form: (0, **X**).
A quaternion (***v***) is purely vectorial if and only if:
(***v***) + (***v***) * = (***0***).

A8.2 Elements of algebra of the quaternions

Multiplication

By applying the definition, one shows immediately that the multiplication, noted $, of two quaternions is:
(*X*, **X**) (*y*, **y**)
(*X* y **x.y**, *X* **there** *y*
O
O
O
O
O
O **X**
X y
$
=
-
+
+).
It is checked that this multiplication is:
· associative
:
([*X*, **X**) + (*y*, **y**)]$ (*Z*, **Z**) = (*X*, **X**) $ (*Z*, **Z**) + (*y*, **y**) $ (*Z*
O
O
O
O
O
O
O, **Z**);
· noncommutative bus:
(*X*, **X**) (*y*, **y**) - (*y*, **y**) (*X*
O
O
O
O, **X**) = (0, 2 **X y**
$
$
).

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Normalizes of a quaternion, noted. :
(
2
X, **X**)
= (*X*, **X**) $ (*X*, **X**) * = *x2* + *x2* + *x2* + *x2*
O
O
O
0
1
2
3 .

A quaternion is **unit** if its standard is equal to the unit. It is checked that:
([*Q*) $ (*Q*)]* = (*Q*) *$ (*Q*
1
2
2
1)*.

A8.3 Representation of a rotation by a quaternion

If (***v1***) is a purely vectorial quaternion and (***U***) a unit quaternion, then the quaternion (***v2***) such as:
(***v***) = (***U***) $ (***v***) $ (***U*** *
2
1
)

éq A8.3-1

is purely vectorial and has even standard that (***v1***). Indeed:
(***v***
*
2) + (***v2***) * =
(***U***) $ (***v1***) $ (***U***) * + (***U***) $ (***v1***) $ (***U***) *
= (***U***) $ ([***v1***) + (***v1***)] *$ (***U***) * = (***0***);
(

2
v2) $ (***v2***) * = (***U***) $ (***v1***) $ (***U***) *$ (***U***) $ (***v1***) *$ (***U***) * = (***v1***).
If one poses:
(***v***) = (0, **X**); (***v***
1
2) = (0, **y**),
it is seen that, by the relation [éq A8.3-1], the unit quaternion (***U***) defines an orthogonal transformation of vector **X** on vector **Y.** This transformation is the rotation whose matrix is defined by [éq A8.5-1].
Opposite rotation is defined by the quaternion (***U***) *, because:
(*U*) * (*v2*) (*U*) = (*U*) * (*U*) (*v1*) (*U*) * (*U*) = (*v1*).
$
$
$
$
$
$
A8.4 successive Rotations
Let us subject the purely vectorial quaternion (***v1***) two successive rotations defined by the quaternions unit (***U***) and (***U***
1
2) :
(***v***) = (***U***) $ (***v***) $ (***U***
2
1
1
1)*
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and:
(***v***) = (***U***) $ (***v***) $ (***U***) * = (***U***) $ (***U***) $ (***v***) $ ([***U***) $ (***U***
3
2
2
2
2
1
1
2
1)]*
éq A8.4-1
One sees on the last member of [éq A8.4-1] that the **unit quaternion defining the product of two rotations is equal to the product of the quaternions of these rotations**.
A8.5 matric Expressions
That is to say (***Q***) a quaternion:
(*Q*) = (*xo*, **X**).
Let us pose:
(
xo
Q) =,
X
(***Q***) is the formed vector of the four components of the quaternion.
Let us define two matrices built on (***Q***):
xo
- xT
xo
- xT
With (=

and
=
.
Q)
(
B *Q*)
X *X* **1**

O
+ **X** "
X *X* **1**

O
- X

"
One checks without sorrow, according to the rule of multiplication, that:
(***q1***) $ (***q2***) = **A** (
= **B**
.
1) (***q2***)
(2) (***Q***
Q
Q
1)
Now let us take the rotation defined by the unit quaternion (***U***) = (*eo*, **E**).
If:
(***v***) = (***U***) (***v***) (***U***
2
1
)*.
Then:
(***v2***) = **A** () (**BT**) (***v***
U
U
1).
In addition:
T
1 **0**
With
T
(***U***) (
B *U*) =
,
0 R

with:
R = **E and** + *e2* **1**
2
O
+ 2nd **E**
O "+ **E**",
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maybe, by using the relation [éq A1-4]:
R = (2 2
E -)
1 **1** + (
2nd **and**
O
+ *E* **E**
O “)**.**

éq A8.5-1
By bringing the preceding relation closer to the equation [éq 4.2-3], one sees that the components of the quaternion
unit defining a rotation than the parameters of Euler of this rotation are not other.
A8.6 Passage of a vector-rotation to the associated quaternion and vice versa
The operator **Q** who makes pass from a vector rotation to the associated quaternion is defined by the relations
[éq 4.2-2]:
1
eo =

cos

éq A8.6-1
2

1

E =

sin

éq A8.6-2
2

The Q-1 operator is less simple bus the reverse of a trigonomitric function does not have a single determination.
But one notices on [éq 10.7.2-1] and [éq 10.7.2-2] that **Q-1** is used to calculate vector-rotation *n+1* deduces from
vector-rotation *N* by the **correction**
N + 1. In general:

n+1 << *N*.
We thus adopted the following strategy: among all the determinations of the vector *n+1* one takes that to which the module is closest to:

n+1 + *N*.
In the plane case, this strategy is rigorous [§ 4.1], but in the three-dimensional case it is not to it parce-that:

n+1 N n+1
+

.
From [éq A8.6-1], one draws:
1
N 1
1
+ = ±
-
cos (*O*
E) + 2 *K*.
2
1
The adopted strategy leads to only one determination of
N 1
+ .
2
[éq A8.6-2] gives then:
1 *n+1*
n+1 = 2
2

.
E
1
sin
n+1

2

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Unilateral contact by conditions kinematics
Summary:
One describes in this document the numerical method used by defect to deal with the problems of contact unilateral in great displacements in operator STAT_NON_LINE. One uses conditions kinematics of not interpenetration which is dualisées. The formulation used is of main type slave (node-facet or nodal) with reactualization of the geometry during iterations, and the resolution of the problem of contact is
carried out by a method of active constraints within each iteration of the total method of Newton of operator STAT_NON_LINE.
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1 Introduction

The key word CONTACT of order AFFE_CHAR_MECA makes it possible to define conditions of contact
unilateral which is treated (in order STAT_NON_LINE) in initial or reactualized geometry, in a nodal formulation or node-facet, adapted better to the incompatible grids and to slip of surfaces one compared to the other. It will replace the key word in the long term LIAISON_UNIL_NO which is valid only for compatible grids undergoing of small slips.

One presents here an algorithm based on the method of the active constraints [bib2]. It is that which is used by defect and which corresponds to METHOD: “FORCED” of the key word CONTACT. Another algorithm is available under METHOD: “LAGRANGIAN”. It is similar to the precedent except for the

detail
that the connections are not activated there one by one (as we will see it), but by package. For more precise details, one will be able to refer to the document [R5.03.51].

1.1 General

Two solids are known as in contact when they “are touched” by part of their borders. To treat it unilateral contact consists in preventing that one of the solids “does not cross” the other: it is the principle of
not interpenetration of the matter, which results in relations of inequality between the variables kinematics (displacements). These relations are written in a discretized form: it is thus necessary to locate the entities between which one writes them (it is what is called pairing).
In *Code_Aster*, the use of the key word CONTACT makes it possible **to pair a** node with another node or
with a mesh: there is then a **potential** couple of contact, i.e. a couple for which one will write relations of nonpenetration. If the contact takes place really (the two nodes find with
even position, or the node is found on the mesh), one will say that the two entities **are associated** centre of an **effective** couple of contact.

Note:

The expression “to make a calculation with contact” wants to say that one writes such relations of not penetration, but does not imply that there is effective contact for the loading considered.

There are four ingredients in an algorithm of treatment of the unilateral contact:
· the location: definition of potential surfaces of contact (cf [§1.2]),
· pairing: determination of the potential couples of contact (cf [§2]),
· the relation of nonpenetration: direction of writing and coefficients (cf [§3]); the relation is written between the main node slave and one or more nodes, according to the formulation used,
· the resolution: one uses a method of active constraints here (cf [§4]); it is an algorithm iterative which determines, step by step, the list of the couples indeed in contact while examining geometrical conditions of contact and the sign of the associated multipliers of Lagrange,
by duality, in these conditions.

1.2

Zones and surfaces of contact

One considers the 3 solids of the figure [Figure 1.2-a], represented in 2D. 3 possible zones were defined of interpenetration enters the solids: a zone enters the solid A and the solid B, and two zones between solid B and the solid C. the user, who defines these zones in the command file, supposes here
that apart from these zones, there is no risk of interpenetration, taking into account the loading.

Solid A
surface 1
zone 3
surface 2
surface 2
zone 1
Solid B
surface 1
Solid C
surface 1
surface 2
zone 2

Appear 1.2-Error! Argument of unknown switch. **: Definition of 3 zones of contact**

Each zone of contact is defined in operator AFFE_CHAR_MECA by an event of the key word CONTACT. A zone is composed by definition of two surfaces which one seeks to prevent interpenetration: first is defined under key word GROUP_MA_1 (or MAILLE_1), the second under key word GROUP_MA_2 (or MAILLE_2), i.e. by the data of the meshs **of edge** which them constitute. These meshs are SEG2 or SEG3 for a grid 2D, TRIA3, TRIA6, QUAD4, QUAD8 or QUAD9 for a grid 3D.

Note:

The meshs of edge necessary to the contact will not be created by the code starting from the elements voluminal and must thus already exist in the file of grid.

It is imperative that the meshs of contact are defined so that the normal is **outgoing**: connectivity of the segments must be defined in order AB, that of the triangles in the order ABC, and that of the quadrangles in order ABCD, as indicated on the figure [Figure 1.2-b]. For one better reading of the drawing, one a little drew aside the mesh of edge being used here in contact with the “face” of the voluminal element 2D or 3D on which it is based.

Particular case: contact for a cable or a beam in 3D

It is possible in 3D to treat the contact between a mesh SEG2 or SEG3 (modelled cables some or

beam) and a surface. In this case, it is imperatively necessary to use the method of pairing "MAIT_ESCL" and to give the segments under key word GROUP_MA_2 (meshs slaves). section of the beam can then be taken into account by the use of key word DIST_2 (cf [§3.3]).

C
N
With
D
N
N
T
With
B
With
C
B
B
B
B
With
With
B
C
With
T
N
N

N
C
D
2D
3D
Appear 1.2-Error! Argument of unknown switch. **: Classification of the meshs of contact to have an outgoing normal**
Note:
One advises to use disjoined zones of contact, i.e. not having no node in commun run.
Chapter 2 details the method of pairing for the formulations node-facet and nodal:
the establishment of the potential couples of contact is made zone by zone. In chapter 3, one gives form relations of nonpenetration (inequations). The imposition of these conditions of nonpenetration is realized by an iterative method, called method of the active constraints, described in chapter 4: the resolution of the problem obtained is total, i.e. it takes into account them couples of all the zones simultaneously.
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2
Establishment of the couples of contact
2.1 Principle
Potential surfaces of contact were defined in operator AFFE_CHAR_MECA like specified in [§1.2]. The effective treatment of the contact is done, him, in operator STAT_NON_LINE. The resolution of a nonlinear problem in operator STAT_NON_LINE is described in detail in document [R5.03.01]. We recall here briefly the principal phases of them, for a calculation comprising two steps of time:
1st step of load
(1/a)

prediction
(1/b1)
iteration of Newton n°1
(1/b2)
iteration of Newton n°2
.....
(1/bm)
iteration of Newton n°m
(1/c)
storage of the results with convergence
2nd not of load
(2/a)
prediction
(2/b1)
iteration of Newton n°1
(2/b2)
iteration of Newton n°2
.....
(2/bp)
iteration of Newton n°p
(2/c)
storage of the results with convergence
The unilateral contact is treated after the phases (1/a), (1/b1), (1/b2),…, (1/bm), (2/a), (2/b1), (2/b2),…, (2/bp) *i.e* after the phase of prediction and each iteration of Newton of STAT_NON_LINE. It is there the essential difference between this algorithm and the algorithm of contact friction (see documentation [R5.03.51]) where the treatment of the contact is effective only at the end of the step of load and not
during iterations.
One calls "master key of contact" each occurrence of treatment of the contact.
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2.1.1 Formulation

node-facet

This formulation, chosen by the key word PAIRING: “MAIT_ESCL”, does not grant a role equivalent on the two surfaces: the surface described under GROUP_MA_1 or MAILLE_1 (S1) is called surface main and the other (S2) surface slave. The conditions of noninterpenetration express that the nodes of surface slave (of stars on the figure [2.1.1-a]) do not penetrate in meshs of surface Master: one can see that, on the other hand, it is possible that the main nodes (rounds) penetrate in surface slave.

surface S2 slave

*

*

*

*

surface main S1

Appear 2.1.1-a: Surface main and surfaces slave

The relation of noninterpenetration will be written between a node and a mesh: one seeks initially it main node of surface nearest to the node slave (cf [§2.2]), then one examines (cf [§2.3]) all the meshs Masters containing this node (the distance obtained by projection of the node slave on each mesh Master makes it possible to choose the mesh nearest). One uses the normal with the mesh Master to write the relation of nonpenetration.

Note:

The nodes slaves are by defect all the nodes belonging to the meshs of contact defining surface slave. Key words SANS_NO and SANS_GROUP_NO make it possible to give, zone by zone, the list of the nodes which must be removed list of the nodes slaves (but they could be used as main nodes). That makes it possible to remove the nodes subjected to boundary conditions of Dirichlet incompatible with the contact.

To symmetrize the role of two surfaces, it would be interesting to use a functionality of the type PAIRING: “MAIT_ESCL_SYME” which would exchange the roles of Master and slave with each pass from treatment of the contact. It is a development under consideration in version 6.

2.1.2 Formulation

nodal

The nodal formulation (PAIRING: “NODAL”) imposes that relative displacement enters a node slave and the main node which is paired to him, projected on the direction of the normal to the node slave, that is to say lower than the initial play in this direction. The use of this formulation is disadvised because it require to have compatible grids (nodes “opposite”) which remain compatible during deformation (assumption of small slips), and for which the normals Master and slave are with little close colinéaires. Without these assumptions, the made approximation becomes hazardous just like

(
the use of LIAISON_UNIL_NO) and it is preferable to use the node-facet formulation.
One chooses to take as surface slave that which comprises less nodes (with an equal number,
it is that described under GROUP_MA_2 or MAILLE_2), in order to maximize the chances to have one injective pairing (a main node is paired only with one node slave). The main node
paired with each node slave is determined by a calculation moreover nearer close explained in [§2.2]. One uses the normal with the main node to write the relation of noninterpenetration (cf [§3]).

Note:

Even in the case of nodal pairing, surfaces of contact are defined in terms of meshs (cf [§1.2]). The nodes slaves and Masters are then the nodes of the meshs thus defined.

2.2

Seek nearer close to a node

The method used to seek the main node nearest to a node slave is very simple:
it is enough to calculate the distance (in current geometry, cf [§2.4]) between the node slave and the nodes
Masters candidates. The only alternative used consists in being able to restrict the whole of the nodes Masters a priori candidates.
The key word SEEKS: “NOEUD_BOUCLE” starts the examination of all the main nodes
belonging to the same zone of contact as the node slave.
One stores with each master key of contact the main node which was closest to each node slave (it is called the former “neighbor”). If the relative slip of two surfaces is small (a mesh or two),
one can choose to examine only the main nodes connected to this old node by meshs of
contact. This method is activated by RESEARCH: “NOEUD_VOISIN”. One chooses among these nodes candidates nearest like “new neighbor”, and one will examine (cf [§2.3]) the meshs containing it
new neighbor. Thus, meshs potentially likely to be paired with the node slave (round
black) in the new configuration are those having stripes on the figure [Figure 2.2-a].

* * *
*
*
* * *
old main node nearest (old “vo
meshs containing the former “neighbor”
* nodes candidates to be the new “neighbor
meshs likely to be paired with the node
Appear 2.2-a: Territory covered by RESEARCH: “NOEUD_VOISIN”
Note:
If the discretization in time is sufficiently fine (what is the case in general out the problems of elasticity), it is reasonable to think that the slip will be small of a step of time to the other. One thus can has minimum to use the option SEEKS: “NOEUD_VOISIN”.

2.3
Research of the mesh nearest (node-facet formulation)

Knowing the main node nearest to the node slave, one examines successively the meshs Masters containing this node. The method of selection is simple: one determines projection M of node slave P on the mesh Master (according to the normal with the mesh Master), one brings back it on the edge of
the mesh if it is outwards, and one calculates the scalar product between vector PM and the normal with the mesh. The mesh carrying out the smallest value of this scalar product (before correction while bringing back
on the edge) is selected to be paired with the node slave.

2.3.1 Projection on a segment (contact in 2D)

One considers the situation described on the figure [Figure 2.3.1-a]. Surface Master is “below” surface slave, therefore the direction of main course of surface must be of *A* towards *B* (the mesh

of main edge is defined as being *AB*, and not *BA*): thus **normal N with the mesh is outgoing**, i.e. point towards surface slave (cf [§1.2]). On the other hand, from a point of view algorithmic, one uses **NR =-n**, the vector opposed to the outgoing normal of the mesh.
One seeks the parametric co-ordinate of the point *M*, projection of the node slave *P* according to entering normal **NR** with mesh *AB*, defined by:
AM = **AB**
MP

= **NR**
(**AB**, **NR**) =

0
where (,) indicates the scalar product.
slave
P
P'
NR
NR
Master
Me B M
With
Appear 2.3.1-a: Projection on a segment
(**AP**, **AB**)
One a: =
.
(**AB**, **AB**)
The point *M* belongs to mesh *AB* if 0
[; 1]. If > 1 (case of *P'* projected in *Me*), one brings back
projection of *A* by posing = 1; if < 0, one bring back projection out of *B* by posing = 0. One evaluate then the scalar product of **PM** with the normal **NR entering** to the mesh (*i.e* opposed to outgoing normal of the mesh Master), whose components are:
yB - *yA*
-

NR =
L
,
2
2
with *L*
length *AB*
of: *L* = (*xB* - *teststemxà*) + (*yB* - *yA*)
X
.

B -

teststemxà

L

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The value of (**PM**, **NR**) is called the play between the node slave and the mesh Master. The direction **NR** is
reserve like direction of writing of the relations of nonpenetration (cf [§3]).

Note:
One could have defined the direction of writing of the relations of nonpenetration by vector **PM***, and thus play by the standard of* **PM***. However, vector* **PM** *tends towards the null vector (direction unspecified) when the points approach (when one tends towards the effective contact) and becomes very sensitive to the errors rounding: to the extreme, when P = M, one can find* **PM** *= (1015; 0) (for a mesh Master horizontal), which is a horizontal direction, perfectly erroneous for the writing of the relations of nonpenetration here. For this reason one chooses to use normal Master which, it, does not vary because of the only bringing together of the solids. The fact of privileging a surface compared to another can generate errors of modeling (loss of symmetry) which one can minimize by refining the grid. Another solution would consist in using the average between the normals Master and slave. This approach is with* the study in version 6.

2.3.2 Projection on a triangle (contact in 3D)

P slave
C
NR
*
With

M
Master
B
Appear 2.3.2-a: Projection on a triangle
One seeks the parametric co-ordinates 1 and 2 of the point *M*, projection of the node slave *P* according to the normal **NR** entering to the triangular mesh *ABC* (one uses the normal with the mesh, but in
the direction slave towards Master), defined by:
AM = **AB**
1
+ **AC**

2
PM

NR = 0
that is to say:

((**AP NR**), **A**)
C

1 = -

(**AB**
)
AC
- AB AC

with **NR** =
((**AP NR**), **AB**)

(entering unit normal).

AB AC
2 =

(**AB**

)
AC
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If one poses 3 = 1 - 1 - 2, the values of the three parametric co-ordinates 1,2,3 allow
to find if the point *M* belongs or not to the triangle *ABC*, like illustrates it the figure [Figure 2.3.2-b].

1 <0
3 <0
6
C
1
1 <0
3
0
3 <0
1 >0
2> 0
3> 0
With
1<0
2 <0
5
2 <0
B
2
2 <0
3 <0 4

Appear 2.3.2-b: Possible zones for the point *M*, prolongation
node *P* according to the direction of the normal to the mesh

If the point *M* is in sectors 1, 2, or 3, one brings back it on the corresponding edge
(*AC*, *AB* or *BC*). If it is in sectors 4, 5 or 6, one brings back it on the corresponding point
(not *B*, *A* or *C)*. That amounts cancelling the parametric co-ordinates which are negative.
Let us take the example of sector 1 where 1 < 0. One brings back the point *M* to the point *Me*, defined by:
AM'

'

= **AC**

2

AM = **AB**
1
+ **AC**
2
(
AM', **MM'**) = 0

AB AC
One finds: '
(
,
)
2 = 2 + 1
.
(
,
AC
)
AC
AB AC
In sector 2, an identical reasoning gives: '
(
,
)
1 = 1 + 2
and **AM'**
'
= **AB**
(**AB**, **AB**)
1
.
-
'
1 (**AB**,
)
BC + 1
(-)(
,

AC
)
BC
In sector 3, there are **AM'**
'
AB (
'
=
2
1
+ 1 -) **AC**
1
with 1 =
.
(
,
BC
)
BC
The play is calculated like the scalar product between vector **PM** and the normal **NR** entering to the mesh
Master.
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2.3.3 Projection on a quadrangle (contact in 3D)

For the quadrangles, the determination of the parametric co-ordinates in the current element would require the return to the element of reference, and thus the resolution of a nonlinear problem, which is

expensive.
An approached solution was initially chosen, which is to cut (virtually) it
quadrangle in two triangles, according to the two possible manners (cf [Figure 2.3.3-a]), to calculate outdistance node slave with each of the four triangles thus defined (cf [§2.3.2]), and to choose it triangle carrying out the smallest distance. The relation of noninterpenetration is then written between node slave and 3 main tops of the selected triangle. If the quadrangle remains plane,
projection on the selected triangle is equivalent to projection on the quadrangle; in the case more general where the quadrangle is left, this operation is a means of taking into account, of one certain way, curve.

Appear 2.3.3-a: Cutting of a linear quadrangle

2.3.4 Case of the quadratic elements

Projection on the quadratic elements is made for the moment while being reduced to the linear case (triangle with three nodes and quadrangle with four nodes). On the other hand, the writing of the relation of not
interpenetration utilizes all the nodes of the main elements with the functions of form associated (cf [§3.2]). It is thus considered that the contribution of the nodes mediums to the result must be
taken into account even if their contribution to the geometrical deformation of the element is neglected.

Note:

For the quadrangles, one uses only the functions of form relating to the three tops of triangle chosen, and that even for the QUAD8 and QUAD9.

Important warning:

The contact in 3D for quadratic elements gives, for exposed theoretical reasons in CR MMN 97/023, results which can surprise the user. Us let us not recommend the use of such elements; if such is the case, however, we advise to refine "sufficiently" the grid on the edges of the structures in contact.

2.4 Reactualization geometrical

In the framework of the modeling of the contact in great displacements, the evolution of the geometry of surfaces plays a fundamental part. Indeed, it is it which conditions the calculation of the normals to the faces
potentially in contact and thus which conditions pairing carried out.
The geometrical reactualization is defined by key word REAC_GEOM_INTE of the key word factor CONTACT. Its operation is as follows:
REAC_GEOM_INTE=0
There is no geometrical reactualization. All calculation is carried out on
initial configuration with initial pairing.
REAC_GEOM_INTE=1
A geometrical reactualization is carried out with convergence of
each step of load *i.e* right before the phases (1/c), (2/c),…
presented to [§2.1]. This reactualization is accompanied again
pairing.
REAC_GEOM_INTE=2
One places oneself at a step of load given. With convergence of this last,
a geometrical reactualization then a new pairing are
carried out. One does not pass to the step of load according to but one begins again **the same** step of load until convergence. A reactualization
geometrical then a new pairing are carried out and one passes to
no the load according to.
REAC_GEOM_INTE=n
It is a generalization of the preceding case. Within the **same** step of
(n>2)
charge, one carries out N time the cycle *iteration until convergence, geometrical reactualization, pairing*.
One can first of all notice that pairing is subjected to the phase of reactualization
geometrical. Moreover, the fact of carrying out several times within the same step of load the cycle *iteration until convergence, geometrical reactualization, pairing* makes it possible to follow the evolution of
geometry of the structure. It should indeed be stressed that this geometrical evolution is one of nonlinear components of a calculation of contact in great displacements.
In practice, one can advise the following values for key word REAC_GEOM_INTE:
· for a calculation in small displacements, the natural value is 0. One works on
initial configuration,
· for calculation in great displacements, the selected value depends of course on the importance on the geometrical evolution of surfaces but values 1 or 2 is generally with advising.
Value 2 is the default value besides of this key word.
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3

Relation of noninterpenetration

3.1 Condition

kinematics

One carries out a idealized modeling of the phenomenon of contact, in the sense that it supposes them borders of the bodies perfectly defined by a line or a surface: one writes a condition then of not discrete and linearized interpenetration [bib3].

That is to say *P a* node slave, *M* his projection on the mesh Master which was given at the time of pairing. In 2D, this mesh Master has 2 nodes (SEG2) or 3 nodes (SEG3). In 3D, it can in to have 3, 4, 6, 8 or 9 (TRIA3, QUAD4, TRIA6, QUAD8, QUAD9). The displacement of the point *M* is one linear combination of displacements of the nodes of the finite element, with for coefficients values functions of form in *Mr.* Plaçons us if the mesh Master is a SEG2 for to simplify the talk. One has then:

U
= (*M*) **U** + (*M*) **U**
M
With
With
B
B

The relation of nonlinearized penetration consists in saying that relative displacement between *P* and *M* according to a given direction cannot exceed the initial play in this direction. One chose to take like direction **NR the** entering normal of the mesh Master (cf [3.1-a]).

B
surface main
M
With
NR

P
surface slave
Appear 3.1-a: Projection of a node slave on a mesh SEG2
The relation of nonpenetration is written then like a scalar sign of product (noted by one**.**) :
PM.N 0, is P-M-. **NR** + (**U**
M - **U**
). **NR**
P
0 ,
if **U** is the increment of displacement since the preceding configuration where displacement was noted **U**.
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One must thus check (**U** - **U**). **NR** P-M-. **NR**
-
-
P
M
. By noticing that **P M**. **NR** is the play *D* -
in the preceding configuration, the relation of nonpenetration is also written:
(**U** - **U**). **NR**
-
P
M
D,
maybe, by using the relation **U**
= (*M*) **U** + (*M*) **U**
M

With
With
B
B:
[**U** - ((*M*) **U** + (*M*) **U**)]. **NR** -
P
With
With
B
B
D
The extension of the formula for a mesh comprising Master *nmaît* noted nodes *Bj*, is immediate:
N

maît

U - (*M*) **U**. **NR**
-
P
B
B
D
J
J

J

=1

If one writes such a relation for all the couples of contact, one obtains the geometrical conditions of nonpenetration in matric form:
With D
Note:
The effective play in the configuration **U** + **U** *is* **d0** - **A** (**U** + **U**)*, is* **D** - **With**. *The condition* of nonpenetration thus expresses that the effective play remains positive or null in any configuration.
Matrix **A**, called matrix of contact, contains 1 line by couple of contact, and as much of columns that physical degrees of freedom of the problem.
Let us suppose that one has 2 meshs of contact of the type SEG2, according to the diagram of the figure [Figure 3.1-b]:
D
B
L/2 *M1*

M
NR
*
2
L/4
C
*
NR
d1
2
D
P
Q
Appear 3.1-b: Writing of the matrix of contact A on an example
If one notes for example **uB the** increment of displacement of the node *B* according to direction *X*, and
X
D and *D*
1
the 2 current plays for the two couples, the two relations of nonpenetration are written
matriciellement:
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U

Px

U

P
y
U
Qx

U
Q
y
NR
NR
0
0
- .
0 **NR**
5
- .
0 **NR**
5
- .
0 **NR**
5
- .
0 **NR**
5
0
0
U *D*
X
y
X
y
X
y
B

X
1

0
0
NR

NR
0
0
- .
0 75N
- .
0 75N
- .
0 25N
- .
0 25N
U

D
X
y
X
y
X
y
B

y
2
U
Cx

U
Cy
U
Dx

U

D y
Note:
One considered here only the degrees of freedom of the nodes implied in the contact; in reality, matrix **A** *is hollower.*
3.2
Coefficients of the relation of nonpenetration
The relation of nonpenetration is written:

nmaît

U -
M **U**
NR *D* -
( )
.
P
B
B
J
J
J

=1

One gives below the values of the functions of form *B* (*M*) of the main nodes to the point *M*
J
for the various treated meshs of contact.

3.2.1 Meshs

SEG2

The parametric co-ordinate of the projection of the node slave on the SEG2 is noted. Values functions of form at the parametric point of co-ordinate are as follows:
(*M*) = 1 -
With

(*M*) =
B

3.2.2 Meshs
SEG3

One projected the node slave by supposing the rectilinear mesh: the point *M* thus does not belong inevitably rigorously with the SEG3 if this one is curved. Nevertheless, one calculates the values of functions of form associated to the SEG3 (tops *A* and *B*, node medium *C)* starting from the co-ordinate parametric paid to the SEG2:

1

$TO(M) = 2(1 -)(-)$

2
1

$B(M) = 2(-)$

2

$C(M) = 4(1 -)$

3.2.3 Meshs
TRIA3

One explained in [§2.3.2] how one finds the co-ordinates parametric 1 and 2 of projection *M* of the node slave in the triangle. The values of the functions of form are in fact those parametric co-ordinates:

$TO(M) = 1 - 1$
$- 2$

$B(M) = 1$

$C(M) = 2$

3.2.4 Meshs
TRIA6

As in the case of the segments, one carried out projection by taking account only of the 3 tops of the triangle: on the other hand, one uses the parametric co-ordinates thus obtained (while posing

To = 1 - 1 - 2, *B* = 1, *C* = 2) to deduce the values from them from the functions of form to the 6 nodes (*A*, *B*, *C* tops, *D*, *E*, *F* nodes mediums respectively on the sides *AB*, *BC* and *CA*):

With (*M*) = *A* (2A -)
1

B (*M*) = *B* (2B -)

1

C (*M*) = *C*
(2C -)
1

D (*M*) = 4AB

E (*M*) = 4BC

F (*M*) = 4 *C*
With
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3.3
Introduction of a fictitious play

One can want to model the contact between structures whose grid did not take account of certain characteristics (“hole” or “bump” nonwith a grid (cf [Figure 3.3-a])).
NB: **N** = NR
N
real structure with a bump

grid
grid
real structure with a hole
Appear 3.3-Error! Argument of unknown switch. **: Holes and bumps**
In this case, it is necessary to correct the value of the play intervening with the second member of the inequation of not
penetration, according to the following model (**NR** is the normal **entering** to the mesh Master):
(**U** - **U**). **NR** -
P
M
D - (*d1* + *d2*)
where *D* and *D*
1
2 are given by the user respectively under key words DIST_1 and DIST_2 for
each zone of contact. These “distances” have a sign: they represent the translation to be applied to node of the grid in the direction of outgoing normal **N** to obtain the point of the real structure
(cf [Figure 3.3-b]).
d1 = 0
d1 = 0
N
N
d2 > 0
d2 < 0
Appear 3.3-Error! Argument of unknown switch. **: Definition of *D* and *D***
1
2
These key words make it possible to also give an account of the contact between hulls of which only them
average surfaces are with a grid: *D* and *D*
1
2 are worth then the half-thickness of the hulls (values
positive) (cf [Figure 3.3-c]).
half-thickness e1
surface average
real edge of the hull
d1 + d2 = e1 + e2 > 0
real edge of the hull
half-thickness e2
surface average
Appear 3.3-Error! Argument of unknown switch. **: Contact between hulls**
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Note:
If one uses DIST_1 and DIST_2, it is necessary to take guard with the visual interpretation of the results. If
D + *D*
1
2 > 0, *the code will be able to announce contact whereas visualization shows one spacing of the two grids. If D* + *D*
1
2 < 0, *the code will be able to announce contact whereas visualization will show two interpenetrated grids.*

Help memory:
To remember the signs, to think of:
D > 0 *or D* > 0*: "matter addition" compared to the grid,*
1
2
D < 0 *or D* < 0*: "ablation" of matter compared to the grid.*
1
2

3.4

Dualisation of the conditions of nonpenetration

To simplify the writing, we in this chapter in linear elasticity place (matrix **C**, loading **F**), by forgetting the boundary conditions of Dirichlet. If one dualise conditions of nonpenetration (cf [bib3]), one must solve the following system, including/understanding equations and inequations:

Cu
+ **AT**M = **F**
With
D
µ
0

J, (**With D**)

J µj = 0
The first line expresses the equilibrium equations: vector **AT** µ can be interpreted like nodal forces due to the contact. The second line represents the geometrical conditions of not interpenetration: the inequality is understood component by component (each line relates to one potential contact couples). The third line expresses the absence of opposition to separation (them surfaces of contact can know only compressions). The last line is the condition of compatibility: when for the connection J the multiplier of Lagrange µ *J* is nonnull, there is contact and thus the play (**D - With**) *J* is null; when the play is nonnull (two surfaces are not in contact), the associated multiplier must be null (not compression).
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4
Resolution of the problem of contact
4.1
Position of the problem
One treats the unilateral contact after the phase of prediction and each iteration of Newton. Thus it field of displacements passes by the following states:
beginning of the step of time I:
ui-1
prediction
~u0i
treatment of the contact
u0i
update:
u0
U

u0
= **-** +
I
I 1
I
iteration of Newton number 1
~u1i
treatment of the contact
u1i
update:
u1
u0
u1
=
+
I
I
I
...
iteration of Newton number N
~uni
treatment of the contact
linked
update:
one
un-1
one
=
+
I
I
I
...
When the contact is not treated, the systems to be solved are as follows (with the notations of [R5.03.01]):
K
BT u~0 Lméca

+ **Lther**

0
I

I
I

~ =

with the phase of prediction
B
0
0
ud

I
I
K *n-1* **BT u~n**
méca
N
T
N

-
-
-

I
I
L
R
I
(**U** 1i) **B** 1i

~ =
with the nth iteration of Newton
B
0
N

I
0
One can write the generic form of the system to be solved when the contact is not treated:
CU = **F**,
where **U** gathers the degrees of freedom of displacement **U** and the associated multipliers of Lagrange with boundary conditions of Dirichlet (the ~ indicates that the contact is not taken into account), **C** is stamp tangent supplements, and **F the** second member.

The relation of noninterpenetration is written:
With D (**D** is the initial play, measured on the grid),
0
0
or: **With D** = **d0 - With** if **U** = **U** + **U** (cf [§3.2]).
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In the presence of conditions of unilateral contact, systems to solve (dualisation of the conditions of not interpenetration) are thus written:
C

U
+ **AT** μ = **F**
WITH U

D
where **A** is the complete matrix of the conditions of contact, μ the vector of the multipliers of Lagrange associated the contact (they must be positive or null), **AT** μ the vector of the nodal forces of contact, and **D the** vector (plunged in the whole of the degrees of freedom, multipliers of Lagrange included/ understood)
containing the play running (for **U**):
D d0 - With
D =
=

0
0

4.2
Method of the active constraints

One will be able to find a description complete of the method with the theoretical justifications necessary in [bib2] and [bib3]. The principle is as follows: a whole of constraints is postulated known as active, which corresponds to a null play (the relation inequality becomes an equality); it is solved
system of equations obtained in this subspace, and it is looked at if the starting postulate were justified. If the selected unit was too small (active connections had been forgotten), one adds with the unit the connection
violating more the condition of noninterpenetration; if the selected unit were too large (connections presumedly active are not it in fact not), one removes from the unit the most improbable connection *i.e* that
of which the multiplier of Lagrange violating condition 3 of the system of [§3.4] to the greatest value absolute. The fact of removing or of adding only one connection with each iteration of the method guarantees
convergence in a finished number of iterations [bib2].

One notes **U the** field of displacements obtained before treating the contact: it is about **ui-1** when one *n-1*
draft the contact at the end of the phase of prediction, and **ui**
when one treats the contact at the end of
the iteration of Newton number N. Increments of displacements (obtained without taking into account it contact) calculated before are thus not taken into account in **U**. One seeks the increment **U** with
to add with **U** to obtain **u0**
N
I or **ui**.

The method of the active constraints is an iterative method uncoupled from the iterations of Newton: with
each iteration of active constraints, the starting solution is noted the **U.K.**, and the increment added by new iteration is noted *K* +1. One thus has in theory: **The U.K.** +1
The U.K.
K +
=
+
1

, and
U = **U** + **U** +
+
K
K

1. One starts from **U0** = **C-1F**, which is the increment obtained without treating the contact
(**U0** = **U0**

N
I given by the prediction, or **U0** = **Ui** given by the nth iteration of Newton) and one
carry out the iterations of active constraints until clean convergence of this algorithm.
convergence within the meaning of the active constraints is obtained when no connection violates the condition
kinematics 2 of [§3.4] and when the associated multipliers of Lagrange are all positive.
In elasticity, at the end of the iterations of active constraints, there is a result converged within the meaning of
Newton. In plasticity or if the geometry is reactualized, it is not the case because several iterations of Newton are necessary to obtain balance. After each iteration of Newton, one launches the algorithm of active constraints to satisfy the conditions of contact. Thus, in elasticity, one will necessarily converge for each step in an iteration if REAC_GEOM_INTE = 0 or REAC_GEOM_INTE = 1 in iterations if REAC_GEOM_INTE = N, N > 1.
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4.2.1 Resolution of the system reduced to the active connections

At the beginning of the algorithm, one evaluates the current play **D**
[
[
]
0] - **A**
J
K (**U** + **u0**)
for all the connections J,
J
by taking account of **U0** displacement = **C-1F** estimated without treating the contact. One calls activates one
connection for which this play running is negative, which indicates an interpenetration. One postulates

that for
the active connections, the effective play will be null, and that thus the inequality **With the d0** becomes an equality for
active connections.

Note:

One could leave the old active connection set obtained to convergence of the master key the preceding one, but if the couples of contact were reactualized, numbers of connections correspond more inevitably. However, whenever this is licit, the iteration count of the method of the active constraints can be decreased by it, as it is the case with the key word LIAISON_UNIL_NO.

If one notes **Ak the** matrix of contact reduced to the active connections with the iteration *K* (one keeps only them
lines corresponding to the active connections), one a:

C

U
K + **C** *k*+
1 + **AT** μ = **F**
K
WITH U

- + **A the U.K.**
K +1
K
K
+ **A**
= **D**
K
0

or:

k+1
-1
K
-1 *T*
= **C F - U**

- C Ak μ

,
-1 *T*
-1
-

-

Ak C Ak µ = **D0 - AkC F - Ak U**
maybe, by taking account of **C-1F** = **U0**:

-
-1 *T*
-
WITH A.C. µ = **D**
0
K
K
- A U

K
K +1

= **U0**

- The **U.K.**

-

-
C 1ATk µ

with **D** = **D**
, where **D** = **D**
0
0 - **With** *ku*- is the updated play corresponding to the field of displacement
U.
The first equation gives the values of the multipliers of Lagrange µ associated the relations of not penetration for the active constraints, and the second equation gives the value of the increment *k+1* of the unknown factors for the kth iteration of the method of the active constraints.
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4.2.2 Validity of the whole of active connections selected

That is to say the connection number *J* (one indicates by [] *J the* jième component of a vector, i.e. that corresponding to the connection *J*). Three situations are possible:

1) relative displacement compensates for the initial play: [**A** (**U**
the U.K.
K +
+
+ 1)] = [**D**
K
0]
J
J

2) relative displacement is lower than the initial play: [**A** (**U**
the U.K.
K +
+
+ 1)] < [**D**
K
0]
J
J

3) relative displacement is higher than the initial play: [**A** (**U**
the U.K.
K +
+
+ 1)] > [**D**
K
0]
J
J

The situation (3) is prohibited: it corresponds to a violation of the condition of noninterpenetration. situation (1) corresponds to a connection known as active, the situation (2) with a nonactive connection. At the beginning of the kth iteration of the algorithm, one had postulated a whole of active connections. One has
found an increment possible *K* +1 of the unknown factors under these assumptions: one now will check

that
this increment is compatible with the assumptions. In practice, that consists in checking:
(I)
that the nonactive supposed connections do not violate the condition of noninterpenetration
(if not one activates one of them);
(II) that the presumedly active connections are associated multipliers of contact μ
positive or null (if not one decontaminates one of them)
Checking (I): (is the whole of the active connections too small?)
One will calculate for all the nonactive supposed connections the quantity:
[
-
K
-
K
d0 - Ak (**U** + **U**
)]
D - With U
J
[
K
]

J
J =
[
=
K +1
K +
With
1
K
]
With
J
[*K*] *J*
· if
[**A** *K*
K +1] is negative, the play for the connection J will increase, and thus the supposed connection
J
not remainder in this state activates when the **U.K.** +1 is written
The U.K.
K +
=

+
1

,
· if
[**A** *K*
K +1] is positive,
J
J should be higher strictly than 1 for a nonactive connection
(situation (b)). One thus examines = *Min J* on the whole of the connections *J* declared not
J
active. If < 1, that indicates that a connection at least is violated (situation (3)) : one adds
then with the list of the active connections the number of the most violated connection, i.e. that which carry out the minimum of
K +1
K
K +1
J, and **U** is written
= **U** +
(that corresponds to a null play
for the added connection). In this case one shunts the checking (II).

Note:

If all the connections are active, the checking (I) does not take place to be. In this case, one poses = 1 *and one passes directly to the checking (II).*

Checking (II): (is the whole of the active connections too large?)
One places oneself now if 1: **the U.K.** +1 is taken
The U.K.
K +
=
+
1

.
· if no connection is active, the method converged towards a state without contact,
· if there are presumedly active connections:
-
if all the multipliers of Lagrange μ are positive or null, one also converged
towards a state with effective contact,
-
if there are negative multipliers of Lagrange μ, the corresponding connections
should not be active: one withdraws from the whole of the active connections the connection of which it negative multiplier is largest in absolute value.

Note:
One removes and one adds the active connections one by one (and not all those which contradict assumptions) in order to ensure the convergence of the algorithm in a finished number of iterations, as shown in [bib2] and [bib3]. However, one could take the risk to add all the connections which seem active of a blow, or to decontaminate all the connections with multiplier negative of a blow, in order to accelerate the convergence of the method (it is what is made for treatment of friction, cf [R5.03.51]). Even if convergence is not theoretically ensured, such an alternative seems to go in practice.

4.3
Recutting of the step of time
On the theoretical level, the convergence of the method of the active constraints is ensured in a number finished iterations. In practice, certain numerical artifacts can return this convergence

delicate. Also a strategy it was developed to ensure the robustness of the algorithm.
During calculations of contact, in particular if the steps of load carried out are too important, of undesirable phenomena can appear:
· matrix of singular contact **A C-1AT**,
K
K
· oscillation of the method of the active constraints: a node is detected alternatively
"stuck" then "taken off".
To mitigate these difficulties, the following strategy was adopted. If:
· the matrix of contact **A C-1AT** is singular,
K
K
· the iteration count of active constraints is higher than a limit which depends on the number potential connections
Then one redécoupe the step of time *i.e* one returns to the preceding step of load and instead of testing to reach the level of loading following in a step as one has just done it, one does several of them (For more precise details on this functionality of operator STAT_NON_LINE, to see documentation [U4.51.03]).
In practice, this functionality is shown very useful for the user.
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5 Precautions of use

An example of use and the associated councils are given in [bib6].
The principal councils or warnings are as follows:
· to check that the normals on the surfaces of contact are outgoing (to be wary in particular if one has used operators of symmetrization in the maillor gibi),
· attention in contact with quadratic 3D if the meshs of edge are QUAD8 (to avoid using

HEXA20 to net volume or to refine “sufficiently”): to use preferably
HEXA27, or many PENTA15 whose TRIA6 sides are the meshs in contact,
· to remove, by boundary conditions of Dirichlet adapted, the movements of body
rigid: it is not necessary that the structure “holds” only by the contact. In other words, that
wants to say that a calculation made in elasticity with order MECA_STATIQUE (without treating it
contact thus) must pass,
· in the event of structure “held” only by the contact, one can add a spring of weak
rigidity to maintain it. This rigidity will not have to disturb the field of deformations of
structure supposed no one (since there is rigid movement of body), but to prevent one
displacement ad infinitum. In practice, its choice proves to be delicate and requires a retiming
precondition,
· to use key word SANS_NOEUD or SANS_GROUP_NO to exclude from the list of the future nodes
slaves those which are subjected in addition to boundary conditions of Dirichlet
(DDL_IMPO, FACE_IMPO, LIAISON_…) in the awaited direction of the contact,
· the calculation of the efforts of contact can be carried out in order POST_RELEVE_T in
calculating the resultant of the nodal forces on the group of meshs representing one of
surfaces of contact,
· the contact and the linear search for STAT_NON_LINE do not do good housework together
when one converges in addition to one iteration. Roughly speaking, that wants to say that one cannot
to use linear research except for elastic designs without reactualization
geometrical, which is rather restricted.

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Titrate:
Contact discrete friction in 2D and 3D

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Organization (S): EDF-R & D /AMA

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Document: R5.03.51

Contact discrete friction in 2D and 3D

Summary:

Discrete modelings of the contact with friction in 2D and 3D, zones of slip being

respectively 1D and 2D, are proposed starting from a mixed variational formulation constraints displacements. The conditions of contact and friction are treated with the nodes of surfaces of contact of
solids implied with taking into account of great displacements. The law of friction of Coulomb is treated
by operators STAT_NON_LINE and DYNA_NON_LINE after definition of the conditions of contact and of
friction under the key word CONTACT of AFFE_CHAR_MECA.

In 2D as in 3D, various methods are usable for the modeling of the problem:

.
taking into account of the contact and friction using multipliers of Lagrange,
.
taking into account of the contact using multipliers of Lagrange and friction using penalization,
.
taking into account of the contact and friction using penalization.

The subjacent algorithms are inspired by the active constraints [bib2] [bib9] and by the predictor-correctors
usually used in plasticity.
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1 Introduction

Discrete modelings of the contact with friction in 2D and 3D, zones of slip being respectively 1D and 2D, are proposed starting from a mixed variational formulation constraints displacements. The conditions of contact and friction are treated with the nodes of surfaces of contact of the solids implied with taking into account of great displacements. The law of friction of Coulomb is treated by operators STAT_NON_LINE and DYNA_NON_LINE after definition of conditions of contact and friction under the key word CONTACT of AFFE_CHAR_MECA.

In 2D as in 3D, various methods are usable for the modeling of the problem:

.
taking into account of the contact and friction using multipliers of Lagrange,
.
taking into account of the contact using multipliers of Lagrange and friction using penalization,
.
taking into account of the contact and friction using penalization.

The subjacent algorithms couple the method of the active constraints [bib2] to determine them

areas of contact and an algorithm of resolution of Newton inspired of the methods of the type predictor-corrector, usually used in plasticity, for friction, in order to determine them zones of slip [bib1] [bib7] [bib10].

In 2D as in 3D, the user has a whole of methods panachant dualisation and regularization by penalization:

.
dualisation of the conditions of contact and friction,
.
dualisation of the conditions of contact and regularization of the conditions of friction,
.
regularization of the conditions of contact and friction.

The compared interests of these methods are well-known: the dualisation introduces news unknown factors but it provides an exact solution; the regularization provides only one approximation of
solution dependent on a parameter chosen by the user but it does not introduce a news unknown factors.

The document begin with a general presentation from the laws from friction. One presents then discretization of these laws like their linearization for their integration within the method of Newton. One details the algorithms then allowing to solve these problems. End of the document draft of the use practises these methods within Code_Aster and of their post front treatment to approach the conclusions.
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2
Formulation of the model

2.1
The criterion of Coulomb

Are 2 solids being able to come into rubbing contact:

T
N

The outgoing normal on the surface of contact, U are ***N*** *=* ***U N***
. displacement following this normal,
N
G existing initial play between 2 solids, = ***N***
. ***N***
. the normal force exerted by one of surfaces
nn
on the other and = ***N***
. - ***N*** *shearing.*
T
nn

Solid
Soli 2
of
N 1 T T
u2
U
T
T
1
2
Solid
Soli 1
of
u1
U

More precisely, for two solids (1) and (2) in contact: the zone of contact is either specific, or linear is surface. The force of shearing then has as a direction in the zone of contact one vector ***T*** *located in the tangent plan (****T****,* ***T****) indicated on the figure above. One defines:*
1
2

R *=*
*(****N***
.) -
*((****N***
.
N
).) ***N*** *= rt,* ***R*** *=*
1
1
1
1
1
1
T

the force of shearing exerted by the solid (2) on the solid (1) per unit of area of contact.

Let us write the system of equations and inequations having to be checked by these sizes:

U *.n +* ***U*** *.n =* ***U U N***
G
1
1
2
2
(-
1
2).

1
0
nn
*(****U*** *.n +u .n - G) = 0*
nn
1
1
2

2

- μ 0

T
nn
u& = u& u& T R
T
(-

2
1)

. =
(- μ) = 0

T
nn

0
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*where **N** is the outgoing normal with the solid (1) and **N the** outgoing normal with the solid (2) opposed to **N**.*
1
2

1
first set of equations and inequations corresponds to the management of the contact; it will not be detailed and
we return to [bib9]. The second corresponds to the description of friction obeying the criterion of Coulomb. It utilizes several fields and binds between them: normal pressure, shearing and tangent displacement. It can be included/understood as follows:

.
If < μ
,
T
nn
-
*= 0 and **u&** = 0*
T
.
If = μ
,
T
nn
-
*> 0 and **u&** = **R***

T

One can give following graphic interpretations:

adhéren
adhére T

nn
N
slipped
glis NT
its
T
T
2
1
T1

t2

In the space of the constraints, the effort of rubbing contact can be only inside the cone of Coulomb: if it is strictly inside, the contact is adherent; if it is on the surface of the cone, it contact is slipping. One can thus give another representation of this criterion for a situation of contact known:

R
μ nn
slipping
member
u&t
slipping
- μ nn

Friction induces the concept of threshold; we will see now how to formulate in manner general of other laws of friction by using this concept.

2.2
General formulation for various criteria

The selected criteria of friction are form:

*G (**R**) 0*

*where G (**R**) is a convex function. The field of nonslip is defined by the interior of the convex one.*

*Two criteria of friction of the form G (**R**) 0 are particularly used:*

.
the criterion of Tresca where:

*G (**R**) = **R** - K 0 and K =constante*

One notes C the convex disc C of ray K centered in the beginning defined by:

*C = {**R***
***R** K}.*

*The condition of nonslip is then defined by the membership of **R** inside C.*
*In the event of slip, for **R** located on the border of C, the direction of slip **T** of **u&***
*is given by the normal to the criterion in **R**, as indicated below:*

t2
T
R
C
T1
- K
+k
T
+k

.
the criterion of Coulomb where:

G (;
***R** µ,) = **R** - K (µ,) 0 and K = µ*

nn
nn
nn

The value of K depends on
= (.

N) .n the 0 normal component of the force exerted by
nn
one of surfaces on the other and µ, the coefficient of friction of Coulomb. In the event of slip, for ***R*** *located on the border of G which is a cone, direction of slip* ***T*** *of* ***u&*** *is not given by the normal to the criterion in* ***R****, but by the normal with the convex disc C of ray K = µ.*
nn
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N
T
R = µ nn
t2
T
T1
T

Determination of the direction of slip for
(T, T
1
2)
the criterion of Coulomb in the reference mark of the vectors

.

the criterion of Mohr-Coulomb where:

*G (**R**, μ, c) = R - K (μ, c) 0 and K = C + μ*

nn
nn
nn

It is particularly used to characterize the behavior of interfaces the géomatériaux one (clays in particular). C is the cohesion of material and μ the coefficient of friction (μ = tan, where is the angle of friction). Again
= (.
***N**) .n < 0 so that the contact remains maintained.*
nn

2.3
Formulation by differential inclusions

*One notes V the whole of displacements kinematically acceptable of the problem. The relation enters the speed of relative slip **u&** and the shear stress **R** translates the two possible states of system: not slip or relative slip following the normal direction to the convex disc C.*
*For the three criteria presented, the function **u&** (**R**) and its reciprocal **R** (**u&**) belongs both with the under-differentials of two combined pseudopotentials, so that one can write:*

***u&** (**R**) and*
***
***R** (**U**.*
C
&)
C

The appearance of included differential comes from the not-differentiable character of the laws from contact-friction. Indeed, the indicating function of the convex disc C of ray K indicates,
C
centered at the origin, previously definite. It is such as:

*0 if **R** C*
*(**R**) =*

C

+ if not
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*(**R**) of as a **R.** It is then the under-differential merges with the normal external with C as a **R.***
C
C
** **U***
( ) =
, where K is the threshold of friction resistance, is combined of Fenchel of
C
&
*K **u&***
indicating function.
***
*(**U** is positively homogeneous degree 1. This function is interpreted*
C
&)
C
like the density of power dissipated in the slip. Using the concepts of under-differential,
*one can establish the following relations for **u&** and **R** associated:*

*(**R**) = 0*
***u&** (R)*
C

;
C
&. (
***U R** - **R**),*

0 ***R***
C
U
& V
*
R

*C (**U***
&)
R
*(.v& - **u&**)*
*

-

*C (**v***
&)
*
*C (**U***
&)
;
*, **v&** V*
*

+
=
=
*C (**U***
&)
*C (**R**)*
&.
***U R** K **u&**.*

Note:

1) The two combined pseudopotentials presented are nondifferentiable.
2) Once known the normal reaction for the criterion of Coulomb, one is reduced locally to a criterion of friction of Tresca whose threshold is worth K = μ
.
nn
*3) The adopted local criteria having a circular form one deduces from it that **u& (R)** implies C*

that there is real positive such as ***u&*** *=* ***R***

.

4) The formulation of the problem of speed suggests an incremental numerical resolution of problem of friction. The resolution of the problem of balance will thus be presented under incremental form.

2.4

Resolution of the problem of balance.

One considers two solids of total volume whose surface of contact is. To simplify, one
C
will suppose the existence of a differentiable deformation energy to characterize the answer of two solids separated with external requests. In fact, one can show that the results given hereafter are independent of this assumption. One notes V the whole of the fields of displacement kinematically acceptable, constrained by the respect of the conditions of contact and friction on the interface.
The balance of the two solids in the absence of friction is written:

To find ***U*** *field of displacement kinematically acceptable such as:*

U *=*
min
arg
*[((****v****))- W (****v****)] {(****U****) - W (****U****) (****v****) - W (****v****),* ***vV****}.*

v *V*

*In elasticity, (****v****) = ((****v*** *D*
))

*is the deformation energy. The function W (****v****) represents it*

work of the external forces. A condition necessary and which becomes sufficient if is strictly convex so that this balance is checked is that:

*D (****U****) - DW (****U****) =*
*D (****U****) -* ***L***
= ***0***
ext.

where D is the operator derived Gâteaux and ***L*** *is the linear form associated the external forces.*

ext.
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With the introduction of friction, the problem must be tackled in incremental form. One is conduit [bib3] [bib4] with the problem of minimization following on the unit V of the fields kinematically acceptable constrained by the respect of the conditions of contact and friction on the interface:

U *known, to find* ***U*** *V such as:*
U *+* ***U***
=
min
arg
*[((****U****+* ***v***
))
***
*+ (****v***
*) - W (****U*** *+* ***v***
) .
C
T
]
vV

U *+* ***U***
is thus solution of:

*min ((****U*** *+* ***v****))*
D + K ***v***
D

*- W (**U** + **v**).*
T
C

***v** V*

C

*where **v***
is the tangential component of the increment of relative displacement of solid 2 compared to
T
solid 1 along the surface of contact, with the conventions adopted with [§2.1].

*Using the relations * (**v***
*) = K **v***
*and * (**v***
*) **R. v***
*if **R** C one deduces from it that **U** + **U***
is
C
T
T
C
T
T
solution of the problem of following MinMax, on space V of the fields kinematically acceptable:

*Min max J (**U** + **v***
*, **R**)*
v
V
R

where:

*J (**U** + **v***
*, **R**) =*
(

*(**U** + **v***
))
D + (.
R v
*- (**R**))*
*D - W (**U** + **v***
)

T
C
C

C

*The presence of the indicating function in this expression indicates that shearing **R** on surface contact belongs to the convex disc of friction C.*
C

2.5 Formulation
variational

If is convex, the problem of MinMax to be solved puts in an equivalent way in the form:

To find U V and R C, together of independent variables such as:
J (U + U, R) 0

This amounts solving the system of equations to following balance:

((U+U))
D + R v
D - L v =,
0

T
C
ext.

C

R U

D - (R),
0

T
C
C
C
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or in an equivalent way:

((U+U)) d+

(L - L v =
frot
ext.)
,
0

U (R),

T
C
R = (.n T

1

). on .c

As in the preceding section, L is the linear form associated the external forces. The form
ext.
linear L
is associated the forces of shearing exerted by solid 2 on the surface of
frot
contact of solid 1. It will be also noted that the variational formulation makes it possible to find not only equilibrium equations of the system but also membership of U
with under
T
differential of.
C
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3
Contact rubbing 2D and 3D in Code_Aster

We saw the formulation previously continues problem of rubbing contact. We go now to examine how it is expressed in discrete form.

3.1
Formulation of the problem

With each step of time N, one seeks to check the total balance of the structure:

L (U) = L - L

- L
int
N
ext.
cont
frot
where:
.
L is the operator of calculation of the internal forces
int
.
L is the vector of the external forces
ext.
.
L
is the vector of the forces of contact
cont
.
L
is the vector of the forces of friction
frot

Moreover, the field of displacement U is subjected to a whole of conditions equality and inequality which
N
include/understand connection by connection:

.
With (U) D
nc
N
nc
.
With (U) = D
C
N
C
.
TO (U - U) = 0
sg
N
n-1
.

WITH (U - U) = L
where 0
G
N
n-1
frot
where
.
L: together possible connections of contact (active and nonactive)
.
NC: together nodes of potential surfaces of contact which are not in contact (nonactive connections)
.
C: together nodes indeed in contact (active connections)
.
SG: together adherent nodes of contact
.
G: together slipping nodes of contact
.
C = SG G, C NC =
I
, L = C NC
.
A is the matrix of the nodes in contact
C
.
A is the matrix of the nodes in adherent contact
sg
.
A is the matrix of the nodes in slipping contact
G

Because of incremental nature of the resolution of balance, one can rewrite these equations and inequations in the form:

i-1
I
I
I
L (U
+ U

+ U
) = L - L

- L

int
n-1
N
ext.
cont
frot

subjected to:
.
i-1
I
WITH (U
+ U

+U) = D is
I
1
-
WITH U = I
D with
C
n-1
N
C
C
C
I 1
-
I 1
-
I 1
-
i-2
I 1
D
= D - With (U + U)
-
= D - WITH U = D - A U
C
C
C
N 1

-
N
C
C
N
C
C
N
.
WITH (U
i-1 +Ui) = 0 is
I
1
-
WITH U = I
D
sg
N
sg
sg
.
I 1
I
I
WITH (U
- +U) = L

where 0
G
N
frot

We have the discretized formulation of a problem of rubbing contact. We will see them various manners of taking into account the whole of conditions (or constraints) equality and inequality
who relate to the field of displacements: dualisation or regularization.
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3.2
Dualisation of the conditions of contact and friction

To take into account the constraints relating to the field of displacements, it is possible of dualiser i.e to utilize them in balance through multipliers of Lagrange (like that is done for the boundary conditions kinematics in the code). 3 sets are introduced of multipliers of Lagrange:

.
μ bearing on the conditions of contact
C
.
μ bearing on the conditions of adherence
sg
.
μ bearing on the conditions of slip
G

Balance is written then in the following form:

i-1
I
T
I
T
I
T
I
L (U
+ U

+ U) + A μ + A μ + A μ = L
int

n-1
N
C
C
sg
sg
G
G
ext.

subjected to:

.
I
1
-
WITH U = I
D
C
C
.
I
1
-
WITH U = I
D
sg
sg
.
I 1
I
I
WITH (U
- +U) = μ where 0 and I
I
I
μ
= K = μμ
G
N
G
G
G

C

This system allows the following interpretation of the multipliers of Lagrange:

T
.
I
With μ is the whole of the nodal forces of contact
C
C
T
.
I
With μ is the whole of the nodal forces of adherence
sg
sg
T
.
I
With μ is the whole of the nodal forces of slip
G
G

Note:

1) In the expression of balance, the condition of contact became an equality. Indeed, this equation is written for the nodes really in contact (for the active connections). It is a logic which takes as a starting point the the method of the active constraints established in code for the treatment of the unilateral contact [bib9]. It is nevertheless imperative to check condition:
μi > 0
C

2) Indeed: For a connection operator A associates the fields displacements U and
C
1
U the sum relative displacements U N
. + U N
. compared to the normals with
2
1
1
2
2

surface contact is the scalar (U - U .n. The operator T
A associates the scalar μ
1
2) 1
C
C
nodal forces μ N and μ N applying to the solids (1) and (2) respectively. These
C
1
C
2
nodal forces are equivalent to external forces - μ N applying to the solids
C
T
(1) and (2) respectively, which amounts transferring the term
I
With μ of the equation
C
C
of balance from left to right. In the surface scalar term close - μ N is equivalent to
C
N what implies the positivity of μ in the event of contact.
nn
C

3) It is also checked that for the nonactive connections one has well:
I
1
-
WITH U I
D.
nc
nc

4) Matrices A, A, A, A vary during reiterated. We will clarify these
nc
C
sg
G
variations more in detail with [§4].
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3.3
Dualisation of the conditions of contact, regularization of the conditions of friction

It is possible of dualiser the conditions of contact and to regularize the conditions of friction. One understands by there the fact of facilitating the treatment of friction by removing of its graph the infinite slope
in 0 i.e.:

R
μ nn
AND
u&t
- μ nn

This graph calls for several observations:

.
the concept of adherence with properly spoken disappeared, all the nodes slip. It is defined nevertheless by:
node I is “adherent” if, being given R = E U, R < μ

T & T
nn
· more the slope T
E is strong, plus the regularized graph approaches the graph not regularized
· in fact of regularization of the conditions of friction, it acts rather of regularization of

conditions of adherence

Taking into account the preceding remarks, one rewrites balance in the form:

i-1
I
T
I
T
i-1
I
T
I
L (U
+ U

+U) + A μ + E A A (U

+ U) + A μ = L
int
n-1
N
C
C
T
sg
sg
N
G
G
ext.

subjected to:

.
I
1
-
WITH U = I
D
C
C
.
I 1

I
I
E WITH (U
- +U) < μμ
T
sg
N
C
.
I 1
I
I
WITH (U
- +U) = μ where 0 and I
I
I
μ
= K = μμ
G
N
G
G
G
C
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3.4
Regularization of the conditions of contact and friction

With same logic that in the preceding paragraph, one defines the graph of the law of contact regularized:

nn
dn
IN

And one deduces the form from it from balance:

i-1
I
T
I
I
T
-1
i-1
I
T
I
L (U
+ U

+U) + E A (A U - D) + E A A (U

+ U) + A μ = L
int
n-1
N
NR
C
C
C
T
sg
sg
N
G
G
ext.

subjected to:

.
E WITH (
I 1
-
U +
I
U) < μ E (
I
I 1
-
WITH U - D)
T
sg
N
NR
C
C
.
I 1
I
I
WITH (U
- +U) = μ where 0 and I
μ = I
K = μ E (
I
I 1
-
WITH U - D)
G
N
G
G
G
NR
C
C

Note:

T

The term E A (
I
1
-
WITH U - I
D) is calculated only for the active connections. The use of
NR
C
C
C
X if X 0
+

the operator left positive [X] =
a compact writing of the law of contact allows
0 if not
T
regularized for all the possible connections in the form E A A U
D
. It would be
NR
C [
I
I
-
1
C
C] +
possible to use it to write the law of regularized adherence. We preferred to preserve partition between the various states of the connections because this presentation is closer to numerical integration in the code.

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4 Resolution algorithmic

4.1 Linearization of the various terms

It is the method of Newton which is used in Code_Aster for the resolution of the problems not linear [bib8]. It is in fact necessary to linearize the various terms appearing in the expression of balance.

4.1.1 Forces interns

The linearization of the operator of forces intern compared to
I
U
conduit with:

-1
-1
L

I
I
I
int
I
L (U
+ U

+ U
) L (U
+ U

) +
. U

int
n-1
N
int
n-1
N
I
U

I
U
+ -1
N
U
-1
N
i-1
I
I
L (U
+ U

) + K. U

int
n-1
N
N

I
K is the coherent tangent matrix which includes non-linearities behavioral and geometrical.
N

4.1.2 Forces of contact

In the absence of regularization of the forces of contact, one does not carry out any linearization. In contrary case, one with the linearization:

T
I
I 1
- T

I
I 1
- T
I 1
-
I
I 1
- T
I 1
-
With μ A
μ E A
WITH U - E WITH
D
C
C
C
C
NR
C
C
NR
C
C
Note:

.
During the linearization of T I
With μ the index i-1 in the matrix of contact I 1 appeared
-
A. In
C
C
C
effect, at the time of the determination of
I
U

, only the state of contact rubbing with the iteration i-1 is known.
T
.
The term
I 1

-
I 1
-
E WITH
A contributes a new share to the tangent matrix of the problem,
NR
C
C
T
while the term
I 1
-
I 1
-
E WITH
D contributes a new share to the second member.
NR
C
C

4.1.3 Forces of slip

Taking into account the definition of the forces of slip, they can be expressed:

With (
I 1
-
I
U

+ U)
I
I
G
N
μ
= μμ
G
C
With (
I 1
-
I
U

+ U)
G
N

With (
I 1
-
I
U

+ U)
I
G
N
= kg A (I 1
I
U

+ U)
G
N

They check the conditions indeed:

I 1
I
I
WITH (U
- +U) = μ where 0 and I
I
I
μ
= K = μμ
G
N
G
G
G
C
and reveal two unknown factors I
μ and
I
U.

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In practice, however, it is considered that the knowledge of the threshold of slip is acquired with the preceding iteration, which amounts being brought back to a criterion of Tresca for each iteration. With
convergence the threshold is obviously fixed: there are not thus more differences between the thresholds with the course
iterations. Another approach, used in [R5.03.52], amounts solving, for each state of
contact, the solution with friction and thus to consider that the threshold of slip is a fixed point. Our approach takes as a starting point this type of method while being less constraining and thus less expensive in
time CPU but can be less robust.
I
μ is thus approximated by:
G
I 1
With - (
I 1
-
I
U

+U)
I
I 1

-
G
N
μ
μμ
G
C
I 1
With - (
I 1
-
I
U

+U)
G
N

I 1
With - (
I 1
-
I
U

+U)
I 1
-
G
N
kg
I 1
With - (
I 1
-
I
U

+U)
G
N
Linearization of I
μ compared to
I

U
in the expression given previously led to
G
following formulation:

I 1
-
I 1
-
I 1
-
I
I 1
-
I 1
-
I 1
-
I 1
-
I 1
WITH U
WITH U
WITH U
WITH U. -
Ad interim
U
I
I 1
-
G
N
I 1
-
G
I 1
-
μ K
+ K
-
G
N
G

N
G
K

G
G
I 1
-
1
-
G
I
I 1
-
1
-
G
I
I 1
-
I 1
-
2
I 1
-
I 1
-
WITH U
WITH U
WITH U
G
N
G
N
G
N
WITH U
G
N

who is still written:

I 1
-

I 1
-
I 1
-
I
I 1
-
I 1
-
I 1
- T
I 1
- T
I 1
-
WITH U
WITH U
WITH U
U
With
Ad interim
U
I
I 1
-
G
N
I 1
-
G
I 1
-
μ
K
+ K
-
G
N
N
G
G
K
G

G
I 1
-
1
-
G
I
I 1
-
1
-
G
I
I 1
-
I 1
-
2
I 1
-
I 1
-
WITH U
WITH U
WITH U
G
N
G
N
G
N
WITH U
G
N

I 1
-
I
I 1
-
I 1
-
I 1

- T
I 1
- T
I 1
-
WITH U
WITH U
U
With
Ad interim
U
I 1
-
I 1
-
G
I 1
-
μ + K
-
G
N
N
G
G
K
G
G
I 1
-
1
-
G
I
I 1
-
I 1
-
2
I 1
-
I 1
-

WITH U
WITH U
G
N
G
N
WITH U
G
N
Note:

I 1
-
I 1
-
I 1
-
I 1
- T
I 1
- T
I 1
-
With
WITH U
U
With
With
•
The terms I 1
-
G
I 1
-
K
-
G
N
N
G
G
K
news brings

G
I 1
-
1
-
G
I
I 1
-
I 1
-
2
I 1
-
I 1
-
WITH U
WITH U
G
N
G
N
WITH U
G
N
contributions to the tangent matrix of the problem.
I 1
-
I 1
-
WITH U
.
In 2D, I
μ is linearized out of I 1
-
G
N
I 1
-
K
= μ which is independent of
I
U

, there is not thus
G
G
I 1
-
1
-
With
G
I
U
G
N
not new contributions to the tangent matrix.

4.1.4 Forces of adherence

In the absence of regularization of the forces of adherence, one does not carry out any linearization. In contrary case, one with the linearization:

T
I
I T
1
-
I
I T
1
-
I 1
-
I 1
-
I T
1
-
I
I
With μ A
μ E A
With

U

+ E A A
1 U
sg
G
sg
G
T
sg
sg
N
T
sg
sg

4.1.5 Notice

The matrices of contact A, slip A and nonslip A are brought to be
C
G
sg
modified during iterations of Newton if contacts change statute or if one geometrical reactualization takes place: they are thus subscripted
I
With, I
With and I
A. In the contrary case
C
G
sg
I
With, I
With and I
A do not vary during not reiterated.
C
G
sg
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5 Resolution

The resolution strictly speaking is approached in this part. Will be presented the total system solved in the framework of the method of Newton and the algorithm of treatment of the rubbing contact
(under-iteration). The initial state with the iteration i=1 for all these methods corresponds to the resolution of one
problem without contact nor friction. If there is indeed no detected contact, this initial state corresponds to the solution of the problem in the linear elastic case for example, if not it is modified as of this iteration in order to take into account the connections which would violate the conditions of contact
unilateral.

5.1.1 Dualisation of the conditions of contact and friction in 2D

Method of Newton:

.
With iteration I, at the total level, resolution of the system (one voluntarily does not make to appear the conditions of Dirichlet):

I
I
I 1
-
1
- T
I
I
1
- T

I
I
1
- T
I
I 1
K U = L - L (U)
-
- A
μ - A
μ - A
μ
N
ext.
int
N
C
C
sg
sg
G
G

Algorithm of rubbing contact:

.
Determination of the connections in contact

- Initial State: NC0 = L,
0
C =,
0
SG =,
0
G = 0. All points of surface
N
N
N
N
potential of contact are lack of contact.
- If i=1 first elastic design without taking into account of the contact. That is to say then
1
C =
. If 1

C = the solution without contact is
N
{connections t.q. 1
0
1
D = D - A U <
nc
nc
nc
} 0
N
valid. If 1
C then
1
1
SG = C, 1
G = 0 and resolution of the system of equations Ci
N
N
N
N
above with the new conditions for iteration 1.
- If geometrical reactualization I
C =
D
,
I
i-1
I
G = G C and
N
{connections t.q I <
nc
} 0 N N
N
I
I
I
SG = C - G.
N
N
N
- If not I

1
-
C = I
C,
I
1
-
SG =
I
SG, I
1
-
G = I
G
N
N
N
N
N
N

.
Resolution of the system

I
I
1
-
T
I
I
I 1
-
1
- T
I
I 1
K U + A
μ
= L - L (U)
-
- A
μ

N
c+sg
c+sg
ext.
int
N
G
G
I 1
-
I
I 1
-
With
U = D
c+sg
c+sg

One carries out for that a resolution per blocks:

1
I 1
I -
-
1 T
I
I
With
K
With
μ
= L - L (I 1
U)
1T
I
I 1
-
- A
μ
c+sg
N
c+sg
c+sg

ext.
int
N
G
G

1
I
I -
U = K (L - L (I 1
U)
1T
I
I 1
-
1 T
I
I
- A
μ - A
μ
)
N
ext.
int
N
G
G
c+sg
c+sg
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.
Checking of the state of the sliding joints

T
- If for a connection J I 1
-
μ
. I 1
-
With (
I 1
-
U +
I
U) < 0, Gi = Gi - {}
J and SGi = SGi + {}
J
G
G
N
N
N
N
N
I
With (
I 1
-
I
U

+U)
- If not I
I
G
N
μ = μμ

G
C
I
With (
I 1
-
I
U

+U)
G
N
- So at least a connection changed state, return to the resolution of the system of equations for same iteration I, but with the connections not slipping located above.

.
Checking of the state of the adherent connections

I
μ
- If for a connection J
I
I
μ
μμ,
I
I
sg
μ = μμ
, SG I = SG I - {}
J and
sg
C
G
C
I
μ
N
N
sg
Gi = Gi + {}
J
N
N

.

Checking of the state of the connections of contact

- If the nonactive connection J supposed is active, the most violated, i.e that whose play is it more negative, is added with the whole of the active connections, C I = C I + {}
J and
N
N
Gi = Gi + {}
J, return to the resolution of the system of equations for same iteration I,
N
N
but with the connections not slipping located above.
- If for a connection J I
μ < 0, C I = C I - {}
J SGi = SGi - {}
J Gi = Gi - {}
J (in
C
N
N
N
N
N
N
function of the type of the connection)

.

Update
I
U
, I
μ, I
μ, I
μ, I
With
and
I
A.
C
G
sg
c+sg

G

5.1.2 Dualisation of the conditions of contact and regularization of friction in 2D and 3D

For this modeling, the algorithm is the same one for the 2D and the 3D.

Method of Newton:

.
With iteration I, the total level, resolution of the system:

I
I
F
I
I 1
-
1
- T
I
I
1
- T
I
I 1
-
I
F
I 1
(K + K N) U = L
- L (U)
-
- A
μ - A
μ
- K naked
N
ext.
int
N
C
C
G

G
N
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Note:

I
.
The matrix
F
K *N contains the contributions of terms of slip and of adherence is:*
I 1
-
I 1
-
I 1
-
I 1
- T
I 1
- T
I 1
-
I
With
With
U
U
With
With
T

F
I 1
-
I 1
-
I 1
-
G
I 1
-
K** N = E **A
With
+ K
-
G
N
N
G
G
K

T
sg
sg
G
I 1
-
1
-
G
I
I 1
-
I 1
-
2
I 1
-
I 1
-
WITH U
WITH U

G
N
G
N
WITH U
G
N
.
In the preceding expression, a contribution is preceded by the sign -. The order of the terms of this contribution is the same one as that of the other terms. The effect of this contribution is particularly destabilizing for the total behavior of the tangent matrix with system, more particularly when one is far from balance and thus with the beginning from resolution with each new step of time. One thus decides to only take it into account partially by affecting it of a coefficient [
]
1
,
0. One advises to use an initial value from 0.5 for this coefficient and to decrease it if convergence is not obtained. In the case where = 0 convergence always seems to be obtained but is particularly slow. When one is close to the solution, it is on the other hand very useful to have a value of it coefficient equalizes to 1 in order to accelerate convergence. That is done automatically in the code when residue RESI_GLOB_RELA is lower than
3
10- .
I
.
The matrix
F
K *N contains the contributions of the new second members of adherence, that is to say:*
I
F
I 1
- T
1
-
K *N =*
I
E ***WITH***
With
T
sg

sg

Algorithm of rubbing contact:

.
Determination of the connections in contact

- Initial State: NC0 = L,
0
C =,
0
SG =,
0
G = 0. All points of surface
N
N
N
N
potential of contact are lack of contact.
- If i=1 first elastic design without taking into account of the contact. That is to say then
1
C =
,
1
1
G = C,
1
SG = 0. If 1
C =
N
{connections t.q. 1
0
1
***D** = **D** - **A** **U** <*
nc
nc
nc
} 0 N N
N
N
solution without contact is valid. If 1
C then
1
1

SG = C, 1
G = 0 and resolution of
N
N
N
N
system with the new conditions for iteration 1.
- If geometrical reactualization I
C =
D
,
I
i-1
I
G = G C and
N
{connections t.q I <
N
} 0
N
N
N
I
I
I
SG = C - G.
N
N
N
- If not I
1
-
C = I
C,
I
1
-
SG =
I
SG, I
1
-
G = I

G
N
N
N
N
N
N

.
Resolution of the system

I
I
F
I
1
- T
I
I
I 1
-
1
- T
I
I 1
-
I
F
I 1
*(**K** + **K** N) **U** + **A***
μ** = **L
*- **L** (**U**)*
-
*- **A***
μ
*- **K** naked*
N
C
C
ext.
int
N
G

G
N
I 1
-
I
I 1
-
WITH U = D
C
C

One carries out like previously a resolution per blocks.

I 1
***With** - (*
I 1
-
I
U

+U)
.
For all the connections, I
I
G
N
***μ** = μμ*

G
C
I 1
***With** - (*
I 1
-
I
U

+U)
G
N
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.
Checking of the state of the connections of contact

- If the nonactive connection J supposed is active, the most violated, i.e that whose play is it more negative, with the whole of the active connections C I = C is added I + {}
J and
N
N
Gi = Gi + {}
J, return to the resolution of the system of equations for same iteration I,
N
N
but with the connections not slipping located above.
- If for a connection J I
***μ** < 0, C I = C I - {}*
J Gi = Gi - {}
J
C
N
N
N
N

.
Checking of the state of the sliding joints and "adherent"

- If for a connection J
i-1

I 1
I
I
I
*E **WITH** (**U***
*- + **U**) < μμ = **μ**, then*
T
G
N
C
G
I
I
***μ** = E **A** (*
I 1
-
I
U

*+ **U***
), SGi = SGi + {}
J and Gi = Gi - {}
J.
sg
T
sg
N
N
N
N
N

I
I
.
Calculation of the tangent matrices
F
K
F
*N and **K** N (if RESI_GLOB_RELA<1.E-3, = 1.)*

.
Update
I

U
, I
μ*, I*
μ*, I*
μ*, I*
With*, I*
With *and I*
A.
C
G
sg
C
sg
G

5.1.3 Regularization of the conditions of contact and friction in 2D and 3D

For this modeling, the algorithm is the same one for the 2D and the 3D.

Method of Newton:

.
With iteration I, the total level, resolution of the system:

I
I
F
I
I 1
-
1
- T
I
I 1
-
I
F
I 1
-
1
- T
I
I 1

(K + K N) U = L - L (U)
-
- A
μ - K naked + E A
D
N
ext.
int
N
G
G
N
NR
C
C

Note:

I
.
The matrix
F
K N contains the contributions of terms of contact, slip and
of adherence is:
I 1
-
I 1
-
I 1
-
I 1
- T
I 1
- T
I 1
-
I
With
With
U
U
With
With

T
T

F
I 1
-
I 1
-
I 1
-
I 1
-
I 1
-
G
I 1
-
K N = E A
With
+ E A
With
+ K
-
G
N
N
G
G
K

NR
C
C
T
sg
sg
G
I 1
-
1
-
G
I

I 1
-
I 1
-
2
I 1
-
I 1
-
WITH U
WITH U
G
N
G
N
WITH U
G
N
I
.
The matrix
F
K N contains the contributions of the new second members of adherence,
that is to say:
I
F
I 1
- T
1
-
K N =
I
E WITH
With
T
sg
sg
T
.
The term
I 1
-
1

-
-
I
E WITH
D contains the contributions of the new second members of
NR
C
C
contact.

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Algorithm of rubbing contact:

.
Determination of the connections in contact

- Initial State: NC0 = L,
0
C =,
0
SG =,
0
G = 0. All points of surface
N
N

N
N
potential of contact are lack of contact.
- If i=1 first elastic design without taking into account of the contact. That is to say then
1
C =
,
1
1
G = C,
1
SG = 0 and
N
{connections t.q. 1
0
1
D = D - A U <
nc
nc
nc
} 0
N
N
N
1
μ = E (1
1
0
WITH U - D).
C
NR
C
C
- If geometrical reactualization I
C =
D
,
I
I
G = C,
I
SG = 0
N

{connections t.q I <
nc
} 0 N N
N
- If not is I
C =
D
, I
μ = E (I
I
I 1
-
WITH U - D), I
I
G = C,
I
SG = 0
N
{connections t.q I <
nc
} 0 C NR C
C
N
N
N
I 1
With - (
I 1
-
I
U

+U)
·
For all the connections, I
μ = μ E (I
I
I 1
WITH U - D -)
G
N

G

NR
C
C
I 1
With - (
I 1
-
I
U

+U)
G
N

.
Checking of the state of the sliding joints and “adherent”

- If for a connection J
I 1
-
E WITH (
I 1
-
U +
I
U) < μ E (I
I
I 1
-
WITH U - D), then
T
G
N
NR
C
C
I
I
μ = E A (
I 1
-
I
U

+ U
), SGi = SGi + {}
J and Gi = Gi - {}
J
sg
T
sg
N
N
N
N
N

I
I
.
Calculation of the tangent matrices
F
K
F
N and K N (if RESI_GLOB_RELA<1.E-3, = 1)

.
Update
I
U
, I
µ, I
µ, I
µ, I
With, I
With and I
A.
C
G
sg
C
sg
G

5.1.4 Dualisation of the conditions of contact and friction in 3D

For the resolution of this problem, one defines the statute of the connections using regularization: initially

all the adherent connections are treated by regularization with a term of penalization E
T
determined by the code. One then repeatedly increases the value of E per E = 10th and one
T
T
T
remove the connections not checking the condition
i-1
I 1
I
I
E WITH (U
- +U) < µµ. When the process
T
G
N
C
is stabilized, the adherent connections and the sliding joints are treated by multipliers of Lagrange and the penalization does not appear.

Method of Newton:

.
With iteration I, at the total level, resolution of the system (one voluntarily does not make to appear the conditions of Dirichlet):

I
I
F
I
I 1
-
1
- T
I
I
1
- T
I
I
1
- T
I
I 1

(K + K N) U = L - L (U)
-
- A
μ - A
μ - A
μ
N
ext.
int
N
C
C
sg
sg
G
G

Note:

I
.
The matrix
F
K N contains the contributions of terms of slip is:
I 1
-
I 1
-
I 1
-
I 1
- T
I 1
- T
I 1
-
I
With
WITH U
U
With
With
F

I 1
-
G
I 1
-
K N = K
-
G
N
N
G
G
K

G
I 1
-
1
-
G
I
I 1
-
I 1
-
2
I 1
-
I 1
-
WITH U
WITH U
G
N
G
N
WITH U
G
N
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Algorithm of rubbing contact:

.
Determination of the connections in contact

- Initial State: NC0 = L,
0
C =,
0
SG =,
0
G = 0. All points of surface
N
N
N
N
potential of contact are lack of contact.
- If
I =1,
1
C =
,
1
1
SG = C,
1
G = 0,
N

{connections t.q. 1
0
1
D = D - A U <
nc
nc
nc
} 0
N
N
N
E = (max {diagonal terms of 1
K}) 25
.
0

T
N
- If geometrical reactualization, I
C =
D
,
I
i-1
I
G = G C and
N
{connections t.q I <
nc
} 0 N N
N
I
I
I
SG = C - G.
N
N
N
- If not I
1
-
C = I
C,

I
1
-
SG =
I
SG, I
1
-
G = I
G
N
N
N
N
N
N

.
Resolution of the system

I
I
1
-
T
I
I
I 1
-
1
- T
I
I 1
K U + A
μ
= L - L (U)
-
- A
μ
N
c+sg
c+sg
ext.

int
N
G
G
I 1
-
I
I 1
-
With
U = D
c+sg
c+sg

One carries out for that a resolution per blocks like previously.

.
Checking of the state of the connections of contact

- If the nonactive connection J supposed is active, the most violated, i.e that whose play is it more negative, is added with the whole of the active connections, C I = C I + {}
J and
N
N
Gi = Gi + {}
J, return to the resolution of the system of equations for same iteration I,
N
N
but with the connections not slipping located above.
- If for a connection J I
μ < 0, C I = C I - {}
J SGi = SGi - {}
J Gi = Gi - {}
J (in
C
N
N
N
N
N
N
function of the type of the connection)

.

Checking of the state of the adherent connections

I
μ
- If for a connection J
I
I
μ
μμ,
I
I
sg
μ = μμ
, SG I = SGi - {}
J and
sg
C
G
C
I
μ
N
N
sg
Gi = Gi + {}
J
N
N

.
Checking of the state of the sliding joints

- If for a connection J
i-1
I 1
I
I
E WITH (U
- +U) < μμ, Gi = Gi - {}
J and
T
G
N
C

N
N
SGi = SGi + {}
J
N
N
I 1
With - (
I 1
-
I
U

+U)
- If not I
I
G
N
μ = μμ

G
C
I 1
With - (
I 1
-
I
U

+U)
G
N
- So at least a connection changed state, i.e one detected an adherent connection among slipping supposed connections, then E = 10th and return to the resolution of the system
T
T
equations for same iteration I, but with the connections not slipping located above.
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I
.
Calculation of the tangent matrices
F
K N (if RESI_GLOB_RELA<1.E-3, = 1).

.
Update
I
U
, I
µ, I
µ, I
µ, I
With
and
I
A.
C
G
sg
c+sg
G

5.1.5 Convergence

The convergence of the algorithm with multipliers of Lagrange for the contact without friction in a finished number of iterations was shown in [bib2]. For the problems with friction, of results of convergence with unicity of the solution to the discretized problem are established in [bib6] for low values of the coefficient of friction of Coulomb. The results are established in

using an algorithm of point fixes associated with a method of multipliers of Lagrange. For each problem of solved contact, one studies the problem of associated friction. Once this one solved, one solves a new problem of contact and so on. These methods are however different from those presented here and one cannot thus have results of convergence theoretical for these last.

The condition of fastening of the points which come in contact is particularly important for to ensure the convergence of the method with multipliers of Lagrange. Indeed when a point returns to the contact during reiterated its tangential displacement remains free. A condition of not slip would be far too constraining. The algorithm would oscillate then between two states with or without
contact in examples of the type of that presented in [V6.04.105]. The point which is attached is thus regarded as free from the point of view of the slip. One can then calculate the normal reaction as well as the tangential reaction by using the assumption of slip and one estimated of the increment of displacement of initial slip.

The use of the penalization alone makes it possible to avoid these oscillations while making it possible to slacken them
constraints on the preceding system. Coupled with a method of multipliers of Lagrange, one find pathology announced above.

5.1.6 Notice

The method of usual research linear RECH_LINEAIRE of STAT_NON_LINE is not usable in this case. Indeed, a correction takes place already on the field of displacement, which is supposed to be optimal for the realization of the conditions of contact. One thus risks if it is corrected field during iterations of Newton not to have compatibility between displacements more and reactions, which makes the method particularly unstable and involves an absence of convergence.

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6
Compatibility with the boundary conditions of Dirichlet

In the case of the methods with multipliers of Lagrange, one can observe incompatibilities with the fact of imposing boundary conditions of the Dirichlet type. Indeed, it is necessary that physically it problem has a direction. One cannot deal with problem of contact in the direction of axis Z if all the points have a null displacement according to Z. As we will see it, to deal with such a led problem T with a singularity of the matrices of type A K 1 - A with the treatment of the boundary conditions of C C Dirichlet by double lagrange of Code_Aster.

6.1
Writing of the boundary conditions

While taking as a starting point the the reference material [R5.03.01] of STAT_NON_LINE, the dualisation of boundary conditions of Dirichlet D DRUNK = U (T) led to the system of equations following to solve:

I T I 1

C U + B = L - L (- U) ext. int N

I D I 1 -

DRUNK = U - BUn

One notes K then the matrix of rigidity of the system such as:

C BT
K =

B
0

This matrix has a reverse of the form:

E
F
-1

K =

T

F
G
such as: EBT = 0.
One checks thus that for each boundary condition I one with property EB T = 0.
I

6.2
Return to the problem of contact

T
T
Matrix A K 1
- A can be also written A EA since the vectors of connection A do not make
C
C
C
C
C
to intervene that degrees of freedom of displacement.

.
It from of results whereas if a vector of connection J of matrix A is a linear combination

C
boundary conditions of the Dirichlet type it checks the following property: EA T = 0.
C J
T
stamp A EA is then singular because it has a column of zeros. In
C
C
practical, without particular treatment, one finishes in the code on a message of stop of the type STOP ON MATRIX OF CONTACT-FROTTEMENT SINGULIERE. The detection of these singular columns was implemented in the code in order to eliminate from the relations from contact-friction this type of relations and to avoid the stop previously described.
.
It from of results whereas if a vector of connection J of matrix A contains a combination
C
linear of the boundary conditions of the Dirichlet type and A is written
= B + A, it checks
C J
I
I
C J
T
following property:
T
T
EA
= EA
. One can then have a singular matrix A EA
C J
C J
C
C
because it has two identical lines. This detection is not for the moment not available in the code and one finishes in the code on a message of stop of the stop type on matrix of contact-friction singular.
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6.3
Illustration on a simple example

The two preceding situations can meet for the same example of study. That is to say one surface being able to slip into the xOy plan. It is supposed blocked in direction X. If direction of blocking corresponds to the one of the principal directions of slip which the user can give in the command file one is found in case 1 of [§7.2]. If the direction of blocking is tilted compared to the principal directions of slip then one finds oneself in case 2 of [§7.2]. One of the two directions of slip is of too much to characterize the physical system.

6.4 Notice

This problem of compatibility between contact-friction and the boundary conditions does not appear with the regularized methods insofar as one adds rigidity with total rigidity and that one does not make elimination as in the calculation of the lagranges.
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7
Implementation in Code_Aster

The call to the routines for the contact with friction in Code_Aster takes place at the same place in STAT_NON_LINE that for the unilateral contact without friction. The phase of prediction to the step of
time N is based on the increment of load between the steps of time n-1 and N.

7.1 Algorithms

These developments are accessible under order STAT_NON_LINE. They are activated by key word CONTACT of the order AFFE_CHAR_MECA with which one defines the zones of contact possible. The whole of the parameters of the model is provided in CONTACT under the key word FRICTION:

FROTTEMENT= “WITHOUT”
“COULOMB”

The various algorithms are chosen by the key word METHOD according to logic:

“LAGRANGIAN” METHODE=
Dualisation contact friction 2D and 3D
“PENALIZATION”
if indicated E_T, dualisation contact, regularization friction
if indicated E_T and E_N, regularization contact and friction

One introduces also the COULOMB key words for the value of the coefficient of friction of Coulomb, and
COEF_MATR_FROT the coefficient of taking into account of the negative component of the tangent matrix
of friction ranging between 0 and 1. The user can thus define the loading in the following way :

CHA =AFFE_CHAR_MECA (MODELE= MO,
CONTACT= _F (GROUP_MA_1 = ISOL1, GROUP_MA_2 = ISOL2

If the method used is “LAGRANGIAN”, no other indication is necessary in 2D and it is necessary to provide COEF_MATR_FROT in 3D. In the case of the penalized methods it is necessary to give the value of
coefficient of penalization E_T, E_N and the COEF_MATR_FROT in all the cases.

The loading thus defined is then used in STAT_NON_LINE:

RESU = STAT_NON_LINE (MODEL = MO, CHAM_MATER = CHMAT,
EXCIT = _F (LOAD = CHA),
NEWTON=_F (REAC_ITER=1),
SOLVEUR = (METHOD = "LDLT") etc…);

It is noticed that one recomputes the tangent matrix with all the iterations of Newton (NEWTON=_F (REAC_ITER=1)).

Note:

With GCPC like method of resolution, one can make only contact without friction for the moment. One does not recommend however to do it because the performances in term of time calculation are not good with this method. One recommends the employment of MULT_FRONT with a renumerotation of the "MONGREL" type.

7.2
Geometrical reactualization

The geometrical evolution is one of non-linearities constitutive of the rubbing contact. Within Code_Aster, its taking into account is done by a method of point fixes (for more precise details, to defer to [bib9]). One points out the use of the key word:

REAC_GEOM= "WITHOUT" step of reactualization
"AUTOMATIC" reactualization managed by the code
"CONTROL" N reactualizations at the beginning of step of time, then afterwards each convergence until the n-1ème. N is defined by:

NB_REAC_GEOM=n

7.3 Post treatment

Different post treatments is possible. For example, the calculation of the efforts of contact can be carried out in order POST_RELEVE_T by calculating the resultant of the nodal forces on group meshs representing one of surfaces of contact.
One draws the attention to the structure of data VALE_CONT which is produced for each calculation implying rubbing contact. It is printed as follows in the form of table:

```
MATABLE=POST_RELEVE_T (ACTION=_F (INTITULE=' INFOS FROTTMNT',
GROUP_NO=' ESCLAVE',
RESULTAT=U,
INST=10.,
TOUT_CMP=' OUI',
NOM_CHAM=' VALE_CONT',
OPERATION=' EXTRACTION',),);

IMPR_TABLE (TABLE=MATABLE);
```

The information printed in each node slave is as follows:

.
CONT: indicator of rubbing contact
- 0: no the contact
- 1: slipping contact
- 2: adherent contact
.
PLAY: value of the play
.
RN: normal reaction of contact normalizes
.
RNX: component according to DX of the normal reaction of contact
.
RNY: component according to DY of the normal reaction of contact
.
RNZ: component according to DZ of the normal reaction of contact
.
GLIX: component according to T1 of the tangential slip (local reference mark)
.
GLIY: component according to t2 of the tangential slip (local reference mark)
.

GLI: normalizes tangential slip

RTAX: component according to DX of the tangential force of adherence

RTAY: component according to DY of the tangential force of adherence

RTAZ: component according to DZ of the tangential force of adherence

RTGX: component according to DX of the tangential force of slip

RTGY: component according to DY of the tangential force of slip

RTGZ: component according to DZ of the tangential force of slip

X-ray: component according to DX of the force of rubbing contact (RNX+RTAX+RTGX)

RY: component according to DY of the force of rubbing contact (RNY+RTAY+RTGY)

RZ: component according to DZ of the force of rubbing contact (RNZ+RTAZ+RTGZ)

R: force of rubbing contact normalizes

Moreover, it is possible to trace this information for a post graphic treatment:

IMPR_RESU (MODELE=MO,

RESU=_F (FORMAT=' CASTEM' or "GMSH" or "IDEAS",
RESULTAT=U,
NOM_CHAM=' VALE_CONT',
NOM_CMP= ("CONT", "RNX", "RNY", "RNZ", etc.),);

One presents below a cylinder in a boring with interaction of rubbing contact. One traced normal forces of contact, tangential forces of adherence and the indicator of contact.

7.4 Precautions of use

These precautions of use are about the same ones as those stated in [R5.03.50]. One them recall here:

- ***to check that the normals on the surfaces of contact are outgoing (to be wary in particular if one has used operators of symmetrization in the maillor gibi),***
- ***attention with the contact friction in quadratic 3D if the meshs of edge are QUAD8 (to avoid using HEXA20 to net volume): to use HEXA27 preferably, or many PENTA15 whose TRIA6 sides are the meshs of contact,***
- ***to remove, by boundary conditions of Dirichlet adapted, the movements of body rigid; it is not necessary that the structure holds only by the contact or friction. In others terms, that wants to say that a calculation made in elasticity with order STAT_NON_LINE without to treat the contact must pass,***
- ***in the event of structure "held" only by the contact, one can add a spring of weak rigidity to maintain it,***
- ***not to use method of research linear, a STAT_NON_LINE incompatible with treatment of the conditions of contact-friction***

8 Conclusion

Discrete modelings of contact-friction with slip surfaces 1D and 2D were established in Code_Aster. These modelings usable with STAT_NON_LINE and DYNA_NON_LINE are accessible under AFFE_CHAR_MECA by the operator from contact with friction in great CONTACT displacements.

Contrary to [bib5] modelings suggested are not pressed on dedicated finite elements. They are based on the grids of surfaces coming in contact and make it possible to retranscribe node with node conditions of contact friction between surfaces after discretization of the formulation variational corresponding. The method extends then without difficulty of small displacements to the case great displacements. Indeed, the absence of use of finite elements, between surfaces being able to come in contact, avoids the great distortion of the latter, in the case of great displacements. One can then use is conditions of direct connections nodes to nodes for grids initially compatible, that is to say conditions of connections nodes to nodes balanced according to one approach by projection of the master-slave type for incompatible grids. The different ones conditions of connection are developed in documentation [R5.03.50] available on the treatment contact without friction in great displacements.

In the case of slip surfaces 1D one could only develop a using algorithm multipliers of Lagrange. The finished convergence of this type of algorithm is proven for the contact unilateral without friction [bib2] and in the case with friction for low values of the coefficient of friction of Coulomb [bib6]. In the case of slip surfaces 2D, the rubbing contact is treated either by dualisation or by regularization with various mixings.

One always advises the use of the dualisation on the contact and friction for the 2D:

method does not utilize of new tangent matrices and it does not present difficulties major in term of use except compatibility with the conditions of Dirichlet, cf [§6]. In 3D, one always advises to use the dualisation on the contact and friction; nevertheless it can afterwards blow being interesting to test the method with dualisation of the contact and penalization of friction: the problems of compatibility with the conditions of Dirichlet are then less and times of calculation can be reduced. One insists on the other hand on the very strong dependence of the result with value of the terms of penalization. For systematic studies, one can nevertheless test validity of the solution regularized compared to the solution with dualisation, used like reference on a standard study.

---

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R5.03 booklet: Nonlinear mechanics
Document: R5.03.52

Elements of contacts derived from a formulation continuous hybrid

Summary:

***This document describes the way in which the elements of rubbing contact are derived from a hybrid formulation
continue problems of contact between solids (2D or 3D) in great transformations and specifies the strategy
of resolution used [bib1], [bib2].***

The approach is implemented in Code_Aster. It is usable with modeling STAT_NON_LINE in assigning to the key word CONTACT, under AFFE_CHAR_MECA, key word METHODE=' CONTINUE'.

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1 Introduction

There exists, for the treatment of the problem of contact-friction, an important "gap" between discrete formulations and the formulation continues. The implementations in the computer codes are (often) founded on discrete models such those developed by [bib3], [bib4], [bib5] or, more recently, [bib6]. The latter are without clear bonds with the formulation continues. That poses question of the precision and the nature of the results obtained: towards what does converge one?

The two essential objects of this document are on the one hand the description of the derivation of elements
of contact starting from a hybrid formulation continues of a problem of contact between solids three-dimensional, deformable, undergoing great transformations, and in addition the detail of strategy of resolution [bib1], [bib2], [bib7].

Section 2, devoted to the continuous hybrid formulation, comprises five paragraphs. In paragraph [§2.1], one points out a Lagrangian formalism of the Principle of Virtual Work for two deformable solids which can come into contact. Thanks to an application of pairing and to Principle of the Action and the Reaction detailed in the paragraphs [§2.2] and [§2.3], respectively, one gives a simplified expression of virtual work of the forces of contact. paragraph [§2.4] is devoted to equivalent writings of the laws of contact and friction which lend to weak formulations. The continuous hybrid formulation of the problem is specified in paragraph [§2.5]. One specifies the strategy of resolution in section 3. The latter is founded on algorithms of fixed point and tangent module. In section 4, one discretizes, by finite element method, the continuous hybrid formulation suggested in the paragraph [§2.5] and one evoke the difficulties due to the incompatibility of the discrete models of interfaces. Key words concerning the implementation of this approach in Code_Aster are given in section 5.

2
Formulation continues hybrid problem of contact

Two solids I are considered
B (I =,
1 2) deformable, presumedly elastic (for the clearness of
document), in rubbing contact. These two solids occupy in their initial configuration adherence of two fields 1

and 2
from 3
R and in their current configuration (at the moment T) the adherence of
1
and 2
, respectively. It is supposed that, in their initial configuration, these two solids are
T
T
in a natural state, such as without residual stresses or predeformations. During their movement, they can come into contact, as indicated on [Figure 2-a]. The border of each solid
I
B is broken up into parts I
, I
and I
in the initial configuration, of which them
0
G
C
intersections are empty 2 to 2, and out of I
, I
and I
, deformed the preceding ones, in the configuration
0
G
C
current. The description of this partitition is given to [§2.1]. Solid I
B is embedded on I
and
0
subjected to a nominal density of surface forces noted I
G on part I
. In addition, one notes
G
I
F the voluminal field of density of efforts applied to solid I
B (I =,
1 2). Parts of
surfaces
I

, likely to come into contact at the time of the deformation of the two solids, are noted
I

. One supposes the existence of noted regular charts I
describing surfaces I
. These charts are
C
C
defined as follows:
I
I
3
: R
(

éq 2-1
,
I
I
p =,
1
2)
(1 2)
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where
I
is a field limited (of reference) contents in 2
R. In addition, one indicates by I

transformation of solid I
B, defined by:

I
I
I
: T éq 2-2
I
I
p X

I is noted
N the unit normal with I
external with I
and I is noted
N its opposite in the configuration
p
C
X
current (cf [Figure 2-a]). One indicates by I
U the field of displacements of solid I
B and by I
F it
tensor gradient of deformation, defined by:

Fi =
I
p, T

éq
2-3
p
(I) = ui +Id
p

Appear 2-a: Description of the mechanical problem

2.1
Principle of Virtual Work

By using the notations introduced previously, the local equations of balance, the conditions initial and the boundary conditions of the problem considered are written in the following form:

2 I

I
I
I
U
I
Div + F
=

in
p
T
2
I I
I
I
N
= G

on
p
G
I
I
U
= 0

on
éq
2.1-1
0
I I
I
I
N
= R

on
p
C
I
U (p, 0)
I
I
I
I
I
U (p 0
,) = U (p) and
U (p 0
,) = v (p)

in

p.p
0
T

0

where
I
indicate the first tensor of the constraints of Piola Kirchoff, I
is the density on
p
initial configuration, I
U is the field of displacements and I
R is the density of the efforts, due to
possible interactions of contact rubbing between the two solids, unknown factor of the problem.
Moreover,
except mention clarifies contrary, exhibitor I takes here, and subsequently, value 1 or 2. Us
let us suppose, inter alia, to concentrate us on the problem of contact, that the density of the forces
voluminal as that of the surface efforts applied to the two solids are null, i.e.,
F I = 0 and gi =.
0
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One indicates by
I
(CA) the space of the fields of displacements kinematically acceptable for
solid I
B. The Principle of Virtual Work is written:

For all
I
I
W (CA),
2 I

U
I
-
I

W
. D =
I I
Tr F S
T
I
W
D
I
p
2

I ((p ())

T

éq
2.1-2
- I
I
R p
(, T W
). di
I
C
C
In [éq 2.1-2], the point in fat (.) represent the Euclidean scalar product in
3
R and
T
(*) is
transposed of (*). The trace of a tensor of order 2 is noted Tr and I
S indicates the second tensor of
constraints of Piola-Kirchhoff, related to the first tensor by the following relation:

I
S = (I
F) 1 - I
éq
2.1-3

Without restricting the general information concerning the mechanics of contact (object of this document), one
that the materials constituting the two solids are hyperelastic, i.e. will suppose,

I

I
I
W (I
F)
=

éq
2.1-4
p

I
F

where
I
W is the mass density of local internal energy definite on solid I
B.

2.2
Pairing and kinematic condition of noninterpenetration

To translate nonthe interpenetration, one proceeds as follows:

1) one couples the points of two surfaces of contact: it is pairing,
2) one imposes between the two points of a couple of points paired nonthe penetration according to one
direction given.

The first stage can be modelled while seeking, for any item 1
X T
( )
1
= (1
p, T) of the border
1
, the point of 2
who is closest to him. This amounts solving the problems of optimization,
C
C
under constraints, following:

For all 1
1
p (thus 1
1
p = ()
1
,) and any T 0,
C

To find (, T) = (, such as:
1
) 2

2

(, T =
1
)
ArgMin (1
(1 (), T) - 2
(2 ( )
2
, T))
2
,
éq 2.2-1
2

The solution of this problem formally makes it possible to define, at any moment T, an application of pairing,

A: 1
2

T

éq
2.2-2

(, T)

1
This application defines, also, item 2
p (1
p, T)
2
= ((1
p, T), noted p, paired to 1
p, at the moment
1
T. It defines also item X, paired as in point 1
X.

1
The condition of noninterpenetration between 1
X of 1
and X of 2
is written in the direction of
C
C
1
N = N
-
, the unit normal with 2
, in X, interior to 2
, in the form:
X (1
2 X)
C
T

D =

éq
2.2-3
N
(1
1
X - X) .n 0

A second approach (cf [bib8] for comments) consists in introducing a direction of acceptable research [bib9]. One of these directions (cf [bib8], [bib2]) is the field speeds standardized V.

Thereafter one will note X the quantity which represents the scalar product X N
. or the projection of
N
X on N.

One however continues to impose nonthe penetration like previously. The influence, in practice, choice of the strategy of pairing on the resolution of the problems of contact is shown in [bib10], [bib8].

Notice 1:

It is pointed out that, under the assumption of small displacements, pairing is made only one only time. In great transformations, pairing depends on the deformation and introduces not geometrical linearity of contact.

Notice 2:

Although pairing seems to introduce a dissymmetry of treatment between the two surfaces constituting the interface of contact, the approach remains "democratic", uninterrupted. dissymmetry is in fact due to the discretization (by the finite element method) and inspired with certain authors the main concept/slave [bib11] or of geometrical surfaces/kinematics [bib1].

2.3
Principle of the Action and the Reaction

By using the procedure of pairing, described in the paragraph [§2.2], the Principle of the Action and of
Reaction (BY) is written in the following local form:

1
R (1
p, T) 1
2

D
+ R p T D
éq
2.3-1
C
(1,) 2 = 0
C
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By taking account of the principle of the action and the reaction [éq 2.3-1] and the assumption made on
pairing, one writes the low balance of the efforts as follows:
(1 2
W, W) (CA) 1 × (CA) 2

2
2 I
2

U
I
-
I

W
.

D
=
I I
R F S
I
W

D

I
p
2
I ((p ())
i=1
(T)

i=1

éq
2.3-2
- R (p, T). [W]

D

C
C

In [éq 2.3-2], one used the following notation:

*[] * (1p) = () 1*
** (1*
p) ()
-
1
2
***
p

éq
2.3-3

Moreover, one posed
1
= and
1
R = R. One announces, finally, that the density of efforts 2
R is prolonged by
C
C
zero at the points of 2

without opposite on 1
.
C
C

2.4 Laws of contact

One breaks up the density of effort of contact R into a normal part which indicates the pressure normal of contact and another tangential
r. Thus, the effort of contact R is written:

R = N + R
éq
2.4-1

where N is the unit normal, defined in the paragraph [§2.2].

The laws of Signorini are written in the following form:

,
0
D,
0
D
= 0
éq
2.4-2
N
N
where D is the directed distance, defined by [éq 2.2-3].
N

By introducing the function characteristic of
-
R, noted:

(
if
X), 1 X 0
=

éq
2.4-3

,
0 if X > 0

and the multiplier (known as of increased contact [bib3]), noted G, defined by:
N

G = - D

éq 2.4-4
N
N
N
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where is a strictly positive reality (noted
N
COEF_REGU_CONT in the fochiers of order), the laws
of contact [éq 2.4-2] are written as follows:

- (G G

éq
2.4-5
N)
= 0
N

For the phenomena of friction, one uses the isotropic laws of Coulomb which are written like follows:

R
μ
(p, T)
(p, T)
If R
< μ
=
(p, T)
(p, T)

then
v (p, T) 0

éq 2.4-6
If R
= μ

=
-
(p, T)
(p, T)

then
0; v (p, T)
R (p, T)

where μ is the coefficient of friction of Coulomb and v are tangent relative speed. One defines this speed v, in a given point of the surface of contact, by:

v (
1

1
1
1
p,
2 p,
p, T) = (Id - N N)
(T)
(T)

-
= (Id - N N) v (1
p, T) éq
2.4-7

T

T

One notes thereafter
X the projection of X on the tangent level on the surface of contact, defined by
X = (Id - N N) X

, where the symbol indicates the tensorial product.
An equivalent formulation of the laws [éq 2.4-6] is as follows [bib12]:

R =

µ

éq
2.4-8
-
G =
éq
2.4-9
0, ()
( )
0
1
G = +

v

éq 2.4-10

In these quantities, definite on, is a strictly positive parameter, is one
C

semi-multiplier (vectorial) of friction, G is the semi-multiplier (vectorial) of friction
increased and
is projection on the ball unit.
(0 1
,)

The laws of friction are supplemented by the equation (of exclusion type) following:

D = 0 or
G
éq
2.4-11
N
(1 - () N) = 0
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2.5
Quasi-static hybrid formulation

In the continuation of the document, one adopts the assumption of quasi-staticity of the movement. They are neglected
terms of inertia. With a stage of loading given (K), corresponding to the fictitious moment T, one
K
suppose known the fields at the moment T
noted I
U*, and*
, and the new ones are sought
K 1
-
K 1
-
K 1
-
K 1
-
fields at the moment T.
K

While using [éq 2.3-2], [éq 2.4-5], [éq 2.4-9] and [éq 2.4-11], one derives the weak, hybrid formulation (with
three fields: ***U*** *= (1*
U*, 2*
U*),*

and
) and quasi-statistics of the problem of contact, following:

*To find (**u1**, **u2***
1
2
, ,
;
K
K
K
*K) (**CA**) × (**CA**) × H × **H***
(
w1**, **w2
*,) (**CA**) 1 × (**CA**) 2 × H × **H***

*2 Tr **F** **S***
W

I
(I I ((I D
G
G
W
D
p
)) - (nk) nk []

C
nk
C
i=1

éq
2.5-1
-
µ (G
G**. **W
D
0
nk) K
0,1 (K) [
]

( )
=

C
K
C
*-1 {- G G *d 0* ***éq***
2.5-2
K
(nk) nk}

=
C
C
N
- µ (G
nk)

K {-
G
. D
K
0,1 (
K
)}

( )

C
C

éq
2.5-3
+
(1 - (G D
,
0
nk)

=
C

K
C
G = - D

éq 2.5-4
nk
K
N
nk
G** = + **Id** - **N N X

éq
2.5-5
K

K
K
(
K
) [K
K
]

where =
is a plus coefficient (is noted
K

COEF_REGU_FROT in the files of
T

K
order Code_Aster) and (K
**) is the increment of (*), at the moment T. It is noted that a diagram of Euler has*
K
summer used to discretize in time the field speeds.

3
Strategy of resolution

The problem describes by the formulation [éq 2.5-1] with [éq 2.5-5] is strongly nonlinear. Indeed, in addition to
not "traditional" linearity due to the framework of the great deformations (and that possibly due to non-linear behaviors of materials), several levels of non-linearities inherent in
phenomena of rubbing contact, can be distinguished:

.
Non-linearity due to the ignorance of the effective surface of contact.
It is solved by an iterative strategy which can be brought closer to the method of
active constraints implemented in Code_Aster (cf [bib13] (tallies linear), [bib14] (tallies non-linear)).
.
The geometrical non-linearity of contact defined notices of them 1 of the paragraph [§2.2]).
It is solved by a fixed algorithm of point on the geometry.
.
Non-linearity related to the threshold of friction (dependent on the field, unknown).
This non-linearity is also solved by the method of the fixed point.

Other non-linearities are treated by an algorithm of the type modulates tangent (or Newton generalized). The diagram general of the algorithm is thus the following:

***I.** Loop on the stages of time:*

*fields **U**, and known at the preceding stage*
***II.** Buckle on the geometry:*
***N** = **Nd**, = D*

and

T = D (new pairing)
***III.** Buckle on the thresholds for friction:*
= S (only for the terms of friction)
***IV.** Buckle on the surface of the contacts with a method of the type forced active:*
= D
***V.** Loop of tangent module:*
generalized linearization
***VI.** End of the loop of tangent module*
***VII.** End of the loop of the method of the active constraints*
***VIII.** End of the loop on the thresholds*
***IX.** End of the loop on nongeometrical linearity*
***X.** Fine of the loop on the stages of time.*

Notice 3:

By defect in Code_Aster the initial statute of the nodes slaves is not contacting. In addition is initialized to zero. That amounts starting by solving the problem without contact with
S
first iteration, then then with contact but without friction, then then to activate it friction with fixed contact pressures, i.e. to solve a problem of Tresca.
S

Notice 4:

The test of stop of the geometrical loops and threshold is based on the relative variations of fields of relative displacements. Three key words: ITER_FROT_MAX, ITER_CONT_MAX and ITER_GEOM_MAX make it possible the user to control the course of calculation.

Notice 5:

The loop of geometry is necessary especially for the problems with surfaces of contact lefts. In practice only one correction often sufficient and is advised (ITER_GEOM_MAX=1) if there are few geometrical modifications and kinematics of surfaces of contact.
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The problem solved by the algorithm of the tangent module is a nonlinear problem of elasticity (it could be of elastoplasticity in great deformations or others) with a term of friction of
Tresca and a reactualized pairing. It is written:
D

To find (1
U, *2*
U,*) (**CA**) 1 × (**CA**) 2 × H × **H**;*
(
w1, ***w2*** *
*,) (**CA**) 1 × (**CA**) 2 × H × **H***

G1 (1
U, *1*
***W**) -*
G w1 D
D
N
( )
C
N
D
D

éq
3-1
-
µ

***G**. 1*
***W** D*
0
D
S
0 1 ( )
( )

( )
=
C
,
D
D
G2 (2
***U**, 2*
***W**) +*
G w2 D
D
N
N (
( )
C
D
D
D

éq
3-2
+
µ
***G**. 2*
W
D
0
D
S
0,1 ()
(
( )

( )
=
C

D
D
D
*-1 (- D G) *d = 0* ***éq 3-3***
C
Nd
N
- µ
D
S (-
G
. D
0 1 ( )

( )
+
C
,
D

éq
3-4
(1-)

D = 0
C
D

where
1
G (. .) and 2
G (. .) virtual work of the internal efforts to solids 1 indicates
2
B and B
respectively and where G
G

and

are such as:
D
N

D

G = - X

éq 3-5
N
N []
D
Nd
G *= +*
*[**U**]*

éq 3-6
D
D

The reference to time "T" is omitted to reduce the writing. Moreover, in the integrals on
K
surface contact one replaced D by
D to simplify the notations.
C

Notice 6:

One solves the problem with iteration I by using the thresholds of frictions μ obtained using
S
contact pressures of the iteration i-1. With convergence, the criterion of stop on the threshold is defined
S
compared to the relative variations enters and.
S
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4 Elements of contact

One describes in this section the space discretization of the problem, defined by the equations [éq 3-1] in
[éq 3-6], within the framework of the finite element method.

Fields I
are supposed to be approached by fields I
H polygonal and thus them
surfaces of contact by

. The border is made up of NR elements:
C
CH
CH
C

Nc
CH
= Uej
J 1
=

Following approximate spaces are introduced:

.
*(**CA**) I: approximate space of () I*
***CA** of dimension Mr. One indicates by I J*
NR, for J =, 1 m
H
I
I
basic functions. Thus, the approximate field of displacements, noted I
***U**, is written it like*
H
follows:
I
m
I
***U** =*
NR,
U

éq 4-1
H
I I J
J
j=1
where I
***U** are the components of the vector of displacements on the basis. Each I*
***U** has two*
J
J
*or three components according to whether the problem is 2D or 3D in the Cartesian base of reference (**E**, **E**, **E**). **E** thereafter will be noted*
***N the** vector which is written:*
1
2
3
D
*= **E N***
*. , **E N***

*. , **E N***
.
*who is not anything else that vector **N** expressed in the base*
D
(1 D 2D 3) T
D
D
*(**E**, **E**, **E**).*
1
2
3

.
H: approximate space of H of dimension NR. J is noted
basic functions. Thus,
H
C
density of normal effort of contact approached, noted breaks up in the form
H
following:
Nc
=
H

J
*J **éq 4-2***
j=1

J being components of
on the basis.
H

.
***H**: approximate space of **H**, dimension NR. J is noted*
basic functions. Thus it
H
m
semi-multiplier of approximate friction is written:
H
Nm
=

éq 4-3
H

J
J
j=1
where are to them the components of on the basis.
J
H
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Discretization of the problem, defined by the equations [éq 3-1], [éq 3-6], the system gives not linear according to:

To find 1
U*, 2*
U*,*

and

such as,

H
H
H
H
for all 1

***W**, 2*
***W**,*

and

with 1 J m 1
, K m 1
, L NR
1

and

m
:
J
K
L
m
1
2
C
Nm
G1 (1
1
,
1 J
***U**, **W** NR*
G
1
***W** .n NR, 1 D*
H
J
)-
H
D
N (
J

) J
D
( )

D
CH

éq
4-4
-
µ
G*. 1*
W
NR, 1 D
0
D
S
0,1 (hd
) (
)
J
J
( )

( )
=

D
CH
G2 (2
2
2, K
U*,* ***W*** *NR*
G
2
W *.n N2, A D*
H
K
)+
H
D
N (
K
) K

D
(( )

D
C
D
H

éq
4-5
+
µ
G.
2
W
N2, A D 0
D
S
0,1 (hd
) (
)
K
K
(( )

( )
=

D
C
D
H

-1(-
H
G D 0

éq 4-6
H

D
N)
L
=

D
C
L
H N
- μ
D
S (-
G
. D
H
0 1 (H
,
)

m
( )
+

D
C
m
H

éq
4-7
(1 -. D 0
D)

m
=
C
H
m
H

where:
H

*G = - **X N***

.

éq 4-8

N
H
N [H]
D
D
H
***G** = +*

U

éq 4-9

H

[ ]
D
H D

That is to say still to find 1
***U**, 2*
***U**,*

and

for all 1 i1 m 1
, i2 m 1
, p NR 1
and Q NR,
i1
i2

p
Q
1
2
C
m
such as for all 1

***W**, 2*
***W**,*

and

with 1 J m 1
, K m 1
, L NR
1

and

m
:
J
K
L
m
1
2
C
Nm

1
1
G
m
1
,
1 I
***U** NR 1 **X**, 1*
W** NR, 1 **X
i1
( )
J

J
( )

I =11

Nc
1
2
p
m
m
1
,
1 i1
2
2, i2

-

-
X *NR*
-
X *NR*
With
1
,
1 J
.n
W *.n NR*
D

C
D p
N i1
( )
i2
(D ()

D (
J
D)
( )

H
p=1
I =11
i2=1

Nm
m1
m2

Q

1
,
1 i1
2
2, i2

-
μ
+
***U** NR*
-
***U** NR*
With
1
,
1 J
.
NR
D = 0

W
C
D
S
0,1
Q
i1 () i2
(D ()
(J)

( )

( )
D
H

q=1
I =11
i2=

1
D
éq 4-4bis
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2
2
G
m
2
2, I
U *N2* ***X***, *2*
W *N2,* ***X***
i2
( )
K

K
( )

i2=1

Nc
1
2
p
m
m
1
,
1 i1
2
2, i2

+

-
***X** NR () -*
***X** NR*
With
2
2, K
.n
***W** .n NR*
With D

C
D
p
N i1
i2
(D ()

D (
K
D)
(D ()

H
p=1
I =11
i2=1

Nm
m1
m2

Q

1
,
1 i1
2
2, i2

+
μ
+
***U** NR () -*
***U** NR*
With
2
2, K
*. **W***
NR
D = 0

With
C
D
S
0,1
Q
i1
i2
(D ()
(K)
(

D ()

( )
D
H

q=1
I =11
i2=

1
D
éq 4-5bis

-1
Nc
1
2
p
m
m
1
,
1 i1
1
,
1 i2

1
(-)
+
***X** NR () -*
***X** NR*
With ()
L
*.n D = 0 **éq***
4-6bis
C

D p
D
N

i1
i2
(D)

D L
H N
p=1
I =11
i2=1

Nm
Nm
m1
m2

- μ
D
S
Q

Q

1
,
1 i1
2
,
1 i2

-
+
***U** NR*
-
***U** NR*
With
m
. D

C
Q
0,1

Q
i1 () i2
(D ()

( )
m
H

q=1
q=1
I =11
i2=

1
D

Nm
+
(1-
. D
0
D)

Q

m
=
C
Q
m
H
q=1
éq 4-7bis

The not-differentiable quantity is projection on the ball unit, defined by:
X if X (0,)
1

éq 4-10
0,1 (X)

( )
= X

if not
X

The "generalized" differential of this application is as follows:
X

if X (0,)
1

1
.
éq
4-11
X
0,1 (X)

( )
X =

X X

I -

X D
2 X if not

X
One writes it in the following generic form:

X (0,) (X) X
= [K

1
(X)] X

éq 4-12
with
Id
if X (0,)
1
[K (X)]

= 1
.
X X
éq
4-13

Id -
2 if not
X

X

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Notice 7:

One can generalize [éq 4-13] as follows [bib15] [bib16]:

Id
if X (0,)
1
[K (X)]

= 1
.
X X

éq
4-14

Id -
2 if not
X

X

where 0 1.

The use of the algorithm of the tangent module for the resolution of the definite nonlinear problem by the equations [éq 4-4] to [éq 4-9], place gives, with each iteration N + 1, the resolution of one linear system of the form:
N
+ N
+ N
NT
NT

n+
U 1 N
L
U
U

H
1

N
N

C
0

n+1

L ***éq***
4-15
H
= N2

N
N

n+1
N

0
F

L
H
3

where
N
indicate the matrix of tangent rigidity traditional of the two solids to state N. Quantities
N 1
+
N 1
+
N 1
U,
+

and
correspond to the increments of the vectors components of the fields
H
H
H
approached displacements, field of density of approached normal effort and field semi-multiplier of friction approached with the iteration N + 1. In addition, [N
], [N
and
C
U]
[N]
[N], [naked] [N

and
F]
the assembled matrices of contact indicate and
matrices assembled of
friction. Second members of the system [éq 4-15], noted N
N
N
L, L

and
L are calculated of
1
2
3
traditional manner by holding account of the contribution of the terms of contact and friction. Their expression is given in this paragraph after that of the tangent matrices.

4.3
Tangent matrices of contact

*The elementary contributions to the tangent matrices of unilateral contact are given below, where, for clearness, the basic functions of spaces (**CA**) 1*
***CA** were renumbered of 1 with*
H
(
and

) 2h

~
m + m and noted I
NR with for convention:
1
2
~
For 1 I m
I
NR ()
,
1 I
= NR ()
1
~
For m I m + m
I
NR ()
2, I 1
m
= - NR
(eA ()
D
)
1
1
2

· *For []*
if the problem is 3D (the version 2D deduces some immediately while not holding count component on ***E****) one has then for I =,*
1 1
, J m
+ m
3
C
1
2
[E
~
*] **
= -
I
E

*NR **N** D **éq***
4.3-1
*, **U***
D
() J ()
I
J
E
D
where the exhibitor E, added with the application of pairing, returns to the fact that into discrete, one
D
work with several charts and that pairing must hold account of it.
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· For [U
] (of penalization type) with 1 < I, J m + m, one a:
1
2
[E
~
~

*NR NR **E E***
***N N** D*
éq

4.3-2

U]
=
I
W
*, **U***
N
D
() J ()
I
J

T
E
D
D

· For [C] with 1 I, J NR:
C

[
1
(
)
E
C]
-
D
I
J

=

éq
4.3-3

,
( ) ( )
I
J
-
D
E

N

4.4
Tangent matrices of friction

*Before detailing the form of the matrices of friction, one indicates per **T** and **T the** two vectors*
1
2
covariants of the tangent plan (in a point) and by ()
** the tangential part of the vector (*).*

*One indicates by [P] the operator of projection on the level defined by normal **N**:*

*[] = [**Id - nn**]*
*which one associates the matrix **P** in the Cartesian base of reference defined by:*
1 - 2
N - N N - N N
1
1 2
1 3
***P** = [*
N N 1 N
N N
éq
4.4-1
1 2
3]

2

= -
-
-
1 2
2
2 3
- N N - N N 1 - 2
N
1 3
2 3
3
where (1
*N, N2, n3) are the components of normal vector **N**.*

[N
K]
While noting
following quantity (cf [éq 4-13]):
[N
K] = [K (hn
***Gd**)]*
with
hn
N
***Gd** = H +*
[N
U
H] D

While noting:

I
= I
*(**T**, **T***

1
2) = (
I
I
1
2)
for 1 I NR

m
éq
4.4-2

I
= I
T
for 1 I NR

m
One has as follows:

NR

NR
NR
NR
m
m
m
m

Q
=
T

1
T

Q

Q
+
Q
=
Q
1
1
2q
2
Q Q =
Q

Q

q=1
q=1
q=1
2q q=1

who allows to have the values of the semi-multipliers in the local Cartesian base associated surface contact.
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By using the notations established previously, one deduces the following elementary contributions with the tangent matrices of friction:

· For [N
B] we have for 1 < I NR and for 1 < J m + m:
m
1
2

[1
~
B]
iT
J
N

= -

() µ
( )

( )

éq
4.4-3
,
*K **P***

NR
D
U
I
J
D
S
[ ]
E

· For the elementary contribution in the tangent matrix of friction [N
B us
U]
let us have for 1 < I, J m + m:
1
2

[
~
~
Bne
= -
( )
( )
(P
)
K ***P***

éq 4.4-4
U]
µ NR I NR J T N D
,

W U
D
S

[ ]
I
J
E

.
[nF]
For
and this whatever 1 < I, J NR we have:
m

D () µs iT
= -

([
N
*) **Id** - K]*
[
J
( )
F

éq
4.4-5
U]

D
E

I, J + (1 -

*() () **M***
D () I J

D
E

***M** whose components are [**M**] =*

T T. is metric current base covariante.

4.5
Expression of the second members of contact-friction

One clarifies here the second members of the system [éq 4-15], noted N

N
N
L, L

and
L by not holding account
1
2
3
that contribution of the terms of contact and friction. One has as follows:

· For N
L:
1
Nc
m +

1 m2
~

N
***L** =*
N
p

-
N
J
***E** ~ I*

***X** .n NR **N** NR D*
wi
C
D p
( )

N (J
D)
() D ()

H
p=1
j=1

éq 4.5-1
Nm
m +

1 m2
~

+
T
N
μ
~
P
+ ***P***
N J
I
U *NR NR D*

0 1

C
D
S
,
Q ()
Q
J () ()

( )

H
q=1
j=1

· For N
L:
2

Nc
m +

1
1 m2
~

N
N
p
N
J
I
L
1
(
)
( )
***X** .n NR*
( )
() D
=

éq 4.5-2

D
p
D
N
J
D
I

-
+

CH

N
p=1
j=

1

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· For N
L:
3
Nm
Nm
m +
µ

1 m2
~

N
D
S
iT
Q
N
Q
N
N
J
L

P
U *NR*
D
=

-
+

Q
0,1
Q

J
I
CH

( )

( )

q=1
q=1
j=

1

éq 4.5-3
Nm
-
(1-
D
D)

iT

Q
N
C

Q
H
q=1

In these expressions, one will note that subscripted quantities N correspond to the values of the fields discrete currents 1
***U**, 2*
***U**,*

and

with iteration N of the step of time to which calculation is carried out. Only
i1
i2
p
Q
N
U
has a a little particular significance, since it corresponds to the increment of displacement since
J
the beginning of the step of time considered, i.e. to the increment of displacement enters iteration 1 and iteration N.

4.6
Calculation of the matrices and the second members

The effective calculation of the solutions of the systems [éq 4-4] with [éq 4-7] requires:

~
· the definition of the basic functions I
I
I
NR
,
and.
· the numerical formula of integration used for the terms of contact and friction.

*The defining choice of basic functions of spaces of approximation and the choice of a formula of numerical integration (joined to the total formulation of the problem of contact) an **element** defines **of contact**. However, this choice is not obvious a priori. Indeed, formulation being of the mixed type, it*

is appropriate to pay attention to considerations of compatibility between the elements of surface of target or main contact and those of the contacting surface of contact or slave. In Code_Aster one has fact the choice of taking the same functions of form for the contact that for the finite elements on which surfaces of contact rest. The discretizations of the fields of densities of efforts are operated in discrete spaces corresponding to the traces of the fields of displacements of the solid known as slave on the surface of contact. This choice, within the model frameworks, is at least supported mathematically [bib17] [bib18] [bib19].
With the configuration of the elements of the surfaces of contact suggested with [Figure 4.6-a] the choice of
surface slave and that of surface Master rests on the stated considerations of compatibility previously. Indeed, if one makes like choice of surface slave, that which is with a grid more finely then the product of the functions of form 1 2
. intervening in integrations of rigidities
and of the second members remains polynomial for this element. A contrario, if one chooses like surface slave the surface with a grid the product of the functions of form 1 2
. its character loses
polynomial there is compact of nonnull measurement on which the product 1 2
. zero are worth - what
makes illicit integrations of the contributions to the rigidity and the second member of the contact. convergence of the algorithm is not then assured any more.
From a practical point of view, one advises with the user whose calculation would not converge to invert it
main role of surfaces and slave, to check that the surface slave which it chooses is discretized with less as far as surface Master and to use the same type of element on both sides of surface of contact.

Appear 4.6-a: Numerical problem of integration on surfaces of contact

---

In addition, even by using linear basic functions, the exact calculation of the terms of the matrices and of the second members resulting from the contact is practically delicate. The quantities which we would have with
to integrate are, from pairing, the characteristic function and projection, nonregular (cf [Figure 4.6-a]). One can however give of it a "approximation" by methods of integration numerical. Various techniques of integration were implemented:

· integration at the tops,
· integration at the points of Gauss,
· integration of the simpson type.

One advises with the user nonsatisfied with these results passing from the one to the other of these methods
in the order given above, the integration of the simpson type being richest but also more expensive.

5 Implementation

This formulation is usable with the method "METHOD": "CONTINUES" under the key word "AFFE_CHAR_MECA". The parameters of the method and are specified by the key words
N

"COEF_REGU_CONT" and "COEF_REGU_FROT", respectively.

Behind the key word "INTEGRATION" one can use either the word "GAUSS" or the word "NODE". axisymmetric modelings are taken into account by another key word "MODEL_AXIS". This last word "YES" or word is followed "NOT".

Notice 8:

The choices "GAUSS" or "NODE" must be always specified even with a modeling axisymmetric since key word "MODEL_AXIS" does nothing but specify modeling. The word key "NODE" is recommended for left surfaces of contact (not-plane).

Let us note that in the implemented formulation, the multiplier of contact and the semi-multiplier of friction are fields defined on surface slave. For that, a modeling "SURF_DVP_2D" in case 2D or "SURF_DVP_3D" in the case 3D, must be added in version 6 of Code_Aster. It will be noted that in version 7, it is not necessary any more to make this addition and that surfaces of contact do not need more to be specified on the level of the assignment of the model.

---

6 Bibliography

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Methods of piloting of the loading
Summary:
This document describes the methods of piloting of the loading available in Code_Aster (by a degree of freedom, by length of arc, increment of deformation and elastic prediction). They introduce one additional unknown factor, intensity on behalf controllable of the loading, and an additional equation, constraint of piloting. These methods make it possible in particular to calculate the response of a structure which
would as well have instabilities, of origins geometrical (buckling) as material (softening).
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1 Principle of the methods of piloting of the loading

In a general way, the functionalities of piloting available in Code_Aster allow to determine the intensity of part of the loading to satisfy a constraint relating to displacements. Their employment is limited to simulations for which time does not play of role physics, which excludes a priori the dynamic or viscous problems. One can distinguish three ranges of use which answer as many methods of piloting (key word factor PILOTING):

· physical control of the forces by the displacement of a point of the structure (for example for to adjust the intensity of the force exerted on a cable so that its arrow reaches a value data): piloting per degree of freedom imposed (STANDARD: “DDL_IMPO”);

· followed geometrical instabilities (buckling), the response of the structure being able exhiber of “soft” snap-back: piloting by length of arc (STANDARD: “LONG_ARC”);

· followed instabilities material (in the presence of lenitive laws of behavior), the answer structure being able exhiber of the “brutal” snap-back: piloting by the elastic prediction or more generally by the increment of deformation (STANDARD: 'PRED_ELAS).

More precisely, the methods of piloting available in Code_Aster rest on both following ideas. On the one hand, it is considered that the loading (external forces and displacements imposed) additivement breaks up into two terms, one known and the other whose only direction is known, its intensity becoming a new unknown factor of the problem:

$$F_{ext} = F_{ext}^{cst} + F_{ext}^{pilo}$$

éq 1-1

$$U = U^{cst}$$

pilo
imp = U imp +
U
imp

In addition, in order to be able to solve the problem, one associates a new equation to him which relates to
displacements and which depends on the increment of time: it is the constraint of piloting, which is expressed
by:
(
P U) =
with P (0) = 0
éq 1-2
where is indirectly an user datum which are expressed via the step of current time T and one coefficient of piloting (COEF_MULT) by = T COEF_ MULT. The condition (
P 0) = 0 is necessary
in order to obtain an increment of all the more small displacement as the step of time is small.
Finally, the unknown factors of the problem become the increment of displacements U = U - U -, them multipliers of Lagrange associated with the boundary conditions and the intensity with the controlled loading
, baptized ETA_PILOTAGE. The nonlinear system to solve is written henceforth:
F
T
cst
pilo
int (U
;) B
F
! +
=
ext. + Fext

B U = Ucst
pilo
imp + U imp
éq 1-3

P (U
) =

Notice 1:
At present, loadings following (i.e which depend on displacements) and them conditions of Dirichlet of the type "DIDI" are not controllable. Nothing is opposed so that they it become in a later version of the code.

Notice 2:
The loading does not depend directly any more on time but results from the resolution of all it nonlinear system [éq 1.3]. That implies that the controlled share of the loading does not have to depend on physical time, contrary to a thermal loading, for example, but corresponds to an effort which one adjusts to satisfy a kinematic constraint additional.

2 Resolution of the total system

The introduction of a new equation does not disturb in addition to measurement the method of resolution of nonlinear system. Indeed, one proceeds as in [R5.03.01] by a linearization of the equations of [éq 1.3] bearing on the interior forces and the conditions of Dirichlet:

$$F_{int}$$

$$($$

$$cst$$

$$N$$

T

N
pilo
U
N)
T

B
U
Fext - Fint (U
) - B
Fext
U
=
+

éq 2-1

cst
pilo
N
B
0

Uimp - B U

Uimp

"$$ #
$
%
$$$

"$$$ #
$
%
$$$$
"#%
pilo
K
R cst
R
T
One can now express the corrections of displacements U and multipliers of Lagrange according to with the help of the resolution of the linear system [éq 2-1] compared to each one of two second members:

U

Ucst

Upilo

Ucst

Upilo
=

where
K -1 cst
-1
pilo

T
R
and
KT R

+

=

=
éq 2-2

cst

pilo

cst

pilo

One can now substitute the correction of displacement U according to his expression [éq 2-2] in the equation of control of the piloting of the system [éq 1.3]; it results a scalar equation from it
in:
~(
P)

=
(PUn +Ucst + Upilo) =
éq 2-3
déf.
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The method of solution of this equation depends on nature on control on piloting adopted cf [§3]. Finally, it any more but does not remain to reactualize the unknown factors displacements and multipliers of
Lagrange:
Un+1 = One + Ucst

+ Upilo

éq 2-4
n+1
N
cst
pilo

= + +

Notice 3:
During iterations of Newton, it can happen that the equation [éq 2-3] does not admit solution, without in so far as there is an error of use. In this case, one decides then
~
~
to determine as the value which minimizes P, provided that one checks (
P)
> ;
as the increment of displacement is all the more small as is small, such a condition impose coarsely that the increment of displacement is at least also large (without entering in precise mathematical definitions) that that prescribed by the equation of piloting
~(
P)
=. Moreover, one imposes has minimum an additional iteration of Newton, in order to to check, with convergence, not only the equilibrium equations and the conditions of Dirichlet, but also the equation of piloting.
Notice 4:
There is no linearization compared to the variable of piloting. This way, one preserves all the methodology of reactualization of the tangent operator already implemented for calculations without piloting. Moreover, the structure "bandages" tangent matrix is preserved.
Notice 5:
This mechanism of resolution is incompatible with the use of linear research. In fact, it would be possible in the presence of a function of linear piloting P, to see Shi and Crisfield [bib4], but it is not true any more in the case general. This is why simultaneous use of research linear and of piloting is prohibited.
3
Equation of control of piloting
3.1
Piloting by control of a degree of freedom of displacements:
DDL_IMPO
For this first type of piloting, the function P is restricted to extract a degree of freedom from the increment from

displacement. In particular, it is thus about a linear function:

(
P U) = L U
with L = 0
0
1
0

0
!
!
éq 3.1-1

node N, ddl I

---

where the nodal vector L is null everywhere except for the degree of freedom being extracted where it is worth 1. The equation
[éq 2-3] is reduced then also to a linear equation which leads to:
- L One - L Ucst
=
éq 3.1-2
L Upilo
It will be noted that there is no solution when the correction of displacement controlled Upilo does not

allow
to adjust the degree of freedom required, which can arrive if, by error, one blocks the degree of freedom in
question.
3.2
Piloting by length of arc: LONG_ARC
Another form of piloting very largely used consists in controlling the standard of the increment of displacement (compared to certain nodes and certain components): one speaks then about piloting by length of arc, to see Bonnet and Wood [bib1]. More precisely, the function P is expressed by:
(
P U) = U = U L U
éq 3.2-1
L
where, again, the nodal vector L makes it possible to select the degrees of freedom employed for calculation
standard (it is worth 1 for the selected ddl, 0 elsewhere). In this case, the equation of piloting is reduced
with a quadratic equation:
[Upilo LUpilo] 2 +2 ([U
N + Ucst) L Upilo]
éq 3.2-2
+ ([U
N + Ucst) L (U
N + Ucst) - 2
] = 0
This equation can not admit a solution. In this case, one chooses the value which minimizes it
~
polynomial [éq 3.2-2]. One checks then well (
P)
>. In the contrary case, it admits two roots
(or a double root). One chooses that of both which minimizes the angle formed by Uavant and Un+1 (where Uavant is the increment of solution displacement of the step of preceding time), i.e. that which maximize the cosine of this angle whose expression is:
(
One + Ucst

+ Upilo

Uavant
Uavant Un+1) (
)
cos
,

=
éq 3.2-3
One + Ucst

+ Upilo

Uavant
3.3
Piloting by the increment of deformation: PRED_ELAS
The last two modes of piloting, controls by increment of deformation and control per prediction rubber band, cf Lorentz and Badel [bib3], are activated by same key word PRED_ELAS. In fact, the second
depends explicitly on the law of behavior and is established only for certain laws (ENDO_LOCAL and BETON_ENDO_LOCAL); when it is available, it is employed. For the other laws, it is piloting by increment of deformation which is activated. It is probable that this mechanism evolves/moves within the framework of
version 6 of Code_Aster and that these two modes of piloting are activables independently one other.
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Piloting by increment of deformation consist in requiring that the increment of deformation of the step current remains close in direction to the deformation at the beginning of the step to time, and this for at least one
not Gauss of the structure. It is required thus qualitatively that minimum a point of the structure has preserve the mode of deformation which it had as a preliminary (for example, traction in a direction data). Mathematically, one can give an account of this requirement with the help of the choice of following function of piloting:
-
(

P U) = max G
-
G
with G = Bg U and G = Bg U
éq 3.3-1
-
G
G
where the index G sweeps the points of Gauss of the structure and where deformation in a point of Gauss
deduced from the nodal vector of displacements via the symmetrical use of the matrices “left the gradient
functions of form” B G (not to be confused with the matrix of the conditions of Dirichlet).
control piloting according to is written then:

-

(0)

G
N
cst
G
With
=
B U
U
-
G (
+
)
~(

0
1

P)
= max (()
( )
G
G
With + G
With) =

with
-
éq 3.3-2
G
"$ #
$
%
$$

( )
1
G
pilo
L
B
U
G ()
G
With =
G
-

G
Such a function is convex and linear per pieces. It generally admits no, one or two solutions, cf [Figure 3.3-a]. When it does not admit solutions, one chooses like previously
~
~
value which minimizes P ()
; it meets the condition min P ()
>. When it admits two
~
solutions, one chooses that which leads to U ()
nearest to U.
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Notice 6:
In the presence of great deformations, one can generalize the function of piloting [éq 3.3-1] in employing deformations of Green - Lagrange (Lagrangian measurement of the deformations in initial configuration):
-
(
E
1
P U) = max G eg
with E =
-
-
(T

F F - I)
D
F = Id + U

E = E - E
G
E
2
G
éq 3.3-3
However, one would not lead any more like previously to a function closely connected per pieces. For y
to cure, one decides to carry out a linearization of E G compared to U. P has one then expression similar to [éq 3.3-2] with:

-
E
With
G
(0)
T
N

cst
G
=
sym F
U

U

-
[gg (+)]

E G

-
éq 3.3-4

E
With
G
( )
1
T
pilo
G
=
sym

F
U

-
[G
]

eg

where U indicates the gradient (not symmetrized) of displacements U evaluated at the point of Gauss of index
G
G.
~
~
~
P ()

P ()

P ()

Appear 3.3-a: Various cases of figure for the equation [éq 3.3-2]: two, one or no solutions
To solve the equation [éq 3.3-2], one proposes the algorithm presented in the table [An1-1], [§5]. It is based on the construction of encased intervals: the terminals of the last of them are the solutions of
~
the equation and, as announced previously, one that which chooses leads to U ()
nearest to
U. This algorithm, rapid, are based on the resolution of G scalar equations linear, where G indicates the total number of points of Gauss. The algorithm can end prematurely when one of
intervals is empty, which means that the equation [éq 3.3-2] does not admit solutions.
When there are not solutions with the equation [éq 3.3-2], it acts then, in accordance with remark 3,
~
to minimize P ()
, problem which can be still expressed like the minimization of a function
linear with two variables under G forced linear inequalities:
min max Lg ()

min
y
with lg (,
y) = Lg ()
- y
éq 3.3-5

G
, y
G lg (, y) 0
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Here the algorithm of the simplex, cf Bonnans and Al [bib2] are employed, whose application to our problem is
presented in the table [An1-2], [§5].
3.4
Piloting by the elastic prediction: PRED_ELAS
If piloting by the increment of deformation proves to be sufficient to follow dissipative solutions in the majority of instabilities materials, the existence of solutions nevertheless is not proven. One him then prefer a method of piloting based on the elastic prediction for which the existence of solutions is shown but which, on the other hand, is specific to each law of behavior (established only for laws ENDO_LOCAL and BETON_ENDO_LOCAL). More precisely, when the law of behavior is controlled by a threshold, one defines P as the maximum on all the points of Gauss of the value of the function threshold in the case of an elastic test (incremental answer rubber band of material).
Thus, let us consider that the state of material is described by the deformation and a whole of variables interns A. Appelons respectively (, has) and (
With, has) the constraints and the forces
thermodynamic associated A. Supposons moreover that the laws of evolution of A are controlled by a threshold F (,
With, has) and a function of flow G (,
With, has) in the following way:
&a = G (,
With, has)
with 0 F

(,
With,) 0 F has (,
With,) = 0 have
éq 3.4-1
Such a formulation includes the majority of the models of behavior dissipative and independent of rate loading. The function threshold is worth then for an elastic test:
F el () = F (A (, -), -
has
has)

éq 3.4-2
One simplifies the problem by linearizing F el compared to in the vicinity of a point which one will define
later:
With

F
F
F el
el

=
+
+
-
éq 3.4-3
L ()
F ()
(
)
def

With

Finally, the function of control of piloting is defined like the maximum of F el compared to
L
all points of Gauss G, function which depends only on U:
(
P U) = max F el

-
L (+ B G U)
with G = BgU
éq 3.4-4
G
It remains to define, not in the vicinity of which the linearization is carried out. It is selected like the point
nearest to - which realizes:
~(
) with ~ ()

B (-

U
One

Ucst Upilo
el
G
)

=
=
+
+
+
and =
~
arg min fL ()
éq 3.4-5

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A graphic interpretation this choice is given on [Figure 3.4-a]. Finally, the equation of control piloting is written:

0
With
( )
F
F
With = F el () +
+
(
-
N
cst

Bg (**U**
U
U))

With

+
+
-
max
1

0

G
With + *G*
With
=
éq 3.4-6
G
[( )
( )]
"$ #
$
%
$$
F *With*

F
( )
1

pilo
F el
With =
+
B U
L ()
(*G*
)

With

One is thus brought back to a problem identical to that of piloting by the increment of the deformation. One
of course employ the same algorithms of resolution as those presented to [§5].

~( )

~
2

2
( )

-
-
F el () = 0
F el () = 0

1
1
Appear 3.4-a: Definition of according to the relative positions of surface threshold F el and of the deformations ~

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Appendix 1: Algorithms of resolution

(1)

Initialization of the interval

I0 =] - + [

(2)

Buckle on the points of Gauss *G*

(2.1)

Root of the active linear function

G tq Lg (*G*) =

(2.2)

Construction of the following interval

(2.2.1)

If the active linear function is increasing

With (1)

G

> 0 *I G = I g-1*] - *G*]

(2.2.2)

If the active linear function is decreasing

With (1)

G

< 0 *I G = I g-1* [*G* + [

(2.3)

To stop if the interval is empty

(3)

The solutions are the terminals of the interval

Fr (*IG*) max Lg ()

=

G

Count An1-1: Algorithm of resolution of the equation refines per pieces

(1)

Initialization with 0 given

(1.1)

Gradient of the function to be minimized

G = (0, -)

1

(1.2)

Initial Summit

S0 = (0

, 0

y)

with
0
y = max Lg (0
)
G
(1.3)
Activated constraint
g0
tq
y0 = Lg
0 (
0)
(2)
Exploration of the successive tops: buckle on *S*
(2.1)
Definition of a direction of descent
G NR
D
S
(1)
S = **G** -
NR *S*
with **NR** = - *A*, 1
NR
S
gs
S **NR**
(
)
S
(2.2)
Found minimum if **D** *S* **G** = 0 (flat bottom)
(2.3)
Acceptable projection *G* for each constraint *G G G* =
max

lg (**Ss** +Ds) 0
(2.4)
Effective projection *S*
S = min *G*
G
(2.5)
Found minimum if *S* = 0

(2.5)
Following Summit
Ss+1 = **Ss** + *S*
Ds
(2.6)
Following activated constraint *gs+1*
gs+1 tq *S = gs+1*
Count An1-2: Algorithm of minimization of the function refines per pieces
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~
P ()

G
L
1( )
S0
D0
L
4 ( )
S1
D
S
1
2
L
3 ( )
L

2 ( )

2
1
0
Appear An1-a: Graphic illustration of the algorithm of minimization
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Nonlocal modeling with gradient of deformation

Summary

This document presents a model of delocalization of the laws of behavior per regularization of deformation. It introduces an additional nodal variable: regularized deformation, dependent on the deformation
local by an equation of regularization of the least type square with penalization of the gradient than one solves
at the same time with the traditional equilibrium equation. The regularized deformations are used for calculation
evolution of the internal variables (and not for the calculation of the constraints!). This method makes it possible to avoid
certain problems involved in the digital processing of the local problems like the dependence with the grid.
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1
Nature of the formulation

In the presence of damage (softening), the local laws of behavior lead to one
badly posed problem which results numerically in a localization of the deformations in one
bandage thickness a mesh: in extreme cases, one breaks without dissipating energy.
There are several extensions to the local models which make it possible to mitigate this problem of localization
*(relieving of the potential energy, enrichment of kinematics, theories with gradient, models not buildings). This document deals with nonlocal model with gradient of deformations, modeling *_GRAD_EPSI, drifting of the model with gradient of equivalent deformation proposed by Peerlings and Al*
(1995). One introduces interactions between the material point and his space vicinity by regularizing them
deformations thanks to an operator of delocalization. The regularized deformations are then used to evaluate the evolution of the internal variable.

It should be noted however that the constraints are calculated starting from the local deformations bus
the use of the deformations regularized in the calculation of the constraints would return to "too to regularize" the problem, which would call into question the existence even of solutions. One is convinced some
easily thanks to the following example:
Let us consider a bar made up of 2 different materials which have different Young moduli. One exert on this bar a simple traction. The 2 elements being gone up in series, the constraint is equal in the two elements:

= 1 1

E = 2 2
E =

E2
E1

With the interface between the two elements, the discontinuity of modulus Young thus imposes one discontinuity of the deformation. Let us consider now either the local deformation but one delocalized deformation. The traditional operators of delocalization cause to make continuous deformation in the structure, which then generates obligatorily a discontinuity of constraint with the interface because of the difference of Young modulus, and this goes against the equation of balance.

The regularization of the deformations leads us to introduce a characteristic length definite by operator DEFI_MATERIAU under the key word NONLOCAL factor which conditions the width of the bands
of localization. The scales thus are not defined any more by the digital processing of the problem but by a parameter material.
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2
Limits of the local models

One initially proposes to illustrate the phenomenon of localization in the simple case of one bar subjected to a uniaxial traction.
One thus considers an assembly of identical elements assembled in series subjected to a traction as represented on [Figure 2-a].

, U
i-1 I
i+1

Appear 2-a: Assembly of identical elements assembled in series subjected to a tensile test

Each element obeys the same law of behavior of the elastic type endommageable with softening [Figure 2-b]. The state of material is described by two variables which are the deformation and the damage characterized by scalar variable D. This variable is worth 0 when the material is healthy and grows up to 1 when it is completely damaged.
We will not enter here in detail of the equations governing such a behavior of material.
Let us specify simply that these equations make it possible to describe the behavior completely of material. They indeed give us access to the constraints and the damage according to the rate of deformation, to see for example [R5.03.18].

peak
E0 peak

Appear 2-b: Law of behavior of material in uniaxial simple traction
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The elements of the studied bar are assembled in series, which implies, because of the equation of balance of the structure, equality of the constraint in all the elements:

I =

One can consequently study the total response of the assembly to a simple tensile test. This answer breaks up into two phases. In a first phase, the behavior of all them elements is elastic and the damage remains null. The response of the structure thus exists and is single. The deformation is identical for all the elements and is worth:

I =

0
E

This phase perdure as long as the peak of constraint is not reached. Microphone-heterogeneities of material imply light fluctuations of the field of elasticity between the various elements, which will involve the damage of a fastener before the others. The second phase start when one of the elements which one notes A damages. The constraint in the whole of structure reached its maximum. While continuing traction, the constraint supported by the structure is to decrease. Element A having passed the peak, it is in the lenitive phase of the behavior of material, which means that it will continue to damage itself during traction. The other elements do not have not reached the critical point, they thus simply will undergo an elastic discharge at the time of decrease of the constraint. This phase finishes when element A is completely damaged. Finally, the damage as well as the deformation thus concentrated in one only element.

One then includes/understands easily the numerical consequences of the localization. The phenomenon described previously on a simple sample will occur whatever the structure with a grid by elements finished. For reasons of stability, the localised solution tends to being selected. The damage and the deformation will concentrate in a band thickness an element and any refinement of grid then will modify the total response of the structure. One includes/understands then well that it is impossible to describe the scale of the bands of localization, the length of the damaged band coming from the grid and not from a physical principle. Moreover, one obtains a result physically inadmissible from an energy point of view. Indeed, the energy dissipated at the time of the damage goes

to depend on the refinement of the grid, and one can even imagine the total rupture of a structure without
expenditure of energy if an extremely fine grid is considered.
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3
Formulation with regularized deformations

3.1 Principle

One considers the state of material defined locally by the deformation and of the internal variables. given free potential energy (,) makes it possible to define the constraint.
In manner general, the law of behavior is given by the expression of the constraint and the law of evolution of the internal variables:

(,)

& = G (&,)

The principle of the method of delocalization of the deformations is to use the deformations regularized in the law devolution of the internal variables:

(,)

& = G (&,)

One thus includes/understands the general information of the method which does not force to reconsider the integration of the law of
behavior. It is indeed the same one as for the local model but by replacing par. It is necessary nevertheless to distinguish well the calculation from the internal variables, which utilizes deformations
regularized, of that of the constraints, which utilizes only the local deformations.

3.2
Choice of the operator of delocalization

The choice of the operator of regularization is purely arbitrary and is not based on any reasoning physics. One however may find it beneficial to choose an operator who is integrated easily and directly in
STAT_NON_LINE by the finite element method. Thus, the use of an integral formulation, where the coupling between the finite elements on the level of the integration of the laws of behavior causes to increase considerably the bandwidth of the tangent matrix and to thus increase it
a many operations to be carried out, are not judicious. The operator of regularization retained, proposed
by Peerlings and Al (1995), a delocalization by least squares with penalization employs of gradient:
1
1
R () = min
(
2
2

-) + (L) D
C
2
2

The term in gradient introduces the interaction between the material point and its vicinity and makes it possible to limit
strong concentration of gradient of deformations. To minimize such an integral amounts solving the following differential equation:
- L2

C =

One sees appearing a major interest of the choice of this operator of regularization. The differential equation

can be integrated classically by the finite element method, and this without introducing news not linearities. It is enough for that to introduce new nodal variables representing the deformations generalized.
There is moreover a tangent matrix of reasonable bandwidth (compared to a formulation integral) but it should be noted that the tangent matrix is not symmetrical, as it further will be seen by clarifying the tangent matrix.
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3.3 Formulation
variational

In the model, two equations control the process of deformation, on the one hand the equation of traditional balance and in addition the differential equation characterizing the regularization of deformations. The integral formulation of our problem is as follows:

v Vad
(v) C: D = v.T D + F FD

V AD
: space acceptable displacements

T: forces imposed on the edge

E [
H1 ()]6

(E + E
. L2
)

D = E
D
C

The limiting conditions for the generalized deformations are the natural conditions rising from the equation of regularization. They are of Neumann type:

.n = 0

One indeed imposes no particular condition on the edge in the equation of regularization.

4 Discretization

4.1 Equations discretized

The equilibrium equation discretized between the external and interior forces is traditional form (cf [R5.03.01]):
T
Fint + D = ext.
F

with F
T
int = BT D and Fext = NR T D

(
T
D: cf T
B of [R5.03.01])

where NR are related to forms associated with the field with displacement and B the derivative with

functions of forms.

The differential equation on the regularized deformations is discretized in the same way:

K
F
=

~ ~
2 ~
with K
= (NT
~
NR + L BT B

~T
C
) D and F = NR D

~
~
where NR are related to forms associated with the field with generalized deformations and B them derived from the functions of forms. It should be noted here that functions of forms associated with generalized deformations are different from the functions of forms associated with displacements.
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The nodal residues associated these two equations are as follows:

F U = Fint + DT - Fext

F = K - F

The tangent matrix associated the resolution of this system by the method of Newton is as follows:

F U F U

U

K =

F
F

U

The various blocks of the tangent matrix are as follows:

Fu
T
=

B

Data base

U

i-1

Fu
T
=
~
B
NR
D

i-1

F
= (~ ~ 2~ ~
NT NR + L BT B
C
)
D
i-1
F
= - ~
NT

Data base

U i-1

It should be noted that the tangent matrix is not-symmetrical.

4.2
Choice of the finite elements

The introduction of new nodal variables forces to use new compatible elements

with the new formulation. One is in the presence of two nodal unknown factors: displacements and regularized deformations. Deformation being the space derivative of a displacement, if one use functions of P2 form for displacement, it is preferable to use functions of form P1 for the deformations regularized for reasons of homogeneity. Quadratic elements, TRIA6 and QUAD8 for the 2D, TETRA10, PENTA15 and HEXA20 for the 3D, were developed. components of displacement are assigned to all the nodes of the element whereas the components regularized deformations are affected only with the nodes tops. For more clearness, element TRIA6 is represented below:

3

6

5
Nodal variables
(U,)
1
(U)
4
2

4.3 Modelings available

These various elements are used in three types of modelings:
Calculation 2D in plane deformations:
D_PLAN_GRAD_EPSI (cf [U3.13.06])
Calculation 2D in plane constraints:
C_PLAN_GRAD_EPSI (cf [U3.13.06])
Calculation 3D:
3D_GRAD_EPSI (cf [U3.14.11])

The axisymmetric mode is not yet available.

5 Interface with the laws of behavior

The use of this method of delocalization requires the calculation of the following terms on the level of law of behavior:

(,)

,

,
,

The last two terms are necessary only for the calculation of the tangent matrix.

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Model of Rousselier with gradient of variables interns

Summary:

One presents here the model of Rousselier in great deformations in a nonlocal version i.e. introducing gradients of variables intern in order to take into account strong space variations of mechanical fields. One activates the nonlocal formulation of the model of Rousselier by one of modelings
"X_GRAD_VARI" of order AFFE_MODELE of the MODEL key word. As for the model even, it is available
in order STAT_NON_LINE via the key word RELATION = "ROUSSELIER" under
key word factor COMP_INCR and with the key word DEFORMATION = 'SIMO_MIEHE.
This model is established for three-dimensional modelings (3D_GRAD_VARI), axisymmetric (AXIS_GRAD_VARI) and in plane deformations (D_PLAN_GRAD_VARI).

One presents the writing and the digital processing of this model.
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1
Recall on the theory of the models with gradient

The models with gradient presented here were developed by E. Lorentz [bib1] in order to be able to describe it
material behavior requested by strong gradients of the mechanical fields which appear
in the damaged zones or in the vicinity of geometrical singularities. Indeed, in
case of strong gradients, the behavior of a material point is not independent any more of its entourage but depends on the behavior of its vicinity, from where the introduction of gradients into the models.
From a numerical point of view, the calculation of a structure with a local law of damage traditional watch which the damaged zone always locates on only one layer of finite elements
and thus that the response of the structure depends on the adopted grid: the models with gradient mitigate it
problem.
In what follows, we make a short recall of this theory. One will find in [R5.04.01] details thorough on this theory.

1.1
Construction of the models with gradient

This formulation is restricted with generalized standard materials. The state is described there by deformation, of the internal variables has and their associated gradient a:

has = has éq 1.1-1

According to the formalism of generalized standard materials, the data of the free energy

(, has, has)
and of the potential of dissipation (&a, a&)
(for the choice of these two energies, one will be able
to refer to [bib1]) allow to deduce from them the laws from state and the laws of evolution:

=
, *A* = -
, *A* = -

éq 1.1-2

has

has

(,
WITH A)
(&a, a&)

éq 1.1-3
If one calls F the threshold of elasticity associated with the potential (&a, a&)
, the preceding equation is
equivalent to:
F
F
a& = &
, a& = &

éq 1.1-4
With

With
The problem here is that the variables are not independent any more and are bound by the constraint not
local [éq 1.1-1] so that one is not sure to check:
F
&a = &
= a& éq 1.1-5
With
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One then proposes to forget the assumption of normal flow in each point of the structure all in preserving the formalism of generalized standard materials but on the scale of the structure, where variables of state are now the field of deformation and the field of internal variables A. One thus defines the total free energy and the total potential of dissipation:

F (, has) = ((X), has (X), has (X))D X éq 1.1-6

D (a&) = (a& (X), a& (X))D X éq 1.1-7

The relation of behavior generalized is written now:

=

F, A = -
F, AD (a&) éq 1.1-8

has

1.2
Discretization in time

While being based on the assumption of convexity compared to A of the potentials F and D and by adopting one diagram of implicit Euler, the temporal discretization of the preceding problem [éq 1.1-8] is reduced to resolution of a problem of minimization relating to the increment has fields of variables interns. This problem is written for behaviors independent of time:
Min [F (, A

+ has
) + D (has
)] éq 1.2-1
has

where -
A is the field of internal variables at the previous moment.

1.3
Space discretization by finite elements

To solve the problem of minimization [éq 1.2-1], one carries out a space discretization by finite elements of the fields of internal variables by means of the nodal unknown factors which one will note A.
has (X) = NR (X
K
) K
With, has (X) = NR (X
K
) K
With
éq
1.3-1
node
node
where Nk and Nk are related to form and their gradients associated with the node K, respectively.
To simplify the writing, one will pose:
R
R
R
R
has (X) = B (X) A

with

has = (has, has and

)
B = (NR, NR) éq
1.3-2
The equation [éq 1.2-1] is written then:
R
R
Min G ((B A)
G
+ (BgA)) éq 1.3-3
With gauss
in which G indicates the weight of integration of the point of Gauss G.

The introduction of new unknown factors R which represent the values and the gradients of the variables
interns at the points of gauss allows to transfer nonthe linearities at the local level and to divide resolution of the equation [éq 1.3-3] in a linear total part (on the structure) and a local part nonlinear (at the points of integration). The problem of minimization is written then:

Min

R

R
R
R
G (

(G) +
(G - -
G) éq 1.3-4
With
R
,
gauss

R

G (B A
G
-

0
=
)
G

By dualisation of the constraint, one builds the Lagrangian one increased problem [éq 1.3-4] for to return to a problem without constraints:
Max Min L
R (A, R, µr) éq 1.3-5
µr
With, R
with
R
R R

R
R
R -
R R
R 2
R R

L (A, µ) =
(
) +
(
-) + B A - R
G
G
G
G
G
G R + µ (B A -
R)
NR

2
G
G

G
G
G

R
éq 1.3-6
R r2
R R
R
> 0
and

X R = X NR
.
xr
.
G
Ng
R
R
The positive matrix NR definite is introduced into the standard so that the coefficient of penalization R is adimensional. This matrix is selected like a diagonal approximation of the derivative second (cf [R5.04.01] for more detail):
R
R
2
=
NR Diag

R R éq 1.3-7

This problem is then solved by a method of Newton to solve the primal problem (calculation of A and R) and a method BFGS with linear research Wolfe to solve the dual problem (calculation of the multipliers of Lagrange).

1.4
Calculation of the variables intern at the points of gauss

At the time of the resolution of the local problem, one seeks to minimize the equation [éq 1.3-6] compared to R,
with A and μr fixed what is equivalent to:
R

R R

R
R R
L
Min R (fixed
With
, fixed
μ
)
fixed
G
-
+ μ
+ RN (
fixed
B WITH
- R)
(R
)
G
G
R
R
G
G
G
G
G éq
1.4-1

G
G

With convergence, the third term in the member of left of the expression above becomes null (the constraint is carried out) and the multipliers of Lagrange μr G seem a force then
R
complementary thermodynamics resulting from the nonlocal condition B
R
G A - G = 0.
From a practical point of view, to write in an incremental way the model of behavior not

room, one will write classically the equivalent of the equation [éq 1.4-1]:
R
F
R

= &

with

Ar
R
= (R
With, R
With) éq
1.4-2
R

With

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with
R R R
R
R
R R
R

R

With = A + μ + RN (BA -
With

with
)
= (,
WITH A) = -
éq
1.4-3
R

where F is the threshold of elasticity associated with the potential of dissipation, calculated by transform of
Legendre-Fenchel, and R
With and R
With the thermodynamic forces associated the internal variables and
, respectively. It is noted here that these two thermodynamic forces must be corrected,
on the one hand, by the multipliers of Lagrange associated with the nonlocal constraint =, and
in addition, by a measurement (balanced) of the variation enters the fields at the points of gauss R = (,)

and nodal field A.

2
Application to the model of Rousselier

We now describe the application of this theory to gradient to the model of Rousselier (cf [R5.03.06] for more detail on this model).

2.1
Some notations of the model of Rousselier

One points out below some definitions and notations used in the model of Rousselier.

F: tensor gradient which makes pass from the initial configuration 0 to the current configuration (T)
F p: "plastic" tensor gradient which makes pass from configuration 0 to the slackened configuration R
Fe: "elastic" tensor gradient which makes pass from the configuration R to (T)
E p
F = F F éq 2.1-1
J: variation of volume

J = det F éq 2.1-2
Be: left tensor eulérien of Cauchy-Green of elastic strain
Be
FeFeT
=

éq 2.1-3
G p: Lagrangian tensor of plastic deformation
G p
F pTF p
=
-
(
) 1 éq 2.1-4
Be
FG pFT
=

éq 2.1-5
E: tensor of the deformations used in the model of Rousselier
1
E = (
E
Id - b) éq 2.1-6
2

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D: rate of deformation
p
D: rate of plastic deformation
p
1
p T
D - FG& F éq 2.1-7
2
: constraint of Cauchy
: constraint of Kirchhoff
=
J

éq 2.1-8
S: tensor of the constraints used for the model of Rousselier
E
= S B éq 2.1-9
R = - isotropic a: work hardening
p: cumulated plastic deformation
F: porosity
1
f& = (1 - F)
p T
tr-
G
F & F éq 2.1-10
2

F: criterion of plasticity of the model of Rousselier
tr S
F (S, A) = eq
S + Df exp
1
+ A (p) -

y
éq
2.1-11

3 1
where is limited there from elasticity and 1, D two parameters materials specific to this law.

2.2 Model
continuous

To preserve a simple model, one will be satisfied to introduce, to control the modes of localization poro-plastic, a quadratic term in gradient of cumulated plastic deformation p in the free energy of the model of local Rousselier. As for the potential of dissipation, there remains unchanged
compared to the local version.
p
2
(

1
2
2L D R
E, p, p
~ ~
B
) =
[K (tre) + µ2 E: E] + R (U) du+
p p
. éq
2.2-1
2
0
13 D p

D

p
tr p
p
p
2
(D, p&, &p)
= p& + tr D ln
-1 + I + (tr D) + I + (p& -
p
D)
y
1

IR
IR
éq
2.2-2
Df p

&
3 eq

This potential of dissipation corresponds to the criterion of plasticity [éq 2.1-11].
B
L is the length characteristic of the material which corresponds to the average distance between two

inclusions, privileged sites of germination and nucleation of cavities.

2.3 Model discretized

To adopt a purely implicit algorithm to integrate this law of led behavior, on the one hand,
the resolution of a rather complex nonlinear system any more, and in addition, does not allow to express it
problem like the minimization of a functional calculus. This is why one prefers to deal with manner
clarify the variation according to porosity in the potential of dissipation [éq 2.2-2] as well as the variation
according to p of the quadratic term out of p of the free energy [éq 2.2-1]. For the other terms, one
employ a diagram of implicit Euler. It will be noted that the discretization of the rate of plastic deformation
p
D is expressed directly according to the elastic strain E:

p
1
p T
D - FG& F = - 1 [E
p T
B - FG F] = 1

1
E - [
p T
Id - FG F]
Tr
= (E - E)/T
éq 2.3-1
2
2 T

T
2

1 4
4 2 4
4 3

T

R
E

So that the free energy and the discretized potential of dissipation are given by the expressions following:

p
2
(
1
2
2L D R
E, p, p) = [K (tre) + µ e~
2
e~
:] + R (U)
B
-

+
(p p
) p
. éq 2.3-2
2
0

13 D p

tr E

2
(,
E p
) = p
+ tr E
ln
-1 + I + (tr E
) + I + (p
-
E
)
y
1

-

IR
IR
éq
2.3-3
Df
p

3
eq

with
-
T = T - T,
tr
E
= E - E,
-
p = p - p and
-
p = p - p éq
2.3-4
-

Q is the quantity known at the previous moment -
T.
In accordance with the paragraph [§1.4] [éq 1.4-1], the integration of the model of nonlocal Rousselier
express yourself like the minimization of the following functional calculus:

Tr
R
min
2
R
(
R
R
R R
R R
E +
-
E p
,
+ p) + (E
, p) + P - p rr +μ (P - Pr)

éq.
2.3-5
E, p
NR

2

What is equivalent to [éq 1.4-1] and [éq 1.4-2]:

F

E =
S = -

E

S

F

(,
E p) p =
éq 2.3-6
R
R
R
R
R R

R
With
R
With = A + μ + RN (P - Pr)

R
To = 0

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R
R
According to the equation [éq 1.3-7], the matrix of weighting ***NR*** *is worth:*
1 0
0
0
2

2
R
R

0 L
0
0
-
C
-
D R
-
2
4 2
=
NR *Diag*
H

=
R R

2
with

H

=
(p)
and
L =
L

éq
2.3-7
p
p
0 0 L
0
D
C
p
13 B

C

2
0
0
0

C
L
In all these equations, one adopted the following notations:
ruffle = (, AA) = (- R, ***R****)*

- éq 2.3-8
R R
With *= (R*
With, R
With*)*
éq 2.3-9
Pr *= (p,* ***p****)*
éq 2.3-10
R R

***P** = **BP** = (P, **P**)*
éq 2.3-11

*The vector **Pr** represents the cumulated plastic deformation and its gradient, calculated at the points of Gauss while **P** represents the plastic deformation calculated with the nodes.*

***μr** = (μ, **μ**)*
éq 2.3-12

The whole of the equations to be solved is thus the following:

Equations of state:
***S** [*
-
*= K tr **ED** + μe~*
2] ***éq 2.3-13***
Ar -
= R (p) μ
+ + Rh (P - p) -
= R (p) - C p +
1
c2
éq
2.3-14
R
- 2
- 2
***With** -*
*= H L **p***
C +μ + Rh Lc (
***P** - **p**) -*
*= C **p** + **C** = 0*
3
4
éq
2.3-15
Laws of flow:
*tr **S***
*tr **e**=*
-
Pd F

exp

éq 2.3-16
1

3
~ 3
s~
e=
p

éq 2.3-17
2
seq
c4
p=

éq 2.3-18
3
C
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Condition of coherence:
(
S
F ***S****, Ar)*
-
tr
= S + D F exp
+ Ar
eq
- y F 0 p 0 FP 0
1
=

éq 2.3-19
3
1

Definition of the various coefficients:

C = -
Rh
0
- 2
2
1
, *C = μ C P*
2
*+ 1****,*** *C = 1*
(+ R) H
0
3
C
L ***and c4*** *=* ***μ*** *+ 1*
C C
L
P éq 2.3-20

2.4
Treatment of the singular points

The expression of the normal cone to which the direction of flow belongs is licit only at the points where the criteria are derivable, i.e. if seq 0. While proceeding classically by a prediction rubber band followed by a plastic correction only if necessary, one can be satisfied to examine singular points confined on the border of convex of elasticity, i.e. points such as:
tr ***S***
~
1
-
R
y
S *= 0*
and

D F

exp
+ A =

éq 2.4-1
1

3
The normal cone with convex of elasticity in such a point is the whole of the directions of flow which carry out the problem of maximization according to:
*sup [**S**: p*
***D** + Ar p& + R*
p
***With** &p - (**D**, p&, &p)]*
p
***D**, p&, &p*
1

p
R
y
p
tr p
D

=
sup
*tr **str D** + A p*

éq 2.4-2
*& - p& - tr **D** ln*
1
-
1
p
D**, p&, **p
3
&
D F p&

-

tr p

***D** 0*
p&-
2
L &p
D p 0
B
- 3
eq
*They are the directions of flow (**Dp**, &p **p***
, &) characterized by:

*tr **S***
p

*tr **D** = D F p&*

exp

*3 1 **éq 2.4-3***
2 p
Deq p

&
3
*Thus, in a singular point of the border of the field of elasticity, the increments of the variables interns (**E***
, p
*, **p**) check simply:*

*tr **S***
-

*tr **E** = D F p*

exp

*3 1 **éq 2.4-4***
2

*(**E**) eq p*
3
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3 Resolution numerical

3.1 Expression of the discretized model

R
*Knowing **μr** and **P**, elastic strain -*
__E__, cumulated plastic deformation -
p, it
gradient of the cumulated plastic deformation -
***p** and displacements **U** and **U**, one seeks with*
to determine (,
***E** p, **p**, F)*

.
*Displacements being known, gradients of the transformation of 0 with -, noted **F**, and of - with, noted **F**, are known.*

The system of equations to be solved is as follows:
-
F = FF éq 3.1-1

J = det ***F éq 3.1-2***

J = ***éq 3.1-3***

E
= ***S B éq 3.1-4***

Be *=* ***Id - E***
2 ***éq 3.1-5***

Tr
1
E *= [****Id - F***
*{****Id - E***
2} T
F

] ***éq 3.1-6***
2
Equations of state:
S *[*
-
= K tr ***eId*** *+ µe~*
2] ***éq 3.1-7***

Ar = - R (p) - C p +
1
c2 ***éq 3.1-8***

Ar=- ***p***
3
C + ***c4= 0 éq 3.1-9***
Definition:
4
C = -
Rh
0
- 2
2
2
1
, *C = µ C P*
2

+ 1, C = 1
(+ R) H
0
3
C
L, c4 = μ + 1
C C
L
P,
2
C
L =
*L **éq 3.1-10***
13 B
*Thereafter, one expresses the laws of flow and the criteria of plasticity directly according to tensor of the elastic strain **E**.*
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Laws of flow:
Tr
-
*K tr **E***
*tr **E** - tr **E***
p

= Df exp (-

*) **éq***
3.1-11
1

~
Tr - 3
~
E
E
p

D

solution

if
gulière
~
2
E
***E** =*
eq
éq
3.1-12

2
0

and

*p (**E**)*
singuli

solution

if
era

eq
3

c4

p=

éq 3.1-13
3
C
Condition of coherence:

-
K tr ***E***
2µe + D F exp -
+ Ar
eq
- y
(
1

D

solution

if
gulière
R

F ***S****, A)*
1
=

-
K tr ***E***
éq
3.1-14
D F exp
R
y
1

-
+ A -

singuli

solution

if
era

1

with F 0 p 0 FP = 0
Porosity:
The law of evolution of porosity is treated same manner as in the model of Rousselier in local version. One obtains (cf [R5.03.06] for more detail):
F = 1 (1 - F exp (treTr
0)
- tr ***E) éq***
3.1-15
where f0 is initial porosity.

3.2
Resolution of the nonlinear system

The integration of the law of behavior is thus summarized to solve only the equations [éq 3.1-11], [éq 3.1-12] and [éq 3.1-14] (the equation [éq 3.1-13] gives ***p*** *directly since* ***c4*** *and 3 C*
are known). Once determined p and ***E*** *by the whole of these three equations, one deduces some constraint* ***S*** *by the equation [éq 3.1-7], the constraint of Cauchy by the equations [éq 3.1-3] and [éq 3.1-4] and porosity F by the equation [éq 3.1-15].*

It is noticed that the three equations to be solved are identical to those of the model of Rousselier room where one changed only thermodynamic force A by R
A. The resolution is thus
identical to this model. For this reason, we give only the broad outline. For more detail, the reader will refer to the document [R5.03.06].
If one poses:
= tr ***E*** *- tr Tr*
X
E *0* ***éq 3.2-1***
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then, the equation [éq 3.1-11] is written:
K tr Tr
E
p
(X) 1
K X
= X exp
with *G =*
-
D F

exp -
éq 3.2-2
G

1

1

After some calculations, the equation [éq 3.1-12] becomes:

3 p (X)
1-
~Tr
E
réguliè

solution

if
Re
~

Tr
2

***E** (X) =*
eq
E

éq
3.2-3

2
0

and

p
Tr
E
singuli

solution

if
era

eq
3

Lastly, if one poses:
Kx
S (X) = R (p (X))

+ y + 1
C p (X) - c2 - 1G

exp -

éq 3.2-4

1

where S (X) is a continuous and strictly increasing function X, then the condition of coherence [éq 3.1-14] is written (while using [éq 3.2-3]):
Tr
(
E
p
X
F X) µ
2 eq - 3
µ - S ()
0
R

solution

if
égulière
=
éq 3.2-5
S (X) =
0

singuli

solution

if
era

One thus brings back oneself to solve this scalar equation in X. Variable X is positive or null X 0 to guarantee a positive cumulated plastic deformation and the elastic solution is obtained for x=0.

3.3
Course of calculation

The general step to determine X is as follows:

1) One seeks if the solution is elastic
· calculation of F ()
0
· if F ()
0 < 0, then the solution of the problem are the elastic solution Sol
X
= 0
· if not one passes into 2)
2) If
S ()
0 > 0, the solution are plastic and regular
· one passes into 4)
3) If
S ()
0 < 0, one seek if the solution is singular
· one solves S (S
X) = 0
2
· if S
X checks the inequality
S
Tr
p
~

Ground
eq
E, then the solution is singular
S

X
= X
3
· if not, S
X is a lower limit to solve F (Ground
X
) = 0, one pass into 4)
4) The solution is plastic and regular
· one solves F (Ground
X
) = 0
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3.4
Resolution of the functions to be cancelled

To solve the equations S (X) = 0, F (X) = 0, one employs a method of Newton with terminals controlled coupled to dichothomy when Newton gives a solution apart from the interval of the two terminals. One now presents the determination of the terminals for each case precedents.

3.4.1 Hight delimiters and lower if S (0) > 0

One solves:

Tr
μ

Kx
2 eq
µe - S (X = 3
)
X exp (
)
F (X) = 0

4
42
1
4
43 G

1

F
1 4
4 2 4
4 3 ***éq***
3.4.1-1
F ()
0 > 0

1
3µ p

F ()
0
1
> 0
where the function p
(X) is continuous, strictly increasing and null at the origin and the function F ()
1 X is
continue, strictly decreasing and strictly positive at the origin (see [Figure 3.4.1-a]).

One poses:
Tr
Kx
F = 2µeeq - R (X) - y - C p + C + G exp (-
) then F

(X) < F (X)

X 0
1
1
2
1
2
1
éq 3.4.1-2
1
4
4
4
4
4
2
4
4
4
4
4
3
1

f2
where the function F ()
2 X are continuous, strictly decreasing. Two cases will be considered.
Case where F (0)
0
2
>
In this case, the successive resolution of the equations:
Inf
Inf
F (p
2
) = 3µ p

éq 3.4.1-3
to deduce p from it
, then
Inf
Inf
Kx
Inf
X
exp (
) = G p

éq 3.4.1-4
1
to deduce X from them a lower limit Inf gives
X
.

Note:

In Code_Aster, the routine rcfonc solves the equation corresponding to the solution of the model to isotropic work hardening and criterion of Von Mises, i.e. el
- R (p) - = 3 p
eq
y

µ. One provides
in entry of this routine el
eq, the Young modulus E and the Poisson's ratio. If one poses
el
=
1
(
2 +)
2 eTr

-
Inf
eq
µ eq - 1
C p + c2 and E =
[3µ+ 1c], the function
Inf
F (p
2
) = 3µ p

3
bring back to solve an equation of the el type
- R (p) - = 3 p
eq
y

µ.
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Case where F (0)
0
2

In this case, the lower limit is taken equalizes to zero: Inf
X

= 0 .

The upper limit Sup
X
is such as:
Sup
Inf
KxSup
Sup
G
3
p

µ
= F (X) X
exp (
)
F (xInf)
1
=

éq
3.4.1-5

3µ 1
1
Kx
The equation of type X exp (
) = constant is solved by a method of Newton.
1

3
Sup
Inf
f1 (X)
p

µ (X)
3
p

µ (X) = f1 (X)
f2 (X)

3
p

µ (Inf
X
) = f2 (Inf
X
)
X
Inf
X
Ground
X
Sup
X

Appear 3.4.1-a: Chart of the hight delimiters and lower

3.4.2 Hight delimiters and lower if S (0) = 0

The system to be solved is as follows:

X
Kx
-
Kx
S (X) = 0
R p + exp (
) + y + C p
1 - c2 = G
1 exp (-

)

G
1
1 éq
3.4.2-1
S (0) < 0

-
-
R (p) + y + C p
1
- c2 < G
1
The part of left is a continuous function, strictly increasing X, the part of right-hand side is one continuous, strictly decreasing of X and strictly positive function in the beginning.
Two cases must be considered.
Case where R (-
p) + y +
-
C p - C > 0
1
2
:
Using the properties of the two functions, a chart (cf [Figure 3.4.2-a]) of these functions shows that the upper limit Sup
X
is such as:
KxSup
Sup

G
1
G exp (-
) = R (-
p)

-
1

+ y + 1
C p - c2
X
=
1
log

K

-
-
1
R (p)

+ y + 1
C p - c2
éq 3.4.2-2
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The lower limit Inf
X
is such as:
KxInf
Sup
Sup
1
G exp (-
) = R (p) + y + 1
C p
- c2
1

+ éq
3.4.2-3

Inf
1

X
=
1
G
log
K

Sup
Sup
R (p
)

+ y + 1
C p
- c2
Case where R (-
p) + y +
-
C p - C 0
1
2
:
In this case, one will take for the lower limit:
Inf
X
= 0 éq 3.4.2-4
The successive resolution of the two equations:
Sup
-
Sup
Sup

Kx
G
Sup
1
+ c2 - C p

1
- R (p
) - y = C p

1
and
Sup
X
exp (
) = G p

éq 3.4.2-5
1
allows to deduce p from it
then X to give an upper limit Sup
X
. To solve the equation of
left above, it is enough (see remark of the preceding paragraph) to use the routine rcfonc in
1
(
2 +)
posing el

-
eq = 1
G - 1
C p + c2, E =
1
C and = 1.
3

R
G
(X) + y +1 p - 2c
1

Kx
1 G

exp -

R (p) +
-

1
y + 1
C p - c2
Inf Ground
Sup
X
X
X
X

Appear 3.4.2-a: Chart of the hight delimiters and lower

3.4.3 Hight delimiters and lower if S (0) < 0 and xs not solution

The following system is solved:

Tr
µ
3
Kx
2µeeq - S (X) =
X exp (
)
4
42
1
4
43 G
1

1 4
4 2 4
4 3
F (X) = 0

f1
3µ

p

S (0) < 0

F (0)
1
> 0

éq
3.4.3-1

S

S (X) = 0
S

Tr
µ
3
S
Kx
2µeeq =
X exp (
)
G

1

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The solution Ground
X
is such as S (Ground
X
) > 0 .
For the lower limit, one takes Inf
S
X
= X. Being given properties of the functions 1f
(strictly decreasing) and 3µ p (X) (strictly increasing), the upper limit Sup
X
is such
that (cf [Figure 3.4.3-a]):
Sup
Sup
Kx
G
2
Tr
X
exp (
) =
eq
E éq
3.4.3-2

3
1
This equation is solved by a method of Newton.

S (X) < 0 S (X) > 0
2µeTr
eq - S (0)
S (X) = 0
3µ
Kx
Tr

X exp (
)
2µeeq
G
1

X
Inf
S
Ground Sup
X
= X X X

Appear 3.4.3-a: chart of the hight delimiters and lower

3.5
Form of the tangent matrix

The resolution of the primal problem (calculation of P and Pr) by a method of Newton, requires calculation
following tangent matrix:
p
p
R

Pr = P
P
H
éq 3.5-1
P
p
p
P
P

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3.5.1 Case
rubber band

It is pointed out that the elastic solution is given by:
-
p = p éq 3.5.1-1

c4
p =

éq 3.5.1-2
3
C
what gives for the tangent matrix:
p

p

= 0
= 0
P

P
I

éq
3.5.1-3
p
I
p
I
R
= 0
=
ij
P

P
J

1+ R

3.5.2 Case
singular

The singular solution is given by the equations:
-
X Kx/1
p = p +
E

éq 3.5.2-1
G

c4
p =

éq 3.5.2-2
3
C
with X which checks:
S (X) = R (p)
- Kx/1
+ C p - C + y - Ge
= 0
1
2
1
éq
3.5.2-3
and
C = µ C P
- 2
2
+ 1, c4 = µ + Rh C
L
P éq
3.5.2-4
The linearization of this system gives:
1
Kx
p
=
1
(+
) eKx/1 X
éq 3.5.2-5
G
1
4
1

4
2 3
coef 1

R
p =

P éq 3.5.2-6
1+ R

S (X) = (H + c)
- Kx/1
p - C P + KGe
X = 0
1
1
éq
3.5.2-7
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By replacing p
in the equation above, one obtains:
C e-Kx/1
X
=
1
P
éq
3.5.2-8
coef 1 (H + c1) + KGe-2Kx/1
what gives for the expression of p
:
1
p
=
C coef 1 P
1
éq 3.5.2-9
(H + C coef 1
1
+ KGe-2Kx/1
)
1
4
4
4
4
2

4
4
4
4
4
3
coef 2

maybe for the tangent matrix:
p

C coef 1
p
1

=

= 0
P

coef 2
P
I
éq
3.5.2-10
p
I
p
I
R
= 0
=
ij
P

P
J 1+ R

3.5.3 Case regular

The regular solution is given by the equations:
-
X Kx/1

p = p +
E

éq 3.5.3-1
G

c4
***p** =*

éq 3.5.3-2
3
C
X checks:
F = 2μeTr
eq - 3
μ p (X) - (
S X) = 0 ***éq***
3.5.3-3
The linearization of the whole of these equations gives:
p
= coef E
1 Kx/1 X
éq 3.5.3-4

R
***p** =*

P éq 3.5.3-5
1+ R

F = -3µp (X) - (
S X) = 0 ***éq***
3.5.3-6

S (X) = (H + C p
1) - C P

1
+ KGe-Kx/1 X
éq 3.5.3-7
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While replacing in the equation [éq 3.5.3-6], the expression of p
[éq 3.5.3-4], one obtains for X
:
(coef1 (h+c1+ µ3 +KGe-2Kx/1
)
) eKx/1 x=c P
1

éq. 3.5.3-8
1
4
4
4

4
4
4
2
4
4
4
4
4
3
coef 2
One finds then:
C coef 1
p
= 1
P
éq 3.5.3-9
coef 2

R
***p** =*

P
éq
3.5.3-10
1+ R
what makes for the tangent matrix:
p

C coef 1

p
1

=

= 0
P

coef 2
P
I

éq
3.5.3-11
p
I
p
I
R
= 0
=
ij
P

P
J 1+ R

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4 Relation
`ROUSSELIER `

One activates the nonlocal formulation of the model of Rousselier by one of modelings "X_GRAD_VARI" of order AFFE_MODELE of the MODEL key word. As for the model even, it is available in order STAT_NON_LINE via the key word RELATION:
"ROUSSELIER" under the key word factor COMP_INCR and with the key word DEFORMATION: "SIMO_MIEHE".
The whole of the parameters of the model is provided under the key words factors `ROUSSELIER `or

`ROUSSELIER_FO `and `TRACTION `(to define the traction diagram) order DEFI_MATERIAU ([U4.23.01]). The characteristic length B
L is given under key word LONG_CARA of
DEFI_MATERIAU.
The constraints are the constraints of Cauchy, thus calculated on the current configuration (six components in 3D, four in 2D).
The internal variables produced in Code_Aster are:

· V1, cumulated plastic deformation p,
· V2 in V4, the gradient following axes X, y and Z of p,
· V5, porosity F,
*· V6 in V11, the tensor of elastic strain **E**,*
· V12, the indicator of plasticity (0 if the last calculated increment is elastic, 1 if solution figure regular, 2 if singular plastic solution).

Note:

*If the user wants to possibly recover deformations in postprocessing of sound calculation, it is necessary to trace the deformations of Green-Lagrange **E**, which represents a measurement of*
deformations in great deformations. The traditional linearized deformations measure deformations under the assumption of the small deformations and do not have a direction into large deformations.

5 Bibliography

[1]
LORENTZ E.: "Laws of behavior to gradients of internal variables: construction, variational formulation and implementation numerical ", Thesis of doctorate of the university Paris 6, April 27, 1999.

Titrate:
Model of Rousselier with gradient of internal variables

Date:
03/02/05

Author (S):
V. CANO Key
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Organization (S): EDF/TESE, SINETICS

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Solution of a differential equation of the second order by the method of NIGAM

Summary:

We present in this document, a method of resolution of the linear differential equation of the second order obtained during the calculation of a spectrum of oscillator.

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Titrate:
Solution of a differential equation of second order (NIGAM)

1 Introduction

During the calculation of a spectrum of oscillator, one is brought to solve a differential equation of second order whose solution is an integral of DUHAMEL.

If this integral can be calculated exactly using the transform of LAPLACE for some simple analytical functions (Dirac, Sine, Cosine, Heavyside,…) [bib1] it must be integrated numerically in the case general.

This document presents an effective method to solve this problem.

This method is implemented in Code_Aster, in the operator CALC_FONCTION, key word factor SPEC_OSCI.

2 Analytical solution of the equation

During the calculation of the spectrum of oscillator of a accélérogramme [R4.05.03], one is brought to solve
the linear differential equation of the second order

q&+ Q + 2
2
&
Q =
- (T)

where
Q (T)
is relative displacement

(T) is the acceleration of the movement imposed on the base

is the pulsation of the oscillator

is the reduced damping of the oscillator

With initial conditions on Q and &q.

The solution of this equation is written in the form:

Q (T) = +t H (T -). () D + Q (0) G (T) + q& (0) H (T) ***éq 2-1***
0

where Q ()
0 and &q ()
0 are displacement and speed at the initial moment.

· Expression of (
H T) and G (T) according to the value of reduced damping.

- If
< 1 (damping under criticality)

- T

(
E
H T) =
2
sin
2
(T 1 -)
1-

éq 2-2
2

G (T) = E T
cos 1
sin 1 2

(T -) + 2 (T -)

1 -

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- If
= 1 (damping criticizes)

(
H T) = you-wt

G (T) = (1 - wt) ewt

- If
> 1 (supercritical damping)

- T

(
E
H T) =
2
.
1
2
(HS T -)
- 1

G (T) = E T
C 2

1
2 1
(
H T
-)+
S
2
(H T -)

- 1

3 Method
numerical

The numerical method established in the Code Aster was proposed by NIGAM and JENNINGS [bib2] in the case of damping under criticality which corresponds to our initial seismic problem [R4.05.03].

By introducing the formulation [éq 2-2] in [éq 2-1] one is thus led to solve the equation differential:

Q (&t) + Q (T) + 2
2
&
Q (T) =
- (T)

with null initial conditions, whose solution is written:

1 T
Q (T)
- (T -)
=
E

[
sin D

(T -)]() D
D
0

with
2
D = 1 -

By supposing that (T) varies linearly inside each interval (T), one can then write
:

() = (T - T
) +
[(T) - (T - T)] for [
0, T
]
T

(T)

(t-t)
T
T
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from where the equation to be solved: (expressed in the new variable)

&q (&) +
2 &q () + 2q () = has + B for [
,
0
T
]

where
= (T has - T
)

B = [(T) - (T - T
)] /T

:
initial

conditions

with
Q (0) = Q (T - T
)
q& (0) = q& (T - T
)

The solution of this equation is the superposition of a particular solution and solutions of homogeneous problem.

has

2 B
B
· a particular solution: qp (T) = -
+
-

2 3 2
Q T = E [C1.cos + C
H
D
2.sin
D

]
· solutions of the homogeneous problem:
( )
( )
( )

has
-
B B.
Consequently: Q () = E
[C1.co (S D) +C2.si (N D)]- +2 -
2
3 2
and by deriving Q (compared to) one a:

B
&q () () E (C cos
1
+ C sin
-
= -
D
2
D
) + E
(- C sin
1
+ C cos
D
D
2D
D
) - 2

The coefficients C1 and C2 are then determined by the initial conditions at the beginning of the interval (i.e. for = 0).

has
2b
C = Q
1
(T - T
) +
-

2
3

1
has
2
2 - 1
C =
Q
2
& (T - T
) +
Q (T - T
) +
-
B
2

D

and by deferring C1 and C2 in the expression of Q and &q one obtains the matric equality for = T:

Q (T)

-
-

= (
Q T
T
T
T
With, T
) (
)

+ (
B, T
) (
)

Q
& (T)
Q

& (T - T
)
(T)

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4
Coefficients of matrices A and B of the system to be solved

Stamp a:

= E T has
sin
cos

11
(T
D
) + (T
D
)
2
1

-

E T

a12 =
(
sin
T
D)
D

has = -
E T

21
(
sin T
D)
2
1 -

= E T cos has
sin
22
(T
D
)-
(T
D

)
2

1

-

Stamp b:

D

2

sin

- T
2
- 1

(T) 2 1
2
B = E
D

.
cos

11
+
+
+

-
2
D
3
2

(T)
T

T
3

T

D
2

sin
- T 2

- 1

(T) 2
1
2
B = E
D
.
.cos

12
+
-
+
2
D
3
(T)
T

T

2
3

T

2
2

-
- 1

B = E
T

. C (
bone
D
D T
) -

sin
T

2
( )
21
+
2
T

1 -

-

2
1

+

1
.
3
2
(if
D
(Nd T) Co (sd T)

T

+
+
2 T

2
- T

2
-

1

b22 = - E

. C
2
(
bone D T
) -
(
sin D T
)

T

1 - 2

-

2

1
.
(if
(T)
3
D
(Nd T)

-
T
+
cos

D

2
T

2
with D
= 1-

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Calculation of Q ()

Knowing Q () and &q (), it is consequently possible to give the analytical expression of acceleration & (&
Q).

&q () = - () E [C1 (
cos D) + C S
2
(
in D)]
1
+ E (- C if
1 D
(Nd) + C C
2D
(
bone D) - 2
2
&q (&) = + () E [C1 (
cos D) + C S
2
(
in D)]
+ () E (- C if
1 D
(Nd) + C C
2D
(
bone D)
- () E (- C if
1 D
(Nd) + C C
2D
(
bone D)
+ E [- C 2 Co

2
1 D
(sd) - C S
2D
(
in D)]
2
&q (&) = [() -2 -
D] E
. [C1c (
bone D) + C2 (
sin D)]
however
2
2
2
D = (1 -), from where
q& (&)
2
= E [C1 (
cos D) + C2 if (
N D)]

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[2]
***N.C. NIGAM & PC JENNINGS: Calculation of Response will spectra from motion earthquake Bull. of the Seismological society of America, Vol.59 n°2 pp 909 - 922 April** 1969.*
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D. SELIGMANN, L. VIVAN: Seismic response by spectral method [R4.05.03].

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Handbook of Reference
R5.05 booklet: Transitory or harmonic dynamics
Document: R5.05.02

Algorithms of direct integration of the operator DYNA_LINE_TRAN

Summary:

*This document describes the diagrams of temporal integration which are used to solve in a direct way of
problems of dynamics in transitory linear mechanics. The diagrams of NEWMARK and WILSON are detailed, as well as the diagrams "centered differences with constant step" and "not of adaptive time".*

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1 Introduction

The goal of the transitory dynamic analysis is to determine according to time the response of one structure, being given an external loading, or boundary conditions functions of time when the effects of inertia cannot be neglected.

In a certain number of physical configurations, one cannot do without a transitory analysis while being satisfied with a modal or harmonic analysis:

· if the history of the phenomenon has an importance in the study,
· if the external loading is complex (seism, excitations multi-components…),
· if the system is nonlinear (plasticity, shocks, friction…).

The methods of transitory analysis dynamic which can then be used are divided into two main categories:

· methods known as of direct integration,
· methods of RITZ including/understanding inter alia the recombination of modal projection.

The methods of direct integration are thus called because of the fact that no transformation is not carried out on the dynamic system after the discretization in finite elements.

We will make a presentation of the algorithms of direct integration used to solve one *dynamic problem in mechanics for linear structures. These algorithms are employed in operator DYNA_LINE_TRAN of Code_Aster.*

The methods of RITZ, on the contrary, proceed to a transformation of the initial dynamic system, very often a projection on a subspace of the space of starting discretization. The resolution dynamics is done then on a modified system, which gives access only one approximation of response of the initial system. They are presented in another document [R5.06.01].

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2 Methods of implicit direct temporal integration and clarify of a dynamic problem

It is supposed that the studied structure has a linear behavior and that equations governing sound balance dynamic were discretized by finished differences or finite elements. One is obtained discrete system of differential equations of the second order which it is a question of integrating in time.

In a general way, these equations take the following form:

$$\mathbf{M}.\ddot{\mathbf{X}} + \mathbf{C}.\dot{\mathbf{X}} + \mathbf{K}.\mathbf{X} = \mathbf{R}$$

$\mathbf{X}^T$, $\dot{\mathbf{X}}^T$, $\ddot{\mathbf{X}}^T$

.
M *is the matrix of mass of the system,*
.
C *is the matrix of viscous damping of the system,*
.
K *is the elastic matrix of rigidity of the system,*
.
R *is the vector of the external forces applied to the viscous system.*

The system is of the second order.

Two classes of methods of integration can be distinguished to integrate them step by step equilibrium equations: they are the methods of explicit and implicit integration.

Let us see what distinguishes them by examining temporal integration from the following linear system:

M.X
& + ***C.X*** +

T
&
K.X = ***R***
T
T
T

This differential connection of the second order can be brought back to a first order system:

A.u& = ***B.u*** + ***F***

éq 2-1

where:

X
X&
I *0*
0
I
0
U = ***U*** = ***A*** =
B =
F =

X

&

X
&
0 ***M***
- K - C
R

To integrate this differential equation, one thus uses a discretization Ti of the interval of study that a formula of differences finished to express the derivative &u.

*One calls methods **of integration clarifies the** methods where, in [éq 2-1] written at time Ti, only the derivative &u utilizes the variable **U** at time ti+1. In this way, determination of sizes sought at the moment ti+1 does not result from an inversion of system utilizing the operator **K**. If moreover, one carries out a "mass-lumping" in order to return the matrix **M** diagonal,*
*determination of **ui+1** is particularly simple. They are the principal characteristics there of methods of explicit integration.*

*The implicit or semi-implicit methods utilize the discretization of **U** in [éq 2-1] to one posterior moment with Ti, generally ti+1, in order to determine the variables of the problem with ti+1. Their determination thus passes by the resolution of a system utilizing the operator **K**.*

Two concepts concerning the diagrams of integration are important: consistency and stability.

---

*The approximations used to obtain the differential operators define **consistency** or **the order of the diagram of integration**. One can indeed consider that the approximation with which one*

displacement with each step of time obtains is related to the order of approximation of derived first and seconds compared to time.

*The study of **stability** of a diagram consists in analyzing the propagation of the numerical disturbances with*
run from time. A stable diagram preserves a finished solution, in spite of the disturbances, whereas one unstable diagram led to a numerical explosion or divergence of the solution.

To make a study of stability of a diagram of integration, one puts this last in the form of one linear recursive system and one determine the particular characteristics of this system. If all them eigenvalues of the operator of recursivity are smaller than 1 modulates some, the diagram is stable, if not it is unstable (cf [bib2]).

The diagrams of integration clarifies are generally conditionally stable, which means that the step of time must be sufficiently small to ensure the stability of the diagram.

Certain implicit algorithms have the characteristic to be unconditionally stable, according to the choice certain parameters, which makes their interest and makes it possible to integrate the dynamic phenomenon with one
no arbitrarily large time.

The diagram of WILSON and the diagram of NEWMARK can be explicit for certain choices of their parameters. In Code_Aster, they are employed for their properties of stability unconditional, clean with the implicit schemes. They will thus be classified here in the category of implicit schemes and one will see under which conditions they give the properties of stability wished.

Two explicit diagrams of integration were also introduced into Code_Aster. It is about diagrams DIFF_CENTRE and ADAPT which are both based on the method of the centered differences. They are conditionally stable and requires to be powerful a matrix of mass diagonalized. Conditional stability leads to a control of the step of time which, exploited in the case diagram ADAPT, allows an adaptation of the step of time according to the speed of modelled phenomena.

3
The diagram WILSON [bib1]

3.1
Presentation of the diagram

It will be supposed in what follows that the solids are elastic linear. This method leaves the assumption that acceleration is linear between T and T +. T
:

&X

= &X +
.
+
(&X

-
+ .
&X
T
T
T
T
T)
. T

éq
3.1-1

&

&
X
X
T +.t
T
T
+
&Xt
T
T + T T +. T

While integrating [éq 3.1-1] according to the variable, one obtains:

2
&X

= &X + &X +
.
+
(&X

-
+ .
&X
T
T
T
T
T
T)

2. T

éq
3.1-2

2
3
***X** = **X** + **X&** +*
X
+
& +

(X

.
&

.
- X
T
T
T
t+
T
& T)
2

6 T

éq
3.1-3

One writes the equilibrium equations at time T +. T
with 1:

M.X
&

+ ***C.X***
t+ T
& +

K.X *=*

R
t+ T
t+ T
t+ T

éq
3.1-4

while expressing &
Xt+*. T and &Xt+. T according to &Xt+. T and of* ***Xt****, &Xt and &Xt by the system*
[éq 3.1-2], [éq 3.1-3], and while replacing in [éq 3.1-4], it comes:

~
~
K. X

=
+ .
R
T
T

where

~
3
6
K = ***K*** + (
. ***C*** +
. ***M***
.t)
(.t) 2

~R = ***R*** *t+. (****R***
t+ T
- ***R*** *T) +* ***Mr.*** *(has.* ***X***
0
T + A.
2 &
X
T + 2. &Xt) + ***C****. (has.* ***X***
1
T + 2. &
X
T + A.
3 &X
T)

6
3
. T

a0 = (
has =

=

has
2.a
=

has

. T
) 2
1
(. T
)
2
1
3
2
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One goes back with displacements, speeds and accelerations to the step T + T
by the relations:

&X =

a.
t+ T

*4 (**X** - **X***
+

t+. T
T)
a.
5 &
***X** + A.*
T
6 &Xt
&X

=
t+ T
&X + A.
T
7 (&X

+
t+ T
&Xt)

X

*= **X** + T*
.
t+ T
T
&X + A.
T
8 (&X

+ 2.
t+ T
&Xt)

has
- has
3
T

t2

has
0
=
has
2
=
= 1 has -
has =
has
4
5
6
7
8 =

2
6

3.2
Complete algorithm of the WILSON method:

) initialization has:

*1) initial conditions **X**, **X***
0
& 0 and &X0
2) choices
of
T and and calculation of the coefficients a1,… a8 (cf above)
*3) to assemble the matrices of rigidity **K** and mass **M***
~
*4) to form the matrix of effective rigidity **K** = **K** + has. **M***
0
*+ has. **C***
1

~
5) to factorize
K

b) with each step of time:

~
1) to calculate the effective loading ***R***

R *=* ***Rt*** *+. (****Rt****+ T -* ***Rt****) +* ***Mr.*** *(has.* ***Xt*** *+ has.* ***X&t*** *+ 2.X&t) +* ***C****. (has.* ***Xt*** *+ 2.X&t + has.* ***X&t***

0
2
1
3
)

~
~
2) to solve ***K****.* ***X***

=
+ .
R
T
T

3) to calculate displacements at time T + T

&X =

a.
t+ T
*4 (****X*** *-* ***X***
+

t+. T
T)
a.
5 &
X *+ A.*
T
6 &Xt
&X

=

t+ T
&X + A.
T
7 (&X

+
t+ T
&Xt)

X

*= **X** + T*
.
t+ T
T
&X + A.
T
8 (&X

+ 2.
t+ T
&Xt)
4) calculation of the step of next time: return to the beginning

3.3
Stability condition of the WILSON diagram

The method is unconditionally stable for WILSON > 1.37, a value usually employed for being 1.4. Moreover, the method presents numerical dissipation for > 1, all the more important what increases.

The key word WILSON factor: (THETA: HT) makes it possible to specify the use of this algorithm and the choice
value of. By defect, the value of is taken to 1.4.
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4
The diagram of NEWMARK [bib1], [bib2]

4.1
Presentation of the diagram

NEWMARK introduced two parameters and for the calculation of the positions and speeds to the step
T + T
:

&X

*= **X** +*
&
T
.
+
[(1). &X +. &X
T
T
T
T
t+ T
]
2

1

***X** = **X** + T*
*. **X&** + T*

-

.

X

. &. ***X***
t+ T
T
T
T
& t+ T

2

+

Let us consider the equilibrium equations at time T + T
:

M.X
&

+ ***C.X***
t+ T
& +

K.X *=*

R
T + T
t+ T
T +t

let us defer the preceding relations while eliminating &
Xt *T*
+ and &Xt
T
+, it comes:

~
~
~
K. X

R** where: **K
K
*. **M***
*. **C***
T
T
has
has
+
=
=
+ 0
+ 1
~
R** = **R

t+ T
*+ **C**. {has. **X***
1
T + A.
4 &
X
T + A.
*5 &Xt} + **Mr.** (A.**X***
0
T + A.
2 &
X
T + A.
3 &Xt)

1

1
1
has =
=
=
=

- 1
0
(
has
has
has
. t2
)
1
(. T
)
2
(. T
)
3
2
with:

T

has =
- 1
has =
- 2
has
T
. 1

.
4
5
6
(
) has
T
7

2

=
-

=

4.2
Complete algorithm of the method of NEWMARK

) initialization has:

1) conditions
initial
X, ***X***
0
& 0 and &X0
2) choices
of
T and, and calculation of the coefficients a1,… a8 (cf above)
3) to assemble the matrices of stiffness ***K*** *and mass* ***M***
~
4) to form the matrix of effective rigidity ***K*** *=* ***K*** *+ has.* ***M***
0
+ has. ***C***
1

~
5) to factorize
K

b) with each step of time:

~
to calculate the effective loading ***R***
2
~
R *=* ***Rt+*** *T +* ***Mr.*** *(has.* ***X***
0
T + A.
2 &
Xt *+ A.*
3 &Xt) + ***C****. {A.****X***
1
T + A.
4 &
Xt *+ A.*
5 &Xt}

~
~
*to solve **K**. **X***

=
+ .
R
T
T

to calculate speeds and accelerations at time T + T

&X =

a.
t+ T
*0 (**X** -*

X
-

t+ T
T)
a.
2 &
***X** - A.*
T
3 &Xt
&X

=
t+ T
&X + A.
T
6 &X + A.
T
7 &Xt+ T

5
calculation of the step of next time: return to the beginning

---

4.3
Stability conditions of the diagram of NEWMARK:

The method of NEWMARK is unconditionally stable if:

(
2 +) 2

1
> .
0 5 and >

4

One introduces a positive numerical damping if >.
0 5 and negative if <.
0 5. When
= .
0 5 and = 0, the formula of NEWMARK are reduced to the diagram centered differences. One
1
combination very often employed is =.
0 5 and =, because it leads to a diagram of order
4
2, unconditionally stable without numerical damping.

This diagram of integration is used in a rather widespread way in the field of mechanics, because it allows to choose the order of integration, to introduce or not numerical damping, and has a very good precision. It is integrated in Code_Aster in operator DYNA_LINE_TRAN. key word factor NEWMARK: (ALPHA: Al, DELTA: of) allows to specify the employment of this algorithm and the choice of the value of and. By defect, the value of is taken to 0.25 and the value of is taken to 0.5.

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5
Numerical damping of the implicit schemes

The numerical advantage of the direct diagrams of implicit integration lies in the fact that the step of time can be substanciellement large compared to the smallest clean period of the system without to be likely to cause an instability of the results.

However, if the contents of the answer reside in a whole of clean modes, of which highest an Eigen frequency has Fmax, one will have to still respect a criterion on the step of time of the form:

1
1
T < (
with

10 F*

*) (100*F
max
max)

For modes of period clean about the step of time or lower than the step of time, them algorithms of integration introduce a strong damping which contributes to erase the contribution of these high modes.

One can see on the graph hereafter the reduction in amplitude of a system to a degree of freedom, without damping, when one integrates it by various methods (WILSON and NEWMARK

1
1

= ,

=) :
2
4

NEWMARK method
1
1
Percentage amplitude decay (AD X 100%)
= , =
2
4

It is checked here that the algorithm of NEWMARK with these parameters does not present any damping numerical.
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On the other hand, the implicit algorithms also have a rather significant effect of elongation of the periods
clean contained in the response of the structure which leads to a dephasing of the calculated solution.
The graph below presents percentages of elongation of the clean period of a system at one
ddl without damping.

Percentage period elongation (PE/T X 100%)
T/T

On these 2 graphs, one notes that to guarantee a precision on the amplitude and the phase of calculated displacements, it is necessary to respect a criterion close to:

.
01
.
0 01
T <
with

F
F
max
max

where Fmax is the high frequency of the movement which one wishes to correctly capture in numerical analysis.
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6
Diagram of the centered differences with constant step

6.1 Principle

The diagram clarifies centered differences with constant step is written:

&X
2

1 = &
X 1 + T &X T
+ O T
N (
, X,
N
N
&Xn) (
)
n+
N
2
2

X
2

1 = X
+ T
T
1
+ O T
n+
N
&X (, X,
N
N
&Xn) (
)
n+ 2

with the following notations:

X
X
N
&x
-1
X
&x
n-12
N
n+12
n+1
tn-1
tn-1/2
tn
tn+1/2
tn+1
T
T
.

The speed is expressed with indices half-entireties of the discretization in time whereas them displacements and accelerations are expressed with the whole indices.
Written this way, the diagram is of order 2.

Acceleration in T is not immediately calculable because speed is known only with the half-not of
N
previous time (in T
), which poses problem to evaluate the terms of damping. For
N 12
to circumvent this difficulty, one calculates acceleration in T per L `following approximation:
N

&X T

T
-
T

1
-
-

N (
, X,
N
N
&Xn) &X
, X,
N
N
&X
M 1 F

N
1
2
=

(N)
K X
C
N
&X
N
N 2
-

what constitutes a valid approximation if damping is sufficiently weak
(&
X =
+ O 1). The diagram loses its precision of order 2 if the damping of the structure is
N
&X
( )
N 12
important.
Other methods of approximation of acceleration can be considered. That selected is revealed a good compromise enters simplicity and stability, like the study described in the reference [bib4] on the precision and the stability of several methods.

The fields are filed at the moment T, T,
N
; +1 L, speed being approximate at these moments by following formula:

&
T
X
= &X
+
&X T, X,

1
1
1
&X
n+
N
N
N
N
+
+
+
+

n+ 1
2

2

1
12
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6.2 Stability

The diagram of the centered differences is conditionally stable. In the case of a system without damping [bib2], the diagram is stable for a step of time checking T < 2/
where
is
max
max
the greatest own pulsation of the system, is T < T
/. One needs a minimum of step of time
min
to describe the smallest period of the system T
.
min
The limiting value for the step of time decreases slowly when damping increases [bib4]. By example, for a damping of 0,5%, the condition becomes T < T
/ 5.
min

6.3 Algorithm

In short, the diagram such as it is introduced into Code_Aster arises in the following way:

0 inialisation
:

T, X, &X given
0
0
&X = M 1
T = 0 -
-

0
(F (
)) K X C
0
&X0
T
&X

1
= &X0 -
&X
-
0
2
2
1
With each step of time X, X
&
, X
& known
N
N
N
- 12
&X

1
= &X 1 + T &X T, X,
N
&X

n+
N
N
N
+
1
2
2

n+ 2
X

1 = X
+ T
n+
N
&Xn+12

&X
= M-1F T -
1 ***-***

n+1

(N) KX
C
N
&X
+
N 12
T

&X 1 =
1
+
N
&X
&X
+
N
N
+

+1
2
2
2
possible filing of X
, X
&
, X
&

n+1
n+1
n+1
then return at stage 1) for the following step.

6.4
Stamp of diagonal mass

The calculation of acceleration requires the inversion of the matrix of mass. This explicit diagram becomes
more powerful if a matrix of concentrated mass is used (`MASS_LUMPING `) so that it
that is to say diagonal. The inversion then does not require any more factorization and is immediate.
This is why in Code_Aster, the diagram of centered differences is licit only with matrices
of mass built in a diagonal way, by option “MASS_MECA_DIAG” of the operator
CALC_MATR_ELEM.
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6.5
Checking of the step of time

It was seen that the diagram of centered differences is stable provided that the step of time, in the absence of damping, is lower than a limiting value, equal to T < 2/
. In practice
max
one employs a step of time which is worth from 5% to 20% of the step of critical time. It was thus introduced one
test on the step of time which checks that:
2
T < 0 0
, 5
where K and m are the diagonal terms of the matrices of stiffness and of
K
II
II
II
max

1 inddl mii
mass.
If this condition is not checked, the user is stopped with a message indicating the step to him of maximum time which it can use.

6.6
Calculation of acceleration

The calculation of acceleration is done as follows:
for each degree of freedom, one tests if the diagonal term of the matrix of mass corresponding is no one.
· if it is not null, the term of acceleration is calculated according to the formula
:
&X
= M-1F T

N
-
N 1 -

+

() KX
C &X
N 1
+
N 12
· if it is null, the term of acceleration is not calculated. It is the case for degrees of freedom said of Lagrange. If they correspond to blocked degrees of freedom, it is licit of not to take account of the line in question and not to calculate its acceleration. If it degree of freedom of Lagrange was introduced to define a connection between two freedom degrees, that does not have any more a smell. The diagram is thus then unusable and a test stops the execution with an explicit message.

7
Diagram with step of adaptive time

7.1 Principle

The methods of calculation clarifies are particularly indicated in simulation of phenomena rapids, such as the wave propagation in the solids. On the other hand, they agree less well with slower phenomena since the stability condition of the diagram imposes a step of time of about a smallest clean period of the system.

The adaptive diagram, based on the diagram of centered differences, was developed to allow it calculation of transitory answers in which fast" and "slow" phenomena ". For example at the time of an impact, initially of the high frequency waves are propagated and dissipate themselves in the structure. Then, the structure does not answer any more but on its modes low frequency, them high frequencies being deadened. The idea is thus to adapt the step of time progressively in function of the phenomena brought into play, by fixing a criterion of precision on the solution.
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7.2 Diagram

The diagram clarifies centered differences with variable steps is written:

T

1 +
T
&X
2

1 = &
X
N
N
1 +
&X T
+ O T
N (
, X,
N
N
&Xn) (
)
n+
N
2
2
2

X
2

1 = X
+ T
T
1

+ O T
n+
N
*&X (, **X**,*
N
N
&Xn) (
)
n+ 2

with the following notations:

T1 +
-
T
N
N
2
X
X
&x
N
&x
-1
n-1
N
N
+1
X
2
2
N + 1
T
T
n+1/2
tn+1
n-1
tn-1/2
tn
T
T

n-1
N

.

It is noted that the step of time varies. It is subscripted: T.
N
That has as a consequence which the diagram is not rigorously any more of the second order, since it is not
“centered more”. More T
and T are different, plus the order of the diagram is close to 1. The strong ones
n-1
N
variations of the step of time are thus accompanied by a fall of precision. The formula speed employee leads to good results when the step of time decreases but cause a drop in the limit of stability when the step of time increases. This is why one it constrained to increase only very gradually.
Lastly, one uses the same approximations as for the differences centered with regard to calculation of accelerations and speeds to the steps of “whole” times:

· acceleration is estimated by &X T

T
and
N (
*, **X**,*
N
N
&Xn) &X
*, **X,***
N
N
&X
N

N

- 12
*&X T, **X,***
-
T -
-
;
N

N
&X
M 1 F

N

=

(N)

K X
C
N
&X
N
N 12
12
T

· and stored speed is evaluated by &X
.
1 = &
***X** 1 +*
&X
n+
N
N
+
+1
2
2

As for the diagram of centered differences, of which it is inspired, the diagram with adaptive step require the inversion of the matrix of mass. This is why the diagonalisation of the matrix is required of mass as well as the same restrictions on the degrees of freedom of Lagrange as for the diagram with centered differences.
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7.3
Estimate of the step of time according to the precision required

To define a criterion on the step of time according to the precision required on the solution, one introduced the concept of ***frequency connect disturbed*** *[bib4]:*

1
X& - X&
F
X
X
=
-1
AP N
2
X - X
X
x-1

This size can be interpreted like the "instantaneous frequency" of the system. It is indeed one approximation of the local slope of the curve forces/displacement. It is related to the error due to truncation in the limited developments of the diagram. It makes it possible moreover to take account of forces external and of their fluctuations in frequency.
In the case of a system with several degrees of freedom, it is necessary to calculate a frequency connect for each degree of freedom. One then employs the maximum on all the frequencies calculated to determine the step of time.
If the denominator tends towards zero, the apparent frequency can become very large and lose its physical significance. One then obtains an unjustified refinement of the step of time when speed cancel yourself. In the case of sinusoidal oscillations, it is the case twice per period. It then is modified criterion by introducing the following condition:

X - X -
&
&
1
1
-
X
X
X
X
X&
F
X
X

min
=
-1
T
AP N

2 p
X&
T
min

One obtains an intermediary between the frequency connect disturbed and the truncation error. The value
of &
X
is not easy to determine a priori and a badly selected value can lead to one
min
artificial moderation of the apparent frequency.
Two methods are proposed.

7.3.1 influences of the close nodes

In the case of a system with several degrees of freedom, one can be useful of the information given by the 1 J nv nodes close to node I:

1
X I
I

& X - X&
F
X
=

-1
max
max
AP N
DX, DY, DZ, DRX, DRY, DRZ
1 inb node 2

Bi

N

1
-15
-1

where Bi
I
J
N =
T
max 10 ms, X& 1,
max &
1

N
n+
N

+
2
100
1 jnv (X
2)

This method requires the census of the nodes close and the estimate “speeds” according to each type of degree of freedom (translation “DX”, “DY”, “DZ”, and possibly rotation “DRX”, DRY'

and
“DRZ”) for these close nodes.
The method programmed in Code_Aster simplifies this formula somewhat and consists, for one degree of freedom given, I, to make starting from this position an ascending research and one downward research on the degrees of freedom in their order of classification defined by NUME_DDL. The first two degrees of freedom, K and L, of comparable nature respectively found before and after the degree of freedom I are regarded as the “neighbors”. To limit the cost of this technique, research is made once for all at the beginning of transitory calculation and the “neighbors”
are recorded in two tables of entireties.
The use of this method is started by key word VITE_MIN: “NORM”.
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7.3.2 use of information at previous time

One can be also based on the information brought by the steps of previous times to estimate minimal speed. One then estimates it by the following formula:

&X I

&
K
X
= max
,
-

10 15 ms-1
min

K <n 100

One has then:

1
X I
I

& X - X&
F
X
=

-1
max
max
AP N
DX, DY, DZ, DRX, DRY, DRZ
1 inb node 2

Bi

N

-15
-
1
1

with Bi
I
J
N =

T
max 10 ms, X& 1,
max &

N
n+ 2

(X K)

K
100 <n

This method is engaged by key word VITE_MIN: "MAXIMUM".

This method cannot be employed if speed varies too much during calculation, bus in this case one would have with each step:
X - X
N
n-1 X I
&
T

min

7.4
Choice of the number of steps per apparent period, NR

Error analyses and criteria of stability established for a system with only one degree of freedom (see [bib4]) allowed to estimate the number of steps NR necessary per period connect to obtain one good precision. These tests showed that a minimum of 20 steps per period is necessary. It a number is skeletal by L `user in the command file thanks to the key word "NB_POINT_PERIODE". Its default value established to 50 leads to a precision on integration temporal of about 1 to 2%.
The step of initial time is useful like step of maximum time in the absolute: T
= T
. Balanced
mac
initial
by a skeletal coefficient by "PAS_LIMI_RELA", it is used as step of minimal time:
T
*= PLR **

min
tinitial

7.5
Heuristics of evolution of the step of time T
N

One defines an indicator, known as "error", on the choice of the step of time:

error = T
NR F

N
AP N

It is necessary that this indicator is lower than 1 to hope to guarantee a good temporal integration of smaller clean period. However the adaptive diagram must concomitantly avoid the use of one no the too small time, which would cause a overcost of calculation then, even appearance of "noises" parasites.
According to the indicator, the algorithm will increase or decrease the step of time. One defines for that two coefficients, CDP, the coefficient of refinement of the step of time (word key "COEF_DIV_PAS", default value: 1,334) and CMP, the coefficient D `amplification of the step time (word key "COEF_MULT_PAS", default value: 1,1).
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At the time of this search of the step of optimal time, one defines a maximum iteration count of reduction of
no time, iter
, to avoid with the step of time to evolve/move in a too brutal way, which is
max
prejudicial with the order of the diagram, and not to launch a too expensive optimization.

· if the indicator of error is higher than its limiting value, that one did not exceed the limiting number

of refinement for a step of time and that the step of time remains larger than its value minimal fixed a priori, the step of time is refined
:
1
T
T >
, iter < iter
and
,
max
T
N
>

min

N
T
T
N
NR F
CDP
N
AP N
· if the indicator shows that since five consecutive steps the step of time appears too fine, i.e.
0 7
, 5
T <
, then semi (
N T, CMP T

N
T
min
)
N
NR F
N
AP N

7.6 Algorithm

the algorithm was programmed in Code_Aster according to the following flow chart:

0 Initialization
:
X, X& given
0
0
&X = M 1
T = 0 -
-
0
(F (
)) K X C
0
&X0
T
&X

1
= &X0 -
&X
-
0
2
2

recovery of the parameters of integration:
T

initial
CMP coefficient of performance of the step of time
CDP coefficient of reduction of the step of time
PLR limits to refinement such as Dt PLR Dt

initial
NR numbers of steps of time per apparent period
itermax a maximum number of reductions of the step of time

1
with each step of time:
X, X&
, X
& known
N
N
N

- 12
T

1 = T
+ T
n+
N
N
1.0
iter=0
1.1: temporal integration
T

1 +
T
&X 1 = &X
N
N
1 +
&Xn
n+
N
2
2
2
X
&

1 = X
+ T X
n+
N
N
1
n+ 2

&X

1 = M 1
- T1 -
1 -

n+
(
F n+)
K X
C
N
&X
+
1
n+

2
T

&X

1 = &
X
N
1 +
&X
n+
n+1
n+
2
2

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1.2 calculation of the apparent frequency and the error on the step of time

1
X I
I

& X - X&
F
X
=

-1
max
max
AP N
DX, DY, DZ, DRX, DRY, DRZ
1 inb node 2
Bi

N

error = T
NR F

N
AP N

1.2 test on the relevance of the step of time
· if error > 1et iter < iter
max
then T/CDP T
N
N
but if T < T
stop of calculation with error message
N
min
iter + 1 iter and return in 1.1

· if error > 1et iter > iter
max
then emission of an alarm and passage as in point 2.
· if error < 1 passage as in point 2
with if error < 0 7
, 5 since 5 consecutive steps:
amplification of the step of time T = semi (
N T
, CMP

max
T
N
N)

2
acceptance of the solution: possible filing of X
, X&, X
&

n+1
n+1
n+1
then N + 1 N: return in 1 for the step of next time

8 Conclusion

operator DYNA_LINE_TRAN allows the choice between several methods of temporal integration. In their parameter setting by defect, the diagrams of WILSON and NEWMARK are implicit schemes unconditionally stable. They thus require a linear inversion of system to each step of times but n the other hand offer a choice of the step time which is restricted only by the smoothness with which one wishes to describe the temporal evolution of the modelled phenomena.

Diagrams DIFF_CENTRE and ADAPT are explicit what avoids to them, in the case of a matrix of mass diagonal, an inversion of expensive matrix. But the conditional stability of this type of diagram generally leads to the use of small steps of times, conditioned by smallest clean period of the system. It is thus not guaranteed that the explicit diagrams are systematically faster. That depends on the simulated phenomena. If the physics of these phenomena requires a fine temporal discretization, the step of time employed is naturally in the interval of stability. In the contrary case, the constraints of numerical stability involves one inflation in the number of steps of time very expensive.

Diagram ADAPT makes profitable information on the frequential contents of the answer to adapt it no time. The discretization of time is not thus imposed any more by the smallest clean period of system but by its answer. That can be an advantage when the frequency of the answer evolves/moves in time, like in the case of the impacts.
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Harmonic answer

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Organization (S): EDF-R & D /AMA

Handbook of Reference
R5.05 booklet: Transitory or harmonic dynamics
Document: R5.05.03

Harmonic answer

Summary

This document presents the theoretical bases of the calculation of the steady operation of the response

of a system
complex mechanics, with linear behavior, subjected to a harmonic dynamic stress. Calculation relate indifferently directly to the system modelled in finite elements, or represented by a base modal; in this last case if the modal base is the product of the technique of under-stucturation one will refer to the document [R4.06.03].

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1 Introduction

In the harmonic problems, the studied system is subjected to an excitation varying like product of an unspecified function of space by a sinusoidal function of time.
To seek the answer consists in calculating the field of the sizes represented by the ddls of modeling in finite elements of the system. When the system has a linear behavior the answer field of the sizes observed tends quickly (because of extinction of its component transient by dissipation interns) towards a steady operation: the resulting field varies finally harmonically like the excitation. It is this steady operation of the answer that one proposes to calculate.

General notations:

T
: time
P
: Not running of the model

:

Pulsation (rad.s-1)

J
: Imaginary pure unit (j2 = -) 1
M
: Stamp of mass resulting from modeling finite elements
K

: Stamp rigidity resulting from modeling finite elements
C
: Stamp damping exit of modeling finite elements
Q
: Vector of the degrees of freedom resulting from modeling finite elements
F *ext.: Vector of the forces external with the system*

: Stamp vectors of the base of the substructures

: Vector of the generalized degrees of freedom

2 Equation harmonic

We establish the dynamic equation in the case of a harmonic request for three kinds of mechanical systems:

· pure structures (without fluid),
· pure fluids (without structure) with linear "acoustic" behavior,
· analog and digital systems structures and fluids in interaction fluid-structure.

2.1 Harmonic equation of the structures

The vibratory behavior of a pure structure results from the external forces which are applied to him. The size to be calculated is displacement in any point P of the model.

2.1.1 Calculation direct

In the case of direct calculation on the model in finite elements we can write:
$$\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + \boldsymbol{KU} = \boldsymbol{F}(P, T)$$
ext.

éq
2.1.1-1
where:

M
is the matrix (real) of mass resulting from modeling finite elements of S,
C
is the matrix (real) of damping exit of modeling finite elements of S,
K

is the matrix (real) of rigidity resulting from modeling finite elements of S,
***F** (P, T)*
is the vector (complex) of field of the external forces applied to S,
ext.
***U**, **u&** and **u&** are the vectors (complex) displacement, speed and acceleration, functions of P and T, resulting from modeling finite elements.*
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In a harmonic problem, one imposes a loading dynamic, spatially unspecified, but sinusoidal in time. One is interested then in the stabilized answer of the system, without holding account transitory part.

The field of the external forces is written:

F** (,) = **F
J T
P T
{
(P)}E

ext.
ext.

The field of displacements is written:

***U** (P, T)*

U
{(P)}E J T
=

The fields speed and acceleration are written:

*&u (P, T) = J {**U** (P)}E J T*
&
***U** (P, T) = - 2 {**U** (P)}E J T*

Finally the structure S checks the following equation:

(
2
***K** + jC - **M**) {}*
***U** = {**F** (P)}*
ext.

éq
2.1.1-2

***Particular case**: if damping is of **hysteretic** type "total" the equation [éq 2.1.1-1] becomes:*
***Mu&** + **K** 1*
*(+ J) **U** = **F** (P, T)*
ext.
éq
2.1.1-3
where is a total loss ratio (cf [R5.05.04]).
Then the equation [éq 2.1.1-2] is replaced by:
(
2
***K** - **M**) {}*
***U** = {*
(P)}
C
fext

éq
2.1.1-4
where:

M
is the matrix (real) of mass resulting from modeling finite elements of S,

K = K + jK is a complex matrix of rigidity.
C

2.1.2 Calculation on modal basis

The calculation of the harmonic response by the method of modal synthesis consists has to seek it field of unknown displacement, resulting from modeling finite elements, on an adapted space, of reduced dimension (transformation of Ritz).
One will refer to the documents [R4.06.02] and [R4.06.03].
If one rather uses this method the equation [éq 2.1.1-2] is projected on the basis of modal S and one leads to the following harmonic equation:

(
2
K + J C - M) {} = {F}
ext.
éq
2.1.2-1
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where:

M = TM
is the matrix (real) of generalized mass of S,
C = TC
is the matrix (real) of generalized damping of S,

K = TK
is the matrix (real) of generalized rigidity of S,

{F}
T
= {F}
is the vector (complex) generalized harmonic external forces
ext.
ext.
applied to S,

is the matrix (real) modal vectors of the base of Ritz of S,
{(P)}

is the vector (complex) generalized harmonic displacements.

Once {(P)} determined by éq 2.1.2-1 one makes a restitution on physical basis (cf [R4.06.02]).

2.2
Harmonic equation of the acoustic fluids

The document [R4.02.01] described modeling by finite elements of a fluid system (without transport) having a linear acoustic behavior.
The fluid system F undergoes a harmonic request acoustic speed on part of its border. The harmonic answer is described by the following equation [éq 2.2-1], where size with to calculate is the acoustic pressure in any point P of the model.

(
2
K + jC - M) {p (P)} = - J {v (P)}
N
éq
2.2-1
where:

M
is the matrix (complex) of "mass" acoustic exit of modeling finite elements of F,
C
is the matrix (complex) of "damping" acoustic exit of modeling finite elements of F, and in the species of the edge F where one
Z
apply an acoustic impedance,
K

is the matrix (complex) of "rigidity" acoustic exit of
modeling finite elements of F,
v (,) = v
J T
P T
{(P) E
}
where {v (P)} is the vector (complex) of field speeds
N
N
N
normal acoustics applied to the border F of F where one
v
apply acoustic speeds,
p (P, T)
p
{(P)}E J T
=

where {p (P)} is the vector (complex) acoustic pressures resulting from
modeling finite elements of F.

2.3
Harmonic equation of the systems fluid-structures

The document [R4.02.02] described modeling by finite elements of a system F + S made up of one fluid part (without transport) F in interaction with a part structure S (interaction out of F S). Fluid and structure have a linear behavior.
The fluid system F undergoes a harmonic request normal acoustic speed on a part of its border. The harmonic answer is described by the following equation [éq 2.3-1], where sizes to calculate are:

· acoustic pressure in any point P of the fluid F,
· displacement in any point P of the structure S,
· as an auxiliary potential of displacement in any point P of the fluid F,
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U
(P)

(
2
3
K - M - J I) p
(P) = + J {v (P)}
N

éq
2.3-1

(P)

where:

M
is the matrix (real) of “mass” fluid-structure resulting from
modeling finite elements of the fields F and S
I
is the matrix (real) of “impedance” fluid exit of modeling
finite elements of the edge F of the field F where one is applied
Z
impedance
K
is the matrix (real) of “rigidity” fluid-structure resulting from
modeling finite elements of the fields F and S
v (,) = v
J T
P T

{(P) E
}
where v (P) are the vector (real) field acoustic speeds
N
N
N
normals applied to the border F of F
v
U (P, T)
U
{(P)}E J T
=

is the vector (complex) field of displacement in the structure
S
p (P, T)
p
{(P)}E J T
=

is the vector (complex) acoustic field of pressure in
fluid F
P T

(,) {(P)}E J T
=

is the vector (complex) field of potential of displacement in
fluid F

2.4
General harmonic equation

With an aim of taking into account all the harmonic cases of equations the operator DYNA_LINE_HARM of Code_Aster solves the following general harmonic equation (cf [U4.53.11]):

3
2

K
J
I
N

(- J I - M + jC + K) {}
Q =
I
180
hi (F) E
gi (P)
éq
2.4-1
i=1

where:

I
Stamp fluid "impedance" possible exit of modeling finite elements,
M
Stamp of "mass" resulting from modeling finite elements,

C
Stamp "damping" exit of modeling finite elements,

K
Stamp "rigidity" resulting from modeling finite elements,
{Q (P}) Vector of the degrees of freedom resulting from modeling finite elements,

{G (P) Vector field with the nodes corresponding to one or more loads of force or

I
} acoustic or potential speed or imposed movement,
H (F)
I
Real or complex function of the frequency F,
=
2 F Pulsation
N
Power of the pulsation when the loading is a function of the pulsation,
I

Phase in degrees of each component of the excitation compared to a reference of
I
phase.

As example if one takes the case of a system of fluid modelled in accoustics, without ddls imposed, simply solicited on part of its border by a field normal speed
v (,) = v
J T
P T
{(P) E
}
, the terms of the equation [éq 2.4-1] become:
N
N

I
non-existent,
M
Stamp of mass resulting from acoustic modeling finite elements,

C
Possibly matrix of damping resulting from modeling finite elements
accoustics if impedance on border,

K
Stamp rigidity resulting from acoustic modeling finite elements,
{Q (P}) = {p (P}), vector of the pressures to the nodes,
{G (P) = {v (P)

N
}
I
}
, vector field normal speed to the faces (finite elements)
H (F)
= -1. (constant),
I
=
2 F Pulsation,
N
= 1,
I

= 0.
I

Note:

In addition to the solution of the harmonic equation [éq 2.4-1], Code_Aster makes it possible to calculate them
derived from this solution compared to the loading {G (P) or to parameters of
I
}
mass, stiffness or damping (M, K, C). The equations whose these derivative are
solutions and the relative theoretical developments are in [R4.03.04].
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Document: R5.05.04
Modeling of damping in dynamics
linear
Summary:
Linear dynamic analyses of the structures subjected to imposed forces or movements require to add characteristics of mechanical cushioning to the characteristics of rigidity and of mass model.
One has several traditional modelings, applicable to all the types of finite elements available:
· the model of viscous damping,
· the model of damping hysteretic (known as also "structural damping") for the harmonic analysis of viscoelastic materials.
For the analyses using the methods of dynamic response by modal recombination, with a base

modal of real clean modes, it is possible to introduce modal damping coefficients.
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1
Concept of mechanical cushioning
1.1 Models
of damping

The movement of the structures subjected to forces or movements imposed, variable with the course time, depends, in particular of the properties of damping, i.e. of the dissipation of energy in materials constitutive of the structure and the connections of the various elements of structure between them and with the surrounding medium.
The physical phenomena intervening in this dissipation of energy are many frictions, interaction fluid-structure in a fluid blade, shocks, viscosity and plasticity, vibratory radiation with the supports.
The models of behavior representing these phenomena are often known little about and it is not possible to explicitly describe them at the elementary level. This is why the most used models are the simple models which make it possible to reproduce on a macroscopic scale the principal effects on the structures [bib1] [bib2]. Those currently available in *Code_Aster* are:
· viscous damping: dissipated energy proportional to the speed of the movement,
· damping hysteretic (known as also “structural damping”): dissipated energy proportional to displacement such as the force of damping of sign is opposed to that speed.
Let us note that the damping of Coulomb, which corresponds to a damping of friction for which dissipated energy is proportional to the force of normal reaction to the direction of displacement is not currently not established in *Code_Aster*.
The values of the parameters of these models are deduced from experimental results. At the stage of design, one is limited to the use of guiding values.

1.2
General standards to characterize damping [bib1]
1.2.1 Loss ratio

The loss ratio is an adimensional coefficient characteristic of definite the shock absorber effect like the report/ratio of the energy dissipated during a cycle with multiplied maximum potential energy by 2:

E
= *D* by cycle

éq 1.2-1
2 *Epmax*
1.2.2 Damping
reduced
By definition reduced damping is equal to half of the loss ratio

=
éq 1.2-2
2
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2
Model of viscous damping
2.1
Physical definition of viscous damping
The traditional cushioning devices (rolling of a viscous fluid through the openings of a piston pulled by the vibratory movement) forces proportional to the speed of the movement deliver and of opposite sign. During a cycle, the work of these forces is positive: it is damping viscous.
K
C
m
F
U
For a simple oscillator of rigidity *K*, mass *m* and viscous damping *C*, the force external applied balance three components: elastic force of recall *K U*, force

of damping *C U*! and inertia *m U*! from where the dynamic equation moving absolute:
m U! + *C U*! + *K U* = *F*
éq 2.1-1
For this model of viscous damping the energy dissipated during a cycle of pulsation is proportional to the vibratory speed - *U*
(*T*
0 sin
) associated displacement *U*
(*T*
0 cos):
2
E
C
U sin *T*
2
0
D (*U* cos *T*
0
) *C U*
D by cycle =
-
=

0
0
and potential energy for a sinusoidal displacement *U*
T
0 cos is:
0
1
E
=
K U cost D

2
0
(*U* cost
0
) = *K U*
p max
/

0
2
2
For a cycle of pulsation and sinusoidal displacement *U*
T
0 cos, the ratio loss is
proportional to the frequency of the movement:
C
=
éq
2.1-2
K
2.2
Harmonic oscillator with viscous damping
Traditional analysis of the model not deadened associated the equation [éq. 2.1-1], put in the form
(
K
K - m2) *U* = 0 gives us *0* =
the own pulsation.
m
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The damping criticizes from which the differential equation [éq 2.1-1] does not have any more an oscillating solution
2k
is given by the formulas *C*
= 2 *km* = 2m
critical

0
= 0 what makes it possible to give one
numerical interpretation of the reduced damping, which is often expressed expressed as a percentage
critical damping:

C
C
= =
=
éq 2.1-3
2
C
2 *m*
critical
0
2.2.1 Response to releasing excitation
F
Starting from a static deformation *U*
0
St =
, to release (release of the system) produced a movement
K
oscillatory free *U* (*T*) = *U*
- *T*
E
0 cos
0
T
L
0 which reveals the own pulsation of the deadened system
'
2
0 = 0
(1-).
In the course of time, the extreme amplitude (*U*, *U*
1
2) decreases at each period of
e-0T
e-2
E
=
= where is the decrement logarithmic curve: = 2
U

u0
u1
u2
T
T

2.2.2 Response to a harmonic excitation

The response to a harmonic excitation of the form *F T*
F
T
( )
0 E J
=
is written with a forced answer
(*T*)

permanent particular solution *U* (*T*)
u0 E J
=
- which is written with the reduced pulsation = *0*
K u0
1
=
H J
H J
2
v (
)
is the complex transfer transfer function of an oscillator
F
where
v (
)
0
1 - + J 2
simple with viscous damping.

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U
K U
1
The module of answer *0*
0
=
= *Hv* (J) =
fact of appearing one
U
F
St
0
(
2
1- 2) + (
2) 2
dynamic amplification compared to the static answer *ust*.

This amplification is maximum for = *0* = (1 - 2)

and the value of displacement gives
0
u0 max
1
maximum
=
. If vibratory speed is observed! *U* (*T*) = J (
U T), amplification
ust
2
(1 - 2

)

vibratory speed is maximum for = *0* =

1 and the maximum amplitude speed is
0
1
! *U* max
Q
0
=
=
2
, where *Q* is the mechanical analogy of the factor of overpressure of the electricians who us make live. These properties are at the origin of the methods of measurement of the characteristics of damping of the mechanical structures.

3

Model of damping hysteretic

3.1

Physical definition of damping hysteretic

For a sinewave excitation applied to an elastoplastic structure or an elastic structure with friction, the curve force-displacement reveals a positive work of the external force which corresponds to an energy dissipated in the structure, which one can at first approximation to represent like below:

F
F
U
U
Elastoplastic
Slip

In both cases the loss ratio believes, in general with the amplitude of the cycle. For values weak of the loss ratio (< 0.2), the form of the cycle does not have an appreciable effect on the movement and one can compare it to an ellipse [bib1].

In the particular case of a relation force-displacement whose cycle is of form elliptic, the expression of the loss ratio is simple. For a force applied *F* and a displacement *U = u0* cos the force of recall is *K u0* cos and damping forces it - *H u0* sin what leads with the relation of balance *F = K u0* cos - *H u0* sin.

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F
K U 0
= 0

= - *H U*
2
0
U
U
0
K
= - *hu0*
1
2
Energies dissipated during a maximum cycle and potential are
2
0
K u2
E
=
- *H U*
2
0
D by cycle
0 sin D (*u0* cos) = *H U*
and *E*
0
=
K U
p has
m *X*
0 cos D (*u0* cos) =
0
/2
2

from where the loss ratio
H u2
H
=
0
=
éq 3.1-1
K u2
K

0
2
2
H
For a sinusoidal cycle = *T*, the coefficient damping hysteretic =
is independent
K
of. It can be given starting from a test under harmonic cyclic loading.
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3.2

Harmonic oscillator with damping hysteretic

The model of damping hysteretic is usable to treat the harmonic answers of
structures with viscoelastic materials.
2
The energy dissipated by cycle in the form *ED* by cycle =
D makes it possible to highlight
0

a YOUNG modulus complexes *E** starting from the relation stress-strain of a material
(
)
viscoelastic

= 0 ej *T* and

=
-
0 E J *T*
where 0 and 0 are the amplitudes and the phase:

0

*E**
J
=
=
E = 0 (cos + *J* sin)
0
0

By noting *E*
0
0
1 =
cos the real part and *E* =
sin the imaginary part one obtains

2
0

0

E
*E** = *E*
1
1 + J *E2* = *E1* (1+ J) with =
= tg, where is also called loss angle.
E2
The traditional analysis of the equation [éq 2.1-1] does not have a direction, with a model of damping hysteretic,

that for a harmonic excitation *F T = F*
J T
( )

0 E
who leads to the equation
m the U.K. (1
) *U m U* (*K*
H) *U F*
T
!
J
!
J

0 E J
+
+
=
+ +
=
éq
3.2-1
where the real part of displacement *U* represents the displacement of the mass and *H = K*.
As previously cf [§ 2.2], the harmonic answer can be written, with the reduced pulsation

K U
1
=
0 =

J
, in the form
H
H J
2
H (
) where *H* () is the transfer transfer function
0
f0
1 - + J
complex of a simple oscillator with damping hysteretic.
U
K U

1
The module of answer *0*
0
=
= *Hh* (J) =
fact of appearing an amplification
U
F
St
0
(
2
1 - 2) +2
dynamics compared to the static answer, amplification which is maximum for = 1 and gives
u0 max
1
1
value of maximum displacement
= =
ust
2 .
In conclusion, damping reduces associated with damping hysteretic is:

H
H
= =
=
éq 3.2-2
2
2k
2m 20
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Other models of damping

One does not treat models here representing the damping “added” by the motionless fluids confined or fluids moving. One will refer to the booklet [R4.07] coupling fluid-structure.

5

Analyze structure with damping

Modelings presented are not easily generalizable with the various analyses of structures cf [§1].

Note:

Two modelings do not have the same field of linear analysis:

· *viscous damping is usable in transitory or harmonic analysis,*

· *damping hysteretic is usable only in harmonic analysis.*

The options of modelings in *Code_Aster* allow the definition:

· of a total damping for the structure,

· of depreciation located on meshs or groups of meshs.

5.1

Total damping of the structure

In the absence of sufficient information on the components and connections creating a dissipation of energy,
a current modeling consists in building a matrix of “total” damping.

5.1.1 Viscous damping proportional “total”

One places oneself within the framework of the traditional equations of the dynamics of the linear structures:

MR. U
! + **C U**! + **K U** = **F** (*T*) **éq**

5.1.1-1

The concept of damping of RAYLEIGH makes it possible to define the matrix of damping **C** like linear combination of the matrices of rigidity and mass:

C = **K** + **M**

éq 5.1.1-2

Advantages:

· easy to implement by using operator COMB_MATR_ASSE [U4.53.01], after having assembled the matrices of rigidity and mass with real coefficients. ;

· useful for the validation of algorithms of resolution;

· historically, its success is attached to the methods of transitory analysis by recombination modal starting from a base of real clean modes.

Properties of orthogonality of the real clean modes solution of the problem to the values clean (**K** -2 **M**) = 0 result in the simultaneous diagonalisation in the passage

in generalized modal co-ordinates of *T* **K** and *T* **Mr.**
The damping of RAYLEIGH is a condition sufficient for diagonaliser *T* **C**.
TC
2
T
The system of modal equations! **q+**
T
T
! **q+ Q** =
F () becomes diagonal then.
M
TM
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T
!
q+ 2
! **q+** 2

Q =
F (*T***)**
éq 5.1.1-3
TM
Disadvantages:
· This modeling does not make it possible to represent the heterogeneity of the structure compared to damping.
· The damping actually introduced into the model strongly depends on the identification coefficients and cf [§ 5.1.2].

5.1.2 Influence damping coefficients proportional

Three simple cases of identification are presented here to illustrate, the effects induced by this modeling:

· damping proportional to the characteristics of inertia: = 0, = *I*

This case was very much used of direct transitory resolution: if the matrix of mass is diagonal, that of damping is still and the saving space memory is obvious in it.

The coefficient can be identified with reduced damping experimental *I* of the mode clean (*I*, *I*) which takes part more in the answer cf [éq. 2.1-1] from where *I* =
2 *II*. For

any other pulsation one obtains a reduced modal damping =
I
I. Modes
raised >> *I* will be deadened very little and the modes low frequency < *I* too deadened.

2 =
I
I

· damping proportional to the characteristics of rigidity: = *J*, = 0.

The coefficient can be identified, like previously starting from *J* associated with mode (*J*, *J*) from where *J* =
2 *J J*. For any other pulsation one obtains one deadened

modal tiny room = *J* sow. The high modes >> *J* are very deadened, etc….
J

2 =
J
J

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· damping proportional complete: = *J*, = *I*
Starting from an identification on two independent modes (*I*, *I*) and (*J*, *J*), us will obtain for any other pulsation a reduced modal damping

=
I
J
+
I,

I
. In the interval [
J], the variation of reduced damping is
J

weak and outwards one finds the combination of the preceding disadvantages: modes outsides with the interval are deadened too much.

2 = *K*+*M*
I
J

I
J

In none the preceding cases, one will be able to reproduce an assumption of equal modal damping for all the modes. Methods were imagined for tending towards this objective [bib1].

5.1.3 “Total” damping hysteretic

The generalization of the equation of the simple oscillator with damping hysteretic leads to system of complex equations or (
F) is a harmonic excitation.

MR. U
! + **K** (1+ *J*) **U** = **F** ()
éq 5.1.3-1

Knowing the matrix of real rigidity, it is possible to build a matrix of damping hysteretic **K**
K

H = J
, with a “total” loss ratio.
As previously of resolution by modal recombination, starting from a base of clean modes realities, T is obtained
M Q
! + TK
T
T
J
H
Q + **K Q** = F (T) where the matrix of damping
hysteretic generalized is diagonal TK
H
= [diag I], as the matrix of rigidity
generalized TK = [diag I].
According to the definition of reduced damping (cf [éq 1.2-2]), modal damping is constant for

all modes from where = 2
Advantages:
· easy to implement by using operator COMB_MATR_ASSE [U4.53.01], after having assembled the matrices of rigidity. ;
· very useful for the validation of algorithms of resolution;
· the damping actually introduced into the model is constant for all the modes of structure, as asks it payments of construction.
Disadvantages:
· this modeling is badly adapted for the industrial studies, because it does not allow to represent the heterogeneity of the structure compared to damping.
· only the harmonic analysis (in complex) is possible.
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5.2 Damping
located

For the analyses requiring a modeling representing the heterogeneity of the structure, it is possible to affect characteristics of damping located on the meshs of the structure, in fact on elements of the model.

5.2.1 Elements
shock absorbers

It is possible to apply elements discrete shock absorbers:
· on meshs POI1: damping is related to the displacement (respectively speed) of node support,
· on

meshs
SEG2: damping is related to relative displacement (respectively relative speed) of the two nodes connected.
Operator AFFE_CARA_ELEM [U4.24.01] allows to define for each discrete element:
· a matrix of damping of the viscous type **adiscret** whose terms are affected to various degrees of freedom of the nodes concerned; several modes of description of matrix are available.
· a hysteretic loss ratio *discrete* multiplying of the matrix of rigidity of the element discrete affected with the mesh support.

5.2.2 Affected damping with any type of finite element

The affected elastic material with any finite element can be defined with parameters of damping by operator DEFI_MATERIAU [U4.23.01]:
· Viscous Amortissement proportional with two parameters of RAYLEIGHT and.
AMOR_ALPHA
:

AMOR_BETA
:

For all the types of finite elements (of continuous, structural or discrete mediums), it is possible to calculate the real elementary matrices corresponding to the option of calculation "AMOR_MECA", after having calculated the elementary matrices corresponding to the options of calculation "RIGI_MECA" and "MASS_MECA".
The elementary matrix of affected element *I* of the material *J*, *J* is then of the form:
-
for a finite element
C
= **K**
+ **m**
elem I

J
elem I
J
elem I
-
for a discrete element
C
= **has**
elem I
discrete I
· Amortissement hysteretic with a coefficient of *E*
AMOR_HYST: coef
For all the types of finite elements (of continuous, structural or discrete mediums), it is possible to calculate the complex elementary matrices corresponding to the option of calculation “RIGI_MECA_HYST”, after having calculated the elementary matrices corresponding to options of calculation “RIGI_MECA”.
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The elementary matrix of affected element *I* of the material *J*, *J* is then of the form:

-

for a finite element

k*

= **K**

elem I

elem I (1+ J *J*)

-

for a discrete element

k*

= **K**

elem I

elem I (1+ J *D*

iscret I)

Important remark:

An anomaly of November 1995 showed that this option is **not**
usable in version 3.6.
It will be corrected in version 3.7.

5.2.3 Construction of the matrix of damping

The assembly of the elementary matrices of damping is obtained with operator ASSE_MATRICE usual [U4.42.02] or by macro order MACRO_MATR_ASSE [U4.31.02]. One must use them same classifications and same mode of storage as for the matrices of rigidity and mass (operator NUME_DDL [U4.42.01]).

Note:

The matrix of damping obtained is nonproportional

C K + **M** or **KH** J **K**

6

Use of the matrix of damping

6.1

Use of the matrix of viscous damping

6.1.1 Analyze dynamic linear direct

The matrix of viscous damping **C**, whatever its mode of development and its character proportional or not proportional, is usable for the direct linear analysis dynamic (key word MATR_AMOR) with the operators:

· of analysis
transient
DYNA_LINE_TRAN
[R5.05.02] and
[U4.54.01]

· of analysis
harmonic DYNA_LINE_HARM
[R5.05.03] and
[U4.54.02]

6.1.2 Analyze dynamic by modal recombination

For the analyses by modal recombination, one must project this matrix in the definite subspace by a whole of real clean modes, obtained on the associated problem not deadened
(**K** -2 **M**) = 0.
This operation is possible with macro order MACRO_PROJ_BASE [U4.55.11] or with operator PROJ_MATR_BASE [U4.55.01].

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For the calculation of the dynamic response in force or imposed in modal space, one have following possibilities:

· use of the matrix of damping generalized $^{T}\mathbf{C}$:

-
in transitory analysis with operator DYNA_TRAN_MODAL [R5.06.04] and [U4.54.03] and it key word AMOR_GENE,
- in seismic analysis by spectral method with operator COMB_SISM_MODAL

[R4.05.03] and [U4.54.04] and key word AMOR_GENE,
-
in harmonic analysis with operator DYNA_LINE_HARM [R5.05.03] and [U4.54.02] and it key word MATR_AMOR.
Let us recall that in the case of heterogeneous damping (use of the options of damping localised), the matrix $^{T}\mathbf{C}$ is not by diagonal.
· use of viscous modal damping by providing a reduced modal damping constant for all the modes or a list of values *I*.
Several methods of identification of these coefficients are possible but there does not exist order automatic construction of the list of values: use of the assumption of

TC
BASILE 2
diag

I I =
, method of payment RCC-G for the seismic analysis

TM
with damping of the ground, exploitation of experimental results,…
-
in transitory analysis with operator DYNA_TRAN_MODAL [R5.06.04] [U4.54.03] and it key word AMOR_REDUIT.
- in seismic analysis by spectral method with operator COMB_SISM_MODAL [R4.05.03] [U4.54.04] and key words AMOR or LIST_AMOR. An evolution is required to generalize key word AMOR_REDUIT.
-
in harmonic analysis a **request for evolution** with operator DYNA_LINE_HARM [R5.05.03] [U4.54.02] is deposited. It is not treated in version 3.6.
For the analyses by dynamic under-structuring, with the use of a modal base (bases RITZ) one will refer to [R4.06.03] and [R4.06.04].

6.2
Use of the complex matrix of rigidity

The matrix of rigidity complexes $\mathbf{K^*} = \mathbf{K} + \mathbf{KH}$, where **KH** is an imaginary matrix (within the meaning of
complexes!), is usable for the direct harmonic analysis with operator DYNA_LINE_HARM [R5.05.03] and [U4.54.02] and key word MATR_RIGI.
For the analyses by modal recombination, no functionality is currently available for the hysteretic use of the model of damping.

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6.3
Complex modal analysis

The matrix of viscous damping **C** is essential for the modal analysis complexes with operators dealing with the quadratic problem with the eigenvalues [R5.01.02]:

· by iterations opposite
MODE_ITER_INV
[U4.52.01]
· by simultaneous iterations
MODE_ITER_SIMULT
[U4.52.02]

For the search for eigenvalues, no functionality is currently available for the use of the model of hysterical damping.

Let us recall that the complex clean modes allow an approach adapted better to the study dynamics of the strongly deadened structures (reduced damping > 20%). To date no tool of dynamic response by modal recombination using a base of complex clean modes is not available in *Code_Aster*.

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Dynamic non-linear algorithm of Code_Aster (operator DYNA_NON_LINE)

Summary:

Operator DYNA_NON_LINE [U4.53.01] of Code_Aster gets busy for the non-linear dynamic analysis of
structures by a direct integration in time. Non-linearities can come from the behavior of material, of the connections (contact-friction), or great geometrical transformations (great displacements and great rotations).

The organization of DYNA_NON_LINE is strongly connected with that of the non-linear quasi-static operator
STAT_NON_LINE [R5.03.01]. *A priori*, all relations of behavior developed within the framework of STAT_NON_LINE function in that of DYNA_NON_LINE.

One presents here the general formulation of the non-linear dynamic problem in order to specify the articulations
between the purely dynamic aspects and those already treated in other operators or formulations available in *Code_Aster*: management of the boundary conditions, the couplings fluid-structure, of damping, of calculation in relative reference mark, then properties of the diagram of numerical integration temporal, which
be worked out independently of any relation of behavior. It is exposed how this one is articulated with the algorithm of Newton to treat material and geometrical non-linearities. *Code_Aster* proposes two implicit schemes in effective times in term of precision and stability: that of Newmark and method "of modified average acceleration" (known as "HHT" in order DYNA_NON_LINE). One gives some councils and choice for a good use.

Count

matters

---

1 Notations

U
field of continuous absolute displacement

K, N
Ki
stamp rigidity, stamps tangent
M
stamp inertia
R
vector of the interior forces
L
vector second member of loadings (linear form)
abso
L
second members respectively due to an absorbing border, the inertial terms
, iner
LGR, non-linear in great rotations of beam, with chainings anelastic (coming from
anél
L

variables of order: temperature…)
C
stamp damping
Q
stamp assembled deformation

T V…
transposed of a vector V: dual linear form…
T; T
time; no time

parameter of the diagram of temporal integration method (HHT)
,
parameters of the diagram of temporal integration of NEWMARK

increment of various sizes during the step of time

virtual variation of a field;
increment of various sizes during iterations of correction
I; N; J
index of the step of time; index of the iteration of NEWTON; index of component
, μ
parameters of LAGRANGE: reactions of connection, reactions of contact

U, U&, U
&

vector ddl successive displacement and derivative compared to time
P
vector ddl of disturbances of fluid pressure barotrope

potential vector ddl of disturbances of fluid displacement barotrope

configuration: vector position: X, y, Z and possibly vector rotation, and others
fields parameterizing the system
&
temporal derivative of the configuration compared to time: speed of traverse and
possibly angular velocity
&
temporal derivative of
& compared to time: acceleration of translation and
possibly angular acceleration

K
K
Convention of the repeated indices: U (T) = U
K
D (T
D
)

K

K

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2 non-linear Dynamics : space discretization of continuous problem

To solve a non-linear problem of dynamics requires to describe the equations first of all of continuous problem, then to present their space discretization, here in finite elements, and finally to describe
method of temporal integration, associated the treatment of material non-linearities and geometrical.

2.1 Discretization of the linear problem of dynamics

One notes U the field of absolute displacements compared to the configuration of reference, and parameterized by the moment T, pertaining to the space closely connected of the acceptable fields Vadm.

The direct method consists in solving the problem resulting from the discretization by finite elements of
formulation in displacement.

The discretization of the virtual variation of the kinetic energy gives the virtual work of the forces of inertia, in a field
0
v Vadm, directing vector space of Vadm:

1

(u&) 2D =
U
.

2
&v
T
D =
V.M.U&

Discretization of the virtual variation of the work dissipated in viscosity (damping brought by one dependence of the constraints according to speeds of deformation) is:

U
C
&.vd =
U
C

&.v
T
D =

U
V.C. &

One specifies with [§2.2.1] how the operator of damping C is built in DYNA_NON_LINE.

The discretization of the variation of elastic energy into linear gives the virtual work of the efforts interiors:
1
(U). A. (U) D =

2
(U). A. (v)
T
D =
V.K.U

Lastly, L designates the second member resulting from the discretization of the virtual work of the external forces.

In linear elasticity, that thus leads to consider the hyperbolic différentio-algebraic system according to, for the ddl U, with the initial conditions:

U IR N
To find
:
M.U& + U
C. & + K.U = L

U (t0) = U0

U& (T
&
0) = U 0

accompanied by boundary conditions.

The initial conditions are provided to the algorithm by key word ETAT_INIT (operands DEPL and QUICKLY).
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If the initial state results from a simulation in linear or non-linear statics, one does not take into account
of initial speed, and displacement, as well as the variables of state (forced, variables intern), are extracted from the result of this simulation at the starting moment considered.

The dynamic system of balance becomes unstable if one can find a pulsation complex which that is to say not real positive for which one can cancel the determinant of: - 2M + I C + K.

2.2 Discretization

problem
of non-linear dynamics

One places oneself now within a non-linear mechanical framework.

Virtual work is noted (U, u&, T). (v)
D of deformation (known as also of the forces intern)

non-linear problem of mechanics, which is written after discretization:

T (
V.R (U, U&, T) + U
C. &) = T (
V.T Q (U). (U, U&, T) + U
C. &)

where one voluntarily distinguished the linear viscous forces (operator C) from the other forces interns. In the case of small displacements, the operator of deformation assembled T Q is constant (and definite on the initial configuration confused with the deformation).

The mechanical assessment of energy is written:

L.v = u.&v
D + U, u&, T. v
D

(
) ( )

The stress field at the moment T is written in a general way (U, u&, Z, T, H), Si one notes Z it field of variables of order, such as for example T the field of temperatures, and H history passed of the structure until the moment T. For the incrémentaux behaviors, the history is the whole of the states (fields of displacements, constraints and variables intern) at the previous moment.

In the linear case (cf [§ 2.1]), one leads to R (U, U&) + U
C. & = K.U + U
C. &, where K is the matrix of
elastic rigidity of the structure and C the matrix of damping.
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2.2.1 Damping

It is permissible to use discrete elements on which one makes carry a behavior of damping via a matrix acting on the ddl, cf [U4.42.01], but damping can too to relate to the massive models or of structures. The operator of damping C of the latter can to be in definite Code_Aster in two ways in DYNA_NON_LINE, cf also [R5.05.04], [U2.06.03]:

1) a total way on a basis of clean modes (K) established as a preliminary on the structure rubber band, expressed on the basis of "physical" modeling by finite elements. One defines thus a coefficient by selected mode. The key word: AMOR_MODAL of operator DYNA_NON_LINE allows to provide him the base of modes and the coefficients damping reduced (according to the assumption of BASILE). Indeed, depreciation is in experiments given by modal analysis on resonances.
Displacements U are thus projected on the modes to obtain their co-ordinates generalized: K T = K. U. The matrix of modal damping is:

C = (K K K) K T . Cmodal. (Kk) with Cmodal = 2
where is the factor
T

K
K. K

. K.K

of modal damping to the pulsation
T
K and kk = K

. K.K is the stiffness generalized of
mode K. Unfortunately this matrix can have a very full profile and make expensive (them matrices C and K not having the same profile), as one will see it with [§ 3.1], his integration in the first member: one will then choose to treat these forces of modal damping -
U
C. & (T)
with the second member by an explicit diagram.

2) a total/local way known as viscous damping proportional (according to the assumption of RAYLEIGH) starting from the matrices of elastic stiffness K and mass Mr. the parameters are given by material on the finite elements of the model (key words AMOR_ALPHA/AMOR_BETA of order DEFI_MATERIAU). The matrix of viscous damping is C = K
+ M
. It
is diagonalisable on the basis of real mode clean, which makes possible to make a calculation transient on modal basis by uncoupling the modes: to see [R5.06.04]. This formulation, in linear, led to a damping ratio related to the frequency F: = F
+/(4 F
). In
non-linear case, this evaluation does not have any more course.
The coefficients in practice are adjusted, so that damping is almost
uniform in the beach [f1, f2] of frequency of interest for the studied structure. From where thus of reasonable manner:

=

.2. F F
and
1. 2
=

2 (f1 + f2)
f1 + f2
If the law of behavior of material is non-linear dissipative, the choice of the parameters of damping the beach concerns where the structure remains almost elastic.
Moreover, it should be noted that, during the integration of non-linearities (cf [§ 3.2] and [§ 3.3]), Produced Code_Aster of the tangent matrices and viscous damping proportional becomes:
C = K

T + M

the matrix of initial mass M, as for it, being preserved. This makes delicate the interpretation of the effect of damping proportional. In particular in the event of appearance of eigenvalues negative of the matrix
T
K (for example in the event of damage of material),
damping can become negative and reinforce instabilities!

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2.2.2 Inertia

One notes T V.M (U, U&, U&) after discretization, virtual work &.
U vd of the inertias of

system.

One notes M the matrix of mass of the system in small transformations. In great rotations of structure (such as for example the beams, cf [R5.03.40]), this virtual work is a function non-linear of U (T) and its derivative temporal (specifically of the degrees of freedom of rotation); one thus reveals the usual term of acceleration with a non-linear correction:
M (U, U&, U&) = M (U) U
. & + Liner
GR. (U, U
&, U&). In the other cases, M (U, U&, U&) = M (U) U
. &, which can
to vary if the geometry is reactualized, or who is constant in small displacements.

2.2.3 Connections

In practice, one can have bilateral or unilateral conditions of connection, or connections of the type "impedance" or "absorbing", cf [R4.02.05].

Bilateral connections

The bilateral connections are written in the form of the following relation:
D
B.u = U (T). Fields of

virtual displacements kinematically acceptable check: B.v = 0. The operator B (U) can to depend on the configuration in the presence of great displacements by reactualization with each step time.

They are perfect: they do not dissipate energy. They are connections "holonomists", where speed & U does not intervene. These connections are in general dualisées by Code_Aster, cf [R3.03.01].

Unilateral connections

The mechanical system object of simulation by finite elements can come into contact (connections unilateral) with "obstacle", which is a solid which one knows a priori the movement, from where definition of a play d0 (T). The unilateral connections (for example the unilateral contact) are written on configuration at any moment T: With (U) .u (T) d0 (T) (nonpenetration or checking that effective play remain positive or null in any configuration). Operator A (U) can depend on the configuration in presence of great displacements per reactualization with each step of time.

One will consider only connections "holonomists", type A (U, T) = 0, who utilize only them values of the degrees of freedom U (T) and time explicitly if the obstacle is mobile. One will not consider connections "non-holonomists", for example of the bearing type without slip, which utilize speed explicitly and are written A1 (U, T) U . & + A2 (U, T) = 0, A2 and dependence direct in time being present only if it there a mobile obstacle.

If the obstacle is motionless, the connection as such will be explicitly independent of time (one says also "scleronomist").

These "loads of contact" are defined by operator AFFE_CHAR_MECA. The presence of connections unilateral requires to define speeds of the solid in a particular functional space in order to ensure the existence of solutions of the system of dynamics. Indeed, at the time of the moments (countable) of impact, speeds & U (T) and &U (t+) can not coincide. It is necessary to guarantee it result that the data (loading, equations of connections) check a property of analycity, which is acquired in practice with the selected discretization by finite elements (see [bib6]).

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One does not introduce a priori a relation constitutive of the impacts (dissipative behavior in rebound simple, expressed using a coefficient of normal restitution E [
0]
1
,), expressed on the differentials
speeds of the two points in impact:

&U+
+
-
-
_norm (T) = &
U _norm (T) - (E &U _norm (T) - &U
2
1
2
_
1 norm (T))

This type of behavior is introduced usually indeed to treat the contact-impact of body rigid, whereas numerical modeling with deformable solids makes it possible to represent directly the vibratory behavior under the shock and material non-linearities. But it is possible to add in the modeling of the discrete elements of contact-shock placed on the interface in contact, provided with law DIS_CHOC, which brings a dissipation of damping (to condition of to suppose small movements…).

One can associate with the unilateral connections a behavior of friction (criterion of COULOMB), which
dissipate energy in relative slip of surfaces in contact.

One refers sometimes to the fact that the dynamic coefficient of friction is lower than that

measured into quasi-static (adherence). That comes owing to the fact that in dynamics from the vibrations high
frequency on the normal reaction appear and weaken the value of the threshold of friction of COULOMB. One would thus not need to provide two values of coefficients to Code_Aster, since one models the deformable solids in contact-friction (on the condition of being able to simulate these
vibrations high frequency…).

It is known that dissipation is a condition necessary for the existence of theoretical solutions to problem of dynamics with impact. Use of the diagram HHT, to see [§ 5], which introduces numerical dissipation can prove to be necessary.

Local dissipative connections

Specific relations (like DIS_CONTACT, DIS_CHOC) are conceived to treat certain types dissipative specific connections, acting directly on the ddls of discrete elements of the system, to see [R5.03.17]. They constitute a law of behavior in generalized forces function of generalized displacements integrated like the whole of the forces intern R (U, U
&, T) of the structures
studied.

Absorbing connections

The "absorbing" connections of the type, cf [R4.02.05], make it possible to simulate the "filtering" of a part
dynamic response, by preventing the exit of diffracted waves, with the profit of an "incidental" field on a border of the model: to see it [§ 2.6]. They introduce damping terms of the type
With
(U
abso &) on a border of the solid considered.

2.2.4 Discretized dynamic problem

The dualisation of the boundary conditions of DIRICHLET
D
B.u = U (T) and of the unilateral conditions
conduit after discretization to define the unknown factors at any moment T: (U, μ), where represents them
"multiplying of LAGRANGE" of the boundary conditions of DIRICHLET [R3.03.01], and μ represent the "multipliers of LAGRANGE" of the unilateral conditions.

The non-linear dynamic problem is written, with the initial conditions [R3.03.01], [R5.03.50]:
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To find the trajectory U (T):
M (U, U&, U&) + R (U, U&, T) + U
C. & +t B. +t A.μ = L (T)

B.U = Ud (T)
.
WITH U d0 (T)

J, μ J 0

éq 2.2.4-1

J, (A.U - d0).
J μ J = 0
U (t0)

= U0

U& (T
&
0) = U 0

L represents the vector of the external forces (mechanical loadings). These forces can
to depend on time and space. It is supposed that, like the connections, they depend on way
regular of the parameters, which ensures the existence of the solution of the problem (theorem of

CAUCHY). One can consider "following" forces L (U, T), for example the pressure, if one takes in account changes of geometry.

The vector T B. is interpreted like the opposite of the reactions of support to the corresponding nodes (B
is the linear operator expressing the passage to the degrees of freedom of the supports). The vector T A.μ
be interpreted as the nodal forces due to contact (A is the linear operator expressing it passage to the degrees of freedom of the zones in contact).

The analysis of stability of the dynamic system of balance [éq 2.2.4-1] is more complex than into linear,
but a sufficient condition of loss of stability is the possibility of finding a pulsation for which one can cancel the determinant of:
T
T
- 2M + I C

+ K, definite on the operators
tangent, at the moment considered.

2.2.5 Conditions initial

Initial conditions U, U
0 & 0 are provided to the code by key word ETAT_INIT (operands DEPL and QUICKLY).

If the initial state results from a simulation in linear or non-linear statics, displacement, as well as variables of state (forced, variables intern), are extracted from the result of this simulation, and initial speed by defect is supposed to be null.

2.2.6 Implicit temporal piloting of external loadings

In general, the loads, defined in AFFE_CHAR_MECA or AFFE_CHAR_MECA_F, are of type "FIXE_CSTE" if their intensity and direction are known a priori.

One can as consider as a share of the external requests is controlled (standard "FIXE_PILO"), pilo
pilo
i.e. its intensity is parameterized: Fimp and/or Uimp, and controlled by a relation scalar, expressed on a node (or groups nodes), on the solution: P (U) = T = ~
P (), this
last being implicit. As it is seen, this equation refers to the time, which contrary to the statics where it is only used to give a chronology on the increments of load, has a physical role

in dynamics. One will have to thus ensure oneself of the precise significance of temporal piloting.
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One thus adds in the non-linear system [éq 3.1-7] this last equation which will be solved with others by the iterative method of NEWTON. One cannot control the forces of gravity, of centrifugal force, of LAPLACE, the thermal or anelastic deformations.

One will be able usefully to refer to [R5.03.80].

2.3
Taking into account of a prestressed initial state

If the dynamic problem to solve "follows" an initial mechanical state, two principal situations are offered to us:

· {}
1 one wishes to calculate displacements U and the dynamic stresses starting from the state "virgin" no initial,
· { }
2 one wishes to calculate displacements U and the dynamic stresses in "differential" starting from a preloaded state.

In the first situation {}
1, it is appropriate to make the calculation of the static state as a preliminary, possibly non-linear (material, great transformations), precondition to dynamics.

Thus, if the structure has a non-linear behavior, one must proceed directly in dynamics non-linear, after having evaluated the state initial by simulation in statics, possibly non-linear (with operator STAT_NON_LINE), and the field of displacement is evaluated since the beginning of the history. It can be necessary to take into account the variations of geometry. Information necessary to describe the initial state (results of a preceding simulation via the concept result EVOL_NOLI or mechanical fields necessary: DEPL, SIGM, VARI) are provided by the key word ETAT_INIT, for example if one is within the framework of an incremental behavior (COMP_INCR), to see [U4.51.03].

The initial state can be also obtained by a simulation in "very slow" dynamics, by having care of to put a "slow" slope of dependence at time on the static efforts applied, like one strong damping (physical, cf [§ 2.2.1] ou/et numerical cf [§ 5]). This manner of proceeding has the advantage of injecting so much into the operator of the phase of prediction [éq 3.2.1-4] of the algorithm of NEWTON, that in that of the phase of correction [éq 3.3-1] of the terms $ K coming from the matrix from mass and of that of damping, to establish the static mechanical state. That is invaluable in situations of contact-friction, damage… to improve convergence.

The second situation {} 2 relates to the case of a structure which underwent a preloading thermomechanical "ordinary", leading to a state of linear elastic balance. If one measures by U displacement starting from this preloaded state, which generated a state of stresses 1, then the elastic deformation energy is supplemented by a geometrical term of stiffness:

(U). A. (v) D + .1u

(v)
T
D =
(
V.K + K G). U

One then assembles simply the matrices K and K G, and one carries out the resolution in dynamics linear, as with [§ 2.1]. That can be for example the case of a seismic study on a stopping arch. With DYNA_NON_LINE, it is necessary to provide by key word ETAT_INIT, the field of constraints SIGM result of the preloaded state and to specify with the key word DEFORMATION under COMP_INCR the taking into account of the non-linear terms of deformation.

It is frequent that the contribution of K G is negligible: one can then be satisfied with an analysis

dynamics on the basis of completely virgin initial state. It will be also noted that the calculation of the matrix
K G is not available for all the finite elements proposed by Code_Aster.
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2.4
Coupled problems vibroacoustic fluid-structure

One will be able for more details to refer to the documents [R4.02.02], [R4.02.04], [R4.02.05].

One considers the small movements approaches eulérienne of a compressible true fluid it, possibly bathing a wall of a solid structure. The fluid is known as barotrope:

· one considers small irrotational disturbances around the initial state (hydrostatic):
R
r0 R
U = U + U
fl
fl
fl, P = P0 + p and F = 0 +, index 0 indicates the permanent part of
fields),
· the law of behavior of the fluid gives the constraints “
fluctuating
»
:

= - Id
p
= - c2Id =
c2
0
0 0 (ur
div fl) Id,

R
R
· fluid speeds derive from a potential v fl = &u fl = & and are modelled using fields (p,): pressure fluctuating and potential of displacement, which are not independent bus
: &p = - c2
0 0 &
(by combining equation of continuity and law of behavior).

R
It is admitted that one does not consider fluctuating forces of volume F being exerted on the fluid.
R
The dynamic balance of the fluid is written: p + &r
F U fl = 0 (equation of linearized Euler), which is valid for a not-heavy fluid compressible or weighing incompressible; on the other hand for a heavy fluid compressible the approaches eulérienne and Lagrangian do not coincide even into small movements: this case is not treated by Code_Aster.

The dynamic balance of the fluid will be written in variational form under the action of a pressure fluctuating pimp imposed on part of the border. In harmonic mode, dynamic balance fluid results in the variational formulation of the equation of Helmholtz.

Code_Aster has a symmetrized formulation, to see [R4.02.02], with elements (P,) noting it vector of the ddl of pressure and fluctuating potential to describe the disturbances in the fluid,
R
knowing that p + &
= 0
0
. An equation out of P solves dynamic balance in the field fluid, that by translating the equation of derived continuity combined with the fluid law of behavior. Boundary conditions out of P and supplement the system of equations to describe the evolutions fluid. Thus, on a border
F _ p of the fluid field, a fluctuating pressure can be applied: P =
(T

imp
P
), however because of the formulation (P,), one must also impose one on it
condition on: (T) = imp (T), checking &
0 imp (T) = imp
P
(T).

R
As one considers a border common fluid-structure
FS, where normal N is defined
outgoing of the structure field towards the fluid, the loading of wall of the fluid is coupled with
R
R 0 R R R
displacement of the structure. Normal displacements are continuous: (U + U
fl
fl) N
. = U N
.
St
on

r0
FS, U fl indicating the permanent part of fluid displacement. By using the fluid potential, one
R R R
a: &
N
. = u& N
.
St
on
FS. In a dual way, the vectors forced are continuous:
- NR
p = N
.r on
FS. The structure thus receives the loading fluctuating of the fluid:

p vr
(. - NR) dS.

FS
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If one considers a free face (subjected to a constant pressure), also treated of description eulérienne, to see [R4.02.04], one notes by Z (and Z ddl associated after discretization) altitude fluctuating (small) of the free face
SL and the fluctuation in pressure eulérienne in the fluid
check:
(p -

gz) zdS 0
0
=, in any virtual altitude Z. It is the only consequence of

SL
gravity which one can take account in this formulation.

In addition, one can need to take into account an artificial border with an infinite medium (which must treat the condition of radiation ad infinitum): Code_Aster proposes finite elements of border
absorbing (paraxial or anechoic elements), to see [R4.02.05]. As they bring a priori one term in derived third of time, because of the introduction of the field, one prefers to treat it with assistance of a transformation into a not-symmetrical term in P& which will be deferred to the second member, of
explicit manner.

On the whole one obtains the différentio-algebraic semi-discrete equations of the coupled problem:

M
0
M
0
U&

C
0
0 0
U& K
0
0
0
U L
structure
FS
St

0
0
M fl
0 P&

0
0
0 0
P& 0 Q
0
0
P 0
fluid
fl

T

T
+
M
M
H
M

+
=

FS
fl
fl
Z
&
&
0
0
0
0

0
fluid
0 A F 0 0

T
0
0
M
0

Z&
0
0
0 0 Z&
0
0
0 K
Z
0
free surfing
Z

Z
.

éq 2.4-1
accompanied by the initial conditions:

U (t0) = U0, U& (t0) = U&0, P (t0) = 0
P, (t0) = 0 and Z (t0) = 0,

and of the boundary conditions: U (T) = Uimp on the edge
S _u of the structure, the possible ones
unilateral conditions, and P (T) = imp
P
(T) with (T) = imp (T), checking &
0 imp (T) = imp
P
(T) on
edge
F _ p of the fluid.

Note:

It is noted that in the non-linear case, one replaces in [éq 2.4-1] term K.U by the forces non-linear interns R (U, U&, Z, T).

The various operators are:

· the matrices K, M, C defined higher for the solid structure,
1
· Q fl is the matrix built from
p.
qd, which has the physical direction of one

2
F C
0 0
elastic energy of the fluid,

· H fl is built from -.
0

D, and described the fluid transport of mass,
F
1
· M fl is built from
.
qd, and described the inertia of the fluid,

2
F c0
· M
R
R
R
FS is built from

0
. v (. - N) dS (N is the normal of the field

FS
structure towards the fluid), and described the mass throughput eulérien with the interface fluid-structure,
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· A F is built starting from 0p
. D, and appoints the operator absorbing border,
F
acting on &P, modifying the equation in absorbing wall: the elements (model
3D_FLUI_ABSO) absorbents die-symmetrize the system resulting from the formulation (P,) and one goes
to defer this term to the second member, who will be noted abso
L
, by temporal discretization
clarify with each iteration, to see the § 3,
· K Z is the “stiffness” of free face, built from
gz
0
.zdS,

SL
· Mz comes from the work of the fluctuating pressure in free face, built from

Z. dS
0
,

SL
· the Lst term contains, inter alia loadings, the effect of the exerted hydrostatic pressure
by the fluid on the structure.

In short, the taking into account of a fluid field in fluctuating evolution barotrope, interacting with the structure results in considering in the non-linear dynamic system [éq 2.2.4-1] enriched on particular ddls (P,) and Z:

· an operator of inertias M (U). U
& enriched by:

U&

0
0
M
0
U&

FS

P&
0
0
M
0
fl
P&
fs

M = T
T

M
M
H
M
;
&
FS
fl
fl
Z &

0
0
T M
0

Z&

Z

Z&

· an operator of interior forces R (U, U&) enriched by:

U
U
0
0
0
0

P

fs

0 Q
0
0
fl

P
K
=

;

0
0

0
0

0 0 0 Kz
Z
Z

· a second member enriched by the carryforward with the second member by temporal discretization clarify - A P&
F
on the ddls.

The fluid must remain in small movements (basic assumption of this modeling), but one can to consider great movements of the structure, bathed by the fluid, via a reactualization of geometry X X + U
(via the key word COMP_INCR, operand DEFORMATION: "PETIT_REAC",
valid if one considers small rotations) borders
FS, which makes recompute term MFS
but also all others since the field F evolved/moved; the scalar fields (P,) are then simply transported to identical on the reactualized geometry. See also case-test FDNV100 [V8.03.100].
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2.5 Taking into account of laws of viscous behavior and

damping

***A law of viscous behavior, to see [R5.03.08], is translated like an elastoplastic law by one evolution of the work of the interior forces R (U, U
&, T). Thus, while bringing a damping
"physical" in dynamic balance, it does not produce a direct taxation at the end
of damping C.U
& of the dynamic equilibrium equation, but however in an indirect way if one
chose a damping of Rayleigh (cf [§ 2.2.1]) via the matrix of tangent stiffness of the diagram
of integration.***

***Indeed with a law of viscous behavior, the tensor of the deformations comprises a part
rubber band, a thermal part, a anelastic part (known) and a viscous, deviatoric part
(diverter of the constraints noted ~
), for example checking:***

***early = E + HT + has + v
= A (T) .E***

***3 ~
&v = G (eq, T) 2 eq***

For the relation of viscous behavior LEMAITRE, the function G is explicit, but it is not always the case. After implicit discretization in time, the viscous flow is:

***~
v
3
-***

= G eq, +

(v), T

***eq
T***

***2
eq***

One can also adopt an semi-implicit diagram, which seems to give better results:

~
~-

+
3

2
v

-

(v) eq

T
= *G* +
-
-

,
, *T*

T

2

2
+
+

2
2
-
eq

+

2 eq

After solution of a local non-linear equation per elimination of v to calculate

-
-
eq = (+
), one thus obtains the constraint at the end of the step of current time = +
.
eq

2.6
Equations of "relative" motion [R4.05.01]

In several applications, in particular in seism, one wishes to calculate the field directly of displacement of the structure deduced from the movement of "drive" coming from displacements imposed Ud (T) of the supports of the structure.

One notes ua then the absolute displacement of the structure: ua = uent + U, uent being displacement of drive (for example, uent can be an incidental field: it is then called "displacement pseudo-statics")) ; it can be rigid body (case MONO_APPUI), but not necessarily (case MULTI_APPUI).
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And one calls U the "relative" displacement (called thus by abuse language if uent is not

rigid body).

Indeed coding (RCC-M, ASME…) introduced the distinction between "primary constraints" due with the vibratory movement "relative" and "secondary constraints" due to the vibratory movement of "drive". The relevance of this distinction disappears a priori obviously as soon as one considers a non-linear behavior of material.

If one deals with the problem of interaction with the ground (which is an infinite half space) in seism by
example, the field of relative displacement U checks: lim U (X) = 0: only the incidental field uent is X
perceptible ad infinitum it is the seismic data of loading. One uses to define this loading
of displacement imposed order AFFE_CHAR_MECA and the key word factor ONDE_PLANE, on one
border given of the grid considered.

In this case of problem of interaction with the ground, one does not know a priori displacement of "drive" directly applied to the structure, since it results from the coupled total answer: also case MULTI_APPUI does not have it a direction.

On the other hand, not being able to simulate in finite elements with Code_Aster the field in all the half
infinite space (ground), one is led to place "absorbing" borders, cf [R4.02.05], at the edge of grid of ground. The virtual work associated these absorbing borders, of outgoing normal N, is treated as a second member (for the finite elements paraxial absorbents of order 0), because it is integrated explicitly in the diagram of temporal integration (cf [§3]) of DYNA_NON_LINE. associated linear form is worth:

$$L^{abso}(uent, u\&ent, u\&).v = \int_{v} (Aabso(u\&) + (uent).n - Aabso(u\&ent)).dS$$

éq 2.6-1

After discretization, one deduces the second member:

$$T^{abso} = V.L^{abso}(Uent, U\&ent, U\&)$$

$$T = V.Aabso\ U.\& + V.(U\&)$$

ent - V.A abso U
. & ent éq 2.6-2

Note: case of a problem with interaction fluid-structure:

For a structure undergoing of imposed displacements, in the presence of fluid interaction structure, where the fluid field is not directly related to the "support" imposing the signal of drive, it is possible to solve the dynamic system of balance in term of "relative" displacement U of the structure and fluid variables (P,) "absolute" and of dimension of free face of fluid Z "absolute", cf [§ 2.4]. Indeed, one can show that (Pent, ent) = 0 and Zent = 0, on the basis of [eq 2.4-1]. One will be able to refer to [R4.02.05].

One will not be able to consider such a type of decomposition field of "drive" field "relative" in the event of loadings of fluctuating the pressure type imposed on a wall of the fluid field.

One seeks to exploit this distinction in the discrete non-linear dynamic system [éq 2.2.4-1] to simplify the taking into account of imposed displacements Ud (T).
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U has
Custom displacement imposed of the supports
U ent
absolute movement
U

Ua
movement of drive U ent
relative movement
U
(R has)

All the conditions of connection, to see [éq 2.2.4-1], are not necessarily conditions of "drive" by the supports:
D
Bs. Ua = U (T); one notes by B. U
L
has = the 0 conditions of
connections which one wishes to impose directly on Ua displacement of the structure (for example of connections intern like "3D_POU"…) and L parameters of associated LAGRANGE. One considers thus thereafter these two families: B S for the movements of "drive" by the supports and BL for the bilateral connections "ordinary".

If the supports are in a finished number (what will be the case in any case after discretization), one notes
Ve the vector space of the fields of displacements of the "involved" structure uent, of dimension finished NS, which one will define hereafter. One breaks up the conditions of DIRICHLET Ud (T), on one
base (X
D
K
ks) of displacements of the supports: U (T) = Ud (T) Xks, K = 1 NS traversing all them "degrees of freedom involved" by the supports.

One builds a "raising elastostatic", i.e. a base of Ve starting from the solutions linear elastic statics of the structure under only basic imposed displacements (Xks) of supports (no loading in imposed force). After discretization by finite elements, that returns to to solve K = 1 NR S problems of elastostatic (matrix of stiffness K):

To find (K
, S,
such as
K

L K
)
K
. K +t B
T

S.

S
+ B
L. L
K
= 0

K
Bs
. K = Xks

éq
2.6-3

BL
. K = 0

One calls “static modes” these NS solutions K (well informed via operand “MODE_STAT” of DYNA_NON_LINE). They are calculated as a preliminary by operator MODE_STATIQUE [U4.52.04] with
option MODE_STAT.

One necessarily has with [éq 2.6-3]: T. K. +t
K
X S
. S
,
L =
L
L
= 0 ,
K 1 NR.
K
S

The field of displacements of the “involved” structure uent, after discretization by finite elements, is thus described by
K
U
K
ent = U ent (T) K
, checking Bs U in particular

. ent = Ud (T) Xks on
supports, traversing the discrete subspace Ve, of the “degrees of freedom involved” by the supports.
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The discrete subspace Ve is thus generated by the base (K). One necessarily has:

The U.K. (T) the U.K.
= D (T)
, K
E

The degrees of freedom of displacements of the "involved" structure thus are directly given by the value of ***Ud*** *(T).*

Having characterized the discrete subspace Ve starting from the static modes, after discretization, let us study a field of displacement ***W*** *of the structure, kinematically acceptable unspecified, but no one on the supports:* ***B W***
0
S.
= and checking the "ordinary" connections ***B W***
.
L
= ***0****. Under the terms of*
[éq 2.6-3], one necessarily has:

T ***W.K***
. K +t ***W.t B****.*
T
T
S
S

*+ **W.B**.*
L L
K
= 0
K

*T **W.Bs***
*. K =t **W.X***
W
ks
*, such as **B W***
.
S
*= **0***
*T **W.B***

L
. K = 0

From where simply:

*T **W.K***
. K = 0

W
*, such as **B W***
.
S
*= **0** and **B W***
.
L
*= **0***
éq
2.6-4
*T **W.X***

ks = 0

*The elastic operator of stiffness **K** being definite positive (having eliminated the rigid modes of body), one*
*note that any field of absolute displacement **Wa** kinematically acceptable of the structure,*
after discretization, can be written by single decomposition:

W = ***W*** + ***W***
has
E on the sum of the additional subspaces Goes = V Ve ***éq***
2.6-5

with Ve generated starting from the static modes (K), and V containing the fields known as "ddls active" such as ***B W***
0
S.
= (null on the supports). One calls V the discrete subspace of the "degrees of freedom credits".

For a loading of the type MONO_APPUI, the modes static are the rigid modes of body of structure: ***K***
. K = ***0****, checking "ordinary" connections* ***BL***
. K = ***0****.*
If the loading is MULTI_APPUI, the static modes (K) are unspecified.

This decomposition on two additional subspaces Goes = V Ve of any field kinematically acceptable built using the operator of elasticity ***K*** *is applicable in all non-linear evolution of the structure, including with shocks…, provided that connections bilateral remain the same ones during the history.*
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Let us exploit this decomposition and project the non-linear dynamic problem now

[éq 2.2.4-1] separately on the subspace V then on the Ve subspace, while exploiting [éq 2.6-3].
The result is simplified (a little only) bus ***B W***
0
S.
= (from where the “active ddls” do not work
in the reactions of the supports ***Bs*** *supports),* ***B****. =* ***0***
L
K
(from where the static modes do not work
in the reactions of connection ***B*** *L),* ***B****. =* ***X***
S
K
ks:

To find ***U*** *=* ***U*** *+* ***U***
has
E
, S, L, µ such as:
Ue *= L*
U D (T)

L
T
(
W.M.U+U*,* ***U&***
E
+U&, ***U&***
E
+U&) +t
E
(
W.C.U&+U&e*) +t* ***W.R*** *(****U****,* ***U&+U&****, T)*
E
=

T
(
W.L *(T) +* ***Labso*** *(****U****,* ***U&****,* ***U&***
E
E
) - tW.tB ***.LL*** *- tW.tA.µ* ***WV***

*T****. M***

K
*(**U+U**, **U&***
E
+U&, ***U&***
E
+U&) +t
E
K (
***. C.U&+U&)** +T*
E
. R** (**U**, **U&
K
+U&, T)
E
=

T
abso
T
T
T

*K (**. L** (T) + **L** (**U**, **U&**, **U&***
E
E
) - X .kss -. A.
K
µ
K

B. U
S
=0

B. U
L
=0

(
***A.U+Ue) d0** (T)*

µ 0
J, ((
***A.U+Ue) - d0)** .jµ J = 0*

*(**U+Ue**) (t0) = **U0***

*(**U&+U&e**) (t0) = **U&0***
éq 2.6-6

It is noted that that is rather complicated.

*We with the dynamic problem restrict initially where operators of inertia **M** and forces interior **R** are linear, and in absence of absorbing borders: **Labso** (**U**, **U&**, **U&***
E
E
*) = **0**.*

*One makes the assumption in Code_Aster that: **C.U***
&
0
E =
, including in multi-support (whereas that is not
*exact that in mono-support, where the static modes are the rigid modes **K**.*
0
K =
without deformation).
That amounts neglecting the contribution of displacements of drive of the structure to viscous damping.
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The static modes K checking [éq 2.6-3], the system [éq 2.6-6] is restricted with:

T ***W.M.U&*** *+t*
U
W.C. *& +t* ***W.K.U*** *=t* ***W.L*** *(T) - T* ***W.t B.***
L L - T ***W.t A.μ*** *- T* ***W.M.U***
& E
W *V*

*T****. Mr.***
K
*(****U&+*** *&l*
U D (T)
T
T
T
T
T
T

μ
L) +
U
. C. *&*
K
+ L
*U D (T)****. K.***
K
*=****. L***
K
(T) - ***X.***
ks
*S -****. A.***
K
K

L
Ue *= U kd (T) K*

B. U
S
=0

B. U
L
=0

(
A.U+U *kd (T) K)* ***d0*** *(T)*

µ 0
J, ((
A.U+U *kd (T) K) -* ***d0) .µ***
J
J = 0

(U+Ue) *(t0) =* ***U***

0

(U&+U&e) *(t0) =* ***U&0***
éq 2.6-7
One notes on the first of these equations [éq 2.6-7], that thanks to the made assumptions, one can to restrict to solve a dynamic problem on the field of "relative" displacement ***U****, having blocked the degrees of freedom on the supports (****B U***
0
S.
=), with the proviso of providing it as a preliminary term T ***W.M.U***
& E, as well as the static modes (K).

The object of the operator
T
CALC_CHAR_SEISME [U4.63.01] is precisely to calculate the term - ***W M***
.
U
. & E,
transformed into a concept of the type "charges", using operator AFFE_CHAR_MECA [U4.25.01]. One can also simply introduce a load of unit "gravity" into the wanted direction, and amplified by the temporal signal of acceleration. The advantage is to exploit the data directly in accélérogramme (for example produced starting from a spectrum), without having to twice integrate it in time with uncertainties which this operation generates. One can produce the constraints directly known as "primary" induced by dynamics in "relative movement".

This advantage is lost if unilateral connections are present, since the condition

***A.** (**U** + the U.K.*
K
D (T) K
*) **D** (T)*
0
ask for the value of U (T
D
) to be expressed correctly, except if
one with the chance that the movement of drive does not have a component on the interface where play of
the unilateral connection is calculated!

*The second equation of [éq 2.6-7] provides the reactions on the supports S (the remainder being given by the resolution of the problem in "relative" displacement **U**); but it is noted there too that it is necessary to know the U.K. (T*
D
) .

*It is the same thing if one wants to reconstitute the complete solution for postprocessing in constraints with their "secondary" contribution known as, related to **U***
K
E = Ud (T) K
.

Now one considers the dynamic problem of a structure interacting with a ground (medium "infinite"), source of an incidental seismic wave, cf [R4.05.01] and [R4.02.05]. One is necessarily within a framework "MONO_APPUI", where the static modes are the rigid modes of the structure, from where:
***K**.*
0
K =
and T K
***. C.U&** = 0. One thus uses elements of absorbing border, and the term*
L
*(**U**, **U&**, **U***
abso
E
E &) is present in [éq 2.6-7]. The incidental wave is provided directly by the signal

---

7.4

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***U** R*
D (X, T). One thus builds the terms of
[éq 2.6-2], as well as the tW.M.U term
& E.
system [éq 2.6-7] becomes:
T
T
T
***W.M.U&** + **W.C.U&** + **W.K.U** =*

T

***W.L** (T) T*
abso
*+ **W.A***
.
0
(
T
T
T
T
T
T
U& - U&e)** + **W. (U&e) - W. BL**. L - **W.A.μ - W.M.U&e
W
V
T
T
T

abso
T
T
T
T
L
K
***. Mr. (U&** *U*
+ & d& (T) L) = K
***. L** (T) + K*
. With
.
0
***(U& - U&e)** + K*
***. (U&e) - Xks.s** - K*
. A.μ
K

K
***Ue** = Ud (T) K*

Bs.U** = **0

B**. **U

L
*= **0***

***A.** (**U**+ K*
Ud (T) K
*) **D** (T)*
0
μ

0

K
*J, (**A.** (**U**+**Ud** (T) K*
*) - **d0**) .μ = 0*
J
J

(U U
*+ E) (t0) = **U0***

(&U+ &Ue) (t0) = &U0
éq 2.6-8
Let us return now to the non-linear problem general [éq 2.6-6]. It is noted that the first equation on the "active" degrees of freedom comprises necessarily a coupling with the field ***U***
K
E = Ud (T) K
in the operator of inertia as in that of internal forces. One thus must, in accordance with the diagram of temporal integration and non-linear resolution, developed with [§ 3], to reconstitute calculation at every moment the value of absolute displacement ***U*** *=* ***U*** *+* ***U*** *has*
E
, them constraints…

In conclusion, one can deal with the dynamic problem moving relative (by admitting that them forces of damping depend only on him):

· if one is in MONO_APPUI or MULTI_APPUI, provided that the behavior is linear in small transformations, without absorbing border, with unilateral conditions such as the plays are modified by the movement of drive, only the fluid field that is to say directly not charged, and with a loading of imposed acceleration, cf [éq 2.6-7];
· if one is in MONO_APPUI, with an unspecified behavior and absorbing border, with unilateral conditions such as the plays are not modified by the movement of drive, and with a loading of imposed acceleration.

If not, one can only deal with the dynamic problem moving absolute.

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3 Diagram of temporal integration: diagram of NEWMARK and method of NEWTON

The mechanical problem to analyze being modelled in finite elements, according to the step described with
[§ 2], one calculates the fields of displacements, speeds and accelerations with the nodes in a continuation
discrete of moments of calculation T, T
T, T
T
1 2K I 1
-
iK NR: {Ti}
.
1i NR
The user of DYNA_NON_LINE can currently choose between two implicit temporal diagrams with two steps: that of NEWMARK (1959) or its alternative known as "modified average acceleration", of HILBER-HUGUES-TAYLOR (HHT, 1977): to see the paragraph [§5]. The state of the structure being known with
the moment Ti-1, one deduces from it his state at the moment Ti by a method of prediction-correction.

Note:

One must note that Code_Aster does not propose method multi-field in time and space, which would allow to define a diagram by zone in the studied solid.

3.1 Diagram of NEWMARK

One presents here this diagram in his traditional form ([bib1] and [bib2]) relating to a movement of translation or of small rotation. For great rotations of elements of structure [bib3], the formulas are more complicated, but they in the same way make it possible to bring up to date speed and acceleration
angular according to the increase in displacement, which is in this case vector-rotation.

One notes hereafter by the configuration, i.e. the parameter setting of the system by the ddl of finite elements: displacements and rotations ***U****, pressure* ***P****, potential…*

The diagram of NEWMARK rests on the following developments of the vector configuration function time, when and are two parameters:
2
(

t+ T
) (T) + T
& ()
T
T +
(1 - 2) & (T) + 2& (t+ T
*) **éq***
3.1-1
2
& (t+ T
) & (T) + T
(1 -) & (T) + & (t+ T
)
éq
3.1-2
The equation [éq 3.1-1] is also written with [éq 3.1-2]:

2
(

t+ T
) ( )
T
T +
(-) & (T) +& (t+)
T
T +
(- 2) & (T)

2

These parameters and are provided respectively via the operands ALPHA and DELTA of the key word NEWMARK of DYNA_NON_LINE.

See it [§ 4] for the characteristics of the diagram according to the values of these parameters.

*The hooks with the second members of the equations [éq 3.1-1] and [éq 3.1-2] are averages balanced & (T) and of & (t+ T
).*

*In practice these expressions are not usable because one will have to express the values at the moment T + T
from those at the moment T.*
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The equation [éq 3.1-1] gives:
& (
1
1
2 -1
T + T
) =
[(T + T) - (T)]-
& (T) +
& (T)
t2

T

2

éq
3.1-3

&
1
=

1
-
& (
1
T) -
& (T)
t2

T

2
And, according to [éq 3.1-2]:

-
2 -
& (

T + T
) =
[(T + T)
- (T)]+
& () (
) T
T +
& (T)
T

2

éq
3.1-4
2 -

& =

- & () (
) T

T +
& (T)
T

2

It is noticed that one cannot have = 0, nor = 0. With the first step of calculation one exploits directly initial conditions: (0) and & (0). Is also needed & (0), except if = 1: to see the remark made with [§ 3.2].

In the case of great rotations of elements of structure homologous expressions with [éq 3.1-3] and [éq 3.1-4] are more complex [bib5], but the relations which follow are rather easily transposable.

During a step of time (of T to T + T
), where the values at the moment T are solidified, equations
[éq 3.1-4] and [éq 3.1-3] the increases speed define
& and of acceleration
&
correspondent with an increase in arbitrary displacement
starting from the position at the moment T,
which one will need at the time of the iterations of correction of NEWTON (within the step of time, to see it
[§ 3.3]) :

& =

éq
3.1-5
T

1
& =

éq
3.1-6
2

T

We place at the moment T = Ti-1, and one writes balance [éq 2.2.4-1] after space discretization with the moment T = T-1 + T
I
I
, possibly with the complementary elements brought in [éq 2.2-6]. One
note ***Ui*** *=* ***U*** *(Ti) the degrees of freedom at the new moment Ti and by exploiting the terms of the diagram*
temporal [éq 3.1-4] and [éq 3.1-3], one leads to the non-linear system of dynamic balance:

^KUi + ***R*** *(****U****, iU& T) + T*
I

B.
+ T
I
µ
A.*I = ^*
,
L *(Ti) - iner*
LGR *(****U****,* ***U&****,*
I
I ***U***
& I)

B.U *= D*
I
U *(Ti)*

A.Ui D *(T)*
0 I

éq 3.1-7

J, µ J 0

*J, (****A.Ui - d0****) µ*
.
I J
J = 0
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with:
1

K^ = ***K*** *fs +*

éq
3.1-8
*2 (****M*** *+* ***M*** *fs) +*
C
T

T

^

L *(Ti) =* ***L*** *(T) + abso*
I
L
(
1
fs
2 1
2
Ti-1) +
MR. M
U
tU&

T
U&
2 (
+
)
-

i-1+ i-1+
i-1

T

2
éq
3.1-9
1

+
2
2
.
C U
i-1 + T (-)
-

U&i-1 *+ T*
U&i-1
T

2

Note*:*

It is noted that the matrix $
K *contributing to the stiffness generalized of the system to solve is*
enriched by terms coming from the matrix from mass ***M*** *from the system (provided that one has affected a density on all the finite elements of the model) and the matrix*
of damping ***C****. One will further see than the linearization of the forces intern* ***R*** *(****U****,* ***U&****,* *T*
I

I
)
also contribute to $
***K**. Thus, even if the mechanical system considered comprises modes*
rigid, for example in a study where a solid is in free fall, the matrix of mass comes
“to prevent” that the matrix of “stiffness” $
***K** is not factorisable. To some extent, they are them*
inertias of the solid body (in its rigid modes) which ensures balance with the forces
external, which could not be done into quasi-static.
However, it is observed that the step of time T
appears too. If it is too large, the terms of
mass will not be important enough vis-a-vis those of stiffness, and quasi stamps it risk to be
non-factorisable (“quasi-null pivots”).

***Note**:*

The terms with the exhibitor fs appear if the system contains fluid fields (see
[§ 2.4] and terms coming from the fluid field in [éq 2.4-1]); it is pointed out that it is not envisaged
of fluid loading fl
***L**.*

***Note**:*

*Contrary to the case of damping of Rayleigh (see [§ 2.2.1]), the matrix of damping **C***
does not appear in the matrix $
***K** in the presence of modal damping (key word AMOR_MODAL),*
*because in this case, force **C.U***
*& i-1 is built directly without storage of the matrix **C**, and is*
deferred to the second member $
***L** (Ti).*
The abso term
L
in the second member appears in the presence of absorbing borders, to see it
[§ 2.6]. It is treated according to an explicit diagram according to the fields solutions with Ti-1.

***Note**:*

Contrary to the diagram used in thermics, cf [R5.02.01], where the derivative are written of
explicit manner, while the conservation equation is written at one fictitious moment resulting from one
combination with the parameter of the values at the moments Ti 1
- and Ti, dynamic balances
are checked over the moments Ti 1
- and Ti, while the derivative are combinations
determined by the diagram [éq 3.1-1] and [éq 3.1-2].

It still remains to treat the non-linear terms: the forces intern ***R*** *(****U****,* ***U***
I & I, T) and the contribution
non-linear of the inertias in great rotations of structures ***Liner*** *(****U****,* ***U&****,* ***U***
GR.
I
I & I) (cf [§ 2.2.2]).

At first moment T1, one sees in [éq 3.1-9] that one needs ***U****,* ***U***
0 & 0, provided by the conditions
initial, but also, because of the diagram of NEWMARK, &U0: to see the remark made with [§ 3.2].

3.2
Phase of prediction

The system [éq 3.1-7] is non-linear and is integrated, after a prediction of EULER by linearization, with assistance of an iterative method of NEWTON, as in non-linear statics [R5.03.01]. The calculation of this prediction can be slightly erroneous, as long as the phase of correction by iterations of NEWTON [§ 3.3] is able to correct with convergence…

3.2.1 No time general

With the phase of prediction, one exploits the solution with the preceding step or the values of the initial state, and one
note the matrix of initial tangent stiffness of the step:

T
R
D Q
T

K I 1
- =
=
.
+ Q
.
éq 3.2.1-1
U (
U
, U&
,
U
I
I I
T -
I
U
U&
T

U
I 1,
-
I 1,
- I 1
-)
(1 1 1)
(1)
(
U
, U&
, T
Ui 1
-)
(I 1 - I 1 - I 1)

These terms (cf [§ 2.2]) are evaluated on the preceding step, the first term of the second member of

[éq 3.2.1-1] appearing only in great displacements (Q not being then constant).

If the behavior is linear, the Ki-1 matrix is simply the elastic matrix of rigidity K of the structure.

One can also decide to save time calculation not to reactualize this matrix, and of to take the elastic matrix of rigidity, to see [U4.51.03], key word NEWTON, operand PREDICTION, "ELASTIC" value, rather than "TANGENT". If one specifies nothing, the choice by defect made by Code_Aster is coherent with that done on the iterations of correction of NEWTON described hereafter with [§3.3].

In addition, in the presence of great rotations (elements of structure: beams…), one must too to derive the non-linear term from inertia iner LGR (Ui, U&i, U&i):

iner
1
M
LGR
K I 1
- =

éq
3.2.1-2
U (Ui 1, - U&i 1, - U&i 1)

The matrix $
K having been established by [éq 3.1-8], this new matrix
1
M
K I 1
- is combined with the matrix
$K + Ki-1, to see below [éq 3.2.1-4].
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One defines also the increases in loading $L (Ti) from [éq 3.1-9], of conditions
imposed
D
U

(T
anél
I), and one gathers in L

(Ti) dependences of the constraints in function
various parameters or "variables of order" Z of the law of behavior of material
constitutive: such as the temperature…:

anél
T
D
L

(Ti) = - Q (Ui-1).
. Z

dZ (U, U&, U&, Z
i-1
i-1
i-1
)

In the presence of modal damping the forces of damping are deferred to the second member.
One then adds to $L (Ti) the corresponding term: -.
C U&i 1
- .

The abso term
L

(Ti) = - A faP& (
abso
Ti 1
-) + L
(U
, U&
,
ent
U&
I 1
-
ent I 1
-
I 1
-) the contribution indicates
integrated into explicit (to simplify) of an absorbing fluid border and an elastic border absorbing, cf [éq 2.4-1] and [éq 2.6-2].

The increase L is noted
^
(Ti), L^ being defined with [éq 3.1-9]:

^
L (Ti) ^
= L (Ti) ^
- L (Ti 1
-)
éq
3.2.1-3

One then calculates predictive values for the step of time in progress (U0
0
0
I, I, µi):

Prediction
^
1
M
0
T
0
T
0

^
anél
(K + Ki-1 + Ki-1) Ui + B
. I + A
. µi = L (Ti) + L
(Ti)
B. 0
U = D
I
U (Ti)

With (. 0
Ui) D (T)
0 I
éq
3.2.1-4

0
J, µi 0
J

J, (. 0
In Ui - d0 .µ
I)
0

I
= 0
J
J

where the prediction is defined
0
0
Ui = Ui 1
- + U
I for the new moment T = T-1 + T
I
I
.

If one chose MATRIX = "ELASTIC" in the key word NEWTON, one does not revalue with each step of

time
1
M
1
^
^
M
K + Ki 1
- + K I 1
- = K + K 0 + K 0, which avoids the cost of re-assembly and inversion,
but the iteration count of correction increases.

After the establishment of a candidate solution of [éq 3.2.1-4] without checking the criterion of contact, one launches
the algorithm of active constraints to satisfy the conditions of contact: one corrects thus
U0 0 0
I, I, µi.
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Note: stamp tangent singular:

One checks on [éq 3.2.1-4] whom if the tangent matrix K I 1
- is singular (case of a rigid mode,
of a damaged material, or ductile plate…), dynamics is controlled by the inertias,
and that, the matrix K

^ being in general regular (cf notices made with [§ 3.1]), one finds in spite of any good a predictor 0 Ui, provided that the precision in the K^ matrix is not lost (pivot quasi-no one) because of a choice of step of too large time which plays in 2 1/ T

in [éq 3.1-8]. It is for example the case in free dynamics of fall.

3.2.2 First step of time

If one is with the first step of time, a study or a recovery (continuation), the predictor is calculated differently to take account of the initial state (U0, U&0, 0):

^ *1 M 0 T 0 T 0* ^ *anél T (K + K0 + K0) . U1 + B . 1 + A . µ1 = L (1t) + L (1t) - Q.0 B. 0 D U1 = X (1t)*

- . B U0

To (0 U1) D (T) 0 1 éq 3.2.2-1

0
J, µ1 0
J

J, (. 0
In U1 - D
.µ
1
0)
0

1
= 0
J
J
with:
^
fs

L (
1
1 2
1
T) = L (1t) +
MR. M
U
tU&
T
U&
2 (
+
)
2 -
0

0 + 0 +
0
T

2
éq
3.2.2-2
1

+
2
C U
0 + T (-)
2
-
0

U&0 + T
U&0
T

2

and acceleration
0
U&0 evaluated by the preliminary resolution of the system (one simplifies by supposing them blocked connections of contact, the iterations of NEWTON will be given the responsability to correct):

0
M.U
T
T
anél
T
& 0 + B.0 + A.µ0 = L (t0) + L (t0) - Q. 0

0
B.U0 = 0
éq
3.2.2-3

0
A.U0 = 0
Note:

It would be more exact to calculate:
0

MU&
T
T
anél
0 +
.
B
T
0 +
.
With µ0 = L (t0) + L
(t0) -.
Q 0 - U
C & 0,
but to simplify knowing that that will have only little influence on the continuation of the solutions, one
neglect U
C & 0 in the second member of [éq 3.2.2-3].
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One must note that:

*· the matrix **M** must be invertible: one will have to affect a density on all them finite elements of the model,*
*· it can be necessary to establish by a static calculation (possibly non-linear) the state of balance under the initial loading, therefore constraints 0, before the imposition of initial dynamic conditions **U**, **U** 0 & 0. Indeed, if not, acceleration 0 **U&0** could be "excessive" and to lead on a nondesired branch of balance,*
*· in the presence of "loads kinematics", cf [U4.44.03], these last are put at zero with this stage of calculation of 0 **U&0**.*

*However, certain finite elements do not have mass on all the degrees of freedom, by example beams with warping POU_D_TG precisely on the ddl of warping, or in fluid coupling/structure. For the vibroacoustic elements of coupling [§ 2.4] indeed, the matrix of mass of the problem [éq 2.4-1] is not invertible. In these cases, the matrix **M** is not invertible, and one is satisfied to take a null initial acceleration on the whole of the model and continuation of the iterations will have to be given the responsability to correct this "good" prediction less. One then will be chosen small step of time to ensure convergence at least the beginning of the transient.*

*The abso term **L** (t0) is evaluated starting from initial speed **U&0** (cf [R4.02.05]).*

3.3
Phase of correction by the method of NEWTON

*One seeks the values (**Ui**, I, µ)*
I of the increments of displacements and parameters of
*LAGRANGE since the values (**U***
, I, µ)
i-1
1
I 1
-
obtained with preceding balance (urgent Ti-1). One
*takes as initial values (**U0***
0
0
I, I, µ)
I obtained at the end of the phase of prediction, before
to begin the iterations of the method of NEWTON.

With each iteration N of NEWTON, one notes by evolutions leading to the estimate of
increments with convergence of the iterations
:
N 1
+
N
N 1
+
***Ui** = **Ui** + **Ui** - **Ui** 1*
- and
n+1
N
n+1
I
= I +i - i-1 (in the same way for the µ). One must then solve a system allowing of
*to determine (**Un+1***
n+1
n+1
I
, I, µi), increments of displacements and the parameters of LAGRANGE
*since the result (**Xn***
N
N
, µ)

I
I
I
preceding iteration:

Correction (iteration n° N)
^
N
Mn
n+1
T
n+1
T
n+1
^
N
abso
N
(K + Ki + Ki) Ui
+ B.i + A.µi = L (Ti) + L
I - I
F

n+1
***B.Ui** = 0*

n+1
A.Ui
***D** (T)*
0 I
éq
3.3-1

J, µ J 0

n+1
n+1
J, (.
In Ui - d0
µ
.
I)

I
= 0

J
J
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with a second member called "residue", because it tends towards zero to convergence. One notes (cf balances [éq 2.2.4-1]):

N
T
I
F *=* ***Q*** *(N*
Ui)
. *(*
&
&
N
N
+ ***U***
C.
+ ***MR. U. U - L***
+

B.
+
µ
A.
éq
3.3-2
***U*, *U&*,**
U*, *U&*, *U&
I
I I)
nor
T
(nor) N iner
I
GR. (N N N
I
I
I)
T
N
T
N
I
I
N
The term $
***L** (T*
abso
N
abso
N
*I) is defined by [éq 3.1-9], the term **L***
*I = - **A** F &i*
***P** 1*
*- + **L***
(Uent*, *U&*, *U&
I 1
-
ent
I

I
1

1
-
-)
corresponds to the fluid and elastic borders absorbing, which is treated into explicit [éq 2.4-1], [éq 2.6-2]; the matrix $
***K** is given by [éq 3.1-8].*

*In the presence of modal damping the forces of damping are deferred to the second member. According to the value given to key word AMOR_MODAL, REAC_VITE, one adds to **L** ^ (Ti) the reactualized term:*
N
*- **C U***
. & I, or not reactualized -.
***C U&i** 1*
- .

*The matrix **K** nor is the matrix of the tangent linear application of the part “forces intern” system of non-linear equations [éq 3.1-7]; it is thus worth:*
méca
N
F
R
Li
***K** I =*
=
-

éq
3.3-3
U
U

&
&
U
(N
***U**, N*
***U**, T)*
(N
***U**, N*
***U**, T)*
I
I I
I

I I
(N
***U**, T)*
I I
In the absence of following forces, the last term is null. The following forces can be: pressure exerted on the edges of solid elements, the loading of gravity for the elements of cable, the centrifugal force in great displacements, the loading of gravity for all them modelings THM of the not-saturated porous environments [R7.01.10].

If one considers the reactualization of the geometry (in great displacements), one has more precisely:

T
méca
N
T

Q U
L
***K** I = **Q** (**U**)*
( )
.
I
+
. -

éq
3.3-4
***U** (N*
U
U
***U**, N*
***U&**,*
I
I Ti)
(naked, nU&,
I
I Ti)
(N
***U**, T)*
I I
The first term is the contribution of the behavior as in small transformations, with difference which this contribution is evaluated here in current configuration. The second term is contribution of the geometry which is not present in small transformations. Within the framework of reactualization PETIT_REAC, this term are not present in the calculation of the tangent matrix.

The matrix
Mn
***K** I is the matrix of the tangent linear application of the part "inertias" of*
system of non-linear equations [éq 3.1-7] which is thus worth:

iner
Mn
L
GR.
***K** I =*
éq
3.3-5
U

(N N N
***Ui**, **U&i**, **U&i**)*

In practice one can use "the true" tangent matrix, but that presents a cost unquestionable calculation (calculation and factorization), or to be satisfied with a reactualization from time to time: to see in [U4.51.03] it
key word factor NEWTON, key word STAMPS.
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Note: stamp tangent singular:

One checks on [éq 3.3-1] whom if the tangent matrix ***K*** *nor is singular (case of a material damaged, or ductile plate…), dynamics is controlled by the inertias, and that, stamp* ***K***
^ being regular, one finds despite everything good a ***Un+1*** *corrector*
I
(as if one were
for example in free situation of fall). However, if the step of time is "too high", stamp (
N
Mn
K^ + Ki + Ki) *can be badly conditioned and the solvor finds a pivot to him quasi-no one.*

After each iteration of NEWTON having established a candidate solution of [éq 3.3-1] without checking the criterion
of contact, one launches the algorithm of active constraints to satisfy the conditions of contact: one correct ***Un+1*** *thus*
I
.

3.4
Update

· Danslecasde small rotations, for usual modelings (solid elements, discrete beams, plates, hulls, elements…), the update is based on the formulas [éq 3.1-5] and [éq 3.1-6].
n+1
N
Ui *=* ***Ui*** *+*
n+1
U

I

n+1
N
Ui *=* ***Ui*** *+*
n+1
Ui

n+1
N

U&

éq
3.4-1
I
*= **Ui** +*
n+1
U

I
T

n+1
N
***U&i** = **U&i** + 1*
n+1
U

I

2
T

· Danslecasde great rotations of the elements of structure (beams…) the update, definitely more complex, is indicated in [R5.03.40].

3.5 Criterion of stop

The criterion of total convergence of the algorithm of NEWTON is identical to that practised in STAT_NON_LINE. It represents the checking of dynamic balance.

At the moment Ti, one stops the iterations with row N, as soon as the following inequality is satisfied:

*T **Q** (N*
U
^
I) N

***.** I +*
N
U
***C.** & I + **M** (N*
***Ui**) N*
U

. & I +t
N

B.*I +t*
N
µ
A.*I - iner*
LGR *(N N N* ***L***
Ui, U&i, U&i) *- (Ti)* ***éq 3.5-1***
L*^ (T) + anél*
I
L
(Ti) - T N

B.*I - T*
N
µ
A.*I*

is a tolerance, introduced in data by the user (key word CONVERGENCE, operand RESI_GLOB_RELA), about 104 to 106, and.
is the standard of the maximum on the ddls.

The denominator of [éq 3.5-1] is a standard of the loading at the moment Ti, to which one brings it back numerator, which is a standard of the forces not (still) balanced.

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Just like in statics, one must attach importance to a correct convergence bus if not, them estimates of interior forces and reactions of contact, checking the relations of behavior and from connection of contact-friction move away from balance, and it is not an inertia
***M** (N*
***Ui)** N*
U
. & I too far away from the “exact” value which will be enough to produce with the step of time following one
good transitory dynamic response.

One will be able to also employ other criteria of convergence as proposed in STAT_NON_LINE.

4
Qualities and defects of the diagram of NEWMARK

4.1
Properties of the diagram of NEWMARK

This paragraph and the following takes again partially certain parts of [bib2]. One will be able too to consult [bib24] and [bib27].

One defines using the methods of numerical analysis several types of properties for a diagram of temporal integration. Here their significance:

· Convergence: the solution tends towards a limit when the step of time T
tends towards 0;
· Précision: rate of convergence when the step of time T
tends towards 0;
· Consistance and order of the diagram: the residue is limited by () 1
+
K
T
(example: K = 2 for
regulate trapezoid): a polynomial of order K + 1 is integrated exactly;
*· Stabilité: a finished disturbance of the initial state exaggeratedly does not involve a disturbance amplified (“numerical explosion”) in a later state; it is necessary that amplification (spectral ray) that is to say lower than 1: for a diagram of general form **Ui** = **WITH***
+
I
L

-1
I, one must have
(A) 1. It is a condition necessary not to diverge! If (A) < 1 there is
numerical attenuation.

The error E induced by the diagram of NEWMARK [éq 3.1-1], [éq 3.1-2] is given by:

T
2
E
U
& I
éq
4.1-1
6

*that one can standardize by the vector position **X** or the amplitude **U**. One can also replace in*
[éq 4.1-1] the standard L
K
2 (
U
& I) by the standard
L:
U
& I = max
. Today

(U&i)
nodes K
Code_Aster does not provide the information of this estimator.

The numerical analysis of the diagram of NEWMARK is done on the treatment of the equation of the oscillator
***linear** with a d.d.l., without friction:*
X
m &+ kx = 0 ***éq***
4.1-2
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*The temporal diagrams do not give the **exact solution** of [éq 4.1-2]:*

X = C

cos T C sint
1
+ 2

K
with =
,
*C depend on the initial conditions, but an **approximate solution**:*
m
1
C and 2
X
~
'
~
'
~
I = [
(
exp
- Ti)](C1
cos Ti + C2
sin Ti)

Two errors are thus brought:

· on the one hand, it is introduced a definite artificial damping by the reduced rate of depreciation,

who is the decrement logarithmic curve divided by
2 ;
· in addition, the pulsation corresponding to the exact period T is replaced by
~
pulsation ~ correspondent at one period T.
T
These errors depend on the report/ratio
and of the diagram itself.
T

By writing the balance of the system with 1 d.d.l. [éq 4.1-2] and equations of the diagram of NEWMARK [éq 3.1-1], [éq 3.1-2] one obtains:

1
X

1
1
-

X

i+1

2

I

1

tx

&
2
2
2
2
1
2
2
i+1
=

- T
1+ T (-) 1 - - T (-2)
tx
2
2
&
+
1
I
2

2

T
T x&

2
2
2
2
2
2 1

2
i+1
- T
- T
- T (-
2
)

T xi&

éq 4.1-3
from where in term of increases:

2
2
1
X

- T
1
-

X

2

I

1

tx
&
=
-
T
- 2
t2 1 - - 1 2
t2 (- 2)
tx ***éq 4.1-4***
2
2
2

&
+
1
I

2
T
2
2
2
2
1
2
2

2
T x&

- T
- T
- T-1

T xi&

2

The properties of this matrix are used to characterize those of the diagram, in linear mode.

The properties of the diagram of NEWMARK are summarized hereafter.

Because of the selected manner to express the diagram (cf [éq 3.1-1] and [éq 3.1-2]), one cannot to take = 0 nor = 0.

***For the linear problems**, the diagram is **unconditionally stable**, even in presence of physical damping - a disturbance is not amplified by the stable diagram i.e. whatever the size of the step of time, if the parameters satisfy the inequalities:*

2 1

2
éq
4.1-5

1
1
If 2 1
and < +
*the diagram is **conditionally stable**: the step of time must be*
4 (
) 22
1
- /
2
min
T

1

chosen lower than: T

(+
- 4
- this relation being valid in absence
2)
2

of physical damping = 0 but also in the presence of physical damping - according to the smallest period min
T
of vibration of the studied system, to see [bib24], [bib27].

In the presence of physical damping > 0, one can allow T slightly higher.

In the presence of contact with impact, one ensures also the stability of the diagram if 2 1 2.

The following table shows some particular cases:

standard method
properties

1/12 1 2 “Fox-
implicit
diagram of order 4; no numerical dissipation

Goodwin”
conditionally stable:
T
3 2/(F

) 1-

max
, fmax being the frequency
vibratory maximum “aiming” in simulation, not
of numerical dissipation
1 6 1 2 implicit acceleration
diagram of order 2, not of numerical dissipation

“linear”
identical to -
conditionally stable:
WILSON method,

-1
T3/(fmax), F
with = 1
max being the frequency
vibratory maximum “aiming” in simulation, not
of numerical dissipation
1 4 1 2 “rule of
implicit
diagram of order 2, not of numerical dissipation

trapezoid” or
unconditional stability in T,
acceleration
the frequencies are shifted to the bottom, but one
average
stamp of consistent mass limits this defect
(since causes the opposite effect),
no numerical dissipation: no attenuation
of amplitude due to the diagram

- implicit method

See [§ 5].

The rule of the trapezoid (= 1 4, = 1 2) is most commonly adopted, associated a mass consistent (default option MASS_MECA). If one wishes to use a diagram where the frequencies are shifted upwards, it is advisable to choose option MASS_MECA_DIAG.

The diagram of NEWMARK is of the second order in displacement (in the worst case) if and only if = 1 2.
As soon as 1
>/2, the diagram of NEWMARK is of a nature 1, and introduced a numerical dissipation proportional to (- 1 2
T. In order to ensure a numerical damping growing with
4)
(+1 2) 2
/
frequency, it is appropriate to choose:
, equality being the best alternative, cf diagram HHT,
4
to see it [§ 5].

If one chose < 1 2, the diagram of NEWMARK would bring a negative numerical damping who would bring an instability.
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By using the increases [éq 4.1-4], one deduces the error [éq 4.1-1] from the diagram, realised on one

period
2/, and standardized compared to the amplitude, in the case of the oscillator with 1 d.d.l.,
cf [bib28]:
3
3
T

enorm
+ T
moy
éq
4.1-6
2 (
4
2
2
1+
T
)
2
2
This expression can be used as error of reference to the diagram applied to the dynamic response of one unspecified structure.

4.2
Energy point of view

In the linear elastic case in small transformations, one can easily evaluate the variations of
various energies during the step of time T
between Ti and Ti 1
+. Thus, respectively for
kinetic energy, the deformation energy, the work of the external efforts:

E = 1
. Mr.
E = 1
. K.

2 (T
déf
(T +1) T
I
- (Ti)) ((ti+1) + (Ti)
2 (T

cin
& (T +1) T
I
- & (Ti)) (& (ti+1) + & (Ti);
T
*Wext= & (ti+1)***. F** *(T +1) T*
I
*- & (Ti)***. F** *(Ti)*

By injecting the expressions of the diagram [éq 3.1-1] and [éq 3.1-2], one finds:

T

E =
(T
cin
& (T +1) T
I
+ & (T) + (1 - 2) T
I
&) (
. Mr. *& (ti+1) + & (Ti)*
4
T
T
2 -

Edéf =

& (T +1) T
I
+ & (Ti) +
(T
T
&
T

- & (ti+1)) (
. K. *(ti+1) + (Ti)*
4

W = 1 (T
T

2 -

ext.
(T +1) T
I
+ (T))
T
I. F
+

& (T +1) T
I
+ & (Ti) +
(T
T
&
T

*- & (ti+1)) (. **F** (ti+1) **F***
+

(Ti)
2
4

By using balance at the moments Ti and Ti 1
+ under the action of the external efforts, one checks that
variation of total energy E
early = E
cin + E
déf - W
ext. of the system is expressed:
T

E =
(1 - 2) T
1
early
&
(
***. Mr.** & (ti+1) + & (T) - (T*

I
(T +1) T
I
+ (Ti))
. F
4
2

éq 4.2-1
T
(2 -)
+
(tt
& (T +1)
T
I
- &) (. (
Mr. *& (ti+1) + & (Ti))*
4
It is checked that when = 1 2 and = 1 4 (rule of the trapezoid), one does not have numerical dissipation of energy brought by the diagram. It is also noticed that a choice different from and/or 2 bring a dissipation proportional to the step of time.
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4.3 Properties of the diagram of NEWMARK for the problems not

linear

For the non-linear problems *(great deformations, non-linearities material), the diagram is*
1
unconditionally stable*, if*
and with = 1 2, cf proof in [bib25].
4

In the presence of ***unilateral contact****, one finds in the literature, cf [bib26], the council to take*
= (for example with = = 1 2) what ensures the compatibility speeds during the phase
where the contact is maintained between two solids. That is obtained directly starting from the equations [éq 3.1-1] and [éq 3.1-2]; the jump []
. *of speed enters the two points remaining in contact on the step T*

([(T)] = 0 and [(t+t)] = 0) at the moment t+ T
is indeed:

[& (t+ T)]= (1 -/) [& (T)]+ (1 -/(2) T [& (T)]

However this choice is not compatible with that of an optimum in numerical term of dissipation, as defined by - method (cf [§ 5]): = 1 would have to be taken
- - 2 which gives one
enormous numerical damping! One thus does not incite the user to follow this recommendation.

4.4
Choice of the steps of time

The step of time to choose must respect a certain number of conditions.

The first, obvious, is that it must ***be adapted to the temporal sampling of the loadings*** *applied to the studied system. Incidentally, it can be convenient to reconsider a modeling" with a temporal dependence too "violent of the loadings applied, while softening changes of incline for example.*

One advises, for ***reasons of precision*** *(to the criterion of the type "Shannon" on the frequency of cut), a step of time to choose such as:*
T 0 1
, T
0 min ***éq***
4.4-1
where Tmin indicates the smallest period of vibration of the system which one wishes to study.

The step of selected space of the meshs finite elements also intervenes: the step of time T maximum with to choose is about H/C, where C is the celerity of the elastic waves of compression of material and H a size characteristic of the meshs, if one seeks to describe partially of the phenomena in

high frequency, for which however this numerical formulation of elastodynamic is not truly not adapted (there are other numerical methods with this intention).

*Finally in the case of solid presenting rigid modes (falls free for example), in accordance with the remark made with [§ 3.1], it is appropriate to choose a step of time T sufficient small so that them terms of mass are of the same order as those of stiffness (in the matrix **K** ^, cf [éq 3.1-8]). Thus, one will be able to choose:*
L
T

éq
4.4-2
50g

where L is the diameter of the solid considered in "free fall" under the acceleration of gravity G. Thus the increment of displacement
2
G T
/2 at the time of this step of time is weak in front of dimensions of solid, or, in a another way, similar to an elastic displacement under a sphere of activity of even width.
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It is possible to make follow one another several dynamic analyses, on intervals of time successive, communicating by recoveries by choosing like initial state the result of the last step studied before, by using steps of very distinct times according to the idea a priori which one has of response of the studied system. One does not have a general result with this type of choice in term of convergence…

One also knows that the treatment of the collisions is sensitive to the clocking of the steps of time, by report/ratio at the "real" moments of shock: one will have to study the sensitivity of the answer obtained.

However, a step of "too small" time can exacerbate the oscillations induced by discontinuity.

It is however advised to maintain constant the step of time during a phase of answer stationary linear dynamics, to keep the properties stated previously.

***Note**:*

One must note that Code_Aster does not propose today method multi-field in time and space, which would allow to define a step of time per zone in the studied solid, nor of criterion of error in dynamics.

5
An alternative of the diagram of NEWMARK: the diagram of modified average acceleration

5.1 Motivation

In mechanical analysis, one wishes that the low frequencies be reproduced most accurately possible.

It is wished on the other hand that the high frequencies be attenuated by calculation because they can generate numerical instabilities and that the associated mechanical constraints are in weak generals.
T
Curves giving the damping ratio according to
(period T = 2/) must
T
thus:

· to leave the origin with a horizontal tangent to give a very weak damping to low frequencies,
· to be increasing functions to attenuate the high frequencies and this more especially as they are higher.

To try to achieve these goals, Hilber, Hughes and Taylor (HHT) proposed in [bib4] defining parameters and of NEWMARK according to a third negative parameter by following relations, which are copied on the stability conditions [éq 3.1-2]:
(1) 2
= 1 - ;
-
=
éq
5.1-1
2
4
This choice offers the best compromise on the precision and damping in high frequencies.

This parameter, negative, is provided via the operand ALPHA of key word HHT of DYNA_NON_LINE.
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5.2
Diagram HHT and method of modified average acceleration

One obtains thus [bib4]:
2
-
& (
4

4
1
- 2
T + T
) =
[(T + T) - (T)]-
& (T) (
)
+
& (T)
(
éq
5.2-1
1 -) 2 T 2

(1) 2 T

(1) 2
2
2
-
+ -

& (
2 4
2
1

T + T
) =
[(T + T) - (T)] (
)
+
& ()
T
T +
& (T)
(
éq
5.2-2
1 -) 2 T

(1) 2
(21) 2

In addition, the dynamic balance [éq 2.2.4-1], discretized in time at the moment T = T-1 + T
I
I
is modified
by introducing a "shift", also controlled by coefficient 0, on the interior forces:

M (Ui) U&i** + (1+) **U
***C** & I + (1+) **R (Ui**, **U&i**, Ti) T*
T
*+ **B.i** + **A μ***
. I =
éq
5.2-3
***L** (Ti 1*
*- +) + **R***
*(**Ui**, **U&***
1
-
I
, T
1
-
I 1
-)
iner
*+ **U***
C** & I - **L
1
*GR. (**Ui**, **U***
*& I, **U&i**)*

where the external forces are evaluated with Ti +
1 = (1 +) T - T
-1 = T + T
I
I
I
.

The system of non-linear equations [éq 3.1-7], récrit thus:

*^KUi + (1+) **R (U**, **U&**,*
I
I Ti) + T.

B *+ T*
I
With *µi = ^*
.
L *(Ti) - iner*
LGR *(**U**, **U&**,*
I
*I **U***
& I)

.
B U *= D*
I
U *(Ti)*

.
In Ui D *(T)*
0 I
éq
5.2-4

µ 0
J, (.
***In Ui - D**).*
0
µ
I J
J = 0

with, while following [éq 3.1-8] and [éq 3.1-9] and and function of the parameter, cf [éq 4.1-1]:
fs
1
fs

***K**^ = (1+) **K** +*
éq
5.2-5
*2 (**M** + **M***
*) + (1+) **C***
T
T
*^L (Ti) = **L** (Ti) + **Labso** (*
fs
Ti 1

*-) + **K***

*. **Ui** 1*
-
1
fs
1- 2

+
M** + **M
***Ui** + tU&i + T*
U&i
*+ **R***
***U**, **U&**, T*
éq
5.2-6
2 (
)
2

1
-
1
-
1
-
(I 1 - I 1 - I 1)
T

2

1

- 2

+
.
C U
I 1 - + T (-)
2
***U&i** 1*
- + T
***U&i** 1*

-
+

U
***C** & I 1*
T

2
-

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Modifications that this diagram brings to the phases of prediction and correction of the method of NEWTON [§ 3.2] and [§3.3] are as follows:

*· phase of prediction [éq 3.2.1-4]: to put the new expression of **K**^ [éq 5.2-5], like*
*(1+) **Ki** 1 - in the place of **Ki-1** in [éq 3.2.1-4] and (-) 1R (**U** -1, **U***
I
& i-1, Ti-1) in the place of
*- **R** (**U** -, **U***
I
& I, T
1
1 i-1) in the expression of $
***L** (Ti) in [éq 3.2.1-3];*
*· phase of correction [éq 3.3-1]: to put (1+) **kN***
N
N
N
*I and **Fi** in the place of **Ki** and **Fi**.*

When = -1 (from where = 1, = 3/2), diagram HHT becomes then explicit (cf [éq 5.2-3]):
T
T
***MU&i** +.*
***B** I +.*
***With µi** = **L** (Ti 1*
*-) - **R** (**U***
*, **U&**, T*

I 1
-
I 1
-
*) **éq***
5.2-7
with:

1
1
3
1
T

***U&** (T T*

+) =
*[**U** (T T*
*+) - **U** (T)]-*
***U&** (T) 1*
*+ **U&** (T) and **U&** (T T*

+) =
*[**U** (T T*
*+) - **U** (T)]- **U&** (T) + **U&** (T)*
T 2

T
2

2 T
2

4

ensuring at the same time greatest possible dissipation in high frequencies.

***Important remark**:*

In this version of Code_Aster, nor terms of [éq 5.2-3] with [éq 5.2-6] "shifted" by (1+)
or on damping and the stiffnesses, nor the evaluation with Ti-1
+ of the second members are not
treaties, which makes lose an order on the diagram (from 2 to 1).
It is thus actually about what one notes in the literature the "method of average acceleration
modified", which is thus limited to define the optimal relation between the parameters of the diagram of

Newmark according to [éq 4.1-1] with [éq 4.1-3].

5.3
Properties of the diagram of modified average acceleration

It is necessary:
0
éq
5.3-1

so that the stability conditions of the diagram are filled, cf [§4.3]: the diagram is ***unconditionally stable****.*
2
The diagram "of average acceleration modified", thus brings = (+ 1/4, which is the choice 2)
optimal to bring damping growing on the high frequencies.

When = 0, diagram HHT (- method) become again the "rule of the trapezoid" and damping is no one [Figure 5.3-a].

The value = 1
-, that is to say = 3 2 and = 1 product strongest dissipation in high frequency, but destroyed much the precision on the low frequency modes.

In practice, in original diagram HHT, one limits to [
-1/ 3,]
0, which ensures the monotony of
the increase in damping according to the frequency. The choice =
10
,
0
-
seem to be
effective.

However, within the framework of the method "of modified average acceleration", which is that proposed
by Code_Aster, it is possible to take values of higher in absolute value.

7.4

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T
The figure [Figure 5.3-a], extracted from [bib5], gives the variations of according to
for some
T
values of. This figure calls for following observations:

· the "rule of the trapezoid" (= 0) is tempting because it does not bring any damping
parasite, but it can be unstable into non-linear,
· when 0, the curves are not with horizontal tangent in the beginning. Like the diagram
HHT (- method) brings an important artificial damping, certain users [bib5]
a step of time chooses, then determine the value of the parameter so that, in
beach of frequencies which interests them, the numerical rate of depreciation is of the same order
that the real mechanical rate of depreciation which, then, is not taken into account.

(%)
30
, = - 1
T
20
H
H
10
HHT, = - 0.3
HHT, = - 0.1
Trapezoid: HHT, = 0
00
0.1
0.2
T

T

Appear 5.3-a: Numerical rate of depreciation according to the step of time of diagram HHT

T

It is noted that one has for the small steps of time following numerical damping:

T

2
T
T
=
+ O

éq 5.3-2

T

T

what makes it possible to estimate in the frequency band concerned average numerical damping brought by the diagram HHT (- method), which comes to be added to physical damping possibly already present.

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:

The figure [Figure 5.3-b], also extracted from [bib4], gives the variations of the relative error in
T
period according to
. The error is an increasing function of.
T

T ~
T %
( )
T
20
T, = - 1
H
10
HHT, = 0
HHT, = - 0.3
H
00
0.1
0.2
T
T
Appear 5.3-b: Relative error over the period according to the step of time of diagram HHT

---

6 Example

It is [Figure 6-a] about the plane movement of pendulaison of an extensible bar AB length unit, rotulée in A with a fixed, free support out of B and given up without speed in the field of gravity terrestrial starting from a position defined by angle 0. All the phenomena of dissipation are neglected mechanics.

1
L
G
With
B
0
trajectory of B

Appear 6-a: Pendulum of great amplitude

As amplitude 0. can be large we will take it 90° the point B undergoes the large ones displacements and the problem is non-linear.

The theoretical period is:
T = 6744
,
1
S

The calculation of the movement of the pendulum by diagram HHT (- method) with = 0 (“rule of trapezoid”) constitutes case-test SDNL100.

The figure [Figure 6-b] represents the evolution for one period of the dimension of the point B calculated by
diagram “of average acceleration modified” with three values of. The period is divided into 40 steps of equal times.
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The curve in full feature relates to = 0. One observes practically no error.

Dimension node B
0.0
ALPHA: 0
- 0.1
- 0.2
ALPHA: - 0.1
- 0.3
- 0.4
- 0.5
ALPHA: - 0.3
- 0.6
- 0.7
- 0.8
- 0.9
- 1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9 1.0
1.1

1.2
1.3
1.4
1.5
1.6
1.7
1.8
time
Appear 6-b: Diagram "of modified average acceleration", 40 steps of 0.0419 S

The curve in feature of axis relates to = - 0 1
. One observes a rate of depreciation from approximately 2%
T
whereas the figure [Figure 6-a], for
= 0 025
,
, 0,8% envisage. It is that this curve was established
T
into linear, whereas the movement of our pendulum is non-linear.

The curve in dotted lines relates to = - 0 3
. The rate of depreciation is approximately 5,8%, then
that that envisaged by the figure [Figure 5.3-a] is approximately 2,2%. The variation is still due to non-linearity
problem.

Lastly, the curves in feature of axis and especially in dotted lines reveal a shortening of the period calculated compared to the theoretical period.

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7 Conclusion

The diagram of temporal integration of Newmark and its alternative known as "average acceleration modified", accompanied by the method of Newton, allow to treat many types of problems of non-linear dynamics, for material or geometrical non-linearities.

Treatment of the highly non-linear and not-regular dynamic problems, such as analysis structures in great displacements or of contact-friction, is prone to numerical instability, even with unconditionally stable methods of temporal integration in the field linear. One then manages to integrate the equations of the movement compared to time only in introducing artificial dissipation. All art is to proportion this dissipation so that, in range of the frequencies which are of mechanical interest, it is about equivalent to natural damping, without shifting the vibratory spectrum of the structure too much.

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[10]
[R3.03.01] Dualisation of the boundary conditions.
[11]
[R5.03.08] Integration of the viscoelastic relations of behavior in the operator STAT_NON_LINE.
[12]
[R5.03.50] unilateral Contact by conditions kinematics.
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[R5.03.80] Methods of piloting of the loading.
[14] [R4.02.02]
vibroacoustic elements.
[15]
[R4.02.04] Fluid Coupling - Structure with Free Face.
[16] [R4.05.01]
Answer
seismic by transitory analysis.
[17]
[R5.03.40] static and dynamic Modeling of the beams in great rotations.
[18]
[R5.03.17] Relations of behavior of the discrete elements.
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STAT_NON_LINE.
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Organization (S): EDF R & D /AMA, EDF/CNPE of Tricastin

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R5.06 booklet: Dynamics in Modal base
Document: R5.06.01

Reduction of model in linear dynamics and not linear: Method of RITZ

Summary:

This document presents the principle of reduction of model by projection on reduced basis (method of Ritz).
The base most usually used is the modal base.

The problems of truncation due to the use of a reduced base are mentioned. Corrections of truncation are proposed.

The description and properties of the algorithms of resolution of the system of differential equations of the second
order obtained in transitory analysis are presented in the document [R5.06.04].

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matters

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1 Introduction

Starting from a description of the geometry and materials of the structures, the method of the elements
stop allows to create a precise and reliable but large-sized model. In the case of one
problem of dynamics, one wishes to calculate the response of a system for various moments
(analyzes transitory) where for various frequencies (analyzes harmonic). Size of the model
finite elements obtained is often irreconcilable with the number of calculations necessary to obtain all desired results.
For a restricted whole of dynamic stresses, there is generally a subspace of
low dimension allowing to describe the dynamic behavior of the structure under
specific requests.
The projection of the model on a restricted basis is called method of Ritz or Rayleigh-Ritz.

This document comprises the following points:

.
a presentation of the methods of Ritz, their use into linear,
.
a detail of the possible corrections of truncation,
.
generalization into nonlinear of the methods of Ritz,
.
two simple examples of illustration.

2
Methods of reduction of Ritz into linear

2.1

General description

2.1.1 Formulation
continuous
The method of Ritz consists in projecting checking displacement on a restricted basis of functions the conditions kinematics of the problem:
N
u~ (M, T) = I T () I (M) éq
2.1.1-1
i=1
Displacement is described by a series of independent forms {(M); I
I
= K
1
}
N multiplied by

amplitudes functions of time {T
(); I
I
= K
1
}
N.
The difficulty consists in defining this family of form {(M); I
I
= K
1
}
N which contrary to
functions of form of the finite element method are nonnull on most of structure.
The quality of the approximation is related to the fact that displacements obtained have good approximation in the subspace generated parVect {(M), I
I
= K
1
}
N.

Projection on modal basis
It is known that the clean modes {(M); I =
I

K
1
}
generate the space of the fields
kinematically acceptable. Displacement breaks up according to:

U (M, T) = (T) (M)
I
I

éq
2.1.1-2
i=1
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The option most usually used for the method of Ritz then consists in taking as bases
projection N first modes:
N
u~ (M, T = ~
)
I T () I (M)

éq
2.1.1-3
i=1
Displacement obtained is an approximation of real displacement.
It can be interesting to add to N first modes, other forms (see [§2.6.2]).

2.1.2 Approximation finite elements

In the case of an approximation of displacement by finite elements displacement is already approximate
in the space of the functions of forms:
Nh
H
U (M, T) = IQ T () Nor (M)
éq
2.1.2-1
i=1
One notes U the vector of the degrees of freedom of displacement: U (T) = [q1 (T), q2 (T), Q K Nh (T)] ;

Method of Ritz in finished dimension
If N < Nh, the method of Ritz applied to the field U (M, T) comes then like a second approximation:
N
~
U T
( ) = ( )
I T
I
éq
2.1.2-2
i=1
with {I
, i= N}
K
1
the base of N vectors independent and kinematically acceptable.

One poses = [
,

,

,
1
2
,
3
N]

U =
K
. From where the matric writing:

éq
2.1.2-3

2.2
Projection on reduced basis

Let us consider the following differential connection obtained by a method finite elements:
Nh
MU& + U
C & + KU =
U

F
R éq
2.2-1

The solution sought in the form [éq 2.1.2-3]. By considering the same form for displacement virtual, it comes:
T
T
T
T
N
M&+
C & + K =

F
R
éq
2.2-2

where: is the vector of generalized displacements, K = T K and M = T M are called respectively matrices of generalized stiffness and mass.

The system [éq 2.2-2] is generally a coupled differential connection, the generalized matrices which compose are in the case general full even if at the beginning the matrices M and K were hollow. One thus loses the structure particular to the profit of a size of problem much more reduced n*n.

In the case general, the system [éq 2.2-2] provides only one approximate solution of the system [éq 2.2-1].
The error which one makes is called truncation error.
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One has no information on the value of this error. It can be very large if the subspace of projection is badly selected. It is known only that this error decreases when the size of the base of projection increases.

If one has information a priori on the form of the solution, one can choose in an effective way base projection in order to minimize this error. For example, if it is known that the solution is not constituted that movements of solid body, one can restrict to 6 the dimension of space.

Thereafter, one chooses the base of the clean modes as bases projection.

2.3
Projection on modal basis

Clean modes
The modes are defined like the couples (
{H H
I, I) i= Nh}
K
1
solutions of the equation:
(
2
K - M) = 0 éq

2.3-1

Note:

It is advisable to check that the modes calculated by approximation finite elements are sufficient representative: (H
H
I, I) (I
, I). One can consider that the approximation
finite elements

is correct when the modal deformations present a length D `wave
higher than the size of the meshs of the grid (the concept wavelength is one generalization of the concept definite on the equation of the waves, one can define it as two time the length between two nodes of the modal deformation).
Thereafter one omits, the index H corresponding to the approximation finite elements.

Quotient of Raleigh: energy interpretation
The own pulsations and forms can be defined as the solutions of the problem of minimization according to:
I
[,
1 Nh]:

RNh - Vect {, J,
0 I -1
J
}
I minimizes in under space
[
]
functional calculus:
X T KX
T
K
R (X) =
one poses: 2
I
I
=
= ( )
éq
2.3-2

X tMX
I
R
I
T
I Semi

Method of reduction
A method of reduction very largely employed for the linear problems is the method of modal recombination. It consists in choosing as bases projection N first modes clean of the structure {I, i= N}
K
1
.
N
~
U T
( ) = ( )
I T
I

éq 2.3-3
i=1

Always let us consider the following differential connection:
Nh
MU& + U
C & + KU =
U

F
R éq
2.3-4
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Clean modes {I, i= Nh}
K
1
have the property to be M and K orthogonal, i.e. one has them
following relations:
T
I M J = I
m ij
T
I K J = ki ij

is the symbol of KRONECKER
I
m is called modal mass or generalized mass of mode I
ki is called modal rigidity or generalized rigidity of mode I

The matrices projected of M and K on the basis of clean mode are thus diagonal; it is one advantages which justified the use of the modal base as bases projection. The system [éq 2.3-4] projected on the basis of clean mode first of the system is written:

\ 0 0
\ 0 0

0 m 0
T
T
I
&
+ C & + 0 K 0
I

= ext.
F

éq 2.3-5

0
0
******
***0 0 ***

The projection of the matrix C does not have any reason in any general information to be also diagonal. If it
system is strongly deadened (presence of shock absorbers on the structure), this matrix will not be diagonal.

Note:

As opposed to what do many software, Code_Aster allows in this case
to integrate the system of modal equations coupled without diagonalisation of the matrix
of generalized damping. The method of integration is in this case an implicit method
NEWMARK or clarifies EULER.

On the other hand, if only damping entering concerned is a structural damping (internal dissipation material for a homogeneous structure) it is then licit to make the assumption of a damping proportional, still called assumption of BASILE, in this case C expresses itself like combination linear of M and K (damping of RAYLEIGH), and its projection on the clean modes is diagonal (cf Doc. [R5.05.04] on the modeling of damping).

In this case, the system [éq 2.3-4] is divided into p linear differential equations of the second order uncoupled. The response of the system is then the recombination of the response of p simple oscillators
associated the clean modes, from where the expression of "modal superposition" used usually.

Each differential equation is written I
m:

I
m i& + ici& + kii = fi

éq 2.3-6

or while dividing by the modal mass:

fi

2
I
& +
2 I I
i& + I
I =

éq 2.3-7
I
m
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with:
I
C
I
C

I amortissem

ent
reduced

modal

=

=

C
2
critical
I
m
. I

This equation can be solved very simply in the frequential field:

)
)
I ()
I =
F

éq 2.3-8
I
m (
2
2
. - + .
2 I I
+ I
)

where $ represent the transform of FOURIER and the frequency of excitation.

Particular numerical methods the such integral of DUHAMEL make it possible to pass this

expression of the frequential field to the temporal field. (see for example Doc. [R5.05.01] on one method of temporal integration).

2.4
Modal truncation error

In the case of the modal recombination with damping proportional, one can put in obviousness the truncation error which one makes while projecting on the basis of clean mode first system. Indeed, if one considers the complete base of N clean modes of the discretized problem, there is equivalence between the initial problem and the projected problem. Thus the exact solution of the problem discretized by finite elements is written:

Nh
U =
I
I
I

where the generalized co-ordinates are solution of:

fi

2
I
& +
2 I I
i& + I
I =

I
m

summation extending on all the clean modes from the system (of finished size).

By solving the problem with a reduced number of clean modes, N < Nh. The solution obtained is the following one:

N
~
U =
I
I
i=1

The error made by truncating the base of representation of the solution is thus:

Nh
E =
~
U - U =
I
I

éq
2.4-1
i=n+1

In the frequential field the expression of the error is:

)
Nh
T
) ~)
I (
F)
1
^E () = U - U =
.

. I

éq 2.4-2
m
2
2
I =n+1
I
I - + 2 J I I

the summation is carried out on all the neglected modes of the system.
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Let us study the response relating static of an oscillator to a purely sinusoidal excitation of variable frequency (diagram below), with statics coefficients of the static answer
correspondent with a static force. One can distinguish three intervals in the spectrum where the oscillator
has a different behavior. In low frequency (<< 0), the oscillator has a static answer.
Around 0 the oscillator has a dynamic response (amplification of the mode), and high frequency
1
the oscillator answers in an inertial way (
dominating term).
2

Response of an oscillator
100
10

/
statics
1
Amplitude
Static answer
Dynamic response
Inertial answer
, 1
0
1
2
Reduced frequency /0

Let us suppose that the excitation of the system, defined by the vector F (), is with narrow band, in particular
that it null for frequencies higher than max is given.

In this case, to represent the response of the linear system correctly, it is necessary undoubtedly to take into account all the modes having a pulsation lower than max, because the latter go to answer in a dynamic way the excitation.

On the other hand, modes such as >>
I
max nevertheless have a static contribution to the answer
system. These are often these modes that one does not take into account.

By making a development limited in in the vicinity of 0. One obtains the principal part of the error who is:
)
N
T
) ~)
I. (
F)

^E () = U - U =

. 1 - 2 J
+ 0

I

. I

éq
2.4-3
K

I = p+1
I

I
I
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The error is all the more small as generalized rigidities of the neglected modes are large. In principle thus, it will be necessary to take all the most flexible modes until residual flexibility of a mode suplémentaire is in negligible relative value compared to the sum of the flexibilities already taken into account:
N
1 << 1
kn+
K
1
i=1 I

However, it is observed that by neglecting the modes of high frequency one makes an error systematic on the response of the system (even in low frequency). There are various possibilities that we will detail now to correct the response in the beach [,
0 max] where one chose
modes.

2.5
Corrections of modal truncation

To mitigate the problem of truncation due to the neglected modes, it is necessary to try to estimate their effect in
the field of frequency [,
0 max] which interests us. We saw that the neglected modes
having an own pulsation such as >>
I
max have a contribution known as static to the answer of
system in the field [,

0 max]. The techniques of correction consist in calculating this static contribution.

2.5.1 Static correction a posteriori

The truncation error, by considering only the static response of the neglected modes (transformed opposite of the principal part of the error) is:

Nh
T
~
I. (
F T)
E (T) = U - U

. I

éq
2.5.1-1
K
i=n+1
I

But a priori the neglected modes as their generalized rigidities are unknown. On the other hand, one can determine the complete static response of the system to a loading F, the latter is worth:

Nh
T

-
. F
1
I
(T)
U = K. (
F T)

. I
K
i=1
I
The correction which should be made is thus:

Nh
T

I. (
F T)
N
T

-
. F
1
I
(T)

. I K. (
F T) -

. I
K
K
i=n+1
I
i=1
I

The corrected solution of the response of the system is thus worth:

N
T
~
~

-
. F
1
I
(T)
U = U + E U + K. (
F T) -

. I éq
2.5.1-2
K
i=1
I

This correction is called a posteriori, because it does not intervene in the dynamic resolution of

linear system and can only be calculated well later on. If (
F T) breaks up into K produced
functions of time by functions of the co-ordinates of space, this correction requires one
factorization of K and K resolutions.
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This method has the advantage of not increasing the number of vectors taken into account in the base.
This method is applicable in the case of an excitation to narrow band, or at least having one known cut-off frequency. The correction is exact in the field low frequency but can
to distort the response of the system in high frequency [§Annexe1].

2.5.2 Addition of static modes to the base

Let us suppose that the loading F (T) is written:

F (T) = J (T) F
. J
J

The second way of correcting the truncation error consists in adding to the base of the clean modes initial of the static modes J defined as the deformation in each Fj effort given:

1
-
J = K F
. J

éq

2.5.2-1

The new base of projection to be considered is as follows:

) = [, ...,
, , , ...,
,] = []
1
2
p
1
2
m

éq
2.5.2-2

The components generalized to use are as follows:

) = [, ...,
, μ, μ…,
, μ] = [, μ]
1
2
p
1
2
m

éq
2.5.2-3

The problem projected on the supplemented basis is:

******
0
0

\ 0 0

T

T

0 m
0
I
. M
.
&
T

0 K
0
I
. K
.
. F
.

+

=

éq
2.5.2-4
0
0
****
µ

&
0 0
****

μ T. F
.

T. M
.
T. M
.
T. K
.
T. K
.

It is noted that one lost the diagonal character of the generalized matrices, but the advantage obtained is that the base supplemented with static modes makes it possible to represent it correctly low frequency behavior of the initial system.

For example it is simple to show that at null frequency the solution of this system is:
=
0 and μ
= which is the exact solution of the initial static problem.

One presents in appendix 1, the comparison on a discrete system with 3 degrees of freedom between exact solution, the solution projected on 1 mode, that projected on a mode with a correction statics and the solution consisted 1 clean mode and 1 static mode.

One realizes that the addition of static modes makes it possible to extend beyond the interval [, 0 = max
max
(J)] the good dynamic representation of the system. This technique thus seems very interesting, it has the merit to carry out the correction immediately what will be interesting for the nonlinear methods where one needs the knowledge of physical displacements with each step of time.
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3 Extension of the methods of reduction of Ritz in non-linear

3.1 Problem general

The non-linear problem of dynamics discretized without damping can be generally put in the following form:

.
Mr. X& + G (X) = F (T)
N
X R éq
3.1-1

G (X) is a non-linear function of X which represents the internal forces of the system like all the other forces which are dependent on displacement, F the vector of the external forces and M the matrix of mass of the system.
The matrix of tangent rigidity of the system is by definition:
tg
G

K (=
X)
(X) éq 3.1-2
X

It makes it possible to define a modal base at every moment by:

2
- tg

M + tg

K
.

tg
= 0
I
I (X)

éq 3.1-3
(X)

The modes thus defined depend on X, therefore moment T.
Knowing that the calculation of the modal base is very expensive in computing times, the idea to want to project
with each step of time the model on a modal basis, then to solve, is without interest by report/ratio with a direct resolution.

The method most usually used consists in defining a base of projection while adding to
modes calculated on an initial configuration of the forms allowing to project nonthe linearity.
Example: if nonthe linearity comes from a specific shock, one proposes to enrich the base
modal with static modes allowing to project the effort undergone by the structure lasting the shock [R5.06.04].

The method of Ritz remains always relevant in nonlinear calculations, if the selected base allows to correctly project displacements and the efforts.

The nonlinear problem projected on an unspecified basis is written:

T
T
T
N

M.&+ G (X)
=
F R

éq 3.1-4

Two possibilities are then possible:

.
nonthe linearities are located and one can evaluate nonthe linearity on the basis of projection : G (X) = G ().
The problem to be solved is a nonlinear differential connection in smaller size.
Various strategies are possible to solve this problem, depending primarily
technique of integration which one wishes to use.
.
nonthe linearities are total, and it is necessary to pass by again in the space of the physical ddls for to calculate the internal forces: G (X).
This second method is more expensive it is much less current.
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One presents in appendix 2, the comparison on a system with 3 degrees of freedom with nona linearity
in 3
X enters the exact solution and the solution obtained by the method above with 1 then 2 modes.
One realizes that it is necessary to take more modes counts some than for the linear problem.
On the other hand, on this example 2 modes are enough very well to describe the system.

3.2
Indication of the error of projection

For the nonlinear problems the physical direction of the number of modes to be taken into account is completely lost, and if the methods of reduction always give a solution it is necessary to know it degree of confidence which one can grant to them. A way of proceeding, which is a little expensive but
essential is to calculate the residue of the initial system to each step of time. It is defined by:

R = MR. X
. & + G (X) - F (T)

This vector residue is unfortunately not null, it is only its projection on the basis used
who is.

A standard can then be calculated for this residue; more the standard of the residue will be small more one will be able
to grant confidence to the solution.

To use a relative value, one may find it beneficial to calculate the following fraction:

R
R =
(

éq
3.2-1
max F (T), G (X), M.X&)

Note:

This indicator is not currently established in Code_Aster.
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4
Use in Code_Aster

In Code_Aster, the methods of Ritz are usable in transient primarily by operator DYNA_TRAN_MODAL [U4.53.21].

A phase of projection of the matrices of rigidity and mass on a basis of vectors is carried out by operators PROJ_MATR_BASE [U4.63.12] and PROJ_VECT_BASE [U4.63.13].

The generalized dynamic problem is then solved in operator DYNA_TRAN_MODAL by one diagram of explicit integration (EULER or DEVOGELEARE) or implicit (NEWMARK). characteristics and properties of the diagrams of integration are presented in the note [R5.06.04]. For the structures for which the assumption of BASILE does not apply (damping not proportional) one will project also the matrix of damping which does not become diagonal. The integration of the coupled system is done then obligatorily with the implicit scheme (NEWMARK) or clarify (EULER).

Localised non-linearities are specified directly in operator DYNA_TRAN_MODAL. One can introduce localised non-linearities of the shock type and friction (see [R5.06.03] Modeling shocks and frictions), modal forces function of displacement or speed (see [R5.06.05] on the modeling of a fluid force of blade). The static corrections of truncation a posteriori are available in the case of an excitation single (see [R4.05.01] seismic Answer).

The addition of static modes can be done by using the operators as a preliminary MODE_STATIQUE [U4.52.14] and DEFI_BASE_MODALE [U4.64.02]. When the problem comprises non-linearities only the explicit diagrams can be used.

For total non-linearities [éq 3.1-4], it is possible to use order DYNA_TRAN_EXPLI [U4.53.03] which calculates with each step of time the internal forces according to the physical ddls, then

projète the problem on a modal basis.

An operation of return to the physical base is then necessary to obtain the sizes physics such as displacement, speed or acceleration on the structure. This operation is carried out by operator REST_BASE_PHYS [U4.64.01].

More generally, the approach of Ritz can be used in harmonic calculation by the order DYNA_LINE_HARM [U4.53.22] and of spectral concentration of power by the order DYNA_ALEA_MODAL [U4.53.23].

Finally the dynamic under-structuring can be regarded as a method of Ritz specific [R4.06.02].

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Appendix 1

Let us consider the discrete system with three masses according to:

m
m
m
K
K
K

The matrices of rigidity and mass are:

m 0 0
K
- K
0

$\boldsymbol{M} = 0\ m\ 0\ \boldsymbol{K} = -\ K\ 2k\ -\ K$

$0\ 0$
m

$0\ -\ K\ 2k$

K
That is to say: 2
$0 =$

m

The clean modes and their pulsation are worth:

1

$2 = 0\ 198$
,
2
1
0
,
1

$m = 1\ 8$
$, 41$
,
m
1

$= 0\ 802$
,

$0\ 4$
$, 45$

1

2 = 1555
,
2
2
0 ,
2
m = 2 8
, 63 ,
m2
= - 0 555
,

-

1 2
, 47

1

2 = 3 247
,
2
3
0
,
3

m = 9 2
, 96
,
m
3

= - 2 247
,

1 802

,

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Let us compare the answers of the system modelled by only one clean mode with or without static correction:

Response of the system
1 mode
Exact solution
1 mode + correc
100
10
1
Amplitude
, 1
, 01
0
1
2
Frequency

It is noted that the static correction makes it possible to correct the low frequency answer, the model with 1
mode plus correction sticks perfectly to the exact solution in low frequency. On the other hand, into high frequency (beyond the first mode), this correction results in over-estimating the answer enormously.
The use of the static correction will have thus to be used with prudence and within the framework of an

excitation with
narrow band.

Let us look at what the method of addition of a static mode gives.

If one applies a unit force to item 1, the static deformation is worth:

3
1
=
S
2
K
1

The projected matrices of mass and rigidity which one obtains are as follows:

049
,
5

841
,
1

m

2

365
,
0
K 1

)
0

)

M *=*

and ***K*** *=*
049

,
5
m

3

1

14

2
2

K
0
K

this system has as Eigen frequencies:

) 2
2
) 2
2
1
=

198
,
0

0

and
1
=

667
,
1

0

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The response of the system modelled with a clean mode and a static mode is as follows:

Response of the system
Exact solution
1 mode + 1 static mode
100
10
1
Amplitude
, 1
, 01
0
1
2
Frequency

One realizes that one corrects very well low frequency, (effect of correction static), one models dynamics of the system well beyond the first mode taken into account. On the other hand the effect of the second mode is
badly represented (shift on the frequency).
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Appendix 2

Let us consider the discrete system with three masses according to:

m
F (X)
m
m
K
K
K

The matrices of stiffness and mass are:

m 0 0
K
- K
0

M = 0 m 0 K = - K 2k - K

0 0
m

0 - K 2k

Let us make this system non-linear by adding a term of force interns between 1
X and x2 cubic:

F = K (. 1
X - X) 3
2

Let us seek to evaluate the response of this system to a forced excitation of frequency close to the first Eigen frequency of the linear system (one chose =

18
,
0

O), with an important amplitude F = K
m
.
3 .

In this configuration, the response of the system cannot be evaluated by the response of the linear system (it
cubic term is well too important), it is necessary to implement a non-linear calculation with pseudo-forces
as one showed in [§3.2].

One can see in the report/ratio [bib8] the curves of the transitory results of this method, while taking in
count one or 2 clean modes of the initial linear system.

With only one clean mode, one realizes that one makes a relatively important error (reaching sometimes
50%), on the other hand it are satisfactory to note that the extrema vibrations are rather well envisaged. One
could have hoped that while exciting in on this side first Eigen frequency it would have is enough to only one clean mode
to model the response of the system, it is seen here that it is not the case. As one often notes it for non-linear systems, the system also answers with the surharmoniques ones of the frequency
of excitation.

On the other hand, by taking 2 clean modes to model the response of this structure to 3 ddl, one obtains one
very satisfactory result (a few % of error on the amplitude), with the eye one has evil to distinguish the difference. This

show that by choosing a sufficiently rich base of projection one can thanks to a method of pseudo-forces to model a dynamic system very well complexes with non-linearities.

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Organization (S): EDF/EP/AMV
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Document: R5.06.03
Modeling of the shocks and friction in
analyze transitory by modal recombination
Summary:
This document describes the physical laws of contact with friction between structures and the modeling which in is
made in the transitory algorithm of analysis by modal recombination of Code_Aster DYNA_TRAN_MODAL
[U4.54.03]. For the various non-linear connections of contact usable, one details the calculation of the sizes
defining the conditions of contact.
The diagrams of use used are described in [R5.06.04].
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1 Introduction

The problems of shock with friction which interest EDF relate to for example modeling tubular vibrations of structures maintained by supports with plays, or separated by plays weak and thus being able to come into contact. Tubes of the steam generators, pencils of control rods, the assemblies of fuel are examples of structures of which one wish to model the vibrations.
The major consequence of the vibrations in the presence of play is to cause shocks as well as friction enters the structure and its supports or between the structures from where risks of wear. It document describes the type of non-linearities introduced by the presence of these plays, as well as modeling used to take them into account in the algorithm of modal recombination.

2 Relations of contact between two structures

Two relations govern the contact between two structures:
· the relation of unilateral contact which expresses the non-interpenetrability between the solid bodies,
· the relation of friction which governs the variation of the tangential stresses in the contact. One will retain for these developments a simple relation: the law of friction of Coulomb.

2.1 Relation of unilateral contact

Are two structures
1/2
1/2
1 and 2. D NR is noted
the normal distance enters the structures, FN

force normal reaction of 1 out of 2.
The law of the action and the reaction imposes:
F 2 1
/= - F 1/2
NR

NR
éq 2.1-1
F 2/1
NR
1
D 1/2
NR
2
F 1/2
NR
Appear 2.1-a: Outdistance normal and normal reaction
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The conditions of unilateral contact, still called conditions of Signorini [bib5], are expressed following way:
D 1/2 0, F 1/2 0, D 1/2 F 1/2 = 0 and F 2 1 = - F 1/2
NR
NR
NR
NR
NR
NR
éq 2.1-2
F 1/2
NR
D 1/2
NR
Appear 2.1-b: Graph of the relation of unilateral contact

This graph translates a relation force-displacement which is not differentiable. It is thus not usable in a simple way in a dynamic calculation algorithm.
If one restricts the study with the case of a tubular structure in the presence of an indeformable support, one notes
D (D = D 1/2
N
N
NR
) the normal distance to the support, and Fn reaction of this last (attention!
F = F 2 1 = - F 1/2
N
NR
NR
to see diagram below).
The expression of the conditions of normal contact, expressing the limitation of displacements due to support is worth:
D 0, F 0, D F
N
N
N
N = 0
Fn
dn > 0
dn = 0
(cf feel N
and of Fn)
N
N
Appear 2.1-c: Outdistance normal and normal reaction between a structure and a support
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2.2

Law of friction of Coulomb

The law of Coulomb expresses a tangential limitation of effort **F** 1/2
T
of tangential reaction of 1 on

1 2
2. That is to say!
/
C the relative speed of 1 compared to 2 in a point of contact and is µ it coefficient of friction of Coulomb, one has [bib5]:

S = F 1/2 - µ *F* 1/2, **U** 1/2
!
= **F** 1/2
0

, 0, .s
T
NR
T
T
= 0

éq 2.2-1

and the law of the action and the reaction:

F 2 1
/= - **F** 1/2
T
T

éq 2.2-2

F1/2

T
Ý
U 1 2
T
Appear 2.2-a: Graph of the law of friction of Coulomb
The graph of the law of Coulomb is him also nondifferentiable and is thus not simple to use in
a dynamic algorithm.
If one restricts the study with the case of a tubular structure in the presence of an indeformable support, only
tangential stress **F** 2 1
/= **F**
T
T is used, the law of friction is expressed in the following way:
S = F - µ *F* 0, **U**! = **F**
, 0, .s
T
N
T
T
= 0
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3
Approximate modeling of the relations of contact between 2 structures by penalization

3.1
Model of normal force of contact
The principle of the penalization applied to the graph of the figure [Figure 2.1-b] consists in introducing

one
univocal relation *F* 1/2 = F (*D* 1/2
NR
NR
) by means of a parameter. The graph of F must tend towards
the graph of Signorini when tends towards zero [bib6].
One of the possibilities consists in proposing a linear relation between *D* 1/2
1/2
NR
and *FN*
:
1
F 1/2 = - *D* 1/2
if D 1/2 0; *F* 1/2
NR
NR
NR
NR
= 0

if not
éq 3.1-1
1
If one notes *kN* = called commonly "**stiffness of shock**", one finds the traditional relation, modelling an elastic shock:
F 1/2 = - *K D* 1/2
NR
NR
NR
éq 3.1-2
The approximate graph of the law of contact with penalization is as follows:
F 1/2
NR
D 1/2
NR
Appear 3.1-a: Graph of the relation of unilateral contact approached by penalization
To take account of a possible loss of energy in the shock, one introduces a "damping of shock " *CN the* expression of the normal force of contact is expressed then by:
F 1/2 = - *K D* 1/2 - *C U* 1/2
NR
NR
NR
NR

! *NR*
éq 3.1-3
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where! /
U 1 2
NR
is the relative normal speed of 1 compared to 2. To respect the relation of
Signorini (not of blocking), one must on the other hand check a posteriori that *F* 1/2
NR
is positive or
+
null. Only the positive part will thus be taken.
expression [éq 3.1-3]:
+
X
= *X* if *X* 0
+
X
= 0 if *X* < 0
The complete relation giving the normal force of contact which is retained for the algorithm is
following:
+
if *D* 1/2 0 *F* 1/2 = - *K D* 1/2 - *C U* 1/2
!
, *F* 2 1
/= - *F* 1/2
NR

NR
NR
NR
NR
NR
NR
NR
if not
F 2 1
/= *F* 1/2
NR
NR
= 0.
éq 3.1-4
3.2
Model of tangential force of contact
The graph describing the tangential force with law of Coulomb is not-differentiable for the phase of adherence (! /
U 1 2
T
=)
0. One thus introduces a univocal relation binding relative tangential displacement
D 1/2
1/2
1/2
T
and the tangential force **F**
= F (**D**
T
T
) by means of a parameter. The graph of F must
to tend towards the graph of Coulomb when tends towards zero [bib6].
One of the possibilities consists in writing a linear relation between **D** 1/2
1/2
T
and **FT**:
1
F 1/2 - **F** 1/2 0 = -
(**D** 1/2 - **D** 1/2 0
T
T
T
T

)
éq 3.2-1
1
If one introduces a “tangential stiffness” *KT* =, one obtains the relation:
F 1 2 = **F** 1 2 0 - *KT* (**D** 1 2 - **D** 1 2 0
/
/
/
/
T
T
T
T
)
éq 3.2-2
For numerical reasons, related to the dissipation of parasitic vibrations [bib7] in phase of adherence, one is brought to add a “tangential damping” *CT* in the expression of the force tangential. Its final expression is:
F 1/2 = **F** 1/2 0 - *K* (**D** 1/2 - **D** 1/2 0) - *C*
T
T **U** 1/2
!
, **F** 2 1
/= - **F** 1/2
T
T
T
T
T
T
T
éq 3.2-3
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It is necessary moreover than this force checks the criterion of Coulomb, that is to say:
U 1/2
!
F 1/2
1/2
µ *F*
if not one appli

that **F** 1/2
1/2
T
= - µ *F*
NR
NR

, **F** 2 1
/= - **F** 1/2
T
T
éq 3.2-4
U
T
T
1/2
! **T**
The approximate graph of the law of friction of Coulomb modelled by penalization is as follows:
F 1/2
T
KT
Ý
U 1 2
T
Appear 3.2-a: Graph of the law of friction approached by penalization

4

Types of modelled connections of contact

Like it was specified in the paragraph [§2.2], the developments presented here relate to the setting in work of non-linear connections with unilateral contact and friction between 1 node and an obstacle or

between 2 nodes given.
The nodes in contact are supposed to belong to two slim structures of beam type or to one beam and an indeformable obstacle. The nodes on which will carry the condition of contact are presumedly carried by the average line of the beams.

4.1
Connections between a node and an indeformable obstacle
4.1.1 Connections of contact node on plane obstacle

One considers a slim structure represented by elements of the beam type. Its displacement is limited in a point by the presence of an obstacle made up of two infinite half-planes in direction Y (see [Figure 4.1.1-a]).

Z
Y
Xloc
X

Appear 4.1.1-a: Slim structures with contact node on plan

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis *Xloc*,
direction of neutral fibre or a generator of the beam. That is to say NO1, the node of the connection considered on the beam, the geometry of the connection contact node on plan (called PLAN_Y in *Code_Aster* [bib3]) is described on the figure below.

Zloc
NO1
Y
Play
loc
ORIG_OBST

1
2
Appear 4.1.1-b: Geometry of the connection node on obstacle plan
Yloc
Are
co-ordinates of
Y, Z
, the origin of this reference mark is it
Z
NODE

NO1 in the reference mark (*loc*
loc)
loc
not ORIG_OBST.
The normal distance *D NR* in this case, by neglecting rotations of the sections is expressed then by:
D
= - Y
+ play
NR
loc
éq 4.1.1-1
The contact in this connection is judicious to take place whatever the shift in *Zloc* between the two structures.
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Normal vector ***N*** in the reference mark (*Y, Z*
loc

loc) has as components:
sign Y
(*loc*)
N =

éq 4.1.1-2

0

Other quantities! *one*, *FN*! *C*, **FT** are calculated in a general way as specified with [§3].

4.1.2 Connections of contact node on concave circular obstacle

One considers a hurled structure, represented by elements of the beam type. Its displacement is limited in a point by the presence of an obstacle made up of a bored infinite plan of a circular hole (see figure below).

Z
X
Y
loc
X

Appear 4.1.2-a: Slim structures with contact node on circular obstacle

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis *Xloc*, direction of neutral fibre or a generator of the beam. Are NO1, the node of the connection considered, geometry of the connection of contact node on circle (called CIRCLE in *Code_Aster* [bib3]) is described on the figure below.

Zloc
NO1
Yloc
Play
ORIG_OBST

Appear 4.1.2-b: Geometry of the connection circular node obstacle

---

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Yloc
Are
co-ordinates of the NODE
Y, Z
, of origin
Z
NO1 in the reference mark (*loc loc*)
ORIG_OBST.
loc
The normal distance *D NR*, by neglecting rotations of the sections is expressed then by:
2
2
D = - (*Y* - *Y*
) + (*Z* - *Z*
) + *play*
NR
loc
ORIGobst
loc
ORIGobst
One poses like normal vector **N the** vector:
ORIG
- *NOEUD1*
N
obst
= *ORIG* - *NODE*
obst
1
play is a strictly positive distance.
Other quantities! *one*, *FN*! *C*, **FT** are calculated in a general way as specified with [§3].

4.1.3 Connections of contact node on concave obstacle discretized by segments

One considers a hurled structure, represented by elements of the beam type. Its displacement is limited in a point by the presence of an obstacle made up of a bored infinite plan of a hole of form concave unspecified being able to be discretized in polar co-ordinates by segments (see figure below).
Zloc
NO1
Yloc
ORIG_OBST

Appear 4.1.3-a: Geometry of the connection node on discretized concave obstacle
Yloc
Are
co-ordinates of the node
Y, Z
, of origin
Z
NO1 in the reference mark (*loc loc*)
ORIG_OBST.
loc
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One seeks the facet of contact nearest to node NO1, normal vector **N** is defined
like the direct orthogonal vector with the facet:
Zloc
N
dn PNO1
NO1
Yloc
ORIG_OBST
Either PNO1 the projection of node NO1 on the facet, the normal distance *D NR* in this case is worth:
D
(*NO1 PNO*
NR =
-
)
1 .n
Other quantities! *one, FN*! *C,* **FT** are calculated in a general way as specified with [§3].

4.2
Connections between two nodes of two deformable structures
4.2.1 Connections of plane contact on plan
The contacts between assemblies fuel, on the level of the grids of mixture, constitute one example of plane contact on plan (see [Figure 4.2.1-a]).
One thus considers two hurled structures, being able to be modelled by beams of section rectangular on the level of the zones of contact.
Z
Y
Xloc
NODE 1
NOEUD2
X
Appear 4.2.1-a: Slim structures with plane contact on plan
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To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis *Xloc*,
direction of neutral fibre of the beams. Are NO1 and NO2, the two nodes of the connection considered, the geometry of the connection plane contact on plan (called BI_PLAN_Y in *Code_Aster* [bib3]) is described on the figure below.
Zloc
NO1
NO2
Y
D1
loc
D2

Appear 4.2.1-b: Geometry of the connection plan on plan
Yi

loc
Are
co-ordinates of NOEUDi in the reference mark (*Y, Z*
loc
loc), of origin ORIG_OBST
Zi

loc
(ORIG_OBST can be provided by the user, by defect ORIG_OBST is selected like the medium of nodes NO1, NO2.
The normal distance *D* 1/2
NR
in this case, by neglecting rotations of the sections expresses itself then by:
D 1/2 = *Y1* - *Y2*
- *D* - *D*
NR
loc
loc
1
2
éq 4.2.1-1
D1 and *D2* are strictly positive distances.
The contact in this connection is judicious to take place whatever the shift in *Zloc* between the two structures.
The normal vector ***n1/2*** in the reference mark (*Y, Z*
loc
loc) has as components:
2
1

sign (*Y loc* - *Y loc*)
n1/2 =

éq 4.2.1-2
0

Other quantities! /

U 1 2
1/2
1 2
1/2
NR
, FN
, ! /
C, **FT** are calculated in a general way [§ 2.4].
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4.2.2 Connections of contact rings on circle

If one considers now two cylinders of circular section, modelled by elements of
beam. The connection of contact between two nodes of the average lines is supposed to take place enters two circles as shown in the figure following:
Xloc

Appear 4.2.2-a: Slim structures with contact rings on circle

One places oneself in the reference mark perpendicular to the axis *Xloc* parallel with a generator of the cylinders.
Are NOEUD1 and NOEUD2, the two nodes of the connection considered, the geometry of the connection contact
ring on circle (called BI_CERCLE in *Code_Aster* [bib3]) is described on the geometry
below:
Zloc
R2
NO2
Yloc
NO1
ORIG_OBST

2
R1
1
Appear 4.2.2-b: Geometry of the connection rings on circle
The normal distance *D* 1/2
NR
has as an expression:
2
2
D 1/2 = (*Y1* - *Y2*) + (*Z1* - *Z2*) - *R* - *R*
NR
loc
loc
loc
loc
1
2
One poses like normal vector of 1 towards 2 the vector:
-
NOEUD2 NOEUD1
n1/2 =
-
NOEUD2 NOEUD1
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5

Use of the localised non-linear forces of shock and friction in modal recombination

The non-linear forces expressed above are explicit functions of the position and speed of the nodes to which the conditions of contact relate.

One chooses to use the technique of pseudo-forces to solve the projected dynamic problem. If it direct dynamic system is written:

MX
! + **CX**! + **KX** = **F** (*T*) + **F**
(**X**, **X**!)
T
T
T
ext.
shock
T
T

The technique of pseudo-forces consists in projecting on the basis of linear system and maintaining them non-linear forces with the second member.

The projected dynamic system takes the form:

MT

T
T
T
T
! *T* + **C**
! *T* + **K**
T = ext.
F (*T*) + *CH*

F *oc* (
T,
! *T*)
The projected problem is integrated numerically by an explicit diagram.

6
Precision on the use of non-linearities of shock with friction

Non-linearities of shock between a structure and an obstacle or two structures were introduced into the algorithms of modal recombination of *Code_Aster*: an algorithm of Euler of order 1 and Devogelaere of order 4 [bib4] [R5.06.04].
These algorithms are used by operator DYNA_TRAN_MODAL [bib1], [U4.54.03]. The type of connection
of shock between the two nodes is specified by a specific order: DEFI_OBSTACLE [U4.21.07].

6.1
Definition of the type of connection of shock

The type of connection of shock is a generic concept, which does not comprise any physical information like a distance or unspecified dimension. The type of connection specifies simply the form geometrical of the connection considered.

The types of connection with shock with two nodes accepted by order DEFI_OBSTACLE are described by the key words following:
PLAN_Y, PLAN_Z or CIRCLE
BI_PLAN_Y, BI_PLAN_Z or BI_CERCLE (see figure below).
Zloc
Z
Z

loc
loc
Yloc
Yloc
Yloc
PLAN_Y
PLAN_Z
RING
Z
Z
loc
loc
Zloc
Yloc
Yloc
Yloc
BI_PLAN_Y
BI_PLAN_Z
BI_CERCLE

Appear 6.1-a: Geometries of the connections of shock

Prefix BI_ specifies that it is about a connection with two nodes.

6.2

Definition of the local reference mark for the conditions of contact

Treated structures, being regarded as cylindrical slim (circular section or rectangular), are modelled by elements of beam. The contact is treated, as one saw with [§3.1] and [§3.2] in a plan perpendicular to the *Xloc* direction of the generator of the cylinders.

To define this change of reference mark completely, one introduces a reference mark local (*X*, *Y*, *Z*
loc
loc
loc).

The Xloc vector is the vector with 3 components provided behind key word NORM_OBST.

Using the first two nautical angles, one passes in a single way of the total reference mark (*X*, *Y*, *Z*) to a reference mark having *Xloc* like first basic vector (see [Figure 6.2-a] hereafter). A third rotation whose angle is provided behind key word ANGL_VRIL gives a single correspondence enters the principal reference mark and the local reference mark.

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Foot-note:
the orientation of this local reference mark is important because it is in this reference mark that they are analyzed
conditions of contact, and are provided the local positions of the nodes of shock.
Z=Z1
Z=Z1
X2=
X2=
X
X
loc
loc
Y1=Y2
Y
Y1=Y2
Y
Y1=Y2
Yloc
X1
X1
Z2
Z2
X
Z
Rotation 1 around Z
Rotation 2 around Y1
loc
Rotation 3d' an angle
ANGL_VRIL around Xloc
Appear 6.2-a: Rotations defining the local reference mark
Operand ORIG_OBST makes it possible to define the origin of the reference mark local (*Orig*, *X*, *Y*, *Z*
loc
loc
loc). This

operand is optional and in theory will not be used in the case of the shocks between two nodes.
code considers whereas the origin is located in the middle of the segment connecting the two nodes.

6.3

Definition of the nodes of the connections

One specifies, behind key words NOEU_1 and NOEU_2, the names of the two nodes of the structures on which will carry the conditions of shock. If it is about a connection between a node and an obstacle, only NOEU_1 is indicated.

6.4

Definition of dimensions characteristic of the sections

The operand PLAY is used for the conditions of contact between a node and an obstacle.
Operands DIST_1 and DIST_2 make it possible to specify dimensions characteristic of
sections of the structures surrounding the nodes of shock. In the case of the connections plan on plan, it are
thicknesses of matter surrounding the node of shock in the direction considered.
In the case of connections rings on circle, it acts of the rays of the sections surrounding the nodes of shock.

6.5

Definition of the parameters of contact

The parameters stiffnesses and damping of shock were introduced with the §3.1 and §3.2, one specifies them here
key words allowing to define them for a given connection.
Operand RIGI_NOR is obligatory, it makes it possible to give the value of normal stiffness of shock *kN*.
The other operands are optional.
Operand AMOR_NOR makes it possible to give the value of normal damping of shock *CN*.
Operand RIGI_TAN makes it possible to give the value of tangential stiffness *KT*.
Operand AMOR_TAN makes it possible to give the tangential value of damping of shock *CT*.
The COULOMB operand makes it possible to give the value of the coefficient of Coulomb.

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Foot-note:

If a stiffness KT is defined and that key word AMOR_TAN misses, the code calculates one damping optimized in order to minimize the residual oscillations in adherence [bib7]:

$C = 2 (K + K) .m - 2 K .m$

T

I

T

I

I

I

I,

where I is the index of the dominating mode in the response of the structure (modal mass more important).

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Document: R5.06.04

Algorithms of temporal integration of the operator DYNA_TRAN_MODAL

Summary

This document describes the diagrams of temporal integration which are used to solve in the space of modes of the problems of transitory dynamics in linear mechanics, with, for certain diagrams, the catch

in possible account of nonlocalised linearities of shocks type, frictions or fluid blade, and the use possible of the under-structuring. Diagrams of NEWMARK, EULER, DEVOGELAERE, and a diagram with step of
adaptive time, ADAPT, are presented.
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1 Introduction

The goal of the transitory dynamic analysis is to determine according to time the response of one structure, being given a loading external or boundary conditions functions of time, in cases where the effects of inertia cannot be neglected.

In a certain number of physical configurations, one cannot be satisfied with a modal analysis or harmonic and one must carry out a transitory analysis. It is in particular the case if:

· the history of the phenomenon has an importance in the study,
· if the external loading is complex (seism, excitations multi-components, etc…),
· if the system is nonlinear (plasticity, shocks, frictions, etc…).

The methods of transitory analysis which can then be used divide into two large categories:

· methods known as of direct integration,
· the methods of Ritz, which include/understand inter alia the recombination of projections modal.

The methods of direct integration are thus called because no transformation is carried out on the dynamic system after the discretization by finite elements. They are presented in document [R5.05.02], algorithms of direct integration of operator DYNA_LINE_TRAN.
The methods of Ritz, on the other hand, proceed to a transformation of the initial dynamic system, by a projection on a subspace of the space of discretization departure. The resolution is done then on a modified system, which, if it is reduced, gives access only one approximation of the answer of real system.

Algorithms of temporal integration on a frame of reference generalized are used to solve the dynamic problems in mechanics for linear structures, with catch in account possible of nonthe localised linearities such fluid shocks, frictions or blades.
Certain algorithms allow moreover the under-structuring.
These algorithms are programmed in operator DYNA_TRAN_MODAL of Code_Aster [U4.53.21].

2
Method of temporal integration of a dynamic problem

2.1 Introduction

It is supposed that the equations governing the dynamic balance of the solids were discretized by finite elements. One obtains a discrete system of equations which it is a question of integrating in time. For
that one chooses a discretization {T, I NR of the interval of time of the study [0, T] and one writes
I
}
the balance of the structure at the moment T.
I

In a general way these equations take the following form:

MR. X
& + CX +
=
T +

T
&
K X
R
T
T
ext. ()
R nl (X, X
T
&, X
T & T)
where
.
MR. X
& + CX +
=
T +
T
&
K X
R
T
T
ext. ()
R nl (X, X
T
&, X
T & T) are the matrix of mass of the system,
.
MR. X
& + CX +
=
T +
T
&
K X
R
T

T
ext. ()
R nl (X, X
T
&, X
T & T) are the matrix of rigidity of the system,
.
MR. X
& + CX +
=
T +
T
&
K X
R
T
T
ext. ()
R nl (X, X
T
&, X
T & T) are the matrix of damping of
system,
.
MR. X
& + CX +
=
T +
is the vector of the external forces,
T
&
K X
R
T
T
ext. ()
R nl (X, X
T
&, X
T & T)
.
MR. X
& + CX +

=
T +
is the vector of the nonlinear forces.
T
&
K X
R
T
T
ext. ()
R nl (X, X
T
&, X
T & T)
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The matrix of damping C is in general difficult to evaluate because damping is often function of the frequency. It is however frequent to simplify the catch in depreciation account in employing the model of damping proportional, or model of Rayleigh: .

The methods of reduction of Rayleigh-Ritz are presented in the document [R5.06.01], Méthodes of Ritz in linear and nonlinear dynamics.
If term R (X, X&, X
& is not null, the technique of the pseudo-forces consists with
nl
T
T

T)
to project on the basis of linear system and to maintain the forces nonlinear with the second member. technique of the pseudo-forces is always associated a diagram of explicit integration. This fact taken into account of nonthe linearities is available only for explicit diagrams. The addition of not linearities does not modify the form of the equations.
In the method of Ritz, the field of displacement X is replaced by its projection on the basis
T
modal such as X = where is the vector of the generalized co-ordinates and is the base
T
T
T
modal, generally reduced.
The projected dynamic system takes the following form:

MT
T
T
T
T
& + C & + K
= R
T + R
,
with
p

T
T T ext. () nl (T &, t&t&)
T

When the assumption of Basile does not apply (damping nonproportional), the matrix of damping projected is not diagonal. The integration of the coupled system is done then obligatorily with one of the three following diagrams: the implicit scheme NEWMARK, the diagram clarify EULER or explicit diagram ADAPT.

The equation obtained is same form as the equation in X. So in the continuation of
T
T
document, one will use as well notation X for displacement in generalized co-ordinates
T
that for displacement in physical space. In the case of operator DYNA_TRAN_MODAL, it acts of generalized co-ordinates.

Two classes of method can be distinguished in integration step by step from the equations

of balance, methods of explicit integration and methods of implicit integration.

That is to say the linear dynamic system according to integrating in time:

MR. X
& + CX +
=
T
T
&
K X
R
T
T
ext. ()

This differential connection of the second order can be brought back to a first order system:

NR U& = H U + F
T
T
T
X
I 0
0
I
0
T
where U =

=
, H =
, F =

T
X&, NR 0 M
- K - C
T
R
T
T

To integrate this differential equation, one uses a discretization {T, I NR, like one
I

}
formulate differences finished to express the derivative U
&.
T
One will call methods of integration clarifies the methods where only the derivative U
& utilizes
T
unknown factors at time T
. In this way determination of the sizes sought at the moment T

I 1
+
I 1
+
do not result from an inversion of system utilizing the operator H. If moreover, one carries out one "mass-lumping" in order to return the matrix M diagonal, the determination of is particularly simple. They are there the principal characteristics of the methods of integration explicit.
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The implicit or semi-implicit methods utilize the discretization of U at one moment
T
posterior with T, generally T
. The determination of the variables thus passes by the resolution of one
I
I 1
+

system utilizing the operator H.

Two concepts are important: consistency, or the order of the diagram of integration, and stability.

The approximations used to obtain the differential operators define consistency, or the order of the diagram of integration. One can indeed consider that the approximation with which one displacement with each step of time obtains is related to the order of approximation of the derivative first and seconds compared to time.

The study of stability of a diagram consists in analyzing the propagation of the numerical disturbances with run from time. A stable diagram preserves a finished solution, in spite of the disturbances, whereas one unstable diagram led to a numerical explosion or divergence of the solution.

To make a study of stability of a diagram of integration, one puts this last in the form of one linear recursive system and one determine the particular characteristics of this system. If all them eigenvalues of the operator of recursivity are smaller than 1 modulates some, the diagram is stable. If not it is unstable.
The diagrams of integration clarifies are generally conditionally stable, which means that the step of time must be sufficiently small to ensure the stability of temporal integration. Certain implicit algorithms have the property to be unconditionally stable, which makes them interest and makes it possible to use a step of arbitrarily large time.

The diagrams retained for operator DYNA_TRAN_MODAL are an implicit scheme, NEWMARK, and three explicit diagrams, EULER, DEVOGELAERE and ADAPT (with step of adaptive time). The choice is done by the key word METHOD: “EULER”, “DEVOGE”, “NEWMARK”, or “ADAPT”.

2.2 Methods of implicit integration

2.2.1 Introduction

The implicit methods utilize the resolution of a matric system with the operator previously definite. If the solids are supposed to be elastic linear, that results in the resolution of a linear system to each step of time.
The advantage of these methods is their unconditional stability, which enables them to integrate the equations dynamics with a step of relatively important time while representing it correctly behaviour of the modes low in frequency of the structure.
An implicit version of the method of NEWMARK, which was programmed in DYNA_TRAN_MODAL

for
linear problems.

2.2.2 Method of NEWMARK [bib1]

2.2.2.1 Presentation of the diagram

NEWMARK introduced two parameters and for the calculation of the positions and speeds to the step
T + T
:

&X

= X +
&
T
.
+
[(1). &X +. &X
T
T
T
T
t+ T
]
2

1

X = X + T
. X& + T

-
.

X

. &. X
t+ T
T
T
T

& t+ T

2

+

Let us consider the equilibrium equations at time T + T
:

M.X
&

+ C.X
t+ T
& +

K.X =

R
T + T
t+ T
T +t
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Let us defer the preceding relations while eliminating &
Xt T
+ and &Xt
T
+, it comes:

~
~
~
K. X

R where: K
K
. M
. C
T
T
has
has
+
=
=
+ 0
+ 1
~
R = R

t+ T
+ C. {has. X
1
T + A.
4 &
X
T + A.
5 &Xt} + Mr. (A.X
0
T + A.
2 &
X
T + A.
3 &Xt)

1

1
1
has =
=
=
=
- 1
0
(
has
has
has
. t2
)
1
(. T
)
2
(. T
)
3
2
with:

T

has =
has =
=

has
T
. 1-
=

.
4
5
6
(
) has
T
7

- 1

2.
- 2

2.2.2.2 complete Algorithm of the method of NEWMARK

) initialization has
1) conditions
initial
X, X
0
& 0 and &X0
2) choices
of
T and, and calculation of the coefficients a1,… a8 (cf above)
3) to assemble the matrices of stiffness K, mass M and damping C
~
4) to form the matrix of effective rigidity K = K + has. M
0
+ has. C
1

~
5) to factorize
K

b) with each step of time
~
1) to calculate the effective loading R:
~
R = Rt+ T + Mr. (has. X
0
T + A.
2 &
Xt + A.
3 &Xt) + C. {A.X
1

T + A.
4 &
Xt + A.
5 &Xt}
~
~
2) to solve
K.
X

t+t = R

3) to calculate speeds and accelerations at time T + T

&X =

a.
t+ T
0 (X -

X
-

t+ T
T)
a.
2 &
X - A.
T
3 &Xt
&X

=
t+ T
&X + A.
T
6 &X + A.
T
7 &Xt+ T

4) calculation of the step of next time: return out of B) 1)

2.2.2.3 Stability conditions of the diagram of NEWMARK

Method of NEWMARK used in a rather widespread way in the field of mechanics, because it allows to choose the order of integration, to introduce or not numerical damping, and has a very good precision.
(
2 +)
1 2
It is unconditionally stable if: >.
0 5 and >

4
One introduces a numerical damping positive if > 12 and negative if < 12.
When = 12 and = 0, the formula of NEWMARK are reduced to the diagram differences centered. It is thus then an explicit diagram.
A combination very often employed is = 12 and = 14, because it leads to a diagram of a nature 2, unconditionally stable without numerical damping. In fact the choice was made in operator DYNA_TRAN_MODAL. The diagram of Newmark of this operator is thus implicit.
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2.2.2.4 Employment

In DYNA_TRAN_MODAL, this diagram allows integration only linear problems. In tally of the dynamic under-structuring, it allows to employ a modal base calculated by under structuring but it does not support direct calculation on the basis of modal substructure.

2.2.2.5 Damping
numerical of the implicit schemes

The numerical advantage of the direct diagrams of implicit integration lies in the fact that the step of time can be substantially large compared to the smallest clean period of the system without to be likely to cause an instability of the results.

For modes of period clean about the step of time or lower than the step of time, them algorithms of integration introduce a strong damping which contributes to erase the contribution of high modes (cf [R5.05.02]).
It there not of numerical damping in the particular case of the algorithm of NEWMARK with = 14 and = 12.
On the other hand, implicit algorithms one a significant effect of lengthening of the periods of the answer
structure.
One notes that to guarantee a good precision on the amplitude and the phase of displacements calculated, it is necessary to respect a criterion close to:

1
1
T < (
with

10 * F

100
max)
*(*Fmax)*

where F
is the high frequency of the movement which one wishes to capture.
max

2.3
Methods of integrations explicit

2.3.1 Introduction

Three methods of integration clarifies are presented: a diagram of modified Euler of order 1, one diagram of Devogelaere-Fu of order 4 and one diagram with step of adaptive time ADAPT. These three
methods are available in operator DYNA_TRAN_MODAL. The diagrams are presented in considering that linear forces. However the taking into account of the nonlinear forces of easily deduced with the technique from the pseudo-forces.

2.3.2 Diagram clarifies of modified Euler of order 1

2.3.2.1 Presentation

This diagram is commonly called "modified Euler" because it is about a very simple alternative but conditionally stable of the diagram of Euler of order 1, which is, him, unstable. It is thus a diagram often employed into explicit for mechanics. In Code_Aster, it is quite simply called
EULER.
This diagram was used in the module STIFF with LICE [bib3], code finite elements of beam, and in code CADYRO [bib4] for the calculation of the lines of trees in rotation.
The diagram uses formulates it of Euler of order 1 to estimate the derivative in time, with a formula of front Euler for the speed and a formula of Euler postpones for displacement, as follows:

&X
1

1
&X
T M -
=
+
-
-

+
+
(R K X C &X O T
N
N
N
N)
( )
X
&

1 = X
+ T X 1 + O T
n+
N
n+
( )
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the algorithm is thus the following:

) initialization has: X, X
& given
0
0
b) with each step of time:
&X

1

1
&X
T M -
=
+
-
-
+
(R K X C &X
N
N
N
N)
X
&

1 = X
+ T X
n+
N
n+1

2.3.2.2 Order and stability of the diagram

the approximations used in obtaining this diagram are of order 1. One can thus consider that the approximation with which one obtains displacement with each step of time is of order 1. It acts of the consistency of the diagram.

If one puts the diagram of integration in recursive form by eliminating the terms speed, one obtains the relation of following recurrence (without external force, nor damping):
X
1
2

2
0
1 + (M - K
T -
+
1 =

+
) X X
N
N
N

The eigenvalue of this diagram are for a system with a degree of freedom:
- -1
2
T ±
-1
2
I
T (- -
2
4
1
2
MR. K
MR. K
MR. K T)

2
=
if T <
.
2
-1
MR. K
The module of the eigenvalues is worth 1. One realizes that one is in a limiting situation but favorable. There will not be uncontrolled increase in the error. Without damping, one is right on the terminal of stability. It can be an asset for the diagram: it does not introduce dissipation numerical.

2
If T >
-
one can show that one of the two eigenvalues has a module larger than the unit
1
MR. K
and thus that the diagram is unstable.
2
The criterion of stability of diagram EULER is thus T <
-

.
1
MR. K
This study can be extended to a system with a finished number of degrees of freedom. In this case, the criterion
of stability becomes:
2
T <.
max

L `analysis can be refined by considering a system with damping [bib13].
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2.3.3 Method of Devogelaere-Fu

2.3.3.1 Presentation

To present the algorithm of Devogelaere-Fu, shortened in DEVOGE in Code_Aster, it is put dynamic problem in the form:

MR. X
& + CX =
T
where the matrix C is supposed to be diagonal.
T
& T
(

G, Xt)

Displacements and speeds are calculated as follows:

) initialization has

2
T
T
X
X
X
1
1
1
&
4
T,
T,
&

1
0
0
(Mr. G (X
0
0)
M
(
G
X
0
0)
M -
=
-
+
-
-
CX0)
-
2
8
2

&
-
-
X
1
1
1
1
4 4
T
4
T
T
T
T

1 =
(I m) C

(I +
M -
) C &X0 - G, X

1
1 +
(
G
, X
0
0)

-
-
-
2

2
2

b) with each step of time

T

T 2

X
1
1
1
&
4
,
,
4 &
&

1 = X
+
X +
M
T
-
-
T
-

N
N

(
G
X
N
N)
MR. G
X
MR. C
X

X
1
1 -
-

N
1
n+
2
24
N
N
N
2

2
2

2

&
-
T

X
1
1
1
4 4

1
(I MT) C &X
G T
T
-
=
+
+
+

N

(, X
N
N)
G
, X
MR. C
1
1
&X

-
N
n+
4
n+
n+
2

2
2

T

T 2

X
1
1
1
&
4
,
2
,
&
2 &

1 = X
+
X +
M
-
-
+

+

(
G T X)
MR. G T
X
MR. C X
X
N
N
N
N
N
1
1 -
+
N
1
2
6
N
N
n+

2
2

2

&
T

X
-
-
1
1
-1
6.6

4
4
1 =
+
+

1
1 +

+
(I MT) C &X

(
G T, X
+
+)
G T
, X
MR. C
1
1
&X 1 &X
N
N
N
N

-
+

N

6
n+
n+
n+

2
2

2

2.3.3.2 Order and stability of the diagram

The diagram is of order 4, the approximations in the writing of the derivative temporal being in (O t4).
It thus has an excellent aptitude for the integration of regular solutions. Its interest is in less manifest revenge if the functions to be integrated present discontinuities (shocks, friction, etc)

One can show that for a linear system not deadened the step of time guaranteeing stability
2 2
is worth: T <.
max
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2.3.3.3 Employment

This method is expensive in computing times because it twice requires the evaluation of the vector of forces intern G, operation particularly heavy. Consequently it is used little in mechanics for direct integration. On the other hand it is employed by the ECA [bib2] in the case of them systems projected on modal basis.
This diagram allows the taking into account of nonlocalised linearities of shocks type and frictions. Within the framework of the dynamic under-structuring, it makes it possible to employ a modal base calculated by under-structuring but it does not support direct calculation on the basis of modal substructure.

2.3.4 Diagram of integration to step of adaptive time

2.3.4.1 Introduction: interest of a step of adaptive time

To carry out the temporal integration of the transient of a structure in a nonlinear phase poses always problems as for the choice of the step time. The estimate of the error is seldom accessible during integration.
The diagrams of explicit integration oblige to respect a step of maximum time for not to diverge. In the case of nonlinear behavior, this step cannot be a priori given and can *to change with each iteration. When rigidity very strongly varies, a step of constant time and very end to preserve the stability of the diagram led to a very large iteration count and to a time of considerable calculation.*
An algorithm of integration to step of adaptive time was thus developed for DYNA_TRAN_MODAL and was named ADAPT. It is based on the diagram of centered differences, of order 2.

One can notice that this type of diagram was also programmed in DYNA_LINE_TRAN (cf [R5.05.02]).

2.3.4.2 Diagram of the centered differences with constant step

One presents initially the diagram of the differences centered at constant step on which diagram ADAPT bases itself.
It is written as follows:
&X
2

1 = &
X 1 + T &X T
+ O T
N (

, X,
N
N
&Xn) (
)
n+
N
2
2
X
2
&

1 = X
+ T X 1 + O T
n+
N
( )
n+ 2

with the following notations:
xn
&x
-1
X
&x
n-1
X
2
N
n+12
N +1
T
T1
T
T
n-1
N
N
1
T
2
N + 2

n+1
T
T

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It is noted that the speed is expressed with indices half entireties of the discretization in time then that displacements and accelerations are expressed with the whole indices. Writing in this way it diagram is of order 2. However acceleration is not immediately calculable because speed is not known that with the half not preceding. To circumvent this difficulty, one can use several approximations speed to the step of whole time.

· method 1: to suppose that &X
T

T what constitutes one
N (X,
N
&X,
N
N)
&X X,
X
N
&X

,
1
N
N

- 2

approximation validates if damping is sufficiently weak (&
X =
1). If
1 + O
N
&X
( )
N 2
damping is important, the diagram loses then its precision of order 2.

· method 2
: to use an approximation of order 1 for speed
:
&
T

X =
what makes it possible to preserve order 2 of the diagram.
1 +
-1 + O
T
N
&X
&X
( )
N
N
2
2
· method 3: to use a diagram of correct the predictor type/
&X p = &X

1 +
T &X
N
N

-1
predictor:
N

- 2

&X p
p
=
T

N
& (
X
, X,
N
N
&Xn)

&
T
X
p
=

1
1 +
+ -
N
&X
(&Xn () &Xn-1)
corrector:
N

-
2

2
&X =
T

N
& (

X
, X,
N
N
&Xn)
where and are two parameter to be chosen. Park and Underwood [bib6] report that to carry out additional iterations does not improve in a significant way the stability of the diagram.

2.3.4.3 Adaptation of the diagram to the variable step of time

When the step of time varies, the expressions of the preceding paragraph are not valid any more,

acceleration &X being more necessarily expressed in the center of the interval &
X
,
,
1
&X

N
1
N
n+

2
2

as one sees it on the diagram below:
T1 +
-
T
N
N
2
X
X
n-1
&x
N
&x
n-1
N + 1

X
2
2
N + 1
T
T
T
n-1
1
T
1
T
N
n+
n+1
2
N
2
T
T

n-1
N

To take account of this, speed is calculated as follows:

T

1 +
T
&X
.
1 = &
X
N
N
1 +
&Xn
n+
N
2
2
2

Complete diagram ADAPT is written then as follows:

1. estimate of &
X according to methods 1, 2 or 3
X

T

1 +
T
2.
&X
2

1 = &
X
N
N
1 +
&X T
+ O T
N (

, X,
N
N
&Xn) (
)
n+
N
2
2
2

3.
X
2
&

1 = X
+ T X 1 + O T
n+
N
( )
n+ 2

The order of the diagram is not rigorously any more equal to 2, the diagram having lost its centered character.
More T and T
are different, more the order of the diagram tends towards 1. Strong variations of the step of
N
n+1
times thus lead to a loss of precision.
It is possible to find expressions more complex, which use speed or acceleration with the preceding iteration [bib7]. However the formula presented here gives satisfactory results when the step of time decreases but it cause a drop in the limit of stability when the step of time increase. The remedy is to control the step so that it increases only slowly.

2.3.4.4 Stability and precision of the diagram

To study the diagram, one was satisfied with the analysis of a system to only one degree of freedom, free and
linear, of own pulsation and reduced damping:
&
X + 2

&x + 2

X = 0.
The approached solution, by using the diagram with step of constant, is obtained by the relation of recurrence
following:
WITH Y + B Y -
0
1 =
N
N

X
& n&
*with **Y** = X*

N
& 1
n+

2
X

n+1

***With** and **B** are two matrices which depend on the selected method to calculate the contribution of term of damping.*
*A solution of the form is sought: **Y** = **Y.***
N
n-1
*is an eigenvalue of **A-1 B** and can be written in the following form:*
= exp
T
- ± I 1 - 2 where and are the calculated pulsation and reduced damping
C
(C (C
c)
C
C
by the algorithm.
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One can compare them with the exact solution, = exp
T
- ± I 1 - 2, which allows
E
((
)
-
-
C

C

D `to evaluate the error on the pulsation and the error on damping:

and
.

One studied [bib 8] and [bib10] the properties of the diagram according to the method employed to estimate
speed with the whole steps. It was empirically found that method 3 is at the same time more precise and more stable than methods 1 and 2. The method, without overcost of calculation, makes it possible to increase the order of
diagram and gives in the majority of the cases a better precision, except in the event of weak damping. It is however less stable than method 1. It is the method 2 which was finally adopted
in diagram ADAPT. These studies made it possible moreover to estimate the number of points per period necessary to guarantee a stable integration. 20 is a value which gives a good margin of
safety. It is the value chosen by defect.

2.3.4.5 Criteria of adaptation of the step of time

The preceding developments make it possible to quantify the errors introduced during the calculation of one
free and linear system. These criteria do not make it possible however to adapt the step of time. They are
indeed delicate to implement in the nonlinear cases and do not take account of the variations excitation.

Another approach consists in studying the site error introduced by the diagram using limited developments.

The exact solution of a system to a degree of freedom checks:

T

T

T
2
T
3
X T +
X (T)
X& (T)
X& (T)
X
& (&t)
(O T3)

2 =
+
+
+
+
2
8
48

T

T

T

2
T3

X T -
X (T)
X& (T)
X& (T)
X
& (&t)
(O T3)

2 =
-
+
-
+

2
8
48

T

T

T
3
X T +
X T
T
X& (T)
X
& (&t)
(O T3)

2 =
-

2 +
+

+
24

The formula of integration of the differences thus leads to a truncation error being worth:

T
3
T
2
E =
X
& (&t)
(X& - X&
N
N
N
n-1)
24
12

One can normalize this error to obtain a relative error:

T
2 X& - X&
E
N
N
=
-1
X 0
N
12
X
N
N

Park and Underwood [bib7] interpreted this error by defining a "apparent pulsation":

X&
2
N
=

With N

X N
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Applied to the diagram with centered difference, this definition makes it possible to interpret the relative error E
N
like a variation of the apparent pulsation:

T
2
E
2

-
2

N
With N
With n-1
12

Many algorithms use a criterion of adaptation of the step of time based on the error of truncation ([bib9], [bib11]). However in the case of a conditionally stable diagram, this method neither to ensure itself of the stability of integration, nor to guarantee a precision for the calculation of
transients.

Other methods use an approximation of the instantaneous own pulsation of the system [bib12], using the matrices of mass and stiffness. They have the defect not to adapt to the forces external and with their fluctuations in frequency.
It is thus useful to find a criterion which takes account of the two approaches. This is why Park and Underwood introduced the concept of "frequency connect disturbed":

1
X& - X&
F
N
N
=
-1
AP N
2
X - X
N
n-1

This size is interpreted like the "instantaneous" frequency of the system.
In the case of a system with several degrees of freedom, it is necessary to calculate a frequency connect for
each degree of freedom and to take the maximum. The step of time can be then selected to respect a minimum of points per apparent period.

If the denominator of the expression of the apparent frequency tends towards zero, this one can become very
large and not to have significance more. This leads to an unjustified refinement when speed cancel yourself. To cure it a criterion of the type is added:
X - X -
&
&
1
1
-
N
N
X
X
< X&
F
N
N
min

=
-1
T
AP N

2

X&
T
min

It is an intermediary between the disturbed apparent frequency and the truncation error. The value adequate of &
X
is difficult to choose a priori and an unsuited value involves a reduction
min
artificial of the apparent frequency. In the case of a system with several degrees of freedom one circumvent this difficulty by employing the “close” degrees of freedom:

1 X I
I

& N - X&
F
N
=

-1
max
AP N

1 inb DLL 2

Bi

N

Bi
J
J
N = max max X N - X n-1, X
&

T

min
I J lb

-
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where lb indicates for example the bandwidth of the matrix of stiffness and where &
X
can be selected
min
very small.
This method appears very effective if X I N indicate physical components (displacements). In the case of a projection on modal basis it is not relevant to employ them close components to calculate the apparent frequency. In this case it is to better return to first criterion and to use one of the two following methods, specified by key word VITE_MIN:
&X
.
N
if VITE_MIN: “NORM” then it is a variable parameter equal to

100
(&
X

I
=
X 2). This method gives good results when the number of degrees
N
& N

1 inb ddl
of freedom is large and is inapplicable with the case with only one degree of freedom. It is not any more indicated if the order of magnitude speed is very different from a degree of freedom to another.
· if VITE_MIN: "NORM" then it is a parameter variable and different for each degree from
&X jm
freedom, &
X
J = max
. This method has the advantage of functioning whatever it
min

1 mn 1001
degree of freedom of the system numbers but it cannot be used if the order of
size speed varies too much during calculation because, in this case, one would obtain
X J
J
N - X n-1
systematically:
X J
&
.
T

min

2.3.4.6 Algorithm of the diagram of the centered differences with adaptive step

The rules mentioned above make it possible to fix a number of steps of time desired per period response according to the wanted precision, NR. It is adjustable by the key word
1
NB_POINT_PERIODE. The step of time T must then be lower than
. The key word
N
NR F
NOT gives
AP N
at the same time the step of initial time, T, and the step of maximum time not to exceed, T

.
ini
max
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The algorithm is described schematically below:

0) initialization: X and &
X given
0
0
T
0, T =
and &X = & (
X T, X, &X
0
0
0
0)
0
T
1 =
-
ini
initialization of &
X

min

with each step of time
1) initialization of the research of the step of time: NR
= 0
iter
2) calculation of &
X
then X
:
1
n+1
n+ 2
T

estimate speed (method 2): &
X = &X

1 +
&X
N
n-1
N
2
2
T

1 +
T
speed with the semi step: &
X

1 = &
X
N
N
1 +
&X T
N (
, X,
N
N
&Xn)

n+
N
2
2
2
displacement: X
&

1 = X
+ T X
n+
N
1
n+ 2
3) calculation of acceleration &X

n+1
4) calculation of the apparent frequency:
X - X -
1
&
&
1
-
N
N
X
X
X&
F
N
N
min
=
-1
T
APn

2

X - X
N
n-1
X - X -

1
&
&
1
-
N
N
X
X
< X&
F
N
N
min
=
-1
T
APn

2

X&
T
min

5) checking of the adequacy enters the step of time and the apparent frequency:
calculation of the indicator err = T
NR F

N
AP N
· if err 1 and NR
< NR
then reduction of the step of time and new iteration of research
itez
iter max
step: T 0 7
, 5t, NR
NR + 1, return in 2)
N
N
iter
iter
· if err 0 7

, 5 since more than 5 steps of consecutive times, then increase in the step of time
T 11
, T
N
N
6) filing of the solution, possible calculation of &
X
and return in 1) for the following iteration.
min

2.3.4.7 Comments on the parameters of the algorithm

The fact of fixing an upper limit NR
by the key word
iter
NMAX_ITER_PAS with the number of
max
reductions of the step of time makes it possible to be ensured of the convergence of the algorithm in the cases
difficult (for example in the event of discontinuity in the external forces).
When the indicator err is higher than 1, the step of time is multiplied by a factor fixes (0,75 by defect but it can be modified by the user thanks to operand COEF_DIVI_PAS). It would have been
1
possible to write directly: T
T = NR F
, which more intuitive. But this strategy
N
err
N
AP N
conduit with an excessive refinement, the calculated apparent frequency being often largely higher at the real frequency, when the error is large. However, in only one step of time, T can be
N
considerably reduced (factor 0 7
, 5Niter).
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On the other hand the increase in the step of time is always much slower (coefficient of multiplication owing to lack of 1,1 definable by COEF_MULT_PAS) and takes place only if the indicator is
lower than 1 during five steps of consecutive time. These restrictions are justified by the risks of loss of stability or precision of the diagram when the step of time varies too quickly. A coefficient of 1, 2 or 1, 3 can allow a faster calculation but exposes sometimes at the risks of error.
In short the default values were validated by many tests and give in general satisfaction in terms of precision and stability [bib8].

2.3.4.8 Performance of the algorithm

With equal precision, the iteration count carried out by diagram ADAPT is at least five times more weak that with a constant step in the phenomena which justify the use of a variable step by the irregular aspect of their evolution (shocks, fluid blade, discontinuous excitations, etc).
The empirical studies showed that the step of adaptive time allows in the successful outcomes of to gain a factor two or three in computing times. This diagram makes it possible moreover to control precision of integration by the method of the control of the number of points per period of the answer.
In the case of the very deadened systems, the profits can be even more important (calculations five with ten times faster).
On the other hand when the step of "ideal" time is about constant, the use of diagram ADAPT appears useless.
It of course allows the taking into account of nonthe localised linearities of shocks type or frictions, thus that fluid blades.
In dynamic under-structuring, it is compatible as well with the transitory analysis on the basis modal restored on the whole system or transitory calculation on the bases distinct from under structures.

3 Conclusion

As a conclusion, summarized here various possibilities of temporal integration which offers the operator:

· Euler ("EULER") clarifies modified to ensure a conditional stability,
· Schéma of Newmark ("NEWMARK") parameterized in order to be implicit,
· Schéma of Devogelaere-Fu ("DEVOGE") of order 4,
· Explicit adaptive Schéma ("ADAPT").

The diagram by defect is EULER but it is not systematically adapted more. The diagram of NEWMARK available in Code_Aster is implicit and guarantees an unconditional stability but is valid only for purely linear problems. Diagram DEVOGE is of order 4 and thus is more precise but it is is expensive in computing times. Diagram ADPAT is more particularly indicated for the problems with nonlocated linearities, where the step of "ideal" time is not constant during the transient. It is thus the experiment of the modeling which makes it possible to choose it
diagram best adapted to the problem according to the report/ratio (computing time) /précision.
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Document: R5.06.05
Forces of fluid blade in transitory calculation on basis
modal
Summary:
This document presents a numerical modeling of the forces of fluid blade which exist when two mechanical systems, plunged in a fluid, vibrate with weak plays between them.
These non-linear forces comprise terms of acceleration which require a particular treatment for traditional explicit diagrams of integration.
An iterative diagram of type not fixes is proposed. It makes it possible to preserve the architecture of the explicit diagrams of
Code_Aster. These forces are established in operator DYNA_TRAN_MODAL [U4.54.03].
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1 Introduction

In the primary education circuit of the power stations REFERENCE MARK the mechanical components are immersed in a fluid.
For some of these materials, put in vibrations by the excitation of the primary education fluid, the

presence of plays
relatively reduced leads to a more or less important closing of these plays even to contacts in fluid environment. Numerical work was undertaken in Code_Aster to model the contact dryness between mechanical structures. This work was established in a transitory operator of calculation by modal recombination [bib3] and were validated by comparison with tests carried out on model SOLID MASS [bib4].
The vibrations with contact in fluid environment show characteristics different from those observed in air. When the play is filled, it creates a all the more important fluid flow as it play is weak. This flow is at the origin of compressive forces acting on the structures antagonists. Contrary to the configuration in air, where the structures interact by forces of contact, only when the play is filled; in fluid environment this interaction is permanent and depends in a non-linear way of the values on the play, normal speed of the structures and of their acceleration. One will qualify the fluid locked up in the reduced type font of ***fluid blade****; forces resulting from*
compression of the fluid will be the ***forces of fluid blade****.*
For materials like the fuel assemblies or the pencils of the bunches of order these forces of fluid blade induce a modification of the mechanical characteristics of structure in air (mass, damping). The damping induced by the fluid blade can be considerable, and it seems interesting to take it into account in a modeling of these materials.
We present, in this report/ratio, a simple geometrical configuration, where one can integrate the flow in the fluid blade realising certain assumptions on the profile of the flow and them pressure losses at the edges. We determine thus the compressive forces exerted by the fluid on structure and let us release a general form of their expression according to the play, speed and of acceleration relative normals between the structures.
We build case-tests of reference on a system to a degree of freedom which illustrate it behavior of a mechanical system subjected to a force of fluid blade.
The numerical establishment of these non-linear forces in Code_Aster is then detailed. It require the use of a fixed algorithm of point to find accelerations generalized.
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2

Analytical expression of the forces of fluid blade in one
simple geometrical configuration

One proposes to determine in an analytical way here the forces being exerted on a structure vibrating in an incompressible fluid in the vicinity of a motionless wall.

2.1 Configuration
geometrical

A problem of plane flow in the case of is placed (invariant in the direction there of [2.1-a]).
A solid body is plunged in a fluid F. The solid has a plane face of width 2 L
parallel in the yOz plan and vibrates in the vicinity of a wall fixes parallel with this plan.

X
2L

.
X

X
F
y O
Z

Appear 2.1-Error*! Argument of unknown switch.* ***: Geometrical configuration of the fluid blade***

2.2
Equations governing the behavior of the fluid

The problem is supposed invariant by translation according to the axis y, one is thus brought back to a problem
two-dimensional.
Speeds in the fluid will be noted:
(
v *T) = U (X, Z, T).* ***X*** *+ (*
W X, Z, T) .z
One will note X, Z, the space co-ordinates eulériennes of the fluid, and X, X! , X! variables
Lagrangian defining the position, speed and acceleration of the solid.
The incompressible fluid being supposed, the components speeds must check:
U W
div ***(v)*** *= 0 oneself*
T
+
= 0
X
Z

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The fluid also checks the Navier-Stokes equations:
v *+ grad (p) + .v.grad (**v**) - µ2v = 0*
T
In the fluid blade one will suppose that the profile according to X of the component W of the field speed is
of invariant form compared to Z. That amounts supposing that it can be written in the form of one function with separate variables:
(
W X, Z, T) = (
W Z, T). (X, T)
One in general considers two rather simple assumptions of profile:
· a ***uniform profile speed****,*
· a ***profile speed parabolic or flow of One tenth of a poise****, valid for speeds*
W weak,
2.3
Resolution of the fluid flow of blade with uniform profile
The flow according to Z does not depend on X:
(
W X, Z, T) = (
W Z, T)
One neglects in that the effects of viscosity of the fluid in the blade.
Let us write the relation of incompressibility of the fluid, integrated on the thickness of the fluid blade:
X U W
X
X
W Z
+
). dx =,

0
that is to say
[U]
( )
(
+
dx =

0 .

0

X
Z
Z
0
0
The boundary conditions are: (
U X) =! X and U (0) = 0,
(
W Z)
! X
one thus obtains
= - which gives by integration and by noticing that W (0) = 0:
Z
X
X!
W (Z) = - Z
X
One then deduces immediately the fields from them speed:
(
X!
W X, Z) = (
W Z) = -
Z
X
(
X!
U X, Z) = (
U X) =
X
X
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By using the Navier-Stokes equation to describe the behavior of the fluid and by projecting it on axis Z, then by replacing the expressions of U and W higher established, and while placing themselves in the assumption of a thin blade of fluid like by considering assumptions of pressure losses [bib2], one can show [bib5] that the fluid force has two expressions different according to the sign from!
X:
if!
X < 0:
2
2
3
X! 4
3
X!
F = -. L. Y. +. L. Y.
3
X 3
X
if!
X > 0:
2
2
3
X! 2
3
X!
F = -. L. Y. -. L. Y.
3
X 3
X

One can give a general expression of the fluid force for the uniform profile in the form:

2
X!
X!
X!. X!
F =. +. +
X
X
X2
For the uniform profile, one a:
= - 2 3
. L. Y
3
= 1 3
. L. Y
3
= - 3
. L. Y

2.4

Resolution of the fluid flow of blade with parabolic profile

One gives at the horizontal speed W a parabolic profile which has as an expression:
(
W X, Z, T) =
(
a.x. X - X). W (Z, T),
W (Z) being the mean velocity in the blade and has = 6
X2
Let us write the relation of incompressibility of the fluid, integrated on the thickness of the fluid blade:
X
U W
X
W Z
.dx = 0, so
[
it U]
( )
0 .
0 +
X
+
0 [a.x. (X - X]
=

0

) dx
X
Z
Z
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The boundary conditions give: (
U X) =! X and U (0) = 0, the expression is thus obtained:
W (Z)
! X
= - which gives by integration and by considering that W (0) = 0:
Z
X
! X
W (Z) = - Z
X
One then deduces immediately the fields from them speed:
(
! X
W X, Z, T) =.
X has. (X - X). W (,
Z T) = -.
6 X. (X - X).
Z
X3

x2. 3X 2x
U (X, Z, T)
(
)
= -
-
. ! X
X3
By using the Navier-Stokes equation to describe the behavior of the fluid and by projecting it on axis Z, then by replacing the expressions of U and W higher established, and while placing themselves in the assumption of a thin blade of fluid as well as assumptions of pressure losses [bib2], one can to show [bib6] that the fluid force in the case of a parabolic profile has two different expressions according to the sign of!
X:
if!
X < 0:
2
2
3
X! 24
3
X!
24
3
X!
F = -. L. Y. -
. L.
Y.
+
. L. Y.
3

3
X
3
X
15
X
if!
X > 0:
2
2
3

X! 24
3
X!
2
3
X!
F = -. L. Y. -
. . .
L Y.
-. L. Y.
3

3
X
3
X
5
X
One can give a general expression of the fluid force in the form:

2
X!
X!
X!
X!. X!
F =. +. +.
+
X
X
X 3
X 2
with the formulated assumptions (parabolic profile), the coefficients are worth:
= - 2 3
. L. Y
3
= 3 3
. L. Y
5
= - 24 3
. L. Y.
3
= - 3
. L. Y

The expression of the fluid force above thus represents the most complete form and is that established in Code_Aster.
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3
Study of the dynamic behavior of a system to one degree of freedom in the presence of a fluid blade

This chapter aims at integrating in an quasi-analytical way a system with fluid blade into a ddl, and serves as references to tests SDND110A [V5.01.110] and SDND111A [V5.01.111].

X
2L

M
.

X
Xo
F
O
Z

Appear 3-Error*! Argument of unknown switch.* ***: Mass deadened by a fluid blade***

One will consider the system without rigidity above nor external force applied. The mass has an initial speed -! X0, and a X0 position. One seeks to determine the position of stop of the mass, evolution of the force of reaction. The equation of the system with the 2 assumptions of profile: uniform and parabolic is as follows:

2
X!
X!
X!
X!. X!
Mr. X! =. +. +.
+

X
X
X3
X2

X (T = 0) = X0

X! (T = 0) = - X!

0

3.1
To launch of a mass slowed down by fluid blade with uniform profile
For the uniform mode, the differential equation governing the movement of stop of the mass is written

2
X!
X!
the following way: Mr. X
! = . + . .
X
X
One can find in [bib1] an analytical resolution whose we will point out the principal results here. By integrating once the differential equation, one obtains an expression the speed of the projectile in function of its position:

2
2
!
X0 +
X
X = -! X.
0
.

where
X0

X +
= M
While integrating once again compared to time this differential equation it comes:
2
1 X

0
X
1
1
T =
.
. X
0
2
0 - X + 2. Log

X! 0 X0 +

X +
-
X X0

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There is thus an implicit definition of the displacement of the mass in the course of time. One can release following properties of this movement:

· the solid can touch the obstacle only at the end of an infinite time,

· the solid approaches at infinitely slow speed of the obstacle.

The total fluid force has as an expression:

4
3

2 *X* 0 +

X
F
= 2 . . ! .
.
0

fluid (*X*)
. *MR. X*
X
5
0
(*X* +)

Its maximum value is obtained by cancelling the derivative of this function. It is reached in

4
8 33
.
Mr. X 2
!
X

X

0
0
F max = 3 and is worth *F*
.
.
.
2
Max fluid =
+
55

X0
The numerical values considered for calculations are:
M = 1000 kg
2L = 100 mm
X0 = 6 mm! *X0* = -0.1 m/s
F = 1000 kg/m3
= 106
The coefficients, are calculated according to the formula of paragraph 2.3 and are worth in this case:
= 0.0833, = 0.1666
One can observe below displacement, the speed of the mass and the fluid force which it undergoes. behavior of the mass is rather similar to that observed for the parabolic mode. The force fluid maximum, given in an analytical way in the preceding paragraph, is worth in this case 8768 NR. The mass in this case approaches in an asymptotic way of the wall and reaches it only at the end of one infinite time.
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Note:
On the graph, at the end of the 0,2 S of calculation it remains a distance from 1.e-6 m to the wall.

3.2

To launch of a mass slowed down by fluid blade with parabolic profile

The analytical resolution of the differential equation governing the behavior of the mass is not any more possible. One proposes to determine in an external way with any computer code, the dynamic response this system with a ddl in the presence of a fluid blade. That gave place to the development of one program dedicated FORTRAN, developed with this occasion.

As we established in the preceding paragraph, the force of reaction of the fluid blade takes the following general form:

2
X!
X!
X!
X!. *X*!
Ffluide =. +. +.
+
X
X
X3
X2

The dynamic equation to which this system is subjected is as follows:

2
X!
X!
X!
X!. *X*!
Mr. *X*! + *K*.*X* = *Fext* +. +. +.
+
X
X
X3
X2

We propose a resolution by a temporal diagram of integration of the dynamic problem.

The expression of the second member is nontraditional because it utilizes acceleration. One proposes to use an explicit diagram of integration, which requires the expression of! *X*, according to *X*, *X*! . It is thus necessary to rewrite the system in the form:

2
X!
X!
X! . *X*!

M -
. *X*! + *K*.*X* = *F*
T
T
T
T
+ . + .
+

X
T
T
ext.

X
X 3
X 2
T
T
T
T
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We will use the diagram of Euler modified to integrate this equation in time:
X, *X*! given to T
0
0
0,
To repeat
! 2

X
! *X*
! *X*. ! *X*
F - *K*.*X* +.
I
+. *I*
T
I
ext.
I
+
X
X 3
X 2
! *X*
I
I
I
I =

M - *Xi*
T +1 = *T* + *dt*
I
I
! *X* +1 =! *X* + *dt*. ! *X*
I
I
I
X +1 = *X* + *dt*. ! *X*
I
I
I +1
as long as *T*
T
i+1 < fine
For the type of non-linearities considered, one does not have criterion of stability *a priori* of the diagram of integration. One thus made sure by a study of convergence by decreasing the step of time of calculation which the results obtained were stable.
The coefficients, and are calculated according to the formula of paragraph 2.4 and are worth in this case: = 0.0833, = 0.19992 and = 0.9996. 106 and masses it M = 1000 kg
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One can observe below displacement, the speed of the mass and the fluid force which it undergoes. One note that it preserves a speed close to that initial before being sufficiently close to
wall. Then it undergoes an important fluid force which dissipates all the kinetic energy of the mass. mass does not touch the wall, but preserves an asymptotic distance at the wall, which is worth 0.098 Misters.

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4 Calculation of a system multi degrees of subjected freedom locally with forces of fluid blade

A means of simulating the forces of fluid blade is to introduce them like non-linear forces into the algorithm of modal recombination DYNA_TRAN_MODAL [bib3] [U4.54.03], allowing to calculate dynamics of a mechanical system by carrying out a projection on the basis of its free mode.
The temporal algorithms of *Code_Aster* treating the non-linear forces are the explicit diagrams of Euler and Devogelaere. We saw in the preceding chapter that the intrinsic form of the forces of fluid blade a problem for the resolution with an explicit diagram poses. A modification of algorithm DYNA_TRAN_MODAL allows a suitable treatment of the forces of fluid blade.
The direct dynamic problem discretized by finite elements is written:

Mr. X
! + **C.X** + **K.X** = **F** (*T*) + **F**
(**X**, **X**! , **X**
!)
T
T
T
ext.
fluid
T
T
T
The technique used in operator DYNA_TRAN_MODAL consists in projecting on the basis of system linear and to maintain the forces non-linear with the second member.
The projected dynamic system takes the form:
T.
Mr.
T
T
T
T
! *T* +.
C. ! *T* +.
K. *T* =. *ext.*
F (*T*) +. **Ffluide** (. *T*. ! *T*. ! *T*) **éq**
4-1
The methods of explicit integration require to determine! *T* knowing *T*! *T* and possibly their former values.
One thus sees in the expression of the system [éq 4-1] above that! *T* is not given in way clarify according to *T*! *T*. From this moment, one proposes to use a method of point fixed to obtain generalized accelerations.
This is obtained by applying the following operations:
!
0t =! ,
T-1
, *T*! *T* given
to repeat until convergence:
-1
!
i+1
T
T
I

T
T
T
T = [.
M]. (. **F**
(.
fluid,
.
T!
, *T*. ! *T*) +. **F** (*T*)
ext.
- . .
C
.
! *T* -.
K
.
T)
convergence is tested by! *i+1*
I
I
T -! *T* <. ! *T*.
Unfortunately this technique of iteration of the fixed point is not necessarily convergent. For
that, it is necessary that the linear operator reiterated either contractor. However for thicknesses of fluid blade low,
the terms of inertias can be very important and thus prevent the convergence of
iterations of fixed point. It is thus not established such as it in operator DYNA_TRAN_MODAL.
We will analyze more in detail the operator *T*. **Ffluide** (. *T*. ! *T*.
! *T*), to extract some
diagonal part and to make it pass to the first member.
In a node *N* comprising an effect of blade fluid, the fluid inertia will be expressed in the form,
linear in acceleration:
X!
F
N
inertia =. Xn
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Sizes! *X*, *X*
N
N are expressed in a local reference mark. They are thus obtained by one
operation of extraction **P** of the total assembled vector, followed by a series of rotations **RN** to obtain in the local reference mark, followed by an extraction **In** normal component.
One will note these operations in a matric way! **X** = **E**. **R**.
P! **X**
N
N
N
.
In a similar way, the local inertia must be turned over in the physical reference mark and to dimension vector assembled, before being projected on the basis. These operations can be noted of
matric way: *T*. {}
F = *T T*
-1
T
. **P. RN**. **In**. **Ffluide**
The vector of the generalized forces representing the component of inertia of the fluid blade is:

t.F =
.t *T*
-1
T
. **P**. **R**. **E**. **E**. **R**.
P
. .
N
N
N
N
!
Xn
In the case general, one cannot determine once for all the value of this matric product because it

thus depends on the local reference mark of the position of the structure compared to the fluid blade. One proposes
to use an approximation of this matrix in the form:

me =
. *T*
. **Pt.P.**
T

T
X N
To return the operator of point contractor it fixes is enough to modify the matrix of mass of the operator of iteration by cutting off the matrix to him **me**. That amounts adding to him in fact of the mass bus is negative.
One will use in the algorithm of Euler modified for the taking into account of the effects of fluid blade, the algorithm of point fixes opposite:
!
0t =! *T-1* to repeat until convergence:
-

1
!
i+1
T
T
I
T =.
.
M -
.diag ('
m)

.
N
(. **F** (
fluid. ,
T. !
, .
T! *T*)

nodes fluid

-
.diag ('
m). *I*
T
T
T
F
C
K

N
!
T +.
(*T*)
ext.
-…! *T* -… *T*
nodes fluid

convergence is tested by! *i+1*
I
I
T -! *T* <. ! *T*, where is a precision given for the stop of
iterations.
The parameter, selected higher than one, is used to guarantee the character contracting of the iterations of not fixes. In practice, one chooses for value = 10 what seems to guarantee convergence in all the cases observed, one can possibly modify this parameter in the event of problems of convergence. There is not theoretical result giving a best alternative for. One will be possibly led to to modify this value according to the importance of the non-linear forces in the response of the system, for to improve convergence of the calculation of acceleration.
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5
Establishment of the non-linear forces of fluid blade
5.1
Specific integration for the forces of fluid blade
The forces of fluid blade compatible with the diagram of integration are named “EULER” and “ADAPT” in operator DYNA_TRAN_MODAL. The treatment of the forces of fluid blade is not activated
that when fluid blades are present in the model what guarantees a maintenance of performances of algorithms EULER and former DEVOGE, and makes it possible to use a blade in the case of
fluid, a specific option but preserving nevertheless non-linearities and the functionalities existing of the initial algorithm.
5.2
Use of the forces of fluid blade in DYNA_TRAN_MODAL
The forces of fluid blade are designed to function like non-linearities of shock, i.e.,
that an effect of blade fluid can act between a point of a structure and a fixed obstacle, or between two antagonistic points of two structures.
The parameters of fluid blade are thus provided in the key word factor SHOCK of the operator DYNA_TRAN_MODAL. Syntax under this key word will be as follows:

SHOCK:
(
....
RIGI_NOR: KN [r8]
....

LAME_FLUIDE: /
“NOT”
[DEFECT]
/
reference mark
[KN]

ALPHA
:
/
0.
[DEFECT]

/

[R8]

BETA:
/
0.
[DEFECT]
/

[R8]

CHI
:
/
0.
[DEFECT]
/

[R8]

DELTA
:
/
0.
[DEFECT]
/

[R8]
)
Key word LAME_FLUIDE makes it possible to specify if the interaction enters the node and the obstacle or between
two nodes takes place in the presence of a fluid blade. By defect the connection is supposed of contact type
dryness.
The key words ALPHA, BETA, CHI, DELTA make it possible to describe the form of the non-linear force of
fluid blade, their values correspond to the coefficients, and are mentioned in the chapter
[§2]. They make it possible to define in the choice a uniform or parabolic profile.
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Model of fluid transition blade - shock

The studies justifying the development of the forces of fluid blade comprise situations where structure vibrates in the presence of fluid blade and can even according to certain conditions of excitation outward journey
until the dry contact with the obstacle in the event of fluid film rupture.
This situation is particularly difficult to manage numerically from the nature of the expression of forces of fluid blade used. In fact, one is obliged to consider a physical limit of validity with the expression of these forces of fluid, limiting blade beyond which it is necessary to forward towards conditions
of mechanical contact (dry) between the structures.
One thus introduces consequently occasion the concept **fluid thickness of blade limits** beyond which the fluid model of blade only is not valid any more, and the blade becomes in fact incompressible.
To preserve at the force of reaction (fluid blade and contact) a continuous character, we introduced a weight function fponder (*dn*) which makes it possible continuously to forward force of repulsion of fluid blade to a force of repulsion of dry the contact type.
fponder (*dn*) = 0 if *dn* 0
fponder (*dn*) = 1 if *dn*
F
0
ponder (*dn*) of C

ontinuitéC

for *D*
N [
,
0]
In the zone of transition *dn* [0,], the force of reaction is written:
F
= fponder (*D*) × Ffluide (*D*, *D*! , *D*!) + (1 - fponder (*D*) × FC (
hoc *D* -, *D*!
reaction

N
N
N
N
N
N
N)
The boundary layer is dynamically given with heuristics according to the formula:
F
(, ! , !
fluid *D*
D D
N
N
N)
=
éq 6-1
KN
If this boundary layer is reached or exceeded *dn* by the wall of the structure one enters one phase of transition towards the shock. The limiting value thickness is then filed and the model of transition is used until *dn* becomes again.
One explains physically the choice of the formula [éq 6-1] to determine the fluid thickness of blade in considering that in this situation the fluid force of blade is such as it can deform the structure on its stiffness of shock and thus the fluid blade in it even becomes incompressible, from where need for to forward towards the model of force of shock.
Note: Use of the forces of fluid blade with the model of transition
The fluid model of transition - shock was introduced in a systematic way. As soon as one introduces a fluid force of blade, one can forward towards the shock. It is thus necessary systematically to introduce a stiffness of shock kN (key word RIGI_NOR). If one never wishes not to forward towards a dry contact (mainly for case-tests) it will be necessary to take a value of kN very large (1015).
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7 Conclusion

This document describes the expression of the fluid forces which are exerted when a structure vibrates with
vicinity of a plane wall, in an incompressible fluid at rest (**put in flow by movement of the structure**). These forces are called **forces of fluid blade**.
For two assumptions of profile of flow in the blade, an analytical form of the force is established. The latter depends in a non-linear way of acceleration, the speed and the position of structure compared to the obstacle.
For a system with a degree of freedom with initial speed deadened by a fluid blade, a calculation analytical could be carried out with a uniform profile. For the other profile, like configurations of system masses spring, a numerical integration was necessary. One could analyze on these calculations the behavior of the fluid blade, which introduces in particular a strong damping.
The treatment of these forces for systems with several degrees of freedom resulted in modifying the explicit algorithm of integration on the basis of modal DYNA_TRAN_MODAL to integrate correctly the dynamic effect of these forces in *Code_Aster*.

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Handbook of Reference
R6.01 booklet: Iterative methods
Document: R6.01.02

Linear Solvor of combined gradient type: study theoretical and establishment in Code_Aster

Summary:

In a number of fields, simulation becomes impossible to circumvent but greedy in processing capacity,

and
more particularly, of resolution of linear systems. The choice of the good linear solvor is thus of primary importance,
on the one hand for its speed, but also for its robustness and the place memory which it requires. Each time, one
compromise is to be operated between these constraints.
For 50 years two types of solveurs have disputed supremacy in the field, direct and those iterative. From where, by precaution, a diversified offer of the codes of mechanics on the matter, not escaping Code_Aster
with the rule. Since it proposes two direct solveurs (Gauss and multifrontal) and iterative (GCPC for Gradient
Combined Packaged).
In this note, one details from a theoretical, algorithmic point of view and Code_Aster, the fundamental ones of
GCPC, its bonds with the direct solveurs, methods of descent, those of continuous optimization and those
spectral. One concludes by the particular difficulties of establishment from the GCPC in the code, his parameter setting and
perimeter like some councils of use.

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1 Problems

Warning:

***The reader in a hurry and/or not interested by the algorithmic and theoretical springs of the gradient combined can from the start jump in the last paragraph, [§5], which recapitulates the principal "aspects
Code_Aster" of the GCPC.***

In a number of fields, simulation becomes impossible to circumvent but greedy in capacity of calculation and more particularly of resolution of linear systems. These inversions of systems are in fact omnipresent and often hidden with deepest of other numerical algorithms: solveurs non-linear, integration in time, solveurs modal…. One seeks the vector of the nodal unknown factors (or their increments) U checking a linear system of the type

$$Ku = F$$

éq 1-1

with K the matrix of rigidity, digs, badly conditioned and often symmetrical definite positive. vector F represents the application of the forces generalized to the mechanical system.

Note:

***In mechanics of the structures conditioning (K) is known much more to be
bad that in other fields, such mechanics of the fluids for example. It can
to vary, typically, from 105 to 1012 and the fact of refining the grid, of using stretched elements or of the structural elements has dramatic consequences on this figure (cf [bib13])***

***The choice of the good linear solvor is thus of primary importance, on the one hand for its speed, but too
for its robustness and the place memory which it requires. Each time, a compromise is to be operated between these constraints.
For 50 years two types of solveurs have disputed supremacy in the field, the solveurs***

direct and those iterative (the border between the two is not absolutely tight besides. One method described as direct can be regarded as iterative thereafter (for example them methods of Krylov). In addition the iterative methods can call punctually upon direct solveurs (preconditionnor of the Cholesky type)) :

1) The purpose of the first are to factorize the initial matrix in a canonical form (LU, LDLT…) allowing a resolution much easier, by descent-increase on triangular systems L or U suitable. In practice, one carries out this operation on one permuted initial matrix in order to limit the filling of the profile of factorized and of *to take account of the hollow character of the operator. These permutations/renumérotations/swivellings can start at various levels of the algorithms and on elements of sizes variables (matrix, super-blocks, blocks…).*
2) The theory of these methods is relatively well tied up (into arithmetic exact as in arithmetic finished), their completion is quasi-policy-holder in a finished number of operations known by advance and their precision is as good as that of the initial problem. Their variations according to grinds standard matrices and software architectures are very complete (packages PETSc, LAPACK, WSMP, MUMPS, UNFPACK… cf [bib4] [§3.1]).
The iterative solveurs start them starting from an initial estimate of the solution *and, with each iteration, try to build vectors which will approach the discretized solution (algorithms based on the splitting operators and the theorem of the fixed point or on minimization of a quadratic functional calculus). This process is stopped when a criterion of convergence is satisfied (in general a relative residue lower than a certain value). In practical, one carries out this operation on a packaged initial matrix (which has better theoretical properties than the initial operator with respect to the algorithm), one holds*

---

count hollow character of the operator and one seeks to optimize in time calculation the tool elementary essence of these algorithms: the product matrix-vector.
The theory of these methods comprises many "opened problems", especially in arithmetic finished. In practice, their convergence is not known a priori, it depends on structure of the matrix, the starting point, the criterion of stop, the parameter setting of the treatments numerical associated…
***Contrary to their direct counterparts, it is not possible to provide the solvor iterative which will make it possible to solve any linear system. Adequacy of type of algorithm to a class of problems is done on a case-by-case basis**.*
Even if, historically, the iterative solveurs always had right of city bus for some applications they function much better than the direct solveurs and they require theoretically (with management equivalent memory) less memory.
For example, in the initial version of the GC, one has right need to know the action of stamp on a vector. It is thus not necessary, a priori, to store entirely the aforementioned stamp to solve the system! In addition, contrary to the direct solveurs, them iterative are not subjected to the "diktat" of the phenomenon of filling ("fill-in" in English) who gangrene the profile of the matrices dig and with the misadventures of the swivelling which requires of
to entirely store (one will see that in Code_Aster, the properties of the operator of work and algorithmic easy ways allow nevertheless a hollow storage optimized even for direct solveurs).

Finally, in spite of its biblical simplicity on paper, the resolution of a linear system, even symmetrical definite positive, is not "a long quiet river". Between two evils, filling/swivelling and prepacking, it is necessary to choose!
In the literature in numerical analysis [bib26] [bib27] [bib14] [bib35] [bib40], one often grants one dominating place with the iterative solveurs in general, and, with the alternatives of the gradient combined in
private individual. ***The most senior authors*** *[bib16] [bib36]* ***agree to saying that, even if sound use gains with the passing of years and of the alternatives, of the "shares of market", certain fields remain still refractory. Among those, the mechanics of the structures, the simulation of circuit electronics…***
To paraphrase these authors, the use of direct solveurs comes under the field of *technique whereas the choice of the good couple iterative method/preconditionnor is rather one art!*

From where a ***diversified offer of the codes of mechanics in the field*** *(cf [bib4] [§8.3]): methods direct (Gauss, multifrontale…) or iterative (Gauss-Seidel, SOR, GCPC, GMRES…). This choice of building blocks (the algorithm of resolution) is declined then according to a whole string of treatments numerical which intervenes at various levels*
: storage of the matrix, renumerotor,
preconditionnor, balancing…

Code_Aster *also does not escape him this principle from precaution by diversity from the offer. Its resolutions of systems linear are structured around three solveurs (via key word SOLVEUR of operators, cf [§5.3], [§5.4]):*

1) One
direct solvor of Gauss type *([bib37], key word "LDLT"), with or without the renumerotation Transfer Cuthill-Mac-kee, but without swivelling (because of its storage SKYLINE). It storage is completely paginated (via key word TAILLE_BLOC of the operator BEGINNING) and thus the passage of large case with a size memory "as small allows as one wants" and a robustness as good as possible into arithmetic finished (with the rounding errors near therefore, cf key word NPREC). However, it with the detriment of the CPU is consumed (the accesses disc are expensive), knowing that calculation intrinsic complexity of the algorithm is already*
3
NR
raised: in

where NR is the size of the system.
3
2) ***A direct solvor of Multifrontal type*** *([bib33], key word "MULT_FRONT"), with renumérotations MANDELEVIUM, MDA or MONGREL and a storage of the matrix MORSE. It is partially*
paginated (only the initial matrix must hold in main memory with a few blocks of stamp in the course of factorization. But this pagination is not skeletal by the user, it is intrinsic with the choices operated by the partitionnor and thus with the structure of

---

the initial matrix) and paralleled in memory divided. It combines robustness, size memory flexible, at a moderate cost CPU (because of its organization per blocks which rests, moreover, on optimized mathematical libraries). ***It is thus quite naturally METHOD BY DEFECT OF THE CODE****.*
3) One ***iterative solvor of Packaged Combined Gradient type*** *([bib22] [bib3], key word "GCPC"), with or without renumerotation Cuthill-Mac-Kee, storage of the matrices (of the matrix initial and of the matrix of prepacking) MORSE and prepacking ILU (p) (by one factorized of Cholesky incomplete of order p of the initial matrix cf [§4.2]). Like all iterative process, its convergence (in a reasonable iteration count) is not acquired in practice. Taking into account its algorithmic process (each matric block one is used great number of times, for the products matrix-vector, contrary to the direct solveurs), the GCPC is not paginated and the matrix must thus hold in only one central memory stack. Its calculation complexity depends on the hollow character of the initial operator, of his conditioning numerical, of the effectiveness of the preconditionnor and the necessary precision.*

Now let us approach the whole of the chapters around of which will articulate this note. After having clarified the ***various formulations of the problem*** *(linear system, minimizations of functional calculus) and in order to better do to feel with the reader implicit subtleties of the methods of the gradient type, one propose a fast overflight the their fundamental ones: traditional and general* ***methods of descent****, like their bonds with the GC. That linear of the resolution of system SPD and that nonlinear of optimization continues.*
These recalls being made, the ***algorithmic unfolding of the GC*** *becomes clear (at least it is hoped for!) and* ***its theoretical properties*** *of convergence, orthogonality and complexity result from this. complements are quickly brushed to put in prospect the GC with concepts and/or recurring problems in numerical analysis: projection of Petrov-Galerkin and space of Krylov, problem of orthogonalisation, equivalence with the method of Lanczos and properties spectral, encased solveurs and parallelism.*
Then one details the "evil necessary" which the ***prepacking of the operator of work*** *constitutes, some often used strategies and that retained by the GCPC of Code_Aster. One insists in particular on the concept of incomplete factorization by levels, its principle and the happy contest circumstances which make it licit.*
One concludes by the ***particular difficulties of establishment from the GCPC in the code*** *(taken into account limiting conditions, obstruction memory, parallelization), its* ***parameter setting*** *and* ***perimeter*** *thus that* ***some councils of use****.*

The object of this document is not to detail, nor to even approach, all the potential aspects of the GC. Several notes HI would not reach that point so much there is plethora of work on the subject. The bibliography proposed at the end of the document a first sample constitutes some.

We nevertheless tried to approach the principal aspects by giving a maximum of tracks, ideas, references, intermingling the visions numericians, mecanicians and Code_Aster. An effort private individual was brought to put in prospect the choices led in Code_Aster compared to last and current research. One moreover tried constantly to bind different the items approached while prohibiting all mathematical digressions.
This note will become the new reference material of the code on the GCPC (Doc. [R6.01.02]).

Figures [Figure 2.1-a] from [Figure 4.1-a] were borrowed from the introductory paper of J.R. Shewchuk
[bib38] with its pleasant authorization: ©1994 by Jonathan Richard Shewchuk.

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2
Methods of descent

2.1 Positioning

problem

That is to say ***K the*** *matrix of rigidity (of size NR) to reverse, if it has "good the symmetrical taste" to be defined*
positive (SPD in the Anglo-Saxon literature), which is very often the case in mechanics of structures, one shows by simple derivation which the following problems are equivalent:

· Résolution of the usual linear system with ***U*** *vector solution (of displacements or*

increments of displacements, resp. in temperature….) and ***F*** *vector representing the application of forces generalized with the thermomechanical system*

(P
éq
2.1-1
1)
Ku *=* ***F***
· Minimisation of the quadratic functional calculus representing the energy of the system, with <, > it usual Euclidean scalar product,
(P2) ***U*** *= Arg min J (****v****)*
v
NR

éq
2.1-2
with (
1
1
v*) =*
:
v, Kv - F*,* ***v*** *=* ***vT***
J
Kv - F *T* ***v***
2
2
Because of "definite-positivity" of the matrix which makes J strictly convex, the vector cancelling J (the cancellation of the derivative is a case particular to the convex and unconstrained case of the famous ones
relations of Kuhn-Tucker characterizing the optimum of a differentiable problem of optimization. It is called "equation of Euler") corresponds to single (without this convexity, one is not ensured of unicity. It is then necessary to compose with local minima!) total minimum ***U****. That is illustrated by following, valid relation whatever the* ***K*** *symmetrical,*
1
*J (****v****) = (****U****) + (****v*** *-* ***U****) T*
J
K *(****v*** *-* ***U****)* ***éq***
2.1-3
2
Thus, for any vector ***v*** *different from the solution* ***U****, the positive definite character of the operator returns*
strictly positive the second term and thus ***U*** *are also a total minimum.*

(A)
(b)
(c)
3 2
2
***Appear 2.1-a: Quadratic example of J in N=2 dimensions with** K =*
:
***and F** =*
: :
2 6
-
8
graph (A), datum lines (b) and vectors gradient (c).
***The spectrum of the operator is** (1; v1) = (7; [1,2] T) and (2; v2) = (2; [- 2,1] T).*
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This very important result in practice is based entirely on this famous definite-positivity property a little "éthérée" the matrix of work. On a problem with two dimensions (NR = 2!) one can of to make a limpid representation (cf [Figure 2.1-a]): the form paraboloid which focuses the single minimum
at the item [2, - 2] T of null slope.

· Minimisation of the standard in energy of the error ***E (v) = v - U****, more speaking for mechanics,*
(P ***U*** *= Arg min E* ***v***
3)
( )
v
NR
éq
2.1-4
with
(v): = Ke (v), E (v) = (Ke (v) *T*
E
E (v) = E (v) *2K*
From a mathematical point of view, it is anything else only one matric standard (licit since ***K*** *is SPD). One often prefers to express it via a residue* ***R (v) = F - Kv***
(v) = R (v) K r1
,
(v) = R (v) *T*
E
K R
1 ***(v) éq***
2.1-5

Note:

· The perimeter of use of the combined gradient can in fact of extending to any operator, not inevitably symmetrical or definite positive and even square! With this intention one defines solution of (P1) as being that, within the meaning of least squares, of the problem of minimization
(P)
2
*'****U*** *= Arg min* ***Kv - F***
éq
2.1-6
2
v
NR
By derivation one leads to the equations known as "normal" which the operator is square, symmetrical and positive

(
éq
2.1-7
2)' '

KT
P
Ku** = **KTf
3
2
1~K
One can thus apply a GC to him or one steepest-descent without too much of encumber.

· The solution of the problem (P1) is with the intersection of NR hyperplanes of N-1 dimension. For example, in the case of [Figure 2.1-a], it is expressed trivialement in form intersections of right-hand sides:
3x + 2x = 2
1
2

éq
2.1-8
2x + 6x = -8
1
2

Appear 2.1-b: Resolution of the n°1 example by intersection of right-hand sides.

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The methods of the gradient type dissociate this philosophy, they are registered

naturally within the framework of minimization of a quadratic functional calculus, in which they were developed and intuitively included/understood.

· When the matrix is not definite positive (cf [Figure 2.1-a] (A)), three cases of figures present themselves: it is definite negative ((b), that does not pose a problem, one works with $\tilde{\boldsymbol{K}} = - \boldsymbol{K}$ *to minimize instead of maximizing), singular ((c), the whole of solutions, if there exists, is a hyperplane) or unspecified ((D), it is about a problem of not saddle on which the methods of the descent type or combined gradient stumble).*

***Appear 2.1-c: Form J according to the properties of** K.*

· The error $\boldsymbol{E}(\boldsymbol{v})$ *which expresses the distance from the intermediate solution to the exact solution and the residue* $\boldsymbol{R}(\boldsymbol{v})$*, the error which this intermediate solution implies on the resolution of linear system, are bound by the relation*

$$\boldsymbol{R}(\boldsymbol{v}) = - \boldsymbol{K}\,\boldsymbol{E}(\boldsymbol{v})$$

éq
2.1-9

More important, this residue is the opposite of the gradient of the functional calculus

$$\boldsymbol{R}(\boldsymbol{v}) = - J(\boldsymbol{v})$$

éq
2.1-10
It is thus necessary to interpret the residues like potential directions of descent making it possible to decrease the values of J (they are orthogonal with the lines isovaleurs cf [Figure 2.1-a]). These basic recalls proves to be useful thereafter.
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2.2 Steepest
Descent

Principle

This last remark precedes the philosophy of the method known as "of the strongest slope", more known under its Anglo-Saxon denomination of "Steepest Descent": one builds the continuation of reiterated ***ui*** *in*
according to the direction whereby more J decrease, at least locally, i.e. I
I
I
D *= - J =* ***R***
with I
J =
: J (I
U*)*
I
I
and ***R*** *=*
: ***F - Ku****. To the ième iteration, one thus will seek to build* ***ui+1*** *such as:*

I +1
I
I I
U
=
: ***U*** *+* ***R***

éq 2.2-1
and
I +1
I
J

< J

éq 2.2-2
*Thanks to this formulation, one thus transformed a quadratic problem of minimization of size NR (in J and **U**) in a unidimensional minimization (in G and)*
I
I
I
To find
that

such
= Arg minG ()
[

m, M]

éq
2.2-3
I
with G () =
: J (I
I
***U** + **R**)*
*The following figures illustrate the operation of this procedure on the n°1 example: on the basis of not **u0=** [- 2, - 2] T (cf (A)) one seeks the optimal parameter of descent, 0, according to the line moreover*
*great slope **r0**; what amounts seeking a point pertaining to the intersection of a vertical plane and of a paraboloid (b), meant by the parabola (c). Trivialement this point cancels the derivative of parabola (D)*
0
G (0
)
2
R
= 0 J (1
***U**)*

0
0
1
0
0
*, **R** = 0 **R**, **R** = 0: =*

éq 2.2-4
0
0

R**, **Kr
This orthogonality between two successive residues (i.e successive gradients) produced a advance characteristic, known as in "zigzag", towards the solution (E). Thus, in the case of a badly conditioned system
*producing narrow and lengthened ellipses (the conditioning of operator SPD **K** is written like*

*the report/ratio of its extreme eigenvalues (**K**)*
max
: = which is themselves proportional to
min
axes of the ellipses. From where a direct and visual bond between bad matric conditioning and narrow valley
and tortuous where minimization is abused), the iteration count necessary can be considerable (cf [Figure 2.2-b] (b)).

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has
B
C

D
E
Be reproduced 2.2-a Illustration of Steepest Descent on the n°1 example: direction of initial descent (A),
intersection of surfaces (b), corresponding parabola (c), vectors gradient and their projection along the direction of initial descent (D) and total process until convergence (E).

Algorithm

From where unfolding

Initialization
0
***U** given*

in

Buckle
I
I
I
)
1
(
R** = **F** - **Ku
residue)

(new
I 2
R
I
(2)

=

descent)

of

optimal

(parameter
I
I
R*, *Kr
I +1
I
I I
)
3
(
U
*= **U** + **R***
reiterated)

(new
(4) Test of

via

stop
i+1
I
J
- J
(for example)

Algorithm 1: Steepest Descent.

To save a product matrix-vector it is preferable to substitute at the stage (1), the update residue following
I +1
I
I
I
R
*= **R - Kr éq***
2.2-5

However, so avoiding inopportune accumulations of round-offs, one recomputes it periodically

residue with the initial formula (1).
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Elements of theory

One shows that this algorithm converges, for any starting point ***u0****, at the speed (definite positivity of the operator ensures us of the "good nature" of parameter I. It is well an attenuation factor because 0 I*
1)

2
2
E (I
U *1*
+)
I
= E (I
U)
K
K
I 4
R

éq
2.2-6
I
with: = 1
1
- I
I

I
I
K R*, *R
Kr*, *R

*By developing the error on the basis of clean mode (J; **vj**) of the matrix **K***

(I
*E **U**) = **v éq 2.2-7***
J
J
J

the attenuation factor of the error in energy becomes
(2 22
I
J)
= 1- (
J
J

éq
2.2-8
23
2
J
)(
J
J
J
)
J
J
In [éq 2.2-8], the fact that the components J are squared ensures the priority ousting of the values clean dominant. One finds here one of the characteristics of the modal methods of Krylov type (Lanczos, Arnoldi cf [bib2] [§5] [§6]) which privileges the extreme clean modes. For this reason, Steepest Descent and the combined gradient known as "coarse" is compared to the iterative solveurs traditional (Jacobi, Gauss-Seidel, SOR…) more "smooth" bus eliminating without discrimination all them
components with each iteration.
*Finally, thanks to the inequality of Kantorovitch (some is **K** stamps SPD and **U** vector not no one:*
1
-
Ku**, the **U.K.U**, **U

*((**K**) 1 + (**K**) 1*
2
) 22
1

) one improves the legibility of the factor largely
4
U
4
2
of attenuation [éq 2.2-6]. At the end of I iterations, in the worst case, the decrease is expressed in the form
I

-
E (I
1
U)
*(**K**)*

E

*(**u0**) **éq***
2.2-9
K
*(**K**) +1*
K

J (i+
***U**) - J (**U**)*
*(**K**)*
2
1
-1
It ensures linear convergence (i.e. lim

. The rate
*I J (**ui**) - J (**U**): = (**K**)*
< 1
+1
of asymptotic convergence report/ratio of Kantorovitch is called) process in a number

iterations proportional to the conditioning of the operator. Thus, to obtain
E ***(ui)***
(

éq 2.2-10
u0
E
) ***K*** *(small)*
K
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one needs an iteration count about
(K) *1*
I
ln

éq 2.2-11
4

A badly conditioned problem will thus slow down the convergence of the process, which one had already "visually" noted with the phenomenon of "narrow and tortuous valley".

For better apprehending the implication of the spectrum of the operator and starting point in unfolding of the algorithm, let us simplify the formula [éq 2.2-8] while placing itself in the commonplace case where N=2.

While noting
1
=
2
μ =
the matric conditioning of the operator and
the slope of the error with ième
2
1
iteration (in the frame of reference of the two clean vectors), one obtains an expression more readable of the attenuation factor of the error (cf [Figure 2.2-b])
I
(2
+ μ) 2
2
= 1- (

éq
2.2-12
2
+ μ) (3
2
+ μ)
As for the modal solveurs, one notes that the importance of the conditioning of the operator is balanced by the choice of a good starting point: in spite of a bad conditioning, the cases (A) and (b) are very different; In the first, the starting point generates almost a clean space of
the operator and one converge in two iterations, if not they are the "sempiternal zigzags".
(E)
Appear 2.2-b: Convergence of Steepest Descent on the n°1 example according to the values of conditioning and of the starting point: large and μ small (A), and μ large (b), and μ small (c), small and μ large (D) and forms total = (, μ) (E).
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Note:

· This method of the strongest slope was initiated by Cauchy (1847) and was given to the taste day by Curry (1944). Within the framework of operators SPD, it is also sometimes called method of the gradient with optimal parameter (cf [bib26] [§8.2.1]). In spite of its weak properties of convergence, she knew her "hour of glory" to minimize quasi-unspecified functions J, makes some only derivable. In this case of appear more general, she then allows to reach only stationary points, with better local minima.
· To avoid zigzag the proceeded various effects of acceleration was proposed (cf Forsythe 1968, Luenberger 1973 or [bib28] [§2.3]) which has a speed of convergence similar to that of the gradient combined but for a calculation complexity quite higher. They thus fell gradually in disuse.
Let us note that alternatives of algorithm 1 were introduced to treat cases not SPD: Iteration of the minimum residue and Steepest Descent with standard of the residue (cf [bib35] [§5.3.2] [§5.3.3]).
· One finds through Steepest Descent a very widespread key concept in analysis numerical: that of the resolution of a problem, by projection of reiterated on under space approximating
NR
*, here Ki: =vect (****laughed****), perpendicular to another under space, here Li: =Ki. They are called, respectively, space of research or of approximation and space of constraint. For Steepest Descent, they are equal and reduced to their simpler expression but we will see that for the combined gradient*

they take the form of particular spaces, known as of Krylov.
Formally, with each iteration I of the algorithm, one seeks an increment thus
I
I I
=
*: **R** such as:*

I
***U** =*
*: **u0** + I*
I

= I I
*: **R** K*

I
éq
2.2-13
0
I
***R** - **K**, **W** = 0 **W** L =*
I
K I
This framework general constitutes the conditions of Petrov-Galerkin what is called
(cf [bib35] [§5]).

Complexity and occupation memory

*The major part of the cost calculation of algorithm 1 lies in the update [éq 2.2-5] and, more particularly, in the product **Kri matrix**-vector. In addition, it was already mentioned that its convergence was acquired and took place in an iteration count proportional to complexity of stamp (cf [éq 2.2-11]).*
*Its complexity is thus, if one takes account of the hollow character of the operator, about ((**K**) Cn) where C is the average number of nonnull terms per line of **K**.*
*The discretization finite elements of the elliptic operators of the second order (resp. fourth order) (case generally encountered in mechanics of the structures) implying conditionings operators in (**K**) = (2D*
*NR/) (resp. (**K**) = (4 D*
NR/)), with D the dimension of space,

2 +
4
1

+1
calculation complexity of Steepest Descent within this framework is written
D
Cn

(resp. D
Cn

).

As regards the occupation memory, only the storage of the matrix of work is possibly
necessary (in the absolute, Steepest Descent as the GC requires only the knowledge of the action
matrix on an unspecified vector and not its storage in extenso. This facility can
to prove to be invaluable for very greedy applications in DDLs (CFD, electromagnetism…)) :
(Cn). In practice, the data-processing installation of hollow storage imposes the management of vectors
additional entireties: for example, for MORSE storage used in Code_Aster, vectors
indices of end of line and indices of columns of the nonnull elements of the profile. From where one
effective memory complexity of (Cn + NR) whole.
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2.3

Method of "general" descent

Principle

A crucial stage of these methods is the choice of their direction of descent (per definition one
T
direction of descent (dd) to the ième stage, ***di****, checks: (I*
J) I
D *< 0) (dd). Even if the gradient of*
functional calculus remains the principal ingredient about it, one can choose another version ***di*** *completely of it that that*
required for Steepest Descent. To the ième iteration, one thus will seek to build ***ui+1*** *checking*
I +1
I
I I
U
=
: ***U*** *+* ***D***

éq 2.3-1
with
I
= Arg min J (I
I
U *+* ***D****)* ***éq***
2.3-2
[

m, M]
It is of course only one generalization of Steepest Descent seen previously and it is shown that its choices of the parameter of descent and its property of orthogonality spread
I
R*, I*
D
I
: = id, I
Kd

éq 2.3-3
I
***D**, i+1*
R
= 0
From where the same effect "zigzag" during unfolding of the process and a convergence similar to [éq 2.2-6] with an attenuation factor of the error undervalued by:
2
2
I
***R**, I*
D
I
1
I
I
R
D
:= 1-

,
éq
2.3-4
1
- I
***K R**, I*
I
***R Kd**, I*
D
(K)
I
I
R
D
This result one can then formulate two reports:

· the conditioning of the operator intervenes directly on the attenuation factor and thus on the speed of convergence,

· to ensure itself of convergence (sufficient condition), at the time of a given iteration, that is needed the direction of descent is not orthogonal with the residue.

*The sufficient condition of this last item is of course checked for Steepest Descent (**di** = **laughed**) and it a choice of direction of descent for the combined gradient will impose. To mitigate the problem*
~
*raised by the first point, we will see that it is possible to constitute an operator of work **K** whose conditioning is less. One speaks then about prepacking.*
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Always in the case of an operator SPD, course of a method of "general" descent is thus written

*Initialization **u0**, **d0** given, **r0** = **F** - **Ku0***
I
in

Buckle
I
)
1
(
***Z** =*
I
Kd

I
I
R, ***D***
I
( )
2
=
descent)

of

optimal

(parameter
I
I
D, ***Z***

I +
)
3
(
U *1 = I*
U *+ I I*
D
reiterated)

(new
I +
( )
4
R *1 = I*
R *- I I*
Z
residue)

(new
I +1
I
I
I +
)
5
(

Test of

via

stop
J
*- J, **D** or **R** 1 (for example)*

I +1
I +1
K
2
K
K

( )
6
dd

of

Calculation
D
*= **D** J, J, **D***

K
Algorithm 2: Method of descent in the case of a quadratic functional calculus.

This algorithm precedes already well that of the Gradient Combined (GC) which we will examine in the chapter
according to (cf algorithm 4). It shows well that the GC is only one method of descent applied in the framework of quadratic functional calculuses and specific directions of descent. Finally, only the stage (6) will be some packed.

Note:

· By successively posing like directions of descent the canonical vectors of
*axes of co-ordinates of space with NR dimensions (**di = I.E.(internal excitation)**), one obtains the method of*
Gauss-Seidel.
· To avoid the overcost calculation of the stage of unidimensional minimization (2) (produced

matrix-vector) one can choose to fix the parameter of descent arbitrarily: it is method of Richardson who converges as well as possible like Steepest Descent.

Complements

With a functional calculus J continues unspecified (cf [Figure 2.3-a] (A)), one exceeds the strict framework of the linear inversion of system for that of unconstrained nonlinear continuous optimization (J is then often called function objective cost or function). Two simplifications which had runs up to now become illicit then:

· The reactualization of the residue stage (4),
· The calculation simplified of the optimal parameter of descent stage (2).

Their reasons to be being only to use all the assumptions of the problem to facilitate unidimensional minimization [éq 2.3-2], one is then constrained to explicitly carry out this linear research on a functional calculus chahutée with this time of multiple local minima. Fortunately, there is a whole panoply of methods according to the degree of necessary information on function cost J:
· J (interpolation quadratic, dichotomy on the values of the function, the gilded section, regulate of Goldstein and Price…),
·
J, J (dichotomy on the values of the derivative, regulates of Wolfe, Armijo…),
·
J, J
2
, J (Newton-Raphson…)
· ...
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As regards the search for descent, there too of many solutions are proposed in literature (combined gradient nonlinear, quasi-Newton, Newton, Levenberg-Marquardt (these two last methods are used by Code_Aster: the first in the nonlinear operators (STAT/DYNA/THER_NON_LINE), the second in the macro one of retiming (MACR_RECAL)) …). Of long date, methods known as of nonlinear combined gradient (Fletcher-Reeves (FR) 1964 and Polak-Ribière (PR) 1971) proved to be interesting: they converge superlinéairement towards one local minimum at a reduced cost calculation and obstruction memory (cf [Figures 2.3-a]).
They lead to the choice of an additional parameter I which manages the linear combination between directions of descent

i+ 2
1

J
(FR)
2
I
J
I +1
I +1
I +1 I
i+1

D
= - J
+ ***D*** *with*
=
:

éq 2.3-5
i+1
J, i+1
J
- I
J
(PR)

2

I

J

has

B
C
Appear 2.3-a: Nonconvex example of J in N=2: graph (A), convergence with
a nonlinear GC of type Fletcher-Reeves (b), plan of cut
the first unidimensional research (c).
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From where the algorithm, by naming r0 opposite of the gradient and either the residue (which does not take place any more to be here),

Initialization u0 given, r0 = - J 0, d0 = r0

in

Buckle
I

)
1
(
To find
I
= Arg min J (I
I
U + D)
descent)

of

(parameter
[

m, M]
I +1
I
I I
(2) U
= U + D
reiterated)

(new
I +1
I
)
3
(
R
= - J +1
gradient)

(new
( )
4
Test of

via

stop
I +1
I
I

I
J
- J
+1
, D
or R
(for example)

I
2
R +1

(FR)
I 2

R
I +1
)
5
(
=
conjugaiso

of

(parameter
N)
I +1
I +1
I
R, R - R
(PR)

I 2

R

I +1
i+1
I +1 I
( )
6

D
= R + D
dd)

(news
Algorithm 3: Methods of the nonlinear combined gradient (FR and PR).

Nonlinear designation combined gradient is rather here synonymous with "nonconvex": there is no more
of dependence the problem of minimization enters (P2) and a linear system (P1). With the sight of algorithm 4, the great similarities with the GC consequently appear completely clear. Within the framework
to a quadratic function cost of type [éq 2.1-2b], it is just enough to substitute at the stage (1) the actualization of the residue and the calculation of the optimal parameter of descent. The GC is only one method of
Fletcher-Reeves applied to the case of a convex quadratic functional calculus.

Now that we started well the bond between the methods of descent, the combined gradient with the linear direction solvor SPD and his version, sight of the end of the spyglass optimization continues not
linear, we go (finally!) to pass to sharp from the subject and to argue the choice of a direction of descent
type [éq 2.3-5] for the standard GC.
For more information on the methods of the descent type, the reader will be able to refer to excellent works of optimization (in French language) of Mr. Minoux [bib28], J.C. Culioli [bib11] or J.F. Bonnans et al. [bib5].
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3
The combined gradient (GC)

3.1 Description
general

Principle

Now that the bases are set up we will be able to approach the algorithm of the Gradient Combined (GC) itself. It belongs to a subset of methods of descent which gather the methods known as "of combined directions" (the methods of Fletcher-Reeves and of Polak-Ribière (cf [§2.3]) form also part of this family of methods). Those recommend to gradually build directions of descents d0, d1, d2… linearly independent of manner to avoid the zigzags of the method of traditional descent.
Which linear combination then to recommend to build, at the stage I, new direction of descent? Knowing of course that it must take account of two crucial information: the value of gradient
I
I
J = R
- and those of the preceding directions 0 1
I 1
D,
-
D
D
K
.
I
?
D =
I
R +
J
D
éq
3.1-1
J <i
J
The easy way consists in choosing a vectorial independence of K-orthogonality type (like the operator

of work is SPD, it defines well a scalar product via which two vectors can be orthogonal)
(cf [Figure 3.1-a])
(D) Ti
J
K D = 0 I J
éq
3.1-2
also called conjugation, from where the designation of the algorithm. It makes it possible to propagate of close relation in
near orthogonality and thus to calculate only one coefficient of Gram-Schmidt to each iteration.
From where a profit in calculation complexity and very appreciable memory.

Appear 3.1-a: Example of pairs of K-orthogonal vectors in 2D: conditioning
K unspecified (A), perfect conditioning (i.e equal to the unit) = usual orthogonality (b).

One can thus be satisfied with a linear combination of the type
I
I
I I 1
D:
-
= R + D

éq 3.1-3
And this, more especially as it checks the sufficient condition [éq 2.3-4] and that because of orthogonality
[|éq 2.3-3b], the unidimensional research [éq 2.3-2] which follows takes place in an optimal space: the plan
formed by the two orthogonal directions (laughed, di-1).
It thus remains to determine the optimal value of the proportionality factor I. In the GC this choice take place so as to maximize the attenuation factor of [éq 2.3-4], i.e. the term
I
I 2
R, D
éq
3.1-4
1

- I
I
I
I
K R, R Kd, D
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It leads to the expression
2
I
R
I
:=
éq
3.1-5
2
I 1
-
R
and the same property of orthogonality of the successive residues as for Steepest Descent induces (but without the zigzags!)
I
R, I 1
-
R

= 0 éq
3.1-6
A condition “residue-dd is added”
2
I
R, I
I
D = R
éq
3.1-7
who forces to initialize the process via 0
0
D = R.

Algorithm

In short, by recapitulating the relations [éq 2.2-5], [éq 2.3-1], [éq 2.3-3] and [éq 3.1-3], [éq 3.1-5], [éq 3.1-7] it
occurs the traditional algorithm

Initialization u0 given, r0 = F - Ku0 d0
,
= r0

in

Buckle
I
I
I
)
1
(
Z = Kd
I 2
R
I
(2) =
(
descent

of

optimal

parameter
)
I
I
D, Z
I +1
I
I I
)
3
(
U
= U + D
(
reiterated

new
)

I +1
I
I I
(4) R
= R - Z
(
residue

new
)
)
5
(
Test of

via

stop
i+1
I
R, R +1
(for example

)
I
2
R +1
I +1
(6)
=
(
conjugaiso

of

parameter
optimal

N
)
I 2
R
I +1
I +1
i+1
I
(7) D
= R + D
dd)

(news
Algorithm 4: Standard combined gradient (GC).
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On the n°1 example, the "supremacy" of the GC compared to Steepest Descent is manifest (cf [Figures 3.1-b]) and is explained by the results of convergence [éq 2.2-9] and [éq 3.2-9]. In two cases, the same points starting and the same criteria of stop were selected: u0= [- 2, - 2] T and
2
I
6
R
< 10-
=
.

has

B
Appear 3.1-b: Compared convergences of Steepest Descent (A)
and of the GC (b) on the n°1 example.
Note:

· This method of the GC was developed independently by M.R.Hestenes (1951) and E.Stiefel (1952) of the "National Office of Standard" of Washington D.C. (seedbed of numericians with also C.Lanczos). First theoretical results of convergence are due to work of S.Kaniel (1966) and of H.A.Van der Vorst (1986) and it has really popularized for the resolution of large hollow systems by J.K.Reid (1971). The interested reader will find a history commented on and a bibliography exhaustive on the subject in papers of G.H.Golub, H.A Van der Vorst and Y.Saad [bib15], [bib16], [bib36].
· For the small history, extremely of its very broad diffusion in the industrial world and academic, and, of his many alternatives, the GC was classified in third position of the "Top10" of the best algorithms of the XXe century [bib10]. Just behind methods of Monte Carlo and the simplex but in front of algorithm QR, them transforms of Fourier and the solveurs multipôles!
· One finds traces in the codes of EDF R & D of the GC as of the beginning of the Eighties with the first work on the subject of J.P.Grégoire [bib18] [bib21]. Since it is spread, with an unequal happiness, in many fields, _ N3S, Code_Saturne, LADYBIRD, Code_Aster, TRIFOU, ESTET, TELEMAC _, and it was much optimized, vectorized and paralleled [bib19], [bib20], [bib22], [bib23].

Note:

Being very depend on the conditioning of the matrices generated by these fields of physics, it is generally more used in mechanics of fluids or in electromagnetism that into thermomechanical of the structures. From where supremacy, in this last field of activities, of the direct solveurs… Code_Aster with its multifrontale [bib33] not escaping the rule. One y cumulate often strong heterogeneities of material, loading and size of mesh, non-linearities and junctions elements isoparametric/elements structural which gangrènent dramatically this conditioning [bib31], [bib13].

· With the test of stop on the standard of the residue, theoretically licit but in practice sometimes difficult to gauge, one often prefers a criterion of adimensional stop, such as the residue laughed relating to the ième iteration: I = : (cf [§5.2]).

3.2
Elements of theory

Space of Krylov

By carrying out a second analysis of Petrov-Galerkin already evoked for Steepest Descent, one can synthesize
in a sentence the action of the GC. It carries out successive orthogonal projections on the space of Krylov
vect
where ***r0*** *is the initial residue*
: with the ième iteration
I (
0
K, ***R****): =*
(0 0
I 1
-
0
R, ***Kr***
K R
K
)
K = L =
. One solves the linear system (P
I
I

I (
0
***K**, **R**)*
*1) by seeking an approximate solution **ui***
in the subspace refines (space of search for dimension NR)
0
*With = **R** +*

éq
3.2-1
I (
0
***K**, **R**)*
while imposing the constraint of orthogonality (space of the constraints of dimension NR)
laughed**: = **F** - **Kui

éq
3.2-2
I (
0
***K**, **R**)*
This space of Krylov to the good property to facilitate the approximation of the solution, at the end of m iterations, in the form
K F
1
***um** = **r0** + P*

éq
3.2-3
m 1
*- **(K) F***
where Pm-1 is a certain matric polynomial of order M-1. Indeed, it is shown that the residues and them directions of descent generate this space
vect (0 1
m 1
R**, **R
***R** -*
K
) =m (0
K**, **R**) **éq
3.2-4
vect (0 1
m 1
D**, **D

D *-*
K
) =m (0
K**,** ***R)***
while allowing the approached solution, ***um****, to minimize the standard in energy on all space*
refine A
m
U
U
U *With*
éq
3.2-5
K
K
This joint result with the property [éq 3.2-4b] illustrates all the optimality of the GC: contrary to the methods
of descent, the minimum of energy is not carried out successively for each direction of descent, but jointly for all the directions of descent already obtained.

Note:

One distinguishes a large variety from methods of projection on spaces of the Krylov type, called more prosaically "methods of Krylov". To solve systems linear [bib35], [bib27] (GC, GMRES, FOM/IOM/DOM, GCR, ORTHODIR/MIN…) and/or of modal problems [bib2], [bib39] (Lanczos, Arnoldi…). They differ by the choice from their space of constraint and by that of the prepacking applied to the initial operator for to constitute that of work, knowing that different establishments lead to algorithms radically distinct (vectorial version or per blocks, tools of orthonormalisation…).

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Orthogonality

*As one already announced, the directions of descents are K-**orthogonal** between them. Moreover, it choice of the optimal parameter of descent (cf [éq 2.2-4], [éq 2.3-3a] or stage (2)) impose, of close relation in*
near, orthogonalities
I
***D**, m*
R
= 0 I < m
éq
3.2-6
I
***R**, m*
R
= 0
One thus notes a small approximation in the name of the GC, because the gradients are not combined (cf [éq 3.2-6b]) and the directions combined do not comprise only gradients (cf [éq 3.1-3]). But “let us not haggle” not, the indicated ingredients are nevertheless there!

At the exit of NR iterations, two cases of figures arise:

*· Is the residue is null **rN=0** convergence.*
· Is it is orthogonal with the NR the preceding ones directions of descent which constitute a base of the finished space of approximation
NR
*(as they are linearly independent). From where necessarily **rN=0** convergence.*

It would thus seem that the GC is a direct method which converges in with the more NR iterations, it is less what one believed before testing it on practical cases! Because what remains true in theory, in arithmetic exact, is put at evil by arithmetic finished calculators. Gradually,
in particular because of the rounding errors, the directions of descent lose their beautiful properties of conjugation and minimization leaves necessary space.
Known as differently, one solves an approximate problem which is not completely any more the projection desired of
initial problem. The method (theoretically) direct revealed its true nature! It is iterative and thus

subjected, in practice, with many risks (conditioning, starting point, tests of stop, precision orthogonality…).

To cure it, one can force during the construction of the new direction of descent (cf algorithm 4, stage (7)), a phase of reorthogonalisation. This very widespread practice in analyze modal (cf [bib2] [§5.3.1] and appendix 2) and in decomposition of field (cf [bib4] [§5.2.6]) can decline itself under various alternatives: total, partial, selective reorthogonalisation… via all a panoply of procedures of orthogonalisation: GS, GSM, IGSM, Househölder, Givens… For methods of the Krylov type, in order to spare the occupation memory and by preoccupations with a robustness, one
recommend réorthogonaliser systematically only compared to the first directions which generate the subspace invariant associated the with greatest eigenvalues (in treatment with signal, this space would define what one call the coherent structure of the field of solution displacement, i.e its substantial marrow).
These réorthogonalisations require the storage of the NR
K
orth first vectors ***D*** *(K =*
NR
K
1
and of
orth)
their product by the operator of work K
K
Z *=* ***Kd*** *(K =*
NR
K
1
. Formally, it is thus about
orth)
to substitute at the two last stages of the algorithm following calculation
max (I, NR
1
orth)
I +
R, ***Kdk***
I +1
I +
D
= ***R*** *1 -*
dk
dd

(news
***K** - orthogonalized)*

éq
3.2-7
K
K
K =0
D**, **Kd
*Through the various numerical experiments which were undertaken (cf in particular work of J.P. Gregoire and [bib3]), it seems that this reorthogonalisation is not always effective. Its overcost due mainly to the new products **Kdk matrix-**vector (and their storage) is not always compensated by the profit in iteration count total. Contrary to what is generally observed in decomposition of field (iterative method of primal Schur and FETI) where installation of this process is recommended. It is true that it intervenes then on a problem of interface much smaller and conditioned better than the total problem.*
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Convergence

*Because of particular structure of the space of approximation [éq 3.2-3] and property of minimization on this space of the approximate solution **um** (cf [éq 3.2-5]), one obtains an estimate of the speed of convergence of the GC*
2
2

2
E (I
***U)** = (I*
) E (0
U)
K
K

éq
3.2-8
I
with: = max (1 - I
m
P 1
- (I
)
1iN
*where one notes (I; **VI**) clean modes of the matrix **K** and Pm-1 an unspecified polynomial of degree with more M-1. Famous polynomials of Tchebycheff, via their good properties of increase in*
the space of the polynomials, make it possible to improve the legibility of this attenuation factor I. At the end of
I iterations, in the worst case, the decrease is expressed then in the form
I

-
E (I
1
U)
(K)
2
*E (**u0**) **éq***
3.2-9
K

***(K)** +1*
K

J (i+1
U) - J (U)
It ensures convergence superlinéaire (i.e. lim
) of the process in one
*I J (**ui**) - J (**U**) = 0*
iteration count proportional to the square root of the conditioning of the operator. Thus, for

to obtain
E ***(ui)***
(

éq
3.2-10
u0
E
) ***K*** *(small)*
K
one needs an iteration count about
(K) *2*
I
ln

éq 3.2-11
2

Obviously as we noticed many times, a badly conditioned problem will slow down convergence of the process. But this deceleration will be less notable for the GC than for Steepest Descent (cf [Figures 3.2-a]). And in any event, the total convergence of the first will be better.

(A)

(b)
Appear 3.2-a: Compared convergences of Steepest Descent (A) and of the GC (b) (with the factor ½ near) according to conditioning.
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Note:

· In practice, benefitting from particular circumstances, _ better starting point and/or advantageous spectral distribution _, the convergence of the GC can be much better than what lets hope [éq 3.2-9]. Methods of Krylov tending to flush out firstly extreme eigenvalues, the "effective conditioning" of the operator of work is some improved.
· On the other hand, certain iterations of Steepest Descent can get the best decrease of the residue that same iterations of the GC. Thus, the first iteration of GC is identical to that of Steepest-Descent and thus with a rate of convergence equal.

Complexity and occupation memory

*As for Steepest Descent, the major part of the cost calculation of this algorithm lies in the stage (1), the product matrix-vector. Its complexity is about (kcN) where C is the number means of nonnull terms per line of **K** and K the iteration count necessary to convergence. For*
3
NR
to be much more effective than simple Cholesky (of complexity

) it is thus necessary:
3
· Bien to take into account the hollow character of the matrices resulting from the discretizations elements
stop (MORSE storage, produced matrix-vector optimized adhoc): C << NR.
· Préconditionner the operator of work: K << NR.

One already pointed out that for an operator SPD his theoretical convergence occurs in, at most, NR iterations and proportionally with the root of conditioning (cf [éq 3.2-11]). In practice, for large badly conditioned systems (as it is often the case in mechanics of the structures), it can be very slow to appear (cf phenomenon of Lanczos of the following paragraph).
Taking into account conditionings of operators generally noted in mechanics

structures and recalled for the study of complexity of Steepest Descent, which one takes them again

1 +
2
1

+1
notations, calculation complexity of the GC is written
D
Cn

(resp. D
Cn

).

As regards the occupation memory, as for Steepest Descent, only the storage of stamp work is possibly necessary: (Cn). In practice, the data-processing installation of hollow storage imposes the management of additional vectors of entireties: for example for storage MORSE used in Code_Aster, vectors of the indices of end of line and the indices of columns of the elements of the profile. From where effective a memory complexity of (Cn) real and (Cn + NR) whole.

Note:

These considerations on the obstruction memory do not take into account the problems of storage of a possible preconditionnor and workspace which its construction can temporarily to mobilize.

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3.3 Complements

Equivalence with the method of Lanczos

In "triturating" the property of orthogonality of the residue with any vector of the space of Krylov (cf [éq 3.2-2]), it is shown that the m iterations of the GC lead to the construction of one factorized of Lanczos of the same order (cf [bib2] [§5.2])
m

m
m
m
m T
KQ = Q T -
Q E
éq
3.3-1
m 1
-
m
4
1
4
2 3
m
R
while noting:

· Rm the specific "residue" of this factorization,
· EM the mième vector of the canonical base,
I
R
·
I
Q =

:
the vectors residues standardized with the unit, called for the occasion vectors of
I
R
Lanczos,
0
M-1
R
R
.
m
Q =
:

K
the orthogonal matrix which they constitute.
0
M-1
R
R

Tm
m

NR
K
Qm
=
Qm
-
0

m

Rm

m
m

Appear 3.3-a: Factorization of Lanczos induced by the GC.

The matrix of Rayleigh which expresses the orthogonal projection of K on the space of Krylov

m (
0
K, R)
takes the canonical form then

1

1
-
0
0

0
0

1
1

1

-
+
...
0
0
1
0

m
T =

éq
3.3-2

M-1

0
...
...
-
m2

M-1
M-1

1

0
0
-
+

m2
M-1
m2

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Almost without additional calculation, the GC thus provides the approximation of Rayleigh of the operator of
work in a form sympathetic nerve, _ symmetrical square matrix tridiagonale of flexible size _,
it will be easy to deduce the spectrum (via, for example, a robust QR, cf [bib2] appendix 2)).
At the end of a linear inversion of system led by a simple GC one thus can, with less expenses,
to know, in addition to the sought solution, an approximation of the spectrum of the matrix and thus of
its conditioning [bib24]. This functionality should be inserted soon in Code_Aster of
manner to control the level of prepacking.

Note:

· The operator of work being SPD, its matrix of Rayleigh inherits this property and one
its decomposition m obtains directly
m
m
T = L D (L) T
m
with
1

1
0
0
0

0
0
0
0

1

1

m
-
1
...
0
m
0
...
0
L =
:
and D =
:

éq 3.3-3
0
...
...
0

1

0
... ...
0
M-1

0
0 -
1

1
0
0
0

M-1

Method of Lanczos not calculating these intermediate terms I and I, but directly the tridiagonaux terms, it does not have access to this information. On the other hand, it can take into account symmetrical matrices not necessarily definite positive.
· Donc, that it is a question of reversing a linear system or of determining part of sound spectrum, these methods of Krylov function on the same principle: to determine gradually vectors of bases generating the space of projection into same time that the result of this projection. In the GC as in Lanczos, one évertue to make the this projection most robust (orthogonality of the basic vectors), more reduced (size of projection = iteration count) and simplest possible (projection orthogonal). For the modal solvor, another constraint is juxtaposed: to approximate it better possible the spectrum of the operator of work by that of the matrix of Rayleigh. In nonsymmetrical, one speaks about either orthogonal but oblique projection and them bringings together is carried out then between GMRES and Arnoldi [bib35].
· Into arithmetic exact, at the end of NR iterations, one completely determined all it spectrum of the operator. In practice, problems of round-offs and orthogonalisation us in prevent. But, if one is sufficiently patient (m>>N!), one is nevertheless able to detect all the spectrum: it is what is called the phenomenon of Lanczos (cf [bib2] [§5.3.1], [§5.3.3] or [bib39]).

Note:

Paradoxically the loss of orthogonality is especially ascribable with convergence of a clean mode more than to the rounding errors! This analysis due to CC.Paige (1980) attests that as soon as a clean mode is captured it the fitting disturbs orthogonal of the vectors of Lanczos (in the case which worries us, them residues and directions of descent). Into modal, digital processing of it parasitic phenomenon was the subject of many palliative developments. For what is GC, it must attach the effectiveness of the methods of reorthogonalisation directions of descent evoked previously.

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Encased Solveurs

As one already pointed out, the linear solveurs are often hidden with deepest of others algorithms, and in particular, of the nonlinear solveurs of Newton type. It is for Code_Aster them cases of application most frequently used: operators STAT_NON_LINE, DYNA_NON_LINE and THER_NON_LINE.
In such a configuration, a linear solvor such as the CG can draw his pin from the play (by report/ ratio
with a direct solvor). Its iterative character proves to be useful to make evolve/move the test of stop of the algorithm
(cf stage (5) algorithm 4) according to the relative error of the nonlinear solvor who encapsulates it: it is
the problems encased solveurs (one also speaks about relieving) [bib12], [bib17], [bib8], [bib9] of which it
CERFACS is the “spearhead” in France.
By slackening at the convenient period the necessary precision on the cancellation of the residue (for example strategy
in the case modal method of Krylov type + GC [bib8]) or, on the contrary by hardening it
(resp. modal method of power type + Gauss-Seidel + GC [bib9]), one can thus gain in calculation complexity on the GC without modifying the convergence of the total process. It is of course necessary to put
at the point of the simple and inexpensive criteria so that the profit is substantial. This functionality should be inserted soon in Code_Aster (internal and automatic piloting of the value RESI_RELA).

Note:

Certain authors also put the question about sequence of these algorithms: “Newton GC” or “GC Newton”? If from a data-processing point of view and conceptual, the question is quickly distinct: one prefers the first solution, more readable, more flexible and which does not disturb existing it, from a numerical and algorithmic point of view, it division is more moderate. That can depend on the “technical tripailles” deployed by

non-linear solvor (tangent matrix, unidimensional minimization…) and of the deployed GC (preconditionnor). Nevertheless, it seems that the natural order, "Newton GC", is more effective and most evolutionary.

Parallelism

The GC as much of iterative solveurs lends itself well to parallelism. It is scalable, i.e. that if a calculation with NR ddls functions on P processors, its during with NR must function on P. Them principal elementary operations which constitutes it (produced matrix-vector and scalar product) break up effectively between various processors, only the parallelization of the preconditionnor (often based on a direct solvor) can prove to be hazardous (problems often called: "parallel preconditioning").
Thus on the CRAY machine, J.P.Grégoire adapted and paralleled, in memory shared [bib20] and in distributed memory [bib23], an algorithm of packaged combined the gradient type. Four types operations are carried out jointly: elementary vectorial operations, products scalars, the products matrix-vector and the construction of the preconditionnor (diagonal). Problems of "parallel preconditioning" which was solved only for the diagonal preconditionnor, do not have unfortunately not allowed the bearing of these models in the official version of Code_Aster.

We besides now will tackle the thorny question of prepacking for the GC. It will become the Packaged Combined Gradient then: GCPC.
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The packaged combined gradient (GCPC)

4.1 Description
general

Principle

Like one could note it (and to hammer it!) in the preceding paragraphs, the speed of convergence of the methods of descent, and in particular that of the combined gradient, depends on conditioning of the matrix (K). More it is close to its value floor, 1, better is convergence.
The principle of prepacking is then "simple", it consists in replacing the linear system of problem (P1) by a system are equivalent of the type
(~
-
-
P
=
éq
4.1-1
1)
MR. K
1
U
1
3
2
1
{
M F
~
~
K
F
such as, ideally:

· Conditioning is obviously improved by it (this theoretical property, just like following, only are very seldom shown. They are often guaranteed only by
~
numerical experiments): (K) << (K).
· Just like spectral distribution: more packed eigenvalues.
· M-1 is inexpensive to evaluate (as for the initial operator, one has often right need for to know the action of the preconditionnor on a vector): Mv = U easy to reverse.

· Voire easy to establish and, possibly, effective to parallel.
· That it is as hollow as the initial matrix because it acts to limit the obstruction memory additional.
~
· That it preserves at the matrix of work K the same properties as that initial: here, it character SPD.

~
In theory, the best choice would be M-1=K-1 bus then (K = I
, but if it is necessary to reverse
NR) = 1
completely the operator by a direct method to build this preconditionnor, it is only of little practical interest! Though, whom will be seen thereafter this idea is not as eccentric as that. Known as differently, the objective of a preconditionnor is to thus pack the spectrum of the operator of work,
as one already mentioned, his "effective conditioning" will be improved of pair with convergence of the GCPC.
Graphically, that results in the more spherical shape of the graph of the quadratic form. Even on a system with N=2 dimensions and with a "rough" preconditionnor (cf [Figure 4.1-a]), them effects are notable.

(A)
(b)

Appear 4.1-a: Effect of diagonal prepacking on the paraboloid of the n°1 example:
~
(A) without, (K) = 3.5, (b) with, (K) = 8
.
2 .
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:

In the absolute, one can préconditionner a linear system by the left (“left preconditioning”), by right-hand side (resp. “right”) or by making a mixture of both (resp. “Split”). It is this last version which goes
to be retained for our operator SPD, because one cannot directly apply the GC to solve
(~P: even if M-1 and K are SPD, it is not inevitably the case of their product.
1)
The easy way then consists in using a matrix of prepacking SPD, M, for which one thus goes
to be able to define another matrix (M being symmetrical real, it is diagonalisable in the form
T
M = UDU with D: = diag (

and U stamps orthogonal. Required matrix SPD
I)
> 0
I
from the associated decomposition M1/2 = Udiag comes then () T
U) M1/2 such as (M1/2) 2=M. From where
I
the new problem of work, this time SPD
*(^*
-
-
-
P
=

éq
4.1-2
1)
1 2
1 2
1 2
1 2
1
M 4
4 2
KM4

4 3M 3
2
1 U m3
2
1 F
*K^*
*u^*
*f^*
on which one will be able to apply the standard algorithm of the GC to constitute one what is called Gradient Combines Packaged (GCPC).

Algorithm

In short, in substituent in the algorithm 4, expressions of the preceding problem (^P and while working
1)
a little with simplification the whole to handle only expressions in K, U and F, it occurs it following unfolding.

Initialization u0 given, r0 = F - Ku0 d0
-
,
= M R
1 0 g0
,
= d0

in

Buckle
I
I
I
)
1
(
Z = Kd
I
I
R, G
I
(2) =
(

descent

of

optimal

parameter
)
I
I
D, Z
I +1
I
I I
)
3
(
U
= U + D
(
reiterated

new
)
I +1
I
I I

(4) R
= R - Z
(
residue

new
)
)
5
(
Test of

via

stop
i+1

I
R, R +1
(for example

)
I +1
1
-
I
(6) G
= M R +1
(
préconditi

ésidu
R
onné)
I +1
I
R, G +1
I +1
(7)
=
(parameter
conjugaiso

of

optimal

N
)
I
I
R, G
I +1
I +1
i+1
I
)
8
(
D
= G + D

dd)

(news
Algorithm 5: Packaged combined gradient (GCPC).
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~
But makes some, the symmetrical character of the initial packaged problem (P is quite relative. It is 1)
indissociable of the subjacent scalar product. If instead of taking the usual Euclidean scalar product, one uses a definite matric scalar product compared to K, M or M-1, it is possible to symmetrize it packaged problem which was not it initially. As for the methods of Krylov into modal,
it is the couple (operator of work, scalar product) which it is necessary to modulate to adapt to the problem!
Thus, M-1K being symmetrical compared to the scalar Mr.-product, one can substitute this new operator of work and this new scalar product in the algorithm of the nonpackaged GC (algorithm 4)
K Mr. K
1

, M
And (surprised Ô!) by working the expressions a little, one finds the algorithm of the GCPC exactly precedent (algorithm 5). One proceeds in the same way with a prepacking on the right, KM-1, via one scalar Mr.-1-product. Therefore, prepacking on the right, on the left or "splitté with mode SPD", all lead rigorously to the same algorithm. This observation will be useful for us thereafter

when one realizes that the matrices provided by Code_Aster (and as the préconditionneurs as one their will associate) are not always in conformity with the ideal scenario (^P.
1)

Note:

· This alternative of the GCPC, which by far is spread, is sometimes called in literature
: packaged combined gradient not transformed (`untransformed preconditioned conjugate gradient'). In opposition to the transformed version which handles clean entities of the new formulation.
· General methods of descent and, particularly nonlinear GC, also préconditionnent (cf [§2.3]). This time with an approximating preconditionnor the reverse of Hessien at the point considered, so as to make spherical the graph of functional calculus in the vicinity of this point.

Overflight of the principal préconditionneurs

The solution often adopted for its simplicity of implementation, his report/ratio "effectiveness numerical/overcost calculation" for problems too badly not conditioned, consists with préconditionner by the diagonal of the initial operator
M =
: diag K éq
4.1-3
J
(II)
It is what is called the diagonal prepacking or of Jacobi (JCG for `Jacobi Conjugate Gradient') by reference to the iterative method of resolution of system linear of the same name.

Note:

· It is the solution often adopted in mechanics of the fluids [Gre97] (N3S, Code_Saturne), where the process of Uzawa naturally produces a well conditioned problem, and which made fame of the JCG, transformed for the occasion into true "animal of race" dedicated to large calculators: vectorization, parallelization, management of the masks and the memories…
· It is a solution suggested, in mechanics of the structures, by good numbers of codes commercial: ANSYS, Zébulon, Nastran (cf [bib4] [§8.3]). It was presents in Code_Aster but its unreliability led to its resorption.
· This principle of preconditionnor "of poor" extends to the methods of descent in taking this time the diagonal of Hessien.

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Another very widespread preconditionnor is the SSOR (for Symetric Succesive Over Relaxation). Like the precedent, it is deduced from an iterative method "ancestral": method of Gauss-Seidel released. By breaking up the initial operator in the usual form
T
K *=*
: ***D*** *+* ***L*** *+* ***L*** *where* ***D*** *is its*
diagonal and ***L*** *its strictly lower triangular part, it is written (according to the parameter relieving)*
M
()
1
:
1
=
-
T
D
L D D
L
éq
4.1-4
SSOR
(
+
+
< <
- 2)(

) (
) 0
2
*It has the advantage of not requiring report storage and of overcost calculation, since it is directly elaborate starting from **K**, while being very simple to reverse (an descent-increase). On model problems (famous "the Laplacian on the square unit") of the theoretical results were exhumed in finite elements*
(K)
1
=
1
1
:
MR. K

éq
4.1-5
2
(-)
=
:

H
H
In addition, it with faculty, for an operator SPD, to fix quotas for his spectrum in the band]]
1
,
0 .
However, it proved in practice industrial, less effective than the préconditionneurs of incomplete Cholesky type which we will approach in the following paragraph. And it poses the problem delicate of the choice of the parameter of optimal relaxation, by nature very "problem-dependent".

Note:

· This preconditionnor was proposed by D.J.Evans (1967) and was studied by O.Axelsson (1974). It is declined in grinds versions: nonsymmetrical, by blocks, with parameter of relieving optimal…
· By posing =1 one finds the particular case of Symmetrical Gauss-Seidel
M
*() = - (**D** + **L**) **D-1***
:
(
T

D *+* ***L***
éq
4.1-6
SGS
)
A string of other préconditionneurs thus was born since about thirty years in literature: especially polynomial [bib7], [bib34] but also ACP, ADI, ADDKR, by elements…. Some are specific of an application, others more general. The "effects of modes" also made them work! For more information one will be able to consult the monumental sum made by G.Meurant [bib27] or books of Y.Saad [bib35] and H.A.Van der Vorst [bib40].

4.2
Incomplete factorization of Cholesky

Principle

One has just seen that the préconditionneurs is often inspired by linear solveurs with whole share: Jacobi for that diagonal, Gauss-Seidel for SSOR. That based on an Incomplete factorization of Cholesky (IC) does not escape the rule! But it is based this time, not on another method iterative, but on his/her "enemy brother", a direct method of Cholesky type. From where denomination ICCG ("Incomplete Cholesky Conjugate Gradient") given to the coupling of this preconditionnor with the GCPC.

The initial operator being SPD, it admits a decomposition of Cholesky of the type
T
K *=* ***DC*** *where* ***C*** *is a lower triangular matrix. One calls incomplete factorization of Cholesky, the search for one stamp* ***F*** *triangular lower as hollow as possible and such as* ***FFT*** *is close to* ***K*** *in one feel to define. For example, by posing* ***B*** *=* ***K*** *-* ***FFT*** *one will require that the relative error (expressed in a matric standard with the choice)*
B
=
:
éq
4.2-1
K
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that is to say smallest possible. With the reading of this "rather evasive" definition one foresees the profusion of
possible scenarios. Each one went there from its own incomplete factorization! The work of G.Meurant [bib27] great diversity shows some: IC (N), MIC (N), released, reordered, by blocks….

*However, to be simplified the task, one forces often a priori the hollow structure of **F**, i.e.*
its graph
*(**F**): = ({I, J), 1 J I -, 1 1 I NR, **F***
éq
4.2-2
ij
} 0
It is obviously a question of finding a compromise: the more this graph will be extended and the more the error [éq 4.2-1]
will be small but more calculation and storage of what is not (in the case which interests us) only one preconditionnor will be expensive. Generally, the préconditionneurs are recursive and in their
*basic level, they impose on **F the** same structure as that of **C**: (**F**) = (**C**).*

Note:

· Initialement, these incomplete factorizations were developed to solve
repeatedly a linear system of type (P1)
T
I +1
I
FF U
*= **F - Drunk***
éq
4.2-3
*[**FF** -1*
T

B]
The "nerve of the war" being then the spectral ray
() that a judicious choice
*of (**F**) can notably contribute to make fall.*
*· This principle of incomplete factorization spreads without sorrow with the standard case where the operator is written **K** = **LU** with this time **B** = **K** - **LU**. One speaks then about factorization Incomplete of type LU (ILU for "Incomplete LU").*

Strategy retained in Code_Aster

It is about a preconditionnor of the type ILU (because we will see in the paragraph according to whether the matrices
*of work of Code_Aster often lose their definite-positivity) inspired of work of H. Van DER Vorst [bib40]. The matrices remaining however symmetrical, one can write **K** = **LDLT** and **B** = **K** - **LDLT**.*

Note:

The matrix not being regular simply symmetrical SPD but one is not, a priori,
*ensured of the existence of one factorized **LDLT** without having recourse to permutations of lines and of columns (**km No** = **LDLT** with **P** stamps permutation). Scenario which was not envisaged in no linear solveurs of Code_Aster and that, in any event, MORSE storage of*
matrices of the prohibited GCPC. Fortunately, it will be seen that a happy combination of circumstance allows to resolve the situation (cf [§5.1])!

In any rigour one should speak about incomplete factorization of type ILDLT but in the literature and in documentations of the codes, one amalgamates already ILU and IC, even their alternatives, it are not thus
not the sorrow to enrich the list by the acronyms!
This incomplete factorization, in the line line of the preceding theoretical recalls, rests on two reports which we now will support:

· concept of filling by levels,
· low magnitude of the terms resulting from this filling.

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Filling by levels

The construction of factorized is carried out, line by line, via the usual formula
1
J -1

***L** =*
K
L D L
éq
4.2-4
ij

-
ij
ik K jk
***D** J*
K =1

*From where the phenomenon of progressive filling of the profile ("fill-in" in English): initially the matrix **L** with*
*the same filling as stamps it **K**, but during the process, the null term of **Kij** can*
*to correspond a term not no one of **Lij**. It is enough that there is a column K (< J) comprising a term not no one for lines I and J (cf [Figure 4.2-a]).*

J
K

J

Ljk

I

Lik

*New **Lij***

Appear 4.2-a: Phenomenon of filling during factorization.

Besides these nonnull terms being able them same to correspond to former fillings, from where a concept of level of recursivity which can be interpreted like as many "levels" of filling.
*One will speak thus about factorized incomplete of level 0 (stored in **L** (0)) if it reproduces with identical the structure (but of course not values which is different) diagonal part lower strict of **K** (i.e the same graph). Factorized level 1 (resp. **L** (1)) will be able it to include filling led by nonnull terms of **K** (noted terms r1 in [Figure 4.2-b]), that of level 2 (resp. **L** (2)) will be able to mingle with it the new nonnull terms (r1) preceding to constitute possible new terms (noted r2), and so on recursively…*
*This is illustrated on the academic case of a matrix digs pentadiagonale resulting from the discretization differences finished of the Laplacian on a uniform grid 2D (cf [Figure 4.2-b]). One only represents any structure: D, terms diagonal spectators, *, terms initially nonnull, laughed, terms filled with the stage n°i.*
*Factorized incomplete the **L** (3) led here to a complete filling, the relative error will be then null = 0 and the GCPC converge in an iteration. The interest of the exercise is of course purely didactic!*
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*

D

* *
sym

* D
0

*0 * *

*0 * D

***K** = 0 0 ***
***L** (0)*

*=

*0 0 * D

* 0 0 *
*

* 0 0 * D

*0 * 0 0
*
*

*0 * 0 0 * D

*0 0 * 0*
0
*

*
*0 0 * 0 0 * D*

D

D

** D*
0

** D*
0

*0 * D*

*0 * D*

***L** ()*

1 = 0
0
*
D
***L** (2)*

=

*0 0 * D*

** r1 0 * D*

** r1 0 * D*

*0 * r1 0 * D*

*0 * r1 r2 * D*

1

1
2

*0 0 * R*
*0 * D*
*0 0 * R*
R
** D*

Appear 4.2-b: Structure of the various factorized incomplete ILU (p) on the academic case of the Laplacian.

Low magnitude of the terms resulting from the filling

In addition, one empirically notices a very interesting property of these new terms resulting from the filling: their absolute value decrease while moving away from the zones already filled by nonnull terms of ***K****. Either, in the preceding case, of the principal under-diagonal and that external. To be convinced some it is enough to visualize the following function (cf [Figure 4.2-c])*
NR K

2
L

- K I, I
y (K)

+
I =1

=
: log
éq
4.2-5
10

NR - K

who represents the orders of magnitude of the terms of the kth under-diagonals. For example y (0) corresponds to the principal diagonal.

Principal diagonal

0

y (K)
***L** =*

Function amplitude
External diagonal

terms

under-diagonal
0

K

Number of the under-diagonal
***Lk+i**, I*

***Appear 4.2-c: Relative importance of the diagonals of** L*
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If one recapitulates, one thus has:

· level of a recursive and skeletal filling,
· a less importance of the terms corresponding to élévés levels of filling.

*From where a certain "full power" left with incomplete factorizations ILU (p) neglecting these terms medians. This approximation "cheaply" of **K-1** will be used then as preconditionnor in algorithm 5 of GCPC (M-1=K-1). To preserve a certain interest at the thing (if not as much to solve the problem directly!), one limits oneself to the first levels of filling: p =0, 1, 2 even 3. All depends on the number of linear systems which one will have to solve with the same matrix.*

Note:

· There exists little of theoretical results on this type of preconditionnor. What does not prevent it not to appear often effective [bib22].
· It is a solution suggested, in mechanics of the structures, by considerable large codes: ANSYS, Zébulon, Nastran, Code_Aster (cf [bib4] [§8.3]).

Complexity and occupation memory

As regards the cost additional calculation with one factorized incomplete, it is very difficult at to estimate and, in practice, depends largely on the way in which it was coded (with our knowledge, there do not exist theoretical results on the thing). With a low level of
3
NR
filling, one hopes only that it is much lower than

of a complete factorization.
3
Because a compromise is to be found between occupation memory and complexity. To constitute these factorized incomplete, it is often necessary to allocate temporary workspaces. Thus, in the establishment of prepacking ILU (p) of Code_Aster, it was selected to facilitate the algorithmic one (sorting, search of indices and coefficients in the profile, management of the recursivity and the filling…)
by creating matrices temporarily representing storage full with the initial matrix and sound filling.

With this temporary workspace, additional storage due is added of course to final preconditionnor. On level 0, it is theoretically at least of the same order as that of stamp. From where minimal effective a memory complexity of the GCPC of (2cN) real and (2cN + 2N) whole. When one goes up in complétude, only the practice can bring a pretence of answer.

In short, one empirically shows with Code_Aster, that an obstruction memory should be envisaged Cn total (
) real and (Cn

+ 2N) whole with
· =2,5 in ILU (0) (level by defect in Code_Aster),
· =4,5 in ILU (1),
· =8,5 in ILU (2).

For more precise details on the data-processing establishment of the GCPC in Code_Aster and its use on
case-tests quasi-industrialists, one will be able to consult the notes [bib22] or [bib3].

Note:

In Code_Aster, the obstruction memory of an entirety is identical to that of a reality and, for the GCPC, no reservation of place or pagination memory could installation.
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5 Establishment in Code_Aster

5.1 Difficulties particular

Taking into account of the limiting conditions

In Code_Aster, there are two manners of taking into account the boundary conditions and this stage be carried out during the effective construction of the matrix of rigidity:

· By double dualisation [bib30] (operators AFFE_CHAR_ACOU/MECA/THER) while using ddls specific, known as of Lagrange, which includes the groups of unknown factors concerned and allow to check all types of linear limiting conditions (Dirichlet generalized)
You = 0

éq 5.1-1
*with **T** stamps real of size p X N. This technique packs the matrix of rigidity in one new matrix, known as "dualized", which becomes the new matrix of work*
K
TT
TT

*) **K = T - Id Id***
éq
5.1-2

T Id
- Id
where and are two strictly positive realities.

· By simple elimination of the unknown factors (operators AFFE_CHAR_CINE) in substituent and in carrying out the setting has zero of the p lines and columns concerned of the matrix of rigidity. This is valid only for blockings of ddls, one cannot thus take into account linear relation. The matrix of rigidity is written then

*) **K** 0*
***K** =*

éq
5.1-3
*0 **Id***
*by noting **K** its unchanged part.*

Each of the two approaches has its advantages and its disadvantages: generics, modularity but increase in the size of the problem, degradation of its conditioning and loss of its definite positivity for the first. Contrary to the second which decreases by it the size but which is circumscribed with certain types of limiting conditions and is, by means of computer, more delicate to implement. In Code_Aster, primacy being given to the facility of use like to the contingencies numérico-data processing of robustness, establishment, of maintenance and evolutionarity, it is clearly the first approach which was privileged.

Note:

Other approaches were possible: simple dualisation, taken into account of conditions limit in the variational formulation, combined gradient projected (GCP cf [bib4] [§6.2.3])...
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Consequence on the GCPC

~
*With AFFE_CHAR_CINE, the operator of work **K** remaining SPD, all the theory recalled in preceding paragraphs applies. On the other hand, with AFFE_CHAR_ACOU/MECA/THER (case more frequent in practice), it is not any more the case, because it becomes simply symmetrical and loses its character*
defined positive. The consequences are then of three types:

· The total convergence of the GCPC (cf [éq 3.2-9]) is not guaranteed any more,
· When it occurs, it is slowed down (cf [éq 3.2-11]) by a degraded conditioning
(
)
***K**) << (**K**),*
*· Prepacking cannot be carried out any more via one IC (p), but rather by a ILU (p) (cf [§4.2]). It is necessary still that factorization **LDLT** is always possible without having with to permute line or column!*

Fortunately, an adequate provision of the multipliers of Lagrange compared to the groups of ddls that they concern (they must include these ddls cf [bib30] [§4]), allows to obtain without blow to férir
this incomplete factorization. B.Nitrosso [bib29] thus showed that it is stable and that, in addition, used as a preconditionnor, it restores the total convergence of the GCPC (ouf!).

Note:

· In this same report/ratio, B.Nitrosso shows that diagonal prepacking is not possible (cf [§4.1]), because it leads to the cancellation of the scalar product to the denominator stage (7) of algorithm 5: calculation of the optimal parameter of conjugation. Therefore, contrary to N3S and Code_Saturne, Code_Aster cannot propose this option not very reliable.
· It does not remain about it less, that with dualized limiting conditions, the conditioning of the operator of work is degraded and thus, the convergence of the slowed down GCPC. Others commercial codes, with the instart of ANSYS [bib31], already made this report.

Obstruction memory

Taking into account the elements of [§4.2] and owing to the fact that in Code_Aster, an entirety has the same one
*obstruction memory that a reality (8 bytes), effective memory complexity of the GCPC is of at least size of **K** with*

· =2,5 in ILU (0) (level by defect in Code_Aster),
· =4,5 in ILU (1),
· =8,5 in ILU (2).

In addition, contrary to the direct solveurs, the GCPC could not profit from a pagination because, on this point, its algorithmic dessert: all the matric blocks are used a great number of times, in fact with each iteration, via the product matrix-vector.
Moreover, contrary has what was made in N3S, with the definition of the calculation case (grid, materials, loadings and conditions limit…), one does not hold, by advance, part of memory to store there the vectors required by the GCPC. Therefore, progressively with the execution of operators of the command file, the memory becomes a "true Gruyere" where it becomes difficult to insert large objects.
The pagination of the direct solveurs enables them to cross without too many difficulties this shelf… it is of course differently of the GCPC. From where, according to the sequence of the orders, an inflation of the sizes
necessary memories to make function its "job" with METHODE=' GCPC'. It is thus advised of to work in mode "CONTINUATION" and to fix quotas for the resolution of the linear system at the head files
of order: the memory to the maximum will be thus preserved.
If the going beyond report is modest, one can also try to fix quotas for this phenomenon of crumbling by modifying the parameter setting of management memory via the parameter setting (key words factor
MEMORY, key words MANAGEMENT, TYPE_ALLOCATION or PARTITION) of the order BEGINNING
[U4.11.01].
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Let us notice all the same that storage MORSE (It is also voted by plebiscite by the multifrontale. solvor of Gauss uses, as for him, a "rough" storage more, SKYLINE ("line of sky" in good

French!), which allows an easier management of the filling but amplifies the occupation memory) matrices and of the vectors chosen for the GCPC makes it possible to manage the hollow character as well as possible of
operators and thus, limit the obstruction memory.

Parallelization

Contrary to the multifrontale, the GCPC was not paralleled. This work was however already carried out [bib20], [bib23], but only for one diagonal prepacking (cf [§4.1-3]) because it poses
problems of effective parallel construction of the preconditionnor in order not to disturb the others stages.
With these problems of "parallel preconditioning" are added strong data-processing contingencies in disrtribuées memories. For more information and references on this parallelism known as "numerical" and his réminicences in Code_Aster, one will be able to consult [bib4] [§3].

5.2 Perimeter of use

List Code_Aster orders which can use GCPC:

· CALC_FORC_AJOU,
· CALC_MATR_AJOU,
· CALC_PRECONT,
Possible · DYNA_NON_LINE symmetrized Approximation (cf [§5.3]),
· DYNA_TRAN_EXPLI possible symmetrized Approximation,
Possible · MACR_ASCOUF_CALC symmetrized Approximation,
· MACR_ASPIC_CALC possible symmetrized Approximation,
· MACRO_MATR_AJOU,
· MACRO_MATR_ASSE,
· MECA_STATIQUE,
· NUME_DDL,
Possible · STAT_NON_LINE symmetrized Approximation,
· THER_LINEAIRE,
· THER_NON_LINE possible symmetrized Approximation,
Possible · THER_NON_LINE_MO symmetrized Approximation.

5.3 Symmetrical character of the operator of work

If the matrix is not symmetrical two cases of figures present. That is to say the linear solvor is inserted in a non-linear process (operators mentioned cf lists [§5.2]), that is to say this one is linear (them other operators).
In the first case, one transforms the initial problem ***Ku*** *=* ***F*** *into a new symmetrized problem*

*1 (**K** + **KT**) **U** = **F**. One supposes by there that the nonlinear solvor including (algorithm of Newton) goes*
2
~
1
*to compensate for the approximation of the initial operator by **K** =*
:
(
T
***K** + **K**). It is not besides only*
2
approximation of this nonlinear process… the choice of the tangent matrix in is, for example, one other. This symmetrized approximation does not harm the robustness and the coherence of the unit and avoid a more expensive nonsymmetrical resolution. This operation is carried out by activating the word key SYME = "YES" (by defect "NOT") of the key word factor SOLVEUR and it is licit for the three linear solveurs of the code: "LDLT", "MULT_FRONT" and "GCPC".
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When the problem is purely linear, one can await no compensation of one unspecified including process and this approximation become impossible. Resorts to the GCPC currently in Code_Aster (it is illicit would be necessary to call upon some of its alternatives: GMRES, Orthomin, double GC, normal equation…) and it is necessary to reverse this nonsymmetrical system via LDLT' or "MULT_FRONT". The nature of the operator of work is detected automatically, no one is not need for to notify via key word SYME.

5.4
Parameter setting and posting

To activate this functionality, it is necessary to initialize the key word METHOD with “GCPC” in the key word factor
SOLVEUR [U4.50.01].
Only the prepacking of incomplete the Cholesky type is available and it is not possible of
to exempt: NIVE_REMPLISSAGE=0 thus does not mean “
without prepacking
” but
prepacking of the type ILU (0).
To minimize the size of the profile of the matrix of work and its preconditionnor, an algorithm of renumerotation is available by defect: “RCMK” for “Reverse Cuthill Mac-Kee” (cf [bib26] [§5.4.2]).
It is nevertheless désactivable.
In “exploiting” the level of complétude of prepacking, one can modify the compromise “time calculation/occupation memory”.
Contrary to the direct methods, it was necessary to fix a maximum iteration count discriminating them iterative calculations converged of not converged. This threshold (skeletal) is arbitrarily fixed at
)
half of the ddls of the problem of work ***K****. If at the end of i=NMAX_ITER stages, the relative residue*
laughed
I
=
:
is not lower than RESI_RELA, calculation stops in ERREUR_FATALE.
F

Key word factor
Key word
Default value
References
SOLVEUR METHODE=' GCPC'
“MULT_FRONT”
[§1]

PRE_COND=' LDLT_INC' “LDLT_INC”
[§4.2]

NIVE_REMPLISSAGE=0,1,2… 0 [§4.2]

RENUM=' RCMK' or “WITHOUT” “RCMK”

NMAX_ITER= I
0
[Algorithm 5]
max, a number
iterations maximum
acceptable.
If imax=0, fixed
automatically with N2

laughed
10-6
[Algorithm 5 and
RESI_RELA=
, value
[§3.3]
F
maximum of the residue relative to
convergence (with ième
iteration).

SYME=' OUI' or “NOT”
“NOT”
[§5.2/3]
Approximation symmetrized of
the operator of work by
~
1
K *=*
:
(
T
K *+* ***K****)*
2
Sell by auction only in not
linear
Table 5.4-1: Summary of the parameter setting of the GCPC.
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To be complete on the parameter setting and posting in the file message (.mess), let us mention:
laughed
· Value INFO=2 traces the evolution of the standards of the residues absolute I
***R** and relative*
, thus
F
that the correpondant number of iteration, I, as soon as it decrease from at least 10% (not skeletal).
· Value INFO=3 traces these evolutions with each iteration.
laughed
· A convergence, i.e. as soon as
< RESI_RELA, one recalls in more the value of
F
absolute residue initial 0 normalizes
r.

5.5 The Councils
of use

Like one recalled it already many times, in Code_Aster a whole series of factors calls into question the principal "historical" advantages concurentiels (cf [§1]) of the GCPC compared to its alter-egos direct:

· Efforts made to paginate as well as possible the resources memories required by the solveurs direct (pagination supplements for "LDLT" and partial for "MULT_FRONT").
· Not required of a strategy of swivelling which allows a hollow storage of the matrix of work (MORSE for "MULT_FRONT" and SKYLINE for "LDLT").
· An optimized management of the filling for "MULT_FRONT" (full frontal matrices).

· Parallélisation in memory shared for "MULT_FRONT" (cf [bib33] or [bib4] [§3.2]).
· The matrix of work is assembled beforehand and stored before resorts to the solvor.
· Mauvais conditioning generally noted in mechanics of the structures worsened by taking into account of Dirichlet generalized via of Lagranges (cf [§5.1]).

All in all, the best compromise robustness/obstruction memory/cost CPU seems to return [bib3] with the method by defect of key word SOLVEUR: the multifrontale. However, for problems conditioned rather well (thermal [bib1], simple geometry with a grid and characteristics materials relatively homogeneous…) or very greedy in memory (several million ddls), it GCPC can prove to be an interesting alternative.

Because of its natural modularity and simplicity of its components (produced matrix-vector and scalar product), it remains nevertheless much simpler to maintain and make evolve/move that the others direct solveurs. It is the solvor "passes everywhere" easy to establish (at least in its version of base) and very teaching. He is often connected in more general processes (solveurs encased, solveurs of interface of the decomposition of fields…) or adapted, on a case-by-case basis, for structures of particular matrices.

In addition, a property not exploited yet in the official version of Code_Aster, its scalability intrinsic, a parallelization accredits on a great number of processors. Whereas for same numbers of processors, the speed-ups of the direct methods worsen largely. It is one of the reasons which led to often retaining the GCPC as solvor of interface for methods of decomposition of fields [bib4].

The small last favours GCPC, which had historically its importance, can lie in the fact that it accumulates less the errors rounding than its direct competitors. They are circumscribed with last iteration whereas the direct methods accumulate them during factorization. This known as they indirectly are found when convergence is long in coming and that the generated residues are not more completely orthogonal!
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In short, the GCPC established in Code_Aster offers a compromise robustness/complexity calculation/memory less powerful than the multifrontale. Nevertheless, this report general is undoubtedly with
to balance, as well it is true as it renders enormous services in other fields of physics: N3S, TRIFOU, Code_Saturne, LADYBIRD, ESTET….
One can try to synthesize, from a point of view "user of Code_Aster" the inventory of fixtures precedent, in the table below.

Solveurs

Cut problem

Robustness Memory
CPU
Parameter setting
Maintainability
Small
Case
Very
case
large standards
(<102
(<109
case
DDL)
DDL)
(>109

DDL)
Excellent MULT_FRONT
Because of Good
Nothing to make
Standard
not
yes
To see
DEFECT
pagination:
weak
LDLT
Excellent
Because of
Expensive Nothing to make
Enough easy
yes
not
prohibited
pagination:
also weak
that one wants
GCPC
Very variable Very variable Very
To adapt to
Easy yes
With
to see
Yes?
variable
individually
Yes in
thermics
Table 5.5-1: Summary of the linear solveurs in Code_Aster.

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Handbook of Reference
R6.02 booklet: Direct methods
Document: R6.02.01
In connection with the methods of decomposition
of GAUSS type
Summary:

*This document presents some aspects related to the method of decomposition of GAUSS.
After a rapid presentation, we recall the principal advantages and disadvantages related to this method direct. Then, we detail the implementation of algorithm LDLT implemented in the Code Aster.
GAUSS (1777 - 1855) is at the origin of all the direct methods of numerical resolution of systems linear. That it is thanked here for it.*

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1
General information on the methods of the GAUSS type
1.1
Presentation of the method.

On the basis of the observation which it is easy to solve the A.x system = B when A is a matrix triangular lower or higher, one seeks to break up the full initial matrix by one factorization of triangular matrices.

The guiding principle is to seek a regular matrix P, known as matrix of permutation, such as product P.A is triangular, then to solve P.A.x = P.b

Note:

In practice, P is determined by products of elementary matrices of permutation
P = P (K)… P (1).
The matrices P (I) depend on the alternative chosen, but one never calculates explicitly

the matrix P but only P.A and P.b
Matrix A being factorized in general form L.U (L stamps triangular lower, U stamps triangular higher), we are brought to solve the two linear systems:
L.
y = B
U.x = y

Note:
*In the method known as **of elimination of GAUSS**, one carries out simultaneously the factorization of With and the resolution of L.y = B*
The following algorithm carries out the elimination of GAUSS and the resolution of L.y = B
at the stage (p+1) we have
(p 1)
+
-
(p)
(p)
1
(p)
has
= has
-
for
p+1 I N
has
(p)
ij
ij (
. app) .a
ij
pj
p+1 J n+1
(p 1)
+
(p)
has
= has
for
1 I p
ij
ij
1 J n+1
(p 1)

+
has
= 0.
for
p+1 I N
ij
1 J p
Note:
In this writing of the algorithm of elimination of GAUSS, the second member B is regarded as an additional column of the matrix which is then treated like one stamp N X (n+1).
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1.2
Concept of pivot
The preceding implementation of the algorithm of elimination of GAUSS supposes implicitly that them diagonal terms called ***pivots*** *are not null in the course of calculation.*
Case of null pivots*: A regular matrix can have a null pivot.*
0 1
example: the matrix

2 1
To cure this disadvantage, one can use a ***strategy of swivelling***
· ***complete swivelling****: this strategy consists in choosing like pivot, the major term in block remaining then to carry out a permutation of line and column.*
There is then system P.A.Q (QT.x) = P.b
where P is the matrix of permutation of the lines and Q that of the columns.
The found solution is then y = QTx, and it is thus necessary to preserve the matrix of

permutation Q to obtain the sought solution X = Q.y
*· **partial swivelling**: the pivot is required as being the term of maximum value, among not yet treated terms, in the current column (the kth one at the stage K) then one carries out one permutation of line.*

1.3
Stability of the method
Definition:
*A numerical method of resolution of system linear is known as mathematically **stable** when "some is regular matrix A, the algorithm succeeds".*
Theorem:
The method of GAUSS with a strategy of swivelling is mathematically stable for any regular matrix.
Corollary:
If during a factorization of GAUSS with swivelling, a null pivot is detected then the matrix is singular and this system does not have a single solution.
Theorem:
The method of GAUSS (without swivelling) is stable for definite real matrices positive.
For more details, one will consult the basic books which are [bib13] [bib14] [bib6].

1.4
Unicity of the decomposition
***Proposal:** The decomposition of GAUSS is not single, but if one specifies the diagonal of L or U then there is unicity of the decomposition.*

1.5
An alternative: the factorization of CROUT
The method of factorization of CROUT [1] is the same algorithm, which requires the same number operations and carries out the same filling of the matrix but calculations are carried out in way different.
We place ourselves if the matrix is factorisable, which is always the case with one permutation close to the lines and the columns since the matrix is regular: one thus has A = L U

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Then one proceeds by identification
i-1

Po
ur I J has = U +
L .u

ij
ij
ik
kj

k=1

J
for I > J has = L .u

ij
ik
kj

k=1
(L and
are the elements of
ij
uij
L and U)
from where values of U and
ij
lij according to aij
u1j = a1j
J = 1,…, N
has
L
i1
=
I = 1,…, N
i1
ui1

i-1
U
= has -
L .u

ij
ij
ik
kj
I J
k=1
1
j-1

L
=
- L .u has

I > J
ij

U
ij
ik
kj
jj

k=1
Note:
The order of calculations is not arbitrary, it is necessary to know the L located on the left and them
ik
ukj with
above of each term to be calculated.
One sees whereas at the kth stage, one defers on the kth line all the former contributions
leaving unchanged the lines k+1 with N.
This alternative of CROUT is also called elimination of GAUSS by columns (or columm activates),
and prévilégie the scalar operation of product.
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1.6
Case of the symmetrical matrices
Proposal: *The decomposition of GAUSS respects symmetry.*
It is enough to note that with each stage the aij terms and aji receive the same contribution.
Indeed:
K
I J
K
by assumption of recurrence, one supposes the matrix
symmetrical at the stage K and consequently one a:
+
has (K 1) = has (K) - has
has
(K) (K) /a (K)
ij
ij
ik
kj
kk
+
has (K 1) = has (K) - has
has
(K) (K) /a (K)
I
ji
ji
jk
ki
kk
J
from where the proposal.

Consequently, a matrix A symmetrical perhaps factorized in the form A = LDLT
where D is a diagonal matrix and L a unit lower matrix (i.e. with unit diagonal)
This decomposition, single since a diagonal was fixed, applies to any symmetrical matrix nonsingular.
If matrix A is definite positive, then the terms of the diagonal are strictly positive and one can to use the form known as of CHOLESKY A = LLT= (LD1/2. D1/2LT).
Let us notice that the decomposition of CHOLESKY requires N extractions of square root (which is one expensive operation in time).
In the case of a factorization LDLT for symmetrical matrices, we can write the algorithm of CROUT in the following form:
Buckle on the columns ic = 2,

,
… N
Buckle on the lines it = 1,

,
… ic - 1
Buckle on the contributions im = 1,

,
… it - 1
L
L

-
L
***
L
it, ic
it, ic
ic, im
im, it
Fine buckles
L
L
/l
it, ic
it, ic
it, it
Fine buckles
Buckle on the contributions im = 1,

,
… ic - 1
L

ic, ic

L
- L
**l*

ic, ic
ic, im
im, ic
Fine buckles
Fine buckles
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2
Disadvantages of the methods of the GAUSS type

The disadvantages of the methods of the GAUSS type are primarily of three types:
1) a high number of operations
2) a filling of the matrix
3) a loss of precision in the course of calculation
The first two points are often qualified major defects whereas third is
regarded as a minor defect.

2.1
The number of operations

For a system full with size N, at the p-ième stage, we must carry out to calculate the new ones coefficients of the matrix and the second member:
· (Np)
divided
· (n-p+1) (Np) additions and multiplications
The number of operations is thus:
N 1
-
N (N - 1)
(N
- p) =
divided
2
p 1
=
N 1
-
N 1
-
N 1

-

-
-
-
-

-
(-)
2
(
N N
) 1 (N) 2 (
N N
) 1
(N
p + 1) N
p =
Q
+
Q

=
+

6
2

additions and as much of
p 1
=
Q 1
=
Q 1
=
multiplications.
1

1
That is to say
N (n-1) N +
3

2
operations for which it is advisable to add N2 operations of the resolution of
triangular system.
1
In short:
For a great full system, the algorithm of GAUSS requires about
3
N

3
operations.
Note:
In the case of a stored matrix band, the number of operations is n.b2 *where* B *is the width of the band.*
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2.2
Filling of the matrix
Let us begin with a traditional example of matrix known as “arrow” which one meets for example in chemistry [bib9].
Maybe With the matrix such as (1, I) 0 have, have (I, 1) 0, have (I, I) 0 and all its other terms
are null, the matrix takes the following form then:
0
0
With =
After the 1st stage of factorization (by the algorithm of GAUSS), the matrix is full with the direction where it y
has more theoretically null terms.

More formally let us look at the phenomenon of filling of to the algorithm; for that let us récrivons the algorithm:
For K varying of 1 to N - 1 to make % buckles on the stages
For I varying of K + 1 to N to make % buckles on the lines
For J varying of K + 1 to N to make % buckles on the columns
has (I, J) = has (I, J) - has (I, K). has (K, J)/has (K, K)
end to make
end to make
end to make
What one can schematize graphically, at the kth stage, for the calculation of the term has (I, J) by:
K (3)
J (2)
I (1)
has (I, J)
has (I, J) = has (I, J) - (1) * (2)/(3)
The term has (I, J) is nonnull at the end of the kth stage:
· if it were nonnull at the beginning of this kth stage,
· or if the terms has (I, K) and has (K, J) are all two the nonnull ones at the beginning of the kth stage,
and this independently of the initial value of the term has (I, J).
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Moreover, one sees that the method of GAUSS fills the profile wraps matrix during
stages of factorization.
In the example of the matrix “arrow”: the profile envelope is the full matrix, from where the result noted.
This example highlights the importance of the classification of the unknown factors of the matrix since matrix can be récrite, after permutation of the unknown factors, in the form:
0

0
whose profile envelope is the matrix it even (there is thus no filling).
We have just seen the importance of the classification of the unknown factors.
We could not insist too much on the fact that algorithms of “optimal” renumerotation must be used to minimize the filling of the matrix.
These algorithms rest on the heuristic ones and are specialized.
Among the algorithms most usually used let us quote

Algorithms	**Objectives**
CUTHILL - Mc KEE	to minimize the bandwidth
Transfer CUTHILL - Mc KEE	to minimize the profile
Degree minimum	to minimize the multiplications by 0

Intrinsic formulation of the filling
One can give an intrinsic formulation of the filling during the elimination of unknown factor in terms of graph [bib5].
That is to say a matrix A which we associate the graph G (X, E), where X is the whole of the nodes and E the whole of the not directed edges.
The problem of elimination of an unknown factor of the matrix is then equivalent to eliminate a node from
graph.
Definition: Are X, there X, one will say that X and are **adjacent** there if and only if {X, y} E
So Y is a subset of nodes of X (X Y) we can define the following sets:
the whole of the adjacent nodes with Y
Adj (Y) = {X/X X there and there Y such as {X, y} E}
the whole of the with dimensions incidents with Y
Inc (Y) = {{X, y}/there Y, X Adj (Y)}
the whole of definition of X
Def (X) = {{y, Z}/y, Z Adj (X), y · Z and Z Adj (Y)}
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def (y)
y
y
· in hatched: what one eliminates,
· into dotted: what one adds (filling)

Operation of elimination of y

The elimination of then consists in there considering the subset
Elim (y, G) = {X - {y}, (E Inc (y)) U Def (y)}
It is thus necessary well to consider the filling which is related to Def (y).
To minimize the filling one can use heuristics consisting in eliminating there from degree minimal, the degree of {y} being the cardinal of the unit Adj (y). It is the governing idea of the use of the algorithm of the degree minimum before a factorization of the GAUSS type.
This approach is used in the multi-frontal method implemented in *Aster* [bib15].

2.3

The loss of precision in the course of calculation

The problem comes owing to the fact that in the course of algorithm the pivots decrease and that they are used
like denominator for the following stages [bib13].

2.3.1 Summary study of the loss of precision

Let us note A (K) the matrix at the stage K (i.e. after the elimination of the kth variable); with by convention A (O) = A.
We can then write that A (1) checks L (1). WITH (1) = A (0)
0
1
0
by taking L (1) =
by identification

Then after (n-1) factorizations of this type we obtain
(L (N -1) ·… · L (1)) U = L.U = A (0)
Because of the errors E on the decomposition, it is advisable to write: L.U = A (0) + E

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The coefficients of the matrix of error E can be evaluated [bib12] by taking account of the error made on the floating operation which we will note fl:

(K)
E
3.e.m .max has

ij
ij
I, J
K, I, J
ij
with
( )
K
(K)
m
:
ij
a many terms such as have .a
0
ik
kj
: relative error of the operations machine

Indeed, while placing itself if the elimination of GAUSS “succeeds” (for example: the matrix is defined positive or one uses a technique of swivelling).

(*K*) 1 *has*
has (*K* -)
1
Let us note
ik
ik = ß
ik

1
for I > K, K = 1,…, N - 1
has (*K* -)
1

has (*K* -)
1
.(1)

+
kk
kk
with
<
1
The term *has* (*K*)
ij
is then evaluated by:
has (*K*) = *ß* (*has* (*K*) 1 - .a (*K*) 1
ij
ij
ik
kj
) for I, J > K, K = 1,…, N - 1
( )
(-)
1
(*K* -)
1
That is to say still *K has* = (
ß has K
- .a
ij
ij
ik
kj
) with, <
2
3
The “disturbance” *ek*
(*K*)
ij undergone by *aij*

can then be evaluated; starting from the definition of *mij* us in
let us deduce the relation:
ek .a (*K*

1)
1
. *has*
ij
ij
ij for i> K, K = 1,…, N - 1
and of the evaluation first of *A* (*K*)
ij
we deduce:
µ
(*K* -)
1
(*K* -)
1
(*K*)
ik .akj
(*ij*
has
- *ij*
has
/ (1+ 3) / (1+ 2)
and finally

1

1

E (*K*)
has (*K*)
(*K* -)

1

1 1

ij
-
-

(1+) (1+) *has*
kj
- *ij*

(1
2
3
+ 2
)
E (*K*).
3 *A*
kj
ij.
however the decomposition L.U = A (0) + E indicates to us that *eij* is the sum of the errors.
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2.3.2 Estimate of the error on the solution

We solved the system
L.U X = A (0) .x + E.x
where
E.x is the term of error induced by the errors rounding/truncation in the operations of factorization.
With (0) X is the second member (makes b) of them.
The found approximate solution ~
X which approximates true solution X is:
~
X = (A + E) - 1b

It is shown whereas the evaluation of the error on X is related to the **conditioning of** A.
Let us pose the problem in the form: (A + A) (X + X) = B
By supposing A.x small one a: X = A-1.A.x
X
With
from where while normalizing
Cond (*A*).
X
With
where
-1
Cond (A) = (*A.A*
) is the conditioning of matrix A.
Note:
It is noted that the error induced on the second member is weak and the solution does not disturb that through a bad conditioning of matrix A.
Indeed, if one considers system A (X + X) = B + B,
because of the equalities: X = A-1.b and Ax = B
-1
there are X A
. *B* E
T *B* < *A.X*
X
B
-1
and thus
With
X
(. With) *B*
Note:
If one considers the variations on A, X, and B simultaneously, one has the following estimate [bib14]:
X
Cond (*A*)
With
B

.
+
X
1 - *To* 1. *WITH A*
B
-

Some remarks on conditioning

· conditioning is defined only for one regular matrix,
· conditioning depends on the standard chosen on IRn,
· some is the standard chosen, we have 1 Cond (A) and a matrix is of as much conditioned better than its conditioning is close to 1.

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If the euclidian norm is selected for standard, then
· the conditioning of an unspecified matrix A is
µn
Cond (A) = µ1
where µ and µ are the extreme singular values of
1
N
With (i.e. smallest and more
large of the eigenvalues of A*.A).
· If matrix A is symmetrical (or square) then
N
Cond (A) = 1
where and are the eigenvalues of minimal and maximum module of
1
N
A.

2.3.3 Estimate of the number of significant figures of the solution

If one has a precision of p figures (decimal) significant, one has then:
With
p
-

10
With
If one wishes a precision of S figures (decimal) significant on the solution
X
S
-
10
X
from where the estimate of the number of exact decimal significant figures of the solution
S p-log10 (Cond (A))

2.3.4 Method to reduce conditioning

The simplest method is that of the scaling:
One “passes” from A to.
with
.
) is better than
1 A.2
I stamps diagonal such as Cond (1 A.2
Cond (A).
This is very theoretical and there is not universal method to determine
.
1 and 2
Let us note that if matrix A is symmetrical and that one wishes to preserve this property, it is necessary then
to take
.
1 2

2.3.5 Example of badly conditioned matrix.

This example, very significant and instructive is with R.S. WILSON.
That is to say the Ax system = B with:
10 7 8 7
32
1

7
5
6
5
23
1
With =

and B = and whose solution is X =
8
6
10
9

33

1

7
5
9
10
31
1
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· If one disturbs the second member of about 0.5% while taking:
~
B = (32.1, 22.9, 33.1, 30)
.9
then the solution is: ~
X = (9.2, -12.6, 4.5, -1.)

1 .
· If the matrix is disturbed of about 1%:
10.
7
8.1
7.2

7.08
5.04
6
5
Ã =

8
5.98 9.89
9

6.99
4.99
9
9.98
then the solution is ~
X = (- 81, 137, -34,
)
22
Remarks on the properties of matrix a:
· *It is symmetrical, definite positive, of determinant 1 and reverse "sympathetic nerve"*.
25.-41 10 -6

- 41
68
-17
10
A-1 =

10
-17
5
- 3

-6
10
- 3
2
· *Its conditioning within the meaning of the euclidian norm is:*
4
L
30.2887
Cond (A) =
=
= 2984.11
1
L
0.01015

2.3.6 A geometrical Interpretation of the bad conditioning

One can give a very simple interpretation of the bad conditioning of a linear system Ax = B in the particular case where A is normal (i.e. A*.A = A.A*).
Are 1 the smallest eigenvalue of matrix A and N its greater eigenvalue and are v1 and v associated clean vectors.
N
X
B
for B = v
=
N and B = .v, one has
cond (A)
X
B
X + X
B + B
, v,
B = vn
U = vn
N

from where if
N
cond (A) = is large, a small disturbance B of B involves a great variation
1
on solution v.

2.4

Criteria of determination of a null pivot

Definition:

A numerically degenerated system is a system for which a pivot is null or
does not have exact significant figure.
Let us note that a system can be degenerated numerically without being it mathematically.
In these two cases, it is advisable not to solve the system from where need for determining one
criterion of stop as soon as one of the pivots does not have any more exact significant figures.
Are With the matrix to factorize and F the matrix of free diagonal resulting from factorization.

Criterion 1: The simplest criterion is to consider that the pivot is null as soon as it is lower, in
absolute value with a given threshold.

F
<

II
1
1 is a “small number” in lower part of which it is considered that the values are arbitrarily null.

Criterion 2:

This criterion applies to the number of significant figures still available. In
noting that one cannot have any more p significant figures on a given machine,
it will be checked that the decrease of the pivot is not carried out in a report/ratio higher than
10-p.

fii
- p
= 10

2
has II
F
Let us note that report/ratio II is always lower or equal to 1 because of decrease of the pivots.
aii
The basic cause of a bad numerical conditioning is the rounding error caused by
the introduction of great numbers without physical significance.
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3 Method

LDLT per blocks implemented in *Aster*

This paragraph details the implementation in *Aster* of the resolution of the linear system A.x = B by method of factorization LDLT of symmetrical matrix A.
Matrix A is stored in profile (or line of sky) per block.

Guiding principle:

A column of the matrix is contained very whole in only one block: us
let us not segment the columns.
The tables allowing the description of the stored matrix profile per block are:
· HCOL height of column of the matrix
HCOL (I) height of the i-eme column
· ADIA addresses diagonal term in its block
ADIA (I) returns the address of the i-eme diagonal term in its block
Pointer · ABLO of block
ABLO (i+1) returns the number of the last equation in total classification
contents in the i-eme block
By convention ABLO (1) = 0 and numbers it equations in the i-eme block is given
by relation ABLO (i+1) - ABLO (I) + 1
The total number of equation results as being ABLO (nombre_de_bloc + 1)

It is also necessary to memorize the total number of blocks used to contain them coefficients of the matrix.

Note:

Formally table HCOL is useless because it results from tables ADIA and ABLO, but it allows to carry out calculations more quickly.

3.1

Implementation of factorization

Principal characteristics of the implementation of the factorization of GAUSS by the alternative of CROUT in the shape LDLT of a stored symmetrical matrix profile per block are:

· factorization is carried out in place, i.e. crushing the initial matrix,

· perhaps partial factorization,

· the criteria of null detections of pivot can be adapted to the factorization of matrices quasi-singular,

· in the event of detection of null pivot, this pivot is replaced by a very great value (1040) it who amounts introducing a condition of blocking by penalization.

Note:

Two tables of work are created:

· a *table which will contain the diagonal of the factorized matrix (minimization of a number of access to the block),*

· a *table for the current column (minimization of the number of calculations carried out).*

Algorithm BEGINNING;
creation of an intermediate table for the current column
creation of an intermediate table for the current column
FOR ibloc VARIANT_DE 1 A nombre_de_bloc TO MAKE
· Détermination of end and the starting columns for the current block.
· Recherche of the smallest equation in relation to an equation contained in the block running. This research is done by exploiting table HCOL
· Recherche of the block of membership of the equation found previously. This research is done by exploiting table ABLO.
· Requête in writing mode of the i-eme block.
FOR jbloc VARIANT_DE plus_petit_concerne A ibloc-1 TO MAKE
Request in reading mode of the j-eme block
FOR iequa CONTAINED IN the i-eme block TO MAKE
calculation of the start address of the column in the block
calculation height of the column
FOR jequa CONTAINED IN the j-eme block TO MAKE
calculation of the start address of the column in the block
calculation length of the column
With (ibloc, jequa) = A (ibloc, jequa) - < A (ibloc, *), A (jbloc, *) >
FIN_POUR
FIN_POUR
release of the j-eme block which was not modified
FIN_POUR
FOR iequa CONTAINED IN the i-eme block TO MAKE
calculation of the start address of the column in the block
calculation length of the column
FOR jequa CONTAINED IN the i-eme block and < iequa TO MAKE
calculation of the start address of the column in the block
calculation height of the column
With (ibloc, lm) = A (ibloc, lm) - < A (ibloc, *), A (jbloc, *) >

FIN_FAIRE
% use of the column iequa (calculation of the pivot)
calculation of the start address of the column in the block
calculation height of the column
safeguard column: wk. (I) A (ibloc, I)
standardization of the column by using the diagonal table:
With (ibloc, *) A (ibloc, *)/diag (*)
calculation of the diagonal term and actualization of the table of work:
tabr8 (iadia) = tabr8 (iadia) - < A (ibloc, *), wk. (*) >
test of the pivot compared to
test of the pivot compared to the number of significant figures
FIN_POUR
release of the block running which one has just modified
FIN_POUR
release of the tables of work
FINE Algorithm;
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Determination of end and the starting column for the current block.

This phase is due to the concept of partial factorization.
IF (last column of the block < beginning of factorization) THEN
request in reading mode of the i-eme block
to fill the table with work containing the diagonal.
release of the i-eme block
ALLER_AU following block
SINON_SI (first column of the block > fine of factorization) THEN
TO LEAVE
IF NOT

IF (first column of the block < beginning of factorization) THEN
% to supplement the “diagonal” table
request in reading mode of the i-eme block
to fill the table with work containing the diagonal.
FIN_SI
IF (last column of the block > fine of factorization) THEN
modification of the last equation to be taken into account
FIN_SI
FIN_SI

Notice on the obstruction:

It is obligatory that one can have at least simultaneously in memory:
· two blocks of the matrix,
· two vectors of work of size: the number of equations of the system to be solved.

3.2

Implementation of the resolution

The implementation of the simultaneous resolution of N second members of the A.x system = B, where stamp A is symmetrical and was factorized in form L D LT (the resolution is in place)

Note:

One creates a table of work which will contain the diagonal of the factorized matrix, for to minimize the access number to the block of the matrix and thus to limit the readings for large matrices not being able to reside completely in memory.

Algorithm BEGINNING;
Creation of a table to store the diagonal to avoid readings at the time of the stage of diagonal resolution.
FOR ibloc VARIANT_DE 1 WITH the nombre_de_bloc
% downward resolution and

% filling of the diagonal table
request in reading mode of the i-eme block
FOR iequa contained IN the BLOCK
Calculation height of the column and
calculation of the start address of the column in its block
FOR each second member TO MAKE
xsol (isol) = xsol (isol) - < X (isol), U) >
FIN_POUR
safeguard diagonal term in the table of work
FIN_POUR
release of the i-eme block
FIN_POUR
FOR each second member TO MAKE
% diagonal resolution
FOR all the equations TO MAKE
xsol (iequa, isol) = xsol (iequa, isol)/diag (iequa-1)
FIN_POUR
FOR ibloc VARIANT_DE nombre_de_bloc to 1 PAR_PAS_DE -1% going up resolution
request in reading mode of the i-eme block
FOR iequa contained IN the BLOCK
Calculation height of the column and
calculation of the start address of the column in its block
FOR each second member TO MAKE
xsol (ixx+i, isol) = xsol (ixx+i, isol) - xsol (isol) *L (ide+i)
FIN_POUR
FIN_POUR
release of the i-eme block
FIN_POUR
Release of the working area (i.e. of the diagonal table)
FINE Algorithm;

Notice on the obstruction:

It is necessary that one can have simultaneously in memory:

· a *block of the matrix,*
· a *vector of work of size: the number of equations of the system to be solved,*
· *N second members.*

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3.3
Scaling

It is possible to make a scaling of the matrix be factorized; this setting on the straight ladder fact in order to obtain a matrix whose diagonal terms are worth 1.
The diagonal matrix is such as:
1
if has

0

II
I =
aii
1
if has

II = 0
It should be noted that at the time of the resolution, one obtains the solution of the system only after déconditionnement.
Indeed: the initial system is Ax = B
after multiplication on the left by, one a:
Ax = .b
However the solved system is:
Ax = B
from where solution X obtained that “déconditionner is needed”.

3.4
Tests on the pivot

Two criteria of null detections of pivot are implemented:
· the test in absolute value *aii* <, with given,
· the test in relative value on the number of exact significant figures.
Let us note that these tests can be reduced to their simpler expression by providing one = 0. and in giving, by convention, a number of exact significant figures no one.
This option is made necessary by the fact that algorithms such as the algorithms of search for eigenvalues [R5.01.01] [R5.01.02] seek to factorize matrices quasi-singular.

3.5
Factorization of complex matrices
The algorithm implemented in *Aster* also makes it possible to treat the **symmetrical** matrices with complex coefficients.
The implemented algorithm does not treat the square matrices, although it is theoretically possible.
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Appendix 1 traditional Methods of storage
A1.1 Stamps full
A nonsymmetrical matrix full with size N has N2 coefficients.
If the matrix is symmetrical, one can store only his triangular the lower or higher is
N (*N* +)
1 values.
2
No table of description of the matrix is necessary.
A1.2 Stamps band
B
B
B + 1 = bandwidth

In this case one stores the band (called sometimes rectified matrix) in a rectangular table N (2b + 1); one includes then B (B + 1) zero values corresponding to the complements of points.
(
B B +)
1
In the case of a symmetrical matrix, one can store only N (B + 1) values of which

2
zero values (useless).
This method only requires to know the bandwidth.

A1.3 Stamps profile or matrix with line of sky

This technique consists in storing the terms of the matrix by columns and lines lengths variables. Terms external with the “line of sky”, which is the envelope of the tops of the columns being supposed not to have any contribution in calculations, are not stored.
Profile of the i-ème line (resp. column) is determined by:
min {J such as 1 J N
aij 0}
(resp
min {J such as 1 J N
aji 0})
If the profile is symmetrical, one speaks about matrix with symmetrical profile.
This method of storage requires tables of storage which we will detail in the case of a matrix with symmetrical profile.
Classically, in this option of storage, the matrix is arranged in the shape of a mono table dimensioned requiring a table of pointer of entry of column ADIA to explore the matrix: the entry is done by the diagonal terms, the number of terms of the column is obtained by differences of two successive terms: ADIA (i+1) - ADIA (I).

If the matrix is nonsymmetrical, but with symmetrical profile, it is necessary to store the i-ème line and the i-ème column. Classically, one put them “ends at ends” and the number of terms of the column or line is (ADIA (i+1) - ADIA (I))/2.

A1.4 Storage per block

The methods of storage seen previously suppose implicitly that the matrix can reside in main memory, which is not always the case.

From where concept of matrix stored per block (or segmented on disc).
All preceding storages can be segmented, but we will state only the case of
stored symmetrical matrix profile.

Stamp profile stored per block

We consider here only the case of the symmetrical matrices, which does not remove anything with the general information of
matter.

1
2
3
4
5
6
7

Appear A1.4-a: Maximum size of a block: 20 elements

In this example, we suppose the of the same blocks cuts, to use blocks of variable size, it
is necessary to introduce an additional table containing the size of each block.
We also consider that a column can belong only to one block: “we do not cross
not columns ".
To manage the matrix, it is always necessary to know the address of the diagonal terms; but
now, this address relates to the block of membership of the column.
This table, it is necessary to join a table giving the equations contained in a block.
This table dimensioned with the number of block plus 1 contains the last equation of the block.

Appendix 2 Variations on the algorithm of GAUSS

As we saw with the alternative of CROUT, there are several implementations of the algorithm
of GAUSS: it consists in carrying out calculations of the coefficients in a different order.

Schematically, it is considered that there are three overlapping loops:
· buckles
I on the lines,
· buckles
J on the columns,
· buckles
K on the stages.
The standard algorithm is characterized by the sequence kij, but there are 5 other permutations of indices which give place to as many alternatives (or of algorithms).
· the algorithm of CROUT is characterized by the sequence jki,
· the algorithm corresponding to the sequence ikj, which works by line, is known under the name of algorithm of “Doolitle”.
Let us give here a chart drawn from [bib10].
U made
U made
U made
L made
not yet
L made
m odifié
L made
not yet
m odifié
With
algorithms kij - kji
algorithms ikj - ijk
algorithms jki - jik
the kth one is calculated
one calculates i-ième
one calculates j-ième
column and K + 1
line of L and U
line of L and U
line and one reactualize
submatrix A
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Summary:

The method multifrontale is a direct method particularly adapted to the resolution of the systems linear whose matrix is hollow. This method includes a preliminary phase of renumerotation intended to minimize the filling of the matrix during factorization.
This phase also makes it possible to gather the variables in “super-variables” or “super-nodes”. Factorization,
as for it, is carried out in the form of a continuation of elimination of super-nodes, in full matrices. These full matrices allow the use of routines optimized like the BLAS, which obtain them better performances on vectorial or scalar machines.
Having the factorized matrix, each resolution of system will require nothing any more but one “gone up descent/” inexpensive.
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1

Description of the method

The method multifrontale is a **direct** method of resolution of linear systems, which is particularly adapted to the systems having a hollow matrix. It factorizes matrices symmetrical or not, not necessarily definite positive. In the case more general, the method multifrontale calls upon the method of Gauss with search for pivots.

The method established in *Code_Aster* is limited to the symmetrical matrices, and uses the algorithm known as "*LDLT*", without search for pivot.

The resolution of a linear system is carried out in three stages:

· renumerotation of the unknown factors,

· factorization of the matrix,

· gone up descent/.

If several linear systems, of the same matrix, are to be solved, only them gone up descents/are to be carried out. The same if several of the same matrices structure are with to factorize, the renumerotation of the unknown factors will not be to remake. This preliminary phase, which the performance of factorization will ensure, has a considerable cost, however its relative weight (in computing time), decreases with the size of the matrices to factorize.

We will follow the following plan:

1) presentation of method *LDLT* traditional adapted to the full matrices. Concept of elimination,

2) extension of the method to the hollow matrices, concept of filling,

3) presentation of the multi-frontal method.

1.1 Method LDLT for the full matrices

Either **A**, an invertible matrix, one knows that there is a triangular matrix lower **L** than diagonal unit and a higher triangular matrix **U**, such as **A** = **L U**. The order of these matrices will be *N*.

If **A** is symmetrical, this decomposition can be written:

WITH = **LDLT**,

éq 1.1-1

where **D** is a diagonal matrix and **LT the** matrix transposed of **L**, with diagonal unit.
2
Are (*I, J*) [
,
1 *N*]; from [éq 1.1-1] one deduces the expression from the coefficients of **L** and **D.** Indeed one can write:
N
N
With
= **L**

D LT
ij
ik
kl
lj
éq 1.1-2
K =1 *l=1*
L being triangular lower than diagonal unit, and **D** diagonal, [éq 1.1-2] becomes:
min (*I, J*)
With
=
L L D
ij
ik
jk
kk
éq 1.1-3
K =1
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There is in many ways to calculate the coefficients of **L** and **D** from [éq 1.1-3]. We go to see the method called “by line” used usually in *Code_Aster*, and the method “by column” used in the method multifrontale.

Method by line

The lines of **L** and **D** are jointly calculated the ones after the others. Let us suppose these lines known until the order (*I* -)
1, and also let us suppose that the coefficients of line *I* are known
until the order *J* - 1; [éq 1.1-3] is written, with *J* < *I*:

j-1
With
= **L**

L
D
+ **L D L**
ij
ik
jk
kk
ij
jj
jj
éq 1.1-4

K =1

One has as follows:

J

-1

L
= **A - L**

L
D D
ij
ij
ik
jk
kk
jj
éq 1.1-5

K =1

and in a similar way:
i-1
D
= **A - L**

L D
II
II
ik
ik
kk
éq 1.1-6
K =1
With this method, each coefficient is calculated in only once (operation [éq 1.1-5]), while going to seek the coefficients previously calculated, and by making the scalar product of (**Ljk**, *K* =, 1 *J*) 1 and (**Tk**, *K* =, 1 *J*) 1, with **T** = **L**. **D**
K
ik
kk
This algorithm is illustrated by [Figure 1.1-a].
D jj
(In gray and hatched, terms necessary
J
with the calculation of Lij)
I
Lij
J
I
Appear 1.1-a
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Method by columns
Let us examine the figure [Figure 1.1-b] following:
One adds to the column in
the column
J
in
, multiplied by the product of the terms
in
I
J
I
Appear 1.1-b
Let us suppose that the column *J*, i.e. the terms **D** *jj* and (**Lij**, *I* = *J*

+ ,
1 *N*), is known and
let us carry out the following algorithm:
for *I* = *J* + 1 **with**,
N **to make**
éq 1.1 - 7
2
Dii = **Dii - Lij D** *jj*
for *K* = *I* + 1 **with** *N* **to fair**
E

,
L *ki* = **Lki - Lkj Lij D** *jj* (saxpy)
éq 1.1 - 8
Let us make three note:
1) the operations [éq 1.1-7] are called the **elimination** of the unknown factor *J*. Indeed, after [éq 1.1-7], one **will make never again** call under the column *J* in the continuation of the algorithm.
method by column is sometimes qualified of “looking forward method”; as soon as they are calculated, the terms of the matrix act on the following terms. On the other hand, them methods by line are called “looking backward methods”; one will seek the terms previously calculated with each new calculation,
2) the operation [éq
1.1-7] is an operation of the type “saxpy”, one withdraws from the vector
(**Lki**, *K* = *I*

+ ,
1 *N*), the product of constant the **L D**
ij
jj and of the vector (**L** *kj*, *K = I*

+ ,
1 *N*),
3) having carried out [éq 1.1-7] for *J* fixed, let us see what it remains to do to know the column
(*J* +) 1,
-
D *j+1*, *j+1* is known (it easily is checked),
-
it is enough to divide the column (**Lk** *J*
K
J 2 *N*
,
, ,
+1
= +
) by **Dj+1**, *j+1* to have the value
final of this one.
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(One wrongly confused above the final terms **Lki** and their name of programming which
contains the values of **Lki** modified during eliminations).
One thus deduces the algorithm general from it from factorization *LDLT* by columns.
for *J* =
1

with,
N **to make**
for *K* = *J* + 1 **with** *N* **to make**

,
L
= **L**/**D**
/standardization/
kj
kj
J
J
éq 1.1-9
for *I* = *J* + 1 **with**,
N **to make**
/elimination/

2
Dii = **Dii - Lij D** *jj*
for *K* = *I* + 1 **with** *N* **to fair**
E

,
L
= **L - L L D**/saxpy/
ki
ki
kj
ij
jj
Before passing to the concept of filling, it is advisable to make as of now a useful remark for the continuation. If one looks at [éq 1.1-9], it appears that one can eliminate the unknown factor *J*, even if the terms
(**Lki**, *K* = *I*

+ ,
1 *N*) and **Dii** are not yet available. Indeed, it is enough to preserve the terms
(- **L L D**
kj
ij
jj), and to then add them under the **Lki** terms. These terms (**L L D**
kj
ij

jj), *I* varying from
J +1 with *N* and *K* of *I* +1 to *N* form a matrix associated with elimination with the unknown factor *J*, that one
will call the **frontal matrix** *J* thereafter.

1.2
Hollow matrix and filling

It is pointed out that if the initial matrix comprises null terms, successive eliminations
cause filling, i.e. certain **Lki** terms are different from zero whereas it
initial term **A** *ki* is.
Let us suppose that before the elimination of the unknown factor *J*, the **Lki** term is null; if **Lkj** and **Lij** are **both**
nonnull, [éq 1.1-9] shows that **Lki** will become him also not no one. This filling has an interpretation graph. Let us suppose in this case that all the unknown factors are represented by the nodes of one graph: one will connect the nodes *K* and *I* if and only if initial term **A** *ki* of the matrix is nonnull.
If **A** *ki* is null with *I* connected to *J* and *K* connected to *J*, the elimination from the graphic point of view of *J*
will consist in connecting *I* and *K* then.
The figure [Figure 1.2-a] illustrates this interpretation. The edges in dotted line are those create by the elimination of *J*. They correspond in the new nonnull terms of the matrix *L*.

I
N
I
N
J
K
K

m
m
L
L
p
p

Appear 1.2-a: The elimination of the node J connects all its neighbors between them

1.3 Method
multifrontale

The method multifrontale is a direct method, of Gauss type, which aims at exploiting to the maximum it hollow of the matrix to be factorized. It seeks, on the one hand, to minimize the filling by using one optimal renumerotation, in addition, it extracts from the structure digs matrix information him allowing **to eliminate** (cf page 5 notices (1)) unknown factors independently from/to each other.
Let us examine the simple case of the figure [Figure 1.3-a], where the matrix has only one null term, **A21**.

Appear 1.3-a

Column 2 of **L** does not undergo the effects of the elimination of unknown factor 1, because the coefficient
With
= 0, then **L**
21
21 = 0 (cf [éq 1.1-9] seen previously). Contributions to columns 3 and 4 of
L, of unknown factors 1 and 2 are independent of their kind of elimination (it is necessary to look in detail
[éq 1.1-9]). Of this observation, one can introduce the concept of tree of elimination presented by I. Duff [bib1].

The tree of elimination of matrix **A** can be represented by the figure [Figure 1.3-b].
4
3
1
2
Appear 1.3-b
This tree structure contains two concepts:
· the independence of certain eliminations (here variables 1 and 2), which will lead to one possible parallelism of the operations,
· the minimization of the operations to be carried out, (one sees on the tree structure that the **L21** term is not
not to calculate).
Being given a hollow matrix, **which one knows the filling**, the tree of elimination can be built as follows:
· all the sheets of the tree (lower ends) correspond to the unknown factors *J*, such that **A** *ji* = 0 for *I* = 1 with *J* -1. Here 1 is well on a sheet because there is no **A1i** term for *I* < 1, 2 is also a sheet bus **A21** = 0,
· one
node
J has as a father *I*, if *I* is the smallest number of line such as **Aij** 0. Here, 3 is the father of 1 and 2.
Note:
1) one employs starting from now the vocabulary of the tree, graph theory, sheet, node… Here the tree is turned over, its sheets are in bottom,
2) one will refer to [bib1] for more details and the demonstrations of the validity of the method,
3) in the example above, the order of elimination between the unknown factors (3) and (4) is fixed by initial classification, one could have permuted the lines and the columns of the matrix and to have 4 like father of 1 and 2,
4) it should be noticed that the manufacture of this tree structure must take into account the ***terms nonnull obtained by filling*** *during elimination. (One will see more details on it subject in [bib2]). One cannot manufacture the tree of elimination only starting from the matrix dig initial: it is necessary to also know the terms of filling like already mentioned previously. The numerical factorization of the method multifrontale is preceded by one important phase: the simulation of eliminations and thus, the creation of the nonnull terms. One calls also this simulation, logical elimination or symbolic system. This simulation takes place during the first phase: the renumerotation. One will see the four phases of method multifrontale, most important being the first and the fourth.*
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First phase: The renumerotation of the “minimum dismantles” (minimum degree).
The purpose of this renumerotation is **first** to minimize the number of operations to be carried out at the time of
factorization. For that one simulates the elimination of the nodes and one chooses like candidate for elimination the node of the graph having the lowest number of neighbors. One uses the concept here of graph seen with the figure [Figure 1.2-a]. The initial hollow matrix defines the graph on which one work. This last is then updated to each elimination of nodes (creation of bonds).
simulation of the filling makes it possible to achieve the **second goal** which is the manufacture of the tree of elimination seen in the preceding paragraph. **The third goal** reached, is the creation of **super-nodes**. It is an important concept that we will develop.
A super-node (SN) is formed of the whole of the nodes which, during elimination, have them same neighbors within the meaning of the graph of elimination. During the simulation of elimination, one
detect that for example, nodes *I*, *J* and *K* are:
· of a share, neighbors between them (the terms **Lij**, **L** *jk*, **Lik**… are nonnull),
· in addition, they have as common neighbors the nodes: {*L*, *m*, *p*, *Q*, *R*, *S*, *T*} (see [Figure 1.3-c]).
They form then the super-node {*I*, *J*, *K*} and will be eliminated all together during factorization numerical.
The MM-F is a method of factorization per blocks, when it uses the concept of “super-node”.
This concept has the double following advantage:
· it reduces the cost (considerable) of the renumerotation,
· it reduces the cost of factorization by gathering calculations (use of routines
of linear algebra of blas-2 type or blas-3).
One sees on [Figure 1.3-c] the structure of the columns {*I*, *J*, *K*} (nonnull terms in grayed) in one **virtual** hollow total matrix. In fact, such a hollow matrix **is never assembled**. One
work in **full** local matrices which one calls the **frontal matrices**. There is one
frontal matrix by SN (One will re-examine this concept in the paragraph “numerical Factorization”).
I
J
K
L

in grayed, nonnull terms of
m
total virtual matrix
p
Q
R
S
T
Appear 1.3-c
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In short, the first phase of the method multifrontale (the renumerotation by the “minimum dismantles”) consist of the three following actions:

· a renumerotation of the unknown factors to be eliminated, in order to minimize the filling. They are simulated
eliminations by updating each time the lists of neighbors of the nodes,

· jointly with the preceding action, one builds the tree of elimination, and,

· one detects the “super-nodes”, which one can describe as class of equivalence for the relation “the same neighbors have”.

Note:
The tree of elimination provided by the renumerotation is expressed in terms of super-nodes, bus numerical elimination to follow will be done by super-nodes.

Second phase: Factorization symbolic system

This phase is not as fundamental as the preceding one. It is an intended technical phase to build certain pointers. They are in particular the tables of total indices and buildings which the correspondences between the unknown factors during the assembly of the frontal matrices establish.

Third phase: The sequence of execution

It is also a technical phase. One saw in the preceding paragraph that the method consisted with

to traverse the tree of elimination, by carrying out an elimination with each node of the tree. The result of this elimination is a **frontal matrix**. The order of this matrix is the number of neighbors of Eliminated SN. The storage of the frontal matrices is expensive in occupation of the memory.
The frontal matrix *J* (result of the elimination of *SNj*) will be assembled in the frontal matrix node *I*, where *I* is the **father of** *J*. One will see this phase more in detail during factorization numerical). All the frontal matrices must be preserved, until they are
used during the elimination of “SN father”. One can then arrange the matrix of “SN father” in the place those of “SN wire”.
There are several ways of traversing the tree by respecting the constraint: “the son must be eliminated front
the father ". The object of the third phase is to find the order of course which minimizes the place in memory necessary for the arrangement of the frontal matrices ([bib2], page 2.12).
Fourth phase: Numerical factorization
This phase is effective factorization, i.e. the calculation of the matrices **L** and **D.** Thereafter, one will confuse these two matrices and from a data-processing point of view **D** will be seen like the terms
diagonal of **L**.
Numerical factorization consists in traversing the tree of elimination; for each “Super-node”, one carries out:
· assembly, in the frontal matrix “mother”, of the frontal matrices “girl”,
· the elimination of the columns of the super-node.
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Let us see the following example with the graph and the tree of elimination of [Figure 1.3-d] (nodes 8,9, 10 are not reproduced on the drawing).
1
4
SN4 (8,9,10)

5
6
SN3 (4,5,6,7)
2
SN1 (1,2)
SN2 (3)
7
3

Appear 1.3-d: Example of graph and tree of elimination

Between brackets, one reads the numbers of the unknown factors of SN.
The elimination of the SN1 consists of:
1) assembly of columns 1 and 2 of the **matrix initial**, in a matrix of work, known as
frontal matrix before elimination. This matrix is of order 5, related to the unknown factors (1, 2, 4, 5, 6). (Because 1 and 2 is related to (4, 5, 6) only),
2) the elimination of the SN1 (of columns 1 and 2, according to the formulas [éq 1.1-7] and [éq 1.1-8] seen
previously),
3) arrangement of two columns 1 and 2 of the matrix, in a table Factor, which contains
columns of the total factorized matrix,
4) arrangement of the frontal matrix 1 of order 3 related to the unknown factors (4, 5, 6).
One makes the same thing with the SN2.
These two eliminations are illustrated by the figure [Figure 1.3-e], where one sees in grayed with hatching them
two frontal matrices.

frontal matrix 1
frontal matrix 2
1
2
4
5
6
3
4
5
6
7
table Factor

Appear 1.3-e

The elimination of the SN3 consists of:
· the addition of the columns (4, 5, 6, 7) of the initial matrix, with the matrix of work of order 7, related to
unknown factors (4, 5, 6, 7, 8, 9, 10),
· assembly of the frontal matrices 1 and 2 in this matrix of work,
· the elimination of the columns (4, 5, 6, 7),
· obtaining in additional Factor of four columns,
· obtaining the frontal matrix 3d' order 3 (columns 8, 9, 10).
It is noticed that the frontal matrix 3 can line up in the place of the frontal matrices 1 and 2.
arrangement of these matrices requires a structure of pile where one piles up at the end of an elimination, and
where one depilates during the assembly. It is the maximum length of this pile which is minimized at the time of
the phase of sequence of execution.
The figure [Figure 1.3-f] illustrates the elimination of the SN3.
initial columns
frontal matrix 1
stamp
frontal 2
4
5
6
7
8
9 10
frontal matrix 3
Appear 1.3-f: Elimination of the SN3
It is noticed that the coefficient **L74** (square white on the figure [Figure 1.3-f]), comes from elimination SN3, and that the initial term **A74** is null (bond in dotted on the figure [Figure 1.3-d].
One saw in paragraph 1.1 that the elimination of a column consisted of an operation of the type “saxpy”, addition with a vector of the product of a vector by a scalar.

It is seen easily that the elimination of a super-node, groups columns, consists of an operation of type “matrix-vector produces”. These operations consume the greatest part in computing times, work of factorization of the matrix. They are carried out by subroutines of library BLAS, provided after being optimized on the majority of the calculators. One sees on appear [Figure 1.3-g] the column *J* updated by the product of the matrix [
With *J* + 1: *N*, 1:]
m and of
vector [
To *J*, 1:]
Mr.
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Factorization being finished, one has the matrix factorized in the shape of a table compacted nonnull terms only. It is a storage of the type “Morse”. The table of index **total** evoked in factorization symbolic system indicates, for each column of **L**, the numbers of line of each stored coefficient.

L
To [*J*, 1: *m*]
With [*j*+ 1: *N*, *J*]
J
N
L
I
m
J
m + *p*
With [*j*+ 1: *N*, 1: *m*]

Appear 1.3-g: Update of the column J by a product matrice*vector

1.4

Descent - Increase

The columns of **L** being stored in a compacted way, the descent is of type “saxpy” and increase of the type produces scalar, these two operations being both indexed. I.e. the algorithm of descent is coarsely the following, (while having initialized beforehand *X* by second member of the system):

for *I* =
1
with *N* **to make**

for *K* Co

lonne *I* **to make**
(**xglobal** (*K*)) = (**xglobal** (*K*))- **Lki** × (**xi**) (saxpy indexed)

for *I* =
1
with *N* **to make**
(
X *I*) = (
X *I*)/**Dii**
The increase, it, is written in the form:
for *I* **variable**
N
of to 1 **making**
(
X *I*) =
(
X *I*) -
ki
L X (*total* (*K*)) : indexed scalar product
K C
olonne I
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Establishment and use in *Code_Aster*

The use of the multi-frontal method is accessible by operator **NUME_DDL**, in the way following:

naked = **NUME_DDL** (**matr_rigi**: matr, **storage**: “MORSE”, or renum: “MDA”
renum: “MANDELEVIUM”); or renum: “MONGREL”

Two other methods of renumerotation are available: Approximate Degré minimum (MDA), which is an alternative of the method of the degree minimum, and a method of encased dissection (MONGREL).

They
are briefly described in appendices 1 and 2. It is enough to replace the value of renum by MDA, or MONGREL.
This method is also available in MECA_STATIQUE, STAT_NON_LINE, DYNA_NON_LINE, THER_LINEAIRE, THER_NON_LINE with same logic.
Then, one will use operator **FACT_LDLT**, the call being the same one as in the case of a matrix stored according to the profile mode.
In **NUME_DDL**, one indicates that the matrix to be factorized is arranged according to the mode "MORSE" and that
one asks for one of the two renumérotations of the "minimum dismantles", or a renumerotation by encased dissection (MONGREL).
In this case, **NUME_DDL** carries out the first three phases seen previously:
1) renumerotation,
2) factorization
symbolic system,
3) sequence
of execution.
These operations take into account the presence of "double-lagrange" and respect the order of the d.d.l. implied by the boundary conditions.
Moreover, **NUME_DDL** prepares cutting per blocks of the factorized matrix. Indeed, one saw that with each elimination, one arranged in a table the columns of the factorized matrix. These columns
will be useful that at the time of the gone up descent/. They are never used for calculation of other columns. There is not
thus no interest to have them all in memory. They are arranged, in *Code_Aster*, under
form of a collection of objects JEVEUX length variable. These objects, blocks of columns, must nevertheless to satisfy the following constraint: each block corresponds to an integer of "supernoeuds". One cannot thus have the columns of same "a supernoeud" on several blocks.
Since one does not know the place in memory available during numerical factorization,
one decides in **NUME_DDL** that the maximum length of each block will be the max. one (on all SN) sum lengths of the columns of SN.

Lb
= max length

max

(*col.k*), (in abrég) é
SNi K S
Ni

Operator **FACT_LDLT** uses the pointers created by **NUME_DDL** and the initial matrix "MORSE". It create the matrix factorized in the shape of a collection of objects JEVEUX, as one saw previously. A structure of data **provisional** is also necessary. It relates to the matrices frontal. Two cases arise:

· the pile of the frontal matrices can hold very whole in memory (in a table monodimensional), in this case, one allocates an object JEVEUX and the algorithm multifrontal manages then itself this space, by arranging the frontal matrix "mother" in the place of the matrices "girls",

· it does not hold and, in this case, it is created in the shape of a collection of objects JEVEUX, each frontal matrix being an object. For the elimination of the "supernoeud" *I*, it is necessary at the same time in memory:

-
the block of factorized to which *SNi* belongs,

-
the object stamps frontal *I*, like all the frontal matrices "girl" of *I*, which **will be destroyed** after their use.

The frontal matrix *I*, it, will be stored until its use and its destruction. One could, as it was made before, to release within the meaning of JEVEUX each frontal matrix after its creation. That can involve then, in the event of weak memory available, a storage on disc crippling in volume. Indeed, a destruction of object JEVEUX does not involve the destruction of sound image on disc, and summons it lengths of the frontal matrices is enormous.

In short, it is necessary to be able to arrange simultaneously in memory:

· a block of factorized,

· the pile of the frontal matrices, in only one object if that holds, several objects if not.

One thus needs **"sufficient"** memory to use the method multifrontale. Second manner of to arrange the pile allows the execution when the memory is sufficient, but émiettée.

When the memory is insufficient, it is necessary to start again the execution of **fact_ldlt** with a memory more large.

Note:
In order to provide an order of magnitude, the large case-tests treated by Code_Aster, required up to 20 or 25 megawords of memory for the pile of the frontal matrices. In the majority of case the ***fact_ldlt*** *resolution is used after many other operators, and does not profit any more that of a émiéttée memory, or little of memory, (in particular during the use of operator MODE_ITER_SIMULT or MODE_ITER_INV, which allocate much place to the vectors LANCZOS). The user wanting to carry out a very large case can have interest to proceed into two stages, the second executant* ***fact_ldlt*** *in mode "continuation" and profiting then from a memory not émiettée power station.*
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3 Bibliography

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Appendix
1
Reference material of the method
“Approximate Minimum dismantles”

The algorithm of the “minimum dismantles” consists has to number the nodes of a graph in the ascending order of
numbers of their neighbor. Here is a simplified but essential description.
1) one forms the initial graph associated with the matrix digs to factorize,
2) one calculates the number of the neighbors of the nodes (their degree),
3) one numbers in first (then eliminates from the graph), the node having less neighbors (minimum degree),
4) one updates the graph of stage 1,
5) one turns over at stage 2.

The calculation of the degree of the nodes is an expensive operation in computing times. The method “Approximate
Minimum dismantles” [bib3] proposes to reduce this cost. For that, instead of calculating the real degree of a node
one is satisfied with one raising (often equal, according to the authors, with the true degree), of easier calculation. Indeed with

run of elimination the true degree of node *I* is equal to *di*, cardinal of the following unit: *WITH* {*L*
I
E}, where
the union is made on the whole of the neighbors *E*, previously eliminated, of *I* (terms of filling), and where *Have*
is the whole of the initial neighbors. In the approximate method *di* is replaced by
D
L
I =card (*Have*) + card {*E*}, i.e. one neglects the intersection as of.
This method is described in [bib3] and the various alternatives of the algorithm of the “minimum dismantles” are
exposed in [bib4].

Appendix 2 Reference material Mongrel

The implementation of the module of renumerotation for matrices dig MONGREL is described in the note [bib5],
provided in the repertory mongrel-4.0/Doc of the software. The algorithm used is described in [bib6]
It is also possible to consult following address Internet: HTTP: /www.cs.umn.edu/~karypis
Mongrel is mainly a cutting tool of graphs (grids), aiming at the 2 following goals:
· to cut out a graph given out of p subgraphs having the closest possible sizes,
· to minimize the size of the borders between the subgraphs.
The first goal aims at a good balance of parallelization on p processors and the second aims at minimizing
communications.
MONGREL uses an algorithm of partition of graphs on several levels, proceeding in the following way: with
each level one seeks separators (together of with dimensions), minimal sizes cutting the graph in equal parts. MONGREL applies this principle to the renumerotation of the nodes of a graph while seeking to each
stage a separator dividing the graph into 2 subgraphs. The nodes of the first at the head are numbered subgraph, then those of the second, then those of the separator. One applies then the same algorithm to the 2
under graphs in a recursive way.

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C. ROSE
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Document: R6.03.01
Resolution of nonregular systems by
a method of decomposition in values
singular
Summary:
This document is devoted to the resolution of the systems of linear equations nonregular. Matrices taken in account can be square noninvertible or rectangular.
After having recalled the theoretical framework of the solutions within the meaning of least squares, we concentrate the talk
on the method by decomposition in singular values which provides, on the one hand, a tool for diagnosis of the degree
of regularity of the system, and, in addition, a family of algorithms of resolution at the same time more general and more
stable that those drifting of the approach by the normal equations.
Lastly, we detail the algorithm implemented in *Code_Aster* which reabsorbs the equivalent functionality of

the bookshop Nag (F04JDF for version 12 and F04JDE for version 15) used for the modeling of metallurgical behavior of steels [R4.04.01].

Contents

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1 Introduction

Being given a real matrix **A** of order $m \times N$ and a vector **B** element of ¤m, we consider
the problem of the determination of a vector **X** element of RN which checks the following linear system:

Ax = **B**

éq 1-1

It is well-known ([bib3] p. 9) that this system admits one, and only one solution, for any **B** element of Rm under the conditions necessary and sufficient that it is equi-constrained ($m = N$) and that its matrix **A** that is to say regular. Also, the investigation of the under-constrained case ($m\ N$) and overstrained ($m\ N$) us
will confront with the one of the three following situations:

1) The linear system [éq 1-1] admits a solution and only one,
2) The linear system [éq 1-1] does not admit a solution,
3) The linear system [éq 1-1] admits an infinity of solutions.

In practice, situation 2) in general meets in the case of an overstrained system then that the singular and under-constrained equi-constrained systems lead in general to situation 3).
To claim to solve a linear system of the type [éq 1-1], we should initially define what us will call **solution**. This is the object of the **paragraph 2** which is based mainly on the concept of **least squares** and on **differentiable optimization** to define, some is the type of system, a solution which is always single.
Paragraph 3 is devoted to the **decomposition in singular values** of the matrices (in summary SVD: Been worth Singular Decomposition), which, not only constitutes a tool to diagnose which of the three preceding situations corresponds to the studied linear system, but also provides a method of determination of the solution defined in paragraph 2.
The method using **decomposition SVD** is presented at **paragraph 4** and y is compared with method of the **normal equations**.
Paragraph 5 details on the algebraic level the application of method SVD to the **resolution of one linear équi or under-constrained system** such as it is implemented in *Code_Aster*.
In the following paragraphs, we will use the notations below:

·

X and (**X, y**) for, respectively, the euclidian norm of vector **X** and the scalar product associated vectors **X** and **y** elements of ¤m or ¤n,

·

MT for transposed of the matrix **M**,

·

Ker **M** and ImM for, respectively, the core and the image of (the associated linear application with) the matrix **M**,

·

X for the orthogonal one of under space **X** of Rm or RN.

Solution of a rectangular linear system

In this paragraph we will define a concept of *solution* for the linear system [éq 1-1] which enjoys properties of **existence** and **unicity.** The step proceeds in two times:

· Initially, by an approach of the type least squares we build a problem of differentiable and convex optimization (section 2.1) which admits always at least a solution (section 2.2). Situation 2) of paragraph 1 is then eliminated,

· Then, analyzing the property of unicity (section 2.3) to note that it is not always guarantee we will impose an additional constraint (section 2.4) on the solution characterized in section 2.1 in order to restore unicity.

2.1

Formalism of least squares

The single solution of a linear system **Ax** = **B** of square and regular matrix carries out the minimum of **Ay** quantity - **B** when **y** described ¤n. This property opens the way to us which leads to a concept of solution for a linear system general of the type [éq 1-1] which confers the same properties to him as those of the particular case of the regular system. We will thus say item **X** of ¤n which it is solution of system [éq 1-1] if it is solution of the **problem of optimization**:

Ax - B = Min **Ay - B**

éq 2.1-1

y *N*

Û

This approach is natural because it defines a solution of which the residue **R** = **Ax - B** is null in the case where the second member is element of Im **A** and is of minimal standard in the contrary case, which constitute best than one can wait.

To analyze the problem [éq 2.1-1], it is convenient to substitute the problem of optimization to him without

constraints are equivalent according to:

to find **X** *N*

Û

J (

such **X**)

that

= MinJ (**y**)

éq 2.1-2

y *N*

Û

where J ()

. is the functional calculus defined by:

J: **y** *N*

1

Û J (**y**) =

Ax - B

2

The interest of the problem [éq 2.1-2] is due to the fact that the functional calculus J ()

. check the following properties:
·
J (). is twice continuement differentiable:
DJ (**X**): **H** *N*
T
T
Û DJ (**X**) **H** = (**A Ax - A B, H**) Û
éq 2.1-3
D2J (**X**): (**H, K**) *N*
N
T
Û × Û DJ (**X**) (**H, K**) = (**A Ah**, **K**) Û
éq 2.1-4
·
J (). is quadratic and convex.
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Thus, the problem [éq 2.1-2] lies within the scope of the differentiable and convex optimization of kind that we have the following results ([bib1] p. 156 and 146):
1) Convexity: any local optimum is in fact a total optimum, i.e. a solution of
[éq 2.1-2],
2) Differentiability: any local optimum checks the equation of Euler DJ (**X**) = 0 on ¤n which, taking into account [éq 2.1-3], led to the characterization by the **equations** known as **normal**:

AT Ax
AT
=
B
éq 2.1-5

2.2 Existence
optima

In [bib1] p. 171 one finds a demonstration of the existence of at least a solution to the equations normals [éq 2.1-5]. This demonstration is based on arguments intended for the taking into account of case of infinite dimension (theorem of projection on convex closed of a space of Hilbert).
Our case being definitely simpler, we give a demonstration of the result which only uses simple algebraic arguments which, moreover, we will be useful in paragraph 3. To show that, for any **B** element of ¤m, the normal equations [éq 2.1-5] admit a solution is equivalent to establishment of inclusion Im **AT**
Im **AT**

A. However, for any real matrix **M** of order *m* × *N* us
let us have ImMT = (Ker **M**) ([bib3] p. 28). Also, the inclusion to be established which is equivalent to
(

Ker **A**) (Ker **ATA**) which is it even equivalent to Ker **ATA** Ker **A.** Is thus
X Ker **AT A**; then **Ax** Ker **AT**, i.e. **Ax** (
With)
Im

. As **Ax** is also element of
Im **A**, it can be only null what means that **X** Ker **A** and completes the demonstration.
This stage of the matter, we can say that any system of the type [éq 1-1] admits at least one
solution within the meaning of [éq 2.1-3] and all these solutions are characterized as solution within the meaning of
Cramer of the normal equations of [éq 2.1-5]. Situation 2) of paragraph 1 is eliminated.
Remain to eliminate situation 3), i.e. to guarantee unicity.

2.3

Unicity of the optimum and row of the system

It is clear that normal equations [éq 2.1-5], characterizing the optima which we seek,
admit a single solution under the condition necessary and sufficient that **AT A** is regular.
As **AT A** is always semi-definite positive, its inversibility is equivalent to its definite positivity, of
left that, taking into account the expression [éq 2.1-4] of the derived second of the functional calculus J ()
. , us
let us find the well-known theorem of unicity of the optimum of the problem [éq 2.1-2] for a functional calculus
convex twice continuement differentiable ([bib1] HT 7.4-3 and 7.4-4).
In any general information, nothing prevents matrix **AT A** from being singular, the solution of the system [éq 1-1]
within the meaning of [éq 2.1-2] is thus not always single. We have nevertheless a criterion for
to detect this situation. With section 2.2 we established that Im **AT**
Im **AT**

With and like
reciprocal inclusion is trivialement true, we can show identity Im **AT**
Im **AT**
=
A.
The introduction of the **row**
(
rg **A**) of matrix **A**, the dimension of its space image, allows us then
of saying that a condition necessary and sufficient so that **AT A** is invertible is that
(
rg **AT A**) = *N*
what is equivalent to
(
rg **A**) = *N* bus (
rg **AT A**)
(
rg **AT**
=
) = rg (**A**).
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The interest of this criterion is due to the fact that it limits the analysis to only matrix **A** without it being necessary of
to form **AT A.** explicitly This criterion also shows us that the normal equations associated one strictly under-constrained linear system always admit an infinity of solutions. Indeed, it row of a matrix is as equal to the number of independent columns as it has; also, for that this row reaches value *N* it is necessary that the columns of the matrix is of order at least *N*.

2.4

Solution within the meaning of least squares

We have just noted that the whole of the points which minimize the residue of the system [éq 1-1] is not not necessarily reduced to only one point. To restore unicity we refine the concept of solution of system [éq 1-1] of section 2.1 by defining the **solution within the meaning of least squares** like the element of minimal standard of the whole of the points which minimize the residue. This solution **X** is
then characterized by:

X Sdef = {**y** *N*
R; **ATAx** = **AT**}
B and **X** = Inf **y**
y S

This characterization is not satisfactory on the practical level because it asks for the resolution of one problem of optimization under constraints. We will substitute another characterization more to him adapted to the direction where it will lead (see section 4.2) to a procedure of calculation definitely simpler.

The unit **S** above is relocated of core of **ATA** by any of the vectors solutions
normal equations [éq 2.1-5]. Also, the additional condition of minimization of the standard be interpreted like a simple projection: the solution within the meaning of least squares of the system [éq

1-1]
is anything else only the projection of the origin of ¤n on the whole of the solutions of the equations normals. Also, we can characterize it like the point of intersection between the unit **S** and the orthogonal one of the core of **AT A.**
The definition of a solution to the system [éq 1-1] can then be summarized as follows:

AT Ax = **ATb**
X is solution of **A**
X = **B**

éq 2.4-1
X
T

(KerA **A**)
The first condition makes **X a** vector of minimal residue while the second selects, among the vectors of minimal residue, that of minimal standard.
The definition [éq 2.4-1] is a traditional generalization of the concept of solution of a system equi-constrained regular and confers on any system of the type [éq 1-1] a solution and only one.
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3 Values
singular

In this paragraph we have some useful results for the design of a method of operational resolution of the system [éq 1-1]. These results derive from the concept of singular values (section 3.1) and allow to build a base of the core and a base of the image of the matrix of system (section 3.2) from which it is possible to give a direction, adapted to the calculation of solution within the meaning of least squares, contrary to an unspecified matrix (section 3.3).

3.1

Decomposition in singular values

Let us start by pointing out the definition of the singular values. One calls **singular values** of one real matrix **A** of order $m \times N$ square roots of the eigenvalues of square matrix **AT A** of order N which, let us recall it, is semi-definite positive.

The concept of diagonalisation of the square matrices (when they are diagonalisables) spreads with rectangular matrices (without restriction) by the concept of decomposition (or factorization) in values singular.

For all real matrices **A** of order $m \times N$, there are two unit square matrices **Q** and **P** of a respective nature m and N such as:

WITH = **Q P**

T

éq 3.1-1

where is a matrix of order $m \times N$ of which the structure is schematized below:

µ1
µ2
=
0
if *m N*
! µn
µ1
µ2
=
!
if *m > N*
µn
0

The µi are the singular values of **A** which we suppose ordered by descending order:

µ1µ2 “µn

One can find a demonstration of this result in [bib1] p.10 for the équi-constrained case and in [bib3] p.73 for the overstrained case, the under-constrained case results some then by transposition.

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Factorization SVD [éq 3.1-1] of **A** gives **AT A**
P *T* **PT**
= and **AAT Q** *TQT*
=

so that,
T and *T* being diagonal square matrices, the matrix **P** is consisted of the vectors
clean orthonormalized of matrix **AT A** while the matrix **Q** is consisted of the vectors
clean orthonormalized of matrix **AAT**.

3.2

Row, image and core

The paragraph [§2] showed the fundamental part which plays the row of matrix **A** and the core of stamp **AT A** for the resolution of a nonregular linear system of the type [éq 1-1]. We will see now how factorization [éq 3.1-1] can be used to determine this row like one base of Ker **AT A.**

That is to say *R the* index of the smallest nonnull singular value. Factorization [éq 3.1-1] is written too **QT AP** = where the taking into account of the null singular values makes it possible to specify the decomposition
in block of:
R
=
0
if *m N*
0
0
R
=
0
if *m > N*
0
0
0
where *R* = Diag (µ, 1µ2, *R*
µ
"
) is the diagonal matrix of order *R* of the nonnull singular values
in the ascending order.

Since the matrices **Q** and **P** are regular, matrices **A** and are equivalent so that them

respective core and image coincide. We thus deduce from it that:
· The row of **A** coincides with the number of nonnull singular values:
rg **A** = *R*
· The vectors columns of **P** of index *R* +1 with *N* form a base of Ker **A**
· The vectors columns of **Q** corresponding to the nonnull singular values form one
base of Im **A**

In addition, with section 2.3 we saw that Im **AT**
Im **AT**
=
A. The ImMT identity = (Ker **M**)
us then Ker **A** = Ker **AT A** gives so that the second condition of the definition [éq 2.4-1] is
simply realized by any vector which is expressed like a linear combination of the vectors
columns of **P** corresponding to the nonnull singular values.

3.3
Pseudo-opposite and solution within the meaning of least squares

Another application of the decomposition in singular values consists of the concept of
pseudo-opposite (or opposite Moore-Penrose) which generalizes the usual concept of reverse of a matrix
square regular with the rectangular matrices on the one hand, and the singular square matrices of other
leaves.
First of all, theopposite one of a matrix of the decomposition in singular values [éq 3.1-1]
is defined by:
+
-1
R
=
0

if *m N*
0
0
+
-1
0
R
=
if *m* > *N*
0
0
0

where -1
1
1
1
R = Diag
,
, # is the reverse with the usual direction of

r.
1
µ
2
µ
R
µ
This being, we use the decomposition [éq 3.1-1] matrix **A** to define its pseudo-opposite **A+** by:
With +
P +
= **QT**
éq 3.3-1
In the same way, from the decomposition [éq 3.1-1] of matrix **A** we draw **AT A**
P *T* **PT**
=, of kind
that theopposite one (**AT** +
With) of matrix **ATA** is defined by:
(
+
+
AT A) = **P** +

(*T*
) **Pt**
éq 3.3-2
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We are able now to provide a simple interpretation of the solution within the meaning of least squares defined by [éq 2.4-1].
The restriction on (Ker **AT**
With) of the linear application associated matrix **ATA** defines one isomorphism of (Ker **AT**
With) on ImATA. Like, on the one hand
(

Ker **AT A**)
(Ker **A**)
=
= Im **AT**, and, in addition, ImATA ImAT
=
, this restriction is in fact an automorphism of (Ker **AT**
With). In the base of (Ker **AT**
With) constituted by the *R* first columns of the matrix **P**, this automorphism is represented by the matrix 2r. Also, sound reciprocal automorphism is represented there by the matrix -2
r. Extension to ¤n of this automorphism is then represented, in the base associated with the matrix **P**, by the matrix
(+
+

T) = *T* (*T*), and thus, in the canonical base, by the matrix (**AT** +
With).
It follows that:
· We find the fact that, for any **B** element of ¤m, there is a single vector

X (Ker **ATA**) solution of **ATAx AT**
=
B, is the existence and the unicity of the solution with
feel least squares [éq 2.4-1) of the **Ax** system = **B**,
· This single solution is given by:
+
X = (**ATA**) **ATb**
éq 3.3-3
Theopposite one of a matrix is defined starting from decomposition SVD of this matrix. Like decomposition SVD is not single, the pseudo-opposite matrix is not single. On the other hand, of point of view of the linear applications associated the matrices, the pseudo-opposite application is single. All the matrices pseudo-opposite associated with various decompositions SVD with a matrix data are not whereas representing matric particular which expresses this application pseudo-opposite relative at the bases induced by the orthogonal matrices of the decompositions SVD. Also, the expression [éq 3.3-3] has a direction: it defines a vector whose **X** represents them components compared to the base of arrival (matrix **P**) of decomposition SVD.
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4
Resolution of a rectangular linear system
The two methods of resolution of the system [éq 1-1] which we present at sections 4.1 (method normal equations) and 4.2 (decomposition in singular values) aim to the resolution of normal equations [éq 2.1-5]. These two methods are characterized not only by the choice of

algorithms which they implement (inversion against pseudo-inversion), but also by their degree of general information and by their numerical properties which are compared with section 4.3

4.1

Method of the normal equations

The resolution of the **Ax** system = **B** by the method of the normal equations consists in calculating solution within the meaning of least squares [éq 2.4-1] in a “direct” way, i.e. while using directly

-1

the relation **X** = (**ATA**) **ATb**. For that, it is initially a question of calculating **ATA** and **ATb**, then to solve

the system obtained either by iterative method or by a factorization of **AT A.**

We can notice right now that this method is limited to regular matrices **AT A**,

what limits its applicability to the system [éq 1-1] whose matrix is of full row (see section 2.3). In particular, the method of the normal equations cannot treat nor the systems strictly under-constrained, nor singular équi-constrained systems (see section 2.4).

4.2

Method by decomposition in singular values

We saw with section 3.3 that the solution of the **Ax** system = **B** within the meaning of least squares

+

defined by [éq 2.4-1] can be characterized by the relation **X** = (**ATA**) **ATb** [éq 3.3-3]. Method of resolution of the system based on this property **is known as method by decomposition in values singular** because it builds theopposite one [éq 3.3-2] of **AT A** via decomposition SVD [éq 3.1-1] of matrix **A.**

As any matrix can be broken up into singular values, it follows that any system of the type [éq 1-1] can be solved within the meaning of [éq 2.4-1] by this method which thus presents, at least, the advantage of the general information compared to the method of the normal equations.

It is not all. Method by decomposition in singular values, contrary to the method normal equations, does not require the explicit construction of matrix **AT A** and the vector **ATb** (we will see with section 4.3 the interest on the numerical level of this property). Indeed, it is easy to check that the matrix of the singular values of factorization [éq 3.1-1] satisfied with

+

identity + (*T*) *T* +

=

, so that, prémultipliant **AT** by theopposite one of **AT A** and taking account of factorization [éq 3.1-1], we obtain

(

+

+

AT A) **AT** = **P** +

(*T*

) *T*

PTP *T*

QT, which, by orthogonality of **P** gives us

(

+
AT A) AT A+
=
. Consequently, the characterizations [éq 3.3-3] and [éq 2.4-1] of the sought solution are equivalent to the characterization:
X is solution of **A**
X = B X = A+b
éq 4.2-1
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4.3 Comparison of the method of the normal equations to the method of decomposition in singular values

In the two preceding sections, we come to note, that algebraically, the method of decomposition in singular values is more general and simpler than the method of the equations normals. We now will note, while following [bib3] p. 336, that it is also higher to him on the numerical level. This superiority is expressed on the one hand, in term of stability not only of resolution, but also of construction of the problem and, in addition, on a less critical level, in term of adaptation to the treatment of the hollow matrices.

4.3.1 Conditioning

The conditioning of a matrix **A** of order $m \times N$ is defined like the report/ratio of its values singular extremes and nonnull:

μ
cond (**A**) = *1*
μr

where *R* is the row of matrix **A.**

Results presented in [bib3] p.184, using the normal equations as a tool for analysis and not like a computational tool, show that the disturbance of the solution of the problem of optimization [éq 2.1-1] due to the rounding errors can be proportional to cond (**A**) 2. But results

traditional of the analysis of stability of the solution of a linear system compared to these same errors show a proportionality with the number of conditioning of the matrix. So that in case of a direct resolution of the normal equations, we obtain an error always proportional with cond (**ATA**) = cond (**A**) 2, which is worse than cond (**A**).
The method of resolution by decomposition in singular values uses only transformations orthogonal (see paragraph 4), so that it does not modify the initial conditioning of problem [éq 1-1] and is thus, from this point of view, more attractive than the method of the equations normals.

4.3.2 Loss of precision

We have just seen that the rounding errors lead to a degradation of the solution more sensitive when it is calculated via the normal equations rather than by a decomposition in singular values. The following example, drawn from [bib 2], shows that construction even system [éq 2.1-5] of the normal equations is disturbed by the rounding errors.
That is to say thus the following matrix:
1 1

1+ 2

1
With =
T
0 leading to **A A** =
2

1
1+

0
whose singular values are µ
2
1 =
2 + and µ2 =, so that the row of **A** is 2 as soon as
0. If *2* < *Mach* checks < where *Mach* is the precision machine, then all coefficients of **AT A** will be calculated with value 1 and the calculated singular values will be, at best, µ1 = 2 and µ2 = 0. It follows that the numerical row, calculated by the normal equations, will be 1, whereas that calculated by a decomposition SVD of matrix **A** would be equal to 2

4.3.3 Structure
hollow

With a less level, the construction of the matrix of the normal equations induced a filling of associated system that the method using the decomposition in singular values avoids.

4.3.4 Conclusion

The following table summarizes the discussion of the preceding sub-sections:

General information	**Conditioning**	**Loss of precision with filling**	**construction of the problem**
Equations	systems of full row	cond (**A**) 2	possible
yes	normals	SVD	any system
cond (**A**)	impossible	not	

5
Algorithm SVD for the resolution of a linear system
équi or under-constrained

In this paragraph, we detail the method of resolution of the not-regular systems put in work in *Code_Aster*. This method applies to the under-constrained or équi-constrained system singular and provides the solution within the meaning of least squares [éq 2.4-1].

The calculation of a decomposition SVD of **A** is equivalent to the calculation of the spectrum of the normal matrix
associated **ATA**. Also, it can be obtained only with the convergence of an iterative process.

Section 5.1 exposes the **principle of the algorithm** and shows in particular how the application of two orthogonal transformations makes it possible **to reduce the problem** to the simple research of

decomposition SVD of a higher matrix bidiagonale. **Sections 5.2** and **5.3** are devoted to **the algorithmic one** of these **reductions**. **Section 5.4** presents the algorithm of **decomposition SVD** matrix bidiagonale.
The algorithms will be described with the convention of notation in which:
.
R (*I, J,*) indicates the rotation of Givens of the plan (*I, J*) and of angle,
.
( )
( )

With *K* indicates reiterated index *K* of a matric iteration and **A** *K L*, reiterated the *L* of an iteration
( )
intern with reiterated **A** *K*.

5.1
Reduction of the problem and principle of the algorithm

In this section, we present the algorithm of resolution of a linear sytème équi or under-constrained by method SVD.
We reduce the problem to the research of decomposition SVD of a matrix bidiagonale like in [bib2] but we carry out the reduction in another way that that proposed in [bib2]: us let us start by reducing the matrix to a higher triangular form, then, we reduce this triangular with a higher form bidiagonale. These two reductions are carried out by orthogonal transformations.
The arithmetic operations **of decomposition SVD** are connected as follows:

5.1.1 Reduction with the higher triangular form

WITH = [**U**
] 0PT1

if $m < N$

éq 5.1-1

WITH = U PT1

if $m = N$

where **P1** is an orthogonal matrix of order N and **U a** higher triangular matrix of order Mr.

5.1.2 Reduction with the higher form bidiagonale

U = Q BPT

2

2

éq 5.1-2

where **Q2** and **P2** are two *orthogonal matrices* of order m and **B a** higher *matrix bidiagonale* of order Mr.

5.1.3 Decomposition SVD of the higher bidiagonale

B = Q

P

3 *T*

3

éq 5.1-3

where **Q3** and **P3** are two *orthogonal matrices* of order m and a *diagonal matrix* of order m of form:

µ1
!
0
R
0
µ2
=
=
0
0
!
0
0
0

Combining the relations [éq 5.1-1], [éq 5.1-2] and [éq 5.1-3], we obtain a **decomposition SVD of matrix a:**

T
T
WITH = Q Q
R 0
P P
3

2
0
T
2
3
0
P
0 0
1
0
éq 5.1-4
I
The solution within the meaning of least squares [éq 2.4-1] of the system [éq 1-1] is then obtained by the application of pseudo-opposite [éq 3.3-1] of **A** deduced from the decomposition in singular values [éq 5.1-4]. We thus obtain:
P
P +
QTQTb
X P
2 3
3
2
= 1

éq 5.1-5

0

The algorithm proposed thus consists of the sequence of factorizations [éq 5.1-1], [éq 5.1-2] and [éq 5.1-3] before the application of the relation [éq 5.1-5].

5.2

Reduction with the higher triangular form

Starting from a matrix **A** of order *m* × *N* for *m* *N* one determines a higher triangular matrix **U** of order *m* and an orthogonal matrix **P** of order *N* such as:

WITH = [**U**

] 0PT

if *m* < *N*

WITH = **U PT**

if *m* = *N*

Algorithmiquement, factorization uses a method of elimination which is interpreted, like

( )

construction of a succession of matrices **A** *K* by:

(*m*)

WITH = **A**

(*K*) 1

(*K*) (*K*)

With

= **A P**

for *K*

= *m*, *m* -

1, 1

#

( )

where each current matrix **A** *K* has the structure schematized below:

Column pivot

Coefficients

(column *K*)

cancelled in

Pivot (coefficient (*K*, *K*))
phase 1
(line *K*)
Coefficients
cancelled in
phase 2
With (*K*) =
(line *K*)
coefficients
Coefficients
null
not changing
more
(*K*)
( )
The coefficients *have*, *J* of matrices **A** *K* of the iteration thus check:
K +1 *I m*
and *m* + 1 *J N*
(*K*)

has
= 0 if *K* +1 *I m*
1
and *J K*
I, *J*
éq 5.2-1
K +1 *J I m*

so that at the end of the recurrence, we will have:
() 1
( )
U = **A**
and
P =
P
K
K =m -
m
,
1, 1
,
#
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( )

The problem is thus reduced to the safeguarding of the structure [éq 5.2-1] at the time of the passage of **A** *K* to

(

)

With *k-1*

( )

by a transformation **P** *K* which must be orthogonal. The problem of orthogonality is regulated by choosing the transformation like a product of rotations of Givens and the problem of safeguarding of the structure is solved by carrying out this product in an order which does not destroy them

zeros created.

( )

(

)

Taking account of the rectangular structure of matrix **A** *K*, we build reiterated **A** *K* -1 in two phases:

(*K*)

.

1

Phase 1 cancels successively the coefficients *ak, J* corresponding to the columns

J = K -1, *K* - 2,

1

, ,

#, which results in:

(*k-1, K*) 1

(*K*)

With
= **A**
(*k-1*, *J*) 1
(*k-1*, *J*)
(*K*) *T*
With
= **A**
R (*K*, *J*, *J*) for *J* = *K* -1, *K* - 2, 1
#
(*K*)
·
1
Phase 2 cancels successively the coefficients *ak*, *J* corresponding to the columns
J = *N*, *N* -,
1
, *m* +,
1
#
, which results in the recurrence:
(*k-1 N*,)
(*k-1*,*0*)
With
= **A**
(*k-1*, *J*) 1
(*k-1*, *J*)
(*K*) *T*
With
= **A**
R (*K*, *J*, *J*) for *J* = *N*, *N* -1, *K*
+1
(*K*)
The angle *J* of the rotation of Givens of the plan (*K*, *J*) is selected to cancel the coefficient in position
(*K*, *J*)
(-1,)
of *A K*
J. The application of each rotation thus modifies only the columns *K* and *J* what
does not destroy the null coefficients produced by the preceding stages. We note that
column *K* plays a particular part (that of pivot) because it only is systematically modified by
each rotation whereas the other columns are modified only by the rotation which cancels them
coefficient with the line *K*.
(*K*) 1
(*k-1*, *K*)
( )

With the exit of these recurrences, we have **A**
= **A**
. The matrix **P** *K* is then given by:
( )
j=n
(
j=k -1
K)
P *K* = R (
K
K, *J*, *J*)
( )
R (*K*, *J*, *J*)
j=m+1
j=1
so that the matrix **P** is worth:
K =
m
j=n
(
J K 1
K)
= -

P = R (
K
K, *J*, *J*)
( )
R (*K*, *J*, *J*)
K =1 *j=m+1*
j=1

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5.3

Reduction with the higher form bidiagonale

To reduce a triangular square matrix higher **A** of order *m* than the higher form bidiagonale consist in finding two matrices orthogonal **P** and **Q** and a higher matrix bidiagonale **B**, all three of order *m*, such as:

WITH = **QBPT**

Algorithmiquement, factorization proceeds like that of the preceding section by using one method of elimination which is interpreted algebraically like the construction of a succession of matrices ( )

With *K* by:

( )

WITH 1 = **A**

(*K*) 1

(*K*) *T* (*K*) (*K*)

With

= **Q**

WITH P

for *K* = 1 2

, *m*

- 2

+

( )

where each current matrix **A** *K* has the diagonal structure per block following:

· The higher diagonal block (indices of line and column varying of 1 to *K* -1) is one stamp bidiagonale higher order *K* - 1,

· The lower diagonal block (indices of line and column varying from *K* to *m*) is a matrix triangular higher of order *m* - *K*.

(*K*)

( )

The coefficients *have*, *J* of matrices **A** *K* of the iteration thus check:

1 *I K* -1 and *I* + 2 *J m*

(*K*)

has

= 0 if 1 *I K* -1 and *I* < *J*

I, *J*

éq 5.3-1
K I m
and 1 *J* < *I*

so that at the end of the recurrence, we will have:
(
K =m
K =m
m+)
1
(*K*)
(*K*)
B = **A**
,
Q = **Q**
and
P = **P**
K =1
K =1
As for the factorization of the preceding section, the problem is reduced to safeguarding
( )
(
)
( )
structure [éq 5.3-1] at the time of the passage of **A** *K* to **A** *K* +1. The orthogonality of the transformations **Q** *K* and
( )
P *K* is obtained by building them like product of rotations of Givens and the problem of safeguarding of the structure is solved by carrying out these products in an order which does not destroy them
zeros created by the preceding stages.
The algorithm thus cancels successively the coefficient (*K*, *J* +)
1 for *J* = *M-1*, *m* - 2, *K*
+ 2 by
the application on the right of a rotation of Givens of the plan (*J*, *J* +)
1. This rotation modifies only them
columns *J* and *J* + 1, which creates a parasitic coefficient in position (*J* +1, *J*). This parasitic coefficient is then eliminated by the application on the left from transposed of a rotation of Givens in the plan (*J*, *J* +) 1.
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( )
(
)
The process of passage of **A** *K* to **A** *K* +1 can then be formalized by:
(*k+1 m*
, -)
1
(*K*)
With
= **A**
(*k+1, j=1/2*)
(*k+1, J*)
(*K*)
With
= **A**

R (*J, J* +1, *J*)
(*k+1, j=*) 1
(*K*)
T
With
=
1, 1/2

R (
K + J
J, J
=
+1, *j=1/2*) (
)

With
for $J = M-1, m - 2,$
$, K$
$\# +1$
$(k+) 1$
$(k+1, K)$
With
$=$ **A**
(K)
(K)
where the angles J and $j=1/2$ are selected to cancel the coefficient in position respectively
$(K, J +) 1$
$(+1,)$
$(+1, =1/2)$
of **A** K
J and the coefficient in position $(J +1, J)$ of **A** $K J$
.
$(+1,)$
The structure of matrices **A** K
J is illustrated in the following figure:
Coefficient to be cancelled by rotation
P current (column $j+1$)
Coefficient cancelled by
Coefficient cancelled by
following rotations **P**
rotations **P the** preceding ones
Bi-diagonal
$K J =$
Line
With (,)
K
Coefficient created by
current rotation **P** and
eliminated by rotation **Q**
current
( )
( )
With the exit of this recurrence, the matrices **P** K and **Q** K are given by:
(
$j=k +1$
$j=k +1$
$K)$
(K)

P
= R (
K
K
J, J +1, *J*)
( )
( )
and
Q
= R (*K, J, j=1/2*)
j=m-1
j=m-1
so that the matrices **P** and **Q** are worth:
K = -
m2 j=k +1
(
K m2 J K 1
K)
= -
= +
P =
R (, +
K
J J 1, *J*)
( )
and
Q =
R (*J, J* +1, *j=1/2*)
K =1 *j*= -
m 1
K =1 *j=m-1*
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5.4

Decomposition SVD of a higher bidiagonale

We present an algorithm of construction of decomposition SVD of a matrix bidiagonale higher **A** of order *Mr*. *the* algorithm thus builds two orthogonal matrices **Q** and **P** and one stamp diagonal **D** such as:

WITH = **QDPT**

The algorithm is drawn from [bib2].

5.4.1 Principle of the algorithm

The calculation algorithm of decomposition SVD diagonalise repeatedly matrix **A** by means of recurrence:

( )

WITH 1 = **A**

(

éq 5.4.1-1

K

) 1

(*K*) *T* (*K*) (*K*)

With

= **Q**

WITH P

for *K*

= 1 2

+

, ,#

( )

( )

( )

where the matrices **Q** *K* and **P** *K* are orthogonal and matrices **A** *K* are bidiagonales higher.

With convergence, we will have:

(

K =

K =

)

(*K*)

(*K*)

D = **A**

, **P** = **P**
and
Q = **Q**
K =1
K =1
The idea of the iteration consists with:
.
( )
To choose
P *K* to make converge algorithm *QR* applied to the diagonalisation of the matrix
(known as normal) **ATA** without forming it explicitly. Indeed, the matrix **P** of the decomposition
SVD of **A** is not anything else that the matrix of the clean vectors of **AT A**,
.
( )
To choose
Q *K* to preserve the higher structure bidiagonale the reiterated successive ones.
( )
( )
As in the case of the factorizations presented at sections 5.2 and 5.3, the matrices **Q** *K* and **P** *K*
( )
(
)
are built like product of rotations of Givens. The passage of **A** *K* to **A** *K* +1 is then
realized by:
(
T
K +)
1
(*K*, 2) (*K*,) 3
(*K m*
,)
With
= [**Q Q**
Q
#
] (*K*) (*K*, 2) (*K*,) 3 (*km*,)
With
= [**P P**
P
#
]
éq 5.4.1-2
( )

( )
where **Q** *K I,* and **P** *K I,* are two rotations of the plan (*I* -1, *I*) of respective angle *J* and *I*:
(*K I,*)
(*K*)
Q
= R (
K I,
K
I -,
1 *I, I*
)
( )
( )
and
P
= R (*I* -, 1i, *I*)
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(
)
Rotations are alternatively applied on the right then on the left so that **A** *K* +1
( )
preserve the higher structure bidiagonale of **A** *K*. With this intention:
(*K*)
( )
Angle 2 is, for the moment, arbitrarily selected; the application of rotation **P** *K*, 2 creates
then a coefficient in position (2)
1

, ,
(*K*)
( )
Angle 2 is selected so that the application of rotation **Q** *K*, 2 cancels the coefficient in position
(2) 1
, which creates a coefficient not no one in position (1)
3
, ,
(*K*)
( )
Angle 3 is selected so that the application of rotation **P** *K*, 3 cancels the coefficient in position
(1) 3
, which creates a coefficient not no one in position (3 2
,) ,
$
(*K*)
(
)
Angle *M-1* is selected so that the application of rotation **Q** *K m*
, -1 cancels the coefficient in
position (*M-1*, *m* - 2), which creates a coefficient not no one in position (*m* - 2,)
m,
(*K*)
( )
The angle *m* is selected so that the application of rotation **P** *K m*
,
cancel the coefficient in position
(*m* 2,)
m, which creates a coefficient not no one in position (*m*, *m* -)
1 ,
(*K*)
( )
The angle *m* is selected so that the application of rotation **Q** *K m*
,
cancel the coefficient in
(
)
position (*m*, *m* -)
1, and stamps it **A** *k+1* is bidiagonale higher.
For any value of the angle, this process ensures maintains it structure bidiagonale higher than reiterated [éq 5.4.1-2]. We will see now how it is possible to choose this angle to make to converge the iteration [éq 5.4.1-1].

5.4.2 Implicit Diagonalisation of the normal matrix

(*K*)
The algorithm of the preceding sub-section leaves unspecified angle 2 of the first rotation of
( )
P *K*
( )
. We will raise this indetermination in order to make matrix **P** *K the* matrix
orthogonal of a step *QR*, with spectral shift, applied to the diagonalisation of the normal matrix
MR. AT
=
A.
With iteration SVD [éq 5.4.1-1] of matrix **A**, we associate an iteration on the normal matrix
MR. AT
=
A:
(*K*) 1
(*K*) *T*
1
(*K*) 1
(*K*) *T*
+
+
+
(*K*) (*K*)
M
= **A**
With
= **P**
MR. P
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Iteration *QR* **for the diagonalisation of the normal matrix**

( )
Transformation *QR*, with spectral shift *K*, applied to **M** *K* is written:
(*K*)
(*K*)
To factorize **M**
- I
K in the form **M**
- I = **P R**
K

(*k*+)
(*k*+)
To build **M**
1
1

by **M**
= **P R** + **I**

K
where **P** and **R** are two matrices respectively orthogonal and triangular higher. Matrices
( )
M *K*
(*k*+)
and **M**
1

are thus tridiagonales and similar:
(*k*+) 1
T
(*K*)
M
= **P MR. P**

From the practical point of view, the matrix **P** is presented in the form of a product of rotations of Givens:
Pt
= R (*N* -,

1 *N*, *N*) R (*N* - 2, *N* -, 1n-1) R
(,
1 2,2)
The angles *K* are selected so that the application on the left of R (*K* -,
1 *K*, *K*) with the matrix

R (1-11
K
, ,1)
( )
(**M** *kI*) the coefficient of position (*K*, *K* -) cancels 1 in the matrix
1

= 1, -2, 2
#

-
K
K
result.
( )
(*k*+)
Francis showed that the passage of **M** *K* to **M**
1

do not require the formation clarifies
(*K*)
stamp **M**
- **I**
K: the shift can be carried out implicitly. The theorem is stated like
follows:
Theorem (Francis): That is to say **X an** orthogonal matrix whose first column coincides with that of **P.** Under the assumptions:
(*k*+) 1
T
(*K*)
1) **M**
= **X MR. X**
(
)
2) **M** *K* +1 is tridiagonale,

( )
( )
3) The under-diagonal elements of **M** *K* are all nonnull (irreducibility of **M** *K*),
one has
(*k*+) 1
(*k*+)
M
= **DM** 1 **D**

where **D** is a diagonal matrix of diagonal coefficients all equal to ± 1.
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Application to algorithm SVD
(*K*)
Consequently, the choice of angle 2 of the first rotation of iteration SVD [éq 5.4.1-1] consists of
(*K*)

(*K*)
T
R (, 12,
( )
2
) = R (, 12,2)
2
=
- 2. Thus
so that the first column of **P** *K* coincides
( )
with the first column of **P.** Donc, if all the under-diagonal elements of **M** *K* are nonnull,

( )
then, the matrices **P** *K* and **P** are identified (with a multiplicative factor ±1 close to the columns) and iteration SVD [éq 5.4.1-1] is equivalent to the application of transformation *QR*, with shift, with
( )
stamp **M** *K*.

Choice of the spectral shift

The shift is usually selected like the eigenvalue of the lower minor of order two of
( )
With *K*
(*K*)
nearest to *amndt m*
. This choice ensures a total convergence which, generally, is
cubic.

Applicability of the theorem of Francis and phenomenon of decomposition

The use of the theorem of Francis supposes nonthe nullity of all the under-diagonal coefficients of
( )
matrices **M** *K*, which of anything is not guaranteed. Moreover, within the framework of method *QR* of diagonalisation of a symmetrical matrix tridiagonale, appearance of null under-diagonal coefficients is:

· Souhaitable: the null coefficients uncouple the diagonal blocks which they frame, which bring back the diagonalisation of the complete matrix to the diagonalisation of its diagonal blocks (this phenomenon is often called “decomposition”),

· Inévitable: the convergence of the algorithm towards an eigenvalue is interpreted algebraically like the appearance of a preceding diagonal block of order 1.

In the following sub-section, we will see which treatment it is appropriate to adopt in the presence of one decomposition.

5.4.3 Analyze decomposition

The analysis of decomposition relates to each reiterated taken independently of the others, also, us will not use the superscript (*K*).

That is to say *D*, *D*,

, *D*

1

2 #

m and *E*, *E*,

, *E*

2

3 # *m* respectively diagonal and on-diagonal elements of **A.**

The under-diagonal elements of the normal matrix **M** = **AT A** are then given by:

m 1 = *D E*

for

I

1

= 1 2

m

i+ *I*

I l+

-1

,

, , ,

Let us suppose, to simplify, that only coefficient *ml L*

1, is null. The matrix **M** presents one then

structure of two diagonal blocks whose meeting of the respective spectra gives the spectrum of **Mr.**

This decomposition takes place either for *e1* = 0 or for *E* 0 and *D*

1

L 1

- = 0 .

The case *e1* = 0 does not raise any difficulty. Matrix **A** then has a diagonal structure of two blocks which provide each one a part complementary to decomposition SVD of **A.** Each block being bidiagonal higher, without null on-diagonal coefficients, its decomposition SVD is calculable by the iteration [éq 5.4.1-1].

The case *E* 0 and *D*

1

L 1

- = 0 are more delicate. Indeed, the iteration [éq 5.4.1-1] cannot be applied nor

with matrix **A**, to violate the assumptions of the theorem of Francis, nor with none under matrices of **A**, to ensure the structure bidiagonale reiterated. This problem is circumvented by postmultiplication of **A** by a series of rotations of Givens in the successive plans

(*L* -1, *L*), (*L* -1, *L* +) 1, (*L* -1,

#

)
m:
· The rotation of the plan (*L* -1, *L*) cancels the coefficient (*L* -1, *L*) and creates a coefficient in position (*L* -1, *L* +) 1,
· The rotation of the plan (*L* -1, *L* +)
1 cancels the coefficient (*L* -1, *L* +)
1 and creates a coefficient in
position (*L* -1, *L* + 2),
· The rotation of the plan (*L* -1, *L* +)
1 cancels the coefficient (*L* -1, *L* +)
1 and creates a coefficient in
position (*L* -1, *L* + 2),
$
· The rotation of the plan (*L* -1,)
m cancels the coefficient (*L* -1,)
m and does not create a coefficient.
So that the matrix produced by this process has the same structure as that corresponding
with the case *e1* = 0.
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5.4.4 Organization of the algorithm

The algorithm isolates successively each singular value, also, it exists an index *K p* such as reiterated
( )
With *kp* breaks up into two diagonal blocks:
( )
(*kp*)
B
0

With *kp* =
( )
0
D *kp*
( )
( )
where **B** *kp* is a higher matrix bidiagonale of order *p* and **D** *kp* is a diagonal matrix
of order *m* - *p* + 1 gathering on its diagonal the found singular values.
( )
From this reiterated, the algorithm applies the iteration of sub-section 5.4.1 to the submatrix **B** *kp*
until the cancellation of the coefficient in position (*p* -1, *p*), signal of the convergence of the *pième* value
singular. Each step of the internal iteration, thus defined, is organized as follows:
· Analyse of decomposition,
· If the found decomposition corresponds to a diagonal element no one, a series of
additional rotations is applied to find the structure of generated decomposition
by a on-diagonal element no one. These rotations are built according to the method presented
with sub-section 5.4.3,
· If the coefficient in position (*p* -1, *p*) then does not produce decomposition the submatrix
( )
B *kp* is the object of a step of the iteration [éq 5.4.1-1] where, in accordance with the analysis of under
section 5.4.2, the angle of the first rotation is selected so that this step is equivalent to
the application of a transformation *QR*, with implicit spectral shift, on the matrix
associated normal.
The complete convergence of the iteration is then obtained with the index *km* for which the submatrix
( )
D *km* is of order *Mr*.
Of course, in practice, a coefficient is regarded as null as soon as it is lower, in value
absolute, with a certain tolerance. The tolerance generally used for the problems of values
singular is selected like the product of the precision machine by **A** 1. Let us notice that in
case of a decomposition produced by a diagonal element no one with the tolerance chosen, the
application of
series of additional rotations described with sub-section 5.4.3 creates under column of elements
nonnull under this coefficient. These elements are not awkward because they all are null with the
tolerance
chosen.

6 Bibliography

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Relation of behavior of Granger for the clean creep of the concrete

Summary:

This document presents the clean model of creep of "Granger", which is a way of modelling creep

clean of the concrete.
One also details there the writing and the digital processing of the model.

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matters

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1 Introduction

Within the framework of the studies of the long-term behavior of structures out of concrete, a share dominating of the deformations measured on structure relates to the differed deformations which appear in the concrete during its life. They comprise the withdrawals at the youth, the withdrawal of desiccation, clean creep and the creep of desiccation.

The model presented here is dedicated to the modeling of the differed deformation associated creep clean. Clean creep is, in complement of the creep of desiccation, the share of creep of the concrete that one would observe during a test without exchange of water with outside. In experiments concrete in
clean creep presents a growing old viscous behavior. The deformation of creep observed is proportional to the constraint of loading, depends on the temperature and the hygroscopy. longitudinal deflection is accompanied as in elasticity by a transverse deformation by sign opposed.
The selected model is that proposed by L. Granger [bib1]. It is model of a viscoelastic type which takes into account the effect of ageing as well as the history of constraint, temperature and of

the hygroscopy. It thus allows this fact of modelling the experimental facts quoted above.

One initially carries out a short recall on the linear viscoelastic models and one present then the model itself like its numerical integration in Code_Aster.

In Code_Aster, 3 versions are available: GRANGER_FP_V the complete model, GRANGER_FP who does not take into account the effect of ageing and GRANGER_FP_INDT, which in more does not depend temperature.

2 Recall on behaviour in creep of a material viscoelastic linear [bib3]

The traditional curve of creep represents the evolution according to the time of the deformation of one material subjected to a constant unidimensional constraint. The deformation of creep fl is, in opposition to the instantaneous strain, the share of deformation which evolves/moves with time. If a material has a linear viscoelastic behavior, then whatever the constant load applied as from the time of loading Tc, the deformation of creep (1D) can be written:
fl (T) = F (T - T)

éq 2-1
C
where J (T, T) = F (T -)
C
Tc is related to creep, increasing function of (T - Tc) and null for (T - Tc) negative.

2.1 Principle of superposition of Boltzmann

The relation [éq 2-1] is valid only for one constant loading. For a history of loading nonconstant the principle of superposition of Boltzmann is applied; history of loading (T) is broken up into increments of load:
N
(T) = H (T -)
I

Ti
i=0

where
H
Heavyside.

of

function

is
N
One can then write: fl
T () = F T (- T)
I
I
i=0
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what uninterrupted gives:
T
fl

(T) =

F (T -)

D = F
= (F)

éq
2.1-1

T
=0
where the product of convolution represents.

2.2
Model of Kelvin in series

One can show that any linear viscoelastic body can be modelled by a series connection models of Kelvin and that the function of creep can then be put in the form

R
T
F (T) = J 1
. (- exp (-
))
S

S 1
=
S

S and Js are plus coefficients identified on the experimental curves of creep.

3
Presentation of the clean model of creep of Granger [bib1]

3.1
Experimental properties of the clean creep of the concrete in loading uniaxial

The clean creep tests on test-tube reveal the following properties:

.
in a range of constraint lower than 50% of the breaking strength, clean creep is proportional to the constraint,

.
the clean creep of a test-tube with hygroscopy ext.
H is almost proportional to ext.
h.
clean creep of a no-slump concrete is almost null and it is maximum for a concrete saturated with water,
.
when the temperature T increases one has an acceleration of creep,
.
clean creep is a strongly growing old phenomenon,
.
a longitudinal deflection of creep is accompanied by a transverse deformation by sign opposite (effect Poisson).

One chooses to model the clean creep of the concrete with a linear viscoelastic model which will have in
to more take into account the dependence of creep with respect to the temperature and the hygroscopy.

3.2
Modeling by a series connection of models of Kelvin

One uses a series connection of models of Kelvin whose coefficients are identified from experimental curves of creep. It is shown in practice that one reproduces in a satisfactory way them curves of concrete creep with R = 8 models in series.
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The following function of creep is thus used:

8
T T
J (T, T) =
C

-
J
. 1 - exp -
C
S

éq
3.2-1

s=1

S

In practice it is very difficult to determine at the same time the J S and S as soon as the number of series of
Kelvin exceeds 2. One thus makes generally a choice a priori on the S,
1
=
S
S
1 10 -

and one
then determine by linear regression the J S.
The expression [éq 3.2-1] is related to basic creep of the model. One shows below how
taken into account of the effect of the temperature, the hygroscopy and ageing is integrated in final model.

3.3
Effect of the temperature

To take account of the effect of the temperature on the kinetics of creep, one defines a "time equivalent" T (T)
eq
who will replace time T in the model.

T
U

1
1
T
T

C
( ) =
exp -

-
ds

eq

éq
3.3-1
R T (S) T

S T

ref.
=

C

Note:

U
.
The temperature and the term of activation of the law of Arrhenius
C are expressed in
R
degrees K.

.
To model thus the effect of the temperature T exploits only the kinetics of creep. For to really utilize T on the amplitude of the phenomenon of creep, in particular on the level of the value ad infinitum of the function of creep, T is also introduced into the expression of J like a multiplicative function of the coefficients of creep such as:

T - T
(
-
R
)
45
T
T
J T
(, T, T) =
ref.

C

-
eq
C
J
. 1 - exp-

S

éq
3.3-2

45
s=1

S

.
T is the temperature of reference. It is chosen by the user. It is
ref.
generally taken equalizes with 20°C. In the continuation of the document T will be taken equalizes with
ref.
20°C.
.
For the version independent of the temperature, there are simply T
T

() = T and
eq
R
T
T
J T
(, T, T) =
C

-
eq
C
J
. 1 - exp-

S

.

s=1

S

3.4
Effect of the hygroscopy

Into the model, H is also introduced like a multiplicative parameter of the coefficients of creep so that:
T -
R
248
T
T
J T
(, T, T, H) = H

J. 1 exp

éq
3.4-1
C

-
eq
C
-
-

S

45

s=1

S

Note:

It is the noted variable drying C that one has at the end of Code_Aster calculation of drying and it is isothermal curve of sorption-desorption which makes it possible to pass from the variable C to the hygroscopy of
ambient conditions h. Are C the isothermal curve of desorption: C = C (H) and H = C-1 (C). The curve
H = C-1 (C) must be indicated by the user.

3.5
Effect of ageing

For a growing old viscoelastic material, the function of creep varies for two times of loading different. Ageing is associated the hydration at the youth and others phenomena like polymerization for the old concrete. The effect of ageing is modelled in multiplying the coefficients of creep by a function of ageing K (T depend on time on c)
loading. Modeling chosen to take into account ageing associated with the hydration is that of the CEB [bib2]:

28 2.
0
+ 1
.
0
K T
( ) =
T

in

expressed

is
day.
C
2
.
0
C
T +1
C

To reveal a sensitivity of the phenomenon of ageing compared to the temperature one also a time of loading are equivalent Tc (T) defines which replaces T in the function of eq
C
C
ageing.

Tc
U 1

1
Tc T

v
( ) =
exp -

-
ds

eq
C

R T (S) T
s=t0

ref.

T: corresponds to the age of the concrete at the youth, it
0
is generally taken equal to 28 days
T: the time or age of loading expressed in
C
days
Note:

U
.
T and v are in degrees K,
R
.
for the old concrete it would be necessary to use another equivalent time and another function of ageing,
.
if one does not take into account ageing, one has simply K (T) = 1.
C

The function of creep, which will be related final to creep of the model, is written then:

T -
N
248
T
T

J T
(, T, T, H) = H
K Tc
(
)
J. 1 exp

éq
3.5-1
C
eq

-
eq
C
-
-

S

45

s=1

S

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3.6 Modeling
3D

The traditional assumption consists in supposing the existence of a Poisson's ratio of constant creep and equal to the elastic Poisson's ratio, that is to say =
2
.
0. From where for, T, H constant:
F

fl
(T) = J (T, T, T, H) 1
[(+ v) - v tr () I]
C
F
F

and thus:

~
~

fl T () = J T (, T, T, H) 1
(+
)

C
F

fl
tr
(
T
()) = J T
(, T, T, H) 1
(-
2)
tr
C
F

3.7
Superposition on the constraint, the temperature and the hygroscopy (1D)

To simplify the demonstration, one takes in this part like function of creep one of components of the series of Kelvin, without taking into account of the effect of ageing, nor of time equivalent parameterizing the temperature, is:

T - 248

T - T
J T
(, T, T, H) = H
J .1 - exp-
C

C
S

45

S

T 248
T T
fl
-

-
deformation of then being written creep:
= H
J .1 - exp-
C
.
S

45

S

It is pointed out that this writing of the deformation of creep is valid for, T and H constant (in this case the model is equivalent in fact to take a Young modulus decreasing according to time).
For a history of loading, temperature and hygroscopy nonconstant one applies it

principle of superposition of Boltzmann.

Let us suppose that for an element of volume given, one knows at time tn the sizes (fl

fl
N, T, H)
N
N
N. At time tn 1
+ the sizes will be (N 1
+ ,
, T
, H
)
N 1
+
N 1
+
N 1
+
.

For T < T <
N
tn 1
+ one proposes to calculate the deformation of creep in the following way:

fl
fl
N 1
+ (T) = N (T) - J (T, T, T, H) +
J (T, T, T
, H
)
N
N
N
N 1
+
N 1
+
N 1
+

N
N

i.e.:

T
- 248
T - T
fl
fl
n+1

N
n+1 T () = N T () +

H
J 1 - exp-
n+1
n+1
S
45

S

T - 248
T - T
N

N
-
H J 1
N
N
S
45

exp

S
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The superposition is thus considered not only on the constraint but also on the temperature and the hygroscopy which is treated mathematically in the same way. From where:

N

T - T
T - 248
fl

I
T () = J .1 - exp-

H
S

i=0

45

S
I

One has then in integral writing, the deformation of creep of a component S of the series of Kelvin:

T

fl
T -
T - 248

T () =
J
. 1 - exp-

éq
3.7-1
S
S

D
H

45
=t0

S

4
Relations of Code_Aster behavior

One introduces into Code_Aster three relations of behavior associated with clean creep:

.
GRANGER_FP_V
.
GRANGER_FP
.

GRANGER_FP_INDT

The first takes account of the whole of the effects (forced, temperature, hygroscopy and ageing), the second does not take account of the phenomenon of ageing and the last does not hold count neither ageing nor of the effect of the temperature. They are available in modeling 2D, 3D and plane constraints.

***The various parameters of the model are indicated in DEFI_MATERIAU. Are well informed under the key word GRANGER_FP, of which the use is common to the relations of behavior GRANGER_FP
and GRANGER_FP_V, the characteristics materials following:***

GRANGER_FP:

***.
(2x8) constant characteristics of
J1: J
1
function of creep J,
I
I
TAUX_1:
1***

***.
.
.
J8: J
8
TAUX_8:
8***

***.
the curve of sorption-desorption giving H
FONC_DESORP: C-1 (C)
according to the variable drying C
.
the constant of energy of activation for
CPU
time-temperature equivalence.
QSR_K:***

R

If one uses the growing old relation of behavior then one informs in more the key word V_GRANGER_FP under which the characteristics associated with ageing are indicated, with to know energy of activation for the calculation of the time of equivalent loading and the function of ageing K (Tc).
eq

V_GRANGER_FP:
U
QSR_VEIL:
v
R
FONC_V: K (Tc)
eq
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For law GRANGER_FP_INDT, the parameters to be informed under key word GRANGER_FP_INDT of DEFI_MATERIAU are:

GRANGER_FP_INDT:

.
(2x8) constant characteristics of
J1: J
1
function of creep J,

I
I
TAUX_1:
1

.
.
.
J8: J
8
TAUX_8:
8

.
the curve of sorption-desorption giving H
FONC_DESORP: C-1 (C)
according to the variable drying C

5
Numerical integration of the model

5.1 Discretization
(1D)

T - 248
Let us pose S = T H
T =

45

The expression [éq 3.7-1] is thus written:

T
-
fl
T

S
T
( ) =
J 1
. (- exp-
.)

.

D
S
S

=t
S
0

The discretization in time is such as for T [T
T] one considers a linear evolution of S
N,
1 N
(decomposition of S (T) in linear functions per piece). One has then:

N

-
fl
S
Ti
T

T
I
( ) =
.
J
N
.1 - exp-

D
S
N

1
i=
T

S

I =t1

S
I -

N
S
S
T
T
T
T
fl

N

-
-

T () =
J
T
J
1
exp
exp

S
N
I -
S
I

I

- N
I
-
- N
I

S S

1
1

i=
T
T
I
i= I

S

S

N
N

T
T
T
fl
-

T () = J
S
S J
exp
1 exp

éq
5.1-1
S
N
S

- I S
N
I
I
I
S

-

-
-

1
1

i=
i=
Ti

S

S

Note:

Notation X = X - X
.
I
I
I 1
-

8
Now let us consider the 8 models of Kelvin in series one a: fl
T () =
T ()
T ()
N
fl
=
S

N
fls N
s=1
S
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T
One can then break up the deformation of creep [éq 5.1-1] on the basis of;
1

exp -

and

S
to carry out a recurrence on the coefficients of this base. According to [éq 5.1-1] one has with T:
N

N
N

-
fl

S
tn Ti
Ti
0
S
( ) =
-

exp-

S
tn
J S
If
If
J S
1 - exp-
= J

S N
With - N
With
T

i=1
i=1
I

S

S
3
2
1
1
4
4

4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
4
4
3
A0
Have
N
N

With T
one can also write:
N 1
+

N
N
1

T
T
T
fl
-

T () = J
S
S
J
1

exp
1 exp
S
N 1
S
-
I

n+
I
I
+
I
S
S

-

-
-

1
1

i=
i=
Ti

S

S

T
+ S J - S
S
J
n+1
1 exp
n+1

S
n+1
S

-
-

T

n+1

S

that is to say:

n+1
N
1

fl
T

T - T

T
T () = J
1
S
N 1
S S -
S
J exp- n+
N
I
I
+
I
I
S
S

exp-

1 - exp-

T

i=1
i=1
I

S

S

S

T

- S

S
J
n+1
n+1
S 1 - exp-

tn+1

S

One can thus write:

fl
T () = J A0 + S J
S
n+1
S
N
n+1
S

T

T

- S
With exp -
n+1
- S

S
J
n+1
1 exp
N

n+1
S

-
-

T

S

n+1

S

8
Let us pose J = J, one has then:
S
s=1

8
8
fl

(T) = J 0
With -
With and
T
J WITH
With

N
N
S
fl (
) = 0 -
S
N
n+1
n+1
n+1
s=1
s=1

with

A0 = A0 + S
n+1
N
n+1

T

T

S
S

With = A exp (- n+1) + S
S
J
n+1
1 exp
n+1
N
n+1

S -
-

T

S
n+1

S

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More precisely, if:

.
one takes into account equivalent time for the temperature and ageing,
.
during a step of time the parameter T is evaluated in the middle of the step of time for calculation of equivalent times T and Tc, its linear evolution being supposed during
eq
eq
this step of time,

then one a:

8
fl
(T) = J 0
With
-
S
With

n+1
n+1
n+1
s=1

with
U 1
1

T (T) - T (T) = dt (T) = exp - C
-
T
eq
N 1
+
eq
N
eq
N 1
+
N 1
+

R T
T

N 1+/2

ref.

U 1
1
Tc (T) - Tc (T) = dtc (T) = exp - v
-
T
eq
N 1
+
eq
N
eq
N 1
+

N 1
R T
293
+

N 1+/2

A0 = A0 + K Tc
(
T
(
)) S
n+1
N
eq
n+1/2
n+1

T

T

eq

eq
S
With
= S
With exp (-
n+1) + S

S
J K Tc
(
T
(
))
n+1
1 exp
n+1
N
n+1
S
eq
n+1/2

-
-

T

S
n+1

S

Note:

N 1
+
.
If one does not take then account of ageing K 0
With
=
S S
,
N
=
1
+
I
N 1
+
I 1
=
X
+ X
.
one noted
N 1
N
X
=
+
N 1
+
,
/2
2

.
T was noted
= T T
.
eq
eq (
1
+)
N 1
+
N

To have at time T
, one should not store that A
With step of previous time, are 9
fl
N 1
+
0 and them
S
variables. In 3D A and
A are tensors. One will associate the two relations then
0

S
clean behaviour of creep (9x6) variable interns corresponding to the components of the tensors A. They characterize the advance of creep.

The writing in increment of deformation, nearer to the programming gives as for it:

teq

fl
(T) = fl
(T) - fl
(T) = S
n+
To 1 exp (
1)
S
n+1
S
n+1

S
N
N -
-

S

teq
+ S
K (Tc (T
)) J
S
N +
1
1
(
exp
1)
n+1
eq
n+1/2
S

-
-
-

T

n+1
S

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5.2
Integration of the relation of behavior

1
That is to say the increment of deformation =
((U
)
T
+ (U
)).
2
If one holds account, in the partition of deformation, of the thermal deformation, the deformations associated the endogenous withdrawal and the withdrawal of desiccation, then:

E
fl
HT
retightens
retdés

=
+

+

+

+

where:

E
= H
: elastic strain
HT

= T
(- ref.
T
I
) D
: thermal deformation

retightens

*= - **I***
: endogenous deformation of withdrawal
D

retdes

= - C
(
-
ref.
*C **I***
) D: deformation of withdrawal of desiccation

with:

: hydration,
C: water concentration.
ref.
T
and Cref: temperature and drying of reference
H,
: characteristics materials

Note:

In the continuation of the document, one will note
With
HT
ret end
ret of

=

+

+

.

~
~
μ
E
2 ~-
~
~ fl
= 2μ =
+ 2μ - 2μ
2 -
μ
and
~ fl

8
~ 0
~ S
(T) = 1
(+
) J ***A***
-
With

n+1
F
n+1

n+1
s=1

it results that:

T T ()
1
2μ
~
S
eq
n+

1+ (2μ) (1+ H T K Tc
J
~
1
1 exp

μ ~
2

F) (
(eq
)

+1/2

-
-
-
=
- +

S
N

2μ
S
T

-
n+1

S

~
T T ()
S
eq
n+1
- (2μ 1
)(+
~
)
***To** 1 exp*
(H T) K Tc

F

-
-
- - - -
N
(eq)

n+

1 2
S

S

T T ()
S
eq
n+1
J 1
1 - exp -

S

S
T

n+1

S

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In the same way:

HT
3K
tr () = 3K tr () =
tr (-) + 3K tr (
E

) - 3K tr (
fl

) - 3K tr (A

)
3K -
and
fl

8
0
S
tr () = 1
(- 2
) J ***A*** *-*
With

F
N

N
s=1

from where:

T (T)
+

K
S
eq
N 1
3
tr () 1
+ 3K 1
(- 2) (H T K (Tc
)
+
J 1
1 - exp -
=
tr (-)
F
eqn 1/2
S

-
S
T +

K
N 1
S
3

T

N +1

T

+ 3K tr (E
) - 3K (1 - 2

***With** - -*
- - H T - K Tc

F)
tr (Sn)
(
)
1 exp
eq
(tr
) (eq
)
N +1/2

S
S

T
(T)
+

J 1
S
-
1 - exp

eq
N 1
-

S

S
tn 1+

S

- 3K tr (
With
)
1
One deduces some then since = ~ + tr
ij
ij
ij
3

5.3 Variables of state

The variables of state of the two relations of behavior are thus:

.
: tensor of the constraints,
.
: tensor of the deformations,
.
T: temperature,
.
C: water concentration,
.
: hydration,

.
A: *tensors characteristic of the advance of creep, are 6x9 variable,*
S
.
Tc: time of equivalent loading, characteristic of the age of the concrete.
eq

A *and Tc are internal variables of the laws of behavior, which thus comprise*
S
eq
55 internal variables.
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5.4 Stamp
tangent

~ 1 (tr)
=
+
I
D

3
~ ~ ~
~ij
=
=

1
-
~

ik
jl
ij kl

3
kl
(tr) (tr) (tr)
( )
=
tr
=
ij

(tr)
ij

Iteration of Newton:

~

T (T)
S
eq
n+

1
+ 2µ 1
(+) .h T K (Tc
)
J

F
eqn+
1-(
1
) (- exp (
1
-
))

$$S$$

$$= 2\mu I$$
$$\sim$$
$$1\ 2$$

$$S$$
$$T$$

$$N\ 1$$
$$+$$
$$S$$

with I
$=$
$ijkl$
ik
jl

(tr)

$$T$$

$$T$$
$($
$)$
$+$

$$1$$
$$+ 3K\ 1$$
$(- 2$
H
$). T\ K\ Tc$
$($
$)$
$+$
J
S
eq
N
$1- ($

1
) (- exp (
1
-
))

= 3KI
F
eq
S
N
(tr)
1 2

S
T

N 1
+
S

Phase of prediction for the step of time [tn, tn+1]

Note:

fl

-

S
With
S
In 1D:
S

=
- J K Tc
(
)

.
T
S
eq

T

S
tn

Writing of speed at the moment T:
N

~

~

-
-

1+2µ 1
(+) (J K Tc
(
) T
H)
2µ
2µ 1
F

=
-

S
eqn

(+ F)
T

S

T

~-
With
T
D
dh

S - J K Tc
(
)~- -
H
- J K Tc
(
)~-
T
S
eq
S
eq

N
N

S
dt
dt
S

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linearization for the phase of prediction of the step of time [tn, tn +]
1 :

~
-
-
1+2µ 1
(+) (J K Tc
(
) T
H)
F

=

S
eqn

S

~-
With

µ~
2
-2µ 1
(+)
T J K Tc
(
~
)
H T J K Tc
(
~

)
T
H
F S -

- - -
- -
S
eq
S
eq
N
N

S
S

Writing of speed at the moment tn:

(tr)
-
-
(tr)
1+3K 1
(-2) (J K (Tc) T H) =3K
T
F
S
eq

S
T
(tr -
With)

S
-
-

T
D
-
- dh
-3K 1
(-2)
- J K (Tc) (tr) H
- J K (Tc) (tr) T

F
S
eq
S
eq

N
N

S
dt
dt
S

dT
D
cd.
-3K 3
(
) +3K 3
(
) +3K 3
(
)
dt
dt
dt

linearization for the phase of prediction of the step of time [tn, tn +]
1 :

-
-
(tr

) 1+3K 1
(-2) (J K (Tc) T H) =3K (tr

)

F
S
eq

N

S

(tr -
***With**)*

-3K 1
(-2)
S

T
- J K (Tc) (tr -
) H T
- J K (Tc) (tr -
) T H

F
S
eq
S
eq

N
N

S
S

-3K 3
(T
) +3K 3
() +3K 3
( )

C

Creep thus introduces a specific term of second member at the time of the phase of prediction which in fact is neglected, without consequence on the results.

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Modeling of the cables of prestressing

Summary

To improve resistance of certain structures of Civil Engineering, one uses concrete prestressed: for that,
the concrete is compressed using cables of prestressed out of steel. In Code_Aster, it is possible to make
calculations of such structures: the cables of prestressing are modelled by elements of bar with two nodes, which are then kinematically related to the elements of volume or plate which constitute the part
structural concrete. To carry out this calculation, there are three order specific to these cables of

prestressed, DEFI_CABLE_BP which makes it possible geometrically to define the cable and the conditions of setting in
tension, AFFE_CHAR_MECA, operand RELA_CINE_BP, which makes it possible to transform the information calculated by
DEFI_CABLE_BP in loading for the structure, and CALC_PRECONT which allows the application of
prestressed on the structure.

Principal specificities of modeling are as follows:

· the profile of tension along a cable is calculated according to payment BPEL 91 [bib1] and takes account of
retreat of anchoring, the loss by rectilinear and curvilinear friction, of the relieving of the cables, creep
and of the shrinking of the concrete and the connection/concrete cables is supposed to be perfect, with the image of the sheaths injected by
a purée
· it is possible to define a zone of anchoring (instead of a point of anchoring) in order to attenuate them
singularities of constraints due to the application of the tension on only one node of the cable (effect of
modeling),
· the behavior of the cables is elastoplastic, thermal dilation being able to be taken into account.
· thanks to operator CALC_PRECONT, one can simulate the phasage setting in tension of the cables and
setting in tension can be done in several steps of time in the event of appearance of non-linearities. Lastly,
final tension in the cable is strictly equal to the tension prescribed by the BPEL.
· the cables being modelled by finite elements, their rigidity remains active throughout the analyses.

Operator DEFI_CABLE_BP is compatible with all the types of mechanical finite elements voluminal and them
elements of plate DKT for the description of the concrete medium crossed by the cables of prestressing. By
against, operator CALC_PRECONT N `is not compatible with the elements of plate.
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Count

matters

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1 Preliminaries

Certain structures of civil engineering are made up not only of concrete and passive reinforcements out of steel, but also of cables of prestressings. Analysis of these structures by the method of EF then requires to integrate not only the geometrical and material characteristics of these *cables but also their initial tension.*
Operator DEFI_CABLE_BP is designed according to the regulations of the payment BPEL 91 which allows
to define the contractual tension of way. The mechanisms taken into account by this operator are them following:

· the setting in tension of a cable by one or two ends,
· the loss of tension due to the frictions developed along the rectilinear ways and curvilinear,

· the loss of tension due to the retreat of anchoring,
· the loss of tension due to the relieving of the cable.

The cables are modelled by elements bars with two nodes, which implies to adopt a layout approached in the case of the layouts in curve. This can be done with more close to reality without major restriction (the nodes of cables must be inside the volume of the elements of concrete) taking into consideration grid of the elements of the concrete. Structural the concrete part can be modelled thanks to all type of voluminal elements 2D and 3D or with the elements plates DKT. Operator DEFI_CABLE_BP with the possibility of creating conditions kinematics between the nodes elements bars and the elements 2D or 3D which do not coincide in space. This has the advantage of to simplify the creation of the grid and to leave free choice to the user in term of provision of elements and of their number. So the connection cables of prestressed/concrete is of perfect type, without possibility of relative slip. The operator also allows to define a cone of diffusion of constraints around anchorings in order to limit to it the stress concentrations much higher than reality and which is due to modeling.
The second principal function of operator DEFI_CABLE_BP is to evaluate the profile of the tension it length of the cables of prestressed by considering the technological aspects of their implementation. At the time of the installation of the cables, prestressing is obtained thanks to the hydraulic actuating cylinders placed at one or two ends of the cables. The profile of tension along a cable is affected by friction (rectilinear and/or curvilinear), by the deformation of the surrounding concrete, the retreat of anchorings at the ends of the cables and by the relieving of steels.
This tension can then be taken into account like an initial state of stress at the time of the resolution complete problem EF. The problem, it is that in this case, under the effect of the tension of the cable, the concrete unit and cable are compressed involving a reduction in the tension of the cable. To avoid this problem and to have exactly the tension prescribed by the BPEL in the structure in balance, tension must be applied by the means of macro-order CALC_PRECONT. In more thanks to this method, it is possible to impose the loading in several steps of time, which can be interesting if the behavior of the concrete becomes non-linear as of the phase of setting in tension of the cables.

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2 the operator
DEFI_CABLE_BP

2.1
Evaluation of the characteristics of the layout of the cables

We present here the method used to obtain a geometrical interpolation of the cables, which is essential to calculate the curvilinear X-coordinate and the angle used in the formulas of loss of prestressing.
One starts by building an interpolation of the trajectory of the cable (in fact an interpolation of two projections of the trajectory in the two plans Oxy and Oxz), then starting from these interpolations, one considers the X-coordinate curvilinear, and the angular deviation cumulated, according to formulas':

$$S(X) = \int_0^X \sqrt{1 + y'^2(X) + z'^2(X)}\, dx \quad \textbf{éq 2.1-1}$$

$$\alpha(X) = \int_0^X \frac{\sqrt{y'^2(X) + Z'^2(X) + [y'(X)\,Z'(X) - y'(X)\,Z'(X)]^2}}{dx}$$

éq
2.1-2

$$\left(\frac{d\theta}{dx}\right)^2 = \left(\frac{d\theta}{dx}\right)^2 + \left(\frac{d\theta}{dx}\right)^2$$

In order to preserve the topology of the cable (and in particular the scheduling of the nodes which it

make) operator DEFI_CABLE_BP works starting from meshs and of groups of meshs, (rather that nodes and groups of nodes), in order to be able to calculate the sizes while following the sequence of the nodes along the cable.
The interpolation used for the calculation of prestressed in the concrete will be a Spline interpolation cubic carried out in parallel on the three space co-ordinates according to the curvilinear X-coordinate. The co-ordinates of the nodes of the cable are the "real" co-ordinates, i.e. the co-ordinates defined by the grid of the cable.

All the calculations presented within the framework of operator DEFI_CABLE_BP are defined from real geometry of the structures and the real positions of the nodes. Calculations of tension to the nodes nodes in nodes will be carried out, in the order given by the topology of the grid, from formulas quoted above [éq 2.1-1] and [éq 2.1-2].
The calculation of the cumulated angular deviation and the curvilinear X-coordinate requires the precise calculation of
derived from the trajectory of the cable defined in the operator in a discrete way by the position of nodes of the grid of cable. The polynomials of Lagrange have instabilities, in particular for irregular grids. Moreover, one significant number of points of discretization will lead to polynomials of high degrees. In addition a small uncertainty on the coefficients of interpolation will have
for consequence an important error on the results, in term of derivative. By choosing one polynomial interpolation of small degree, one will obtain derivative second null or not continuous (according to the degree).
The interest of a cubic interpolation of Spline type is to obtain drifts second continuous and costs of calculations of order N, Si N is the number of points of the function tabulée to interpolate, with polynomials of small degree. The principle of this method of interpolation is described exclusively in the case of a function of the form xf (X).
One supposes that one carries out an interpolation of the tabulée function, starting from the values of the function
at the points of discretization x1, x2,…, xn, and of its derivative second. One can thus build one polynomial of order 3, on each interval xi, xi+1, whose polynomial expression is as follows:

$$
\begin{array}{l}
X+1-X \\
X-X \\
y \\
J \\
= \\
y \\
J \\
+ \\
y+1+Cy+Dy \\
X \\
+1 \\
+1-X
\end{array}
$$

J
X +1 - X
J
J
J
J
J
J
J

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with:

3
1
X +1 - X X +1 - X
2
C
J
J
=

-

(X +

1 - X
J
J)
6
X +1 - X
X +1 - X
J
J
J
J

3
1
X - X X - X
2
D
J
J
=

-

(X +1 - X
J
J)
6
X +1 - X
X +1 - X
J
J
J
J

One can check easily that:

y (X

J) = y
and
y
J
(X J) = y J

y (X

j+1) = y
and
y
j+1

(X j+1) = y j+1

It is then necessary to estimate the values of the derived second with the points of interpolation. By writing the equality
interpolations on the intervals [xi-1, xi], and [xi, xi+1] from derived from order one, as in point xi, one obtains
the following expression:

X - X
X
-1
+1 - X
X
-1
+1 - X
y +1 - y
y - y
J
J
y
J
J

-1
-1
+
y
J
J
+
y
J
J
J
J

+1
=
-
J
J
J
6
3

6
X +1 - X
X - X
J
J
J
j-1

One obtains (N2) equations thus connecting the values of the derived seconds to the points of discretization
x1, x2,…, xn. By writing the boundary conditions in x1 and xn on the values of the derived seconds, one obtains a system (N, N) which one can determine in a single way the value of all the derivative, and to obtain the function of interpolation thus. Two solutions arise then for the establishment of boundary conditions:

· to arbitrarily fix the value of the derived second at the points x1, and xn, to zero by example,
· to allot the actual values of the derived second in these points, if this data is accessible.

One obtains a system of equations having for unknown factors N derived seconds from the function tabulée to interpolate. This linear system with the characteristic to be tri-diagonal, which means that resolution is about O (N). In practice the interpolation breaks up into two stages:

· the first consists in calculating the values estimated of the derived second with the points, operation which is carried out only once,
· the second consists in calculating, for a given value of X, the value of the function interpolated, operation which can be repeated time as many as one wishes it.

Tests carried out on the function sine, over three periods, show that the results are strongly dependent on the number of points, as well as distribution of the points of the curve to be interpolated, (awaited result), but that even in delicate situations (little points and curve very irregular) the interpolation does not diverge. In other words, even if the correlation concerning trajectory of the cable is not the very good (interpolation with very few points) interpolation will be roughly located in a fork close to the real trajectory. This case will not arise not in practice, but allows to check the stability of the method of interpolation.

For the problem that we consider here, one cannot always write the trajectory of the cable under the form [y (X)], [Z (X)], whenever this curve is not bijective, in particular when projection of the trajectory in one of the two plans Oxy or Oxz cyclic or is closed (case of one circular concrete structure).
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By taking an intermediate variable of the type U =
X
, parameter always growing and
of increase identical in absolute value to that in variable X, one can be reduced to
expressions [y (U)] bijective functions of the variable U. The cubic interpolation Spline described above
is then applicable to the function y (U) (like with function Z (U)). In practice, that led however
with problems of connections of tangent (angular points) at the points where variable X changes
feel variation, and with specific irregularities.

One describes the trajectory of the cable like a parametric curve. Knowing a whole of points
curve, the parameter most easily accessible is then the curvilinear X-coordinate. One writes
trajectory of the cable in the form [X (p), y (p)], in the Oxy plan, (resp. [X (p), y (p), Z (p)] in a space
with
three dimensions).

The cumulated cord "p" discretized at the tabulés points of the function which one interpolates P1, P2,
…, Pn
calculate in the following way:

p (1) = 0 at the P1 point,
p (K) = p (k-1) + distance (km No-1Pk) to point km No

One thus has two curves defined by a whole of couple [X (I), p (I)] and [y (I), p (I)] to which
one can directly apply the cubic Spline interpolation presented before, and which allows
to free itself from the difficulties encountered previously. The interpolation is made for both

co-ordinates, (or three co-ordinates, in dimension 3), independently one of the other.

2.2
Determination of the profile of tension in the cable according to BPEL 91

2.2.1 Formulate
general

Operator DEFI_CABLE_BP allows to calculate the tension F (S) along the curvilinear X-coordinate S of
cable. This one is given starting from the rules of the BPEL 91 [bib1].
All in all, one leads to the following formulation:
~
~

5
F (S)
~
F (S) = F (S) - X flu × F0 + xret × F0 + R (J) ×
×
µ
1000
- 0 × F (S) ***éq***
2.2.1-1

100

Its × y

where S indicates the curvilinear X-coordinate along the cable. Parameters introduced into this expression
are:

.
F0 initial tension,
.
X flu standard rate of loss of tension by creep of the concrete, compared to the initial tension,
.
xret standard rate of loss of tension by shrinking of the concrete, compared to the initial tension,

.
relieving of steel at 1000 hours, expressed in %,
1000
.
Its surface of the cross-section of the cable,
.
y elastic stress ultimate of steel,
.
µ adimensional coefficient of relieving of prestressed steel.
0

~
In this formula, F0 indicates the initial tension with anchorings (before retreat), F (S) represents tension after the taking into account of the losses by friction and retreat of anchoring, X
F
flu × 0
represent the loss of tension by creep of the concrete, X
F
ret × the 0 loss of tension by shrinking of the concrete,
~
5
F S
~
R (J)
( )
×
×

-

0 × F (S) losses by relieving of steels
100
1000 S

× has
µ
y

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Note:

The introduction into these elements of losses of tension is optional. Thus, if one considers to make a calculation of creep and/or shrinking of the concrete by using a suitable law with STAT_NON_LINE, one should not introduce these elements into the losses calculated by DEFI_CABLE_BP.

The evaluation of the losses requires the knowledge of the curvilinear X-coordinate S and the deviation angular cumulated calculated as from derived the first and second from the trajectory of the cable. precise calculation of these derivative requires an interpolation between the points of passage of the cable. This
interpolation is carried out using Splines, better than the polynomials of Lagrange which present instabilities, in particular for irregular grids (cf preceding paragraph).
In what follows each mechanism intervening in the calculation of the tension is detailed.

2.2.2 Loss of tension by friction and retreat of anchoring

We start by calculating the tension along the cable by taking account of the losses per contact between the cable and the concrete: F () = 0exp - -
C S
F
(F
S)
where indicates the cumulated angular deviation and the introduced parameters are:

.
F coefficient of friction of the cable on the partly curved concrete, in rad1,
.

coefficient of friction per unit of length, in M-1,
.
F tension applied to one or the two ends of the cable.
0

To take into account the retreat of anchoring, the following reasoning is made:
the tension along the cable is affected by the retreat of anchoring at a distance D which one calculates in
*solving a problem with two unknown factors: the function F * (S) which represents the force after retreat of*
anchoring and the scalar D:

Tension (F)

1 D
without retreat of anchoring
=
[F (S) - F (S) ds,*
E S
]
has
0 have
F (S)
(- -)
F E F is worth
S
with passing of anchoring
0

is the value of the retreat of anchoring
(it is a data)
X-coordinate (S)
D

*F * (S), the force after retreat of anchoring, is given starting from the formula [bib1]:*

[F (S). F (S)]= [F (D)]2,*

The length D will be given in an iterative way thanks to the preceding integral. Other authors use different relations such as:

*[F (S) F (D)] [F (D) F**
-

=
-
(S)]

For the calculation of D, three particular cases can arise:

1) This loss by retreat of anchoring is localised in the zone of anchoring. If the cable is curve, and the sufficiently short length of the cable, it can arrive that D is larger than the length of the cable. In this case, the loss of prestressing due to the retreat of anchoring apply everywhere. It is necessary to calculate the surface ranging between the two curves F (S) and F * (S), which must be equal to E S, and which thus make it possible to calculate F * (S).
has
has
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2) If a tension is applied to each of the two ends of the cable, let us call F (S) the distribution of initial tension calculated as if the tension were applied only to
1
the first anchoring, and F (S) the distribution of initial tension calculated like if the tension
2
was applied only to the second anchoring. The value which must be retained in any point of cable as initial tension is F (S) = max (F (S), F (S).
1
2
)

3) Lastly, if D is larger than the length of the cable, and when a tension is applied to each of the two ends of the cable (superposition of the two preceding cases), one must apply the following procedure:
- calculation of F (S) initial tension calculated as if the tension were applied only to 1 the first anchoring and by taking account of the retreat of anchoring (as in the case private individual 1),
- calculation of F (S) initial tension calculated as if the tension were applied only to 2 the second anchoring and by taking account of the retreat of anchoring (as in the case private individual 1),
- calculation of F (S) = Min (F (S), F (S) 1 2).

2.2.3 Deformations differed from steel

The loss by relieving of steel, for an infinite time, is expressed in the following way:

~ *5 F (S)* ~ *R (J)* × × *- μ × F (S) 100 1000 0 S* × *has y*

(relieving with 1000 hour in %; μ the coefficient of relieving of prestressed steel and

1000
0
y
guaranteed value of the maximum loading to the rupture of the cable).
This relation expresses the loss by relieving of the cables for an infinite time. The BPEL 91 proposes
J
following formula: R (J) =
where J indicates the age of the work in days and R a 0 ray
J + 9 r0
.
m
m
characteristic obtained by submitting the report/ratio of the section of the structure out of concrete, m
², by
perimeter of the section (in meters) of concrete.

2.2.4 Loss of tension by instantaneous strains of the concrete

The instantaneous losses are not taken into account in the formula [éq 2.2.1-1] used in Code_Aster. What the BPEL calls loss of instantaneous tension is in fact the loss of tension induced in cables already posed by the installation of a new group of cables. To model it phenomenon, it is necessary to represent the phasage of setting in prestressed in Aster calculation, i.e. not to tighten the whole of the cables at the same time but in a successive way by connecting them CALC_PRECONT (see test SSNV164).

2.3 Determination of the relations kinematics between steel and concrete

Since the nodes of the grid of cable do not coincide inevitably with the nodes of the grid of concrete, it is necessary to define relations kinematics modelling perfect adhesion between cables and concrete.
The following paragraphs describe in the allowing order the space geometrical considerations to define the concept of vicinity enters the nodes of elements of cable and concrete, then the method of calculation of the coefficients of the relations kinematics.
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2.3.1 Definition of the close nodes

The calculation of the coefficients of the relations kinematics requires to determine the nodes "close" to
each node of the grid of the cable. The diagram which follows symbolizes a node cables and a mesh concrete:

1
Nodes
neighbors
Node cables
Element
concrete
2
4
Nodes concrete
3

The mesh defined by nodes 1, 2, 3, 4 contains the node cables. The close nodes are thus them tops 1, 2, 3, 4. If the node cable is located inside an element at p nodes P1, P2,…, Pn, then the nodes P1, P2,…, Pn are called "nodes close" to the node cables.
One treats in the same way, the elements of plate without offsetting, and the solid elements. calculation of the offsetting of each node of the grid cable is necessary for the calculation of coefficients of the relations kinematics.

In the case of elements of plate, when the node cable is characterized by a offsetting not no one, one defines the nodes close as the unit to the nodes top of the element which contains projection of the node cables in the tangent plan with the grid concrete. If the node cables (or well its projection in the tangent plan with the grid concrete) belongs to a border of an element, it are the tops of this border which form the whole of the close nodes.

2.3.2 Calculation of the coefficients of the relations kinematics

In the whole of descriptions which follow the sizes are systematically expressed in total reference mark of the grid. The connections kinematics are thus expressed according to the degrees of freedom expressed in this base. The normals and vectors rotation are expressed in the reference mark total, except explicit contrary mention.
In modeling finite elements of the structure cable-concrete, the displacement of a material point of the structure concrete can be expressed easily using the functions of form of the element or of net concrete whose tops form the close nodes, according to displacements of the nodes neighbors of the discretization "concrete". In the same way, a size or a displacement of a point of the cable, (or of its projection on the tangent level of the grid concrete) is identical to the value of this size at the material structural concrete point which occupies this same position (perfect connection between the concrete and steel), and is thus expressed according to the value of this same size at the tops of the element, using the functions of form.
If (X, y, Z) are the co-ordinates of the node cables, or those of its projection, and N1, N2,…, Nn functions forms associated with the nodes concrete P1, P2,…, Pn tops of an element of the grid concrete (or tops of a border of an element of the grid concrete), and (xi, yi, zi) the co-ordinates of node I, then the interpolation of a variable U on the element is written:

$$U(X, y, Z) = \sum_{i=1}^{N} NR(X, y, Z)\, U_I$$

U which can be a co-ordinate, or any other nodal data.
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The connections kinematics make it possible to express the identity of displacement between the node of the grid
cable, and the material point concrete which occupies the same position. This corresponds to the assumption of one
perfect connection between the concrete and the cable.

2.3.2.1 Case where the concrete is modelled by massive finite elements

By taking again the preceding notations and while considering
C
C
C
dx Dy
,
, dz displacements of the node
cable, and dxb, dyb, dzb
J
J
J displacements of the nodes J (J = 1, N) of the structure concrete neighbors of the node
cable we obtain the following relations:

N
C
C
C
C
B
dx = NR (X, y, Z) dx

I
I

i=1

N
C
Dy =
C
C
C
B
NR (X, y, Z) Dy
I
I

i=1

N
C
C
C
C
B
dz = NR (X, y, Z) dz

I
I
i=1

N being the number of nodes of the element concrete neighbors of the node of the cable, or that of one of its
borders. For each node of the cable one obtains 3 relations kinematics between displacements
nodes of the two grids cables and concrete.

2.3.2.2 Case where the concrete is modelled by finite elements of plate

Pb
P
3
C
N
P
P2

P1

That is to say C
P the initial position of a point of cable in the not deformed geometry and is C
P
0
the position
this same point after deformation. Let us call p
P the projection of C
P on the surface of the layer
0
0
means of the hull of concrete not deformed and p
P the projection of C
P on the surface of the average layer
concrete hull deformed. That is to say N the normal in the average plan of the concrete hull out of p
P and
0
0
N that out of p
P.
p
X C
X C
X
0
0
0 -
B
X
0
N
N
0 X 0 X

p
P is given by: p
y
.

.

0 =
C
y0 - C
y0 - B
y0 N
N
O

0y 0y
PC C
B

z0 z0 z0 - z0 N
N
0 Z 0z
p
X C
X C
X - B
X N
N
X
X

p
P is given by: p
y = C
y - C
y - B
y .n
. N
y y
PC C
B

Z Z Z - Z N
N
Z Z
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The point
p
P belongs to a mesh of concrete plate whose nodes are noted Pb, Pb and Pb.
O
1
2
3
One defines the offsetting of the cable compared to the concrete hull as the distance
p
C
E = P P and
0
0
the assumption is made that this offsetting does not vary when the structure becomes deformed
:
p
C
p
C
E = P P

0
0

= P P
One introduces displacements of the points of the cable and his projection:

N
p
p
p
p
B
dx = NR (X, y, Z) dx

I
0
0
0
I
C
dx

i=1
N
C
C

p
p

P P
P P
(,
,
)
0
=
p
Dy =
p
p
p
B
NR X y Z Dy
0
= C

Dy

I
0
0
0
I
C

i=1
dz
N

p
p
p
p
B
dz = NR (X, y, Z) dz

I
0
0
0
I
i=1

One introduces the vector “rotation” of the plate at the point p
P and degrees of freedom of rotation of

N
B
p
p
p
B
drx = NR (X, y, Z) drx
I
0
0
0
I

i=1

N
nodes of the plate: =
B
dry =
p
p
p
B
NR (X, y, Z) dry
I
0
0
0
I

i=1

N
B
p
p
p
B
drz = NR (X, y, Z) drz

I
0
0
0
I
i=1
R
By definition of, one a: nv - nv = nv
0
0
One can then write:
P pPc
0
0 = en0
P pPc = in

By withdrawing these two equations, by taking account of the definition of one finds:

C
dx -
p
dx = E (
p
. dry N
.
.
0 -
p
drz N
Z
0 y)

C
Dy -
p
Dy = E (
p
. drz N
.
.
0 -
p
drx N
X
0z)
C
p
p
p
dz - dz = E (.drx N
.
dry N
.
0 y -
0 X)

By injecting into this last equation the functions of form, one has finally:

C
N
N

N
dx - NR X, y, Z dx
E.
NR X, y, Z dry
N
.
NR X, y, Z drz
N
.
I (p
p
p) B

I
= I (p p p) B

I
0 Z -
I (p
p
p) B

I
0 y
i=1

i=1

i=1

C
N
N
N
Dy - NR X, y, Z Dy
E.
NR X, y, Z drz
N
.

NR X, y, Z drx
N
.
I (p
p
p) B

I
= I (p p p) B

I
0 X -
I (p
p
p)
B

I
0 Z
i=1

i=1

i=1

C
N
N
N
dz - NR X, y, Z dz
E.
NR X, y, Z drx
N
.
NR X, y, Z dry
N
.
I (p
p

p) B

I
= I (p p p) B

I
0 y -
I (p
p
p)
B

I
0 X
i=1

i=1

i=1

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2.3.2.3 Case where the node of the cable is projected on a node of the grid concrete

The distance enters projection
p
P of the node cables C
P and a node concrete B
P is given by:
0
0
I

xc - xb
xc - xb
I
I

D = P P Pb
R
R
0
= yc - yb -
yc - yb.n .n
I
I
I

0 0

zc - zb zc - zb
I

I

If it happens that this distance is null (in practice lower than 10-5), it is that the node cables project at the top of a concrete mesh, and then the relations kinematics are simplified:

C
dx -
p
dx
E. dry N
.
drz N
.
I =
(pi 0z - pi 0y)

C
Dy -
p
Dy
E. drz N
.
drx N
.
I =
(pi 0x - pi 0z)
C
p
p
p
dz - dz
E. drx N
.
dry N
.
I =
(I 0y - I 0x)

These relations are the general relations in which: NR (p
X, p
y, p
Z)
if J I.
J

= 0
2.4
Treatment of the zones of end of the cable

The modeling of a cable of prestressed such as it is made in Code_Aster consists with to represent the unit cables, sheath of passage, and all the parts of anchoring, only thanks to a succession of elements of bar. The bond between the elements of cables and the concrete medium is ensured by conditions kinematics on DDLs of each node of the cable, and those of the elements concrete crossed.

When the setting in tension of the cable is applied, it is observed that the reactions generated with ends of the cables on the concrete create levels of constraints much higher than reality, and cause the damage of the concrete. As example, in certain studies, one could observe compressive stresses of more than 200 MPa, which largely exceeds the value experimental observed (40 MPa). In reality, this phenomenon is not observed thanks to the setting in place of a cone of diffusion of constraint (see drawing below) which distributes the force of *prestressed on a great surface of the concrete. In the case of model EF this surface does not exist, since the force is directly taken again by a node.*

With

Real situation
Model EF without cone

This way of modeling has several disadvantages:

· the concentration of this effort crushes the concrete,
· the space discretization of the model changes the results.
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To cure this problem, the key word CONE of operator DEFI_CABLE_BP makes it possible to distribute this force of prestressed either on a node, but on all the nodes contained in a volume (all the nodes of this volume are dependent between them to form a rigid solid) delimited by a cylinder of ray R and length L, representing the equivalent of the zone of influence of the cone of blooming of an anchoring (see figure below).

ray
length

The identification and the creation of the relations kinematics between the nodes of the concrete and the cable are done in an automatic way by order DEFI_CABLE_BP, where the new data R and L will be with to provide by the user.

2.5 Note: calculation of the tension of the cable as a loading mechanics

We made the choice leave the elements of cable in the mechanical model support of calculation by finite elements (linear or not). So there is no calculation of equivalent force to defer to nodes of the grid. One is simply satisfied to say that the cables of prestressing have a state of initial constraint not no one. This state of stress is that deduced from the tension as calculated by DEFI_CABLE_BP.

For reasons of simplicity, the data-processing object created by operator DEFI_CABLE_BP is a table memorizing values with the nodes of the cable. Then let us consider two related elements of the cable:

e1 of N1 tops and N2, and
e2 of top N2 and N3.

We suppose that L and S are the length and the section of an element e1 and that L and S are
1
1
2
2
length and the section of the element e2.

N2
e1
e2
N3
N1

DEFI_CABLE_BP will calculate with the node N2 a tension T
defined by:
N2

T (S) ds T (S) ds
1 E
E

TN =
2
1
+ 2

2
L
L
1
2

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Conversely, for calculation finite element, operator STAT_NON_LINE will consider that the constraint
T + T
NR
NR
initial in the element e1 is e1
1
2
=
0
2S1

Note:

It will be always considered that the law of behavior of the cable is of incremental type.

3
macro-order
CALC_PRECONT

3.1
Why an macro-order for the setting in tension?

It is possible to transform the tension in the cables calculated by DEFI_CABLE_BP into one loading directly taken into account by STAT_NON_LINE thanks to the order AFFE_CHAR_MECA operand RELA_CINE_BP (SIGM_BPEL=' OUI'). In this case, the tension is taken
in account like an initial state of stress at the time of the resolution of complete problem EF.

Initially
f0
f0
With balance
F
F

The resolution of the problem makes it possible to reach a state of balance between the cable of prestressed and it
remain structure after instantaneous strain. Indeed, under the action of the tension of the cable, the unit cables (S) and concrete will be compressed compared to the initial position (cable in tension, grid not deformed). The length of the cable thus will decrease, and the initial tension also goes, by consequence sees, to decrease. One thus obtains a final state with a tension in the cable different tension calculated initially. It is then essential to increase proportionally
tension applied in situ to the level them anchorings to take account of this loss.

The use of macro-order CALC_PRECONT makes it possible to avoid this phase of correction, in obtaining the state of balance of the structure with a tension in the cables equalizes with the tension lawful. In addition because of adopted method, it allows in addition to applying the tension in several steps of time, which can be interesting in the event of plasticization or of damage of concrete. It makes it possible moreover to tighten the cables in a nonsimultaneous way and thus of manner more
near to the reality of the building sites.

To profit from these advantages, the loading is applied in the form of an external loading and not like an initial state, which allows the progressive loading of the structure. In addition, for to avoid the loss of tension in the cable, the idea is not to make act the rigidity of the cables during phase of setting in tension (cf [bib3]).
The various stages carried out by the macro-order are here detailed.

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3.1.1 Stage 1: calculation of the equivalent nodal forces

This stage consists in transforming the internal tensions of the cables calculated by DEFI_CABLE_BP into
an external loading. For that, one carries out a first STAT_NON_LINE only on the cables that one wishes to put in prestressing, with the following loading:

· embedded cable
· the tension given by DEFI_CABLE_BP

T
T
T

Appear 3.1.1-a: Loading at stage 1

One calculates the nodal efforts on the cable. One recovers these efforts thanks to CREA_CHAMP. And one
built the vector associated loading F.

3.1.2 Stage 2: application of prestressed to the concrete

The following stage consists in applying prestressing to the concrete structure, without making take part
rigidity of the cable. For that, one supposes for this calculation that the Young modulus of steel is null. One

can choose to apply the loading of prestressed in only one step of time or several steps time if the concrete is damaged.
The loading is thus the following:

· blocking of the rigid movements of body for the concrete,
· nodal efforts resulting from the first calculation on the cable,
· the connections kinematics between the cable and the concrete.

F
Ecable = 0

Appear 3.1.2-a: Loading at stage 2

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3.1.3 Stage 3: swing of the external efforts in interior efforts

Before continuing calculation in a traditional way, it is necessary of retransformer the efforts outsides which made it possible to deform the concrete structure in interior efforts. This operation is done
without modification on displacements and the constraints of the whole of the structure, since balance was reached at stage 2: it is about a simple artifice to be able to continue calculation.
loading is thus the following:

· blocking of the rigid movements of body for the concrete,
· the connections kinematics between the cable and the concrete,
· tension in the cables.

T
T
T

Appear 3.1.3-a: Loading at stage 3

4
Procedure of modeling

4.1
Various stages: standard case

To manage to model a concrete structure prestresses the procedure to be followed is as follows:

· to model the concrete elements (DKT, 2D or 3D),
· to model the cables of prestressed by elements bars with two nodes (BAR),
· to allot to the elements bars the mechanical characteristics of the cables of prestressing,
· thanks to operator DEFI_CABLE_BP to calculate the data kinematics (relations kinematics between the nodes of the cable and those of the concrete elements) and statics (profile of tension along the cables),
· to define the data kinematics like mechanical loading,
· to call upon operator CALC_PRECONT,
· to solve the problem with operator STAT_NON_LINE by integrating only them data kinematics and loadings other than prestressing.

For more practical information, to refer to the document [U2.03.06].

4.2
Particular case: DKT

For the moment, macro-order CALC_PRECONT does not function if the elements concrete are of type DKT. In this case, it is advisable to adopt the following procedure:

· to model the concrete elements (DKT),
· to model the cables of prestressed by elements bars with two nodes (BAR),

· to allot to the elements bars the mechanical characteristics of the cables of prestressing,
· thanks to operator DEFI_CABLE_BP to calculate the data kinematics (relations kinematics between the nodes of the cable and those of the concrete elements) and statics (profile of tension along the cables),
· to apply these data kinematics and statics like a mechanical loading,
· to solve problem with operator STAT_NON_LINE by integrating all the loadings.

For the exit of this calculation it is necessary to determine the coefficients of correction to apply to the initial tensions applied to the cables (on the level of the declaration of operator DEFI_CABLE_BP) allowing to compensate for the loss by instantaneous strain of the structure.

Once the command file modified by these coefficients of correction, the modeling of the cables of prestressing is accomplished.

Attention, in the case of sequence of STAT_NON_LINE, it is appropriate starting from the second call, of to include in the loading only the relations kinematics and not the tension in the cables, under pains to add this tension, with each calculation.

4.3
Precautions of use and remarks

It is recommended to limit the recourse to a great number of relations kinematics under sorrow to weigh down the computing time. However, when a node of the elements of bar constituting the cables

coincide topologically with a node concrete, it does not have there a kinematic addition of relation.
If one carries out a first STAT_NON_LINE before putting in tension in the cables, it is preferable to decontaminate the cables, either by not taking them into account in the model, or in their affecting a tension constantly null (law of behavior WITHOUT), and while including in loading the relations kinematics binding the cable to the concrete.
If one carries out a phasage setting in prestressing, it is necessary to think of including them relations kinematics in the loading for the cables already tended at the preceding stages.

---

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Law of behavior to double Drücker criterion
Prager for cracking and compression

concrete

Summary

The model presented in this document is a nonlinear law of behavior for the concrete. It rests on *theory of plasticity, it is valid for the three-dimensional states of stress. Assumptions of modeling retained are as follows:*

· a field of reversibility of the constraints delimited by two criteria of the type Drücker Prager,
· a work hardening of each criterion,
· in compression, a positive work hardening to a peak, then a negative work hardening,
· in traction, a negative work hardening exclusively,
· a dependence of the shape of the curves post-peak in both cases (traction/compression) with the size finite element (the shape of this curve is related on negative work hardening and the energy of cracking),
· of the normal plastic rules of flow (associated plasticity) and a formulation of work hardening isotropic,
· the taking into account of the dependence of the thresholds of elasticity compared to the temperature,
· the taking into account of the dependence of the Young modulus compared to the temperature.
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Count

matters

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1 Notations

indicate the tensor of constraint, arranged in the form of vector according to convention:

11

22

33

12
13

23

One notes:

I

1 = Trace ()

= 1 tr

the hydrostatic constraint
H
()
3
1

S = - tr () I

the diverter of the constraints
3

= 1 tr
voluminal deformation
H
()
3

~ = - 1 tr () I
the diverter of the deformations
3

&
~
3

the rate of deformation is equivalent
eq =
trace (~.
2
& &~)
1

J = trace (2
S
the second invariant of the constraints
2
)
2

eq
3
= 3J =
trace (2
S
the equivalent constraint
2
)
2
2
trace (2
S)

=
J =

Oct.
3 2
3
I
trace

1
( )

Oct. = H =
=
3
3

F

C
initial limit of rupture in simple compression
F
initial limit of rupture out of Bi compression
DC
F

C
elastic limit in compression
F

T
initial limit of rupture in traction
F

T
=

relationship between rupture limit in traction and compression
F C
F

DC
=

relationship between rupture limit in Bi-compression and simple compression
F C
p

plastic deformation in traction
T
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T
plastic multiplier in traction
p

plastic deformation in compression
C

plastic multiplier in compression

C

F (p

C
curve of work hardening in compression
c)

F (p

T
curve of work hardening in traction
T)
U

T
ultimate plastic deformation in traction
U

C
ultimate plastic deformation in compression
F

G
C
energy of rupture in compression (characteristic of material)
F

G
T
energy of rupture in traction (characteristic of material)

the maximum of the temperature during the history of loading

2 Introduction

2.1
Principal characteristics of the model

The model presented in this document is a nonlinear law of behavior for the concrete. It be based on the theory of plasticity, it is valid for the three-dimensional states of stress. assumptions of modeling selected partly take again the models developed per G. Heinfling

[bib2] and J.F. Georgin [bib1] and are as follows:

· there exists a field of reversibility of the constraints delimited by two criteria of the Drücker type Prager,
· each criterion is hammer-hardened, the field of rupture corresponds to the maximum of the field of reversibility,
· in compression, work hardening is positive to a peak, then it becomes negative,
· in traction, work hardening is negative exclusively,
· the curves post-peak in both cases (traction/compression) vary with the size of the finite element (the shape of this curve is related on negative work hardening and the energy of cracking),
· the plastic flow is governed by a rule of normality (associated plasticity) the formulation work hardenings is of isotropic type,
· the modulus of elasticity and the thresholds of reversibility vary with the temperature.

Note:

The terminology of criterion of traction and criterion of compression is debatable. Us will use by practice, while being quite conscious that a state of tensile stresses can lead to the activation of the criterion known as of compression.

2.2
Why two criteria of Drücker Prager

The authors of the theses referred to [bib1] and [bib2] use a criterion of Drücker Prager in compression and a criterion of Rankine in traction. They justify these choices by considerations physics by showing that the field of reversibility thus obtained is close to reality experimental. On the other hand they limit their modelings in states of two-dimensional stresses. We preferred to also replace the criterion of traction by a surface of the type Drücker Prager. By this choice, one frees oneself from certain difficulties particularly in the formulations three-dimensional. Surface 3D defining the working states of stresses with respect to traction is not any more one pyramid (Rankine 3D) but a conical surface whose top is located on the hydrostatic axis. The trace of the criterion "known as of traction" on the plan deviatoric is not any more one triangle,
but a circle. The formulation obtained is simpler. The difference between the two criteria is tiny for states of stress close to states of plane stress. On the other hand, for the states of constraint strongly confined, the two approaches (of Rankine and Drücker Prager) are different, what is a limit of the model suggested.
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3
Field of reversibility and functions thresholds

3.1
Pace of the field and the thresholds of reversibility

The field of reversibility is the field of the space of the constraints inside of which ways of constraint are reversible. In the space of the principal constraints (, they are two
1
2
3)
cones whose axis is the trisecting one of equation = =. [Figure 3.1-a] one gives some
1
2
3
chart.

2
1
3
Appear 3.1-a

25
Oct.
C

20
15
10
Field of
reversibility
5
B
Oct.
0
-
-
-
0
10
P
P
T
C

Appear 3.1-b
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In a plan (,
the field of reversibility is determined by two lines as indicated on
Oct.
Oct.)
[Figure 3.1-b].
For a state of stress planes, the field of reversibility is the cut of the three-dimensional field
by a plan of equation = cste, as indicated on [Figure 3.1-c], the result in a plan
3
(, being represented on the figure [Figure 3.1-d].
1
2)

2
1
3
Appear 3.1-c

2
1

Appear 3.1-d

3.2
Mathematical expression of the field of reversibility

It is defined by the inequation:

$F(, \mathbf{A}) \; 0$

éq 3.2-1
in which ***A*** *represents the thermodynamic forces associated with the variables intern (us let us note the whole of the internal variables).*
For the model concrete that we present here, the equation [éq 3.2-1] takes the particular form

has
F
With
F
With

éq

3.2-2
comp (
c)
Oct. +.
,
=
Oct. - C + C 0
B
C
F
With
F
With
éq
3.2-3
trac (
T)
Oct. +.
,
=
Oct. - t+ T 0
D
has
H
F
(A).
,
=
Oct.
F
With

éq
3.2-4
C
- C + C 0
comp
B
C
H
F (A)
.
,

=
Oct.
F
With

éq
3.2-5
T
- t+ T 0
trac
D

The equations [éq 3.2-2] and [éq 3.2-3] correspond respectively to the thresholds of "compression" and of "traction". The equations [éq 3.2-4] and [éq 3.2-5] limit the threshold of reversibility in the field isotropic traction, they amount excluding the x-axis on [Figure 3.1-b] beyond
points P or P. It is clear that only one of these two last conditions is enough. For material not
T
C
hammer-hardened, the choice of the coefficients is such as COp < COp and the condition [éq 3.2-5] involves [éq 3.2-4].
T
C
We will see later that work hardening can reverse the order of the points P and P, returning the condition
T
C
[éq 3.2-4] more constraining than [éq 3.2-5].
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3.3
Criterion of rupture. choice of the coefficients has, B, C and D

When the state of stress reaches the edge of the field of reversibility, of the plastic deformations develop and the thresholds move: they are hammer-hardened. The threshold of compression "increases" in
the first time, then decreases, whereas the threshold of traction can only decrease. The threshold of rupture
corresponds to the maximum field being able to be reached, it is represented on [Figure 3.3-a] in one diagram of plane constraint:

Initial threshold of reversibility
in compression
2
1
Threshold of rupture
in compression

Appear 3.3-a

The work hardening of the thresholds results mathematically in the evolution of quantities A and A, them
C
T
thresholds of rupture corresponding to the maximum of the functions F = F - A and F = F - A. In
C
C
C
T
T
T
models selected, these functions are such as: Max F = F and max F = F;
C
C
T
T

The coefficients has, B, C, and D are thus defined from:

· ft': resistance in axial traction plain of the concrete,
· fc': resistance in axial compression plain of the concrete,
· fcc': resistance in axial compression Bi of the concrete,
F
F
One defines moreover coefficients:
T
=
and
DC
=

F
F
C
C
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To determine the coefficients has, B, C and D it is necessary to give itself 4 equations which express in fact that them
criteria are reached for states of stresses particular and judiciously selected.
A first possibility consists in writing that the two criteria are cut on the axes compression simple (points C of [Figure 3.3-b]).

With
C
D

Appear 3.3-b

By recalling that:

2
In simple compression:
<
0 ;

Oct. =
; Oct. = -

3
3

2
Out of Bi compression <
0 ;

Oct. = 2
; Oct. = -

3
3

2
In simple traction >
0 ;

Oct. =
; Oct. =

3
3

The following relations then are obtained:

Number
State of stress

Criterion reached
relation obtained
of condition
1 Compression
simple
Compression
+ 3b = 2 has

3
2
Bi compression
Compression
2a + B = 2

3 Traction
simple Traction - C + 3D = 2
4 Compression
simple
Traction C + 3D = 2
Table 3.3-a
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F
Who gives, while posing:
T
=
and
DC
=

F
F
C
C
-1
2
has =

2
B =

éq
3.3-1
2 -1
3 2 -1
1-
C = 2
D = 2 2 1 ***éq***
3.3-2
1+
3 1+
But this choice is problematic.

Indeed, after work hardening of the criterion of traction, and for a limit of traction become null it field of admissibility takes the form indicated on [Figure 3.3-c], making nonacceptable of Bi compressions states.

Appear 3.3-c

Moreover, with this choice of the coefficients, certain ways of simple traction compression presented snap-back as indicated in appendix.

We then preferred to replace the condition number 4 of [Table 3.3-a] by a condition expressing that, after the limit of traction fell down to zero, the field of reversibility is that represented on [Figure 3.3-d].

Appear 3.3-d
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This resulted in replacing the relation C + 3D = 2 by C = 2
The choice of the coefficients has, B, C and D is finally:

-1
2
has =

2
B =

éq
3.3-3
2 -1
3 2 -1
2 2
C = 2D =

éq
3.3-4
3

model selected

Appear 3.3-e

[Figure 3.3-e] shows the difference between the two models for a state of plane constraint.

3.4
Analyze field of reversibility retained

In this chapter, we give indications on the order of magnitude of working stresses to feel criterion selected. We endeavour to give indications on tensile stresses, in particular for three-dimensional states of stress.
[Figure 3.4-a] shows the initial fields (i.e. before work hardening) for the values following of the parameters materials:

F

F = 40 Mpa
C
initial limit of rupture in simple compression:
C
F

F = 44 Mpa
DC
initial limit of rupture out of Bi compression
DC
F

DC
=

relationship between rupture limit in Bi-compression and simple compression
= 1.1
F C
F

=

C
elastic limit in compression;
33
,
0
F

F = 4 Mpa
T
initial limit of rupture in traction
T
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2
1
3
Appear 3.4-a

Figures [Figure 3.4-b], [Figure 3.4-c] and [Figure 3.4-d] the cuts of the field show three-dimensional by plane = 0 and = -
Mpa

25
.
3
3

Plan = Mpa

0
3
2
1
3

Appear 3.4-b
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Plan = - Mpa

25
3
2
1

3

Appear 3.4-c

Plan = Mpa

0
3
Plan = - Mpa

25
3
2
1
3

Appear 3.4-d

[Figure 3.4-e] shows the fields of reversibility in a plan (, for states of
1
2)
constraint constant, fields parameterized by the value of. We represent the fields
3
3
for = -25 Mpa, = 0 Mpa, = 4 Mpa, = 10 Mpa, = 15 Mpa. One sees there that for one
3
3
3
3
3
containment of 25 Mpa of compression, stresses tensile can reach 15 Mpa, and that,
in parallel, the field of reversibility for = 15 Mpa N `is not empty and corresponds to
3
compressive stresses and -25 Mpa. It is also seen, that, for a value
1
2
data of, the maximum value of traction
3
Obtained for and is reached with the intersection of the criteria of traction and compression.
1
2
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Field of rupture = - Mpa

25
3
Field of rupture = Mpa

0
3
Field of rupture = Mpa

4
3
Field of rupture =
Mpa

10
3
Field of rupture
= Mpa

15
3
Field of rupture = - Mpa

25
3
Appear 3.4-e

We thus study the place of intersection of the criteria of traction and compression. We note
(0 eq
, the point of intersection of the two criteria in the plan (
eq
, (not C of [Figure 3.4-f]).
H
)
H
0)

eq

C
eq
0
Field of
reversibility
H
0

P
P
H
T
C

Appear 3.4-f
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The place of intersection of the two criteria in the space of the constraints is given by:

2 eq

sin
1 =

+ + 0
0
H

3

6

2 eq

sin

2 =

- + + 0
0
H

3

6
= 0
3
3
H - 1 -

2

Where is a parameter.

Appear 3.4-g

[Figure 3.4-g] shows projections of this place in the plans (, and (.
2
3)
1
2)
One can easily calculate the maximum value of the constraint along this curve:

F
2
C

éq
3.4-1
max =
+
F T
3

3

This equation shows that, whatever the value chosen for the rupture limit in traction,
maximum constraint attack in traction is higher than the third of the rupture limit in compression.
[Figure 3.4-h] shows the three principal constraints according to the parameter.
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Appear 3.4-h

It is seen that one can reach a level of traction of 15 Mpa, but for a containment of
= - Mpa
25
and = - Mpa
25
.
2
3

To try to avoid this disadvantage, which is important, one can try to exploit the values of resistance in compression and the parameter.
As example, we chose the following play of parameters:

F = 20 Mpa
C
F = 40 Mpa
DC
= 2
F = 4 Mpa
T

[Figure 3.4-i] shows the criteria with this choice of parameters. [Figure 3.4-j] the value shows of principal constraints with the intersection of the two criteria for this new choice of parameters. maximum traction obtained is weaker (8 Mpa), but it is reached for a level of containment

also low (- 7Mpa).
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Initial criterion of traction with = Mpa

0
3
Initial criterion of traction with = - Mpa

8
3
Limit of the initial elastic range
in compression with
= Mpa

0
3
Criterion of the initial peak of compression with
= Mpa

0
3
Criterion of the initial peak of compression with
= - Mpa

8
3
Appear 3.4-i

Appear 3.4-j
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3.5 Work hardening

As we already mentioned in the paragraph [§3.3], when the state of stress reaches the edge field of reversibility, the plastic deformations and the variables intern develop, them thresholds move: they are hammer-hardened. For our model, the variables intern are two, they are noted p
for the internal variable "known as of compression" and p
for that "known as of
C
T
traction
". These variables determine the evolution of the thresholds of compression and traction respectively, the thermodynamic forces theirs are connected by the relations:

With = F - F

éq
3.5-1
C
C
C (p
c)
and
With = F - F

éq 3.5-2
T
T
T (p
T)
where F and F represent the values of resistances in compression and traction
T (p
T)
C (p
c)
respectively.

3.5.1 Functions of work hardening

The function F is initially increasing then decreasing, the decreasing part being is linear
C (p
c)
[Figure 3.5.1-a], that is to say quadratic [Figure 3.5.1-b],

F
p

C (c)
f' C
F C
PC

U
E
C

Appear 3.5.1-a

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F
p

C (c)
f' C
F C
p

C
E
CPU

Appear 3.5.1-b

is a data of the model. Shape of the curve between E
and U
(negative work hardening) depends on

C
C
*the element, and more precisely of its dimensions, according to a criterion similar to that chosen by G. Heinfling, [**Error! Source of the untraceable reference.**] for the taking into account of the localization of*
deformations.
In traction, shape of the curve giving the value of the elastic limit F according to
T (p
T)
cumulated plastic deformation p
do not comprise a part “pre-peak”, the part “post-peak” being is
T
linear [Figure 3.5.1-c], that is to say exponential [Figure 3.5.1-d].

F
p

T (T)
ft
G ft
Pt
C

Appear 3.5.1-c
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F
p

T (T)
ft
U
p
T
T

Appear 3.5.1-d

3.5.2 Curves
of work hardening and modules post peak

3.5.2.1 Models cracking distributed

The introduction of a behavior softening post-peak into the relations stress-strains
pose a major problem. Under statical stress, beyond a certain level of constraint,
corresponding to the starter of the lenitive behavior, equations governing the balance of
structure lose their elliptic nature. These equations of the mechanical problem form one then
system of partial derivative equations evil posed of which the number of solutions is multiple. It
problem results in an not-objectivity compared to the grid. It results from this a sensitivity
pathological of the numerical solution to the smoothness and the orientation of the grid.
In order to solve this problem, or at least, to limit the consequences of them on the reliability of the solution
predicted, it is necessary to use techniques known as of regularization. The object of these techniques is
to enrich the mechanical description of the medium, to be able to describe nonhomogeneous states of deformation, and to preserve the mathematical nature of the problem. One operates this regularization in
introducing, in the law behavior, a characteristic length or internal length, connected to
width of the zone of localization. Several techniques are possible to improve description
mechanics of the lenitive medium. They constitute limitings device of localization. The implementation of
these techniques requires in general, of the delicate numerical developments. An approach
intermediary enters the use of the traditional models and the placement of these limitings device of localization consists in making depend the slope post-peak on the relation stress-strain, of the size *element, so as to dissipate with the rupture a constant energy. This approach constitutes one*

not towards a nonlocal description of the continuous medium.
Let us consider initially a real crack of surface S whose measurement is A [3.5.2.1 Figure - has]. S is one
surface discontinuity of the field of displacement ***U****. It is supposed that to create this discontinuity, it is necessary to spend an energy W whose expression is: W = G (****X****) dS, G being a property of*
S
F
F
material.
Let us consider now that one wants to represent the same phenomenon, while representing not a discontinuity of displacement but a plastic deformation uniformly distributed in one volume V.
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tr
D
ij
p
Dissipated energy will be: W = FD
ij dt, where one noted T the "time-to-failure".
V

dt
R

0
S
V

Lver

Appear 3.5.2.1 - has
By making the series of following assumptions:

· the crack is plane,
.
G is constant along the crack and thus W = A G
.
,
F
F
.
V is a basic cylinder S and a height L,
worm
tt
D p
.
ij
ij
G
dt is constant in V.
F =

dt
0

One leads finally to the relation:
tt
p

D
ij
ij
W = Vg = V

dt

= AG

.

éq

3.5.2.1-1

F

F

dt

0

Or:

tt

p

D

G

F

ij

ij

F

G =

dt

=

éq

3.5.2.1-2

dt

L

0

worm

It is seen easily that: G F = U F

, writing in which the quantities (F

G, F,

U)

0

() D

represent (F respectively

U

p

G, F, in traction and (F

U

p

G, F, in compression.

C
C
C
c)
T
T
T
T)
data of F
G thus determines, this in traction as in compression:
U
U
C

G
F
G =
F

=

C
(p D
C
c)
F
p
C

L
0
worm
U
T

G
F
G =
F

=

T
(p D
T
T)
F
p
T

L
0
worm
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The quantity F
G is thus related to the slope of the curve post peak in a diagram constraint-variable
of work hardening, which is related to the forced slope post peak in a diagram deformation.
Let us suppose for example that the forced relation deformation is linear in mode post peak.
Let us call E
the slope post peak in the diagram (,) and H < the 0 corresponding slope in
T < 0
EE
He
of diagram (F,) [3.5.2.1 Figure - B]. There is the relation H

T
=
E =
who show
E - E
T
E + H
T
that one must have:
- H < E, or else the diagram (,) presents a snap back.

ft

ft
EE
E
H
T
=
T
G F
E
E - E
T
T
T

Appear 3.5.2.1 - B
The condition - H < E is known as condition of applicability, it will result in an inequality on F
G and thus
on L.
worm
Within the framework of a resolution by the finite element method, representative elementary volume fissured medium can be compared to an element of the grid. The characteristic length (noted by continuation lc) introduced into the method of the energy of equivalent rupture corresponds to the Lver length.
During a calculation corresponding to an unspecified structure, determination this length characteristic is delicate. It depends on the position of the plan of crack, dimensions and the type elements…
A simple estimate for the two-dimensional cases can be expressed in the form:
L = R A where A
C
E

E is the surface of the element considered, and R, a correct factor, being worth 1 for quadratic elements, and 2 for the linear elements.
One can extend this formulation to the case 3D:
3
L = R V where V
C
E
E indicates the volume of the element.
Concerning the evolution of work hardening with the temperature, we regard as in [bib2] that energies of rupture and resistances to rupture not depend on the current temperature T material point considered at time T, but of the maximum temperature reached in this point since the beginning of the loading until time T. When we need to show the dependence of quantities compared to the temperature, we will note:

· the maximum of the temperature since the beginning of loading,
·
F
resistance in compression,
C ()
·
F (indicates resistance in traction,
T)
·
F, the curve of work hardening in compression,
C (
p
c)
·
F, the curve of work hardening in traction.
T (
p
T)
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3.5.2.2 Behavior of the concrete in traction and linear curve post-peak

In this modeling, the concrete is supposed to be elastic until its resistance in traction F.
T
curve F in traction is represented on [Figure 3.5.1-c] and is entirely defined by
T (p
T)
resistance in traction of material, the energy of cracking
F
G, and the characteristic length lc.
T
The mathematical expression of this curve is:
F
,
F
1

éq
3.5.2.2-1
T (
p
Pt) = ()

-
T
T

C ()

The equivalence of dissipated energy makes it possible to write:

C
U
F

G
L
F
,
L F
1
D

T (
T
p
) = C T (Pt)

= C T ()

T
p

-
0
0
C ()

T
from where

L F
F
G

éq
3.5.2.2-2
T (
U
)
.
C
T () .t ()
=
2
and
F

G
.
2
U
éq
3.5.2.2-3
T ()
T ()
=

L. F
C
T ()
The condition of applicability is written:
.
2nd (). F
WP ()
L

éq
3.5.2.2-4
C
2
ft ()

3.5.2.3 Behavior of the concrete in traction and exponential curve post-peak

In this modeling, the concrete is supposed to be elastic until its resistance in traction F.
T
curve F in traction is represented on [Figure 3.5.1-d] and is entirely defined by
T (p
T)
resistance in traction of material, the energy of cracking
F
G, and the characteristic length lc.
T
The mathematical expression of this curve is:

F
,
F
.exp
has

éq
3.5.2.3-1
T (
p
Pt) = ()

-
T
T

C ()
The equivalence of dissipated energy makes it possible to write:

F
G
L
F
L F
has
D

T (
p
) = C T (, Pt)

= C T ()

T
p

exp -
U
T
0
0

T ()

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From where:

U
.
.

G ft () L F
C
T () T ()
=
éq
3.5.2.3-2
has
and
C ()
F
WP ()
=

éq
3.5.2.3-3
has

lc. ft ()

p
p
That is to say still: F
,
F
.exp
L. F

T (T) =
T (
)

- C T () T

F
WP ()

2
L. F
C
T

The maximum slope of the curve is then H

max ()
( )
= -
F
WP ()
and the condition of applicability is written:

E (). F
WP ()
L
éq
3.5.2.3-4
C

2
ft ()

3.5.2.4 Behavior of the concrete in compression and linear curve post-peak

In this modeling, the behavior of the concrete is supposed to be elastic until the elastic limit, data by a proportionality factor (noted) expressed as a percentage of resistance to the peak F

.

C ()

For the standard concretes is about 30%. The curve F in compression is represented

C (p

c)

on [Figure 3.5.1-a] and is entirely defined by resistance in traction of material, the energy of cracking

F

G, and the characteristic length lc.

C

The mathematical expression of this curve is:

U

p 2

FC (, PC)

C

= F 'C () + (2 -

2)

C

+ -1

if p

C E

éq 3.5.2.4 - 1

2

E ()

(

)

( )

E ()

p
U
F
,
F '
(C - c)
C (c) = C ()
( )
if E PC U

U

C
éq 3.5.2.4 - 2

(E () - C ())
( )
( )

FC

Resistance in maximum compression is reached when:
2 2

E () = (
-) ( )
E ()
The equivalence of dissipated energy makes it possible to write:
U
C
F
G = L
F, D
C ()
C C (
p
c)
p
C
0
from where
F
2
1
1
G
L. F

éq
3.5.2.4-3
C ()

= C C () + E () + CPU ()

6
2

and
F
U
G
.
2
2

1

éq
3.5.2.4-4
C ()
C ()
+
=
-
E ()

L. F
3
C
C ()

FC
The slope of the curve is then H ()
( )
= -

CPU () - E ()
and the condition of applicability is written:

E (). F
Gc ()
L

éq
3.5.2.4-5
C
6

2
FC (
2
) 11-
4 -
4

3.5.2.5 Behavior of the concrete in compression and nonlinear curve post-peak

In this modeling, the behavior of the concrete is supposed to be elastic until the elastic limit,

data by a proportionality factor (noted) expressed as a percentage of resistance to the peak F
.
C ()
For the standard concretes is about 30%. The curve F in compression is represented
C (p
c)
on [Figure 3.5.1-b] and is entirely defined by resistance in traction of material, the energy of cracking
F
G, and the characteristic length lc.
C
The mathematical expression of this curve is:

p
p 2
FC (, PC)
C

= F C () + (2 -
2)
C

+ -1
if p
C
E
éq 3.5.2.5 - 1

2
E ()
(
)

( )

E ()

2

(PC - E)
FC (, c) = F C ()
( )
1-
if E PC CPU
éq 3.5.2.5 - 2

U
2

(C () - E ())
( )
( )

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FC
Resistance in maximum compression is reached when:
2 2

E () = (
-) ( )
E ()
The equivalence of dissipated energy makes it possible to write:

U
C
F
G = L
F, D
C ()
C C (
p
c)
p
C
0
from where:
F
2
G

L. F

éq
3.5.2.5-3
C ()

= C C () CPU ()
+ E ()

3
3

and:
F
U
3 G

éq
3.5.2.5-4
C ()
C ()

=
- E ()
2

L F
2
.
C
C ()

.
2 FC
The maximum slope of the curve post-peak is then H

max ()
( )
= - CPU () - E ()
and the condition of applicability is written:
F
3rd (). G ()
L
C

C
1

éq
3.5.2.5-5
2
2
FC (
2
) 4 - -

4 Flow
plastic

In this paragraph, we give the expression speeds of plastic deformation, while distinguishing the case says general where the state of stress is located on a "regular" zone of the edge of the field of reversibility and the case where it is at the top of one of the cones.

4.1
General form of the rule of normality

*In space (**, A**), the inequalities [éq 3.2-2], [éq 3.2-3], [éq 3.2-4], [éq 3.2-5], define one convex field which we will note C*
the indicating function of this convex:
(
. We will note
, A)
C

(
if

0
***, A)** C*
(, A)

éq
4.1-1
C
=
(, A)
if not

When the border of the field of reversibility is reached, of the irreversible plastic deformations develop, according to the classical theory of plasticity.
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For a standard material, [bib4] the law of flow checks the principle of maximum plastic work, what results in the equation: (p) ***éq 4.1-2***
C
&, &
where
note under differential of the function. We point out [bib3] that under differential of one
C
C
convex function in an item X is the whole of vectors Z such as
:
*F (**
X) F (X)
*
**
+ Z, X - X X

It is then seen easily that [éq 4.1-2] involves:

éq

4.1-3
*C (**
***
, A) (, A) + (**
*-)+ (**
WITH - WITH)
***
***

With
and

&p
C
&

Taking into account the definition of the characteristic function, one to see easily that [éq 4.1-3]
is
equivalent with:

p
p
+ ***A***
***
+
***
With
***
and

***
***WITH** C (*
éq
4.1-4
, A)
& & &
&

*In other words the plastic flow is such as the couple (, **A**) carries out the maximum of plastic dissipation among the acceptable thermodynamic forces.*

4.2
Expression of the plastic flow partly current

When the function F is differentiable at the point considered (,) the rule of normality is written simply
p
F
=
*& & **éq***
4.2-1

F
=
*& & **éq***
4.2-2
With

& and F checking the conditions of Kuhn-Tucker:
0
&
*F 0 **éq***
4.2-3

. F = 0
&

The variable of work hardening is related to the plastic multiplier by the law of work hardening. By using it
plastic work, one can write:
p
F

& = &.
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F

If F is a homogeneous function of order 1 compared to the tensorial variable, one has
= F, it

who leads to the equality: =
& & and thus finally with the equations:
F
p

éq
4.2-4
C = p
comp
C
& &

F
p
p

éq
4.2-5
T =
trac
T
& &

4.3

Expression of the plastic flow at the top of a cone

We give two presentations of the same result. The first presentation uses the theory of standard materials generalized and under differentials, the second share of an equality posed a priori on plastic work.

4.3.1 Demonstration by the general theory of standard materials

The field C (

consists of two cones. The function
is not differentiable is with
, A)
C
the intersection of these two cones, is at the top of each one of these cones. When the point (, A) belongs to the intersection of the two cones, the preceding equations remain valid, with precision that the deformations figure of compression and traction develop into same time. This case known as "multi criterion" is remainder treated in [bib4]. We will be satisfied here to treat
the case where (, A) is at the top of a cone, and we will choose the most frequent case of the top of cone of traction, knowing that the case of the top of the cone of compression is treated exactly even way.
The criteria are rewritten by using the variables eq
and, more practical in the development
H
analytical.
2
C
F
(, A)
F
With

4.3.1-1
trac
=
eq
T
+ H - t+ T 0
3D
D
H
F (

C
, A
F
With

4.3.1-2
T) =
H - t+ T 0
trac
D
We thus consider a case where:
eq = 0

C

4.3.1-3

F
With
H -
t+ T = 0
D

On the basis of [éq 4.1-4], we will calculate plastic dissipation as being the maximum of
p ***
***

With
& +& for all the couples ***, *A** *C (*
, A)

p
D = max (p ***
***

With)
4.3.1-4

,
C, A &
+ &
**** A****
(
)
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By writing whereas this maximum is finished and reached when ******* *= and* ***A**** *=* ***A****, we will find conditions on p*
& and &. In fact, the finished character will be enough.
By using the partly isotropic decomposition of the tensors and déviatoire, and the particular form of variables of work hardening, one finds easily:
p *******
****
*p **
***
p
p
***
+ ***A*** *= ~*
S *+ 3 + A*
&
&

&

4.3.1-5

H H
T
T
& &
Then let us consider the whole of the vectors forced of null trace and of which the constraint
1
equivalent of Von Mises 1 is worth: = eq =
=
1
{ ,
,
1
(
trace)
} 0
=
eqs
I
1 +

H
s1
1
*(, **A**)*

2
C
C (

4.3.1-6

, A)

eq +
F
With
H -
t+ T 0
3D
D

C
F
With
H -
t+ T 0
D
In other words, the "direction" of the diverter of the constraints is unspecified for a couple
*(, **A**) C (.*
, A)
One can thus write:
p

*
eq
*p **
*
p
*p **
D =

Max ~
Max

4.3.1-7
S
*& **s1** +3H + A*
H
T
T
& &
*

1 1

eq
*
* *

,
,
*, **S***

T

With
H
1
1
2
**
eq

*+ C **
*- F + *0*

3D
D H
T
With
T

*C **
**

- F + 0
D H
T
With
T
~
It is clear that the maximum of
**
PS
*& when *sest is reached “parallel” with p*
&~ and that one has then:
1
1
*p **
2 p
Max ~
S
.
1 =

1
S
1
&

~eq
3 &

[éq 4.3.1-7] can thus be written:
p
2
*
~ p eq*
D =
eq +
Max
&
Max
*(*
3
pH
H
)

+
&
*
3*

*
eq*
*
*
eq
*
*

,
,

,
,
T
With
H

T
With
H

2
*

eq
*C **
*
2
*

F
0

+ - +

*eq + C * - F +*
**0*

3D
D H
T
With
T
3D
D H
T
With
T

C

*
*
*C **
*

- F +

0

- F +

0
D H
T
With
T
D H
T
With
T

4.3.1-8
+
Max
*(p *A*
T
T)

&
*

*eq **
*

,
, T
With
H

2
*
eq
*C **
*

+ - F +

0
3D
D H
T
With
T

*C **
**

- F +

0
D H
T
With
T

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Like ~

peq 0
&
, one has for the first term:
2
***
p
eq
2
~
p
3
~
*D * 3D*
*3c **
Max

With
F
4.3.1-9
eq
= eq -
+
T

&
&
***
3
3

2 T
2
2 H
*eq **
***
,
, T
With
H
2
***

*eq + C * - F +*
**0*

3D
D H
T
With
T

*C **
*

- F +

0
D H
T
With
T
[éq 4.3.1-9] deferred in [éq 4.3.1-8] gives:

p
~ p
D = 2D F +
*
3
-
C ~
2
+
A -*
D ~
2
4.3.1-10
eq
T
&
Max ((p
p
H
eq
**
**
&
&
Max
**

*
*
&
&
H
)
(T (p
p
T
eq)
, your
,
H
T
With
H
C *
*
C
- F +

0
* - F +
*0
D H
T
With
T
D H
T
With
T
Let us pose then:
p
~ p
m =
3
- 2c,
p
~ p
N = - 2D and Q =
D p

eq F

H
eq
&
&
T
eq
&
&
&
~
2

With these notations, [éq 4.3.1-10] becomes:

p
D = Q +
*Max (**
*
m + Na
4.3.1-11
T
H
)
*
**
, your
H
*C **
*
- F +

0
D H
T
With
T

*It is about a problem of the type “simplex”. The field of **
*
, A is represented on
H
T
[Figure 4.3.1-a].

*
At
F T
*
*
Field, A
H
T
*
D
H
F T
C

Appear 4.3.1-a
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*Like the field **
*
*, A extends towards - at the same time for **
*and **
With, so that p
D is finished, it is necessary

H
T
H
T
that m and N are positive. The maximum of
*
*
*m + Na is reached for a couple **
*
, A located on
T
H
H
T
*edge of the field of **
*
, A.
H
T
p
*
C
One has then: D = Q + nf

T + max m - N

H

*

D
H
So that
p
D is finished, it is necessary that:
C
m = N
D
Taking again the definitions of m and N, this relation gives:
p
C p
=

H
T
&3

éq 4.3.1-12
D &
In addition, the constraints m 0 and N 0 give:
p
~ p

3
2c

éq 4.3.1-13
H
eq
&
&
and
p
~ p
2D

éq 4.3.1-14
T

eq
&
&
these two last inequalities being equivalent because of [éq 4.3.1-11].
The equations [éq 4.3.1-11] and [éq 4.3.1-12] define the plastic flow in the top of one of cones of the field of reversibility.

4.3.2 Demonstration by plastic work

The starting point is to consider that compared to the developments made in regular points, they are primarily the relations [éq 4.2-1] and [éq 4.2-2], known as rules of normality, which cannot to be written more. However the relation [éq 4.2-1] implies the equality p
F
, which can, it, being
T
T (p
T)
p
&
= &
maintained.
We will thus leave the equation:
p
F

éq
4.3.2-1
T
T (p
T)
p
&
= &
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We use the partly isotropic decomposition and déviatoire tensors and find:
p
F = ~
S +
3

&
&
éq
4.3.2-2
T
T (p
T)
p
H H
&
At the top of the cone of traction, there are the relations [éq 4.3.1-3], which, carried in [éq 4.3.2-2] give,
while also using [éq 3.5-2]:
p
D
F =
F
T
T (p
T)
T (p
T) H
&
3

éq
4.3.2-3

C
&
C
And one thus finds the relation [éq 4.3.1-12]:
p
=
H
T
&3
.
D &

4.4
Together equations of behavior (summarized)

One notes H the matrix of elasticity:

+ µ
2

0
0
0

+ µ
2

0
0
0

+ µ
2
0
0
0
H =

0
0
0

μ
2
0
0
0
0
0
0
μ
2
0

0
0
0
0
0
μ
2
With:
=
E
E
3 + 2μ
(
and μ =
, and K =

1 +)(1-
2)
(21+)
3

The forced relations deformations are written finally:

= H (
p
p
- -
éq
4.4-1
C

T)

For a regular point of the cone of compression:
has
F
With
F
With

éq
4.4-2
comp (
) 2
,
=
eq
C
+ H - c+ C 0
3b
B
F
p F
0
éq
4.4-3
C
comp =
PC = p comp
C
&
;
& &

For a regular point of the cone of traction:
2
C
F
(, A)
F
With

éq
4.4-4

trac
=
eq
T
+ H - t+ T 0
3D
D
F
p
p
p
F
0

éq
4.4-5
T
trac =
T =
trac
T
&
;
& &

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For a point at the top of the cone of compression:
S = 0

éq 4.4-6
H
F
(
has
, A
F
With
éq
4.4-7
c) =
H - c+ C = 0
comp
B
p
p has

=
C H
C
&3

éq 4.4-8
B &
p
~ p

3
2a

éq 4.4-9
C
&
&
H
C eq
For a point at the top of the cone of traction:

S = 0

éq 4.4-10
H
F (
C
, A
F
With

éq
4.4-11
T) =

H - t+ T = 0
trac
D
p
C p

=
T H
T
&3

éq 4.4-12
D &
p
~ p

3
2c

éq 4.4-13
T
&
&
H
T eq

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5
Numerical integration of the law of behavior

5.1
The total problem and the local problem: recalls

For a given structure (geometry and material), and for a given loading, fields of displacement, constraint and variables intern are by solving a whole of equations with nonlinear derivative partial formed starting from and the law equilibrium equations of behavior. The document [bib5] presents the algorithm of which we give a summary here:

U and known
0
0
Buckle urgent T: loading L = L T
I
(I)
I
U known; calculation of the prediction
0
U

I 1
-
I
Iterations of balance of Newton N

N
U known; N
U
U
U
I =
N
I -
I
I 1
-
Buckle elements el
Buckle points of gauss G
N
calculation
el
el

=
U

G
G
I
(nor)
law of behavior:
N
N
N
el
calculation of: el
and el
starting from el

, el

and

G I
G I
G I 1

-
G I 1
-
G I
N
el

calculation of
G I (according to option)
N
el

G I
Accumulation in vectors and matrices assembled:
el
N
Accumulation of T
el
Q
T
G
.
in
N
Q
.
G I
I
N
el
el

Accumulation of calculation of T
G I
el
Q G
Q in N
K (according to option)
N
G
el

I

G I
Calculation of
N
U
by:
I
N
N
T
N
K. U
= Q
-

. + L
I
I
I
I
linear iteration of research to determine
Actualization:
N
N
N
U
+1 = U
+ U

I
I
I
IF test convergence OK
fine Newton: no time following I = i+1
If not
N = n+1
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N
The calculation of the constraints and variables intern el

N
el
with the iteration of Newton N and time T with
G I
G I
I
N
el
to leave the constraints and variables intern el

, el

at time T and value and

of
G I 1
-
G I 1
-
I 1
-
G I
the increase in deformation in the interval of time estimated with the iteration of Newton N consists with
to integrate the equations [éq 4.4-1], [éq 4.4-2] with [éq 4.4-5] or [éq 4.4-6] with [éq 4.4-9] or [éq 4.4-10] with

[éq 4.4-13] according to the cases with the initial conditions:
(T

éq
5.1-1
I
= el
1
-)
G I 1
-
(T

éq
5.1-2
I
= el
1
-)
G I 1
-
p
(T
= el
p

éq
5.1-3
I 1
-)
G I 1
-

With the condition of loading in imposed deformation:
(T =
éq
5.1-4
I)
N
el
G I
The result of this integration will provide:
N

el

= T
G
(I)
I
N
el
p
p
G = T
I
(
I)
N
el

= T
G
(I)
I

The object of this chapter is to present the numerical integration of these equations. It is about a system
nonlinear differential equations which we solve by a method of implicit Euler. To leave
of now, the quantity at the beginning of the step of time (known) will be noted with an index -, then N
that unknown factors at the end of the step of time (all unknown factors except
el
=) will be noted without index.
G I
For an unspecified quantity one has notes
-
= has - A. has.
One always starts by calculating an elastic solution E
, by supposing that there is no evolution
plastic deformations and internal variables. So at least one of the criteria is violated by this elastic solution, it is necessary to calculate plastic flows. The cases should then be distinguished regular for which the solution is on the regular part of one of the cones or with their intersection cases known as singular where the solution is at the top of one of the two cones. Logic allowing to examine and choose these various cases, and the algorithm which results from this are relatively complex.
We thus present in first the treatment of each case and explain their sequence

subsequently, in the chapter [§5.7].
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5.2
Digital processing of the regular case.

One presents in detail only the case where at the same time plastic deformations develop in traction and in compression and where thus the solution belongs to the intersection of the two cones. Let us note
however that, even if E
violate at the same time the two criteria, for as much the final solution can very to belong well finally only to one of the cones hammer-hardened. One is thus brought to seek balanced which one postulates that they belong to one of the two cones or both. The case or it belongs to only one of the two cones results easily from the case plus general presented here. Equations that us let us have to solve are finally:

$\mu = -S + \mu \sim 2$

éq
5.2-1
$\mu - E K =$

- + 3K
éq
5.2-2
H
-
H
H
K
E
***S** = **S** - 2μ (~ p*
~ p
+
éq
5.2-3
C
T)
E
= - 3K

+

éq
5.2-4
H
H
(p
p
H C
H T)
has
F

F

éq
5.2-5
comp (
PC)
2
,
=
eq + H - C (PC) = 0
3b
B

F

p

éq
5.2-6
C =

p comp
C

p
2
C
F
(,
F

éq
5.2-7
trac
)
T
=
eq + H - (tp)
T
= 0
3D
D
F
p
p

éq
5.2-8
T =

trac
T

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By taking the isotropic and deviatoric parts plastic deformations, the equations [éq 5.2-6] and [éq 5.2-8] give:

$$\tilde{p}$$

$$p$$

$$\boldsymbol{S}$$

$$C$$

$$=$$

éq
5.2-9

$$C_{eq}$$

$$B$$

$$2$$

$$p$$

p has

$$=$$

éq 5.2-10
H C
C
B
3
p
~

p
S
T
=

éq 5.2-11
T
eq
2D
p
PC

=

éq 5.2-12
H T

T
D
3
While deferring [éq 5.2-9] and [éq 5.2-11] in [éq 5.2-3], one finds:
p
p

E
S
C
T
S *=* ***S*** *- μ*
2
+

éq
5.2-13
eq

B
2
2D
who shows that ***S*** *is parallel to E*
S *from where one deduces:*
E
S *=* ***S éq 5.2-14***
eq
eq
E

While deferring [éq 5.2-14] in [éq 5.2-9] and [éq 5.2-11] one finds:

p
E
~

p
S
C

=

éq 5.2-15

C
eq
E
B
2
p
E
~

p
S
T
=

éq 5.2-16

T
eq
E
2D

One defers then [éq 5.2-15] and [éq 5.2-16] in [éq 5.2-3] and [éq 5.2-4], and one expresses the criteria [éq 5.2-5] and [éq 5.2-7] with these new results. That led to two equations having like unknown factors

p

and
p

:

C
T

2
eq
has

E
E
p
2
2
μ Ka
p 2μ
Kac
+
F
éq
5.2-17
H -

C
+
-
T
+
- C -
p
C
+
PC =

2
2

(
) 0
3b
B
3b
B
3bd
data base

2
eq
E
C E
p 2µ
Kac

p
2
2
µ
Kc
+
F
éq
5.2-18
H -

C
+
-
T
+
- T -
p
T
+
Pt =

2
2
(
) 0
3D
D
3bd
data base
3D
D
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It is this system of two equations to two unknown factors which should finally be solved. If the functions F and F are linear, i.e. if one is in linear mode post-peak in
C
T
compression as in traction, it acts of a linear system which will thus be solved in an iteration. In the case, is mode pre-peak in compression (which is always nonlinear), that is to say of modelings with nonlinear modes post peak, the system [éq 5.2-17] and [éq 5.2-18] is solved by one method of Newton:
*One notes F **

,

the criterion of compression regarded as function of the only variables
comp (
p
p
C
T)
p

and
p

, in the same way for traction:
C
T

*
2
has
μ
2
Ka2
μ
2
Kac
F

,

=
+ -

+
-

+
- *F* - +

comp (
p
p
C
T)
eq
E
E
p
p
H
C
2
2

T
C (p
p
C
c)

B
3
B
B
3
B
data base
3
data base
*
2
C
µ
2
Kac
µ
2
Kc2
F

,

=

+ -

+
-

+
- *F* - +

trac (
p
p
C
T)
eq
E
E
p
p
H

C
T
2
2
T (p
p
T
T)
D
3
D
data base
3
data base
D
3
D

The ième iteration of Newton for system [éq 5.2-17] - [éq 5.2-18] is:

i+1
I
p
p

,
-1
F

comp

C
C
(p p
C
T) I

=
- J

p
p
I

T
T
F

,

trac (p
p
C
T)
The jacobien J is worth:
I
I
F

F

comp
comp

p
p

C
T
J

=

I
F

F

trac
trac

p
p

C
T
With:

F

μ
2
2

-
comp
Ka
F
+

C (p
p
C
c)
= -
+
-

p

2
2
p

B
3
B

C
C

fcomp
μ
2
Kac
= -
+

p
3
T
data base
data base

F
2
trac
μ
Kac
=
-
+

p
3
C
data base
data base
F

μ
2
Kc2
F
- +

trac
T (p
p
T
T)
= -
+
-

p

2
2
p

D
3
D

T
T

Initial Jacobien of the system results from the values of derived in
p
and
p

, which
T = 0
C = 0
amounts solving the nonlinear system on the basis of the null solution. Nonthe linearities are introduced by the curves of softening. In the post-peak part, when they are linear, convergence is done in an iteration. When they are nonlinear, convergence only requires some iterations. To leave the null solution thus does not pose a problem of convergence. That returns starting from the linearization of the criteria in the vicinity of the elastic prediction.

5.3
Existence of a solution and condition of applicability

We point out that the solution of the problem [éq 5.2-17] and [éq 5.2-18] must check the conditions [éq 4.2-3] and thus inter alia the positivity of the increases in the plastic multipliers.

p
éq
5.3-1
C 0

p
éq
5.3-2

T 0
Let us suppose that we are in a case of linear behavior post peak in traction as in compression and let us call H and H respectively the slopes of the parts post peak. Increases
C
T
plastic multipliers are obtained by solving the linear system:

μ
2
Ka2
μ
2
Kac
p

+
+
2
has
H
F
2
2
C
+

C

eq
-

E + eH -
p

C (c)
B
3
B
data base
3
data base

B
3
B

éq
5.3-3
2

=
μ
2
Kac
μ
2
Kc

2
C
p
eq
-
E
E
p

+
+
+ H
F

2
2
T
data base
3
data base
D
3
D

T
+ H - T (T)

D
3
D

Since the criteria of traction and compression were activated in traction as in compression, the second member of this system is positive. But nothing ensures in so far as the solution of [éq 5.3-3] will be positive.
If one poses:
µ
2
Ka2
µ
2
Kac

+
+ H
2
2
C
+

***HTC** = B*
3
B
data base
3
data base

éq
5.3-4
2

µ
2
Kac
µ
2
Kc

+

+
+ H

2
2
T
data base
3
data base
D
3
D

One a:
p

2
has
C

eq
E
+ E
F
H -
-
p

C (c)
-1 B
3
B

*= **HTC***

éq
5.3-5

2
C

p
eq
-

F
T

E + eH - T (Pt)
D
3
D

With:
H
9
3μ
T
***HTC** = 3*
+ μ -
+ μ +
+
+
+
+
+
éq
5.3-6
2 (3
) H
K
T
6
(3K 2) H H
H K 9h K
16
9
C T
C
T
(H
H
K

C
T
)

4
4

3
9K
9K
μ
3
HT + (3K + μ)

-
+
- μ
3

-
HTC 1 *=*
1

4

2
2

2
éq
5.3-7
HTC *9K*
9K
μ
3
K
(-)

1
3
H
9K 18
12

-
+
- μ
C +
-
+ μ

2
2

2

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It is seen that conditions of positivity [éq 5.3-1] and [éq 5.3-2] lead to relations relatively complicated. If the solution of the problem [éq 5.2-17] and [éq 5.2-18] does not check the conditions of positivity [éq 5.3-1] and [éq 5.3-2], that can correspond is with the fact that the coefficients H and H are
C
T
such as there is no positive solution (that would correspond to a snap-back in a diagram (,)), that is to say with the fact that the solution activates finally only one of the two criteria.

Let us examine the simpler case of only one activated criterion. Let us suppose to fix the notations that only
activated criterion is the criterion of traction.
One must have:

2
p
µ
2
Kc

2
C
H
F
éq
5.3-8
T
+
+
=
eq
E
2
2
T
+ E
H
-
p

T (
-
T)
D
3
D

D
3

D
One sees reappearing the condition known as of applicability:
2
2
µ
Kc

+
+ H

éq
5.3-9
T
> 0

3 2
2

D
D

This condition is the generalization of the condition - H < E presented at the paragraph [§ 3.5.2.1] in a particular case of axial request plain.
The following strategy will thus be retained:
2
eq
has
-
2
eq
has
-
If
E + E
F
and
E + E
F

H -
C (PC) > 0
H -
C (PC) > 0

3b
B
3b
B
Activation a priori of the two criteria: resolution problem [éq 5.2-17] and [éq 5.2-18]
So not convergence or so not checking conditions of positivity [éq 5.3-1] and [éq 5.3-2]
Seek solution with only one activated criterion
So not convergence or not checking condition positivity
Stop on diagnostic of nonchecking of condition of applicability of the type [éq 5.3-9]

5.4 Treatment of the nonregular cases

In this paragraph, we describe the discrete treatment of the equations corresponding to projection at the top of the cone of traction, [éq 4.4-10] with [éq 4.4-13], knowing that projection at the top of cone of compression is done in the same way. The equations [éq 4.4-10] and [éq 4.4-12] define the plastic flow in this case, whereas the equation [éq 4.4-13] is a condition of acceptability projection at the top of the cone.

5.4.1 Calculation of the constraints and plastic deformations

Discrete forms of [éq 4.4-10] with [éq 4.4-13], are:
S = 0 éq
5.4.1-1
D
=
F - +

éq
5.4.1-2
H
T (p
p
T
T)
C
p
C
p

3
=

éq
5.4.1-3
T H
T
D
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The relation [éq 5.2-4] established in the regular case is always valid, one jointly uses it with [éq 5.4.1-3] in [éq 5.4.1-1] and one obtains:

E
C
p
D
- K

=
F - +

éq
5.4.1-4
T
T

H
(p
p
T
T)
D
C
The relation [éq 5.4.1-4] is a nonlinear equation compared to the variable
p

that one solves by
T
an algorithm of Newton, which makes it possible to calculate
p

by [éq 5.4.1-3] and by [éq 5.4.1-2].
T H
H
Taking into account [5.4.1-1], the constraints are thus completely known. [éq 5.4.1-1] gives still:
E
~
S *=* ***S*** *- 2*
µ
p

éq
5.4.1-5
T
= 0
This last equation makes it possible to calculate
p
~
and the plastic deformations are completely
T
known.

5.4.2 Acceptability

The discrete form of the relation [éq 4.4-13] is:
p
p

3

c~
2

éq
5.4.2-1
T H
T eq
[éq 5.4.1-5] gives:
eq
~ p
E

éq
5.4.2-2
T
=
eq
µ
2
While using [éq 5.2-4] and [éq 5.4.2-2], [éq 5.4.2-1] is written:
eq
E
cK
E

- éq
5.4.2-3
H
H
µ
2
5.4.2.1 Acceptability a priori and a posteriori
D
-
For the criterion of traction and the part post peak of the criterion of compression, =
F +

H
T (p
p

T
T)
C
is a decreasing function of the variable of work hardening
p

. One deduces from it that
T
E
-
E
- - and thus that:
H
H
H
H
eq cK
eq
E
E
-
E
cK
E

-

-
H
H
H
H
μ
2
μ
2
eq
The condition E
cK
E
-

- is known as condition of acceptability a priori because it can be calculated

μ
H
H
2
eq cK
as of the elastic prediction. The condition E
E

- is known as condition of acceptability has
H
H
μ
2
posteriori.
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Direction of projection
eq

(eq
E
E
,

H
)
zone of projection
at the top of the cone
Of traction
H
Pt

Figure 5-1

These conditions have a simple graphic interpretation. One can see easily that, in the case of one regular solution, one a:
eq
E
eq
-
μ
= 2
E
-
cK
H
H
eq
That shows that the solution in constraint is obtained by projecting the point (E
E
,
parallel to one
H
)
direction (cK,
μ
2) in a diagram (
eq
, as indicated on [Figure 5.4.2-a]. Zones
H
)
of acceptability of projection at the top are cones of which the top and that of the cone of reversibility and delimited on the one hand by the axis > COp and a half-line resulting from the same point and
H
T
of direction (cK,

μ
2).
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5.4.3 Existence of a regular solution and a singular solution.

If projection at the top of the cone is acceptable a posteriori, it may be which exists too a regular solution as one can see it on [Figure 5.4.2-b].

zone of projection
at the top a posteriori
eq

zone of projection
at the top a priori
(eq
E
E
,
H
)
H
2
1

-
P
P
P
T
T
T
Direction of projection for
regular case

Appear 5.4.2-b
The top of the cone of traction before work hardening is noted -
P, that of the cone hammer-hardened with one
T
1
increase in variable of work hardening
p

1 is noted
P, that of the cone hammer-hardened with one
T
T
2
1
increase in variable of work hardening
p
p

>

is noted 2
P. It is seen that there is a solution
T
T
T
1
2
regular with
p

and a solution with projection at the top of the cone for

p

. Since
T
T
the regular solution corresponds to a less work hardening, in the process of evolution, it will be met before the solution with projection at the top: it is thus the regular solution which it is necessary to retain.
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For this reason the sequence of regular research of solution and with projection at the top is the following:
eq
I projection at the acceptable top a priori: E
cK
E
-

-
μ
H
H
2

Calculation of the solution with projection at the top:
p

by [éq 5.4-4]
T
So not
Seek regular solution
So not convergence or not checking condition positivity
Calculation of the solution with projection at the top:
p

by [éq 5.4-4]
T
eq cK
Checking of acceptability a posteriori: E
E

-
H
H
µ
2
eq cK
If not acceptable: E
E

> -
H
H
µ
2
Stop on diagnostic of nonchecking of condition of applicability

5.4.4 Inversion of the tops of the cones of traction and compression

A priori, the top of the cone of compression corresponds to a hydrostatic pressure of traction much larger than that the top of the cone of traction. But, as one can see it on [Figure 5.4.4-a], one can find a history of loading which never activates the criterion of traction, who activates and strongly hammer-hardens the criterion of compression until it to return strictly included in field of reversibility of the criterion of traction.

Way of constraint
eq

Criterion of compression
H
Pt
Criterion of compression hammer-hardened

Appear 5.4.4-a
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When the two criteria were thus reversed, but the criterion should not intervene a priori any more of compression. It may be whereas the solution is a projection at the top of the cone of compression, who treats himself exactly like projection with the top of the cone of traction.

5.4.5 Projection at the top of the two cones

If the two cones were inverted and if the elastic prediction violates both criteria, it may be which are finally acceptable at the same time the solution of projection at the top of cone of compression and at the top of the cone of traction. In these situations, no criterion allows to select a solution rather than the other and one will thus seek a simultaneous projection with top of the two cones, which will have to thus share the same top, as indicated on [Figure 5.4.5-a].

eq
Direction of projection
Criterion of

Criterion of
the criterion of compression
compression
traction
Elastic prediction
initial
initial
(E
H, eq E) to project
Solution with projection
on the criterion of traction
Criteria of
compression and
traction hammer-hardened
H
Solution with projection
at the top of the two cones

Appear 5.4.5-a
The solution with projection at the two tops is obtained by solving the system:
E
has
p
C
p
B
- K
- K

= F - +

éq
5.4.5-1
C
T
C
H
(p
p
C
c)
B
D

has
E
has
p
C
p
D
- K
- K

=
F - +

éq
5.4.5-2
C
T
T
H
(p
p
T
T)
B
D
C
The state of stress is given by:
S = 0
D
-
B
=
F +

= F - +

éq
5.4.5-3
H
T (p
p
T

T)
C (p
p
C
c)
C
has
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5.5
Determination of the tangent operator

During iterations of the algorithm of Newton-Raphson, it is necessary to calculate the matrix of tangent stiffness. The construction of this one plays an important part in stability, the speed and precision of the method of resolution. To preserve these properties, the matrix of tangent stiffness must be built starting from an operator binding the increment of constraint to the increment of deformation
in a precise way at the end of the process of return on surfaces of load. The matrix of Hooke, thus that the thermal deformations intervene like constants at the time of the determination of the coherent tangent operator, built at the end of the iteration in the increment concerned.
The calculation of the operator of coherent tangent behavior takes into account the deformations plastics. For reasons of simplicity, we chose to calculate the operator of behavior tangent of speed.

5.5.1 Operator tangent in speed with only one active criterion

In the case of an only active criterion, for example, the criterion in compression, the calculation of the operator of
tangent behavior speed is as follows:
One thus uses the equations of speed, in elastoplastic load:

F
p

comp
***- H** - C*
= 0

&

éq
5.5.1-1
& &

T
F
F
comp

+ comp p

éq
5.5.1-2
p
C = 0

&

&
C
The tangent operator of speed is defined by:

D
& = &
éq
5.5.1-3
While identifying [éq 5.5.1-3] with [éq 5.5.1-1] and [éq 5.5.1-2], one finds classically:
1

T
F
F
comp

D = H - H

comp ***H éq***
5.5.1-4

with:
T
F

F

F

comp
comp
comp
=
H
-

éq
5.5.1-5
p

C
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5.5.2 Operator
tangent
in
speed with two active criteria

If the two criteria are activated, the criterion in compression and the criterion in traction, it calculation of the operator of tangent behavior speed is as follows:
One leaves:

F
F
p
comp
p

*- **H** - C*
-
trac
T
*= **0***

&

éq
5.5.2-1
& &

&

T
F
F
comp

+ comp p

éq

5.5.2-2

p
C = 0

&

&
C

T
F
F
trac

+ trac p

éq
5.5.2-3

p
T = 0

&

&
T
One leads to:
F

F
F
F
F
comp

T
T

comp

F
trac

T

T
D* = *H* - *H

trac
comp
trac

éq
5.5.2-4
DC
+ ct
+
Tc
+

tt

with:

T
F
F
F
trac
trac

H
- trac

p

T

DC =
T
F
F
F
F
F
F
F
F
F
F
comp
comp

comp

T
trac
trac

T
trac
comp

trac

T

H
-

H

-

-
H

trac
H *comp*

p

p
C

T

éq 5.5.2-5

T
F

comp
ftrac
-
H

ct =
T
F
F
F
F
F
F
F
F
F
F
comp
comp

comp

T
trac
trac

T
trac
comp

trac

T

H
-
H
-
-
trac
comp

p

p

H

H

C

T

éq 5.5.2-6

T
F
F

*- trac **H** comp*

Tc =
T
F
F

F
F
F
F
F
F
F
F
comp
comp

comp

T
trac
trac

T
trac
comp

trac

T

H
-

H
-

-
H

trac
H *comp*

p

p
C

T

éq 5.5.2-7
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T

F
F
F
comp
comp

comp

H
-

p

C

tt =
T
F
F
F
F
F
F
F
F
F
F
comp
comp

comp

T
trac
trac

T
trac
comp

trac

T

H
-

H
-

-
H

trac
***H** comp*

p

p
C

T

éq 5.5.2-8

The expression seems expensive to express in term of products of matrix and calculation. But, when

the operations are made in the order which is appropriate, it is enough to calculate terms little. Moreover, it is
the same terms which intervene on several occasions. It is necessary to calculate the derivative of the criteria by
report/ratio with the constraint, and the plastic multipliers, then sums and products
with the actual values, to finish by the constitution of the matrices and theirs let us be.
Lastly, the resulting matrix with the advantage of being symmetrical, which is appropriate for the standard resolution
with Code_Aster.

5.5.3 Derivative successive of the criteria in traction and compression

5.5.3.1 successive Drifts of the criteria compared to the constraint

The derivative of the isotropic and deviatoric components of the constraints compared to the tensor of constraints are expressed in the following way:
1

1
1
By defining the vector

0 *=*
0
0

0
the derivative of the criteria compared to the tensor of constraints are expressed in the following way:
F
comp
S
has
=
+

0

B eq
2
B
3
F

S
C
trac =
+

0

2D eq

D
3

5.5.3.2 successive Drifts of the criteria compared to the plastic multipliers

Derived from the criterion of compression in the case of a linear curve post-peak:
p
F

4

1

comp
C
p
= -. F.
-
if

p
C
2
C
E

3

C
E
E

F

1

comp = F.
if
p
C

-
C
(C
E)
p
U
E
C
C

Derived from the criterion of compression in the case of a nonlinear curve post-peak:
p
F

4

1

comp
C
p
= -. F.
-
if

p
C
2
C
E

3

C
E
E

p
F

comp
-
= .
2 F

.
if
p
C
2
C
(C
E
U
-
C
E)
p
U
E
C
C

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Derived from the criterion of traction in the case of a linear curve post-peak:
F

F
trac
T
p
U
=
if
p
U
T
T

T
T
Derived from the criterion of traction in the case of an exponential curve post-peak:
a. p
-
T
F
has
trac

U

T
= F

E

p
T U
T
T

5.6
Variables intern model

We assemble here the internal variables stored in each point of Gauss in the implementation of model

Internal number of variable
Feel physical
1
p
: plastic deformation cumulated in compression
C
2
p
: plastic deformation cumulated in traction
T
3
: maximum temperature attack at the point of gauss
4 Indicator
of
plasticity

5.7
Flow chart general of resolution

The flow chart includes/understands the various stages of the resolution, with the treatment of projections
at the tops of the cones of compression and traction in the following way:
at the beginning of algorithm,
one carries out a projection at the top of the cone of traction:
· when the elastic prediction checks the condition of projection a priori in traction,
· when the elastic prediction checks the condition of projection a priori in compression and that the tops of the cones of traction and compression were inverted on the axis hydrostatic,
one carries out a simultaneous projection with the tops of the cones of traction and compression:
· when the elastic prediction checks the condition of projection a priori in compression and that the tops of the cones of traction and compression were inverted on the axis

hydrostatic, and that projection at the top of the cone of traction did not give a solution validate,
one carries out a projection at the top of the cone of compression:
· when the elastic prediction checks the condition of projection a priori in compression and that the tops of the cones of traction and compression were inverted on the axis hydrostatic, and that projection at the top of the cone of traction did not give a solution validate, and that simultaneous projection with the tops of the two cones did not give valid solution,
in medium of algorithm,
one carries out one, two or three standards resolutions with projection on the criterion of compression or
on the criterion of traction or the two criteria at the same time,
and at the end of the algorithm,
when that the standards resolutions with activation of a criterion (traction or compression) or two criteria at the same time did not give a solution,
one carries out a projection at the top of the cone of traction:
· when the elastic prediction checks the condition of projection a posteriori in traction,
· when the elastic prediction checks the condition of projection a posteriori in compression (and that the tops of the cones of traction and compression were inverted on the axis hydrostatic,
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one carries out a simultaneous projection with the tops of the cones of traction and compression:
· when the elastic prediction checks the condition of projection a posteriori in compression

and that the tops of the cones of traction and compression were inverted on the axis hydrostatic, and that projection at the top of the cone of traction did not give a solution validate,
one carries out a projection at the top of the cone of compression:
· when the elastic prediction checks the condition of projection a posteriori in compression and that the tops of the cones of traction and compression were inverted on the axis hydrostatic, and that projection at the top of the cone of traction did not give a solution validate, and that simultaneous projection with the tops of the two cones did not give valid solution.

With the exit of each resolution having converged, one carries out the checks of conformity of the solution
following:

· validity of the solution compared to the second criterion, when the resolution was made with one only criterion. In all the cases, it is enough to check that the two computed criterions with final constraint, are negative or null,
· conformity of the solution: one calculates in the course of resolution the final equivalent constraint. It
arrive sometimes that the solution is beyond the top of the cone which is hammer-hardened, which leads to
an equivalent constraint "negative". Numerically, that results in a final criterion strictly positive. To check the conformity of the solution, it is enough to check that both computed criterions with the final constraint, are negative or null,
· validity of projections at the tops of the cones. It should be checked that after resolution, when one knows the work hardening of the criterion, the slope of the right-hand side connecting the elastic prediction to
projection is lower than the slope of the direction of projection. (condition of projection has posteriori at the top of the cones). In the contrary case, that means that there is a solution with standard resolution.

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Note:

In the case of projections at the tops of the cones, one starts systematically with projection at the top of the cone of traction. If this solution is valid, this one is preserved. In the contrary case, if the criterion of traction is activated, one carries out a resolution with projection with tops of the two cones. If the new solution is valid, that one is preserved. If not, one carry out a resolution with projection at the top of the cone of compression alone.
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Note:

If the conditions of projection at the tops of the cones are activated, one of the three solutions must be valid, but for particularly important elastic jumps, it may be that the resolution does not succeed. The solution is then to carry out a recutting of the step of time.

Note:

Projection at the tops of the two cones simultaneously supposes that the criterion of traction is activated. It may be very well that in the event of permutation of the tops, only the criterion of compression finds itself activated. It is necessary to then make a projection at the top of the cone of compression alone.
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Appendix 1 snap-back with the initial values of the coefficients C and D

We show in this appendix the problem of snap-back met in the simulation of a tensile test follow-up compression simple, if the choice of the coefficients C and D of the criterion of traction corresponds to
a situation where the criterion of traction cuts the axes in a diagram of constraint plane, i.e. it choice of the coefficients [éq 3.3-1] and [éq 3.3-2] leading to a field of reversibility represented on [Figure 3.3-b]:
The assumptions are as follows:

· one takes into account only the criterion of traction,
· the marrow of work hardening is of the type: F = F + H,
T (p
T)
p
T
T
· one notes simply:
p
= so that the curve of work hardening is written:
F
,
T = F
T +

H
T
· the null Poisson's ratio,
· work hardening is negative,
· the condition of applicability is filled: - E < H < 0.

One considers an axial plain test controlled in deformation according to X, as indicated on Figure 5 -

xx
xx
1
xx
P0
0

P1
xx
Time T
2
xx
0
xx

1

xx
xx
2
xx
P2

Appear 5-a

Under the other the imposed directions y and Z, conditions are conditions of null constraints:

.
yy = zz = 0
One starts by imposing a deformation of traction 1
such that there is a plasticization in traction, but
xx
without the limit of traction falling down to 0. It is the point P in the diagram stress-strain. One notes
1
0
the deformation for which the rupture limit in traction appears for the first time. Beyond the point
xx
P, one imposes a decrease of the deformation, which involves an elastic unloading of material,
1
up to value 2

deformation for which one has a plasticization again, but this time under
xx < 0
compressive stress. The object of this appendix is primarily to study the behavior of the model
not retained (that corresponding to the formulas [éq 3.3-1] and [éq 3.3-2]) beyond the point P.
2

A1.1 Calculation of the constraints and the deformations during the loading

Taking into account the assumptions pointed out higher, the invariants of constraint are worth:

I

éq A1.1-1
1 = xx
xx
J =

éq A1.1-2

2
3
eq
=

éq A1.1-3
xx
The plastic flow is calculated by:
p
C +
=
2
&

éq A1.1-4
xx
&
D
3
A1.1.1 Ways 0P0 and P0P1

Taking into account the relations [éq A1.1-1] with [éq A1.1-4], and the fact that along this way them constraints are positive, one finds easily
xx

C + 2 '

H xx +
F
1
3D

T

éq A1.1.1-1
xx = E
1

(c+ 2) 2
E

H +
2

9d

A1.1.2 Way P1P2

By definition, P1P2 is an elastic way of discharge, the P2 point being such as the criterion is there with
new reached
3D
=

éq A1.1.2-1
2
xx
1
C - 2 xx
3D 1
xx
=

+

éq
A1.1.2-2
2
xx
1
C - 2
xx
E

A1.1.3 Beyond the P2 point

One is interested now in the slope of the curve at the P2 point in the reference mark (. More
xx
xx)
precisely one is interested in the slope of this curve for a dissipative solution.
By writing that the material remains plastic beyond the P2 point, i.e. that the state of stress remain on criterion-which is hammer-hardened, one finds:
=
D
3

éq A1.1.3-1
xx

&
h&
C - 2
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In addition, by calculating the increase in flow plastic, and by deferring it in the calculation of the increase in constraint, one finds:
xx

&
&
xx =
+
(C 2)
&

éq A1.1.3-2
E
3D
One can then eliminate & between [éq A1.1.3-1] and [éq A1.1.3-2] and one obtains:

He
xx
& =

éq
A1.1.3-3
&
(
= AND
xx
C - 2) 2
2

H + E
2
9d
This formula gives the slope of the response in the plan (,

xx
xx)
The numerator is always negative since H is negative.

The sign of xx
& = E depends on the sign of the denominator, thus two cases are posed:
2
T
xx 2
&
(C 2) 2

If H < - E
then xx
& = E is positive one has a configuration of snap-back.
2
9d
2
T
xx 2
&

(C 2) 2

If H > - E
then xx
& = E is negative and there is no snap-back.
2
9d
2
T
xx 2
&

A new condition appears to avoid the snap back, condition which we already compare with that evoked, but which related to in fact possible the snap back at the P1 point.
He
Slope at the P1 point in the reference mark (:

E =

,

xx

xx)

T1

H + E

He

Slope at the P2 point in the reference mark (:

E =

.

xx

xx)

2

T

(C 2) 2

H + E

2

9d

(C 2) 2

2

'

F

However

2

= 3 = 3 T

.

2

'

9

D

FC

One can for example express E according to E by eliminating H:

2

T

1

T

E E

1

E

.

T =
T
2
2
3rd + AND -
1 (
2
1 3
)
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For example,
3 2
E
E =
for E
.
T = -
2
T

1
1
(- 3 2

)
As example, for E=32000 Mpa, F '= Mpa
3
and F '=
Mpa
3
,
38
E is found
. Thus
T
601
-
T
C
1
E is very weak compared to E. as illustrated on [A1.1.3-a Figure].
1
T

xx
xx

Appear A1.1.3-a

Thus, a condition implying that there is no snap back at the P2 point would be too restrictive and would lead to practically choose a material not fragile in traction.
For this reason we preferred to modify the expression of the coefficients C and D like indicated at the paragraph [§3.3].

This said, and even if the adopted solution, consisting in modifying the coefficients C and D seems reasonable, the example treated in this appendix shows that a very simple problem can finally be a problem of structure: there is in this example of the equilibrium conditions, they are the conditions:

.
yy = zz = 0
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Law of behavior ENDO_ISOT_BETON

Summary:

This documentation presents the theoretical writing and the numerical integration of the law of behavior
ENDO_ISOT_BETON which describes an asymmetrical local damage mechanism of the concretes, with effect of
restoration of rigidity. In addition to the local model, the nonlocal formulation with regularized deformation is also
supported to control the phenomena of localization.
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1 Introduction Applicability

The law of behavior ENDO_ISOT_BETON aims at modelling the most simply possible one behavior of elastic concrete fragile. It can be seen like an extension of the law ENDO_FRAGILE [R5.03.18] (with which it keeps a proximity of unquestionable formulation) for applications of Civil Engineering.

As for law ENDO_FRAGILE, material is isotropic. Rigidity can decrease, the loss of rigidity measured by an evolving/moving scalar of 0 (healthy material) to 1 (completely damaged material).
On the other hand, contrary to ENDO_FRAGILE, the loss of rigidity distinguishes traction from

compression, to privilege the damage in traction. Moreover this loss of rigidity can to disappear by return in compression, it is a question of the phenomenon of restoration of rigidity with refermeture. It as should be noted as this law of damage aims at describing the rupture of the concrete in traction; it is not thus adapted at all to the description of the nonlinear behavior of the concrete in compression. It thus supposes that the concrete remains in a moderate compactness.

Law ENDO_ISOT_BETON present of softening, which generally involves a loss of ellipticity of the equations of the problem and consequently a localization of the deformations, from where one pathological dependence with the grid. To mitigate this deficiency of the model, a formulation not local must be adopted: for law ENDO_ISOT_BETON, modeling GRAD_EPSI [R5.04.02], based on the regularization of the deformation is usable. In this formulation, it should be noted that only the relations of behavior are faded compared to a traditional local modeling; consequently, the constraints preserve their usual direction.

Lastly, that one activates or not the nonlocal formulation, softening character of the behavior also involve the appearance of instabilities, physics or parasites, which result in snap-backs on the total answer and returns the piloting of the essential loading in statics. piloting of the type PRED_ELAS [R5.03.80] then seems the mode of control of the loading it more adapted.

2 Local law of behavior

2.1 Writing theoretical

If one seeks to take account of the effect of refermeture, it is necessary to pay a great attention to continuity of the constraints according to the deformations (what is an essential condition for a law of behavior in a computation software by finite elements), cf [bib1]. Indeed, if one model this effect in a too simplistic way, the law of behavior is likely great to present a discontinuous answer.

To take account of the refermeture (i.e the transition between traction and compression), it is necessary to start by finely describing what one calls traction and compression, knowing that in traction (resp. compression) the crack will be considered “open” (resp. “closed”). A natural solution is of to place itself in a clean reference mark of deformation. In such a reference mark, the elastic free energy is written (and μ indicating the coefficients of Lamé):

( ) =

() 2
tr
+ µ 2

éq 2.1-1
I
2
I

One can then define:

· a traction or voluminal compression, according to the sign of tr,
· a traction or compression in each clean direction, according to the sign of I.
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According to the rather reasonable principle according to - in a case of traction (“open crack”), one corrects
the elastic energy of a factor of damage; in a case of compression (“closed crack”),

one keeps the expression of elastic energy -, the free energy endommageable is written:

(

2
1
2
1
, D) = (
tr)
- D
D

H (-
tr) +
H (
tr)

+

µ H
H
éq 2.1-2
I
(- I) -
+
(I)

2

1+ D

1
I

+ D

It is noticed that the free energy is continuous with each change of mode. It is even continuously derivable compared to the deformations, since it is nap of derivable functions (function X 2 H (X) is derivable) and the continuity of the partial derivative at the points tr = 0 and = 0

I
is
immediate. One then clarifies the constraints (by knowing that they will be functions everywhere continuous of the deformations). As in elasticity, the clean reference mark of the constraints coincides with
clean reference mark of the deformations, result shown in appendix.

One writes the constraints in the clean reference mark:
1 D
1 D

tr
H

tr
H
tr
2μ H
H
éq
2.1-3
II =
( )
-

(-
)+
( )

+

II
(- II) -
+
(II)

1+ D

1+ D

In this form, the continuity of the constraints with respect to the deformations is clear. The figure opposite
show the constraint
, 1
1 in the plan (
2) with constant damage (case 2D, deformation
plane). The effect of refermeture as well as the continuity of the constraints are quite visible.

Appear 2-a: illustration of continuity
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The thermodynamic force F D associated with the variable interns damage is written:

1

D

+

F = -

=
2
tr
tr

éq
2.1-4
2
() H () + µ 2H
I
(I)
D
(1+ D
)

2
I

It remains to define the evolution of the damage. The diagram selected is that of the standard models generalized. A criterion should be defined, that one takes in the form:

F (Fd) Fd
=
(, D) - K éq
2.1-5

where K defines the threshold of damage. In order to take into account, on the level of the evolution of
the damage, the effect of containment, the threshold K depends on the state of deformation, in the form:

K = K + K
H -

éq
2.1-6
0
(
tr
1
) (
tr)
One compels oneself to remain in the field:

F (Fd) 0 éq 2.1-7

The evolution of the variable of damage is then determined by the conditions of Kuhn-Tucker:

d& = 0 for F < 0

éq 2.1-8
d& 0 for F = 0
Note:

From a formal point of view, the generalized standard materials are characterized by one potential of dissipation function positively homogeneous of degree 1, transformed Legendre-Fenchel of the indicating function of the field of reversibility, which is thus worth here:

(&d) = sup Fd &d = K &d + I

éq
2.1-9
IR+
D
D
(&d)
F/F (F) 0
One will note the presence of an indicating function relating to &
D, which ensures that the damage is
growing.

It still remains to take into account the fact that the damage is raised by 1. From a point of view intuitive, that seems easy. To keep a writing completely compatible with the formalism generalized standard, it is enough to introduce an indicating function of the acceptable field into the expression of the free energy:

2
1

(
D

, D)
= (tr) H (- tr) + - H (T
R) +
2

1 + D

éq
2.1-10
µ2
1 D

H (- + -
I)
H (I) + I
D
I
]-;]
1 ( )

1 +

I
D

The introduction of this indicating function prevents the damage from exceeding 1, indeed, for

D = 1, F D = -
= -, and the damage does not evolve/move any more.
D
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2.2
Taking into account of the withdrawal and the temperature

The law of behavior takes into account a possible withdrawal of dessication, a possible withdrawal endogenous and a possible thermal deformation. The deformation in question in it document being then “elastic strain” ~
HT
rd
Re
= - - - .
On the other hand, the parameters materials in question in the next paragraph are considered like constants (in particular, they cannot depend on the temperature, in the state of current development)

2.3
Identification of the parameters

The parameters of the law of behavior are 4 or 5 (see following paragraphs). They are classically provided in operator DEFI_MATERIAU.

2.3.1 Parameters
rubber bands

They are simplest: it is about the two traditional parameters, Young modulus and coefficient of Poisson, provided under key word ELAS or ELAS_FO of DEFI_MATERIAU.

2.3.2 Parameters
of damage

According to whether the user wants to use the dependence of the threshold with containment or not, 2 should be provided
or 3 parameters to control the law of damage.

2.3.2.1 Use without dependence with containment

In this case, one considers that the parameter K is null. It should be noted that the compactness of 1
concrete must remain moderate so that the law remains valid (compressive stress about

some times the constraint with the peak of traction, in absolute value).
The user must inform, under key word BETON_ECRO_LINE of DEFI_MATERIAU, the values of:

· SYT: limit of simple tensile stress,
· D_SIGM_EPSI: slope of the curve post-peak in traction.

2.3.2.2 Use with dependence with containment

In this case, the dependence with containment makes it possible the concrete to keep a realistic behavior in
compression until the order of magnitude of appearance of nonthe linearity in compression, given by SYC, cf below (classically, a compressive stress of about ten times the constraint
with the peak of traction, in absolute value)

The user must inform, under key word BETON_ECRO_LINE of DEFI_MATERIAU, the values of:

· SYT: limit of simple tensile stress,
· SYC: limit of the simple compressive stress,
· D_SIGM_EPSI: slope of the curve post-peak in traction.
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2.3.2.3 Passage of the values "user" to the values "models"

For information, one obtains the values of, K and possibly K (if the user informed
0
1
SYC) by the following formulas:

E
= -

D _ SIGM _ EPSI

2 1 +

1+ -
2 2
K
SYT

0 = (
)

2nd

1+

(1+) 2
E
K = SYC
-

1
(1+)(1-
2) k0 (1 -
2) (SYC)

2.4 Integration
numerical

Two points are to be regulated before establishing the model: the first relates to the evaluation of the damage; the second consists in calculating the tangent matrix, calculation made a little more delicate
that usually by the passage in a clean reference mark of deformation.
One places oneself here within the framework of the implicit integration of the laws of behavior. Dependence of
criterion according to containment [éq 2.1-6] is taken into account in explicit form, i.e the threshold K is entirely determined by the state of deformation of the preceding step, this to simplify integration model.

2.4.1 Evaluation of the damage

As one will see it, a simple scalar equation makes it possible to obtain the damage, which allows to avoid a recourse to the iterative methods.
One notes D - the damage with the preceding step and D + the evaluation of the damage to the step running to the current iteration which will be the damage with the current step when convergence is attack. Simplest to evaluate the damage of the current iteration is to suppose that one reached the criterion at the current moment, which results in:

1
2
2

F (Fd) = 0
+

µ

éq

2.4.1-1
2
(tr) H (tr) +
H
I (I) K
(
=
1+ D
) 2
I

what gives:

1
1
2

D test =
+

(tr) H (tr) +
2
µ H

1 éq
2.4.1-2
I
(I) -

K

2

I

3 cases arise:

.
D test D -: that wants to say that at the moment running, the criterion is not reached, one concludes from it that

D +
D -
=

,
.
D - D test 1: the criterion is thus reached, the condition of coherence implies D + = D test,
.
D test 1: the material is then ruined in this point, from where D + = 1.
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2.4.2 Calculation of the tangent matrix

The tangent matrix is the sum of two terms, the first expressing the relation constraint/deformation with constant damage, the second being resulting from the condition F = 0. In effect, one can write:

ij ij
ij D
=
+

éq
2.4.2-1

D
kl
kl
you

=

D C
kl F =0

If the user asks for calculation with tangent matrix (cf documentation of STAT_NON_LINE, [U4.51.03]), the law of behavior provides the expression given by [éq 2.4.2-1]. On the other hand, if the user asks for calculation with the matrix of discharge, the law of behavior provides the matrix secant, i.e. the first term of the member of right-hand side of [éq 2.4.2-1].

2.4.2.1 Stamps tangent with constant damage

As we underlined previously, the calculation of the tangent matrix is a little delicate fact of the writing of the model in the clean reference mark of deformation. Thus, one knows easily stamp tangent with constant damage in the clean reference mark of deformation, but what one seek is this same tangent matrix in the total reference mark.

If the damage does not evolve/move, in the clean reference mark of deformation, the matrix sought a simple relation of degraded elasticity expresses:

~
1 D
1 D
I

-
~
= H (- tr) +
H (tr)

+

2μ H
H

éq
2.4.2.1-1
ij
(- J) -
+
(J)

1
D

1
D

J

you

+

+

D =c

It is now necessary to express the passage of the total reference mark to the clean reference mark of deformations, at least
if the eigenvalues of deformation are different. The tangent matrix not being
necessary that the algorithms of ***numerical*** *resolution (diagram of Newton), one will allow oneself, at the time of*
calculation of the tangent matrix (and ***only*** *in this case) to disturb the possible ones numerically identical eigenvalues (in order to make them distinct). It will be noticed in particular that that allows, null damage, to find the matrix of elastic rigidity.*

One notes with a tilde the tensors in the clean reference mark of deformation (which, one points out it, is too
the clean reference mark of constraints). By definition, by noting U the clean vector associated the i-ème I
eigenvalue, the matrix basic change Q = (U U U, one a:
1
2
3)

~ T
= Q Q
= Q Q ~ + Q Q ~ + Q Q ~
ij
im
jm
m
im
jm
m
im
jm
m

If the eigenvalues of deformation are distinct, evolution of the clean vectors and eigenvalues is given by (cf previously [§2]):

~&jk
U& U =
for J K ***éq***
2.4.2.1-2
J
K
~ - ~
J
K
&~ = &~ for J K ***éq***
2.4.2.1-3
I
II
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One deduces Q easily from it:

~
~
Q =
jk
U
*Q **éq***
2.4.2.1-4
ij
~
~ (K) =
jk
I
~
~ ik

K J
J -
K
K J
J -
K

While using then (the last expression being used only to obtain a clearly symmetrical matrix)
:

=
1
~
Q Q =
Q Q + Q Q
ij
ki
lj kl

(ki lj Li kj) kl
2

One thus obtains:
~

m
= Q Q

~ + *Q Q* ~ + *Q Q* ~
ij
im
jm
~
N
im
jm
m
im
jm
m

,

m N
N
m
~

1
Q Q Q Q
m
in
km
ln
jm
= Q Q Q Q

+

ln

~

im
jm
kN
~
kl
~
~
m
kl

-
, , ,

2 ,

m N K L
N
K L
m
N
Nm
1
Q Q Q Qln
1
Q Q Q Q
1
Q Q Q Q
im
jn
km
in
lm
kN
jm
im
jn
lm
kN
+
~ +
~ +
~

~
~
m
kl
~
~
m
kl
~
~
m
kl
2
-
-
-
,

2 ,

2 ,

K L
m
N
K L
m
N
K L
m
N
Nm
Nm
Nm
éq 2.4.2.1 - 5

The tangent matrix with constant damage is thus written:

~

1

Q Q
Q Q
Q Q
Q Q
ij

m
(km ln + lm kN) (in jm + jn im)
With
Q Q Q Q
~

ijkl =
= im jm kN ln ~
+

~
~

2

kl
m, N

N
m
you
m, N
N -
D =c

m

Nm
éq 2.4.2.1 - 6

2.4.2.2 Term of the tangent matrix due to the evolution of the damage

The expression to be evaluated is written:

ij D
éq
2.4.2.2-1

D kl F =0

One writes the equation [éq 2.4.1-1] in the form:

1 +
W = K
éq
2.4.2.2-2
2 [( )]
(1+d)

with: W () = (tr) 2 H (tr) + µ 2 H.
I
(I)
2
I
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While differentiating this expression, it comes:

- 2 (1+) W

D
el

éq
2.4.2.2-3
3
( ) + 1+

= 0
(1+ D)
(1+ D) 2
W
with: el

=

One uses then the following equality:

1 el
=
+
éq
2.4.2.2-4
D (1+ D) 2

One concludes:

ij
1

=
+
el el
ij
éq
2.4.2.2-5

kl
2 1 + D W
kl

F =0
( ) 0

2.4.3 Case of completely damaged material

In the case of the completely damaged material, D = 1, the rigidity of the material point can to cancel itself. That poses problem for the constraint by no means; on the other hand, that can involve
null pivots in the matrix of rigidity. To mitigate this difficulty, one allows oneself to define a rigidity minimal, for the tangent matrix or the matrix of discharge. This minimal rigidity does not affect value of the damage (which can reach 1) or the constraint (which can reach 0).

To preserve a reasonable conditioning of the matrix of rigidity, minimal rigidity is taken with 5
10 - rigidity initale. An indicator specifies the behavior during the step of current time:

.
= 0: no evolution of the damage during the step,
· = 1: evolution of the damage during the step,
.
= 2: damage saturated D = 1.

2.5
Description of the internal variables

The model has two internal variables:

· VI (1): damage D,
· VI (2): indicator.
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3
Formulation with regularized deformation

3.1 Formulation

The approach with regularized deformation [R5.04.02] also makes it possible it to control the phenomena of
localization and for this reason seems an alternative to the formulation with gradient
of damage. But at the difference in the latter, this formulation has the advantage of
to resort to the standard algorithms for the nonlinear problems. Indeed, the only difference by report/ratio with the local law of behavior lies in the data of two deformations instead of one, local deformation which intervenes in the forced relation deformation and the deformation regularized which controls the evolution of the damage. This one results from the local deformation by resolution of the system of partial derivative equations according to:

- 2
B
L = 0
structure

in

éq
3.1-1

N = 0
on
normal

of

edge

N

where the characteristic length L is again indicated under the key word
B
LONG_CARA of
DEFI_MATERIAU. Finally, the relation of behavior is written in the following way, the equation (2-3) remains identical, while the equation [éq 2.1-5] takes into account the regularized deformation:

1 D
1 D

tr
H

tr
H
tr
2μ H
H
éq
3.1-2
II =
( )
-

(-
)+
( )

+

II
(- II) -
+
(II)

1+ D

1+ D

F (F D) = F D (, D) - K

éq 3.1-3

3.2
Integration of the law of behavior

One of the advanced advantages for the nonlocal formulation with regularized deformation is the little of
modifications which it involves in the construction of the law of behavior. Indeed, the integration of internal variables is completely controlled by the regularized deformation.

The method of integration is exactly that described in the paragraph [§2.4.1], in condition of to replace the deformation by the deformation regularized in the equations.

For the calculation of the tangent matrix, the expressions are the same ones as those given to paragraph [§2.4.2], certain expressions of the deformation are to be replaced by the deformation regularized (when the deformation concerns the criterion), while others do not change (when deformation raises of the forced relation deformation).

3.3 Variables
interns

They are the same internal variables as for the local law:

· VI (1) damage D,
· VI (2) indicating.
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4
Piloting by elastic prediction

The piloting of the type PRED_ELAS controls the intensity of the loading to satisfy some equation related to the value of the function threshold el
F during the elastic test. Consequently, only them
points where the damage is not saturated will be taken into account. The algorithm which deals with this mode of piloting, cf [R5.03.80], requires the resolution of each one of these points of Gauss of the following scalar equation in which
is a data and the unknown factor:

~el
F () =

éq 4-1
~
The function el
F provides the value of the function threshold during an elastic test when the field of displacement breaks up in the following way according to the scalar parameter:

U = U + U

éq 4-2
0
1

where U and U are given. Thanks to the linearity in small deformations of the operators deformation
0
1
(calculation of the deformations starting from displacements) and regularized deformation, one also obtains
following decompositions:

= 0 + 1
and
= 0 + 1
éq
4-3

The function el
F presenting the good property to be convex, the equation [éq 4-1] presents zero, one or two solutions, which are required as follows:

· Détermination of the number of solutions per study at the boundaries ± and possibly (if value at the two boundaries is each time positive) determination if el
F presents a minimum
negative;
· Détermination of a framing of each solution starting from the preceding study
· Détermination of the solution (for a convex function knowing the framing, this research is simple and fast)

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Appendix 1 Demonstration of the clean reference mark of constraint

The term traces some in energy does not pose a problem: it is invariant by any change of reference mark.

2
1 D
Remain the term out of H -
+ - H
I
(I)
(I).

1 + D

I

Notation: one writes with an index (for example) the i-ème eigenvalue of a tensor which is written (while clarifying
I
its two indices).
kl

· If the eigenvalues of the deformation are all distinct, one shows whereas & =
I
&ii, with
&kl components of & in the fixed reference mark coinciding with the clean reference mark of deformation with
the moment considered (in this reference mark one thus has =).
kl
K
kl
Indeed, let us write the deformations in the form:

= U U
I
I I
I
While differentiating this expression, it comes:

& = & U U
U
U
U
U
I
I
I

+ &
I
I
I + I
I & I
I
By using the fact that the clean vectors are orthonormés:

U U = U& U + U U& = 0
I
J
ij
I
J
I
J

one obtains the variations of the eigenvalues and the clean vectors:

& =
jk

=

I
&ii and &
&
U U
J
K
for J
K
-
J
K
This is obviously valid only if the eigenvalues are distinct (as one can see it
clearly on the expression of the variations of the clean vectors). That comes owing to the fact that them
clean vectors are not continuous functions of the elements of the matrix.

· If two eigenvalues of deformations are equal (and apart from the case very
private individual where they are also null), they are either positive, or negative. Let us take the case where they are positive (the other case lends itself to a demonstration in any similar point). Energy 3
concerning these two eigenvalues is written then: 2
(the two equal eigenvalues are
I
i=2
considered to have indices 2 and 3). By differentiating this expression, one obtains:

23
3
D

D by noting the common eigenvalue.
I
I = 2
I
i=2
i=2
By invariance of the trace of a matrix, here the restriction of the deformation on the clean plan considered, one obtains:

3
3
D
D, whatever the evolution which underwent the clean reference mark at this time.
I =
II
i=2
i=2

For the remaining eigenvalue (distinct from both others and index 1 with the selected notations), one a: D = D.
1
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By gathering these expressions, one obtains:

D 3 2
H
D H
D
H
H D
I

(I)
3
3

= (2
2
1
( )

+
2
1
I
(I)

= II (II)

II
i=1

i=2

i=1

In conclusion, that the eigenvalues are distinct or not, one obtains:

D
2 H () = 2

H
II
(II) D
I
I
II with the adopted notations.

I
I

This reasoning spreads easily with the case of three equal eigenvalues.

The differential of energy with constant damage is written then:

1 D

D (, D)
=
- + -
+
D =cte
(tr) D (tr) H (tr)
H (tr)

1 + D

1 D
2µ

D H - + -

II
II
(II)
H (II)

1 + D

I

On this expression, one observes well that the clean reference mark of deformation is also reference mark
clean of constraint.

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Relation of behavior of Bazant for the intrinsic creep of desiccation of the concrete

Summary:

Contrary to the clean creep which is the share of the creep measured on a test-tube protected from external desiccation, the creep of desiccation is calculated on a mechanically charged test-tube and subjected to drying simultaneously.
This document presents the model of intrinsic creep of desiccation of Bazant (1985). One details there also the writing and digital processing of the model.

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matters

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1 Introduction

One points out the deformations differed from a concrete structure to locate the share of the deformation
calculated in this document:

· at the youth:

- endogenous withdrawal (1j - 1 year),
caused by a reaction of thermohydration
- thermal withdrawal (1h 1j).

· in the medium term without load: withdrawal of desiccation (qq m qq year) according to dimensions' of
structure caused by the drying which results in an evaporation of part of water not
used in the process of hydration.

· in the long run under load:

- clean creep (without exchange of moisture with outside thus without drying),
- the creep of desiccation (with drying which assigns the behavior of the concrete to the scale microscopic, which is translated the macroscopic scale by creep of desiccation).

The differed deformations constitute a significant part of the deformations which appear in concrete during its life. Among its differed deformations, withdrawals endogenous and thermal with short
term, withdrawal of desiccation caused by medium-term drying. One quotes also them deformations differed under long-term load like clean creep and creep from desiccation.

The model presented here relates to the modeling of the deformation differed associated creep from intrinsic desiccation. The creep of desiccation in complement to clean creep is the share of total creep directly related to the water departure affecting the concrete which undergoes a mechanical loading on the one hand and drying on the other hand. In other words, the deformation which one measures in one test-tube which dries is directly related to the drying under constraints which carries not of creep of desiccation.

The model suggested here is that of Bazant (1985) and adopted by L. Granger in its thesis (1995). It is a law of the viscoelastic type linear which holds in account of the effect of the variation of the hygroscopy. One presents the details of the numerical integration of this law in Code_Aster.

In Code_Aster, this model is used under the name of BAZANT_FD.
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2 Partition of the deformation

Into small deformations, the increment of the total deflection is broken up into several terms relating to the mechanisms considered. If one holds account in the partition of the increments of deformations, thermics, associated the thermal, endogenous withdrawal and with the withdrawal of desiccation, then:

E
HT
Re

Re
fl

=
+

+
***end* +**
***dess* +**

éq
2-1
The increment of the deformation of creep
fl
breaks up into two components, corresponding
with clean creep and the creep of desiccation:

fl
fl
fl

=
***Pr* +**
dess éq 2-2

The creep of desiccation
fl

dess as for him, breaks up into two intrinsic and structural part:

fl
fl
fl

=

+

dess
dess _ int
dess _ struc

It is agreed that the structural deformation is not a component of deformation in oneself, thus in this document the only component of the creep of desiccation relates to the part intrinsic:

fl
fl

=

dess
dess _ int

éq 2-3

with:
E =
H
HT
= (T - Tref) I
Re

= - I
end
: hydration
Re

= - Ci
dess
C: water concentration

H: stamp elastic, thermal dilation, coefficients related on the withdrawals endogenous and the withdrawal
of desiccation are data material.

Here, one wants to model
fl

dess.

Note:

This partition of the deformations is purely numerical. For the calculation of each one of these components, the experimenters consider a combination different from components of deformation (See [bib1] and [bib2]).

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3 Law
constitutive

Bazant et al. (1985) suggest that the drying and the application of a loading in compression at the same time are responsible for the microphone-diffusion of the molecules between the macro-pores and them micropores. The microphone-diffusion of the water molecules would support the rupture of the connections between particles of freezing inducing the deformation of creep of desiccation. It is one of the phenomena physicochemical most complicated to model resulting from a coupling between the constraint, it clean creep and drying. They propose the following equation to take into account the creep of intrinsic desiccation at the elementary level:

fl
&
= &

éq 3-1
dess
H
with:
fl

,
dess deformation of the intrinsic creep which evolves/moves in time,
, a parameter material [1
Pa],
,
H the relative humidity which evolves/moves in time, fact of the case of evolution.

This expression is similar to the rheological model of the shock absorber:

fl

éq 3-2
dess
&
=
Note:

By preoccupation with a lightening of notations, one uses fl
to replace fl
dess in the continuation of
document.

4 Discretization

The evolution of the relative humidity is approached by a function closely connected per pieces (Benboudjema and
Al, 2001d). This discretization according to (Bazant, 1982) makes it possible to increase the precision

of calculations
numerical in a considerable way compared to an approximation by stage (Heaviside function)
especially if the size of the step of time is important:

,
H (T)
(T - T
T
T T
N)
[N n+1]
= N
H +
N
H
with

éq
4-1
tn
N
H = N
H +1 - N
H

according to the equation [éq 3-1], one can write:

fl = fl
N
N +
+
hn+ - H.
N tn +
T with
1
1
(
)
[] 1
,
0
éq
4-2

For = 1 2, semi-implicit diagram which makes it possible to have a better quadratic convergence of solution, one obtains:

fl
fl

fl
fl
(N + N 1+)

=

or

N
N +
N
H
- H.
N N +

=
N
N +
N
H
- H.
1
+
1
+
éq
4-3

2
1
+
1
+

N
2

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5
Integration of the law of behavior

As in this document one is interested in integration of the intrinsic creep of desiccation, one goes to consider for creep the only component
fl

dess_int but to simplify the writing one goes
to call
fl

. One poses in the same way:

With
HT
Re
Re

=

+
end +
dess ***éq***
5-1

By employing the following notations: Has, A, A
for quantity A evaluated at the known moment tn, with
the moment tn 1
+ and its increment T
, respectively.

It is a question of expressing the constraint at time + according to the constraint at time and of the increment
of deformation at time -. One seeks initially the expression of the deviatoric component and then the expression of the hydrostatic component of the constraint.

5.1.1 Part
deviatoric

One seeks a relation between the deviatoric constraint ~ and the variation of the deformation deviatoric ~
at time +:

The constraint at time + is written:

~
~e
2μ ~-
~e
= 2μ =
+ 2μ

éq
5.1.1-1
2μ -
The elastic prediction of the deviatoric constraint is written:
E
2
~
μ
=
~ - + μ ~
2
éq
5.1.1-2
2μ -
Like the component
With

do not have a deviatoric part, one can write:

2
~
μ ~-
~
~ fl
=
+ 2μ - 2μ ***éq***
5.1.1-3
2μ -
While using [éq 4-3], one obtains:
2
~
μ

=
~ - + μ ~
H
H
2 - 2μ
~ - - 2μ
~ ***éq 5.1.1-4***
2μ -
2
2
1 4
4 2 4
4 3
~e
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from where,

2μ

~
H
-
~
~-
E

+ 2
μ - 2μ

~
H ~
-
-
-
2μ

~
= 2μ
2
=
2
éq
5.1.1-5

H

H

1 + 2μ

1 + 2μ

2

2

5.1.2 Part
hydrostatic

One seeks a relation between tr () and tr (
) at time +:

The constraint at time + is written:

3
tr () = 3
(E) K
Ktr
=
tr (- 3
éq
5.1.2-1
-
) + Ktr (E
)
3K
The elastic prediction of the hydrostatic constraint is:

() = 3K
tr E
tr (- 3Ktr

éq
5.1.2-2
-
)+
(
)
3K
from where,
3
tr () = 3
(E) K
Ktr

=
tr (- 3
3
3
éq
5.1.2-3
-
) + Ktr (
) - Ktr (fl

) - Ktr (A

)
3K
According to [éq 4-3], one can express the hydrostatic part of
fl

:

3

tr (
K
H
H
) =
tr (-
éq 5.1.2-4
-
) +3Ktr () - 3Ktr (A
) - 3K
tr (-) - 3K
tr ()
3K
2
2

from where,

3K tr (
H
H
-

3Ktr
3Ktr
With

3K
tr
tr E

3Ktr
With

3K
tr
-
)+
( )-
( )

-
(-) ( )- ( )

-
*(-) **éq 5.1.2-5***
tr () = 3K
2
=
2

H

H

1 + 3K

1 + 3K

2

2

One thus deduces the total constraint from it by combining the two components deviatoric and

hydrostatic at time +:

tr
~
( )
= +

éq
5.1.2-6
ij
ij
3
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6 Matrix
tangent

6.1
Phase of prediction

The option used is RIGI_MECA_TANG, the tangent operator calculated in each point of Gauss is known as
in speed:

&ij = D
ijkl &kl,

in this case, Dijkl is a viscoelastic operator calculated starting from the not discretized equations.

6.2
Reactualization of the tangent matrix

The option used is FULL_MECA, when one reactualizes the tangent matrix with each iteration in updating the internal constraints and variables:

D ij = ijkl
With dkl,

in this case, Aijkl is a viscoelastic operator calculated starting from the discretized equations implicitly.

~ 1 tr
() D
=
+
I

éq
6.2-1

3

~
~ 1 tr
() tr
() D
=
+

éq
6.2-2

~

3 tr
( )
I

~

1 tr

ij
ij
(ij)
1

=
-
= -
ik
jl
ij
kl

3

3
kl
kl
kl

tr
(ij)

=
ij
kl

kl

According to [éq 5.1.1-5]:

~

H
1 + 2

μ
= 2μ ***éq***
6.2-3
~

2

According to [éq 5.1.2-5]:

tr
( )
H

1 + 3

= 3
éq
6.2-4
tr
( )
K
K
2

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Writing of speed:
~
H

~
1 + 2µ
= 2µ.

éq
6.2-5
T

2
T

tr
( )
H

tr
( )
1 + 3K

= 3K.

éq
6.2-6
T

2
T

Thus while returning to [éq 6.2-2], one can deduce the writing of speed:

2µ
D 1 D
D
1
K
=
3
I
éq
6.2-7
4 -
I I +
(D D
I I)

H

3
3
H

1 + 2µ
1 + K

3

2

2

Linearization:

H
~
1 + 2

μ
= 2μ ~
.

éq
6.2-8
2

tr () 1+ 3 H
K
= 3K.tr () ***éq***
6.2-9

2

Like
H
is independent of the constraint, it is the same writing that one finds afterwards linearization from where the form of the tangent matrix:

K
4μ
K
2μ
K
2μ

+
-
-
0
0
0

H

H
H

$$H$$
$$H$$

$$H$$
$$1+3$$
$$K$$
$$3\ 1+2\mu$$
$$1+3$$
$$K$$
$$3\ 1+2\mu$$
$$1+3$$
$$K$$
$$3\ 1+2\mu$$

$$2$$

$$2$$
$$2$$

$$2$$
$$2$$

$$2$$

$$K$$
$$2\mu$$
$$K$$
$$4\mu$$
$$K$$
$$2\mu$$

-
+
-
0
0
0

H

H
H

H
H

H

1+ 3
K
3 1+ 2μ
1 + 3
K
3 1 + 2μ
1+

3
K

3 1+ 2μ

2
2
2
2
2

2

K
μ
K
μ
K
μ

11
2
2
4
11

-
-
+
0
0
0

H
H
H
H
H
H

22

1 + 3
K
3 1+ 2μ
1+ 3
K
3 1 + 2μ
1+ 3
K
22

3 1+ 2μ

2
2
2
2
2
2

33

$$=$$

$$\frac{33}{2}$$

$$\frac{2\mu}{}$$

$$\frac{2}{12}$$

$$\frac{0}{0}$$

$$\frac{0}{0}$$

$$\frac{0}{12}$$

$$\frac{2}{H}$$

$$\frac{2}{23}$$

$$1 + 2\mu$$

$$\frac{23}{2}$$

$$\frac{2}{2}$$

$$\frac{2}{31}$$

31

2μ

0
0
0
0
0

H

1+ 2μ

2

2μ

0
0
0
0
0

H

1+ 2μ

2
1
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
4
4
4

4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
3

In this case, the tangent operator is the same one for RIGI_MECA_TANG and FULL_MECA:
With = D. It has a writing similar to the elastic matrix with dependent coefficients
ijkl
ijkl
of H
and of.
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6.3 Variables of state

The variables of state are:

.
: tensor of the constraints,
.
: tensor of the deformations,
.
C: water concentration.

The internal variables of this law of behavior is the value of the hygroscopy at the current moment.

7 Implementation of a calculation of creep of desiccation

In a way similar to the clean model of creep of Granger, GRANGER_FP, established already in Code_Aster, this law constitutive depends on H, the relative humidity, which evolves/moves in time.

1)
To make a mechanical calculation of creep of desiccation with this law, it is necessary to have relative humidity. The user can confront himself with two situations:

Has
The user knows moisture H or the water content C of the structure at various moments, initial and final in the majority of the cases.

In this case, it can with CREA_CHAM and key word AFFE to affect the field of temperature "TEMP" with the structure. It must repeat order CREA_CHAM with each desired moment. Then, with order CREA_RESU, creates a structure of data result starting from the fields already defined in the corresponding moments.

B
The user does not know the distribution of the field of moisture of the structure.

In this case, it must carry out a calculation of drying. The field of drying is given thanks to the order THER_NON_LINE, but which is comparable in term of variable with a temperature (standard TEMP) of field NOEU_TEMP_R.

Once defined (A) or calculated (B) a field of temperature "compared to a field of drying", it is necessary to begin mechanical calculation:

2)
Initially by creating the loading corresponding under AFFE_CHAR_MECA and the key word SECH_CALCULEE. On the level of STAT_NON_LINE, which one put in SECH_CALCULEE is regarded from now on as a field of drying with variable "SECH".

However the law is written according to the hygroscopy H and not according to the water content C, They is the same the case of the clean law of creep of Granger. One proceeds in the same way, it is necessary:

3)
To define the curve sorption-desorption which allows the passage of the water content C the hygroscopy h. This curve must be indicated by the user with DEFI_FONCTION and NOM_PARA = SECH.

4) To define
under
DEFI_MATERIAU, the key word BAZANT_FD in which it is necessary to give like words obligatory keys: LAM_VISC which is a parameter material and FONC_DESORP which is one function defined before and which connects H the hygroscopy to C the water content.

5)
Mechanical calculation is carried out thanks to order STAT_NON_LINE with like relation in the key word COMP_INCR = _F (RELATION = "BAZANT_FD").

One of the evolutions to be envisaged is the use in RELATION_KIT of the two laws of creep of Granger:
GRANGER_FP and BAZANT_FD.
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Organization (S): EDF-R & D /MMC

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R7.01.06 document

Relation of behavior UMLV for creep clean of the concrete

Summary:

This document presents the clean model of creep UMLV, which is a way of modelling the clean creep

of
concrete.
One also details there the writing and the digital processing of the model.

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1 Introduction

Within the framework of the studies of the long-term behavior of structures out of concrete, a share dominating of the deformations measured on structure relates to the differed deformations which appear in the concrete during its life. They comprise the withdrawals at the youth, the withdrawal of desiccation, clean creep and the creep of desiccation.

The model presented here is dedicated to the modeling of the differed deformation associated creep *clean. Clean creep is, in complement of the creep of desiccation, the share of creep of the concrete that one would observe during a test without exchange of water with outside. In experiments concrete in clean creep presents a growing old viscous behavior. The deformation of creep observed is proportional to the constraint of loading, depends on the temperature and the hygroscopy.*

Models of creep of the existing concretes (e.g.: model Granger to see [bib4] and [R7.01.01]) have summer developed in optics to predict the longitudinal deflections of creep under uniaxial constraints. The generalization of these models, in order to take into account a state of multiaxial constraints, is done then via a Poisson's ratio of creep arbitrary, constant and equal, or close, of the elastic Poisson's ratio. However, determination a posteriori of Poisson's ratio of effective creep shows his dependence with respect to the way of loading. In addition, concrete of certain works of Park EDF, the such containments of nuclear engine, is subjected in a state of biaxial stresses. This report led to the setting to not law of deformations of clean creep UMLV (University of Marne-the-Valley, partner in the development of this model) for which the Poisson's ratio of creep is one direct consequence of the calculation of the principal deformations.

In Code_Aster, the model is used under the name of BETON_UMLV_FP.

2 Assumptions

Assumption 1 (H.P.P.)
The law is written within the framework of the small disturbances.

Assumption 2 (partition of the deformations)
Into small deformations, the tensor of the total deflections is broken up into several terms relating to the processes considered. Being the description of the various mechanisms of deformations differed from the concrete, it is admitted that the total deflection is written:

E
FP
fd
Re
rd
HT
= {

+ {
+ {

+ {

+ {

+ {

éq
2-1
N
déformatio
creep
of

creep
withdrawal
of
withdrawal
N
déformatio

rubber band
clean
one
dessiccati
endogenous
one
dessiccati
thermics

Within the framework of this documentation, one will limit oneself to the description of clean creep. With ends of
simplification of writing, the exhibitor F will indicate the clean deformation of creep so that [éq 2-1] reduces to:
E
F
= +

éq 2-2
N.B.:

In the continuation the term "creep" will indicate clean creep exclusively.
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Assumption 3 (decomposition of the components of creep)
In a general way, clean creep can be modelled by combining the elastic behavior of solid and the viscous behavior of the fluid. For the law presented, creep is described like combination of the elastic behavior of the hydrates and the aggregates and the viscous behavior water.
In the case of law UMLV, one carries out the assumption that creep can be broken up into one process uncoupling a spherical part and a deviatoric part. The tensor of the deformations total of creep is written then:
F
fs
fd
=
1
1 +

with fs
F
= tr
éq
2-3
{
{
3
part
part
spherical
ue
déviatoriq
The tensor of the constraints can be developed according to a similar form:

S
D
= 1 +

éq 2-4
{

{
part
part
spherical
ue
déviatoriq

The law of creep UMLV supposes a total decoupling between the spherical components and deviatoric: the deformations induced by the spherical constraints are purely spherical and the deformations induced by the deviatoric constraints are purely deviatoric. To hold count effect of internal moisture, the constraints are multiplied by internal relative moisture:

S
= H F (S
)
D
= H F (D
)
and

éq 2-5

Or H indicates internal relative moisture.
The condition [éq 2-5] makes it possible to check a posteriori that the deformations of clean creep are proportional to the relative humidity.

3
Description of the model [bib1]

3.1
Description of the spherical part

The spherical constraints are at the origin of the migration of the water adsorbed with the interfaces between
hydrates on the level of the macroporosity and absorptive within microporosity in porosity capillary. Diffusion of water interlamellaire of the pores of hydrates towards capillary porosity be carried out in an irreversible way. The total spherical deformation of creep is thus written like summon of a reversible part and an irreversible part:

fs
fs
fs
=
+
{
R
éq
3.1-1
{
I
part
part
reversible

irréversib
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The process of deformation spherical of creep is controlled by the system of coupled equations according to (equations [éq 3.1-2] and [éq 3.1-3]):

fs
& = 1 [
S
S
fs
H - Kr R]

fs
- I
éq
3.1-2
S
&
R
where S
Kr indicates rigidity connect associated with the skeleton formed by blocks with hydrates on the scale mesoscopic;
and S
R viscosity connect associated with the mechanism with diffusion within capillary porosity.

fs
= 1
&

I
[sk fs -
R
(sk + S
R
I
K) fs
I] - [
S
H
- S
fs
Kr R] +

éq
3.1-3
if

where
S
ki indicates rigidity connect intrinsically associated with the hydrates on the scale microscopic;
and S

I viscosity connects associated with the interfoliaceous mechanism of diffusion.
+
+
1
In [éq 3.1-3], hooks
appoint the operator of Mac Cauley: X
= (X + X)
2

Appear 3.1-a: Phenomenologic model associated the spherical part of clean creep

3.2
Description of the deviatoric part

The deviatoric constraints are at the origin of a mechanism of slip (or mechanism of quasi dislocation) of the layers of HSC in nano-porosity. Under deviatoric constraint, creep
be carried out with constant volume. In addition, the law of creep UMLV supposes the isotropy of creep
deviatoric. Phénoménologiquement, the mechanism of slip comprises a contribution reversible viscoelastic of the water strongly adsorbed with the layers of HSC and a contribution irreversible viscous of interstitial water:

fd
fd
fd

=

+

éq
3.2-1
{
{
R
{
I
contribution
N
déformatio
one
contributi
water

ue
déviatoriq
water
absobée
total
free
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The principal component jème of the total deviatoric deformation is governed by the equations [éq 3.2-2] and [éq 3.2-3]:
D
D, J
D
D, J
D, J
& + K = H éq
3.2-2
R
R
R
R
where D
Kr indicates rigidity associated with the capacity with water adsorbed to transmit loads (load bearing toilets);
and D
R viscosity associated with the water adsorbed by the layers with hydrates.
D D, J

D J
I
& =
I
H
,

éq 3.2-3
where D
I indicates the viscosity of interstitial water.

Appear 3.2-a: Phenomenologic model associated the deviatoric part of clean creep

4
Discretization of the equations constitutive of the model

4.1
Discretization of the equations constitutive of spherical creep

One carries out a linearization with the first order of the product of the constraints and moisture:

T - tn
T () H T () N N
H +
(N nh +n nh)
éq
4.1-1
T
N
After discretization of the constraints and relative humidity by functions closely connected, the deformation
spherical of clean creep is discretized by the following equation:

fs

= S
+ S has
B S
+ S

C S
F
S
S
S
N
N
N
N
N N 1
+
(tr) =3a +b tr +c
N
N
N tr

éq 4.1-2
N
N 1
+
where S
and S

N
N 1
+ are the spherical constraints at the beginning and the end of the step of current time.
It is necessary to distinguish two cases from figures according to whether the unrecoverable deformation must be taken into account or
not.

1st case: the deformation of spherical creep irreversible is not taken into account, the equation [éq 4.1-2] can put itself in the form (simple chain of Kelvin):
S
fs
& (T) + K S fs
(T) = H (T) S
(T)
R R
R R
R

éq
4.1-3
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After discretization, the preceding equation can be put in the form:

fs
S
S
S
S
S
R, N = rear, N + R
B, N N + Cr, N N 1
+
éq
4.1-4
With:

T
S
has
= exp
1
R, N

-
N

- fs

D
R, N

R

1

2

T

2

T
S

S

S

S

S -

R
R
N
R
R
N

B =
1 H
H
exp
1 H
H
éq

4.1-5
R, N
D -
+
+
N
n+1
-
+
-
-
N
n+1
S

K
T
T

T
T
R

N

N

R

N

N

1 S

T
S
- T

S
C
=
exp
H
H
R, N
R
-
N
- R
N
N
N

D

S

K
T

T
R

N

R

N

The unrecoverable deformation, as for it, does not vary:

S
has
= 0
fs
I, N

= 0
S
B

éq 4.1-6
I, N

= 0
I, N
S
C = 0
I, N
2nd case: the deformation of spherical creep irreversible must be taken into account.
Using the linearization [éq 4.1-1],], the system of coupled equations is written:

1
T - T

fs
& T () + fs

2 & T () =
H
H

H
K
T
( )
R
I
S

+
N
N
N
(+
N
N
N
N) -
S
fs
R
R

T
R
N

+
éq 4.1-7

1
T - T
fs
& T () = -
-
S
fs
2k
T
() + S fs
K
T

(
)
H +
N
H

H
I
R
R
I
I
N
N
(+
N
N
N
N)

S

T
I
N

This system can be put in the form:

&
&

fs
fs T ()
fs
S
fs
S
S
S
S
T
() has

T
() has
T
() B
C T T
& T () = R
A:
T
() B
T T C

fs
=
fs
+ +
R
(- N)

=
+
+
+
R
rr
R
laughed I
R
R (- N)

& T ()
& T () has T () has T () B C T T
I

fs
= S fs
+ S S
+ S + S
I
ir
R
II I
I
I (- N)
éq 4.1-8

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With, B and C are defined as follows:

S
S
S
K
K
K
4
2
S
S
- R -
R
I
S
S
S
has
has

With = rr
ir

S
S =
R
I
I
has
has

S
S
K
K
ir
II

I
2
- I
S
S

I
I
1
2
S

+
S
S
B

B = R
H
éq
4.1-9
S =
R
I
N
N

B
1
I

-

S

I

1
2
S

+
S
S
C H + H

C = rs =
N
N
N
N R
I
C

T
1
I
N

-

S

I

The preceding system of equations can be uncoupled and solved in space of the clean vectors. The system of equations is written indeed:

*
*
*
*
*
*
& (T) = (T) + B + C

éq
4.1-10
K
K
K
K
K (T - T N)

&

with

& = 1
P
**** =***
-1

&
&2

Thus, in the space of the clean vectors, the model of creep becomes equivalent to a double chain of Kelvin. It is necessary to know the solution of the homogeneous equation (without second member),
as well as a particular solution in order to solve the preceding differential equation. The solution homogeneous of each of the two equations is as follows:

*
T
K
T () = µ E
éq
4.1-11
K

K

where μ is a parameter depend on the initial condition. A particular solution is obtained by
K
method of variation of the constant (μ = μ (T)). The following solutions then are obtained:
K
K

****
1

****
****
1
T () =
T
μ E K -
B
C T T

éq
4.1-12
K
K
+
K
K -
-
N

K

K

The spherical deformations of reversible and irreversible creep are then equal to:

H +
H
fs

(T) = N
N
n+1
N +

1tn 1
2tn 1
X µ E
µ E
R
n+1
(
+
+
+
1
1
2
)

S
K

R

éq
4.1-13

H +
H
fs
(T) = N
N
n+1
N +

1tn 1
2tn 1
µ E
X µ E
I
n+1

(
+
+
+
S
1
2
2
)

ki
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After simplification, one then obtains the following expressions for the values of μ:
K

1

H +
H
H +
H
μ =
(
)

(
)
1

fs
X
T
- N
N
n+1
N
-
fs T
- N
N
n+1
N

2
R
n+1
I
n+1

(X X -1 1
1
2
) T
E N
S

S

K
K
R

I

1

H +
H

H +
H
fs
N
N
n+1
N
fs
N
N
n+1
N
μ =
(
)
(
)
2
(
T
X
T
X X -1 2
1
2
)
-
-
+
-
R
n+1
1
I
n+1

tn

S

S

E

K
K
R

I

éq 4.1-14

The equation [éq 4.1-2] can thus be put in the form, after discretization:

fs

= S
has
+ S
B
S
+ S
C
S

R, N
R, N
R, N
N
I, N
n+1
éq
4.1-15

fs
S
S
S
S
S

= + B has + C
I, N
I, N
I, N
N
I, N n+1

With:

S
X X

1
E
tn -

2
E
tn

1 tn
2 tn
1
2
fs
E
-

E

has
=

fs
R, N

-
1
- X
R, N
1
max (I, K)

X X -1
1
2

X X -1
1
2

kN

H

S
- X X

1
E
tn +

2
E
tn
H

1 tn
2 tn
N
1

2
N
E
-

E

B
=

R, N

+
1 +
X
1
éq
4.1-16

K Sr
X X -1
S
1
2

ki

X X -1
1
2

H

S
- X X

1
E
tn +

2
E
tn
H

1 tn
2 tn
N
1
2
N
E
-

E

C =

R, N

+
1 +
X
1

S
S

Kr
X X -1
1
2

ki

X X -1
1
2

1
S
E
tn -

2
E
tn

fs
X X

2
E
tn -
1
E
tn

has
= X
fs
I, N
2

-
R, N
1 2
-
1 max (I, K)

X X -1
1
2

X X -1
1
2

kN

H

2
S
N
E
tn -
1
E
tn H - X X

2
E
tn +
1
E
tn

B =
X
N
I, N
2
+
1 2
+
1
éq
4.1-17

K Sr

X X -1
S
1
2

ki

X X -1
1
2

H

2
S
N
E
tn -
1
E
tn H - X X

2
E
tn +
1
E
tn

C =
X
N
I, N
2
+
1 2
+
1

S
S

Kr

X X -1

1
2

ki
X X -1
1
2

In the equations [éq 4.1-16] and [éq 4.1-17] the parameters, X and X are a function of
1
2
1
2
intrinsic parameters of material. With each step of calculation, it is necessary to safeguard two variables intern fs
, last reversible spherical deformation obtained and max (fs

, i.e.
I, K)
R, N
kN
fs
, greatest reversible spherical deformation obtained in the history of the element. The choice of
I, N
to retain the expressions [éq 4.1-5] and [éq 4.1-6] (not of deformation unrecoverable), or the expressions
[éq 4.1-16] and [éq 4.1-17] (existence of unrecoverable deformations) to determine the increment of total spherical deformation is carried out a posteriori according to the sign of
fs
in [éq 4.1-15].
I, N
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1
3
1
2
4
1
1
2
4
3
3
4
2

Illustration of the numerical answers obtained by using the discretized expressions [éq 4.1-3] to [éq 4.1-17] for four stories of loading: 1 level of unit constraint to moisture constant (100%), 2 level of unit constraint to linearly decreasing moisture of 100% with 50%, 3 level of unit constraint during half of the duration of the calculation followed by a recouvrance with half of the initial constraint on the second part of calculation; moisture is supposed to be constant (100%), 4 the mechanical loading is identical to 3; moisture decrease linearly of 100% to 50%. Appear 4.1-a

To carry out simulations of [Figure 4.1-a] the following parameters were retained:
K S =,
2 0e + 5 [MPa]; S
= ,
4 0e +10 [MPa.s]; K S =,
1 0e + 4 [MPa]; S
= ,
1 0e +11 [MPa.s].
R
R
I
I
calculation comprises 200 intervals of 5000 [S].

4.2
Discretization of the equations constitutive of creep deviatoric

After discretization of the constraints and relative humidity by functions closely connected, the tensor diverter of the deformations of clean creep is discretized by the following equation:

fd
D
D
D
D
D

= + N has
B + N
C

éq
4.2-1
N
N
N
N 1
+

where D
and D

are the tensors of the deviatoric constraints at the beginning and the end of the step of time
N
N 1
+
running.
The stages carried out are:

.
One calculates the parameters compared to the deformation of clean creep deviatoric reversible, whose model is:

D
fd
& (T) + kd fd

(T) = H (T) D
(T)
R
R
éq
4.2-2
R
R
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After discretization, the preceding equation can be put in the form:

fd
D
D
D
D
D

= has
+ R
B, N + R
C, N

éq
4.2-3
R, N
R, N
N
N 1
+
With:

D
T

has

=

exp -
N -
1 F, D

R, N

D

,
R

R N

D
1
D

2

D

T D

2

D

-
R
R
N
R
R
tn

B
=
R, N
-
+
1 H +

N
N
H +1

exp -
+
-
1 H -
N
N
H +1 ***éq 4.2-4***
D

D

K
T
T

T
T
R

N

N

R

N

N

D
D

1

D
T

-
C
=
R

exp -
N H - R
tn
R, N

N
N
H
D

D

K
T

T
R

N

R
N

Note:

The equation [éq 4.2-4] (left reversible creep deviatoric) is similar to the equation [éq 4.1-5] (left reversible creep in the absence of unrecoverable deformations). They correspond to the discretization of a single chain of Kelvin.

One calculates the parameters compared to the deformation of clean creep deviatoric, of which the

model
is:
D
F, D
& (T) = H (T) D (T)
I

éq
4.2-5
I

After discretization, the preceding equation can be put in the form:
F, D
D
D
D
D
D

= has + ib, N + ic, N
éq
4.2-6
I, N 1
+
I, N
N
N 1
+
With:

D
has
=

0
I, N
D
T H
B
=
N
n+1
R, N

éq
4.2-7

D
2 I

D
T H
C
=
N
N
R, N

D
2 I

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1
1
3
2
2
4
1
1

3
2
3
4
2
4

Illustration of the numerical answers obtained by using the discretized expressions [éq 4.2-1] to [éq 4.2-7] for four stories of loading: 1 level of unit constraint to constant moisture (100%), 2 level of unit constraint to linearly decreasing moisture of 100% to 50%, 3 unit level of constraint during half of the duration of the calculation followed by a recouvrance to half of the initial constraint on the second part of calculation; moisture is supposed to be constant (100%), 4 the mechanical loading is identical to 3; moisture varies linearly decreasing 100% to 50%.
Appear 4.2-a

To carry out simulations of [Figure 4.2-a], the following parameters were retained:
K D =,
5 0e + 4 [MPa]; D
= ,
1 0e +10 [MPa.s]; D
= ,
1 0e +11 [MPa.s]. Calculation comprises 1000
R
R
I
intervals of 1000 [S].

5 Matrix
tangent

By introducing the elastic modulus of rigidity μ, the diverter of the constraints at the moment N + 1 is written according to the diverter elastic strain:

D
ED
D
D
F, D

= 2μ
= + 2μ

- 2µ

éq
5-1
N 1
+
N 1
+
N
N
N

In substituent the deviatoric part of the clean deformation of creep by the expression [éq 4.2-1], it rise the following relation:

D

(1+2d
µc)
D
= (1 - 2D
B
µ) + 2
D
µ

- 2
D
µa 1 ***éq***
5-2
N 1
+
N
N
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Expression which induces by derivation compared to D

:
N 1
+

*dn 1+ (1+2µ cd.) = 2 1µ **éq 5-3***
dn 1+
By taking a similar step for the spherical part and by introducing the module of rigidity with dilation K, it follows the three following relations:

E
tr
= 3K tr
= tr + 3K tr (
) - 3K tr (F

*) **éq***
5-4
n+1
n+1
N
N
N
tr
(
S
1+ 3Kc) = tr (
S
1 - 3Kb) + 3K tr ()
S
*- Ka **éq***
5-5

n+1
N
N
(tr
)
+

éq 5-6
(
N 1
S
1+ 3
= 3
tr
) (Kc) K
N 1
+
The tangent matrix is written finally:

D
D
D
1 (tr)

1 (tr) (tr)
=
+
1 =
+

éq
5-7

3
D
(tr)
1
3

I.e.:

2μ

1

=
K
1- 11 +
11
éq
5-8

1+ 2μ D
C
3
1+ 3 S
Kc
4
1 4
2 3
4
1 4
2 3

After linearization, the tangent matrix develops as follows:

2
1
1

+ - - 0 0 0
11

3

3
3
11

1
2
1
22 - + - 0 0 0 22

3
3
3

33
=
1
1
2
33
éq
5-9
2

0 0 0
12 -
-
+
2
3
3
3
12

2
0
0
0
0
0
13
213

2
0
0
0
0
0

23
2 23

0
0
0
0
0

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6
Description of the internal variables

The following table gives the correspondence between the number of the internal variables accessible by Code_Aster and their description:

Number of the variable	***Description***
1	*Reversible spherical deformation*
2	*Irreversible spherical deformation*
3	*Reversible deviatoric deformation, component 11*
4	*Irreversible deviatoric deformation, component 11*
5	*Reversible deviatoric deformation, component 22*
6	*Irreversible deviatoric deformation, component 22*
7	*Reversible deviatoric deformation, component 33*
8	*Irreversible deviatoric deformation, component 33*
9 -	
10 -	
11 -	
12	*Reversible deviatoric deformation, component 12*
13	*Irreversible deviatoric deformation, component 12*
14	*Reversible deviatoric deformation, component 13*
15	*Irreversible deviatoric deformation, component 13*
16	*Reversible deviatoric deformation, component 23*

17
Irreversible deviatoric deformation, component 23
18 -
19 -
20 -

7 Notations

tensor of the total deflections
F
tensor of the deformations of clean creep
E
tensor of the elastic strain
fs
1 spherical part of the tensor of the deformations of clean creep
fs
1 reversible spherical part of the tensor of the deformations of clean creep
R
fs
1 irreversible spherical part of the tensor of the deformations of clean creep
I
fd
deviatoric part of the tensor of the deformations of clean creep
fd
reversible deviatoric part of the tensor of the deformations of clean creep (contribution of water
R
absorptive)
fd
irreversible deviatoric part of the tensor of the deformations of clean creep (contribution of
I
interstitial water)
tensor of the total constraints
S
1 spherical part of the tensor of the constraints
D
deviatoric part of the tensor of the constraints
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H relative humidity internal
K modulates elastic rigidity with dilation
S
Kr rigidity connects associated with the skeleton formed by blocks with hydrates on a mesoscopic scale
S
ki rigidity connects intrinsically associated with the hydrates on a microscopic scale
D
Kr rigidity associated with the capacity with water adsorbed to transmit loads (load bearing toilets)
µ elastic modulus of rigidity
S
I viscosity connects associated with the mechanism with diffusion interlamellaire
S
R viscosity connect associated with the mechanism with diffusion within capillary porosity
D
I viscosity of interstitial water.
D
R viscosity associated with the water adsorbed by the layers with hydrates

X, X, X respectively indicate a scalar, a vector and a tensor of order 2.
N
X, N
X

+ ,
1
N
X indicate respectively the value of quantity X at time tn, at time tn 1
+ and
variation of X during the interval [T;
]
N tn 1

+ .

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Model of damage of Mazars

Summary:

***This documentation presents the model of behavior of Mazars which makes it possible to describe the behavior
rubber band-endommageable of the concrete. This model is 3D, isotropic and is based on a criterion of damage
writing in deformation and describing dissymmetry traction and compression. It does not take into account the possible ones
plastic deformations or viscous effects which can be observed during deformations of a concrete.***

***Two versions of the models are established: the local version (with risk of dependence to the grid) and not-local version where the damage is controlled by a not-local deformation. It is also possible
to take into account the dependence of the parameters of the law with the temperature, the hydration and drying.***
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Count

matters

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---

1 Introduction

1.1
A law of behavior élasto-endommageable

The model of behavior of MAZARS ([bib1]) is a model simple, considered robust, based on mechanics of the damage [bib2], which makes it possible to describe the reduction in the rigidity of material
under the effect of the creation of microscopic cracks in the concrete. It is based on only one internal variable
scalar D, describing the isotropic damage of way, but distinguishing despite everything
the damage of traction and the damage of compression.
Contrary to model ENDO_ISOT_BETON, this model does not allow to translate the phenomenon of refermeture of the cracks (restoration of rigidity). In addition, the model of Mazars does not take in count the possible plastic deformations or viscous effects which can be observed with the course deformations of a concrete.

1.2
Limits of the local approach and methods of regularization

Like all the lenitive laws, the model of Mazars raises difficulties related to the phenomenon of localization of the deformations.
Physically ([bib3]), the heterogeneity of the microstructure of the induced concrete of the remote interactions
between the formed cracks. Thus, the deformations locate in a metal strip, called band of localization, to form macrofissures. The state of the constraints in a material point cannot any

more
to be only described by the characteristics at the point but must also take into account sound environment. In the case of the local model, no indication is included concerning the scale of *cracking. Consequently, no information is given over the bandwidth of*
localization which becomes null then. This leads to a mechanical behavior with rupture without dissipation of energy, physically unacceptable.
Mathematically ([bib4]), the localization returns the problem to be solved badly posed because softening
cause a loss of ellipticity of the differential equations which describe the process of deformations. The numerical solutions do not converge towards physically acceptable solutions in spite of refinements of grid.
Numerically, one observes a dependence of the solution to the network extremely prejudicial (cf [R5.04.02]).

A method of regularization thus becomes necessary. Several are possible. The choice which was fact here is to take again one of the developments already made for models ENDO_FRAGILE and ENDO_ISOT_BETON, by using a tensor of nonlocal deformation which checks the equation characteristic [R5.04.02]:
=
2
-
C

where the scalar a.c. the dimension a length to the square one.

Note:

Let us announce that this model not-room does not correspond to the version initially proposed by J. Mazars and G. Pijaudier-Pooch [bib5] and which is in particular established in Castem 2000. The delocalization is obtained while using as equivalent deformation, the average local equivalent deformation on a volume V:

(X) = 1 -

Vr (X)
(X S) eq (S) ds

where is the volume of the structure
Vr (X) is representative volume as in point X: Vr = (X - S) ds

---

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2
4 X - S
(X - S)

is a weight function: (X - S) =

exp -

2

lc

lc is the internal length of the not-continuous medium (traditionally estimated at three times cut larger aggregate).

Numerical tests made it possible to connect the 2 parameters of delocalization lc and C in the case of the model of Mazars. The following relation was obtained:

4 C lc

The model of Mazars is thus available in Code_Aster under 2 versions:

· the local version of the model for which the dependence of the solution to the network is observable as for all the lenitive models.
· a nonlocal version which uses a tensor of nonlocal deformation.

1.3

Coupling with thermics

For certain studies, it can be interesting to be able to take into account the modification of parameters materials under the effect of the temperature. This is possible in Aster (MAZARS_FO compound or not with ELAS_FO). The assumptions made for the coupling with thermics are them following:

· thermal dilation is supposed to be linear is HT
*= (T - T **I** with = constant or*
ref.) D
function of the temperature,
· one does not take into account thermomechanical interactions, i.e. one does not model not the effect of the mechanical state of stress on the thermal deformation of the concrete,
· concerning the evolution of the parameters materials with the temperature, one considers that those depend not on the current temperature but on the maximum temperature seen by material during its history,
· only elastic strain (mechanical) induced of the damage.

Note:

Because of data-processing constraints, the initial value of Tmax is initialized to 0. In consequence, one cannot use the parameters materials defined for temperatures negative (if necessary, one can however circumvent this problem while returning all them temperatures in Kelvin instead of °C)

One initially presents the writing of the model then some data on the identification of parameters. To finish, one exposes the principles of numerical integration in Code_Aster.

1.4
Law of Mazars in the presence of a field of drying or hydration

The use of ELAS_FO and/or MAZARS_FO under operator DEFI_MATERIAU makes it possible to make depend
the parameters materials of drying or the hydration.
In addition, the deformations related on the withdrawal of dessication and the endogenous withdrawal are taken in
count in the model, in the following form:
Re
*= - **I** and = - C*
(
*- C) **I***
D
rd
ref.

D
where is the hydration, C, the water concentration (field of drying in the terminology Code_Aster), Cref initial water concentration (or drying of reference). Finally is the coefficient of endogenous withdrawal and the coefficient of withdrawal of desiccation to be informed in DEFI_MATERIAU,
key word factor ELAS_FO, operands B_ENDO and K_DESSIC. As one said to the paragraph precedent, the choice which was made in the establishment of the model of Mazars, it is that only elastic strain induced of the damage. Consequently, if a test-tube is modelled
out of concrete which dries freely or which is hydrated freely, one a field of deformation will obtain well not no one and a stress field perfectly no one.
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2
Description of the model

2.1
The model of Mazars

The model of Mazars was elaborate within the framework of the mechanics of the damage.
The constraint is given by the following relation:

E
= 1
(- D) ***a:***

éq 2.1-1
with: ***With the*** *matrix of Hooke*
D the variable of damage
E
elastic strain E
HT
rd
Re
= - - -
HT

*= (T - T **I** thermal dilation*
ref.) D
Re
*= - **I** endogenous withdrawal (related to the hydration)*
D

= - C
(
*- C) **I** withdrawal of desiccation (related to drying)*
rd
ref.
D

The variable of damage D results from a combination of a damage of traction T
D and
of a damage of compression C
D:
D

= T T
D + 1
(- T) C
*D **éq***
2.1-2
· The coefficient is a parameter material which makes it possible, when it is higher than 1, to improve the response in shearing.
· The damages of traction and compression are defined by the equations
following since eq d0:
d0 (1 - C
With)
With
D = 1
C
-
-
(limited between 0 and 1)
eq
[C
exp B (
-)
C
eq
D 0]

d0 (1 - T
With)
With
D = 1
T
-
-
(limited between 0 and 1)
eq
[T
exp B (
-)
T
eq
D 0]
where A, A, B, B,
C
T
C
T D 0 are parameters materials to be identified.

· The damage is controlled by the equivalent deformation eq which makes it possible to translate one triaxial state by an equivalence in a uniaxial state. As the extensions are of primary importance in the phenomenon of cracking of the concrete, the introduced equivalent deformation is defined in to leave the positive eigenvalues of the tensor of the deformations, is:

eq = +: + or in the principal reference mark of the tensor of deformations
2
2
2
=
1

eq
+ 2 + 3
+
+
+

knowing that the positive part < >

+ is defined so that if
I is the deformation
principal in direction I

I = I if I 0

+

I
= 0 if I < 0
+
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Note:

In the case of a thermomechanical loading, only elastic strain
E
HT
= - contributes to the evolution of the damage from where: eq =
E
E

:
.
+
+

· The coefficient T which carries out the coupling between traction and damage is such as in the total absence of traction,
and in the total absence of compression
. It is
T = 1
T = 0
defined by the following relation:
3
[
I
Ti
+
]
I 1

=
=
(limited between 0 and 1)
T
2
eq

where Ti is the deformation created by the positive principal constraints is:

1+

T =
+ - tr (+)
E
E

2.2
Example in 1D

One shows here the answer obtained with the model of Mazars (local or not-local version) when one an element of volume subjects to a uniaxial loading.
The parameters materials used are as follows:

E = 32.000 MPa, = 0.2, d0 = 9.375 105, C
With = 1.15, T
With = 0.8, C
B = 1391.3, T
B = 10.000, = 1

The loading is applied in several stages:

1) compression up to 0.3%
2) discharge
3) compression up to 0.4%
4) discharge then traction up to 0.035%
5) discharge
6) traction up to 0.07%
7) discharge

On the following figure, one represented axial stress according to the axial deformation thus that according to the lateral distortion. Initial rigidity was also represented in order to better to visualize the effect of the damage.
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6
7
4
2
3
1

Appear 2.2-a: Forced answer deformation of the model of Mazars for a request 1D

The preceding figure makes it possible to visualize a certain number of characteristics of the model of Mazars,

with knowknowing:

· the damage affects the rigidity of the concrete,
· it does not have there unrecoverable deformations,
· the answers in traction and compression are quite dissymmetrical,
· although one distinguishes the damage due to compression and that due to traction, effective variable of damage is well a linear combination of both. In consequence, when one préendommage the material in compression (resp. in traction), and that one charges then in traction (resp. in compression), it is well rigidity with damage which is observed and not initial rigidity.

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3 Identification

In addition to the thermoelastic parameters E, the model of Mazars utilizes 6 parameters material: WITH, B, A, B,
C
C
T
T D 0 and.

.

d0 is the threshold of damage. It acts obviously on the constraint with the peak but also on the shape of the curve post-peak. Indeed, the fall of constraint is of as much less brutal than d0 is small. In general d0 is included/understood into 0.5 and 1.5 104.

The coefficients A and B make it possible to modulate the shape of the curve post-peak:

.

A introduces a horizontal asymptote which is the axis of for A = 1 and the horizontal one passing by the peak for A = 0 (cf [Figure 3-a]). In general, C
A lies between 1 and 1.15 and
T
With between 0.7 and 1.
.

B according to its value can correspond to a brutal fall of constraint (B > 10.000) or one preliminary phase of increase in constraint followed, after passage by a maximum,
of a more or less fast decrease as one can see it on [Figure 3-b]. In
general C
B lies between 1000 and 2000 and T
B between 10.000 and 100.000.

Appear 3-a: Influence parameters A [bib1]
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Appear 3-b: Influence parameters B [bib1]

· is a corrective factor making it possible to improve the response in shearing compared to

initial version of the model (which corresponds to =1). The advised value is 1.06.

A means simple to obtain a set of parameters is to have the uniaxial test results in compression and of deflection tests (for several sizes of beams to determine the parameter C in not-room)

---

4 Resolution numerical

4.1 Evaluation of the damage

The procedure of obtaining the damage is simple. The stages are as follows:

· calculation of the elastic strain and thermal
· calculation of the equivalent deformation
· calculation of T, C
D and T
D for this state of deformation
·

calculation of
test
D

= T
T
D + 1
(- T)
C
D
- if
-
Dtest D, then the criterion is not reached and +
-
D = D
- if
-
Dtest > D, then the criterion is reached and +
test
D = D

- if test
D
= 1, then the material is completely damaged and +
D = 1 (makes some (1 -) for
to avoid the numerical problems).

4.2
Calculation of the constraint

After evaluation of D, one calculates simply:

E
= 1
(- D) A

4.3
Calculation of the tangent matrix

*One seeks the tensor **M** such as & = **M&** knowing that = 1*
(- D)
A. The matrix is thus
summon of two terms, one with constant damage, the other due to the evolution of
the damage:

& = (1 - D) A& - A D
&

The first term is easy, it acts simply of the operator of Hooke, multiplied by (1 - D).
The second requires the evaluation of the increment of D& damage.
Knowing that the damage is defined by the relation D

= D

+ 1
(-) D, it comes
T
T
T
C
.
.
D& =
D& + 1
(-

) D& +
D + 1
(-

) D
T
T
T
C
T
T
T
C
If it is supposed that the loading is radial &
([bib1]), it is then enough to express D&.
T = 0
C T,
D
~
C T,

&Dc T, =
Tr

~

&

~

with
+
=

~
D 1
(
0
- A)
WITH B

C, T
C, T C, T
from where &
D

C, T =
+

+
Tr

~

2
~
~ &

[
exp B (
C, T
- D)
0]

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Note:

1. Being given made simplifications, in the case general the tangent matrix is not consistent. Also, it can happen that the reactualization of the tangent matrix at the course iterations of Newton does not contribute to convergence. In this case, it is enough to use only the secant matrix by imposing ***STAT_NON_LINE (NEWTON = _F (REAC_ITER =0)).***
2. In the case general, the tangent matrix is not-symmetrical. In version 6, the matrix tangent was symmetrized. It is not any more the case starting from version 7 (even if it is always possible to do it thanks to key word ***SOLVEUR=_F (SYME = "YES")*** *of* ***STAT_NON_LINE****).*
3. Concerning the nonlocal approach, the treatment of the boundary conditions is such as one could be brought, in the case of symmetrical structures, to treat the calculation of the whole of the structure and not of the "representative" part (cf [R5.04.02]).
4. The analytical expression of the tangent matrix is valid only for loadings radial (dt = cd. = 0). In the other cases, the quadratic convergence of the method is not guaranteed any more.
Knowing that in any case, the use of the law of damage in form directly integrated ([éq 2.1-2]) is theoretically valid only under this same assumption, in the case of nonradial loadings, one will préfèrera other models with model of Mazars, in particular of modelings taking into account anisotropy of the behavior or the effect of refermeture of the cracks (requests

alternated), adapted to this type of loading.

4.4
Stored internal variables

We indicate in the table according to the internal variables stored in each point of Gauss for the model of Mazars:

Internal variable
Feel physical
V1
D: variable of damage
Indicating V2
of damage
(0 so elastic, 1 if damaged i.e. as soon as D is not null any more)
V3
: maximum temperature attack at the point of gauss

4.5
Modelings compatible with the law of MAZARS

The law of Mazars is usable in Aster with various modelings:

· traditional version: 3D, D_PLAN, AXIS, C_PLAN,
· not-local version: 3D_GRAD_EPSI, D_PLAN_GRAD_EPSI, C_PLAN_GRAD_EPSI,
· coupled with the models of THHM (cf [R7.01.11]).

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Organization (S): EDF-R & D /AMA

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Law of behavior ENDO_ORTH_BETON

Summary:

This documentation presents the theoretical writing and the numerical integration of the law of behavior
ENDO_ORTH_BETON developed by [bib1], which describes the anisotropy induced by the damage in the concrete,
as well as the unilateral effects (different behaviour in traction and compression). Validation of the model
compared to experimental results is also proposed in this document.
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1 Introduction

1.1
Characteristics of the damage of the concrete

The concrete is a complex material, formed of aggregates and a paste ensuring cohesion between these
aggregates, and in which preexist of the microscopic cracks formed at the time of the various stages of
manufacture. The concrete is generally regarded as an initially isotropic material, them microscopic cracks not having privileged orientation. This isotropy is preserved if the loading applied remains in the elastic range. Starting from a certain level of loading, microscopic cracks will develop in particular directions, which induces the appearance of the anisotropy in non-linear phase. The cracks develop preferentially orthogonally with the directions of greater traction or of smaller compression. The process of damage is thus translated by a loss of rigidity generated by the decoherence of the matter. Rigidity can be found when the cracks are closed (unilateral effect). It is added to that another strong dissymmetry of behavior enters traction and compression: the constraints supported in compression are 10 times (even more) higher than the constraints supported in traction. Let us announce finally others
phenomena like the formation of unrecoverable deformations (caused for example by blocking lips of cracks by friction or the presence of matter degraded between these lips) or of phenomena of dissipation of energy by friction of the lips of cracks.

1.2
Objectives of law ENDO_ORTH_BETON

In spite of the existence of various models of anisotropic damage for the concrete, the models isotropic are always exclusively used in the engineering and design departments to account for behavior of the concrete structures. This is due, for the anisotropic models, with the complexity of their implementation numerical, with the difficulty in identifying their sometimes many parameters, with not
agreement of the objectives of the model and the industrial study, with the difficulty of coupling the model with
other physical phenomena (creep, plasticity), and in the majority of the cases at the computing times important needs by the anisotropic models. The use of the anisotropic models is not moreover not necessary whenever the isotropic models describe the same behavior of

the structure. There is however cases where the anisotropic models can prove to be interesting, as of at the time they predict a behavior different from the isotropic models.
The objective is to have a simple anisotropic model (low number of parameters), numerically robust, and complying with the rules of thermodynamics (positive dissipation). It is obvious that a certain number of phenomena observed in experiments could not be taken in account. A schedule of conditions was thus defined before the development of the model so to define the framework of it.

Two categories of requirements can be distinguished, the objective being to obtain a reasonable result whatever the loading. One relates to the physical coherence of the prediction of the model (1) with 4)), and the other relates to the numerical robustness (5) and 6)). The framework of our model is made up
following points:

1) Taking into account of the anisotropy grace the introduction of a symmetrical tensor of order 2 representing the effects of the damage. It is thus more exact to speak about model orthotropic insofar as the use of such a tensor makes it possible to define only three clean directions of the damage. A tensor of a higher nature (4 even 8) is necessary to account for the complete anisotropy.
2) Cancellation of the constraint to the ruin. That leads us to define a free energy, function deformations, rather than a free enthalpy, function of the constraints, because it seems more easy to obtain a null constraint starting from deformations finished rather than the reverse.
3) Increasing and limited eigenvalues of the damage. This point accounts for irreversible character of the process of damage (growth of the eigenvalues) of which the ruin constitutes the limit (limited eigenvalues). It makes it possible of more than reach the ruin in several directions.
4) Taking into account of the unilateral behavior of the concrete: refermeture of the cracks in compression, dissymmetry of the thresholds of damage between traction and compression.
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5) Continuity of the forced answer/deformation, particularly with the open-closed passage cracks. In addition to a discontinuity would be physically doubtful, it would involve problems of convergence of the numerical algorithm.
6) Respect of the framework of generalized standard materials. That makes it possible to ensure coherence
thermodynamics of the model (positive dissipation) and that provides properties pleasant mathematics for the resolution numerical (existence and unicity of the solution of problem of the calculation of the constraints and the final damage with increment of deformation fixed, said “local problem of projection” by analogy with plasticity).

Note:

The framework of generalized standard materials (CSMG) such as it here is heard is not strictly that defined by Halphen and Nguyen [bib11]. Indeed, the strict CSMG ensures the existence and unicity total solution of the problem if energy is convex compared to all the variables at the same time. This cannot in the case of be checked the lenitive laws of behavior. The “degraded” CSMG that we define ensures only the existence and the unicity of solution of the local problem of projection (calculation of the evolution of the damage with deformation fixed). To respect the CSMG, one must check the convexity of the free energy, on the one hand by report/ratio with the deformation, and in addition compared to the internal variables:

· Convexity compared to the deformation is necessary to ensure the stability of elastic problem.
· ***Convexity compared to all the variables intern simultaneously*** is *necessary to have the good mathematical properties for the local problem of projection. If several internal variables are used, separate convexities compared to each one of these variables are not sufficient.*
· Total convexity compared to the deformation and with the variables intern simultaneously is not necessary since the increment of deformation is fixed for local projection allowing to calculate the evolution of the internal variables. It seems impossible besides to obtain this total convexity in the case of lenitive laws of behavior.

The framework which one defined omits a certain number of physical phenomena associated the damage of the concrete:

· appearance of unrecoverable deformations
· voluminal dilation of material in compression
· behavior hysteretic for cycles load-discharge, generated by friction between the lips of cracks.

2
Expression of the free energy

2.1
Taking into account of refermeture of the cracks

If one seeks to take account of the effect of refermeture, it is necessary to pay a great attention to continuity of the constraints according to the deformations (what is an essential condition for a law of behavior in a computation software by finite elements), cf [bib2]. Indeed, if one model this effect in a too simplistic way, the law of behavior is likely great to present a discontinuous answer. A solution is to finely describe what one calls traction and compression, knowing that in traction (resp. compression) the crack will be considered “open” (resp. “closed”). A natural solution is to place itself in a clean reference mark of deformation. In such a reference mark, the elastic free energy is written (and μ indicating the coefficients of Lamé):
()

= (*tr*) 2 + μ 2
I **éq**
2.1-1
2
I
One can then define:

· a traction or voluminal compression, according to the sign of *tr*,

· a traction or compression in each clean direction, according to the sign of *I*.

The elastic free energy can then be written:

(

) = ([*tr*) 2+ + (*tr*) 2 -] + µ [*tr* (2
+) + *tr* (2
)]
éq
2.1-2
2
-

with the following definitions for the parts positive and negative:

(*tr*) = *H* (*tr*) *tr*
2
2
2
2
+
; (*tr*) = *H* (- *tr*) *tr*
-
; *tr* () =
+
H (*I*) *I*; *tr* () =
-
H (- *I*) *I*
I
I
where *H* is related to Heaviside.

Note:

A more detailed study of the properties of the parts positive and negative of a tensor is made in appendix 1.

2.2
Introduction of the variables of damage

Taking into account the complexity of the damage mechanisms, and having noted that it was difficult to describe the behavior of the concrete by using only one variable of damage, we chose to introduce two variables of damage:

· A tensor **D** of a nature 2 relating to the damage created in traction
· A scalar D relating to the damage created in compression
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Note:

The choice of a tensor of order 2 to model the damage of traction is relatively
traditional and intuitive. It makes it possible orthogonally to describe the privileged orientation of the cracks with
direction of greater traction. The question of the damage created in compression
(orthotropic or isotropic?) is much less clear.
Let us suppose to simplify that the cracks can be generated only in the plans
orthogonal with er, er or er. In the case of a simple traction in the direction er, the cracks
1
2
3
1
are created in the orthogonal plan with er, which involves a loss of rigidity in the direction
1
er. If one exerts then a simple traction in the direction er, one “does not see” not the crack bus
1
2
the loading is parallel to the plan of the crack and rigidity is not affected.
The damage is thus clearly anisotropic in traction. In the case of a compression

simple in the direction er, one creates cracks in the orthogonal plans with er and er. If one
1
2
3
charge then in compression in the direction er, one "sees" the orthogonal cracks with er.
2
3
rigidity in the direction er is thus lower than not degraded normal rigidity, but it is
2
stronger than rigidity in the direction er because compression in the direction er is sensitive to
1
1
all cracks, i.e. orthogonal with er and er. Consequently, the damage in
2
3
compression seems anisotropic than in traction, without being however completely isotropic. In the absence of physical argument clearly on the isotropic or anisotropic character of the damage in compression, we chose to take it isotropic for reasons of
simplicity.

One poses **B** = **I** - **D** representing the integrity of material in traction. One introduces the damage of traction in the "positive" terms of the free energy [éq 2.1-2] and the damage of compression in the negative terms. The free energy is now defined as follows:

(

, *D*) = ([*trB*) 2 +
2
2
1
2
2
2
+
(1 - *D*) (*tr*) -]
+ µ
tr ((+ B)+) + (1 - *D*) *tr* (-)

éq 2.2-1
2
4

Note:

The convexity of the free energy opposite, of the deformation on the one hand, and the variables of damage simultaneously in addition, is respected strictly. One will refer to [bib2] and [bib1] for a demonstration.

2.3
Terminals of the damage

Ruin, or creation of a crack crossing the matter element completely considered, imposes a limit upper than the damage. This terminal is imposed on each eigenvalue of the damage of traction (*D*
where *D* indicate the eigenvalues of **D**), which
I
[] 1
,
0
I
allows to reach the ruin in 3 orthogonal directions. A convex indicating function by report/ratio with the damage, *I*]
I
-] for *D* or
for **B**, is thus used to control each one of
1
,
[0,[
eigenvalues of the damage (cf [bib2]). In the same way, one uses an indicatrix on the value of the scalar damage of compression. One obtains the expression of the following free energy:

(

1

, **B**, *D*) =
([*trB*) 2+ + (1 - *D*) 2 (*tr*) 2] + µ *tr* (

(**B+B**) 2+) + (1-d) 2tr (2)
2
4

éq 2.3-1
+ *I* [0, [[min (*Bi*)]+ *I*] -] 1, [*D*]
Handbook of Reference

Note:

It results from this expression which once the damage is worth 1 in a clean direction, this clean direction from now on is blocked and the damage cannot evolve/move any more but in the plan perpendicular to this direction. The demonstration of this point is in appendix 2.

2.4 Introduction of a blocked energy

We propose to introduce an energy blocked in order to better control the evolution of the damage according to the loading, in the form used by [bib4]. The idea consists with to introduce an additional term into energy depending only on the damage, and not of the state of deformation. It then results from it an additional term in derivation from the forces thermodynamic which controls the evolution of the damage (cf section [§3]). This term additional on the other hand no modification of the expression of the constraint implies.

Energy is written in the following way:

(

2
2
2
1

, **B**, *D*) =
([*trB*) +
+
(1 - *D*) (*tr*) -] + μ *tr* (

(**B+B**) 2+) + (1-d) 2tr (2)
2
4

éq 2.4-1
+ *I* [0, [[min (*B)*] + *I*] -] 1, [*D*]
blocked
I
+
(**B**, *D*)

where
blocked

(**B**, *D*) is a convex function of the damage. One chooses to take this
null additional energy when the material is healthy. It must moreover be expressed with the means invariants of the tensor of damage. One wishes finally that the additional term in
the expression of the thermodynamic forces depends on the damage, which makes it possible to eliminate it
choice of a linear term in damage for *blocked* energy

(**B**, *D*).

We chose the following expression:

blocked

(

B, *D*)
B
=
tr ((**I** - **B**) 2)
D
2
+
D

2
2

éq
2.4-2
B
=
tr (2
D)
D
2
+
D
2
2

where *B* and *D* are parameters of the model.
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Note:

One can put the question of the physical significance of this additional energy.
The evolution of the damage indeed gives place in energy to the appearance of an energy "blocked" by the system, and this energy is not null when the material is ruined. By consequent, if one makes the energy balance of the creation of a crack, a part only of

the power consumption corresponds to the dissipation of energy in the form of heat, whereas one another part remains blocked.
One can find an origin physical of this energy blocked in the case of compression. Indeed, if the macroscopic constraint is slackened, cracks opened by compression are not closed again completely, as the existence of a residual deformation proves it macroscopic. There are thus probably deformations and residual stresses local which stores energy (cf [Figure 2.4-a]), by analogy with plasticity. (One recalls that law ENDO_ORTH_BETON *does not describe the unrecoverable deformations.).*

Appear 2.4-a
Apart from the physical considerations, interest of this blocked energy, equivalent to one energy of work hardening, is that the model remains well within the framework of standard materials generalized. The advantage, compared to an energy of work hardening, is that this energy will have an effect different on the two variables from damage (we will explain this point in following section).

2.5 Final expression of the free energy, the constraints and the forces thermodynamic associated the variables of damage

The final expression of the free energy is written:

(

2
2
2
1

, **B**, *D*) =
([*trB*) +
+
(1 - *D*) (*tr*) -] + µ *tr* (

(**B+B**) 2+) + (1-d) 2tr (2)
2
4

éq 2.5-1
+ *I* [0, [[min (*B)*] + *I*] -] 1, [*D*]
blocked
I
+

(**B**, *D*)

The expression of the constraints results from energy by derivation compared to the deformations:

(, **B**, *D*)
=
= ([*trB*) **B** +
2
+
(1 - *D*) (*tr*) - **I**]

éq 2.5-2

+
1
µ
(**B** + **B) B** +
2
+
B (**B** + B)+) + 2 (1 - *D*)

-
2

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The continuity of the constraint results from the continuity of the eigenvalues of a matrix with respect to this matrix (cf theorem of Ostrowski in [bib4]).

Note:

A defect appears however in the "open-closed" passing. Fields of activation and of desactivation of the terms of opening and closing of the cracks do not coincide. This is due with the fact that the opening and closing are not associated positive separation in parts and negative of the same sizes (opening: damage combined with the deformation, closing: deformation alone). This does not affect however the property of continuity. It appears moreover than this defect is limited to a zone close to the origin of the deformations, where it does not generate no the physical aberration, and which it has no incidence apart from this interval (cf [bib2]). formalism suggested in [bib3], allowing to ensure the continuity of the constraint while taking in account the effect of refermeture of the cracks, presents the same defect.

One in addition deduces from the free energy the expression of the thermodynamic forces associated damages:

B

µ
F (, **B)** = -
= - (*trB*) -
+
([**B** + **B)** +
+
(**B** + B)+] + *B* (**I** - **B) éq 2.5-3**
B

2

F D (, *D*) = -
= (1 - *D*) (*tr*) 2 + 2µ
2
-
(1 - *D*) *tr* (-) - *D* **éq**
2.5-4
D
D

Each thermodynamic force is consisted of the two parts:

· A part depending on the deformation and the damage, which is derived from the part rubber band of energy.
· A part depending only on the damage, which is derived from blocked energy. It term will play the part of a work hardening, and makes it possible to control the answer stress-strain. It is seen that the terms deriving from the energy blocked in each one thermodynamic forces are independent one of the other, which makes it possible to control more easily evolution of each variable of damage.

3
Law of evolution of the variables of damage

3.1 Law of evolution

So that model ENDO_ORTH_BETON enters within the framework of generalized standard materials, one must obtain a potential of dissipation. For reasons of simplicity, one defines the field rather of reversibility, as for law ENDO_ISOT_BETON (cf Doc. [R7.01.04]).

Note:

From a formal point of view, the generalized standard materials are characterized by a potential of dissipation function positively homogeneous of degree 1, transformed of Legendre-Fenchel of the indicating function of the field of reversibility. One can thus choose to define, that is to say a potential
of dissipation, that is to say a field of reversibility.

The first idea is to define two criteria of evolution corresponding to each variable interns. This solution is completely possible insofar as the thermodynamic forces are dissociated one of the other. It proves however that this solution has two disadvantages:

· First is of a "physical" nature. Let us consider a sample subjected to a compression uniaxial. One can imagine that the criterion of compression alone is reached and that the criterion of traction is not activated. Only the damage of compression evolves/moves then. If one subjects this sample, after discharge, with a traction in an orthogonal direction with the axis of compression precedent, the material behaves according to the model like a healthy material, in spite of creation, actually, microscopic cracks parallel to the axis of compression, therefore perpendicular to the axis of traction.
· The second disadvantage is of order practical. It is indeed easier to treat numerically only one criterion, utilizing only one multiplier of Lagrange (cf [Figure 3.1-a]), rather that two separated criteria, utilizing two multipliers of Lagrange and being able to create zones where the direction of flow is not *a priori* defined (cf [Figure 3.1-b]).

Appear 3.1-a
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Appear 3.1-b

One thus decides to introduce a single criterion coupling the evolution of the two variables of damage:

G (
2
2
B
F, *F D*) =
B
F
+
D
-
(1-)(+
F) - *K* () 0 **éq**
3.1-1

where *K* () is a threshold depend on the state of deformation (this point will be commented on in the section
[§3.2]).

The evolution of the variables of damage is then determined by the conditions of Kuhn-Tucker:

d& = 0

for *G* < 0

&B = **0** &D = **0**

éq
3.1-2
d& 0

for *G* = 0

B& 0 *D&* 0
I
I

Note:

The criterion utilizes only the positive part
D
F+ of D
F and negative B
-
F *of B*
F *so*
to impose the growth of the damage. This condition is ensured in the potential of dissipation by the introduction of the indicating functions I
I - B&
+ (*d&*) *and*
(*(equivalent to*
I)
IR
IR
I + (*D&).*
I)
IR

Note:

The elliptic criterion is convex in the space of the thermodynamic forces, which ensures convexity of the potential of dissipation.
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Within the framework of generalized standard materials, the evolution of the internal variables follows the law
of flow associated with the criterion via the principle with normality:

$$\dot{B}_G = -\dot{\lambda}\frac{\partial F(\mathbf{B}_\&, R_\&)}{\partial \mathbf{B}_\&} = \dot{\lambda}\frac{\partial F}{\partial \mathbf{B}}$$

éq 3.1-3

$$\dot{B} = 2\left(\mathbf{F}\cdot B - \dot{\lambda}\left(D_{F^+}\right)^2\right)\cdot \mathbf{F}$$

$$\dot{G} = \dot{\lambda}\left(1 - D_{F^+}\right)$$

D
+
& = &
= &

éq
3.1-4
D
2
F

B
-
F
+ (1 -) (*D*
F+) 2

One can integrate the common denominator in the multiplier of Lagrange to have a relation simpler:

G

B
B& = &
= & **F éq**
3.1-5
B
-
F

G

d& = &
= & (1 -) *D*
F **éq**
3.1-6
D
+
F

This system utilizes a single plastic multiplier &.

The equations of evolution ensure the positivity of the potential of dissipation:

2
B
D

2
B
D

F: **B&** + *F d&* = & **F**
+
-
(1-)

(+F) 0

éq
3.1-7

3.2
Function threshold depend on the deformation

In order to better control the dissymmetry of the behavior enters traction and compression (report/ratio 10
rupture limits), we introduced a function threshold depend on the state of deformation
in the criterion [éq 3.1-1]. The role of this function threshold is to push back the elastic limit in compression. One wishes moreover than the breaking stress in simple traction cannot be exceeded during a biaxial test (cf [bib1] for a detailed study of the function threshold).
The function that we propose is as follows:

K ()
tr
= *K* -
0
1
K (*tr*)
( )
-
- arctan -

éq

3.2-1

k2

This function threshold introduces 3 parameters for the model.
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Note:

This function was not optimized for the case of the loadings of triaxial compression. One recall for this reason that law ENDO_ORTH_BETON *was conceived to describe in a finer way the damage of traction, the description of the remaining damage of compression isotropic. Another law of behavior must thus be used for applications making to intervene of the loadings of strong triaxial compression.*

When the trace of the deformations is positive, the threshold remains constant: $K() = K$. The threshold increases
0
when one passes in compression, which makes it possible to push back the elastic limit, and consequently
rupture limit. It is noted that the function "*arctan*" was introduced for represented the envelope better of rupture in the case of biaxial tests (a detailed study is in [bib1]).
[Figure 3.2-a] the comparison between the elastic limit shows us provides by a constant threshold and that obtained with a threshold depend on the trace of the deformations in the case of biaxial tests in plane constraint.

Appear 3.2-a: Wrap field of elasticity for biaxial loadings in plane constraint.

4
Study of the parameters

In addition to elastic parameters traditional E (Young modulus) and (Poisson's ratio), it model utilizes 6 additional parameters:

Code_Aster	Dimension*	function	Identification **
ALPHA	Without	Parameter of coupling	1
k0	MPa	Constant part of the threshold	K0

2
k1
K1
Parameter of the threshold
MPa
3
k2
K2
Parameter of the threshold
Without
3
B
ECROB
Blocked energy relating to traction
MJ/m3=MPa 2
D
ECROD
Blocked energy relative to
MJ/m3=MPa 3
compression

* One multiplies the parameters in MégaPascals (MPa) by 106 if one works in Pascals (Pa).
** The parameters must be gauged in the following order:

· 1 one fixes the parameter
· 2 identification of k0 and B on a simple tensile test
· 3 identification of k1, k2 and D on an unconfined compression test and a biaxial test
(= with =-0,2 to check that the breaking stress in traction is not
1
2
exceeded)

Let us study now a little more in detail the influence of the various parameters on the answer of model.

4.1
Influence parameter

The role of the parameter is to control the report/ratio of influence of the two thermodynamic forces associated in the criterion of evolution. A parameter close to 1 privileges the evolution of the damage of traction and a parameter close to 0 privileges the evolution of the damage of compression. We decided to take a constant parameter for reasons of simplicity.

If one damages in simple compression in direction 1, one creates cracks in the plans orthogonal with *er* and *er*. If one makes then a traction in direction 2 or 3, one “sees” these
2
3
cracks. To obtain this effect in the model, it is necessary that a compression generates not only one damage of compression, but also of traction. If one starts on the other hand with one traction in direction 1, the cracks in the plan perpendicular to *er* will be not very open
1
(weak deformation due to the rupture), therefore one can think that it them “will not be seen” not if one does then one
compression in direction 2 or 3. One concludes from it that it is necessary to take 0 and 1 to have one coupling, and close to 1 to support the evolution of the damage of traction at the time of compressions. There will be also a small evolution of the damage of compression at the time of tractions, deprived of physical direction, but that will not be awkward if this damage remains weak.

4.1.1 Tests uniaxial

One proposes here to observe the influence of the parameter in the case of the simple traction and of simple compression. The other parameters of the model are taken constant for our series of tests:
E = 28800 MPa, = 0.2, k0 = 3.10-4 MPa, k1 = 10 MPa, k2 = 2.10-4, B = 0 MJ/m3, D = 0..06 MJ/m3.

The parameter has an influence
relatively weak on the peak of constraint
in traction for the range of values
regarded as one can observe it on
[Figure 4.1.1-a]. One remains indeed in
the case where the thermodynamic force
associated the damage of traction
is dominating in the criterion (it
would not be the case if one took near
from 0)

**Appear 4.1.1-a: Influence parameter has
in simple traction**

For compression, one observes an important difference of peak of constraint (cf [Figure 4.1.1-b]). The closer is to 1, the more the constraint threshold is high. This phenomenon is accentuated when one takes a threshold depend on the deformations as it is the case on [Figure 4.1.1-b].

Note:

*The fact that the constraint threshold is more sensitive in compression than in traction comes owing to the fact that
one takes a value of close relation of 1, privileging the damage of traction. The effect would be reversed if one took near to 0.*

The parameter also influences the relative speed of evolution of damages via the law of normality of the flow. The closer is to 1, the more the side damage of *Dyy* traction increase quickly compared to the damage of compression *D* as one sees it on [Figure 4.1.1-c].

Appear 4.1.1-b: Influence parameter on a simple compression

Appear 4.1.1-c: Evolution of the damage of traction per report/ratio with the damage of compression for a simple compression

4.1.2 Warning

In an unconfined compression test, cracks are created orthogonally with the directions of positive deformation, and influences later behaviour in traction. We introduced it coupling in order to represent this phenomenon. One however sees on the unconfined compression test that the side damage of traction does not reach the ruin (lim *D* < 1) when the damage of *yy* compression *D* tends towards 1. This represents a limitation of the model.

It is however possible to reach the ruin for a value moreover nearer to 1. The damage of traction then will evolve/move more quickly than the damage of compression (cf [Figure 4.1.2-a]). Unfortunately the answer stress-strain then reveals one snap-back (cf [Figure 4.1.2-b]), deprived of physical direction, so that it is necessary to exclude these values from.

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Appear 4.1.2-a: Effect of a parameter very close to 1 on the answer stress-strain in simple compression

Appear 4.1.2-b: Effect of a parameter very close to 1 on the relative evolution damages of traction and compression in simple compression

4.1.3 Identification of the parameter

There is a breaking value of parameter beyond which one falls on the disadvantages statements in the section [§4.1.2]. This breaking value depends on the other parameters but us let us not have an empirical formula allowing to find it. One will thus prefer to use a value of around 0.9 which makes evolve/move the damage of compression more quickly than the damage of traction in the unconfined compression test, so that one then cannot to observe the ruin in traction in the directions perpendicular to that of compression.

4.2
Influence parameters B and D

The introduction of a blocked energy depend on the variables of damage makes it possible to control the speed of evolution of the damage, and this fact makes it possible to control the shape of the curve stress-strain.

4.2.1 Simple tensile test

In the simple tensile test, only the part of blocked energy relating to the damage of traction has a true influence. The stress-strain curve is represented on [Figure 4.2.1-a] for various values of B. One sees that the larger B is, the more the constraint of rupture is large, and more the peak is broad. This is due to the fact that blocked energy slows down the evolution damage, as shows it it [Figure 4.2.1-b].

Appear 4.2.1-a: Influence energy blocked on a simple tensile test

Appear 4.2.1-b: Influence energy blocked on the speed of evolution of the damage

---

4.2.2 Unconfined compression test

In the unconfined compression test, only the part of blocked energy relating to the damage of compression a true influence has. The parameter D is however less easy to gauge fact that the threshold of the criterion depends on the state of deformation. We immediately will explain this point in the following warning.

Note:

Theoretically, the warning that we will expose is also valid in traction. In practical, we let us not be confronted there because the damage of traction is always privileged by taking a parameter of coupling close to 1, and one does not take a value of D too much large (the behavior of the concrete in traction is quasi-fragile).

4.2.2.1 Warning

The introduction of blocked energy must make it possible to slow down the evolution of the damage of compression, and of this fact must make it possible to round the shape of the peak in a test of compression simple. A warning must however be formulated concerning the use of this energy blocked. The thermodynamic force which controls the evolution of the damage of compression is summon of two terms: one corresponding to the derivation of elastic energy, depend on deformation and of the damage, positive, and the other corresponding to the derivation of energy blocked, depending only on the damage, negative. However, when the damage increases, it may be that the term corresponding to blocked energy is too large in absolute value compared to that corresponding to elastic energy, which can prevent the evolution of the damage.

This problem appears when one considers a constant threshold in the criterion. [4.2.2.1 Figure - has] us show in this case the influence of the introduction of energy blocked on the forced curve deformation in an unconfined compression test. It is observed that the introduction of this energy, in slowing down the evolution of the damage of compression, allows well initially to increase the height of the peak of constraint as well as the width of the peak. One notices however what appears a snap-back on the two curves where a blocked energy was added. This is due to fact that one slows down the evolution of the damage of compression but which one does not act on the damage of traction. This is illustrated on [4.2.2.1 Figure - B], which shows a stabilization of the damage of compression combined to a fast increase in the damage of traction.

Appear 4.2.2.1 - has: Influence energy blocked in an unconfined compression test

Appear 4.2.2.1 - B: Evolution of the side damage of traction according to the damage of compression

4.2.2.2 Combination of blocked energy and the threshold depend on the deformations

When one uses a threshold depend on the state of deformation, the force associated with elastic energy remain more important than that associated with blocked energy because, for the same state of damage, the criterion is reached for more important levels of deformation. One sees in this case the influence of the introduction of an energy blocked on the response of the model in compression.
That makes it possible well to control the shape of the peak (cf [4.2.2.2 Figure - has]), i.e. the constraint of rupture as well as the strain at failure. One in addition sees on [4.2.2.2 Figure - B] that more it parameter associated with blocked energy is large, plus the side damage of traction evolves/moves by report/ratio with the damage of compression. It is however obvious that if a parameter is taken D too large, one finds the problems stated before.

Appear 4.2.2.2 - has: Influence energy blocked in an unconfined compression test

Appear 4.2.2.2 - B: Evolution of the side damage of traction according to the damage of compression

4.2.3 Identification of the parameters

The parameter B is identified on the simple tensile test. It makes it possible to regulate the height and the width of peak for this test. It must be regulated at the same time as the parameter k0 threshold.
The parameter D is identified on the unconfined compression test. It makes it possible to regulate the height and width of the peak for this test. It must be regulated at the same time as the parameters k1 and k2 of the threshold.

4.3
Influence parameters of the function threshold

The function threshold which one uses utilizes three parameters:

K ()
tr
= K -
0
1
K (*tr*)
( )
-
- arctan -

éq
4.3-1

k2

In the case of a simple traction, only the parameter k0 intervenes. Since the trace of deformations becomes negative, as it is the case in simple compression, the three parameters intervene. The parameter k0 must thus be gauged beforehand on a simple tensile test. parameters k1 and k2 will be then gauged on an unconfined compression test and a biaxial test, with k0 fixed.

4.3.1 Simple traction: influence parameter k0

If one takes a blocked energy not depending on the damage of traction (B=0), the value breaking stress is completely determined by the value of k0, and the parameters rubber bands:

K E
2
0

=

éq
4.3.1-1
rupture
4

+ 2 (1 -) (1+) 2

Appear 4.3.1-a: Dependence of the breaking stress in traction with respect to the parameter k0 (E=32000 MPa, =0.2, =0.87)

In an obvious way, more k0 is large, more the breaking stress in traction is large like show it [Figure 4.3.1-a].

There does not exist unfortunately of analytical expression of the breaking stress when one B0. dependence of the response with respect to the parameter B is studied in the paragraph [§4.2.1]. parameters k0 and B must be gauged simultaneously.

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4.3.2 Compression: influence parameters k1 and k2

The role of the function threshold, which depend on the negative trace of the deformation, is to increase rupture limit in compression, in order to better control the dissymmetry of the behavior of the concrete between traction and compression.

Moreover, we made the choice take a function threshold allowing to describe the envelope of rupture of the concrete in the case of biaxial loadings. This choice was made for two reasons:
We have for these tests experimental results (cf [bib5]). These tests concern certainly of course only the concretes used by [bib5], but present common characteristics that one seems it can to generalize.
That makes it possible to widen the range of use of the model. The uniaxial tests are indeed insufficient to ensure the relevance of the model in the case of calculations in 3D.

Note:

*The model should not in the case of be used strong triaxial compressions, the free energy and the function threshold not having been established to treat this case. It is pointed out that the principal objective of
model is to describe the damage of traction in the concrete.*

4.3.2.1 Role of the parameter k1

The parameter k1 is the parameter which makes it possible to increase the rupture limit in compression. [4.3.2.1 Figure - has].

Appear 4.3.2.1 - has: Dependence of the response in simple compression with respect to k1 (E=32000MPa, =0.2, k0=3.10-4MPa, k2= 6.10-4, d=6.10-2MJ/m3)

4.3.2.2 Role of the parameter k2

To include/understand the role of the parameter k2, let us reconsider the advance which led us to choose the function threshold [éq 3.2-1].

The first function threshold having been tested is the linear function:

$K() = K - K$

**éq
4.3.2.2-1**

0
1 (*tr*) -

This function makes it possible well to increase the breaking stress in simple compression, but it poses problem when one is interested in biaxial tests. [4.3.2.2 Figure - has] the envelope shows us of elastic range for biaxial tests.
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Note:

The envelope of the field of elasticity is different from the envelope of rupture (for which us let us have experimental results). We study the envelope of the field of elasticity because it can be calculated analytically, contrary to the envelope of rupture, and bus it is from this one which we chose our function threshold. The user will have to gauge however well its parameters on the envelope of rupture.

Appear 4.3.2.2 - has: Wrap field of elasticity for biaxial tests with a linear function threshold

The linear variation of the threshold with the deformation does not seem not adapted since one sees appearing one
swelling in the zone (>,
0 < 0 which gives ultimate stresses in traction too much
1
2
)

important.

We thus modified the function threshold:

$$K(\text{tr}) = K_0 - \frac{1}{k_1}\left(K(\text{tr}) - K_0\right) - k_2 \arctan\left(-\text{tr}\right)$$

éq 4.3.2.2-2

k2

The fact that the function arctan presents a stage makes it possible to find a linear threshold when the trace
deformations increases in absolute value. The fact that it is null in the beginning makes it possible to slow down
increase in the threshold close to this origin.

One can see on [4.3.2.2 Figure - B] the effect of the introduction of the new function threshold compared to
linear function on the field of elasticity for the same value of the parameter *k1*. This news
function threshold makes it possible to avoid the phenomenon of swelling observed with the linear function when one
increase the value of the parameter *k2* (the case *k2=0* corresponds to the linear function). The other effect of
parameter *k2* is to decrease the constraint of initiation of the damage, which implies one
reduction in the breaking stress (opposite effect of *k1*) as shows it it [4.3.2.2 Figure - C].

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Appear 4.3.2.2 - B: Field of elasticity with the function threshold with two parameters

Appear 4.3.2.2 - C: Dependence of the response in simple compression with respect to k2 (E=32000MPa, =0.2, k0=3.10-4MPa, k1= 10.5 MPa, d=6.10-2MJ/m3)

4.3.3 Identification of the parameters

The parameters D, k1 and k2 are identified simultaneously. The parameters k1 and D make it possible to regulate
the breaking stress in simple compression and the parameter k2 make it possible to avoid the "swelling" of
the envelope of the elastic range (and thus of the envelope of rupture) for the biaxial tests. More one
takes k1 large, more it is necessary to take k2 large.

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4.4
Assessment on the study of the parameters

In spite of the relative simplicity of the model, and the low number of parameters to be identified (6), the number of
experimental data available to the engineer is often, even always, lower than the number of parameters to be identified. This implies that the arbitrary character of the choice of certain parameters.
Here, in short the order in which the parameters must be selected:

· For the parameter, a value between 0,85 and 0,9 is recommended.
· The parameters k0 and B must be identified simultaneously on a simple tensile test (not of analytical formula in the case B0).
· The parameters D, k1 and k2 must be identified simultaneously. Simplest is to fix it parameter k2 to 6.10-4 (value for which it is probable that one does not observe swelling envelope of rupture (cf [§ 4.3.2.2]) for biaxial tests), and to gauge D and k1 on one unconfined compression test. One checks then if the envelope of rupture is correct, one modify k2 if need be and one starts again for D and k1.

Examples of sets of parameters are in section 6 (validation on tests
experimental). One will find moreover in the document [V6.04.176], the case-test allowing to identify
parameters.

5 Establishment
numerical

For the integration of the law of behavior in *Code_Aster*, we placed ourselves in tally of the implicit integration of the laws of behavior.

5.1
Evaluation of the damage

One notes -
deformation and -
B and -
***D* variables of damage at the end of the step of time**

precedent (after convergence). One wishes to determine the evolution of the damage when one apply an increment of deformation
. Final deformation = - +
is thus fixed.
The criterion of damage is evaluated:

2
***F* (, B**
2
, *D* -) = BFR +
-
(1) (*F d*+) - *K* ()
éq
5.1-1

If *F* 0, the variables of damage do not evolve/move and one can pass to the following iteration for mechanical balance.

If *F* > 0, the variables of damage evolve/move, by respecting at the same time the criterion and the flow
normal. One thus seeks (B, *D*,
) solution of the system:
- B +
B
-
F (-
B + B) 0

R (B, *D*,) = - *D* + (1 -) *d*+
***F* (-**
***D* + *D*)**

0
éq
5.1-2

***F* (**
-
, B +
-
B, *D* + *D*)
=

0

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This system is nonlinear and requires the use of an iterative method. We chose
to use a method of Newton-Raphson. Index *N* represents the iterations of Newton. That is to say
N
R (
N
N
N
B, *D*
,
) the residue of the system to iteration *N*, the linearization of the system are written:

***N* +1**
N
B
- B
n+1
R (
n+1
n+1

n+1
N
N
N
N
N
N
N
N
B, *D*,
) = R (B, *D*,) + (R (B, *D*,)

:
***N* +**
D
1 - *N*
D

***N* +1**
N

-

éq 5.1-3

- *N*
B + *N*
B *N*
F

-

where
N
R = - *N*
D* + *N
(1 -) *D N*
F

***F* (-**
, B + *N*
-
B, *D* + *N*

***D*)**

B
B
***N* F**
F
-
B

- I +
:
0
F
B
-

F
B

N

F D
N
F D
D
and R
=
0
-1+ (1-)
+

(1-)

F
.
D
+

F D

B
F
B
F
1 ***F D***
F D
-
(-)

+
:
0

2
B
F
+
2
2
2
1 ***F D***
B
B
F
1
D
D
F
-
(-)

+
+
-
(-)

+

One solves then the system linearized Rn+1 (B
n+1, n+1
D
, *n+1*
) = 0. One obtains
(*N* 1+ *N* 1+ *N* 1
B, *D*,
+

). This procedure is reiterated until the residue is lower than one
parameter of convergence.

5.2
Calculation of the tangent matrix

The tangent matrix is the tensor *M* of a nature 4 defined by:

ij
= M
M =

éq
5.2-1
ij
ijkl
kl
ijkl

kl

This matrix is not calculated on the continuous problem but on the incremental problem. One seeks
thus effects of a variation of the increment of deformation between two steps of successive times on variation of final constraint, taking into account the fact that the internal variables can also evolve/ move. One
has with the first order:

***ij* (, B, *D*)**

ij
ij
=

+
B
ij

+
D
kl

éq
5.2-2

B
kl

D
kl
kl

B, D
, D
, B

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that is to say:

B

D
, B, =

+
+

***ij* (**
***D*)**
ij
ij
mn
ij

***kl* éq 5.2-3**

kl
B

mn

kl
D

kl

B, D

, D
, B

With

The notation
employee here means that one derives A compared to B for C and D constant.
B
C, D

One thus breaks up the tangent matrix into two parts, one with constant damage, and the other representing the evolution of the internal variables:

cst
evol
M = M + M
éq
5.2-4

B

D

with
cst
ij
M
=
and
evol
ij
mn
ij
M
=
+

ijkl

ijkl
B

D

kl B, D
mn
kl
, D
, B
kl

5.2.1 Term with constant damage

The matrix
cst
M
is the derivative of the constraint compared to the deformation with damage constant:

cst
M
= H trB B B + F (D) H - tr
ijkl
(
) *ij kl*
(
) *ij kl*
µ *A*
+ip
+
(*B + B + B + B B*

mk nl
mk
nl
ml nk
ml
nk*) *pj
4 *A*
mn
µ
With

éq
5.2.1-1
+ *pj*
+ *B*
B* + *B* + *B* + *B
IP
(*mk nl mk nl ml nk ml nk*)
4
With
mn

- *ij*
+ 2µf (*D*) *kl*

With

where A = B + B
, *H* is related to the Heaviside and the derivative
- and
+ are defined in

With

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5.2.2 Term related to the evolution of the damage

The matrix
evol
M
is calculated by differentiating the criterion and the law of normality. This derivation is done on the incremental problem and not on the continuous problem.
The criterion first of all is differentiated:

B
B
D
D
B
B
F F

***F* (**
2
D
D
+ - *F F*
F, *F*) = F
+
F
K
K
-

(1-)(+)2 - ( )
:
-
(1)
0
+
-
=
diff
2
B
F
+
F D
-
(-)(+)
0
1
2
éq 5.2.2-1

One differentiates then the law of flow of the incremental problem, discretized in an implicit way:

B = B
F

(1 -) *F dB* = B
F
D

+
-
D
= (1 -) *F D*
+

éq 5.2.2-2
(1 -) *F dB* + 1
B B
F
B
F
+

(-) *Fd* =
D
+
D

+
-
-
diff

One seeks the relation between B, *D* and. One can express the variations of the forces thermodynamic according to the variations of the deformations and the variables of damage:

B
BFR BFR
BFR

F =
- :
:
: B
-
B

+

F

B
éq
5.2.2-3
D
F D F D
F D

***F* =**
+
:

D
+
D
F

D

The system of equations defined by [éq 5.2.2-1], [éq 5.2.2-2] and [éq 5.2.2-3] leads to the expressions following:

B
= -1 : :

éq
5.2.2-4
-1

D **= -**
+ : : :

with

D
B
B
B
B
B
F

B

F

F

F

F

F

1
+
-
B
=
-

F

-
:
:
+ *D*

- :
- 1 - *F* +
ijkl
-

(
) *D*
D
D
D
+ (
B
B
ik
jl
it
jk)
F

F

2
+
(1-)

D
F
F
B
F
B

kl

ijkl
F

D
ij

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B
D
B

B

F

F

B

2
2

-
+
B
F
F
F
K
=
-

F
-

:
:
+ 1 - *F*
- *F*
+ 1 - *F*
ijkl
-
(
)
D
D
B
+
-
(
)
D
D

D
B
(*d*+)
(

1-)

D
F
F F
F
+
kl
F

D

ij
+ (1-)
D
D
B
B
F

F

F

F

+
B

- *D*

- :
D
ij

B

F

F

kl

ijkl

B
B

2
2

B
F
F
F
K
= F
-

:
:
+ 1 - *F*
- *F*
+ 1 - *F*

ij
-
(
)
D

D
B
+
-
(
)
B
(*d*+)
F

ij

B
B

B
F
F
= *F*
-
:
:

ij
-
B

F

B

ij

= (1-)

D
F D
F+

D

-1

1
1
such as -
= +

ijkl
klmn
(*im jn in jm*)
2

B

D
One thus obtains the expressions of
and of
.

Moreover, according to the definition of the constraint and the thermodynamic forces, which derive from one
even energy, one a:
B

F

D

=
F
and
=

éq
5.2.2-5
B

D

, *D*
B
, *B*
D

what enables us to calculate the part of the tangent matrix relating to the evolution of the damage:

B

evol
ij
D
mn
ij
M
=
+

éq
5.2.2-6
ijkl
B

D

mn
kl
, D
, B
kl

Note:

It should be noted that the tangent matrix is not symmetrical. This is due to the fact that the threshold on damage depends on the deformation. In the case of a constant threshold, the matrix is well symmetrical, since the model quite standard is then generalized and a diagram is used of implicit integration.

---

6 Validation

The first stage of the validation of the model is the comparison between the prediction and different

experimental results for simple tests. Taking into account the simplifying assumptions which have summer made for our modeling, we will not be able of course to reproduce all them experiments, in particular experiments of triaxial compression. One concentrates here on the tests of simple traction, simple compression, like on biaxial tests.

6.1 Identification of the parameters

The identification of the parameters must be done in three successive phases:

· One must choose a value for the constant of coupling. It must be taken of such kind that the scalar damage of compression remains negligible in a tensile test. One decides to take it equal to 0,87 for all the tests of this section in order to avoid it phenomenon of snap-back in compression (cf [§4.1.2]).
· The second phase is the identification of the parameters k0 and B. These parameters can be identified directly on the tensile test simple bus other parameters of the model do not intervene in this test.
· The third phase is the identification of the parameters D, k1 and k2 on the tests of simple compression and biaxial tests.

In any rigour, the identification of the parameters of our model for a material requires to have the experimental curves in simple traction, simple compression and under loading biaxial. Unfortunately, all these results are generally not available simultaneously for a material. We will have to thus make a certain number of assumptions to gauge our parameters. For example, we will choose the parameter k2 for all the tests in such a way that tensile stresses in the biaxial tests do not exceed the breaking stress in traction. Moreover, we do not have for the tests of compression the breaking stress in traction for studied materials. We will thus take in an arbitrary way of the parameters k0 and B equal to those which we calculated for the simple tensile test.

6.2 Traction simple

The experiments aiming at observing the behavior of the concrete under loading of traction are extremely difficult to realize, which explains the relatively low number of results of tests in simple traction. The difficulty lies in the fact that the damage concentrates in bands of localization corresponding to cracks, which causes to make inhomogenous the specimen studied. Since strong heterogeneities appear, it becomes impossible to deduce one stress-strain curve starting from the curve force-displacement, and thus to establish a law of behavior for material. Apparatus FOOT the measuring ([bib6], [bib7]) allows to measure one relatively homogeneous deformation and stress field by limiting the localization. It is why we use the results obtained by [bib6] testing our model.
In the case of simple traction, only 3 of the 6 parameters of the model will play a part:

· The parameter of coupling.
· The constant of threshold k0.
· The constant of blocked energy associated the damage of traction B.
· The parameters k1 and k2 do not have any influence on this test and the influence of the parameter D is
negligible.

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The parameter k0 governs the height of the peak of constraint and the parameter B controls its width as well as
slope post-peak. The larger B is, the less the damage evolves/moves quickly, which reveals non-linearity before the peak and increases the deformation with the peak and its width. [Figure 6.2-a] shows
response of the model, compared with the experimental data of [bib6]. Calculation is carried out on one only element not to meet a phenomenon of localization. The parameters are as follows:

k0 (Mpa)
B (kJ/m3)
0.87 3.10-4 7

Appear 6.2-a: Simple tensile test, comparison with the experimental data of Bazant and Pijaudier-Pooch [1989]

6.3 Compression simple

The fact that the experimental data are more numerous than for the tensile tests is due to fact that they are easier to realize. The phenomenon of localization is much less important there that for a loading of traction, at least when the damage remains relatively weak. One use the results of Hognestad and Al [bib8] and the test of Ramtani [bib9] to validate our model on unconfined compression tests. In spite of the fact that the results of [bib8] are relatively old, we use them because the experiment was undertaken for several different concretes. We use too results of [bib9] to show that the model remains valid for more recent experiments.

As we said to the paragraph [§6.1], we do not have the breaking stress in traction for these various tests, this is why we use the same parameters as those obtained in the paragraph [§6.2]: = 0.87, k0 = 3.10-4 Mpa, B = 7 kJ/m3.

6.3.1 Hognestad and Al [1955]

[Figure 6.3.1-a] shows us the comparison between the experimental results of [bib8] and prediction of our model in the case of three materials of which the maximum constraint in value absolute FC is noted:

We used the following parameters:

Ultimate constraint FC
20.7 Mpa
32.1 Mpa
42.8 Mpa
E
17000 27000 36000

0.2 0.2 0.2
D (kJ/m3)
60 60 60

k1 (Mpa)
4.8 10. 18
k2
7.10-4 7.10-4 7.10-4

Appear 6.3.1-a: Unconfined compression tests of Hognestad and Al [1955]

6.3.2 Ramtani
[1990]

The test of [bib9] is a cyclic test of compression. It highlights the creation of deformations irreversible and the phenomenon of hysteresis. We do not describe these phenomena. Us thus let us satisfy for our part to calculate the response under monotonous loading. Parameters used are as follows:

E

D (kJ/m3)
k1 (Mpa)
K2
33700 0.2
60
20.5 7.10-4

Appear 6.3.2-a: Unconfined compression tests of Ramtani [1990]
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6.4
Simple traction followed by a simple compression

One proposes here to compare our model with the experimental results of [bib9] on a test of traction simple follow-up of a simple compression, in order to highlight the restoration of rigidity generated by refermeture of the cracks.

The parameters used are as follows:

E

k0 (Mpa) B (kJ/m3) D (kJ/m3)
k1 (Mpa)
k2
16400 0.2 0.87
7.
10-5 0.3
40
5.5
6.10-4

The experimental results show the appearance of unrecoverable deformations in the phase of simple traction (cf [Figure 6.4-a]). These unrecoverable deformations are not described by our model, this is why the loss of rigidity generated by the damage seems over-estimated. It problem does not seem to have of incidence when the cracks are closed. One observes indeed on [Figure 6.4-b] a good correspondence of the model with the experimental results in the phase of compression. It seems as well as the restoration of the rigidity obtained thanks to the model is very
near to that obtained in experiments.

Appear 6.4-a: Simple phase of traction in the test of Ramtani [1990]

Appear 6.4-b: Tensile tests followed by a simple compression (Ramtani [1990])

6.5 Tests
biaxial

This paragraph is devoted to the study of the biaxial tests of [bib5]. One seeks on the one hand to observe
the answer stress-strain in the case of a uniaxial loading and of two loadings
biaxial in plane constraint, and in addition to describe the envelope of the field of rupture in space constraints in the case of biaxial loadings in plane constraints.

Note:

The experiments of [bib5] are relatively old. These biaxial tests require
to carry out a great number of tests, this is why one finds few more recent results on
this kind of tests. We use them because they always represent a reference for
modélisateurs.

The maximum constraint in absolute value of the uniaxial pressing, noted p in [bib5] is worth 4650 psi (32.1 Mpa). We standardize our response by this breaking stress. Parameters that
we use are as follows:

E

k0 (Mpa)
B (kJ/m3)
D (kJ/m3)
K1 (Mpa)
K2
32000 0.2 0.87 3.10-4 1
60 10.5 6

6.5.1 Answer
stress-strain

One traces the answer stress-strain initially in the case of a loading uniaxial and of two biaxial loadings in plane constraint:

= with = (0, 0.52, 1) and
2
1
= 0
3
.

The model enables us to obtain the answers represented [Figure 6.5.1-b] which one compares with results of [bib5] represented [Figure 6.5.1-a]. As one saw in the preceding paragraph, it is possible to correctly describe behaviour in uniaxial pressing in the direction of compression. One however sees for this test that the lateral distortion predicted by the model decrease when the damage occurs whereas it actually increases. This is due to the fact that one did not take into account the existence of unrecoverable deformations dependent on the damage, which seems to be at the origin of voluminal dilation in compression. It phenomenon is even more important in the case of the tests of bicompression. One observes indeed with the model that the threshold of rupture in bicompression is higher than that in simple compression
(less than the experimental results), but that the strains at failure are less important in the case of bicompression that for compression, which does not correspond to experimental results.

Appear 6.5.1-a: Test biaxial of Kupfer and Al [1969]
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Appear 6.5.1-b: Response of the model for the biaxial tests

6.5.2 Wrap field of rupture

One is interested now in the envelope of the field of rupture for biaxial tests in constraint plane. The experimental results obtained by [bib5] for various concretes are represented on [Figure 6.5.2-a]. One observes a relative similarity of the shape of the envelope of standardized rupture
for various materials.
We took again the parameters used in the paragraph [§6.5.1] and we compared the prediction of our model for the envelope of rupture of the biaxial tests with the experimental results (cf [Figure 6.5.2-b]).

Appear 6.5.2-a: Wrap rupture for biaxial tests
in plane constraints Kupfer and Al [1969]

Appear 6.5.2-b: Prediction of the model for the envelope of rupture
biaxial tests in plane constraints

One observes a relatively satisfactory correspondence of the prediction compared to the results

experimental. The most important variation is in the zone of bicompression. This problem is not not astonishing insofar as we took the party to use only two parameters for threshold, which controls the pace of the envelope of rupture as well as the response of the model in compression.
A thorough study of the function threshold would probably make it possible to better approach them experimental results, at the price of the introduction of new parameters.

Appendix 1

A1.1 Definition of the spectral decomposition and the positive parts and negative of a tensor

That is to say X a tensor of order 2 symmetrical. Not to weigh down the notations, X will be also noted,
~
wrongly, the matrix of this tensor in the base fixes observer. Maybe moreover X the matrix (diagonal) this tensor in its own base:

~
X
0

0
1

~
X =
~
0
X
0

éq A1.1-1
2

~
0
0
X

3

While noting *U the* clean vector associated the ième eigenvalue, and Q = (*U, U, U the* matrix of passage enters
1
2
3)
I
the base fixes and the clean base of X, one with the relation:
~ *T*
X = Q X
. Q
.

éq A1.1-2

The parts positive and negative of tensor X are defined by:
***H* (~ ~**
X X
0
0
1)

~

1

T
~
~ ~
X = P: X = Q X
.
Q
.
X =
0
H X X
0
+
(
éq
A1.1-3
2)

+
+
+
with
2

0
0
***H* (~ ~**
X X
2)

3

***H* (- ~ ~**
X X
0
0
1)

~

1

T
~
~ ~
X = P: X = Q X
.
Q
.
X =
0
H
X X
0
-
(-
éq A1.1-4
2)

-
-
-
with
2

0
0
***H* (- ~ ~**
X X
2)

3
where *H* is related to Heaviside.

A1.2 Calculation of the derivative

For the calculation of the tangent matrix, like for the calculation of the evolution of the damage, we have
need to evaluate the derivative of the parts positive and negative of a tensor compared to this last. It is enough for
that to imagine that tensor X depends on time and to calculate the tensors M
M
+ and
- defined by:
X& = M: X
X& = M:
-
-
&
+
+
& and
X

éq A1.2-1

The differentiation of the equation [éq A1.1-2] gives us:
~
T
~
T
~
T
X = Q X&
&
.
Q
.
+ Q& X
.

Q
.
+ Q X
.
Q
. &
+
+
+
+

éq A1.2-2
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Assumption:
It is necessary to establish the expression of Q
&. The demonstration which will follow is valid only if the eigenvalues of X
are distinct. Insofar as the calculation of M
M
+ and
- will be used only in algorithms of
numerical resolution, we will allow ourselves to disturb possible eigenvalues numerically

identical in order to make them distinct and to be able to use the results below.

To calculate Q
&, one needs to express the derivative of the clean vectors *U&*. For that, according to the step of *I*
[bib11], the expression of X is differentiated:

X = ~
~
~
***X U U* X =**
X&
&
~
U
U
X U&
U
X U
***U&* éq**
A1.2-3
I
I
I

+

+

I
I
I
I
I
I
I
I
I
I
I
I

Moreover, one has the following relations between the clean vectors and their derivative:

U U

.
=, *U&*. *U* +U. *U&* = 0

éq A1.2-4
I
J
ij
I
J
I
J

By contracting the equation [éq A1.2-3] on the left and on the right by the clean vectors, and by using the relations
[éq A1.2-4], the following relations are obtained:
~
***U*. &**
~
XU
.
X* = *X&
&

éq A1.2-5
I
I
() *II I*

~
~
~
~
~
***U*. &XU**
.
&

=
. & +
&.
=
-
&.
for
éq A1.2-6
J
K
(*X*) *X U U X U U*
jk
K
J
K
J
J
K
(*X X*
J
K*) *U U
J K
J
K

(~X&)
In these expressions, there is no summation on the indices, them
the components of X indicate
&
jk
(~X&) =U. X&U. , and *X*&~
jk
J
***K*)**
in the fixed base coinciding with the clean base of X at the moment considered
I
indicate the derivative of the eigenvalues of X (not the eigenvalues of derived X
&.

One deduces from the relation [éq A1.2-6] the expression of *U*&:
J
(~X&)
***U*& =**

U& U
.
U
.
~
~ *U*

éq A1.2-7
J
(*J K*) =
K

jk
K
K J
K J* (*X* - *X
J
***K*)**
This enables us to express Q
&:
(~X&)
&Q = (*U&*, *U&*, *U&*
&

éq A1.2-8
1
2
3) *Q*
=
jk
~
~ *Q*
ij
ik
K J* (*X* - *X
J
***K*)**
(~X&)

, components of X

& in the fixed base coinciding with the clean base of X at the moment considered,
jk
~
can express itself according to the components of X
& in the fixed base. Thus, X& indicating the matrix of
(~X&), one a:
jk
~
X& QT
=
X
. & Q
.

éq A1.2-9
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One deduces from the expression [éq A1.2-8] the expression of Q
&:

Q Q X&
Q& =
~
~
Q

éq A1.2-10
ij
mj nk mn ik
K J m, N (X - X
J
K)
There are finally the obvious relations between the eigenvalues of X+ and X:
~
~ ~
~
~
X = H X X X&
~
= H X X&
+
éq
A1.2-11
I
(*I*) *I*
+i
(*I*) *I*

One deduces from the relations [éq A1.2-5], [éq A1.2-9] and [éq A1.2-11], the following relation:

~
~ ~
~ ~
X&
~
= H X X& = H X X& = H X Q Q X&
+

éq
A1.2-12

I
(*I*) *I* (*I*) () *II* (*I*) *ji ki jk*

The relations [éq A1.2-10], [éq A1.2-11] and [éq A1.2-12] allow us to express the relation [éq A1.2-2]:

~
Q Q
~ ~
km
ln
X&
Q Q Q Q H X
X&
~
~ *H X X Q Q*
Q Q
X&
+
éq A1.2-13
ij

=
***im jm km lm* (*m*)**
+
kl
(*m*) *m* (
+
in
jm
im
***jn*)**
K, L m

kl
K, L
m, N X
-

X

m

N
mn

As X is a symmetrical tensor, one a:
1
1
X = (
T
X + X) X& = (
T
X& + X&)
éq
A1.2-14
2
2

This allows us récrire the equation [éq A1.2-13]:
~
X&
Q Q Q Q H X
X&
+ij

=
***im jm km lm* (*m*)**

kl
K, L m

~ ~

éq
A1.2-15
+ 1
H X X
m
m
&
2
(*Q Q* + *Q Q*
~
~

Q Q
Q Q
X
km
ln
kN
***lm*)**
( ) (
+
in
jm
im
***jn*)**
kl
K, L
m, N
X -

X

m
N
mn

One deduces the expression from it from the components of M+:
~
1

~ ~
H X X

M
=
Q Q Q Q H X +
Q Q +
m
m
Q Q
~
~
Q Q + Q Q

+

ijkl
***im jm km lm* (*m*) (*km ln kN lm*) () (*in jm im jn*)**
m
2 *m, N*
X - X
m
N

mn
éq
A1.2-16

By analogy, one easily deduces the expression from it from the components of m:
~
1

~ ~
H - X X

M
=
Q Q Q Q H - X +
Q Q +
m
m
Q Q
~
~
Q Q + Q Q
-

ijkl
***im jm km lm* (*m*) (*km ln kN lm*) (**
) (*in jm im jn*)
m
2 *m, N*
X - X
m
N

mn

éq A1.2-17
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Appendix 2

One wishes to show that when an eigenvalue of the damage reaches 1 in a direction, then this direction is blocked and the damage cannot evolve/move any more but in the orthogonal plan with this direction.

One places oneself in the clean base of the damage:
~
***D* = *D E E* éq A2-1**
I I
I
I
~
where *E* indicates the clean vector associated with eigenvalue *D*.
I
I

Adopted energy is the sum of an elastic energy and an indicating function of the eigenvalues of the damage:
(, D)
el

= (, D)
~
+ *I* - 1, [max (*D*

éq A2-2
***I*)]**
]
]

The criterion of evolution of the damage is written:
***F* (D**
F) = *tr* ((D
F) 2
K
+) -
0

éq A2-3

The evolution of the internal variable follows the following law of flow obeying to the principle of normality:

D
F
D& = &
=
F+
&
, .
0

éq A2-4
D
D
F
F: D

F
+
+

The derivative D
& is thus colinéaire with D
F+.

The thermodynamic force derives from the free energy [éq A2-2]:
~

el

***I*] -] (max**
1
(*Di*)
FD = -
= -
-

éq A2-5
D

D

D

Assumption:
~
It is supposed that the damage is worth 1 in direction 1: *D* = 1 and which it is different from 1 in the others
1
directions. The demonstration would be similar if the damage is worth 1 in two directions orthogonal.

The derivative of the indicating function compared to the damage is written:
***I* (**
(~
max *Di*)
***I* (~**

~
~
D
I D
D
D
1)
(1)

=
=
1
11
~

éq A2-6
D

D

D
D
D

1
11

~
***I* (~**
D
~
D

D
~
1)
However one a:
~
= + bus *D* = 1;
11

= ;
1 =1 bus (*D*)
D&
&
=
(see Appendix 1).
D
1
I
1
1 *J*
D
D
11
1
1
ij
11
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This enables us to write the derivative of the indicatrix compared to the damage:

T 0 0
I (~
Di)

-
= 0 0 0 with *T* = -

éq A2-7
D

0 0 0

The derivative of the elastic part of energy is with finished values. One writes it in the shape of a matrix 3x3
symmetrical:
has
has
has
11
12
13
el

-
=a
has
has

éq A2-8
12
22
23
D

has
has
has
13
23
33

This enables us to write the expression of the thermodynamic force:
T + has
has
has
11
12
13

D
F = *has*
has
has

éq A2-9
12
22
23

has
has
has
13
23
33

One now seeks to calculate the positive part of the thermodynamic force. For that, one must calculate them
eigenvalues of **D**
F and associated clean vectors. To carry out this calculation, us conveniently
let us regard the term *T* as very large negative (tending towards -) but not strictly infinite.

There are then *T* >> *has* for all indices I, J. One can thus write the matrix **D**
F in the form:
ij

1
O (1/T) O (1/T)
D

F *T* O (1/T) O (1/T) O (1/T)

éq
A2-10
O (1/T) O (1/T) O (1/T)

That is to say an eigenvalue of **D**
F and *U the* associated clean vector, then the components of *U* are solutions
following system:

U + O 1 *T U* + O 1 *T U* = *U*
(*I*)
1
(

) 2
(
) 3
1
T

D
F. *U* = *U*
(1/T) *U* + O 1/T *U* + O 1/T *U* = *U*
(*II*) **éq A2-11**
1
(
) 2
(
) 3
2
T

O (1/T) *U* + O 1/T *U* + O 1/T *U* = *U*
(*III*)
1
(
) 2
(
) 3
3

T
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Among the clean vectors, there are some at least for which the *U1* component is nonnull, because them clean vectors form a base of **R3**. Let us consider this clean vector. Then equations of the system [éq A2-11] impose:

(*I*)
= *T*
(*I*) and (*II*)
U = 0
2

éq A2-12
(*I*) and (*III*)
U = 0
3

One deduces from it that *T* is an eigenvalue of **D**
F and that the vector **e1** is the associated clean vector.

Moreover, the clean base of a symmetrical matrix being orthogonal, two other clean vectors of **D**
F
are in the plan defined by **e2** and **e3**.

Thus the U1 component of these clean vectors is null. Under these conditions, the equation (I) does not provide, in
limit *T* -, that the identity 0=0, and the equations (II) are reduced, after the multiplication per *T*, with:

U + has U = U has

22 2
23
3
2

éq A2-13
U + has U = U has

23
2
33
3
3

Thus clean vectors of **D**
F distinct from **e1** and the associated eigenvalues are the clean vectors and

has
has
has
11
12
13
has
has
22
23

the eigenvalues of projection *has*
=
matrix *has = has*
has
has
in the plan (**E**
2D

has
has
12
22
23
2, **e3**).

23
33

has
has
has
13
23
33
It results from what precedes that the positive part of **D**
F is worth:
0 0
0
D

F = 0
+

éq A2-14

0
(*a2D*)
+

Like **D**
& is colinéaire with **D**
F+, this implies that only the components D22, D23, D33 still can
to evolve/move, the D11 components, D12, D13 from now on being fixed.

---

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Modelings THHM. General information and algorithms
Summary
Modules THM of *Code_Aster* are those which treat the equations of the mechanics of the continuous mediums in
using the theory of the porous environments possibly unsaturated and by considering that phenomena
mechanics, thermics and hydraulics are completely coupled. We present the equations here
of balance, or conservation equations solved by these modules. We give a definition of generalized constraints and generalized deformations, allowing to define way rather general what is
law of behavior THM - at least what the modules considered consider thus - and allowing

to treat the nonlinear equations exhibées within the framework of the algorithms of operator STAT_NON_LINE.
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1 Introduction

Modules THM of *Code_Aster* are those which treat the equations of the mechanics of the mediums continuous by using the theory of the porous environments possibly unsaturated and by considering that them
mechanical, thermal and hydraulic phenomena are completely coupled.
We present here the equilibrium equations, or conservation equations solved by these modules. We give a definition of the generalized constraints and deformations generalized, allowing to define way rather general what of behavior THM - at least is a law what modules considered consider thus - and allowing to treat the nonlinear equations exhibées in tally of the algorithms of operator STAT_NON_LINE.
The laws of behavior THM strictly speaking are not developed in this document, but in the document [R7.01.11].

Phenomena chemical (transformations of the components, reactions producing of the components etc…), just as the radiological phenomena are not taken into account at this stage of development of *Code_Aster*. The mechanical, hydraulic and thermal phenomena are taken in count or not according to the behavior called upon by the user in order STAT_NON_LINE, according to the following nomenclature:

Modeling
Phenomena taken into account
KIT_HM
Mechanics, hydraulics with an unknown pressure
KIT_HHM
Mechanics, hydraulics with two unknown pressures
KIT_THH
Thermics, hydraulics with two unknown pressures
KIT_THM
Thermics, mechanics, hydraulics with an unknown pressure
KIT_THHM
Thermics, mechanics, hydraulics with two unknown pressures

The document present describes the laws of conservation for the case more general says THHM. Cases more
simple are obtained starting from the case general by simply cancelling the quantity absent.

2 Presentation of the problem: Assumptions, Notations

In this chapter, one mainly endeavours to show the porous environment and his characteristics.

2.1 Description of the porous environment

The porous environment considered is a volume made up of a more or less homogeneous solid matrix, more
or less coherent (very coherent in the case of concrete, little in the case of sand). Between solid elements, pores are found. One distinguishes the closed pores which do not exchange anything with theirs
neighbors and the connected pores in which the exchanges are numerous. When one speaks about porosity, it is many connected pores about which one speaks. Inside these pores are at the maximum
two components present at more under two phases. The system is regarded as closed.

2.2 Notations

The sizes associated with a component C present under a phase p are noted
p
X**. The index of**
C
component C can vary from 1 to 2 and that of the phase also.
The porous environment at the current moment is noted, its border
, and it is noted,
at the initial moment.

0
0
N
N

D
indicate the normal in a point of
, image of the normal with
. We will note (
)
0
0
(resp *D* (

) the element of surface of
(resp
).
0)
0
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The medium is defined by:
· of the parameters (vector position X, time T),
· of the variables (displacements, pressures, temperature),
· of the intrinsic sizes (forced and deformations, contributions mass, heat,
hydraulic enthali, flows, thermics…).
For the solid phase, one makes the assumption of small displacements.
The various notations are clarified hereafter.

2.2.1 Descriptive variables of the medium

These are the variables whose knowledge according to time and of the place make it possible to know
completely the state of the medium. These variables break up into two categories:

· variable
geometrical,
· variable of thermodynamic state.

2.2.1.1 Variables
geometrical

In all that follows, one adopts a Lagrangian representation compared to the skeleton (within the meaning of
[bib1]) and the co-ordinates X = X (*T* are those of a material point attached to the skeleton. All them
S
)
space operators of derivation are defined compared to these co-ordinates.
U

X
Displacements of the skeleton are noted U (,
X *T*) = *U*.
y

uz

2.2.1.2 thermodynamic Variables of state

The thermodynamic variables are:

· pressures of the components: since we consider that there is to more both
components, there will be with more the two conservation equations of the mass, and thus by duality
with more the two variables of pressure,
· the temperature of the medium *T* (,
X *T*).

2.2.1.3 descriptive Fields of the medium

The principal unknown factors, which are also the nodal unknown factors (noted U (,
X *T*) in this document)
are:

· 2 or 3 (according to the dimension of space) displacements *U* (,
X *T*), *U* (,
X *T*), *U* (,
X *T*) for
X
y
Z

modelings KIT_HM, KIT_HHM, KIT_THM, KIT_THHM,
·
temperature
***T* (,**
X *T*) for modelings KIT_THH, KIT_THM, KIT_THHM,
· two
pressures
***p* (,**
X *T*), *p* (,
X *T*) for modelings
1
2
KIT_HHM, KIT_THH, KIT_THHM,
· one
pressure
***p* (,**
X *T*) for modelings
1
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2.2.2 Sizes

The equilibrium equations are:
· conservation of the momentum for mechanics,
· conservation of the masses of fluid for hydraulics,
· conservation of energy for thermics.
The equilibrium equations utilize directly the generalized constraints. Constraints

**generalized are connected to the deformations generalized by the laws of behavior.
generalized deformations are calculated directly starting from the variables of state and their gradients
space temporal.
The laws of behavior can use additional quantities, often arranged in the variables interns. These quantities are not described in this document which does not treat laws of behavior strictly speaking.**

2.2.2.1 sizes characteristic of the heterogeneous medium

· Porosity eulérienne: .

If one notes the part of volume occupied by the vacuums in the current configuration,

one a:

=

éq 2.2.2.1 - 1

The definition of porosity is thus that of porosity eulérienne.

· The saturation of the phase p: *p*
***S*.**

If one notes
p
the total volume occupied by the phase p, in the current configuration, one has by definition:

p

***S p* =**
éq 2.2.2.1 - 2

This saturation is thus finally a proportion varying between 0 and 1. In the equations of assessment, it is clear that it is the product of porosity by saturation
p
S
**who will intervene. One
can thus legitimately wonder why it is not this quantity which is taken as
unknown factor. The answer comes from what it is saturation *p***
***S* which intervenes more simply**

in the laws of behavior.
· Density eulérienne of the component C in the phase p: ***p***
.
C

If one notes
p
M **mass of the phase p of the component C, in volume of the skeleton in**
C
the current configuration, one has by definition:

p
Mc = p
D p
p
C
=
PS p
C

D
=
PS p
C

D

éq 2.2.2.1 - 3
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The density of the phase p is simply the sum of the densities of its components:
p = PC

C

· Lagrangian homogenized density: *r*.
At the current moment, the mass of volume, *M* is given by:
M =
rd

0
0
éq 2.2.2.1 - 4
2.2.2.2 sizes
mechanics
1
· The tensor of the deformations **U**
() (**X**,) = (**u+T**
T
U),
2
· The tensor of the constraints which are exerted on the porous environment: .
This tensor breaks up into a tensor of the effective constraints plus a tensor of constraints of pressure =
+
'
1 .

and
'

are components of the generalized constraints. This cutting
p
p
is finally rather arbitrary, but corresponds all the same to an assumption rather commonly allowed, at least for the mediums saturated with liquid.

2.2.2.3 sizes

hydraulics

· Mass contributions in components *p*
m (unit: kilogramme per cubic meter). They
C
represent the mass of fluid brought between the initial and current moments. They form part generalized constraints.

p
p
p
p
p
m = J S
- S

0
C
C
C 0 0
éq 2.2.2.3 - 1

The mass contributions make it possible to define the total density seen compared to configuration of reference: *R = R*
, where *R* indicates the density
0 + *m*
+ m + m
lq
vp
have
0
homogenized in an initial state.

· Hydraulic flows:

p
W (unit: kilogramme/second/square meter) of representation eulérienne
C

p
M (unit: kilogramme/second/square meter) of Lagrangian representation
C

P is noted
v the speed of the component *C* in the phase *p,* J Jacobien of the material transformation
C
and the **v** speed of the skeleton. *p*
,
,
p
S indicate the densities, porosity and them
S
0
C 0
0
saturations at the initial moment. By definition:

p
p
p
W = *S*

v - v
C
C
(*PC S*)

éq 2.2.2.3 - 2
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The Lagrangian form of
p
W noted
p
M is obtained while writing:
C
C

p
M N
. *D*
W N
.
C
0 (
0) = *p D*
C
(
)

éq 2.2.2.3 - 3

The variables *m*, **M** and *m*, **M** refer each one to a component **of conservative mass**.
1
1
2
2

One poses by principle:

1
2
1

2
m = *m* + *m*
;
M = **M** + **M**
1
1
1
1
1
1
1
2
1
2
m = *m* + *m*
;
M = **M** + **M**
2
2
2
2
2
2

What we will write:

m
m
component =
phase
component
Nb phasedu
component
M
M
component =
phase
component
Nb phasedu
component

In the applications, one could for example have:

2 components: air and water

2 phases for water

1 phase for the air

One would have then: 1
1
m and
M: contribution of mass and liquid water flow
1
1
2
2
m and
M: contribution of mass and vapor flow
1
1
1
1
m and
M: contribution of mass and flow of dry air
2
2
2
2
m and **M**: non-existent
2
2

· Pressures:
Since we consider that there can be two components other than the solid, there are two conservation equations of the mass, and thus two multipliers associated, i.e. two pressures *p* and *p*. No assumption is made on what these two pressures *p* and *p*. mean.
1
2
1
2
That will depend on the laws of behavior and the way of writing them. For example one can choose:
p =
(p (gas)

capillary
pressure
-
)
p (liquid)
1
p pressure
=
ga

of
Z
air)

+
(vapor

2
2.2.2.4 Sizes
thermics
· Not convectée heat *Q* (see further) (unit: Joule),
p
· Mass enthali of the components *m*
H C (unit: Joule/Kelvin/kilogramme),
· Heat flow: **Q** (unit: Square J/s/meter).
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2.2.3 Data

external
· The mass force *m*
F (in practice gravity),
· Heat sources,
· Boundary conditions relating either to variables imposed, or on imposed flows.
2.3
Derivative particulate, densities voluminal and mass
Description that we make of the medium is Lagrangian compared to the skeleton. One will find in [bib1] a definition of the concept of skeleton: “the matrix (left occluded solide+porosity) component the material part of the skeleton and the connected porous space of elementary volume in question the material point of the skeleton or particle of the skeleton constitutes”.
Either *has an* unspecified field on, or **X** (*T the* punctual coordinate attached to the skeleton that
S
)
we follow in his movement and is **X** (*T the* punctual coordinate attached to the fluid. One notes
fl
)
D its
= dt has
! the temporal derivative in the movement of the skeleton:
D its
(**X** *has* +, + - **X**
,
S (*T*
T) *T*
T) *has* ((*T*) *T*)
has
S
=
= lim
dt
T
0
T

!
da
has
! is called particulate and often noted derivative (for example in [bib1]). We prefer
dt
to use a notation which recalls that the configuration used to locate a particle is that of skeleton by report/ratio to which a particle of fluid has a relative speed. For a particle of fluid location **X** (*T* is unspecified, i.e. that the particle of fluid which occupies position **X** (*T* with

S
)
S
)
the moment *T* is not the same one as that which occupies position **X** (*T* at another moment *T*.
S
)
That is to say then *A* =
AD a quantity related to a *voluminal* density *has*, which density is it even

p
range *partly by the solid matter constituents and the fluids*. That is to say *m*
has C the mass density of carried by
the liquid phase p of the component *C* and is *with the voluminal* density of bound to the solid matter constituents. All
S
these definitions finally amount writing:

p
With =
D =A + *A has* =
D has +
D has =
has
p
+
S p
amndt

éq 2.3-1

C D
S
fl
S
fl
S
C

p, C

D fl A
While following [bib1], we note
fl the derivative of *A* if we follow in the movement of
dt
fl
D S.A.
fluid and
S the derivative of *A* if we follow in the movement of the skeleton.
dt
S
We define then:
DA
D S.A.
D fl A
S
fl
=
S +
fl = D
D

p
p
p m
D has
S +

Cs has Cd
Dt
dt
dt
dt
dt p, C
éq 2.3-2
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p
The density *m*
C has is transported with a relative speed of (*p*
v - **v** compared to the skeleton.
C
S)
D its
Taking into account the definition of *A* =
p
p
p
p
W = *S*

v - **v**, one see easily
dt
how the total derivative of *A* per R!
, and of the definition *C*
C
(*C S*)
contribution at time is written finally:
DA
p

=
m
p

+ *Div has* (*cwc has*)

D
Dt
!

p, C

éq 2.3-3
Note:
Insofar as we made the assumption of small displacements of the skeleton,
D its
has

has =
and **v** *can be*
dt
! *can merge with the partial derivative compared to time T S*
regarded as null. In the same way, in the continuation of the note we will confuse them
Lagrangian representations and eulériennes of flows,
p
M *and p*
W.
C
C
3 Equations
continuous
3.1
Mechanics: conservation of the momentum
We note the tensor of the constraints of Cauchy and ***S the*** second tensor (symmetrical) of Piola Kirchoff.
We note **P the** gradient of the transformation **X** = **X** 0 **X X**,
0
S ()
(*T*
S
0
)
X
(**X**, 0)
P =
T
S
X
0
One a:
-1
- *T*
S = det **P P**

.
P
.
.
The equilibrium equations mechanical are written in the configuration:
0
Div **P** ***S*** +
m
RF =
0 (.)
0
We noted Div the operator of divergence compared to the variables of space **X** of
0
0
configuration.
0
Insofar as we make the assumption of small displacements and the small deformations of
skeleton, this equation can be approached by:
Div +
m
RF = 0
éq 3.1-1
We will further see we always adopt the decomposition =
+ *I*, where indicates
p
effective constraint. It is thus with the load of the module of integration of the equilibrium equations to
make
summon: =
+ *I*.
p
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3.2
Hydraulics: conservation of the mass
The writing eulérienne of the conservation of the fluid mass for the component *C* is written:
D fl pcSp
D = 0
dt p
p
One can then apply [éq 2.3-1] while taking: *has*
and *m*
C = 1 *has*
S = 0
and [éq 2.3-3] will give:
S
D p S
C

p + *Div* (*p*
WC) = 0
dt
p
p
By using the definition of the mass contributions [éq 2.2.2.3 - 3], the definition of Lagrangian flows [éq 2.2.2.3 - 2] one finds the form Lagrangian of the conservation of the fluid mass:
m Div **M**
1 +
0 (
1) = 0

m
Div **M**
2 +
0 (
2) = 0
!
!
éq 3.2-1
3.3
Equation of energy
For the function thermodynamic, we adopt a decomposition of the type systematically

[éq 2.3-1]. That corresponds to the fact that various energies have a whole a part carried by the solid and a part carried by the fluids. The part carried by the solid is characterized by a density voluminal whereas the parts carried by the fluid are characterized by mass densities, as we showed in the paragraph [§2.3].

p
Total internal energy: *E = E*
éq 3.3.1
S + GCV pemc D

p, C

p
Total entropy: *S =*
S
éq 3.3.2
S + GCV psmc D

p, C

p
Total enthalpy *H =*
H
éq 3.3.3
S + GCV phmc

D

p, C

= E - TS

Free energy:
E

Ts
éq 3.3.4
S =
S -
S

p
p
p
mc = m
E C - m
Ts C
G = H - TS

Free enthalpy: *G*
H
Ts
éq 3.3.5
S =
S -
S

p
p
p
m
G C = m
H C - m
Ts C
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Lastly, by noting *Q* ()
! the rate of heat received by a volume, one has by definition:
Q () = **qn**.
D +
D
!

éq 3.3.6
It is pointed out finally that the enthalpy of the fluids is calculated by the formula:
H = + *p*
E
éq 3.3.7
3.3.1 The first principle
With the definitions given higher, he is written:
- *E* - *Div* (*H pmcMp*: +
c)+
M *p.*
Fm
C
+ -
Q
Div = 0
!
!
p, C
p, C
éq 3.3.1-1
This writing corresponds to the equation (22) chapter III-2-3 of [bib1], in which we have neglected the terms of inertia. For the homogeneous mediums, it corresponds to the equation (31) of paragraph IV-3-2 of [bib3].
3.3.2 The second principle
Its form rather well known is:
Q
S + *Div* (*S p*
m
p
C **Mc**)

+ *Div* - 0
! *p, C*
T T

éq 3.3.2-1
By using the traditional thermodynamic considerations [bib1] related to the introduction of the enthalpy free [éq 3.3.5], it is shown that one must necessarily have:

-
= 0

éq 3.3.2-2
p
m

G C -
= 0
éq 3.3.2-3
p
mc

S +
= 0
T
éq 3.3.2-4
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3.3.3 Equation of energy

Enough often, one considers that, the transformations being reversible, the second principle provides finally an equality. Moreover, one replaces in [éq 3.3.2-1] the unknown temperature *T* by one value constant known as temperature of reference. It is finally about a linearization of [éq 3.3.2-1] justified if the variations in temperatures are “small”. Let us note that the term of transport
Div (*pm p*

C M **M** complicates the treatment of nonthe linearity due to the presence of the temperature in
c)
p, C
denominator of the other terms of [éq 3.3.2-1].
We work in enthalpy in order to overcome this difficulty. One leaves the equation of the first principle [éq 3.3.1-1] into which one injects the equations [éq 3.3.2-2], [éq 3.3.2-3], [éq 3.3.2-4], and the definition free enthalpy [éq 3.3-5] and one obtain:
Ts + (*H p*
mc m p - Ts p
mc m p = -
+ . + -
éq 3.3.3-1
C
c)
Div (*H p*
mc **M** *PC*)
M *p F m*
Q
Div
C
!
!
!
p, C
p, C
p, C
One poses then:
Q = Ts - T p
m
C M p
mc
p, C
éq 3.3.3-2
The quantity *Q* with the dimension of an energy per unit of volume. It represents the heat received by the system in a transformation for which there are no contributions of heat per entry of fluid having a enthalpy. Although *Q*
is not an exact differential, we take this quantity
like variable of state.
Finally, the equation of energy retained with the following form:
pm p
H C m
Q
Div H **M**

Divq
M. **F**
C + +
(*pm p*
C
c)+
- *p m*
C
=
! !
p, C
p, C
p, C
éq 3.3.3-3
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4
Variational writing of the equilibrium equations

4.1 Mechanics

We note *U*
the whole of the fields of displacement kinematically acceptable, i.e.
AD
elements of (*H* () 3
1 checking the boundary conditions in displacement on the part of

supporting such conditions [bib3].
The variational form of [éq 3.1-1] is:

= '

+

p I

éq 4.1-1
. (**v**) *D* =
m
RF. *D*
v +
ext.
F
D
v
v

U

AD

4.2 Hydraulics
We note *P* (resp. *P*
) the whole of the acceptable fields of pressure, i.e. them
1ad
2ad
elements of
1
H () checking the boundary conditions in pressure *P* (resp. *P*) on the part of

1
2
supporting such conditions [bib3]. The variational form of [éq 3.2-1] is:
- *m1 m2 D*
1
2
M
Mr.
D
(1 +
1)1 + (
1 +
1)1 =

! ! 1 2
M
M
. *D*
P

(
1ext +
1 *ext.*) 1

1 1ad
éq 4.2-1
1
2
1
2

- *m m*
D
M
Mr.
D
(2 +
2)2 + (
2 +
2)2

! ! = 1 2
M
M
. *D*
P

(
2
+
ext.
2ext)

1

2 2ad
4.3 Thermics

We note *T the* whole of the acceptable fields of temperature, i.e. the elements of
AD
1
H () checking the boundary conditions in temperature on the part of
supporting of such
conditions. [bib3]. The variational form of [éq 3.3.3-3] is:

Q' D
+
M Q.

p
p
m
p
m
p
H C m D
-
H C
+
D

=
C
C

!
!
p, C
p, C

p

p
m

m
p
+
M. **F**
D

-
H C
M +q. *D*

éq 4.3-1
C
C ext.
ext.

p, C
p, C

Tad
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Let us note that, contrary to other presentations, and in particular [bib8] we did not inject them conservation equations of the mass, and we integrated by part the term of transport
Div (*pm p*
H C **Mr.** This last point has the advantage of not revealing of derived from order
c)
p, C
superior, and, contrary to revealing naturally boundary conditions relative to
p
the entry of heat related to hydraulic flows:
hm
p
C **M**

. D
cext

.

p, C
One will be able in makes consider that the conditions of heat flux define directly:
p
m
p
Q
~ = H cm +q
ext.
C ext.
ext.
5
Discretization in time
In this chapter, we are satisfied to take again the variational formulations in their applying a discretization compared to the time of the type téta diagram. It is about a method general of integration of the differential equations [bib12] and [bib13].
is a numerical parameter ranging between 0 and 1. For the linear differential equations (what is not our case…) this diagram is unconditionally stable for 1 2, it is of order 1 for 1 2 and of order 2 for = 1 2. Nevertheless, it can be preferable to use a value different from 1 2, and this for parasitic reasons of oscillations [bib12].
The subscripted quantities by + are the quantities at the end of the step of time, and those subscripted by are
those of the beginning of the step of time. One notes:

+
-
T = T - T
has = has
+ + (1 -) *A*
has

5.1 Mechanics

+
+
+

= '+ *I*
p

éq 5.1-1

+

. (**v**)
+ *m*+
ext+

D = R **F v**
.
D +
F
v
D
v

U

AD

5.2 Hydraulics

- (+
m1
M
Mr.
1 +
+
m21) *D*

1 +

+
+
T
1
2
D

(1 + 1)1 =

- (-
m1
1
M
Mr.
1 +
-
m21)

D
1 - (-)
-
-
T
1
2
D

(1 + 1)

1

+ *T* (1
2
M
M
. *D*
P
ext.
1
+

ext.
1
)

1

1

1ad

éq 5.2-1
- (+
m1
M
Mr.
2 +
+
m22) *D*
2 +

+
+
T
1
2
D

(2 + 2)

2 =

- (-
m1
1
M
Mr.
2 +
-
m22)

2

D - (-)
-
-
T
1
2
D

(2 + 2)2

+ *T* (1
2
M
M
. *D*
P
2ext +

2ext)

2

2

2ad

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5.3 Thermics

(+
Q′ - -
Q′)

p +

D -

m
p+
T
H C **M**
Q
D
C
+ +

p, *C*

- (1-)

p

m
p
T
H C **M**
Q
D
H
m
m
D

C
+ -
+

p
m
p
p
C

+
+
-

(*C* - *c)*

p, C

p, C

+ (1 -) *p*
m
H C (*p*+
m
m
D
éq 5.3-1
C
- *p*
C
)

=

+

p +
m
T
M. **F** *D*
1
T
M. **F** *D*
C
+ (-)

p -
m
C

p, *C*
p, *C*

p

m
p

+ *T*
D - *T*
H C **M**
Q
. *D*
T

cext +
ext.

AD

p, C

One can again consider that the conditions of heat flux define directly:

p

~
m
p

Q
H
M
Q
ext. =
C
C
+
ext.
ext.
p, C
6
Principle of virtual work, deformations and forced generalized, laws of behavior
6.1
Generalized constraints and deformations
While referring to the variational formulations [éq 4.1-1], [éq 4.2-1] and [éq 4.3-1], it appears that one can choose:
For the generalized constraints:

, ;

p

1
m, **M1**, 1
hm
m
1;
2
m, **m2**, 2
H 1;
= 1 1
1
1

éq 6.1-1
1
m, **M1**, 1
hm
m
2 ;
2
m, **m2**, 2
H2;
2
2
2
2

Q' **Q**
,

For the generalized deformations:
= {**U**, (**U**); *p*, *p*
1
; *p*,
1
2 *p T*
; ,
2
T}
éq 6.1-2
The fact is noticed that the generalized deformations contain displacements. That is due to term **Fm**
R
v
. variational formulation of the conservation equation of the quantity of

movement [éq 4.1-1], which term couples finally the generalized constraints and displacements because of [éq 6.3.4-1]. The generalized deformations contain the pressure and the temperature parce that the associated equations are parabolic.

6.2
Principle of virtual work

The whole of the nonlinear equations to solve can be put in the form:

R (**U**)
meca
= **L**
éq 6.2-1

where **U** indicates generalized displacements, i.e.: **U** = {*U U*
, *U*
, *p*, *p*, *T* in the case more
X
y
Z
1
2
}
general. The forces intern **R** express themselves starting from a principle of virtual work generalized. In the case of the mechanics of the continuous mediums “traditional”, i.e. when there is no other component that the solid, one is accustomed to defining the forces intern by:

T
W. **R** = **W**
.
D **W**, field of displacement kinematically acceptable.
( )

In this formulation, the field of deformation depends only on the field of displacement and on its derivative space, possibly in a nonlinear way if one takes into account deformations finished. One writes symbolically:

R = *T*
Q

The law of behavior connects the constraints to the deformations.

Within the framework of the theory of the porous environments developed here, we try to bring us closer it
more possible of this formulation by introducing generalized constraints and deformations
generalized **E**; The generalized deformations depend only on the field of displacements
generalized **U** and of its derivative space. The operator **U**

E (**U**) is an operator of derivation
compared to the field of co-ordinates.
The law of behavior makes it possible to calculate according to **E**.
On the other hand, we cannot write directly
T
W R
. = **W**
.

D, for the reasons

( )
following:
· the equations which we treat are evolutionary equations in times and the derivative by
report/ratio at the time of the quantities intervene,
· the equations are nonlinear because of the terms of transport related to the representation
eulérienne of the fluids: these nonlinear terms appear only in the equation of thermics,
· the choice of the unknown factors makes that the nonlinear terms of transport intervene in
generalized constraints. That is to say a term of transport in the equation [éq 4.3-1],

p
m
p

H C **M**
Q.
, since one took as principal unknown factors for
C +

D

p, *C*

hydraulics pressures, quantities
p
M related to speeds of the fluids belong to
C

p
generalized constraints, just as the enthali *m*
H C, and the term of transport given in
example is linear in deformation generalized and quadratic in generalized constraint. This
fact a difference with the formulation of the theory of the traditional continuous mediums where
terms of great deformation are quadratic quantities of the deformations.
For all these reasons, we introduce a field noted such as:
R QT
=

éq 6.2-2
T
Q is defined by:
WT. **R** = **E** (**W**).

D

éq 6.2-3
W

cinématiqu

generalized
T
déplacemen

of

field
acceptable
ement
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It is very seen easily and classically that *T*
Q is transposed of the operator **Q** such as:
E = **QU**
éq 6.1-4
The field is a linear function of
! and nonlinear of:
= (,)
!
éq 6.1-5
After discretization in time, +
will become a nonlinear function of +
and -
:
+
+
= (+ -
,)
éq 6.1-6
Let us note finally that for algorithmic reasons (inter alia), one needs to know the derivative forces intern compared to generalized displacements:
R

R

E

T

=
= **Q**
Q
U

E

U

E

It is clear that
depends only on the form of the equilibrium equations.

6.3
Laws of behavior
A law of behavior will be simply defined like an unspecified relation between constraints generalized and generalized deformations. The variables intern are defined like fields necessary to the calculation of the constraints, whose evolution is given by the laws of behavior, but who do not intervene directly in the equilibrium equations.
Moreover, we consider that the laws of behavior are written in incremental form and that they are local. By noting the internal variables, a law of behavior is thus one relation:
, **E**
,

!

! !
After discretization in time, the law of behavior becomes a relation:
-
-
-
+
+
+
, **E**, **E**

,
The law of behavior will have to also provide the only term which in the expression
R

T

= **Q**
Q depends on it, namely
. Finally a relation of behavior is one
U

E

E

relation:
+
-
-
-
+
+
+
, **E**, **E**

, ,
éq 6.3-1
+
E
In the following paragraphs, we specify certain aspects of the laws of behavior in distinguishing the mechanical, hydraulic and thermal contributions.
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6.3.1 Mechanical law of behavior

6.3.1.1 Writing

general

The variables intern are noted. A law of behavior of mechanics, within framework THM

is written:

+
= +
(+ + + + - - - - - -
, *p*, *p*, *T*;
p p T
1
2
, , , ,
1
2
,)

éq 6.3.1.1 - 1

+
= +
(+ + + + - - - - - -
, *p*, *p*, *T*;
p p T
1
2
, , , ,
1
2
,)

6.3.1.2 Case of the effective constraints

In the case of the assumption of the effective constraints, one with the decomposition: =
+ **I** where is
p
the tensor of the effective constraints and is a scalar.
p
The variables intern are separate in two parts: the variables intern mechanical and them
variables intern hydraulic. The mechanical law of behavior is divided then into two laws,
H
whose first can be an already existing law within the usual framework of thermomechanics.

+
= +
(+ + - - - -
, *T*; , *T*,
,)

éq 6.3.1.2 - 1

+

, *T*; , *T*,
,
=
+
(+
+
-
-
-
-
)

+
p, *p*; *p*, *p*,
1
2
1
2
p = +p (+
+
-
-
-
H)

éq 6.3.1.2 - 2

+

p, *p*; *p*, *p*,
1
2
1
2
H =
+
H (+
+
-
-

-
H)
The dependences indicated by the equations [éq 6.3.1.2 - 1] and [éq 6.3.1.2 - 2] do not have a justification
theoretical a priori. It is simply a question of showing the most general possible dependences point of view of the data-processing programming. One notices in this decomposition that dependence compared to thermics was left in the effective constraints; typically, one think that the laws on the effective constraints are written as in traditional thermomechanics:
+
+
= (+ + + - - - - -
- *T* **I**, - *T* **I**,
,)

6.3.1.3 Choice of the constraints

Because of rather frequent use of the assumption of the effective constraints, one decides that it vector of the constraints for the mechanical part contains in all the cases the tensor of the constraints effective and the scalar. In the case general where the assumption of the effective constraints is not
p
reserve, one will have simply:
. It is thus with the load of the module of integration of the equations
p = 0
of balance (and not of the laws of behavior) to make the sum: +
+
+
=
+ **I**.
p

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6.3.2 Hydraulics

The hydraulic law of behavior provides the following relations:

+
p
m
m
p p T p p T m **M**
C
=
+
p
C
(
-
-
+
+
+
+
-
-
-
-
p
p
–
,
,
,
; ,
,
,
,
,
,
1
2
1
2
C
C
H)

+ +
, *p*, *p*, *p*, *p*, *T*, *T*;
C
1 +
+
1
2 +
+
2
+

+
+

p
M
M
p
p p
p
p
C
=
p -
-
,
,
,
,
;
C
1 -
-
1
2 -

2

-
+
-
-
p
–
m

T, *T*, **M**; **F**
C
H

-
+

p p T p p T m
H =
+
H (+
+
+
+
-
-
-
-
p
–
,
,
,
; ,
,

,
,
,
1
2
1
2
C
H)

éq 6.3.2-1

It is noticed that the field of gravity is a data of the hydraulic law of behavior parce that the evolution of the vector of flow follows relations of the type:

fl
M = - *P* + **F**.
H
[
fl
m]

6.3.3 Thermal law of behavior

The laws of behavior give:

Q
'+ = Q'+ (+
, *p*+, *p*+, *T* +; -
1
2
, *p*, *p*, *T* -, Q'-
1
2
)

p+
p+
p
hm
m
+ + + + - - - - *m*
C
= *H C*, *p*, *p*, *T*;

1
2
, *p*, *p*, *T*, *H*
1

2
C
C and *p*

éq 6.3.3-1
+
+
Q

= **Q** (+
, *p*+, *p*+, *T* +, *T* +
1
2
; -
, *p*, *p*, *T* -, *T* -
1
2
, -
Q)

+
+
+

=

(, *p*+, *p*+, *T*+, *T*+; -, *p*, *p*, *T*, *T*, -

1
2

T)
T
T
1
2
Let us note that we introduced possible internal variables related to thermics.

6.3.4 Homogenized density

By definition, the homogenized density, which intervenes in the equilibrium equation of mechanics [éq 3.1-1] is given by:

+
+
+
+
+
$R = R$
m
m
m
m
0 +
1
1
+ 21 + 12 + 22

éq 6.3.4-1

This equation is not a law of behavior, but it belongs to the conservation equations.
It is integrated in the module of calculation of the equilibrium equations, the modules of calculation of the laws of
behavior not having to treat it.

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7
Algorithm of resolution
7.1
Nonlinear algorithm of resolution of the equilibrium equations

In the case general of modeling (variable coefficients, desaturation, convection) the problem

variational presented above [éq 4.1-1] with [éq 4.3-1] is nonlinear compared to the fields of displacement, pressure and temperature. After discretization by finite elements, one obtains a system matric nonlinear. The matrix of the tangent operator contains moreover one term nonsymmetrical treaty as tel. One uses in all the cases of modeling nonlinear solvor STAT_NON_LINE of *Code_Aster* resting on a method of Newton-Raphson, described in [bib5]. One introduces vectorial functional calculus:
F (**U**) = **R** (**U**)
meca
- L
éq 7.1-1
F

The associated tangent operator is noted: **F**
D = **U**

For the modules THM, objects of this note, the operator *meca*
L
does not depend on displacements
generalized. All the terms depending on generalized displacements were introduced into **R**, and it is precisely for this reason that displacements are found in the deformations
generalized. Let us note on this subject the very particular treatment of the term **Fm**
R
v
. equation [éq 4.1-1].

+
+
+
+
According to [éq 6.3.4-1], *R m*
F v
.
D =

R
m1
m2
m1
m2
m
F. *D*
v

(0 + 1 + 1 + 2 + 2)

We chose to divide this term into two:
The term *R m*
F v
.
is a contribution to *meca*
L
0

D
if the user informed the operand GRAVITY

loading used (defined by order AFFE_CHAR_MECA), whereas the term
(+
m1
, which depends on the generalized constraints is a contribution to
1 +
+
m21 + +
m12 +
+
m22) *m*
F v
.
D

R.
7.2
Passage of the nodal values to the values at the points of Gauss
As in all the codes of finite elements, the terms are calculated by loop on the elements and buckle on the points of gauss. While noting
el
R and
el
DF values at the point of gauss G of the element el
G
G
nodal forces and of the tangent operator, and *el*
W the weight of integration related to this point of gauss, one a:
G
(
R U) = *el el*
W **R U**

G
G (
)
el
G
éq 7.2-1
DF (**U**) = *el el*
W DF **U**
G
G (
)
el
G
éq 7.2-2
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Let us note *el* then
U the vector of the nodal unknown factors on a finite element el. One can thus have:
U
v
W
1

node

p1
p
2
T

U
v
W
el

example
by
U =
2

node

p1
p
2
T
U
v
W
3

node

p1
p
2
T
.
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Let us note also *el*
E the vector of the deformations generalized at the point of gauss G of the element el and *el*

G
G
the vector of constraints generalized for the point of Gauss G of the element el. In the case more complete one has as follows:
el

p
m1
1
M11
el
U
m

H 1
1
(**U**)

m21

p
2

1
M1

m
p1
H 21
el
el
E =
;
=

G
G
1
p2
m

2
p

2
M1

21
m
T

H2
T

m2
G
2
M22
m
H2
2
Q'

Q *G*
The functions of form of the finite elements make it possible to calculate the matrix then
el
Q of passage of
G
nodal unknown factors with the deformations generalized at the points of gauss defined by:
el
el
el
E = **Q U**
.

éq 7.2-3
G
G
7.3
Vectors and matrices according to options'
The presentations of the two following paragraphs are made in the case more general where one has one equation of mechanics, two equations of hydraulics and an equation of thermics. The indices G and el from now on are omitted, but it is clear that what is described applies to each point of gauss of each element.
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7.3.1 Residue or nodal force: options RAPH_MECA and FULL_MECA
One distributes the terms of the variational formulation according to the following principle:
el
el
If
*
E
*
E *G* = (**v**, **v**,
calculated with
1

,1 ,2 ,2,)
G indicates a virtual field of deformation,
( )
el
to start from a vector of virtual nodal displacements

*
U, one can define
:
T
*
E *elg* .elg (**U**) = 1v + 2 (**v**) + 3 + 4
. They then are taken again
1
+ 5
1
+ 6
2

+ 7
2
+ 8
discrete variational formulations [éq 5.1-1], [éq 5.2-1], [éq 5.3-1], and one replaces the integrals there *fd* by *el el*
W F for all integral *F*. One distinguishes the terms multiplying

G
G
el
G
respectively **v**, (**v**), and, and one finds:
1
1
2
2
Index
associated
1
- (+
+
+
+
v
m11 + *m21* + *m12* + 22) +m
m
F
2
+
+

+ *I*
(**v**)
p
3
+
+
-
-
- 1
m
m
m
m
1
1
- 21 + 11 + 21
4

T (+
+
1
M
M
1
T **M**
M
1
1
+ 21)+ (-) (-
-
1
1
+ 21)
5
+
+
-
-
- 1
m
m
m
m

2
2
- 22 + 12 + 22
6

T (+
+
1
M
M
1
T **M**
M
2
2
+ 22)+ (-) (-
-
1
2
+ 22)
7
- Q'+ +Q'-

+
-
+
-
m
- *H* 1
+
-
+
-

1 + (1 -) *m*
H 11 (*m1* - *m1* -
+ -

-
1
1)
m

H 21
(1) *m*
H 21 (*m2 m2*
1
1
)

+
-
+
-
m
- *H* 1
+
-
+
-

2 + (1 -) *m*
H 12 (*m1 - m1* -
+ -

-
2
2)
m
H 22
(1) *m*
H 22 (*m2 m2*
2
2)

+ *T*
(
M1+ + M2+ + **M1+** + M2+
-
-

-
-
+ -
+
+
+
1
1
2
2) *m*
F
.
T (1
) (**M1 m2 M1 m2**
1
1
2
2) *m*
F
.
8

+
+
+
+

+

1
+
m
m
m
m
T H 1 **M1**
H
H
H
1
+ 2
+
1 **m2**
1

+ 1
+
2 **M12** +
2
+
2 **M** 22 + +
Q

+

-
-
-
-
+ (1-)

1
-
m
m
m
m
T H 1 **M1**
H
H
H
1
+ 2
-
1 **m2**
1
+ 1
-
2 **M12** +
2
-
2 **M** 22 + -
Q

Note:
In first term 1 does not appear the term

m
- *R* **F** because it is put in the loading
0
outside *meca*
L
and calculated by the option of calculation of the loading external of gravity.
By using the definition [éq 7.2-1] of *el*
R, one a:
G
T
T
el
*el
*el
*el
*el
U
R
.
= **E**

. *G*, which still gives us:
G
G
T
el
el
Rel = **Q**

. *G*
G
G
This last equality is only the local form on the level of a point of gauss of [éq 6.2-2].
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7.3.2 Tangent operator: options FULL_MECA, RIGI_MECA_TANG

In what follows, if *X* indicates a vector of components *X I* and *Y a* vector of components *Y J*,
X
I
X

a matrix will indicate whose element occupying line I and the column J is
.
Y
J
Y

To calculate the tangent operator, the following quantities will be calculated:
[**DRDE**] =
DR1U
DR1E
DR1P1
DR1GP1
DR1P2
DR1GP2
DR1T
DR1GT
DR2U
DR2E
DR2P1
DR2GP1
DR2P2
DR2GP2
DR2T
DR2GT
DR3U
DR3E
DR3P1
DR3GP1
DR3P2
DR3GP2
DR3T

DR3GT
DR4U
DR4E
DR4P1
DR4GP1
DR4P2
DR4GP2
DR4T
DR4GT
DR5U
DR5E
DR5P1
DR5GP1
DR5P2
DR5GP2
DR5T
DR5GT
DR6U
DR6E
DR6P1
DR6GP1
DR6P2
DR6GP2
DR6T
DR6GT
DR7U
DR7E
DR7P1
DR7GP1
DR7P2
DR7GP2
DR7T
DR7GT
DR8U
DR8E
DR8P1
DR8GP1
DR8P2
DR8GP2
DR8T
DR8GT
Where one noted:
F

F
DRiU
I
=
DRiGP
I
1 =
U
p

1
F
F
DRiE
I
=
DRiGP
I
2 =

p2
F
F
DRiP
I
1 =
DRiT
I
=
p

T
1

F
F
DRiP
I
2 =
DRiDT
I
=
p

T
2

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To make these calculations one considers that the laws of behavior provide, for the options corresponding, all the derivative following:
'
'
'
'
'
'
'
'

U

p

1

p1 p2

p2
T
T

p
p
p
p
p
p
p
p
U

p

1

p1 p2

p2
T
T

m1
1
1
1
1
1
1
1
1
m1
m1
m1
m1
m1
m1
m1
m1
U

p

1

p1 p2

p2
T
T

M
1
M1
1
1
1
1
1
1
1

M
1
1 **M1 M1 M1 M1 M1**
U

p

1
1
p
2
p
2
p
T
T

1
1
1
1
1
1
1
1
1
hm
1

hm
1
hm
1
hm
1
hm
1
hm
1
hm
1
hm

U

1
p
1
p
2
p
2
p
T
T
2
2
2
2
2
2
2
2
1
m
1
m
1
m
1

m
1
m
1
m
1
m

U

1
p
1
p
2
p
2
p
T
T
M2
M2

M2

M2

M2

M2

M2

M2
1
1
1
1
1
1
1
1

U

1
p
1
p
2
p
2
p
T
T

2
2
2
2
2
2
2
2

1
m
1
m
1
m
1
m
1
m
1
m
1
m
1
H
H
H

H
H
H
H
hm

U

p
p
p
p
T
T

D D
E =
1
1
2
2

1
1
1
1
1
1

1
1
2
m
2
m
2
m
2
m
2
m

m

m
2
2

U

1
p
1
p
2
p
p
T
2

T

1
1
1
1
1
1
1
1
M

2
M2 M2 M2 M2 M2 M2 M2

U

p
1

p
p
1
2

p
T
2

T
1
1
1
1
1
1
1
1

hm2
hm2 hm2 hm2 hm2
hm2
hm2

hm2

U

p

1

p
p
1
2

p
T
2

T

m2

m2

m2

m2
2
2
2
2
2
2
2
2
2
m
2
m
2
m
2
m
U

p

1
1
p
2
p
2
p
T
T

2
2
2
2
2
2
2
2
M2 M2 M2 M2 M2

M2 M2 M2
U

1
p
1
p
2
p
2
p
T
T

2
2
2
2
2
2
2
2
2
m
2
m
2
m
2
m
2
m
2
m
2
m
2
H
H
H
H
H

H
H
hm

U

1
p
1
p
2
p
2
p
T
T
Q'
Q'
Q'
Q'
Q'
Q'
Q'
Q'

U

p1 p1 p2 p2
T
T

Q

Q

Q

Q

Q

Q
Q
Q
U

p

1
p1 p2
p2
T
T

Note:
In these expressions, the derivative compared to **U** *are all null, but we keep the writing taking into account the definition of the matrices*
el
Q *which we adopted.*
G
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The call to the laws of behavior will provide the pieces of the matrix **D OF**

according to the equations
present:

[
]
DMECDE =
p
p
p
p
;
[
]

DMECP1 = 1

1 ;
[**DMECP2**] = 2

2 [**DMECDT**]

= *T*

T
p
p
p
p
p
p
p

p
p
p
p

1

1
2

2
T

T
1

m
m11
m1

1
m11
m1
1

1
m11
m1

1
p
p
p
p

1

1
2

2

1
1
1
1
1
T

T
1
1
[**DP11DE**] **M**
M
M
M
M
M
1
=
;

[**DP11P1**] 1
=
1 ;
[**DP11P2**] 1
=
1 [**DP11DT**] 1 1
=

p1

p1
p2

p

2
T

T
1

m
1
1
m1
m1
m1
m1
m
m
H 1
H 1. 1
H 1. 1

H 1. 1

p1

p1
p2

p
T

T

2
2

m
m21
m2

1
m21
m2
1

1
m21
m2

1
p
p
p
p

1

1
2

2

2
2

2
2
2
T

T
2
2
[**DP12DE**] **M**
M
M
M
M
M
M
1
=
;
[**DP12P1**] 1
=
1 ;
[**DP12P2**] 1
=
1 [**DP12DT**] 1 1
=

p1

p1
p2

p

2
T

T
2

m
2
2

m2
m2
m2
m2
m
m
H 1
H 1. 1
H 1. 1

H 1. 1

p1

p1
p2

p
T

T

2
1

m
m12
m1

2
m12
m1
2

2
m12
m1

2
p
p
p
p

1

1
2

2

1
1
1
1
1
T

T
1
1
[**DP21DE**] **M**
M
M
M
M
M
2
=
;
[
] 2
DP21P1 =
2 ;
[**DP21P2**] 2
=
2 [**DP21DT**] 2
2

=

p1

p1
p2

p

2
T

T
1

m
1
1
m1
m1
m1
m1
m
m

H2
H2 H2
H2 H2

H2 H 2m

p1

p1
p2

p
T

T

2
2

m
m22
m2

2
m22
m2
2

2
m22
m2

2
p
p
p
p

1

1
2

2

2
2
2
2
2
2
T

T
2
2
[**DP22DE**] **M**
M
M
M
M
M
2
=
;
[**DP22P1**] 2
=
2 ;
[**DP22P2**] 2
=
2 [**DP22DT**] 2 2
=

p1

p1
p2

p

2
T

T
2

m
2
2
m2
m2
m2
m2
m

m
H2
H2 H2
H2 H2

H2 H2

p1

p1
p2

p
T

T

2
'
Q
Q
'
Q'
Q'
Q'
Q' Q'
[**DTDE**]
=
p
p
p
p

;
[
]

DTDP1 = 1

1; [**DTDP2**]

= 2

2 [**DTDT**]

= T T
Q
Q
Q
Q
Q
Q
Q

p1

p

1
p2

p

2
T
T
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In addition, by deriving the expression from the residue compared to the constraints, one defines:

1 1 1 1
1
1
1
1
1
1
1
1
1
1
1 1

1
1
1
2
2
2
1
1
1
2
2
2

p m
hm
m
hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

2 2 2 2 2 2 2
2

2
2
2
2
2
2
2 2

1
1
1
2
2
2
1
1
1
2
2
2

p m
hm
m
hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

3 3 3 3 3 3 3
3
3
3
3
3
3

3
3 3

1
1
1
2
2
2
1
1
1
2
2
2

p m
hm
m
hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

4 4 4 4 4 4 4
4
4
4
4
4
4
4
4 4

1

1
1
2
2
2
1
1
1
2
2
2

p m
hm
m
hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

D D = 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5

1
1
1
2
2
2
1
1
1
2
2
2

p m
hm
m
hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

6.6 6.6 6.6 6
6
6
6
6
6
6
6
6

6
1
1
1
2
2
2
1
1
1
2
2
2

p m
hm
m

hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

7 7 7 7 7 7 7 7 7 7 7 7 7 7 7

7
1
1
1
2
2
2
1
1
1
2
2
2

p m
hm
m
hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

8 8 8

8
8
8
8
8
8
8
8
8
8
8
8

8
1
1
1
2
2
2
1
1
1
2
2
2

p m
hm
m
hm
m
hm
m
hm
Q'
1
M1 1 1 **M1** 1 2 **m2** 2 2 **m2** 2
Q

All these quantities not being inevitably calculated, one will note, for *I* from 1 to 8:
[
D *I*
D] *I I*
=
,

[
D *iDP*] *I I I*
21 =
,
,
1
1
1

m
p
2 **m2** *m*
H2
[
D *iDP*] *I I I*
11 =
,
,
2
1
2
[
D *iDP22*] *I I I*
=
,
,
2
2
2
m1 **M1** *m*
H 1
m2 **M2** *m*
H2
[
D *iDP12*] *I I I*
=

,
,
2
1
2
[
D *iDT*] *I I*
=
,

m1 **M1** *m*
H 1
Q' **Q**
It is then clear that:
D OF

= **D D**
D
. **OF**

And the contribution of the point of gauss to the tangent matrix
el
DF is obtained by:
G
T
el
el
el
DF = **Q**
D
.
OF

Q
.
G
G
G
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7.4 Algorithm

total

The algorithm becomes then:
Initializations:
+
Calculation of *meca*
L
(option CHAR_MECA)
Calculation of
-
F
D
(option RIGI_MECA_TANG)
+
-
Calculation of **U**
by:
-
F
. **U0** = *meca*
D
L
- *meca*
L
0
Iterations of balance of Newton N
Buckle elements el
Buckle points of gauss G
calculation
el
HQ
-

-
+
+
calculation *el*
E
Q U
.
and *el*
E
Q U
.
G
= el el
G
= el el
G
G
+
el
+
+
el
el
G
-
-
-
+
calculation of:
N
, ,
(according to option) starting from *el*
el
el
el
E, **E**
G
G
N
+
el
G
G

G
G
E
N
G N
+
el
+
+
+
T
el
calculation of
el
el
el
G
from
; **R**
Q

.
G
=
N
G
G
N
G
N
N
+
+
+
el
el

el

G
+
+
T

G
el
el
G
calculation of
N starting from *el*
;
N
N
el
DF
= **Q**.
.
Q
.
(
+
according to option)
el

G N
G
G
+
+
G
N
el
el

E

G N
G
G
N
N
Calculation of **U**
by:
N 1
+
+
+
+

DF. **U**
R
L
N
N 1 = -
N +
meca
+
Actualization:
U
U
U
N
= *N* +
1
+
N 1
+
IF test convergence OK
fine Newton: no next time
If not
N = n+1
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---

8 the option
FORC_NODA

To the level of the continuous equations, option FORC_NODA corresponds to the calculation of the operator

T
el
R QT
=

el
el
. At the discrete level, option FORC_NODA amounts calculating the vector **R** = **Q**

. *G*.
G
G
As we already noted that depends not only on, but also of
! , one should not
to be astonished to see appearing the step of time *T* and the constraints at the same time at time + and time -.
The algorithm of Newton-Raphson of order STAT_NON_LINE uses option FORC_NODA for
the calculation of the prediction at the beginning of each step of time. It is not thus pain-killer to calculate
correctly all terms for this option, including those which depend on the step of time. Us
let us illustrate this question for a simple example corresponding to the only equation of hydraulics.
That is to say a simplified version of the hydraulic equation:
DM
-
p D
* +
p D
* =

*F p**
M
dt

ext.

After discretization in time:
-
mp D
*
+

T (+ + (1 -) - *p D*
*

*F p**

M
M

) =

ext.
Revealing
+
-
+
-
M = **M** - **M**

and

p = *p* - *p*, and writing a law of
behavior: *m*
= *NR p*
, one finds:
-
* +
* =
* -

*

M
M

-
NR pp D
T
p D
F p
T
p D
ext.

By definition the phase of prediction STAT_NON_LINE is written:
0
1
K U = *F* - *T*
Q
ext.

0
0
It is then clear that one must take **QT** *p D*
* = - *T*
M-p *D*
*

0

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9 Discretization
space

Finite elements THM of Aster are mixed elements, in the sense that they have at the same time unknown factors of displacements, pressures and temperatures. A choice of discretization where displacements, the pressures and the temperatures are interpolated with the same order of approximation conduit with oscillations, especially for choices of step of too small times compared to

discretization spaces some One will consult on this subject for example [bib10]. This problem is also in relationship with the manner of calculating the matrix known as of mass, and one will be able to consult on this subject [bib14].
We give in addition in appendix, to illustrate our matter, the solution for the first step of time of a problem of mono consolidation dimensional with an interpolation P1P1. It is seen that for a small step of time, it is very oscillating.
For this reason, quadratic elements THM are elements in P2P1, i.e. the interpolation of displacements is quadratic and that of the temperatures and pressures is linear. We nevertheless kept all the unknown factors on all the nodes, including the nodes mediums, but we imposed in the calculation of the matrices of rigidity that the pressure of a node medium of segment is equal to the half summons nodes top of the segment to which it belongs.
In addition, in the programming, we took account of the following property:
That is to say S a node top, 1
W its function of form as pertaining to a linear element (by
S
example QUAD4), and 2
W its function of form as pertaining to a quadratic element (by
S
example QUAD8). That is to say Na the number of edges having S like end and 2
W the function of form of
my
the quadratic interpolation attached to a node medium of edge, one has the relation then:
Na
1
2
w2
W
W
S =
+ my
S
ma=1 2
This said, and including in interpolation P2P1, of the conditions of nonoscillation exist on the step of time. [bib10] the relation gives:
x2

T
>
C
20 *v*
kE (1 -)
where *C* is the coefficient of consolidation: *C*
, K

$v =$
v

being the permeability measured in
lq (1 +) (1 -

2)
m/s.
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Appendix 1 Problem mono dimensional P1P1

A unidimensional problem of consolidation is considered whose unknown factors vary only according to only variable of space *X*.

A rectangular field length L, is filled with a porous material of coefficients of Lamé and μ, of

1

coefficient of biot *B* and module of biot *NR* =

and of hydraulic conductivity. Density of
M
H
U

fluid is noted. One notes the constraint, *U* displacement in direction *X*, =
deformation,
xx
X

p pressure, *m the* mass contribution in fluid, **M the** flow of fluid.
The boundary conditions are:
in $X = 0$: $= 0$; $p = 0$
in $X = L$: $U = 0$; $p = 1$ for $T > 0$
The initial conditions in $T = 0$ are $= U = p = 0$.
$X = 0$;
$= 0$;
$p = 0$
$X = L$;
$U = 0$;
$p = 1$
The setting in equation gives:
Balance mechanical and linear elasticity: $= (+ 2\mu) - LP = 0$
m **M**
Conservation of the mass:
+
$= 0$
T
X
p

Law of Darcy: **M** = -
H
X

m
Couplings:
$= Np +$
B

We make a discretization in implicit time **and we are interested in calculation of the first step of time**:
The system of two equations is obtained:
$= (+ \mu$

2) $(U) - LP = 0$

$Np + B - htp = 0$

The mixed variational formulation of this problem is:

*
*
*
2
(*U*) *U*
LP U
0
U
(
[+ µ) () - ()]=

*
*
*
*
Npp
B (*U*) *p*
T p p
0
p
[
+
+ *H*] =

éq Year 1-1
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Concerning the space discretization, we cut out the mediums out of *N* finite elements. Nodes of element I are I and i+1. 1 is noted
U and 1
p the displacement and pressure of the first node of the element E, 2
U and 2
p it
E
E
E
E
displacement and pressure of its second node. One supposes that one uses finite elements P1P1, it is with to say that displacements as the pressures are linearly interpolated. Discretization of the first equation gives then:
2
1
*
*
U.E. - *U.E.*
U
U
p
p
E -
E
E +

(
2
1
1
2
+ 2μ)
-
E
B
= 0

X
X
E

2

From where one deduces:
(2

1
1
+ 2
U - *U* =

E
E)
B X p
p
E
E
E
+ µ
2
2
éq Year 1-2
The discretization of the second equation gives:
2
1
2
1
*
*
2
1
1
2
*
*
2
1
p E + p E U

U
p
p
p
p p
p
E -
E
E +

E
E
- *E E* -

B
+ *NR*
+ *T*
H
E = 0
X
X
X
E
2

2

E

In there bearing [éq Year 1-2], one finds:
2
1
*
*
2
1
2
p E + p E
B
p
p

T
E +
E

NR +
+
p E p p p
H
- *E E - E* =

2
(2 1
*
*) (2
1) 0
E
2
+ 2μ
2

X E
éq Year 1-3
So now, we make tighten the step of time towards zero with step of constant space,
T

2
B

, and [éq Year 1-3] is reduced to:
H
<< *NR*
2

+

X

+ μ
2

(2 1
*
*
p E + *p E*) (1
2
p
p
E +
E) = 0
E
A total classification of the nodes and unknown factors of pressure are introduced:
1
2
p
p
p
p
J =
;
J
J =
J +1
One can see that this whole of relations gives finally:
p1 = 0

p + 2p + + *p* + = 0
I
I
I 1
I 2
[
1, *N* -]
1
p
n+1 = 1
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The solution of this continuation is:

J -1
p2 j-1 =

N

2 *J* -1
p2 J = -

2n

What gives the distribution of following pressure:

Pressure with the first step of time

1,5
1
0,5
P
0
-0,5 0
5
10
15
20
25
-1
-1,5
X

As an indication, we give Ci below a comparison between numerical results obtained and elements P1P1, P2P2 and P2P1.

It is seen that element P2P1 does not remove the oscillation, but attenuates it appreciably.

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Document: R7.01.11

Models of behavior THHM

Summary:

This note introduces a family of laws of behavior THM for the saturated and unsaturated mediums. One y

described the relations allowing to calculate the hydraulic and thermal quantities, by taking account of
strong couplings between these phenomena and also with the mechanical deformations. Relations presented here
can be coupled with any law of mechanical behavior, subject making the assumption
said effective constraints of Bishop and that the mechanical law of behavior defines constants
rubber bands (useful for the coupled terms). The purely mechanical part of the laws is not presented.
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1 Introduction

We introduce here a family of laws of behavior THM for the saturated and unsaturated mediums. We describe the relations allowing to calculate the hydraulic and thermal quantities, in taking account of strong couplings between these phenomena and also with the mechanical deformations.
The relations presented here can be coupled with any law of behavior mechanics, subject making the assumption known as of the effective constraints of Bishop and that the law of mechanical behavior defines constant rubber bands (useful for the coupled terms). For this reason, the purely mechanical part of the laws is not presented here.
Modelings selected are based on the presentation of the porous environments elaborate in particular by O. Coussy [bib1]. The relations of behavior are obtained starting from considerations thermodynamic and with arguments of homogenisation that we do not present here, and which are entirely described in the document of P. Charles [bib2]. In the same way the general writing of equilibrium equations and conservation is not detailed, and one returns the reader to the documents [R5.03.01] [bib3] and [R7.01.10] [bib4], which contain definitions useful for the comprehension of present document.

The mechanics of the porous environments gathers a very exhaustive collection of physical phenomena concerning with the solids and the fluids. It makes the assumption of a coupling between the mechanical evolutions solids and fluids, seen like continuous mediums, with the hydraulic evolutions, which regulate the problems of diffusion of fluids within walls or volumes, and the evolutions thermics.

Each component of the porous environment thus has a mechanical, hydraulic behavior and thermics. The theory tries to gather all these physical phenomena. Phenomena chemical (transformations of the components, reactions producing of components etc…), in the same way that the radiological phenomena are not taken into account at this stage of the development of Code_Aster. The mechanical, hydraulic and thermal phenomena are taken into account or not according to the behavior called upon by the user in order STAT_NON_LINE, according to following nomenclature:

Modeling
Phenomena taken into account
KIT_HM
Mechanics, hydraulics with an unknown pressure
KIT_HHM
Mechanics, hydraulics with two unknown pressures
KIT_THH
Thermics, hydraulics with two unknown pressures
KIT_THM
Thermics, mechanics, hydraulics with an unknown pressure
KIT_THHM
Thermics, mechanics, hydraulics with two unknown pressures

The document present describes the laws for the case more general says THHM. Simpler cases are obtained starting from the case general by simply cancelling the quantity absent.
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2
Presentation of the problem: Assumptions, Notations

In this chapter, one mainly endeavours to present the porous environment and its characteristics.

2.1
Description of the porous environment

The porous environment considered is a volume made up of a more or less homogeneous solid matrix, more or less coherent (very coherent in the case of concrete, little in the case of sand). Between solid elements, pores are found. One distinguishes the closed pores which do not exchange anything with theirs
neighbors and the connected pores in which the exchanges are numerous. When one speaks about porosity, it is well of these connected pores about which one speaks.

Inside these pores are a certain number of fluids (one excludes solidification from these fluids), present possibly under several phases (liquid or gas exclusively), and
presenting an interface with the other components. To simplify the problem and to take in count the relative importance of the physical phenomena, the only interface considered is that enters liquid and the gas, the interfaces solid fluid/being neglected.

2.2 Notations

We suppose that the pores of the solid are occupied by with more the two components, each one coexisting in two phases to the maximum, one liquidates and the other gas one. Sizes X associated with the phase J (j=1,2) of fluid I will be noted: X. When there are two components in addition to the solid, it
ij
are a liquid (typically water) and a gas (typically dry air), knowing that the liquid can be present in gas form (vapor) in the gas mixture and that the air can be present under
form dissolved in water. When there is one component in addition to the solid, that can be one liquid or a gas.
The porous environment at the current moment is noted, its border. It is noted, at the initial moment.
0
0
The medium is defined by:

· of the parameters (vector position X, time T),
· of the variables (displacements, pressures, temperature),
· of the intrinsic sizes (forced and deformations, contributions mass, heat,
hydraulic enthali, flows, thermics…).

The general assumptions carried out are as follows:

· assumption of small displacements,
· reversible thermodynamic evolutions (not necessarily for mechanics),
· isotropic behavior,
· the gases are perfect gases,
· ideal mixture of perfect gases (total pressure = nap of the partial pressures),
· thermodynamic balance between the phases of the same component.

The various notations are clarified hereafter.

2.2.1 Descriptive variables of the medium

These are the variables whose knowledge according to time and of the place make it possible to know completely the state of the medium. These variables break up into two categories:

· variable geometrical,
· variable of thermodynamic state.
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2.2.1.1 Variables
geometrical

In all that follows, one adopts a Lagrangian representation compared to the skeleton (within the meaning of
[bib1]) and co-ordinates X = X (T
S) are those of a material point attached to the skeleton. All them
space operators of derivation are defined compared to these co-ordinates.
ux

Displacements of the skeleton are noted U (X, T) = U Y.

uz

2.2.1.2 Variables of thermodynamic state

In a general way, the following indices are used:

W for liquid water
AD for the dissolved air
have for the dry air
vp for the steam

The thermodynamic variables are:

· pressures of the components: p
,
X
, p
,
X
, p
,
X
, p
,
X
,
have (
T)
vp (
T)
AD (
T)
W (
T)
· the temperature of the medium T (X, T).

These various variables are not completely independent. Indeed, if only one is considered component, thermodynamic balance between its phases imposes a relation between the pressure of vapor and pressure of the liquid of this component. Finally, there is only one pressure independent by component, just as there is only one conservation equation of the mass.
The number of independent pressures is thus equal to the number of independent components. choice of these pressures is free (combinations of the pressures of the components) provided that them pressures chosen, associated the temperature, form a system of independent variables.

For the case known as saturated (only one component air or water) we chose the pressure of this

single
component.
For the case says unsaturated (presence of air and water), we chose like variables independent:

· total pressure of the gas p
,
X T =p + p,
gz (
) vp have
· capillary pressure p
,
X T = p - p = p - p - p
C (
) gz
lq
gz
W
AD.
These pressures have a very strong physical interpretation, the total gas pressure for reasons obvious, and pressure capillary, also called suction, because capillary phenomena are very important in modeling presented here. It would have been possible also to choose steam pressure or the percentage of relative moisture (relationship between the steam pressure and the pressure of vapour saturated at the same temperature) physically accessible. Modeling becomes then more complex and in any event, capillary pressure, gas pressure and percentage of relative moisture (relationship between the steam pressure and the saturating steam pressure) are connected by the law of Kelvin.
For the particular case of the behavior "LIQU_GAZ_ATM" one says makes the assumption known as of Richards:
pores are not saturated by the liquid, but the pressure of gas is supposed to be constant and only variable of pressure is the pressure of liquid.
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2.2.1.3 descriptive Fields of the medium

The principal unknown factors, which are also the nodal unknown factors (noted U (X, T) in this document)
are:

· 2 or 3 (according to the dimension of space) displacements U (,
X T), U (,
X T), U (,
X T) for
X
y
Z
modelings KIT_HM, KIT_HHM, KIT_THM, KIT_THHM,
· the temperature T (X, T) for modelings KIT_THH, KIT_THM, KIT_THHM,
· two pressures p (X, T), p (X, T) (which are p
,
X
, p
,
X
in the case studied) for
gz (
T)
C (
T)
1
2

modelings KIT_HHM, KIT_THH, KIT_THHM,
· a pressure p (X, T) (which is p
,
X
or p
,

X
according to whether the medium is saturated by one
gz (
T)
W (
T)
1
liquid or a gas) for modelings KIT_HM, KIT_THM.

2.2.2 Derived
particulate

This paragraph partly shows the paragraph “derived particulate, densities voluminal and mass” of the document [R7.01.10]. Description that we make of the medium is Lagrangian by report/ratio with the skeleton.
Either has an unspecified field on, or X (T
S) the punctual coordinate attached to the skeleton that
we follow in his movement and is X (T
fl
) the punctual coordinate attached to the fluid. One notes

D its
= dt has
& the temporal derivative in the movement of the skeleton:

D its
(xs (T + T has
), T + T
) - has (X (T), T)
has
S
=
= lim
dt
T
0
T

&

da
has
& is called particulate and often noted derivative. We prefer to use a notation which

dt
recall that the configuration used to locate a particle is that of the skeleton by report/ratio to which a particle of fluid has a relative speed. For a particle of fluid location X (T is
S
)
unspecified, i.e. that the particle of fluid which occupies position X (T at the moment T is not
S
)
even as that which occupies the position xs (you) at another moment you.

2.2.3 Sizes

The equilibrium equations are:

· conservation of the momentum for mechanics,
· conservation of the masses of fluid for hydraulics,
· conservation of energy for thermics.

The writing of these equations is given in the document [R7.01.10] [bib4], which defines also what we call in a general way a law of behavior THM and gives the definitions of generalized constraints and deformations. This document uses these definitions. Equations of balance utilize directly the generalized constraints.
The generalized constraints are connected to the deformations generalized by the laws of behavior. The generalized deformations are calculated directly starting from the variables of state and theirs temporal space gradients.
The laws of behaviors can use additional quantities, often arranged in the variables interns. We gather here under the term of size at the same time the constraints, the deformations and additional sizes.
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2.2.3.1 Sizes characteristic of the heterogeneous medium

· Porosity eulérienne: .
If one notes the part of the volume occupied by the vacuums in the current configuration, one a:

=

The definition of porosity is thus that of porosity eulérienne.
· Saturation in liquid: S
lq
If one notes total volume occupied by the liquid, in the current configuration, one has by
lq
definition:

S = lq
lq

This saturation is thus finally a proportion varying between 0 and 1.
· Densities eulériennes of water, the dissolved air, the dry air, of
W
AD
have
vapor, of gas.
vp
gz
If one notes (resp
,) water masses (resp of dissolved air, dry air and of
W
AD
have
vp
vapor) contents in a volume of the skeleton in the current configuration, one has by definition:

W = S D
AD

$$W_{lq} = S_D AD_{lq}$$

$$have = 1 \frac{vp}{1} \frac{have - S_D}{lq} = \frac{vp - S_D}{lq}$$

( )

( )

The density of the gas mixture is simply the sum of the densities dry air and vapor:

$$= + gz \; have \; vp$$

In the same way for the liquid mixture:
= +
lq
W
AD

0 are noted
0
0
0
, initial values of the densities.
W
AD
vp
have
· Lagrangian homogenized density: r.
At the moment running the mass of volume, M
M =
rd
, is given by:

.
0
0

2.2.3.2 Sizes
mechanics

1
· The tensor of the deformations U
() (X,) = (U
+T
T
U).
2
One will note V = tr ().
· The tensor of the constraints which are exerted on the porous environment: .
This tensor breaks up into a tensor of the effective constraints plus a tensor of constraints of pressure = +
“1. ”
and
are components of the constraints

p
p
generalized. This cutting is finally rather arbitrary, but corresponds all the same to an assumption rather commonly allowed, at least for the mediums saturated with liquid.
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2.2.3.3 Sizes
hydraulics

· Mass contributions in components m, m m
,
m
,
(unit: kilogramme per meter
W
AD
vp
have
cubic).
They represent the mass of fluid brought between the initial and current moments. They form part generalized constraints.
· Hydraulic flows M, M, M, M (unit: kilogramme/second/square meter).
W
AD
vp

have
One could not give very well no more precise definition of the contributions of mass and of flow, considering that their definition is summarized to check the equilibrium equations hydraulics:

m
m
Div M
M
W +
vp +
(W + vp) = 0
& &

éq
2.2.3.3-1

m
m
Div M
M
have +
AD +
(have + AD) = 0

& &
We nevertheless will specify the physical direction as of the these sizes, knowing that what we write now is already a law of behavior.
Speeds of the components are measured in a fixed reference frame in space and time.
One notes v W the speed of water, vad that of the dissolved air, vvp that of the vapor, go that

dry air, and v S =
that of the skeleton.
dt
The mass contributions are defined by:

m =
+ S - S
W
W (1
V)
0
0

0
lq
W
lq
m =
+ S - S
AD
AD (1
V)
0
0
0
lq
AD
lq

éq
2.2.3.3-2
m =
+ - S -
- S
have
have (1
V) (1
lq)
0
0
have
(0
1
lq)
m =
+ - S -
- S
vp
vp (1
V) (1
lq)
0
0
vp
(0
1

lq)

Mass flows are defined by:

M = S v - v
W
W
L (W
S)
M
= S v - v
AD
AD
L (AD
S)
éq
2.2.3.3-3
M = 1 - S v - v
have
have
(
L) (have
S)
M = 1 - S v - v
vp
vp
(
L) (vp
S)
The mass contributions make it possible to define the total density seen compared to configuration of reference: R = R
, where R indicates the density
0 + m
+ m + m + m
W
AD
vp
have
0
homogenized at the initial moment.
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Other intermediate hydraulic sizes are introduced:

p
· concentration of the vapor in gas
vp
C =
,
vp
pgz
M
M
M
· flow of gas:
gz = (1 - C
+ C
. This equation specifies that the speed of
vp)
have
vp
vp

gz
have
vp
gas is obtained by making an average (balanced sum) speeds of different

gas according to their concentration,
· the steam pressure p.
vp

2.2.3.4 Sizes
thermics

· not convectée heat Q (see further) (unit: Joule),
· mass enthali of the components
m
H
(m
m
m
m
H, H, H, H) (unit
:
ij
W
AD
vp
have
Joule/Kelvin/kilogramme),
· heat flow: Q (unit: Square J/s/meter).

All these sizes belong to the constraints generalized within the meaning of the document [R7.01.10] [bib4].

2.2.4 Data
external

· the mass force m
F (in practice gravity),
· heat sources,
· boundary conditions relating either to variables imposed, or on imposed flows.

3 Equations
constitutive

3.1
Conservation equations

It is here only about one recall, the way of establishing them is presented in [R7.01.10] [bib4].

3.1.1 Balance
mechanics

By noting the tensor of the total mechanical constraints and R density homogenized of medium, mechanical balance is written:
(
Div) + m
RF = 0 éq
3.1.1-1
We point out that R is connected to the variations of fluid mass by the relation:

R = R
éq
3.1.1-2
0 + m
+ m + m + m
W
AD
vp
have
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3.1.2 Conservation of the fluid masses

D its
For the fluid the derivative has = dt
& in fact a derivative eulérienne is and the equations which we write
for the fluid comprise terms of transport, even if they can be hidden by the choice of
unknown factors. The conservation equations of the fluid masses are written then:

m
m
Div ***M***
M
W +
vp +
(W + vp) = 0
& &

éq
3.1.2-1

m
m
Div ***M***
M
have +
AD +
(have + AD) = 0

& &

3.1.3 Conservation of energy: thermal equation

m
H m
H m
H m
H m
Q' Div H M
H M
H M
H M
Div Q
W
W +
m
AD
AD +
m
vp
vp +
m
have
have +
+
&
(MW W + mad AD + mvp vp + farmhouse have) + () =
(&
&
&
&

M
M
M
MR. F
W +
AD +
vp +
) m
have
+
éq 3.1.3-1

3.2
Equations of behavior

3.2.1 Evolution of porosity
dp
S dp
D = (B -)
gz -
lq
C

D
3 dT
V - 0
+

Ks
éq
3.2.1-1

In this equation, one sees appearing the coefficients B and K. B is the coefficient of Biot and K is
S
S
the module of compressibility of the solid matter constituents. If K indicates the module of compressibility
0
“drained” of the porous environment, one with the relation:
K
B
0
=1- Ks éq
3.2.1-2

3.2.2 Evolution of the contributions of fluid mass

By using the definition of the contributions of fluid mass and while putting forward arguments purely geometrical, one finds:

m =
+ S - S
W
W (1

V)
0
0
0
lq
W
lq
m =
+ S - S
AD
AD (1
V)
0
0
0
lq
AD
lq

éq
3.2.2-1
m =
+ - S -
- S
have
have (1
V) (1
lq)
0
0
have
(0
1
lq)
m =
+ - S -
- S
vp
vp (1
V) (1
lq)
0
0

vp
(0
1
lq)
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As example, we show the first relation in the case saturated S
(with =).
lq = 1
lq
W
That is to say an elementary field of porous environment of volume. One notes the volume occupied by
S
solid matter constituents and the volume occupied by the liquid and gas. 0 are noted
, 0
, 0
the same ones
L
S
L
volumes in an initial state. We point out that note the variation of volume of the porous environment and us
V
let us note the voluminal variation of the solid matter constituents.

S
V

One has by definition: =
L

= - = 1- = 0 1+
S
L
(
)
(Vs)
S

But (1 -) = 0 (1+

V) (1 -)
One deduces some:
0(1+ 1- = 0 1+
V) (
)
(Vs)
S

It is enough to write then
0
0
=
to obtain:
S
(0
1-)

0(1+ 1- = 0 1- 0 1+
V) (
)
(
) (Vs)
From where one deduces:
Vs (
0
1 -) = V (1 -) - (

0
-)

One uses the definition eulérienne density homogenized R (not to be confused with the Lagrangian definition of the equation [éq 3.1.1-2]):

R = 1
S (-) + lq

and the definition of the mass contribution in liquid:

R
= (R + mlq) 0

0

One obtains:

m
S (1 -)
0
+ lq
= S (
0
1 -) 0
0
0
0
0
+ + lq
lq

that is to say still:

m
ss + lq (
1+ V) 0
0
0
0
0
0
0
= S + + lq

S
lq

Using the conservation of the mass of the solid matter constituents:
0
0
= one obtains finally:
S
S
S
S

+ = 0
1
0 + m
lq (
V)
lq
lq
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3.2.3 Laws of behavior of the fluids

Liquid 3.2.3.1
D

dp
W
W
=
-
3
dT éq
3.2.3.1-1

K
W
W
W

One sees appearing the module of compressibility of water K and his module of dilation.
W
W

3.2.3.2 Gas

For the equations of reaction of gases, one takes the law of perfect gases:
pvp
R
=
T

M ol
vp
vp
éq
3.2.3.2-1
p
R
have =
T éq
3.2.3.2-2

M ol
have
have
One sees appearing the molar mass of the vapor,
ol
M, and that of the dry air,
ol

Mr.
vp
have

3.2.4 Evolution of the enthali

3.2.4.1 Enthalpy

liquid
dp
m
p
dh = C dT + 1 -
3 T

éq
3.2.4.1-1
W
W
(
) W
W
W

One sees appearing the specific heat with constant pressure of water:

p
C.
W

By replacing in this expression the pressure of the liquid by its value according to the pressure capillary and of the pressure of gas, one a:

-
-
dhm = 1 -
3
+

éq
3.2.4.1-2
W
(
T
W
) dp dp dp
gz
C
AD

C pdT
W
W
While noting
p
C specific heat with constant pressure of the dissolved air, one a:
AD
dhm = C p dT
éq
3.2.4.1-3
AD
AD

3.2.4.2 Enthalpy of gases

dhm = C p dT
vp
vp

éq
3.2.4.2-1
dhm = C p dT éq
3.2.4.2-2
have
have

One sees appearing the specific heat with constant pressure of the dry air
p
C and that of the vapor p
C.
have
vp
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3.2.4.3 Contribution of heat except fluids

It is the quantity Q
who represents the heat received by the system except contribution enthalpic of fluids.

$$Q = 3\,K\,Td_m + 3\,Tdp_0 - 3 + 3 + 0_0 V_{lq} C_{(m_m gz_{lq})} Tdp C\,dT_{gz}$$

éq 3.2.4.3 - 1

One sees appearing several dilation coefficients:
m_m,
. The coefficient is a data:
0,
lq

gz
0
it corresponds at the same time to the dilation coefficient of the porous environment and to that of the solid matter constituents (which
find being inevitably equal in the theory which we present here).

m
m
, are given by the relations:
lq
gz
1-
m
= 1-
- +
gz
(Slq) (b)
(Slq)
0
T
3

éq
3.2.4.3-2
m
= S B -

éq
3.2.4.3-3
0 +
S

lq
lq (
)
lq
lq

One also sees appearing in [éq 3.2.4.3 - 1] the specific heat to constant deformation of porous environment 0
C, which depends on the specific heat to constant constraint of the porous environment 0
C by
the relation:
0

0
2
C = C - 9TK
0
0
éq
3.2.4.3-4

0
C is given by a law of mixture:
C 0 = (1 -)
S
p
p
C + S

(C + C) + 1 - C M + C éq
3.2.4.3-5
S
lq
W
W
AD
AD
(lq) (
p
p
vp
vp
have
have)

where
S
C

represent the specific heat to constant constraint of the solid matter constituents and
mass
S
voluminal of the solid matter constituents. For the calculation of, one neglects the deformation of the solid matter constituents, one
S
thus confuses with its initial value 0
, which is calculated in fact starting from the specific mass

S
S
initial of the porous environment R by the following formula of the mixtures:
0

(0
1 -) 0
0
0
0
0
= R - (+) S - 1 - S + ***éq***
3.2.4.3-6
0
W
AD
lq
S
(0lq) 0 (0 0
vp
have)

3.2.5 Laws of diffusion

3.2.5.1 Diffusion of heat

One takes the traditional law of Furrier:
T
Q = -
. T
éq
3.2.5.1-1

where one sees appearing the thermal coefficient of conductivity T.

T
is a function of porosity, saturation and temperature and is given in the form of product of three functions plus a constant:

T
T
T
T
T
=
(

). (S

).
T
( ) +
éq
3.2.5.1-2
S
lq
T
cte

3.2.5.2 Diffusion of the fluids

They are the laws of Darcy, to which one adds the law of Fick in the presence of vapor. The laws of Darcy are written for gas and the liquid:

Mlq
H
= -
+ F
éq

3.2.5.2-1
lq (
m
plq
lq
)
lq
M gz
H
= (
m
- pgz + gzF) éq
3.2.5.2-2
gz
gz
where we see appearing hydraulic conductivities H
and H
for the liquid and gas
lq
gz
respectively.
M
One makes the approximation that
W
H
= -
+ F
W (
m
plq
lq
)
W

The diffusion in the gas mixture is given by the law of Fick thanks to the relation:

M
M
D
p
vp
have
vp

vp
-
= -

éq
3.2.5.2-3

C
1
(
C) p
vp
have
vp
- vp gz

where D is the coefficient of diffusion of Fick of the gas mixture (L2.T-1), one notes thereafter F such
vp
vp
that:
vp
D
vp
F =

éq
3.2.5.2-4
C
1
(- C)
vp
vp

and with
pvp
Cvp =

éq
3.2.5.2-5
pgz

One thus has:

Mvp Farmhouse
-
= - F C éq
3.2.5.2-6
vp
vp

vp
have
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Moreover, one a:
M gz = (
M
M
1 - Cvp) have
vp
+ Cvp
éq
3.2.5.2-7
gz
have
vp
and:

gz = vp + have éq
3.2.5.2-8

For the diffusion of the liquid mixture, the usual writing is as follows:
M - M = - D
éq
3.2.5.2 - 9
AD
W
AD
AD

where D is the coefficient of diffusion of Fick of the liquid mixture. In order to keep a writing
AD
homogeneous with that of the gas mixture one notes F thereafter such as:
AD
F = D
éq
3.2.5.2-10
AD
AD
And the concentration C corresponds here to the density of the dissolved air:
AD
C =
éq
3.2.5.2 - 11
AD
AD

M - M = - F C
éq
3.2.5.2 - 12
AD
W
AD
AD

Concerning the liquid, one admitted that liquid Darcy applies at the speed of liquid water. There is not thus not to define mean velocity of the liquid.

M W
H

= - p + F
éq
3.2.5.2-13
lq (
m
lq
lq
)
W
and:
= +
éq
3.2.5.2-14
lq
W
AD

By combining these relations, one finds then:

M have
H
= -
+ F +

éq
3.2.5.2-15
gz (
m
p
C F
C
gz
gz
) vp vp vp
have
Mvp
H
= -
+ F - 1
(-
)

éq

3.2.5.2-16
gz (
m
p
C
F
C
gz
gz
)
vp
vp
vp
vp
M W
H
= - p + F
éq
3.2.5.2-17
lq (
m
lq
lq
)
W
H
M
= - p + F - F C éq
3.2.5.2-18
AD
AD
lq (
m
lq
lq
) AD AD
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Hydraulic conductivities H
lq and H
are not directly data and their value is
gz
known starting from the formulas:

K int
(
K rel
).
H
lq (Slq)
=

éq
3.2.5.2-19
lq
µw (T)
K int
(
K rel
).
,
H
gz (S
p
lq
gz)
=
éq

3.2.5.2-20
gz
µgz (T)
int
K is the intrinsic permeability, characteristic of the porous environment and user datum, function unspecified of porosity;
µ is the dynamic viscosity of water, characteristic of water and user datum, function
W
unspecified of the temperature;
µ is the dynamic viscosity of gas, characteristic of gas and user datum, function
gz
unspecified of the temperature;
rel
K is the permeability relating to the liquid, characteristic of the porous environment and user datum,
lq
unspecified function of saturation in liquid;
rel
K is the permeability relating to gas, characteristic of the porous environment and user datum, function
gz
unspecified of saturation in liquid and the gas pressure.

Note:

Here definite hydraulic conductivities are not inevitably very familiar for
mechanics of grounds, which usually use for the saturated mediums the permeability K,
which is homogeneous at a speed.
H
K
The relation between K and H
is as follows: =
where G is the acceleration of gravity.
lq
lq
G
W

The coefficient of diffusion of Fick of the gas mixture vp
F is a characteristic of the porous environment,
unspecified user datum function of the steam pressure, the gas pressure, of
saturation and of the temperature which one will write like a product of function of each one of these variables: F (P, P, T, S) = F vp (P). F gz (P). F T (T). F S (S)
vp
vp

gz
vp
vp
vp
gz
vp
vp
one will neglect the derivative by
report/ratio with the steam pressure and saturation. Same manner for the coefficient of diffusion of Fick of the liquid medium: F (P, P, T, S) = F AD (P). F lq (P). F T (T). F S (S)
AD
AD
lq
AD
AD
AD
lq
AD
AD
, one does not take
in account that the derivative according to the temperature.

3.2.6 Balance water-steam

This relation is essential and it results in to reduce the number of unknown factors of pressure.
M is noted
H mass enthalpy of water, m
S its entropy and m
m
m
G = H - Ts its free enthalpy.
W
W
W
W
W
M is noted
H mass enthalpy of the vapor, m
S its entropy and m
m
m

G = H - Ts its enthalpy
vp
vp
vp
vp
vp
free.
Balance water vapor is written:

m
m
G = G éq
3.2.6-1
vp
W
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Who gives:

m
m
H - H = T S - S
éq
3.2.6-2
vp

W
(m m
vp
W)
dp
In addition, the definition of the free enthalpy teaches us that: dg =
- sdT, which, applied to

vapor and with water, compound with the relation
m
m
dg = dg and while using [éq 3.2.6-2] gives:
vp
W

dpvp
dpw
=
+ (hm
m
-

éq
3.2.6-3
vp
W) dT
H

T
vp
W

This relation can be expressed according to the capillary pressure and of the gas pressure:

dp
vp
=
-
-
+
-
éq

3.2.6-4
vp
(dp dp dp
gz
C
AD)
vp (H m
m
vp
W) dT
H

T
W
3.2.7 Balance air dissolved dryness-air

The dissolved air is defined via the constant of Henry K, who connects the molar concentration of dissolved air
H
ol
C (moles/m3) with the pressure of dry air:
AD
p
ol
have
C =

éq
3.2.7-1
AD
K H

with ol
AD
C =
éq
3.2.7-2
AD
ol
M AD

Molar mass of the dissolved air,
ol
M is logically the same one as that of the dry air

ol
Mr. For
AD
have
the dissolved air, one takes the law of perfect gas:

pad
R
=
T
éq
3.2.7-3

M ol
AD
have

The pressure of dissolved air is thus connected to that of dry air by:
p
p
have
=
RT
éq
3.2.7-4
AD
K H

3.2.8 The mechanical behavior

One will write it in differential form:
D = D +
'D 1
p éq
3.2.8-1
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By using a formulation of Bishop [bib10] extended to the unsaturated mediums one writes:

D = - B dp - S dp
éq
3.2.8-2
p
(gz lq c)

In the relation [éq 3.2.8-1] the evolution of the tensor of effective constraint is supposed to depend that displacement of the skeleton and variables internal. Usual terms related to thermal deformation are integrated into the calculation of the effective constraint:

of
= F (
D - IdT,
0

D)
éq
3.2.8-3
The reason of this choice is to be able to use any law of traditional thermomechanics for calculation of the effective constraints, laws which, in more the share of the writings are in conformity with [éq 3.2.8-3].

3.2.9 The isotherm of sorption

To close the system, there remains still a relation to be written, connecting saturation and the pressures. Us
chose to consider that saturation in liquid was an unspecified function of the pressure capillary, that this function was a characteristic of the porous environment and provided in data by the user.
Since the user can provide a function S very well
p refines per pieces, and being

lq (c)
S

given that the derivative of this function,
lq, plays an essential physical part, we chose
p
C
to ask the user to also provide this curve, remainder with its load to ensure itself of coherence of the data thus specified.
It is noticed that in the approach present, one speaks about a bi-univocal relation between saturation and
capillary pressure. It is known that for the majority of the porous environments, it is not the same relation which
must be used for the ways of drying and the ways of hydration. It is one of the limits approach present.

3.2.10 Summary of the characteristics of material and the user data

· The Young modulus E and the drained Poisson's ratio make it possible to calculate it
0
0
modulate compressibility drained of the porous environment by
0
=
E
K
,
0
(31- 20)
K
· the coefficient of Biot B
0
= 1-
allows to calculate the module of compressibility of the grains
Ks
solids K,
S
· the module of compressibility of water K,
W
· the dilation coefficient of water,
W
· the constant of perfect gases R,
· molar mass of the vapor
ol

M,
vp
· molar mass of the dry air
ol
M, (=
ol
M)
have
AD
· specific heat with constant pressure of water p
C,
W
· specific heat with constant pressure of the dissolved air p
C
AD
· specific heat with constant pressure of the dry air p
C,
have
· specific heat with constant pressure of the vapor p
C,
vp
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· the dilation coefficient of the porous environment which is also that of the solid matter constituents,
0
· specific heat with constant constraint of the solid matter constituents S

C,
· the thermal coefficient of conductivity of the solid matter constituents only, T
, unspecified function of
S
the temperature,
· the thermal coefficient of conductivity of the liquid, Tlq, unspecified function of
temperature,
· the thermal coefficient of conductivity of the dry air, Heap, unspecified function of
temperature,
· the coefficient of diffusion of fick for the gas mixture, F, unspecified function of
vp
temperature, of the gas pressure, the steam pressure and saturation
· the coefficient of diffusion of fick for the liquid mixture, F, unspecified function of
AD
temperature and of the pressure of liquid, the pressure of the dissolved air and saturation.
· The constant of Henry K function unspecified of the temperature,
H
· the intrinsic permeability, int
K, unspecified function of porosity,
· the dynamic viscosity of water, µ, unspecified function of the temperature,
W
· the dynamic viscosity of gas, µ, unspecified function of the temperature,
gz
· the permeability relating to the liquid, rel
K, unspecified function of saturation in liquid,
lq
· the permeability relating to gas, rel
K, unspecified function of saturation in liquid and of
gz
gas pressure,
· the relation capillary saturation/pressure, S p, unspecified function of the pressure
lq (c)
capillary,
· in a general way the initial state is characterized by:
-
the initial temperature,
-
initial pressures from where one deduces initial saturation 0
S p
lq (
0
c),
-

initial specific mass of water 0
,
W
-
initial porosity 0
,
-
initial pressure of vapor 0
p from where one deduces the initial density from
vp
vapor 0
,
vp
-
initial pressure of the dry air 0
p from where one deduces the initial density from L `dry air
have
0
.
have
-
initial density of the porous environment R,
0
- density of the solid matter constituents 0
. This data is useful only for
S
modelings including of thermics, and one will have to take care that it is coherent with the other data, through the relation
:
(0
1 -) 0
0
0
0
0
= R - (+) S - 1 - S +,
0
W
AD
lq
S
(0lq) 0 (0 0
vp

have)

-

initial enthali of water, the dissolved air, the vapor and the dry air.
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3.3
The state of reference and the initial state

The introduction of the initial conditions is very important, in particular for the enthali.
In practice, one can reason by considering that one has three states for the fluids:

· the state running,
· the state of reference: it is that of the fluids in a free state. Very often one will take for
pressures of water and air atmospheric pressure. In this state of reference, one can
to consider that the enthali are null,
· the initial state: it is important to note that, in an initial state of the porous environment, water is in a
state
hygroscopic different from that of interstitial water. For the enthali of water and vapor one
will have to take:

init
init
m
p
p
p
p
W
- ref.
init
L
W

-
hw =
=
atm

W
W
init
m
H
L T

vp =
(init) =
ion
vaporisat
of

latent
heat
init
m
has = 0
init
m
had = 0

Note:

The initial vapor pressure will have to be taken in coherence with these choices. Very often, one leaves the knowledge of an initial state of hygroscopy. The relative humidity is the report/ratio between the steam pressure and the steam pressure saturating at the temperature considered. One then uses the law of Kelvin which gives the pressure of the liquid according to the pressure of vapor, of the temperature and the steam pressure
0
p
p
R
p

W -
saturating:
W =

T ln
vp

ol
sat

. This relation is valid only for
M
W
vp
p (T)
vp

isothermal evolutions. For evolutions with variation in temperature, the formula [éq 4.1.4-1] further established gives a more complete expression but in fact incremental. Knowing a law giving the steam pressure saturating to the temperature T, by
0

0
T -
5
.
273

2
+
7858
.
sat

31 559
.
+ 1354
.
(0T-
)
example:

p (T) 10
, and a degree of hygroscopy HR, one deduces some
vp
0
=

5
.
273
the steam pressure thanks to p (T) = HR psat (T).
vp
0
vp
0

3.4
Nodal unknown factors, initial values and values of reference

We approach here a point which is due more to choices of programming than to true aspects of formulation. Nevertheless, we expose it because it has important practical consequences. principal unknown factors which are also the values of the degrees of freedom, are noted:

ux

uy

{U}
U
ddl

Z

=
ddl

1
PRE

ddl
PRE2
ddl
T

According to modeling, they can have different significances:

LIQU_SATU LIQU_VAPE LIQU_GAZ_ATM
GAS
LIQU_VAPE_GAZ
PRE1
p
p
p = - p
p
p = p - p
W
W
C
W
gz
C
gz
W
PRE2

p
gz

LIQU_GAZ LIQU_AD_GAZ_VAPE
PRE1
p = p - p
p = p - p - p
C
gz
W
C
gz
W
AD
PRE2
p
p
gz
gz

One will then define the real pressures and the real temperature by:

ddl
init
p = p + p for the pressures
ddl
PRE1 and PRE2 and
init
T = T
+ T for the temperatures, where init
p and
init
T
are defined under key word THM_INIT of order DEFI_MATERIAU.
The values written by IMPR_RESU are the values of {} ddl
U
. The boundary conditions are defined
for {} ddl
U
. Key word DEPL of the key word factor ETAT_INIT of order STAT_NON_LINE defines
initial values of {} ddl
U
. Initial values of the enthali, which belong to the constraints
generalized are definite starting from key word SIGM of the key word factor ETAT_INIT of the order STAT_NON_LINE. The real pressures and the real temperature are used in the laws of

dp
D dT
behavior, in particular laws of the type S = F p or
=
+
. Initial values of
lq
(c)
p

T
densities of the vapor and the dry air are defined starting from the initial values of the pressures gas and of vapor (values read under key word THM_INIT of order DEFI_MATERIAU). One notice that, for displacements, the decomposition
ddl
init
U = U + U is not made: the key word
THM_INIT of order DEFI_MATERIAU thus does not make it possible to define displacements initial. The only way of initializing displacements is thus to give them an initial value by key word factor ETAT_INIT of order STAT_NON_LINE.

3.5 Effective constraints and total constraints. Boundary conditions of constraint

The partition of the constraints in constraints total and effective is written:

= +
*'**1***
p

is the total constraint, c.a.d that which checks:
(
***Div**) + m*
RF = 0
is the effective constraint. For the laws of effective constraints, it checks:
1
D = F (
D -
= +T
0dT,), where
*(**U U**) and represents the internal variables.*
2
is calculated according to the water pressures. The adopted writing is incremental and, if one

p
wants that the value of is coherent with the value init
p defined under the key word
p
THM_INIT, it is necessary
to initialize by the key word
p
SIGM of the key word factor ETAT_INIT of order STAT_NON_LINE.

In the files results, one finds the constraints effective under the names of components
SIXX… and under the name
p
SIP. The boundary conditions in constraints are written in constraints
total.
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3.6
Some numerical values

We give here some reasonable values for certain coefficients. These values are not
programmed in Code_Aster, they are provided here as an indication:

For perfect gases, one retains the following values:
1
R
3144

.
8
J. -
=
K
3
-
-1
M ol

vp = 18. 10
kg.mole
3
-
1
M ol

have =
96
.
28
10 kg.
-
mole
For CO2, the value of the constant of Henry with 20°C is of:
3
1
K
3162
.
-
H =
Pa m mole
For liquid water, one a:
3
= 1000 kg/m
W
K = 2000 MPa
W

The thermal dilation coefficient of water is correctly approached by the formula:
5
= 52

.
9
10 - Ln (T
(
1
-
K)
W
-
)
-4
273 - 19
.
2
10

The heat-storage capacities have as values:
S
1
-
1
C = 800
-

JKg K
p
1
-
1
Clq = 4180
-
JKg K
p
1
-
1
Cvp =1870
-
JKg K
p
1
-
1

Case =1000
-
JKg K

One also gives a law of evolution of the latent heat of liquid phase shift
vapor:
L (T) = 2500800 -
(
2443 T - 273 15
.) J/kg.

4
Calculation of the generalized constraints

In this chapter, we specify how are integrated the relations described into chapter 3. More precisely still, we give the expressions of the constraints generalized within the meaning of document [R7.01.10] [bib4] when the laws of behaviors THM are called for the option RAPH_MECA within the meaning of the document [R5.03.01] [bib3]. So that this document follows of readier the order of
programming, we will consider two cases: the case without dissolved air and that with.

The generalized constraints are:

*; m, **M**, hm; m, **M**, hm; m, **M**, hm; m, **M**, hm; Q*
p
W
vp
vp
have
AD
AD
AD
*'**Q**,*
W
W
vp
have
have

The generalized deformations, from which the generalized constraints are calculated are:

***U**, (**U**); p, p; p, p T*
; , T

C
C
gz
gz

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The variables intern that we retained are:

In the case without vapor:
, S
W
lq
In the case with vapor and without dissolved air:
, p, S
W
vp
lq
In the case with dissolved vapor and air:
, p, p, S
W
vp
AD
lq

In this chapter, we adopt the usual notations Aster, namely the indices + for the values quantities at the end of the step of time and indices - for the quantities at the beginning of the step of time.

Thus, the known quantities are:

· generalized constraints, deformations and internal variables at the beginning of the step of time:
-
-
-
-
-
-
-
-
-
m
-
-
m
-
-
m
-
-
m
-
-
*; m, **M**, H; m, **M**, H; m, **M**, H; m, **M**, H; Q*

p
W
W
W
vp
have
AD
AD
AD
*'**Q**,*
vp
vp
have
have

-
-
***U**, (-*
***U**) -*
-
-
-
-
-
; p, p; p, p T
; , T
C
C
gz
gz
-
-
-
-
-
, p, p
AD
W
vp
· deformations generalized at the end of the step of time:
-
+
***U**, (+*
***U**) +*
+
+
+
+
+
; p, p; p, p T
;
, T
C
C
gz
gz
· The unknown quantities are the constraints, and variables intern at the end of the step of time:
+

+
+
+
-
+
+
+
+
m
+
+
m
+
+
m
+
+
m
+
+
*; m, **M**, H; m, **M**, H; m, **M**, H; m, **M**, H; Q*

p
W
W
W
vp
have
AD
AD
AD
*'**Q**,*
vp
vp
have
have
-
+
+
+
+
, p, p
AD

W
vp

4.1
Case without dissolved air

4.1.1 Calculation of porosity and it it density of the fluid

The first thing to be made is of course to calculate saturation at the end of the step of time
+
S
S p. porosity is by integrating on the step of time the equation [éq 3.2.1-1].
lq =
lq (+
c)

One obtains then:
+

B -
+
-
+
+
-

p
p
S p
p
ln
= -
+
-
+
-
gz
gz
lq
C
C

3

T
T
éq
4.1.1-1
-
(V - V) + 0 (-) (-) - (-)
B -

-
K

S

The density of the liquid is by integrating on the step of time the equation [éq 3.2.3.1 - 1].

What gives:

+
+
-
+
-
p - p - p + p
ln W = gz
gz
C
C -
3
T

éq
4.1.1-2
W
- T
-
(+ -)

W
Kw

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4.1.2 Calculation of the dilation coefficients

It is about a simple application of the formulas [éq 3.2.4.3 - 2] and [éq 3.2.4.3 - 3], which are evaluated
at the end of the step of time:
+
+
+
1 S
m
1

éq
4.1.2-1
vp
= farmhouse = mgz = (
-
+
- Slq) (B -)
+
+
(+lq)
0 +
+

T
3
+
m
+

S B

éq
4.1.2-2
0
S
W
= lq (
+
-
)
+
+
+ lq
lq

4.1.3 Calculation of the fluid enthali

The fluid enthali are calculated by integration of the equations [éq 3.2.4.1 - 1], [éq 3.2.4.2 - 1], [éq 3.2.4.2 - 2].
+
-
1

3 T
m
H
H
C T
T
p
p
p
p éq
4.1.3-1
W
= MW + pw (+ -) (
+

-
-
W
)
+
gz -
gz -
+
+
(+ - + - C
C
)
W

+
-
hm
éq
4.1.3-2
vp
= hmvp + C pvp (+
-
T - T)
+
-
hm
éq
4.1.3-3
have
= hmas + C not (+
-
T - T)

4.1.4 Air and steam pressures

On the basis of the relation [éq 3.2.6-4] in which one carries the law of reaction of perfect gases
dp
M ol
vp
vp 1
1
m
m dT
[éq 3.2.3.2 - 1], one finds

=

dp
that one integrates by one
gz -
dpc + (hvp - hw)

p
RT

vp

T
W
W

way initially at constant temperature (one then considers the density of constant water),
then of -
T with +
T with constant pressures.

+
p
+
ol
ol
T
ln
M
1
M
vp =
vp
vp
m
m dT
p
p
p
p
H
H

-
+
+ (
+
-
+
-

[gz - gz) - (C - c)] + (vp - W)
p
RT
R
2

T
vp
-
T
W

The first term corresponds to the way at constant temperature, the second with the way with pressures constants. By using the definitions [éq 3.2.4.1 - 1] and [éq 3.2.4.2 - 1] of the enthali, one sees that for an evolution with constant pressures:

hm - hm
hm - - hm-
-
-
-
vp
W
vp
W
(C p CP
vp
W) (T
T)
=
+

2
2

2
T
T
T

One thus has for such a way:

+
T
+
1
1
1
1
m
m dT
-
-
m
m
p
p
T

(H
ln

vp - hw)
=
2
(hvp - hw)

-
+ C
C
T
-
+
(vp - W) -

-

+

-

-
+
-
T
T
T
T
T
T
T
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That is to say finally:

+
p
ol
vp
M
=
vp
1

ln
p
p
p
p
-
+
+ (
+
-
+
-

[gz - gz) - (C - c)] +
p

vp
RT
W
éq
4.1.4-1
M olvp (
ol
+
-
-
M
hmvp -
-
hmw) 1
1
vp
p
p
T
T

-
+
C
C
ln
1

-
+
(vp - W)

+
-

-
+

R
T
T
R
T T

One can then calculate the densities of the vapor and the air by the relations [éq 3.2.3.2 - 1] and [éq 3.2.3.2 - 2]:

ol
+
M
p
+

éq
4.1.4-2
vp =
vp
vp
+
R T
M ol p
have
gz - p
+
(+ +vp)
have =
+
R
T

éq
4.1.4-3

4.1.5 Calculation of the mass contributions

The equations [éq 3.2.2-1] give null mass contributions to moment 0. Way is written incremental the equations [éq 3.2.2-1]:

+
-
+
m
m

S

S
W =
W +
W (1
+
+ V) + +
-
lq -
W (1
-
+ V) - - lq
+
-
+
m
m

S

S
have =
have +
have (1
+
+ V) + 1
(
+

-
)
-
lq
- have (1 -
+ V) - 1
(
-
-
)
lq

éq 4.1.5-1
+
-
+
m
m

S

S
vp =
vp +
vp (1
+
+ V) + 1
(
+
-
)
-
lq
- vp (1 -
+ V) - 1
(
-
-
)
lq

4.1.6 Calculation of the heat-storage capacity and Q' heat

There are now all the elements to apply at the end of the step of time the formula [éq 3.2.4.3 - 5]:

+
0
+
S
+
+
+
p
C = 1
(-) C + C M + 1
(- S +
+
+

)
C
+
+ C éq
4.1.6-1
S
lq
W
W
lq
(
p
p
vp
vp
have
have)

One uses of course [éq 3.2.4.3 - 4] who gives:
0+
0+
+
2
C = C - 9T K
0
0
éq

4.1.6-2
Although variation of heat Q
is not a total differential, it is nevertheless licit of
to integrate on the step of time and one obtains while integrating [éq 3.2.4.3 - 1].

+
-
1
1
1
Q' = Q' +
3 K T 2
3
3
3
0
0
(+
0
2
2
V - V)
+
- + m T
lq
(+ -
p - p
C
C) - (
+
+
mgz + mlq) T (+
-
p - p
gz
gz)
+
+ C T -
(+
-
T)
éq 4.1.6-3
1

+
-
T +
where we noted:
2
T
=
T. We chose here a formula of "point medium" for
2
variable temperature.
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4.1.7 Calculation of the mechanical constraints

The calculation of the effective constraints is done by calling upon the incremental laws of mechanics selected
by the user. One integrates on the step of time [éq 3.2.8-2] and one a:

+
-

B p
p
bS p
p ***éq***

4.1.7-1

p = p -
(+ -
gz -
gz)
+
+ lq (+
-
C -
c)

4.1.8 Calculation of hydrous and thermal flows

It is necessary well could calculate all the coefficients of diffusion:

The coefficient of Fick
+
F = F (+ + +
T, p, p

C
gz)
Thermal diffusivity T +
T
+
T
+
T
+
T
=
(

). (S

).
T
(
) +
S
lq
T
cte
Hydraulic permeabilities and conductivities:

int
+
rel
int
rel

(
).

+
K
K
(
).
,
H
W (+
Slq)
+
+
K
kgz
H
(+ +
S
p
lq
gz)

lq
=
μ
μ
W (
gz
=
+
T)
gz (+
T)
+
p

Vapor concentrations:
+
C = vp
vp
+
pvp

It does not remain any more whereas to apply the formulas [éq 3.2.5.1 - 1], [éq 3.2.5.2 - 15], [éq 3.2.5.2 - 16] and
[éq 3.2.5.2 - 17] to find:

+
+
T
+
Q = - T éq 4.1.8-1
+
M
+
have = H

p

F
C F
C éq
4.1.8-2
gz
-
+
+
m + vp vp
+
[+gz (+ +
have
vp)
] + + +vp
have
+
Mvp
+
= H

p

F
1
(
C) F
C éq
4.1.8-3
gz
-
+
+
m - - vp vp
+
[+gz (+ +
have
vp)
]
+
+
+
vp
vp
M+w
H +
=
- p+
+
+ F
éq
4.1.8-4
+
lq [
m
lq
W
]
W

4.2
Case with dissolved air

4.2.1 Calculation of porosity

Same manner, the first thing to be made is to calculate saturation at the end of the step of time

+
S
S p. porosity is by integrating on the step of time the equation [éq 3.2.1-1]. One
lq =
lq (+
c)
thus recall that:

+

B -
+
-
+
+
-

p
p
S p
p
ln
= -
+
-
+
-
gz
gz
lq
C
C

3
T
T
-
(V - V) + 0 (-) (-) - (-)
B -

-
K

S

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4.2.2 Calculation of the dilation coefficients

It is about a simple application of the formulas [éq 3.2.4.3 - 2] and [éq 3.2.4.3 - 3], which are evaluated
at the end of the step of time:
+
1 - S
m
m+
m+

1

éq
4.2.2-1
vp
= aas = agz = (
+
- Slq) (B -)

+
+
(+lq)
0 +
+
T
3
+
m
+
W = Slq (
+
B -)
+ +
0 + lq Slq
éq
4.2.2-2
+

m
+
S
AD = S (
+
B -)
+ +
0 +
lq
lq

éq
4.2.2-3
+
T
3

4.2.3 Calculation of density and dissolved and dry air, steam pressures

On the basis of the relation [éq 3.2.6-4] in which one carries the law of reaction of perfect gases [éq 3.2.3.2 - 1], one finds:

dp
M ol

vp
vp
=
1 dp

éq
4.2.3-1
W + (H m
vp - H m
W) dT

p
RT

vp

T
W

Contrary to the case without dissolved air p is not now known any more.
W
RT
RT
p = p - p = p - p -
p = p - p -
(p - p)
W
lq
AD
gz
C
have
gz
C
gz
vp
K
K
H
H
thus

RT
R
dp = dp - dp -
(dp - dp) -
(p - p) dT éq
4.2.3-2
W
gz
C
K
gz
vp
K
gz
vp
H
H

One integrates [éq 4.2.3.1] while including there [éq 4.2.3.2] by a way initially into constant temperature (one
then consider the density of constant water), then of -
T with +
T with constant pressures. With
final one obtains:

+
p
ol
ol
ol
ln
M
1
1
M
M
vp = vp (
-
vp
vp
) p
p
p

p
p
p
-
-
+
(+gz - - gz) + - (+vp - - vp) - - + (+c - - C) +

p

RT
K
K
RT
vp
W
H
W
H
W
éq 4.2.3-3
ol
ol
T
MR. R
T
M
vp
vp
dT
m
m
p
p ln
H
H
-
(
+
+
+
vp
- +gz)

+
-

(vp - W)

K
T
R
T 2
W
H

-
T
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Contrary to the preceding case, there is here a nonlinear equation to solve. One will make for that a method of the corrector-predictor type. P is posed
~ such as:
vp

p~

ol
ol
ln
M
1
1
M
vp = vp (
-
vp
) p
p
p
p
-
-
+
(+gz - - gz) - - + (+c - - C)

p

RT
K
RT
vp
H
W
W
éq
4.2.3-4
+
ol
T
M
+ vp
dT

(m
H
H
vp -
m
W)

R
2
-
T
T

and thus

+
ol
ol
T
M
1
1
M
~
-
p
p.

dT
exp
(
) p
p
p
p
H
H

vp =
vp
vp
-
vp
m
m
-
+
(+gz - - gz) - - + (+c - - C) + (vp - W)
RT
K
RT

2
-

T
T
H
W
W

éq 4.2.3-5
Moreover, as in the section [§4.1.4], one recalls that
+
T
+
1
1
1
1
m
m dT
-
-
m
m
p
p
T

(H
ln
vp - hw)
=
2
(hvp - hw)

-
+ C
C
T
-
+

(vp - W) -

-

+

-

-
+
-
T
T
T
T
T
T
+
p
p~
+
p
+
+
~
p

p
p
p
vp
vp -

+
Like ln vp
ln vp ln vp
=
+

vp

vp

and that by D.L ln
= ln 1+

-1
-
-
~
~

~

~
p
p
p
p
p
p
vp
vp
vp
vp

vp

vp

+
pvp will thus be given by the following linear expression:
+
ol
ol
+
p
M
MR. R
vp = 1+
vp
vp
T
p
p

p
p
ln
~
éq 4.2.3-6
-
(+vp - - vp) + - (+vp - - gz)

-
p
K
K
vp
H
H
W
W
T
from where

+
ol
T
-
-
-
K
M
p
p R ln

H -
vp
W

vp + gz

-
T
+

p

éq
4.2.3-7
vp =
-
KH

+
W
ol
T

- M 1
ln
vp
~

+ R -
pvp

T

From there the other pressures are calculated easily:

+
+
+
p
p
p
have =
gz -
vp
+
+
not
+
p

AD =
RT

K H
+
+
+
+
p
p
p
p
W =
gz -
C -
AD
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One can then calculate the densities of the vapor and the air by the relations [éq 3.2.3.2 - 1], [éq 3.2.3.2 - 2] and [éq 3.2.7-3]:

ol
+
M
p
+

éq

4.2.3-8
vp =
vp
vp
+
R T
M ol p
have
gz - p
+
(+ +vp)

éq
4.2.3-9
have =
+
R
T
+
ol
+

éq
4.2.3-10
AD = p
M
AD
have
+
RT
The density of water is by integrating on the step of time the equation [éq 3.2.3.1 - 1].

What gives:

+
+
-
+
-
+
-
p - p - p + p - pad + p

ln W =
AD
gz
gz
C
C
-
3
T

éq 4.2.3-11
W
- T
-
(+ -)

W
Kw

4.2.4 Calculation of the fluid enthali

The fluid enthali are calculated by integration of the equations [éq 3.2.4.1 - 1], [éq 3.2.4.1 - 3], [éq 3.2.4.2 - 1], [éq 3.2.4.2 - 2].
+
-
1

3 T
m
H
H
C T
T
p
p
p
p
p
p

éq 4.2.4-1
W
= MW + pw (+ -) (

+
-
-
W
)
+
gz -
gz -
+ C - AD +
+
(+ - + - + - AD
C
)
W
+
-
hm
éq
4.2.4-2
AD
= hmad + C pad (+
-
T - T)

+
-
hm
éq
4.2.4-3
vp
= hmvp + C pvp (+
-
T - T)
+
-
hm
éq
4.2.4-4
have
= hmas + C not (+
-
T - T)

4.2.5 Calculation of the mass contributions

The equations [éq 3.2.2-1] give null mass contributions to moment 0. Way is written incremental the equations [éq 3.2.2-1]:

+
-
+
m
m

S

S
W =
W +
W (1
+
+ V) + +
-
lq -
W (1
-
+ V) - - lq
+
-
+
m
m

S

S
AD =
AD +
AD (1
+
+ V) + +
-
lq -
AD (1
-
+ V) - - lq

éq
4.2.5-1
+
-
+
m
m

S

S
have =
have +
have (1
+
+ V) + 1
(
+
-
)
-
lq
- have (1 -
+ V) - 1
(
-
-
)
lq
+
-
+
m
m

S

S
vp =
vp +

vp (1
+
+ V) + 1
(
+
-
)
-
lq
- vp (1 -
+ V) - 1
(
-
-
)
lq
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4.2.6 Calculation of the heat-storage capacity and Q' heat

There are now all the elements to apply at the end of the step of time the formula [éq 3.2.4.3 - 5]:

+
0
+
S
+
+
+
p
+
p
C = 1
(-) C + S (C + C) + 1
(- S +
+
+

)
C
+
+ C éq 4.2.6-1
S
lq
W

W
AD
AD
lq
(
p
p
vp
vp
have
have)

One uses of course [éq 3.2.4.3 - 4] who gives:

0+
0+
+
2
C = C - 9T K
0
0
éq
4.2.6-2

Although variation of heat Q
is not a total differential, it is nevertheless licit of
to integrate on the step of time and one obtains while integrating [éq 3.2.4.3 - 1].

+
-
1
1
1
Q' = Q' +
3 K T 2
3
3
3
0
0
(+
0
2
2

V - V)
+
- + m T
lq
(+ -
p - p
C
C) - (
+
+
mgz + mlq) T (+
-
p - p
gz
gz)
+
+ C T -
(+
-
T)
éq 4.2.6-3
1
+
-
T +
where we noted:
2
T
=
T. We chose here a formula of “point medium” for
2
variable temperature.

4.2.7 Calculation of the mechanical constraints

The calculation of the effective constraints is done by calling upon the incremental laws of mechanics selected
by the user. One integrates on the step of time [éq 3.2.8-2] and one a:

+
-

B p

p
bS p
p éq
4.2.7-1
p = p -
(+ -
gz -
gz)
+
+ lq (+
-
C -
c)

4.2.8 Calculation of hydrous and thermal flows

It is necessary well on calculating all the coefficients of diffusion:

Coefficients of Fick
+
F (+
P, +
P, +
T, +
S) and +
F (+
P, +
P, +
T, +
S)
vp
vp
gz
AD
AD
lq

Thermal diffusivity T +
T
+
T
+
T
+

T
=
(

). (S

).
T
(
) +
S
lq
T
cte

Hydraulic permeabilities and conductivities:

int
+
rel
int
rel

(
).

+
K
K
(
).
,
H
W (+
Slq)
+
+
K
kgz
H
(+ +
S
p
lq

gz)

lq
=
μ
μ
W (
gz
=
+
T)
gz (+
T)

+
p
Concentrations out of vapor and air dissolved:
+
C = vp and +
+
C

AD =
vp
+
p
AD
gz
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It does not remain any more whereas to apply the formulas [éq 3.2.5.1 - 1], [éq 3.2.5.2 - 15], [éq 3.2.5.2 - 16],
[éq 3.2.5.2 - 17] and [éq 3.2.5.2 - 18] to find:
+
+
T
+
Q = - T éq 4.2.8-1
+
M
+
have = H

p

F
C F
C éq
4.2.8-2
gz
-
+
+
m + vp vp
+
[+gz (+ +
have
vp)
] + + +vp
have
+
Mvp
+
= H

p

F

1
(
C) F
C éq
4.2.8-3
gz
-
+
+
m - - vp vp
+
[+gz (+ +
have
vp)
]
+
+
+
vp
vp
M+w
H +
=
- p+ + (+
+
+) F
éq
4.2.8-4
+
lq [
m
lq
W
AD
]
W
+
+
M

p
(
) F

F
C
éq 4.2.8-5
AD =
H
AD
lq [
+
+
- lq + W +
m
AD
] + +
- AD AD

5
Calculation of derived from the generalized constraints

In this chapter, we give the expressions of derived from the constraints generalized by report/ratio with the deformations generalized within the meaning of the document [R7.01.10] [bib4], C `be-with-statement terms who are calculated when the laws of behaviors THM are called for option RIGI_MECA_TANG within the meaning of the document [R5.03.01] [bib3].
In order not to weigh down the talk, we give the expression of the differentials of the constraints generalized, knowing that the derivative partial result some directly.

5.1
Derived from the constraints

The calculation of the differential of the effective constraints is left with the load of the law of behavior purely mechanical, that we do not describe in this document. Differential of the constraint is given directly by the expression [éq 3.2.8-2].
p

5.2
Derived from the mass contributions

While differentiating [éq 3.2.2-1], one a:
DM
D
W

W
=
(1+ S + D S + 1+ dS + 1+ dS
V)
lq
V
lq
(V) lq (V) lq

W
W
DM
D
AD
AD
=
(1+ S + D S + 1+ dS + 1+ dS
V)
lq
V
lq
(V) lq (V) lq

AD
AD
éq
5.2-1
DM
D
have
have
=
(1+ 1 - S + D 1 - S + 1+ D 1 - S - 1+ dS
V) (
lq)
V
(lq) (V) (lq) (V) lq

have
have
DM

D
vp
vp
=
(1+ 1 - S + D 1 - S + 1+ D 1 - S - 1+ dS
V) (
lq)
V
(lq) (V) (lq) (V) lq

vp
vp
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5.2.1 Case without dissolved air

by taking account of [éq 3.2.1-1] and of [éq 3.2.3.1 - 1], [éq 3.2.3.2 - 1], [éq 3.2.3.2 - 2] and while supposing
1+
one finds:
V 1

DM
S
S

S 2 B
(
)

(
)
W
lq
lq
lq
-

B -

= bS D

3
lq
V +
-
-
dpc + Slq
+
dpgz -
m dT

W
P
K
K

W

C
W

S

K
K
W
S

DM
1
(
)
(
)
(
1
)(
)
vp

Slq
- S S B
lq
lq
-

B -
- Slq
dpvp
m

= B 1
(- S) D

1
(
)

3
lq
V +
-
-
dpc +

dpgz +
- Slq
-
dT
vp

vp

P
K
C
S

K S

pvp

DM

S
1
(
)
(
)
(
1
)(
)
have
lq
- S S B
lq
lq
-

B -
- Slq
dpas
m

= B 1
(- S) D

1
(
)

3
lq
V +
-
-
dpc +
dpgz +
- Slq
-

dT
have
P
K
K
p
have

C
S

S

have

éq 5.2.1-1

One sees appearing the derivative of saturation in liquid compared to the capillary pressure, quantity which
play an essential part.

The expression [éq 3.2.6-4] of the differential of the steam pressure also makes it possible to calculate pressure of dry air:

M ol p
M ol
M ol
dp
vp
vp
= 1
(-
) dp
vp
-
dp
vp
-
-
éq
5.2.1-2
have
gz
C
(hm m
vp
W) dT
H
RT
RT
RT
T
W
W

One defers [éq 5.2.1-2] and [éq 3.2.6-4] in [éq 5.2.1-1] and one finds:

DM

S

S
S 2 B
(-)

(-)
W
lq
lq
lq
B
= bS D

+

-
-
dp + S
+
dp
m
-
3
dT
lq
V

P

K
K
C
lq

K
K
gz
W

W

C
W
S

W
S

éq 5.2.1-3
DM

1-
-
1-

vp = B (1 - S + -
-
-

lq)
Slq (Slq) Slq (B
) (Slq)
D
vp
dp
V

p

K
p
C

vp

C
S
vp
lq
éq 5.2.1-4
(B -) (1 - S
1-

1-
-

lq)
(Slq) vp
m
vp (
Slq) (hm hm
vp
lq)
+
+
dp + -
3
+
dT

K
p
gz
vp

p T

S
vp
lq

vp

DM

1-
-
1-

have = B (1 - S
+ -
-

+

lq)
Slq (Slq) Slq (B
) (Slq)
D
vp
dp
V

p

K
p
C

have

C
S
have
lq

(B -) (1 - S
1-
-

1-
-

lq)
(Slq) lq vp
m
vp (
Slq) (hm hm
vp
lq)
+
+
dp + -
3
-
dT

K
p
gz
have

p T

S
have
lq

have

éq 5.2.1-5
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5.2.2 Case with dissolved air

As previously, by taking account of [éq 3.2.1-1] and of [éq 3.2.3.1 - 1], [éq 3.2.3.2 - 1], [éq 3.2.3.2 - 2], [éq 3.2.7.3] and by supposing 1+
one finds:
V 1

DM
S
2 (-)

(
)
-

W
lq
p
S
B
S

W
lq
lq
p

W
B
Slq p
= bS D +
-
+
dp + S
+
dp
W
m
+
-
3
dT
lq
V

K
P

K
P
C
lq

K P

K
gz

K T
W

W
W
C
S
C

W
gz
S

W

éq 5.2.2-1

DM
S M ol
2 (-)

(
)
-
AD
lq
have
p
S
B

S

have
lq
lq
M ol
p

have
have
B
= bS D +
-
+
dp + S
+
dp
lq
V

K
P

K
P
C
lq

K P

K
gz

AD
AD
H
C
S
C

AD H
gz
S

éq 5.2.2-2
M ol p

+ S
have
have - 3 B
(

0
-) dT
lq K
T

AD
H

DM

S

1
(- S) S B
(-) 1
(- S)
B
(- 1
) (- S) 1
(- S)
vp = B 1
(- S) D
lq
lq
lq
lq
vp
+ -
-
-
dp
lq

lq
vp
+
+
dp +
lq
V

P

K
p
C

K
p
gz

vp

C
S
vp
lq

S
vp
lq

1
(- S) (hm - hm)
m
vp
lq
vp
lq
3

+ - +
dT
vp

p T

vp

éq 5.2.2-3

DM

S

1
(- S) S B
(-) 1
(- S)
have = B 1
(- S) D
lq
lq
lq
lq
vp
+ -
-
+
dp +
lq
V

P

K
p
C

have

C
S
have
lq
éq

5.2.2-4

B
(- 1
) (- S) 1
(- S) -

1
(- S) (hm - hm)
lq
lq
lq
vp

+
dp
m
vp
lq
vp
lq
3

+ - -
dT

K
p
gz
vp

p T

S
have
lq

have

The derivative partial are given in [§Annexe 3].

5.3
Derived from the enthali and Q' heat

There still, we do nothing but point out expressions already provided to chapter 2:

5.3.1 Case without dissolved air

-
dhm = 1 -
3
+

W
(
T
W
) dp dp
gz
C
C pdT
W
W
dhm = C p dT
vp
vp
dhm = C p dT
have
have
Q
'= 3 K Td
m
+ 3 Tdp
0
- 3 + 3
+
0
0
V
lq
C
(m
m
gz
lq) Tdp

C dT
gz

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5.3.2 Case with dissolved air

-
-
dhm = 1 -
3
+
W
(
T
W
) dp dp dp
gz
C
AD
C pdT
W
W
1-

3 T
p

p

1-
3

W
AD
AD
p
T p
=
1-
dp - 1+
dp + C
W
AD
-
dT

p
gz

p
C
W

T

W
gz
C

W

dhm = C p dT
AD
AD
dhm = C p dT
vp
vp
dhm = C p dT
have
have
Q
'= 3 K Td
m
+ 3 Tdp
0
- 3 + 3
+
0
0
V
lq
C
(m
m
gz
lq) Tdp
C dT
gz

5.4

Derived from the heat flow

One leaves [éq 3.2.5.1 - 1] and [éq 3.2.5.1 - 2].

While differentiating [éq 3.2.5.1 - 2] and while using [éq 3.2.1-1], one finds:

D T
= B
T
T
(-) '(
). (S
T
). T
() D
S
lq
T
v
B
(-) T
T
+
'(
). (S
T
). T
() dp
K
S
lq
T
gz
S

S

(-)

T
T
T

lq
B
+ (

). '(S). T
( ).
- S
T
T
'(
). (S
T
). T
() dp
S
lq
T
p
lq

K
S
lq
T
C

C
S

+ (T
T
(
). (S
T
). 'T
() - B
T
T
(- 3
). '(
). (S
T

). T
( )
S
lq
T
0
S
lq
T
) dT

That is to say finally:

dq = - B
T
T
(-) '(
). (S
T
). T
(
) Td
S
lq
T
v
B
(-) T
T
-
'(
). (S
T
). T
(
) Tdp
K
S
lq
T
gz
S
éq

5.4-1

S

(-)

T
T
T
lq
B
- (
). '(S).
T
( ).
- S
T
T
'(
). (S
T
). T
() Tdp
S
lq
T

p
lq

K
S
lq
T
C

C
S

- (T
T
(

). (S
T
). 'T
() - B
T
T
(- 3
). '(
). (S
T
). T
( )
0

S
lq
T
S
lq
T
) TdT

5.5
Derived from hydrous flows

It is of course necessary to set out again of the equations [éq 3.2.5.2 - 15], [éq 3.2.5.2 - 16], [éq 3.2.5.2 - 17] and [éq 3.2.5.2 - 18]
that one differentiates.
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Models of behavior THHM

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5.5.1 Case without dissolved air

M

M - C F C
have
H
m
have
have
vp
vp
vp
H
DM =
+ F D +
D
have
have
gz
have

H
gz

have

lq
H
+ - dp + D F
éq
5.5.1-1
have
gz (
m
gz

vp
)
F
F

vp
vp
+ C
dT +
p

C + cd. F C + C F cd.
have
vp
gz
vp
have
vp
vp
vp
have
vp
vp
vp
T
p

gz

M

M + 1 - C F C
vp
H
m
vp
vp (
vp) vp
vp
H
DM =

+ F D +
D
vp
vp
gz
vp
H
gz

vp

gz
H
+ - dp + D F

vp
gz (
m
gz
have
)
F
F

vp
vp
- 1
(- C)
dT +
p

C + cd. F C - 1
(- C) F cd.
vp
vp
gz
vp
vp
vp
vp
vp

vp
vp
vp
vp
T
p

gz

éq 5.5.1-2
M

M
W
H
m
W
H
DM =
+ F D +
D
W

W lq

W
H
lq

W

lq
éq
5.5.1-3
H
- dp - dp
W lq (
gz
c)

In order to clarify these differentials completely, it is necessary to know the differentials of the masses
p

voluminal of the fluids, as well as the differentials of
vp
C =
and of its gradient. Knowing
vp
pgz
[éq 3.2.6-4], one can then calculate the differential of the pressure of dry air:

-

dp = dp - dp
W
vp
=
dp
vp
+
dp -
-

éq
5.5.1-4
have
gz
vp
gz
C
vp (H m
m
vp
W) dT
H

T
W
W

D
dp
dT
D
dp
vp

vp
dT
By deriving the relation from perfect gases one a:
have
have
=
-
and
=
-
, which, in

p
T

p
T
have
have
vp
vp
using [éq 3.2.6-4] and [éq 5.5.1-4] gives:

2
2
2
-

vp
vp
vp (H m
hm
vp
W)
D =
dp -
dp
vp

+
-
dT
éq

5.5.1-5
vp
p
gz
p
C

Tp
T
W
vp
W
vp
vp

-

-

have
W
vp
have
vp
have
vp (H m
hm
vp
W)
D =
dp +
dp
have

+ -
-
dT éq 5.5.1-6
have
p
gz

p

C

Tp
T
have
W
have
W
have

[éq 3.2.6-4] allows to express the gradient of the steam pressure:

p
vp
=

-
+
-
éq

5.5.1-7
vp
(p
p
gz
c)
vp (hm
m
vp
lq)
T
H

T
lq

While differentiating [éq 5.5.1-7] one finds:

m
m

-

dp
vp
=

-
+
-

-
+

vp
(D p D
gz
c)
D

p
vp
vp D
2
W (p
p
gz
c)
H
H
vp
W
D T
vp

T

W
W
W

éq
5.5.1-8

hm - hm
+d
vp
W

T
vp

T

The last term of [éq 5.5.1-8] is written:

hm - hm
hm - hm

D
vp
W

T
vp
W

=

T D
vp
+
T
-
-
-

vp
vp
(dhm dhm
vp
W)
vp (H m
hm
vp
W)
dT
T 2

T

T

T
T

éq 5.5.1-9

Knowing the differentials of its gradient and steam pressure, the expressions of differentials of C and its gradient are easy to calculate:
vp

dp
p
p
p
vp
vp
cd. =
-
dp which gives:
vp
vp
C =
-
p and which one differentiates in:
vp
2
gz
p
p
vp
2
gz
p
gz
p
gz
gz
gz

p
p
dp
p
p
p
p
vp
vp
vp
gz
vp
vp
vp
cd. = D
-
p =
-
dp

+ 2
p

-
dp -
dp
vp
2
gz
2
vp
3
gz
2
gz
2
gz
p

p
p

p

p
p
p
gz
gz
gz

gz
gz

gz
gz

dp is given by [éq 3.2.6-4] and dp by [éq 5.5.1-8].
vp
vp

5.5.2 Case with dissolved air

M

M - C F C
have
H
m
have
have
vp
vp
vp
H
DM =
+ F D +
D
have
have
gz
have

H
gz

have

gz
H
+ - dp + D F
éq 5.5.2-1
have
gz (
m
gz
vp
)
F
F

vp
vp
+ C
dT +
dp

C + cd. F C + C F cd.
have
vp
gz
vp
have
vp
vp
vp
have
vp
vp
vp
T
p

gz

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M

M + 1 - C F C
vp
H
m
vp
vp (
vp) vp
vp
H
DM =
+ F D +
D
vp
vp
gz
vp
H
gz

vp

gz
H
+ - dp + D F

vp
gz (
m
gz
have
)
F
F

vp
vp
- 1
(- C)
dT +
dp

C + cd. F C - 1
(- C) F cd.
vp
vp
gz
vp
vp
vp
vp
vp
vp
vp
vp
vp
T
p

gz

éq 5.5.2-2

M

M
W
H
m
W
H
H
DM =
+ F D +
D + - dp + D F éq 5.5.2-3
W

W lq

W
H
lq
W lq (
m
lq
AD
)

W
lq

DM
= - p + F D + -
(p + F) D
AD
(H
H
m
lq
lq
lq
lq
) AD AD (H
m
lq
lq

lq
) Hlq
H
+ - dp + D F

AD
lq (
m
lq
W
)
F
F

AD
AD
-
dT +
dp C - F cd.
C
AD
AD
AD

T
p

C

éq 5.5.2-4

It is necessary to know the differentials of the densities of the fluids, as well as the differentials of
pvp
C =
, C
= and of their gradient. One first of all will calculate the differentials of the masses
vp
p
AD
AD
gz

voluminal by using the derivative partial of pressures given in [§Annexe 3].

D
dp
dT
D
dp
vp
vp
dT
By deriving the relation from perfect gases one a:
have
have
=
-
and
=
-
, that one

p
T

p
T
have
have
vp
vp
can express in the form:

p

p

p

D
have
have

=
dp
have
+
dp
have
have
have
+
-
dT
have
éq 5.5.2-5
p p
C

p
gz

p
T

T
have
C
gz
have

p

p

p

D
vp
vp

=
dp
vp

+
dp
vp
vp
vp

+
-
dT
vp
éq 5.5.2-6
p p
C

p
gz

p
T

T
vp
C
gz
vp

By using the relation [éq 3.2.3.1 - 1], one obtains:
p

p

p

D
W
W
=

dp
W
+

dp
W
W
+
- 3 dT éq
5.5.2-7
W
K p
C

p
gz

K
T
W
W

W
C
gz
W

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ol

M
And like
have
dad =
dpas,
K H
M ol
have
not
not
p

D
éq
5.5.2-8
AD =

dpc +
dpgz +
have dT

K H PC
pgz
T

As previously, the expressions are used

dp
p
vp
vp
cd. =
-
dp which gives
vp
2
gz
p
p
gz
gz
1 p

p

1 p

p
vp
vp
vp
vp
cd. =
dp +
dT

+
-
dp éq
5.5.2-9
vp
C
2
gz
p p

T

p
p

gz
C
gz
gz
p

gz

p
p

and
vp
vp
C =
-
p and which one differentiates in:
vp
2
gz
p
p
gz
gz
p
p
dp
p
p
p
p
vp
vp
vp
gz
vp
vp
vp
cd. = D
-
p =
-
dp

+ 2
p

-
dp -
dp
vp
2
gz
2

vp
3
gz
2
gz
2
gz
p

p
p

p
p
p
p
gz
gz
gz

gz
gz

gz
gz

éq 5.5.2-10

with
p

p

p

p
vp
=
p
vp
+
p
vp

+
T
éq
5.5.2-11
vp
p
gz

p
C

T

gz
C
and dp that one differentiates in the following way:
vp
p

p

p

p

p

dp = D
vp p + D vp p + D vp T
vp
+
dp
vp
+
dp
vp
+
dT
vp
p
gz

p

$$C$$

$$T$$

$$p$$
$$gz$$

$$p$$
$$C$$

$$T$$
$$gz$$
$$C$$
$$gz$$
$$C$$

$$p$$

$$p$$

$$p$$

$$vp$$

$$=$$
$$p$$
$$vp$$
$$+$$
$$p$$
$$vp$$
$$+$$
$$T\ dp$$
$$p$$

$$p$$
$$gz$$

$$p$$

$$p$$
$$C$$

$$p$$

T
C

C
gz
C
C
C

p

p

p

vp

+
p
vp
+
p
vp
+
T dp

p

p
gz

p

p
C

p

T
gz

gz

gz
gz
C
gz

p

p

p

vp

+
p
vp
+
p
vp
+
T dT
T

p
gz

T

p
C

T

T

gz
C

p

p

$$\frac{\partial p}{\partial v_p}$$

$$+$$

$$\frac{\partial p}{\partial d_p} \frac{\partial d_p}{\partial v_p} + \frac{\partial p}{\partial d_p} \frac{\partial d_p}{\partial v_p} + \frac{\partial p}{\partial dT} \frac{\partial p}{\partial g_z}$$

$$\frac{\partial p}{\partial C}$$

$$\frac{\partial T}{\partial g_z} \frac{\partial}{\partial C}$$

éq 5.5.2-12

The derivative partial of the second order are developed in [§Annexe 4].

For the dissolved air, one proceeds with the same stages:

p
C = = M ol
AD
.
AD
AD
AD
RT
thus
ol dp
p
AD
AD

cd.
.
who gives
AD = M AD
-
dT
RT
RT 2

ol
1 p
1
1
AD
p

AD
p
P
AD
AD
cd.
.

éq
5.5.2-13
AD = M AD
dpc +

dpgz +
-

dT

2
RT PC
RT pgz
T
RT plq

ol p
p
AD
AD

and C
.
and that one differentiates in:
AD = M AD
-
T
RT
RT 2

ol
1

1 T
pad
pad
pad

cd.
2

AD = M AD
dpad -

dp
2
AD +
T -
dT -
dT
3
2
2

RT
RT
RT
RT
RT

éq 5.5.2-14
with
p

p

p

p
AD
=
p
AD
+
p
AD
+
T éq
5.5.2-15
AD
p
gz

p
C

T

gz
C
and dp that one differentiates in the following way:
AD

p

p

p

AD
=
p
AD
+
p
AD
+
T dp
p

p
gz

p

p
C

p

T
C

C
gz
C
C
C

p

p

p

AD
+
p
AD
+
p
AD
+
T dp
p

p
gz

p

p
C

p

T
gz

gz
gz
gz
C
gz

éq
5.5.2-16
p

p

p

AD
+
p
AD
+
p
AD
+
T dT
T

p
gz

T

p
C

T

T

gz
C

p

p

p

AD
+
dp
AD
+
dp
AD
+

$$\frac{dT}{p} = \frac{gz}{p}$$

$$\frac{T}{C} = \frac{gz}{C}$$

The derivative partial of the second order are developed in [§Annexe 4].

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Internal appendix 1 generalized Constraints and variables

Constraints:

Number	***Name of component Aster***	***Contents***
1	***SIXX***	
	xx	
2	***SIYY***	
	yy	
3	***SIZZ***	
	zz	
4		

SIXY

xy
5
SIXZ

xz
6
SIYZ

yz
7
SIP

p
8
M11
m
W
9
FH11X
M
W X
10
FH11Y
M

W y
11
FH11Z
M
W Z
12
m
ENT11
H
W
13
M12
m
vp
14
FH12X
M

vp X
15
FH12Y
M

vp y
16
FH12Z
M

vp Z
17
m
ENT12
H
vp
18
M21
m
have
19
FH21X
M

have X
20
FH21Y
M

have y
21
FH21Z
M

have Z
22
m
ENT21
H
have
18
M22
m

AD
19
FH22X
M

AD X
20
FH22Y
M

AD y
21
FH22Z
M

AD Z
22
m
ENT22
H
AD
23
QPRIM
Q'
24
FHTX
Q
X
25
FHTY
Q
y
26
FHTZ
Q
Z
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The variables intern if one takes account of the saturation (of type ". .HH." or THV):

Number
Name component Aster
Contents
1 V1
0
-
2 V2
0
-
lq
lq
3 V3
0
p - p
vp
vp
4 V4
S
lq

And in the other cases:

Number
Name component Aster
Contents
1 V1
0
-

2 V2
0
-
lq
lq
3 V3
0
p - p
vp
vp

Appendix 2 Data material

One gives here the correspondence between the vocabulary of the Aster orders and the notations used in
present note for the various sizes characteristic of materials.

A2.1 key Word factor THM_LIQU

RHO
0

lq

UN_SUR_K
1
Klq

ALPHA

lq

CP
p
C
lq

VISC
μ (T
lq
)

D_VISC_TEMP

μ
lq (T)
T

A2.2 key Word factor THM_GAZ

MASS_MOL
ol
M
have

CP
p
C
have

VISC
μ (T
have
)

D_VISC_TEMP
μ
have (T)
T

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A2.3 key Word factor THM_VAPE_GAZ

MASS_MOL
ol
M
VP

CP
p
C
vp

VISC
μ (T
vp
)

D_VISC_TEMP
μ
vp (T)
T

A2.4 key Word factor THM_AIR_DISS

CP
p
C
AD

COEF_HENRY
K
H

A2.5 key Word factor THM_INIT

TEMP
initT

PRE1
init P
1

PRE2
init P
2

PORO
0

PRES_VAPE
0
p
vp

One recalls that, according to modeling, the two pressures P and P
1
2 represent:

LIQU_SATU LIQU_VAPE LIQU_GAZ_ATM GAS
LIQU_VAPE_GAZ
P
p
p
p = - p
p
p = p - p
1
W
W
C
W
gz
C
gz
W
P
p
2

gz

LIQU_GAZ LIQU_AD_GAZ_VAPE
P
p = p - p
p = p - p - p
1
C
gz
W
C
gz
W
AD
P
p
p
2
gz
gz

A2.6 key Word factor THM_DIFFU

R_GAZ
R

RHO
R
0

CP
S
C

BIOT_COEF
B

SATU_PRES
S p
lq (c)

D_SATU_PRES

S
lq (PC)
p
C

PESA_X
m
F
X

PESA_Y
m
F
y
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PESA_Z
m
F
Z

PERM_IN
int
K ()

PERM_LIQU
rel
K
S
lq (lq)

D_PERM_LIQU_SATU
rel
K
lq (S
lq)
S
lq

PERM_GAZ
rel
K
S, p
gz (lq
gz)

D_PERM_SATU_GAZ
rel
K
gz (S, p
lq
gz)
S
lq

D_PERM_PRES_GAZ
rel
K
gz (S, p
lq
gz)
p
gz

FICKV_T
F T (T)
vp

FICKV_S

F S (S)
vp

FICKV_PG
gz
F (P)
vp
G

FICKV_PV
vp
F (P)
vp
vp

D_FV_T
F T
vp (T)
T

D_FV_PG
gz
F
vp (P)
gz
P
gz

FICKA_T
F T (T)
AD

FICKA_S
F S (S)
AD

FICKA_PA
AD
F (P)
AD
AD

FICKA_PL
lq

F (P)
AD
lq

D_FA_T
F T
vp (T)
T

LAMB_T
T
(T)
T

T
T
( )

DLAMB_T
T

T

T

(

LAMB_PHI
)

T
( )

DLAMBPHI

LAMB_S
T
(S)
S

T
(S)

DLAMBS
S
S

LAMB_CT
T

CT

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Note:

For modelings utilizing it thermal, and for the calculation of the specific heat homogenized, the relation is used: C 0 = (1 -)
S
p
C + C M

+ 1 - S C + C.
S
lq lq
lq
(lq) (p
p
vp
vp
have
have)
In this formula, one confuses with his initial value 0 whose value is read under the key word
S
S
RHO of the key word factor ELAS.

Appendix 3 Derived from the pressures according to the deformations generalized

One details here the calculation of derived from pressure according to the generalized deformations. It is pointed out that
dpvp
dpw
dT
the equation [éq 3.2.6.3] is
=
+ L
with
m
m
L = H - h. Moreover

T
vp
W

vp
W
R
RT
dp = dp - dp =
p dT +
dp and dp = dp - dp. By combining these equations one obtains:
AD
lq
W
have
have
K
K
have
gz
vp
H
H

RT

dT
RT

dp
L
p
1 dp
dp
vp
- W = (-
W +
AD)
+
-
gz +

C
K

T
K
H
vp

H

vp RT

vp
p
RT

dp
1
LR
dT
1 dp
dp
W
- = -
+ AD
+
-
gz +

C

K
K
T
K
W

H

H

H

One can thus write the derivative partial of water and the vapor according to the generalized deformations:

-
vp
LR
+ p
RT
AD
-1
p
K
T
p
K
p
W
H
W
H

1
=
;
=
;
W =

T

vp RT
p
RT

p
RT
gz
vp

-1
-1
C
vp
-1
K
K
K
W
H
W
H
W
H
RT -1
p

- L + p
1
p

K
p

vp
(
1
W
AD)
vp
vp
H
=
. ;
=
;
=

T

RT

T
p

RT

p

RT

W
gz
W
C
W
-
-
-
K

K

K

H
vp
H
vp
H
vp

The relations dp
= dp - dp and dp = dp - dp - dp makes it possible to derive all the pressures,
have
gz
vp
AD
gz
C
W
since one will have
p

p

p

p

p

p

have
vp
have
vp
have
vp
= -
;
= 1-
;
= -

T

T

p

p

p

p

gz
gz
C
C
and

p

p

p

p

p

p

AD
W
AD
W
AD
W
= -
;
= 1-
;
= -1-

T

T

p

p

p

p

gz
gz
C
C
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Appendix 4 Derived seconds of air and steam pressures
dissolved according to the generalized deformations

One calculates here the derivative partial of the second order of the steam pressure necessary to the section [§5.5.2].
One will note thereafter:

RT

RT

W
A1 =
-
and 2
With = W
-1
K

K
H
vp
vp
H

1
p
vp 1
1
p

W
A3 =
- -
-
3

W
p
T

T
K
T

vp
W
1
p
vp 1
1
p
W
A4 = -
+ +
-
3

W
p
T

T
K
T

vp
W

Derived seconds from the steam pressure:

p
A2 1
p
1

p
vp

=

W -
vp
2

p p
A1
K
p
p
p
C gz
W.C.
vp C
p
A2 1
p
1
p
vp

=

W -
vp
2

p
p
A1
K
p
p
p
gz gz
W gz
vp gz

pvp
R
1 RT

R

=
-
-1
- W 4
With
2

T p
K
1
With
1
With

gz
H
K H

K H
vp

p

1 1
p
vp

1
p
W
vp

= -

-
W
2

p
p
A1 p
p
K
p
C C
vp
vp C
W.C.
p

1 1
p
vp

1
p
W
vp

= -

-
W
2

p
p
A1 p
p
K
p
gz C
vp
vp gz
W gz
pvp
1 R

= -

- W 4
With
2

T p
1
With

C
K H
vp

p

1 1
p

1
p
1
p
vp

W
W
W

= -
A1 1
1
(+ L
) + (p
L)

2
AD -

W
W
-
vp
p
T
T A1
p
K

K
p
p
p
C

C
W

vp
W.C.
vp

C
p

1 1

p

1
p
1
p

vp

W
W
W

= -
With
1 1-
1
(+ L
) + (p
L)

2
AD -

W
W
-
vp
p
T
T A1
p
K

K
p
p
p
gz

gz
W
vp
W gz

vp

gz
p

vp
1
p

p
RT

R

AD
W
W
1
=

- L
- 3
W
-

-
+ T (
W
-
With)
4 (p - L)
T

T

T. 1
W

W

2
With
T

K
T

T. 12
AD
W
WITH K

K

W

H
vp
H
vp

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Derived seconds from the pressure of dissolved air:

p
RT A2 1 p
1
p
AD
vp

=

-
W
2

p p
K
A1
p
p
K
p
C gz
H
vp C
W.C.
p
RT A2 1 p
1
p
AD
vp

=

-
W
2

p
p
K
A1
p
p
K
p
gz gz
H
vp gz
W gz
2
p

1
2
AD
R
W
W
R WITH
= -
+

+

2 (1
A3 T
.)
T

p

K A1

K
A1
gz
H
vp
vp
H

p
RT
1 1 p
1
p
AD
W
vp

=

-
W
2

p
p
K A1
p
p
K
p
C C
H
vp
vp C
W.C.
p
RT
1 1 p
1
p
AD
W
vp

=

-
W
2

p
p
K WITH
p
p
K
p
gz C
H
vp
vp gz
W gz
p

R
1
AD
W
=
+

2 (1
WITH T
.
3)
T

p

K A1
C
H
vp

p

1 LR
p

AD

W
vp
vp
1 p
RT
1
LR
p
AD
W
AD
=

-
-
-
. 3
With
p

T

1
With

K
p
p

T p

K
12
WITH K
vp
T
C
vp
H
vp
C
C
H

vp
H

p

1 LR
p

AD
W
vp
vp
1 p
RT
1
LR
p
AD
W
AD
=

-
-
-
. 3
With
p

T

1
With

K
p
p

T p

K
12
WITH K
vp
T
gz
vp
H
vp
gz
gz
H
vp
H

p
1
LR 1
1
1

1
1
AD
W
vp
p

vp
pad
pad
RT

W
LR
p
AD

=

-

+
-
-

. 3

2
2
vp -
With +
T T
A1

K
p

1
vp
H
vp
T
T
T
T T

K
With
H
vp
K
T
H

T
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Appendix 5 Equivalence with formulations ANDRA

In order to be able to fit in the platform ALLIANCE, it is necessary to be coherent with the formulations
posed by the ANDRA in the document [bib11]. We propose here an equivalence between the notations which
would be dissimilar. These differences relate to only the writing of:

· The equation of energy
· The law of Henry
· Diffusion in the liquid
· Diffusion in gas

Notice concerning the enthali:

It is required to have coherence between the two models which the user of Aster returns:
m
=
lq
H
= 0 and hm
L,

0
vp
0

0

A5.1 Equation of energy

The table above points out the two formulations:

Notations Aster
Notations ANDRA
m

lq
H
W
=

S N
W
W
m

vp
H
v
=

1-
v (
Sw) N
m

have
H
have
=

1-
have (
Sw) N
M
=
lq
W fw
M
=

have
have fas
M
=
vp
v fv

By rewriting the equation of energy of Aster with these notations, one finds:

D

F
F
F

+
W
have
v
Div
Div
W
+ have
+
Q
dt

S N
1 S N
1 S N
W
(-)
v
W
(- W) +

( )

- (T - 0) D
T
([1 - N) C
S
S]

dt

D
dp
dp
+
2 dT
v

3 K T
3
3
9
0
0
+
m
C
T

T
TK
lq
- (mlq + mgz)
gz -
0
0
=
dt
dt
dt
dt
(F F F
W W +
v v +
have
have) ***G*** *+*

The first line being that of the ANDRA and others being a priori negligible.
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A5.2 Law of Henry

ol
P M
In the formulation of the ANDRA, the formulation of Henry is given by
has
have
have
=
with
L
W
H M W
concentration of air in water that have it can bring back to a density such as A
=. H
L
AD
express yourself out of Pa.

In the ASTER formulation, one recalls that the law of Henry is expressed in the form:

p
ol
AD
C =
with ol
have
C =

. K is expressed in Pa.m3.mol-1.
AD
ol
M
AD
K
H
AD
H

There is thus equivalence:
M W
K = H

H
W

A5.3 Diffusion of the vapor in the air

In formulation ANDRA the steam flow in the air according to the concentration of steam d' in the air or the relative humidity is noted:

E
F
= - D
.

_

Diff
v
v
G
N
with the concentration defined as the molar report/ratio in gas: E

=
.
G
ng

In Aster, this same flow is written
: F
with the coefficient of Fick vapor
_ = F C

Diff
v
vp
vp
D

vp
F =
vp
and D
C 1
(- C)
vp the coefficient of diffusion of Fick of the gas mixture. Cvp is defined like
vp
vp
p
the report/ratio of the pressures such as:
vp
C =
.
vp
pgz

The law of perfect gases makes it possible to write that
E
C = thus
E
= C and F
.
_ = D
. C
vp
G
G
vp
Diff
v
v
vp
Thus equivalence ASTER/ANDRA is written simply:

F = D.
vp

v

A5.4 Diffusion of the air dissolved in water

In formulation ANDRA the flow of air dissolved in water is expressed

has
F

_
=D
.
_

ds E has
has
L

with A
AD
=
.
L
ol
M AD
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In Aster, this same flow is written: F
with the coefficient of air-dissolved Fick

_
_ = F
C
ds v has
AD
AD
D

AD
F =
AD
and D
C 1
(- C)
AD the coefficient of diffusion of Fick of the liquid mixture. Cad is definite such
AD
AD
that:
has
C = W. Thus:
AD
L
F = D.
AD

has

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Organization (S): EDF-R & D /AMA, IRSN

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Modeling of thermohydration, drying and of the shrinking of the concrete

Summary

One describes two types of phenomena here occurring at periods distinct from the life of a concrete:

.
on the one hand a reaction of thermohydration generating a withdrawal known as endogenous, appearing with
youth of the concrete (the first 100 days),
.
in addition an evaporation of part of the water not used in the process of hydration,
phenomenon called drying and involving a withdrawal of desiccation. This phenomenon can last, according to dimensions of the structure of concrete implemented, a few months to several years.

These phenomena are modelled in Code_Aster in the form of equations of diffusion whose solution is represented by new variables allowing to calculate the deformations of the endogenous withdrawal directly
(of with the hydration) and of the withdrawal of desiccation (of with drying).
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Count

matters

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1 Introduction

The behavior of the concrete, fragile in extension, heterogeneous and porous material is governed by many and complex physicochemical phenomena. Losses of prestressings induced by behavior differed from the concrete (withdrawal and creep) reduce in the course of time the field of loading which the structure can support. These differed deformations which appear in concrete during the life of this one, are composed by the withdrawal at the youth (endogenous withdrawal specific to the hydration and thermal withdrawal), by the withdrawal of desiccation with the modeling of drying, and as soon as it undergoes constraints, by clean creep and the creep of desiccation.
In the rules of dimensioning, the deformations differed from the concrete are generally based on empirical rules fixed on a great number of results resulting from the literature, fascinating in count the principal parameters, like the temperature, moisture, the content of aggregate, proportion water/cement. The kinetics of the phenomena uses times equivalent calculated to the assistance of a law of Arrhenius to take into account ageing and the temperature.
A fine analysis of the physicochemical phenomena which are at the origin of the various deformations differed from the concrete allows to propose a modeling on the basis of model of the medium type continuous equivalent [bib2], which was introduced into Code_Aster (clean creep and the creep of desiccation are not treated here).

1.1 Phenomenologic aspects of the behavior of the concrete to the young person age: thermohydration

One defines the youth as the first 100 days of the life of the concrete. Endogenous withdrawal or withdrawal of hydration, and the thermal withdrawal intervene as of the first moments of the catch (at the youth), for one duration going from a few hours to a few days, for the thermal withdrawal, and of some month at one year, for the withdrawal of hydration, in general finished at the time of the setting in prestressing.
phenomena of prevented withdrawals or differential withdrawals, under formwork, can be at the origin of constraints or of cracks which should be evaluated. In liquid phase, the concrete is a viscous fluid in which the solid matter constituents are in suspension in the hydraulic binder containing of the solid

particles
(cements…). Following the formation of the first hydrates, the catch of the concrete intervenes, ten hours after its manufacture, which corresponds to the establishment of related bridges hydrates between
cement grains in the totality of material. With the whole beginning, the grains are relatively dispersed in mixing water. In the course of time, the hydration of the cement grains is accompanied by one consumption of this mixing water. In experiments, it is noted that the voluminal assessment of reaction is negative; it is the contraction of Chatelier. Known as simply, the total volume of the hydrates
is lower of almost 10% than the total volume of these components. Mechanically, on a scale cement grains, the phenomenon stops when the bridges of hydrates formed between the grains are sufficient rigid to prevent a possible relative bringing together of the grains. Consequences macroscopic on the works are practically non-existent since in all the duration of this
phase, the concrete is still deformable, and that any contraction is compensated by a readjustment granular of material against the walls of the formwork. Although of relatively weak width, and effect
mechanics insufficient to generate a real cracking of the concrete, constraints generated with
the interface of two consecutive liftings can start of 50% the margin of resistance in traction of material.
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The catch of the concrete accompanied by the hydration of cement involves an exothermic reaction. In massive structures the temperature can then rise with more 50°C. The hydration is one thermo-activated reaction, i.e. the speed of hydration increases with the temperature.
When the speed of hydration decrease, the temperature decreases, involving a thermal withdrawal. Moreover
the mechanical properties of the concrete vary according to its degree of hydration, and finally water consumption occurring during the hydration involves a capillary withdrawal. The different ones

withdrawals can cause constraints much higher than resistance (weak) in traction of the concrete and to bring to a cracking of material.

The calculation of the fields of temperature and degree of hydration is available with the order THER_NON_LINE (cf [U4.30.02]). The calculation of the mechanical fields taking of account withdrawal
endogenous is carried out with order STAT_NON_LINE.

1.2
Drying and withdrawal of desiccation

To model drying is important owing to the fact that the physicochemical and mechanical properties of material are strongly dependent on moisture inside this last. The objective is to propose
a macroscopic modeling of the drying of the concrete starting from a restricted number of parameters, easily measurable in experiments, starting from a law of transitory diffusion nonlinear of
the moisture, chained at the temperature, while freeing itself from the mechanical complexes couplings, physics and chemical, on a material scale.
To the dismantling, the concrete is plunged in an external environment which presents a degree in general
of moisture of about 60 to 80% HR (relative humidity = report/ratio of the steam pressure on steam pressure saturating for a given temperature). It undergoes a true hydrous shock then (by analogy with a thermal shock). The concrete is then in thermodynamic imbalance with atmosphere. Drying will enable him to find a hydrous balance with the external medium.
Physically, drying brings into play complex phenomena closely coupled the ones with others, depend on the heterogeneous and granular structure of the concrete. With the macroscopic scale, it is
possible [bib2] to model drying like a nonlinear phenomenon of diffusion, with diffusion in liquid phase of ***Darcy*** *type, as long as there is continuity of the liquid phase, and with diffusion in phase*
gas of ***Fick*** *type, for the steam.*

The withdrawal of desiccation is the macroscopic consequence first of the drying of the concrete. It is direct prolongation of the phenomena of capillary tension which are at the origin of the endogenous withdrawal. By
its intensity, deformations being about 400.106 to 800.106 per 50% of hygroscopy and for current concretes, it is of one to three times the more important than the elastic strain for one loading close to 10 MPa.

One initially presents the modeling of thermohydration in the operator of
nonlinear thermics of Code_Aster, then the modeling of drying, and finally, the introduction of endogenous withdrawal and of the withdrawal of desiccation in the nonlinear operator of mechanics.
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2
Formalization of thermohydration

2.1
Equation of thermohydration

The modeling of the phenomenon of thermohydration consists in enriching the equation by the heat of following way:

$$\frac{dT}{D(T)}\,C + \operatorname{div}\mathbf{Q} = Q + S\,\rho\,\frac{dt}{dt}$$

éq
2.1-1

$$\mathbf{Q} = -\operatorname{grad} T$$

where Q is the heat of hydration per degree of hydration (presumedly constant in J/m3), S is one source interns heat (J/m3s), C voluminal heat with constant pressure (J/m3°K) and

p
thermal conductivity (W/m2°K), these two last quantities being independent of the temperature T (°K).

The law of evolution of the degree of hydration is given by:

D
E
= A
has

() exp (-
)
éq
2.1-2
dt
RT
E
with
with the constant of Arrhenius (variable rather empirical parameter between 4000 and 7000°K, and being
R
regarded as being equal to 4000°K in the absence of additional information [bib2]), temperature T being expressed in Kelvin degree.

With () is a function depending on the degree of hydration and the composition of the concrete, given with
assistance of a calorimetric test. The determination of function A () is done starting from the data of the calorimetric test. The test being adiabatic, the only data of the change of the temperature is enough to determine function A (). The heat of hydration Q is determined by the difference of temperature of the sample at the end and the beginning of the test:

Q = C (T - T)
p

0

The originality of the treatment compared to a traditional thermal calculation is the joint presence of one
term of source depending on the temperature (equation [éq 2.1-1]) and on a law of evolution of one parameter (in fact the degree of hydration) (equation [éq 2.1-2]).
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2.2 Exploitation of the calorimetric test for the determination function A ()

Function A () must be provided in data by the user. The goal of this paragraph is to show how one can exploit a test quasi-adiabat to determine A (). A concrete sample expenses is plunged in a calorimeter and one measures the change of the temperature in the course of time until hardening of the concrete. The test being adiabatic, div ***Q*** *= 0, and one has then:*

C (Tad (T) - T)
Tad (T) - T
(T)
p
=
0
, i.e. (T) =
0, where Tad
Q
is the final temperature of
Tad - T

0
the adiabatic test, T0 the initial temperature (one makes the assumption that (0) =0). One determines then required function:

1
dTad (T)
E
With
has

() =
exp (
) .
Tad - T
dt
RTad (T)

0

When
AD
T = T, = 1.

In fact, one can generally define the degree of hydration in
each moment T as being the report/ratio of the quantity of heat released until the moment T on
Q (T (T))
quantity of total heat released at the end of the process of hydration: (T) =
.
Q (T)

3
Discretization of the problem of thermohydration

3.1
Choice of the method of resolution

The selected method consists in solving overall the nonlinear equation [éq 2.1-1] while putting at profit the nonlinear algorithm of thermics of Code_Aster and to locally solve the equation [éq 2.1-2] which represents the law of evolution of a kind of variable interns representing the degree of hydration,
this law expressing itself by a function of the thermal state of the system. Indeed, there is no operator differential spaces some for the variable in the equations and thus not need for finite element. relation [éq 2.1-2] represents a local law as in plasticity. The same number then is preserved degrees of freedom that for traditional thermics. Such an uncoupled process involves nevertheless the calculation of the same quantities several times. Indeed, let us suppose that is discretized with the nodes
elements. Let us consider the example schematized by [Figure 3.1-a].

1
3
1
2

4

Appear 3.1-a
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On node 1, the equation of evolution [éq 2.1-2] will be integrated four times. A possible solution would have
summer that local calculations can be done on fields with the nodes (concept Aster CHAM_NO) and not on fields of nodes by element (concept Aster CHAM_ELEM, option ELNO), which is currently impossible.
The solution which was finally adopted, consists in calculating at the points of gauss of the element, it who is all the more natural as for mechanical calculation the Young modulus depends explicitly of. This generates nevertheless much local calculations except strongly under-integrating the element finished. For example, if one considers a grid comprising NR hexahedral elements with 20 nodes, it exist about 4N nodes and 27N points of Gauss.

3.2
Algorithm of resolution

By taking again the notations of documentation Aster [bib5], the weak formulation of the equation [éq 2.1-1] is written in the following way:
Ea
-
*& (T). **
T D +
(T) T.

*
T
D =
*. **
S T D +
(
*QA) E RT. **
*T d+. **
T D,
*
T

'
1
( )

The development of thermohydration within the algorithm general of thermics not linear in Code_Aster thus consists in discretizing in an explicit way in the second member
E
- has
the term
*QA () E RT T **
.

D. While noting respectively -
-
T +
+
,
,
, T, variables of hydration

and of temperature at the beginning and the end of the step of time, one calculates in each point of Gauss
- Ea
quantity
-

-
RT
QA () E
who is integrated directly in the second member. After each resolution
step running, the variables is reactualized (+
- , +
-
=
T = T). The test of convergence is not
credit that on the temperature, the variable not entering the iterative process of Newton used
in nonlinear thermics. The taking into account of the hydration is in fact only the taking into account
of a heat source known at the beginning of the step of time. This purely explicit discretization
thus require to use steps of sufficiently small times.
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4
Formalization of drying

This part refers to the document of specification of the development of drying in
Code_Aster [bib3], like with the thesis of L. Granger [bib2].

4.1
Modeling and equations of drying

Modelings of thermics or thermohydration and drying are uncoupled at the time of
the resolution. Drying is then presented like an operation chained at thermics. Like
equations making it possible to solve drying and thermal nonlinear are similar for it to

coefficients close, this decoupling makes it possible to integrate the resolution of the calculation of drying in
Code_Aster, by directly using the module of resolution of nonlinear thermics, without adding new phenomena, new types of elements nor new options of calculation, and in
thus minimizing the volume of added and duplicated code.
The concentration or water content, variable of calculation in the modeling of drying, is comparable, in term of the type of variable, at a temperature (standard TEMP). The transitory field of temperature, intervening in the equation of drying, is only one auxiliary parameter on which depends
possibly the coefficient of diffusion.

Phenomena of thermics and drying, within the framework of a modeling uncoupled between thermics and drying, is governed by the following equations:

.
equation of "traditional" thermics:

dT

D (T)

C
+ div ***Q*** *= Q*
+ (
S T)
p dt
dt
éq
4.1-1
Q *= - (T) grad T*

(C voluminal heat with constant pressure, thermal conductivity, Q heat
p
of hydration per degree of hydration and S the second member).

.
equation characterizing drying:
C - [
Div (
D C, T) C] =

0
éq
4.1-2

T
where
C (m3/m3 or l/m3) is the variable of calculation (concentration or water content voluminal),
T is the variable of entry of calculation (the temperature), variable auxiliary of the resolution drying,
D (m2/s) is a coefficient of diffusion, characterizing nonthe linearity of the equation, and depending at the same time on the variable on calculation, C, and auxiliary variable, T. This law of diffusion is given in various forms, according to the model selected, (law of ***Bazant****, law of* ***Mensi****, cf [§4.3] and [bib2]).*

The equations [éq 4.1-1] and [éq 4.1-2] correspond to a thermal chained calculation/drying. One can thus to calculate T without knowing the water concentration, then to calculate the latter, for which T is then a parameter, (by making the assumption that thermal conductivity does not depend on water concentration C). Also let us note that the phenomenon of drying is uncoupled from the evolutions mechanics of the concrete.
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4.2
Coefficient of diffusion

The material is described by the coefficient of diffusion D, characteristic of material, dependent with time of the temperature T and the water C concentration. The equation of the migration of moisture in the concrete is resulting from those of the mechanics of the porous environments. One will refer to [bib2] for
more precision. Classically, a law of diffusion expresses a flow like the product of one size characteristic of material by the gradient of an intensive size. The different ones sizes considered are defined by an average on representative elementary volume, for as far as one can define this average for material considered, so that operators of derivation a direction has. One thus makes in general the assumption which consists in supposing that them
phases liquid and gas are related:

.
*for the diffusion of the vapor, one leaves the positivity of dissipation associated with transport gas phase, by differentiating two phenomena, a phenomenon of permeation type (**Darcy**), related to gradients of pressure, and a phenomenon of type diffusion (**Fick**), related to gradients of concentration,*
.
*for the diffusion of liquid water, the positivity of dissipation associated with transport with water liquid, and the law of **Darcy**, makes it possible to express the flow of liquid according to the pressure of liquid. The law of **Kelvin** describing the coexistence of the two phases liquid and gas by the writing of the equality of the mass free enthali leads to the expression of flow in function gradient of the percentage of moisture.*

From the two preceding results, one obtains the expression of total flow according to the gradient of the degree of

water concentration. Traditional experimental methods in the problems of drying generally access to the water concentration gives, and very seldom with the relative humidity. It is thus preferable to express flow according to the water content, while using classically the isotherm of desorption of the concrete, which connects the water content, C, and the relative humidity, h. moisture relative is the relationship between steam pressure and saturating steam pressure.
The postulate of the local state stipulates that the current state of a homogeneous system in unspecified evolution can be characterized by the same variables as with balance, and than it is independent speeds of evolution. In other words, water content C, and the relative humidity H, are well connected by even relation that with balance. What leads to the traditional equation of the diffusion:

C - Di [v (cd., T) C] = 0

éq
4.2-1
T

*This equation highlights the nonlinear character of the diffusion of moisture in the concrete. In the industrial cases, the temperature is in general not uniform in the structure. It is thus necessary to take into account a coefficient of diffusion of the moisture which depends on temperature. In practice, in the literature, the most known authors (**Bazant** cf [bib2]) propose an expression of the coefficient of diffusion of the type:*

- Qs 1 1
(
T -
D C T) = (
D C T
0

éq
4.2-2
0)
E R T T
,
,

T0
with Qs/R = 4700 K-1 and T in °K
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Note:

Way in which the things are presented, it would seem that one did not use the fact that drying is a phenomenon coupled with the mechanics, (i.e. it is the cause of one withdrawal of desiccation). Actually, we made the assumption of a decoupling of phenomena, when we used the curve of sorption/desorption. In fact, at the time of measure loss in weight with balance according to H, the body of test carries out a withdrawal. At the microscopic level, all occurs like if the withdrawal, modifying porosity, went to interact on the relative hygroscopy inside the sample, since pressure of vapor and H increase. This withdrawal of desiccation being very weak, it is usual of to neglect in calculations of the water content. There is thus only one chaining between the calculation of water content and the mechanical calculation of withdrawal of desiccation.

4.3
Usual laws of diffusion

The law of diffusion, function of the two parameters, C and T, can be freely defined by the user in the form of a tablecloth. However, usual expressions of the law of diffusion, which one finds in the literature are as follows:

Law suggested by Granger:

- Q 1 1
S
(
-
,)
(B C

.)
T
D C T = A.e
E R T T0
éq
4.3-1
T0
With (m2/s), B, T0, Qs, and R (Qs/R in °K) are coefficients chosen by the user. D is a function of temperature and of the water concentration.

Law of Mensi:

( )
(
)
D C
With E B C
= . .
éq
4.3-2

With and B are coefficients chosen by the user. D is a function only of the concentration out of water.

Law of Bazant:

The law of Bazant is expressed starting from the percentage of moisture H, which is connected to the water concentration by
curve of sorption/desorption. The form of this law is as follows:

1
D (H)

-

= D

éq
4.3-3
1
+

N

1 - H C
( )

1+

1- 0.75

Usually,
D1 = 3.10-10 m2/s
lies between 0.025 and 0.1,
N is about 6.
(
H C) is the percentage of moisture, which is expressed according to the water concentration with assistance of the curve of sorption/desorption.
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The curve of sorption/desorption can be introduced in the form of a tabulée standard function, knowing that actually, this curve has a hysteresis, but can be regarded as being invertible, if one takes account only of one direction of course.

4.4
Modeling of the boundary conditions

The boundary conditions are expressed in general by a nonlinear relation between the flow of

water concentration (l/m3 X ms1) W fl and concentration out of water. These conditions are thus analogues in the conditions known as of exchange in thermics. One will be able for example to use the formula
proposed by L. Granger [bib2] page 181:
Its expression is as follows:

0 5
.
W fl = (
C
.
2
- 2 C - C
C - C

éq
4.4-1
0
eq
eq
C - C
0
eq) [
(
)](
)

where
Ceq is the water concentration for a moisture of 50%HR,
C0 is the water concentration for a moisture of 100%HR,
(l/m3 X m/s) is a coefficient, which can be defined in experiments and can evolve/move according to the cracking of the heat-transferring surface ([bib2]),
and
C is the current concentration (unknown) on the heat-transferring surfaces.

5
Integration of drying in Code_Aster

These developments relate to the axisymmetric elements 2D and elements, as well as elements 3D isoparametric, of a number of nodes unspecified, linear and quadratic.

5.1 Introduction of the concept of behavior into the operator of nonlinear thermics

Operator THER_NON_LINE was reserved exclusively for the nonlinear thermics, which will remain ***the option of calculation per defect****. But one uses the same module of resolution to solve them problems of drying and hydration, because of analogy of the equations.*

The concept of behavior was added in the nonlinear operator of thermics, with one nomenclature and a syntax analogues with those of the operator of nonlinear mechanics. It imply for drying a concept of entity topological, to which this behavior is applied.
This can be useful, when there are several types of possible laws of diffusion, or when one wants to make a purely thermal calculation on part of the grid, whereas on another part one does one calculation of thermohydration (on the other hand, the simultaneous use on the same grid of behaviors of the drying type, and behaviours of the thermal type or hydration would not have direction).

A behavior "drying" is associated each law of diffusion, such as one can find them in the literature, just as a specific material is associated each law of diffusion, to define the characteristic coefficients of them. The resolution of drying is identical, with coefficients close, with that of nonlinear thermics, and no modification was made to the algorithm of resolution.
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For drying, four distinct behaviors are defined under key words "SECH_GRANGER", "SECH_BAZANT", "SECH_MENSI", or "SECH_NAPPE", to characterize each law of possible diffusion. They can be allotted to parts complementary to the grid, during one even calculation. The simultaneous definition several behaviors "drying" associated with entities topological different several occurrences of the key word "BEHAVIOR requires". Then, entity topological will have to be identified by informing one of key words GROUP_MA or MESH.

In parallel of the four behaviors "drying", in the operator DEFI_MATERIAU, four materials initially make it possible to define the values of the coefficients of the laws of diffusion, nonlinear functions of the water content and the temperature. The user can choose (or them) law (S) of its choice, and defines the value which it wishes for each one of these coefficients.
Key word SECH_GRANGER makes it possible to define the law of diffusion of liquid and gas water under its form most traditional among the expressions of the literature. Four coefficients like one temperature of T0 reference characterize this law.
Key words SECH_MENSI and SECH_BAZANT make it possible to define the laws of ***Mensi*** *and* ***Bazant****, with assistance of the coefficients which are appropriate. The law of* ***Bazant****, expressing itself starting from the percentage of moisture, require to define a curve of desorption allowing to convert the water content into degree of moisture within the framework of this modeling.*
Lastly, key word SECH_NAPPE makes it possible to use a law of diffusion, starting from a function tabulée of two variables, which will be interpolated in calculations starting from the values of the water concentration and temperature. This last possibility presents the disadvantage of not raising ambiguity enters these two variables associated with an identical type, "TEMP".

It is necessary, for drying, to introduce in entry of calculation a concept of the type [evol_ther], representing the evolution of the field of temperature of the concrete structure, within the framework of a calculation chained thermal/drying. Indeed, the calculation of drying requires the preliminary calculation of temperature and possibly of the hydration, because the coefficient of diffusion (D C, T) depends on temperature.

5.2 Implementation of the boundary conditions for drying

5.2.1 Expression of the boundary conditions

The boundary conditions are expressed in the form of flow of moisture on surfaces in contact with external medium following the expression [éq 4.4-1].

5.2.2 Delimitation of the calculation of drying using the boundary conditions

The calculation of drying is defined on the totality of the grid where finite elements are affected. For to make effective the calculation of drying that on a portion of the grid (this with an aim of preserving it even model for calculations of drying and mechanical calculations and to facilitate them "continuations" of calculation Aster [bib4]), one will use the boundary conditions. Indeed, drying

does not take place
that if there is exchange with outside. It is thus the attribution of the boundary conditions which allows
“to locate” calculation. The absence of drying on a portion of the structure will be expressed by the absence of
boundary conditions on the heat-transferring surfaces concerned.
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5.2.3 Implementation in Aster

The boundary conditions can be defined, as in thermics, in the form of normal flow not
linear formulated starting from a function tabulée of the variable of calculation, and interpolated during calculations.
That makes it possible to avoid creating new options of calculation, similar to the options of thermics not
linear char_ther_flunl and resi_ther_fluxnl which calculates the first and the second member, and who can be used directly for drying. It is then enough to choose a tabulée function corresponding to the expression of flow, given by the equation [éq 4.4-1].
Using a preset function (FORMULA), the expression of flow, given in form polynômiale and function of the variable of calculation, is transformed into tabulée curve, via the operators Aster (CALC_FONC_INTERP). One thus does not create a new option of calculation for the treatment of
boundary conditions.

The calculation of new options would have the advantage of being optimal in term of result (because of absence
interpolations and because of “exact” calculations of derivative), but would require to develop two

new options of calculation, similar to the options char_ther_flunl and resi_ther_fluxnl.

5.2.4 Example of working of the boundary conditions

The sequence of orders, described in the example which follows and whose numerical values are fictitious, will be adopted, for the creation of a boundary condition charsech on a group of meshs limit.

Note:

The "FORMULA" Aster is the numerical expression of flow of the normal water concentration which the equation [éq 4.4-1] begins again.

beta

=
DEFI_VALEUR

(R8:
0.25)
c_0

=
DEFI_VALEUR

(R8:
0.30)

c_eq = DEFI_VALEUR
(R8:
0.70)

! FORMULATE (REAL: (fon_humi
(REALITY: temp) =
(0.5*beta
/
((c_0
-
c_eq) ** 2)

**** (temp - (2*c_0 - c_eq))* (temp - c_eq))));***

pas0

=
DEFI_VALEUR
(R8: EVAL (1. /20.));

list0 = DEFI_LIST_REEL (BEGINNING: 0.0

INTERVAL: (JUSQU_A: 1.0
NOT: pas0)) ;

flu_humi = CALC_FONC_INTERP (
FUNCTION: fon_humi
LIST_PARA

:
list0
PROL_GAUCHE:
“EXCLUDED”
PROL_DROIT
:
“EXCLUDED”
Interpol

:
“LINE”
TITRATE

:
'flow
of humidité');

charsech = AFFE_CHAR_THER_F (MODEL:
modether

FLUX_NL
:
(
GROUP_MA
: limit
FLUN
:
***flu_humi**)) ;*

Note:

It is important that the interpreted function and the tabulée function do not bear the same name, so that the interpolations on the right and on the left are suitably defined, because them exclusions on the right and on the left "do not overload" not the prolongations of a function interpreted, transformed using operator CALC_FONC_INTERP.

In the example above, the tabulée function fon_humi is defined outside the interval [0.0, 1.0] but it is not defined apart from the interval in the interpreted function flu_humi.
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5.3
Numerical integration of drying

dT
The equation of heat C
- Div
=

or &
-
[
Div (T) T] = (
S T) led,
p
(grad T) (St)
dt
in the case of a boundary condition in normal flow on the border with the variational formulation:

(T)
T
.d + (T) T. .d =
(St). .d + (T). .d

éq
5.3-1
T
N

C
In a similar way, the equation governing drying
-
[
Div (
D C, T) C] =

0 conduit, in the case
T
of a boundary condition in normal flow on the border with the variational formulation:

C
C
.d +
(
D C, T) C. .d = 0 +
(
D C, T). .d

éq
5.3-2

T
N

The resolution of drying is integrated into operator THER_NON_LINE, by replacing C
by
p
constant function equalizes with the identity, and conductivity by the diffusion (
D C, T), the temperature
intervening like a constant in calculations (auxiliary variable). According to the law of diffusion chosen, it is necessary to calculate the value of the coefficient of diffusion like its derivative, according to the temperature
and water concentration at the moment running, the current point.

One will refer to the documentation of the nonlinear operator of thermics [R5.02.02] for moreover full details on the numerical integration of nonlinear thermics.

Within the framework of drying, the boundary conditions are given in term of normal flow, and lead, as in thermics, with a term in the first member, associated the option of calculation rigi_ther_fluxnl, and in the term in the second member, associated the option char_ther_fluxnl.

6
Formalization of the endogenous withdrawal and desiccation

6.1
Withdrawal in Code_Aster

In the framework of a formalization of the withdrawal in term of deformation, the total increment of deformation
can break up thermal component all in all, of a component representing it
endogenous withdrawal, and of a component representing the withdrawal of desiccation, added to mechanical component (elasticity, creep,…).

One can model the withdrawal of desiccation in the form:

= - (C

C
0 -
) D
one
dessiccati
. I
éq
6.1-1

where C is the water concentration, C0 the initial water concentration.
and a coefficient characterizing the withdrawal, depending mainly on the water concentration.
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One can model the endogenous withdrawal in the form:

D

=
- .
I. ***éq***
6.1-2
endogenous

where is the hydration,
and a coefficient characteristic of the material whose dependences are badly known.

The withdrawals of desiccation and endogenous can thus intervene in any law of

behavior by replacing the usual terms there - T I D
.
by
- T.I D -
-
. In the current state of Code_Aster, this is done only for
desiccation
endogenous
linear elasticity and for the behaviors elastoplastic of Von Mises type with work hardening
linear or kinematic. One has then for example in elasticity 1D:

1
=
. T
.
C
éq
6.1-3
E (
+
- -
)
(

)

The mechanical parameters E (Young modulus) and (thermal dilation) depend
mainly of the variable of hydration.

This formulation of the withdrawal of desiccation and the endogenous withdrawal has the advantage of using directly
water content C, which one can connect to the loss of weight by simple integration on volume. If one used the relative humidity H, it would have to be retranslated in term of water content by the means of the isotherm of desorption of each various concrete.

For Code_Aster, these parameters can be defined within a framework relatively general, like functions of the various variables of calculation and variables auxiliary (temperature, hydration, water concentration, or constants) to leave the choice to the user to define them freely dependences of the parameters. It remains with the load of the user to use the functions of Code_Aster to reproduce the expression of the Young modulus given in the equation [éq 6.1-3].

For more detail on these formulations, and the means of calculating the coefficients and, one will defer to the thesis of L. Granger, [bib2], on pages 99 and following, and pages 210 and following.

For mechanical calculation the variables (the hydration) and C (water concentration) are data, like the east the temperature during a thermomechanical calculation.
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6.2
Integration of the withdrawal in the law of behavior mechanics

Thermics and drying are uncoupled from the mechanical resolution, just like drying is an operation chained with thermics and the hydration. This decoupling makes it possible to integrate the withdrawal in the operator of resolution of nonlinear mechanics, without adding new phenomena, behaviors, types of elements and options of calculation. Moreover, it makes it possible to introduce the withdrawal in way simple in all the nonlinear laws of behavior. The syntax of the operators of mechanics STAT_NON_LINE and MECA_STATIQUE are not modified.
In the current version of the nonlinear operator of mechanics, the withdrawal was integrated into elastic behavior (ELAS), with the elastoplastic behavior of linear work hardening type and kinematics (VMIS_ISOT_TRAC and VMIS_ISOT_CINE) and with the models specific to the concretes: MAZARS, ENDO_ISOT_BETON, BETON_DOUBLE_DP, GRANGER, BETON_UMLV_FP, BAZANT_FD. It consist in removing the terms of withdrawal to the total deflection, before the resolution of the equations of balance at the points of Gauss, in the same way which is taken into account thermal dilation.

The coefficients and characterizing the withdrawals endogenous and of desiccation are defined under the word key "ELAS_FO", like constants. Other mechanical characteristics, coefficient of Poisson, modulus Young, thermal dilation coefficient can also be defined like

functions of new variables HYDR and SECH, which were added to the catalogues of both operators DEFI_FONCTION and DEFI_NAPPE.

Two new options of loading were added to operator AFFE_CHAR_MECA, in order to to define under key words SECH_CALCULEE and HYDR_CALCULEE, the concepts results, of type [evol_ther], resulting from a nonlinear calculation of thermics, or thermohydration, and a calculation of drying. They correspond respectively to the fields thermo-hydrous of type "TEMP/HYDR", and with field drying of the type "TEMP", calculated previously with the mechanical resolution. These options (as their syntax) are similar to the option TEMP_CALCULEE, which defines the thermal field. They allow:

.
to calculate the withdrawals endogenous and of desiccation, if characteristics material associated will have been before defined in DEFI_MATERIAU,
.
to interpolate the Young modulus, the Poisson's ratio, and the dilation coefficient thermics, when those are functions of the variables hydration or drying.

Note:

In the presence of a field of drying, it is necessary to inform key word SECH_REF in order AFFE_MATERIAU. This value defines the value of SECH for which withdrawal of desiccation is null.
It is thus necessary to take care to be coherent with values SECH_CALCULE used (in particular at the moment
initial!).

6.3 Stamp
tangent

The calculation of the tangent matrices of the various laws of nonlinear behavior is not affected by the addition of the endogenous withdrawal and withdrawal of desiccation, because one neglects the derivative compared to
variables of hydration and drying, of the terms of the equilibrium equations, just as are usually neglected the derivative compared to the temperature of these same terms. These derivative intervene with the second order.

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Organization (S): EDF-R & D /AMA, EDF-DIS/CNEPE

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Document: R7.01.13

Law CJS in géomechanics

Summary:

One presents here the law CJS which applies to the soil mechanics. One specifies:

.
the description of the model,
.
the integration of the law in Code_Aster,
.
the description of the introduced routines.
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1 Notations

The notations used here are the usual notations of the soil mechanics, to which are added notations suitable for the writing of the parameters of law CJS.

One also gives the correspondence, if it takes place, between the parameters of the law and their notations
in Aster.

With
parameter of the model
A_CJS
B
parameter of the model
B_CJS
C
parameter of the model
C_CJS
N
parameter of the model
N_CJS
K
modulus of voluminal deformation elastic

E
K
parameter of the model

O
p
K
parameter of the model
KP
O

G
elastic modulus of rigidity

E
G
parameter of the model

O
D
G
function controlling the evolution of the plastic deformations déviatoires

S
diverter of the tensor of the constraints

I
first invariant of the constraints

1
p
pressure of initial criticism
PCO
Co
P
pressure of reference of the model
Pa
has
I
D
F, F
thresholds of the plastic mechanisms isotropic and déviatoire

Q
variable interns model corresponding to the acceptable limit of the plan
Iso
déviatoire
Q Q
,
tensors of the model

R, X
variables intern model corresponding to the average radius and the center
surface of load in the déviatoire plan
R

parameter of the model
RM
m
R
parameter of the model
RC
C

I
D

,
plastic multipliers of the mechanisms isotropic and déviatoire

E
IP
dp

,

, tensors of the respectively total, elastic, plastic deformations
isotropic and plastic déviatoires

voluminal deformations

v

parameter of the model
BETA_C
JS

parameter of the model
GAMMA_
CJS

angle of Lode

function limiting the evolution of X

μ
parameter of the model
MU_CJS
Q

parameter of the model
Q_INIT
init

Note:

Foreword: Contrary for the use of géomechanics, the convention of
sign reserve is that of the mechanics of the continuous mediums, i.e tractions are counted positively.
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2 Introduction

Model CJS is an elastoplastic law of behavior adapted to the modeling of materials
granular. It was developed at the Central School of Lyon ([bib1], [bib2], [bib3]).

Version CJS established in Code_Aster is a model arranged hierarchically including/understanding several
levels of complexity. In its most complete expression, the model has two surfaces of
charge: one is activated by the isotropic requests, the other by the requests déviatoires.
first undergoes an isotropic work hardening and the second a mixed work hardening (isotropic and kinematic).
The elastic law is of hypoelastic type nonlinear.

3

Description of law CJS

3.1
Partition of the deformations

The increment of total deformation breaks up into three parts, relating to each one of mechanisms brought into play:
E
IP
dp
& = & + & +
ij
ij
ij
&ij
where E
&, IP

ij
& and dp
ij
ij
& are respectively the increments of elastic strain, of deformation isotropic plastic and of plastic deformation déviatoire.

3.2 Mechanism
rubber band

The elastic part of the law is of hypoelastic type, whose general expression is:
s&
I&
E
ij

1
& =
+

ij
ij
G
2
9K

where I is the first invariant of the constraints: I = tr
, S is the déviatoire part tensor of
1
()
1
constraints, and where K and G are respectively the voluminal modulus of deformation and the module of
shearing rubber bands. Those depend on the state of stresses according to:
N

N
I + Q
I + Q

E
1
init
K = K
,
E
1
init
G = G

O
P

O

3
3P
has

has

E
K, G, P and N are parameters of the model. P is a pressure of reference equal to -100
O
O
has
has
kPa.

3.3
Isotropic plastic mechanism

The surface of corresponding load I
F is, in the space of the principal constraints, a plan
perpendicular with the hydrostatic axis, is:
I + Q
I
F (Q
,
= - 1

+ Q
Iso)
(
init)
Iso
3
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where Q is the thermodynamic force which depends on the variable interns Q according to:
Iso
N
Q
Q& = K p q& = K p

Iso
Q
Iso
O

&
P
has

p
K, P and N are the parameters of the plastic mechanism déviatoire (P and N are identical to
O
has
has
those of the elastic mechanism). The rule of normality makes it possible to express the evolution of the deformation
plastic and of the variable of work hardening according to the evolution of the plastic multiplier I
:
I
F
1
I
F
IP
I
I
& = &
= - & and
I
I
q& = - &
= - &
ij
ij

3
Q
ij
Iso

Taking into account the second equation, the law of work hardening can be also put in the form:

N
I

p

Iso
Q
&iso
Q
= - & KB

has
P

3.4
Plastic mechanism déviatoire

The surface of load of this second plastic mechanism is a convex surface with ternary symmetry defined by the equation:
D
F (, R, X) = Q H + R I + Q

II
(Q) (1 init)
with Q = S - I X
ij
ij
1
ij
Q = Q Q
II
ij
ij
1/

det Q
H (

.
Q) = (1 +
cos (3 Q) 1/6
() 6
= 1+
54

3

qII
The scalar R and tensor X respectively represent the average radius and the center of surface of load in the déviatoire plan.
S, Q and X are tensors déviatoires. is a parameter which translates the behavior dissymmetrical of the grounds in compression and extension. is the angle of Lode.
This surface of load evolves/moves according to two types of work hardening: isotropic work hardening and work hardening
kinematics.
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Note:

The expression of the angle of Lode is found in the following way:
In a reference mark (H, I,) J of the déviatoire plan vector HM can be given from outdistance HM = and angle of Lode (cf [Figure 3.4-a]). Co-ordinates of HM
S
are:
HM = (sin, cos
S
S)

s1
M
J

I
H
s2
s3

Appear 3.4-a: Angle of Lode in the déviatoire plan

The principal components of the diverter are thus:

4

2

S =

cos, S =
and S =

3

cos
- S
2

cos
- S
1
S
3

3

3
Consequently, one a: S =
and
II
2
(S) 1

det
= 3

cos
cos2
2
-

1
sin
3
= 3cos
3

S (
S
S)
(S)
4
4
one deduces the relation then from it:
(
S
cos 3 =

S)
1 2 3/2 det ()
2 3
3
sII
The angle calculation in the same way.
Q

3.4.1 Work hardening
isotropic

The isotropic law of work hardening is written as follows:
2
AR R
R
m
=
&
& (

R + R
With
m
) 2
The thermodynamic force R is related to R whose evolution is given by:

- 5
.
1
1
- 5
.
D
F I Q
I
Q
D
+

init
D
+

1
r& = - &
= - & (I + Q

init)
1

init
1
R
3

P
3

P
has

has

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By direct integration of the law of work hardening, it comes:
AR R
RR
R
m
=
, that is to say also R
m
=

R + R
With
(
WITH R - R
m
)
m

The law of work hardening can thus be also expressed by:
2
1
- .5

+

D
R
R& = - & A 1
(I +Q
1
= &
,
1
init) I
Qinit
D G R (R)

R

3P

m

has

2
1
- 5
.

R
I
Q
R
+

with G (, R) = - A 1 -
(I + Q

init)
1

init
1

R
3

P
m

has

and where R (which are the average radius of the elastic range in rupture) and A are parameters of
m
model.

3.4.2 Work hardening
kinematics

The kinematic law of work hardening is given by:
1
X& =
ij
&ij
B
The thermodynamic force X is a function of the variable whose nonlinear evolution is given
by:
1
- 5
.

D
F
& = -
I
Q
D
& Dev.
I

Q
X

ij

- (+ init)
+

1
ij
init
1

X
P
ij

3

has

The term - (I + Q
allows to obtain nonlinear kinematic work hardening, translating
1
init) X
limitation of the evolution of the surface of load.

D
D
D
F
F Q
F

D
F
By taking account of
kl

=
= - (I + Q
, and while posing: Q = Dev.
, it
init1)
X
Q X
Q
ij

Q
ij
kl
ij
ij

ij
comes finally for the law from work hardening:
1
- 5
.
I + Q
D 1
X& = &
Q + X
I + Q
= & G

ij
(ij
ij) (
init)
1
init
D
X
1
ij (, X)
B

3P

has

- 5
.
1
I
Q
X
1
+

with G
X
Q
X I
Q
.
ij (,
) = (+
ij
ij) (
+ init) 1
init
1

B
3

Pa

where a function which limits the evolution of X and is a parameter of the model.

The tensor Q is calculated according to the formula:

1

Q
54
det Q
Q =
1
cos

3
Dev.

ij
+
() ij
( )

+

H ()

5
2

2
Q
6q
Q
II
II

ij

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The preceding expression is obtained in the following way. One a:
D
F

= H (
Q
H

+ Q

Q)
(
II
Q)
II
Q
Q
Q
ij
ij
ij
Q
H (Q)
where
II and
are respectively given by:
Q
Q
ij
ij
Q
Q
II
ij
=

Q
Q
ij
II

H (
1

det Q
-
3
cos () Q
Q)
( )
Q
ij
54

=
Q
1+ 54
=
+
det ()
Q
H
6
5

Q
q3
2h () 5
q2
6h () 5 q3
Q

ij
(Q) ij
II

Q
II
Q
II
ij
from where

D

F
1

Q
54 det Q

=
ij
1
cos
3

Q
H 5
2
Q
6q2
Q
ij
(Q) +
(Q)
( )

+

II
II

ij

The function, as for it is given by:
= H Q
O
(S) II
1/

det S
where Q = Q Q and H (

. The term is expressed in
S) = (1 +
cos (3 S) 1/6
() 6
= 1+
54
II
ij
ij

3

S
O
II

function of characteristic to the rupture of material.

3.4.3 Law of evolution of the plastic mechanism déviatoire

In granular materials, a variation of volume can occur for a loading purely déviatoire. This variation of volume is related to the discontinuous aspect of material and on conditions kinematics which result during the loading. This particular phenomenon does not allow to define the plastic deformations déviatoires starting from the only rule of normality. This is why it plastic mechanism déviatoire is nonassociated. There is thus a potential function controlling evolution of the deformations:

dp
D
D
& = & G

ij
ij
The potential function is defined starting from the following kinematic condition:

dp

S.E.
S

ij &ij
dp
II
& = -
-1

v

Cs S
II

II

where is a parameter of the model and C
S represents the characteristic state of stress. A surface,
II
from form identical to the surface of load in the space of the constraints, separates the contracting states
dilating states. This surface, known as characteristic, has as an equation:

C
C
F = S H + R I + Q

II
(S) C (1 init)
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---

where R is a parameter corresponding to the average radius of this characteristic surface. The condition
C
kinematics can be also put in the form:

S

dp
S.E. &
S.E.
ij ij
S

dp
dp
&
S.E. &
dp
& +
II

-1
= dp
& +
II

-1 ij ij
ij ij
v

C

v

C

S
S
S
S.E.
S

II

II
II

dp
ij &ij
II

= dp
& +
dp
S.E.

v
ij &ij
sII

= dp
& +
dp
S & = 0
v
ij
ij
sII
S

where
II
=
-1 sign (
dp
S

&.
C
ij
ij)
S
II

It is then possible to seek to express this kinematic condition starting from a tensor N under form:
dp
& N = 0
ij
ij
i.e., after decomposition of each term in déviatoire parts and hydrostatic:
dp
dp 1
& N = e& + dp
& N S
N
N S.E. &
N

ij
ij
ij
v
ij (

+
=
dp +
dp
& =
1 ij
2
ij)
0

3
1 ij ij
2

v
n1

One deduces the relation from it
=
, which added to the condition of standardization N: N = 1, led to

N
S
2
II
expressions:

S
ij +
S
1
ij
S
N =
II
and N =
, N is =
II

1
2
ij
2
+ 3
2
+ 3
2
+ 3
The law of evolution of dp
ij& must be such as the kinematic condition is satisfied. It is thus proposed
to take the projection of dp
ij& on the hypersurface of deformation of normal N, is:
D
D
F
F

dp
D
D
D
& = &
-
N N = & G
ij

kl
ij
ij

ij

kl

D
D
F
F

with
D
G =
-
N N.
ij
kl
ij

ij

kl

In addition, for the calculation of the potential, one can note that:

D
D
F
F qkl
=
+ R ij

Q
ij
kl
ij
D
D

F 1 F

1

= Dev.
+
- + X + R
kl

ik jl
ij
kl
kl

ij

Q
3 Q

3

kl
mm

D
1
1 F
1

= Q - Q + Q X +

- + X + R
kl
ik
jl
ij
kl
kl
kl
kl
ik jl kl
ij
kl
kl
kl
kl

ij
3
3 Q
3

mm

= Q - Q X - R
ij
(kl kl
) ij
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3.4.4 Rough surface

The state of rupture results from the nonlinear nature of the laws of work hardening and the existence of values
limits associated with the variables with work hardening R and X. The limit of R, noted R, is reached when
m
R tends towards the infinite one. The limit of X is reached when X& becomes null.
ij
ij
Under these conditions:
Q = X and Q =
1
X

X
=

ij
ij
II
II lim
II lim
H
O
(S)
In the state of rupture one thus has [Figure 3.4.4-a]:

S + I X

cos
II
Q =
1

II
cos (IIlim
-
S
Q)
By replacing this expression and the value of R in rupture, in the equation of the surface of load in rupture, one obtains the equation of a limiting envelope for surfaces of load:
R
F = S H
R I
Q

II
(S) + R (+ init) = 0
1

cos
H (S)
with R =
+
R cos -, average radius of the envelope, which is determined from
R

H
O
(Q) m (S Q)
mechanical characteristics with the rupture of material. The value of can then be deduced from it:
O

=
cos

O
H (S)
R -
R cos -
R

H (Q) m
(S Q)
q2 - S 2 - I X
2
II
II
(1 II)
with cos =

2s I X
II 1
II

s1
rough surface
caracterisic surface
q1
S

s2
s3
Q
surface of load to the rupture
q2
q3

Appear 3.4.4-a: Representation of the rough surfaces, characteristic and of load in the déviatoire plan
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In addition, R is related to the maximum angle of friction and depends on the average constraint and on
R
relative density. To take into account the dependence of the maximum angle of friction in function average constraint and relative density, the relation is considered:
3p

$$R = R + \mu \ln$$

R

C
C

I + Q
1
init
where R and μ are parameters of the model. p is the average constraint criticizes, i.e.
C
C
minimal average constraint (it is negative with our convention of sign) known by material during its history. It depends on the initial relative density according to the traditional concept
of right-hand side criticizes in the plan (E ln
, p):

$$p = p \exp - C$$

C
Co
(
v)
where p is the initial critical pressure and 1 C is the critical line slope of state in the plan
Co
(ln
, p.
v
)

3.5

Hierarchisation of the model

3.5.1 Summary description of three levels CJS

Starting from the complete description of the model given above, one deduces three levels from complexity
increasing whose characteristics are summarized in the following table:

Elastic mechanism
Plastic mechanism
Plastic mechanism
isotropic
déviatoire
CJS1
linear
not activated
activated, perfect plasticity
CJS2
nonlinear
activated
activated, isotropic work hardening
CJS3
nonlinear
activated
activated, work hardening
kinematics
Table 3.5.1-1: Various mechanisms used by the various levels of model CJS

3.5.2 Assessment of parameters CJS

In addition, one can also summarize the correspondence between the various levels of the model and parameters associated with each one of them:

N
E
K
E

μ
O
Go
p
K

C
R With B
Rm
pco C
Pa
CJS1

CJS2

CJS3

Table 3.5.2-1: Assessment of the various parameters according to levels CJS

In Code_Aster, the elastic parameters of model CJS (
E
K and G) are directly taken in
O
O
count in the elastic characteristics of material, i.e. through the Young modulus E and the NAKED Poisson's ratio.
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In Code_Aster, the user explicitly does not indicate selected level CJS only it. It is in effect the choice of the various parameters which determines the corresponding level. We have for to summarize the following logical tests which are integrated in the code:

.
if N = 0 then level CJS1,
.
if (N 0 and A 0) then level CJS2,
.

if (N 0 and A = 0) then level CJS3.

Note:

The user must fix the value of Pa equal to -100 kPa according to the selected units. In in addition to, for CJS3, the value of pco must be negative.

3.5.3 Correspondence with the cohesion and the angle of friction

The mechanics of the grounds are accustomed to using the concepts of cohesion Cohésion C, of angle of friction and of angle of dilatancy: . These parameters are used in the law of Mohr Coulomb. Level 1 of law CJS makes it possible to find a very nearby behavior by making the following choice parameters:

1 6
1-
3 - sin ()
=

1+
3 + sin ()

2
2
sin () (1 -) 1/6
=
3
m
R

3 - sin ()

Q
= -3 .
C cotan
init
()

2 6 sin ()
= -

3 - sin ()
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4
Integration of law CJS

We detail below the integration of law CJS according to or of the activated mechanisms:

.
nonlinear rubber band,
.
nonlinear rubber band and isotropic plastic
.
nonlinear rubber band and plastic déviatoire
.
nonlinear rubber band, isotropic plastic and plastic déviatoire.

In each case, the goal is to calculate, starting from the fields known with the state less -
, -
and of
the increment of deformation
, the new state of stress +
.

In the sequence of calculations, one starts by making the assumption that only the elastic mechanism

nonlinear intervenes. An elastic prediction is thus carried out. This prediction is then used to calculate the functions of load I
F and D
F, one seeks to know if one goes then beyond
thresholds:

.
if I
F 0 and D
F 0, the elastic prediction is regarded as new state of stress,
.
if
I
F > 0 and D
F 0, one makes the integration of the mechanisms elastic nonlinear and plastic isotropic,
.
if
I
F 0 and D
F > 0, one makes the integration of the mechanisms elastic nonlinear and plastic déviatoire,
.
if
I
F > 0 and D
F > 0, one makes the integration of the mechanisms elastic nonlinear, plastic isotropic and plastic déviatoire.

At exit of elastoplastic calculation, when only one plastic threshold was initially exceeded, one recompute each function of load. Indeed, it is possible that while seeking to bring back itself on one of the thresholds, one then exceeds the other threshold not activated initially by the elastic prediction. In
this case, one solves then by integrating all the mechanisms.

4.1
Choice of the internal variables

The variables Q, R and are equivalent to the associated thermodynamic forces Q, R and X.
Iso
For this reason and since their geometrical significance is more obvious, we will retain like variables intern for the integration of law CJS, the sizes Q, R and X.
Iso
In addition, we add to the number of the internal variables:

.
the sign of the product
dp
S
ij
ij
.
the elastic or elastoplastic state of material, while noting:
- 0: elastic state
- 1: elastoplastic state, isotropic plastic mechanism
- 2: elastoplastic state, plastic mechanism déviatoire
- 3: elastoplastic state, plastic mechanisms isotropic and déviatoire
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Finally, the variables intern are stored in a vector VI in the following order:

Internal index of variable
CJS1
CJS2
CJS3
3D 2D CJS1
CJS2
CJS3
1 1 Q =

Q
Q
Iso
Iso
Iso
2 2 R = R
R
R = R
m
m
3
3 0
0
X

11
4 4
0
0
X
22
5 5
0
0
X
33
6 6
0
0
2X
12
7

0 0 2X
13
8

0 0 2X
23
9 7 Q H ()
Q H ()
Q H ()
II
Q

II
Q

II
Q

R I + Q
R (I + Q
R I + Q
m (1
init)
1
init)
m (1
init)
10 8

R
X

II
R
lim
X
m
II
11 9

Q
3
Q
3

I + Q
I + Q
1
init
1
init
12 10
Numbers
iterations

Iteration count
Iteration count
interns
interns
interns
13
11
local test reached
local test reached
local test reached
14
12
no. of recutting no. of recutting no. of recutting
15 13
sign (
dp
S)
sign (
dp
S)
sign (
dp
S)
ij ij
ij ij
ij ij
16
14
0,1,2,3 state of
0,1,2,3 state of
0,1,2,3 state of
material
material
material

4.2
Integration of the nonlinear elastic mechanism

In the elastic case, the new state of stress +
, checks simply:
+ = - + D
+

ij
ij
ijkl (
) kl
The dependence of the nonlinear tensor of elasticity according to the state of stresses is summarized in
fact with:
I +
+
+ Q
D
= D
1

ijkl (
)
N
linear
init
ijkl

3P

has

where
linear
D
is the tensor of isotropic linear elasticity traditional, obtained from
E
K and G or by
ijkl
O
O
equivalence starting from E and Naked.
From this relation, one deduces in particular that the first invariant of the constraints satisfied:
+
N
+

-
E
I + Q
I - I - 3
1
K
init
tr =
1
1
O
() 0

3

Pa

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This nonlinear equation is solved by a method of the secant for CJS2 and CJS3, in differentiating the cases following the sign from tr (). With regard to the model CJS1, for which parameter N is null, the explicit resolution is immediate, since one has then
+

I = -
I + K E
3
tr

1
1
O
()
N
I + + Q
In the case general, the knowledge of +
I and thus of term 1
init
allows to define
1

P

3 A

the nonlinear operator of elasticity D (+
. Obtaining the new state of stress is then direct.
ijkl
)

4.3 Integration of the mechanisms elastic nonlinear and plastic isotropic

In this case, the new state of stress +
, checks:
+ = - + D
+

-

ij
ij
ijkl (

)(
IP
kl
kl)
Being given the simple form, plastic deformations of the isotropic plastic mechanism:
IP
1
I

= -
ij
ij
3
the nonlinear system to solve is composed of:
+
-
+
1
I

.
: the elastic law state: - - D

ij
ij
ijkl (
) +
kl
kl = 0
ij

3

.
LQ: the law of work hardening of the variable interns Q: +
Q - -
Q - I Q
G Iso

Q

Iso
Iso
(+iso) = 0
Iso
+
I + Q
.
FI: the equation of the isotropic surface of load:
1
-
init + +
Q = 0
3
Iso

Schematically, one thus seeks to solve the system R (Y) = 0, where the unknown factor Y is given by Y = (+
+
I
Q
,
,

and where R = (IT, LQ, FI. The resolution of R (Y) = 0 is done by the method
ij
)
ij
Iso
)
of Newton:
.
0
initialization and calculation of a solution of test Y
DR.
.
iterations of Newton: resolution of
(p
Y)
1
+
p
DY
= - R (p

Y)
DY
.
test of convergence: if convergence
p
Y = Y; if not
1
+
p
p
1
+
p
Y
= Y + DY
and p = p + 1

We detail these three stages below.

4.3.1 Initialization and solution of test

Y 0 = (
0
0
0
, Q, I

, following values:
ij
Iso
)
We take simply for
0
elas
=
: constraints given by the elastic prediction,
ij
ij
-
Q0 = Q: variable interns with T
Iso
Iso
0

I
= 0: plastic multiplier no one

Contrary to the other elastoplastic mechanisms, here a solution of test is not calculated.
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4.3.2 Iterations of Newton

DR.
The resolution of
(p
Y)
1
+
p
DY
= - R (p
Y) naturally requires the calculation of derived from,
DY
ij
LQ and FI compared to each component of Y. One a:

ij
ij

ij

I

Q

kl
Iso

DR.
LQ
LQ
LQ
=

I
DY

Q

kl
Iso

FI
FI
FI

Q
I

kl
Iso

with:
N

D
1
-
ij

+

ijmn
1
1
N
I
Q
I
linear
I
1
init
= -

+

= - *D*

+

ik
jl
mn
mn
ik
jl
ijmn
mn
mn
kl

3

3
P
3
P
3

kl
kl
has
has

LEij = 0
Qiso
LEij
1
= - D
I
ijmn
mn

3
LQ = 0
kl
N 1
-
Q
p
LQ
G Iso
nK Q
= 1 - I

= 1+ I
O
Iso

Q
Q
P
P
Iso
Iso
has
has

LQ
Iso
Q
= G
-

I

FI
1
= -
kl

3
kl
FI =1
Qiso
FI = 0
I

4.3.3 Test of convergence

1
+
p
DY
The iterations of Newton are continued as much as the relative error
remain higher than
1
+
p
0
Y
- Y
tolerance allowed by the user and defined by key word RESI_INTE_RELA. The standard used here is the vectorial standard: X = x2.
I
I
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4.4 Integration of the mechanisms elastic nonlinear and plastic
déviatoire

In this case, the new state of stress +
, checks:
+ = - + D
+

-

ij
ij
ijkl (
)(
dp
kl
kl)
The plastic deformations of the plastic mechanism déviatoire are given by the potential
D
G:
dp
D
D

= G
ij

ij
One deduces from it that the nonlinear system to solve is composed of:

· IT: the elastic law state: +
- -
- D

ij
ij
ijkl (+) (
- D Gd
kl
kl (+,
+
R, +
X) = 0
ij
· LR: the law of work hardening of the variable R: +
R - -
R - D
GR. (+
, +
R) = 0
· LX: the law of work hardening of variable X:
+
X -
-
X - D X
G

ij
ij
(+
, +
X) = 0
ij
ij
· FD: the equation of the surface of load déviatoire: +
Q H
R I
Q

II
(+q) + + (+ + init) = 0
1

As in the preceding paragraph one solves by the method of Newton the system R (Y) = 0, where the unknown factor Y is given by Y = (+
+
+
D
, R, X,

and where R = (IT, LR, LX, FD.
ij
ij
)
ij
ij
)

4.4.1 Initialization and solution of test

Starting from the state at the moment T (-
-
-
, R, X, we seek a solution of test which brings us closer
ij
ij)
the final solution. For that we solve the following equation:
D
F (-
+ -
D

G
R
G
X
G

ij
ijkl (
- D D
kl

kl),
- + D R, - + D X -
ij
ij
) = 0
with
-
D
= D
,
D -
G
= Gd
,
R
G
= G R (- -
, R), X
G
= X
G
and where
ij
ij (-
-
*, **X**)*
kl
kl (-
-
-
*, R, **X**)*
ijkl
(-

ijkl
)
the unknown factor is the plastic multiplier
D

, by only one iteration of Newton, i.e.
finally of we let us have:
D
F

D
D
F

=
D
= - D
0
F
D
D
=0 is still = -

D

D
F
D
=0
D
D =0
with:
D
F

= H () Q
H (Q)
R
I
II
+ Q
+ I + Q
+ R
1

D
Q
D
II
(
D

1
init)
D
D

Moreover,
I
one a:
-
-
I = I + K
3
then:
1
-
= -3K tr ***G***

D
(D)
1
1
(tr () - dtr (D
G)

R
one a:
-
D
R
R = R + G then:
R
= G
D

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:

-
-
1
D
D -
-
-
D
D -

-
D
X -
one a: Q = + D

G
I
3K tr
tr ***G***

X
G

ij
ij

ijkl (
-
kl
kl) - [
+
1
(( )- ( )] + +
ij
ij
ij

3

qij
-
1
D -
-
D -
-
then:
= - D G + 3K tr ***G***

3

D
ijkl
kl
() + X
ij
ij -
X -
Gij (-
I +
-
K tr
1
(
)

3
D
=0

Q
Q Q
Q Q
H (
H Q
Q)
(Q)
one a:
II
II
ij
ij
ij
=
=
and
ij
=

D
D
D

Q

Q

D
D

Q

ij
II
ij

Ultimately, we take for the solution of test: Y 0 = (0
0
0
D 0
, R, X,

, with the values
ij
ij
)
following:

D 0

: the value found according to the preceding formulation.
0
-
-
= + D
0

G

ij
ij
ijkl (
D
D -

-
kl
kl)
0
-
D 0
R
R = R + G
0
-
D 0
X -
X = X + G

ij
ij
ij

4.4.2 Iterations of Newton

DR. is given here by:

DY

ij
ij
ij
ij

D

R
X

kl
ij

LR
LR
LR
LR

D
DR.

R
X

=

kl
ij

DY
LX
LX
LX
LX
ij
ij
ij
ij

D

R
X

kl
ij

FD
FD
FD
FD

R
X
D

kl
ij

with:

-1
ij
N I + Q
G
linear
= - D

-
G
1
+ D

ik
jl
ijmn
(
D
D

mn
mn)
N
D
init
D
mn
kl
ijmn

P
3
P

3

kl
has
has

kl
LEij

D
G D
= D
mn

R
ijmn
R
D
LEij
G
D
mn
= D

ijmn
X

X
kl
kl
LEij
D
= D G
D
ijmn
mn

R

2
- ,
1 5
LR
G
With
R I + Q
D
D
1
init
= -

= -

1-

kl

2
R
P
3

kl
kl

m
has

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- ,
1 5
R
LR
G
With
R
I
Q
D
D 2

+

= 1-
= 1-
1-
(I + Q

init)
1
init
1

R
R
R
R
3

P
m
m

has

LR = 0
X kl
LR
R
= G
-

D

X
LX
G
ij
D
ij
= -

kl
kl
LX ij = 0
R
X
LX
G
ij

D
ij
= -

ik
jl
X
X
kl
kl
LX ij
X
= G
-

D
ij

D
FD F
=
= Q - Q X - R
kl
(mn mn
) kl

kl
kl FD = I 1
R
D
FD
F
=

X
X
kl
kl
FD = 0
D

D

G
Gd
D
G
X
G
X
G
D
F
In addition, the calculation of the terms
mn,
mn,
mn,
ij
,
ij
and
is detailed

R
X

X
X
kl
kl
kl
kl
kl
hereafter, as well as the calculation of useful intermediate terms:
D
F
.
calculation of
:
X kl
D
F
H (

Q)
Q
= Q

+ H

II
() II
Q
X
X
X
kl
kl
kl
H (

Q)
Q
Q
Q
mn
= Q
+ H

II
(Q) II mn
Q
X
Q X
mn
kl
mn
kl
H (

Q)
Q
Q
Q
mn
= Q
+ H

II
(Q) mn mn
Q

X
Q X
mn
kl
II
kl

H (

Q)
Q
Q
= Q
+ H

II
(Q) mn
mn

Q
Q X

mn
II
kl

H (

Q)
Q
= - I Q
+ H

1
II
() mn
Q
mk
nl

Q
Q

mn
II
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D
F
= - I

1

qkl
One will notice for the continuation that:
D
F
Dev.
= - I Q
1
kl

X

kl

D
F

.
ij

calculation of
:
kl
D
D
F
F

ij

ij qmn
=

Q

kl
mn
kl
(Q - Q X - R
ij
(rs rs
) ij) qmn
=

Q

mn

kl
Q

ij
Q
Q
rs
mn
=

-
X

rs
ij
Q

Q

mn

mn

kl
Q

ij
Q

rs
mn

=
-
X

X

rs ij

-
mk
nl
kl
+ mn

Q

Q
3
mn

mn

Q
.
calculation of
ij:
qmn
*Au préalable, one definite the tensor **T** and its part déviatoire D*
***T** while posing:*
*det (**Q**)*
*det **Q***
D
( )
T =
and T = Dev.

ij
Q
ij

Q
ij

ij

One has as follows:
T
Q Q
Q Q
11

-
22
33
23

23

T
Q Q
Q Q
22

-
11 33
13 13
T Q Q - Q Q
***T** = 33 = 11 22*
12 12
T
Q Q
Q Q
12

-
13
23
12
33
T Q Q - Q Q
13 12 23
13
22

T
Q Q
Q Q
23

-
12 13
23 11

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ij
Q
- 5

qij 54
(H Q)
=

qmn
H (

Q) 6 1 + cos (3 Q)
+
2 Dev. (tij)
(

2
Q
6q
II
II
qmn
Q
D
T

ij
ij

1

2
Q

II
1
Q
cos
ij
(3 Q)

1
54 qII
+

H (

Q) 5 1 + cos (3 Q)
+
+
2
Q
H
mn
(Q) 5 2q Q
H
II
mn
(Q) 5 6 qmn

- 5

qij 54

(H Q)
1

Q Q
im jn
ij mn
=

H (

Q) 6 1 + cos (3 Q)
+
2 Dev. (tij)
( )

+
1 + cos 3
-

Q
2
Q
6q
II
II
Q
H
mn
(Q) 5
( )
3

2

Q
Q
II
II

D
1

Q
54
ij

det Q

1

54 tij
Q

D
+
H (Q)
mn
5

T-3
4
mn
2 qmn +
- 2tij
2 qII
qII
H ()

5
2
2
6 Q
Q
II Q
Q
mn
II

D
T
The expression of
ij
clarify yourself as follows:
qmn
- 1(

1

1

Q + Q
(- Q + 2q
(- Q + 2q
11
22)
11
33)
22
33)

3

3

3

1(-
1
1
Q + 2q
- (Q + Q
(2q - Q
11
22)
11
33)
22
33)
D
3

D
3
D
3

T

= 1(
T
1
T
1
2q - Q
,
= (2q - Q
,
= - (Q + Q
,
11
22)
11
33)
22
33)
Q

Q
q33

22

11
3

3
3
0

0

- Q

12

0

- Q
0
13

- Q
0
0
23

2

2

4

Q
Q
- Q
12

13

23
3

3

3

2
4
2
Q
- Q
Q
13
23
D

12
3

D
3
D
3
T

= 4

T
T
,
= 2
,
= 2

Q
- Q
Q
Q
12 Q

13 Q

23
12
3

13
3
23
3
- Q

Q

Q

33

23

13

Q
- Q
Q
23

22

12

Q
Q
- Q
13

12

11

D
G
.
calculation of
mn:
kl
One a:
D
D
F
F

D

D

D
G
F
N

F N
mn
mn
mn
rs
rs
=
-
N
-

N +
N
rs
rs
mn

kl
kl

rs

kl

kl
rs
kl

S
*One definite tensor **N***
~ by
ij
n~ =
+
ij
ij
sII
n~
*i.e. that **N** is then given by*
ij
N =
with ~
2
N = + 3
ij
n~
II
II
In practice, for the calculation of, one uses
in the place of
dp

, i.e. one a:
ij
ij
S

II
=
-1 sign (S

C
ij
ij)
S

II

D
G
One has then for
mn:
kl
D
D
F F

1

D

D
~
~
D
~
D
G

F N N
F
N
1
F

n~ 2
mn
mn
rs
rs
mn
~
mn
~ ~
II
=
-
N N -
-
N
-
N
N

rs
mn
~ 2
rs
~ 2
rs
mn

N

N

kl
kl
kl
rs
kl
II
rs
kl
II

rs

kl

with:
1

1

S
S

2 2
-1

n~2

2

+ 3
1
2

S

S
II
(
)
( )
II
II
C
C
II
II
=
= -
= -

+ 2
2
3

+ 2
2
3

kl
kl
(
)
kl
(
)
kl

S
II
Cs

.
calculation of
II
:
kl

S
II
Cs

1

II
(S
S
S
II)
II
(cII)
=
-

C
C

S
2
kl
II
kl
S
kl
II
R I + Q

C (1
init)
-
1 (

S S
S

H

II)

(

mn
II
S)

=
-

C
C
S S

2

II
mn
kl
S
kl
II
1 S
1
S
R
I
R I
Q
H
mn

II
C
1
C (
+
1
init)
(S)
=
-

C
mk
nl

mn
kl -
C 2
-
S S
3
S
H
2

II
II

(S)
+
H
II

kl
(S)

kl

1 S
1
S
R
R I
Q
H S
mn

II
C
C (
+
1
init)
(S)

=
-

C
mk
nl
mn
kl -
rs
C 2
-
S S
3
S
H
H2
S
II
II

(S) +
kl
II

(S)

rs
kl

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:

n~
.
calculation of
mn:
kl
1
1

n~
1
1
C
S
S
S
mn =
-
sign (S
sign S S

C
ij
ij)
mn +
(
ij
ij)

II
II

mn
-

S
S

kl
II
II
kl

kl
kl

1
1
C
1
1
1
=
-
sign (S
S
S

sign S S
C
ij
ij)

-
mk
nl
mn
kl +
(

ij
ij)

(II)

II

mn
S
S
3

-
2
C
S
2
II
II

II
kl
S
kl
II

Gd
.
calculation of
mn:
R
D
D
F
F

D

D

D
G
F
N

F N
mn
mn
mn
rs
rs
=
-
N
-

N +
N
rs
rs
mn
R
R

R
R
R

rs

rs

D
D
F F

mn

rs
=
-
N N
rs
mn
R
R

= - N N
mn
(rs rs) mn
S
2
- 3 mn
mn
S
=

II
2
+ 3

D
G
.
calculation of
mn:
X kl
D
D
F
F

D

D

D
G
F
N

F N
mn
mn
mn
rs
rs
=
-
N
-
N +
N

rs

rs
mn
X
X

X
X
X
kl
kl

rs

kl
kl
rs
kl

D
D
F F

mn
rs
=
-

N N

rs mn
X
X
kl
kl

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D
F

.
calculation of
mn
:
X kl
D

F

mn
Q
Q
X
mn
rs
rs
=
-
X + Q

rs
rs

mn
X
X
X
X
kl
kl

kl
kl
Q Q
Q Q

mn
ij

rs
ij
=
-
X + Q

rs
rs
Kr
ls
mn
Q X
Q X

ij
kl

ij
kl

Q

Q

mn

rs
= - I
- - I
X + Q

1
ik
jl
1
ik
jl
rs
rs
Kr
ls
mn
Q

Q

ij

ij

X
G
.
calculation of
ij
:
kl
-
-
X
G
I
Q
I
Q

ij
1
= -
(Q +
I
Q
X
X
I
Q

ij
ij)
,
1 5
,
1 5
+

1

init
ij

+

1
1 +
+
ij (
+ init) 1

init
1

2b
3

P

B

3

P
kl

has

kl

kl
kl

has

-
-

1
I
Q
Q
= -
(Q +
Q
I
Q
X

I
Q
ij
ij)
,
1 5
,
1 5
+

1
init
ij
+

1
+
mn
kl
(+ init) 1

init
1
2b
3

P
B Q
3

P
has

mn
kl

has

- ,
1 5
1

+
H (
H
Q
I
Q
Q
Q
H
X I
Q
S)
O
(S)
+
+
II
O
II
O (
)

II
S
ij (
+ init) +

1

init
1

B

3

P
kl
kl
kl

has

H (S)
·
calculation of
:
kl
H (
H S
S)
(S) mn
=

S

kl
mn
kl
54

3
cos (

)
Q

1

=
T -
S

5
mn
3
5
mn
mk
nl
mn kl
H
6 (Q
2h
q2
3
S)
II
(S)

-

II

Q
.
calculation of
II:
kl
Q
Q Q Q

II
II
rs
mn
=

Q Q
kl

rs
mn
kl
Q Q
1
rs
rs

=

-

X

mk
nl
mn
+
kl
kl

Q Q
3
II
mn

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.
calculation of
O:
kl

O
1
cos
=

H (

kl
S)
R -
R

R
H (Q) cos
m

(-
S
) kl
Q
R
H
H
H
H

R
1
cos (-
S
S
S
S
S
Q)

-
R
-
+
R
-
-
R

H

H

H

kl
(Q) cos
m
(S Q) () ()
kl
(Q) cos
m

(S Q) () () m

2
kl
(Q)
kl

- cos

H (S)
2

R -
R
-
R
H (Q) cos
m
(S Q)

with:
cos
1

Q
2 I1
S
=
II
II

2q
- 2I X
- 2s
II
1
II
II

2s I X

kl
II 1
II
kl
kl
kl

q2 - I X
2
s2
1
I1
S
II
(
II)
-

-
II
II

S X
+ I X
II
II
1
II

S I X

II 1
II

kl
kl
1

2
2
2
S
=
(Q - I X - S
Q
I X
S 2 S X
I X

kl
1
II
kl
kl) - (
-
II
(1 II) - II)

kl

+
II
II
kl
1
II

S I X
S
II 1
II

II
R
μ
R = -

kl

I + Q
kl
1
init
cos (-

S
Q) = - sin (-

S
Q)
S
Q

-

kl

kl
kl

X
G
· calculation of
ij
:
X kl
- ,
1 5
X
G
Q
X
ij
1
ij
ij
=
+

(I +
I
Q
Q

init)
+

1

init
1

X
B X
X
3

P
kl

kl
kl

has

- ,
1 5
1 Q
ij
Q

=
I
Q
mn +
I
Q

ik
jl (

+ init) +

1

init
1

B Q X
3

P
mn
kl

has

- ,
1 5
1
Qij

I + Q
=
- I
+
I +

init
Q

1
mk
nl
ik
jl (
init)
1

1

B
Q
3

P
mn

has

4.4.3 Test of convergence

1
+
p
DY
The criterion of convergence remains
RESI_INTE_RELA.
1
+
p
0
Y
- Y
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4.5 Integration of the mechanisms elastic nonlinear, plastic isotropic and plastic déviatoire

In this case, the new state of stress +
, checks:
+ = - + D
+

-

-

ij
ij
ijkl (
)(
IP
dp
kl
kl
kl)
Taking into account what precedes, one deduces from it that the nonlinear system to solve is composed of:

.
: the elastic law state:
ij
+
- -
- D

ij
ij
ijkl (+)

1

+ I
- D Gd
kl
kl
kl (+
+
+

*, R, **X**) = 0*

3

.
LQ: the law of work hardening of the variable interns Q: +
Q - -
Q - I Q
G Iso

Q

Iso
Iso
(+iso) = 0
Iso
.
LR: the law of work hardening of the variable R: +
R - -
R - D
GR. (+
, +
R) = 0
.
LX: the law of work hardening of variable X: +
X -
-
X - D X
G

ij
ij
ij (+
, +

X) = 0
ij
ij
+
I + Q
.
FI: the equation of the isotropic surface of load:
1
-
init + +
Q = 0
3
Iso
.
FD: the equation of the surface of load déviatoire: +
Q H
R I
Q

II
(+q) + + (+ + init) = 0
1

As in the preceding paragraphs one solves by the method of Newton the system R (Y) = 0, where the unknown factor
Y is given by Y = (+ +
+
+
I
D
Q
,
, R, X,
,

and where
ij
Iso
ij
)
R = (IT, LQ, LR, LX, FI, FD.
ij
ij
)

4.5.1 Initialization and solution of test

Starting from the state at the moment T (-
-
-
-
Q
,
, R, X
, we seek a solution of test which us
ij
Iso
ij)
bring closer the final solution. For that we solve the system of equations according to:
I -
+
1
I
D
D
-
I
Q -
F + D

G
Q
G Iso
ij
ijkl
+
-
kl
kl
kl,
+
=

Iso

0

3

D -
+
1
I
D
D
-
D
R
-
D
X -
F
+ D

G
R
G
X
G
ij
ijkl
+
-
kl
kl
kl,
+
,
+
=

ij
ij

0

3

with:
-
D
= D
,
D -
G
= Gd
,
Q -
G Iso = Q
G Iso (-
Q

R
G
= G R (- -
, R),
Iso)
kl
kl (-
-
-
*, R, **X**)*
ijkl
(-

ijkl
)
X -
G
= X
G
and where the unknown factors are the plastic multipliers
I

and
D

, by one
ij
ij (-
-
*, **X**)*
only iteration of Newton, i.e. finally that we have:
I
I
F

F
I
+
D
= - I
F I =0, D =0
I
D

I
D

=0, =0
I
=0, D
=0

D
D
F

F
I
+
D
= - D
F I =0, D =0
I
D

I
D

=0, =0
I
=0, D
=0
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that is to say still:
I
D
F
F
D
I
F
F
D
I
F -
F
I
D
F -

F
D
D

I
I

I

=
and
D

=

I
D
I
D
F F
F F
I
D
I
D

-
F
F
F
F
-
I
D
D
I

I
D
D
I

with:
I
F = - (-
p
K + K
)
I

I
F
-
= K tr ***G***

D
(D)

D
F

= H (
Q
H
R
I

+ Q
+ I + Q

+ R
1

I
Q)
(
II
Q)
I
II
(
I
1
init)
I
I

D
F

= H (
Q
H
R
I

+ Q
+ I + Q
+ R
1

D
Q)
(

II
Q)
D
II
(
D
1
init)
D
D

D
F
It is known that
is calculated in the same way that previously when only the mechanism
D

D
F
plastic déviatoire was activated. In addition, one has, for the calculation of
and when I
= 0 and
I

D
= 0, following relations:
qij
1 -
E 1
-
= D - K
3

X

I
ijkl
kl

+
ij
ij

3
3

Q
Q Q
Q
ij
ij 1
1
II
II
-
E
-
=
=
D
K
3

X

I
I

-
ijkl
kl

+
ij
ij

Q
Q

3
3
ij
II

H (
H Q
Q)
(Q) ij
=

I
I

Q

ij
R = 0
I

I1
-
= 3K
I

Ultimately, we take for the solution of test: Y 0 = (0
0
0
0
i0
D 0
, Q, R, X,
,

, with
ij
Iso
ij
)
following values:
i0

: the value found according to the preceding formulation.
D 0

: the value found according to the preceding formulation.
1
0
-
-
i0
D
D -
= + D
0

G

ij
ij
ijkl
+
-
kl
kl
kl

3

0
-
i0
-
Q = Q +
Iso
Q
G

Iso
Iso
0
-
D 0
R

R = R + G
0
-
D 0
X -
X = X + G

ij
ij
ij
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4.5.2 Iterations of Newton

DR. is given here by:
DY

ij
ij
ij
ij

ij
ij

I
D

Q
R
X

kl
Iso
kl

LQ
LQ
LQ
LQ
LQ
LQ

I
D

Q
R
X

kl
Iso
kl

LR
LR
LR
LR
LR
LR
DR.

Q
R

X
I

D

=
kl
Iso
kl

DY
LX
LX
LX
LX
LX
LX

ij
ij
ij
ij
ij
ij

Q
R
X
I

D

kl
Iso
kl

FI
FI
FI
FI
FI
FI

I
D

Q
R
X

kl
Iso
kl

FD
FD
FD
FD
FD
FD

I
D

Q
R
X

kl
Iso
kl

where the new terms are null:
LQ

LX
=
LQ
LQ
LR
LR
0 ,
= 0,
= 0 ,

= 0 ,
= 0 ,
ij = 0,
R
X
D

Q
I

Q
kl
Iso
Iso
LX

ij =
FI
FI
FI
FD
FD
0 ,
= 0 ,
= 0,
= 0 ,
= 0 ,
= 0
I

R
X
D

Q
I

kl
Iso

and where the already definite terms remain unchanged, except for

ij which becomes:
kl
n-1
D
LEij

1
N I + Q
G
linear
I
D
D
1
init
D
mn
= - D

+

-
G
+ D

ik
jl
ijmn
mn
mn
mn
kl
ijmn

3
P
3
P

3

kl
has
has

kl

4.5.3 test of convergence

1
+
p
DY
The criterion of convergence remains
RESI_INTE_RELA
1
+
p
0
Y
- Y

4.6
Procedure of relieving based on an estimate of the normals with surface of load déviatoire

When the plastic mechanism déviatoire intervenes, a procedure of relieving inside iterations of Newton is taken into account. This one makes it possible to avoid certain problems of oscillation in
the calculation of the solution p 1
+
Y
who lead finally to nonthe convergence of integration numerical.

Thus, with the iteration p + 1, instead of bringing up to date the unknown factor p 1
+
Y
by a complete increment
p 1
+
Y

p 1
+

p
p 1
+
Y
= Y + Y

one poses
p 1
+
p
p 1
+
Y
= Y + Y

m
m
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and one seeks, by carrying out a loop on under-iterations m, to determine an optimal value scalar. This value is required by considering the rotation of the normal, in the plan
m
déviatoire, on surface
D
F, during under-iterations. This normal, noted ***n~m****, is expressed with*

to leave the constraints contained in the term
p 1
+
Y
by
m
F
6
~
5
*det (**Q***
***Nm** =*
)
2 H (Q)
D
qII
= (2 + cos (3q) qij +

qij
qII
Q
ij

Starting from the initial value = 0
.
1, the process set up consists of the following stages:
0

.
calculation of the normals ~
***Nm** 1*
*- and **N***
~m
N
~
N
~
:
.
calculation of the swing angle
m 1
-
m
=

m between these normals: cos
m

N
~
~
m
N
1
-
m
.
test on the evolution cos:
m
if cos TOLROT then
= DECREL and m = m +1
m
m+1
m
if not end of the under-iterations and
p 1
+
1
+
Y
= p
Y

m

4.7
Recutting of the step of time

As for other relations of behavior (CHABOCHE, VISCOCHAB, TAHERI, LMARC), it was introduced for model CJS the possibility of redécouper locally (at the points of Gauss) the step of time in order to facilitate numerical integration. This possibility is managed by the operand ITER_INTE_PAS of the key word CONVERGENCE of operator STAT_NON_LINE. If itepas, the value of
ITER_INTE_PAS, is worth 0, 1 or -1 it has no recutting there (note: 0 are the default value). If itepas is positive recutting is automatic, if it is negative recutting is not taken in count that in the event of nonconvergence with the step of initial time.

Recutting consists in realizing, after the phase of elastic prediction, the integration of

plastic mechanisms put in plays with an increment of deformation of which components correspond to the components of the initial increment of deformation divided by the absolute value of itepas.

4.8 Remarks
any other business

4.8.1 Calculation of the cos term (-
S
Q)
The cos term (- appears in the expression of. We adopted for his calculation
S
Q)
O
even method that that used with the ECL. I.e. we determine the angles and of
S
Q
the manner which follows:

1
1 cos2
-
(
3
1 cos
1
2
-
(3q)
S)

= Arctan

and = Arctan

S
3

(
cos
3
Q
3

(
cos
3 Q)
S)

then we take the cosine of the difference.
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These expressions of and are also useful for calculation of:
S
Q
cos (-

S
Q) = - sin (-

S
Q)
S
Q

-

kl

kl
kl

1
54
2

det (Q)

with
S = -
1 - cos 3
( )
T
3
Q

S
3
-
kl
2
kl

3
Q
Q
kl
II
II

4.8.2 Calculation of R

The ray of rupture introduced into model CJS3 is given by the formula
3p

R = R + μln

R

C
C

I + Q
1
init

I + Q
In fact, when 1
init > p, one must block R with the value of R. the field of dilatence
C
3
R
C
disappears and one does not admit that R can decrease in on this side R. Consequently, one introduces, with
R
C
place preceding formulation, the following expression

3 p
R = R + μ max,
0 ln

R

C
C

I + Q
1
init
4.8.3 Traction

Non-cohesive, the field of traction which corresponds to positive constraints is inadmissible for the grounds. From the point of view of the integration of model CJS, when the state of the constraints tends towards
the top of the cone of the surface of load, the numerical risk to rock in this prohibited field increase. However when that one projects oneself or when one makes a prediction in a point of it field, numerical calculation ends either in an erroneous result, or with a fatal error. Indeed, traction appears numerically by a positive value of I. This value poses then
1
- .
1 5
+
I + Q
problem at the time to evaluate certain quantities like
1

init
; in addition it
3

Pa

would generate from a theoretical point of view a negative value Q according to the equation of the surface of
II
charge déviatoire.

Such a phenomenon was detected on several levels: in a particular way in the elastic prediction with model CJS1, and in a general way in the local iterations of Newton utilizing it mechanism déviatoire. The same answer was brought in order to free itself from this pathology: it acts to virtually project the constraints in the elastic range on the hydrostatic axis in posing:
= = = 1
- kPa
11
22
33

= = = 0
12
13
23
One thus repositions the state of stresses in the field of compression while moving away little of inadmissible initial prediction considered, and by hoping that considerations of structures will allow total calculation to converge.

Moreover internal variables do not evolve/move and one supposes being returned in the elastic range

---

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5 Operator tangent

The tangent operator called by option RIGI_MECA_TANG corresponds to the tangent operator deduced from
problem of speed and calculated starting from the results known at the moment T.

The tangent operator called by option FULL_MECA should correspond to the tangent operator with discretized problem in an implicit way. Actually, we did not carry out this calculation. We take
then, when option FULL_MECA is retained, the tangent operator deduced from the problem of speed and
calculated starting from the results known at the moment t+dt.

We detail below the tangent operator deduced from the problem of it speed according to or of mechanisms brought into play.

5.1 Tangent operator of the nonlinear elastic mechanism

We have simply the following nonlinear elastic relation:
N
I + Q
& = D
& = 1

D

ij
ijkl ()
init
linear
kl
ijkl

&kl
P
3 A

from where immediately the tangent operator:
N

.
+

elas nl
I
Q
1
init
linear
H
=
D

ijkl

ijkl
P
3 A

5.2
Tangent operator of the mechanisms isotropic rubber band and plastic

In this case, we have the following relation:

1
& = D &
&
D
&
&

ij
ijkl () (

- IP
kl
kl) =
ijkl ()
I

+
kl
kl

3

it comes: I& = K
3

1
(
I
& + &
v
)
By taking account of this relation and the law of work hardening of Q, condition I
f& = 0 becomes:
Iso
I&
I
1
f& = -
+ Q& = - K & &
K &

Iso
(+ I
v
) - p I = 0
3
K
that is to say: I
& = -
&
p
v
K + K

By deferring this result in the expression of &, one finds:
ij

1
K

1
K

& = D & -
& = D -
D

ij
ijkl
kl
&
p
mm
kl
ijkl
p
ijmn
mn
kl
kl

3 K + K

3 K + K

from where the tangent operator:
1
K
IP
H
= D -
D

ijkl
ijkl
p
ijmn

mn
kl
3 K + K
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One can also write in matric form:
- + 2μ
-
-
0
0
0

-
- + 2μ
-
0
0
0
I N -
-
- + 2μ
0
0

0
IP
H =
1

3Pa
0
0
0
2μ
0
0

0
0
0
0
2μ
0

0
0
0
0
0
2μ
E
K 2
where for this formula only and μ are the coefficient of Lamé and
O
=
.
E
p
K + K
O
O

5.3
Tangent of the mechanisms rubber band and plastic operator déviatoire

The condition
D
f& = 0 is written:
D
D
D
D
D
D
F
F
F
F
F
F
D
f& =
& +
R& +
X& =
& +
D
R
& G +
D
X
& G = 0
ij
ij
ij
ij

R
X

R
X
ij
ij
ij
ij

D
F
The tensor
X
G being purely déviatoire, the product
X
G is reduced to:
ij
X ij
D
D
F
F
X
X
X
G = Dev.
G

= - I Q G
ij
ij
1
ij
ij
X
X
ij

ij
The plastic multiplier can thus be put in the form:
D
F
D
&
1
=
&
Dev.
ij
H
ij
while revealing the plastic module

Dev.
H
, given by:
- ,
1 5

2
Dev.
2
I + Q

R
1
H
=
1
init
I
To 1
+ Q Q
X

1
ij (
+
ij
ij)

P
3
R
B
has

m

The relation stress-strains then makes it possible to write:
D
D
F
F
F
F
& =
D
& - & G =
D & - &
D G
ij
ijkl (
D
D
kl
kl)
D
D
D
D
ijkl
kl
ijkl
kl

ij
ij
ij
ij
what gives finally for the plastic multiplier:
D
F D
ijkl &kl

D
ij
& =

D

F
Dev.
D
H
+
D G
ijkl
kl
ij
By deferring this result in the expression of &, one finds:
ij

D
F

D

&

pqmn
mn
pq

& = D &
G

ij
ijkl
-
D
kl
D
kl

F
Dev.
H
+
D
D G

rstu

you

rs

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from where the tangent operator:
D
F Dpqkl

dp
D
pq
H
= D - D G

ijkl
ijkl
ijmn
mn
D
F
Dev.
D
H

+
D G
rstu
you
rs
The tangent operator thus obtained is not symmetrical. However for the moment law CJS rests on finite elements which claim a symmetrical operator. Ultimately, we retain not
dp
H but
ijkl
~ dp
H which is given by:
ijkl
~
dp
dp
H
+ H
dp
ijkl
klij
H
=
with ij and kl taken in (,
11
,
13
,
12
,
33
,
22
23)
ijkl
2

5.4 Tangent operator of the mechanisms rubber band, plastics isotropic and déviatoire

One must satisfy the two following conditions: I
f& = 0 and D

f& = 0. Taking into account the relation stress-strains which is written:

1 I
D
D
& = D &
&
& G
ij
ijkl
+
-
kl
kl
kl

3

the first condition gives:
I
f& = - K (& + I
& - D D
& G
K &

v
v) -
p
I = 0
where one posed
D
D
G = G = tr G.
v
kk
(D)
The second condition led to:
D
F
1
D
D

D & +
F
F
I
&
D - D
D
&
D G -
Dev.
D
H
& = 0
ijkl
kl

3
ijkl
kl
ijkl
kl

ij
ij
ij
Thus, plastic multipliers I
& and D
& are obtained by solving the system:
- (K + p
K) I
& +
D
D
KG & = K&

v
v

D
1 F

D

F

D
F

-
I
D & +
D
D G +
Dev.
H
D
& =
D
ijkl
kl

ijkl
kl

ijkl &kl
3

ij

ij

ij
that is to say:
D
D
F
F

D
D
Dev.
KG
D & -
D
G + H
K
v

ijkl
kl
mnpq
pq
&v

I
ij

mn

& =

(
F
1
F
p
K + K)
D
D
D
Dev.
D
D G + H
- KG
D

rstu
you
v
wvxy
xy

3

rs

VW
(
F
1
F
p
K + K)
D
D
D & - K
D

ijkl
kl
mnpq
pq &v

3
D
ij
mn
& =

(
F
1
F
p
K + K)
D
D
D
Dev.
D
D G + H
- KG
D

rstu
you
v
vwxy
xy

3

rs

VW
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These expressions are still written:
I
& = T and D
& = T
1kl &kl
2 kl &kl
where the tensors T and T are given by:
1
2
D
D
F
F

D
D

Dev.
KG
D
-
D
G + H
K
v
ijkl
mnpq
pq
kl

ij

mn

T
=

1kl
(
F
1
F
p
K + K)
D
D
D
Dev.
D
D G + H
- KG
D

rstu
you
v
vwxy
xy

3

rs

VW
(
F
1
F
p
K + K)
D
D
D
- K
D

ijkl
mnpq
pq
kl

3
ij
mn
T
=

2 kl
(
F
1
F
p
K + K)
D
D
D
Dev.
D
D G + H

- KG
D

rstu
you
v
vwxy
xy

3

rs

VW
By deferring expressions I
& and D
& of in the formula of &, one finds:
ij

1
D
& = D &
T

1
&
T

2
& G
ij
ijkl
+
-
kl
Nm
Nm
kl
pq
pq
kl

3

from where the tangent operator:
idp
1
D
H
= D + D T - D G T
ijkl
ijkl
ijmn
mn 1kl
ijpq
pq
2 kl
3
~
This tangent operator not being symmetrical, we retain not
idp
H
but
idp
H
who is given
ijkl
ijkl
by:
~
idp
idp
H
+ H
idp
ijkl
klij
H
=
with ij and kl taken in (,
11
,
13
,
12

,
33
,
22
23)
ijkl
2
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6 Sources
Aster

6.1
List modified and added routines

Only the routine nmcomp.f was modified. It makes it possible to call, when behavior CJS is chosen, the routine nmcjs.f, starting point of the integration of the law.

The whole of routines FORTRAN developed within the framework of the integration of law CJS in Code_Aster is as follows:

cjsc3q.f,
cjsci1.f,
cjsdtd.f,
cjsela.f,
cjside.f,

cjsiid.f,
cjsjde.f,
cjsjid.f,
cjsjis.f,
cjsmat.f,
cjsmde.f,
cjsmid.f,
cjsmis.f,
cjsnor.f,
cjspla.f,
cjsqco.f,
cjsqij.f,
cjssmd.f,
cjssmi.f,
cjst.f, cjstde.f,
cjstel.f,
cjstid.f,
cjstis.f,
lcdete.f,
nmcjs.f,
cjsinp.f,
cjsncn.f,
cjsncv.f,
cjsnvi.f,
cjsqq.f.
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6.2
Flow chart general of the principal routines

Principal routines FORTRAN for the integration of law CJS are connected in the following way:

nmcomp.f
nmcjs.f
cjsmat.f
cjsela.f
cjssmi.f
cjssmd.f
cjspla.f
cjsmis.f
cjsmde.f
cjsmid.f
cjstel.f
cjstis.f
cjstde.f
cjstid.f

6.3
Details of the functionalities of developed routines FORTRAN

6.3.1 Routine
:
CJSC3Q

Objective: calculation of cos (3
Q)

Variables of entry and exit:
IN
SIG: CONSTRAINTS

X: VARIABLES HAMMER-HARDENED MOVIES

Pa: ATMOSPHERIC PRESS (DATA MATERIAL)

OUT
*Q: DEV. (SIG) - TRACE (SIG) *X*

*QII: SQRT (QIJ*QIJ)*

*COS3TQ: SQRT (54) *DET (Q)/(QII ** 3)*

6.3.2 Routine
:
CJSCI1

Objective:
+ N
E
I
resolution of the equation +
I - -
I - 3
1
K
tr = by the method of the secant,
1
1
O
() 0

3
Pa
for the nonlinear elastic behavior

Variables of entry and exit:
IN
CRIT: CRITERIA OF CONVERGENCE

MATER: COEFFICIENTS MATERIAL A T+DT

LIFO: INCREMENT OF DEFORMATION

SIGD: CONSTRAINT A T
OUT
I1: TRACE SIG A T+DT

LEAFLET: LOGICAL VARIABLE INDICATING TRACTION

6.3.3 Routine
:
CJSDTD

Objective:
calculation of derived from the tensor D
T compared to Q

Variables of entry and exit:
IN
MOD: MODELING

Q: TENSOR (6 COMPONENTS)
OUT
DTDDQ: TENSOR RESULT (6 COMPONENTS)

6.3.4 Routine
:
CJSELA

Objective:
nonlinear elastic design of the constraints

Variables of entry and exit:
IN
MOD: MODELING

CRIT: CRITERIA OF CONVERGENCE

MATERF: COEFFICIENTS MATERIAL A T+DT

SIGD: CONSTRAINT A T

LIFO: INCREMENT OF DEFORMATION
OUT
SIGF: CONSTRAINT A T+DT
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Organization of CJSELA
·
calculation of the first invariant of the I1 constraints to t+dt:
- call of CJSCI1

.
calculation of the coefficients of the elastic matrix and assembly of the matrix
.
calculation of the increment of the constraints and the constraints with t+dt:
- call of LCPRMV and LCSOVE

6.3.5 Routine
:
CJSIDE

Objective:
for the integration of the plastic mechanism déviatoire, calculation of a solution of test so to start the local iterations of Newton then.

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL A T+DT

EPSD: DEFORMATION A T+DT

LIFO: INCREMENT OF DEFORMATION

YD: VARIABLES A T = (SIGD, VIND, LAMB)
VAR
GD: TENSOR OF THE LAW D FLOW PLASTIC DEV.
OUT
DY: SOLUTION D TEST

Organization of CJSIDE
.
calculation of the elastic operator,
.
calculation of laws of work hardening
R
G and X
G,
.

calculation of the law of flow of the plastic mechanism déviatoire
D
G,
D
F
.
calculation of the threshold
D
F, of its derivative
and of the plastic multiplier
D

,
D

.
calculation of the solution of test

6.3.6 Routine
:
CJSIID

Objective:
for the simultaneous integration of the plastic mechanisms isotropic and déviatoire, calculation of one
solution of test in order to start the local iterations of Newton then.

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL A T+DT

EPSD: DEFORMATION A T+DT

LIFO: INCREMENT OF DEFORMATION

YD: VARIABLES A T = (SIGD, VIND, LAMB)
VAR
GD: TENSOR OF THE LAW D FLOW PLASTIC DEV.

OUT
DY: SOLUTION D TEST
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Organization of CJSIID
·
calculation of the elastic operator,
·
calculation of laws of work hardening
R
G and X
G,
·
calculation of the law of flow of the plastic mechanism déviatoire
D
G,
I
F
I
F
D
F
D
F
·

calculation of thresholds I
F and D
F, of their derivative
,
,
and
, and of
I

D

I

D

plastic multipliers
I

and
D

,
.
calculation of the solution of test

6.3.7 Routine
:
CJSJDE

Objective:
DR.
calculation of DRDY and R for the resolution of
(p
Y)
1
+
p
DY
= - R (p
Y) (mechanism

DY
plastic déviatoire)

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL A T+DT

EPSD: DEFORMATION A T

LIFO: INCREMENT OF DEFORMATION

YD: VARIABLES A T = (SIGD, VIND, LAMBDD)

YF: VARIABLES A T+DT = (SIGF, VINF, LAMBDF)
VAR
GD: TENSOR OF THE LAW D FLOW PLASTIC DEV.
OUT
R: SECOND MEMBER

SIGN: SIGN S: DEPSDP

DRDY: JACOBIEN

Organization of CJSJDE
·
calculation of the elastic operator,
·
calculation of laws of work hardening
R
G and X
G,
·
calculation of the law of flow of the plastic mechanism déviatoire
D
G,

.
calculation of multiple derivative intermediaries
R
G
G R
X
G
X
G
D
G
Gd
D
G
.
calculation of the terms
,
,
mn,
mn,
mn,
mn,
mn

R

X

R
X
ij
ij
ij
ij
ij
.
calculation of the components of DRDY and R
.
assembly of DRDY and R

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6.3.8 Routine
:
CJSJID

Objective:
DR.
calculation of DRDY and R for the resolution of
(p
Y)
1
+
p
DY
= - R (p
Y) (mechanisms
DY
plastics isotropic and déviatoire)

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL A T+DT

EPSD: DEFORMATION A T

LIFO: INCREMENT OF DEFORMATION

YD: VARIABLES A T = (SIGD, VIND, LAMBDD)

YF: VARIABLES A T+DT = (SIGF, VINF, LAMBDF)
VAR
GD: TENSOR OF THE LAW D FLOW PLASTIC DEV.
OUT
R: SECOND MEMBER

SIGN: SIGN S: DEPSDP

DRDY: JACOBIEN

Organization of CJSJID
.
calculation of the elastic operator,
.
calculation of laws of work hardening
Iso
Q
G
,
R
G and X
G,
.
calculation of the law of flow of the plastic mechanism déviatoire
D
G,
.
calculation of multiple derivative intermediaries
Q
G Iso
R
G
G R
X
G
X

G
D
G
Gd
D
G
.
calculation of the terms
,
,
,
mn,
mn,
mn,
mn,
mn
Q

R

X

R
X
Iso
ij
ij
ij
ij
ij
.
calculation of the components of DRDY and R
.
assembly of DRDY and R

6.3.9 Routine
:
CJSJIS

Objective:
DR.
calculation of DRDY and R for the resolution of
(p

Y)
1
+
p
DY
= - R (p
Y) (mechanism
DY
isotropic plastic)

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL A T+DT

LIFO: INCREMENT OF DEFORMATION

YD: VARIABLES A T = (SIGD, VIND, LAMBDD)

YF: VARIABLES A T+DT = (SIGF, VINF, LAMBDF)
OUT
R: SECOND MEMBER

DRDY: JACOBIEN

Organization of CJSJIS
·
calculation of the elastic operator,
·
calculation of the components of DRDY and R
·
assembly of DRDY and R
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6.3.10 Routine: CJSMAT

Objective:
recovery of data materials, the component count of the fields, the number of internal variables and of selected level CJS.

Variables of entry and exit:
IN
IMAT: ADDRESS MATERIAL CODES

MOD: TYPE OF MODELING

TEMPF: TEMPERATURE A T+DT
OUT
MATERF: COEFFICIENTS MATERIAL A T+DT

NDT: NB TOTAL OF COMPONENTS TENSORS

NDI: NB DIRECT COMPONENTS TENSORS

NVI: NB INTERNAL VARIABLES

NIVCJS: LEVEL 1, 2 OR 3 OF LAW CJS

Organization of CJSMAT

.
recovery of the component count of the fields and the number of variables intern in function of modeling chosen,
.
recovery of data materials,
.
recognition of level CJS chosen according to the parameters given.

6.3.11 Routine: CJSMDE

Objective:
elastoplastic calculation of the constraints with the plastic mechanism deviatoire activated: resolution by the method of Newton of R (Y) = 0

Variables of entry and exit:
IN
MOD: MODELING

CRIT: CRITERIA OF CONVERGENCE

MATER: COEFFICIENTS MATERIAL A T+DT

NVI: NB INTERNAL VARIABLES

EPSD: DEFORMATIONS A T

LIFO: INCREMENT OF DEFORMATION

SIGD: CONSTRAINT A T

VIND: INTERNAL VARIABLES A T

STOPNC: STOP IN THE EVENT OF NOT CONVERGENCE
VAR
SIGF: CONSTRAINT A T+DT

VINF: INTERNAL VARIABLES A T+DT

NOCONV: NO CONVERGENCE

Organization of CJSMDE
.
initialization of YD by the state with T
.
calculation of a solution of test with CJSIDE
.
buckle on the iterations of Newton
- incrementing YF = YD + DY
- calculation of DRDY and R: CJSJDE
- resolution of the system by the method of Gauss: MTGAUS
- actualization of the solution DY
- test of convergence
.
update of the constraints and internal variables
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6.3.12 Routine: CJSMID

Objective:
elastoplastic calculation of the constraints with the plastic mechanisms isotropic and deviatoire activated: resolution by the method of Newton of R (Y) = 0

Variables of entry and exit:
IN
MOD: MODELING

CRIT: CRITERIA OF CONVERGENCE

MATER: COEFFICIENTS MATERIAL A T+DT

NVI: NB INTERNAL VARIABLES

EPSD: DEFORMATIONS A T

LIFO: INCREMENT OF DEFORMATION

SIGD: CONSTRAINT A T

VIND: INTERNAL VARIABLES A T

STOPNC: STOP IN THE EVENT OF NOT CONVERGENCE
VAR
SIGF: CONSTRAINT A T+DT

VINF: INTERNAL VARIABLES A T+DT

NOCONV: NO CONVERGENCE

Organization of CJSMID
.
initialization of YD by the state with T
.
calculation of a solution of test with CJSIID
.
buckle on the iterations of Newton
- incrementing YF = YD + DY
- calculation of DRDY and R: CJSJID
- resolution of the system by the method of Gauss: MTGAUS

- actualization of the solution DY
- test of convergence
.
update of the constraints and internal variables

6.3.13 Routine: CJSMIS

Objective:
elastoplastic calculation of the constraints with the activated isotropic plastic mechanism: resolution by the method of Newton of R (Y) = 0

Variables of entry and exit:
IN
MOD: MODELING

CRIT: CRITERIA OF CONVERGENCE

MATER: COEFFICIENTS MATERIAL A T+DT

LIFO: INCREMENT OF DEFORMATION

SIGD: CONSTRAINT A T

VIND: INTERNAL VARIABLES A T

STOPNC: STOP IN THE EVENT OF NOT CONVERGENCE
VAR
SIGF: CONSTRAINT A T+DT

VINF: INTERNAL VARIABLES A T+DT

NOCONV: NO CONVERGENCE

Organization of CJSMIS
.
initialization of YD by the elastic prediction
.
buckle on the iterations of Newton

- incrementing YF = YD + DY
- calculation of DRDY and R: CJSJIS
- resolution of the system by the method of Gauss: MTGAUS
- actualization of the solution DY
- test of convergence
.
update of the constraints and internal variables
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6.3.14 Routine: CJSNOR

Objective:
D
F
calculation of a vector parallel with

qij
Variables of entry and exit:
IN
MATER: MATERIAL

SIG:
CONSTRAINTS

X:
VARIABLES INTERN KINEMATICS
OUT
NOR:
ESTIMATE OF THE DITRECTION OF THE NORMAL

ON SURFACE DEVIATOIRE IN PLAN DEVIATOIRE
PERPENDICULAR
With

TRISECTING

THE NOR VECTOR (1: NDT) NR IS NOT STANDARD
SA
NORMALIZES
EAST
NOR (NDT+1)

6.3.15 Routine: CJSPLA

Objective:
elastoplastic calculation of the constraints.

Variables of entry and exit:
IN
MOD: MODELING

CRIT: CRITERIA OF CONVERGENCE

MATER: COEFFICIENTS MATERIAL A T+DT

SEUILI: FUNCTION OF ISO LOAD. CALCULEE WITH PREDICT ELAS

SEUILD: FUNCTION OF LOAD DEV. CALCULEE WITH PREDICT ELAS

NVI: A NUMBER OF INTERNAL VARIABLES

EPSD: DEFORMATIONS A T

LIFO: INCREMENT OF DEFORMATION

SIGD: CONSTRAINT A T

VIND: INTERNAL VARIABLES A T
VAR
SIGF: CONSTRAINT A T+DT (IN - > ELAS, OUT - > PLASTI)
OUT
VINF: INTERNAL VARIABLES A T+DT

MECANI: MECHANISM (S) ACTIVATES (S)

Organization of CJSPLA
.
assumption on the plastic mechanisms activated according to the values of thresholds I
F
and
D
F calculated starting from the elastic prediction,
.
treatment of the possible recutting of the step of time
.
safeguard elastic prediction,
.
elastoplastic calculation,
- isotropic plastic mechanism: CJSMIS
- plastic mechanism déviatoire: CJSMDE
- plastic mechanisms isotropic and déviatoire simultaneously: CJSMID
.
calculation of the thresholds starting from the constraints with t+dt
- call of CJSSMI and CJSSMD
- if (assumption of an isotropic mechanism and D
F positive) or (assumption of a mechanism
déviatoire and I
F positive): return to elastoplastic calculation with plastic mechanisms

isotropic and déviatoire simultaneously,
- if not end of routine
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Titrate:
Law CJS in géomechanics

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6.3.16 Routine: CJSQCO

Objective:
utility routine of CJS allowing the calculation of standard sizes listed below

Variables of entry and exit:
IN
GAMMA: PARAMETER MATERIAL

SIG: CONSTRAINTS

X: VARIABLES HAMMER-HARDENED MOVIES

PREF: PRESS REF. FOR STANDARDIZATION

EPSSIG: DEVIATIVE EPSILON FOR NULLITY

I1: TRACE TENSOR OF THE CONSTRAINTS

OUT
S: DEV. (SIG)

SII: SQRT (S: S)

SIIREL: SII/PREF

COS3TS: LODE (SIG)

HTS: FUNCTION H (TETHA_S)

DETS: DETERMINANT OF S

Q: Q (SIG-X)

QII: SQRT (Q: Q)

QIIREL: QII/PREF
COS3TQ

HTQ: FUNCTION H (TETHA_Q)

DETQ: DETERMINANT OF Q

6.3.17 Routine: CJSQIJ

Objective:
calculation of the tensor Q
ij

Variables of entry and exit:
IN
NR: DIMENSION OF S, X, Q

S: DIVERTER

I1: FIRST INV.

X: CENTER SURFACE OF LOAD DEVIATOIRE
OUT
Q: TENSOR RESULT

6.3.18 Routine: CJSSMD

Objective:
calculation of the threshold of the plastic mechanism déviatoire.

Variables of entry and exit:
IN
SIG: CONSTRAINT

WINE: INTERNAL VARIABLES
OUT
SEUILD: THRESHOLD ELASTICITY OF MECHANISM DEVIATOIRE

6.3.19 Routine: CJSSMI

Objective:
calculation of the threshold of the isotropic plastic mechanism.

Variables of entry and exit:
IN
SIG: CONSTRAINT

WINE: INTERNAL VARIABLES
OUT
SEUILI: THRESHOLD ELASTICITY OF THE ISOTROPIC MECHANISM

Titrate:
Law CJS in géomechanics

Date:
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Author (S):
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6.3.20 Routine: CJST

Objective:
det S
calculation of T =
.
S

Variables of entry and exit:
IN
S: STAMP
OUT
T: T (IN VECTORIAL FORM WITH RAC2)

6.3.21 Routine: CJSTDE

Objective:
calculation of the tangent matrix for the plastic mechanism déviatoire

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL

NVI: NB INTERNAL VARIABLES

EPS: DEFORMATIONS

SIG: CONSTRAINTS

WINE: INTERNAL VARIABLES
OUT
DSDESY: STAMP TANGENT SYMETRISEE

Organization of CJSTDE
.
calculation of the elastic operator,
.
calculation of laws of work hardening
R
G and X
G,
.
calculation of the law of flow of the plastic mechanism déviatoire
D
G,
.
calculation of intermediate terms
.
calculation of the tangent matrix
.
symmetrization of the tangent matrix

6.3.22 Routine: CJSTEL

Objective:
calculation of the tangent matrix for the elastic mechanism

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL

SIG: CONSTRAINTS
OUT
HOOK: ELASTIC OPERATOR RIGIDITY

Organization of CJSTEL
.
calculation of the elastic operator

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6.3.23 Routine: CJSTID

Objective:
calculation of the tangent matrix for the plastic mechanisms isotropic and déviatoire

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL

NVI: NB INTERNAL VARIABLES

EPS: DEFORMATIONS

SIG: CONSTRAINTS

WINE: INTERNAL VARIABLES
OUT
DSDESY: STAMP TANGENT SYMETRISEE

Organization of CJSTEL
·
calculation of the elastic operator,
·
calculation of laws of work hardening
R
G and X
G,
·
calculation of the law of flow of the plastic mechanism déviatoire
D
G,
·
calculation of intermediate terms
·
calculation of the tangent matrix
·
symmetrization of the tangent matrix

6.3.24 Routine: CJSTIS

Objective:
calculation of the tangent matrix for the isotropic plastic mechanism

Variables of entry and exit:
IN
MOD: MODELING

MATER: COEFFICIENTS MATERIAL

SIG: CONSTRAINTS

WINE: INTERNAL VARIABLES
OUT
DSDE: STAMP TANGENT

Organization of CJSTEL
.
calculation of the tangent matrix

6.3.25 Routine: LCDETE

Objective:
calculation of a determining matrix 3×3

Variables of entry and exit:
IN
A: STAMPS
OUT
LCDETE: DETERMINANT
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6.3.26 Routine: NMCJS

Objective:
realization of the integration of law CJS: calculation of the constraints with t+dt and/or the matrix tangent, according to the selected option of calculation.

Variables of entry and exit:
IN

STANDARD TYPMOD OF MODELING

IMAT ADDRESSES MATERIAL CODES

COMP BEHAVIOR OF L ELEMENT

CRIT LOCAL CRITERIA

URGENT INSTAM T

URGENT INSTAP T+DT

TEMPM TEMPERATURE A T

TEMPF TEMPERATURE A T+DT

TREF TEMPERATURE OF REFERENCE

EPSD TOTAL DEFLECTION A T

LIFO INCREMENT OF TOTAL DEFLECTION

FORCED SIGD A T

VARIABLE VIND INTERN A T + INDICATING STATE T

OPT OPTION OF CALCULATION TO BE MADE
OUT
FORCED SIGF A T+DT

VARIABLE VINF INTERN A T+DT + INDICATING STATE T+DT

DSDE STAMPS TANGENT BEHAVIOR A T+DT OR T

Organization of NMCJS
.
recovery of data materials, the component count of the fields, the number of internal variables and of selected level CJS:
- call of CJSMAT
.
blocking of variables intern according to selected level CJS
.
calculation of the constraints with t+dt
- elastic prediction: CJSELA
- isotropic calculation of the thresholds of the mechanisms and déviatoire: CJSSMI and CJSSMD
- if one of the thresholds is exceeded, elastoplastic calculation: CJSPLA
.
calculation of the tangent matrix according to the mechanism brought into play
- rubber band: CJSTEL
- isotropic plastic: CJSTIS
- plastic déviatoire: CJSTDE
- isotropic plastic and déviatoire: CJSTID

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Law of behavior CAM-CLAY

Summary:

The Camwood-Clay model one of the elastoplastic models known and the most are the most used in mechanics of
grounds. It is especially adapted to argillaceous materials. There are several types of models Camwood-Clay, that
presented here is most current and is called modified Camwood-Clay. This model is characterized by surfaces of

load hammer-hardenable in the shape of ellipses in the diagram of the first two invariants of the constraints. With
the interior of these surfaces of reversibility, the material is elastic nonlinear. There exists moreover, in a point of
each ellipse, a critical state characterized by a null variation of volume. The whole of these points constitute a line separating the zones from dilatancy and contractance from material as well as the zones
of negative and positive work hardening. Work hardening is governed by only one scalar variable and the rule of flow
normal is adopted.

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1 Notations

indicate the tensor of the effective constraints in small disturbances defined as being
difference between the total constraints and the pressure of water in the case of water-logged soils, noted under
the shape of the following vector:
11

22

33

212

2 23

2 31

One notes:

1

P = - tr ()
3
constraint of containment

S = + pi
diverter of the constraints

1

I = tr

second invariant of the constraints
2
(s.s)
2

Q = = 3I
equivalent constraint
eq
2

= 1

(U T
+ U)
total deflection
2

= + +

partition of the deformations (elastic, plastic, thermal)
E
p
HT

= - tr + -

v
() 3 (T T0) voluminal total deflection

p
= - tr
voluminal plastic deformation
V
(p)

= 1
~
+ I
diverter of the deformations
v
3

~

E
~ ~ p
= -
diverter of the elastic strain

p
p
1
~
p
= + I
deviatoric plastic deformation
v
3

E

2
=
tr ~ ~
.

equivalent elastic strain
eq
(E E)
3

p

2
=
tr ~ ~
.

equivalent plastic deformation
eq
(p p)
3

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E index of the vacuums of the material (report/ratio of the volume of the pores on the volume of the solid matter constituents)

E initial index of the vacuums
0
porosity (report/ratio of the volume of the pores on total volume)

coefficient of swelling (elastic slope in a hydrostatic test of compression)

M slope critical line of state

1
(+ E)
0
K =

0

P variable interns model, critical pressure equal to half of the pressure of consolidation
Cr
P

IDIOT

coefficient of compressibility (plastic slope in a hydrostatic test of compression)

1
(+)

0
=
E
K

(-)
μ elastic coefficient of shearing (coefficient of Lamé)

F surfaces of load

plastic multiplier

D
I tensor unit of order 2 whose term running is
ij
D
I tensor unit of order 4 whose term running is
4
ijkl
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2 Introduction

The model describes here is the model known as of modified Camwood-Clay. The initial Camwood-Clay model was
developed by the school of soil mechanics of Cambridge in the Sixties. It predicted

too important deviatoric deformations under weak loading deviatoric, and was modified by Burland and Roscoe in 1968 [bib1].

2.1 Phenomenology of the behavior of the grounds

The materials poroplastic such as certain clays are characterized by the behaviors following:

· the strong porosity of these materials causes unrecoverable deformations under loading hydrostatic corresponding to an important reduction of porosity. This mechanism purely contractor is sometimes called "collapse",
· under loading deviatoric, these materials show a contracting phase followed of one phase where the material becomes deformed with constant plastic volume or dilates.

For the two types of loading, the energy blocked in material evolves/moves according to the number of contact between the grains. For a hydrostatic loading, the number of contact increases, thus that blocked energy, one thus has positive work hardening. For a loading deviatoric, the material can become deformed without variation of volume to a number of intergranular contacts constant. Moreover,
one can observe in the tests of the localizations of deformations accompanied by strong dilatancy. In these zones, the number of grains in decreasing contact, there is reduction in energy blocked and thus softening.

These behaviors are highlighted primarily by triaxial compression tests of revolution. These observations bring to postulate that there is a plastic threshold whose evolution is controlled by two mechanisms: one purely contractor associated with the hydrostatic constraint, and a mechanism deviatoric controlled by internal friction being held with constant volume and possibly dilating with the approach of the localization.
All the interest of the Camwood Clay model lies in its faculty to describe these phenomena with one minimum of ingredients and in particular only one surface of load and a work hardening associated with one
only scalar variable.

2.2 Behaviour under hydrostatic compression

During a hydrostatic test of compression (E the initial index of the vacuums under loading equal to 0
atmospheric pressure P), the grounds present an index of the vacuums which decrease logarithmiquement
has
with the exerted hydrostatic pressure (cf [Figure 2.2-a]). Until a pressure 0

P
called
IDIOT
pressure of consolidation, the behavior is reversible, the slope of the diagram (,
E Ln P) is
called elastic coefficient of swelling. 0
P
corresponds to the maximum pressure which underwent it
IDIOT
material during its history. Beyond this preconsolidation, the diagram presents one
new slope (coefficient of compressibility) more marked and appearance of deformations
irreversible. 0
P
thus corresponds to an evolutionary elastoplastic threshold.
IDIOT
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E

e0

Ln Pa
Ln 0
P
1
Ln P

IDIOT
IDIOT
Ln P

Appear 2.2-a: Hydrostatic test of loading and unloading

Note:

The diagram above corresponds to a whole of measurements where the effective constraint is stabilized. Indeed, in the process of consolidation of the grounds, it is the water contained in the pores which takes again initially the hydrostatic pressure with very little deformation, front to run out and let the skeleton become deformed. After consolidation of material and stabilization of the pressure of water, the effective constraint (forced total minus pressure water) is stabilized and deferred on the graph. Relations of behavior in saturated porous environments are generally expressed with the effective constraints according to the assumption of Terzaghi.

2.3
Behavior under loading deviatoric

The triaxial compression tests of revolution make it possible to control at the same time the deviatoric component Q and
spherical component P of the loading. According to the report/ratio of these two components, one observes
Q
Q
a plastic behavior purely dilating (
> M) or contracting (< M), the line Q = MP
P
P

Cr
representing the whole of the critical points on surfaces of load where the mechanical state evolves/ moves
without plastic change of volume. The basic Camwood Clay model makes the assumption that the rates
~

p
F
p
F
plastic deformations are normal on the surface of load F (& = &
, & = &
). Of
v
P

Q

more, plastic work in an unspecified point of the surface of load is considered equal to work
plastic in a critical state. These considerations bring to the following equation for the plastic threshold:

Q
P
F (P, Q, P)
Ln
éq
2.3-1
Cr
=
+
(
) = 0
MP
Pcr
Note:

In Code_Aster, the adopted criterion is that of modified the Cam_Clay model [éq 3.2-1].
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3
Camwood Clay law modified

The criterion of plasticity formulated above is not satisfactory for certain ways of loading.
In particular, for low values of Q/P, the model predicts deviatoric deformations too much
important. To cure it, a new expression of plastic work was adopted, which
conduit with the model known as of Camwood modified Clay [bib1].

3.1
Assumptions of modeling

The model is written in small disturbances.
The coefficients of the model do not depend on the temperature.

3.2
Surface of load

The new assumptions lead to the following expression of the surface of load:

F (P, Q, P) = Q2 + M 2P2 - 2M 2PP

0
éq
3.2-1
Cr
Cr

In the plan (P, Q), the expression represents a family of ellipses, centered on P which is equal to
Cr
half of the pressure of consolidation (cf [Figure 3.2-a). P will be the parameter of work hardening of
Cr

model.

Q

Q=MP

Pcr1 Pcr2
Pcon1
Pcon2
P

Appear 3.2-a: Family of hammer-hardenable surfaces of load

When F = 0 and P < P the material is dilating (p
&
) and P is decreasing (softening).
v < 0
Cr
Cr
When F = 0 and P > P the material is contacting (p
&
) and P is increasing (hardening).
v > 0
Cr
Cr
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3.3
Elastic law and law of work hardening

One makes the assumption of the decoupling of the partly hydrostatic and deviatoric elastic law and the additional assumption that the modulus of rigidity is constant.

One thus considers an isotropic elastic law, with a linear deviatoric part and a part voluminal non-linear:

Déviatoire part:

~e
S
=

éq 3.3-1
2μ
Voluminal part:
E

&
&

ln

éq 3.3-2
v = -
E
or E = e0 -
P if P < Pconsolidation
1+ e0
The law [éq 3.3-2] is in fact derived from a test oedometric where one measures the variation of the index of
vacuums according to the loading [Figure 2.2-a]. Let us recall that a homogeneous test oedometric consist in increasing the axial effective constraint all while maintaining the constraint radial null on a cylindrical test-tube.

Note:

The pressures P correspond to tests drained or not. Nevertheless, in one modeling with Code_Aster constraints handled in the laws of behavior are effective i.e. that one does not take into account the hydrostatic pressure of the fluid who can circulate in the pores, this one being calculated in modelings THM.

The tests of voluminal loading (cf [Figure 2.2-a]) bring us to the following elastic law:

1 + E
P = P exp K (
p
-
)
with K =

éq
3.3-3
0
[0 v v]
(0)
0

In the same way, growth of the surface of load in phase of contractance (and decrease for the experimental dilatancy) and results suggest writing:

p
p
+ E

0
P
with

éq
3.3-4
Cr = Pcr
[
exp K (v -)
K
v
=
0]
(1 0)
,
(-)
p
and E correspond to the voluminal deformation and the index of the initial vacuums, determined by
v0
0
extrapolation of the oedometric curve of the test to pressure 0
P (cf [Figure 2.2-a]).
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3.4

Plastic law of flow

The two plastic variables are the plastic deformation voluminal p
and the tensor deviatoric
v
plastic deformations p
~. The internal variable is also p
but associated the force
v
of P. work hardening the material standard is not generalized. The rule of flow is written:
Cr

p
F
p
F
& = &
, & = - &
,
éq
3.4-1
v

P
Cr
by breaking up the first term, one obtains:
F

F
~

F

p
p
p
& = &
& = &
& = - &

éq 3.4-2
v
v

P

S

P
Cr
knowing that:
1
P = - tr () and = - tr + -

éq
3.4-3
v
() 3 (T T0)
3
F is the plastic potential associated the phenomenon of work hardening. Let us note that the third part of [éq 3.4-2] is only formal. Indeed, p is known
& by the first relation thus one knows
v
evolution of P.
Cr

3.5
Energy writing and plastic module of work hardening

One is thus within the not generalized "standard" material framework (one uses three then potentials: the surface of load F, plastic potential F, and free energy. Even in this configuration less favorable than the traditional framework of not generalized standard materials, one is ensured to satisfy the second principle of thermodynamics [bib4]. Using the condition of consistency (expressing that the point representative of the loading "follows" the surface of load) which is written in the following way:
F
F

df =
dP +
+ F
dQ
dP

éq
3.5-1
Cr = 0,

P
Q
Pcr
the expression of the plastic multiplier is determined [bib4]:

1
F

1
F

=
D = -
dP ***éq***
3.5-2
Cr
H
H
P

p
p
Cr
with [bib4]:
F
2
F

H =
, where H
of

modulate

is
E
écrouissag ***éq***
3.5-3
p
2
p
p
P

P

Cr
Cr
v
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The identification of the first and third part of [éq 3.4-2] makes it possible to calculate F which is written:
F

F = -
dP = M 2P
- 2

éq
3.5-4
Cr
Cr (P
P
Cr
)
P

Concept of work hardening being associated that of blocked energy:

2
p
P
=
thus
dP =
D
éq
3.5-5
Cr
p
Cr
2
p
v

v
v
where is the density of free energy:

3
0
P
P
= µ (E
) 2
0
+
exp (
E
K)
Cr
+
exp (K (p
p
-))
éq

3.5-6
2
eq
0 v
v
v0
K
K
0

By using them [éq 3.4-2], [éq 3.5-4] and [éq 3.5-6], one can draw according to [éq 3.5-3] the expression from
modulate plastic work hardening:
F
2 F

H =
= 4kM 4PP P - P
éq
3.5-7
p
2
Cr (
Cr)
p
P

P

Cr
Cr
v

The module of work hardening is positive in phase of contractance (P > P and negative in phase of Cr)
dilatancy (P < P. For P = P, the behavior is plastic perfect and proceeds with volume
Cr)
Cr
constant plastic.

3.6 Relations
incremental

The equation [éq 3.4-3] and the condition of consistency give the relations of flow:

1 1
1
p
Q

D

éq
3.6-1
v =

-
dP +

dQ
2

K P
P
Cr

Mr. PPcr

1
2
p
Q
Q

D
éq
3.6-2
eq =

dP +
dQ
K M 2PP
M 4 PP
Cr

Cr (P - Pcr)

~
3
p
p
S
D = D

éq 3.6-3
eq 2 Q
The rearrangement of [éq 3.6-1] and [éq 3.6-2] led to:
p
deq
Q
=

éq 3.6-4
p
D
M2

P - P
v
(
Cr)
i.e. with the equation [éq 3.6-3],

~ p
D
3
S
=

éq 3.6-5
p
D
M2
2
P - P
v
(
Cr)
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Particular case of the critical point:

For F = 0 and P = P: P&
, p

. One deduces some, by considering the elastic law: P& = K P
.
0

V
& =0
Cr = 0
Cr
V
&
The condition of consistency gives us Q& = 0.

3.7
Summary of the relations of behavior and the data of the model

3.7.1 Data and critical of the model

The data specific to the model are five:

The slope criticizes M,
the initial index of the vacuums E associated with an initial pressure equalizes in general with the pressure
0
atmospheric,
the elastic coefficient of swelling (which leads to K),
0
the plastic coefficient of compressibility: (which leads to K),
the initial critical pressure P equalizes with half of the pressure of preconsolidation,
Cr 0
which it is necessary to add the traditional coefficient of Lamé µ and the thermal dilation coefficient
. The coefficient of Lamé µ is in fact calculated starting from the two elastic coefficients E
, provided
in data.

The number of data is relatively low, which makes the model very simple. One of the limitations more visible of the model is the assumption of the alignment of the critical points on a line of slope Mr. This is besides the expression of the concept of internal friction. One can also interpret the size
M
3
M by connecting it to the angle of repose natural of Coulomb by the relation: sin =
. However one knows

6 + M
that for very cohesive materials, this angle varies when the average constraint decreases. One note besides that for a chock of M on a triaxial compression test with a certain average constraint, one simulates well with this model the triaxial ones realized with a average constraint step too different
but one cannot correctly consider the stages plastic for a broad range of pressure of containment (cf [bib2]). It is thus necessary to readjust M for several beaches of constraint average.
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3.7.2 Summary of the relations of behavior of the model

Elasticity

~e
S = 2μ

éq 3.7.2-1
P = P exp

éq
3.7.2-2
0
(E
k0v)
Plasticity

The criterion: F (, P
Q
MR. P
MR. PP
with (Q =

eq)
Cr)
2
2
2
=
+
- 2 2 Cr = 0
F
1 F D 3 F S
= -
I +

éq
3.7.2-3
3 P
2 Q Q
1

4
4
4
4
2
4
4
4
4
3

thus:
~ p
& = 3 & S

éq 3.7.2-4
p
& = & m2
2
P - P éq
3.7.2-5
v
(
Cr)
Work hardening
P
= P
K -

éq
3.7.2-6
Cr (p
v)
exp
p
p
Cr 0
((v v 0)

Elastic behavior: If F < 0 or (F = 0 and f& 0) then:
P&

éq 3.7.2-7
Cr = 0
~ p
&

eq =,
0 p
&v = 0
éq
3.7.2-8
&s = µ&~
2

éq 3.7.2-9
P& = K

éq
3.7.2-10
0& P
v
Elastoplastic behavior: If F = 0 and f& = 0 then:
p
P& 0
;
P& = k& P
éq
3.7.2-11
Cr
Cr
v
Cr
~ p

& = 3& S
if
P P
éq
3.7.2-12
Cr
p
& = & m2
2
P - P
if P P

éq 3.7.2-13
v
(
Cr)
Cr

Note:

· From only unknown p

P&.
v
&, one can deduce the other unknown factors p
&~ and Cr
· If P = P: p
&
, Q& = P& =,
0 P& = K P.
0

v = 0
Cr
Cr
&v
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4
Numerical integration of the relations of behavior

4.1
Recall of the problem

For an increment of loading given and a whole of variables given (initial field of displacement, constraint and variable intern), one solves the discretized total system (2.2.2.2 - 1 of [bib3])
who seeks to satisfy the equilibrium equations.

The resolution of this system gives us U
, therefore
. One thus seeks locally (in each
not Gauss) the increment of constraint and variable interns correspondent with
and which satisfies
law of behavior.

The following notations are employed: Has, A, A
for the quantity evaluated at the known moment T, with
the moment T + T
and its increment respectively. The equations are discretized in manner
implicit, i.e. expressed according to the unknown variables at the moment T + T
.

4.2
Calculation of the constraints and variables internal

The elastic prediction of the deviatoric constraint is written:

= -
S + μ ~
2 ***éq***
4.2-1
however one can always write S at the moment + as being:

-
~e
S = S + 2μ

éq 4.2-2
These two equations enable us to deduce S according to E
S:

E
~
~e
S = S - 2μ
+ 2μ

éq
4.2-3

E
~ p
or S = S - 2μ

éq 4.2-4
While replacing
p
~

by its expression according to
p
, one obtains:
v

E
S
S =

éq 4.2-5
p
3
µ
1+ m2 (v
P - Pcr)
from where,
E
Q
Q =
éq
4.2-6
p
3
µ
1+ m2 (v
P - Pcr)
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One writes P according to the hydrostatic elastic prediction:

There is the equation:
P = P exp

éq 4.2-7
0
(k0 (
p
-
v
v)

By supposing that K is independent of the temperature, the incremental writing of this
0
equation is:
P = P exp [
-
-
E
K

éq 4.2-8

0
k0
v -
E
v
]
P = p exp [
E
K

éq 4.2-9

0 v]
P = -
P (exp [K E

éq
4.2-10

v -
0
]) 1

In the same way one can write the expression of E
P according to -
P:

E
P = p exp [K

éq 4.2-11
0 v]

from where the expression of P at the moment + is:

E
P = P exp [
p
- K

éq 4.2-12
0 v]

In the incremental writing of P, the coefficient K does not depend on the temperature, one thus finds
Cr
the following expression:
P = P exp
p
p
K -

éq
4.2-13
Cr
Cr 0
[(v v 0)]
P = p exp K

éq 4.2-14
Cr
Cr
[statement]
P
P
K
éq
4.2-15
Cr =
-
Cr [exp (p
v) -]
1

Summary:

F (E
S, E
P, -
P
in this case P
that is to say
-
E
S = S + S = S
Cr = 0
Cr) 0
E
P = P

F (E
S, E
P, -
P
in this case P
, ~
p
0 and p

v 0
Cr > 0

Cr) > 0
that is to say
E
~ p
S = S - 2μ

P = P exp - K

0
E
[
p
v]
P = p exp K

Cr
Cr
[statement]
Note:

The principal unknown factor is
p
.
v
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4.3
Calculation of the unknown factor
p

v

By deferring in the criterion the expressions of P and Q according to E
P and of E
Q and while using
the equation [éq 4.2-6]:
p
2

μ

2
3
v
Q = - 1
+
M 2P P - 2P ***éq***
4.3-1
E
2
(
Cr)
M (P - P)

Cr
p

2
2
μ
2
3

Q = - M 1+
exp
2
-

- 0

M

(
v
P
K
E
P exp - K
éq
4.3-2
0

- P exp K

E
[
p
v]
Cr
[statement]) E [
p
v]

(P exp - k0

- 2P- exp K

E
[
p
v]
Cr
[statement])

In under following paragraph one determines limits with this function which facilitate the resolution equation [éq 4.3-2] with for example the method of the cords or by the method of Newton.
Some examples of paces of the preceding function are given in the following figures for several data files.

Particular case: *Q = 0 (hydrostatic test of compression)*
The criterion is reached for P = 2P

Cr
From where: P exp - K

0
= 2P- exp K
E
(
p
v)
Cr
(statement)
E
1
P
Thus p

Ln

v =
-
K + K
2P
0
Cr
For: M = 9
,
0; μ = 4000; -
P
; K = 10; K = 30 and P
; then
-3
p

v = 2.310
E = 1
Cr =
,
0 2
0

***Appear 4.3-a: Pace of the function [éq 4.3-2] for** Q = 0*
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Example for: *-*
-
Q < MP (contractance)
That is to say following data: M = 9
,
0; μ = 4000; -
P
; K = 10; K = 30
Cr =
,
0 2
0
E
Q = 2; E
P = 6
,
0 ; ~
4

.
2 10- ;
-4

ν = 3.10

eq =

Appear 4.3-b: Pace of the function [éq 4.3-2] for -
-
Q < MP

Example for: -
-
Q > MP (dilatancy)
That is to say following data: M = 9
,
0; µ = 4000; -
P
; K = 10; K = 30
Cr =
,
0 2
0
E
Q = 2; E
P =,
0 2 ; ~
-4

;
-4

v = 3.10
eq =
.
2 10

Appear 4.3-c: Pace of the function [éq 4.3-2] for -
-
Q > MP
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4.4
Determination of the terminals of the function

One poses
p
= X the unknown factor of the problem.
v
One thus has:
P (X) = EP exp (- K X ***éq***
4.4-1
0)
P (X)
P -
=
exp (kx)

éq 4.4-2
Cr
Cr
X
(X) =

éq
4.4-3

2M 2 (P (X) - Pcr (X)
Qe
Q (X) = 1+6μ

éq 4.4-4
(X)
F (X)
2
= Q (X)
2
2
+ MR. P (X) - 2 2
MR. P (X) P
éq
4.4-5
Cr (X) = 0

(X) 0 then two cases arise:

X 0 ***and*** *P P P (0) P*
and *0 X X****;*** *P (X*
= P X
éq
4.4-6
sup)
Cr (sup)
Cr (0)
Cr
sup
X 0 ***and*** *P P P (0) P*
and *X*
*X 0****;*** *P (X*
= P X
éq
4.4-7
inf)
Cr (inf)
Cr (0)

Cr
inf

The first terminal is the 0 which is the terminal inf in the case of the contractance and limits it sup in case of dilatancy.

Calculation of the second terminal X = X
= X:
B
sup
inf
P (X = P X P exp - K X
P
=
exp kx
b)
Cr (
)
E
B
(0 b) Cr
(b)
E
P = exp K + K X
-
(
0) B
Pcr

éq
4.4-8
E

1
P
X
Ln
B =
-
K + K
P
0
Cr

One will thus distinguish between the two fields:
Dilatancy: X [X 0
; and Contractance: X [0: X
B]
B
]
Values of the function at the boundaries:
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At the point
E
X = 0; P ()
0 = P; P ()
0 = p; 0 = 0; Q 0 = Q
éq
4.4-9
Cr
( )
() E
Cr
F (0) = E
Q 2 +
2
E
MR. P (E

-
P - 2P
éq
4.4-10
Cr)
At the point
X = X P = P;
=
= and
= -

éq 4.4-11
B
Cr
(xb)
; Q (xb) 0
F (xb)
2
2
MR. P

4.5
Particular case of the critical point

Q

Q=MP

T

t+

Pcr

Pcon

P

Appear 4.5-a: Mechanical State around the critical point

If at the moment -
T one reaches the critical state, then +
P
P

and
-
-
Q = MP. If F =,
0 f& = 0,
Cr =
- ,
Cr
statement = 0
then the point (P, Q)
at the moment +
T moves on the initial ellipse (cf [Figure 4.5-a]). One deduces immediately from the law rubber band and of the condition p

:
v = 0

-
P = K

éq 4.5-1
0 P
v

The criterion being checked at the moment +

T, one has while using [éq 4.5-1]:

+2
2
Q = Mr. P+ (2P- - P+)
2
= M (p + P
) (p - P
)
2
-(2)
= MR. P
1
(
2
2
- K
)
-(2)
= Q
1
(
2
2
- K
)
Cr
0
V
0
v
éq 4.5-2
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In addition the diverter of the constraints can be written:

~

E
p
E
F
S = S - 2μ = S - 2μ
= - 6μ S ***éq***
4.5-3
S

One deduces some:
Qe
1+ 6 μ =
, éq
4.5-4
Q
and:

Q 1
(- K 2
2

0
V
E
S =
S
éq
4.5-5
E
Q

4.6 Summary

The discretization of the equations and the law of implicit behavior of manner leads to the resolution equation [éq 4.3-2].

If -
-
P P, then one solves the equation [éq 4.3-2] whose unknown factor is
p
.
Cr
v

One deduces then:
E
-
-
S
p
p
P = P exp (K), P = P exp (K (
), then

0
-
S =
éq 4.6-1
Cr
Cr
v
v
v
p
3µ
1+ m2 (v
P - Pcr)
One deduces finally:
p

p
3

~
v
=
S
éq
4.6-2
2 m2 (P - P)
Cr
At the critical point:
p
-

,
0 P
P
éq
4.6-3
v =
Cr =
Cr
In this point, there is no evolution of work hardening, on the other hand the state of stress can continue with
to evolve/move either in contractance, or in dilatancy (the tangent with the criterion is horizontal). The new state
constraints moves on the surface of load of the preceding state.
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5 Operator
tangent

If the option is: RIGI_MECA_TANG, option used at the time of the prediction, the tangent operator calculated in
each point of Gauss is known as of speed:

elp
& = D
ij
ijkl kl
&

In this case,
elp
D is calculated starting from the not discretized equations.
ijkl

If the option is: FULL_MECA, option used when one reactualizes the tangent matrix with each iteration
by updating the internal constraints and variables:

D = A D
ij
ijkl
kl

In this case, A is calculated starting from the implicitly discretized equations.
ijkl

5.1
Nonlinear elastic tangent operator

The elastic relation of speed is written:
&
P
S
K Ptr

éq
5.1-1
ij = - & ij + &ij =
ij + µ &

&
~
2
0

2
& = (K P -
+

éq
5.1-2
0
µ tr
) &
µ
2
ij
ij
&ij
3
The tangent operator in elasticity of the law noted Cam_Clay
E
D is thus deduced from the matric writing
following:

4
2
2

K P
µ K P
µ K P
µ
0
0
0
&

11
0 +
0
-

0
-
11
&

3
3
3

2
4
2
&

22
K P
μ K P
μ K P
μ
0
0
0
&
0
-
0
+
0
-
22

3
3
3

&

33

33
&

=
2
2
4

K P
µ K P
µ K P
µ 0
0
0

éq 5.1-3
2
0
-
0
-
0
+

&
2
12

3
3
3

12
&

0
0
0
2μ
0
0

2&
2
23

&23

0
0
0
0
2μ
0
2

&
2

31

&31

0
0
0
0
0
2μ
1
4
4
4

4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
4
4
4
3
E
D
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5.2
Plastic tangent operator of speed. Option RIGI_MECA_TANG

The total tangent operator is in this case K (the option
I 1
-
RIGI_MECA_TANG called with the first
iteration of a new increment of load) starting from the results known at the moment T [bib3].
I 1
-
If the tensor of the constraints with T is on the border of the field of elasticity, the condition is written:
I 1
-
f& = 0 which must be checked jointly with the condition F = 0. If the tensor of the constraints with T
I 1
-
is inside the field, F < 0, then the tangent operator is the operator of elasticity.

F

f& =
F

& +
P&cr = 0

éq
5.2-1

Pcr
P

like
Cr
p
P& =
, then:
Cr
p
v
&
v
F
F
f& =

P

& +
Cr
p

éq
5.2-2
p
v = 0

&

P
Cr v

In addition E
p
& = & - &

thus:
-
1 & =

& - F
Of
&
,
éq
5.2-3

i.e.:

E
E
F
& = D & - &D

éq
5.2-4
ij
ijkl kl
ijkl kl

The plastic module of work hardening is written according to the equation [éq 3.5-7] and by using the rule
of flow:

F

P

F

1
F

P

Cr
Cr
p
H =
= -

éq
5.2-5
p
p
p
v
P

&

P

& P

Cr
v
Cr
Cr
v
The equations [éq 5.2-1] and [éq 5.2-5] give:

F

&
& H

éq
5.2-6
ij -
p = 0
ij
F

Multiplication of the equation [éq 5.2-4] by
give:
ij
F

F

F

F

E
E

& =
D & - &
D

éq 5.2-7
ij
ijkl
ijkl

ij
ij
ij

kl
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The two preceding equations make it possible to find:
F

E
F
E
F
H & =
D & - &
D

éq
5.2-8
p
ijkl kl
ijkl

ij
ij

kl

from where the expression of the plastic multiplier:

F

E

D
ijkl kl

&
ij
& =

éq
5.2-9
F

F

E

D
+ H
ijkl
p

ij

kl

That is to say H the definite elastoplastic module like:
F

F

E

H =
D
+ H
éq
5.2-10
ijkl
p

ij

kl
The plastic multiplier is written:

F

Of
ijkl kl

&
ij
& =

éq
5.2-11
H
While replacing & by his expression in the equation [éq 5.2-4], one obtains:

1 F

F

E
E
E

& = D & -

D
& D

.

éq
5.2-12
ij
ijkl kl
mnop COp
ijkl
H

mn

kl
One thus deduces the elastoplastic operator from it
elp
E
p
D
= D - D:

F

F

E
E
E

& =
1
D -

D D

éq 5.2-13
ij
ijkl
ijop
mnkl
&kl

H

COp
mn

1
4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
3
Delp
with,
1 F

F

p
E
E

D
=

D D

éq 5.2-14
ijkl
ijop
mnkl
H

COp

mn
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Calculation of H:
F

2

= - m2 (P - P + S
3 ,
éq
5.2-15
Cr) ij
ij

3
ij
who is written in vectorial notation:

2
2
- M (P - Pcr)

+ 3s11
3

- 2 2
M (P - Pcr) + 3s

22
3

- 2 2
M (P - P
éq
5.2-16
Cr)

+ 3s33
3

3 2s12

3 2s

23

3 2s31

from where the expression of:
-
2
2k MR. P P P
6µs
0
(- Cr) +

11
2
- 2k MR. P P P
6µs
0
(- Cr)

+
22

µ
E F
-
2
2k MR. P P P
6 S
0
(- Cr) +

Dijkl
:
33

éq
5.2-17

µ
kl

6
2s12

µ

6
2s23

6µ 2s31

and
F

E
F
4
2

2

D
= 4k MR. P (P - P) +12 Q
µ éq
5.2-18
ijkl
0
Cr

ij
kl
According to the equations [éq 3.5-7] and [éq 5.2-17], one can deduce the expression from H:
4
H = 4M P (P - P
-
+
+ µ
Cr) (K (P
Pcr) kPcr)
2
4 Q

éq 5.2-19
0
While posing:
To = 2
- K M 2P P - P
+
,
éq
5.2-20
0

6 S
µ
ij
(
Cr) ij
ij

one can write the following symmetrical plastic matrix:

2
With
WITH A
WITH A
6 2μA S
6 2μA S
6 2μA S
11
11
22
11
33
11 12
11 23
11 31

2
.
With
WITH A
6 2μA S
6 2μA S
6 2μA S
22
22
33
22 12
22 23
22 31

2

μ
μ
μ
D p = 1
.
.
With
6 2 A S
6 2 A S

6 2 A S

33
33 12
33 23
33 31 éq
5.2-21
H
2 2
2
2
.
.
.
36µ S
36µ S S
36µ S S
12
12 23
12 31

2 2
2

.
.
.
.
36µ S
36µ S S
23
23 31

2 2
.
.
.
.
.
36µ s31
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5.3
Tangent operator into implicit. Option FULL_MECA

To calculate the tangent operator into implicit, one chose by preoccupation with a simplicity to separate in first
place treatment of the deviatoric part of the hydrostatic part for then combining them in order to
to deduce the tangent operator connecting the disturbance from the total constraint to the disturbance of total deflection.

5.3.1 Treatment of the deviatoric part

It is considered here that the variation of loading is purely deviatoric (P =)
0 .
The increment of the deviatoric constraint is written in the form:
S
= μ ~
2
- ~

éq
5.3.1-1
ij
(
p
ij
ij)
Around the point of balance (- +

), one considers a variation S of the deviatoric part of
constraint:
S
= μ
~
2
-
~ éq
5.3.1-2
kl
(
p
kl
kl)
Calculation of ~ p
:
kl

It is known that:
~ p
=
3 S éq
5.3.1-3
kl
kl
By deriving this equation compared to the deviatoric constraint, one obtains:
~ p
= 3 S +3 S
éq
5.3.1-4
kl
kl
kl
Calculation of:
One a:
1 F

1 F

F

=

=

S

+
P

H
mn

H

S
mn

P

p
mn
p
mn
éq 5.3.1-5
= 1 [S
3
S

+ m2
2
-

mn
mn
(P Pcr) P]
H p
If one considers only the evolution of the deviatoric part of (P =)
0, then:

(H) = H + H =
3 S S + 3s S
- M2
2
PP éq

5.3.1-6
p
p
p
[mn mn
mn
mn]
Cr
However:
P
P = kP.
Cr
Cr
v

p
2
p
Like

= 2M (P - P),
has

one
=
2 m2 (P - P) - 2M 2
P, éq
5.3.1-7
v
Cr
V
Cr
Cr
From where:

2
1
2 M (P - P) =
+ 2
2
M

P.
éq
5.3.1-8
Cr
Cr
kP
Cr

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In addition,
H = 4
4
km PP (P - P) and H = 4
4
km P (P - 2P) P. éq 5.3.1-9
p
Cr
Cr
p
Cr
Cr

By injecting this last equation in the equation [éq 5.3.1-6], one obtains:

H + 4kM 4P (P - 2P) + 2M 2P P =

3 S S + 3s S
éq
5.3.1-10
p
[
Cr
] Cr [mn mn mn mn]

While using the relation [éq 5.3.1-8], it comes then:

[3s S +3s S
mn
mn
mn
mn]
=
éq
5.3.1-11
(H +)
With
p

2

M (P P)
4
2
-
with A = [4k MR. P (P - 2P)
Cr
+ 2M P]
Cr

1
2

+ M
2kPcr

One then obtains immediately the variation of the deviatoric part of the plastic deformation:
~ p
=

9
9
6
S
S S + S S S +
S
S
S +
M2 P - P
P
S
kl
(mn mn kl mn mn kl)
mn
mn
kl
(
Cr)
kl
(H +
p
With)
H p
H p

éq 5.3.1-12
S is written then:
ij
18
~
µ
18µ
12µ
S = 2
µ -
S
S S + S S S -
S
S
S -
M2 P - P
P
S

ij
ij
([kl ij kl kl ij kl)]
kl
kl
ij
(
Cr)
ij
(H + A)
H
H
p
p
p
éq 5.3.1-13

who becomes by separating the terms in variation from constraints and the term in variation of deformation
total:

12µ

2
18µ
18µ
+
-
+

+
+

= 2µ~
ijkl
ijkl
M (P
Cr
P) P
(sklsij sklsij)
smn smn ijkl skl
ij

H

+
p
H p A
H p

éq 5.3.1-14
or in tensorial writing:

D
12µ

I 1
2
+
MR. P - P
éq 5.3.1-15
Cr P +
S S +
S + S S S =
4
(
)
18µ
18µ
:
(
)

µ~
2

H
H
(H
p
p

p +
)

With

that one can still write by symmetrizing the tensor (S + S
) S:

D
12µ

I 1
2
MR. P P
P
S S
+
- Cr +

+

S =
4
(
)
18µ
18µ
:

µ~
2

éq 5.3.1-16

H
H
(H
p

p
p +
)

With

1
with: = [
T
((S + S
) S) + (S (S + S
))]
2
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Calculation of, while posing:
=
+
ij
T
sij
sij
T S

T S
T S
2T S
2T S
2T S
11 11
11 22
11 33
11 12
11 23
11 31
T S
T S
T S
2T S
2T S
2T S
22 11
22 22
22 33
22 12
22 23
22 31

éq 5.3.1-17
T
T S
T S
T S
2T S
2T S
2T S
S = 33 11
33 22
33 33
33 12
33 23
33 31
2T S
2T S
2T S
2T S

2T S
2T S
12 11
12 22
12 33
12 12
12 23
12 31

2T S
2T S
2T S
2T S
2T S
2T S
23 11
23 22
23 33
23 12
23 23
23 31

2T S
2T S
2T S
T S
2T S
2T S
31 11
31 22
31 33
31 12
31 23
31 31

1
= [
T
T
(S) + T
(S)]

éq 5.3.1-18
2
That is to say:

D
1
6
2
9

C =
9
I
MR. P P
P
S S
4
+
(- Cr)

+
: +

2µ

éq 5.3.1-19
H
H
(H
p
p
p +

)
With

one poses:
9
C =
(S
: S)

éq 5.3.1-20
H p
and
6
D =
M2 (P - P
Cr) P

éq 5.3.1-21
H p

The symmetrical matrix C of dimensions (6,6) is too large to be presented whole, one break up into 4 parts C, C, C and C:
1
2
3
4

C
C
1
2
C =

C
C
3
4
with

1

9
9
9

+ C + D +
S T
(T S
T S)
(T S
T S)
11 11
11 22 + 22 11
11 33 + 33 11

2μ
(H p +)
With
2 (H p +)
With
2 (H p +)
With

9
1
9
9

1
C =
(22
T
11
S + 11
T s22)
+ C + D +
22
T s22
(22
T s33 + 33
T s22)
2 (H
μ

p +
)
With
2
(H p +)
With
2 (H p +)
With

9
9
1
9

(33
T 11
S + 11
T s33)
(22
T s33 + 33
T s22)
+ C + D +
33
T s33
2 (H
µ
p +
)
With
2 (H p +)
With
2
(H p +)
With

éq 5.3.1-22
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9 2
9 2
9 2

(11
T 12
S + S T)
(
11 12
11
T s23 + S T)
(
11 23
11
T 13
S + S T)
11 13
(
2 H p +)
With
(
2 H p +)
With
(
2 H p +)
With

C2 =
9 2
9 2
9 2

(22
T 12
S + S T)
(
22 12
22
T s23 + S T)
(
22 23
22
T 13
S + S T)
22 13
(
2 H p +)
With
(
2 H p +)
With
(
2 H p +)
With

9 2
(33
T 12
S +
9 2
S T)
(
33 12
33
T s23 +
9 2
S T)

(
33 23
33
T 13
S + S T)

33 13
(
2 H p +)
With
(
2 H p +)
With
(
2 H p +)
With

éq 5.3.1-23
C =
3
C2

éq 5.3.1-24
1
18
9
9

+ C + D +
S T
(T S
T S)
(T S
T S)
12 12
12 23 + 23 12
12 23 + 23 12

2μ
(H p +)
With
(H p +)
With
(H p +)
With

9
1
18
9

C4 =
(23
T 12
S + 12
T s23)
+ C + D +
23
T s23
(23
T 13
S + 13
T s23)
(H
μ
p +
)
With
2
(H p +)
With
(H p +)
With

9
9
1
18

(13
T 12
S + 12
T 13
S)
(13
T s23 + 23
T 13
S)
+ C + D +
13
T 13
S
(H
μ
p +
)
With
(H p +)
With
2
(H p +)
With

éq 5.3.1-25

Calculation of the rate of variation of volume:

p
2
p
2
2
3B

= 2M (P - P), = 2M (P - P) - 2M P = B =
(S + S

). S
v
Cr
v

Cr
Cr
(H + A)
p
éq 5.3.1-26
2
M (P - P)
with: B = 2
2
M (P - P)
Cr
- 2 2
M
Cr
.

éq 5.3.1-27
1
2
+ M
2kPcr
or while using [éq 5.3.1-11]
p
3B
=
(S + S
). S éq
5.3.1-28
v
(H + A)
p
One thus has:
B
= (
-
(+)
ij
C
)
ijkl
S
S kl ij

skl
éq
5.3.1-29
(H +
p
With)

5.3.2 Treatment of the hydrostatic part

It is considered now that the variation of loading is purely spherical (S = 0).
The increment of P is written in the form:
-
P = P exp (K

éq
5.3.2-1
0 E
v)
-
- P
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The derivation of this equation gives:
P = K P
0

(
p
-
v
v)

éq 5.3.2-2
Calculation of
p
:
v

It is known that:
p
=
2
2
-
v
M (P
Cr
P)

éq 5.3.2-3
By differentiating this equation, one obtains:
p
=
2
2
-
+ -
v
M (
(P Cr
P)

(P
Cr
P)
éq
5.3.2-4

One knows the expression of:

2M 2 (P -
+ 3
Cr
P) P
S S
B
=
=

éq
5.3.2-5
H p
H p
while posing
B = 2M 2 (P - P + 3
éq
5.3.2-6
Cr) P
S S

While differentiating, it comes:
2M 2
=
([
4kM 4b
P -
+ -
-

-
+
-
Cr
P) P (P
Cr
P) P]

[Cr
PP (2P
Cr
P)
Cr
P P (P 2 Cr
P)]
H
2
p
H p
éq 5.3.2-7
One seeks the expression of P according to
:
Cr

One has
p
P = kP
éq
5.3.2-8
Cr
Cr
v

One can write:
Cr
P = 2M 2 (P -
+
2
2
-
Cr
P)
M (P
Cr
P)
éq 5.3.2-9
Cr
kP
1+ 2M 2kP
P
Cr

= 2M 2
2
-
+ 2

Cr
(P Pcr)
MR. P

éq 5.3.2-10
kP

Cr

2M 2 (P - P

2 2

Cr) kP
M kP
P
Cr
Cr
=

+

P
Cr

éq 5.3.2-11
1+ 2kP
M2

1+ 2kP
M2

Cr

Cr

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One poses

2M 2 Cr
kP (P - Cr
P)
C = [
,

éq 5.3.2-12
1 + 2M 2 Cr
kP]
2M 2 Cr
kP
has = [

éq 5.3.2-13
1+ 2M 2 Cr
kP]
One has then:
P

éq 5.3.2-14
Cr =

p + C has

By replacing the expression of P in
[éq 5.3.2-7], one finds:
Cr
= [2
2
1
2M (P - P P + 2M P - C - aP P
.
Cr)
(
)] HP éq 5.3.2-15
4kM 4b
-

-
+ +
-
2
[PP 2P P
C
P P P 2P has
Cr (
Cr)
(
) (
Cr)]

H p
By gathering the terms in
and those out of P, one finds:
F
= P

éq 5.3.2-16
E
with,
1
F =
[2M 2 (P - P + 2M 2P -2aM 2P
Cr)
]
H p
éq
5.3.2-17
4kM 4b
-
-
+
-
2
([2P P P aP P 2P
Cr) Cr
(
Cr)]
H p
2cM 2 P

bckM
4
4
E = 1+
+
P
-

éq

5.3.2-18
2
(P 2Pcr)
H
H
p
p
The expression of
p
thus becomes:
v
p
2
F
F

= 2M - has - C +
-

v

(P Pcr) P éq 5.3.2-19

E
E

from where the expression of according to P:
v
K P
P
0
=
v éq 5.3.2-20
G
2

G = 1+ 2M K P
0
- - F
has
+ F
C

(P - Cr
P)

éq 5.3.2-21

E
E

Calculus of the variation of deviatoric deformation:
~
~ p
F
= = =
ij
3
S 3 Psij

éq 5.3.2-22
E

One thus has finally:
= F P
ij

éq 5.3.2-23
ij
with
3 F
G
D
F =
S -
1
éq
5.3.2-24
E
K
3 P
0

5.3.3 Operator tangent

The tangent operator connects the variation of total constraint to the variation of total deflection. Being
given that the increment of the total deflection under loading deviatoric is written:
B
= C
(
-
(S + S
) D1
)
,
éq
5.3.3-1
ij

ijkl
kl
ij
klmn
mn
(H + A)
p
with:
2/3 -1/3 -1/3 0 0 0

-1/3 2/3 -1/3 0 0 0

1
-1/3 -1/3 2/3 0 0 0
D =

éq
5.3.3-2
0
0
0
1 0 0
0
0
0
0 1 0

0
0
0
0 0 1
projection in space deviatoric,

and that under spherical loading one a:

2
=

ij
ij
F Dkl
kl

éq
5.3.3-3
with:
-1/
3

-1/
3

2
-1/
3
D =

éq
5.3.3-4
0
0

0
hydrostatic projection, one has then:
=
ij
ijkl
With kl éq
5.3.3-5
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with:
1
-

B

1
2

ijkl
With
= (Cijmn -
(S + S)
)
mn ij Dmnkl +
ij
F Dkl éq
5.3.3-6

(H p +)

With

the discretized tangent operator.

5.3.4 Tangent operator at the critical point

If the point of load is at the critical point (P = P, the general expression of the tangent operator is not Cr)
more valid. This appears in particular by divide by 0 (see the equations of [§ 5.3.1]). One
detail in what follows the coherent tangent operator to the critical point while passing as for the case general by the partly deviatoric and partly hydrostatic decomposition.

5.3.4.1 Treatment of the deviatoric part

Let us recall that to the critical point, the expressions of the plastic multiplier and its derivation are written in the following way:
Qe

E
E
=
-
Q
Q Q
1/6µ

and =
-
éq
5.3.4.1-1
Q

2
6µQ 6µQ
with,
E
E
3
3
E
S S

S S
Q =
and Q =

éq
5.3.4.1-2
E
2 Q
2 Q
from where the expression of:
1 3 sese
Qess
=

-

éq
5.3.4.1-1

E
3

6μ 2 Q Q
Q

Let us point out in the same way the expression of S:

S = μ ~
2
-
3 S -
3 S
ij
(ij
ij
ij)

While replacing and by their expressions, one can write:
E
E
E
E
3 S
~
S
3 Q
Q

kl
kl
S = 2μ -
S +
S S S -
-1 S
éq
5.3.4.1-2
ij
ij
E
ij
3
kl
kl ij

ij
2 Q Q
2 Q
Q

E
E
E

Q
3 Q

3 S .s
kl
ij
S

+
- -
S .s = 2μ

-

~ éq 5.3.4.1 - 5
kl
ijkl
ijkl
ijkl
3
kl
ij
ijkl
E
kl
Q
2 Q
2 Q Q

or in tensorial writing:

E
E
E
Q

D
3 Q
D
S
S
S
I -
S S

=
I -
éq
5.3.4.1-6
4
µ
3
2
3
4

~
Q
2 Q
2
E
Q Q
1

4
4
4
2
4
4

4
3
1

4
4 2 4
4 3
G
H
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As S does not depend on, one can confuse ~ with.
v

By using the tensor of projection in the space of the deviatoric constraints 1
D [éq 5.3.3-2], one
can write:
1
1
-
D.G.
=
H

.

éq
5.3.4.1-7
2µ

5.3.4.2 Treatment of the hydrostatic part

In tensorial writing, one with the following relation:

D
I P = K P
.

éq 5.3.4.2 - 1
0
v
according to the equation [éq 5.3.2-2] with
p

at the critical point.
v = 0

As P then does not depend on ~ one can confuse with.
v
I dP = K

éq
5.3.4.2-2
0

P
By using the tensor of projection in the space of the hydrostatic constraints
2
D [éq 5.3.3-4], one
can write:

I D
=
D
2
éq

5.3.4.2-3
K P
0

5.3.4.3 tangent Operator

By combining the contributions of the two parts deviatoric and hydrostatic, one finds the writing of the tangent operator who connects the variation of the total constraint to the variation of the total deflection to
not criticizes:

1
-1
D G H
I D

. .
2
=
+
D
.

2

µ
K P
0

or
= A
éq
5.3.4.3-1
ij
ijkl
kl
with
1
1
1
-
D.G. -
H

I D

2
Aijkl =
+
D
2

µ

éq
5.3.4.3-2
K P
0

6
Examples of implementation of the model

6.1
Initialization of calculation

In model CAM_CLAY, the non-linear elastic law reveals a hydrostatic constraint

for a null voluminal deformation [éq 3.3-4]. One thus needs at the beginning for calculations to initialize
hydrostatic constraint with a strictly positive value. With this intention, one can proceed of two different ways:

To carry out a linear elastic design by affecting boundary conditions such as the field of forced in the structure is a uniform hydrostatic compression equal to pressure Pa data in DEFI_MATERIAU. Pressure Pa corresponds to the initial index of the vacuums and is generally equalizes with the atmospheric pressure (the latter is given positively to be coherent with conventions of the civil engineering). One extracts from this calculation the stress field with
points of Gauss. This stress field is regarded as the initial state of the constraint hydrostatic necessary to law CAM_CLAY in calculation STAT_NON_LINE using the model CAM_CLAY.

To use operator CREA_CHAMP to create with operation "AFFE" a stress field hydrostatic to the points of Gauss of value Pa, the constraint in this case is given of sign negative (convention Aster for compressions) and the initial state in the STAT_NON_LINE constitutes
according to.

Numerical results for triaxial compression tests.

The following figures show triaxial ways of loading with evolutions of axial deformation according to the diverter Q. They result from numerical calculations carried out with model CAM-CLAY established in Code_Aster. These test were carried out by using one modeling of the type KIT_HM in not drained condition (this condition allows us easily to charge in a purely deviatoric way, the hydrostatic part of the loading being taken again by pressure of water). The shapes of the curves obtained numerically with Code_Aster are very with fact comparable with the diagrammatic curves presented in the paper of Charlez [bib2].

In the first test, the material is normally consolidated, i.e. hydrostatic pressure of departure is equal to the pressure of consolidation (in this case
5
10
.
6
Pa). Work hardening (positive)
start at the beginning of the deviatoric phase, without preliminary elastic phase. Hardening continue to a stage of perfect plasticity when the critical point is reached (Q=MP).
As for the three other tests, the deviatoric phase starts for a value of the constraint effective average lower than the pressure of consolidation, the material is of this surconsolidé fact. If P is higher than P equal to

5
10
.
3
Pa, the specific point of the loading cross
Cr
surface of load before the critical line. There will be thus three specific phases: a phase rubber band, a contracting plastic phase then a perfect plastic phase.
If P = P, the behavior is plastic perfect right after the elastic phase.
Cr
In the case where P is lower than P, the point representative of the loading cuts the critical line
Cr
before the surface of load which it reaches during a purely elastic way. In this configuration, the behavior is lenitive and dilating and blocked energy decreases. The point representative of the loading joined then the critical state where the material will enter in perfect plasticity.
The Cam_Clay behavior cannot produce a behavior continuement contractor/dilating.
not representative of the loading is obliged to pass by the critical state where the whole of the parameters
of work hardening (plastic voluminal deformation, critical pressure, blocked energy) become stationary [bib2].
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6,00E+05
6,00E+05

5,00E+05
5,00E+05
Q=MP
4,00E+05
4,00E+05

Critical State
)
has)
3,00E+05
3,
***P** 00E+05*
Q (Pa
Q (
2,00E+05
2,00E+05

hardening
1,00E+05
1,00E+05

0,00E+00
0,00E+00

0
100000 200000 300000 400000 500000 600000 700000
0, E+00 5, E-02
1, E-01
2, E-01
2, E-01
3, E-01
3, E-01 4, E-01
P (Pa)
eps1

6,00E+05

6,00E+05
5,00E+05
5,00E+05

4,00E+05
4,00E+05

radoucissement
critical state
3,00E+05
3,00E+05
Q (Pa)
Q (Pa)
2,00E+05
2,00E+05
1,00E+05
1,00E+05
rubber band

0,00E+00
0,00E+00

0
100000 200000 300000 400000 500000 600000 700000
0, E+00
5, E-02
1, E-01
2, E-01
2, E-01
3, E-01

P (Pa)
eps1

6,00E+05
6,00E+05

5,00E+05
5,00E+05

Q=MP
4,00E+05
4,00E+05
)
critical state
3,00E+05
Pa *3,00E+05*
Q (Pa)
Q (
2,00E+05
2,00E+05
hardening
1,00E+05
1,00E+05
rubber band
0,00E+00
0,00E+00
0
100000 200000 300000 400000 500000 600000 700000
0, E+00
5, E-02
1, E-01
2, E-01
2, E-01
3, E-01
P (Pa)
eps1
6,00E+05
6,00E+05
5,00E+05
5,00E+05
4,00E+05
4,00E+05
Q=MP
)

critical state
3,00E+05
3,00E+05
Q (Pa)
Q (Pa
2,00E+05
2,00E+05
1,00E+05
1,00E+05
rubber band
0,00E+00
0,00E+00
0
100000 200000 300000 400000 500000 600000 700000
0, E+00 5, E-02 1, E-01 2, E-01 2, E-01 3, E-01 3, E-01 4, E-01
eps1
P (Pa)
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Law of behavior of Laigle

Summary:

The rheological model of Laigle makes it possible to analyze the rock mechanics behavior. development of this model of behavior was initiated following the difficulty in apprehending correctly response of the solid mass during the excavation of an underground cavity, with an aim:

· to define the need and the nature of possible supportings to implement;
· to determine the extent of the ground around a work influenced by the digging.

The implementation of this elastoplastic model was mainly focused on the simulation of behavior post-peak of the rock. It is supposed, accordingly, that there is no work hardening of the rock before the rupture of this one. That results in a linear elastic behavior to the peak of resistance (it can nevertheless y have damage of the rock whereas the material is not yet in rupture). The definite criterion of plasticity is of type generalized Hoek and Brown and accounts for the influence of level of constraint on the shear strength. Radoucissement of material is associated one progressive reduction in the properties of cohesion and angle of friction accompanied by a change by volume. It is controlled by the plastic deformation déviatoire cumulated considered as only variable of work hardening.

To facilitate the integration of this model in Code_Aster, the law initially developed in the formalism principal constraints was rewritten with invariants of constraints on a basis of the model Cambou-Jafari-Sidoroff (CJS). The numerical formulation is implicit compared to the criterion and explicit by report/ratio with the direction of flow.

The convention of sign used for the formulation of the equations, within the framework of this note, is that of mechanics of the continuous mediums.
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Count

matters

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1 Notations

1.1 General

indicate the tensor of the effective constraints in small disturbances, noted in the form of following vector:
11

22

33
212

213

2 23

One notes:

I = tr

first invariant of the constraints
1
( )
I

S
1
= -
I
tensor of the constraints déviatoires
3

S = S S
.
second invariant of the tensor of the constraints déviatoires
II

major principal constraint
1

minor principal constraint
3
Tr ()

E = -
I
diverter of the deformations
3
= Tr

voluminal deformation
v
()
(
S

cos 3)
1 2 3 2 det ()
= 2 3

being the angle of Lode
3
sII

p
2 p p
=
E E
cumulated plastic deviatoric deformations
ij ij
3
N
normal of the hypersurface of deformation
G
function controlling the evolution of the plastic deformations and describing

direction of flow
~
Tr (G)

G = G -
I
diverter of G
3
G = Tr (G)
trace G
~
~ ~
~
G = G G
.
G normalizes
II

angle of dilatancy

angle of friction
F
surface of load
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1.2

Parameters of the model

Notation Description
m
Slope of the criterion in the plan (p', Q) for the very strong constraints (function of mineralogical nature of the rock)
S
Cohesion of the medium. Representative of the damage of the rock.
has
Characterization of the concavity of the criterion, function of the level of deterioration of the rock. It the influence of the component of dilatancy in the behavior defines in large deformations.
ult
Plastic deformation déviatoire corresponding to the ultimate criterion
E
Plastic deformation déviatoire corresponding to disappearance supplements cohesion
ult
m Value of
m of the ultimate criterion reached in ult
me Value of
m of the intermediate criterion reached in E
ae Valeur of
has intermediate criterion reached in E
m peak
Value of m of the criterion of peak reached with the peak of constraint
peak has
Value of A of the criterion of peak reached with the peak of constraint

Exhibitor controlling work hardening
C
Compressive strength simple

First parameter regulating dilatancy

Second parameter regulating dilatancy
cjs
Parameter of form of the criterion of plasticity in the déviatoire plan
E
Young modulus

Poisson's ratio

1p
Intersection of the intermediate criterion and the criterion of peak
p2
Intersection of the intermediate criterion and the ultimate criterion
Pa
Atmospheric pressure

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2 Introduction

The object of this note is to present the rheological model to analyze the behavior rock mechanics, adapted to the simulation of the underground works, introduced into Code_Aster and developed by the CIH [bib1]. The finality of this model is to be able to be implemented, in manner rapid and industrial in order to answer the principal interrogations that is posed the engineer at the time of analysis and of the design of an underground cavity. The rheological law must for that remain relatively simple, as well during the identification of the parameters as in its implementation and during the interpretation of the results.

2.1 Phenomenology of the behavior of the grounds

One of the characteristics of a rock, compared to a ground, is that its mechanical behavior is, on a beach of important constraint, controlled by cohesion. This cohesion is associated one cementing of the medium, induced during the geological history of the solid mass, and is primarily of

epitaxic nature. On the contrary, the resistance of a ground is more particularly governed by the term of
friction and/or of dilatancy. Cohesion, of primarily capillary origin, does not have an influence then that for very weak states of stresses of containment.

This distinction between a ground and a rock is important because it directs the choice and the assumptions of
base model of behavior.

The principal rheological phenomena associated this context are as follows:

· In the field of the small deformations, the response of a rock, in particular under weak states of containment, can be comparable with a linear elastic behavior, slightly depend on the state of the constraints. Non-linearities of the behavior are likely to appear the peak of resistance before, in the case of tender rocks, for a level of constraint of about 70 to 80% of the maximum value. This threshold decreases with the increase in the average pressure for almost cancelling itself when the constraint of surconsolidation is reached (course-model). Under very low constraints of containment representative of those reigning near the underground works, these non-linearities are generally weak, more especially as cementing is important, and thus the level of surconsolidation of the rock high.

· Dilatancy (increase in volume) is initiated when non-linearities appear on stress-strain curve. This dilatancy increases until there is localization with centre of the sample. At this time, the rate of dilatancy (or the angle of dilatancy) is maximum, for then gradually decreasing and cancelling themselves with the very great deformations.

· The peak of resistance is reached for constraints describing a criterion of rupture, generally curve in the plan of Mohr or the plan of the principal constraints major and minor. The assumption of a linear criterion of Mohr-Coulomb is thus only one simplifying assumption, having tendency, for low constraints of containment, with to raise the cohesion of the medium.

· Once maximum resistance reached, the resistance of the rock decreases. It radoucissement post-peak is all the more fast and important (in intensity) the constraint of containment is weak. This decrease is related to a damage more or less located rock, according to the level of containment. Whatever this constraint, beyond the peak, the rock cannot be regarded any more as continuous. Its behavior is then controlled by the conditions of deformation and strength to the level of the zone of localization of the deformations.

· The appearance of one or more discontinuities kinematics within the rock is associated a loss of cohesion. The behavior post-peak is then governed by the conditions of friction and of dilatancy along the plans of discontinuity or within a band of

localization of the deformations. It comes out from this reasoning that for the very large ones deformations, the behavior of the comparable rock to a "structure", is only rubbing, and is characterized by an ultimate angle of friction. This angle is a data intrinsic of material, function of minerals constitutive of the rock. It thus does not depend directly conditions of cohesion, and it can especially be regarded as independent dimensions of the sample.
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· When the behavior only becomes rubbing, it is associated no deformation voluminal. Dilatancy was thus cancelled, and does not exist any more with the great deformations.

· The evolution enters the resistance of peak and the state criticizes corresponding to large deformations, is more or less progressive according to the state of the pressures applied. For a state of null containment (simple compression), the behavior is only controlled by cohesion, and the rupture results in an immediate and brutal loss of any resistance. Radoucissement will be more progressive as the constraint of containment will increase, to become non-existent beyond of a certain constraint of limiting containment ductile and fragile fields of behavior.

2.2
Context of study and simplifying assumptions of the model

The will to develop a model easy to implement is necessarily accompanied by simplifications, resulting from a compromise enters the awaited objectives, the conditions of use of model (quality of the data input, times and cost available…) and means implemented for to ensure these developments. These compromises are primarily the following:

· A linear elastic behavior to the peak of resistance. This amounts supposing that there is no work hardening of the rock before the rupture of this one.

· Seul a criterion of rupture in shearing is retained. This means that if the rock is crushed in an isotropic way, the behavior remains elastic, and that there is not damage and work hardening of material under this type of way. During the phases of excavation of an underground work with implementation of a light supporting, average pressure in the solid mass located in the vicinity can only decrease (or remain constant in the ideal case of a circular cavity subjected to an isotropic request, for one linear elastic behavior). Plasticization under isotropic constraint, which one can to find on a Cape-Model or a law of the Camwood-Clay type did not seem to us essential taking into account the sought objectives, and in the case of a request isotherm and short-term.

During the development of this model, we voluntarily focused ourselves on the study and simulation of the behavior post-peak of the rock. In this field of behavior, the resistance of material is supposed to be controlled, according to the state of the constraints and the level of damage rock, by cohesion, dilatancy or friction.

Cohesion defines the resistance of material as long as this one remains continuous. It is active until peak of resistance, and has only little influence on the radoucissant behavior, unless cohesion is representative of a ductile "adhesive" (case of the grounds injected by silicate freezing, …).

As cohesion worsens by damage, dilatancy increases, for to reach its maximum value at the time of the loss of continuity of the medium. At this time, under the effect of shearing of induced discontinuity, this dilatancy is degraded gradually and slowly. rheology of the rock evolves then to a behavior purely rubbing.

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3
The continuous model

3.1 Behavior
rubber band

The elastic behavior is controlled by a linear law, with a constant module independent of the state of stresses. The 2 parameters characterizing this behavior are the modulus of elasticity E and the Poisson's ratio.
s& = μ (
p
2e& - e&)

éq 3.1-1
1
I = 3K (
p
v - v)
&
& & éq 3.1-2

3.2
Criterion of plasticity

The adopted formulation is that of [bib2].

3.2.1 Surface of load

3.2.1.1 Expression of the criterion of Laigle in major and minor constraints

1
2 A (p

)

1
1

p
3
m

F =
(-
1
3)
( )
(p has
) - (c)

p

has (p)
(- 3) + S ()
éq
3.2.1.1-1

C

C

3.2.1.2 Expression
general

One transforms the preceding expression according to the first invariant and of the diverter of constraints, by a retiming of the criterion on triaxial in compression, to obtain:
1
G (S) has (p
)
F =

- U (, p
) 0

éq
3.2.1.2-1
0
C C
H
with:
1/
S
H () = (

1+ cos

éq 3.2.1.2 - 2
cjs
(3) 1/6
det () 6
= 1+
54

cjs
3

sII
0

= = = -
C
H

H
(1
)1/6

3
cjs

0 = +
T
H
(1 cjs) 1/6
G (S) = S H

éq
3.2.1.2-3
II ()

S

U (,)
m (p
p
) K (p) G () m (p) K (p)
= -
-
I + S

éq 3.2.1.2 - 4
0
1
(p
) K (p
)

6
H

3
C
C
C
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Note:

· One shows [Appendix 1] the equivalence of the two expressions
· One shows that a second formulation of the criterion with a retiming on triaxial in compression and in extension is possible but we do not choose it. It is however presented at the chapter [§9].

3.2.1.3 Pace of the thresholds

One traces the pace of the thresholds to the criterion of peak and the ultimate criterion.

S
Threshold with the peak
2
1
S
Ultimate threshold

3.2.2 Work hardening

To translate radoucissement post-peak of the rock laws of variations of the parameters are defined m, S and have criterion according to the internal variable of work hardening p (it is about the deformation déviatoire plastic cumulated, proportional to the second invariant of the tensor of the deformations déviatoires, corresponding to the plastic distortion).

p

p
p

S () = 1
if
< E

E

éq
3.2.2-1

S (p
)=
p

0
if
E

If p
> ult (
3
1 10-
-
) # one chooses to take an epsilon of 3
10 - to avoid the errors
 # numerical during division by in the equation [éq 3.2.2-2]
ult

= 1 has
m = ult
m
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If not

( )
p
has - has -
p
E
peak
ult
E
=

éq
3.2.2-2
p

1 - has
-
E
E
ult

+
has (
has
p
)
(p

peak
)
=

éq 3.2.2-3
1+ (p
)

apic

p

p
C
p1
has

m ()
( )
=
m
+1
-
peak
S (p
)
p
if
<

E

p1
C

éq
3.2.2-4

ae

2
m (
p
p
)
C
p has (
)
p

=
m
if

E

E

p2
C

1
(p 2 2

K)
(p
has
)
=
.

éq 3.2.2-5
3

These laws of evolutions for each of the 3 parameters are dependent from/to each other and observe the conditions of intersection of the criteria during the phase of work hardening [bib1].

Note:

The condition of coherence to respect door on the continuity of the parameter m in:
E

apic

p

lim
m
(p
)

C
p1
has ()

=
m
+1
- S

peak
(p)
p

E
p1
C

that is to say:
apic

has

1
E
C
p
m =
m
+1

éq
3.2.2-6
E
peak

p1
C

3.2.3 Law of dilatancy

3.2.3.1 Writing generalized

The law of dilatancy (one admits that the value of dilatancy is inversely proportional to that of cohesion) can be generalized while writing:

- m -
sin = sin ((')
'
1
=
ult

éq
3.2.3.1-1
'+m +1
ult
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R7.01 booklet: Modelings for the Civil Engineering and the géomatériaux ones
HT-66/05/002/A

Code_Aster ®
Version
7.4

Titrate:
Law of behavior of Laigle

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with:
~
'= '(
-

I, G S,
T
=

éq
3.2.3.1-2
1
() t0) 1
0
~
-
3
T 0

2
2
2
2
2
S =
S cos (
);
S =
S cos (+

);
S =
S cos (-

);
1
where
is
3 II
2
3 II
3
3
3 II
3
the angle of Lode
I
I
I

1
1
1
=
+ S
; =
+ S
;
=
+ S
1
1
2
2
3
3
3
3
3

~
= with I

that

such
= max
1
I
I
(, j=12,
J
)

3

~
= with I

that

such

= min
3
I
I
(, j=12,
J
)

3

Note:

A condition to respect is that the report/ratio
remain lower than 1. In the case of rocks

hard very resistant, subjected to constraints of containment relatively low, the law of dilatancy can thus tend towards this report/ratio. If the two parameters are unit one find the expression of the law of Rowe describing the law of dilatancy for grounds powders. This approach amounts preserving the same expression as for a rock strongly damaged, by comparing the effect of cohesion to that of a containment additional of value t0.

Characterization of t0 according to the parameters (has, m, S) characterizing the rock

· Cas where (p
S) = 0
Disappearance of cohesion, one poses
0
0 =
T

· Cas where (p
S) 0
1 - sin
t0 = 0 (0, C
T
0)
0
= 2C0

éq
3.2.3.1-3
1 + sin0

with:

=

,
m S, has
2 arctan 1
has
AMS
0
(
) =
(+ -1
0
)-

2

has

C =
S
C
,
m S,
C
has
0
0 (
) =

1+
a-1
AMS

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3.2.3.2 Determination of the intersection of the intermediate criterion and the ultimate criterion

By writing the continuity of m in ult one obtains the following relation:

ae

C
p2 has

m (
m

ult)
( )
=
ult
E

p2
C

ae

has

2 ult

C
p
m =
m

ult
E

p2
C
1 has
-

has
p E

2
m =
E
m

ult
E

C
1
1 has
-
m
= ult E
éq
3.2.3.2-1

p2
C
ae
me

3.2.4 Flow
plastic

The adopted formalism is rewritten on the basis of model CJS [R7.01.13]. When constraints
reach the edge of the field of reversibility, plastic deformations develop. For
to calculate, there is a function potential controlling the evolution of the deformations and defined by the relation
*= **G***
&p & where & is the plastic multiplier and

F
F
***G** =*
-
N N
.
éq
3.2.4-1

The potential function is obtained starting from the following kinematic condition:
p
p
S
= - &
éq
3.2.4-2
v
&
.
sII
The parameter of dilatancy is calculated starting from the angle of dilatancy (defined by [éq 3.2.3.1 - 1])
by the formula:

2 6 sin

= ()
( )
= -

3 - sin ()

éq 3.2.4-3

=
p

if

0
> (1- -

3
10
ult
)

Note:

is positive when P=0 and in compression, then it becomes negative when plasticity develop. It is always negative in traction

It is then possible to seek to express the kinematic condition [éq 3.2.4-2] starting from a tensor N in the form:
. p
N = 0
& éq
3.2.4-4
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After decomposition of each term in déviatoire parts and hydrostatic, one finds the expression:

(
p
p

N S + N
e&

N S.E.
N

ij
ij
+
ij
v
ij =
p +
p =
1
2
)
1
.
0

3
1 ij ij
2
&
&

v
&
N
'
One deduces the relation from it 1 =
who added to the condition of standardization of tensor N leads to
N
S
2
II
the expression:
S + I
S
N =
II
éq
3.2.4-5
2
+3

The law of evolution of p
& must be such as the kinematic condition is satisfied. It is thus proposed
to take the projection of p
& on N (normal of the hypersurface of deformation), that is to say:

p
F
F
= G
=
& &

-
nn
&

One also deduces the condition from it relating to the plastic voluminal deformation:

p
= G éq
3.2.4-6

v
& &

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4
Calculation of the derivative

4.1
Derived from the criterion

4.1.1 Derived compared to the constraints

4.1.1.1 Derived intermediary compared to the diverter

G
S
H
One leaves:
= H () II
( )
+ S

II
S
S

S
ij
ij
ij
S
H ()
where
II and
are respectively given by:
S
S
ij
ij
S
S
II
ij
=

S
S
ij
II
H ()
1

det (S)
=
1+
54

S
5
3
ij
6h ()

cjs

sij
sII

- cjs cos (
3)

54
cjs
det (S)
=
S +

2h ()
ij
5 s2
6h 5 s3
S
II
( )

ij
II

Finally:

G
1

cjs
S

54
ij
cjs
det (S)
=
1+
cos (
3)
+

S
5
2
ij
H ()

2
S

6s
S

II
II
ij

And consequently:

G
1

cjs
S
cjs 54 det (S)
=
1+
cos (
3)
+

éq 4.1.1.1 - 1
S
H () 5

2

2
sII
6s

II

S

4.1.1.2 Derived intermediary compared to the constraints

G
One poses by definition: Q = Dev.

ij

sij
G
G S

G 1 G

1
kl

=
= Dev.
+

kl
-
ik
jl
ij
kl

S
S
3 S
3
ij
kl

ij

kl
mm

G
1
1 G
1

= Q - Q +

kl ik
jl
ij
kl kl
-
ik
jl
kl
ij kl kl

3
3q
3
ij
mm

G = Q ij

ij
G
It is then enough to take the deviatoric part of
to obtain:
sij
G
G

1

cjs
S

54

ij
cjs
det (S)
= Q = Dev.
=
1+
ij
cos (
3)
+
Dev.

S
5
2
ij

ij
H ()

2
S

6s
S

II
II
ij

7.4

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And consequently:

G
1

cjs
S
cjs 54
*det (**S**)*
***Q** =*
=
1+
cos (
3)
+
Dev.
éq 4.1.1.2 - 1

H () 5

2

2
sII

6s

II

S

4.1.1.3 final Expression of derived from the criterion compared to the constraints

The derivative of the criterion compared to the constraints is then:

1
F
1 1 A ()
-
1 A (p
p
)

=
U
G
p

éq
4.1.1.3-1

has
(p has
)

( )
0
() Q -

H

C C
with
U
m (p) K (p) 1
1

= -

Q + I
éq
4.1.1.3-2
0

C
6h
3
C

4.1.2 Derived compared to the variable from work hardening

1
2
F

1 G (S) has (p
)
G (S) has

U

=
-

Log

-
éq
4.1.2-1
p

(p has
)
0

0
p
p

C C
H

C C
H

with
U

1
(km)
= -
(p) G 1 (km)
-
(p) (ks)
I +
éq
4.1.2-2
0
1
(p)
p
p
p
p

6

H

3

C
C
C

4.2
Total derivative of the criterion compared to the plastic multiplier

Let us consider the function:
~
2
*
F (
)

-
~
=
E
E
p
F S - 2

μ G, I - K
3

G, +

G
éq
4.2-1
1
II

3

Where G is a fixed tensor independent of
. It is of this function of which we seek the zero
to find the state of stress:

*
F

F

= -
(~

. 2µG + KGI)
F
2 ~
+
G
éq
4.2-2
p
II

3
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4.3
Derived from the parameters compared to the variable of work hardening
S = - 1
p

if

<

E
p

E

éq 4.3-1
S =
p
0
if

E
p

m

= - C
p
if
<

E
S

p1

éq 4.3-2
m
p

= 0
if

E
S

apic
m

has

C
p1
peak
p1
has

= -
Log m
+1
m
+
p
1
if
<

peak

2 peak

E
has

1
has

p

C

C

éq
4.3-3

ae
m

C
p2 has
E
p2 has
= -
p
Log m
m
if

E

2nd

E
has
p2

a.c.
C

(- has has

ult
E)
-

=
E
peak

éq
4.3-4
p

(p
p
) -1 + (p)

1

( )
1 - has
2

E
E

-
ult
(- p

ult
)
has
1 - has

=
peak

éq 4.3-5

(+) 2
1
m
m has
m S

=
+
p
if
< E
p

p has

S p

m
m has

=
if
3
ult (1 -
-
10) > p
E
éq

4.3-6
p

p has

m
-3
p

= 0
if ult (1 -10) <
p

1
K
2 2a
2 1

= -
has
Log
ult (1 - -
10 3)
p

>
p

3
3 2 2
p has

éq
4.3-7
K

= 0
if not
p

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5
Tangent operator of speed

The condition
f& = 0

éq 5-1
is written:
.

&
=
F
F
& + F p
= 0
ij
p

ij
2
From the expression of the cumulated plastic deviatoric deformation p
p p
=
E E and of
ij ij
3
p
~
relation E
G

& = &, the condition then is found:

F
F
2 ~
f& =
& +
G = 0
ij
&
p

3
II
ij

What gives us for the plastic multiplier:

F &ij
ij
&= -

2 F ~
G
p
II
3

By then considering the relation forced/deformations:

F
F
F
F
F
& =
D & =
D =
D & - &
D G
ij
ijkl
kl
ijkl
ijkl
kl
ijkl
kl

ij
ij
ij
ij
ij

and by deferring it in the expression of & one can write:

F
F
D & - &
D G
ijkl
kl
ijkl
kl

ij
ij
&= -

2 F ~
G
p
II
3
That is to say:
F D
ijkl &kl
ij
= -
&

éq 5-2
2 F ~
F
G -
D G
p
II
ijkl
kl
3
ij
By deferring this result in the expression of & one finds:
ij

F

D

&

ijkl
kl
ij

& =D
&

G

éq
5-3
ab
abcd
+
Cd
Cd

2 F ~
F

G -
D G
p
II
ijkl
kl

3
ij

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6
Digital processing adapted to the nonregular models

The law of evolution of the plastic mechanism, defined in the chapter [§3], must satisfy the condition kinematics [éq 3.2.4-2]. The projection suggested on the normal of the hypersurface of deformation can
to lead to a "not-solution" which results in a failure of the digital processing (see
the graphic interpretation of the chapter [§ 6.1.3.3]). One proposes in this chapter to define rules of projection allowing to manage the models known as "not-regular" in their imposing projection known as
"at the top of the cone".

Moreover, as for other relations of behavior, one adds the possibility of cutting out
locally (at the points of Gauss) the step of time to facilitate numerical integration.

6.1
Projection at the top of the cone

6.1.1 Definition of the jetting angle

One places oneself in this chapter within the framework of finished increase. Equations translating it elastic behavior are written:

S = S + μ
2 (
p
E
- E
) E
p
= S - 2nd

μ

éq
6.1.1-1
I = I - + 3K
éq
6.1.1-2
1
1
(
p

-

= I - 3K

v
v)
E
p
1
v

One can also express the kinematic condition starting from tensor N (cf paragraph [§3.2.4]):

.
N p
= 0
éq
6.1.1-3

By deferring the two equations translating the elastic behavior in the preceding expression one find:
1
E
p =
(- S) éq
6.1.1-4
2µ
p
1
=
-

éq
6.1.1-5
v
(IE I
1
1)
3K
One then expresses the kinematic condition by the following relation:
S + I
1 E

E

S
.
N
(S - S) 1 1
+

(I
II
1 - I1) I = 0 with N =

2μ
3K 3

K

2
+ 3

Maybe by combining the two preceding relations where N indicates the normal of the hypersurface of deformation:
S + I
1
sII
(- S) 1
+
(IE - I Tr N =
1
1).
() 0
2
2
μ
9
+ 3
K
1
S (. E
S - S)
1

+
(IE - I =

1
1)
0
2μ
S
3K
II
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---

*This last equation defines the point (I, **S** like a projection of the point (E E*
*I, **S**) on the criterion.*
1
)
1
not (I, S
will be the oblique projection of the point (E E
I, S), projection whose direction varies with. One
1
II)
1
II
can give the chart of it of the chapter [§ 6.1.3.3].

The preceding relation can then be rewritten as follows:

*3K **S** (E -*
E
S
***S**)*
I - I =
-
éq
6.1.1-6
1
1
.

µ
2
sII
One defines the jetting angle then S by the relation:

***S** (.se - **S**)*
cos =

éq
6.1.1-7
S
*sII (- **S**) (- **S**)*

By deferring the definition of the angle S in the relation of projection one finds the relation:

E
I - I
3K
(1 1
=
-
cos
éq
6.1.1-8
E
***S** - **S**) (**S** - **S**)*
S
E
µ
2

6.1.2 Existence of projection

The principle of this paragraph is to discuss on the question the existence the angle such as
S
projection of the point (E E
I, S) always belongs to the surface of load. These problems appear
1
essential for projections around the top of the surface of load, in other words when
S 0. There is by definition the relation:
S (.se - S)
S (.se - S)
cos =

éq
6.1.2-1
S
S
S
II
(
=
- S) (- S)
- S
II

E
p
E
~
By combining this equation with the expression: S = S - 2μ E

= S - 2μ G

One obtains:
~
.
S G
cos =

éq
6.1.2-2
S
~
S G
II
II
An estimate of coss is sought.
~
S G
.
Stage 1: estimate of

sII

One places oneself in this paragraph under the conditions: S 0 and F = 0.
~

~

Tr (G)
F

By definition of G and G one a: G S
. =
F
G -
I S. = G S. =
-
N N
S.

3

For preoccupations with a simplification of calculation one brings back the resolution of F to the resolution of the equation:

1
G (S) has (p
)
p

F =

- U (
G

has

S
, p
)
( )
= 0 F =

p
2
- U (,) () = 0

éq 6.1.2.3
0

0
C C
H
C C
H

By derivation of this new function one finds the relation:

F

1
G
p
has -
2
p

p
1 U
p
1
has -
=

-
1
() U has (

p
p
U
,) ( )
=
Q
- () U has (,) ()

0
H

0
H

C C
C C
U
m (p
) K (p
) 1
1
with:
= -

Q + I
0

H
C
6
3
C

Who gives after simplification:

F
2 = Q
WITH + I
B
éq
6.1.2.4

Where:

1
(p has
) m (p
) K (p
)
p
has -1
With =
1+
U (, p
) ( )

0

C C
H
6

éq
6.1.2.5

(p has
) m (p
) K (p
)
p
has -
B =
U (, p
) () 1

0
3 C C
H
S + I
F
sII

With
3B
One has as follows:
2 .N = (AQ + BI).
=
.
Q S +

2

+ 3
2
+ 3
2
sII
+ 3

And consequently:

~
F
F
G S
. =
-

N N
S.

S

+ I

With
3

=
B
S
Q
WITH + I
B -
Q S
. +
.

II
S.

2
+ 3
2
sII
+ 3
2
+ 3

3A
3B
=
.
Q S -
S
2
2
II
+ 3
+ 3

From where it is deduced that:
~
G.s
3A
.
Q S
3
B
=
-
éq
6.1.2.6
2
sII
+ 3
2
sII
+ 3

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By definition of Q one a:
G
1
cjs
S

54
cjs
det (S)
Q S
. = Dev.
S. =
1+
cos (3)
+
Dev.
.

S

H ()
S
5
2

sII
6 2
S

II

S

1

cjs
=
1+
cos (
3)

H ()
S
5
II

2

= H () S
II

One expresses finally:
~
G.s
3A

B
=
H ()
3
-
éq
6.1.2.7
2
sII
+ 3
2
+ 3

1
When S 0 then U (, p
) 0
and
With
, B 0

0
H
C C
And thus:

~
G.s
3h ()
When S 0 then

éq
6.1.2.8
0
S
S
II
0
C C
H (2
+ 3)

~
Stage 2: estimate of G
II
1
One places oneself in this paragraph under the conditions: S,
0 A
, B 0
0
H
C C
~
F
F

G =
-
nn

S

+ I

1

1
3B

sII

=
Q + I
B -
Q S

0
H

. +
0
2
2
H S
C C

.
2

+3 C C II
+ 3
+

3

$$
\begin{array}{l}
2 \\
1 \\
H() \\
= \\
Q - \\
\\
h0 \\
2 \\
+ 3\ h0s \\
C\ C \\
(\\
) \\
S \\
C\ C \\
II
\end{array}
$$

$$
\begin{array}{l}
2 \\
4 \\
2 \\
\sim \\
Q \\
H\ S \\
\\
2 \\
\sim\ \sim \\
H \\
II \\
() \ 2 \\
2 \\
2 \\
II \\
( ) \\
G = G.G = \\
+ \\
- \\
\\
II \\
(H \\
+ H\ S \\
+ H \\
C\ c)
\end{array}
$$

2
2
0
(
) 2
2
3 (C C) 2
0
2
II
(2) 3 (C C) 2
0

2
2
2
1

H
2
(+6) ( )

= (
Q

II
H
C C) 2
0

-
2

(2 +)
3

7.4

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It is shown [Appendix 2] that:

2
2
1

2

cjs

cjs

cjs

Q =
1+
cos
éq 6.1.2.9
II
(
3) +
+ cos
cjs
(
3) 1+
cos (
3)
H ()

10

2

4

2

and thus like H () = (1 + cos
:
cjs
(3) 1/6

2
~
H
cjs
H

H
2
1

1
1
( )6
2

6
1

6
2 2
2
G =

+
+
+
6
H
1
II
2
10
(
-)
( )
(+) ( )
(0
H
C c)
( )
+
-

H ()

2
2
4
2
2
2
2

(+)3

~
1
3h

1

1
cjs

H
2
() 2
2
2
2
2

-
(+6) ( )
G =

II
(hcc) 2
0

+
4
2h () +
4
4h () -
10
2

(2 +)
3

2
2
~
H

2
( )

1
2 -1

cjs

3

G =

+
+
- 1
II

0
H
C C 2h () 6
4h ()

12

2

+ 3
4

And consequently:
~
H ()
2
2

3

1
1
-1
G
cjs
=
- +
+
II

0

2
H

éq
6.1.2.10
+
C C

3
4

2h () 6
4h () 12

Stage 3: estimate of cos
S

One deduces from the two paragraphs precedent the expression of the following jetting angle:
When S 0 then:

3
cos

éq 6.1.2.11
S
2
2

1
-
(
3 1
1

cjs
2
+)3
- +
+
2
3

+ 4
(
2 1+
(
cos 3))
cjs
(
4 1+
(
cos 3)) 2
cjs

It is noticed that depends on the angle of Lode, and that consequently limit of
S
the jetting angle when S 0 does not exist. However a framing of cos allows us
S
to determine a zone of projection at the top a priori (demonstration of the framing in [Appendix 3]):

cos min

cos cos max
S
S
S

min
3
cos
=
S
2
with
:

2

2
3

cjs

éq 6.1.2.12

(+3)
+

2

+ 3
4 (1 - 2
cjs)

cos max

= 1
S
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6.1.3 Rules of projection

0 are called
I the intersection of the field of reversibility with the hydrostatic axis. One obtains:
1
0

3
S (p

C
)
I =
.

éq
6.1.3-1
1
m (p
)
By deferring 0
I and the framing of cos, when S 0, in the relation
1
S
E
I - I
3K
(1 1
=
-
cos, one deduces the following rules of projection from them according to
E
S - S) (S - S)
S
E
μ
2
sign parameter of dilatancy, and for values of E
I and of E
S given:
1
II

6.1.3.1 Case where the parameter of dilatancy is negative

E
0
I - I
3K
If 1
1

min
< -
cos
then projection will be regular;

2
S
μ
II
E
0
I - I
3K
If 1
1
max
> -
cos
then projection will be at the top.

2
S
μ
II

6.1.3.2 Case where the parameter of dilatancy is positive

E
0
I - I
3K
If 1
1
max
< -
cos
then projection will be regular;

2
S
μ
II
E
0

I - I
3K
If 1
1
min
> -
cos
then projection will be at the top.

2
S
μ
II

6.1.3.3 Interpretation
graph

(E E
I, S)

1
II

sII
3K
min

-
cos
3K
2
S
μ
max
-

cos
2
S
μ

Zone of regular projection

Intermediate zone

(E E
I, S)

1
II

Zone of projection at the top

0
I
I
1
1
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6.1.3.4 Equations
of flow

In the intermediate zone one solves the equations corresponding to a regular projection. If this resolution does not give a solution one then solves the equations of flow of projection with top.

In the case of projection at the top there are the relations:

S = 0 éq
6.1.3.4-1
0

3
S
. (p

C
)
I
=

éq
6.1.3.4-2
1
m (p
)
p
1
2nd

=
S éq
6.1.3.4-3
II
2µ 3

6.2
Local Recutting of the step of time

As for other relations of behavior (model CJS for example) one added possibility for the model of LAIGLE of redécouper locally (at the points of Gauss) the step of

time in order to facilitate numerical integration. This possibility is managed by the operand ITER_INTE_PAS of the key word CONVERGENCE of operator STAT_NON_LINE. If the value of ITER_INTE_PAS (itepas) is worth 0,1 or 1 it N `has no recutting there (note: 0 are the value by defect). If itepas is positive recutting is systematic, if it is negative recutting is taken in account only in the event of nonnumerical convergence.

Recutting consists in carrying out the integration of the plastic mechanism with an increment of deformation whose components correspond to the components of the increment of deformation initial divided by the absolute value of itepas (cf Doc. STAT_NON_LINE [U4.51.03]).

7
Internal variables

For implementation the data-processing we retained the 4 following internal variables:

7.1
V1: the plastic deformation déviatoire cumulated

The variable of work hardening p
is proportional to the second invariant of the tensor of the deformations déviatoires.
2
p
p p

=
E E
ij ij
3
tr
p
p
(pij)
with E = -

ij
ij
ij
3

7.2
V2: cumulated plastic voluminal deformation

The plastic voluminal deformation is defined by the relation presented at the paragraph [§3.2.4] on law of evolution of the plastic mechanism: p
= G
v
& &

7.3
V3: fields of behavior of the rock

Five fields of behavior, numbered from 0 to 4 (cf appears), are identified to make it possible to have a relatively simple representation of the state of damage of the rock, since the rock
intact to the rock in a residual state. These fields are a function of the deformation déviatoire figure cumulated p
and of the state of stress. Each increment of number of field defines it
passage in a field of higher damage.

· If the diverter is lower than 70% of the diverter of peak, then the material is in the field 0 ;
· If not:
- If p
= the 0 then material is in field 1;
- If
p
0 < < E then the material is in field 2;
- If
p

< < then the material is in field 3;
E
ult
- If p
> then the material is in field 4.
ult

State field of the rock
0 Intact
1 Damage
pre-peak
2 Damage
post-peak
3 Fissured
4 Fractured

7.4
V4: the state of plasticization

It is an internal indicator in Code_Aster. It is worth 0 if the point of gauss is in elastic load or in discharge, and is worth 1 if the point of gauss is in plastic load.

8
Detailed presentation of the algorithm

One retains a formulation implicit compared to the criterion and explicit compared to the direction of flow: the criterion will have to be checked at the end of the step, whereas the direction of flow is that
calculated at the beginning of the step (and thus the value of dilatancy will be also that calculated at the beginning of
no time).

One places oneself in a material point, and one considers that are given:

· The tensor of increase in deformations
from where E is deduced
and
;
v
· Constraints at the beginning of the step -
from where one deduces -
S and -
I;
1
· The values of the variables intern at the beginning of the step of time (only the plastic deformation -
cumulated p is necessary).

It is a question of calculating:

· Constraints at the end of the step of time;
· The variables intern in end of the step of time (p
, p
, fields of behavior);
v

· The tangent behavior at the end of the step:

8.1
Calculation of the elastic solution

E

=

- - T

E
S
=
-
S
+ 2μe
E
I
= I -
+ 3K

1
1
v

8.2
Calculation of the elastic criterion

Calculation of E
E
G = S H (E
)
II

m = (-
-
p
m
) S = (-
-
p

S
) has = (-
-
p
has
)
Calculation of
,
,
and -
K = K (-
has)

- -
E
- -
E
m K G
m K
Calculation of
E
-
-
U = -
-
I + S K
6 h0

1
3
C
C
C
1
E

has
G
-
Calculation of E
E
F =
- U

h0
C C
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8.3 Algorithm

If E
F > 0
Calculation of:
-
-
S
0

3 .
C
-
I
=
;
G = G S
1
-
(-)

m
-
= m S.A.
C
C m S.A.
C
0 (
-
-
-
, ,
)
-
;
= 0(- - -
, ,
)
-
;
= t0 (- -
,
)
0
0
T 0
0
0
-
= '(- - -
I, G,)
-
;
= (-
')
-
;

1
T 0
= (-
)
Calculation a priori of projection at the top
-

1
2
3 S
. (p

C
)
S = 0
top
; Calculation of p
p
E
p
=
+
S =
and of I =
= I
.
II
2μ 3
1
m ()
top
p
1
(I.E.(internal excitation) - top) < -3K
I
- E
S
S
1
1
cosmax
-

if

;

II
<
0

μ
2
If (

E
I - top) < - 3K
I
- E
S
S
1
1
cosmin
-

if

;

II

0

μ
2
Projection at the top is not retained a priori. The regular solution is calculated.

Q (-

F

G (- -
,) -
if
F

N (- -
,) -
-
)
-

if 0
if 0
0
Q =
N
=
G
=

Q (E)
-
if = 0

N (E, E)
-
if = 0

G (E, E)
-
if = 0
-
If p

= 0
0
-
Initialization
0
p
p
0
E
0
E
0
E
= 0;
=
; S = S; I = I; F = F
1
1

1p

= 1 max E

10
ij
And
1

p

1

p

=
3

~

Bfr
G
2
II
If not
Calculation of the increase in the plastic multiplier
by Newton:
0
-
Initialization
0
p
p
0
E
0
E
0
E
= 0;
=
; S = S; I = I; F = F
1
1

U
0
U
-
m K -
-
- m

=
= -
Q - K
I

6 h0

3
C
C
C
U 0
1
(km)
E
= -
G
1
km
ks

I

p
(- p
p
)
( )
-
0
(- p
p

) E ()
+
1
(- p
p
)

6

H

3

C
C
C
1
F 0
1 1
- -
-
1 A
has
0
E
U
F
-
-

-

=
G has

-
0
() Q -

has
H

C C
1
2
0
E
-
E
has
F

1 G
G has

0
-
U

F
-

=
-
log

-

-

p

0

0
(p
p
) p

p

H has
H

C C
C C
F 0*

F 0

~
F 0

2 ~

= -
(F
F
. 2µG + KG I)
F
+
G
p
II

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Buckle iterations N
N
F

n+1
N
= - F

N 1
+
N
N 1
+

=
+
N 1
+
2 ~
N 1
p
N 1
F
p
p +
+
F

=

G
;

=

G
II
v
3
N 1
+
+
~
N 1
N 1
E
p
F
N 1
E
p +
+
F
S
= S - 2

μ
G
;
I
= I - K
3

G
1
1
N 1
If
+
p

< 0 Not convergence
Calculation
N 1
+
Q
N 1
+
G
= G (N 1+
S);

N
m
= m (N 1
1
+
+
p

); NS = S (N 1
1
+
+
p

); Na = has (N 1
1
+
+
p

); N 1+
K
= K (N 1+
has);

N 1
+
N 1
+
N 1
+
N 1
+

N 1
+
N 1
+
m K
G
m K
N 1
+
N 1
+
N 1
+
U
= -
-
I
+ *S*
K

0
1
6
H
3
C
C
C
1
N 1
+
N 1
G
+
has
N 1
+
N 1
+
F
=
-

U

0
H
C C
n+1
n+1
n+1
N
U

m
K
1
+1
+1 m +
= -
Qn
N
- K
I

6 h0

3
C
C
C
n+1
U
1
(km)
n+
n+
1
1
p
G
1

(km) n+1

p
n+

(ks)
= -
n+

-

I
+

1
1
p

p
6

p
0
H

C
C
C

p
1
3

p

1
N 1
N 1
+
N 1
F
1 1

- +
+
has
n+
has
n+
N
U
+
N

1
+
=
has
G
Q
-

N 1
+
0
(1) 1
1
1

has
H

C C
1
n+
2
1
n+1
n+1
N
has
F

1 G

G +1 has

+1
n+1
U
=
-
log
+

-
p
N 1

0

0
() N
p
p
p

H has
H

C C
C C
N 1*
+
N
F

F 1
+
+
= -
(~
1

F
2 ~
. 2μG + KG I)
N
F
F
F
+
G
p
II

3
If
N
F +1/>

C
prec
n=n+1
If N > no. ite interns max

(I.E.(internal excitation) - top) > -3K
I
E
S

1
1
cos min

if

;

S
II
< 0

2µ
If

(I.E.(internal excitation) - top) > -3K
I
E
S

1
1
cos max

if

;

S
II
0

2µ
top
One retains projection at the top:
top
p
p
S = 0; I = I
;
=
1

1
If not
Not convergence
If not
Not convergence
If not
Convergence
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If FULL_MECA
Calculation of:
T
N
+1
F
F
H G
.
.
H
N
+1

*= **H***

+

T
n+1
N

2 F

~
+1
F
F
F
G -
HG
p
II

3

Mechanical symmetrization:
n+
1

n+1
+ T
N 1
sym = 1

+

2

9
Alternative on the expression of the criterion of plasticity

In this alternative proposal, one expresses the criterion of plasticity according to the first invariant and diverter of the constraints, by a retiming on triaxial in compression and extension by following relations:

9.1 Formulation
general
S.A. p
II
(1)

F =
-
p

U (,) 0 ***éq***
9.1-1
C
Where the expression of (
p
U,) is:

If
0
cjs

+ 0 - 0
2

U (,)
m (p K
H
H
H
p
) (p) ()
m
K
T
C
(p) (p)
= -
-
I + S

éq 9.1-2
0
0
1
(p

) K (p
)

6
H - H

3
C

T
C

C
If
= 0
cjs

3 1

U (,)
m (p
p
) K (p)
= -
+ cos (
3)
m (p) K (p)
-
I + S
1
(p) K (p) ***éq***
9.1-3
6
2 2

3
C
C

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9.2
Pace of the thresholds

One places oneself if
=

;
7
.
0
m =

;
21 S =
;
1 A = 1, then one traces the pace of the thresholds in
cjs
the plan perpendicular to the hydrostatic axis (known as plan), one standardizes compared to C and one consider the two values of containments such as I = 0 [Figure 9.2-a] and I = -
3

1
1
C

[Figure 9.2-b].
I1/3SIGC=0
0,1
0,05
)
T
has
(
T
E
0
Formulation
S
O
(a)
-0,15
-0,1
-0,05
0
0,05
0,1
0,15
Version 2
)
****
C
Formulation
C
Version 1 (a)
compression
ig
-0,05
II/s
(
S
-0,1
-0,15
(SII/sigc) *Sin (teta)
Appear 9.2-a: Pace of the thresholds for a null containment

I1/3SIGC=-1
2
1,5

1
)
T
has
0,5
(you
Formulation
S
O
0
(a)
Version 2
Formulation
*) *C -2*
-1
0
1
2
C
-0,5
Ve
Co rsion
mpre 1
ssibi
O S
N
I
G
I
I
/
S
-1
(S
-1,5
-2
-2,5
*(SII/sigc) *Sin (teta)*
Appear 9.2-b: Pace of the thresholds for a null containment in compression
One notes in these charts that the (a) formulation has the disadvantage of having one nonconvex pace in the plan.
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Appendix 1 Retiming of the criterion on the triaxial one in compression

By taking the general expression of the criterion under the conditions of triaxial in compression, one finds:

1
G (S) (p) m (p
has
) K (p
) G (S) m (p
) K (p
)
F =
- -
-
I + S
0
0
1
(p
) K (p
)

H

6

H

3
C C

C
C
C

1
2
(p has
)

2

- H

H
1
3
p
p
-

3

1 m () K ()
1
3
3
m (p
) K (p
)

=
+
+
(+
2

0
0

1
3) - S (p
) K (p
)

6

3

H
H
C C

C
C
C
C

1
has (p)
2

-
1
3
(
m p) K (p) 2
(
m p) K (p)

=
3

+

- +
(+2) - S (p) K (p)

1
3
1
3

6
3
3
C

C
C

1
has (p)
2

1
m p K p
m p K p

=
3
(
-
)
(
) (
)
(
) (
)

has (p)

+

-
+
(+2) - S (p) K (p)

1
3
1
3
1
3
3
3
C

C
C

1
has (p)
2

1
m p K p
m p K p

=
3
(
-
)
(
) (
)

has (p)

+
(
-)
(
) (
)
+
(+2) - S (p) K (p)

1
3
3
1
1
3
3
3
C

C
C

1
has (p)
2

1
m p K p

=
3
(
-
)
(
) (
)

has (p)

+
() - S (p) K (p)

1
3
3
C

C

1
2 A (p
)

1
3
1
p
p
2

=
(-
1
3)
m
(p has
)
has () ()
-

(- 3) + S (p
)

C
3
C

1
2 A (p
)

1
1

p
3
m

=
(-
1
3)
( )
(p has
) - (c)

p

has (p)
(- 3) + S ()

C

C

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Appendix 2 Standardization of Q

1

cjs
S

54
cjs
det (S)
Q =
1+
cos (
3)
+
Dev.

H () 5

2

2
sII
6.sII
S

det (S)
det (S)
T= is posed
and T D = Dev.
(cf reference document CJS R7.01.13)
S

S

1

3
54
2
cjs
2
2

cjs
D
D
cjs

cjs

Q =.

Q Q =
1+
cos 3
.
T .t
1
cos 3
.
S T

II
( ) +
+
+
() D
H ()

10

4
3

2

2 S
3 S
.
2
II
II

To evaluate this expression, one places oneself if S is diagonal by preoccupations with a simplification of
calculations.

S

S
2 S - S S - S S
2 3
1 2
1 3
1
S

S
2 S - S S - S S
2
1 3
1 2
2 3
S
D
1 S
2 S - S S - S S
3
As follows: s=
1 2
1 3
2 3
and T =

0
3
0

0

0

0

0

By using the property of S: S + S + S = 0

4
2 2
2 2
2 2
1
2
3
, it is shown that S
= 4
II
(1ss2 + 1ss3 + s2s3) and by
consequent:
S
2 S - S S - S S
S
2 S - S S - S S
2 3
1 2
1 3
2 3
1 2
1 3
S
2 S - S S - S S
S
2 S - S S - S S
1 3
1 2
2 3
1 3
1 2
2 3
-
-
-
-
D
D
1
S
2 S
S S
S S

S
2 S
S S
S S
1 2
1 3
2 3
1 2
1 3
2 3
s4
T .t =

.

II
=

9
0
0
6
0
0
0
0

One also shows starting from the property S + S + S = 0
3
3
3
1
2
3
that S + S + S = 3s S S =.
3 det (S)
1
2
3
1 2 3
and by
consequent:

S
S
2 S - S S - S S
1
2 3
1 2
1 3
S
S
2 S - S S - S S
2
1 3
1 2
2 3
. 54
. 54
cjs
D
cjs
S
S
2 S - S S - S S
. 54
.
S T =
3
.
1 2
1 3
2 3 = cjs
det (S) = .cos

3
3
3
cjs
(
3)
S
.
3
S
.

9
0
0
S
II
II
II
0
0
0
0
One deduces some as follows:

2
2
1

2

cjs

cjs

cjs

Q =
1+
cos

II
(
3) +
+ cos
cjs
(
3) 1+
cos (
3)
H ()

10

2

4

2

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Appendix 3 Framing of the jetting angle

3
It is pointed out that coss

S 0
(

3

1
1

-1
2
+)
2
2
3
cjs

- +
+

2
+ 3
4
2

(1+ cos
cjs
(3)) 4 (1+ cos
cjs
(3))2

2
1
-1
One poses: X () =
+
where [,
0 2
[

2(
cjs
1+ cos
+

cjs
( )
(41
cos () 2

cjs

It is noted that: X (-) = X (), function X being even one restricts the interval of study at [, 0 [

.

dX
sin
cjs
( )
The resolution of
= 0 give
.
+
=
3
cjs (
.
cos
cjs
) 0
D
2 (1+ cos
cjs
( )
( )

One deduces from it that the limits lower and higher of function X are:

X (=) = 1
0

4

(cjs) =
1
X
4(
where
cos

that

is such

1 - 2cjs)
cjs
(cjs) = -

cjs

One can thus give the framing of cos according to:
min
max
cos
cos cos
with:
S
S
S
S

min
3
cos
=
S
2
2

2
3

cjs

(+)3
+

2

+ 3

4 (1 - 2
cjs)

cos max

= 1
S

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Law of behavior of the porous environments: model
from Barcelona

Summary:

The model of Barcelona [bib1] described the behavior soil mechanics unsaturated coupled with hydraulic behavior (this model must thus be used in an environment THHM [bib7]). In the case private individual of a ground completely saturated with water, it is reduced to the model Camwood modified Clay, also
implemented in Code_Aster [bib5]. He is particularly adapted to the study of the behavior of clays.

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1 Notations

T
indicate the tensor of the total constraints in small disturbances, noted in the shape of the vector according to:

T
11

T
22

T

33

2 T

12
2 T
23

T

2 31

The behavior is described in a space of constraints to two variables:

T
= + p I
=
gz and p
-
C
pgz plq,

with plq, pgz, PC respectively pressure of liquid, gas pressure, capillary pressure (or suction)

One notes:

I the tensor unit of order 2 whose indicielle notation is
ij
I

4 the tensor unit of order 4 whose indicielle notation is

ijkl

1

P = - tr ()
constraint of containment
3

S = + pi
diverter of the constraints

1

I = tr

second invariant of the constraints
2
(s.s)
2

Q = = 3I
equivalent constraint
eq
2

= 1

(U T
+ U)
total deflection
2

= + +
partition of the deformations (elastic, plastic, thermal)
E
p
HT

= - tr + -

v
() 3 (T T0)
voluminal total deflection

p
= -

tr
v
(p)
voluminal plastic deformation

= 1
~
+ I
diverter of the deformations
v
3
~e ~ ~p
= -
deviatoric elastic strain

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p
p
1
~
p

= + I
deviatoric plastic deformation
v
3

E

2
=
tr ~ ~
.

equivalent elastic strain
eq
(E E)
3

p

2
=
tr ~ ~
.
equivalent plastic deformation
eq
(p p)
3

E index of the vacuums of the material (report/ratio of the volume of the pores on the volume of the solid matter constituents)

E initial index of the vacuums
0

porosity (report/ratio of the volume of the vacuums on total volume (pores plus grains))

E
p
, ,
lq
lq
lq content of total, elastic and plastic liquid

coefficient of swelling (elastic slope in a hydrostatic test of compression)

S elastic coefficient of rigidity in a test of variation of suction
1
(+ E)
0
K =

0

(1+ E)
K
0
=
0s

S

(p)
C coefficient of compressibility (plastic slope in a hydrostatic test of compression)

*
coefficient of compressibility in conditions of saturation

S coefficient of compressibility plastic in a test of variation of suction
1
(+)
0
=
E
K

(-)
(1+ E)
K =
0
S

(-)
S
S

M slope critical line of state

coefficient of correction of the normality of the plastic flow

P
(
)
idiots PC pressure of consolidation

P (
)
critical Cr PC pressure, variable interns model, equal to half of the pressure of consolidation

****
Cr
P pressure criticizes in conditions of saturation

S
P cohesion (hydrostatic traction limits to suction given)
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P confining pressure of reference generally equal to the atmospheric pressure
0
has
P

kc slope of cohesion according to suction

parameter controlling the increase in (p)
C with PC

R parameter defining the peak of (p)
C with PC

μ elastic coefficient of shearing (coefficient of Lamé)

f1 surface of load in space (P, Q)
f2 surface of load out of PC
pc0 threshold of irreversibility of suction
plastic multiplier
S
lq
water saturation, S
=

lq
lq

p
voluminal plastic deformation due to a loading in hydrostatic pressure
vp
p
voluminal plastic deformation due to a loading in suction
vs
p
~ deviatoric plastic deformation due to a loading in hydrostatic pressure
p
B coefficient of Biot
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2 Introduction

The concepts of plasticity used for the water-logged soils are extended on the unsaturated ground. The model
original of Barcelona is described using the variables, PC, which distinguishes it from the models from
mechanics coupled to a thermohydraulic behavior which is described using one only
effective constraint (constraint of Bishop). One can notice that this model is rewritten within a framework
poroplastic with the introduction of an additional poroplastic variable which is the water content [bib2], allowing to collect the phenomena of hystereses which appear on the drying cycles
damping. This phenomenon is not taken into account in the here exposed original model.

2.1
Phenomenology of the behavior of the unsaturated grounds

2.1.1 Curve of retention of water

In addition to the principal common mechanical aspects with the water-logged soils [bib3], the porous environments
comprising liquid and gas phases (grounds unsaturated with water) have as a characteristic
specific to be very sensitive to the phenomena of capillarity. The latter correspond to
localization of meniscuses of liquid (increasingly small as ground désature)
in which the water pressure is lower than the pressure of air (and all the more low as it
meniscus is small and thus the désaturé ground). One thus sees appearing the concept of pressure capillary or
suction p = (p - p). While drying, a ground unsaturated has a water content lower what
C

gz
lq
lq
corresponds to a higher suction. The correspondence between these two sizes is the curve of retention of water (cf [Figure 2.1.1-a]). This one is obtained by drying of a ground initially saturated (suction is then null) and damping starting from the dry state.
PC
lq

Appear 2.1.1-a: Curve of retention of water

2.1.2 Extension of the definition of the effective constraints on the unsaturated ground

The behavior soil mechanics unsaturated is primarily observed in laboratory with the assistance apparatuses with controlled suction (oedometers and triaxial). The modeling of this behavior mechanics was initially tried by extending the concept of effective constraint to the unsaturated mediums.
This one is a function of the total constraint and intersticielle pressure: '
= F (T
, p)
lq. In
the saturated case, one has simply additivity of the pressure and the constraint:
T
'= - p I
lq because
pressure of water acts in the same way in water and the solid in all the directions.
The widening of this concept in the mediums unsaturated in the years 1950 (holding account with

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pressures of the two liquid phases) brought the following form of the effective constraint:
'
= (T
- p I) + G (p)
gz
C of which there remained the constraint suggested in the form:
'
T
D = D (- pgz) + bSlq dpc
where S is the degree of saturation out of water and B the coefficient of Biot [bib7].
lq
As of the years 1960, the experimental observation clarifies certain limitations of the concept of effective constraint extended on the unsaturated ground. In particular, the test of collapse to the oedometer
at fault the constraint of Bishop puts: This test consists in consolidating a sample unsaturated with maintaining suction constant, then with the remouil er with constant loading. One then is observed collapse of the ground. If the consolidation is continued, the curve corresponds to a standard test in saturated. However, if one refers to the effective constraint, this one decreases during the remoistening (since
suction p = (p - p) is cancelled) and as it is supposed to control the deformation, it would owe y
C
gz
lq
to have swelling what is contradictory with the experimental observation. Majority of mechanics of the grounds agree now on impossibility of describing it completely behavior of the grounds unsaturated using one only constraint and note the need for using two independent variables (constraint and suction).

3
Description of the original model of Barcelona

In this model, the curve of retention of water does not have hysteresis, and it is not modified by the mechanical deformation as it is the case in the presentation made by Dangla and coll [bib2]. It exist nevertheless a threshold in capillary pressure pc0 with beyond which unrecoverable deformations
appear. In this paragraph one distinguishes a mechanical part which treats deformations mechanics induced by a mechanical loading and a hydro-mechanical part which treats effect suction on mechanics before writing the equations of the complete behavior.

3.1

Purely mechanical behavior

The assumption is made that suction p remains constant during the mechanical transformation.
C
deformations resulting from the variation of the constraint are subscripted.
p
One examines the behavior, under successively spherical and deviatoric loading, it
behavior being considered isotropic.

3.1.1 Spherical loading

3.1.1.1 Elasticity

The mechanical state of a ground unsaturated under hydrostatic request is determined by tests oedometric with controlled suction. As for the water-logged soils, volume v of the sample varies logarithmiquement with the load with a slope of way reversible until a pressure of consolidation
idiots
P
(PC). One will choose independent of p, the experiment showing weak
C
dependence of the elastic slope with respect to p.
C

The elastic component of the voluminal deformation varies then like:

&
E
P
& =
if P
P
(p)
vp
<

éq 3.1.1.1 - 1
1
idiots
C
+ e0 P
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The preceding expression is in fact derived from a test oedometric with constant suction where one measure the variation of the index of the vacuums according to the loading, from where the following elastic law:

P = P0
[
exp k0 (
p
-
vp
vp)]

éq 3.1.1.1 - 2
(1+ e0)
with K =
, where P is the value of reference corresponding to E

and E = E, index of
vp = 0
0

0

0
vacuums initial.

3.1.1.2 Plasticity

Beyond the pressure of consolidation, the behavior of the ground is plastic and the slope (p is c)
dependent on suction (cf [3.1.1.2 Figure - has]), this dependence being estimated way semi-empirical following: (p = 0 1 - exp - +
c)
() ([R) (PC) R]
(p)
where R =
C
is a constant connected to the maximum of the rigidity of the ground and a parameter which
(0)
control the evolution of rigidity according to suction.

(PC) P&
The voluminal rate of deformation is then: vp
& =
if P > P,
1+ E P
idiots
0
p
((PC) -) P&
from where the plastic component: &vp =
.
1 + E
P
0
The expression of P is thus written:
P = P

éq
3.1.1.2-1
0
[
exp K (p
vp)]
(1+ E)
with K =
0

-

Note:

The two expressions [éq 3.1.1.1 - 2] and [éq 3.1.1.2 - 1] are similar to those of the model of Camwood-Clay [bib5] with the parameter (or K) depend on the capillary pressure. compressibility of the ground decreases with suction.

---

v

(pc1)
(pc2)
(p = p
C
c1)
(p > p
c1
c2)
(p = p
C
c2)
2
1
ln P

P
P0 P
p
P
p
idiots (
1
c)
idiots (c2)
0

Appear 3.1.1.2 - has: Variation of specific volume under loading oedometric

3.1.2 Loading
triaxial

3.1.2.1 Elasticity

The elastic component of the deviatoric deformation is proportional to the loading:
~e
S
=

éq 3.1.2.1 - 1
2µ
µ is independent of suction.

3.1.2.2 Plasticity

Into a triaxial compression test of revolution, one introduces the shear stress Q = - (one will be able
1
3
to extend the formulation which follows with the 3D by using the standard within the meaning of von Mises of the constraint).
When suction becomes null (saturated medium), the model is supposed to be reduced to the Cam_Clay model
modified [bib5]: the threshold of plasticity is then an ellipse of center (*, 0)
Cr

P
who cuts the axis of
hydrostatic constraints in zero and a value of pressure of consolidation *
*
P
= 2P.
idiots
Cr
surface of load associated with a nonnull suction p is also an ellipse of center
C
P
(P (p)
S
-
)
0
,
Cr
C
(cf [3.1.2.2 Figure - has]) which cuts the hydrostatic axis out of P
(p) = 2P (p)
2
idiots
C
Cr
C
and - P, P representing a cohesion varying linearly with suction: P = K p. the line
S
S
S
C
C
representing the critical states (null voluminal variation) the same slope M preserves as that in saturated but shifted condition S
P. The equation of the surface of load in the diagram (P, Q)
for p given is written:
C
2
2
Q - M (P + P
éq
3.1.2.2-1
S) (2Pcr - P) = 0

The plastic flow in the plan (P, Q) thus with p constant is not associated the surface of
C
charge. If it were the case, one would have:

1
~

p
F
p
F
& = &
& = 1
&

vp
P

S

and the following report/ratio:
~ p
&eq
Q
2

=

,

éq

3.1.2.2-2

p
2
&vp
M (2P + S
P - 2 Cr
P)

similar to the report/ratio obtained in the Camwood-Clay model (with S
P = 0). In fact in this model, one
introduced a parameter of correction which destroys the character of normality, so that:

~ p
&eq
Q
2
=

. is given by the authors of the model [bib1] as being:

p
2
&vp
M (2P + S
P - 2 Cr
P)

M (M - 9) (M - 3)

1

=
(

éq

3.1.2.2-3

9 6 - M)

1-

(0)

This corrector allows to better take into account the experimental results, and in particular of to better estimate the coefficient of thorough grounds.

Q
(PC > 0)
M
(PC = 0)
M
P
- P
***
S
idiots
P
P (p)
idiots C

***Appear 3.1.2.2 - has: Criterion in space** (P, Q)*

3.2
Hydro-mechanical coupling or effect of suction on mechanics

The variations of suction (with constant load) involve deformations (those will be then subscripted by) reversible when p <
and irreversible when suction exceeds the threshold p.
S
C
pc0
c0
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3.2.1 Part reversible

The tests oedometric with constant constraint and controlled suction give us the variation of the index of the vacuums according to suction [Figure 3.2.1-a] reversible in lower part of the threshold in suction:
PC
E - E = - Ln
if
0
p < p
S
,
C
c0
patm
with S independent of the state of containment.
E - E
Deformation being able to be written:
0
- = -
, one a:
v
v0
1+ e0

E
S
p&c
1
p&c
& =
=
vs

éq** **3.2.1-1

1+ E p
0
+
0
+
C
patm
K S (PC patm)
The evolution of suction is written then:
1 + E
p = p exp K (-
***,** *with K*
0
=
éq
3.2.1-2
C
atm
(E E
0s
vs
v0)
0s
S
E
e0
S
S
Lnpc
Lnp c0
Lnpa

Appear 3.2.1-a: Evolution of suction

3.2.2 Part irreversible

Beyond threshold PC, unrecoverable deformations appear, the slope in the test oedometric
0
becoming S. This slope can actually depend on the hydrostatic constraint applied to the sample, but it is considered constant in the original model of Barcelona. Like one can note it on [Figure 3.2.2], the pressure of consolidation increases with suction. [Figure 3.2.2 (A)]) shows two compression tests in condition saturated (p and unsaturated

C = 0)
(p
*. A relation between **
P
(point 3) value of the preconsolidation with of saturated and P
(point 2)
C > 0)
idiots
idiots
pressure of preconsolidation in unsaturated is established by comparing specific volumes obtained on ways according to items 1, 2, 3 [Figure 3.2.2 (has)] who describe a discharge of P
*with **
P

idiots
idiots
*with constant suction followed by a remoistening of a value p to 0 with constant pressure **
P
, from where
C
idiots
the following equation:
v + v

+ v

= v
1
pressure
suction
3
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The assumption is made that the reduction of suction 2 3 is accompanied by recoverable deformations.
dp
The elastic relation is as follows:
C
FD = -
, where p
is the atmospheric pressure.
S (p + p
atm
C
atm)
One writes for item 1 and 3 the expression of volume as follows:

P
v = NR (P) - p ln

0
(c) P0
where P is a pressure of reference corresponding to an initial volume NR (P). One combines this
0
0
expression and elastic relations:

+
NR (
P
P
p
p
P
P - p ln idiots + ln idiots + ln C
atm = NR 0 - 0 ln idiots
0)
(c)
S
( ) ( )
*

*
P
P
p
P
0
idiots
atm
0

By eliminating initial volumes by the elastic relation:

0
+
v
(P

0)
= NR (0) - NR (P0)
p
p
C
atm
=
ln
S
PC
patm

one then determines the following evolution of the threshold of consolidation in unsaturated condition:

(0)-

P P (PC)*
-
idiots
=

idiots

P0 P0
Like P

= 2P,
idiots
Cr

One finds:
(0)-

P 2P (PC)*
-
P = 0 Cr

éq
3.2.2-1
Cr

2 P0

*[Figure 3.2.2] visualizes way 1-2-3 in the plan **(P, p)***
C.
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v
NR (0)
G O N fl EM in T
NR (PC)
PC

3
v
v
3
suction
v
1
S I
2
v pressure
2
2
1
v1
(p, S
C
)
L C
effo N D R EM in T
(PC = 0, S = 1)
3
P
*
P*
LnP
*
P*
ref.
P
P
idiots
idiots
idiots
idiots
(A)
(b)

Figure 3.2.2: (A) Curves of compression for water-logged soils and not-saturated (b) criterion in the diagram *(P, p*
c)

The total component of the voluminal deformation due to the evolution of suction is:
S

p&c
& =
if p > p
vs

éq
3.2.2-2
C
atm
1
(+ E) p
0
+
C
patm
from where the plastic component which is written:

p
(-
S
S)
p&c
1
p&c
& =
=
vs
éq
3.3.2-3
1
(+ E) p
0
+
+
C
patm ks PC patm

Note:

The variation of suction does not generate deviatoric deformations.

3.3
Complete behavior (mechanical and hydrous loading)

3.3.1 Behavior reversible

Under spherical loading, the evolution of the total voluminal elastic component is thus written:

1 &
E
E
E
P
1
p&c
& = & + & =
+
v
vp
vs

éq 3.3.1-1

K P
K
0
0
+
S (PC
patm)

The evolutions of the parts hydrostatic and deviatoric of the constraint are thus written:

P&
K
p
E
0
&c
= K
,

éq 3.3.1-2

0 & -
v
P
K
p
0
+ p
S (C
atm)
~e
s& = 2
µ &,

éq 3.3.1-3
ij
ij
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3.3.2 Thresholds of flow

The two thresholds of the reversible field are such as:

Mechanical criterion: F (P, Q, P (p), p)
2
2
Q
MR. P K p P
P p
éq 3.3.2-1
Cr
C
C
=
+
+ C C
- Cr C
1
(
) (2 ()) 0
Hydrous criterion: F p p
p
p

éq 3.3.2-2
C
C
= C - C
2 (
,
0)
0
0
The three-dimensional field of reversibility in space (P, Q, p) is represented on
C
[Figure 3.3.2-a].
These two criteria are reduced in the plan (P, p) to curves called LC (loading collapse) and IF
C
(suction increase) (cf [Figure 3.3.2-b]).

Q
f1
f1
*
Pcons*
f2
PC
IF
LC
F
P
2

Appear 3.3.2-a: Surface of load in space *(P, Q, p*
c)

PC
p
IF
c0
LC
- PS
Pcons
P

Appear 3.3.2-b: Surface of load in space *(P, PC)*

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3.3.3 Laws of flow

The plastic flow is controlled by the two criteria in pressure and suction:

p
F
1
2
&
&
&
v =
= M (2P - 2 Cr
P + kc PC)

éq 3.3.3-1
P

~ p
F

F
1

Q
1
& = &
= &
=
3 &s

éq 3.3.3-2
S

Q

S

p
1
p&c
~
or &

v =
, p
& = 0

éq 3.3.3-3
ks PC + patm

3.3.4 Laws
of work hardening

The evolution of surfaces of load is controlled by the forces of work hardening: Cr
P and pc0.
The laws of work hardening of each surface are:
P&
On F, Cr
p
= K

éq 3.3.4-1
1
&v
C
P R

p&
On F,
c0
p
= K

éq 3.3.4-2
2
S &v
p 0 +
C
patm

3.3.5 Inventory

configurations
of mechanical and hydrous coupling

One examines the various configurations of loading in space (P, p)
C.

3.3.5.1 Reversibility
total

The loading represented by the point M (cf [3.3.5.1 Figure - has]) is inside the field of reversibility: elasticity, and hydrous reversibility. That results in:

F < 0
F =, f& <
p&
).
C <
1
, or (
0
0
1

1
), and p <
C
pc0, or (p =
C
pc0,
0

The relations expressing this reversibility are:

P&
K
p
0
&c
= K

0 & -
v
P
K
p
0
+ p
S (C
atm)

i.e.:
(
***exp** K -*
*0 **(**v*
*v0)**)***
P = P0
,

éq 3.3.5.1 - 1
K/K
0
0s
p +

C
patm

p

atm

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and
S =

µ~
2

éq 3.3.5.1 - 2
C
p
PC
C
p 0
pc0
M
field of reversibility
P
Slq

Appear 3.3.5.1 - has: Field of reversibility in the plan** (P, p **- curve of retention of water
c)

Elastoplastic 3.3.5.2 Behavior

The point M touches the criterion of mechanics alone (cf [3.3.5.2 Figure - has]):

F = 0, f& = 0, and p < p (or p = p and p&
)
C < 0
1
1
C
c0
C
c0

The elastic evolution is thus written:

P&
K
p
E
0
&c
= K
,
0 & -
v
P
K
p
0
+ p
S (C
atm)
i.e.:
(E E
exp K -
0 (v
v0))
P = P0

éq 3.3.5.2 - 1
K/K
0
0s
p +

C
patm

p

atm

and
S =

µ~
2

éq
3.3.5.2-2

The evolution of the components of the plastic deformation is:

~ p
& =
3
S

p
2
&v = M [2P - 2 Cr
P + kc PC]

The evolution of the mechanical threshold is written:
p
2
Cr
P& = K Cr
P &vp = K Cr
P
M

[2P - 2 Cr
P + kc PC].
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A specificity of the original model of Barcelona is the assumption that mechanical work hardening is completely coupled with the hydrous work hardening (cf [3.3.5.2 Figure - has]) from where the relation:

p&
&
C
K
0
S
Cr
P
=

éq 3.3.5.2 - 3
p 0 +

C
patm
K Cr
P

C
p
p
2
C 0
1 M
P

Appear 3.3.5.2 - has: Coupling of mechanical work hardening to hydrous work hardening

Hydrous 3.3.5.3 Behavior generating of the unrecoverable deformations

The point M reaches the threshold in suction (cf [3.3.5.2 Figure - has]):

p = p and p&

C > 0
C
c0

The mechanical behavior is elastic:

(E E
exp K -
0 (v
v0))
P = P0
, S =

µ &
&
~
2
éq
3.3.5.3-1
K/K
0
0s
p +

C
patm

p

atm

but as the mechanical threshold is coupled with that of suction, there is also mechanical work hardening:

&cr
P
K
p&c
=
0
éq
3.3.5.3-2
P
K p 0 + p
Cr
S
C
atm
The rate of plastic deformation is written:
p
1
PC
==
&
&v

K
+
S PC
patm
~ p
& = 0
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C
p
M
p
1
c0
2
P

Appear 3.3.5.3 - has: Coupling of hydrous work hardening to mechanical work hardening

3.4
Data of the model of Barcelona

The model requires the following parameters:

1) Elastic parameters provided under key word ELAS:
The thermal dilation coefficient, two elastic coefficients E
, provided in
given from which the coefficient of Lamé µ is calculated.

2) Under key word CAM_CLAY:
·
0
P Initial hydrostatic pressure equal to noted atmospheric pressure Pa under the key word CAM_CLAY
· instead of giving the initial index of the vacuums E one gives the initial porosity which must be of 0

value equalizes with that given under key word THM_INIT, noted PORO.
*· Parameters associated with surface threshold LC (forced isotropic): **
P, equalizes with half
Cr
*pressure of preconsolidation **
P
noted
idiots
PRES_CRIT,
***
, the coefficient of
compressibility for a saturated state and the elastic coefficient of compressibility, noted LAMBDA and KAPA.
· The slope criticizes M,

3) Under the key word BARCELONA:
· R and, coefficients allowing to calculate (p, noted R and BETA.
c)
· parameters related to a variation of suction: , coefficient of compressibility related to one
S
variation of suction in the plastic range, coefficient associated with the change with
S
suction in elastic range, noted LABDAS and KAPAS.
· K the parameter which controls the increase in cohesion with suction
C
· the initial threshold of suction p, noted PC0_INIT
c0
· the coefficient of normality, noted ALPHAB.

Here a set of values of some of these parameters, resulting from [bib1]:
(0) =
;
2
.
0 =
;
02
.
0
R =
;
75
.

0
=
5
.
12
1
-
MPa; P = 10
.
0
MPa;
0

G
MPa M
K
S =
;
08
.
0
S =
;
008
.
0
= 10
;
= ;
1 C = 6
.
0
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4
Numerical integration of the relations of behavior

4.1
Recall of the problem

The numerical integration of the model is similar to that carried out for the Camwood-Clay law [bib5], in
operating a translation on the axis of the capillary pressures.
This model is obligatorily coupled with the hydraulic behavior, contrary to Camwood-Clay which can be used within a purely mechanical framework (one simulates a drained behavior then).
The model of Barcelona is thus usable only within the framework of established behaviors THHM in Code_Aster [bib7] and [bib8]. It will be more particularly employed with modelings KIT_HHM and KIT_THHM (in this last case, there is not for the moment of dependence of mechanical characteristics specific to the model of Barcelona with the temperature).
The variables of entry of the model are U
or
and P1, P2, P1 and P2 being equal in
quoted modelings with p
, p
, p and p that it is with hydrous modelings
C
gz
C
gz
LIQU_VAPE_GAZ or LIQU_GAZ.
The variables of exit of the model are: '
, P, p, P.
Cr
c0
S
The following notations are employed: Has, A, A
respectively for the quantity evaluated at the moment
known T, at the moment T + T

and its increment. The equations are discretized in an implicit way,
i.e. expressed according to the unknown variables at the moment T + T
.

One will note:
-
-
Cr
P the quantity P (-)
Cr PC,
(-
PC) -

-
P 2P (PC)
-
P (
)
0
Cr
Cr PC quantity

and
2 P0
P (p) = p (p) exp K

Cr
C
Cr
C
(statement)

4.2 Relations
incremental

The rules of flow and the condition of consistency give the following relations of flow:
If the threshold f1 is reached, the increment of plastic deformation voluminal is written:

p
1
(2P - 2 Cr

P + kc PC)
Q
kc

v =
(2
)

éq 4.2-1
K (
P
Q
P
P
p
P + K
2
C PC)

+
-
Cr -
C
Cr
P
2
M
2

The increment of the standard of the equivalent plastic deformation is then:

2

p

Q
Q
2
kcQ (2 Cr
P -

P)
eqp =
P

Q
p

2
4
2
Cr
kP (P + kc PC)
+
-
C

M
M (2P - 2 Cr
P + kc PC)
M (2P - 2 Cr
P + kc PC)

éq 4.2-2

and the tensor deviatoric is written:
~ p
3 S

p

=

v
éq
4.2-3
M2 (2P - 2 Cr
P + S
P)
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If f2 is reached, the increment of plastic deformation voluminal is determined by:

p + p
p
1
=
Ln (c0
atm)
éq
4.2-4
v
K
p + p
S
c0
atm
plastic deformation being purely voluminal (~
p
= 0)
p
will be the principal of the problem and given unknown factor while solving
v
F (-
P, -
Q, -
P (p), p,
Cr
C
C p
) 0
1
v
=, or F (p,

C p
)
C
= 0
2
0
, the increment of plastic deformation
voluminal being obtained starting from p
c0.On then deduces the evolution from it from the constraints and the thresholds.

4.3
Calculation of the constraints and the internal variables

The elastic prediction of the deviatoric constraint is written:
= -
S + μ ~
2

éq 4.3-1
One chooses the elastic prediction E
P in the following way:

-
exp K
E
(0 v)
P = P

éq 4.3-2
K K
0/0 S
p + p

C
atm

p + p
C
atm
(-
PC) -
P
-
P (PC)
E
E
p
2

-
If F < 0
F <
0
P = P S = S
= P =
Cr
p
,
Cr
C =
1
and
0
2
, then
,
,
,
0
,
0
2

0
P0
If not:

E
~ p
S = S - 2μ

éq 4.3-3
E
P = P exp [
p
- K

éq
4.3-4
0 v]
(p -
c)

-

2
(

PC)
P
P

-
0
Cr
P =
exp K

éq 4.3-5
Cr

[statement]
2
P
0
(p
éq
4.3-6
0 + p
) = (p-0 + p) exp K
C
atm
C
atm
[p
S
v]

The principal unknown factor is thus
p
.
v

If F > 0
1
, then

While replacing
p
~

by its expression according to
p
[éq 4.2-3] one obtains:
v
E
S
S =

éq 4.3-7
p
6

µ ν
1+ m2 (2P-2 crP +kcpc)
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and:
E
P = P exp [
p
- K

éq 4.3-8
0 ν]

The unknown factor is given by solving F (P, Q, P, p)
Cr

C = F (E
P, E
Q, -
P (p), p,
Cr
C
C p
)
v
= 0
1
1
,

I.e.:
p
2
2

μ

2
6
E
v
Q = - M 1
+

+
- 2
,
M2

(
P K p P
P
2P - 2P + K p
Cr
C
c)
(
C
c) (

Cr)

or:
p
2

2
6
2

E
μ
Q
= - M 1 +

M

2 (
v
E
2P exp (
p
- k0
-
v) - 2 Cr
P (PC) exp (
p
K v) + kc PC)
[eP exp (
p
- K
éq
4.3-9
0 v) + kc PC]
[eP exp (
p
- k0
-
v) - 2 Cr
P (PC) exp (
p

- K v)]

If F > 0
2
, then: p 0 =
C
PC, the unknown factor are immediately given by:

p
1
p
+ p
=
Ln (c0
atm)
v
, éq
4.3-10
K
-
S
PC 0 + patm
from where
E
S = S and
E
P = P exp [
p
- K

éq 4.3-11
0 v]
(p -
c)

-

2

(

PC)
P
P

-
One has moreover
0
Cr
P =
exp K.

éq 4.3-12
Cr

[statement]
2
P
0

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5 Operator tangent

If the option is: RIGI_MECA_TANG, option used at the time of the prediction, the tangent operator calculated in
each point of Gauss is known as of speed:

elp
& = D
ij
ijkl kl
&,

i.e.
elp
D is calculated starting from the not discretized equations.
ijkl

If the option is: FULL_MECA, option used when one reactualizes the tangent matrix while updating internal constraints and variables:

D = A D
ij
ijkl
kl

In this case, A is calculated starting from the implicitly discretized equations.
ijkl
The tangent operator of the generalized constraints is implemented in THHM under name DDE and partitionné in several blocks. The blocks concerned with the model are [DMECDE], [DMECP1] [bib8].
One calculates the contribution of the model to each one of these blocks for the tangent operator in elasticity,
the operator of speed and the coherent operator.

5.1 Nonlinear elastic tangent operator

The elastic relation of speed of the model of Barcelona is written:

K
p
~
0
&c
&
&
2 &
ij = - P ij + sij
& = K Ptr
0
&ij + μ ij +
P
ij
éq
5.1-1
k0s
PC + patm
2
K
p
0
&c
&ij = (K P
0
- μ) tr& ij + μ
2 ij
& +
P
ij éq
5.1-2
3
k0s
PC + patm
The tensor of the constraints used in the model of Barcelona (and the tests determining them given model) is a function of the total constraint and of the gas pressure and is written:
T
= + p I
gz

éq 5.1-3

The tensor of the constraints of Bishop '
used in Code_Aster is such as: & = & '+& I with
T
P
&
= -
-
P
B (p&gz Slq p&c) éq
5.1-4
From where the expression of the constraint of Bishop according to the constraint of the model of Barcelona:
& '
= & + (B
(-)
1 p& - bS p
gz
lq & C) I
éq
5.1-5
Note:

The constraint of Bishop is generally regarded as an effective constraint
(controlled only by the deformation). It is not the case of the model of Barcelona where it
is necessary two constraints ((, p)
C to describe the behavior. Consequently, in
'
&
& p
the tangent operator, the term
does not summarize itself with -
.
p
C
p
C
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'
Part [DMECDE] of matrix DDE corresponding to is such as:

4
2
2

'

K P
µ K P
µ K P
µ 0
0
0
&

11
0 +
0
-
0
-
11

&

3
3
3
'

2
4
2
&
&
22
K P
µ K P
µ K P
µ 0
0
0
0
-
0
+
0
-
22

éq 5.1-6
'

3
3
3

&
&
33

33

2
2
4
′ =

K P
μ K P
μ K P
μ 0
0
0
0
-
0
-
0
+

2
2

&
&
12

3
3
3

12

′

0
0

0
2μ
0
0

2
2

&
&
23

23

0
0
0
0
2μ
0
'

2
2

&
&

31

31

0
0
0
0
0
2μ
1

4
4
4
4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
4
4
4
3
E
D

'

Part [DMECP1] of matrix DDE is reduced to
with (p
who is such as:
1 = PC)
p
1
'

11
&

'
&

22

'

&

33
k0
P
k0
P
k0
P

=
- bS
- bS
- bS
0 0 0

lq
lq
lq
{p

&}
1
'
2 K
0 p + p
K
S
C
atm
0 p + p
K
S
C

atm
0 p + p

12
S
C
atm
&

'
2

&
23

'
2

&
31
éq 5.1-7

5.2
Plastic tangent operator of speed. Option RIGI_MECA_TANG

The total tangent operator is in this case obtained starting from the results known at the moment T (the option
I 1
-
RIGI_MECA_TANG called with the first iteration of a new increment of load).
If with T the border of the field of reversibility is reached, the condition is written: f& = 0 which must be
I 1
-
checked jointly with the condition F = 0. If with T one is strictly inside the field,
I 1
-
F < 0, then the tangent operator is the operator of elasticity.

If the mechanical criterion is reached:
f& = 0
1

f1
f1
f1
f& =
& +
Cr
P& +
p&
0
1
C =

éq 5.2-1

Cr
P
PC
P

P

like
Cr
p
Cr
Cr
P& =
&v +
p&c, then:
p

p

v
C
f1

f1 Cr
P
p
Cr
P

f& =
F

& +
(
&v +
p&)
1
C +
p&
0
1
C =
éq
5.2-2

Cr
P
p

p
p
v
C
C
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p&
One has in addition:
E
E
C
&ij = D & kl + 0

ijkl
K P

éq 5.2-3
k0s (PC + p
) ij
atm

i.e.:
E
E
F

1 F
1

p
1
&c
&ij =
& -

-

+ 0

ijkl
D
kl
ijkl
D

kl
K P
éq
5.2-4
S

kl
3 P

K

0s (PC + p
) ij
atm

By writing the plastic module of work hardening:

F
1 P

F
Cr 1
H = -
,
p

éq 5.2-5
P
p

P
Cr

v
The equations [éq 5.2-2] and [éq 5.2-5] give:

f1
F
F P

&ij - H p + (1
1
+
Cr) p&c = 0 éq 5.2-6

ij
PC Cr
P PC
F
1
Multiplication of the equation [éq 5.2-4] by
give:
ij
F
1
F
1
E
F
1
E
F

1 F
1

1
F
1
p&c

&
éq 5.2-7
ij =

& -

-

+

ijkl
D
kl
ijkl
D

kl

K P

0
ij
ij
ij
ij
S
kl
3 P

ij
k0s (PC + patm)
The two preceding equations make it possible to find:

F
1
E
F

1
E
F

1 F

F

p
1
1
1
&c
F

F
1
1 Cr
P
H

p =

-

-

+

+
+
ijkl
D
&kl
ijkl
D

kl
K P

0
ij
(
) p&
ij
ij
S
kl
3 P

K
p + p
p

P

p

kl
ij
0s (C
atm)
C
C
Cr
C
éq 5.2-8
from where and to deduce the expression from it from the plastic multiplier:

F

1
E
F
1
1
F

F
1
1 Cr
P

& +
0
+ (
+
)
ijkl
D
kl
K P ij
p&
ij

ij
k0s (PC + patm)
C
p
C Cr
P
p
C
=
éq
5.2-9
F
1
E
F

1 F
1

-
1
+
ijkl
D

kl

H

p
ij
S
kl
3 P

That is to say H the definite elastoplastic module like:
F
1
E
F

1 F
1

H =

-
1
+
ijkl
D

kl
H

p

éq 5.2-10
ij
S

kl
3 P

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The plastic multiplier is written:

F

1
E
F
1
1
F

F
1

P
1

D & +
K P
Cr

ijkl kl
0 ij
+ (
+
) &

k0
+

ij

ij
S (p
p
C
atm)
p
p
P
p
C
C
Cr
C
=
éq
5.2-11
H
While replacing by his expression in the equation [éq 5.2-4], one obtains:

E
1 F
1
E
E
F

1 F
1

&

1
ij =
-

.

-

-
ijkl
D
&kl
Dmnop &op
ijkl
D

kl
H

mn

S

kl
3 P

1 f1
1
F

F

1
1

Cr
P
E
F

1 F
1

K P

K P
0

+ (
+
) D

-
1
-
0
p

&
H mn
k0s (PC + patm) mn
ijkl
kl
p
C Cr
P
p
C
S

kl
3 P

K

0s (PC + patm) ij
C

éq 5.2-12
One thus deduces the elastoplastic operator from it
elp
E
p
D
= D - D:

E
1 F
1
E
E

F

1 F
1

&
1

ij =
-

-

-
ijkl
D
ijop
D
Dmnkl
mn
&kl
H
3

COp
S
mn
P

1
4
4
4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
4
4
3

D p

ijkl

1

F

1 F

1
1
E
F
1
1
F

F
1
1

Cr
P
K P
0

-
COp
ijop
D

(

K P
0 mn
- (
+
)) -
p&
H
S

3

COp
P

mn
k0s (PC + patm)
p
C Cr
P
p
C
k0s (PC + patm) ij
C

1
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
4
2
4
4
4
4
4
4
4
4
4
4

4
4
4
4
4
4
4
4
4
4
3

D PC

ij

éq 5.2-13
with,
p
1 f1

E
E
f1
1 F

D
=

1
ijkl
ijop
D
Dmnkl
-
mn
H COp
smn 3 P

and

p
1
F

1 F
1
1
E
F
1
1
F

F
1
1

Cr
P
K P
D C
0

***ij* = -**

-

0
+ (
+
)
COp
ijop
D
K P mn
+

H
S

3

COp
P

mn
k0s (PC + patm)
p
C Cr
P
p
C
K

0s (PC + p
) ij
atm
éq 5.2-14
Calculation of
p
D:
ijkl
F

1
1

= - m2 (2P - 2P + K p + S
3 ,
éq
5.2-15
Cr
C
c) ij
ij

3
ij
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who is written in vectorial notation:

- 1 2
M (2P - 2Pcr + K p
C
c)

+ 3s11
31
-
2
M (2P - 2Pcr + K p
C
c)

+ 3s22
3

- 1 2
M (2P - 2P

éq
5.2-16
Cr + K p
C
c)

+ 3s33
3

3 2s12

3 2s23

3 2s31

from where the expression of:
-
2
K MR. P 2P 2P
K p
6 S
µ
0
(- Cr + C c)+

11
-
2
K MR. P 2P 2P
K p
6 S
µ
0
(- Cr + C c)

+
22
µ
E
F

-
2
K MR. P 2P 2P
K p
6 S
0
(- Cr + C c)+

D

éq
5.2-17
ijkl
:
33

µ
kl

6
2s12

6µ 2s

23

6µ 2s31

and
F

E
F
1 1 1f

4
2
2

D

-

= K
-
+
+ μ
ijkl
kl
0M P (2P
2P
K p)
12
Q
éq
5.2-18

S

3 P
Cr
C C
ij

kl

However the plastic module H is written in the form:

F

E
F

1 F
1

H =

-
1
+

ijkl
D

kl
H

p
ij
S
kl
3 P

4
H = M (2P - 2P + K p
Cr
C c) [K P (2P - 2P
+ K p
Cr
C c) + 2kPcr (P + K p
C c)]
2
0
+12µQ éq 5.2-19

While posing:
2
2
ij
With = - K MR. P
0
(2P - 2 Cr
P + kc PC) ij + 6 S
µ ij, A' ij = - K MR. P
0
(2P - 2 Cr
P + kc PC) ij + 6 S
µ ij,
éq 5.2-20
with: tr ()
With = -3

2
K MR. P (2P - 2P + K p)
0
Cr
C
C
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In A'
In A'
In A'
6 2
With
µ S
6 2
With
µ S
6 2
With
µ S

11
11
11
22

11
33
11 12
11 23
11 31
A' A
In A'
In A'
6 2
With
μ S
6 2
With
μ S
6 2
With
μ S

11
22
22
22
22
33
22 12
22 23
22 31
1 A A'
In A'
In A'
6 2
With
μ S
6 2
With
μ S
6 2
With
μ S
p
D =

33

113
22
33
33
33
33 12
33 23
33 31

H 6 2 A
μ 'S
6 2 A
μ 'S
6 2 A
μ 'S
2
36μ s2
2
36μ S S
2
36μ S S
11 12
22
12
33 12
12
12 23
12 31
6 2 A
μ 'S
6 2 A
μ 'S
6 2 A
μ 'S
2
.
36μ s2
2
36μ S S

11
23
22

23
33
23
23
23 31
6 2 A
µ 'S
6 2 A
µ 'S
6 2 A
µ 'S
2
.
.
36µ S

2
11
31
22
31
33
31
31

SYM
éq. 5.2-21

'
One can write the components
D
piece [DMECDE] of the matrix
Which is those of

the operator
elp
E
p
D
= D - D.
'

According to the equation [éq 5.2.14]. Components
with (p

piece [DMECP1] of
1 = PC)
p
1
stamp DDE are:

****

2

M
2P
( )
0
Cr
-
K (2P
C
Cr - P) - 2P (P
Cr
+ K p) Ln
C C

'

- tr ()
With
P

((p)
C -
2
0
)

K P
A11
' +
'
With +
0
11
- bS
3Hk
lq
0s (PC + patm)
H
k0s (PC + patm)

****

2

M
2P
( )
0
Cr
-
K (2P
C
Cr - P) - 2P (P
Cr
+ K p) Ln
C C

'

- tr ()
With
P

((p)
C -
2
0
)

K P
'
To 22+
'
With
+
0
22
- bS
3Hk
lq
0s (PC + patm)
H
k0s (PC + patm)

*

2
2P
( )
0
Cr
-
'

M K (2P
C
Cr - P) - 2P (P
Cr
+ K p) Ln
C C

2

P

- tr ()
With
0

((p)
C -)

k0P

'
With
'
With
bS
3Hk
lq
0s (PC + patm)
33+
33+
H
k0s (p
p
C
atm) -
+

*

2
2P
( )
0
Cr
-
'

6 2µM K (2P
C
Cr - P) - 2P (P
Cr
+ K p) Ln
C C

2

- 2 2µtr ()
With
0
P

((p)
C -)

S

S

Hk0s (PC + patm) 12 +

12

H

****

2
2P
(0)
Cr
-

6 2µM K (2P
C
Cr - P) - 2P (P
Cr
+ K p) Ln
C C

'

- 2 2µtr ()
With

P ((p)
C -
2
0
)

S

S

Hk0s (PC + patm) 23 +
23

H

****

- 2 2µtr ()
With
2
2P
( )
0
Cr
-

13
S + 6 2µM K (2P
C
Cr - P) - 2P (P
Cr
+ K p) Ln
C C

'
13
S

Hk0s (PC + patm)

P ((p)
C -
2
0
)

éq 5.2-22

with '
=
= -([)
0 1
(- R) exp (- p
)
C]
p
C

If the hydrous criterion is reached:
p
One leaves again the equation [éq 5.2.3] with this time p
C
=
&
&
,
ks (PC + patm)
One finds a relation direct enters & and &, p&c of the form:
E
p&
p
&
= D & + K P
C
&c
(
+
)
0

éq
5.2-23
ks (p + p
C

atm)
k0s (PC + atm) I
p
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One deduces then the constraint from Bishop
'
E

K P
K P

&
= D & + (
0
+
0
) - bS
p
I

& éq
5.2-24
K
S (PC + patm) k0s (PC + patm)

lq
C

'

Components
D
piece [DMECDE] of the matrix
OF are nothing other than those

matrix
E
D.
'

The only components of piece [DMECP1] of matrix DDE are thus those of
with
p
1
(p
:
1 = PC)
'

11
&

'
&

22

'

&
33
K

0P
1
1

k0P
1
1
k0P
1
1

=
(
+
) - bS
(
+
) - bS
(
+
) - bS
0 0 0 p

&
'
lq
lq
lq
2 (p + p
C
atm) K
K
S
0s
(p + p
C
atm) K
K
S
0s
(p + p
C
atm)
{} 1
K
K
S

0
12
S
&

'
2

&
23

'
2

&
31
éq 5.2-25

5.3
Tangent operator into implicit. Option FULL_MECA

To calculate the tangent operator into implicit, one chose as for the model Cam Clay separating initially treatment of the deviatoric part of the hydrostatic part for then them to combine in order to deduce the tangent operator connecting the disturbance from the total constraint to disturbance of the total deflection.

5.3.1 If the mechanical criterion is reached

5.3.1.1 Treatment of the deviatoric part

It is considered here that the variation of loading is purely deviatoric (P =) 0 .
The increment of the deviatoric constraint is written in the form:
S
= μ ~
2
- ~

éq
5.3.1.1-1
ij
(

p
ij
ij)
Around the point of balance (- +
), one considers a variation S of the deviatoric part of
constraint:
S = μ ~
2
- ~ éq
5.3.1.1-2
kl
(
p
kl
kl)
Calculation of ~ p
:
kl

It is known that:
~ p
=
3
kl
skl
éq
5.3.1.1-3
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By deriving this equation compared to the deviatoric constraint, one obtains:
~ p
=
3
+ 3
kl
skl
skl
éq
5.3.1.1-4
Calculation of:
One a:
1 F

F

F
Cr
P

1 F

F

F

F
Cr
P

=

mn + (
+
) p
C =

S
mn +
P

+ (
+
) p
C
H p mn
p
C Cr
P
p
C
H

p S
mn
P

p
C Cr
P
p
C

= 1 [S
3
2
2
mn S
mn + M (2P - 2 Cr
P + kc PC) P
- M [kc (2 Cr
P - P) + (
2 P + kc PC) P' Cr] p
C]
H p
éq 5.3.1.1 - 5
If one considers only the evolution of the deviatoric part of (P =)
0, then:

(H
2
p) = H p + H p = [
3 smnsmn + 3smnsmn] - 2M
P
Cr

P +

M 2k
2
2
C Ppc - 2M kc Cr
P PC - M [kc (2 Cr
P - P) + 2P' Cr (P + kc (PC + PC))]PC
éq. 5.3.1.1 - 6
P

with
Cr
P' Cr =

p
C
However:
P
P = kP.
Cr
Cr
v

Like
p
2
= M

(2P - 2P + K p), one a:
v
Cr
C
C
p

2
2
2
v = M (2P - 2 Cr
P + kc PC) - 2M Cr
P + kcM p

C
éq.
5.3.1.1-7
From where:

1
M2 (2P
2
2
- 2 Cr
P + kc PC) =
+ 2M

Cr
P - kcM PC
éq
5.3.1.1-8
Cr
kP

In addition,
H = 2kM 4P P + K p 2P - 2P + K p
p
Cr (
C
c) (
Cr
C
c)
and

H = 2kM 4 P + K p (2P - 4P + K p
) P + 2kP M 4k 3P - 2P + 2k p p
p
(
C
c)
Cr
C
C

Cr
Cr
C (
Cr
C
c)
C
éq 5.3.1.1 - 9

By injecting this last equation in the equation [éq 5.3.1.1 - 6], one obtains:

H
4
2
2
p + [2 km

(P + kc PC) (2P - 4 Cr
P + kc PC) + 2M
P
+ 2M kc p
C] P =
-[
Cr
2
4
2
Cr
kP M kc 3
(P + 2kc PC - 2 Cr
P) + M [kc (2 Cr
P - (P + P
) + 2P' Cr (PC + p
c)] + 2P' Cr P] PC +
[
3 smn S
mn + 3smnsmn]
éq 5.3.1.1 - 10
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While using the relation [éq 5.3.1.1 - 8], it comes then:

[3s S

+ 3s S
mn
mn
mn
mn]
Zpc
=
-

éq
5.3.1.1-11
(H +)
With
(H +)
With
p
p

2
M (2P 2P
K p)
with
With = [

K 4

M (P + K p
C c) (2P - 4Pcr + K p)
C C +
2
MR. P +
2
M kcpc]
- Cr + C C

1
2

+ M

2kPcr

2
2 kkc M (P + kc PC) (2P - 4P
+ K p)
Cr
C C

Z = M
+ 2
2
kk P M
3
(P - 2P + K p)

1
C Cr
Cr
C C

+

2

2
km

Cr
P

2
(2
2
2
M kcP + kc M
p
c)
2
- M kc P
+
+ M kc (2 Cr
P - P) + 2 2
Mr. P' (
Cr P + kc (PC + PC))
1
+
2
2
km
C
P R

One then obtains immediately the variation of the deviatoric part of the plastic deformation:

p

= 9
~
(

+

+ 9

kl

smn smnskl smn smnskl)
smn smn skl
H p + A
H p

3
2

3
2

+
3
éq
5.3.1.1-12
M (
Z
2P - 2 Cr
P + kc PC) P
skl -
M kc (2 Cr
P - P) p
cskl -
pcskl
H p
H p
H p + A

- 6 m2 (P + kc PC) P' Cr p
cskl
H p
S is written then:
ij
18
~
µ
18µ
sij = 2µij -
([sklsijskl + sklsijskl)]-
skl S
klsij
(H p + A)
H p
6µ

2
6µ
2
6µ
-
éq
5.3.1.1-13
M (
Z
2P - 2 Cr
P + kc PC) P
sij +
M kc (2 Cr
P - P) p
csij +
sijpc
H p
H p
(H p + A)
12µ
+
M2 (P + kc PC) P' Cr p
csij
H p

i.e.:

6µ
2
18µ
18µ

ijkl + ijkl
M (2P - 2 Cr
P + kc PC) P +
(sklsij + sklsij) +
smnsmnijkl
H

p
H p + A
H p

S
kl =
6µ
2
12µ

-
2
- 2 -
2
+
'

ijkl
kcM (Cr
P
P) PC
M (P kc PC) P Cr PC

H

p
H p

6
~
µ Z
2µij +
sijpc
H p + A
éq 5.3.1.1 - 14
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or in tensorial writing:

D
6µ
2
18µ
6µ
I 1+
M (2P - 2
2

+
+

-
-

4
Cr
P
kc PC) P
S: S
M kc (2 Cr
P
P) PC

H

p
H p
H p

S =

18µ
12µ

éq 5.3.1.1 - 15
+
(S + S
) S -
M2 (P + K

C PC) P' Cr
p
C
(H

p + A)
H p

6
Z
~
µ
2µ +
spc
(H p + A)
that one can still write by symmetrizing the tensor (S + S
) S:

D
6µ
2
6µ
2
18µ
12µ
I 1+
M (2P - 2

2

+
-
- +

-
+

4
Cr
P
kc PC) P
kcM (2 Cr
P
P) PC
S: S
M (P kc PC) P' Cr PC

H

p
H p
H p
H p

S =

18μ

+

(H

p + A)

6
Z
~
μ
2μ +

spc
(H p + A)
éq 5.3.1.1 - 16
1
with: = [
T
((S + S
) S) + (S (S + S
))]
2
Calculation of, while posing:
=
+
ij
T
sij
sij
T S
T S
T S
2T S
2T S
2T S
11 11
11 22
11 33
11 12
11 23
11 31
T S
T S
T S
2T S
2T S
2T S
22 11
22 22
22 33
22 12
22 23
22 31

T
T S
T S
T S
2T S
2T S
2T S
S = 33 11
33 22
33 33
33 12
33 23
33 31
2T S
2T S
2T S
2T S
2T S
2T S
12 11
12 22
12 33
12 12
12 23
12 31

2T S
2T S
2T S
2T S
2T S
2T S
23 11
23 22
23 33
23 12
23 23
23 31

2T S
2T S
2T S
T S

2T S
2T S

31 11
31 22
31 33
31 12
31 23
31 31
1
= [
T
T
(S) + T
(S)]
2
That is to say:

D

C =
1
I
+ 3
2
kc
4
M (2P - Pcr + K p
C C) P + 9
2
S S - 3
:
2
M (2Pcr - P PC - 6
)
2
M (P + K p
C c)

P' Cr p
2µ
H
H
H
H
C

p
p
p
p

+
9

(H p +)
With

one poses:
9
C = (S
: S)
H p
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3
D =
M2 (2P - 2P + K p
Cr
C c) P

H p
- 3
G =
M 2k 2P - P p

C (
Cr
) C
H p

6
H = -
M2 (P + K p P' p
C
c)
Cr
C
H p

The symmetrical matrix C of dimensions (6,6) is too large to be presented whole, one break up into 4 parts C, C, C and C:
1
2
3
4
C
C
1
2
C =

C
C
3
4
with

1
9
9
9

+ C + D + G + H +
11
S 11
T
(11
T s22 + 22
T
11
S)
(11
T 33
S + 33
T 11
S)

2µ
(H p +)
With
2 (H p +)
With
2 (H p +)
With

9
1
9
9

1
C =
(22
T
11
S + 11
T s22)
+ C + D + G + H +
22
T s22

(22
T
33
S + 33
T s22)

2 (H p +)
With
2µ
(H p +)
With
2 (H p +)
With

9
9
1
9

(33
T 11
S + 11
T 33
S)
(22
T
33
S + 33
T s22)
+ C + D + G + H +
33
T
33
S

2 (H p +)
With
2 (H p +)
With
2µ

(H p +)
With

éq 5.3.1.1 - 17

9 2
9 2
9 2

(11
T 12
S + S T)
(
11 12
11
T 23
S + S T)
(
11 23
11
T 13
S + S T)
11 13
(
2 H p +)
With
(
2 H p +)
With
(
2 H p +)
With

éq 5.3.1.1 - 18
C2 = 9
2
(22

T 12
S +
9
2
S T)
(
22 12
22
T
23
S +
9
2
S T)
(
22 23
22
T 13
S + S T)
(
2 H p +)
With
(
2 H p +)
With
(
2 H p +)
22 13
With

9 2

(33
T 12
S +
9
2
S T)
(
33 12
33

T
23
S +
9
2
S T)
(
33 23
33
T 13
S + S T)
(
2 H p +)
With
(
2 H p +)
With
(
2 H p +)
33 13
With

C =
3
C2

éq 5.3.1.1 - 19

1

18
9
9

+ C + D + G + H +
12
S
12
T
(12
T s23 + 23
T 12
S)
(12
T s23 + 23
T
12
S)

2μ
(H p +)
With
(H p +)
With
(H p +)
With

9
1
18
9

C4 =
(23
T
12
S + 12
T s23)
+ C + D + G + H +
23
T s23
(23
T

13
S + 13
T s23)

(H p +)
With
2µ
(H p +)
With
(H p +)
With

9
9
1
18

(13
T 12
S + 12
T 13
S)
(13
T s23 + 23
T 13
S)
+ C + D + G + H +
13
T 13
S

(H p +)
With
(H p +)
With
2µ
(H p +)
With

éq 5.3.1.1 - 20

Calculation of the rate of variation of volume:
p

= m2 (2P - 2P + K p),
v
Cr
C
C
p
= M
2 (2P - 2P + K p) - 2M 2 P + M 2k p éq 5.3.1.1 - 21
v
Cr
C
C
Cr
C
C
= B + Dpc
B
3
BZ
=
(S + S
). S + (D -

) PC
(H + A)
(H + A)
p
p
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2
M (2P - 2Pcr + K p)
with:
2
B = M (2P - 2P

Cr + K p)
2
C
C
- M
C
C.
1
2
+ M
2kPcr
2
K M
C

and
2
2
D = K M
C
- M
.
1
2
+ M
2kPcr
One thus has:
p
3B
BZ

v =
(S + S

). S - (
- D
) PC éq
5.3.1.1-22
(H p + A)
(H p + A)
and finally:
B
= (C -
(S + S)) S -
ij
ijkl
kl
ij
kl
(H + A)
p

éq
5.3.1.1-23
BZ
D
ij
3Z
(-
+ +
+
S
) p
ij
ij
ij
C
(
3 H + A)
3
K
3
(p + p) (H + A)
p

0s
C
atm
p

5.3.1.2 Treatment of the hydrostatic part

It is considered now that the variation of loading is purely spherical (S = 0).
The increment of P is written in the form:

E
-
exp (k0v)

P = P
-
1
éq
5.3.1.2-1

K
K
PC + p
0/0s

atm

-

PC + p

atm

The derivation of this equation gives:
P = K P
0
(
p
k0

P
v - v) -
PC éq 5.3.1.2 - 2
k0s PC + patm
Calculation of
p
:
v
It is known that:
p

= m2

2P - 2P + K p

éq 5.3.1.2 - 3
v
(
Cr
C
c)

By differentiating this equation, one obtains:

p

2
v = M ((2P - 2 Cr
P + kc PC) + (
2 P -
2 Cr
P + kcpc)

éq 5.3.1.2 - 4
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One knows the expression of:

M2 (2P - 2
2
Cr
P + kc PC) P + 3ss - M [kc (2 Cr
P - P) + 2 (P + kc PC) P' Cr] PC
B
=
=

H p
H p
éq 5.3.1.2 - 5
while posing
B = m2 (2P - 2
2
Cr
P + kc PC) P + 3ss - M [kc (2 Cr
P - P) + 2 (P + kc PC) P' Cr] PC

While differentiating, it comes:
2

= M ([P
2 - P
2
K p P

2 P
2 P

K p

P K P
2
(
P) p

K 2
(P

P
) p
P
(
2
K p P
) 'p

(
2 P
K p
P
) '
p
Cr + C c)
+(
- Cr + C c) - C Cr -
C - C
Cr -
C -
+ C C Cr C -
+ C C Cr C]
HP

4

3

km
2
b2 PP

P
2
P

K p
P

P
2 2
PP
4
P
4 K p
Km No
3
p k2p2
Cr
- Cr + C c+ Cr (- Cr - Cr C C + C C + C c)
-
2

2

HP +k P P3 P
2
K
2 p p

C Cr (
- Cr + C c)

C

éq 5.3.1.2 - 6
One seeks the expression of P according to:
Cr

One a:
p
P = kP
éq
5.3.1.2-7
Cr
Cr
v
One can write:
Cr

P = m2 (2P - 2
2
Cr
P + kc PC) + M (
2 P -
2 Cr
P + kcpc) éq
5.3.1.2-8
Cr
kP
1+ 2M 2

Cr
kP

2
2
2

Cr
P
= M (2P - 2 Cr
P + kc PC) + 2M P + M

kcpc

Cr
kP

M2 (2P - 2 +

2 2

2

Cr
P
kc PC) Cr
kP
M
Cr

kP
M kc Cr
kP

Cr
P =
+
P +
PC

2

2

2

1 + 2 Cr
kP
M

1+ 2 Cr
kP
M

1+ 2 Cr
kP
M

éq 5.3.1.2 - 9
One poses

M 2kP 2P 2P
K p
2M 2
K

2
C M
Cr
kP
Cr
kP
Cr (
- Cr + C c)
C =
[
, has =
, D =

1+ 2M 2kP
[1+2M 2
[1+2M2 Cr
kP]
Cr
kP]
Cr]
One has then:
Cr
P = aP + C + dpc

éq 5.3.1.2 - 10
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By replacing the expression of P in [éq 5.3.1.1 - 6], one finds:
Cr

(2P - 2
2
Cr
P + kc PC + 2P + kcpc - 2aP - 2akcpc - 2P Cr
'PC) P - 2c (P + kcpc
) M
=
.
+ (K

C (P + P - 2 Cr
P) - 2D (P + kcpc) - 2P Cr
'(P + kc (PC + PC)))PC
H

p

2kM 4b
-
[
2kM 4b
Cr
P (4P - 2 Cr
P + 3kc PC) + has (2P - 4 Cr
P + kc PC) (P + kc PC)]P -
[C (2P - 4 Cr
P + kc PC) (P + kc PC)]
H2
2
p
H p
2kM 4b
-
[kc Cr
P (3P - 2 Cr
P + 2kc PC) + D (2P - 4 Cr

P + kc PC) (P + kc PC)]PC
H 2p
éq 5.3.1.2 - 11
By gathering the terms in and those out of P, one finds:
F
H
= P + PC éq 5.3.1.2 - 12
E
E
with,
2
M
F =
[2P - 2P + K (p + 1 (- 2a) p) + 2P - 2aP - 2P'
Cr
C
C
C
Cr PC]
H p

4
2kM B
-
(4P - 2P + 3k p P + has 2P - 4PP - 4P K p + 3Pk p + K p
2
[
Cr
C c) Cr
(2
2 2
Cr
Cr C C
C C
C c)]
H p
M2
H =
[- 2dP - 2dkcpc + kcP - 2kc Cr
P + kcP - 2P' Cr (P + kc (PC + PC))]
H p

2kM 4b

-
[D (2P - 4 Cr
P + kc PC) (P + kc PC) + kc Cr
P (3P - 2 Cr
P + 2kc PC)]
H 2p
2
4
2cM (P + K p)
C C
2bckM
E = 1 +
+
2P - 4PP - 4P K p + 3Pk p + K p
2
(2
2 2
Cr
Cr C C
C C
C
c)
H p
H p
The expression of
p
thus becomes:
v
p
v = XP + Ypc éq 5.3.1.2 - 13
with,
2
X =
F
F
M (2 - 2a - 2c
+ (2P - 2P
K p
Cr +
))
C
C
E

E
2
Y =
H
H
M ((2P - 2P
K p
C
D K

Cr +
C
c)
- (2
+ 2 -)
c)
E
E

from where the expression of P according to and p
:
v
C

1

P 1
(+ K PX) = K P (- Y
+
)
0
0
V
p

éq
5.3.1.2-14
K (p + p
)
C

0s
C
atm

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Calculus of the variation of deviatoric deformation:
~
~ p
F
H
ij = =
3 S = 3 Psij + 3 pcsij

éq 5.3.1.2 - 15
E
E
One thus has finally:

= ij
F P + Kijpc

éq 5.3.1.2 - 16
ij
with
3 F
1 + K PX
0
D

F =
S -
1 ,
E
K
3 P
0

éq
5.3.1.2-17
H
3
K PY
K P
0
0
D
K =
S - (
+
1
)
E
3
K
3
(p + p)
0s
C
atm

5.3.1.3 Operator
tangent

The tangent operator connects the variation of total constraint to the variation of the deformation and of suction. Since the increment of the total deflection under loading deviatoric is written:
B

1
ij + H ijpc = C
(ijkl -
(S + S) kl ij) Dklmn mn,

éq 5.3.1.3 - 1

(H p + A)
with:
2/3 -1/3 -1/3 0 0 0

-1/3 2/3 -1/3 0 0 0

1
-1/3 -1/3 2/3 0 0 0
D =

éq
5.3.1.3-2
0
0
0
1 0 0
0
0
0
0 1 0

0
0
0
0 0 1
projection in space deviatoric,

and that under spherical loading one a:

2
ij - K ijpc = ij
F Dkl kl ***éq***
5.3.1.3-3
with:
-1/
3

-1/
3

2

-1/
3
D =

éq
5.3.1.3-4
0
0

0
hydrostatic projection, one has then:
ij = ijkl
With kl + Bijpc ***éq***
5.3.1.3-5
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with:
-1

B

1
2

ijkl

With
= (Cijmn -
(S + S)
)
mn ij Dmnkl +
ij
F Dkl ***éq***
5.3.1.3-6

(H p +)

With

1

-
B

B = (C
-
(S + S
))
1
2
D
+ F D (H - K)
ij
ijmn
éq
5.3.1.3-7
(H +)
mn ij
mnkl
ij
kl
kl
kl

p
With

The constraint of Bishop is thus written:

'= A + B - bS p
ij
ijkl
kl
(ij
lq)
C

5.3.2 Tangent operator at the critical point

As for model CAM_CLAY one writes a tangent operator specific to the critical point. Like for the case general, one makes a treatment of the deviatoric part and another for the part hydrostatic.

5.3.2.1 Treatment of the deviatoric part

According to the equation [éq 4.3.3] one finds:

~

E
p
E
F
S = S - 2
μ = S - 2μ
= - 6
μ S ***éq***
5.3.2.1-1
S

The expressions of the plastic multiplier and its derivation are written in the following way:
Qe

E
E
=
-
Q
Q Q
1/6μ

and =
-
éq
5.3.2.1-2
Q

2
6
Q
µ
6
Q
µ
with,
E
E
3
3
E
S S

S S
Q =
and Q =

E
2 Q
2 Q
from where the expression of:
1 3 sese
Qess
=

-

éq
5.3.2.1-3
E
3

6µ 2 Q Q
Q

Let us point out in the same way the expression of S:

S = μ ~
2
-
3 S -
3 S
ij
(ij
ij
ij)

While replacing and by their expressions, one can write:
E
E
E
E
3 S
~
S
3 Q
1 Q

kl
kl
S = 2μ -
S +
S S S -
-1 S ***éq***
5.3.2.1-4
ij
ij
E
ij
3
kl
kl ij

ij

2

Q Q

2 Q
Q

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E
E
E

1 Q
1
3 Q

3 S .s
kl
ij
S

+
- -
S .s = 2μ

-

~ éq 5.3.2.1 - 5

kl
ijkl
ijkl
ijkl
3
kl
ij
ijkl
E
kl

Q

2 Q

2 Q Q

or in tensorial writing:

E
E
E
Q

D
D
1
3 Q
S
S
S
I + I 1
D
- -
S S

=

I -
éq
5.3.2.1-6
4
4
μ
3
2
3
4

~
Q

2 Q

2
E
Q Q
1
4
4
4
4
4
4
2
4
4
4
4
4
4
3
1 4
4 2
4
4
4 3

G
H
As S does not depend on, one can confuse ~ with.
v

By using the tensor of projection in the space of the deviatoric constraints 1
D [éq 5.3.1.3 - 2], one
can write:
1
1
-
D.G.
=
H

.

éq
5.3.2.1-7
2µ

5.3.2.2 Treatment of the hydrostatic part

In tensorial writing, there is according to the equation [éq 5.3.1.2 - 2] the following relation:
K
D
P
0
D
I P = K P
éq
5.3.2.2-1
0
-
I p
v
C
K
p
0
+ p
S
C
atm

knowing that at the critical point,
p

.
v = 0

As P then does not depend on ~ one can confuse with.
v
K
D
P
0
D
I P = K P

éq 5.3.2.2 - 2
0
-
I PC
K
p
0
+ p
S
C
atm
By using the tensor of projection in the space of the hydrostatic constraints
2
D [éq 5.3.1.3 - 3], one
can write:
2
K
D
P
0
D
I D = K P

0
-
I PC
K

p
0
+ p
S
C
atm
from where
D
2
D
I D
I
=
+
p

éq 5.3.2.2 - 3
K P
K
p
0
0
+ p
S (C
) C
atm
5.3.2.3 Operator
tangent

By combining the contributions of the two parts deviatoric and hydrostatic, one finds the writing of the tangent operator who connects the variation of the total constraint to the variation of the total deflection to
not criticizes:
1
-1
D
2
D
D G
. H

I D
I
=
+

.
+
p

2µ
K P
K
p
0

0
+ p
S (C
) C
atm

= A - B p
éq
5.3.2.3-1
ij
ijkl
kl
ij
C
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with
1
1
1
-
D
2
-
D. G.H
I D
With
éq
5.3.2.3-2
ijkl =
+
2

µ
K P
0

and
D
I
B = -

éq 5.3.2.3 - 3
ij
K
p
0

+ p
S (C
atm)

As it is necessary to deduce the variation from the constraint of Bishop, one finds:

D
I
B = -
- bS

éq 5.3.2.3 - 4
ij
K
p
0
+ p
S (C
)
lq
atm

5.3.3 If the hydrous criterion is reached

The variation of the elastic strain is written in the form:
E
E
1
~
E
= - éq
5.3.3-1
kl
kl
v
kl
3
that is to say:

S
P
p
E
kl
C
=
-
-
éq
5.3.3-2
kl
kl
2µ
K
3 P
K
3
p
0
0
+ p
S (C
) kl
atm
In this case the plastic deviatoric deformation is null thus the plastic deformation has the following expression:
p
1
p
= - ***éq***
5.3.3-3
kl
v
kl
3
that is to say:
p
p
C
= -

éq

5.3.3-4

kl
K
3
p 0 + p
S (C
) kl
atm

By combining each component rubber band and plastic one finds:

S

1
1
1

E
p
P
kl
= + =
-
-
+
p

éq 5.3.3-5

kl
kl
kl
kl
2µ
K
3 P

3 K
p
0
0
+ p
K p 0 + p

S (C

atm)
S (C
atm)
C kl

By using the matrices of projection in the space of the deviatoric and hydrostatic constraints one leads to the following expression:

D1
D
2

1
1
1

ijkl
ij
kl
=
-

-
+
p
éq 5.3.3-6
kl
ij
2µ
K
3 P

3 K
p
0
0
+ p
K p 0 + p

S (C

atm)
S (C
atm)
C kl

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thus one can write:

-1
1
1
2
1
2
-

ijkl
D
ij
D kl
1
1
1

ijkl
D
ij
D kl

éq 5.3.3-7
ij =
-
kl +
+
-
p

2µ
K
3 P
3 K
+
+

0
0s (C
p
atm
p
) ks (cp0 atm
p
)
kl
C
2

µ
K
3 P
0

D1
D2
A is posed
= ijkl
ijkl
- ij kl

2μ

K
3 P
0

2
2μ
1
2μ
1
2μ

+
- +
- +
0 0 0
3 9k P
3 9k P
3 9k P
0
0
0

1
2μ
2
2μ
1
2μ

- +

+
- +
0 0 0
K P
K P
K P
1 3 9
3
9
3 9
0
0
0

or A
μ
μ
μ
ijkl =
- 1 + 2
- 1 + 2
2 + 2
0 0 0
2μ

éq 5.3.3-8
3 9k P
3 9k P
3
9k P

0
0
0
0
0
0
1 0 0

0
0
0
0 1 0

0
0
0
0 0 1

and by deducing the constraint from Bishop, one finds:

1
1
1

'= A-1 +
+
A-1 - bS

p ***éq 5.3.3-9***
ij
ijkl
kl
3 K
p
0
+ p
K p 0 + p
S (C
atm)
S (C
atm)
ijkl
kl
lq
C

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6
Summary of the model of Barcelona

Modelings THHM:

KIT_HHM and KIT_THHM (in this last case, there is no dependence of the characteristics mechanics with the temperature).

Variables of entry:

'-
, +
p, +
p, -
P, -
p,
C
gz
Cr
c0

, p
C and p
gz

Variables of exit:

.
'

, more tangent operators (necessary to operator STAT_NON_LINE).
· Variables interns +
P, newer variables +
p: threshold in suction and +
P: pressure of
Cr
c0
S
cohesion, and indicators of mechanical work hardening I1 and hydrous I2.

Elastic prediction:
exp

E
-
(K
0
v)
P = P
~
, = -
S + 2μ

k0/ks0
p +

C
patm

p

C + p

atm

.
F < 0 and F < 0 (p <

): reversible behavior
1
2
C
pc0

E
P = P, S = E p
S, = 0
-
-
, Cr
P = Cr
P, p
p
c0 =
c0

.
F > 0
F >
1
or
0
2
plasticization and mechanical and hydrous work hardening

E
P = P exp [
p
- K
,
0 v]
E
S
S =
p
6

µ v
1 + m2 (2P - 2P + K p
Cr
C
c)

P = p exp K
Cr
Cr
[statement],
p 0 + p
= p 0 + p
exp K

C
atm
(C
atm)
[p
S
v]

p
Q
2

The single unknown factor is
p

F =
~ p
v
=
v determined by
0 (one have then:
)
1
M2 (2P +
-
S
P
2 Cr
P)
or F = 0
~
p
=
2
(and

0)

Note:

The constraint resulting from the data of the model of Barcelona east
D
= +
early
pgz1, it will be
thus the variable used in the routine describing the behavior, the constraint of exit
provided to STAT_NON_LINE being the constraint of Bishop: '=
-
early
p.
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Tangent operators:

The tangent operator of the generalized constraints is implemented in THHM under name DDE and

'&
& '

partitionné in several blocks. The components concerned with the model are
and

p
&c

& '
'

'

&
&

p

&
p
C
&
blocks [DMECDE] and [DMECP1] correspondent with:
&,
C

.
&
'
'

p
& p
& p

& p&
p
C
&c

7
Implementation of the model

7.1 Data
material

The use of the model of Barcelona requires to enrich the data of the model by Camwood-clay by additional data specific to the unsaturated grounds. This is concretized by the simultaneous adoption of the two key words Camwood-clay and Barcelona under order DEFI_MATERIAU.

7.2
Initialization of calculation

It is necessary that the initial state of material is plastically acceptable (the constraint and the pressure capillary are thus such as the point of initial loading is inside the surface of load). It
is necessary thus on the one hand that suction is lower than the hydrous threshold, and on the other hand that the constraint
maybe inside the ellipse defined in the plan of initial suction. In particular, if the loading mechanics initial is purely hydrostatic, it must lie between the terminals represented by cohesion (- K p) and pressure of consolidation (2P). The constraint used to describe it
C
C
Cr
behavior (forced total plus gas pressure) is different from the constraint to initialize in ETAT_INIT (forced of Bishop '). The relation between the two types of constraint is:

&
'= & + [(B -) 1 p&gzI - bSlq p&c]

7.3
Variables intern at exit

The model produces five internal variables:

V
: critical pressure
1 = Pcr
V2 = I1: mechanical indicator of irreversibility
V = p: hydrous threshold of irreversibility
3
c0
V4 = I2: hydrous indicator of irreversibility
V
: pressure of cohesion

5 = PS
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8
Development prospects of the model

One of the phenomena not studied in the original model of Barcelona is nonthe reversibility of capillary curve of pressure [Figure 8-a] and its dependence with the state of stress. This is treated by Dangla and coll [bib2] by integrating the model of Barcelona within a framework poroplastic with the introduction of the water content like additional poroplastic variable, whose evolution is directly connected not only to the capillary variation of pressure via the curve of drainage-imbibition, but also with the mechanical evolution of the medium. Two aspects should be distinguished there distinct but nevertheless coupled phenomenon. Nonreversibility of the curve drainage-imbibition is a phenomenon purely hydraulic and thus independent of the mechanical law adopted in one modeling THHM, but this curve thus depends on the index of the vacuums of the mechanical state on medium. The partition of the water content partly elastic and plastic and of the considerations thermodynamic [bib2] allows to deduce the evolution at the same time from the water content (and thus from the degree of saturation) and constraint according to the deformation and of the capillary pressure. By example, the evolution in the field of reversibility is given by:

E
D = - NR (E
, p) dp + B (E
, p) dtr (E

*)
lq
C
C
C

dP = B (E
, p) dp + K (E
, p) dtr (E
)
C
C
C

PC
Curve of drainage
Field of reversibility
Curve of imbibition

E
p
lq
lq lq

Appear 8-a

Where (NR, b) are the generalized coefficients of Biot [bib6]. To enrich the model by Barcelona in it direction thus implies two separate developments:

1) The introduction of a curve of drainage-imbibition into developments THHM.
2) The complétude of the model of Barcelona by the calculation of the degree of saturation in addition to constraint.

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Date:
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Document: R7.01.21

Law of behavior (in 2D) for the connection steel concrete: JOINT_BA

Summary:

The law of behavior JOINT_BA describes the phenomenon of degradation and rupture of the

connection between
bars of steel and concrete, in the reinforced concrete structures. This documentation presents the theoretical writing
in the thermodynamic framework and the numerical integration of the law, as well as the parameters which manage it
model.
For his use, one will be pressed on the finite elements of joint type (see Doc. [R3.06.09]) already existing in
the code.
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matters

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Titrate:
Law of behavior (in 2D) for the steel-concrete connection: JOINT_BA

1 Introduction

The law of behavior JOINT_BA describes the phenomenon of degradation and rupture of the connection
existing enters the steel bars (smooth or ribbed) and the concrete surrounding it. Key point for the purpose of structural design out of reinforced concrete, the modeling of the steel-concrete connection is
representation as well as simplification of this phenomenon complexes interaction between the two materials which develops in the interface and which undergoes an increasing degradation when one exceed certain thresholds of resistance, specific for each material. The structural models which do not take into account the linkage effects, are generally unable to predict
localization of the cracks as well as the networks created. In addition, the degradation of the rigidity of connection increases the period of vibration, reduced the capacity of dissipation of energy and conduit to one
significant redistribution of the internal forces (according to Bertero, 1979, cf [bib2]).

The law of behavior JOINT_BA is described within the framework of the thermodynamics of the processes
irreversible: the writing and L `use of a "traditional" material model coupling cracking and friction makes it possible to integrate in a robust way of the fine mechanisms nonlinear concomitant into particular description of the kinematics of slip. This last point allows us not
to resort to traditional modelings of the type "contact" very often used in this context in spite of many sources of numerical instabilities. Thus, in monotonous loading the taking into account coupling normal effort shearing makes it possible to treat cases of strong multiaxial pressures; in cyclic, the behavior hysteretic and corresponding dissipations are expressed thanks to coupling between the state of damage and kinematic work hardening. Use of an implicit scheme allows to obtain a robust implementation.

The paragraph [§2] described in short form the phenomenon of the connection steel concrete. The paragraph [§3]
present the thermodynamic writing of the law of behavior, while the paragraph [§4] precise the numerical stage of integration of the law. The parameters which manage the model and which could be
obtained starting from the properties of implied materials, are described in the paragraph [§5].

2
Short description of the steel-concrete connection

Conceptually, the phenomenon of connection corresponds to the physical interaction of two materials different, which occurs on a zone of interface by allowing the transfer and the continuity of the efforts and of the constraints between the two bodies in contact. In the case of reinforced concrete structures, it phenomenon is also known as the "rigidity of tension" which develops around an element of reinforcement, partially or completely drowned in a volume of concrete. Forces of traction which appear inside the reinforcement are transformed into shear stresses on surface, and are transmitted directly to the concrete in contact which will balance them finally, and vice poured. The response of the unit will depend on the capacity of the concrete to become deformed as much as steel, since steel will tend to slip inside the concrete surrounding it. The phenomenon of connection corresponds to this capacity of the concrete to become deformed and to degrade themselves locally by creating a species of layer, or wraps, around the reinforcement, of which the properties kinematics and material differ from those of the remainder from the concrete or reinforcement employed.

The phenomenon can be broken up into three well defined mechanisms:

.
a chemical adherence of origin,
.
a mechanism of friction between two rough surfaces (steel-concrete or concrete-concrete),
.
a mechanical action created by the presence of the veins of the steel bar on the concrete bordering.

According to this decomposition, one can clearly deduce that for a smooth bar, the mechanism dominating is friction between two materials, while for a bar ribbed (in French usually called "braces ha: High Adherence"), the mechanism dominating is the mechanical interaction between surfaces. When the reinforcement is consisted the strands with steel wire ropes, it is possible to control or combine the various mechanisms since they are function directly of the surface of the cables.
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The connection will undergo a degradation different according to the type of loading applied, that is to say monotonous,
that is to say cyclic. In addition, among the most important parameters which influence the behavior of the connection, one can quote:

· characteristics of the loading,
· geometrical characteristics of the steel bar,
· spacing between active bars,
· characteristics of the concrete,
· containment by passive reinforcement,
· side pressure.

At the time of the study of a cylindrical bar drowned in an infinite medium, one can identify the surface of
discontinuity where one will place the linkage effects, which develops in a certain concrete zone fissured and crushed around the steel bar. At a given moment, this surface will correspond to cylindrical crack created during the coalescence of the cracks of shearing. By looking at the network cracks, one can suppose that, in ideal conditions, the plan of cracking is always perpendicular (normal direction) on the surface of the bar and parallel (tangential direction) with sound longitudinal axis (see [Figure 2-a]). That enables us to project the components of displacement on normal and tangential direction of the plan of cracking, and consequently to obtain the deformations and corresponding constraints.

BODY A
Sd
A1
N
F
A2
B1
T
crushed concrete
B2
slipped
BODY B

Appear 2-a: real description of the phenomenon of connection and simplification finite elements: co-ordinates in the local reference mark of the element of interface used like support of law JOINT_BA

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3 Writing theoretical

The formulation presented here was developed within the framework of the thermodynamics of the processes
irreversible; it gives the constitutive relation between the normal effort, the shear stress and it slip by considering the influence of the cracking of the concrete, friction and the various couplings in the phenomenon. For that, the constitutive relations which connect the tensor of the constraints and it tensor of the deformations must include:

.
the cracking of material of interface by shearing
.
inelastic deformations because of the slip
.
the behavior hysteretic due to friction
.
coupling between the tangential answer and the normal constraints

3.1

Presentation of the model

One places oneself within the framework of a plane formulation in 2D, in the definite local reference mark [Figure 2-a].
tensors of the constraints and the deformations are written:

NR

NR

=

and =

éq 3.1-1

0

0

where NR is the normal constraint and is the tangential constraint of the element of interface;
NR
corresponds to the normal deformation and the tangential deformation. Normal deformation in the tangential direction with the interface is regarded as null. This mode of deformation for an element of adherence is with null deformation energy.

Normal and tangential behaviors being regarded as uncoupled on the level from the state, it thermodynamic potential obtained starting from the free energy of Helmholtz is expressed way following:
= 1

[E + E 1 - D
NR -

NR -
NR +
(
NR)
NR +
2
+ G 1 - D + -
D -
2
+ +
éq
3.1-2
T
(
T) T
(
F
T
T) G
T (
F
T
T)
] (Z)

where is the density, is the D NR, Young modulus is the internal variable of normal damage and DT the variable interns tangential damage, both being related to cracking and ranging between 0 and 1. G is the module of rigidity or shearing, F is the deformation
T
irreversible induced by slip with friction of the cracks, is the internal variable of kinematic work hardening, is a parameter material and Z, the variable of pseudo “work hardening isotropic” by damage, with its function of consolidation (Z).
and
define
-
+
respectively positive and negative parts of the tensor considered.

One can notice in the equation [éq 3.1-2] that in the normal direction, the damage will be activated during the appearance of the positive deformations produced by forces of traction, while if the deformations are negative because of the effects of compression, the behavior will remain rubber band. With regard to the tangential part of the behavior, one can recognize a coupling

traditional élasticitéendommagement as well as a new term allowing to associate the state elasticity-endommageable, a state of slip with friction. Coupling between slip and cracking is possible thanks to the presence of the variable of damage like multiplier in the second element of the right part of the equation [éq 3.1-2].

The laws of state are obtained classically by derivation of the thermodynamic potential, and thus allow to define the associated thermodynamic variables. The normal constraint is expressed like:

E
if
0

NR
NR
=
=
NR

éq
3.1-3
NR
(

1 - DN) E

if
> 0
NR
NR

and the total tangential constraint like:

T =
= G 1
(- DT) T + G DT (
F
T - T) ***éq***
3.1-4

T
One can also define the tangential constraint due to the slip with friction (deformation
S
) :

F

= -
= D -
T
G
T (
F
T
éq
3.1-5
F
T)

T

Note:

Such a formulation moves away amply from a traditional formulation of coupling plasticity damage. The assumption bringing to the introduction of the damage into the constraint by slip bases itself on an experimental observation which is that all the phenomena inelastic in a fragile material come from the growth of the cracks.

The rate of energy restored by damage-friction can be written like:

1
1
- = -
= T G T - (
F
T - - = -
+
T) G (
F
T
T)
(DT fT) ***éq***
3.1-6
DT
2
2

In this last expression, DT corresponds to the rate of energy restored by damage and fT at the rate of energy restored by friction of the cracks.

The law of state of kinematic work hardening brings to the definition of the constraint of recall:

=
= ***éq***
3.1-7

Concerning the law of work hardening of the isotropic damage, it is expressed by:

=
='(Z)

éq 3.1-8

Z

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Titrate:
Law of behavior (in 2D) for the steel-concrete connection: JOINT_BA
Date:
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Author (S):
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It is necessary for us now to clarify in a more detailed way the evolution of the mechanism of damage in the connection, in other words to specify the expression of (Z). For weak value of damage, the mechanism which prevails is the interaction of the concrete with the veins of the steel bar, while for a value much larger, it is friction between the concrete and the steel which prevails. During the evolution of the damage, 2 principal phases could be identified:

· the first phase corresponds to a stable growth of transverse cracks related to presence of veins on steel (positive apparent work hardening of the law of evolution),
· the second does not utilize any more but the coalescence of these transverse cracks bringing to not to more consider but the mechanisms of friction (negative work hardening towards a constraint of friction residual).

3.2
Analyze damage in the tangential direction

The law of evolution of the damage is divided into three stages:

1) area of perfect adherence,
2) area of passage of small deformations to the great slips,
3) area of maximum resistance of the connection and degradation until residual resistance ultimate.

To identify these areas, two thresholds are established:

· the threshold of perfect adherence 1

,
T
· the threshold of continuity before coalescence of cracks 2

.
T

Thus, by taking again the expressions related to the damage with knowing that of the rate of refund of energy [éq 3.1-6] and that of the variable interns associated with isotropic work hardening [éq 3.1-8], one can
to note:

· a true separation between L `damage and the friction of the cracks (what allows
to amend only the law of evolution of the damage without affecting the part
“friction”),
· the partition in two parts of isotropic work hardening since one has two different stages in
the damage.

From now on we will write for work hardening related to the variable of damage:

,
if 1 < 2

T1
T
T
T

=
=
T
'(Z)
=
éq
3.2-1
Z

2
, if
<
T1
T 2
T

T

Components and
express themselves in the following way:
T1
T 2
2

1

G

=

T
+

+ Z

T1
T1
ln (1
T)

éq
3.2-2
With

1

1
2
DT

T

1 - Z

=
T 2

+
T
T 2

éq
3.2-3

With
1 Z
2DT + T
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The function threshold DT is also defined which depends on DT and which S `writes like:

=
DT
D -
T
(+
T1
T) 0 ***éq***
3.2-4

The thresholds which manage the law of evolution of the damage are also expressed in terms of T
D

(see [Figure 3.2-a]). The first expression corresponds to the threshold of perfect adherence and S `written:

1 1
1
T1 = elas = G ***éq***
3.2-5
T
2 T
T

Where T1 is the initial threshold of damage defined according to the limiting deformation of adherence perfect 1
T, which will correspond to the limiting deformation of shearing or traction of the front concrete the initialization of the damage. In addition, T 2 is the threshold of initiation of coalescence of microscopic cracks which is defined according to the initial tangential deformation of the great slips
2
T:
1 2
2
T 2 = G
éq
3.2-6
2 T
T

2
1
0
0

2
T
2t

Appear 3.2-a: construction of the functions thresholds in terms of energy

The laws of evolution of the internal variables within the framework of the standard associated laws allow
to obtain the derivative of the multiplier of damage D:

.

.
.
.
.
.
D
D
D = D
= D and Z = D
= - D

éq 3.2-7
D

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By using the condition of consistency in addition, one obtains the expression of the damage:

B

1 T
T

2
D

D = 1
1
-
exp
T
1
With

-

DT
(

DT
T1)
***
DT

G

1

éq 3.2-8
1 + A

-
B2D
2

DT
DT
T1 +
T

In this expression, one can identify the part which corresponds to the area of the passage of small deformations with the great slips with two parameters: A1
B
DT and 1DT, as well as the part
of damage finale in mode 2, with the A2 parameters
B
DT and 2 DT. It should be noted that the relation

-
DT
T1 is managed by a function of Macaulay, i.e. this difference in energy must
to be always positive or null.

The functions which manage isotropic work hardening in the tangential direction are expressed like:

Z
= Y
- Y;
T1
DT
T1

éq 3.2-9

,
0
if
<

=

1
D
2
T 2

T
T
T
éq
3.2-10

-, if
<
DT
T 2
T 2
DT

According to these expressions, one can notice that
is not taken into account in the area of
T 2
transition from the small deformations to great slips.

3.3
Analyze damage in the normal direction

The two most important mechanisms which can appear on the normal direction are it
detachment between the concrete and the bars of steel, and the penetration of the reinforcement in the body of
concrete. These two conditions can be interpreted respectively like an opening or one
closing of crack, and can be described by a particular law of behavior in
normal direction uncoupled from the tangential behavior.

In order to simplify the resolution for compression between surfaces, one decided to allow small
penetration between those, which implies that 0, and by adopting a law of behavior
NR
rubber band, one will have:

= - if 0
NR
E
NR
NR

éq
3.3-1

The case of the decoherence of the interface can be described by a behavior endommageable in normal direction, is:

=
NR
(1 - DN) E + if > 0
NR
NR

éq
3.3-2
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with D NR variable scalar of the damage in the normal direction, calculated with the expression following:

0
if
1

NR
NR
D =
NR

éq
3.3-3

1

if 1 <
NR
B
NR
1+

- D NR
ADN
D NR
N1 +

In this expression, two parameters material, AD NR and data base NR, control decoherence by the damage in traction of the concrete. In addition, is the threshold of damage defined in
NR 1
term of energy, are equivalent to the elastic threshold in the normal direction
and which is expressed
elas NR
like:

1 1
1
N1 = elas
= E
éq
3.3-4
NR
2 NR
NR

1
being limiting deformation of perfect adherence, which corresponds to the limiting deformation of the concrete in
NR
traction before the initialization of the damage. It should be mentioned that when the detachment or the opening of crack reaches the maximum value of resistance to traction, no force of shearing will not have to be transmitted between two materials: it east is the single condition in which the scalar variable of damage in the tangential direction becomes 1 because of

the damage in the normal direction

3.4 Analyze contribution of the friction of cracks by slip

With regard to the part "slip" of the formulation, one supposes that it has a behavior pseudo-plastic, with nonlinear kinematic work hardening. Initially introduced by Armstrong & Frederick, 1966, cf [bib1], nonlinear kinematic work hardening makes it possible the formulation to surmount
the principal disadvantage of the kinematic law of work hardening of Prager, namely, the linearity of the law
of state which connects the forces associated with kinematic work hardening. Here, the nonlinear terms are
additions in the potential of dissipation. The criterion of slip takes the traditional form of function threshold of Drucker-Prager which takes into account the effect of radial containment on the slip:

= F
- + C I
0
F
T
1
éq
3.4-1

Here is the constraint of recall, C is a parameter related to material, translating the influence of containment, while I corresponds to the first invariant of the tensor of the constraints, which for our
1
case is expressed like:
1
1
I =
1
***Tr** [] = NR*
éq
3.4-2
3
3

In addition, the initial threshold for the slip is 0. Moreover, by considering the principle of dissipation figure maximum, the laws of evolution can be derived from the expression of the plastic potential which is:

p
F
3
2
= - + C I
F
T
1 +
has
éq
3.4-3
4

Where A is a parameter material. It should be mentioned that the quadratic term allows some to introduce the non-linearity of kinematic work hardening. Laws of evolution for the deformation of slip as for kinematic work hardening take the following forms:

.
.
p
.
.

p

F
=
F
F
and = ***F***
F
T

éq 3.4-4
F

T
.
The multiplier of slip ***F*** *is calculated numerically by imposition of the condition of consistency.*

3.5
Summary of the equations

We show here, a summary of the equations which constitute the law of behavior of the connection steel-concrete:

1
. = [
-

E
-

+
+

E
+
NR
NR
NR
. (1 - DN)
Free energy

NR
2

of Helmholtz
+ G
2
T
(1 - T
D) T + (
F
T -
- + +
T) G.
T
D (
F
T
T)
] H (Z)

= Y
DT
D -
T
(Y + Z
T1
T) 0;
Function threshold

= F
- X + C. I 0
F
T
1

E. NR
if NR 0
NR = (
;
1 - DN). E. NR if NR > 0
Laws of state
T = G (1 - T
D) T + G. T
D (

F
T - T);
F
=. D -

G
T (
F
T
T)

- Y = -
= - (D
Y + Yf);
D

Dissipation
X =
=
;

'
Z =
= H (Z)
Z

D& = &.
D = &; z& = &. D = - &;
D
Y
D
D
Z
D
Laws
D

of evolution
p
F

p

& = &. F; & = &. F
T
F
F
F

X
T
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3.6
Form of the tangent matrix

In order to ensure the robustness and the effectiveness of the model in the numerical establishment and for the analysis
total of the massive structures, it is necessary to calculate the tangent matrix, which can be given with to leave the following expression:
.
.
=
T
T

éq 3.6-1

After some analytical calculations, one can deduce the expression from the tangent module while using condition of consistency and respective laws of evolution:

G (1 - (G ()
T
T) F
T)
H =

éq
3.6-2

(
F
T)
p

(
F
T)

1+ G DT (
F
) 2
2
p

(

)(
F
)
With

G
(T)
DT DT F G H -
'F “G H - F G” H

=

=
G

éq
3.6-3
2

T

T
DT

T

H

Where F, G and H are the following functions, obtained thanks to [éq 3.2-8]:

T
F
1
=

éq 3.6-4
DT
B

2
1DT
G = expA

1

éq
3.6-5
DT
(
-
DT
T1)

G

B2DT
H = 1 + A2

-
DT
DT
T1 +
éq
3.6-6

Note:

In practice in Aster, the tangent matrix was not established, only the secant matrix is
E 1
(- D)
0
NR

used either H =

.

0
G 1
(- D)
T

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4 Integration numerical

Separation in two parts in the formulation: damage slip, allows us to treat each one of it separately. Thus, the integration of the damage part is carried out of explicit way by the definition of two surfaces threshold. On the other hand, the part "slip" is solved in an implicit way by a traditional method with knowing the algorithm of the type "return-mapping" proposed by Ortiz & Simo, cf [bib4], which will ensure the effective convergence of way.

4.1 Calculation of the part "friction of the cracks" with a method of implicit integration

The effects on the connection associated with the phenomenon of friction with the cracks can be calculated in
the framework of a behavior pseudo-plastic with a nonlinear kinematic work hardening. For the establishment with the method of integration suggested, we will carry out a linearization of function threshold around the current values of the variables intern associated. With the iteration (i+1), surface threshold is written:
(I)
(I)

(i+)
1
(I)

(I)

F
F
F

= +
:
éq
4.1-1
F

F

-
F
T
T
+
: ((i+) 1 - (I)) 0
F

T

According to the equations [éq 3.1-7], [éq 3.1-8], and [éq 3.6-5], one a:
.
.
· p

= = -
F
F
éq
4.1-2

.
.
p
·

F
F

= - D = - D
F
T
G
T
T
G
T
F

éq
4.1-3
F

T
That one can discretize in the following way:
p

= (i+) 1 - (I) = = -
F
F

éq
4.1-4

p
(i+)
1
(I)

F
F
F
F

=
-
= - D

= - D
F
T

T
T
G
T
T
G
T
F
éq
4.1-5
F

T

By combining these expressions with the expression of surface threshold and by writing that is equal to
F
zero, one can deduce the increment from multiplier
with each iteration I:
F
(I)

=
F
F

éq
4.1-6
(I)
(I)
(I)
p
(I)
p

F
G D
F
F
F
T
+

F
F

T
T

After obtaining the value of
, one can substitute it in the equations [éq 4.1-4] and [éq 4.1-5]
F
in order to bring up to date the thermodynamic forces
F
T and. The iterations will have to continue until
moment when the condition of consistency is checked.
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4.2
The algorithm of resolution

In a general way, one seeks to check the balance of the structure at every moment, in a form incremental. As clarified previously, for the damage a simple scalar equation
allows to obtain the corresponding value, which makes it possible to avoid a recourse to the iterative methods.
On the other hand, an iterative method is applied for integration of the friction part of the cracks.
Then, the algorithm is as follows:

(I) Reactualization

geometrical:

(
=
*+ **U***
T) +
(
N 1
T)
S
N
T
(II) Prediction

rubber band:

(F () =
T) 0
n+1
(fT) N;
(E () =
-
T) 0
n+1
(T) n+1 (fT) n+1;
(0
) =
n+1
N;
(0)

=
,
n+1
((and) (0) (0)
n+1
n+1)
(III) Evaluation of

threshold:

*(**F**) (0) 0?*
n+1

if SO, end of the cycle; so NOT, beginning of the iterations

YES:
(F =
(E =
(0)

=
(0)

=
T) n+1
(and) (0)
T) n+1
(fT) (0) n+1;
n+1; n+1
n+1; n+1
n+1
NOT:
I = 0
(iv) Correction

plastic:

(F) (I)
= (
N 1
+
F
F
F
/

+

T) (I) G. D
/
/X
. .
/X

N 1
+
T (
p
F
F
T) (I)
(
1
+
F
) (I)
N
N 1
+
(PF) (I) n1+

(i+) 1
(I)

=
-

-

N 1
+

N
G.
1
+
T
D.
.
F
(p F
/
F

T) (I)
.
.
F
(p/X
F
) (I)

(i+1)
(I)

=
+

n+1
n+1
F
(PF) (I)
(v) Checking of

convergence:

*(**F**) (i+1) TOL*
+
*(**F**) (0)?*
N 1
n+1

if SO, end of the cycle; so NOT, to continue the iterations in (iv)
YES:
(i+1)

=
n+1
n+1;
(i+1)

=
n+1
n+1;

(E =,
T)
E
n+1
T (n+1
n+1);
(F =
-
T) +
(
N 1
T) n+1
(and) n+1
NOT:
I = I + 1
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4.3
Variables intern model

We show here the internal variables stored in each point of Gauss in the implementation of model:

Number of
Feel physical
variable
intern

1
D NR: Scalar variable of the damage in the normal direction
2
DT: Scalar variable of the damage in the tangential direction
3
zT1: Scalar variable of isotropic work hardening for the damage in mode 1
4
zT2: Scalar variable of isotropic work hardening for the damage in mode 2
5
F
: Deformation of slip cumulated by friction of the cracks
T
6
: Value of kinematic work hardening by friction of the cracks

5
Parameters of the law

The law of behavior presented here is controlled by 14 parameters, of which 3 manage the answer in the normal direction and the others affect the response in the tangential direction. In addition, it Young modulus is recovered starting from the elastic data provided by the operator ELAS, who must to always appear in the command file.

These parameters, or the analytical expressions which make it possible to obtain them, were obtained or determined starting from the digital simulation of the experimental tests carried out by Eligehausen and Al, 1983, cf [bib3]. The realization of multiple simulations made it possible to determine a relation enters
geometrical and material characteristics of materials in question (steel and concrete) and them parameters which manage the model of the interface.

5.1
Initial parameters

5.1.1 The parameter “hpen”

The element joint functioning on the concept of jump of displacement, it is necessary to introduce one dimension characteristic of the zone of degraded interface allowing to define the concept of deformation in the interface. With this intention it was introduced the principle of penetration between surfaces:
parameter “hpen” makes it possible to define this zone surrounding the bar of steel. This parameter corresponds
with the possible maximum penetration which depends on the thickness of the compressed concrete - crushed. Into same

time, "hpen" manages the dissipation of energy in the element as well as the kinematics of the slip.

In order to give a reference to the user for the choice of this parameter, one proposes to calculate it with
to leave the diameter of the bar D

B and the relative surface of the veins
defined by:
Sr

K FR

sin
Sr =

éq
5.1.1-1
D C
B
where K is the number of veins on the perimeter; R
F the transverse surface of a vein; is the angle
between the vein and the axis longitudinal of the steel bar; and C is the measured distance between veins
center in center. Finally, "hpen" will be calculated with the expression:

hpen = dB Sr
éq
5.1.1-2
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*According to Eligehausen and Al, the reinforcements usually used in the United States have values of Sr
between 0.05 and 0.08. For the smooth bars, since one needs a small value for
“hpen”, one proposes values of between 0.005 and 0.02.
Sr
The following table gives the values of “hpen” according to the diameter of the bar:*

Diameter (mm)
Relative surface
Hpen (mm)
Description
*8 0.01
(0.08) 0.1
Smooth commercial bar
8
0.08
0.64
Ribbed commercial bar
20
0.08
1.50
Ribbed commercial bar
25
0.08
2.00
Ribbed commercial bar
32
0.08
2.54
Ribbed commercial bar*

The unit of “hpen” must of course correspond to the unit used for the grid.

5.1.2 The parameter G or module of rigidity of the connection

Generally, because of difficulty in measuring the deformations by shearing, the module of rigidity of a material is calculated starting from Young and the Poisson's ratio modulus, parameters currents obtained in experiments. However, for our case, the interface is a pseudo-material whose characteristics must depend on the properties corresponding to materials in contact, steel and concrete. Since the material which one expects to damage is the concrete, one proposes to initially use for the connection the same value of G that for the studied concrete but it can be

higher up to a value similar to the value of the Young modulus E, when one increases value of "hpen". In the case of reinforcements with rigidities higher than those of the bars commercial current (because of a provision or special geometry of the veins), one can make a correction of the value chosen, by multiplying the module of rigidity by a coefficient of correction calculated starting from the relative surfaces of the commercial bars, with the expression:

(SR) bars
Carm = (

éq
5.1.2-1
SR) barrecomm

Then, the module of rigidity of the connection G will be:

Gliai = Carm Gbeton
éq
5.1.2-2

In the last expressions, Glia is the module of rigidity of the connection; Gbeton is the module of rigidity of the concrete; Carm is the coefficient of correction per reinforcement; () bars SR , relative surface of veins of the bar concerned; and (SR) barrecom, relative surface of the veins of the bar commercial of the same diameter (preferably, 0.08).

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5.2
Parameters of damage

5.2.1 Limit of elastic strain 1 or threshold of perfect adherence
T

To define the threshold of perfect adherence, it is considered that the damage by shearing must to initiate itself at the time of the going beyond of a certain threshold of deformation. So one proposes to adopt them
limiting deformations of the concrete in traction, i.e., between 1x10-4 and 0.5x10-3, which corresponds to
shear stresses between 0.5 and 4 MPa in perfect adherence.

5.2.2 The parameter of damage A for the passage of the small deformations
1DT
with the great slips

In this area, the law of evolution of the damage is expressed in term of deformations and its construction depends on the definite elastic slope for the linear behavior (constraint on shearing vs. deformation) in the area of perfect adherence: this parameter controls the value of the constraint compared to the slip in the passage of small deformations to large slips.

The determination of the value of this parameter is a key and delicate point model, since the evolution damage must be carried out with certain conditions noticed by several researchers;
for example:

.
the resistance of the connection is directly proportional to the compressive strength of concrete. However, as the resistance of the concrete is increased, it
behavior becomes more rigid, bringing to the brittle fracture of the connection,
.
the particular rigidity of the reinforcement, which is related on the diameter and the quantity of the veins on
surface, must increase the resistance of the connection,
.
the relation between the moduli of elasticity of two materials concerned must manage directly the kinematics of the connection.

From the digital simulations that one carried out, one observed that this value is located between a minimum of 1 and one maximum of 5, and which it will have to be adjusted according to the test of
reference selected. Optionally, one proposes an expression which makes it possible to adopt an initial value

and which depends on the particular characteristics of materials:

1
F 'C
Ea
A1
=

DT
(

éq
5.2.2-1
1 + SR)
30
Eb

In the last expression, Eb will be calculated with the expression provided in the section A.2.1, 2 of BAEL'91:

E = 11000
B
× (F 'C) 1 3 éq
5.2.2-2

In the two last expressions, one a:

.
F 'C, compressive strength of the concrete in MPa;
.
Ea, modulus of elasticity of steel, in MPa;
.
Eb, modulus of elasticity of the concrete, in MPa;
· Sr, relative surface of the veins of the bar concerned.
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[Figure 5.2.2-a] gives some a graphic comparison.
COMPARISON OF A1DT
- Local Law of the connection -
40
20
ad1=1.0
35
ad1=2.19
ad1=3.8
-
O
30
15
MT
ent
25
)

has
)
Pa
L
E
m
P
20
M
10
Y (M

C
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0
0.5
1
1.5
2
0
0.05
0.1
0.15
defo EXY
defo EXY

Appear 5.2.2-a: Comparison of A1DT: growth of the resistance of the connection

5.2.3 The parameter of damage B
1DT

The purpose of this parameter is to soften the shape of the curve of behavior, like facilitating transition from the elastic slope towards the nonlinear area. It can have a value included/understood enters
0.1 and 0.5 (never higher than 0.5 since it is the equivalent of the square root of the formula). One can
to advise to adopt the value of 0.3 for ordinary calculations. (See [Figure 5.2.3-a]).
VARIATION DE Bd1
- Local Law of the connection -
25
20
Bd1=0.1

Bd1=0.2
Bd1=0.25
20
Bd1=0.3
Bd1=0.5
-
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0
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1
1.5
2
0
0.05
0.1
0.15
defo EXY
defo EXY

Appear 5.2.3-a: Comparison of B1DT: Modification of the curve
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5.2.4 Limit of deformation 2 or threshold of the great slips
T

According to several authors, the great slips are overall higher 1 mm from displacement,
but that is an indicator which depends on the form and dimensions of the specimens tested; therefore, one
propose that this deformation never exceeds 1.00 (adimensional value). In way more
specify, one proposes to apply the following expression:

N
With

With

2
1
(
=

1DT)
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-
=

1DT) 4
T
(

éq 5.2.4-1
hpen)
1
-

2 C + (
N

1
WITH T) (hpen)
1
2
D

+ (1
WITH T)
1.0
9
4
D

In this expression, one applied a sigmoid function whose coefficients C and N allow to adjust the kinematic effect of A1DT on the slip, i.e., when the connection becomes more resistant because of an increase in rigidity, the slip is reduced gradually. One adopted values 9.0 and 4.0 respectively, but they are always optional.

The choice of the value of the limit of deformation 2

is very important because it introduces one more or less
T
great brittleness of the response by translation of the threshold of passage of the small deformations to large
slips. This brittleness is related to the stiffness of the concrete via parameter A1DT. It is necessary to note that the following parameters which manage the damage must be also adjusted on the level room to ensure the correct continuity of behaviour in shearing of the connection and to thus be able to obtain the desired or awaited response of a system real steel connection concrete.
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5.2.5 The parameter of damage A
2DT

The damage, such as it was conceived in the model, obeys two laws of evolution which are expressed using one only variable traditional scalar which will ensure the coherence of the damage. The parameters of each of the 2 laws are independent and numerically stable, but they are likely to generate serious errors in the continuity of the behavior if one does not pay attention to the shape of the local curve stress-strain: to see the case of curve shown in graphics of [Figure 5.2.5-a], with a value A
= 1x10-3 MPa-1. Us
2 DT
let us not be able to propose an analytical relation for the choice of this parameter, but the gained experience enables us to affirm that the value of this parameter must be included/ understood enters
1x10-3 and 9x10-2 MPa-1 roughly.

COMPARISON OF A2DT

- Local Law of the connection -
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35
-
Ad2 = 1e-3
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Ad2 = 3rd-3
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Ad2 = 6th-3
Ad2 = 9th-3
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1.5
2
defo EXY

Appear 5.2.5-a: Comparison of A2DT: damage and rupture of the connection

5.2.6 The parameter of damage B
2DT

This parameter, which supplements the law of evolution of damage in great slips, controls not only growth of the resistance of the connection or shape of the curve of behavior to the peak and in the area post-peak, but also the kinematics of the answer, which implies the determination of slip for the maximum shear stress as well as the amplitude of the curve to the peak of behavior. Then, although values of the parameters of A2 damage
B
DT and 2 DT
will have to adjust itself at the same time when one builds the curve of behavior of the connection in order to
to respect the continuity of the pace, one can say that the value of B2DT is inversely proportional the amplitude of slip at the top, i.e., a value of 0.8 allows great slips
broader in the top than a value of 1.2, for example.

For practical cases, one recommends to use a value ranging between 0.8 and 1.1 to reproduce a coherent curve of behavior (See [Figure 5.2.6-a]).
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VARIATION OF B2DT
- Local Law of the connection -
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defo EXY

Appear 5.2.6-a: Comparison of** B2DT**: damage and rupture of the connection

5.3
Parameters of damage on the normal direction

5.3.1 Limit of deformation 1 or threshold of great displacements
NR

In a way similar to the elastic behavior in the tangential direction, it is considered that decoherence must be initiated at the time of the going beyond of a certain threshold of deformation. We propose to adopt a value between 10-4 and 10-3.

5.3.2 The parameter of damage A
DN

This parameter controls primarily the slope of degradation of the normal constraint compared to deformation due to the opening of the interface. We propose to use a minimal value of 1x10-1 MPa-1, which corresponds to a degradation similar to that of the concrete. Nevertheless, if one wishes to have a behavior of the connection even more fragile, it is enough to increase this value.

5.3.3 The parameter of damage B
DN

In combination with the preceding parameter, this parameter controls the damage of the connection, in particular shape of the curve of behavior in phase post-peak.
We propose to use a value equalizes to 1, or 1,2 for more marked curves.
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ADN = 1e-3
ADN = 1e-2
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2
Normal deformation

Appear 5.3.3-a: behavior of the connection on the normal direction at the time of the opening interface (normal traction on the connection).

5.4
Parameters of friction

5.4.1 The parameter material of friction of the cracks

One of the assets of the model suggested here is that it is able to take into account the effects of friction of the cracks, which, in the case of monotonous loading, appears by a contribution positive with the shear strength of the connection; in addition, in the cases of loadings cyclic, it is obvious that the pace of the loops of hysteresis depends directly on the choice of the value this parameter material. However, the corresponding values were not gauged, since we did not simulate tests with cyclic loadings yet to validate them.
Temporarily, one proposes to use values lower than 10 MPa, with a maximum value of *equal to 1.0 MPa-1.*

5.4.2 The parameter kinematic material of work hardening

On [Figure 5.4.2-a], one can appreciate that the reduction in the value of increases dissipation hysteretic, but also the resistance of shearing and the residual deformation pseudo-plastic. That is very important for the cyclic modeling of the connection since in reality, when one

exceed the peak of maximum resistance, one notices that at the time of the discharge there is no more elastic contribution of the slip, i.e. the residual deformation pseudo-plastic
corresponds exactly to the total slip reached. In other words, once connected all them
cracks in the potential of rupture, longitudinal and tangential layer with the steel bar, the single one resistance which will prevent the displacement of the reinforcement is the friction resistance of the connection,
produced by the contact and the tangle of the asperities between surfaces concrete concrete.
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As previously, our experiment is limited: one proposes to use a maximum value of
0.1 MPa-1 which gives correct results for applications in monotonous loading, and which seem suitable for cyclic loadings.

EFFECT OF A ON the LOOPS OF HYSTERESIS
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- Friction of the cracks -
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100 A 4
-
Co
100 A 1
100 A .5
100 A .2
-5
0
0.5
1
1.5
2
defo EXY

Appear 5.4.2-a: Comparison of a: effects on the loops of hysteresis into cyclic

5.4.3 The parameter of influence of containment C

In our model, the influence of containment was taken into account thanks to the application of it parameter which controls these effects on the connection, and which appears by an increase in maximum shear stress like by the increase in maximum displacement to the peak when containment increases.

For the calibration, we carried out simulations with containments of 0, 5, 10 and 15 MPa, by always using a value of 1.0 for this parameter. It was noticed that if one wants to produce one

kinematic translation of the slip caused by containment, it is enough D `to adopt a value of 1.2 or 1.5 (adimensional). Optionally, it is advised to maintain the value of 1.0 for ordinary calculations.

5.5
Summary of the parameters

To facilitate the use of the law, the suiant table presents a synthesis of the whole of parameters of the model of behavior.

It is pointed out that the values or the expressions suggested have only one indicative value, and that arbitrary combination can give inaccurate and unexpected results compared to the behavior hoped for connection; in other words, a bad choice of the parameters can produce a strong rigidity or a weak response of the interface steel concrete.

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Value
Variables
Parameter Unit
Analytical expression
proposed
concerned
Diameter of
dB
the bar
H

H
= D
PEN
mm -

PEN
B
Sr
Relative surface
Sr
veins
Coefficient of
Carm correction by
reinforcements
G
G
= C
G
bound
MPa -
bound
ARM
concrete
Modulate
Gbeton rigidity of
concrete
1

min 1.0x10-4

T
-
max 1.5x10-3
Resistance to

f' C,
compression
concrete
(MPa)
1
F 'C
E

With
min 1.0
has
With
=

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Modulate
DT
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max 5.0
DT
(1+
30
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SR)
B
Ea
of elasticity of

steel
Modulate
Eb
of elasticity of
concrete
B
min 0.1
1

DT
-
max 0.5
With
2
1

(1 TD) 4
2

=

1 -

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(H
With
PEN) 2

+ (1 T
D)
1.0
9
4

With
min 1.0x10-4
2

DT
MPa-1 max 9.x10-2
B
min 0.8
2

DT
-
max 1.5

MPa
max 10.0

has
min 0.01

MPa-1

max 1.0
C
- 1.0
(value
recommended)

1

min 10-4
-

NR
max 0.9 10-3
With
(value recommended, not gauged)
DN
MPa-1 min 1.0x10-1

B
(value recommended, not gauged)
DN
- 1.

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Rate of refund of energy in linear thermoelasticity

Date:
26/05/05
Author (S):
E. GALENNE, O. BOITEAU, E. SCREWS Key
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Organization (S): EDF-R & D /AMA, SINETICS, UTO/SIS

Handbook of Reference
R7.02 booklet: Breaking process
R7.02.01 document

Rate of refund of energy in thermoelasticity linear

Summary:

One presents the calculation of the rate of refund of energy by the method theta in 2D or 3D for a problem thermoelastic linear. It is explained how the field theta is introduced into Code_Aster and how it rate of refund of energy is established.
Studies mechanic-reliability engineers of evaluation of probability of starting of the rupture requires, moreover, its derivative compared to a variation of field controlled by another field. The establishment of this option is detailed in the code.
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1
Calculation of the rate of refund of energy by the method
theta in linear thermoelasticity

1.1
Relation of behavior

One considers a fissured elastic solid occupying the field of space R2 or R3. That is to say:

.
U the field of displacement,
.
T the field of temperature,
.
F the field of voluminal forces applied to,
.
G the field of surface forces applied to a part S of,
.
U the field of displacements imposed on a Sd part of.

F
S
G
Sd

Appear 1.1-a: Fissured elastic solid

To simplify, one places oneself in linear elasticity and small deformations, but this approach generalize without sorrow with plasticity [R7.02.07], the great deformations, dynamics [R7.02.02]…
One indicates by:

.
the tensor of the deformations,
.
° the tensor of the initial deformations,
.
HT the tensor of the deformations of thermal origin,
.
the tensor of the constraints,
.
° the tensor of the initial constraints,
.
(, °, °, T) density of free energy,
.
the tensor of elasticity.

is connected to the field of displacement U by:

1
(U) =
(U +u
I, J
J I,)
2
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The density of free energy *(, °, °, T) are identified by a dilation and tensile test in small deformations. (, °, °, T) is a convex and derivable function.*

(
1
1
, °, °, T)
(HT °) (HT °) (HT
=
-
-
-
-
+
-
- °)°+ °°
2
2

The law of behavior of an elastic material is written in the form:

=
(, °, °, T) = (- HT - °) + °

with HT =
ij
(T - Tréf) ij

1
The constant term ° ° has a null contribution on the calculation of the rate of refund of energy, but
2
from a numerical point of view it makes it possible to find exactly the same value for a calculation rubber band of G by having any intermediate elastic initial state: ° = °.

1
(, °, °, T) = (- HT) (- HT)
One finds:
2

= (- HT)

If the initial strains ° and the initial stresses are null, density of energy free is written:

(
1
2
9
2
, T) =
() + μ - 3
K (T - T) +

K 2 (T - T
II
ij
ij
réf
kk
réf)
2
2

The relation of behavior *is written:*

(
)
ij
= kk ij + 2μ ij - 3K T - Tréf ij

and μ is the coefficients of BLADE.

is the thermal dilation coefficient.
Tréf is the temperature of reference.
K, module of compressibility voluminal, is connected to the coefficients of BLADE by :
3K = 3 + 2μ.

The relation of behavior starting from YOUNG E and the Poisson's ratio modulus is:

( )
E (
)
ij
=
E

tr
T - T

1 +
ij +

1 - 2
ij - 1 - 2
réf
ij

with:

=
E
1
(+) 1
(- 2)
μ
E
=

2 1
(+)
E

1.2
Potential energy and relations of balance

One defines spaces of the fields kinematically acceptable V and Vo.

V =
v
{acceptable, ***v*** *=* ***U*** *on S}*
D
V
{
}
O
=
v *acceptable,* ***v*** *= 0 on Sd*

With the assumptions of [§1.1] (and for ° = ° = 0), ***relations of balance in weak formulation*** *are:*

U
V

ij VI, J D
=
F
I VI D + G
I VI D, ***vVo***

S

They are obtained by minimizing the total potential energy of the system:

W ***v***
( ) =

*((****v****), T) D - F*
I VI D - G
I VI D

S

Indeed, if this functional calculus is minimal for the field of displacement **U***, then:*

W =

D - F v
v

ij
I I D G
I I D

ij

S
=
1

(
)
ij
v

D - F
v
v
2
I, J + vj, I
I
I D -
gi I D

S
=

ij VI, J D F
I VI D - G
I VI D = 0

S

We thus find the equilibrium equations and the relation of behavior while having posed:

ij =.
ij

1.3
Lagrangian expression of the rate of refund of energy

By definition [bib1] the rate of refund of local energy G is defined by the opposite of derived from potential energy compared to the field:

W
G = -

This rate of refund is calculated in Code_Aster by the method theta, which is a ***method Lagrangian of derivation of the potential energy*** *[bib4] [bib2]. Transformations are considered*

F ***M***
:
M *+ (****M****) of the area of reference in a modelling field of the propagations*
crack, which at a material point ***P*** *make correspond a space point* ***Mr.*** *These transformations must modify that the position of the bottom of crack O. The fields must thus be tangent with , i.e. by noting* ***N the*** *normal with:*

=
μ
*{such as μ **N** = 0 on}*
Notice

*This family of functions of transformation must be sufficiently regular. In particular, it must be at least twice derivable per pieces out of **P** and in (so that the derivative partial seconds commutate) and to carry out a diffeomorphism for each value of the parameter (that ensures the reversibility of the process).*
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*That is to say **m the** unit normal with O located in the tangent plan at (i.e. tangent at the plan of fissure) and returning in.*

O
m
Plan of
fissure

Appear 1.3-a: Melts of crack in 3D

According to proposal 7 of [bib4], the rate of refund of local energy G is solution of the equation variational:

*G **m***
G (),

=

O

where
(( ))

G
*() is defined by the opposite of derived from the potential energy W **U** in balance by report/ratio*
with the initial evolution of the bottom of crack:
*D W (**U** ()*
G () = - &
W = -

D
=0

*The quantity **m** represents the normal speed of the bottom of crack. In addition, G*
() with same
value which it is of a right propagation [Figure 1.3-b] (A) or about a curved propagation
[Figure 1.3-b] (b) insofar as that Ci with the same tangent at the beginning (then one can anything of it to say). On the other hand, one can nothing say case of the propagation in a direction marking one angle [bib5] [Figure 1.3-b] (c).

has
B
C

Appear 1.3-b: Various geometries of propagations
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Rate of refund of energy in linear thermoelasticity

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*Thereafter, when no confusion is possible, one will indicate par. the Lagrangian derivative in a virtual propagation of crack speed. That is to say (, **M**) a space field (or eulérien) unspecified definite on R+ ×, we will note his material representation (or Lagrangian) (**P**) = (*
,
*, F (**P**)) and its derivative particulate (or Lagrangian) compared to this propagation*

virtual &
=

.
=0

Remarks [bib6]:

*· The fact of adopting two different visions (eulérienne and Lagrangian) introduced structurally concepts of cross derivabilities. Thus, this particulate derivative of a space field called Lagrangian derivative consists in deriving (, **M**) by fixing the material point*
-1
***P** = (**F**) (**M**). One transposes the field of Lagrangian representation, then it is derived compared to before reconverting it of representation eulérienne.*

· One recalls that this Lagrangian derivative is related to derived the eulérienne by the relation

& =
+
.

Notice [bib4]:

The derivative eulérienne does not depend that on restricted with, i.e. trace of on melts of crack.

With these notations, the rate of refund of energy in this propagation is written (by using it

theorem of transport of Reynolds *cf [§4.2.1]):*

.

.
6 7
4 8
4

G ()

F U
(F U)
}
-
=
-

D -
,
G U + G U
-
N D
I
I
I
I
K K
I
I
I
I
K, K
K

+
-

nk

S

However

&

(, °, °, T) =
& +
& ° +
&
° +
T&
ij
ij
ij

°
°
T

ij
ij
ij

*T, **F**, **G**, °, ° being supposed independent of, i.e. being the restriction on (or) of fields defined on R3, there are the following relations:*

&T =
,
T K K
&f = F
I
I, K K
&g
= G
I
I, K K
&°

°
ij = ij, K K
&°
°
ij = ij, K K
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Indeed, if one considers loadings and materials which are the ***restriction on geometry*** *(or part of its border) of fields defined on entire R3:*

/ =

Derivation compared to the parameter commutating with this restriction, one has the result

=

= 0
= 0

Note:

· This assumption is checked only for sufficiently regular fields (for example

belonging to spaces of Sobolev of). Their definition should not be impacted by variation of border.
· In the case of the derivation of the rate of refund of energy compared to a variation of field (cf [§4]) the derivative eulérienne of the field of temperature could not be neglected any more.

In addition, one as supposed as the ***derivative eulériennes of the characteristics materials*** *are null, which is true only on the problem discretized with the current functionalities of operator DEFI_MATERIAU. Their gradient on each element is also null by construction (they are discretized P0 i.e. constant by finite elements), it results from this that the derivative Lagrangian is null:*

& =
+. for {E, Tref}

{123
= 0
= 0
Caution:

With characteristics variable materials within finite elements of the crown theta of calculation, this simplification is not licit any more.

Like (,
°
1
1
,
° T) = (
HT
- - °)(
HT
- - °)+ (
HT
- - °) ° +
° °
2
2

has

one
=
- - ° + ° =

ijkl (
HT
kl
kl
kl)
ij
ij
ij
= - - - ° 1
- °
1
= ° -
°
ijkl (
HT
kl
kl
kl)
ij
ij
ij

2
2

ij
=
°
(
HT
- - °
1
+ ° = -
1
- °
ij
ij
ij)
HT
ij
ij
ij
ij

2
2
ij
.
1 °
°

1 ° °

HT
of or
& = & + -

+ - -

+
T
ij ij
ij
ij
ij, K
K
ij
ij
ij
ij, K
K
, K
K
2

2

T

}
·

In addition, according to proposal 2 of [bib4]:

= & -

I, J
I, J
I, p
p, J

1
1
&
=
(&u, + &u,) - (U, +u,
ij
I J
J I
I p
p J
J p
p, I)
2
2
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And one can eliminate &u from the expression of G
() by noticing that &u is kinematically acceptable
and by using the ***equilibrium equation****:*

u& D =
F U
,
& D + G u& D + N U
ij
I J
I
I
I
I
ij
J
I, K
K D

S
Sd

from where:

- G () =
&
- fi &u - &f U +

I
I
I
(- F U
I
I)
D -
K, K
(gi &u +
I
&g U
I
I) D

S

- G U
-
N

D
I
I

K, K N
K
K

S
1
=
ij &u -
I,
F
J
I &
U D - G
I
I &
U D -
&f U D -

I
I I
ij (U +
I, p
p,
U
J
J,
p
p, I) D
2

S

+
&T +
(-

I)
- &
+

-

F U
D
G U
G U
N
D
T
I
K, K
I
I
I
I
K, K
K

N

S
K
1

HT
1
+

°
°
°
°
ij - ij ij, K K
ij ij
ij ij, K K D

2

+
-
-

2

and finally:

G () =
U
-
-

T
ij
I, p
p, J
K, K

, K
K D
T

1
1
+ -
° °

HT
°

°

ij
ij
2

-
-
-
ij, K
K

ij
ij
ij
2

ij, K K D

+ F U
+

F

I
I
K, K
I, K
K ui D

+ G U + G U
-

N
I, K K I I I
D
K, K
K
nk

S
- ij nj Ui, K K D
Sd
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Note:

· In deformations of Euler-Lagrange the first intégrande becomes H U
with
I, p
ij
I, p
p, J
H
= + U
I, J
I, J
I, J.
· In axisymetry, one has the formal analogy (X, y) (R, Z) and all components of

gradients implying the component orthoradiale are null except, = R. Moreover
R
the element of surface is multiplied by R to take into account the calculation of the integral for one unit of radian.
· The possibility of taking into account fields of imposed displacements was not developed. Those are not constrained besides by the propagation of crack since they appear via the equilibrium condition.
· In the surface term one has normal derivations on the surface which do not have a direction for the elements of skin used in Code_Aster. One thus has recourse to the geometry differential and with derived the contravariantes for better apprehending this intégrande on surface calculation (cf [Appendix 2]).
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2
Discretization of the rate of refund of energy

2.1
Method theta in dimension 2

It is pointed out that the rate of refund of energy G is solution of the variational equation:

G (S) (S)
(
***m** S) ds*
G (),

=

O

where:

.
***m** is the unit normal at the bottom of crack O located in the tangent plan at and returning in,*
.
{
= μ such as

*μ **N** =*
0 on

}
.

*In dimension 2, the **bottom of crack** O **is brought back to a M0 point**, and one can choose a field unit in the vicinity of this point, so that: G (**M0**) = G ()*

m
O

Appear 2.1-a: Melts of crack in 2D

2.2
Method theta in dimension 3

Dependence of G
() with respect to the field on the bottom of crack is more complex. The field scalar G S
() can be discretized on a basis which we will note p
(( ))
J S
.
1 J NR
0
S
O

Appear 2.2-a: Discretization of the bottom of crack in 3D (curvilinear X-coordinate)
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That is to say Gj components of G S
() in this base:

NR
G S
() = G ()
J pj S
j=1

In the same way, fields I (pertaining to) can be discretized on a basis which we will note
Q
(
)
( )
I
K S
( )
. Let us indicate by I the trace of field I on the bottom of crack
=
S
() and
1k M
O: I S

by I
( )
K components of I S in this base:

M
I (S) = ik qk S ()
k=1

G S
() being solution of the variational equation G (S) (S) ***m*** *(S) ds G (),*

=
, Gj
O

check:

NR
M
G p (S)
I
I

J
(Q (S)
K
K
) ***m*** *(S)*

ds = G (), I [, 1 P
J
]

j=1
K =1
O
that is to say:
NR
M

I
p (S) Q (S) ***m*** *(S)*

I
K

J
K
ds G J
G (),
I [1, P]

=

j=1 k=1

O

Gj can thus be given by solving the linear system with P equations and NR unknown factors:

NR

G has
= B
, I = 1, P
ij
J
I
j=1

M

with A
I
=
p (S) Q (S) ***m*** *(S) ds*
ij
K J
K

K =1
O

B
I
I
= G ()

This system has a solution if one chooses P independent fields I such as: P NR and if MR. NR. It can comprise more equations than unknown factors, in which case it is solved within the meaning of least
squares.

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2.3 Choice
in
Aster *of the discretization of G in dimension 3*

In dimension 2*, there is no problem bus by choosing a unit field in the vicinity of*
melts of crack, one obtains the relation G = G (). The rate of refund of energy is independent of field.

In dimension 3*, dependence of G*
() with respect to the field on the bottom of crack is more
complex. In Code_Aster, one can calculate:

.

The value of G
() for a field theta given by the user (cf orders
CALC_G_THETA_T [U4.82.03]). It is interesting to choose the unit field theta with
vicinity of the bottom of crack and such as:

(S) ***m*** *S*
() = 1, S X-coordinate curvilinear of O

O

Appear 2.3-a: Discretization of the bottom of crack in 3D (normal)

One obtains in this case a total rate of refund G corresponding to a uniform progression of the crack such as:
Gl = G (S) ds = G ()

O

where L is the length of the upper lip or lower of the crack.

.
The rate of refund of energy room G S
() solution of the variational equation

G (S) (S) ***m*** *(S) ds G (),*

=

O

In this case, the user does not give a field theta, fields I necessary to calculation
G S
() are calculated automatically (cf orders CALC_G_LOCAL_T [U4.82.04]).

In Code_Aster, one chose ***two families of bases*** *(cf [§2.2]):*

.
Polynomials of LEGENDRE *()*
J S of degree J (0 J Degmax).
.
Functions of form *of the node K of O: K S*
() (1 K NNO = a number of nodes of
O) (of degree 1 for the linear elements and of degree 2 for the quadratic elements).

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Let us recall that the polynomials of LEGENDRE constitute a not normalized orthogonal family. They are obtained by the relation of recurrence:

N
(+1) P () (
)
n+1 T - 2n + 1 T Pn T
() + N Pn-1 T () = 0

In particular:
P0 T () = 1
P1 (T) = T
P
(
)
2 T
() = 3 t2 -1/2
P
(
)
3 T
() = 5 T3 - 3t/2

In Code_Aster, one normalizes them in the form:

( )
2
S

J S
=
2 J + 1 P
-1
L
J L

where:
.
S is the curvilinear X-coordinate of O,
.
L the length of the bottom of crack O.

( )
2 (S)
1 S
O (S)
0
S = 0
S = L O
L
0
L

Appear 2.3-b: Polynomials of Legendre
In Code_Aster, one limits oneself to Degmax = 7 like maximum degree.

Functions of forms K S
() are associated the discretization of O.

3 (S)
( )

1 S
2 S
( )
K = 1
K = 2
K = 3

Appear 2.3-c: Functions of form of the bottom of crack (linear elements)
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Let us recall that one is brought to discretize G S
() and fields I (S) (trace of field I on the bottom
of crack O).

NR
G (S) = Gj pj S ()
J =1

M
I (S) = I ()
K qk S
k=1

There is thus several possible choices of discretizations, summarized in the table below:

Polynomials of LEGENDRE
Functions of form
G S
( )
NDEG
NNO
G ()

( )
J J S
Gj J S

j= 0
j=1
I (S)
NDEG

NNO
I
( )
I
K K S
( )
K
K S
k= 0
K =1
Table 2.3-1: Choice of the discretization

NNO:
a number of nodes of the bottom of crack O
NDEG: maximum degree of the polynomials of LEGENDRE chosen by the user

(NDEG deg = 7
max
)

In order CALC_G_LOCAL_T (cf [U4.82.04]) key words LISSAGE_THETA and LISSAGE_G allow to choose the discretization of I and G.

The options available in Aster are summarized in the following table:

I (S)

Polynomials of LEGENDRE
Functions of form

Polynomials of *LISSAGE_THETA: "LEGENDRE"*
LISSAGE_THETA: "LAGRANGE"

LEGENDRE
LISSAGE_G: "LEGENDRE"
LISSAGE_G: "LEGENDRE"
G S
( )
(1st case)

(2nd case)

Functions of
Nonavailable
LISSAGE_THETA: “LAGRANGE”
form
LISSAGE_G: “LAGRANGE”
or “LAGRANGE_NO_NO”
(3rd case)
Table 2.3-2: Options of discretization of Code_Aster
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First case:

G S
() and fields I (S) are broken up according to the polynomials of LEGENDRE.

NDEG
G (S) =
Gj J S ()
J =0

NDEG
I (S) =
I ()
K K S
K = 0

The component NDEG Gj are given by solving the linear system with P equations:

NDEG

G has
= B
, I = 1, P
ij
J
I
j=0

NDEG

has

=

(S) (S) I ***m*** *(S) ds*
ij
J
K
K
with
K =0
O

B
I

I = G ()

One makes the choice in Code_Aster take, like fields I, the NDEG fields I such as:

I (S) ***m*** *(S) = (S)*
I

where I S
() is the polynomial of LEGENDRE of degree I.

The linear system is simplified then in a system of P = NDEG equations with unknown NDEG:

NDEG

G has
I
ij
J
=
(
G), I = 1, NDEG
j=

0

with A
=
(S) (S) ds
ij
J
I
=
I
J

O

because the polynomials of Legendre form a base orthonormée on 0.
NDEG
Thus G
J
J

J = G () and thus G (S) =
G () (S)
J
.
j=0
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Second case:

G S
() is broken up according to the polynomials of LEGENDRE.
I (S) is defined by the functions of form of the nodes of the bottom of crack.

NDEG
G (S) =
G (S)
J
J
j=0

NNO
I
(S)
I
=
(S)
K
K
K =1

One makes the choice in Code_Aster take, like fields I, the NNO fields I such as:

I (S) ***m*** *(S) = (S)*

I

where ()
I S is related to form of node I of the bottom of crack.
That is to say:
NNO
I
( )
K K S
*() **m** S = I S ()*
k=1
and there are NNO equations with unknown NDEG:

NDEG

G has
= B
, I = 1, NNO
ij
J
I
j=0

has

=
(S) (S) dS

ij
J
I

with
O

B

I
I
=

G ()

In this case, one must have NDEG NNO, that is to say NDEG min (7, NNO) where NNO is the number of
nodes of the bottom of crack.

Third case:

G (S) and I (S) are defined by the functions of form of the nodes of the bottom of crack.

NNO
G (S) = G (S)

J
J

j=1

NNO
I
I
(S) = (S)

K
K
K =1
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The system to be solved is as follows:

NNO
G (

G has =
ij
J
)
(I =,
1 NNO)
I
j=0

with A =
(S) (S) ds

ij
I
J
0

NNO: a number of nodes of the bottom of crack
I: function of form of node I

If there are linear elements:

(X) 1
= (1 - X)
1
1
1
2
2

Element of reference
(X) 1
= (1+ X)

0
2
2
-1 0 1

has
= has
= ,
0 if J 2
I (I J)
I (i+ J)
has
=
S S ds = if S S ds
I (I)
1
I () i-1 ()
I ()
-1 ( )

(
S
I
O
I -1
S - S
1
S - S
I
I
I
I
1
1
1) +
=
X X dx
1 X dx
S
S
1 ( ) 2 ( )
(

-1)

+
=
1
- 2
=
-
-1
(
)
(I I)
-
-
1
1
(2
2
4
6

S - S
1
S
S
I
I 1
2
-
-) +
has
=
I
I
X dx
X dx
II
2 ( )
(+1
) +
+
1
2

1 ( )
-1
-1
(2
2
S - S
1
S
S
I
I
1
I
I
1
1
1
2
-
-) +
=
(1+ X)
(+1
) +
dx +
1
(1 - X) 2dx = ([S - S + S - S
i+1
I)
(I I)]
-1
-
-
1
1
2
4
2
4
3

I -1 I
I + 1

Appear 2.3-d: Linear functions of form

The Aij matrix is thus written:
(
2

2
S - 1
S) (2
S - 1
S)
0
0

L
(2
S - 1
S)
(2 3s - 1s) (3s - 2s)
0

L
1
0
(

3
S - 2
S)
(2 4s - 2s) (4s - 3s)

6

L

0
0
(

4
S - 3
S)
(2 5s - 3s) L

M
M
M
M

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If there are quadratic elements:
(X) 1
= X (X -)
1
1
2
(X) 1
= X (X +)
2
1

2
(X) = (1 - X) (1+ X)
3

Appear 2.3-e: Quadratic functions of form (Element of reference)

It is necessary to distinguish node top and node medium

.
I = node top:

has
= has
= ,
0 if J 3
I (I J)
I (i+ J)

has
=
S
S ds = if S
S ds
I (i-2)
I () i-2 ()
I ()
-2 ( )
O

(
S
I
I -2
S - S
1
S - S
I
I
1
2) +
=
I
I
X X dx
X X
1 dx
1 ( ) 2 ( )
(
-2)

+
=
1
2
2 -
-1

(
)
-1
(2
2
4
S - S
I
i-2)
= -
30
S

I
S -
has
=
S S ds =
S S ds =
S
1
I
I 2
X X dx
I (I)
1
I () i-1 ()
I ()
-1 ( )
(
-) +
2 ( )
( )

S
I

O
I -
-1
3
(
2
2
S - S
1
S
S
I
I
1
2
2
-
-) +
=
X (X +)
1 (1 - X)
(I i-2)

dx = +
-1
2
2
15
S - S
+1
S
S
I
I
I
I
I
2
2
2
-
-
has
=

X dx
X dx
S
S
II
2 ( )
(+2
) +
+
1
2
1 ( )
=
(-
i+2
I
)
-1
-
-
1
2
2
2
15

i-2 i-1 I i+1 i+2
Appear 2.3-f: Node top
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.
I = node medium:

has
= has
= 0 if J 2
(II J)
I (i+ J)

has
=
si+1 S S ds
I (I)
1
I ()
-1 ( )
S
I
I -1
+1 (S
- S
S
- S
i+
I
I
I
1
1
-1)
=
X X dx
X X 1 1 X 1 X dx
3 ( ) 1 ( )
(+1
) +
=
1

(-)(-)(+)
-1
-1
(
2
2
2

S
- S
i+1
i-1)
=
15
has
=
si+
S
S
1
2
S ds
X
X dx
S
S
II
( )
(-
i+
I
8
1
) +
=
1(1-)2(1+)2 = (-
i+1
I
)
S I
I -1
-
-

1
1
2
15

i-2 i-1
I i+1
i+2
Appear 2.3-g: Node medium

The Aij matrix is written:

(
4

3
S - 1
S)
(2 3s - 1s) - (3s - 1s)
0
0

L
(
2 3
S - 1
S) 1 (
6 3
S - 1
S)
(2 3s - 1s)
0
0

L
1 - (

3
S - 1
S)
(2 3s - 1s) (4 5s - 1s) (2 5s - 3s) - (5s - 3s) 0

30

0
0
(2

5
S - 3
S) 1 (
6 5
S - 3
S)
(2 5s - 3s) 0

0
0
-

(5s - 3s) (2 5s - 3s) (4 7s - 3s)
L

0
0
M
M
M

Particular case: S
S = cste = L
i+ -
2
I
= length of an element
+
+
L

4
2
- 1 0
0

L
node top of edge

2
16
2
0
0

L
node medium
L - 1 2
8
2
- 1
node top

L

30 0
0
2
16
2

L
M
0 0 -1 2 8

L
M
M
M

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Method "node by node":

This method results from the Lagrange-Lagrange method but it is simplified: one replaces resolution of the linear system by multiplying the values G (I) by a weighting coefficient.

G =
I
I

I G (
)
3
4
5
...
2
1
NR
Appear 2.3-h: Method "node by node"

Moreover if G () = cte = B, I and that one considers one G constant per element (this method does not have
no vectorial significance), one a:

L
6
.
node top of edge:
(4+ 2 -) 1G = bsoitG = B
30
L

L
3
.
node top:
(- 1+ 2 + 2 + 8 -) 1G = bsoitG = B
30
L
L
3
.
node medium:
(2+16+ 2) G = bsoitG = B
30
L
2

L
6b
3b
3b

L
L
2l

What gives if the elements do not have constant lengths:

6
6
.
node top of edge: = (
or
=

S - S)

1
NR
3
1
(S - S
NR

NR -2)
6
6
.
node top: for example 3 = (
=

3
S - 1
S) + (5
S - 3
S) (5
S - 1
S)

6
that is to say: I = (

I
S
-
+2
I
S -2)
3
or: '= (

S
- S
+1
)
I
I
I
I

3
.
node medium: I = (

2 is -
+1
I
S)

To activate this method it is necessary to specify in CALC_G_LOCAL_T:

LISSAGE_G: “LAGRANGE_NO_NO”
LISSAGE_THETA: “LAGRANGE”
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2.4
Establishment of G in linear thermoelasticity in Aster

2.4.1 Types of elements and loadings

In Code_Aster, it is possible to calculate in linear thermoelasticity:

.
the rate of refund of energy G
() in 2D and 3D, associated a field of propagation
virtual of the crack (given by the user using order CALC_THETA
[U4.82.02]): order CALC_G_THETA_T [U4.82.03],
.
the rate of refund of local energy G S
() in 3D, where S is the curvilinear X-coordinate of the bottom of
fissure: order CALC_G_LOCAL_T [U4.82.04].

These calculations are valid for following modelings:

.

D_PLAN
.
C_PLAN
.
AXIS
.
3D

and for the following thermomechanical loadings applying to a two-dimensional medium (affected to triangles with 3 or 6 nodes, quadrangles with 4, 8 or 9 nodes and segments with 2 or 3 nodes) or on a three-dimensional medium (affected with hexahedrons with 8, 20 nodes or 27 nodes, pentahedrons with 6 or 15 nodes, of the tetrahedrons with 4 or 10 nodes, of the faces with 3 or 6 nodes and of the faces to 4, 8 or 9 nodes):

.
F, field of voluminal forces applied to (mechanical loads of the type GRAVITY, ROTATION, FORCE_INTERNE),
.
G, field of surface forces applied to a part S of (including on the lips crack: PRES_REP, FORCE_FACE),
.
U, field of displacements imposed on Sd part of (Not developed to date),
.
T, field of temperature (TEMP_CALCULEE),
.
, initial field of defomation (EPSI_INIT).

These loadings can depend on time and space.

The characteristics of the material (E, and) can depend on the temperature T and on space while remaining constant by elements.

2.4.2 Environment necessary

For the calculation of the rate of refund of energy G () by the method in the case of a problem thermoelastic, the field must obligatorily be created before (either by the order CALC_THETA [U4.82.02], is by order AFFE_CHAM_NO [U4.44.11]).

For the calculation of the rate of refund of local energy G S (), fields I necessary to calculation are

generated automatically.

In both cases, it is about a postprocessing only starting from the field of solution displacement calculation on the model considered. In particular, the density of free energy and the constraints are calculated starting from the field of displacement and the characteristics of material.

For calculation in 3D, it is necessary to define, starting from an ordered list of nodes, a bottom of crack of one
grid 3D, and starting from two lists of meshs, the upper lip and the lower lip of this
fissure order DEFI_FOND_FISS [U4.82.01]. This operator creates a concept usable by
operators CALC_THETA and CALC_G_LOCAL_T. In 2D, the bottom of crack is tiny room to a point and this
operator is not necessary for the calculation of G
( ).

2.4.3 Calculations of the various terms of the rate of refund of energy

The expression supplements G
() is given to [§1.3]. We will detail each term. The field
is null apart from a disc of ray Rsup S
() defined in chapter 3 [Figure 3.3-a].
Let us notice that as all the terms utilize or its gradient, the elementary terms
are null apart from this disc of ray Rsup S
(). In orders CALC_G_THETA_T and
CALC_G_LOCAL_T, it is thus not necessary to specify the loadings which do not apply

in this zone.

Elementary 2.4.3.1 traditional Term

TCLA = U

- ((U), T
ij
I p
p J
)
,
,
K, K

The density of energy elastic ((U), T) is written in linear thermoelasticity:

.
in 3D and AXIS:
(
1
(U), T) =
(2
II) + µ -
ij
ij
HT
2
.
in DP:
1 - E

(
E
E
(U), T)
(
)
=
(2 +2
2
xx
yy)
(

+
+
-
2 1 +)(1 -
2)
(1+)(1-
2) xx yy
(1+) xy
HT
.
in CP:
E

(
E
E
(U), T) =
(
2 + 2 +
+
2 -
2 1 - 2) (xx
yy)
(1-2) xx yy (1+) xy HT

9
with
= 3
K (T - T) -
2
2
HT
réf
II
K
(T - rTéf)
2
where:
E
E
E
3K =
;

=
1 - 2
1
(+) 1
(- 2); 2μ = 1+

E: YOUNG modulus
: Poisson's ratio
, μ: coefficients of BLADE
: thermal dilation

The density of energy elastic ((U), T) can be written in a general way in the form:

1
2
2μ
((U), T) = K (-
3 (T - T
2
kk
réf)
+
eq
2
3

3
1
2
with
= D D and D = -
eq
ij ij
ij
ij
kk ij
2
3
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2
1
that is to say eq = (
2
3ijij - kk)
2
and ((
μ
U), T) 1
2
= K
- 3
kk
K (T - réf
T
)
9
2
kk + K (T - réf
T
) 2
2
+ μ -

2
2
ij ij
3 kk

2
=
+ μ - 3
kk
ij ij
K (T - réf
T
)
9
2
kk + K (T - réf
T
) 2
2
2

2.4.3.2 Terme forces voluminal

TFOR =
fi ui K, K + fi, K K ui

2.4.3.3 Terme forces surface

TSUR = G

I, K K ui + gi ui
K, K -
N

N K
K

Note:

In this surface term there are normal derivations on the surface which do not have a direction for elements of skin used in Code_Aster. One thus has recourse to the differential geometry and with derived the contravariantes for better apprehending this intégrande on the surface of calculation (cf [Appendix 2]).

2.4.3.4 Term thermics

THER = -
T
T, K K
with:

(
1 dK T

D T

(U), T)
( )
=
(- 3 (T - T)
( )
- 3K +
(T - T)
- 3
-

2

(
(T T
kk
réf
réf
kk
réf)
T
dT
dT

2.4.3.5 deformations Term and initial constraints

1 ° °

1 °

TINI
HT
=
°
ij - ij ij
, K
I
J
I
J
ij
ij, K K

2

-
-
-

2

One can notice that if ° = ° then:

= (- HT -) ° + °= (- HT) and TINI = 0

Note:

Taking into account the various digital processings carried out at the time of the establishment in the source of
the operator, it is not licit to cumulate stress fields and initial deformations,
because this term is then not cancelled. The user will have to return either of the initial constraints, or of
initial deformations but not both.
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2.4.4 Standardization of the rate of refund of energy in Aster

2.4.4.1 Axisymétrie

G
() such as it is established here, calculates the restitution of energy in definite kinematics par. It can be necessary to standardize it (with the hand! it is not done automatically in the code) to be able to compare with an intrinsic value with material, in particular into axisymmetric. Let us consider the case of an inclined crack, whose bottom of crack is at a distance R of the axis of symmetry:

Y
R
L
X
Appear 2.4.4.1 - has: Melts of crack in axisymetry

In Aster, axis OY is the axis of symmetry in modeling "AXIS" and the rate of refund of *energy calculated is:*

G
() = - dW

D L

where W is the potential energy per unit of radian.

However the intrinsic value of the rate of refund of energy is:

$$G_{total} = -\frac{dW}{dA}$$

where:

.
Wtotale is total potential energy,
.
dA is the variation of surface of the crack.

$$W = 2W_{total}$$

with:

$$dA = 2Rdl$$

$$dW_{totale} = dW\,D\,L$$

$$1 = \frac{dW}{dA}$$

from where:

$$= 2 =$$

$$\frac{dA}{D\,L\,dA} = R\,D\,L$$

$$1 = G$$

and thus

$$= G()$$

in axisymetry.
R
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Different 2.4.4.2
case

***In dimension 3**, the value of G*
() for a field given by the user is such as:

$$G(\ ) = \int_{(S)} (S)\ \mathbf{m}\ S\ ()\ ds$$

O

In order CALC_THETA [U4.82.02], the user defines the direction of the field in bottom of fissure. By defect, it is the normal at the bottom of crack in the plan of the lips. By choosing one unit field in the vicinity of the bottom of crack, one a:

*(S) **m** S*
() = 1, S X-coordinate curvilinear of
O

and:

$$G(\ ) = \int_{(S)} D$$

O

That is to say G the total rate of refund of energy. To have its value per unit of length, it is necessary to divide the value obtained by the length of the crack L:

G ()
G =
in 3D
L

***In dimension 2** (C_PLAN and D_PLAN), the bottom of crack is tiny room to a point and the value of G () is*
independent of the choice of the field (with and unit in the vicinity of the bottom of crack).

G = G (),

2.5
Paramètrage of the orders

The table below proposes a summary of the paramètrage orders CALC_G_LOCAL_T CALC_G_THETA_T. For more precision one will refer to [U4.82.03] and [U4.82.04].

Orders Key word
Value
by
Ref.
defect

CALC_G_LOCAL_T
MODEL

[§2.4]

“D_PLAN”

“C_PLAN”

“AXIS”

“3D”

CHAM_MATER

*[§2.4.1] Def.
materials*

MELTS

*[§2.4.1]
Def. bottom of
fissure*

DEPL

*Recup.
of one
field of depl.*

RESULT

EXCIT

*[§2.4.1] Standard
charg.*

SYME_CHAR *“WITHOUT”*

“WITHOUT”

“SYME”

“ANTI”

LISSAGE_THETA *“LEGENDRE”*
[§2.2]

“LEGENDRE”

“LAGRANGE”

LISSAGE_G *“LEGENDRE”*
[§2.2]

“LEGENDRE”

“LAGRANGE”

“LAGRANGE_NO_NO”

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***DEGREE** 5*
[§2.2]

***INFORMATION** 1*

TITRATE

***OPTION** "CALC_G"*

"CALC_G"

[§2.4]

"CALC_G_LGLO"

R_INF

[§3.2]

R_SUP

[§3.2]

R_INF_FO

[§3.2]

R_SUP_FO

[§3.2]

COMP_ELAS

COMP_INCR

ETAT_INIT

[§2.4.3]

CALC_G_THETA_T
MODEL

[§2.4]

“D_PLAN”

“C_PLAN”

“AXIS”

“3D”

CHAM_MATER

[§2.4.1] Def.
materials

THETA

*[§2.4.2] Def.
theta*

MELTS

*[§2.4.1]
Def. bottom of
fissure*

DEPL

*Recup.
of one
field of depl.*

RESULT

EXCIT

*[§2.4.1] Standard
charg.*

SYME_CHAR *“WITHOUT”*

“WITHOUT”

“SYME”

“ANTI”

INFORMATION *1*

TITRATE

OPTION *“CALC_G”*

“CALC_G”

[§2.4]

“CALC_G_LAGR”

“CALC_K_G”

“G_BILINEAIRE”

“CALC_G_MAX”

“CALC_DG”

[§4] Behavior

COMP_ELAS

COMP_INCR

ETAT_INIT

[§2.4.3]
Table 2.5-1: Paramètrage of the orders
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3
Introduction of the field theta into Aster

3.1
Conditions to fill

The field theta is a field of vectors, definite on the fissured solid, which represents the transformation field during a propagation of crack within the meaning of [§1]. The transformation should only modify the position of the bottom of crack and not the edge of the field, i.e.: ***N*** *= 0 on (****N***
normal with). Moreover, the ***field theta must be regular*** *on [bib4].*

Because of the singularity of the field of displacement, it is interesting from the numerical point of view to use constant fields in a vicinity of O, thus cancelling in this vicinity them
singular terms
- U
K, K
ij
I, p
p, K in G
( ).

3.2
Choice of the field theta in dimension 3

3.2.1 Method of construction

One must build a checking field:

N = N = 0 on the edge of the field (N is the normal with)

= O given on the bottom of crack O

where represents the trace of on O.
One gives oneself two volumes T and S (deformed cylinders) surrounding the bottom of crack O.

Appear 3.2.1-a: Construction of the field theta in 3D (overall picture)
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R is noted
(S)
inf
the variable ray of T and R
(S)
sup

that of S.

P
S
T
R
O
Rinf
Rsup

Appear 3.2.1-b: Construction of the field theta in 3D (plane of cut)

In any point of O, located by his curvilinear X-coordinate S, one can define a normal plan P in which the field is introduced in the following way:

.
N (R (S)) = (S)
O
for 0 R (S) R
(S)
inf

.
N (R (S)) = 0 for R (S) R (S)
sup

.

( )
S (R
S
sup
) \ T (R (S)
inf
)
N varies linearly compared to the ray R S in the crown
( )

.

S (R
S
sup
)
N is continuous in

( ) .

This manner of introducing is geometrical. It amounts giving itself two rays R ()
inf S and
Rsup S
(), and to carry out calculations of distance from a point running at the bottom of crack to determine value of in this point.

3.2.2 Calculation algorithms

The method requires the data of the field O on the bottom of crack O and of the two rays R (S)
inf

and R
(S)
sup
who can depend on the position of the point on O. The user introduces these data
node by node on O in the following way:

Nodes of O
O
Rinf
Rsup
N1

M

NR
R
R
I
O I
inf I
sup I

M

Table 3.2.2-1: Data for construction of the field theta in 3D
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The program is given the responsability to calculate the field in any point of according to the following procedure:

.
Calculation of the field theta O in each point of O: The module O being given (by the user or by the method theta, to see [§2.3]), the problem is to determine the direction of O. must be locally in the tangent with the lips of the crack and normal plan to the edge with which it belongs. being calculated with the nodes, in the case general (bottom of crack not plan) the direction of will be realised on the 2 edges of O having the joint node.

n1
N2
N
O
T1
n1 N
F
2
1
T2
Mr. F2

Appear 3.2.2-a: Construction of the field theta in 3D (normals)

Are F1 and F2 two faces belonging to the lips of the crack and including/understanding the successive edges

T1 and T2 of O. One calculates initially the normal n1 with edge T1 in the plan of the F1 face then normal N2 with the T2 edge in the plan of the F2 face.
N + N
N
1
2
1 and N2 being unit normals, one deduces some N =
then (M) = (M) N for
2
M O.

It is considered that the Fi faces are right:

.
If Fi is a triangle, the plan of the Fi face is defined.
.
If Fi is a quadrangle, one cuts out Fi in 2 triangles Fi1 and Fi2. One must
then to calculate the equations of the two plans containing the faces Fi1 and Fi2 and to make two calculations of normal per edge Ti.

This calculation requires to know the faces belonging to the lips of the crack and including/ understanding one
edge of O. In Code_Aster, the user returns all the surface elements belonging to
lips of the crack. These faces appear in one or more groups of meshs and are described in connectivities of the elements of surfaces. The algorithm sorts these faces to preserve only those having 2 tops on O. The stages of the algorithm are as follows:

1) For each node of O, one extracts the meshs belonging to the lips from the crack,
2) Of these meshs, one tri those having two nodes on O,
3) One recovers the type of the face (SORTED or QUAD) and one calculates the equation of the plan (S)
tangent (S),
4) For each edge of O of tops NR, NR calculation of normals N
N and
I
I 1
+
I, 1, ni+1,1,
I, 2
N
.
I,
1
+ 2

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Lastly, is calculated according to the following algorithm:

Buckle on the tops Nor of O:
1
nor = (N + N
I,
I,)
2
1
2
(Ni) = (Neither) nor

End of the loop on the tops Nor of O

Algorithm 1: Calculation of

ni+1,1
nor, 2
N
N
i+1,2
I, 1
NR
NR

I
i+1

Appear 3.2.2-b: Notations of the normals at the bottom of crack

.
Calculation of the field in each point of:

Buckle on the nodes M

Calculation of projection M of M on O
(Gives in fact the nodes M I and M i+1 such as M [
M, M
I
i+1] and
S [0]
1
, such as MR. M = S MR. M
I
I
i+1
D = D M
(, M)
- Calculation
of

- Calculation
of
(M) by linear interpolation:
(M) = (1 - S) (M) + S (M
I
i+1)
- Calculation
of
R (S)
inf
and R
(S)
sup
by linear interpolation:
R (S) = (1 - S) R
+ S R
inf
inf I

inf I

+1
R
(S) = (1 - S) R
+ S R
sup
S
I
I
up sup
+1
-
/If D > R
(S), (M)
sup
= 0
/If D < R (S), (M)
inf
= (M)
D - R
/
S
If
R (S) D R
(S),
inf ()
inf
sup
=
and (M) = (1 -) (M)
S
R up (S) - I
R nf (S)
Finsi

Algorithm 2: Calculation of the field theta in 3D
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M
O
M I
D
M
S
M i+1

Appear 3.2.2-c: Calculation of the field theta in 3D

We detail below the calculation of projection M of M on O:

For each node M:

-
Recovery of the co-ordinates of M
-
Buckle on the Mi nodes of O (I = 1, NNO -)
1
Recovery of the co-ordinates the Semi one and Mi+1
MR. M
. MR. M
Calculation of S
I
I 1
I
I
=
+

MiMi+1

/if < 0: if = 0
/if > 1: if = 1
Calculation of the co-ordinates of M I: OMi = OM + S
I
I Semi Mi+1
Calculation of D
= D (M, M
I
I)
Fine buckles

- Recovery
of
J such as D = min (D
J
I)
I
- Knowing
J one recovers M, M
, S
J
j+1
J and projection M of M on O such as:
MR. M = S MR. M
J
J
J
j+1

Algorithm 3: Calculation of projections on the bottom of crack

M
di-1
di
if-1
if
Mi+1
M
M I
I

Appear 3.2.2-d: Projection of the points on the bottom of crack
Handbook of Reference

3.3
Choice of the field theta in dimension 2

It is about a particular case of dimension 3. O is limited to a point, the user chooses the rays Rinf and Rsup, the module in bottom of crack O and the field are built so that:
(R) = 0 if R R
sup
(R) = N if R R
0
inf
R
- R
sup
(R) =
N if R R R
0
R
- R
inf
sup
sup
inf

Rsup
0
Rinf N
0
0
R
R
inf
sup
^
Appear 3.3-a: Calculation of the field theta in 2D

3.4 Other method

The user can enter itself the field, by using order AFFE_CHAM_NO [U4.44.11] Code_Aster which makes it possible to affect node by node or group of nodes.

4
Derived from the rate of refund of energy compared to one variation of field

Initially one points out the problems mechanic-reliability engineer justifying the introduction of this option then one summarizes, through an example, its procedure in Code_Aster. Afterwards some preliminaries on the theoretical implications of the derivation installation (supplementing those G

(F)
[§1.3]) one details the calculation of each integral term of
where F
is the field
S

s=0
theta used in the preceding paragraphs. To conclude, one is interested in the establishment of this functionality in the code and with its perimeter of use.

4.1 Problems

Studies mechanic-reliability engineers require the derivative of the rate of refund of energy compared to
a variation of field. By coupling Code_Aster with software PROBAN, one can thus to know the probability of starting of the rupture for a distribution of variation of field data. For example, within the framework of project PROMETE [bib7], one sought to determine probability of rupture of a tank REFERENCE MARK by regarding the thickness of its lining as a random variable.

Until now this type of application required expensive parametric studies to determine, with each step of calculation of PROBAN, sensitivity of the thermomechanical fields and rate of restitution of energy to a variation thickness of the coating. From now on, with this option of Code_Aster, one determines in only one calculation the value of these derivative.
Beyond the aspect performance, that largely simplifies the process of obtaining of derived and improve their reliability. One thus avoids having to re-mesh and requalify infinitesimal alternatives of the initial structure. There are not any more states of heart to have as for the relevance of the parameter of variation
of thickness. Indeed, calculation by finished differences (paradoxically, to validate the step analytical on real cases, one is well obliged y to have recourse!) can depend on the variable with to differentiate, to be sensitive to the grid and, in a general way, to the errors of any kind (elements stop, discontinuity, conditioning, programming…).

The technique of derivation selected is completely analytical (taking into account its architecture software, Code_Aster cannot be differentiated by automatic tools (such ODYSSEY) for to solve this type of problem) and rests on the direct derivation of the equations expressed in form variational. The variation of field is then modelled by a function theta sensitivity noted S, not to confuse with the function theta fissures noted F. In practice, although one be interested that with derivative eulériennes, one handles also derivative Lagrangian because they intervene naturally in the results of derivation of integral (theorems of transport of Reynolds). Moreover, one calculates the first using the seconds.

These studies of sensitivity are for the moment accessible only in 2D for plane modelings or axisymmetric in linear thermoelasticity and with loadings (and materials) independent of the temperature and the variation of field. But they can spread with 3D, with non-linear elasticity, plasticity…

Thus let us consider a plane structure subjected to a pressure distributed on its edge higher and than displacements and of the imposed temperatures. For carrying out thermomechanical calculation, it is necessary
to define the field theta sensitivity. In our example, it decrease between the X-coordinates x1 and x2 of sound
vertical support and it are directed along the x-axis. It gathers all the material points of configuration which will move virtually according to the transformation:

F: MR. M
S
+

S
S
(M)
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This one answers the same properties of regularity as the transformation related to the field theta fissures
that we will note F henceforth: MR. M
F
+
(cf [§1.3]). The variation thus is materialized
F
F
(M)
of field on the left edge of the structure.

y
Field S
sensitivity
Field F
fissure
X
F
Fissure
x1
x2

Appear 4.1-a: Derived from G (F) compared to a variation of field controlled by S

Then, one provides this field thermal theta sensitivity to the operators and mechanics which go

to solve, in addition to their problem direct, of the "pseudo" assistant systems built by derivation terms with terms of the first [R4.03.01]. The resolution of these systems makes it possible to exhume the derivative Lagrangian of the temperature and displacement, noted respectively, & T and &U.
By assembling these derivative Lagrangian during the calculation of the rate of refund of energy, one deduces some then the derivative compared to the variation of field. Well-sure only the parts intervene of supports of the fields included in the crown of calculation.
On the figure above this crown is centered on the bottom of crack F and it corresponds to one linear decrease of the module of theta fissures, radially, from the center towards the circumference.

Note:

This technique of derivation is related deployed technique of representation Lagrangian of variation of field [R7.02.04]. In both cases, one avoids the expensive ones parametric studies by using a grid fixes reference and by modelling the variations virtual of field by suitable functions theta.
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The procedure (cf [§4.4.1]) of a calculation of sensitivity can be thus schematized in the form following:

Calculation of S
CALC_THETA

S
S
Calculation
T, T&
Calculation
Thermics
Mechanics
THER_LINEAIRE
MECA_STATIQUE
Calculation of F
T, T&
CALC_THETA
U, U&
F
G

(
F)
Calculation of G (
F) and of
S

s=0
CALC_G_THETA_T

Appear 4.1-b: Procedure of the derivation of G

4.2 Remarks preliminaries

4.2.1 Theorem of transport

The expression of G (F) established in the code comprises five integral terms in accordance with definition of [§2.4.3] of the type (or its during into surface):
Is =
v
D

S

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To derive them compared to their support (in the vicinity of the support of reference), one uses one of alternatives of the theorem of transport of Reynolds by supposing that all conditions of regularity are checked: it is necessary that the Fs transformation modelling the movement of the border of
volume moving

S, and the intégrande (tensor of order 0, 1 or 2) are all of C1 class.
One has then, while noting &
S the Lagrangian derivative (in the vicinity of the origin) compared to the field
theta sensitivity:

$$I_S$$

$$S$$
$$F$$

$$S$$
$$\&$$

div

=
+

v
D

S

S
=0

=0
S
S

This theorem is declined in several versions, according to whether one considers a material or space volume
and that one places of Lagrangian description or eulérienne. However like one derives with

I S
vicinity of the origin

all these alternatives are equivalent (“Philosophically” it

S s=0
result is reassuring because it makes it possible not to privilege neither the matter elongation (volume geometrical), nor
appearance of matter (material volume), in the interpretation of this variation of field).
The necessary theoretical regularities are far from being checked in practice, but these flat are majority of the times “drowned” in the errors due to arithmetic finished, the method of the elements stop and with numerical integrations. Thus the border moving, as in example Ci
above, often presents “corners” and the transformation FF is not always C1 (Fs is, but not FF, which has two surfaces of discontinuity on the Rinf borders and Rsup of the crown).
Indeed, the field theta crack is defined in the form of a first order polynomial in
crown and of a constant polynomial outside (it is thus C0) whereas the field theta sensitivity

is a combination of third order students'rag processions which return it safe C2 in the middle of its support
(where it is right C1). During the calculation of G one calls directly upon the derived first of the field theta fissures, whereas for obtaining his derivative one uses the derivative indirectly second theta sensitivity (for obtaining the Lagrangian derivative of the tensor of the deformations, cf p.44). A compromise was thus found between the theoretical order required by derivations and the precision of
finite elements modelling calculation (One did not have to penalize the calculation of G with elements linear). One uses functions theta of an order of regularity just lower than the theoretical order.

Note:

· During numerical tests one substituted for the functions theta sensitivity and crack a spline cubic natural particular (with condition of connection of the derived type first null with edges) due to R.Wodicka (R.Wodicka. Carryforward of the institutes für Geometry and Praktische mathematik, RWTH Aachen, 1977), which filled all desired conditions of regularity. But this one brings only marginal profits unless refining to the extreme the grid and to circumscribe the zone of calculation around discontinuities of the fields theta used.
· In practice, for better apprehending the cubic variations of the function theta sensitivity and to ensure a better convergence of the solution, the user is obliged to lead his calculation of sensitivity with complete or incomplete quadratic finite elements (SEG3, TRIA6, QUAD8 and QUAD9). Whatever their order, these elements of the Lagrange type us guarantee that a C0 regularity at the borders. The use of elements of Hermit would have been more adapted to bring this continuity to the level of the derivative first.
· The derivation of the integral reveals two terms: the first corresponds to derived from the intégrande calculated as if its paramètrage were distinct from that defining the support of the integral; the second evaluates the rate of through the mobile border (term of convection particulate derivative).
· The result is unchanged when the integral is surface (or linear in a PLANE problem or AXIS). It is just necessary to replace the voluminal divergence by surface. From a point of view

---

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numerical, it is besides to better calculate the latter via derivative contravariantes on surface:
divs =

that to approximate it, by a voluminal divergence and a normal derivative brought back to surface calculation:
div
= div
S
- (N) .n
Indeed, this term reveals normal derivations on the surface which do not have a direction for elements of skin used in Code_Aster. They would have to be approximated while projecting voluminal calculations on the surface element. To cure it one has recourse to the derivative contravariantes which makes it possible to express this divergence with the assistance only of sizes surface. One finds these problems in all the surface calculations set up in the rate of refund of energy and its derivative.

The terms of the rate of refund of energy, except function theta, can pose problem. It is for this reason that the function theta fissures revêt the shape of a crown of constant value in its center. That makes it possible to dam up discontinuities of the gradient of field of displacement on melts of crack likely to penalize the term traditional elementary.

Thereafter, as long as confusion will not be possible, we will note by a simple point &

Lagrangian derivative related to the variation of field. One will not be interested any more but in this transformation.
On the other hand one will continue to distinguish the various fields theta. Before approaching calculations us
let us close these rather qualitative remarks by examining the loadings and materials.

4.2.2 Loadings and materials

Let us take again the same remarks as those formulated with [§1.3]. We thus point out that:

&f = F S

,
S
=
I

I
&g
G
,
I
I
&°
°
S

= ,
°
°
S

=
ij
ij
&
,
ij
ij
&
S
E
= E
, &
S

= ,
&
S

=
,
&
S
T
= T
ref.
ref.

Indeed, that is to say the loading or the material considered, then there exists a field of R3 such as:

/ =

Derivation compared to the parameter commutating with this restriction, one has the result:

=

= 0
= 0

Note:

This assumption is checked only for sufficiently regular fields (for example belonging to spaces of Sobolev of). Their definition should not be impacted by variation of border.

On the other hand, for this derivation, the derivative eulérienne of the field of temperature is nonnull, because
the trace of S on the edge of the structure moving cannot be neglected any more [bib4].
The Lagrangian derivative of the characteristics materials are null for the problem discretized taking into account the preceding remark and owing to the fact that one defines them constant by elements

finished. When one allows them to depend on the temperature, their Lagrangian derivative will not be any more
null. One will have, for example, for the Young modulus:

E& (T)
E
E
=
+ E
. =
(
S
T& - T
.)

3
2
1
T
=0

Note:

· One can use characteristics variable materials within finite elements or dependent on the temperature, provided that that is apart from
(
supp F)
(
supp S
I
).
· For the calculation of derived from G, the derivative Lagrangian of the loadings (and even those their gradients) do not intervene apart from
(
supp F)
(
supp S
I
).
· For the moment, one does not take into account loadings whose definition is impacted by variation of field. It is for example the case for a function whose support is defined according to geometrical characteristics of the moving border, or, for a field of

initial deformations builds starting from displacement resulting from a thermo calculation mechanics. One could envisage specific options of calculations in the operators concerned to exhume the derivative eulériennes missing and to instill them into the operator CALC_G_THETA_T via a second operand of the key word SENSITIVITY.

4.2.3 Form

Setting with share the relation between the Lagrangian derivative and eulérienne:

& =
+. S

éq
4.2.3-1

one uses only the formula giving the Lagrangian derivative of the gradient of a field according to gradients of the field and its Lagrangian derivative:

& = & - S éq
4.2.3-2

In Cartesian co-ordinates, one can apply these formulas component by component when the field is represented by a vector or a matrix. The second term is then a simple product matrix-vector (in theory, it is about the contracted product of two tensors). For example in the case of a vector or a tensor one a:

}
.
= & - S
I, J
I, J
I, K K, J éq
4.2.3-3
}
.

= &
- S
ij, K
ij, K
ij L, L, K

from where the Lagrangian derivative of the divergence:
}

.
div = & - S
I, I
I, K K, I éq
4.2.3-4
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In cylindrical co-ordinates, the things become complicated a little if the field considered is one tensor of order 1 or 2. It is then necessary to take account of the component orthoradiale gradients. By

example, for a tensor of order 1 the first relation comprises the complementary term R which
R
finally does not intervene because it is multiplied by the component orthoradiale field theta sensitivity who is null. On the other hand, in the second, a complementary term appears due to the derivation of R preceding:
R

.
S
S
S
S

0

0

R R
,
R Z
,
&
0
R R
,
&r Z, +
+
R R
, R R
,
R Z
, Z R
,
R R
, R, Z
R Z
,
Z Z
,

}
.

&

S
=
R

0
0 =
R
0
0 -
R R
0

0
éq
4.2.3-5

R

R

r2

0
Z R
,
Z Z
, &
0
Z R
,
&z, Z S + S
0

S
S
Z R, R R,
Z Z
, Z R
,
Z, rr Z +z Z
,
, Z Z
,

Note:

· In the calculation of derived from G, this relation intervenes only for the particulate derivative of gradient of theta fissures (thus that of its divergence) and for that of the gradient of displacements (thus for those of the tensors of the strains and the stresses), because all the other gradients are multiplied by the component orthoradiale theta fissures which is

null.
· In axisymetry, with the help of the complementary terms, the Cartesian formulas can to apply directly with the formal analogy (X, y) (R, Z). Moreover the element of surface is multiplied by R to take into account the calculation of the integral for a unit of radian.

When one is interested in the Lagrangian derivative of the gradient of a loading such as it is taken in currently count, derivative second appear. Thus, in the case of a vector it comes:

}
·
= & - S
I, J
I, J
I L,
L, J

I
=
+ S - S
éq
4.2.3-6

I, K
K
I L,
L, J

{

= 0
, J
=
S
I, K J
K

The derivative second of the functions of form of the quadratic elements not being available in the code, they thus should have been set up. Their introduction on the element of reference is quasi-immediate, but their transcription on the element real 2D is harder (cf [Appendix 1]). On the elements 1D (for the surface loadings), one has recourse to the differential geometry and to

derived contravariantes for better apprehending the intégrandes on the surface of calculation (cf [Appendix 2] and [§4.2.1]).
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Note:

· One could have been freed from this establishment by carrying out integrations by parts (via theorem of Green), but those having to apply to factors made up of three terms, that complicated much the formulation (without counting the taking into account of integrals of border).
· In spite of the low regularity of the elements used, of the numerical tests showed the maid quality of these derivative second (for polynomial fields).
· The analytical calculation of these derivative second was set up for the elements quadratic in modelings plane or axisymmetric related to the mechanical phenomena and thermics. This calculation concerns only the first family of points of Gauss and, for reasons of data-processing stability, values of these derivative second (at the points of Gauss) were stored at the end of object JEVEUX dedicated to the functions of forms.

4.3 Calculations of the various terms of derived from the rate of refund of energy

G

(F)
One wishes to calculate the various terms of
. One takes again the nomenclature of [§2.4.3]

S

s=0
by handling only the intégrandes and by detailing each term in co-ordinates first of all
Cartesian (indifferently 2D or 3D) and in small displacements. Thereafter, on a case-by-case basis, one
specify the possible modifications justified by the axisymetry and great displacements.
Let us notice that they utilize all F or its gradient: they are thus null apart from the disc of
Rsup ray. In order CALC_G_THETA_T it is thus not necessary to specify them
loadings which do not apply in this zone.

4.3.1 Derived from the elementary traditional term

According to the theorem of transport of [§4.2.1], the intégrande corresponding to derived from the term
traditional elementary is written:

.
TCLA

6
7
44444
4 444448
4

(F)
= U F -
F
S
ij
I, p
p, J
((U), T)
div
+ TCLA D
iv
S
s=0

.
}
.
}

= & U + U F + U F
ij I, p
ij
I, p p, J
ij
I, p
p, J

.
678
- & div F - div F + TCLA
S
div

.
.
}
.
}
678
F
It is thus necessary to calculate &
,
,
,
F
U,
,
&
and di
ij
I p
p J
v
.

First of all, taking into account the regularity of Fs, one can show [bib4], [bib9] that there is a field Lagrangian (this remark simplifies much calculations and consists in changing theta formally fissure for each S) F representing theta fissures such as:
F (P)
F
= ((P)

S
F
) P
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thus the Lagrangian derivative of theta crack is null:

F

F
F P
P
F

(( )
S
)

( )
& =

=
0

=
S
S

=0
S =0
S

[éq 4.2.3-6] led then to:

}
.
.
678
F = F
-
S and di F
v
= - F S
p, J
p L, L, J
K L, L, K

While applying [éq 4.2.3-3] to the field of displacement it occurs:
}
.
U, = (u&) - U
S
I p
I

, p
I, K
K, p

from where the calculation of the tensor of the deformations which will enable us to obtain the derivative Lagrangian
stress field and density of free energy:

.

678
PD
1
1
S
S
ij
=
(u&i) + u&
U
U
(small deformations)
, J
(J)
(

,
,
J, K
K I,)
2
I
I K
K J
, -
+
2
.
.

678 678
GD
1
= PD

S

S

ij
ij
+
(&
,

,
,
&
(great deformations)
2 (ui) - U

U
+ U
- U
U
, K
I p
p K) J K (J)

, K
J, p
p, K
I, K

Since one limited oneself to linear elasticity and the preceding remarks on loadings and of [éq 4.2.3-3]:
}
·

0
S
0
S
ij = ijkl (& - &
T
-

kl
kl
kl, m m) +

ij, m m
4µ
& = K (& -

kk
3 T&) (-
kk
3 (T - R
T ef) +

eq &eq
3
3 6 D
&

ij
&eq =
(tensor are equivalent)
12
D
with
ij
D
1
& = & - &
ij
ij
kk kl
(tensor deviatoric)
3

In axisymetry, in accordance with the remarks of the paragraph [§4.2.3], one applies the formulas Cartesian on the first two variables with the formal analogy (X, y) (R, Z), that one supplements by the "orthoradiaux" terms following:
F
S
F

R
U
=
, S
R
R

,

, =
,

U, =
R
R
R
}
.
.

F S
}
S
1

F

U

R
R
R R

and

, = -
2

U, = &ur -

R
R
R

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from where the Lagrangian derivative of the component 3x3 of the tensor of the deformations:

.
678
}
.
PD

=

U, (small deformations)
.
.

678 678 }
.
GD
PD
ur

=
+
U,
(great deformations)
R

4.3.2 Derived from the thermal term

According to the theorem of transport of [§4.2.1], the intégrande corresponding to derived from the term
thermics is written:
.

6 7
4
8
4
THER

(

F)
= -
T F + THER
S
div

T, K K
S
s=0
}
·

= -
T
S
F
S
, K +
(T& - T
,
,
+ THER

div
T
T (), K
L L K) K

It thus remains us to calculate the Lagrangian derivative of derived compared to the temperature from
density of free energy:
}
·
= -
3 K (& -
3 T
kk
&)

T

4.3.3 Derived from the forces terms voluminal and surface

According to the theorem of transport of [§4.2.1], the intégrande corresponding to derived from the term forces
voluminal is written:
·

64447
4 4448
4
TFOR

(F)
= U
F
F
S
I (F
+ F

I, K K
I div
) + TFORd
iv
S
s=0
·
·

678
=

U
F
F
F
S
F
F
&i (F + F

I, K K
I div
)
}
+ U F + F D
iv
+ F

I I, K K
I L, L
I div

+ TFOR
S
div

The only term which it remains to calculate is the Lagrangian derivative of the gradient of the voluminal forces:

}
.
F
= F
S
I, K
I, jk J
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Note:

The surface theorem of transport led to replace the voluminal divergence by one surface divergence. This one and the Lagrangian derivative of the intégrande reveals normal derivations on the surface which do not have a direction for the elements of skin used in Code_Aster. One thus has recourse to the differential geometry and derived the contravariantes for to better apprehend the intégrandes on the surface of calculation (cf [Appendix 2]).

4.3.4 Derived from the deformations term and initial constraints

According to the theorem of transport of [§4.2.1], the intégrande corresponding to derived from the term
"initial strains and stresses" is written:
.
64444444444 7
4
8
44444444444
TINI

1 0 0

1 0

(F)

=
-

0
F
S
ij
ij
ij, K
ij
(T Tref)

+ TINI

div

ij
ij
ij, K K

2

+
-
-
-

2

S

S =0

}
.

1
1
1

= F

0
& - 0
& 0

0 0
- &
T
- & 0
K
ij
ij
ij, K
ij
ij
ij, K

&
+

2

+
-

2

+ ij
ij
ij
ij, K

2

.
F
1
-
0
0
S
K
ij
(T - Re
T F)
}
-
TINI

ij
ij
ij, K

div
2

+

Only the derivative which was not exhumed yet are those of the gradients of the tensors of deformations and of the initial constraints which, according to [éq 4.2.3-6], are written:

}
.
0 = 0 S
ij, K
ij L, K
L
}
.
0 = 0 S
ij, K
ij L, K
L

Note:

Taking into account the various digital processings carried out at the time of the establishment in the

source of
the operator, it is not licit to cumulate stress fields and initial deformations,
because this term “strains and stresses initial” is then not cancelled. The user will have
to return either of the initial constraints, or of the initial deformations but not both.
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4.4
Establishment in Code_Aster

4.4.1 Perimeter
of use

The calculation of derived from the rate of refund of energy is obtained in supplement of the value of the rate
of energy with option “CALC_DG”. This option enriched the total card by a fourth noted field “DG”.

ASTER 6.00.19 CONCEPT G CALCULATES THE 07/12/2000 OF TYPE TABL_CALC_G_TH
NUME_ORDRE INST G DG
1 0.00000E+00 3.622622E-01 1.889340E-03

Example 1: Trace total card

Its perimeter of application limits to linear thermoelastic calculations 2D resting on
complete or incomplete quadratic finite elements (SEG3, TRIA6, QUAD8 and QUAD9). Options

sensitivity allowing preliminary calculations of the Lagrangian derivative of the temperature and of field of displacement were installation in operators MECA_STATIQUE and THER_LINEAIRE.

Caution:

.
The calculation of sensitivity in thermics is restricts with the linear 2D, stationary case or transient, with voluminal sources and conditions of imposed temperature, flow normal imposed and of convectif exchange. Conditions of exchange between wall and of radiation are not taken yet into account [R4.03.01] [U4.54.01].
.
In mechanics, the calculation of sensitivity is restricted, for the moment, with linear case D_PLAN or AXIS with conditions limit of imposed displacement type, connections uniform and pressure external [R4.03.01] [U4.51.01].

This calculation of sensitivity is based on modelings 2D: D_PLAN and AXIS. They are taken in count in the entirety of the process of derivation (THER_LINEAIRE, MECA_STATIQUE and CALC_G_THETA_T). On the other hand, configuration C_PLAN is taken into account only in postprocessing
calculation of mechanics. In fact, it should appear only after the calculation of sensitivity of MECA_STATIQUE which supports only modelings D_PLAN and AXIS (with this option). data-processing developments corresponding to this taking into account in a calculation of sensitivity were not still carried out. In such a configuration, the user is of course an only judge of relevance of its results.

Taking into account the preceding remarks, it is clear that one is interested only in materials isotropic rubber bands independent of the temperature. They can be heterogeneous provided that their characteristics remain constant by finite elements.

One can use the same loadings as for the rate of energy provided that they are independent of the variation of field in their intrinsic definitions as in those of their supports. In other words, their derivative eulérienne must be null.
In addition, only loadings of the pressure type distributed (PRES_REP) and calculated temperature (TEMP_CALCULEE) are usable in the totality of the process. This software restriction is not due that with the limited development of the option SENSITIVITY in operator MECA_STATIQUE. Like for modeling C_PLAN, the other types of loading (FORCE_INTERNE, FORCE_CONTOUR, EPSI_INIT, GRAVITY and ROTATION) are taken into account only in postprocessing of the calculation of
mechanics. They cannot and they should intervene only for the assembly of the terms of derived from G. They are thus modelled by inserted AFFE_CHAR_MECA or AFFE_CHAR_MECA_F
between MECA_STATIQUE and CALC_G_THETA_T.
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One can summarize these operations by the following table:

Modeling
D_PLAN - AXIS
Elements
SEG3 - TRIA6 - QUAD8 - QUAD9
Materials Rubber band
isotropic
Loading
PRES_REP TEMP_CALCULE

THERMAL MECHANICS

C.L.
EPD. Imposed - uniform Connections
C. exchanges between wall: refused
Chgt. thermics: TEMP_CALCULEE
C. of radiation: refused
Calculation
MECA_STAT
THER_LINE

Configuration

Modeling
C_PLAN
usable
only in
postprocessing
(relevance
left with the free one
choice of
the user)

Type of loading FORCE_INTERNE
FORCE_CONTOUR
EPSI_INIT
GRAVITY
ROTATION

CALC_G_THETA_T

Via key word ETAT_INIT one can also take into account a stress field or of
initial displacements in the calculation of the rate of energy. This possibility was extended to calculation of
its derivative with the same restrictions as for the loadings. For the same reasons these
initial fields are taken into account only in postprocessing of the calculation of mechanics.

For more information on the field of validity of the options of calculation and to take as a starting point examples
of use one will be able to refer to the user's manual [U4.82.03] and the case test HPLP100B [V7.02.100].

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4.4.2 Environment
necessary

As for the calculation of the rate of refund of energy, the field theta fissures F must obligatorily to be created before (either by order CALC_THETA [U4.82.02], or by the order AFFE_CHAM_NO [U4.44.11]). For obtaining its derivative, it is necessary in more to have constituted the field
theta sensitivity S before thermomechanical calculation (since it is provided in entry of these operators
via the operand SENSITIVITY).
It is him also a field of vector 2D in each node of the grid. It is directed along the axis of X-coordinates. It can be affected directly with order AFFE_CHAM_NO but, in practice,
it results generally from specific order CALC_THETA with the option THETA_BANDE which allows to seize the module (key word MODULATES) and the X-coordinates x1 and x2 (key word R_INF and R_SUP) of
points delimiting its vertical support. It is pointed out that this field decrease value MODULATES with
zero value between the X-coordinates x1 and x2, and that it is null everywhere else. These X-coordinates can be
negative but one must have x1 < x2. [Figure 4.1-a] an example of this type of field theta illustrates.

Caution:

The field theta sensitivity is thus for the solidified moment, colinéaire with the unit vector of the axis of
X-coordinates (and in the same direction). This preliminary construction of S by the operator CALC_THETA corresponded to the specifications of project PROMETE [bib7]. But nothing prevents taking into account of unspecified directions to be able to simulate derivations compared to tilted variations of fields.

4.4.3 Standardization

In axisymetry, to carry out comparisons, it is necessary to standardize with the hand (it is not

automatically not made) the derivative provided by Code_Aster. As for the rate of refund of energy (cf [§ 2.4.4.1]) it is necessary to divide the numerical value obtained by the ray R of the bottom of crack (equal to its distance to the axis of symmetry y):

Gintrinseque

(F)
1 G

(F)

=

R

S
S

=0
=0
S
S

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Appendix
1
Calculation of the derived seconds of the elements
quadratic 2D

Initially one expresses the derivative second functions of form on the element of
reference, then are used they to determine those of the real element which are the only ones to intervene
indeed in the calculation of the elementary terms. One preserves here the notations of the code for isoparametric elements [R3.01.01]. We will carry out the exercise only in 2D but it spreads without sorrow with the 3D.
To calculate the derivative second on linear elements one has recourse to the geometry differential and with derived the contravariantes (cf [Appendix 2]). They make it possible to better apprehend
intégrandes on the surface of calculation so that they do not reveal normal derivations who do not have a direction for the elements of skin used in Code_Aster.

A1.1 Derived seconds on the element from reference

A1.1.1 Segment (element of edge)

The derivative second of the three functions of forms are written:

2
2
1
NR (
NR
)
2
= 1

() = 1
2
2
N1
N3
N2
2 NR
-1
0
1
3 () = 2
-
2

Appear A1.1.1-a: Segment of reference

A1.1.2 Triangle (element of face)

The derivative second of the six functions of forms are written:

N1

N4

N6
N2
N5
N3

Appear A1.1.2-a: Triangle of reference
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2 NR
2 NR
2 NR
I
I
I
I
2
2

1
0
1
0
2
1
1
1
3
1
0
0
4
0
- 2
- 1
5
- 2
0
- 1
6
0
0
1
Appear A1.1.2-b: Derived seconds from the triangle of reference

A1.2 complete or incomplete Quadrangle (element of face)

N1
N8
N4

N7
N5

N9
N2
N6
N3

Appear A1.2-a: Quadrangle of reference
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The derivative second of the eight (resp. nine) functions of forms of the incomplete quadrangle (resp. complete) are written:
2 NR
2 NR
2 NR
I
I
I
I
2 NR
2 NR

2 NR
2
2

I
I
I
I
2
2

1 +
- 1
(
2 +)
1 (
2 -)
1
1

1 +
1 -
- 1
2
2
4
1
-
2
2
2
4
- 1
- 1
(
2 -)
1 (
2 -)
1
1-
1 -
- - 1
2

2
+
2
2
4
2
2
2
4
- 1
1 +
(
2 -)
1 (
2 +)
1
1-
1 +
- + 1
3

3
-
2
2
4
2
2
2
4
1 +
1 +
(
2 +)
1 (
2 +)
1
1 +
1 +
+ 1
4

4
+
2
2
2
4
2
2
4
5
1 - 2
(1-)
(1-
2)
5
0
- 1

6
(1-)
1 - 2
(
1-
2)
6
- 1
0

7
1 - 2
- (
+)
1
- (
2 +)
1
7
0
- - 1
-
8
- (+)

1
1 - 2
- (
2 +)
1
8
- - 1
0
-
9
(22 -) 1
(22 -) 1

4

Appear A1.2-1: Derived seconds from the complete and incomplete quadrangle of reference

A1.3 Derived seconds on the real element

Problematic A1.3.1

The elementary terms to discretize are written in the real field, even if they are transcribed on the element of reference via the change of variable using the jacobien. Their intégrandes uses thus derivative in X. However the nodal approximation on the real element being often too complicated (
geometrical function of interpolation: X admits a reciprocal bijection but its construction is hard as of the QUAD4. One will note [J] his matrix jacobienne and det (J) his jacobien. Of other code, such N3S, chose however, for reasons of performance, to work exclusively on the real element), one prefers his expression to him on the element of reference:

1
() N1 ()… NR E ()…
N

noted () NR () {N}
with:

1, 2,…., them values of at the points of interpolation and
N1 (), N2 (),…, Nne () their functions of form associated on the element with reference.

It is thus necessary to transcribe these derivative compared to X in derived compared to via

description
direct of the geometrical interpolation.
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A1.3.2 two-dimensional Case

By using derivation in chain, one first of all writes the derivative in = (,) from those in x= (X, y):

$$\frac{\partial}{\partial X}$$
$$\frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial X} = X$$
$$\frac{\partial}{\partial y}$$

éq A1.3.2-1

y
noted {} = [J] {X}

By reversing this system (as is bijective) one can thus deduce the derivative in X from it from those in:
{
-1
X} = [J]
{} éq A1.3.2-2

and by deriving them formally one obtains:
2
2

2
X

2

2

2

= T
+ T

2
[1] [2]

y

2

éq A1.3.2-3
2

2

X y

noted {2} = [
2
1
T] {} + [2
T
X
]{}

In addition while deriving [éq A1.3.2-1] compared to, while taking account of [éq A1.3.2-2], it comes:

{2
2
} = [1
C] {X} + [C2] {X}

= [
1
-
2
1
C] [J] {} + [C2] {X}

by deferring the expression obtained in [éq A1.3.2-3]:
{2
1
-
2
X} = [

(T1] + [T2] [C1] [J]) {} + [T2] [C2] {X}

[
-
T
1

2] = [C2]
[T
1
-

1] = [
- T2] [C1] [J]
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The matrices [C1] and [C2] being obtained easily, via, there are thus a constructive process allowing to deduce matrices [T1] and [T2] required:

X 2

y 2

X y

2
X
2
y

2

2
2

2
2
$[$
X
y
$X\ y$

2
X
2
y
C

$2\ ] =$

2
$[C1] =$

2
2

X X there there there X

X y
2

X
2
y

+

y 2
y 2

y y

- 2

2
2

[
1

X
X
X X
T2]

=

2
2
det (J)

-

y
X
y X
y X X y
-
-
+
+

y
y
[
1

T

1]
-
-
=
[T2] [C1]
det (J)
X
X
-

*Thus, for example, the first derived second in **X** expressed on the element of reference is written:*

2
2
2
2
() = T (1,)
1
() + 1T (12,) () + 2
T (1,)
1
() + T (12,) () + T (1,)
3
()
2
1
X

2
2
2
2

For further information, one will be able to refer to the excellent work of G.Dhatt and G.Touzot [bib8] pp51-57.
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Appendix 2 Calculation of the term forces surface and of its derivative in 2D

A2.1 Introduction

According to the paragraphs [§ 2.4.3.3] and [§4.3.3] the term forces surface and its derivative compared to
variation of field are written (before projection of the operators on the Cartesian basis):

TSUR = (
F
F
F + F div
S
) .u D
S
.

TSUR

678

(F) =

(
S
F) F

S
F
F
+ F div + F div .u

S
S

+
S
=0 S
S

(F
F
F + F div
S
).(
S
u& + udiv
S
) D

They thus reveal clearly derivative normals on the surface of calculation. However in Code_Aster, one chose to calculate these elementary terms (had with the surface efforts) on "elements of skin" for which this normal variation does not have a direction. To cure it one has resort to the differential geometry which makes it possible to express these intégrandes only using surface sizes.
We will carry out the exercise only in 2d-PLAN but it spreads without sorrow with the 3D. In our case, the surface of calculation is reduced to a curve (in the plan (X, y) of calculation) and the forces are not
more than linear. In addition, according to whether modeling is plane or axisymmetric, it is necessary to take into account complementary terms, because in the first case it is a question of a calculation per unit
of length, whereas in the second, it is per unit of radian.

We now will introduce a curvilinear paramètrage vicinity of the curve of work S and of its associated fundamental reference marks. That is to say an acceptable paramètrage of S. to describe it
volume made up of a vicinity of this curve by using an orthogonal reference mark, one associates two to him
other variables and.

M ()
G
G
1
2
S
y
, g3
X
O
Z

Appear A2.1-a: Curvilinear Paramètrage of the vicinity of the curve of work
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The figure above illustrates the natural base covariante (g1, g2, g3) associated the parameters, and. The vectors of this curvilinear base are written in the total reference mark (O, X, y, Z) in the form (in one
not M (X (), y ()) unspecified of S)
X
y

0

y
X
G =
G = -
G
1
2
3 = 0

0
0
1
From where the metric tensor G and its reciprocal tensor g-1, by noting J the jacobien transformation:

2
2
J
0
0
-
J
0
0
G = [G
G G
G
G G
ij]
= [

-
-
I
J

-

.i J]
2
1

= 0
J
0
=
ij
ij

[1
gij] = [.]
2
= 0
J
0
ij
ij

0
0
1
0
0
1

2
2
X
y
with J =
+

Reciprocal metric tensor one deduces bases it contravariante (g1, g2, g3) which proves very useful for

to calculate the derivative covariantes:
1
1
gi = g-1 G
g1 =
G, g2 =
G, g3 = G
ij
J

J 2 1
J 2 2
3

Note:

That modeling is plane or axisymmetric, these tensors remain diagonal since them selected bases are orthogonal. On the other hand the value of the elementary element of integration differ
J D in 2D - PLAN
D =

R J D

in 2D - AXI
to take account of integration per unit of radian in axisymetry. Taking into account the analogy
R 2 2
Z
formal (X, y) (R, Z), the jacobien of the transformation is written: J =
+

A2.2 Terme forces surface

Let us break up this term of into two intégrandes:
TSUR = (TSUR + TSUR
1
2) D
S

with
TSUR =

1
(F
F) .u and TSUR =
2
(
F
F div
S
) .u
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A2.2.1 Calculation of TSUR1

By expressing the gradient by derivative covariantes and by breaking up the vector theta fissures and it
vector displacement on the basis covariante, one obtains (after some tensorial operations of base)
I, K, L,
1 2
TSUR = F
.u =
G G
G. G
with

1

(F
)
(I
J
fk
F

U
J
I
K) L
{ }
L
j=1
I
fk
L
= F
U G

J
jk
it
2
I
fj
I
= J F
U
J

It remains to determine I
fj
I
F

,
and
U via the base contravariante to obtain:
J
F

F

F

TSUR = J 2
1
1
1
2
2
1
(F .g) .g (.ug) + .g (.ug)

X
y

F

F
= J -2
F
F

+
U
X + U
y
X
y
X
y

Note:

In axisymetry, taking into account the nullity of the component orthoradiale of the field theta fissures,

it
does not have there complementary term.

A2.2.2 Calculation of TSUR2

By expressing the surface divergence as the trace of the surface gradient
div
F
= tr

G
G

S
(F
S
) = tr (fi J
J
I
) with I, J = 1

and by breaking up the vector theta fissures and the vector displacement on the basis covariante, one obtains (after some basic tensorial operations (to take the trace of a tensor of the second order amounts carrying out its contraction)) :
K, L
,
1 2
TSUR =

F tr
.u =
G tr
G G. G
with

2
((F

F

U
S
) (K K (fi
J

J
I
) L
{ }
L
I, j=1
K
fi
L
= F
U G

J
ij
kl
2
K
fj
K
= J F
U
J

It remains to determine K
fj
F
,
and K
U with the base contravariante to obtain:
J
F

TSUR = J 2
1
1
1
2
2

.g
2
{(f.g) (.ug) + (f.g) (.ug)}

F
X
F

-
y y
= J 2
X

+

(F U + F U
X X
y y)

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Note:

In axisymetry it is necessary to take account of the nonnull component orthoradiale gradient at the time of
calculation of the surface divergence:
F

Fr
F
div =
+
S
R

F
F

-2
R Z
1
R
Z
F
= J
+
+ R

R

A2.3 Dérivée from the term forces surface

Let us break up this term of into five intégrandes:

TSUR

(F) = (DTSUR + DTSUR + DTSUR + DTSUR + DTSUR D
1
2
3
4
5)

S
=0
S
S

with
DTSUR = (F S) F) .u
1
DTSUR = (F S) .u div F
2
S

.
678
DTSUR = F .u div F = F .u tr
S
(F S
S
S
)
3
DTSUR = (F F + F div F .u
S
) &
4
DTSUR = (F F + F div F .u
S
) div S
5
S

A2.3.1 Calculation of DTSUR1

By expressing the double gradient by derivative covariantes and by breaking up the vector theta fissure, the vector theta sensitivity and the vector displacement on the basis covariante, one obtains (afterwards
some basic tensorial operations):
I, L
, m, N,
1 2
DTSUR = F
.u =
G G G
G
G. G with

1
(
S

) F
)
(I
J
K
L
F

U
jk
I
S
L) m
F
m) N
{ }
N
J, k=1
I
sl
Fm
N
= F
U G

jk
kl
jm
in
2
I
sk
fj
I
= J F
U
jk

It remains to determine I
fj
sk
I
F

,

,
and
U with the base contravariante to deduce some:
jk
S
2
2

F

F

TSUR = J 2
1
1
1
1
2
2
.g .g

.g.
U G

.g
1
(F) (S)
+
.
U G
2
(
) 2 ( )

X
y
X
y
2

F
2
F
= J -4
F
F
S
S

+

+
U
X + U
y
X
y
X
y
X

2
y
2

Note:

In axisymetry, taking into account the nullity of the component orthoradiale of the field theta

sensitivity, it does not have there a complementary term.
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A2.3.2 Calculation of DTSUR2

It is obtained immediately by taking again the result of the calculation of TSUR1 (after having replaced the field
theta fissures by the field theta sensitivity) and by multiplying it by the expression of the divergence surface of the field theta fissures TSUR2.
4
S X
S y

F
F
X
y F
X
X fy y
DTSUR = -
J

+
U
+ U

+

2

X
y

X
y

Note:

In axisymetry it is necessary to take account of the nonnull component orthoradiale gradient at the time of
calculation of the surface divergence
F
R
F
Z
F

div
F
= J -2
R
Z
R

S
+

+ R

A2.3.3 Calculation of DTSUR3

By expressing the surface divergence as the trace of the surface gradient and by breaking up it vector forces linear and the vector displacement on the basis covariante, one obtains (after some basic tensorial operations (to take the trace of a tensor of the second order amounts carrying out its contraction):
K, L

,
1 2
TSUR =

F tr

.u =
G tr
G G G G
. G with

3
((F
S

F

U
S
S
) (K K (fi Sm
J
N (
J
I
)(
N
m
)) L
{ }
L
I, J, m, n=1

K
fi
Sm
L
= F

U G

J
N
jm
in
kl
2
K
fi
sj
K
= J F

U
J
I

It remains to determine K
fi
sj
K
F
,

,
and
U with the base contravariante
J
I
F
S

2

1

TSUR = J
1
.g
.g
f.g
.
U G
f.g
.
U G
3
({ 1)(1)+ (2)(2)}

F
F
S
S

4

-
X
y

X
y
X
y
X
y

= J
+

+
(F U + F U
X
X
y
y)

Note:

In axisymetry it is necessary to take account of the nonnull component orthoradiale derivative Lagrangian of the surface gradients (cf [§4.2.2])
.
F
678

-
R
F
Z
S
R
S
Z
F
F

div F
= J 4
R
Z
R
Z
R

S

+

+
-

S

R 2
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A2.3.4 Calculation of DTSUR4

It is enough to summon the expressions of TSUR1 and TSUR2 by replacing the components of the field of
ux
&ux
displacement U
U
U by those of its Lagrangian derivative &
.
y
&uy

A2.3.5 Calculation of DTSUR5

It is enough to multiply sum TSUR1+TSUR2 by the expression of the surface divergence of the field theta sensitivity
S
X
S

y y
div S
= J -2
X

+

S

Note:

In axisymetry it is necessary to take account of the component orthoradiale not no one of the gradient at the time of
calculation of the surface divergence
S
R
S
Z
S

div S
= J -2
R
Z
R

+
+

S

R

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Organization (S): EDF/IMA/MMN
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R7.02.03 document
Rate of refund of energy in thermoelasticity
non-linear
Summary:
One presents the calculation of the rate of refund of energy by the method théta in 2D or 3D for a problem
thermoelastic non-linear. The relation of nonlinear elastic behavior is described in [R5.03.20].
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Contents

1
Calculation of the rate of refund of energy by the method théta in nonlinear thermoelasticity

1.1
Relation of behavior

One considers a fissured solid occupying the field of space R2 or R3. That is to say:

.

U the field of displacement,

.

T the field of temperature,

.
F the field of voluminal forces applied to,
.
G the field of surface forces applied to a part S of,
.
U the field of displacements imposed on a Sd part of.
S

F
G
Sd
The behavior of the solid is supposed to be elastic non-linear such as the relation of behavior coincide with the elastoplastic law of Hencky-Von Mises (isotropic work hardening) in the case of one loading which induces a radial and monotonous evolution in any point. This model is selected in orders CALC_G_THETA [U4.63.03] and CALC_G_LOCAL [U4.64.04] via the key word RELATION: “ELAS_VMIS_LINE” or “ELAS_VMIS_TRAC” under the key word factor COMP_ELAS
[R5.03.20].
One indicates by:
.
the tensor of deformations,
.
° the tensor of the initial deformations,
.
the tensor of the constraints,
.
° the tensor of the initial constraints,
.
(, °, °°, T) density of free energy.
is connected to the field of displacement U by:
1
U
( ) =
U
(
)
2 I, J + uj, I
Density of free energy (,
, °, °°, T) are a convex and differentiable, known function for one
state given [R5.03.20 éq 3]. The relation of behavior of material is written in the form:

ij =
(, °, °, T)

ij
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It derives from the potential free energy. For this hyperelastic relation of behavior, one knows to give a direction to the rate of refund of energy within the framework of the global solution in mechanics of
rupture. It is not the case for a plastic relation of behavior.
1.2
Potential energy and relations of balance
One defines spaces of the fields kinematically acceptable V and Vo.
V =
v
{acceptable, v = U on Sd}
Vo =
v
{acceptable, v = O on Sd}
With the assumptions of the paragraph [§1.1] (with ° = ° = 0), relations of balance in formulation weak are:
U
V

ij v

v
v

I, J D
=
fi

I D +
gi
S
I D, v Vo
They are obtained by minimizing the total potential energy of the system:
W v
( ) =

((v), T) D F
v
v

I

I D -
gi
S
I D
The demonstration is identical to that in linear elasticity [R7.02.01 §1.2].
1.3
Lagrangian expression of the rate of refund of energy
That is to say m the unit normal with O located in the tangent plan at in.

O
m
That is to say the field such as:
=
µ
{such as µn = 0 on}
by noting N the normal with.
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The rate of refund of energy *G* is solution of the variational equation:

G **m** *dS*
G (),

=

O

where G () is defined by:

G () =
ij iu, *p p*, *J - K*, *K -*

T, *K D*

K
T
1
1
+

HT
ij -
°
°

°

,
,

ij
2

ij kk - ij - ij -

ij
2

ij K kd
+
fi I
the U.K., K +

fi, K K I
U D

+
gi, K U + G the U.K., K -
nk

D
S
K
I
I
I
nk

- ij N U D
S
J
I, K
K
D

The demonstration is identical to that of the calculation of *G* in linear elasticity [R7.02.01]. The expression is
the same one, postprocessing is thus identical.

1.4 Establishment of G in nonlinear thermoelasticity in

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The types of elements and loadings, the environment necessary are the same ones as for establishment of *G* in linear thermoelasticity [R7.02.01 §2.4].

For the calculation of the various terms of G
(), in a given state, one recovers the density of free energy

,
(, *T*), strains and stresses, calculated for the relation of behavior
non-linear (routine NMELNL).

It is supposed that ° = ° = 0 (identical term in linear or non-linear thermoelasticity). Density of free energy is written then [R5.03.20 §1.5]:

· in linear thermoelasticity:

1
2
2μ
(, *T*) = *K*
3
2
2 (
- (*T* - *T*
kk
réf) +
eq
3

with

3
2
D D
3
1

1
eq =
ij ij =
ij - *kkij ij*
kkij
2
2

3

-

3

3
2

1
2
eq =
ijij - *kk*
2

3

· in non-linear thermoelasticity (2µ *eq y*):
() 1
2
1
2
3
2 (
(
)

p
, *T*
K
T T
R p
eq
=
-
-
+
+
R S ds
kk
réf
6µ
((*eq*)

( )

( )
0
with *R* ((
p eq): function of work hardening.
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For a linear isotropic work hardening (RELATION: “ELAS_VMIS_LINE”) one a:
E E
R (*p*) = + *p*
T
= + *p has*
y
E - E
y
T
eq - y
E E
p =
with *A*
T
=
µ
3 + *has*
E - AND
(

p
1
1
p
With p) =
R (*S*) *ds* = + *has p2* = *p*

YP
y
+
0
(+a *p*
y
)
2
2
2
(
p
With p) = (+ *R*
y
(*p*)
2
Postprocessing is then identical to the problem in linear elasticity except for the term thermics:

THER = -
,
T K
T
K
If coefficients of BLADE (*T*) and µ *T*
() are independent of the temperature, this term is null.

In the contrary case, it is necessary to calculate

(, *T*) at a given moment.
T
For a linear isotropic work hardening, one a:
(
1 *dK T*

D

T
, *T*)
( )
=
kk -
3 *T* - *T*
- 3K
réf
+
T - *Tréf kk* -
3 *T* - *T*
T
2 *dT* (
(
)
( )(
) (
réf)

dT

R (*p*)
Dr. (*P*) *dµ* (*T*)
dA p
+
2µ
-
R
2
(*p*)
( )
6µ
dT
dT

+ *dT*
Dr. (*p*) *D y* (*T*) *D has* (*T*)
D (
p T)

with
=
+
p + has
dT
dT
dT
dT
D has (*T*)
1
dET (*T*)
D E
2
(*T*) 2
=
E -
E
dT
(
2
T
E - E

T)
dT
dT
D (
p T)
1

dμ T
dA T
D T
=
(-
y
eq)
( )
( )
3
+

(µ
3
has)
( )
dT
(

µ
2
y
3 + *has*)

dT
dT -
+
dT

dA (*p*) 1 (
dp T)

=
(
1
D T
dRp T

y

y + *RP*)
( )
( )
+ *p*
+
dT
2 *dT*
2 *dT*
dT
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1.5
Warning

Caution! By definition, in the case general:

(, *T*):
Although it is possible to not carry out an elastoplastic calculation followed by the calculation of *G* in elasticity
linear, it should well be known that that does not have any thermodynamic direction and that it is normal that the result
depends on the field.

2
Calculation of the rate of refund of energy by the method
théta in great transformations

One extends the relation of behavior of [§1] to great displacements and great rotations, in measurement where it derives from a potential (hyperelastic law). This functionality is started by key word DEFORMATION: “GREEN” in orders CALC_G_THETA [U4.63.03] and CALC_G_LOCAL [U4.64.04].

2.1
Relation of behavior

One indicates by:
·
E the tensor of deformations of Green-Lagrange,
·
S the tensor of the constraints of Piola-Lagrange called still second tensor of Piola-Kirchoff,
·
E
() density of energy internal.
The behavior of the solid is supposed to be hyperelastic, namely that:
·

E is connected to the field of displacement **U** measured compared to the configuration of reference *O* by:
1
E
(
)
ij **U**
( ) =
U
2 *I*, *J* + *uj*, *I* + the *U.K.*, *I the U.K.*, *J*
.
S is connected to the tensor of the constraints of cauchy *T* by:
S
-1
-1
ij
= det **F**
() *Fik Tkl Fjl*
F being the gradient of the transformation which makes pass from the configuration of reference *O* to current configuration, connected to displacement by:
F
(
)
ij
= *ij* +ui, *J*
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The relation of behavior of a material hyperelastic is written in the form:

Sij =
=
= *S*
E
ji
ij
Ejj
This relation describes a non-linear elastic behavior, similar to that of [§1.1]. It offers
possibility of dealing with the problems of breaking process without integrating plasticity into it. And in case of a monotonous radial loading, it makes it possible to obtain strains and stresses of
structure similar to those which one would obtain if the material presented an isotropic work hardening.
material hyperelastic A a reversible mechanical behavior, i.e. any cycle of
loading does not generate any dissipation.
This model and selected in orders CALC_G_THETA [U4.63.03] and CALC_G_LOCAL [U4.64.04]
via the key word:
RELATION: “ELAS”
for an elastic relation “linear”, i.e. the relation between the deformations and them
constraints considered is linear,
RELATION: “ELAS_VMIS_LINE” or “ELAS_VMIS_TRAC”
for a “nonlinear” relation of elastic behavior (law of HENCKY-VON PUT at
linear isotropic work hardening).
Such a relation of behavior makes it possible in any rigour to take into account the large ones
deformations and of great rotations. However, one confines oneself with small deformations to ensure
the existence of a solution and to be identical to an elastoplastic behavior under one
monotonous radial loading [R5.03.20 §2.1].

2.2

Potential energy and relations of balance

The loading considered is reduced to a nonfollowing surface density **R** applied to a part
O of the edge of *O* (assumption of the dead loads [R5.03.20 §2.2]).
One defines a space of the fields kinematically acceptable V:
V
=
v
{acceptable, **v** = **0** on *O*}
The relations of balance in weak formulation are:
F
ik Skj VI, *J D* =
R
I VI D, **vV**

O

They can be obtained by minimizing the total potential energy of the system:
W **v**
( ) =

(**v** ())*D - R*
v

I I D
O

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Indeed, if this functional calculus is minimal for the field of displacement **U**, then:

W =
E
v

ij D -
IH I D = 0, **v** V
O E

ij
=
S
(
)
ij

1 *v*
D - R
v

I, J + vj, I + vp, I up, J + U p, I vp, J
I
I D
O
2

=
S

(
)
ij
v
v

IP + up, I
p, I D -
IH I D
O

=
F
pi S
v

ij vp, J D -
IH I D
O

=
F
ik S
v

kj VI, J D -
IH I D = 0, **v** V
O

We thus find the equilibrium equations and the relation of behavior while having posed:

Sij =
E
ij

2.3 Lagrangian expression of the rate of refund of energy in non-linear thermoelasticity and in great transformations

By definition, the rate of refund of energy *G* is defined by the opposite of derived from energy potential with balance compared to the field [bib1]. It is calculated by the method théta, which is one Lagrangian method of derivation of the potential energy [bib4] and [bib2]. One considers transformations *F*: **MR. M** + **M**
() of the field *O* in a field which corresponds to
propagations of the crack. With these families of configuration of reference thus defined correspond of the families of deformed configurations where the crack was propagated. The rate of restitution of energy *G* is then the opposite of derived from the potential energy *W* **U**
(
()) with balance
compared to the initial evolution of the bottom of crack:
W **U**
(
( ))
G = -
D
=0

One notes as in [bib 4] par. Lagrangian derivation in a virtual propagation of crack of speed. That is to say (, **M**) an unspecified field (real positive and **M** pertaining to the field *O*), we will note:

(, **M**) = (
, *F* (**M**)) and
! = =0

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Definite potential energy on is brought back on *O*, **R** is supposed to be independent of, derivation compared to the parameter of propagation is then easy and the rate of refund of energy in this propagation is solution of the variational equation:
G **m** *dS*
G (),

=

O

with:

" #
$ %
$

- G () = ((**E**, *T*) + (**E**, *T*) *K*, *D*
K
- *IH*! *ui* + *IH*, *K U*
K
I + *R U*
I
I K, *K* -
nk

D
O

nk
However:

(" #
$ %
$

(**E**, *T*) =

! *E* +
! *T*
E
ij
T
ij

Thereafter, we will consider only the term
!

E, the thermal term being treated the same one
E
ij
ij
way that into small displacement [R7.02.01].
And according to proposal 2 of [bib4]:
1
! *Eij* = 2 (! *ui*, *J* +! *uj*, *I* +! *the U.K.*, *U*
I
K, + *U*
J
K, *I*!
the U.K., *J*)
1
-

2 (*ui*, *p p*, + *U*
J
J, *p*
p, + *U*
I
K, *p*
p, *U*
I
K, + *U*
J
K, *U*
I
K, *p*
p, *J*)

One can eliminate! **U** of the expression of *G* as in small deformations by noticing that! **U** is kinematically acceptable (cf [bib3] for the problems of regularity) and by using the equation of balance:

"#%
((**E**))
D -
R U!

D
I
I
=
O

1 *U*! , +u! , +u! *U*
,
, + *U*
U
, !
D

,
O
E 2 (*I J*
J I
K I
K J
K I
K J)

ij
1
- *R U*
I! *I*
D -
U,
, + *U*,
, + *U*
U
,
,
, + *U*

U
D

,
,
,
O
E 2 (*I p p J*
J p p I
K p p I
K J
K I
K p p J)
ij
= -
S U,
, + U
U
D
ij (*I p p J K I, K, p p, J*)
O
= -
S

(
K I,)
ij
+ U
U

D

ki
K, p
p, J
O
= -
S
ij kFi the U.K., p p, J D
O
= - *ik*
F Skj I

U, p p, J D
O
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Finally, one obtains:

G () =
F S
, ,
E,
,
,

(*U*
) - () *D + R U + R the U.K.K -*

D
ik
kj
I p
p J
K K
I K
K
I
I

I
O

N
K
The expression supplements for the following loadings:
· nonfollowing surface density **R** applied to part of the edge of *O*,
· nonfollowing voluminal density **F** applied to the field,
and by taking account of thermics:

G () = *F S*
ik
kj

E

(*ui*, *p p*, *J*) - () *K*, *K* -, *T D*
O
T
K
K

+ F U
I
I K, *K* + *fi*, *U D*
K
K
I

O

+ IH, *U*
K
K
I + *R U*
I
I K, *K* -
nk D

nk

2.4
Establishment in *Code_Aster*

The comparison of the formulas of G
() of [§1.3] and of [§1.4] shows that the terms of G
() are very
close relations. The introduction of the great transformations requires little modification in postprocessing.
The presence of the key word DEFORMATION: “GREEN” under the key word factor COMP_ELAS of orders CALC_G_THETA and CALC_G_LOCAL indicates that it is necessary to recover the tensor constraints of Piola-Lagrange **S** and the gradient of the transformation **F** (routines NMGEOM and NMELNL).
The types of finite elements are the same ones as in linear elasticity [R7.02.01 §2.4]. They are them isoparametric elements 2D and 3D.
The supported loadings are those supported in linear elasticity provided that it is
dead loads: typically an imposed force is a dead load while the pressure is one
following loading since it depends on the orientation of surface, therefore of the transformation.

2.5 Restriction

With the relation of behavior specified to the 2, there is a formulation of *G* valid for the large ones deformations for materials hyper-rubber bands, but… if one wishes a coherence with
actual material which, let us recall it, is elastoplastic, it is imperative to be confined with deformations small, displacements and rotations being able to be large.
Conditions of loadings proportional and monotonous, essential to ensure
coherence of the model with actual material, lead to important restrictions of the field of
capable problems being dealt with by this method (thermics in particular can lead to
local discharges). It can thus be a question only of one palliative solution before being in measurement of to give a direction to the rate of refund of energy within the framework of plastic behaviors.

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Code_Aster ®
Version
3.6
Titrate:
Lagrangian representation of variation of field
Date:
29/11/96
Author (S):
G. DEBRUYNE
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Organization (S): EDF/IMA/MMN

Handbook of Reference

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Document: R7.02.04
Lagrangian representation of variation of field

1 Goal

· Calculer mechanical fields relating to a field with variable geometry, by using one field fixes reference in way, for example, to carry out parametric studies on several fields by using one grid.
· Within the framework of the breaking process, to calculate the rate of refund of energy for various lengths of crack (in 2D and 3D) by using only one grid representing one length of crack fixes reference.

These developments are available in linear elasticity, for the elements of continuous medium 2D and 3D, in the situations where the variations of geometry do not affect the edges charged. possible initial deformations are treated only in the medium 2D.
Any calculation using this method requires, to ensure the passage of the real field studied area of reference, the preliminary creation of a field, using order CALC_THETA [U4.63.02].
The formulation developed in *Code_Aster* does not take account of the thermal terms, of loadings on the lips of the crack nor of the forces of volume in general, except deformations initial which is taken into account in 2D only.

---

2 Principle of the method

The principle is double:
· to use a bijective geometrical transformation making correspond the field of reference (that which one nets) with the real field. For that, one is useful oneself of the field and of a real parameter. In theory, any type of transformation is authorized provided that it is

regular but the shape of the field théta currently established authorizes roughly speaking only translations of under-fields, except in 3D where by differentiating the modules from the field théta along a border, one can simulate changes in form of this border
(application to the parametric study of cracks 3D),
· to write the new equations of the elastic problem on the field fixes reference,
variability (translated by the parameter) appearing in the equations and either in
field (from where a single grid to analyze all the configurations).
With regard to the applications to the breaking process, one calculates moreover the rate of restitution of energy relating to various lengths of cracks by using the configuration of reference. For that, one writes the potential energy in form Lagrangienne and one derives sound expression compared to the parameter.

2.1

Transformation of field by the field théta.

One considers a succession of fields, and an area of reference
I
0. Each point
material pi of the various fields is located by a point M of the area of reference in the way following (cf [Figure 2.1-a]):
P = F (*M*

) = *M* + (*M*
I
)
I
I
Field of
reference
M
f3
f1
f2
0
P1
P2
P3

1

2
3

Appear 2.1-a: Representation of a succession of fields by a field of reference.

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2.2
Lagrangian representation of the problem of elasticity.
Mechanical fields relating to the current configuration and written on the configuration of reference express themselves in the following way, that one will call Lagrangian expression:
~
U (*M*) = *U* (*M* +), ~
(*M*) = (*M* +), where *U* and are respectively the fields of
displacements and of constraints. In a general way any field C will have its representation Lagrangian ~
C with ~
C (*M*) = *C* (*M* +).
The principle of virtual work (P.T.V) is written classically, in linear elasticity, on each field in the following way:
A.U
. v
D =
T.v dS +
A. v
D

I
,

with *U* field solution of the problem, *v* field of displacement kinematically acceptable, *I*. one field of initial deformations, *A the* tensor of elasticity and *T* forces imposed on the border of field.
By noticing that the field of deformation is written: (*P*) = ~ (*M*) (*F*

-1
), and that
$D = \det F \, D$

0, the P.T.V are written on the area of reference (knowing that the support of transformation is strictly included in, i.e. this transformation does not affect them edges charged):
$A.u\sim$ (*F*
) 1
-. *v*

(*F*
) - 1 det

F D
=
T.v dS +
A.~
. *v*
(*F*
) 1
- det *F*

D

I

0
0
0
The discretized problem arises then in the following form (with the usual notations of finite elements:
~
K (*F*) ~
~
U = *F* + *Q* (*F*)

,
~
~
~
where *U* is the Lagrangian field solution of the problem, *K the* matrix of rigidity, *Q the* vector corresponding to the initial deformations, both depend on the transformation (and thus on the field

and of the parameter).
Parametric study thus consists in dealing with several problems by using the same grid and in calculating each time the elementary matrices of rigidity depending on the parameter using option “RIGI_MECA_LAGR” (available **in 2D and 3D**) of operator CALC_MATR_ELEM [U4.41.01] and
option “CHAR_MECA_LAGR” (available only for the initial deformations and in 2D) of operator CALC_VECT_ELEM [U4.41.02], with in data the field and the parameter. The remainder of calculation continues in a usual way: assembly and resolution (This development are not integrated in total order MECA_STATIQUE [U4.31.01], it is thus necessary to break up calculation with the elementary operators). The field of displacement obtained is a **Lagrangian** field. Of even, the Lagrangian constraints are obtained thanks to operator CALC_CHAM_ELEM [U4.61.01], with options “SIEF_ELGA_LAGR” and “SIGM_ELNO_LAGR” [U4.61.01].
The calculation of the constraints takes account of the possible presence of a field of initial deformations. In particular, one can calculate with these options the field of the real constraints with predeformation without variation of field (i.e. by taking =0): = *A* (- *I*).
The initial deformations are obligatorily defined in the form of constants or of functions of space, via operator AFFE_CHAR_MECA [U4.25.01] with the key word “EPSI_INIT”. The development does not take into account initial deformations given like fields with the nodes or the points of Gauss.
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2.3
Calculation of the rate of refund of energy
The field of Lagrangian displacement ~
U being available, one can evaluate written potential energy in Lagrangian form:
~
~

W (~
U (), *T*, ~i) *W* (*U* (), *T*, *I*

=

), like its derivative.
The expression of the rate of refund of energy is then:
1
1
G =
With ~i
. *U*
-1
-
-
~

(*F*), det *F*
D +
With
I

(*F*
) .u (*F*
) det *F*
D

0
0
+
-1
-1
-1
With ~i
. *U*

(*F*) (det *F*
) *D* -

A.U

,

(*F*). *U*
(*F*) det *F D*

0

,
0
-
1
1

0 (
-
-
With U
~
(*F*
)
I
-
). *U*
(*F*) det *F*
D

,
-
-1
~i
-1
With

.
det

0 (*U* (*F*) -
) *U* (*F*) (*F*) *D*

,
3
Some examples of use
3.1 Parametric study of a structure having an inclusion with variable positioning.
The goal is to calculate the mechanical fields of a structure containing a heterogeneity (material of different characteristic or cavity for example,) with different positions, by using one grid corresponding to a position of reference (cf [Figure 3.1-a]).
Support of
field théta
T
T
r2
r1
P
P
M

Position of
Position
reference
current
U
U
Appear 3.1-a
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The first stage consists in choosing a support of the field théta compatible with the kinematics of inclusion. In practice, if the current position of inclusion underwent a translation compared to position of reference, with =1, one will build the field théta so that the small disc of crown (r< r1) includes inclusion completely (=1 for any point of inclusion) and that <r2-r1.
From now on, in all the analysis, the points M attached to the grid represent in fact the points materials P, determined by P = M +.
With these considerations, all the positions of inclusion are not possible to analyze and depend on the geometry of the structure.
The second stage consists in formulating the problem of Lagrangienne representation by calculating them elementary matrices using operator CALC_MATR_ELEM [U4.41.01] and the option RIGI_MECA_LAGR, with like data the field and the kinematic parameter.
The remainder of the analysis continues in a traditional way, but the fields obtained at the points of grid (nodes or points of Gauss) represent the mechanical state of truths material points of variable configuration:
~
U (*M*) = *U* (*M* +), ~
(*M*) = (*M* +).

3.2 Calculation of the rate of refund of the energy of a fissured structure in a field of initial deformations

One considers a springy medium 2D containing a rectilinear crack AO and being propagated (until not O') in a field of initial deformations (cf [Figure 3.2-a]). The problem consists in calculating the rate of refund of the energy of the structure for all the positions of the crack.

T
field where is
defined a field
deformations
initial I
D
With
O
O'
U = 0

Appear 3.2-a

One starts by defining the field of initial deformations in the field D:

```
CH = AFFE_CHAR_MECA_F (
MODEL .....,
EPSI_INIT: (
GROUP_MA: D
EPXX: f1
```

EPYY: f2
EPZZ: f3
EPXY: f4)
.....);
The fi quantities are functions of space defined as a preliminary by DEFI_FONCTION [U4.21.02].
Let us notice that even in plane deformations one can define a component EPZZ.
components EPXZ and EPYZ can be defined but will not be used for nothing in the continuation treatment
since one limits oneself to the 2D. It is necessary that the crack is not propagated apart from the field predeformed.
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More precisely, it is necessary to be able to define the quantity:
~
(*M*)
I
= (
I M +), **and thus to make sure that the material point M + is in the field**
D. If it is not the case, the user will prolong the field D with zero values of *I*.
The second stage consists in defining the support of the field théta, by taking care that this one is such that: *M*, *M*

+
D, with
= *OO'*
max
max
.

This is carried out with operator CALC_THETA [U4.63.02]. The order of the first two stages is indifferent.
One calculates then the elementary matrices of rigidity:
mel = CALC_MATR_ELEM (OPTION: “RIGI_MECA_LAGR”
THETA: théta
PROPAGATION: alpha .....
and second elementary members:
vel = CALC_VECT_ELEM (OPTION: “CHAR_MECA_LAGR”
THETA: théta
PROPAGATION: alpha .....
The user will endeavour to indicate the same field théta and the same parameter alpha in both operators under penalty of serious vexations.
The field théta must be compatible with the maximum extension of the crack, i.e.
max = OO'< $R - R$
2
1, with R and R
2
1 the higher and lower ray of the support crown of
field théta (this precaution must be taken independently of the presence of deformations initial).
The assembly and the resolution are carried out in a traditional way.
One then can, if required to calculate the residual stresses for each elongation of crack, for example in the absence of external forces T, this thanks to operator CALC_CHAM_ELEM [U4.61.01], with options “SIEF_ELGA_LAGR” (forced at the points of Gauss) or “SIGM_ELNO_LAGR” (forced with the nodes).
The calculation of the rate of refund of energy is carried out with operator CALC_G_THETA [U4.63.03] and
option “CALC_G_LAGR”. The same precaution that previously must be taken, namely coherence of the field théta and the parameter alpha (by defect this parameter is worth zero and one calculates G
traditional with initial deformations). It is also necessary to take care not to forget the key word “CHARGES”
so that the field of initial deformations is taken into account.
The elastic behavior being linear, the history of the variation of the field does not intervene. One can thus very well on the basis of the configuration of reference (crack length AO), to analyze all them cracks of which the length varies between AO-OO' and AO+OO'. For the cracks of which the length is lower than AO, one will consider a negative field théta in bottom of crack, positive remainder (one “shortens” the crack to some extent).
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3.3 Parametric study of a structure 3D fissured, the parameter being the fissure shape.

One considers a linear elastic body comprising a three-dimensional crack of which geometry of bottom is a priori unspecified but regular. The goal is to calculate the rate of refund of energy total and room for the various shapes of crack. Let us consider the face of crack of reference 0 and it face running (cf [Figure 3.3-a]). The transformation binding the two faces is built using one definite field théta local on all the face.

For that, one defines initially the face with operator DEFI_FOND_FISS [U4.63.01], then the field théta with operator CALC_THETA [U4.63.02] and the key word THETA_3D [U4.63.02], the module being
defined in each node in order to build from 0.

0

Appear 3.3-a

The treatment takes place then as in 2D, i.e. by the construction of the elementary matrices with option “RIGI_MECA_LAGR” of operator CALC_MATR_ELEM [U4.41.01]. The option “CHAR_MECA_LAGR” of CALC_VECT_ELEM [U4.41.02] (only to treat the deformations initial) is not planned for the 3D. The form of the various faces is defined by the field théta, it parameter alpha creating a homothety for a given form.

The total rate of refund of energy for the configuration is calculated with the option “CALC_G_LAGR” of CALC_G_THETA [U4.63.03].

The rate of refund of energy room is calculated with option “CALC_G_LGLO” of CALC_G_LOCAL [U4.63.04]. An important restriction of this development relates to this option: normals between the various faces must be preserved (for example the parametric study of cracks circulars does not pose problems on this subject). This restriction does not relate to the calculation of G total.

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Calculation of the coefficients of intensity of constraints in plane linear thermoelasticity

Summary:

One presents the method of calculation of the coefficients of intensity of constraints KI and KII and in thermoelasticity
linear plane. The formulation regards the rate of refund of energy as a symmetrical bilinear form field of displacement U and uses the explicit expressions of the fields of singular displacements known in plane linear elasticity.

Key words:

Breaking process, coefficient of intensity of constraints, thermoelasticity.

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1
Expressions of the stress intensity factors KI and KII
in linear thermoelasticity 2D

1.1 Presentation

x2

R

0

x1

Are the axes of Cartesian co-ordinates Ox1 in the prolongation of the crack and Ox2 perpendicular with the crack. The problem is plane. We will express the Cartesian components displacements and constraints according to the polar co-ordinates R and.

In linear elasticity, the system of the equilibrium equations, without voluminal force, and the conditions
in extreme cases homogeneous on the crack, the null constraints ad infinitum, admit a noncommonplace solution
1
-
form U = R G ()
2

I
I
. The constraints are infinite at the bottom of the crack like R
[bib3].

For an unspecified problem in plane linear elasticity (plane strains or plane stresses), the field of displacement U can break up into a singular part and a regular part. singular part, also called singularity, is that clarified above, it contains them coefficients of constraints. In linear elasticity, the modes of rupture I and II are separate:

U = U + K uI + K uII
R
I
S
II
S

with:

1+ R 1/2

uI

=
cos
K
S1
- cos

E 2
(2)(
)

1+ R 1/2

uI
=
sin
K
S2

- cos

E 2
(2)(
)

1+ R 1/2

uII

=
sin
K
S1
+ cos + 2

E 2
(2)(
)

1+ R 1/2

uII
= -
cos
K
S2
+ cos - 2

E 2
(2)(
)
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with:

K = 3 - 4
in plane deformations D_PLAN
K = (3 -)/(1+)
in plane constraints C_PLAN

and:

E modulus YOUNG
Poisson's ratio

The distribution of the singular constraints in the vicinity of the crack is given by the formulas:

S

= K I + K II
11
I
11
II
11

S

= K I + K II
12
I
12
II
12
S

= K I + K II
22
I
22
II
22

with:

1

3
I

11 = (
cos
1 sin
sin

2 R) 1/2

2 -

2 2

1

3
I

12 = (
cos
sin
cos

2 R) 1/2

2

2

2

1

3
I

22 =
cos
sin
sin

(
1

2 R) 1/2

2 +

2

2

1

3
II

11 = - (
sin
2 cos
cos

2 R) 1/2

2 +

2

2

1

3
II

12 = (
cos
1 sin
sin

2 R) 1/2

2 -

2

2

1

3
II

22 =
sin
cos

cos

(

2 R) 1/2

2

2

2

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1.2
Formulate IRWIN and rate of refund of energy G

In plane linear elasticity, the stress intensity factors are connected to the rate of refund of energy G by the formula of IRWIN:

2
1-
G =
(2 2
K I + KII)
déformatio

in
(DP)

plane

NS
E

1
G =
(2 2
K I + KII)
constraint

in

(CP)

plane

S
E

The demonstration of these formulas can be made starting from the expression of the rate of refund of energy G established in Code_Aster and known under the name of the method theta [bib5].
Let us recall that G is defined by the opposite of derived from the potential energy compared to the evolution
bottom of crack.

F

F
F
U

In the Lagrangian method of derivation of the potential energy, one considers transformations MR. M
+ (M) of the area of reference 0 in a field which corresponds to
propagations of the crack. With these families of configuration of reference thus defined correspond of the families of deformed configurations whose crack was propagated. Energy potential definite on is brought back on 0.
*One considers the surface **F** and voluminal forces **F** respectively applied to F and 0. One note ((**U**)) density of free energy, **U the** field of displacement, T the field of temperature and the field of vectors describing the direction of transport in = 0, then the general expression of rate of refund of energy G [bib5] is:*
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G =
*[(**U**): (**U**) - ((**U**) div] D*

Traditional term

-
(T) D

Term
thermics

with

had

T
+
*([**F**) **U** + **F** **U** div] D*
Term
voluminal

forces
with

had

***F** on*

+
(F)

U + ***F***
U *div* - ***N***
D Term
surface

forces
with

had

S ***F*** *on*

F
F

N
-
*(****N****) (****U****) D*
Term
déplacemen
with

had

on

imposed

ts

U
U

In linear elasticity, G can be regarded as the symmetrical bilinear shape of the field of displacement ***U****. Density of energy elastic ((****U****)) is written:*

(
1
1
*(****U****)) =*
*(****U****): : (****U****) =*
*B (****U****,* ***U****)*
2
2

while noting:

the tensor of elasticity
*B the symmetrical bilinear form defined by: B (****U****,* ***v****) = (****U****): : (****v****)*

and the bilinear form G (,) associated G is defined by:

1
B
B

*G (****U****,* ***v****) =*
*(****v****) +*
*(****U***
) - B

*(****U****,* ***v****) div D*
2 ***U***

v

1
*+ ([****F****)* ***v*** *+ (****F****)* ***U*** *+ (****F*** ***v*** *+* ***F*** ***U****) div] D*
U
v
U
v

2

by limiting under the terms traditional and due to the voluminal forces ***F****.*

*There are G = G (****U****,* ***U****) if* ***U*** *is solution of the elastic problem.*

1.3
Decoupling of the modes of rupture I and II

In the method established in Code_Aster, to uncouple the modes from rupture I and II and to calculate coefficients KI and KII, one uses the bilinear form symmetrical G (,) and decomposition of the field of displacement ***U*** *in parts regular and singular [bib7].*

G
(
I
I
II
I
I
I
I
II
I
U*,* ***custom****) = G (****U*** *+ K* ***U*** *+ K*
R
I
S
II ***custom****,* ***custom****) = G (****U*** *R,* ***custom****) + K G*
I
(custom*,* ***custom****) + K G*
II
(custom*,* ***custom****)*

G

(
II
I
II
II
II
I
II

II
II
U*, *custom*) = *G* (*U* + *K* *U* + *K
R
I
S
II ***custom*, *custom*) = *G* (*U R*, *custom*) + *K G***
I
(custom*, *custom*) + *K G
II
***(custom*, *custom*)**
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One shows in plane linear thermoelasticity that ***uI***
II
S and ***customs*** *are orthogonal for the product*
scalar defined by the bilinear form G (,), that terms utilizing the regular part
cancel themselves and finally:

G (
I
I
I
U*, *custom*) = *K G
I

(custom, custom)
G (
II
II
II
***U, custom)** = K G*
II
(custom, custom)

Moreover, by writing the rate of refund of energy in the form:

G
*G (**U, U)***
*G (**U** + K I*
***U** + K*
II
***U, U** + K*
I
***U** + K*
II
=
=
R
I
S
II
S
R
I
S
*II **custom)***

and like:

G (I
II
II
I
***custom, custom)** = G (**custom, custom)** = 0*
G (R I
R
II
***U, custom)** = G (**U, custom)** = 0*

the formula of IRWIN is found:

G (,) = K2 G (I
I
2
II
II
U U
***U*, *custom*) + K G**
I
S
II
***(custom*, *custom*)**

with:

1
2
-
G (I I
***custom*, *custom*) = G (II II**
***custom*, *custom*) =**

in D_PLAN
E

1
G (I I
***custom*, *custom*) = G (II II**
***custom*, *custom*) =**

in C_PLAN
E

Finally:

=
E
K I
G (
I
U U
, S)

1 - 2

=
E
K II
G (
II
U U
, S)
in D_PLAN

1 - 2

K
=
I
E G (
I
U U
, S)

K
=
II
E G (
II
U U
, S)
in C_PLAN

Establishment of the calculation of the coefficients of intensity of constraints in plane linear thermoelasticity
in Code_Aster is realized starting from the expression of the rate of refund of energy G in elasticity linear 2D, written in symmetrical bilinear form, by introducing the known expressions of singular displacements, and by using the method theta.
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2
Establishment of KI, KII in linear thermoelasticity 2D in Aster

2.1
Types of elements and loadings

To calculate the coefficients of intensity of constraints KI and KII in linear elasticity 2D, it is necessary to use option CALC_K_G of order CALC_G_THETA. This option is available for all thermomechanical loading applying to a model of affected two-dimensional continuous medium to triangles with 3 or 6 nodes, quadrangles with 4, 8 or 9 nodes, and segments with 2 or 3 nodes.
It is valid for a modeling "C_PLAN" or "D_PLAN".

Note:

One does not take account of the term due to the displacements imposed on U, one thus should not to impose conditions of DIRICHLET on the lips of the crack.

2.2
Environment necessary for the calculation of KI, KII

Order CALC_G_THETA makes it possible to recover the model of the problem, the characteristics of material, the field of displacements and the field theta. For the calculation of the coefficients of intensity of
constraints, it is necessary to add the key word FOND_FISS, which makes it possible to recover a concept of

fond_fiss type where the basic node of crack and the normal with the crack are defined.

When that the crack is laid out along an axis of symmetry, one can also specify symmetry of the loading by key word SYME_CHAR. By defect one supposes that there is no symmetry.
If one assigns value "SYME" to key word SYME_CHAR, that means that only mode I of rupture acts (opening of the lips of the crack) and one automatically assigns the zero value to KII. If one affects the "ANTI" value, then only mode II is active (slip of a lip compared to the other) and KI is no one.

Let us insist on the need for assigning to all the elements (including those of edges) the values of YOUNG E and Poisson's ratio moduli, because they are used in the calculation of singular displacements. These values must be homogeneous on all the support of the field theta.

2.3
Bilinear form symmetrical G (. .)

Note:

Routine GBILIN calculates the rate of refund of energy G in the bilinear form symmetrical G (U, v) in linear thermoelasticity planes (plane strains or stresses) for the isoparametric elements 2D.

2.3.1 Elementary traditional term

TCLA = (U): (U) - ((U)) div

Density of energy elastic ((U)) is written in linear thermoelasticity:

---

in D_PLAN:

1 - E
E
*((**U**))*
(
)
=
(2 +2
xx
yy)
(
+

2 1+)(1-
2)
(1+)(1-
2) xx yy
E
+
2 -
1+ xy
HT

in C_PLAN:
E
E
E
*((**U**)) =*
(
2 + 2 +
+
2 -
-2) (xx
yy) (-2) xx yy (+) xy HT
2 1
1
1

with HT = Density of energy due to thermics:

= 3
HT
K (T - D
T F) tr

where:

3
=
E
K
1-
2

=
thermics

dilation

= tensor
déformatio

of

NS
T
= température
reference

of

E
ref.

and in a general way, one can write:

*2 ((**U**)) = C (2*
2
+
xx
yy) + 2 C
+ 4

2

C - 2
1
2 xx yy
3

xy
HT

with:

(1 -) E

C =
= + μ
C =
E
1 (1+)(1-)
2
2
1
(1 2
-)

E

E
C =
=
C =

2
(

1+)(1- 2)
in
D_PLAN

;

in
.
2
C_PLAN
1
2

-

E

E
C =
= μ
C =
3

(
3
2 1+)
(
2 1+)

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*Therefore, while noting ((**U**)) = (,*
***U U**), one has 2 (**U**, **v**) = S1- S1TH with:*

U

v
U

v

U

v
U

v

1
X
X
y
y
X
y
y
X
S
=
1
C

+

+ C2

+

X

X

y

y

X
y

y

X

U

U

v

v

X
y
X

y
+
3
C
+

+

y

X

y

X

1
*S TH = 3K (**U***
T - réf
*T) tr (**v**) + (**v***
T - réf
*T) tr (**U**))*

*where You is the temperature associated with the field with displacement **U** by the relation:*

*= ((**U**) - HT)*
where
HT = (T - D
T F)
ij
ij

and the equilibrium equations check.

*In the same way, the term (**U**): (**U**) can be written:*

*(**U**): (**U***

) = S2 - S2TH

with:

U U
U

X
X
X
X
y
uy uy
U
X
y
y
S2 = C1
+
+

+

X
X X
y X

y X y
y y
U U

X
y
U
X

y
y
uy U
U
X
X
X
y
+ C2
+
+

+

X
X y
y y

y X X
y X
U
U

X
y
U
U
U
X
X
X
y
y
U
X

y
y
+ C3
+

+
+
+

y
X X y
y y
X X
y

X

U U
U v
Terms
X
X
X
X

become in the symmetrical bilinear form
and terms of
X X
X X
U U
1 U v
v U
type
X
y
X
y
X
y

+

become
.
y X
2 there X
y X
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TH
v
v
X
X
y
y
v
v
X
y
y
S TH =
1
2

3K (T - T
U
réf)

+
+
+
X
2
X X
y y
y X
X y
TH
U
U
X
X
y
y
U
U
X
y
y
+
13K (T - T
v
réf)

+
+
+
X
2
X X
y y
y X
X y
TH1 =
1
in

D_PLAN
where

1 - 2
TH1 =

in C_PLAN

1 -

and finally the traditional term is written:

1
TCLA = (S2 - S2TH) - (S1- S TH
1
) div
2

2.3.2 Term forces voluminal

TFOR = (F) U + F U div

In any rigour, the symmetrical bilinear expression of TFOR is written in (U, v):

1
TFOR (U, v) =
([fu) v + (fv) u+ (fu v +fv U) div]
2

where fu is the voluminal forces associated the field of displacement U for the elastic problem.

but as the expressions which we are brought to calculate are of type TFOR (U, U) and
TFOR (
S
U, U*), where* ***U*** *and* ***customs*** *are respectively the field of displacement and the singular field, and that:*

F
S
div
= 0 on
custom
=

*((**U**)*

One limits oneself to write:

S
TFOR (
CS
5
.
0
***U**, **v**) = CS (*
*[**F**.*
*U) **v** + **F**. **v** div*
U
]

=
v** = **U
with

CS = 1
v** = **U
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Finally:

F
F

*TFOR (**U, v**) = CS v*
X
X

+
+ F
X
div
X X
y y
X

F
F

+ v
y
y

+
+ F
y
div
X X
y y
y

The same remark is valid for the thermal traditional term, the additional term due to thermics and terms due to the surface forces.

2.3.3 Term thermics

One makes the assumption that the characteristics of the material (E
,
,) do not depend on

temperature.

1
T
T
TTHE = -
(T) =
3 K tr
+

T
2
X X there y

2.3.4 Term forces surface

In 2D, for the isoparametric elements of edge, one introduced the loadings of the pressure type shearing and force distributed of real type.

The term forces surface is written in the same way that the voluminal term from:

TSUR = (F) U + F U div - N

.
N

2.4
Fields of singular displacements and their derivative

Singular fields uI
II
S and customs, respectively associated with modes I and II, are known
explicitly like their derivative. They are written according to the polar co-ordinates in
reference mark related to the crack. The knowledge of the co-ordinates of the basic node of crack and of its
normal makes it possible to calculate them in the total reference mark 0xy.

The successive introduction of these fields uI
II

S and custom allow, as indicated in [§1], calculation
elementary of the coefficients of intensity of constraints KI and KII.

2.5
Postprocessing of the results of KI and KII

Knowing the values of the coefficients of intensity of constraints KI and KII for a crack
data, formulas of AMESTOY - BUI and DANG-VAN, allow the calculation of the angle of
propagation of the crack according to 3 criteria (maximum KI, KII and G maximum) [bib6].

That is to say m a field identical to except that the crack is prolonged in the direction of angle m
of a segment of right-hand side length.
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U

m
F
F
F
m

O
m

Are K (
)
m
I,
, K (
)
m
II,
, G (,)
m stress intensity factors and the rate of refund

of energy of m subjected to the same loading as.

One poses:

K ()*
m
= lim K
I
I (,
)
m
0
K ()*
m
= lim K
II
II (,
)
m
0
G ()*
m
= lim G (,)
m
0

Criteria quoted by AMESTOY - BUI and DANG-VAN [bib6] are:

.
to choose m
***

O such as K (m
I
O) is maximum,
.
to choose m
***
O such as K (m
II
O) is null,
.
to choose m
***
O such as G (Mo) is maximum.

These criteria give very nearby results [bib8].

*The results are given in the form of a table of 4 coefficients K11, K21, K12, allowing K22 to calculate K**
***
I and KII in all the cases of loading:

*K**
K
K
11
12 K
I
I

=

*K**
K
K
21
22 K
II
II
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Angle m (°)
K11
K21
K12
K22
0
1
0
0
1
10
0,9886
0,0864
-- 0,2597
0,9764
20
0,9552
0,1680
-- 0,5068
0,9071
30
0,9018
0,2403
-- 0,7298
0,7972
40
0,8314
0,2995

-- 0,9189
0,6540
50
0,7479
0,3431
-- 1,0665
0,4872
60
0,6559
0,3696
-- 1,1681
0,3077
70
0,5598
0,3788
-- 1,2220
0,1266
80
0,4640
0,3718
-- 1,2293
-- 0,0453
90
0,3722
0,3507
-- 1,1936
-- 0,1988

K (-)
m = K ()
m, K (-)
m = - K ()
m, K (-)
m = - K ()
m, K (-)
m = K ()
m
11
11
21
21
12
12

22
22

The research of the angle Mo in CALC_G_THETA is made of 10 degrees in 10 degrees. The angle of propagation is not calculated and is printed out (in the file MESSAGE) only if INFORMATION is worth 2.

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Organization (S): EDF/MMC

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Models of Weibull and Rice and Tracey

Summary

One first of all points out the bases of these two models of local approach of the rupture allowing of to model, for one, brittle fracture (model of Beremin known as of WEIBULL), for the other ductile

starting
(model of Rice and Tracey). Concerning the model of Beremin, one describes how the probability is calculated of
rupture of a structure starting from knowledge of the mechanical fields requesting it. While placing itself in
case general of a nonmonotonous way of thermomechanical loading and by supposing that parameters of
this model do not depend on the temperature, one establishes the general expression of this probability of rupture
cumulated, including the case of a correction of plastic deformation. Then, one presents the model leading to
law of growth of the cavities of Rice and Tracey as well as the ductile criterion of starting being referred to it. Lastly, of
indications concerning the implementation of these two models in Code_Aster are summarized.
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1 Introduction

One is interested here in a metal structure requested thermomécaniquement. One seeks with to determine criteria of rupture of this structure, representative of the two mechanisms met on certain steels:

· at low temperature, certain metallic materials (as the steel of tank) can

to comprise like fragile materials while breaking brutally by cleavage,
· at higher temperature, appears the ductile tear.

In opposition to the global solution, the models of Beremin and Rice - Tracey introduced here are based on the knowledge of the mechanical fields in the zones most requested for to obtain a local criterion of rupture representative of the physical mechanisms brought into play (instability of microscopic cracks of cleavage or increase then coalescence in porosity).

2 The model of Beremin (or Weibull)

The generally allowed mechanism of rupture by cleavage is as follows: the plasticization of material conduit with the starting of microscopic cracks. Taking into account the metallurgical heterogeneity of material, these microscopic cracks have a random size and a position. The total rupture is then obtained when the normal constraint with one of these microscopic cracks becomes sufficiently large to return it unstable.

The model of Beremin (cf POST_ELEM [U4.61.04]), proposed at the beginning of the years 1980 [bib1] begins again these ideas while being based on the knowledge of the local mechanical fields requesting the structure considered. We present here the broad outline of them by adopting the framework plus general established in [bib2]. By abuse language, we call this model, in what follows, model of Weibull, in reference to the law of probability to which it leads.

For that, one considers a structure subjected to a history of thermomechanical requests with to leave the moment t=0 fixed arbitrarily. This structure is made up (at least partly) of a steel, likely to break by cleavage at low temperature. One seeks to determine the probability of *cumulated rupture of this structure at any moment.*

2.1 Bases

2.1.1 Assumptions general

Let us consider first of all an elementary volume representative Vrep of material considered. One suppose that the microstructural heterogeneity of material led to the existence of sites of damage (microdéfauts) appearing with plasticity. One notes V0 the volume of each Vrep site, so that in a volume plasticized Vrep, the number of sites of damage is

.
V0
For each one of these sites, one notes G () D the probability of having a critical stress of cleavage included/understood in [; +
D]. Probability that one of the sites of damage has one
forced cleavage lower than I (c) is thus:
Ic
G (
) D
0
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In practice, one postulates a form of G for the positive constraints
:
G
m
() = '-1 (G () = 0 if < 0).

If, one considers now the structure, we suppose that 3 V
3
V 0, where V is a volume
elementary whose characteristic dimension is lower than the macroscopic fluctuations of mechanical fields.

Lastly, the events of rupture of the sites of damage are supposed to be independent the ones

others, rupture of one of the sites involving the rupture of the whole of the structure (assumption of the weakest link).

2.1.2 Probability of cumulated rupture of the structure

***One supposes here to know the probability of cumulated rupture of each site, noted Pr (sit)
E. One can
then successively to write the probability of cumulated rupture of an elementary volume, then of complete structure. The stress field being homogeneous in V, the first is worth:***

***1 - p (V
) =
1
(- p
R
R (site)) ,
site V***

***that is to say:
V***

***p (V
) = 1 - (1 - p
R
R (site)) V0.
The probability so that at the end of the loading, our structure (volume) is not broken raises then with:
V***

***V
1 - P
V
R =
1
(- p (V
R)) =
1
0
(- Pr (if)
you)
= exp L (
N 1 - Pr (if)
you)***

,

V

V

V

0
V
Knowing that p (site) remains small, in front of the unit, the preceding expression can be simplified for
R
to give finally:
V

V

P 1 - exp (- p (sit)
E
) = 1 - exp- p (site)

R
R
R
.
V
V

V

0

0

That is to say:
V

P
1 ex (
p
X
R = -
-) with X = p

R (site)
V0

2.2
Expressions of the probability of cumulated rupture of the sites.

At any moment, the evolution of the mechanical fields in each element V is supposed to be radial and not necessarily monotonous. This evolution is characterized in any point by a history of principal constraint maximum I (U) 0ut.
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2.2.1 Parameters of cleavage independent of the temperature

The loading being radial, the direction of maximum principal constraint is supposed to be constant. When there is plasticization, the sites of damage appear. We suppose that one condition necessary of rupture of a site of damage is that plasticity is active. So that this volume did not break at the moment T, it is necessary and it is enough that:

Ic I (U), U
< T,
such as p& (U
) > 0,

&p (U) being the rate of plastic deformation cumulated at the moment U.

Foot-note:

Let us stress that this condition of active plasticity is different from the condition classically adopted (p > 0, to see document [R7.02.06] Index A Version 5.0). It is clear that these two conditions are equivalent in the case of a monotonous way of loading.
For more general ways of loading, this condition of active plasticity leads in revenge with much better results [bib7].

Only times U are considered for which plasticity is active, since the rupture is not possible that at these moments there. One notes {U < T, p
& (U) >}
the 0 whole of these moments for the element V considered. The preceding condition is thus written:
Ic
max
( )
{
I U.
u<t, p& (U) >}
0
Its probability of rupture being equal, as in the preceding section, with the probability so that it have a critical stress of cleavage lower or equal to the member of right-hand side of the inequality the preceding one, it is thus written:
{max
(U
I
)
m
u<t, p
& (U) >}
0

max
(U)
{
I
u<t, p& (U) >}
0

p (site) =
G () D
R
=
,

U_0

$\frac{1}{m}$

m
$U =$
indicating the constraint of cleavage (forced for which probability of rupture

'

cumulated potential sites of cleavage is worth 1).

The probability of rupture of the structure is written then according to [§2.1.2]:
m

P_W

$R = 1 - \exp -$

U
where the constraint of Weibull at the moment T is given by:
1

m

V
$\sim m$

$\sim$
$W(T) = I$
with =
$\max_I$
$( )$

V
I U

0
{u<t, &p (U) >}

0
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Let us notice that in the case of a monotonous way of loading, the preceding expression of constraint of Weibull is reduced to:
1

m
~

if the element is plasticized
m

V
I
W = I

with ~
=
.

V
I

0
0
if not

2.2.2 Constraint of cleavage depend on the temperature

That is to say (U) 0ut evolution of temperature in V.

For any moment (U), we suppose that in the vicinity of each site of damage,
normal constraint "microscopic" checks:

=
I (micro) (U)
F. I (U),

F being a parameter of localization depending only on the average temperature (U) in V.
So that the site of damage did not break, it is necessary thus that:

Ic
, <, such as &
>
I (micro)
U T
(
p U)
.
0

that is to say:

Ic F
.
, <, such as &
>

I (U)
U T
(
p U)
,
0

so that the probability of cumulated rupture of a site rises with:

m

.

I
R (site)
(U) F ((U))
p
= max

[

,
u<t, p& (U)

>0]

U

or:

m

U
p
I
R (site)
( )
=
max
,
[

u<t, p& (U)

>0] U ((U))

with: (
U
U) =
,
a function of the temperature. The introduction of the parameter of localization
F ()
F thus leads to an apparent dependence of the constraint of cleavage.

In a general way, the probability of cumulated rupture of the structure rises with:

m

I (U)
FD
p

R = 1 - exp -
max

{

u<t, p& (U)

> }
0 U ((U))
V

0
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The constraint of Weibull has nothing any more but the following conventional smell then: while noting or a value
chosen arbitrarily, one can write:

O m

p

1 exp

R = -
-
,
O

U

O being defined by:

1

O
~

°
U I (U)
O

= ~°m V m

I
with

=
max

V

I
u<t, &p U >0

U (U)
0
[
( )]

2.3
Correction of deformation

A great deformation of the sites resulting in ' decreasing harmfulness (relative contraction by it of microscopic cracks in the transverse plan with the axis of traction), the constraint criticizes cleavage at one moment
U increases under the effect of this deformation (U) according to:
1
(U) = (U =)
0 exp ((U)) with (U) = N. (U) .n
I

Ic
Ic

I
2

where N (U)

indicate the direction associated with the maximum principal constraint at the moment U.

The probability of rupture of a site at one moment U given is written now:

m
I (U)
1

p (site) =
max

.exp - U
R
.
{
I
u<t, p
& (U) >}
0

2

U ((U))
( )

For a monotonous way of loading (constant temperature and uniform), the preceding relation conduit with the traditional expression [bib2]:
m

m
p
I
R (site) =
exp - I.

2
U
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2.4 Establishment in Code_Aster

Let us consider a field C of the studied structure which can be the whole of the studied grid, one group of mesh or a mesh. Following an elastoplastic thermomechanical calculation, one knows evolution of the plastic deformation and deformation, stress fields cumulated in it field and one wish to determine his probability of cumulated rupture.

Let us stress that for calculation with correction of deformation (option CORR_PLAST: "YES"), one preliminary calculation (simple postprocessing) of the field of deformation of Green - Lagrange from field of displacement on the zone of the studied structure (cf CALC_ELEM [U4.61.02]) is necessary. In the contrary case, postprocessing stops.

Corresponding numerical integration in Code_Aster is carried out in two times:

· one calculates in each point of Gauss ~
I if the rate of plastic deformation cumulated in it not is strictly positive,
· by squaring on each mesh then simple summation on the field C concerned, one in deduced the constraint from Weibull as well as the probability of associated rupture. The summation is balanced by a multiplicative coefficient which takes account of possible symmetries and the type of modeling selected (axi,…). One will take care well to define this coefficient (COEF_MULT) in accordance with the indications given in [U4.61.04].

The first stage makes it possible to introduce an alternative (key word SIGM_ELMOY instead of SIGM_ELGA)
leading to appreciably different results in the case of a fissured structure (presence of gradient): in each mesh, ~
I is given starting from the average on this mesh of the field
of constraint (and, possibly, field of deformation of Green - Lagrange). It is nonnull if the rate of plastic deformation cumulated at the moment considered is strictly positive in a point of Gauss at least.

3
The Model of Rice and Tracey

One is interested now in the case of ductile starting. By considering an element of volume initially healthy, ductile this element tears it results from the following elementary mechanisms:

· nucleation of cavities caused by the decoherence of inclusions present in material,
· growth then coalescence of these cavities.

3.1
Cavity insulated in an infinite plastic rigid matrix

In an analytical step of comprehension, Rice and Tracey [bib3] studied the behavior of a cavity, initially spherical (Sv surface), insulated in an infinite isotropic medium (volume V), of behavior of plastic rigid Von Mises (elastic limit 0), incompressible, ad infinitum subjected to a speed of deformation & unspecified (noted constraint with L `infinite).

They show that the field rate of travel, solution of the posed mechanical problem, minimize the functional calculus:
Q (u&) = [S (&) - S] & FD -

N U
& dS
ij
ij
ij
ij
I J

V
Sv

3.2
Approximate law of the growth of the cavities

While managing to minimize this functional calculus in various situations, Rice and Tracey have then

m
shown the dominating influence of the rate of triaxiality (=

, m: trace and equivalent

eq
eq
(von Mises) of the constraint imposed on the element of volume considered) on the growth rate of cavities.

They exhibent even a law of growth of the cavities, certainly approached, but very near to the results preceding model. Thus, in each principal direction (K) associated at the speed of deformation &, the rate of elongation of a cavity rises with:
&R = {& + &

} DR.
K
K
eq
K
(R

K: ray of the cavity in the direction (K), &
K, &eq: principal value in the direction (K) and
equivalent (von Mises) of the speed of deformation imposed ad infinitum), relation in which
coefficients and D depend on the situation considered:
5
.
= for a linear matrix of work hardening or a perfectly plastic matrix with weak
3
rate of triaxiality or = 2 in the case of a perfectly plastic matrix atstrong rate of
triaxiality,

3

.
D
m
= exp
for a perfectly plastic matrix or D
m
=
for one

2

0

4 eq
stamp linear work hardening. = 0 283
,
is the value given by Rice and Tracey whereas
more precise calculations (cf [bib4]) showed that this coefficient is higher (= 1 2
, 8).

Mudry [bib6] then proposed to apply these theoretical results to the case of the steel of tank, i.e.:

· intermediate behavior enters the extreme cases of behavior studied by Rice and
Tracey with a reasonable work hardening not no one but,
· fissured structures (high rate of triaxiality).

It deduced from it the approximate law following, valid for sufficiently high rates of triaxiality (superiors with 0,5):

3
&

R = & p

exp
m R
eq

,

2 eq
expression in which:

.
&
p
eq was substituted by &
eq (equivalent (von Mises) of the plastic part the speed of
deformation) in order to extend the law of Rice and Tracey to the elastoplastic case,
· elastic limit

0 were substituted by eq in order to take account of the hardening of
stamp around the cavity.
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Titrate:
Models of Weibull and Rice and Tracey

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20/08/02
Author (S):
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Experimental measurements of growth of porosity for various rates of triaxiality allowed
to validate this expression (cf Appears following). These results show that, when the proportion of air voids
initial weak remainder, exponential character of the relation between the ray of the cavities and the rate of
triaxiality is well confirmed. On the other hand, the coefficient depends on material considered as well as
initial fraction of porosity.

Experimental results of measurement of the growth of cavities in various metallic materials (figure extracted the ref. 6, *eq* ***indicating the equivalent of the plastic deformation noted*** *p*
eq ***in***

body text) according to the rate of triaxiality
m

eq

3.3
Ductile criterion of starting

R0 and R (T) indicating the initial ray of the cavities and at the moment T considered, the criterion of starting
ductile adoptee here is:
R (T)
R
=
,
R
R
0
0 C

expression in which the first member results from the integration of the law of growth,
in accordance with the indications of the preceding paragraph.
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One can object several arguments of principle against the direct use of this law of growth cavities of Rice and Tracey like ductile criterion of starting. As follows:

· inclusions, and thus the cavities, are not actually insulated. Worse, they are often gathered in cluster,
· the coalescence of the cavities undoubtedly results from interactions which, they also, are not described in the established model,
· in a fissured structure, the presence of gradients in bottom of crack returns less directly applicable the preceding analysis which relates to an infinite medium subjected to boundary conditions homogeneous.

Nevertheless, by using the preceding criterion, one hopes that this law remains realistic, on average, even in clusters or zones of strong gradients (average on an element of dimensions comparable with that of the model of Beremin). In addition, one makes the assumption that the critical size reserve, in general readjusted on geometries given (test-tube CT, for example), translated coalescence, which amounts supposing that coalescence does not depend too much on nature on mechanical requests imposed on the element of volume (triaxiality, shearing,…).

Let us notice to finish that the model of Rice and Tracey is only one approached law, valid for important rates of triaxiality (i.e higher than 0,5).

3.4 Establishment in Code_Aster

Let us consider a field C of the studied structure which can be the whole of the studied grid, one

group of mesh or a mesh. Following an elastoplastic thermomechanical calculation, one knows evolution of the plastic deformation and deformation, stress fields in this field and one wishes to determine the space and temporal variations growth rate of the cavities in this field.

In each point of Gauss of the field C, one assimilates the constraints and speeds of deformation calculated at every moment with the quantities applied to the infinite medium considered previously. The law of growth of Rice and Tracey is thus integrated step by step using the approximate formula following:

R (T)
R (T
)
(T)

(T)
Log
N

= Log
N 1

- +, 0283 sign m N

Exp
m
N
p
, 1 .5
(T
p
) - (T
)
R
R

(T)

(T) (eq N
eq
N 1
-)
0

0
eq
N

eq
N

R
The values of the report/ratio are thus at every moment obtained
in each point of Gauss of the field
R0
C, the sign of the rate of triaxiality allowing the taking into account of evolutions as well in traction
that in compression. Two functionalities are then offered in Code_Aster:

3.4.1 Research of the maximum value of the growth rate

At every moment, one seeks on the whole of the field C the point of Gauss (and the volume of
R
associated under-mesh) maximizing
.
R0
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3.4.2 Calculation of the average value of the growth rate

By squaring on each mesh then moyennation on the field C concerned, one deduces with each

R
moment the average value of
on
R
C
0

As in the case of the model of Weibull, an alternative is introduced: temporal integration
the preceding one is then carried out starting from the constraint and of the average plastic deformation
by
net.

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Note EDF HT-26/00/028.

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Author (S):
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Organization (S): EDF/IMA/MMN
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R7.02.07 document
Rate of refund of energy in thermo-élasto-plasticity
Summary:
One presents the calculation of the rate of refund of total mechanical energy by the method théta in 2D or 3D
for a thermoelastoplastic problem. The relations of thermoelastoplastic behavior are described in detail in the document [R5.03.02].
This rate of refund of total mechanical energy makes it possible to analyze the situations of loadings not
monotonous of the defect, for irreversible material behaviors.
Let us note that the problem of the thermoelastoplastic rupture is a delicate problem. It is advised of to consult the references before a first use.
Caution:
The defect must be modelled by a notch and not by a crack [§5].
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1

Choice of the formulation of the rate of refund of energy in thermo-elastoplasticity

One considers a notched elastic solid occupying the field of space R2 or R3. That is to say:

·
U the field of displacement,
·
T the field of temperature,
·
the tensor of the constraints,
·
F the field of the voluminal forces applied to,
·
G the field of the surface forces applied to a part S of,
·
U the field of displacements imposed on a Sd part of.

F
S
G
Sd
In linear or non-linear thermoelasticity, the rate of refund of energy G is defined by the opposite derivative of the potential energy compared to the field [bib1]:
W
G = -
Total potential energy with the balance of the system is:
W (U) =
D - F U D - G U

D
I
I
I
I

S
where is the density of free energy. In elasticity, is equal to the density of free energy elastic [R7.02.01]
One extends this definition for the thermoelastoplastic problem, while choosing to replace by
~
total mechanical energy. This choice is justified in the document [bib2].
~
is a function of the following variables of state:
·
the tensor of the total deflections,
·
p the tensor of the plastic deformations,
·
T the field of temperature,
·
p the variable interns scalar isotropic work hardening (cumulated plastic deformation),
·
one or more tensorial or scalar variables of kinematic work hardening.
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~
(
D
T
T
D
, p
T
, T,
p) = () D =

(, p, p T,) + Dp () D + S T () D

0
D
O
O

D

where
is the density of free energy

Dp is the density power density of plastic dissipation
S
(T) be
T density of entropy

~

It is noted that is the density of increased free energy of dissipated voluminal energy
T
D
plastically during all the evolution, and to which energy is added
S
T () D
O D
(contribution of the temperature to the variation of free energy).
Caution:
One limits oneself to a notched solid (cf [§5]).
2
Relation of behavior
The behavior of the solid is supposed to be thermoelastoplastic associated a criterion of Von Mises with isotropic or kinematic work hardening. This type of behavior is currently treated in operator STAT_NON_LINE [U4.32.01] under the key word factor COMP_INCR. Treated relations in this document are:
VMIS_ISOT_LINE: Von Mises with linear isotropic work hardening,
VMIS_ISOT_TRAC: Von Mises with isotropic work hardening given by a traction diagram,
VMIS_CINE_LINE: Von Mises with linear kinematic work hardening.
For more details, to consult the documents [R5.03.02] and [U4.32.01].
is connected to the field of displacement U by:
(
1
U) = (U, + U,)
2
I J
J I
The density of free energy is written:
(, p,) = E (, p
T p
, T) + H (, p, T) + Z (T)
where
H is the density of energy of work hardening
Z an arbitrary function of the temperature
E density of energy thermoelastic defined by:
E (
1
1
, p, T) = E
p
p
ij ij =
Aijkl

,
2
2
(ij - ij - (T - réf
T) ij) (kl - -
-
kl
(T réf
T) kl)
with the thermal dilation coefficient, and (Aijkl) the tensor of elasticity.
In the particular case where there no was plastic evolution, one finds the expression of the density of elastic energy for a linear thermoelastic problem [R7.02.01 §1.1].
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The free energy of work hardening H is deduced from:
H (, p T) = R (, p T), for isotropic work hardening where R (, p T) is the ray of the surface of load
p
H
and
(, T) = X (, T), for kinematic work hardening where X (, T) is the translation of surface

of load in the space of the constraints (in the case of linear kinematic work hardening = p)
For the relation of behavior of Von Mises with linear isotropic work hardening:
E E
R (,
p T)
T

=

p
E - AND

AND
y
E

Traction diagram
Characteristics of the material (Young Modulus E and D_SIGM_EPSI: AND) can depend on temperature [R5.03.02 §3.2.1].
Plastic dissipation for a law of behavior of Von Mises checks:
T Dp () D p

=
O
y
where is the initial linear elastic limit there.
~
Finally the density of total mechanical energy is written:
~(
T
T
, p, T,
p) = (, p, p, T) + Dp

() D + S! T () D

O
O
= 1
p

p
ij (ij - ij - (T - réf
T) ij) + R
2
(, p T) dp + X
(, T) D + p
O
y

O
For a linear isotropic work hardening:
~
~
= (
1
1 E E
, p, T, p) =
p
T
2
ij

2
(ij - ij - (T - réf
T) ij) +
p +
p
2nd -
y
T
E
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Lagrangian expression of the rate of refund of energy
in thermo-élasto-plasticity
The rate of refund is calculated in Code_Aster by the method théta [R7.02.01 §1.3]. One notes by

! Q the Lagrangian derivative of the quantity Q in a virtual propagation of the notch of,
being a small real parameter and a field of vector representing the direction of propagation of
Q
the notch (one thus has!
Q (X (),

) =
+ Q.).

The rate of refund of total mechanical energy in this propagation is:

-
~
~
G () = -

F U
I
I + (- F U
I
I) K, K D

-
G U
I
I +

G U
I
I K, K -
nk D

N
K

S
~
~
~
~
~
~
However!
(

p

p

,
ij
, T,
p
ij
I) =
. ! ij +
.
T
p
T
p
! ij +
! +
! +
. ! I + S!
ij
ij
T
p
I

~
=

ij
ij
~

= -

p
ij
ij
with ~
= - S

~T

= R (p, T) + p
p
Y
~
= Xi (, T
I
)

I
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~
~

that is to say - G () =

p
!
~
ij! ij - ij!
!
ij +
T + R p!
I
D

+
+

T

K, K
I
+ traditional terms (F, G)

1 ui, J
U J, I 1
! ij =
+
- (ui, kk, J + uj, kk, I)
2

2
with
p

p
ij
p
! ij =
+

ij, K

K

T
T! =
+ T

, K K

p
p! =
+

p, K K

ij
! ij =
+ ij, K

K

One can eliminate! U of the expression of G () by noticing that! U is kinematically acceptable and by using the equilibrium equation [R7.02.01 §1.3]. In the same way, terms
p
p

p
~ T
T
(-

.
+ R
+ X.
) + are eliminated as well as the terms
.
- S

.

Y
T

The following expression then is obtained:
~
-
~
p

G () =

K, K - ij iu, K K, J - ij K +
T K

ij, K
,

K
T
~
+ (

R (,
p T) + y), km No K +
ij, K D

K
ij
+ traditional terms (F, G)
and finally:
~
~

~
G () = -

p

K, K + ij
I
U, K

K, J -
,
T K + (R+ y) p K
ij K ij D

,
, -

+
ij,
T

K
K

ij

+ traditional terms (F, G)
~
p

For a radial and monotonous loading: ij ij K = (R+ y) p
,
, K +
and one finds

ij, K
ij
the expression of G () in nonlinear thermoelasticity [R7.02.03].
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4
Establishment in Code_Aster
The comparison of the formulations of G () in linear thermoelasticity and thermo-élasto-plasticity show that the terms of the two formulations differ only by terms from transport from internal variables.
The presence of the key word factor COMP_INCR, and the key word factor RELATION: "VMIS_ISOT_LINE"
(or "VMIS_ISOT_TRAC" or "VMIS_CINE_LINE") indicates that it is necessary to recover the field displacements U, constraints, and characteristics of elastoplastic material. It is also necessary to recover the fields of the tensors of plastic deformation by the operator CALC_ELEM [U4.61.02].
The types of finite elements which support these options are the same ones as in elasticity [R7.02.01 §2.4].
They are the isoparametric elements 2D and 3D.
The supported loadings are the same ones as in the elastic case.
5 Restrictions
Caution:
This formulation of G for a thermoelastoplastic relation is valid only for one solid notched and not for a fissured solid. One will choose for example (but the user will be able to choose its own regular notch):
OK
OK
NOT
Indeed, the principal difficulty in the establishment of this formulation is impossibility of showing the existence of derived from total mechanical energy for a field comprising a crack, and this mainly by the absence of knowledge of the singularities of the fields in plasticity. For to circumvent the problem, one regularizes the field by representing the defect in the form of notch. For more details, it is advised to consult [bib2].
The validation of this formulation is carried out in test SSNP102 [V6.03.102] - Calculation of the rate of restitution of energy for an elastoplastic problem.
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Titrate:
Calculation of the factors of intensity of the constraints

Date:
26/05/05
Author (S):
***E. GALENNE**, J.M. PROIX Key*
:

Organization (S): EDF-R & D /AMA

Handbook of Reference
R7.02 booklet: Breaking process
R7.02.08 document

Calculation of the factors of intensity of the constraints by extrapolation of the field of displacements

Summary:

One describes here a method of calculation of K1, K2 and K3 in 2D (plane and axisymmetric) and 3D by extrapolation of
jumps of displacements on the lips of the plane crack. This method is applicable only to the case of the cracks
plane, in homogeneous and isotropic materials. It is usable using the order
POST_K1_K2_K3.
The method used is less precise than calculation starting from the bilinear form of the rate of refund of
the energy and of singular displacements [R7.02.05] (option CALC_K_G of operator CALC_G_THETA_T in
2D). It however makes it possible to easily obtain approximate values of the factors of intensity of constraints, in particular in the case 3D, for which the method of singular displacements is not operational that with linear elements (option CALC_K_G of operator CALC_G_LOCAL_T).
One can have an idea of the precision of the results by recomputing G by the formula of Irwin, from values of K1, K2 and K3, and by comparing this value with that obtained by the energy method (method
"THETA"). The precision of the results is clearly improved if the elements touching the bottom of crack
(quadratic elements) nodes mediums located at the quarter of the edges have (elements known as of "Barsoum").

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Position of the problem

The method recommended for calculation of the factors of intensity of the constraints is the use of bilinear form of the rate of refund of energy and singular displacements (option CALC_K_G CALC_G_THETA_T [R7.02.05]). This method is operational and precise, but usable only in 2D (forced plane and plane deformations) and in 3D for linear elements. This method is not available into axisymmetric.

Indeed, in the case general in 3D and axisymmetric, one does not experience the development asymptotic of the stress field, and thus not the form of the singularity. K1 and K2 (and K3 in 3D) then do not have clear significance.

One can however give a significance to K1 and K2 (and K3) under certain assumptions ([bib1], [bib2]):

1) the crack is plane
2) the behavior is elastic, linear, isotropic and homogeneous
3) one places oneself in a normal plan at the bottom of crack

Under these assumptions, one can show that in the normal plan at the bottom of crack in a point M, them expressions of the stress fields (or displacements) are identical to the case of plane deformations; in this case, one can thus give a precise significance to K1, K2 and K3. It is the same definition as in plane deformations, and moreover, the formula of Irwin who allows to calculate G with to start from K1, K2 and K3 remains valid. This situation is easily transposable with the axisymmetric case, if the crack is plane.

These expressions are, in 3D, for a plane crack, in a point M:

$$K(M) = \lim_{R \to 0} \frac{E}{2} \left(\frac{U}{\sqrt{R}} \right)$$

$$2$$
$$8\ 1 -)[$$
$$] R$$

$$\lim$$
$$E$$
$$2$$

$$K (M) =$$

$$U$$

$$2$$

$$R\ 0$$
$$N$$

$$($$
$$2$$
$$8\ 1-)[$$
$$] R$$
$$\lim$$
$$E$$
$$2$$
$$K (M)$$

$$3$$
$$=$$
$$[C]$$
$$R\ 0\ ($$
$$8\ 1 +)$$
$$R$$

with:

T, N in the plan of the crack
T tangent vector at the bottom of crack in M,
N normal vector at the bottom of crack in M,
m normal vector in the plan of the crack
[U] jump of displacement enters the lips of crack,
R = MP where P is a point of the normal plan at the bottom of crack in Mr.

Note:

· One can note that the signs of K2 and K3 depend on the orientation on T and N. This is not not too awkward insofar as the criteria of rupture or tiredness use only them absolute values of K2 and K3.
· One can also give expressions according to the stress fields, but them values of the vectors forced on the lips of the crack are less precise than them displacements (bus exits of a transport of the points of Gauss to the nodes).
· To have an idea of the precision of the calculation of K1, K2 and K3 by extrapolation of the jumps of displacements, it is enough to recompute G by the formula of Irwin starting from the factors of intensity of
constraints and to compare this value with that obtained with G_THETA.
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2
Implementation of the methods of extrapolation:

The methods of extrapolation of displacements are implemented in POST_K1_K2_K3, afterwards extraction of the fields of displacements on the two lips of the crack.

They are tested with or without a grid of the type “Barsoum” (the nodes not tops on sides of the quadratic elements concerning the bottom of crack are moved with the quarter of with dimensions).

Three methods are programmed:

.
Method 1: one calculates the jump of the field of displacements squared and one divides it by R.

Various values of K ² are obtained (except for a multiplicative factor) by extrapolation in R =0 of the segments of right-hand sides thus obtained. If the solution were perfect (asymptotic field analytical everywhere), one should obtain a line. Actually, one obtains almost a line with a grid of the type "Barsoum", and a nonright curve if not:

.

Method 2: one traces the jump of the field of displacements squared according to R. Them approximations of K are (always except for a multiplicative factor) equal to the root of slope of the segments connecting the origin to the various points of the curve.

.

Method 3: one identifies the stress intensity factor K starting from the jump of displacement [U] by a method of least squares. Retiming is done on a segment length dmax, where dmax is the parameter fixed in operand ABSC_CURV_MAXI of the operator POST_K1_K2_K3:
dmax
1
2
K minimizes J (K) =

([U (R)]- K R) Dr.
2 0
That is to say thus the formula clarifies to calculate K:
2 dmax
1 nbno 1
-
K =
[U (R)]

rdr =
(R - R) ([U] R - U R
2
2
I 1
+
I
I 1
+
I 1
+
[ ])
I
I
R
R
m
0
m
i=0

where nbno is the number of nodes on the segment of retiming [0, dmax]. It is noticed that in this expression K is, for a fixed dmax, the linear shape of the field of displacement.

These three methods were validated on tests SSLP313 (crack inclined in an infinite medium) and SSLV134 (plane crack, in the shape of disc, in a space infinite 3D). The reference [bib2] recommends the use of method 1, with grid of the type "Barsoum".

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3
Validity of the methods suggested

One evaluates the precision of the methods of extrapolation on tests whose analytical solutions are known. One compares the results with the more precise methods founded on calculation of the rate of restitution of energy and on the singular functions.

For highlighting well the inaccuracy of the methods of extrapolation, POST_K1_K2_K3 provides systematically for the first two methods the values maximum and the values minimum (on the whole of the calculated points) of the factors of intensity of the constraints, as well as the value of G recomputed by the formula of Irwin. One will thus qualify precise method which provides values maximum and close relations minimum, and to which the rate of refund of energy is close to that calculated by the method "THETA".

Method 3 provides as for it only one value for each stress intensity factor and for G. the precision of the method will thus be estimated only by comparison between the value G recomputed by the formula of Irwin and the rate of refund of energy calculated by the method "THETA". Method 3 can be regarded as a weighted average of the rates of refund of energy extrapolated in each nodes. It will thus be checked that the error obtained with this method is lower to the maximum of error of the two preceding methods.

One has here briefly the results obtained on a test 2D and a test 3D.

3.1

Test SSLP313: 2D C_PLAN

It is about a crack inclined in an infinite medium subjected to a uniform stress field in a direction (analytical reference solution in plane constraints). The crack opens in mode mixed (K1 and K2).

For the test Aster, the crack is with a grid in a rather large plate. The grid is very fine. results are as follows:

Reference solution (CALC_G_THETA_T)

K1 aster
K2 aster
Gtheta
3.6037E+06 2.7000E+06 1.0013E+02

POST_K1_K2_K3: grid without nodes of edges to the quarter

K1_max method
K1_min
K2_max
K2_min G_max G_min difference
difference
between Gmax between Gmin
and Gtheta
and Gtheta
1 3.54E+06
3.19E+06
2.63E+06 1.92E+06 9.73E+01 6.94E+01 3,33% 30,70%
2 3.51E+06
3.33E+06
2.61E+06 2.25E+06 9.57E+01 8.08E+01 4,50% 19,32%
3 3.50E+06
2.59E+06
9.47E+01 -5,47%

POST_K1_K2_K3: grid with nodes of edges to the quarter

K1_max method
K1_min
K2_max
K2_min G_max G_min difference
difference
between Gmax and Gmin and

Gtheta
Gtheta
1 3.61E+06
3.60E+06
2.70E+06 2.69E+06 1.01E+02 1.01E+02 1,29% 1,07%
2 3.60E+06
3.53E+06
2.69E+06 2.65E+06 1.01E+02 9.75E+01 1,02% 2,67%
3 3.56E+06 2.66E+06 9.88E+01 -1,42%

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On this test one notes that the grid of the type "Barsoum" is essential if results are wanted precis. With "Barsoum" method 1 is more stable. It provides values of G (starting from K1 and K2) with approximately 1% of solution G_THETA. Methods 2 and 3 lead to errors from 1 to 2,5%.

On the other hand, with a normal grid, the results vary much (between 3% and -30% of the solution). It is the same with linear elements. In the case of a grid without elements of "Barsoum", method 3 is most precise.

3.2
Test SSLV134: 3D

It is about a plane crack in the shape of disc in an infinite medium 3D subjected to a field of constraints uniform in a direction (known analytical reference solution under the name of

“penny shape ace”). The crack opens in mode 1 (K1 alone) [V3.04.134].

For this test, the crack is with a grid in a block parallelepiped. The grid is relatively coarse. One compares the results with those of the method “THETA”.

Reference solution:

K1 G
room
1,6 106 11,59

The results are as follows:

POST_K1_K2_K3: grid without nodes of edges to the quarter

K1_max method
K1_min
G_max
G_min
difference
difference
between Gmax and Gmin and
Gtheta
Gtheta
1 1.56E+06
1.45E+06
1.11E+01 9.63E+00 4,32%
-16,91%
2 1.53E+06
1.49E+06
1.06E+01 1.01E+01 8,35%
-13,08%
3 1.52E+06 1.05E+01 9,51%

POST_K1_K2_K3: grid with nodes of edges to the quarter

K1_max method
K1_min
G_max
G_min
difference
difference
between Gmax and Gmin and
Gtheta

Gtheta
1
1.61E+06 1.59E+06 1.18E+01 1.16E+01 1,32%
-0,06%
2
1.59E+06 1.53E+06 1.15E+01 1.07E+01 0,42%
-7,87%
3 1.55E+06 1.10E+01 5,16%

On this test one still notes that the grid of the type "Barsoum" is essential if one wants precise results. With "Barsoum" method 1 is most stable. It provides values of G (to to start from K1 and K2) to 1% solution G_THETA.

3.3 Conclusion

It should well be noticed that the asymptotic expression of displacements is valid only for R tending towards 0. This is why the grids of the type "Barsoum" are definitely preferable (and almost obligatory). It is also necessary to take care not to choose a field of extrapolation too much large (distance dmax of operator POST_K1_K2_K3 of about 4 to 5 elements).

With Barsoum, method 1 gives the most precise results and most stable, that it is in 2D or in 3D. If the grid does not comprise elements of Barsoum, one then advises to use results of method 3.
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Summary
One tackles here the problem of the identification of the parameters of the model of WEIBULL on a sample of tests
representative of behaviour with rupture of a fragile material (typically, ferritic steel with low temperature). The method of regression linear and the method of the maximum of probability are both adopted methods. One details of it the principle as well as the associated methods of resolution, resting in
two cases on an iterative process. Lastly, one shows their extension if one of the two parameters of it model (the constraint of cleavage) depends on the temperature.
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1 Introduction

When they call upon the model of WEIBULL (cf POST_ELEM [U4.81.22]), the study of modeling brittle fracture of steels in general require a preliminary identification of the parameters of this model. In order to avoid a hard identification “with the hand” of these parameters which would require to start again repeatedly operation POST_ELEM with option WEIBULL, a procedure of retiming automatic was established in *Code_Aster*.

In this document, one briefly recalls the equations of the model of WEIBULL then one defines it problem of identification posed. One then describes the principle of the two adopted methods of resolution (linear and maximum regression of probability) by including the case where one of the two parameters of model depends on the temperature.

2 Recalls

2.1 The model of WEIBULL

One considers a structure of behavior elastoplastic subjected to a request thermomechanics. It is supposed that the probability of cumulated rupture of this structure follows the law of WEIBULL [bib1] with two parameters following:

$$F(W) = 1 - \exp\left(-\left(\frac{W}{P}\right)^{m}\right)$$

éq 2.1-1

U

expression in which the module of WEIBULL $m > 1$ described the tail of the statistical distribution sizes of the defects at the origin of cleavage, U is the constraint of cleavage and W is the constraint of WEIBULL which depends on the history of the principal stress field in the zone plasticized on structure. For example, in the case of a monotonous way of loading, it is written:

$$W = \sum_{p} V_p \, (I_p)^m$$

éq 2.1-2

V_p = V_0 : summation relates to volumes of V_p matter plasticized, I_p indicating the principal constraint maximum in each one of these volumes (V_0 is a volume characteristic of material).

2.2 Identification of the parameters

In a very general way, one considers an experimental base made up of tests the different ones natures (type 1, 2,…, N), each type of test being carried out N_j time so that the total number tests rises with:

$$NR = \sum_{j=1}^{j=n} N_j .$$

This experimental base could for example consist of tests on axisymmetric test-tubes notched different rays of notch led to various temperatures. Taking into account nature random of the properties with rupture of material considered, this base constitutes only one sample. The more important the number of these samples will be, the more it will be representative of the behavior of material considered.

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Among the various methods of identification suggested in the literature (see for example [bib2]), we retain two of them: method of regression linear, often used, like that of maximum of probability recommended by the “Structural European Integrity Society (ESIS)” [bib3].

Note:
A comparative systematic study of the results given by these two methods [bib2] in function of the number of sample taken by chance on a theoretical distribution showed that method of the maximum of probability led to a better estimate of parameters of the model of WEIBULL. Method of regression linear remaining nevertheless very much used, we integrated it into our developments.

In the two adopted methods of retiming, one carries out the first calculation of the constraints of WEIBULL with a play of parameter given (typically, m=20, =3000 MPa). These NR is classified U tests using their constraint of WEIBULL reached at the instant of the failure. One thus lays out of an increasing list of constraints of WEIBULL (1 I NR W,…, W,…, W), such as for each (I), it a number of test-tubes broken with a constraint of WEIBULL lower or equal to I I W is nw (in general $nor = I$ I W). Among the various possible estimators of the probability of cumulated rupture PF I correspondent with I I W [bib2], we choose that generally recommended: $PF =$. $NR + 1$

Note:
In the particular case where the constraint of WEIBULL depends on the temperature, it preceding classification must be made temperature by temperature, each temperature correspondent with a different statistical law. The estimator of the probability of rupture I *precedent thus becomes: Conk =* *, if the test-tube* (I) *were broken with*

NT + 1
temperature T, for which there was NT tests.
The two adopted methods of retiming are valid as long as [éq 2.1-1] remains true. If identification is carried out on test results anisothermes whereas the constraint of cleavage is supposed to depend on the temperature, this condition is not checked any more (cf POST_ELEM [U4.81.22]). In it particular case, one will not be able to thus apply the developments which follow.

3
Method of the linear regression
3.1.1 Principle
The variation theory-experiment is measured by the expression:
2

1
1

LogLog (
)

- *LogLog* (
)

éq 3.1.1-1
1

- *Pi*
1 - *P*
I

I
F
F (*W*)
("Log" indicates the Napierian logarithm). One wants to minimize this variation compared to (m, *U*).
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3.1.2 Resolution

The method of retiming usually used is based on successive linear regressions: with
the iteration (K), the values (*mk*, *U* (*K*)) of the module and the constraint of cleavage are known. It is thus possible, with these values, to calculate the constraints of WEIBULL *iW* (*K*) at the various moments of rupture thanks to [éq 2.1-1]. One then classifies these new constraints of WEIBULL per amplitude increasing and one deduces the new estimates from them from the probability of rupture *Pif* (*K*) to the iteration (K).
For these values of constraints of fixed WEIBULL, the minimization of [éq 3.1.1-1] is brought back to one
1
simple linear regression on the group of dots (*Log*
I
(*W K*)
(), *LogLog* (
)) since if one
1 - *Conk* (*K*)
1
defer *LogLog* (
)
(), one obtains a line of slope m which cuts the axis
1-
according to *Log*
P
W
F
X-coordinates in (*Log* ()). The new values (*m*
,
) of these parameters are thus
U
K +1
U (*K* +1)
data by (cancellation of the derivative partial of [éq 3.1.1-1] compared to each parameter):
1 *X Y*
( )
() - *Y*
X
NR
I K

J K
I (*K*)
I (*K*)
I, J
I
mk+1 = 1
éq 3.1.2-1
X X
( )
() - *X*
2
NR
I K
J K
I (*K*)
I, J
I
1
1

the U.K.
exp
X
+1 =
I K - iY
(
)
( )
(*K*

,
éq 3.1.2-2
NR

m
)
I
I

1
with *X*

= Log I
I (*K*)
(*W* (*K*)) and *Y*
= LogLog
I (*K*)
(
) .
1 - *Conk* (*K*)
One repeats these iterations as long as the difference between the plays of parameter obtained with the iterations (K) and
(k+1) is significant (typically, five iterations). The measurement of this variation is given
m

1 - *m*

the U.K. +1 - the *U.K.*
by: *Max*
K +
K

,
(
)
( )
m

K

.
U (*K*

)

Note:
If m is fixed, U (*k*+) 1 *is always given by [éq 3.1.2-2]. On the other hand, if U is fixed,*
X
Y
I (*K*) *I* (*K*)
m
I
K +1 *is not given any more by [eq 3.1.2-1] but: mk* +1 =
.
X

2
() - log ()
X
I K
U I (*K*)
I
I
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4

Method of the maximum of probability

4.1 Principle

Let us note *p F* (*W*) the density of probability associated with the probability of cumulated rupture *PF* (*W*):
M-1
m
m

p
W
W

F (*W*) =

exp -

U U
U

The quantity *p* () *D*
F
W
W is equal to the probability of breaking a test-tube subjected to one
request corresponding to a constraint of WEIBULL included/understood in the interval [*W*, *W* + *dW*].
The probability so that all the test-tubes of the base broke thus raises with:
p (*m*,) *D* = *p*
I
() *D*
U
W
F
W
W,
éq 4.1-1
I
p being related to probability. The method of the maximum of probability consists then with
to choose the parameters of the model of WEIBULL so that the function of definite probability
by [éq 4.1-1] (in practice rather its Napierian logarithm) that is to say maximum.

4.2 Resolution

An iterative process again is used. There still, with the iteration (K), (*m*
I
K, *U* (*K*)) as well as *W* (*K*)
are known. For these values of constraints of fixed WEIBULL, the maximization of *Log* (*p*) led
with a new couple (*mk+1*, *U* (*k+1*)) given by:
i= *NR*
I
mk
(
) +1 *Log*
I
(
)

NR
W (*K*)
W (*K*
i= *NR*
)

F (*m*
)
1
1 =
+
Log
I
(
)

- *NR i=*
K +
= 0 **éq 4.2-1**
m
W (*K*)
i= NR
K +1
i=1
I
mk
(
+

1
W (*K*)
)
i=1
i= NR
1

m
=

(*I*
) *mk*
K
+
+
1
1
.
éq 4.2-2
1

(*K* +)
W (*K*)
NR i=1
With each step, the resolution of [éq 4.2-1] can be carried out using the method of Newton, it gradient of *F* (*m*) being given by:

i= NR
i= NR
i= NR

2

I
m
() *Log2 I*
I
m
I
m
() () - () *Log I*

W
W
W
W
(*W*)
df
1 =1
=1
=1

(*m*) = - *NR*
I
I
I

+
.
DM
m2
i= NR

2

I
m

(

W)

i=1

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Note:
If m is fixed,
is given by [4.2-2]. On the other hand, if is fixed, m
is not any more
U (*K* +)
1
U
K +1
solution of [4.2-1] but of:
NR
I
I
i= NR

F m

=
+
Log
W (*K*)
W (*K*)
mk
(
)
(
) (1
1
0
1

- (
) +) =
K +
m

.
K
I
+
=
1
1
U
U
This equation can be again solved using the method of Newton, the gradient being now given by:
df
NR
I
I
i= NR

(*m*
W
m
) = -
2
2 -
(

) *Log*
W
(
)

DM
m

.
i=1
U
U
5
Dependence of the parameters with the temperature
If one wishes to fix independently the two parameters temperature by temperature, it is enough to to break up the base of tests into as much of under - bases by temperature and to apply to each one of these subbases preceding methods. If, on the other hand, one only wishes to vary forced cleavage with the temperature, one proceeds in the following way.
U
5.1 Regression
linear
The estimate of the probabilities of rupture being now carried out temperature by temperature (cf notices [§2.2]), it is enough to fix the constraint of cleavage on each associated group of dots at the various temperatures (T). The equation [éq 3.1.2-2] thus becomes:
1
1

the U.K. 1 = exp
X
+
I K - iY
(
)
( )
(*K*
NR
)

T
m

I T

I T

(*NT* indicating the number of tests for the subbase corresponding to the temperature (T)), the module WEIBULL being given by:

1 *X Y*
( )
( )
Y X
NR
I K
J K
I (*K*)
I (*K*
-
)
T
T I T
, *J T*

I
mk+1 =
.

1 *X X*
( )
( )
X 2
NR
I K
J K
I (*K*
-
)
T
T I T
, *J T*

I
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5.2
Maximum of probability

The constraint of cleavage is given for each temperature (T) considered by:

1
I
m
m
+1
the U.K.
(*T*)
1
= +1
(*W* (*K* (*T K*
K
(
)
))
+
,
NR
)
T I T

m being solution of:
K +1
I
mk
(
) +1 *Log*
I
(
)

NR
W (*K*)
W (*K*
i= *NR*
)
F (*m*
)
1 =
+
Log
I
(
)

-
NR I T

K +

= 0.
m
W (*K*)
T
I
m
K
K
+
+
1
1
i=1
T
(

W (*K*))
I T

6 Conclusion

Order RECA_WEIBULL of *Code_Aster* makes it possible to carry out the chock of the parameters of the model
WEIBULL [U4.82.06].
The user gives in entry of this order the concepts results associated with various calculations nonlinear carried out. The possible dependence of the constraint of cleavage with the temperature is implicitly specified when different temperatures are associated each one of these concepts results (if all these temperatures are identical or if they are not specified, there is not dependence with the temperature of this parameter).
The user can carry out this retiming by the method of the maximum of probability (METHOD: "MAXI_VRAI") or that of the linear regression (METHOD: "REGR_LIN").
The sizes determined by order RECA_WEIBULL are deferred in a table in which one finds the value of the identified parameters, the probabilities of rupture estimated from experimental results as well as the probabilities of theoretical rupture calculated with the parameters identified.

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Organization (S): EDF-R & D /MMC, AMA

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Document: R7.02.10

Analyze simplified harmfulness of defect by the method
K-beta

Summary:

The method of analysis presented (method K-beta) is applied to the analysis of harmfulness of a defect located under
the coating of the tanks REFERENCE MARK. The purpose of it is codified in the RSE-M and is to evaluate the factors of intensity
constraints corrected plastically for the coating (in first point of the defect) and for the metal of base or welded joint (in second point of the defect).
With this intention, one calculates the stress intensity factors elastic to the two points of the defect, with the assistance
constraints with the nodes resulting from the mechanical resolution and residual stresses given by the user. The reports/ratios of critical tenacities on the stress intensity factors obtained determine factors of margins.
The theoretical aspects of the method K-beta and its implementation data-processing make the objects of

following paragraphs.
This method corresponds to the Rupt1D approach in the nomenclature of project EDF Epicure.

Count

matters

1
Theoretical aspects of the method K-beta

1.1
Validity of the method K

The method suggested applies to a defect under coating located partly current of one ferritic steel tank covered by subjected austenitic stainless steel either:

.
with a thermal transient applied on the surface possibly interns combined with one loading of pressure limited
.
with a loading of direct compression.

The method is valid only for defects under coating of which the point, side coating, penetrate slightly in the coating. This is why for calculation, with the initial size of the defect considered ***prof_def,*** *one adds the penetration in the coating |deca|. [Figure 1.1-a the] precise one the difference between the initial defect (dimensions sunken in POST_K_BETA) and the defect considered in calculation (defect taking account of the penetration in the coating) by method.*

Initial defect
Defect considered
in calculation
2b
2a
- decaf
prof_def

Coating
Base metal
ray_int
ep_rev
ep_mdb
Center tank

Appear 1.1-a: Diagram of the defect under coating considered

Conditions of validity of the method:

· penetrating defect in the coating,
decaf
2a
2a
1
.
,
0 2 and
3 and

.
ep _ rev
ep _ rev
(ep _ rev + ep _ mdb)
10

By convention in order POST_K_BETA one selected decaf 0. The default value selected is decaf = -2. 10-4.
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:

1.2 Stage n°1: Calculation of the stress intensity factors elastic of a defect bandages in a plate of infinite size

*The stress intensity factors **elastic** of a defect **bandages** in a plate of **dimensions infinite** are given by the following relations:*

+a (X) has - X
K
=
dx

IA
has has +

X
- has
***K01**:*

+a (X) has + X
K
=
dx

IB

has

- X has
- has

where 2a is the bandwidth (depth of the defect), A and B is the two ends (respectively has some and +a).
The constraint (X) is the normal constraint useful for the plan of the crack (forced elastic added the residual stress).
The configurations "defect circumferential" and "longitudinal defect" are defined by the two sketches hereafter.

y
Defect bandecirconférentiel
X
With
B
- has
+a
Z

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---

y
Defect bandages longitudinal
X
With
B
- has
+a
Z

For the defect bandages circumferential, one takes (X) =

yy (X)
For the defect bandages longitudinal, one takes (X) =

zz (X)

1.2.1 Change of reference mark

1) Basic change

.

Case 1: passage of the local Cartesian base (in the plan of cut of the model axisymmetric) at the cylindrical base

y
Z

X

R
Z

One a: E =
=
Z = -
X
er ey eZ E
E
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The basic change for the tensor of the constraints is written:

rr R rZ 1
0
0 xx xy xz 1
0
0

R Z =

0
0 -
1 xy yy yz 0 0

1
rZ Z
ZZ
0
1
0 xz yz
zz
0 - 1
0

=
rr

xx
= -
R

xz
One obtains finally: =
and
zz

=
rZ

xy

=
ZZ
yy

= -
Z
yz

.

Case 2: passage of the total Cartesian base (model 3D) at the cylindrical base

Z

Z
y
R

X

er = cos E
X + sin E
Y
eX = cos E
R - sin E

One a: E = - sin E

cos of
X +
eY
where eY = sin E
R + cos E

eZ = eZ
eZ = eZ

The basic change for the tensor of the constraints is written:

rr R rZ cos
sin
0 XX XY XZ cos

- sin
0

R Z =

- sin
cos
0 XY YY YZ sin
cos
0
rZ Z
ZZ
0
0
1
XZ YZ
ZZ
0
0
1

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One obtains finally:

2
2
rr = cos XX + 2 sin cos XY + sin

YY

2
2
R = - sin cos XX + (cos - sin)

XY + sin cos YY

rZ = cos XZ + sin

YZ

2
2
= sin XX - 2 sin cos XY + cos YY
Z = - sin XZ + cos

YZ

ZZ = ZZ

.
Synthesis: components used for the calculation of the stress intensity factors

Circumferential defect: in the cylindrical base is
zz
with an axisymmetric model
yy
with a model 3D
zz
Longitudinal defect: in the cylindrical base is
with an axisymmetric model
zz

2
sin - 2 sin cos
2
+ cos with a model 3D
xx
xy
yy

2) Translation of the origin

The origin of the reference mark must be relocated radially to coincide with the point medium of bandage:
R R r0 with r0 = (ray_int + ep_rev + decaf) + has

With:
ray_int: ray interns tank
ep-rev: thickness of the coating
|deca|: penetration of the defect in the coating
half a: length of the defect considered for calculation
All these sizes are schematized [Figure 1.1-a].
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1.2.2 Method of calculation

Integrals giving K
K
and

are calculated per pieces: the decomposition comes from one
IA

IB
subdivision of the interval [A/2; +a/2] in NR elementary subintervals on which the constraint useful (X) is linearized:

(X) = I X + I for X II = [have; ai+1] [A/2; +a/2]

The meeting of the NR subintervals II per 1 I NR reconstitutes the band [has; +a].
The contributions of the subinterval II = [have; ai+1] with the calculation of the FIC is given by:

ai+1 X
X has
I
+
I
I
-
K
=
dx

IA

has

+ X has

I
has
K02:

ai+1 X
X has
I
+
I
I
+
K
=

dx

IB

has

- X has
I
has

These integrals can be calculated analytically. One obtains finally the K03-a relations:

ai+1

NR

2

2
has
has
X X

X

X
X
K
=
×

Arc sin
1 has
Arc sin
1

IA
I -
+ - - + I
+
-

i=1

2
has 2

has
has
has

have

have 1
NR

2

2 +

= has
K
×
has
X X

X

X
X

Arc sin
1 has
Arc sin
1

IB
I
- + - + I

-
-

i=1

2
has 2

has
has
has

have

N.B.

There are formulas equivalent to the relations, established above after the changes of variables.

have
= Arc sin
I

has

ai+1
= Arc sin
i+1

has

The FIC are then given by the new K03-b expressions:

NR
has

ad interim
have

K
=
×

cos
cos
sin 2
sin 2
IA
-
I
(
-
i+1
I) + (
-
I
ad interim) (
-
i+1
I) +
(
-
i+1
I)

i=1

2
4

NR
has

have

K
=
×
ad interim

cos
cos
sin 2
sin 2
IB
+
I
(
-
i+1
I) - (
+
I
ad interim) (
-
i+1
I) -

(
-
i+1
I)

i=1
2
4

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Note:

In practice, the calculation of K
K
and
be carried out on the segment of support of the postulated defect. On
IA

IB
this segment, points A (side coating) and B (side base metal or welded joint) of the crack necessarily do not coincide with nodes of the grid.
A first stage thus consists in positioning A and B on the way of radial support on the basis of external skin and finishing in external skin. This positioning takes account of the shift of defect compared to the localization of reference of a DSR, and also depth of the defect.

A translation of the origin is then carried out, the new origin being located in the middle of segment [A, B] (cf preceding paragraph concerning the change of reference mark).
The NR subintervals on which the calculation of the FIC is broken up are defined by the succession [A, NO1], [NO1, NO2],…, [NON-2, NON-1], [NON-1, B]. The nodes of the grid determine them terminals. The linear interpolations of the useful constraint (X) are thus carried out on these subintervals; for the first and the last, one respectively uses the interpolations on [NO0, NO1] and [NON-1, NOT], which will thus be used for calculation of the FIC only on part of their field of definition (NO0 is the immediate predecessor of A on the radial way, NOT is it immediate successor of B).
The formulas K03-a or K03-b are then applied for the calculation of K
K
and
.
IA

IB
It is important to note that this calculation uses the constraints with the nodes of the grid, to leave whose the linear interpolations per pieces are given.

1.3 Stage n°2
: Geometrical corrections for a Defect Under Elliptic coating

Stress intensity factors K
K
and
determined at the end of the stage n°1 concern
IA

IB
a defect bandages in a plate of infinite size.
The postulated defect is a Defect Under elliptic Coating of profile. Factors of intensity of constraints determined for this type of geometry are obtained by application of corrections geometrical on K
K
and
.
IA

IB
[Figure1.1-a] allows to define the geometry of the DSR considered for calculation.

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Certain conventions are fixed:

.
The depth 2a of a longitudinal or circumferential DSR corresponds to its radial dimension, i.e according to the direction carried by er.
.
The length 2b of a longitudinal DSR corresponds to its axial dimension, i.e according to direction carried by eZ.
The presence of DSR of longitudinal orientation is postulated in the base metal.
[Figure 1.1-a] thus precisely this configuration of defect represents.
.
The length 2b of a circumferential DSR corresponds to its dimension orthoradiale, i.e according to direction carried by E.
The presence of DSR of circumferential orientation is postulated in the welded joint. By report/ratio with [Figure 1.1-a], this configuration of defect would be obtained by carrying out one rotation of 90° of the face of crack around the small axis of the ellipse.

1.3.1 Correction by the factors of edge

This first correction holds account owing to the fact that the defect is located in a noninfinite medium.
localization of the DSR defined by [Figure 1.1-a] implies corrections in points of crack side coating and side base metal.
One defines beforehand the reduced variable of space Z = has (has + (ep _ rev+ decaf), where ep_rev is
the thickness of the coating and decaf is the penetration of the DSR in the coating (see [Figure 1.1-

a]).

Point A side coating: formulate K04
2
3
4
5
F
= 0 998742
,
+ 0142801
,
Z -1133379
,
Z + 5 491256
,
Z - 8 981896
,
Z
5 765252
,
Z
Ba
+

Point B side base metal (or welded joint): K05 formulas

1 - 0 012328
,
Z + 0 395205 2
,
Z - 0 527964 3
,
Z + 0 432714 4
,
Z

if 0 Z
F
=
0 92
,

bB

- 414 20286

,
+1336 75998

,
Z -143611970 2

,
Z + 51514949 3

,
Z

if 0 92

,
< Z 1

1.3.2 Correction by the factors of ellipticity

This second correction holds account owing to the fact that the defect found an elliptic profile. It must be
applied to the estimates determined for a defect bandages.
Two cases are distinguished, according to the preponderance of one or the other of two dimensions of the profile
elliptic.

First case: has B Depth of the defect Length
1
K06: F = F =

With
B
has, 165
1 + ,
1 464
B
Second case: B has Length of the defect Depth
B
1
K07: F = F =
×

With
B

has
B, 165
1 + ,
1 464
has
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1.3.3 Stress intensity factors of an elliptic DSR

Stress intensity factors of a Defect Under elliptic Coating, obtained by correction
FIC of a defect bandage in a plate of infinite size, are given by the relations
Point A side coating:
K08-a: K
= F × F × K

IA
With
Ba

IA
Point B side base metal (or welded joint): K08-b: K
= F × F × K

IB
B
bB

IB

1.4
Stage n°3: Plastic correction known as “correction”

1.4.1 Formulation of the correction

The stress intensity factors determined by the relations K08-a and K08-b are those of one Elliptic DSR, under the assumption of an elastic behavior of materials.
The correction, specific to the DSR stuck to the interface, makes it possible to take account of plasticization
with the two points of the crack side coating (point A) and side base metal or joint welded (point B).
The corrective factors are defined by the following relations:

R

36 yA
= 1+ 3
,
0 × tanh
2
With

ep _ rev
1 K

K09:
where R

=
IA
yA

R

36 yA

6 yA
= 1+ 5
,
0 × tanh
B

ep _ rev

ep_rev is the thickness of the coating, is the yield stress of the coating at the temperature of
yA
point A.
From where FIC corrected with the two points of the crack:

K = × K

K10: With
With
IA
K = × K
B

B
IB

1.4.2 Plastic correction progressively with the history of the loading

The plastic correction is calculated according to the formulas K09 and K10 above for a phase of charge considered separately in the history of the loading.
To evaluate the plastic correction progressively history of the loading, one must retain with one moment given the maximum correction obtained on all the preceding phases of load.

Principle
With each new phase of load, one revalues a plastic correction
K11: K = K K I = (1) × K I
(even calculation with two points A and B of the crack, from where the omission of the indices). If this news
plastic correction is higher than the maximum correction Kmax obtained hitherto, one updates Kmax. The correction finally applied is written

K12: K CP = K I + Kmax
In phase of discharge, the plastic correction applied is the addition of Kmax obtained on all them preceding phases of load:

.
no plasticization in phase of discharge,
.
the correction corresponds to the plasticized residue of the preceding phases of load.
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Algorithmic
One initializes Kmax = 0
One initializes K I_ast with a high arbitrary value

.
at the first moment one will be in phase of discharge per comparison to K I_ast
.
no plasticization at the first moment

Buckle over the moments of the history of the loading
If K I (tn) K I_ast then (phase of discharge)
K CP (tn) = K I (tn) + Kmax
If not (phase of load)
If (tn) × K I (tn) > K I (tn) + Kmax then
K CP (tn) = (tn) × K I (tn)
Kmax = K CP (tn) K I (tn)
If not

K CP (tn) = K I (tn) + Kmax
End If
End If
K I_ast = K I (tn)
Fine Buckles

The same algorithmic one described above is implemented for the plastic corrections of the FIC with two points A and B of the crack progressively with the history of the loading.

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Law of behavior of Barenblatt

Summary:

***The law of behavior of Barenblatt is a law of interface making it possible to model the opening of a crack
while taking account of a force of cohesion enters the lips of this one. An energy of surface allows to take into account the energy cost of the opening of the crack. The latter will be represented by finite elements of joint type. An energy of penalization will make it possible to take into account the condition of not
interpenetration of the lips of the crack.
We present here the form of energy and the constraint which in drift according to the jump of displacement
as well as the internal variables. The existence of instability requires a piloting by the elastic prediction of which one
will detail the elements specific to this law.***

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1
Law of Barenblatt behavior

To raise the problem of infinite constraints on the face of crack Barenblatt introduces forces of cohesion enters the lips of the crack. They are gravitational attractions being exerted between the particles of
leaves and other of the plan of separation of the crack.

It is considered that the opening of the crack costs an energy defined on the surface of discontinuity of the crack. It is called energy of surface. The problem is solved by minimizing the sum of this energy and of elastic energy.
The surface of discontinuity is modelled by a finite element of joint (Doc. [R3.06.09]) only element of code supporting this law of behavior.
The condition of noninterpenetration of the lips of the crack is taken into account by a method of penalization.

To preserve a local treatment of the conditions of opening, one carried out a regularization of the energy of surface for jumps close to zero. With this intention a small parameter was introduced (key word: SAUT_C) defining the width of these “small” jumps.

1.1
Energy of surface

On the surface of discontinuity () modelled by elements of the type joint (QUAD4)
(see Doc. [R3.06.09]) one defines an energy of surface Es depend on the standard of the jump on displacement.

*E = (**U**) D + I + U*
S
[ ]

([] N)
R

The first term corresponds to energy necessary for the opening of the crack, the second is one energy of penalization which will make it possible to take into account the condition of noninterpenetration
lips of the crack.

+ if U < 0
The function is defined:
N
I
U
+
[( ]

N)
[ ]
=
R

0
if [U] 0
N

In other words, to want to impose the interpenetration of the lips will cost an infinite energy. In practical, one will approach this function by a continuous function which tends quickly towards the infinite one
when the normal jump becomes negative:

1
2
I
U
~

C U
D
.
+ ([ ]
with C coefficient of penalization and
negative part.
N)
[ ]

R

N -
2
-

Note:

C is adjusted starting from critical jump SAUT_C entered by the user, it is the same coefficient that that taken for the regularization: cf slope of the constraint in 0+.

2
Moreover one poses [U] =
[U] + U so that a negative normal jump does not influence
N
[] 2t

+
the energy of surface and does not make evolve/move the threshold of the criterion.

One defines the energy of surface according to the cases of figure:

.
If** [U] **[SAUT_C, + [and** [U] 0**:
N

*- C **U***
*E = (**U**)*
*with ([**U**]*
[ ]

C
G

) =G 1 - E

S
[] D

C

G

C is the rate of critical refund of energy and
C the constraint criticizes in the beginning.
K having the following properties (choice carried out starting from article [F&M]):

K

GC

[ ]

G
U
C

(0) = 0

concave

SAUT_C

*[**U**]*

.
If** [U] **[0, SAUT_C [and** [U] 0**:
N

*E = (**U**)*
with K continuous and derivable in
K' 0 =:
S
[] D

SAUT_C and such as
() 0

1
2
~
~
*([**U**]) = C [U] + C*
with C constant
2

G
K
C

SAUT_C
*[**U**]*
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.
If *[U] < 0* ***and*** *[U] = 0****:***
N
T

1
2
Energy of penalization: E =
C

S
[U]

D

N -
2

K
GC

[U] N

1.2
Constraint in the element of joint

The constraint in the element of joint derives from the energy of surface. It is given by:

U
N
[
]
N
=
=

T

[
U]

T

.
***If** [U] **[SAUT_C, + [and** [U] 0:*
N

[

*U] - C [**U**]*

N

C
G

E
*C [**U**]*

=

C

[U]

*- C [**U**]*

T
G

E
C [
C
U]

SAUT_C
[ ]
U

.
If** [U] **[0, SAUT_C [and** [U] 0**:
N

C

_

C
U.E.

C []
-
JUMP C
G

C
=
N

C

_
C
U.E.
C []
-
JUMP C
G

T

SAUT_C
[ ]
U
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.
If *[U] < 0* ***and*** *[U] = 0****:***
N
T

N
C [U] N
=

C
0

SAUT_C
[ ]

U N

Note:

The regularization of energy in zero makes it possible to define a constraint in the element for one null jump. Physically in other words, as soon as the constraint in the element of joint will increase, a small jump will appear. The behavior of the Barenblatt type will be carried out that when the standard of the jump in the element exceeds SAUT_C, before the joint comprise as a spring.

1.3 Variables

interns

The law of Barenblatt behavior at summer implemented with three internal variables.
First 1
v is used for piloting and for the discharges, it is a threshold which corresponds to
greater jump (in standard) ever reached (see: [§2] Piloting).
*The second v2 makes it possible to know if the element is in elastic mode ([**U**] < SAUT_C) or softening ([**U**] > SAUT_C).*
The third corresponds to the percentage of energy of surface dissipated during the loading:
C
v
1 exp
v
3 =
-
-
1
Gc
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Titrate:
Law of behavior of Barenblatt and piloting

Date:
12/03/04
Author (S):
J. LAVERNE *Key*
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2 Piloting

The piloting recommended with the law of Barenblatt behavior is of type PRED_ELAS [R5.03.80], it allows to follow dissipative solutions presenting of instabilities.

The implementation of this technique of piloting requires the resolution of the following local equation:

F U + U
- +
=
+ +
éq
2-1
el
[0] [1] (N JUMP _C)
(L
N
JUMP _ C)
C

with unknown factor which one obtains by the solution of a quadratic equation, lc length characteristic of the model, and N
threshold which will make it possible to keep in memory the standard of the jump with the previous moment (stored in the variable interns No 1 of the law of Barenblatt behavior).

The threshold evolves/moves as follows:

+
If [U] N 1 + JUMP _ C then
+
= -
If not
+
= [U] N 1
+ - JUMP _ C

Piloting imposes ultimately that the jump of displacement continues to increase some share length potential crack.
The implementation of this piloting makes it possible to follow unstable branches of balance of the curve
total force/imposed displacements (see [Figure 2-a]).

Appear 2-a: Total curve Forces/Déplacements imposed with unstable branch
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3 Words
keys

The law of Barenblatt behavior is used in STAT_NON_LINE and DYNA_NON_LINE with the word key BARENBLATT. This law of behavior is usable on elements of joint with modeling PLAN_FISSURE and AXIS_FISSURE.

Two parameters are to be seized in DEFI_MATERIAU:

SIGM_C: critical stress
SAUT_C: small parameter of regularization

Orders

STAT_NON_LINE COMP_INCR
RELATION
BARENBLATT
DYNA_NON_LINE COMP_INCR
RELATION
BARENBLATT
AFFE_MODELE MODELING PLAN_FISSURE

AXIS_FISSURE

DEFI_MATERIAU RUPT_FRAG

SIGM_C

SAUT_C

This law of behavior was validated by the case test SSNP118 (see Doc. [V6.03.118]).

4 Bibliography

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[2]
J. LAVERNE, E. LORENTZ, J.J. MARIGO: "A model of rupture with energy of Barenblatt: theoretical considerations and numerical establishments. 16th French congress from Mechanics, Nice September 1, 2003.

Handbook of Reference
R7.04 booklet: Evaluation of the damage
R7.04.01 document

Estimate of the fatigue life to large a number of cycles and in fatigue oligocyclic

Summary:

The majority of the industrial structures are subjected to variable efforts in the time which, repeated one great number of times can lead to their rupture by tiredness. One presents in this note the principal ones functionalities of orders POST_FATIGUE [U4.83.01] and/or CALC_FATIGUE [U4.83.02] and/or CALC_ELEM
[U4.81.01] which makes it possible to estimate the limit of endurance and the office plurality of damage of a part.

The various methods available are:

.
linear office plurality: methods based on tests uniaxial (methods of Wöhler, Manson-Whetstone sheath and Taheri).
These methods have as a common point to determine a value of damage starting from the evolution with run from the characterizing time of a ***scalar*** *component, for the calculation of the damage, the amplitude of*
constraints or of structural deformations.
With this intention, it is necessary to extract by a method of counting of cycles, the elementary cycles of loading undergone by the structure, to determine the elementary damage associated with each cycle and to determine the total damage by a linear rule of office plurality;

.
nonlinear office plurality: method of Lemaître and method of Lemaître-Sermage
These methods make it possible to calculate the damage D at every moment T, starting from the data of

tensor of the constraints (T) and the cumulated plastic deformation p (T);

.

limit of endurance: criteria of Crossland and Dang Van Papadopoulos
These criteria apply to uniaxial or multiaxial loadings in ***periodic*** *constraints. They provide a value of criterion indicating if there is tiredness or not. Definite equivalent constraints for these criteria can also be used for to calculate the office plurality of damage.*
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Count

matters

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1 Introduction

The industrial experiment shows that the ruptures of structure or machine components in normal operation are generally due to tiredness. Its masked progressive character conduit very often with a brutal rupture.

One understands by tiredness the consecutive modification of the properties of materials to the application of
cycles of efforts, cycles whose repetition can lead to the rupture of the parts made up with these materials [bib1].

Various methods are available for the evaluation of the damage. The second part of it document is devoted to the presentation of oldest which is methods based on uniaxial tests: method of Wöhler, method of Manson-Whetstone sheath and more recently methods *proposed by S. Taheri (EDF-R & D /AMA).*

These methods have as a common point to determine a value of damage starting from the evolution with run from the characterizing time of a scalar component, for the calculation of the damage, the state of constraints or of structural deformations.

The evaluation of the damage is based on the use of curves of tiredness of the material (Wöhler or Manson-whetstone sheath), associating a variation of constraint of amplitude given to a number of cycles
acceptable.

To use these curves starting from a real uniaxial loading, it is necessary to treat the history of constraints or of the deformations by identifying elementary cycles (cf [§2.2]).

The difficulty in defining a cycle for a complex signal explains the profusion of the methods of counting appeared in the literature [bib2].

Two methods among most usually used were introduced into Code_Aster:

.
counting of extended in cascade or method RAINFLOW,
.
regulate RCC_M.

One adds to it a third method which we will appelerons method of "natural" counting and which respect the order of application of the cycles of loading.

For each elementary cycle, one evaluates an elementary damage using methods founded on curves of Wöhler, Manson-Whetstone sheath or both simultaneously.

For the method of Wöhler (cf [§2.3]) the user can correct the constraint to be integrated in the curve of Wöhler by:

.
a concentration factor of constraints KT, to take account of the geometry of part,
.
an elastoplastic coefficient of concentration Ke,
.
a correction of Goodman or To stack in the diagram of Haigh to take account of average constraint of the cycle.

In addition, one proposes to define the curve of Wöhler in three different forms, a form discretized point by point and two analytical forms.

The method of Manson-Whetstone sheath (cf [§2.4]) applies to loadings in deformations. The curve of Manson-Whetstone sheath is defined in a single form, forms discretized point by point.

The methods of Taheri (cf [§2.5]) also apply to loadings in deformations and require the data of the curve of Manson-Whetstone sheath and possibly of the curve of Wöhler. Their characteristic is to hold account about application of the elementary cycles of loading with structure, contrary to the two other methods.

Note:

Three methods of extraction of the elementary cycles are available: method of Rainflow, regulate RCC_M and "natural" counting.

The first two methods do not hold account about application of the cycles what is of no importance for the calculation of the damage by the methods of Wöhler or Manson-Whetstone sheath.

For the calculation of the damage by the methods of Taheri, it is necessary to use the method of extraction of cycles by "natural" counting [§2.2.3] which respects the order of application of the cycles.

For the whole of these methods calculation of the total damage undergone by the structure is determined by a method of office plurality, the rule To mine.

The third part of this document presents the methods of Lemaître and Lemaître-Sermage which are "analytical" methods allowing to calculate the damage D (in each moment T) from data of the tensor of constraints (T) and of the cumulated plastic deformation p (T). These two methods apply to loadings in unspecified constraints (uniaxial or multiaxial).

A linear rule of office plurality can be used to determine the total damage undergone by the structure.

Lastly, the criteria of Crossland and Dang Van Papadopoulos are presented in fourth and last part of this document. They apply to unspecified loadings (uniaxial or multiaxial) in constraints and periodicals. They provide a value of criterion indicating if there is tiredness or not.

From the value of the criterion, one can specify a scalar component characterizing the state of structure for calculation of the damage and to determine a value of damage by using the curve of Wöhler of material.
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2
Methods of Wöhler, Manson-Whetstone sheath and Taheri

2.1
Extraction of the peaks

The user provides to Code_Aster a function which defines the history (scalar) loading in one not given. For that, it has the key word HISTORY.

On this history of the loading, which can be complex, a first operation of extraction of the peaks is realized. This operation consists in reducing the history of loading to the only fundamental peaks.

Note:

In fatigue, one names loading in a point given the value of the response of the structure in this point.
In the use of the curves of Wöhler, it is about constraint in this point.
In the use of the curves of Manson-Whetstone sheath, it is about deformation in this point.
The history of loading is thus the evolution in the course of the time of a constraint, or one deformation.

If the function remains increasing or decreasing on more than two consecutive points, they are removed intermediate points to keep only the two extreme points.

One also removes history of the loading the points for which variation of the value constraint or deformation is lower than a certain level chosen by the user. That amounts applying a filter to the history of the loading. The value of the level of the filter is introduced by the user under key word DELTA_OSCI.

For illustration let us consider the following history of loading:

N°
not
1 2 3
4 5 6
7 8
9
10
11
12
13
14
Moment 0.
1.
2.
3.
4.
5.
6.
7.
8.
9.
10. 11. 12.
13.

Loading 4. 7. 2. 10. 9.6
9.8
5.
9.
3.
4.
2. 2.4 2.2
12.

N°
not
15 16 17 18 19 20
21 22
23 24 25 26 27 28 29
Moment 14.
15.
16.
17.
18.
19. 20.
21.
22.
23.
24. 25. 26.
27. 28.
Loading 5. 11. 1. 4. 3. 10.
6.
8.
12.
4.
8. 1. 9.
4. 6.

The extraction of the peaks of this history of loading, with a value of delta of 0.9 conduit with to destroy all the oscillations of amplitude lower than 0.9. What leads to the history of loading following:

N°
not
1 2 3 4 7 8 9
10
11 14 15 16 17
Moment 0.
1.
2.
3.
6.
7. 8.
9.
10. 13. 14. 15. 16.

Loading 4. 7. 2. 10. 5. 9. 3.
4.
2. 12. 5. 11. 1.

N°
not
18 19 20 21 23 24 25 26 27 28 29

Moment 17.
18.
19.
20.
22. 23. 24.
25.
26. 27. 28.

Loading 4. 3. 10. 6.
12. 4. 8.
1.
9. 4. 6.

Note:

Let us note CH the value of the loading; CH can be a constraint or a deformation.
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History of loading was removed:

.
item 5 bus CH =
CH
)
5
(

- CH (4) <0.9,
.
item 6 bus CH =
CH (6)
- CH (4) <0.9,
.
item 12 bus CH =
CH 12
(
)
- CH)
11
(
<0.9 ,
.
the point 13 bus CH =
CH
)
13
(

- CH)
11

(
0
< .9 .

In the same way one removes the point 22 bus the history of loading is increasing between the items 21, 22 and
23 and thus one keep only the extreme points.

2.2
Methods of counting of cycles

During their life, the industrial structures are generally subjected to loadings complexes whose levels of requests are variable.

The methods of counting of cycles make it possible to extract from the history of loading, of the cycles elementary according to various criteria.

Code_Aster proposes three distinct methods including two nonstatistical methods among methods most usually used.

2.2.1 Method
RAINFLOW

Method of counting of extended in cascade more often called method of RAINFLOW, defines cycles which physically correspond to loops of hysteresis in the plan stress-strains. In the literature, one counts several alternatives of this method.

The algorithm implemented in Code_Aster is essentially that proposed by recommendation AFNOR A 03-406 of November 1993 [bib3] (with characteristics which is specified during the presentation of the detail of the algorithm) and breaks up into three stages:

.
A first stage *which consists in rearranging the history of the loading (T) or (T) of such left that the loading begin with the maximum value, in absolute value, of the loading.*

Note:

In recommendation AFNOR A 03-406, it is not made state of a rearrangement of history of loading. This rearrangement is however carried out in the software POSTDAM [bib2] and included in Code_Aster.

.
The second stage *consists in extracting the elementary cycles from the history of loading*

thus rearranged.

The method consists in being based on four successive points of the history of loading (CH (I), i=, 1Nbpoint).

One notes:

X = CH (I +)
1
- CH (I) and Y = CH (I +)
2
- CH (I +)
1
and Z = CH (i+
)
3
- CH (i+)
2 .
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As long as Y is strictly higher than X
with

or Z, one traverses the history of the loading in
moving of a point towards the line (what amounts incrementing the value of I).

As soon as Y is lower or equal than X and inferior or equal to Z, it is considered that one has met an elementary cycle which is defined by the two points (I +)

1 and (I + 2).
The amplitude of the cycle is given by CH = CH (I +
)
1
- CH (I + 2).

When the cycle is extracted one removes the two points of the history of loading and one the algorithm continues.

.
The third stage *consists in treating the residue, i.e. the remaining history of loading after the stage of extraction of the cycles.*

With this intention, one adds the same residue with his continuation possibly realising some precautions on the level of connection following the values of the extrema considered thus that value of the first and the last slope of the residue.

The last point of the residue the first point of the cycle succeeds. So points considered can not seem extrema more. If that occurs, it is appropriate
to eliminate. Eight different cases are encountered. To treat them explicitly, let us call
1
R and R2 the first two points of the residue and RN and R
1
-
N its last two points.

Note:

Recommendation AFNOR A 03-406 fact also state of a possible preprocessing of signal, which would consist of a filtering of the signal (suppression of the parasites) and of one quantification of the history of loading.

The filtering of the signal is possible, at the request of the user (see [§2.1]. Extraction of peaks).

The quantification of the signal can be useful for the speed of the analysis of the results of analysis of tiredness. Practically, the quantification of the signal consists in cutting out maximum extent of the signal in classes of intervals of constant width called not, and to bring back to a value representative of a given class (its average value in general) all values located in this class. This possibility of preprocessing signal as for it, is not available in Code_Aster.
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Encountered case
Connection

1) (R
).(
-
)
-
).(
-
)
N
Rn1 R2
R1 > 0 and (RN
Rn1 R1
RN < 0
R
RN
R
N
R
2
2
Rn1
R
R
n1

1
R1
R
R
n1
1
R
R
n1
1
R
R
N
R
2
RN
2
has) Connection without problem: transition (RN, R1)
Encountered case
Connection

2) (R
).(
-
)
-
).(
-
) > 0
N
Rn1 R2
R1 > 0 and (RN
Rn1 R1
RN · 0
R
R
2
2
RN
R1
Rn1
Rn1
Rn1

Rn1
R1
RN
R
R
2
2
b) Connection transition (Rn1, R2), one eliminates R1 and RN

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Encountered case
Connection

3) (R
).(
-
)
-
).(
-
)
N
Rn1 R2
R1 < 0 and (RN
Rn1 R1
RN < 0

R
R
N
N
R1
R
R
n1
n1
R2
R2
R
R
2
2
Rn1
Rn1
R1
RN
RN
c) Connection transition (RN, R2), one eliminates R1
Encountered case
Connection

4) (R
).(
-
)
-
).(
-
) > 0
N
Rn1 R2
R1 < 0 and (RN
Rn1 R1
RN · 0
R
R
1
1
RN
R

R
2
2
R
Rn1
n1
Rn1
Rn1
R
R2
2
RN
R
R
1
1
D) Connection transition (Rn1, R1), one eliminates RN

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In order to illustrate the method and to clarify the points which would remain obscure, one considers the history of
loading following (which for the example is considered of type forced):

N° not
1

2
3
4
5
6
7
8
9
10
11
12
13
14
15
Moment 0.
1.
2.
3.
4.
5.
6. 7.
8.
9.
10.
11. 12. 13. 14.
Loading 0. 40. 10. 60. 20. 50.
30. 80.
-70.
30.
-50. 20. -30. 25. 0.

(T)
8
60.
4
6
2
10
7
14
12
5

1
15
T
3
13
11
9

The method of RAINFLOW thus leads, on this example, (see [§Annexe1], for detail of the stages of the algorithm) with the determination of 7 elementary cycles defined by the maximum value and the value
minimal of the loading, for each cycle.

Cycle 1:
VALMAX = 20.
VALMIN = 30.
Cycle 2:
VALMAX = 25.
VALMIN = 0.
Cycle 3:
VALMAX = 30.
VALMIN = 50.
Cycle 4:
VALMAX = 40.
VALMIN = 10.
Cycle 5:
VALMAX = 50.
VALMIN = 30.
Cycle 6:
VALMAX = 60.
VALMIN = 20.
Cycle 7:
VALMAX = 80.
VALMIN = 70.

Note:

.
The calculation of the damage not holding account about appearance of the elementary cycles of loading, it is without consequence to rearrange the history of the loading.
.

For the methods of Taheri, the order of application of the elementary cycles of loading is taken into account, also very vigilant with the use of such a method of counting is necessary it to be cycles. It is advised, for the calculation of the damage by the methods of Taheri, to use method of "natural" counting known as [§2.2.3].
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2.2.2 Method RCC_M

This method consists in forming the elementary cycles of request while starting with those which the greatest variations cause.

Thus for a history of loading comprising NR points, one determines NR elementary /2 cycles if NR is even and NR /2 +1 if NR is odd.

The algorithm breaks up into two stages. The first stage consists in ordering the history of loading of smallest with the greatest value of the constraint, or the deformation.

The second stage consists, as for it, to form the elementary cycles with the greatest variation value of the constraint, or deformation.

On the history of loading CH (T) rearranged, the elementary cycles are defined by:

VALMAX = CH
for I =,
1 NR/2

NR + -

1 I

VALMIN =
I
CH

If NR is odd one determines a definite additional cycle by:

VALMAX = CH
if CH

> CH

NR/2+1
NR/2+1
m
VALMIN
= CH
+
NR/2+
*2 **
1
m
CH

and

VALMAX = CH
if not

NR/2+1

VALMIN
= CH
+
NR/2+
*2 **
1
m
CH

NR
1
where
m

CH = constraint average or average deformation of the loading =

I
CH.
NR 1

To illustrate method RCC_M let us consider the same example as that used for the method RAINFLOW (of which the loading was considered of type forced).

N° not
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Moment 0.
1.
2.
3.
4.
5.
6. 7.
8. 9.
10.
11. 12. 13. 14.
Loading 0. 40. 10. 60. 20. 50.
30. 80.
-70. 30.
-50. 20. -30. 25. 0.

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(T)
8
4
6
2
10
7
14
12
5
E 7
cycl
E 6
E 5
E 4
1
cycl
cycl
cycl
E 3
T
15
E 2
3
cycl
E 1
cycl
cycl

13
11
9

The first stage which consists in ordering the history of the loading, of smallest with largest value of the loading, conduit to following storage:
N° not
9
11
13
3
1 15
5 12
14
7
10
2
6
4
8
Loading 70. 50. 30. 10. 0. 0.
20. 20. 25. 30.
30. 40. 50. 60. 80.

The history of loading being composed of 15 points, method RCC_M determines 8 cycles elementary:

Cycle 1:
VALMAX = 80.
and
VALMIN = 70.

Cycle 2:
VALMAX = 60.
and
VALMIN = 50.

Cycle 3:
VALMAX = 50.
and
VALMIN = 30.

Cycle 4:

VALMAX = 40.
and
VALMIN = 10.

Cycle 5:
VALMAX = 30.
and
VALMIN = 0.

Cycle 6:
VALMAX = 30.
and
VALMIN = 0.

Cycle 7:
VALMAX = 25.
and
VALMIN = 20.

NR
1

Cycle 8:
VALMAX = 20.
and
VALMIN = 6.
because
=
m
= 6.

I

NR 1

Note:

This method of counting of cycles does not hold absolutely account about appearance of cycles, and systematically orders the elementary cycles by decreasing amplitude. This

method must be used with vigilance for the calculation of the damage by the methods of Taheri whose characteristic is to hold account about application of the cycles of loading. For calculation of the damage by the methods of Taheri, it is strongly advised to use the method of "natural" counting of cycles known as [§2.2.3].
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2.2.3 Method "natural"

This method consists in generating the cycles in the order of their appearance in the history of loading.

Thus for a history of loading of NR +1 points, one determines NR elementary /2 cycles if NR par and NR elementary /2 +1 cycles if NR odd.

The method consists in being based on three successive points of the history of loading.

One notes X = CH (I +)
1 - CH (I) and Y = CH (I + 2) - CH (I +)
1 .

If X Y one considers that one met an elementary cycle which is defined by the two items (I) and (I +)
1 .

The amplitude of the cycle is given by CH
= CH (I +)
1 - CH (I).

If X < Y one considers that one met an elementary cycle which is defined by the two points (I +) 1 and (I + 2).

The amplitude of the cycle is given by CH = CH (I + 2) - CH (I +) 1 .

When the cycle is extracted one removes the two items (I) and (I +) 1 of the history of loading and one the algorithm continues.

If the number of points (NR +) 1 of the history of loading is odd, the algorithm described previously allows to discuss all the items.

If the number of points (NR +) 1 of the history of loading is even, it remains to discuss the two items remainders.

It is considered that these two points form a cycle defines by the two points NR and (NR +) 1 .
The amplitude of the cycle is given by CH = CH (NR +) 1 - CH (NR).

To illustrate this method let us consider the same example as that used for the methods RAINFLOW and RCC_M.

N° not
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Moment 0.
1.
2.
3.
4.
5.
6. 7.
8. 9.
10.
11. 12. 13. 14.
Loading 0. 40. 10. 60. 20. 50.
30. 80.
-70. 30.
-50. 20. -30. 25. 0.

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(T)
8
4
6
2
10
7
14
12
5

1
T
15
3
13
11
9

The history of loading being composed of 15 points, the method "naturalness" determines 7 cycles elementary:

Cycle 1:
VALMAX = 40.
and
VALMIN = 10.

Cycle 2:
VALMAX = 60.
and
VALMIN = 10.

Cycle 3:
VALMAX = 50.
and
VALMIN = 20.

Cycle 4:
VALMAX = 80.
and
VALMIN = 70.

Cycle 5:
VALMAX = 30.
and
VALMIN = 70.

Cycle 6:
VALMAX = 30.
and
VALMIN = 50.

Cycle 7:
VALMAX = 25.
and

VALMIN = 30.

Note:

This method is that which it is strongly recommended to use in the case of the calculation of damage by the methods of Taheri.

2.3
Calculation of the damage: method of Wöhler

The number of cycles to the rupture is determined by interpolation of the curve of Wöhler of material for a level of alternate constraint given (to each elementary cycle a level corresponds of amplitude of constraint

= max -
min and an alternate constraint S
= 1/ 2

alt
).

The damage of an elementary cycle is equal contrary to the number of cycles to the rupture D = 1 NR.
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In the case of a uniaxial homogeneous test with an alternate constraint pure (or symmetrical), it a many cycles to the rupture are given starting from a diagram of endurance, still called curve of Wöhler or curve SN.

In the case of geometrical defects or of elementary cycles of nonnull average constraint, of corrections of the curve of Wöhler are necessary before the determination of the number of cycles to rupture and thus of the elementary damage.

2.3.1 Diagram of endurance

The diagram of endurance, also called curve of Wöhler or curve SN (curve forced a number of cycles to the rupture) is obtained in experiments by subjecting test-tubes to periodic cycles of efforts (generally sinusoidal) of normal amplitude and frequencies constants, and by noting the number of cycles NR to the end of which the rupture occurs.

The curve of Wöhler is thus defined for a given material and is presented in the form:

Salt
zone 1
zone 2
zone 3
105
106
107
ln NR
NR: Numbers of cycle with the rupture

1
where Salt = the alternate constraint of the cycle = max -
min
2

One distinguishes three zones on this curve:

.
a zone of oligocyclic fatigue, under strong constraint, where the rupture occurs after one very small number of alternations,
.
a zone of tiredness or limited endurance, where the rupture is reached after a number of cycles which grows when the constraint decrease,
.
a zone of unlimited endurance or zone of safety, under low constraint, for which rupture does not occur before a number given of cycles superior to the lifespan under consideration for the part.
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There are many expressions of the diagram of endurance:

.
Oldest is that of Wöhler:

ln (NR) = has - B Salt ***éq***
2.3.1-1

where NR is the number of cycles to the rupture,

Salt the alternate constraint applied,
has and B two characteristics of material.

This analytical expression does not return account well, of a horizontal branch or asymptotic of the curve SN supplements, but it often gives a representation very good of average part of the curve.

.
Since 1910, Basquin proposes the formula:

ln (NR) = has - B ln (Salt) ***éq***
2.3.1-2

to take account of the curve of the curve of Wöhler which connects the branch downward with the horizontal branch.

D **=** *damage of an elementary cycle = 1 NR =*
S
With
where
With e-a
=
and B
alt
=

.
Another analytical shape of the curve of Wöhler is proposed in POSTDAM to hold count curve out of the singular zone:

S
=
alt
1 2 (E/E
C
)
éq
2.3.1-3

where
E =
of

Modulate

material,

tire

of

curve

with

associated

Young
C
E =
of

Modulate
for

used

Young
to determine

constraint

S.
X =LOG10 (Salt)

NR = a0+ 1
X has +
2
a2X +
3
10
a3X
1 NR

if S

S

where S
of

limit

is
material

endurance
D =
alt
L
L
.
0
if not

Note:

If one takes a2 = a3 = 0 and E/E =1
C
one finds the formula of Basquin.

The user can introduce the curve of Wöhler into operator DEFI_MATERIAU [U4.43.01] under three distinct forms:

.
a point by point discretized form *(key word WOHLER under the key word factor TIRES in DEFI_MATERIAU).*

The curve of Wöhler is in this case a function which gives the number of cycles to the rupture NR according to the alternate constraint Salt and for which the user chooses the mode of interpolation:
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- "LOG" ----> interpolation logarithmic curve on the number of cycles to the rupture and on alternate constraint (formula of Basquin per pieces),
- "FLAX" ----> linear interpolation on the number of cycles to the rupture and on the constraint alternated (this interpolation is disadvised because the curve of Wöhler is not absolutely not linear in this reference mark).
- "FLAX", "LOG" interpolation in logarithmic curve on the number of cycles to the rupture and in linear on the alternating load, which leads to the expression given by Wöhler.

The user must also choose the type of prolongation of the function on the right and on the left (if it is necessary to interpolate the function in an unauthorized point by the definition of the function there is stop of
program by fatal error).

.
***an analytical form of Basquin** (key words A_BASQUIN and BETA_BASQUIN under the word key factor TIRES in DEFI_MATERIAU)*

D = ASalt They are constant A and used in this formula which are with to introduce by the user (in accordance with code POSTDAM).

.
an analytical form except singular zone

S
=
alternated

constraint
=1/
alt
(

2nd/
C E)
X = LOG10 (Salt)
NR =
a0+ 1
X has +
2
a2X +
3
10
a3X

1 NR

if S
S

where S
of

limit

is
material

endurance
D =
alt
L
L
0.
if not

The user must introduce:

EC. = Young Modulus associated with the curve with tiredness with the material (key word E_REFE under the word
key factor TIRES in DEFI_MATERIAU)
E =
Young modulus used to determine the constraints (key word E under the key word factor

ELAS in DEFI_MATERIAU),

constants of the material a0, a1, a2 and a3 (key words A0, A1, A2 and A3 under the key word factor TIRE in DEFI_MATERIAU)
and Sl the limit of endurance of the material (key word SL under the key word factor TIRES in DEFI_MATERIAU).

Note:

This expression of the damage is available in the same form in the software POSTDAM.

2.3.2 Influence geometrical parameters on the endurance

2.3.2.1 Coefficient of stress concentration

According to the geometry of the part, it can be necessary to balance the value of the constraint *applied by the coefficient of stress concentration KT .KT is a coefficient function of geometry of the part, the geometry of the defect and the type of loading.*
This coefficient is given by the user under key word KT of the key word factor COEF_MULT.

It is used to apply to the history of the loading, a homothety of report/ratio KT, which returns to to multiply all the values of the history of loading by coefficient KT.

(The calculation of the damage will be made on a history of loading (T) =KT × (T)).
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2.3.2.2 elastoplastic Coefficient of concentration

It can also be necessary to balance the value of the pressure applied by the coefficient of elastoplastic concentration Ke.

The elastoplastic coefficient of concentration Ke (aimed to the B3234.3 articles and B3234.5 of the RCC_M
[bib4]) is defined as being the relationship between the amplitude of real deformation and the amplitude of
fictitious deformation determined by the elastic analysis.

An acceptable value of the Ke coefficient can be determined by [bib4]:

K =1
if
<
S
3
E
m
K =1+
E
(1 - N) (/S
3
-
m
) 1 (N (m -) 1 if S
3
< <
m
m
3 Sm

K =1/N
if
m
3 S
<
E
m

where Sm is the acceptable maximum constraint,

and N and m two constants depending on material.

The elastoplastic factor Ke is a report/ratio of homothety of the loading. This factor dependent on the amplitude of the loading. It is applied, cycle by cycle to the values of the maximum constraint and minimal of each cycle.

Data S, N
m
m
and
are introduced under key words SM_KE_RCCM, N_KE_RCCM and
M_KE_RCCM under the key word factor TIRES in DEFI_MATERIAU.

The user asks for the taking into account of the elastoplastic concentration factor while indicating CORR_KE: “RCCM” in POST_FATIGUE [U4.83.01].

2.3.3 Influence average constraint

If the part is not subjected to pure or symmetrical alternate constraints, i.e. if
average constraint of the cycle is not null, resistance to the dynamic stresses of material
(its limit of endurance) decreases.

One thus balances the curve of Wöhler to calculate the number of effective cycles to the rupture with the assistance
various diagrams.

The diagram of Haigh makes it possible to determine the evolution of the limit of endurance according to
average constraint m and of the alternate constraint Salt.

Salt
parabola To stack
right-hand side
Diagram of
of
HAIGH
Goodman
Known
m

Starting from a cycle (Salt
, m) identified in the signal one calculates the value of the alternate constraint
corrected '
Salt.
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'

If
Goodman

of

right-hand side

use

one
S = Salt
alt

1 - m
Known

'
S =
Salt

If
To stack

of

parabola

use

one
alt
2

1-
m

Known

It is noticed that this last does not differentiate the average constraint in traction and compression.

where Su is the limit with the rupture of material.

The influence of the average constraint is taken into account only on request of the user (key word CORR_HAIG).

Note:

If the curve of Wöhler is defined by the analytical form except singular zone [éq 2.3.1-3], of extended from variation of constraints being in lower part of the limit of endurance can to find higher than this one. To avoid that, one corrects the limit of Sl endurance while taking a limit of corrected endurance [bib5]:

'
S = Sl
for
Goodman

of

right-hand side

L

1 - m
Known

'

S =
Sl
L
for
To stack

of

parabola

2

1-
m

Known

2.4
Calculation of the damage: method of Manson-Whetstone sheath

The applicability of the method of Manson-Whetstone sheath [bib1] is oligocyclic plastic tiredness, who as his name indicates it shows two fundamental characteristics:

.
it is plastic, i.e. a significant plastic deformation occurs with each cycle,

.
it is oligocyclic, i.e. the materials have an endurance finished with this type of request.

To describe the behavior of materials in fatigue oligocyclic plastic, one uses tests with alternate imposed deformation.
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In the case, of a uniaxial homogeneous test with an alternated deformation, the number of cycles with rupture is given starting from a diagram of resistance, which connects the variation of deformation to a number of cycles involving the rupture.

In the diagram of resistance, one separates the deflections total, elastic and plastic. These diagrams are still known under the name of Whetstone sheath-Manson which proposed them in 1950.

Variation of deformation
10
(%)
T
p
E
0,1
ln (a number of cycles with rupture NR)
2
3
4
5
6
10
10
10
10
10
10

p

Relations
E ln
- (NR) and
ln
- (NR) are lines. The relation
T ln
- (NR) presents, as
2
2
2
with it, a curve towards the positive deformations.

It was shown that a relation power connected the plastic deformation (
p) and deformation
rubber band (
E) with the number of cycles to the rupture, which leads to the following relations:

- has
p=AN
B
-
e=BN

- has
B
-
t=AN +BN

where has and B are two characteristics of material (in general A is close to 0,5 and B close to 0,12); With and B, two constants of material.
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The user can introduce the curve of Manson-Whetstone sheath in a single mathematical form: form discretized point by point. It is a function which gives the number of cycles to the rupture NR in

function of the amplitude of deformation T.

2

As for the curve of Wöhler, the user can choose the mode of interpolation on the number of cycles with the rupture and on the amplitude of deformation.

The type of prolongation of the function on the right and on the left is also with the choice of the user.

The damage of an elementary cycle is equal contrary to the number of cycles to the rupture D 1 =/NR.

2.5
Calculation of the damage: method of Taheri

The methods of calculation of the damage proposed by Taheri [bib12] are two: one them will name respectively Taheri-Manson and Taheri-mixed. These methods apply to loadings characterized by a scalar component of deformation type.

These methods have as a characteristic to hold account about application of the elementary cycles of loading to the structure. For this reason, it is advisable to be vigilant for the choice of the method of
counting of the cycles. It is strongly advised to use the method of counting known as method "natural" [§2.2.3].

2.5.1 Method
Taheri-Manson

Are N cycles elementary of half-amplitude
1 ,
N

L
.
2
2

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-whetstone sheath of material.

The calculation of the elementary damage of the following cycles is carried out by the algorithm:

+

.
I 1
if
I

2
2

the value of the elementary damage of the cycle (I)
1
+ is determined by interpolation on
curve of Manson-Whetstone sheath of material.

+

.
I 1
if
I
<

2
2

one determines:

i+1

i+1
J
=F

, Max

TABLECLOTH

2
2
j<i 2

then

****
i+1
i+1
= FONC
F

.
2
2
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FNAPPE is the cyclic curve of cyclic work hardening with préécrouissage of material.

FONC
F
is the cyclic curve of work hardening of material.

***
I 1
+
The value of the damage of the cycle (I)
1
+ is determined by interpolation of
on the curve of
2
Manson-whetstone sheath of material.

Note:

If all the cycles applied are arranged by ascending value of the amplitude of deformation, this method is identical to the method of Manson-Whetstone sheath.

2.5.2 Method
Taheri-mixed

Are N cycles elementary, of half-amplitude
1 ,
N
L
.
2
2

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-whetstone sheath of material.

The calculation of the elementary damage of the following cycles is carried out by the algorithm:

+

.
I 1
if
I

2
2

the value of the elementary damage of the cycle (I)
1
+ is determined by interpolation on
curve of Manson-Whetstone sheath of material.

+

.
I 1
if
I
<

2
2

one determines:

i+1
i+1
J
=F

, Max

TABLECLOTH

2
2

j<

I 2

where FNAPPE is the cyclic curve of cyclic work hardening with préécrouissage of material.

I 1
+
The value of the damage of the cycle (I)
1
+ is obtained by interpolation of
on the curve
2
of Wöhler of material.

Note:

If all the cycles applied to the structure are arranged by ascending value of the amplitude of deformation, this method is identical to the method of Manson-Whetstone sheath.

The damage of an elementary cycle is equal contrary to the number of cycles to the rupture D 1 =/NR.
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2.6
Calculation of the total damage

The simplest approach and most known to determine the total damage of a part subjected to
I
N cycles of alternate constraint Salt or alternate deformation Ealt is the linear rule of
damage proposed by Mining:
I
N
Di=

Ni

Under operation, the structures are subjected to various loadings of different amplitudes.
tire undergone is due to the accumulation of the elementary damage and the total damage is
calculated with
assistance of the rule of office plurality To mine [bib6]:

N
D
= I
total

NR
I
I

In the case of Wöhler and Manson-Whetstone sheath, this law supposes that the damage increases
linearly
with the number of imposed cycles and which it is independent of the level of loading and the order
of application of the levels of loading (whereas in experiments, it is shown that the order
of application of the loading is an important factor for the lifespan of material).

The calculation of the total damage is required by the user with the key word OFFICE PLURALITY.

The methods suggested by Taheri hold account about application of the loading, in
calculation of the elementary damage associated each cycle.

2.7 Conclusion

For the methods based on uniaxial tests, the calculation of the total damage undergone by a part
subjected to a history of loading breaks up into several stages:

.
extraction of the peaks of the history of loading, to lead to a simpler history,
.
extraction of the elementary cycles of the history of loading by a method of counting

cycles,
·
calculation of the elementary damage associated each elementary cycle resulting from the real history loading,
- possibly (and for the method of Wöhler), correction of the loading by one coefficient of stress concentration KT,
- possibly (and for the method of Wöhler), correction of the loading by one elastoplastic coefficient of concentration Ke,
-
possibly (and for the method of Wöhler), correction of Haigh to take account of the nonnull value of the average constraint,
·
calculation of the total damage, by a linear rule of office plurality.
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3
Calculation of the damage of generalized Lemaître

This law of damage relates to the study of the starting of a macroscopic crack, using one post-processor of mechanics of damage based on a unified formulation of the laws of evolution of the damage. This one uses, on the one hand, of the laws of evolution of the damage specific to the various mechanisms considered, and, in addition, a model plus general based on a micromechanical analysis of the phenomenon of starting.

This law offers a single formalism which supposes that the various damage mechanisms all are controlled by the plastic deformations, elastic deformation energy and one process of instability.

3.1
The law of Lemaître generalized

The law of Lemaître generalized consists of an enrichment of the method of calculation of damage of Lemaitre [bib7] by the introduction of a law in power (model of Lemaître-Sermage). It is written [bib14]:

S

Y
D& =

p& if p>

p
S
D

éq 3.1-1

D =
0

if not

with:

2
2
eq
2

Y =
R and R =

(

1+)+ (
3 1-2) H.
2nd (1-D) 2
3

eq

Y is the rate of refund of density of elastic deformation energy.

H

R is related to triaxiality,
the rate of triaxiality.
eq

3D D
eq = ij ij is the equivalent constraint of von Mises.
2

D

1
ij
= ij - kk Sij is the diverter of the constraint.
3

p is the threshold of damage, S and S of the characteristics material.
D

p (T) cumulated plastic deformation.

This law thus makes it possible to calculate the damage D (T) starting from the data of the tensor of the constraints
(T) and of the cumulated plastic deformation p (T).

The integration of the equations [éq 3.1-1] led to:

1
D (
2s 1 2s
1
S
S

2s 1
ti+1)

+
+
=
-
1 (-
1 D (Ti)

-
(+
C) + (-
C)
+

(p (ti+1) - p (Ti))

if p > Pd
D (
2
ti+1)

=
0
if not

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---

with:
+
1
2
3
2
C =

+ T + T
+
-
+
T + T
3rd (
I
eq I
I
H I
T
+
I 1
+) S (Ti 1
+) (1
(1) (1) 2nd (Ti 1+) S (Ti 1+) (1 2 (1) (1) ***éq 3.1-2***
-
1
2
3
2

C =

+ T
T +
- T
T
3rd (Ti) S (Ti) (1 (I) eq (I)
2nd (Ti) S (Ti) (1 2 (I) H (I).

It is supposed that D (T) 0
= .
O

Note:

.
It is considered that the characteristics material E (Young modulus), (coefficient of fish) and S (parameter material) depend on the temperature T.
.
The value of the Young modulus and the value of the Poisson's ratio are defined in DEFI_MATERIAU [U4.43.01] under the key word factor ELAS_FO.
.
The values of S, Pd and of S are defined in DEFI_MATERIAU under the key word factor DOMMA_LEMAITRE and operands S, ESPS_SEUIL and EXP_S. Parameters S and Pd can depend on the temperature TEMP.
.
The law of Lemaître is obtained by assigning the value S = 1

Knowing the value of the damage D (Ti), i= N
,
0, one can calculate a value of total damage:

N
D= D (Ti).
I 1
=

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4
Criteria of Crossland and Dang Van Papadopoulos

The criteria [bib9] and [bib13] allow for metal structures subjected to constraints forced following a great number of cycles to distinguish the loadings damaging from others.

One can classify the criteria in two categories according to nature of their approach:

.
macroscopic approach: criterion of Crossland,
.
microscopic approach: criterion of Dang Van Papadopoulos.

The criteria of Crossland and Dang Van Papadopoulos apply to uniaxial loadings or multiaxial periodicals.

The goal of these criteria is not to determine a value of damage, but a value of criterion Rcrit such as:
R
0 step of damage
crit

R
> 0 damage
(tiredness).

possible
crit

4.1
Criterion of Crossland

The criterion of Crossland is empirical and is written only starting from variables macroscopic.

In fact, starting from trial runs, one could note that the amplitude of cission as well as the pressure hydrostatic played a fundamental part in the mechanisms of tiredness of the structures.

This is why, Crossland postulated the criterion:

R
= + aP
- B
crit
has
max

where
= 1
has
Max Max (Dt
amplitude of cission
1) -
(Dt0) =
2 0t T
T T
0
0 1

with
D
diverter of the tensor of the constraints.

1

P
= max
max
trace = maximum hydrostatic pressure.

0 tT 3

d0
d0

has =

0 -
and B =

0

3
3

with:

=
0 limit of endurance in alternated pure shearing,

D =
0
limit of endurance in alternate pure traction and compression.
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4.2
Criterion of Dang Van Papadopoulos

It appeared that the crack initiation of tiredness is a microscopic phenomenon occurring with a scale about the grain. This is why, of the criteria of tiredness, starting from variables microscopic local were postulated.

The implemented criterion [bib8], [bib9] and [bib10] in Code_Aster is the criterion of Dang Van Papadopoulos, which is written in the form:

*Rcrit = K * + has Pmax - B*
where:

R

D
D
k =*

if
R = max ((T) - C)
: ((T) - C)
**
2
0tT

k = R*
1

if
R = max J (T) = max
D
T - C
D
T - C
2
(( )
):(( )*
*)**
0tT
0tT
2

with:

.
R, the ray of the smallest sphere circumscribed with the way of loading in the space of diverters of the constraints;
.
J (T), the second invariant of the diverters of the constraints;
2
.
C = Min max (D*
T () - C): (D
T () - C), the center of the hypersphère.
C
T

Note:

It is the definition of R which uses J (T) which is programmed.
2

1

P

max
= maximum hydrostatic pressure =
Max
trace
0 T T3

D
D
0
0
has = -
and B =
0

3 3
0

with:

0 = limit of endurance in alternated pure shearing,

d0 = limit of endurance in alternate pure traction and compression.

The basic idea of Papadopoulos is to write that the grain obeys a criterion of plasticity of the type von Mises instead of the criterion of plasticity of the Tresca type used by Dang Van.

Papadopoulos conducted a campaign of comparisons between the results provided by its criterion and experimental results, which shows that the predictions of the criterion of Papadopoulos are excellent for the loadings closely connected; they are a little less precise for the ways not closely connected.

In its thesis [bib10] Papadopoulos shows that the criterion of Crossland and the criterion of Dang Van Papadopoulos give the same results for radial loadings.

The algorithm employed for the calculation of the ray of the smallest sphere circumscribed with the way of
loading in the space of the diverters of constraints, is that proposed in [bib11]. It is about one recurring algorithm which rests on the second invariant of the diverters of the constraints.

Let us note If the value of the diverter of the constraints at the moment Ti, One centers it hypersphère with

the iteration N, RN the ray of the hypersphère to iteration N and X the "isotropic parameter of work hardening" of
the algorithm.

.
Phase of initialization of the algorithm:

1 NR
1
O
=
S
NR
I
I 1
=

R
= 0.
1

.
Iteration of stage N at the stage
1
+
N:

one supposes One and RN known. One calculates then:

D =
Si-O
+1
N
P = D - RN

- If
P 0
>

RN 1 = R +x.
+
N
P
O -

N If
O

N 1
+ = If 1
+ + RN 1
+
O -
N If 1
+

- If 0
<
P

N
R =
+1
N
R
N
O =
+1
N
O

The algorithm ends when all the points If are in the hypersphère of centre One and of RN.

Ti
P
if
If
O
O
N
N + 1
Ti +1
RN
S i+1
- of VI
If +1
- of vi+1

RN +1
0

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4.3
Calculation of a value of damage

These two criteria applicable to multiaxial periodic loadings make it possible to say if there is damage or not:

R
0 step of damage
crit

R
> 0 damage
.
(tiredness)

possible
crit

*These criteria do not provide a value of damage. It can however be interesting to calculate a value of damage by using the curves of Wöhler of material. With this intention, it is necessary to define an equivalent constraint *, value to be interpolated on the curve of Wöhler.*

The curves of Wöhler can be built starting from shear tests in which case the limit

of endurance is 0, but are more generally built starting from tests of traction and compression for which the limit of endurance is d0 (d0 < 0).

So that there is coherence between the criterion and the curve of Wöhler it is necessary that:

*
no damage
0

for
Wöhler
of

curve

one

cisailleme

in

defined

NT,
* >
too bad

0

* D
no damage
0

for
Wöhler
of

curve

one

traction

in

defined

- compression.
** > D*
too bad

0

It thus seems possible to us to take:

** = R*

+
for
Wöhler
of

curve

one

cisailleme

in

rare),

enough

is

who

(it

NT
crit
0

** = (R*

crit + 0) (d0/0) for

Wöhler
of

curve

one

traction

in

- compression.

*In a general way, the user can take * = (Rcrit + 0) corr where corr is a coefficient of correction introduces by the user.*

By defect, this coefficient corr is taken equal to (d0/0) (case of the curve of Wöhler introduced in traction and compression).

***Note**:*

In the literature, one does not find presentation of a step of use of a criterion to calculate a value of damage. It is known however that certain industrialists use one such step, but without knowing the adopted form of it.

The step implemented in Code_Aster is proposed by department AMA.
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5 Conclusion

In this note the various methods of calculation from the damage available are exposed is in operator POST_FATIGUE either in operator CALC_FATIGUE, or in the two orders at the same time.

One can classify these methods in two great classes:

.
estimate of the damage to great numbers of cycles,
.
estimate of the damage in fatigue oligocyclic plastic.

In the first class of problems, one finds the method of Wöhler, based on tests uniaxial, and which applies to loadings in constraint. One also finds in this class, the criterion of Crossland, which is an empirical criterion being based on macroscopic sizes and the criterion of Dang Van Papadopoulos which is based on microscopic phenomena. The two criteria are addressed to loadings in constraints which can be uniaxial or multiaxial but periodic.

In the second class of problems, one finds the method of Manson-Whetstone sheath and the methods of Taheri, which applies to loading in deformations.

The whole of the methods based on uniaxial tests (method of Wöhler, method of Manson-whetstone sheath and methods of Taheri) are available in two operators POST_FATIGUE and CALC_FATIGUE.

The criteria, as for them, are only available in POST_FATIGUE.
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Appendix 1

The following history of loading is considered (which for the example is considered of type forced):

N° not
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Moment 0.

1.
2.
3.
4.
5.
6. 7.
8.
9.
10.
11. 12. 13. 14.
Loading 0. 40. 10. 60. 20. 50.
30. 80.
-70.
30.
-50. 20. -30. 25. 0.

(T)
8
60.
4
6
2
10
7
14
12
5
1
15
T
3
13
11
9

The stage of rearrangement of the history of loading leads to the following loading:

N°
not

8 9
10 11
12 13
14 15 2 3 4 5 6 7
Loading 80. 70. 30. 50. 20. 30.
25. 0. 40.
-10.
60. 20. 50. 30.

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(T)
8
4
6
2
10
14
7
12
5
1
15
T
3
13
11

9

The second stage consists in extracting the elementary cycles. The first extracted cycle is the cycle defined by
items 12 and 13 since 12
(
)
-
)
13
(
is lower than 14
(
)
-
)
13
(
and 12
(
)
-
)
13
(
is lower than
12
(
)
-
)
11
(
.

Cycle 1: VALMAX = 20. and VALMIN = 30.

The cycle having been extracted one removes these two points of the history of the loading, and one starts again on
remaining history.

(T)
8

4
6
2
10
14
7
5
1
=
15
T
3
11
9

The following cycle extract is the cycle defined by items 14 and 15.
Cycle 2: VALMAX = 25. and VALMIN = 0.
The remaining history, after suppression of these two points is:
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(T)
8
4
6
2
10

7
5
T
3
11
9

One extracts then the cycle defined by items 10 and 11.
Cycle 3: VALMAX = 30. and VALMIN = 50.
One sets out again on the following history of loading:

(T)
8
4
6
2
7
5
T
3
9

The following cycle extract is defined by items 2 and 3.
Cycle 4: VALMAX = 40. and VALMIN = 10.
The remaining history of loading is (it is the residue of the history of the loading):
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(T)
8
4
6
7
5
T
9

One cannot extract any more from cycles, because all the history of the loading was traversed.

One thus passes at the third stage, which consists in treating the residue:

(T)
8
8
4
4
6
6
7
7
5
5
T
9
9

One adds the same residue with his continuation, and one starts again the second stage on this loading.

The following cycle extract is defined by items 6 and 7.
Cycle 5: VALMAX = 50. and VALMIN = 30.
The remaining history of loading is:
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(T)
8
8
4
4
6
7
5
5
T
9
9

The following cycle extract is defined by items 4 and 5.
Cycle 6: VALMAX = 60. and VALMIN = 20.
The remaining history of loading is:

(T)
8

8
4
6
7
5
T
9
9

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The last extracted cycle is a cycle defined by items 8 and 9.
Cycle 7: VALMAX = 80. and VALMIN = 70.

(T)
8
4
6
7
5
T
9

It is noticed well that when one applies counting RAINFLOW to the unit made up of the two residues, one

obtains in end counting again the initial residue.
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Document: R7.04.02
Estimate of tiredness under random request
Summary:
This note presents two methods of counting of cycles of constraints which lead to an expression analytical of the mechanical damage generated by a random loading:
· method of counting of the peaks of constraints,
· method of counting of the goings beyond of a given level.
The first method calls upon the signal, its derivative first and its derivative second. The second requires only the knowledge of the signal and its derivative first.
The cycles of constraints being known by these methods, one determines the average damage over the duration of
signal, using the method of Wöhler.
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1 Introduction

The evaluation of the damage is based on the use of curves of endurance of material, associating one variation of constraints of amplitude given to a number of acceptable cycles defined by a curve of tiredness.

The curves of tiredness of material are established by subjecting test-tubes to requests sinusoidal of constant amplitude since the beginning of the test until the rupture.

To use these curves starting from a real loading, it is necessary to identify cycles in the history of the constraints, which is done by methods of counting of cycles. Many methods exist: the document [R7.04.01] presents two methods of counting of cycles in case of a deterministic real loading.

However, of many real mechanical loadings affecting the nuclear components

present a randomness which results in privileging the use of statistical methods for to evaluate the damage undergone by these structures.
Certain methods of counting of cycles of constraints were the interpretation object statistics:
· method of counting of the peaks of constraints
· method of counting of goings beyond of a given level.
The field of application of these two methods [bib1] [bib2] is limited to random loadings ergodic (the analysis of only one sample is enough to characterize the parameters of the process) and Gaussian (the values of the measured signal are distributed according to a normal law).
In addition, the evolution of the signal is comparable with a random process characterized by its statistical parameters (spectral moments of order 0, 2 and 4) [R7.10.01].
In both cases, the statistical event to take into account is simple to define:
· a peak of constraints is defined by a null slope and a negative acceleration for a peak positive, a positive acceleration for a negative peak,
· a going beyond of level of S0 constraints is characterized by a value of the signal equalizes with S0 and by a positive slope.
The cycles of constraints being known by these methods, one passes then to the calculation of the number of
cycles with the rupture starting from a curve of tiredness. The curves of Wöhler which are established in experiments were approached by various mathematical expressions characterizing more or less correctly various zones of these curves.
Three mathematical expressions are available in Code_Aster: a discretized form and two analytical forms.
Knowing the number of cycles of constraints (given by one of these two methods of counting cycles) and elementary damage associated (determined by interpolation on the curve with Wöhler with material), one can calculate an average damage over the duration of the signal.
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Evaluation of the damage

For a structure without defect geometrical and subjected to a pure alternate constraint, the number of cycles with the rupture is given starting from a diagram of endurance, still called curve of Wöhler or curve SN.

The number of cycles to the rupture is determined by interpolation of the curve of Wöhler of material for a level of alternate constraint (unidimensional) given (to each elementary cycle a level of amplitude of constraint corresponds =

-

max

min and an alternate constraint

Salt = 1 2).

The damage of an elementary cycle is equal contrary to the number of cycles to the rupture:

1

D = NR (Salt)

2.1 Diagram of endurance

The diagram of endurance, also called curve of Wöhler or curve SN (curve forced a number of cycles to the rupture) is obtained in experiments by subjecting test-tubes to periodic cycles of efforts (generally sinusoidal) of normal amplitude and frequencies constants, and by noting the number of cycles NR R to the end of which the rupture occurs [R7.04.01].

The various mathematical shapes of the curve of Wöhler are described in the document "Estimate of tiredness to great numbers of cycles", [R7.04.01] as well as the way of introducing them in Code_Aster.

2.2 Elastoplastic coefficient of concentration

It can also be necessary to balance the value of the constraint determined by the method of counting by the elastoplastic coefficient of concentration Ke.

The elastoplastic coefficient of concentration Ke (aimed to the B3234.3 articles and B3234.5 of the RCC_M

[bib4]) is defined as being the relationship between the amplitude of real deformation and the amplitude of

deformation determined by the elastic analysis.

The value of the Ke coefficient is given in the document [R7.04.01].

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3
A number of cycles of constraints
3.1
Recalls: spectral moments and factor of irregularity

One calls spectral moment of order I the following quantity [R7.10.01]:

$$I = \int_{-\infty}^{+\infty} \omega^I G_{SS}(\omega)\, d\omega$$

where is the pulsation and GSS the spectral concentration of power or DSP.
One has in particular:

$$m_0 = S^2$$

2 = S 4 = S which is the standard deviations of S and of its first derived.

The factor of irregularity *translates the frequential pace of the signal. Ranging between 0 and 1, it tends towards 1 when the process is with narrow band, on the other hand it tightens towards 0 for a broad band process. Its expression is:*

$$\gamma = \frac{m_2}{\sqrt{m_0 m_4}}$$

4
We point out these definitions because the evolution of the signal is comparable with a random process characterized by its statistical parameters (spectral moments of order 0, 2 and 4).
For the method of counting of the peaks of constraints, the random signal is entirely characterized by the three spectral moments of order 0, 2 and 4.
In the case of the method of counting of the goings beyond of level, the spectral moments of order 0 and 2 are enough to characterize the random signal.
In a practical way, these values are determined by the order POST_DYNA_ALEA [U4.76.02] which operate statistical processing on a random loading. The definition of the various parameters is given in the document [R7.10.01].
The operator of fatigue analysis in random field POST_FATI_ALEA [U4.67.05] uses them values three spectral moments calculated by POST_DYNA_ALEA and calculates the average damage and
the standard deviation of the damage by the methods described in this document.

3.2

Method of counting of the peaks of constraints

The principle of this method consists in counting the local maxima (in absolute value) located leaves and other of the average value of the constraints.
The stationary Gaussian signal, being centered compared to its average value, the distribution of the peaks
is symmetrical compared to this average. One is thus interested in the ***distribution of the positive peaks****.*
In the case general, the distribution of the peaks of amplitude S positive is written in the form:

T

2

2

2
-
S
S

-
E 2 dt
-

2
+
2

2
2
2
2
2
-
1 I
S

S
-
peak ()
() IS
P
S =
1 -
+

2

S (
I
E
E
1+ I)
S

2

'
I
S
=

SS "
with
S
I
=

S

1 - I 2
This distribution of the positive peaks is in the case of simplified the signals for which the factor of irregularity I = 0 or I are worth = 1. :
· Signal with broad band: law of Gauss or normal law (I = 0)
S2
-
2
2
P+ (S) =
E 2 S
peak
2
2
S
· Signal with narrow band: law of Rayleigh (I =)
1
S2
S
-
2
P+ (S) =
E 2 S
peak

2
S
The method of counting of peaks of constraints associates each peak of positive amplitude S a cycle of amplitude S = 2 S (there is thus directly S = Salt).
The number of peaks of positive amplitude is written: n+ (S) = P+ (S) × NR +
peak
peak
peak
1
1 "
where NR + =
1
(+ I
S
peak
) ×
4
= an average number of positive peaks per unit of time.

1+ I “
From where the number NR of cycles to be taken into account is: NR (S
S
) =
P+pic (S)
4
Note:
It is noticed well that the expression of the number NR of cycles to be taken into account does not depend that of S (for the calculation of the factor of irregularity I), and S ".
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3.3

Method of counting of the goings beyond of level

This method requires the variation division of constraint in classes of amplitude.
The number of cycles *NR* (*S*) is obtained starting from the difference of the numbers of goings beyond of level with a positive slope between two successive classes, on the basis of the class of amplitude maximum.

S2
'
-
1
2

For a Gaussian process centered the expression of *NR* (*S*) is: *NR S*
S
=
S.E. 2 *S*
( )
2
3
S

Note:

The expression of the number NR of cycles to be taken into account requires only knowledge of S and (independence with respect to S ").
In this method the coefficient of irregularity I does not intervene.

4

Statistical estimate of the damage

The mechanical damage is calculated by using the linear rule of office plurality To mine.
NR
The damage *D* generated by *NR* cycles of half amplitude *S* is expressed by *D = NR* (*S*)
where *NR* (*S*) is the acceptable number of cycles determined by the curve of endurance of material.
The mechanical damage is a random variable which one determines the average.

4.1 Too bad
means
The average damage is written in the form of the expectation:
Smax NR (*S*)
E (*D*) = *T*
dS

where *T* is the duration of the signal
NR R (*S*)
Smin
The two methods of counting suggested calculate the number of cycles *NR* (*S*) from
forced positive amplitude from where *Smin* = 0 (except when the curve of Wöhler is given under form “current zone”, in which case *Smin* = *Sl* with limiting *Sl* of endurance of material).
In addition, laws of distributions used being continuous, *Smax* =. However, the experiment show that the expression to be integrated attenuates quickly and one thus takes *Smax* = 10. *S* where *S* is the standard deviation of the signal.
The calculation of *E* (*D*) is carried out by numerical integration, by the method of the trapezoids while taking
S
- *S*
for step of integration max
min.
.
300
Note:
In the case of the method of counting of goings beyond of level and for a curve of Wöhler expressed in the mathematical form suggested by Basquin, the average damage by unit of time to an analytical expression (this expression is not used in order POST_FATI_ALEA).
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5 Conclusion

In the case of a Gaussian loading, ergodic and stationary, two methods of counting of cycles find an interpretation statistical and provide an analytical expression of the damage means, utilizing only the formulation of the curve of endurance of material and standard deviations signal and of its derivative first and seconds.

In *Code_Aster*, the calculation of the damage under random request is carried out by the order POST_FATI_ALEA [U4.67.05].

The user can determine the damage by the method of counting of the peaks of constraints (COUNTING: “PEAK”) or by the method of counting of goings beyond of a given level (COUNTING: “LEVEL”).

According to the adopted method, the random signal will have to be introduced by the data of the moments
spectral of order 0 and 2 or by the data of the spectral moments of order 0, 2 and 4 (key words MOMENT_SPEC_0, MOMENT_SPEC_2 and MOMENT_SPEC_4). Values of the spectral moments can also be recovered in a table created by POST_DYNA_ALEA [U4.76.02] (key word COUNT).

The curve of Wöhler of material can be introduced in three distinct forms (in accordance with order POST_FATIGUE [U4.67.01] (calculation of the damage to great numbers of cycles) and the software
POSTDAM).

The given size is the average damage over the duration of the signal which is stored in a table of type POST_FATI_ALEA.

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Summary:

Operator POST_RCCM allows to check the criteria of level 0 and certain criteria of level A of RCC-M-B3200, for modelings of continuous mediums 2D or 3D.

It also allows the calculation of the criteria of tiredness of the §B3600 in postprocessing of

calculations of pipings.

The criteria defined in the B3200 chapter of the RCC-M utilize significant sizes that one compare with limiting values.

The criteria of level 0 aim at securing the material against the damage of excessive deformation, of plastic instability and elastic and elastoplastic instability. These criteria require the calculation of equivalent constraints of Pm membrane, local membrane Pl and membrane plus Pm+Pb inflection. order POST_RCCM calculates Pm or Pl and Pm+Pb.

The criteria of level A aim at securing the material against the damage of progressive deformation and of
tire. Except tiredness, they require the calculation of the amplitude of variation of linearized, noted constraint Sn, and
possibly of the Sn quantity. For tiredness, they require in more calculation of the amplitude of variation of*
constraint in a point, noted Sp.

Order POST_RCCM [U4.67.04] carries out Sn calculations, Sn, Sp and of the number of acceptable cycles*
in fatigue. In postprocessing of analyses of pipings, option FATIGUE_B3600 allows calculation factor
of use in fatigue by taking into account all the calculated situations.
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matters

1
Criteria of the RCC-M B3200. Adaptation to Code_Aster

The B3200 chapter of the RCC-M [bib1] described the general rules of analysis of the behavior of materials of level 1 of the Nuclear thermal power stations. These rules make it possible to ensure the safeties necessary with respect to the damage to which these materials are subjected. For that one defines various levels of criteria in each category of situation which make it possible to compare significant sizes with limiting values. The adaptations necessary to Code_Aster are described here, and justified in [bib2].

1.1 Data geometrical

The user of the RCC-M must distinguish in his structure the zones of major discontinuity, the zones of minor discontinuity and zones comprising of the geometrical singularities. The RCC-M defines

“segments of support” which are used to linearize the constraints. These segments are, out of the zones of
discontinuity, of the generally normal segments on the median surface of the wall, and in zones of discontinuity, shortest segments allowing to join the 2 faces of the wall.

The user of ASTER must thus define the whole of the sections of the structure where calculations of post-
treatment will be made (it is him which knows if these sections pass by current zones, or zones of geometrical discontinuity). In practice, one works on a segment provided by INTE_MAIL_2D or INTE_MAIL_3D. One systematically calculates all the criteria at the two ends of the segment, or more precisely with the two intersections of the segment with the edges of the structure.

1.2
Data of loading

The user of the RCC-M must give the number of occurrences of each situation of operation (for example: heating of the boiler, hot stop, etc.). A situation of operation can to be broken up into transients, i.e. of the evolutions of the parameters of operation total (pressure, temperature) according to time.

In ASTER, one treats mechanical results (produced by MECA_STATIQUE or STAT_NON_LINE), therefore transients. For each transient, the stress fields are provided to the moments of discretization of calculation.

1.3 Data
material

The data material required are as follows:

· Sm: acceptable value (tabulée in the RCC-M Annexes Z1).
· m, N: constant material for the calculation of Ke (defined in the RCC-M B3234.6)
· EC., E: moduli of elasticity (for the correction of the curve of tiredness, annexes Z1).
· Courbes of tiredness of material: according to RCC-M'S annexes Z1.

1.4 Assumptions
simplifying

In the RCC-M, the user must be able to say, after analysis of the results of calculation, if them principal directions in a given point are fixed or if they turn in the course of time.

On the other hand, in order POST_RCCM, one can not make an assumption. One will not consider that the case where the principal directions are unspecified.
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Moreover, the user must be able to classify the constraints in the following categories:

· General Primaire of membrane: Pm
· Primaire of local membrane: Pl
· Primaire of inflection: Pb
· Thermal Expansion: EP
· Secondaire: Q
· Of point: F

This choice cannot be made by POST_RCCM. Only the user can qualify a stress field ("Primary education", "secondary", or summon it of both). The criteria which are to be checked are calculated to leave stress fields (constant or function of time) provided by the user. It is him which ensures coherence enters the calculation of these fields and the criteria applied.

However, to fix the ideas, classification is simpler in the following cases:

· a constant or variable loading with imposed force or pressure is primary, except for certain very particular structures,
· a constant or variable loading with imposed displacement is in theory, secondary (except in the case of "the effect spring"),
· a permanent or transitory thermal loading is in theory secondary.

On the other hand, the combination of these types of loadings leads to a result which cannot be any more qualified of primary education or secondary. According to the criteria, the user could thus be brought to

to break up its loadings.

1.5
Calculations carried out by POST_RCCM

One describes here the operation of order POST_RCCM allowing to carry out the calculation of certain criteria RCC-M B3200 of levels 0 and A. the realization described here does not take into account
touts the criteria of B3200 and could be supplemented (for example for other levels of criteria, or for criteria of the RCC-MR,…).

The principal data is the segment (of support) where calculations will be carried out. It is the user who
the segment chooses and which with the responsibility to find that for which quantities intervening in the criteria are maximum. The automatic search for this segment carrying out the maximum is one difficult problem, and is not programmed.

After having calculated one or more results by MECA_STATIQUE or STAT_NON_LINE, and having defined it
segment by INTE_MAIL_2D or _3D, the user requires the calculation of the criteria by the operator POST_RCCM.

Three types of criteria are accessible each one by a key word factor:

· of the criteria of level 0 by key word PM_PB,
· of the criteria of level A (except tiredness) by the key word SN,
· of the criteria of tiredness (also of level A) by the key word TIRES.
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2
Criteria of level 0 (key word PM_PB)

2.1
Criteria of level 0 specified by the RCC-M

The criteria of level 0 aim at securing the material against the damage of excessive deformation, of plastic instability and elastic and elastoplastic instability. They must be checked by the situation of reference (see B3121 and B3151). These criteria require the calculation of the constraints equivalent P, P, P
m
I
B which is below defined.

2.1.1 General primary equivalent constraint of Pm membrane

Being given the primary constraint of the situation of reference (1e category) and a segment located out
of a zone of major discontinuity. In each point end of this segment length L, one
calculate:

L
1
P = my (moy
moy
X ij)

=
ds
m
ij
ij,
T
Eq T
. resca
L 0

where (ij)
= max -

are the principal constraints
Eq Tresc

.
has
I
J
(I I=, 13
)
I, J

The criterion is written:

P S
m
m

2.1.2 Primary equivalent constraint of local membrane Pl

Being given the primary constraint of the situation of reference (1e category) and a segment located in a zone of major discontinuity, the definition of pi is identical to that of Pm on it segment.

The criterion is written:

P 15
. S
I
m

2.1.3 Primary equivalent constraint of membrane+flexion Pmb (or Plb)

Being given the primary constraint of the situation of reference (1e category) and a segment (directed).
In each point end of this segment length L, (ends corresponding to the skins external and intern), one calculates:

P
moy
fle
flax
m = max ij
B = max ij
mb = max ij
T (
)
P
Eq T

. resca
T (
)
P
( )
Eq T
. resca
T
Eq Tr
. ESCA
L
L
moy
1
fle
6
L

=

ds

=
S
2
-
ds
flax
moy
fle
ij

L
ij
ij
L
ij
ij
ij
ij

2
=
±
0
0

flax
moy
fle
ij (S =)
0 =ij
- ij
flax
moy
fle
ij (S = L) =ij +ij

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The criteria are written:

P
15
. S
mb

m
P 15
. S
Ib
m

2.2
Calculations carried out by Aster

It is with the user to know if one calculates Pm (forced general of membrane: out of the zones of geometrical singularity) or Pl (forced local of membrane: in the singularities). To leave stress fields provided (in the result), a membrane stress is thus calculated.

The concept result comprises either only one stress field, or of the fields resulting from one evolution. In this last case, one will seek the maximum compared to the list of the sequence numbers terms intervening in the criteria.

The algorithm is as follows:

Impression of the segment (cf POST_RELEVE)

· On the whole of the sequence numbers n=1, nbmax
- extraction of the moment T
- on each end of the segment
- calculation of Pm and Pmb by integrations on the segment

P
moy
m = max ij
T (
) EqTres
.
Ca
L

moy

1
=
ds
ij

,
L

ij
0
L
fle
fle
6
L
Pb = max ij

ij = 2 - ij,
T (
)
S
ds
Eq Tresca
.
L

2
0
P (S
moy
fle
moy
fle
mb
=)
0 = max ij
- ij
mb (=) = max ij
+ ij
T (
)
P
S L
Eq Tres
.
Ca
T (
) EqT.resca

- Research of the maximum of Pm, Pmb (s=0), Pmb (s=l).
- Exit and storage in the table of the result.

The values limit are Sm and 1.5 Sm, Sm being working stress function of material and of temperature, given by mot_clé the SM_KE_RCCM of the behavior TIRES in DEFI_MATERIAU.

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3 Criteria of level A (except tiredness) (key word SN)

3.1 Criteria of level A specified by the RCC-M

This option makes it possible to calculate the criteria of level A (except tiredness) which aim at securing the material
against the damage of progressive deformation. They require the calculation of the amplitude of variation
linearized more secondary primary constraints, noted Sn.

3.1.1 Sn calculation

One takes into account the more secondary primary constraints and the constraints resulting from opposed thermal dilations: Pl + Pb + EP + Q which thus represents the linearized constraints associated all the loading (mechanical and thermal).

The points of calculation are the two ends of the segment (given by the key word WAY). In each not end of this segment length L, one calculates:

S
flax
flax
N = max my (
X ij (T1) - ij (t2)

T1 T
Eq Tre
.
sca
2

L
L
moy
1
fle
6
L

=

ds

=
S

2
-
ds
flax
moy
fle
ij
L
ij
ij
L
ij

ij
ij
ij

2
=
±

0
0
flax
moy
fle

ij (S =)
0 =ij
- ij
flax
moy
fle
ij (S = L) =ij +ij

The criterion (of total adaptation) is written:

S 3S
N
m

Sm being the working stress function of material and the temperature, given by mot_clé SM_KE_RCCM of the behavior TIRES in DEFI_MATERIAU.

If this criterion is not checked, one can practise the simplified elastoplastic analysis of B3234.3. It is necessary
to carry out the three following operations:

· to check the criterion:

*S * 3S*
N
m

· to make an elastoplastic correction (Ke > 1) in the analysis with tiredness,
· to check the rule of Bree (B3234.8) in the current parts of the cylindrical hulls (and pipes) subjected to a pressure and a variation in cyclic temperature. This relates to one

very particular situation and will thus not be described here.

*3.1.2 Calculation of Sn**

One notes S *
N the amplitude Sn calculated without taking into account bending stresses of origin thermics. One calculates for each end:

S *
flax
fleth
flax
fleth
= max max
1 -
1
-
2 -

n*
ij
ij
ij
ij
2
T1 T (
(T)
(T) ((T)
(T))
2
Eq Tres
.
Ca
L

fleth
6
L

=
S
HT

2
-
ds
ij
L
ij

2
0
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thij coming from a calculation carried out with the thermal loading only (i.e. which one removes from complete calculation, having led to the Sn value, all the loadings other than the loading thermics).

3.2
Calculations carried out by Aster

3.2.1 Sn calculation

From the moments of calculation selected in the result, one thus calculates Sn at each end of segment. If it y does not have calculation at the moment T = 0, one creates a stress field identically no one with
T = 0.

The algorithm is as follows:

Impression of the characteristics of the segment (cf POST_RELEVE)

· on the whole of the sequence numbers, n1=1, nbmax
- Extraction of moment T1
- calculation of flax
flax
ij (T, S =)
1
0 and ij (T, S L)
1
=
- For varying N2 of n1+1 with nbmax

- Extraction of the moment t2
- calculation of flax
flax
ij (T, S =)
2
0 and ij (T, S L)
2
= and of
flax
flax
flax
flax
ij (T, S =)
0 - ij (T, S =)
2
1
0 and ij (T, S = L) - ij (T, S = L)
2
1

- calculation of the principal directions and the criterion of Tresca:
(flax
flax
(flax
flax
ij
T, S = L - ij T, S = L
2
1
)
ij (T, S =)
0 - ij (T, S
2
1
=)
0
and
(
)
(

Eq Tre
.

sca
Eq T
. resca
- research of the maximum thus of Sn

Exit and storage in the table of the result.

*3.2.2 Calculation of Sn**

This calculation is carried out if operand RESU_SIGM_THER is present. Only the user ensures coherence, i.e. this result must be produced by a thermomechanical calculation under thermal loading only, knowing that the result given by RESULT can be due to one combination of this thermal loading with other loadings. It is necessary thus that the moments of this result correspond to those of the result associated with the key word RESULT.

The algorithm is identical to the precedent but relates to two stress fields.
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4
Criteria of tiredness (of level A) (key word TIRES)

In the case of tiredness it should be made sure that the factor of total use - which integrates the effect of
combinations of situations 2 to 2 - is lower than 1. For this analysis one needs:

· on the one hand, of Sn previously defined, which requires the linearization of the constraints, this to calculate the elastoplastic concentration factor Ke,
· in addition, of Sp which is the amplitude of variation of total constraints (Pl + Pb + EP + Q + F), who does not require linearization, and who whose definition follows. This Sp is used to calculate alternate equivalent constraint Salt which, via the curves of tiredness, makes it possible to determine it factor of use.

We present two methods:

· the first makes it possible to calculate the factor of use for only one transient (according to the RCC-M
B3234.5). It is supposed here that the transient does not comprise secondary fluctuations (each quantity varies between an always identical minimum and a maximum),
· the second method makes it possible to combine several transients and to take account of factors of use specific to the secondary fluctuations (additional RCC-M ZH210).

It is also supposed that one is not located in zones comprising of the singularities geometrical (if not, it is necessary to apply the methods of calculation to the starting of the singular zones which
are the subject of appendix ZD of the RCC-M).

4.1
First method: maximum amplitude in a transient

It is a first approach of the calculation of the damage of tiredness of the RCC-M B3200, limited to treatment of only one transient (not of combination of transients) and without consideration of under-cycles. This method is the only available one in version 3 of Code_Aster (it is activated by key word “FATIGUE_SPMAX” in version 4). One calculates the amplitude of variation of constraint in one
, not noted Sp, and the amplitude of variation of linearized constraint Sn for the calculation of the factor of
elastoplastic correction Ke (according to the RCC-M B3200).

In each point end of the segment length L, one calculates:

S
flax
flax
p = max max ij 1 - ij
2
N = maxmax ij 1 - ij 2

T1 T (
(T)
(T))
S
((T)
(T)
Eq T
. resca
2

T1 T
Eq T
. resca
2

L
L
moy
1
fle
6
L

=

ds

=
S

2
-
ds
flax
moy
fle
ij
L
ij
ij

L
ij
ij
ij
ij

2
=
±
0
0

then

1 E
S
C
=
Ke (Sn) S
alt
and by the curve of tiredness of Wöhler: NR
= F (S
adm
alt).
2nd
p

An acceptable value of Ke can be given according to the RCC-M B3200 as follows:

Ke (Sn) = 1
if S < 3S
N
m
1 - N S

K
N
E (Sn) = 1 +

-
1
if
3S < S < 3m S

N (M-1) S
m
N
m
3 m

1
Ke (Sn) =
if
S > 3m S
N
N
m
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The values of m and N are given in B3234.6 of the RCC-M. These parameters and the curve of tiredness are introduced into order DEFI_MATERIAU [U4.23.01] under the key word factor TIRE.

.
N corresponds to N_KE_RCCM
.
m corresponds to M_KE_RCCM
.
Sm corresponds to SM_KE_RCCM

· The Young modulus of reference of the curve of tiredness EC. corresponds to E_REFE.
Young modulus corresponding to calculation carried out is defined classically under the key word factor ELAS.
· The curve of tiredness NR
= F (S
adm
alt) is a function defined by DEFI_FONCTION, and
introduced into DEFI_MATERIAU by key word WOHLER of the key word factor TIRES.

This algorithm is directly deduced from calculation from Sn and maximum Sp for only one transient and
does not take into account the under-cycles. It thus does not correspond to that of POST_FATIGUE.

If the user wishes at the same time the Sn and fatigue analysis for the transient, it can occur to use the key word Sn, because the calculation of Salt implies two calculations:

· that of Sp
· that of Sn carried out previously.

As for the criterion Sn < 3Sm, as from every moment of calculation, one calculates Sp with each end of the segment. If only one moment ago, one creates a fictitious transient between this moment and the state
identically no one.

The algorithm is almost identical to that used for the Sn calculation, without linearization. It is written:

· Impression of the segment (cf POST_RELEVE)
· on the whole of the sequence numbers, n1=1, nbmax

4.1.1 Calculation of Sp

· Extraction of moment T1
· For varying N2 of n1+1 with nbmax
- Extraction of the moment t2
- calculation of ij (T, S =)
0 - ij (T, S
2
1
=)
0 and ij (T, S = L) - ij (T, S = L
2
1
)

- Calculation of the principal directions and calculation of
-
(
(ij T, s=l - ij T, s=l
1
2
)
ij (T, S =)
0 - ij (T, S
1
2
=)
0)
and
(
)
(

Eq Tres
.
Ca
Eq Tre
.
sca
- research of the maximum to obtain:

S (S =)
0 = max max ij,
1
= 0 - ij,
2
= 0

T1 T (
(T S)
(T S
p
))EqTres
.
Ca
2

S (S = L) = max max ij,
1
= - ij,
2
=

T1 T (
(T S L)
(T S L
p
))EqTres
.
Ca
2

4.1.2 Sn calculation by the algorithm describes previously

S (S =)
0 = max my (flax
flax
X ij (T, S
1
=)
0 - ij (T, S
N
2
=)
0

T1 T
Eq Tres
.
Ca
2

S (S = L) = max my (flax
flax
X ij (T, S

1
= L) - ij (T, S
2
= L
N
)

T1 T
Eq Tr
. ESCA
2

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Note:

The moments which maximize Sp are not inevitably identical to those which maximize Sn.

· Sortie and storage in the table of the result.

After the calculation of Sp and Sn, one obtains the number of acceptable cycles by:

- The calculation of Ke for each end, as by the preceding formula.
- The calculation of Salt starting from Sp, EC., E, Ke:

1 E

S
C
=
Ke (Sn) S
alt

2nd
p

- One deduces then Nadm de Salt and from the curve of tiredness.

4.2 Second method: combination of several transients and under-cycles, method ZH210

The first method does not take into account the possible under-cycles, and does not combine them transients between them. One describes here another possibility, available only in version 4 of Code_Aster (key word "FATIGUE_ZH210").

The algorithm is similar to that of POST_FATIGUE. More precisely, the algorithm used in POST_FATIGUE is a restriction on the uniaxial case of method ZH210. Indeed, the data of order POST-FATIGUE is a scalar function of time (whereas POST_RCCM treats tensors of constraints functions of time).

This method resulting from appendix ZH210 of the RCC-M was preferentially selected with others methods described in the RCC-M [bib1].

The principal advantage of this method is to consider all the under-cycles automatically possible. Its disadvantage is the number of calculations to be carried out if one does not restrict the whole of moments used in calculation.

Indeed, one defines for each transient a whole "of states of loading", which are the moments significant where the constraints pass by a local extremum. By defect, in Code_Aster, all them moments of calculation are used. One associates each one of them the number of occurrences of the transient.
definition is thus:

State of loading = {urgent, tensor of constraints, numbers occurrences}.

Then, one builds the whole of all the states of loading by sweeping all the transients. With boils of the account, the concept of transient is forgotten: one does not work any more but on one whole of states of loading. One then calculates all the elementary factors of use associated all them combinations taken two to two. One uses then a method of office plurality of the factors of use elementary, based on the assumption of the linear office plurality of the damage, to obtain the factor

of use
total.

4.2.1 Calculation of the elementary factors of use

At each end of the segment, for any couple of states of loading K and L, one calculates the quantities Sp (K, L) and Sn (K, L) by:

S (K, L) =

-

((K) - (L))
S
flax
flax
N (K, L)

= (ij (K) ij (L
p
ij
ij
))

Eq Tres
.
Ca
Eq T
. resca

From Sn (K, L), one calculates Ke (K, L) like previously, then:

1 E
S (K, L)
C
=
Ke (K, L) S p (K, L
alt
) and by the curve of tiredness of Wöhler: NR (K, L) = F (S

adm
alt).
2nd
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The method to determine Ke is identical to the preceding one.

From the number of acceptable cycles Nadm (K, L), one calculates the factor of use of the combination:

U (K, L) = Nkl/Nadm (K, L), with Nkl = min (Nocc (K), Nocc (L)).

This calculation is carried out for each combination of two states of loading. One thus obtains (always for each end of the segment) a symmetrical matrix U (K, L), of order the number of states of loading.

4.2.2 Algorithm of office plurality

For each end:

Data: Numbers total states of loading NR
stamp U (K, L), K, l=1, NR
vector of entireties Nocc (I), i=1, NR

U (M1) =0 (factor of total use)

· research of the maximum of U (M1, K, L) on all the combinations (K, L) such as Nocc (K) >0 and Nocc (L) >0. That is to say U (M1, m, N).
· U (M1) = U (M1) + U (M1, m, N).
· If Nocc (m) < Nocc (N) then
Nocc (N) =Nocc (N) - Nocc (m)
Nocc (m) =0
and conversely
If Nocc (N) < Nocc (m) then
Nocc (m) =Nocc (m) - Nocc (N)
Nocc (N) =0
· If there are still states of loading I such as Nocc (I) >0, return in 1.

Two remarks can be made:

If the number of moments defined for each transient is large, calculation can be prohibitory. It is thus necessary to be able to restrict it. It is what is made in POST_FATIGUE, by a sorting preliminary of the moments. One eliminates the moments such as the scalar function is linear for *to keep only the ends of the segments of right-hand side. One eliminates also the very small ones variations. Here, in multiaxial situation, the sorting is more delicate. Concept of constraints proportional could be used, but it is necessary to envisage in more one possibility for the user to define itself the list of the moments (key word NUME_ORDRE)*

By this method, one is sure not to forget no under-cycle. On the other hand, it is desirable to eliminate the moments which do not correspond to local extrema, because they could to generate factitious under-cycles, augmentatnt the factor of use (these moments are only used for the numerical discretization of the mechanical or thermal problem).
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5
Criteria of tiredness for the simplified analysis of pipings according to the RCC-M B3600

Vocabulary used: compared to the preceding options, which treat complete transients (mechanical modeling of the structure subjected to stories of temperatures and loading mechanics), it is of use in B3600 of defines each situation as the passage of a stabilized state With (correspondent with a pressure interns given in the line of piping, a uniform temperature data, and of the fixed mechanical loadings) in a state stabilized B (with constant loadings different from the precedents). One associates this situation a thermal transient.

The treatment which is described here is carried out for each node of each mesh of the line of piping considered. The result obtained will be thus a factor of use (total or partial) for each node of each mesh requested by the user.

5.1
Calculations of all the states of loading

For each node of each mesh, the present stage consists in calculating, for all the situations, the moments relating to each stabilized state (by cumulating the various loadings which intervene).

5.1.1 Calculations of the static states of loading

One treats the results of static calculations (field EFGE_ELNO_DEPL or SIEF_ELNO_ELGA) for stabilized states of the list of the situations undergone by the line. Torques for each stabilized state are obtained by algebraic summation of the torques corresponding to the various loading cases of situation (signed).

M
M
M

I =
I TANK +

I TANK
+ I

...

–

1
_
2
{
X;
}
Z

y;

(the loadings are for example opposed thermal dilation, the displacement of anchoring).

5.1.2 Calculation of the seismic loadings

The seismic loading breaks up into 2 parts:

· An inertial part

It is calculated by imposing on the whole of anchorings the same movement characterized by spectrum envelope of the various spectra of floor, in the horizontal directions X and Y on the one hand, and vertical Z on the other hand (in the total reference mark). With this intention, the order is used
COMB_SISM_MODAL, which produces generalized efforts which correspond to each direction of seism as well as the quadratic office plurality of these efforts:
The inertial contribution of the seism to component I of the moment is written:
M
=
2 I, J; ; ; ; ;

_ _
I _S _ DYN
J
J (M
(spectrum) () {X y} Z {X Y Z
I S DYN
}}
with: Mi_S_DYN (spectrej) moment in direction I resulting from the dynamic loading in direction J. This office plurality is already made by COMB_SISM_MODAL.
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· A quasi-static part

It is estimated by imposing static differential displacements corresponding to the maximum one differences of the seismic movements of the points of anchoring in the course of time. Calculations are thus realized for each unit loading (a calculation by displacement in one direction given for an end of the line).
The loadings must then be combined by quadratic average by POST_RCCM_B3600 (this calculation is not carried out as a preliminary).

The quasi-static contribution of differential displacements of anchoring to component I of moment is written:

$$M_{NR\_ANC} = \left(M_{K} \right)^{2}_{I\_S\_ANC}$$

with: $M_{I\_S\_ANC}$ $K=1$
the component iéme of the moment corresponding to the kéme displacement of anchoring.

Combination of the inertial components and differentials due to the seism:

The iéme component resultant is obtained by quadratic average of the inertial component iéme and differentials:

M
=
2 +
2 I; ; what amounts carrying out it average quadratic

_
(M _ _) (M _ _)
{X y} Z
I S
I S ANC
I S DYN
from every inertial and differential moment,

M
2
2
=

+
I
; ;
I _
(Mk
S
I _ S _ ANC)
(Mi_S_DYN)
{X y Z}
k=,
1 NR _ ANC
The result of this office plurality is to be stored in the table above.

Each one as of the these loadings (inertial answer, displacement of anchoring) is defined by one occurrence of key word RESU_MECA.

5.1.3 Calculation of the thermal transients

In the § B3653 of the RCC-M which describes the method of analysis to tiredness for a line of piping, the loadings of the type “heat gradient in the thickness” are taken into account by the intermediary of four variables:

T1: amplitude of the variation enters the two stabilized states of the difference in temperature enters walls internal and external, for an equivalent linear distribution of the temperature.

T2: nonlinear part of the distribution in the thickness of wall of the amplitude of variation of the temperature enters the two stabilized states.

Your and Tb: amplitude of variation between two stabilized states of the average temperatures in respective zones has and B located on both sides discontinuity of material or structure.

Methodology: For each one of the transients and each section of piping of the line (and each junction), one realizes as a preliminary, according to the geometrical complexity of the problem studied a calculation
thermics 2D or 3D.

Each calculation is stripped using two calls to POST_RELEVE_T (OPERATION =EXTRACTION and OPERATION=MOYENNE) in order to extract, for each moment I, variation in the temperature on
the selected section and average values (moments of order 0 and 1).
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From these values, one calculates the quantities T1, T2, Your and Tb in the following way:

For each moment of the transient, one calculates (by the same routines as in POST_RELEVE_T):
T

T y Dy: average value of T () on the ligament
moy ()
T
= 1 2 (,).
- T
T

2
Possibly (material discontinuity or junction):
B
T

T y Dy: average value of T () on the ligament corresponding to the node B located
moy ()
T
= 1 2 B (,).
- T
T
2
on the other side of the junction
V

y T y Dy: variation of a linear distribution equivalent to T ().
moy () = 12
T
. (,).
2 2
- T
T
2
then:
T1 () = Vmoy ()
T
-
-

ext. ()
Tmoy () 1

T
1( )
2
T
=
T - T
-

moy
T
2 ( )

max
int ()
() 1

1( )
2
0

In the case of a discontinuity of material or a junction, one calculates:

T
= T
has ()
moy ()
T
= T

B ()
B
moy ()
T
- T
has has ()
B B ()

by using the possibly different dilation coefficients for the two convergent meshs with the treated node.
In practice, the zones has and B will correspond to segments chosen by the user in POST_RELEVE, and the produced tables will be associated the two adjacent meshs having jointly the node which corresponds to the junction.

5.2 Calculations of the amplitudes of constraints

5.2.1 Calculation of the combinations of loading (I, J) inside each group of situations

The first phase consists in calculating the amplitudes of constraints which correspond to combinations of all the stabilized states pertaining to the situations of a given group, in choosing the moments of the thermal transients which maximize these amplitudes of constraints. In effect, the thermal transients defined in the File of Analysis of the Behavior are associated situations. During the analysis of the behavior to tiredness, we are brought to define cycles of fictitious loadings by associating stabilized states pertaining to different situations.
In this case, the thermal transient associated the fictitious cycle corresponding to the stabilized states

I and J
will be selected in order to maximize the amplitude of constraints.

The whole of the combinations (I, J) is thus considered with (I, J) ({1, 2. , NR}, {1, 2. , NR}) (NR being it
a number of states stabilized except seism, i.e. 2 times the number of situations of the group), and one built a matrix [NR; NR] of the values alt (I, J).
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For each combination, alt (I, J) was obtained in the following way:
Are two stabilized states, I and J belonging respectively to the situations p and Q:

One calculates
Sp (I, J): amplitude of variation of the total constraints (eq. (11) of the §B3653 of the RCC-M) by:

P I, J. D
0
0
D
1
S I, J, T = K C
. .
+ K C
. 0 M
.
I, J +

. K. E. T

T, T
p (
p)
( )
1
1
2
2
I (
)
3
1 (p
p
K
L)
2 T.
2.I
2 (.1-)

1
+ K C
. E. T
. T, T - T
. T, T
+
. E. T
T, T
3
3
ab
has
(p has
p
K
L)
B
B (p
p
K
L)
2 (p

p
K
L)
1-
(p p
T, T indicates two unspecified moments of the transient associated with the situation p.
K
L)

One calculates also the same type of term, with the thermal transient associated the situation Q:
P I, J. D
0
0
D
1
S I, J, T = K C
. .
+ K C
. 0 M
.
I, J +
. K. E. T

T, T
p (
Q)
( )
1
1
2
2
I (
)
3
1 (Q
Q
K
L)
2 T.
2.I
2 (.1-)

1
+ K C

. E. T
. T, T - T
. T, T +
. E. T
T, T
3
3
ab
has
(Q has
Q
K
L)
B
B (Q
Q
K
L)
2 (Q
Q
K
L)
1-
then:
S I, J
max max S I, J, T, max S I, J, T
p (
)

=

Q
Q (
p (
p)
Q
Q (
p (
Q)
(T T,
,
K
L)

(T tkl)

with:

· C1, C2, C3, K1, K2, K3 indices of constraints provided to the §B3680 of the RCC-M
· E: modulus of elasticity of piping at ambient temperature
·: Poisson's ratio
·
: dilation coefficient of piping at ambient temperature (with T_REF)
· Eab: average modulus of elasticity enters the two zones separated by a discontinuity to ambient temperature (TEMP_REF).
· D0: diameter external of piping
· T: nominal thickness of the wall

I =
2
2 2
·
·
64 (D0 - (D0 - T))
· I: moment of inertia of piping

· Mi (I, J): variation of moment resulting from the various loadings of the situations to which belong the stabilized states I and J:
M =
-
+
-
+
-
I
(MR. X (I) MR. X (J) 2 (MY (I) MY (J) 2 (MZ (I) MZ (J) 2
·

· P0 (I, J): difference in pressure between states I and J.

The terms utilizing the temperature are:

1
1
1
T
T, T =

2 T T, T (y Dy
).
=
2 T T (y Dy
2
).
T T (y Dy
).
T
=
T - T
T
moy (Q
Q
-
-

K
L)
T
Q
Q
T
(K L)
T
Q
T
(K)
T
Q
T
-
(L)
(Q
moy
K)
(Q
moy
L)
T
2
T
2

T
2
12
12
12
T

,
.
,
( ).
.
( ).
.
( ).
1 (Q
Q
T T =
2 y T T T
y Dy =
2 y T T
y Dy -
2 y T T
y Dy V
=
T - V
T
2 -
2 -
2
K
L)
T
Q
Q
T
(K L)
T
Q
T
(K)
T
Q

T
-
(L)
(Q
moy
K)
(Q
moy
L)
T
2
T
2
T
2

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Q
Q
Q
Q
Q
Q

T
T T - T
T T -
T
T T
max (,
K
L)
moy (K
L)
1
,
1(,
K
L)
2
and
Q
Q
Q
Q
Q
Q
Q
Q
T
T T =
T
T T - T
T T -
T
T T
2 (,
K
L)
max min (, K L) moy (K L) 1
,
1(,
K
L)
2
0

One also calculates:

Sn (I, J): amplitude of variation of the linearized constraints (eq. (10) of the §B3653 of the RCC-M)

P I J D
D
S I J T = C
+ C
M I J +
E T
T T + C E T T T - T T T
N (, p)
0 (,
).
.
0
. 0 .
p
p
p
p
p
p
1
2
I (
)
1
,
. . .
1(
,
K
L)
.
. .
3
ab
has
has (
,
K
L)

.
B
B (
,
K
L)
.
2 T
.
2 I
(.
2 1-)
P I J D
D
S I J T = C
+ C
M I J +
E T
T T + C E T T T - T T T
N (, Q)
0 (,
).
.
0
. 0 .
Q
Q
Q
Q
Q
Q
1
2
I (
)
1
,
. . .
1(,
K
L)
.
. .

3
ab
has
has (,
K
L)
.
B
B (,
K
L)
.
2 T
.
2 I
(.
2 1-)
S I J =
S I J T
S I J T
N (,
) max (max N
Q
Q
(, p), max N
Q
Q
(, Q)
(T, tkl)
(T, tkl)
)

One calculates then S p, Q = max S I, J, for I and J sweeping the whole of the stabilized states of both
N (
)
N (
)
I, J
situations p and Q (in general, 6 possible combinations).

One obtains finally the amplitude of constraint between states I and J, by:

1

,
= .
.
,
.
,
alt (I
)
E
J
C Ke (Sn (p Q) S p (I J)
2nd

with:
EC.: Young modulus of reference for the construction of the curve of Wöhler, provided by the user in DEFI_MATERIAU, under key word E_REFE, of the key word factor TIRES.
E: Smaller of the Young moduli used for the calculation of states I and J, i.e. evaluated with temperatures of these stabilized states.

Ke the elastoplastic concentration factor defined in the §B3234.6 of the RCC-M.

1

if
S

N (p, Q)
3.Sm
-

K
=
+
-
<
<

E (S N (
N
S p Q
p, Q)
1
N (,

)
1

N (
S p Q
m
. m -).
1 3.S

if

,
3. S

m
N (
)
m
1
3.S

m

1

if
S

N (p, Q)
3. S
m m
N

The values of m and N depend on material, and are provided by the user in DEFI_MATERIAU, under key words M_KE and N_KE, key word factor TIRES. The value of S is smallest of
m
values corresponding to the stabilized states I and J. If key words TEMP_REF_A and TEMP_REF_B are
present, S is interpolated for this temperature (which must correspond to the average temperature
N
transient). If not, S is taken at ambient temperature.

N
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Note:

In the case of mixed loadings mechanics and thermics, the RCC-M (from modifying of June 1994) the decomposition of the concentration factor authorizes élasto-plastic in a mechanical component (Ke_meca) and a thermal component (Ke_ther). This method of calculation is generally (but not in all the cases) a little less penalizing that method above. We chose here not to use this possibility for two reasons. On the one hand the decomposition of Ke is profitable only if the share of thermal loading of origin is important (and it complicates the analysis with tiredness). In addition the expression of Ke_ther proposed in the RCC-M is valid only for steels austenitic. In the case of ferritic steels, coefficients of the expression of Ke_ther must be the subject of a validation on a case-by-case basis, which seems not very compatible with objectives of our schedule of conditions.

One builds thus, for each group of situation, a symmetrical square matrix containing the whole of the alt (I, J) thus obtained. In this unit, one identifies the combination (K, L) corresponding to the greatest value of alt. One associates this matrix a vector containing it a number of occurrences of each stabilized state

5.2.1.1 Case of the under-cycles

The under-cycles correspond either to the taking into account of the under-cycles related to the seism, or with situations for which key word COMBINABLE=' NON' was indicated. In both case, one calculates the amplitude of constraints while utilizing only constraints related to these under-cycles (not of combination of states of loading apart from this situation). For the calculation of alt, it is necessary to use the Ke factor which corresponds to the principal situation from which the under-cycle results.

5.2.2 Calculation of the combinations of loading (I, J) for the situations of passage between group of situations

Two states of loading are combinable only if they belong to the same situation or if it exist a situation of passage between the groups to which they belong. In this last case, one will associate combination I, J the number of occurrences of the situation of passage. If situation of passage belongs to the one of the two groups (what is not excluded a priori), it is naturally combined with the other situations of this group, then is used for the combination of the situations
of its group with the situations of the group in relation.

For each situation of passage of a group with another, one thus considers the whole of combinations (I, J) with I pertaining to the first group (of dimension NR) and J pertaining to second group (of dimension M). For each combination, alt (I, J) was obtained in the same way that previously and one associates to him the number of occurrences of the situation of passage. One builds
still a matrix (rectangular) containing all the alt (I, J),

5.3
Calculation of the factor of use

One notes:

nk the number of cycles associated with the situation p to which belongs the state stabilized K;
nl the number of cycles associated with the situation Q to which belongs the state stabilized L;
NS the number of occurrences of the seism (only SNA is considered in second category)
NS numbers under-cycles associated with each occurrence with the seism.
npass a number of cycles associated with a possible situation with passage between p and Q if these situations do not belong to the same group, but if there exists a situation of passage enters both.
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For the whole of the combinations of states of loading (inside a group of situations or associated a situation of passage):

If NS 0, one selects Ns/2 combinations of stabilized states K and L more penalizing, i.e. Ns/2 combinations (K, L) driving with the greatest values of alt (K, L).

For each one of these Ns/2 combinations:

· one superimposes the loadings of seism to the variation of moment resulting from different loadings of the stabilized states K and L:

2
2
2
M = (M

1 (K) - M1 (L) +
M 1) + (m2 (K) - m2 (L) + m2) + (m3 (K) - m3 (L) + M
I
S
S
S 3)
with:

MX (K) and MX (L): components in the direction X (X {1 2
; ; }
3) of the moments associated with
states K and L
MSx: Total amplitude of variation in direction X of the moments due to the seism.

· One calculates then Sp and Sn with the new value the Semi one and one calculates:

1

, = .

.
,
.
,
alt _ S (K
)
E
L
C Ke (Sn_ S (m N) Sp_ S (K L)
2nd

· one calculates the number of acceptable cycles NR (K, L) for the amplitude of constraint alt_S (K, L).
NR (K, L) corresponds to the X-coordinate of the point of ordinate alt_S (K, L) in the curve of Wöhler associated material.
1
· one calculates finally U
=

1 (K, L)
NR (K, L)
· one takes into account the under-cycles due to the seism while calculating:

E
D

K,
C
L =
. K S
K, L. K. C.
.
M

+ M

+ M

alt _ SC (
)
E (N _ S (
)
0

2
2
2
2
2
S1
S 2
S 3
E
4.I
2 N
. -1
· one calculates with this value: U
=
with:
NR
2 (K, L)
(S)
NR
,
SC (K, L) a number of cycles
SC (K L)
acceptable for the amplitude of constraint alt_SC (K, L). It should be noted that one uses value K
,
previously calculated for the principal cycle.
E (S N _ S (K L)

· One then cumulates these factors of use partial in the factor of total use: U = U + u1 (K, L) + u2 (K, L)

One starts again this calculation until exhaustion of Ns/2 combinations more penalizing.

The calculation of the factor of use is then continued without taking into account the seism:

If NS = 0, or after having taken into account the seism for Ns/2 combinations more unfavourable:

· One selects the combination (K, L) leading to the maximum value of alt (K, L), on the whole of the combinations, such that the number of occurrences n0 is nonnull.
With n0 = min {nk, nl, npass,} if npass is nonnull, or n0 = min {nk, nl,} if npass is null.
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· one calculates the number of acceptable cycles NR (K, L) for the amplitude of constraint alt (K, L). NR (K, L) corresponds to the X-coordinate of the point of ordinate alt_S (K, L) in the curve of Wöhler associated material.

1

, = .
.
,
.
,
alt (K
)
E
L
C Ke (Sn (p Q) S p (K L)
2nd
N
· one calculates U (K, L)
0
=

NR (K, L)
· one replaces then
nk by (nk - n0)
nl by (nl - n0)
if it is about a situation of passage, npass by (npass - n0)

then:

if nk = 0, the column and the line corresponding at the state stabilized K of the matrix alt (I, J) are settings with 0.
if nl = 0, the column and the line corresponding at the state stabilized L of the matrix alt (I, J) are settings with 0.

The loop is repeated until exhaustion of the number of cycles. The factor of use U of the line G
for the group considered is then defined by:

U
. It is cumulated with the factor of total use: U = U +U
G = U (K L)
early
early
G

Note:

Appendix ZI of code RCC-M defines the curves of Wöhler until an amplitude of constraint minimum corresponding to one lifespan of 106 cycles. If the value alt calculated for a combination (I, J) of stabilized state is lower than this amplitude minimum, the factor of use is equal to 0 for the combination (I, J) considered. This returns implicitly to to consider the existence of a limit of endurance to 106 cycles.

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The treatment which is described here (cf [bib2]) is to be carried out for the segment considered. The result obtained
will be thus a factor of use (total or partial) in each end of this segment.

6.1
Calculations of all the states of loading

6.1.1 Linear combination of the tensors of constraints

The present stage consists in reconstituting, for all the situations, (including the situations of passage) tensors of constraints in each node of the segment relating to each state stabilized (in cumulating the various tensors of the constraints which intervene).

For each calculation of unit loading, one extracts the tensor from the constraints along the segment of analysis. The tensors all of the constraints must be expressed in the same reference mark (the reference mark
total related to modeling 2D or 3D).

This reference mark must be coherent with that in which the total efforts resulting from calculation are expressed
beam.

One notes _ () with
U

{XX, YY, ZZ, XY, XZ,
}
YZ components of the tensor of
constraints associated with the unit loading. The calculation of the tensor of constraints corresponding to
mechanical loading pertaining in a stabilized state is then obtained in the following way:

that is to say F
,
,
,
,

,
the torque of effort associated with the loading (I).
X (I) Fy (I) Fz (I) M X (I) M y (I) M Z (I)

one has then, by linear combination:

=

+

+

(I)
F I.
F
F I.
F
F I.
F
+
X ()
_U (X _U)
y ()
_U (Y _U)
Z ()
_U (Z _U)

M I.
M
+ M I.
M
+ M I.
M
X ()
_U (
X _ U)
y ()
_U (
Y _ U)
Z ()
_U (
Z _ U)

One then linearly cumulates the tensors of constraints for all the loadings of the state stabilized considered.

One stores, on this level, the tensors of constraints for each node of the segment.

6.1.2 Calculation of the thermal transients

As the thermal transients lead to statements of constraints (on a section
corresponding to the node studied) variable according to time, it is necessary to store all these values in order to maximize the amplitudes of constraints correctly.
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6.1.3 Case of the seismic loadings

The seismic loading breaks up into 2 parts:

has) An inertial part

It is calculated by imposing on the whole of anchorings the same movement characterized by spectrum envelope of the various spectra of floor, in the horizontal directions X and Y on the one hand, and vertical Z on the other hand (in the total reference mark). With the exit of this calculation beam, one obtains generalized efforts which with the quadratic office plurality of these efforts for each direction of seism, therefore not signed efforts. One stores these constraints in table 1 above, for

to seek thereafter the combination of sign which maximizes the amplitude of constraints.

b) A quasi-static part

It is estimated by imposing static differential displacements corresponding to maximum of the differences of the seismic movements of the points of anchoring in the course of time. Of even, the efforts are combined by quadratic average, therefore not signed. The result is with to store in table 1 above.

6.1.4 Calculations of the amplitudes of constraints inside each group of situations

The first phase consists in calculating the amplitudes of constraints which correspond to combinations of all the stabilized states pertaining to the situations of a given group, in choosing the moments of the thermal transients which maximize these amplitudes of constraints.

Indeed, the thermal transients defined in the File of Analysis of the Behavior are associated situations. During the analysis of the behavior to tiredness, we are brought to to define cycles of fictitious loadings by associating stabilized states pertaining to situations different. In this case, the thermal transient associated the fictitious cycle corresponding to the states stabilized I and J will be selected in order to maximize the amplitude of constraints.

The whole of the combinations (I, J) is thus considered with (I, J) ({1, 2. , NR}, {1, 2. , NR}) (NR being it a number of states stabilized except seism, i.e. 2 times the number of situations of the group), and one built a matrix [NR; NR] of the values alt (I, J).

6.1.4.1 Calculation of the alt (I, J) without taking into account of the seism

The calculation of alt (I, J) was carried out, for each couple of stabilized states (I, J), and each end of segment, starting from the tensor of the constraints S , and of the tensor of the linearized constraints p (I J)
S
, :
N (p Q)

1

,
= .

.
,
.
,
alt (I
)
E
J
C Ke (Sn (p Q) S p (I J)
2nd

with:

EC.: Young modulus of reference for the construction of the curve of Wöhler, provided by the user in DEFI_MATERIAU, under key word E_REFE, of the key word factor TIRES.
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Ke the elastoplastic concentration factor defined in the §B3234.6 of the RCC-M.

1

if
S

N (p, Q)
3.Sm
-

K
=
+
-
<
<

E (S N (
N
S p Q
p, Q)
1
N (,
)
1

N (
S p Q
m
. m -).
1 3.S

if

,
3. S

m
N (
)
m
1
3.S

m

1

if
S

N (p, Q)
3. S
m m
N

The values of m and N depend on material, and are provided by the user in DEFI_MATERIAU, under key words M_KE_RCCM and N_KE_RCCM, key word factor TIRES.

Note:

In the case of mixed loadings mechanics and thermics, the RCC-M (from modifying of June 1994) the decomposition of the concentration factor authorizes élasto-plastic in a mechanical component (Ke_meca) and a thermal component (Ke_ther). This method of calculation is generally (but not in all the cases) a little less penalizing that method above. Mechanical decomposition - thermics leads in effect with more important values of Ke for certain Sn values.

We chose here not to use this possibility for two reasons. On the one hand decomposition of Ke is profitable only if the share of thermal loading of origin is important (and it complicates the analysis with tiredness). In addition the expression of Ke_ther proposed in the RCC-M is valid only for the austenitic steels. In the case of them ferritic steels, the coefficients of the expression of Ke_ther must be the subject of a validation on a case-by-case basis, which seems not very compatible with the objectives of our schedule of conditions.

One builds thus, for each group of situation, a symmetrical square matrix containing the whole of the alt (I, J) thus obtained. One associates this matrix a vector containing the number occurrences of each stabilized state. In fact, the process this calculation of the factor of use requires to calculate two matrices alt (I, J): one without taking into account the seism, the other with taking into account seism. One is interested here in the matrix without seism.

For each combination, alt (I, J) was obtained in two stages: Sn (p, Q) should initially be calculated, then Sp (I, J).

It is necessary to seek the maximum of alt (I, J) for each combination of states of loading I, J. But it is necessary to obtain these values to take account of the variation of the constraints due to the transients thermics, which are variable according to time. The step presented here thus consists with

to seek the moment of the thermal stresses which make maximum alt (I, J).

The following paragraphs thus correspond to calculations to carry out for any couple of state of loading I, J pertaining to the situations p and Q of the same group of situations, and for any couple moments T p and tq transitory respectively associated with the situations p and Q.

Sn calculation (p, Q) without seism

One calculates the equivalent amplitude of the linearized constraints Sn for any couple of stabilized states
belonging to the situations p and Q of couple I and J current (in practice there are two states stabilized by
situation, there are thus 4 combinations to calculate).
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All along the ligament of analysis, one calculates the tensor of amplitude of the constraints (I, J)

ij (I, J) = ij (I) - ij (J)

with () the tensor of the constraints associated with the state with loading.

Moreover, it is necessary to tarnish account of the thermal transients associated with the situations p and Q. One superimposes
thus with (I, J) the tensors of constraints corresponding to the thermal transients associated situations p and Q:

I, J, T = I, J + T and I, J, T = I, J + T
ij (
Q)
ij (
) ij (Q)
ij (
p)
ij (
) ij (p)

T and T represent two moments of the transients respectively associated with the situations p and Q.
p
Q
One then linearizes (I, J, T and (I, J, T, component by component, along the ligament:
Q)
p)
T
T
(
2
moy

fle
6
y Dy and (
y
y Dy
ij)
= .
.
.
2
(ij
mn
) ( )
ij
)
2
= 1. (ij
mn
) ( ).
mn
T

mn
-
T
T
- T
2
2
(flax

=
+

ij)
(moy
ij
mn
) (fleij
mn
) mn

with: T thickness of the wall

y radial position of the point considered (is null there with mid thickness and positive on the surface interns)

For each combination (I, J), one obtains two values
: S I, J, T, S I, J, T which are
N (
Q)
N (
p)
respectively equal to the equivalent constraint (within the meaning of Tresca) of the two tensors flax

I, J, T,
flax

I, J, T
ij
(Q)
ij
(p)
One retains finally largest of these two values, for each 4 combinations I, J and
then the greatest value among the 4 combinations:

S p, Q
max max S I, J, T, max S I, J, T

N (
)

=

N (
p)
N (
Q)

I, J T
T
p
Q

One stores, for the couple of situation of the group considered, the values of S
,
. Indeed,
N (p Q)
maximization over the moments could not be made on this level, but only on the level of value of alt (I, J). In fact one chooses here to return maximum Sn (p, Q) so that the factor of correction
elastoplastic Ke is maximum (what is in conformity with the RCC-M, to see ZH210 for example).

Calculation of Sp (I, J) without seism

The calculation of Sp (equivalent amplitude of the total constraints) is to be realized for each of both ends of the segment, in the following way:
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For the couple (I, J), at each end of the segment, one calculates the tensor of amplitude of the constraints
(I, J)

ij (I, J) = ij (I) - ij (J)

with () the tensor of the constraints associated with the state with loading.

Moreover, it is necessary to tarnish account of the thermal transients associated with the situations p and Q. One superimposes
thus with (I, J) the tensors of constraints corresponding to the thermal transients associated situations p and Q:

I, J, T = I, J + T and I, J, T = I, J + T
ij (
Q)
ij (
) ij (Q)
ij (
p)
ij (
) ij (p)

S (I J
I, J
p,) is equal to the constraint equivalent (within the meaning of Tresca) to the tensor (
):
That is to say 1, 2 and 3 components of (I, J) in the principal reference mark,

S (I J
1 2 2 3 3
p,) = my {
X
-
,

-

,
- 1 }

For each combination (I, J), one obtains two values
: S I, J, T, S I, J, T which are
p (
Q)
p (
p)
respectively equal to the equivalent constraint (within the meaning of Tresca) of the two tensors
I, J, T, I, J, T
ij (
Q)
ij (
p)

Final expression of alt (I, J)

The value retained for alt (I, J) is that which makes maximum the expression:

1 E

I, J, T =.
. K S p, Q S
.
I, J, T
alt (
)
C
p
E (N (
) p (
p)
2nd

That is to say: S I, J
max max S I, J, T, max S I, J, T
alt (
)=

(alt (p)
(alt (Q)

T
T
p
Q

One then stores this value in a symmetrical square matrix containing the whole of the alt (I, J) without seism. One stores also the symmetrical matrix square containing the values of K
,
, which
E (Sn (p Q)
could be used for the under-cycles.

Case of the under-cycles

The under-cycles correspond either to the taking into account of the under-cycles related to the seism, or with
situations for which key word COMBINABLE=' NON' was indicated. In both case, one calculates the amplitude of constraints while utilizing only constraints related to these under-cycles (not of combination of states of loading apart from this situation). For the calculation of alt, it is necessary to use the Ke factor which corresponds to the principal situation from which the under-cycle results.

6.1.4.2 Calculation of the alt (I, J) with taking into account of the seism

The second phase consists in calculating the amplitudes of constraints which correspond to combinations of all the stabilized states pertaining to the situations of a given group, in choosing on the one hand, moments of the thermal transients which maximize these amplitudes of constraints, and in addition, combination of signs for the not signed loadings which provide also the maximum.
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Sn calculation (p, Q) with taking into account of the seism

The seismic loadings are not signed. Each component of the tensor of the constraints can thus to take two values (positive and negative). At the time of the superposition of a loading not signed with a signed loading, the RCC-M forces to retain on each component a sign such as calculated constraint (makes alt of it) is raised. The tensor of the constraints cash six components, it thus exist 64 combinations of signs to be examined.
After having reconstituted the stress fields corresponding to the stabilized states, on the one hand and with
seism, in addition, one proceeds in the following way:
one considers the combination of two stabilized states I and J, and one calculates (I, J, T and (I, J, T, Q)
p)
as above. One defines, for each node of the segment, and each of the two transients thermics, 64 Sk tensors of seismic loadings corresponding to the 64 combinations of sign possible, for the 6 components of the tensor of the constraints corresponding to the loading seismic, and one numbers them from 1 to 64.
One calculates then the equivalent amplitude of constraint linearized on the segment: S I, J, T, S
N (
p
K)
corresponding to each of the 64 combinations:
S I, J, T, S = forced equivalent of Tresca (
flax

I, J, T, S).
ij
(p K)
N (
p
K)
S

S p, Q
max max max S I, J, T, S, max max S I, J, T, S

N
=

I, J
T
[N p K
K
] T [N Q K
K
]
One retains finally
( )
(
)
(
)
p
Q

One stores, for the whole of the couples of situations of the group considered, the values of S S
,
N (m N)

6.1.4.3 Calculation of Sp (I, J) with taking into account of the seism

Just as previously, the seismic loadings not being signed, it is necessary to retain on each component a sign such as the calculated constraint is raised. The tensor of the constraints cash six components, there are thus 64 combinations of signs to be examined. For each combination of sign and each moment, two values are obtained: S I, J, T, S, S I, J, T, S which are
p (
Q
K)
p (
p
K)
respectively equal to the equivalent constraint (within the meaning of Tresca) of the two tensors I, J, T, I, J, T. These values are stored.
ij (
Q)
ij (
p)

Final expression of alt (I, J)

The value retained for alt (I, J) is that which makes maximum the expression:

1 E

I, J, T, S =.
K
.
S p, Q S
.
I, J, T, S
alt (
p
)
C
K
E (S
N (
) p (p K)
2nd

That is to say: S I, J, S
max max max S I, J, T, S
, max max S I, J, T, S
alt (
)=

T
([alt (p K)]
K
) T ([alt (Q K)]
K
)
p
Q

One then stores this value in a symmetrical square matrix containing the whole of the alt (I, J) with seism.

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6.1.5 Calculation of the amplitudes of constraints for the situations of passage enters group situations

For each situation of passage of a group with another, one considers the whole of the combinations (I, J) with I pertaining to the first group (of dimension NR) and J pertaining to the second group (of dimension M). For each combination, alt (I, J) was obtained in the same way that previously. One still builds a matrix (rectangular) containing all the alt (I, J), which one associates it a number of occurrences of the situation of passage.

Indeed, two states of loading are combinable only if they belong to the same situation or if there is a situation of passage between the groups to which they belong. In it last case, one will associate combination I, J the number of occurrences of the situation of passage. If the situation of passage belongs to the one of both groups (what is not excluded has priori), it is naturally combined with the other situations of this group, then is used for the combination situations of its group with the situations of the group in relation.

6.1.6 Storage of the amplitudes of constraints for all the combinations

To carry out the calculation of the factor of use, amplitudes of the constraints calculated previously and the associated numbers of occurrences are stored in a square matrix containing all them amplitudes of constraints alt except seism, for all the possible combinations of situations (with interior of each group of situations, and between two groups if there is a situation of passage). The matrix has as a dimension the sum of the number of situations of all the groups:

Alt

Group 1
Group 2

Group 3

Etat1
...
State J
...
...
...
State L
...
...
..
State N
State NR
Group 1
State 1
Alt
... ... 0 0 0
0 ... ...
...
State
I

Alt (I, J)
...
0 0 0
0…… alt (I, N)
...
...

... 0 0 0
0 ... ...
...
...

... 0 0 0
0 ... ...
...
Group 2
....

...
...
...
...
0 0 0
0
State
K

...
Alt (K, L)
...
0 0 0
0
...

...
...
0 0 0
0
....

SYM

...
0 0 0
0
Group 3
....

...
...
...
...
State
m

… Alt (m, N)
...
...

...
...
State
NR

...

In the table above, one associates value 0 the combinations of states between groups 1 and 2 and groups 1 and 3, because there is not situation of passage between these groups. On the other hand it in
exist between groups 1 and 3, one thus associates herring barrel combination states of groups 1 and 3 the value of alt.
The number of the states of loading will have to be built starting from the number of situation (single) and of
relative number (1 or 2) of the stabilized state of the situation, to obtain a univocal classification of

states of loading.

In the case of the under-cycles, only the diagonal term is filled.

This matrix is also to build for the values of S
, ,
who take into account
alt (I J S)
seism. There are thus two matrices giving all the possible values of alt.
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In correspondence of this matrix, one can associate a table giving the number of occurrences, it number of situation and the number of group of each stabilized state, and a table giving the number occurrences of the situations of passage:

State
Situation groups
Under-cycle
nocc

group 1 group 2
Npass
State
1
1 1 0 N1 1 1 0
… 1 1 0 N1 1 2 0
State I
2
1
0
Ni

1
3
npass
… 2 1 0 Ni.……
State
K

3 2 0 Nk 2 1 0
... ... 2 2 0
State
m
... 3
nsous
0 2 3 0
... ... 3 1
npass
State NR
NR
3
0
NR

...
...
...

6.2
Calculation of the factor of use

For the whole of the combinations of states of loading (inside a group of situations or associated a situation of passage), and for each end of the segment:

The general step described by the §ZH210 of code RCC-M is as follows:

One considers the whole of the NR stabilized states, and one builds:

· the matrix [NR, NR] of the equivalent amplitudes of constraint alt (I, J, T, S) corresponding to superposition of the transient of passage of the state stabilized I in a state stabilized J, of the transient thermics associated and with the seismic loading.
· the matrix [NR, NR] of the equivalent amplitudes of constraint alt (I, J, T) corresponding to superposition of the transient of passage of the state stabilized I in a state stabilized J and of the transient
associated thermics (without seism).
One notes:
nk
the number of cycles associated with the situation p to which belongs the state stabilized K;
nl
the number of cycles associated with the situation Q to which belongs the state stabilized L;
NS
the number of occurrences of the seism (in general only of SNA is considered in second

category)
NS
a number of under-cycles associated with each seism.
npass a number of cycles associated with a possible situation with passage between p and Q if these situations do not belong to the same group, but if there is a situation of
passage enters both.

If NS 0, one selects Ns/2 combinations more penalizing, i.e. Ns/2 combinations (K, L) driving with the greatest values of alt (K, L), without taking into account it seism.

For each one of these Ns/2 combinations:

1
· One calculates the factor of use u1 (K, L) associated alt (I, J, T, S): U
=
, with NR (K, L) it
1 (K, L)
NR (K, L)
a number of acceptable cycles associated alt (I, J, T, S).
· One takes into account the NS under-cycles of the seism by:
, K and L being both
alt _ SC (K L)
extreme states of the under-cycle. Then:
2. -1
U
=
with NR
2 (K, L)
(NS)
NR
,
SC (K, L) a number of acceptable cycles for the amplitude of
SC (K L)
constraint alt_SC (K, L).
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· One obtains then: U (K, L) = u1 (K, L) + u2 (K, L)

· One starts again this calculation until exhaustion of Ns/2 combinations more penalizing.

The calculation of the factor of use is then continued without taking into account the seism:

If NS = 0, or after having taken into account the seism for Ns/2 combinations more unfavourable:

· One selects the combination (K, L) leading to the maximum value of alt (K, L), on the whole of the combinations, such that the number of occurrences n0 is nonnull.
With n0 = min {nk, nl, npass,} if npass is nonnull, or n0 = min {nk, nl,} if npass is null.
· one calculates the number of acceptable cycles NR (K, L) for the amplitude of constraint alt (K, L).
NR (K, L) corresponds to the X-coordinate of the point of ordinate alt (K, L) in the curve of Wöhler associated material.
· One calculates the factor of use U (K, L) associated the amplitude of constraint alt (K, L):
U (K,)
N
L
0
=

NR (K, L)
· One increments the factor of total use: U = U + U (K, L)
· One replaces:
nk by (nk - n0)
nl by (nl - n0)
if it is about a situation of passage, npass by (npass - n0)
if nk = 0, the column and the line K of the matrix of the alt (I, J) were put at 0.
if nl = 0, the column and the line L of the matrix of the alt (I, J) were put at 0.

· The loop is repeated until complete exhaustion of the number of cycles.

Note:

Appendix ZI of code RCC-M defines the curve of Wöhler until an amplitude of constraint minimum corresponding to one lifespan of 106 cycles. If the value alt calculated for a combination (I, J) of stabilized state is lower than this amplitude minimum, the factor of use is equal to 0 for the combination (I, J) considered.

The loop is repeated until exhaustion of the number of cycles.

The factor of use U of each end of the segment for the group considered is then defined by:
G

U
. It is cumulated with the factor of total use: U = U +U
G = U (K L)
early
early
G
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Multiaxial criteria of starting in fatigue to large numbers of cycle: models of DANG VAN and MATAKE

Summary:

In this note we propose a formulation of the criteria of MATAKE and DANG VAN within the framework of office plurality of damage under periodic and nonperiodic multiaxial loading.

The first part of this document is devoted to the criteria of MATAKE and DANG VAN adapted to periodic multiaxial loadings. In this part after having approached the concepts of endurance and office plurality of damage and the general form of the criteria of tiredness, we describe the two models of DANG VAN and MATAKE (Plane criticizes) designed to carry out calculations of office plurality of damage under multiaxial loading. One details there the definition of the various plans of shearing associated with the points of Gauss or the nodes, thus that the definition of an amplitude of loading through the circle circumscribed with the way of the loading in plan of shearing. Finally the criteria available in Code_Aster are presented.

In the second part we propose a formulation of the criteria of MATAKE and DANG VAN in tally of the office plurality of damage under nonperiodic multiaxial loading. To define a cycle in the case variable amplitude, we reduce the history of the loading to a unidimensional function of time in projecting the point of the vector shearing on an axis, and we use a method of counting of cycles. Here we choose method RAINFLOW. Criteria of MATAKE and DANG VAN adapted to the office plurality of damage under nonperiodic loading are established in Code_Aster.

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matters

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1 Introduction

The models of endurance in fatigue multiaxial under periodic loading are models of following type:
VAR _ amplitude + has VAR _ average < B,

where B is the threshold of endurance in simple shearing, and has a positive constant without dimension.
VAR _ amplitude is a certain definition of the amplitude (half of the variation) of the cycle of loading and VAR _ average are a variable in connection with the constraint (or sometimes deformation) or constraints (or sometimes deformations) average. The models are distinguished by definitions different of VAR _ average amplitude and VAR _.
To pass from the endurance to the office plurality of the damage, a definite equivalent constraint is introduced
by:
= VAR _ amplitude + has VAR _ average.
eq

This equivalent constraint gives us a unit damage on the curve of tiredness. Like second member of the inequation B corresponds to the threshold in shearing, one needs a curve of tiredness in
shearing. But the curves of tiredness in shearing are rare since difficult to obtain, one

thus try to use the curves of tiredness in traction alternate compression. For that it is necessary to multiply the equivalent constraint by a corrective coefficient of about 3.

The macroscopic models of MATAKE (plane criticizes) and macro microphone of DANG VAN are described.
It is shown that under certain assumptions the model of DANG VAN is similar to the model macroscopic of MATAKE. The only difference lies in the average variable VAR _: DANG VAN uses the hydrostatic pressure, while MATAKE employs the normal constraint on the plan of maximum amplitude of shearing.

After having defined the plan of shearing, we express the shear stress in this plan.
The plans of shearing are then explored according to a method described in the reference [bib4] which
consist in cutting out the surface of a sphere of pieces of equal sizes.

The normal vectors being known we then determine for each plan the points which are them more distant from/to each other. Among those we find the two points which are most distant one of the other. That being made we use, if necessary, the method of the circle passing by three points in order to obtain the circle circumscribed with the way of loading.

In the first part of this document we present the models of endurance in fatigue multiaxial under loading periodicals As well as the concept of office plurality of damage. The passage of
the endurance with the office plurality of damage is also approached.

In the second part the criteria of MATAKE and DANG VAN are then presented under aspects limits endurance and office plurality of damage under periodic loading.

The third part is devoted to the definition of the plan of shearing, of the expression of the constraints of shearing in this plan and finally, with the manner of exploring the plans of shearing.

The fourth part is dedicated to the determination of the circle circumscribed with the way of shearing in
plan of the same name. Finally we describe the criteria and the sizes which are introduced into Code_Aster.

After having extended the models of MATAKE and DANG VAN to the office plurality of damage under loading
periodical, we present the adaptation of these models at the office plurality of damage under loading not periodical. Thus, the fifth part is devoted to the definition of the equivalent constraint elementary.

The sixth part deals with the manner of selecting the axis (or the two axes) on which is projected the history of the cission.

The seventh part is dedicated to projection itself of the point of the vector cission on this center or these two axes. Lastly, concerning the criteria of MATAKE and DANG VAN formulated in office plurality of damage under nonperiodic loading, we describe the sizes which are introduced into Code_Aster.

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2 Preliminaries

In this part we treat the concepts of limit of endurance and office plurality of damage. Us also let us present the general form of the criteria of tiredness.

2.1
Limit of endurance and office plurality of damage, uniaxial case

In the uniaxial case the rigorous definition of a threshold of endurance it is the half-amplitude (half variation) of loading defined in constraint in lower part of which the lifespan is infinite. However, as in practice the lifespan cannot never be infinite, one defines limits of endurance with 107, 108, etc cycles of loading. There is another way of seeing the things: since in practice the infinite lifespan does not exist, one uses the concept of office plurality of damage. The approach by the office plurality of damage consists in defining a limit in a number of cycles beyond which the cumulated damage is equal to one. Thus the limit with 107 wants to say that after 107cycles it cumulated damage is equal to 1.

2.2

Criterion of tiredness, multiaxial case

In the literature a certain number of criteria were proposed to define the threshold of endurance under multiaxial cyclic loading. The general form of these criteria is:

VAR _ amplitude + has VAR _ average < B éq
2.2-1
where B is the threshold of endurance in simple shearing, A is a positive constant without dimension. VAR _ amplitude is a certain definition of the half-amplitude (half of the variation) of the cycle and VAR _ average is a variable in connection with the constraint (or sometimes deformation) or them
constraints (or sometimes deformations) average. Various models are characterized by definitions different of VAR _ average amplitude and VAR _.
To pass from the endurance to the office plurality of the damage, one can define a constraint (or one deformation) equivalent:
= VAR _ amplitude + has VAR _ average
éq
2.2-2
eq

This equivalent constraint gives us a unit damage on the curve of tiredness. Like second member of the inequation [éq 2.2-1] corresponds to the threshold in shearing, one needs a curve of
tire in shearing. But the curves of tiredness in shearing are rare since difficult with to obtain, one thus tries to use the curves of tiredness in traction alternate compression. For that it is necessary to be coherent at least for the level of the threshold of endurance i.e. to multiply eq by one
constant of about 3 to be able to use the curve of tiredness in traction. Value 3 is exact value for a criterion of the type Put, in experiments this coefficient is smaller than 3.

2.3
Definition of an amplitude of loading in the multiaxial case

In Code_Aster, there are two definitions of amplitude of loading in the multiaxial case:

A: ray (half diameter) of the sphere circumscribed with the way of the loading;
B: half of the maximum of the distance between two unspecified points of the way.

It is clear that in the case of a loading being defined on a sphere A and B give the same one amplitude. On the other hand, if one takes a way (two-dimensional) in the form of a triangle equilateral of
dimensioned L, definition A gives us L
3, while the definition B gives us L/2. To work within a conservative framework we take as definition of the amplitude (half-variation) of a way of

loading the ray of the sphere (or rings for the case 2D) circumscribed.
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2.4
Definition of the plan of shearing

In a point M of a continuous medium we express the tensor of the constraints in a reference mark orthonormé (O,
X y, Z). With unit normal N of components (N, N, N)
X
y
Z in the reference mark
orthonormé, we associate the vector forced F = .n of components (F, F, F)
X
y
Z. It
vector F can break up into a normal vector with N and a scalar carried by N, is:

F = NR N +

éq 2.4-1

where NR represents the normal constraint and the vector the shear stress. In the reference mark (O,
X y, Z), the components of the vector are noted: (,)
X
y
Z. The vector results
directly of [éq 2.4-1] and the normal constraint:

NR = F.n; from where = F -.
F N.
N
éq
2.4-2

y
F

N
X
O

Z

Appear 2.4-a: Representation of the vectors of constraint F and shear stress
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3
Model of MATAKE (plane criticizes) and model of DANG VAN

Here we clarify the criterion of MATAKE and DANG VAN at the same time from the limiting point of view
of endurance and the point of view of the office plurality of damage.

3.1
Criterion of MATAKE

In this type of criterion the calculation of the deformation and stress fields is made under the assumption
of elasticity, cf reference [bib1]. Like it was known as in chapter 2, in the multiaxial case the criterion of endurance is generally written in the form:

VAR _ amplitude + has VAR _ average < B éq
3.1-1

Amplitude of loading: In the case of the criterion of MATAKE at each point of the structure (or not Gauss for a calculation with the finite elements) to calculate VAR _ amplitude one proceeds of following way:

· for each plan of normal N one calculates the amplitude of shearing by determining the circle circumscribed with the way of shearing in this plan;
*· one seeks the normal N * for which the amplitude is maximum. This amplitude is*
indicated by
*(*)*
N.

Average constraint: For the calculation of VAR _ average one proceeds in the following way:
*· within normal N * one calculates on a cycle the indicated maximum normal constraint*
by NR
*(*)*
max N
.

The criterion of endurance is written: (*)
N + has NR (*)
max N
B,
2
where has and B are two positive constants and B represents the limit of endurance in simple shearing.
Identification of the constants: to determine the constant ones has and B it is necessary to use two

tests
simple. Two possibilities exist:
A pure shear test plus an alternate tensile test compression. In this case constants

D D
are given by: B =
0
0
0 A = 0 -

, where the limit of endurance represents in

2
2
0
alternate pure shearing and d0 limit of endurance in alternate pure traction and compression.
Two tensile tests compression, alternated and the other not. The constants are given by:

(
2 -
1)
has = (
1 -
2)
,
- 2 m

m

1
B = (
×

2 -
1) + 2
2
m

where

1 is the amplitude of loading for the alternate case and
2 for the case where the constraint
average is nonnull.
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3.2
Criterion of DANG VAN

It is supposed that the material remains overall elastic while it is plasticized locally.
The interesting physical assumption of the model is that the material adapts locally (it becomes rubber band after having passed by plasticity) below the limit of endurance, which corresponds to nonthe initiation of crack. Above the limit of endurance there is locally accommodation plastic thus initiation of crack.
The basic assumptions of the microphone-macro interaction, Flax-Taylor, make it possible to write:

Loc

$$(T) = (T) + (T)$$
$$ij \quad ij \quad ij$$

$$(T) = 2\,p - \mu\,(T)$$
$$ij \quad ij$$

One indicates the local constraint by
Loc

$$(T)$$
$$ij$$

, the total constraint by (T)
$$ij$$

, the residual stress
local by (T
p
ij) and by
(T)
ij
local plastic deformation. As soon as there is adaptation the deformation
figure local becomes constant and thus the local residual stress also.

Criterion of plasticity:
In a point of the continuous medium (where there is a distribution of the crystallographic directions random of
grains), it is supposed that there is only one grain which is plasticized and this, following only one system of
slip. This system of slip will be that which will be most favorably directed, i.e., it
grain in which the greatest scission (the projection of the vector shearing will occur on one direction given). The slip is done in the plans of normal ***N*** *= (N, N, N)*
1
2
3 and direction of
slip is defined by the vector ***m*** *= (m, m, m)*
1
2
3. The two vectors ***N*** *and* ***m*** *are*
orthogonal.
The law of ***Schmid*** *says that so that there is no irreversible slip (plastic deformation) it is necessary that the scission, does not exceed a certain threshold, that is to say:*

m
Loc
N
*(****N****,* ***m****, T) - Loc*

(T)
y
0
éq
3.2-1

where
Loc
loc
1

(T) = has
and has
=
(m N
+ N m)
ij
ij
ij
2
I
J
I
J
The drawing of [Figure 3.2-a] shows that the maximum value of Loc

, indicated by Loc
max, is obtained
by the orthogonal projection of
Loc
Loc
F
= ij N J within normal ***N****. The relation [éq 3.2-1] must*
in particular to be checked if one replaces Loc

by its raising Loc
max, this one is written
then:

Loc
N

*(****N****, T) - Loc*

(T) 0
max
y

éq
3.2-2

where
Loc

(T)

Loc
y
is the threshold of the microscopic or local scission.
(T) depends on the variables

y
of work hardening.
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N
FLoc
Loc
max
Loc m

Appear 3.2-a: Projection of *Loc*
F
within normal N

One chooses a microscopic work hardening of the linear isotropic type. That makes it possible to show the existence
of a field of adaptation [bib2], [bib3].
In the state adapted by analogy with the formula:

Loc
***

(T) = (T)
ij
ij
+ ij

*one has, if one places oneself in the plan (**N**, **m**) in such a way that the scission is maximum, the formula following:*
*Loc (**N**, T) = (**N**, T)*

+ ( )
max
N
*where (**N**, T) is the vector macroscopic shearing defined in [the Figure 3.2-b] and where*
*(**N**) is it*
vector microscopic residual shearing (independent of time since we are with the state
adapted).

Criterion of tiredness
Introduction of the maximum pressure: DANG VAN uses in the place of the normal constraint on one plan, as that is done in model MATAKE, the maximum hydrostatic pressure on a cycle.
The criterion is thus written:
MAX (Loc

*(**N**, T*
max
) + P Loc has
max) B
***N** T,*

As the hydrostatic pressures local and total are identical the criterion becomes:

MAX (Loc

*(**N**, T*
max
) + Pmax) B has
***N** T,*

For a positive maximum pressure we have:

MAX (Loc

*(**N**, T*
max
)) + Pmax B has

For an ***always negative pressure*** *one can take P*
0
max =
to remain conservative.
Assumption on
(N)
In the radial case where the direction of maximum shearing is defined in advance one can calculate exact way
(N)*. In the case general DANG VAN proposes the following method to do one calculation simplified of*
(N)*. One gives for a plan* ***N the*** *macroscopic way of the vector shearing defined previously. The vector residual shearing taking into account the preceding assumption is defined by MO, where M is the center of the circle circumscribed with the way of the end of the vector shearing in the plan of shearing.*
Macroscopic way
Lo (N
c)
max
M
Lo (N
C, Ti)
max

P
(N)
(N, T
Plan of
I)
Loc
shearing
(N)
max
0
Microscopic way

Appear 3.2-b: Ways macro microphone/in the plan of shearing

Final formulation: *taking into account the two formulas:*
Loc

*(**N**, T*
max
*) = (**N**, T) +*
*(**N**) and MAX (Loc*

*(**N**, T*
max
)) + Pmax B has
***N** T,*
one finds oneself with
*MAX (**MP**) + has Pmax B*
***N** T,*
*where P is a current point of the way of shearing in the plan of normal **N**.*

Identification of the constants*: to determine the constant ones has and B it is necessary to use two tests simple. Two possibilities exist:*
*1) **A pure shear test plus a tensile test alternate compression**. In it*
cases the constants are given by: B =
has = (- D/)
2/(D/)
3
0
0
0
0
.
*2) **Two tensile tests compression, alternated and the other not**. The constants are*

data by:
3
(
2 -
1)
m

1
= × has
B =
×
.
2 (
1 -
2) - 2 m
(
2 -
1) + 2
2
m
with
1 the amplitude of loading for the alternate case and
2 for the case where the constraint
average is nonnull.
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3.3
MATAKE and DANG VAN modified for the office plurality of damage

The models of MATAKE and DANG VAN were proposed initially for the study of the limit of endurance. As the infinite lifespan does not exist one uses limits of endurance with, 106, 107, 10n cycles of loading. Thus the initial criteria of MATAKE and DANG VAN are presented like criteria of going beyond of a threshold and do not give an office plurality of damage. The use, in particular criterion of DANG VAN, in the car industries is suitable since the objective sought is nonthe going beyond of a threshold of endurance contrary to the problems of EDF where one wishes to follow the damage.
Thus we use for the office plurality of damage an equivalent constraint of MATAKE or DANG VAN defined by:

MATAKE
eq =
*(N) *+ has Nmax (N) *,*
2

DANG VAN
= MAX
eq
*(**MP**) + a.*
max
***N**, T*

The taking into account of the surface treatment east summarized with the taking into account of the harmful effect of
pre-work hardening over the lifespan in controlled deformation [bib5]. In the models of MATAKE and DANG VAN the effect of pre-work hardening is taken into account by multiplying the half-amplitude of constraint
of shearing by a corrective coefficient higher than the unit, noted C p:

MATAKE
=

C
eq
p
*(N) *+ has Nmax (N) *,*
2

DANG VAN

= C MAX
eq
p
*(**MP**) + a.*
max
***N**, T*

These equivalent constraints are to be used on a curve of tiredness in shearing. For the use on a curve of tiredness in traction compression it is necessary to multiply these equivalent constraints by one corrective coefficient, noted here:

MATAKE
eq = CP
(N)

** + has Nmax (N)*
** ,*

2

DANG VAN eq = C MAX
p
*(**MP**)*

+ aP
.

max

***N**, T*

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4
Calculation of the plan of maximum shearing

We use here the definition of the plan of shearing introduced in the paragraph [§2.4]. Practically, for us the point M of the continuous medium will be a point of Gauss.

4.1
Expression of shear stresses in the plan

*For reasons of symmetry we vary unit normal **N** according to a half-sphere with the assistance angles and, cf [Figure 4.1-a].*
In the reference mark (O,
***X y**, **Z**), normal vector unit **N** is defined by:*

$$nx = \sin \cos$$
$$ny = \sin \sin$$
$$nz = \cos.$$

éq
4.1-1

*We introduce a new reference mark (O, **U**, **v**, **N**) where **N** is perpendicular to the plan of shearing and where **U** and **v** are in this plan, cf [Figure 4.1-a]. In the reference mark (O,*
***X y**, **Z**) vectors*
*unit **U** and **v** are respectively defined by:*

$$ux = - \sin$$
$$U y = \cos$$
$$uz = 0,$$

éq
4.1-2

vx = - cos cos
vy = - cos sin
vz = sin.
éq
4.1-3

Appear 4.1-a: Location of normal N in a plan by the angles and
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In the plan, the components U and v of the vector representing the shear stress are obtained by the relations:
U = ***U*** *= ux X + uy y + uz Z,*
éq
4.1-4
v = ***v*** *= vx X + vy y + vz Z.*
éq
4.1-5
On [Figure 4.1-b], we represented shear stresses in the plan as well as locate (O, ***U****,* ***v****,* ***N****).*
y
F

N
U
v

X
O

Z

Appear 4.1-b: Representation of the vector stress shear in the plan

*Now our problems are to determine for each point of Gauss or each node of one grid the plan of normal **N** such as is maximum. With this intention we vary the normal unit **N**.*

4.2
Exploration of the plans of shearing

*The method that we present here results from the reference [bib4]. Its principle is as follows. As indicated in the paragraph [§4.1], for reasons of symmetry we vary the normal unit **N** according to a half-sphere using the angles and, cf [Figure 4.1-a]. The question which comes immediately is which must be the step of variation of the angles and. Indeed, one should be found optimum between the smoothness of exploration and a reasonable time computing insofar as it is necessary to make this operation at each point of Gauss of the grid. The author of the reference [bib4] propose to divide the surface of the half sphere into facets of equal surfaces to the center of which unit normal **N** is positioned, cf [Figure 4.2-a]. In practice surfaces are not strictly equal but of the same order of magnitude.*
The step value of variation of,
is worth 10 degrees. The angle varies according to a step
who is
function of the angle. The closer is weak or to 180 degrees and the more must be large for
to preserve a surface of about constant facet. It is in the vicinity of =
°
90 that
is more
small. [Table 4.2-1] the cutting summarizes which was retained.
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With this method the number of facet thus the number of normal vectors to explore is equal to 209 for a half sphere.
X

Appear 4.2-a: Division of the surface of the half sphere in facets

°
0°
10°
20°
30°
40°
50°
60°
°
180°
60°
30°
20°
15°
°
857
,
12
,
11
°
25
A number of facets
1
3
6
9
12

14
16

°
70°
80°
90°
100°
110°
120°
130°
°
°
588
,
10
10°
10°
10°
10 588
,
°
,
11
°
25 12 857
,
°
A number of facets
17
18
18
18
17
16
14

°
140°
150°
160°
170°
180°

°
15°
20°
30°
60°
180°

A number of facets
12
9
6
3
1

Table 4.2-1: Numbers of facet according to and of

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*In order to determine the normal vector **N** which will give the plan of maximum shearing except for the degree,*
the author recommends to resort to two additional successive refinings. The first consists with
to explore eight new facets around the initial normal vector, like illustrates it it [Figure 4.2-b].

max
Facet Fm
max
=2°
=2°

Appear 4.2-b: Representation of the eight additional facets around N

In this case
is equal to two degrees and for] 0°, 180 [
°,
=
sin.

***Particular case.** If the facet m*
*F is perpendicular to **y**, one considers the six facets*
all around it located at = °
5 and respectively definite by = °
0 , =
°
60 , =
°
120 ,
=
°
180 , =
°
240 and =
°
300, cf [Figure 4.2-c].

***Appear 4.2-c: Localization of the explored facets when** m*
*F **is normal with y***
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*For each point of Gauss or each node we explore 209 normal vectors **N**. With each normal vector is associated a history of shearing concretized by a certain number of points located in the plan of shearing from axes U and v. Now it is a question of finding the circle circumscribed*
at the points belonging to the plan of shearing so as to deduce to it half amplitude from it from shearing.

5
Calculation of the half amplitude of shearing

The problems are thus to find the circle circumscribed with a certain number of points located in one plan. The half amplitude of shearing will be equal to the radius of the circumscribed circle.

5.1
General presentation of the calculation of the circumscribed circle

The method that we use is an exact method which breaks up into four stages.

Stage 1
We frame the points and we determine the co-ordinates of the four corners of the framework in locate (,
*0 U, v), and co-ordinates of the center of the framework O cf [Figure 5.1-a] and [Figure 5.1-c]. **In***

particular case *where the framework summarizes with a horizontal line or vertical it half length of the line*
is equal to the half amplitude of shearing.

Stage 2
The objective of the second stage is to select the two most distant points. In order to not
to examine the distance between all the possible pairs of points, we build four sectors,
cf [Figure 5.1-a] and [Figure 5.1-c]. These sectors are at the four corners of the framework and are delimited
on the one hand, by the contour of the framework and on the other hand, by an arc of circle whose center is the opposite corner
and the ray the large side of the framework which in fact undervalues the distance between the two most distant points.
Finally, we evaluate the distances between the points of the four sectors two to two:

distances between the points of sector 1 and the points of sector 2;
distances between the points of sector 1 and the points of sector 3;
distances between the points of sector 1 and the points of sector 4;
distances between the points of sector 2 and the points of sector 3;
distances between the points of sector 2 and the points of sector 4;
distances between the points of sector 3 and the points of sector 4.

In the particular case *where the report/ratio on the small side of the framework on the large side is strictly lower than*
3 4 we do not evaluate the distances between the points belonging to sectors 1 and 2 nor them distances between the points of sectors 3 and 4, case of the example of [Figure 5.1-a].

Stage 3
In the third stage we build the fields 1 and 2 in which we will seek them
points which are apart from the initial circumscribed circle, cf Etape 4. The constitution of fields 1 and 2 be to reduce the number of points to be explored at the time of stage 4. Principles of constructions of these two fields are as follows.

1) From the points mediums on the two large sides of the framework (Omi and Omi, cf [Figure 5.1-b]
1
2
and [Figure 5.1-d]) we trace an arc of circle whose ray corresponds to undervaluing
value of the half amplitude of shearing and is equal to the half length on the large side of tally.
2) Center of the framework O we trace four arcs of circle whose ray is also it
undervaluing value of the half amplitude of shearing.

If O the center of the circle circumscribes initial A a component according to the axis U which places it between Omi and

I
1
O, then if there is a point whose distance to O is higher than R the radius of the circumscribed circle
I
I
initial, it can be only in field 1, cf [Figure 5.1-b].
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v
v
(Umin, Vmax)
(Umax, Vmax)
(Umin, Vmax)
(Umax, Vmax)
1
1
Sector 1
Sector 2
FIELD 1
16
16
13
FIELD 2
13
12
14

12
14
2
11
9
2
11
9
10
10
15
15
5
5
3
3
OMI1
OMI2
OMI1
OMI2
O
U
O
U
4
4
6
6
FIELD 2
FIELD 1
Sector 4
Sector 3 7
8
7
8
(Umin, Vmin)
(Umax, Vmin) (Umin, Vmin)
(Umax, Vmin)
Appear 5.1-a: Exemple1, localization
Appear 5.1-b: Exemple1, localization
sectors
fields
v

v
OMI2
OMI2
(Umin, Vmax)
(Umax, Vmax)
(Umin, Vmax)
(Umax, Vmax)
9
8
9
8
7
7
Sector 1
Sector 2
FIELD 1
O
O
U
U
1
6
1
6
5
5
FIELD 2
2
Sector 4
Sector 3
2
4
4
3
3
(Umin, Vmin)
(Umax, Vmin)
(Umin, Vmin)
(Umax, Vmin)
OMI1

OMI1

Appear 5.1-c: Exemple2, localization
Appear 5.1-d: Exemple2, localization
sectors
fields
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Stage 4
The goal of the fourth stage is to find the circle circumscribed by the method of the circle passing by three points, cf [§5.2]. With this intention, we calculate the point medium 1
O associated with the two points more
moved away that we note 1
P and 2
P, we deduce the value from it from a first ray noted 1
R. In
function of the position of 1
O compared to the large axis of the framework passing in its center, us
let us seek either in field 1, or in field 2, if there is a point located at a distance
higher than the half measured between the two most distant points 1 outdistances
P and 2
P. Let us note 3
P
such a point. If there is no point such as 3
P then it half amplitude of shearing is equal to 1
R,
cf [Figure 5.1-c]. On the other hand, if 3
P exists we seek the co-ordinates of the point located at equal
distance from 1
P, 2
P and 3
P; we note this point O2. We thus obtain a new ray, R2
thus new a half amplitude of shearing. Again, according to the O2 position by
report/ratio with the large axis of the framework passing in its center, we seek either in field 1, or in field 2, if there is a point located at a distance higher than O2 R2. Let us note 4
P such

not. If there is no point such as 4
P then it half amplitude of shearing is equal to R2. In
revenge, if 4
P exists we seek the smallest circle circumscribed at the four points: 1
P, 2
P, 3
P
and 4
P by using the method of the circle successively passing by three points, cf [§5.2]. That us
provides a new center 3
O and a new ray 3
R. As previously, according to
position of 3
O compared to the large axis of the framework passing in its center, we seek is in
field 1, is in field 2, if there is a point located at a distance higher than 3
R of 3
O.
Let us note 5
P such a point. If there is no point such as 5
P then it half amplitude of shearing is
equalize to 3
R. On the other hand if a point such as 5
P exists we have five points, if we want to use
the preceding method, where there are only four points concerned, it is necessary to eliminate one from the five
points. That cannot be the last: 5
P, therefore we preserve preceding iteration the three
points which made it possible to determine 3
O and 3
R, i.e. the smallest circumscribed circle. Let us suppose
that 1
P is thus eliminated. We thus seek the smallest circle circumscribed at the four points: 2
P,
3
P, 4
P and 5
P by using the method of the circle successively passing by three points, cf [§5.2].
That provides us a new O4 centre and new R4. According to the position of O4
compared to the large axis of the framework passing in its center, we seek either in field 1, or
in field 2, if there is a point located at a distance higher than R4 de O4. If it is not the case
the half amplitude of shearing is equal to R4 and the circle circumscribes has as a O4 center,
cf [Figure 5.1-f]. Contrary, if such a point exists we remake an iteration identical to
the preceding one.

Appear 5.1-f: Exemple2, research of the circumscribed circle

Appear 5.1-e: Exemple1, research of the circle circumscribed

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5.2
Description of the method of the circle passing by three points

In this paragraph we will treat the case general, then the particular cases.

5.2.1 Case
general

To determine the circumscribed circle at three items 0
P, 1
P and 2
P, cf [Figure 5.2.1-a], we proceed
in three stages.

Appear 5.2.1-a: Determination of the circle passing by three points

Stage 1
We calculate the co-ordinates of the three points mediums: M, M and M, cf [Figure 5.2.1-a].
0
1
2

Stage 2
We determine the normals passing by the three points mediums
: M, M and M,
0
1
2
cf [Figure 5.2.1-a]. These normals are of the right-hand sides of the type v = has U + B where has and B are constants
that it is possible to calculate with the co-ordinates of the points P, P, P, M, M and Mr.
0
1
2
0
1
2
Let us describe, now, the manner of determining these normals.

***1) Normal with segment** 0*
P 1
*P **passing by** M1*
We determine the punctual coordinates

'

M1 by rotation of 90° of segment 0
P M1:

U M1 = UM1 + (VM1 - VP0)
éq
5.2.1-1
VM1 = VM1 + (U P0 - U M1)

where U
=

K and Vk with K
M1, M,
1 P0 account for the components U and v of the points
'
M, M and
1
1
P. We deduce the constant ones has and B of the right-hand side representing the normal with the segment
0
0
0
P P passing by M:
0
1
1
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a0 = (VM1 - VM1) (U M1 - UM1)

éq
5.2.1-2
0
B = (U M1 V
M1 - U M1 VM1) (U M1 - U M1)

In the particular case *where (U M1 - U M) = 0*
1
, we force a0 and 0
B with zero and we obtain them
coordinated center O of the circle circumscribed as in points 0
P, 1
P and 2
P by a specific method
described in the paragraph [§5.2.2].
2) Normal with segment *0*
P 2
*P **passing by** M 0*
We determine the punctual coordinates
'
M 0 per rotation of 90° of segment 0
P M 0:

U M 0 = U M 0 + (VM 0 - VP0)
éq
5.2.1-3
VM 0 = VM 0 + (U P0 - UM 0)

where U
=

K and Vk with K
M 0, M,
0 P0 account for the components U and v of the points
'
M 0, M 0 and
0
P. We deduce constants 1 from them

has and 1
B of the right-hand side representing the normal with the segment
0
P 2
P passing by M 0:
1
has = (VM 0 - VM 0) (UM 0 - UM 0)
éq
5.2.1-4
1
B = (UM 0 V
M 0 - U M 0 VM 0) (U M 0 - U M 0)

In the particular case *where (U M 0 - U M) = 0*
0
, we force 1
has and 1
B with zero and we obtain them
coordinated center O of the circle circumscribed as in points 0
P, 1
P and 2
P by a specific method
described in the paragraph [§5.2.2].

3) Normal with segment *1*
P 2
P ***passing by*** *m2*
We determine the punctual coordinates
'
M2 per rotation of 90° of segment 1
P M2:

U M2 = U m2 + (VM 2 - V 1
P) ***éq***
5.2.1-5
VM 2 = VM 2 + (U 1
P - U m2)

where U
=

K and Vk with K
M2, M,
2 1

P account for the components U and v of the points
'
M2, m2 and
1
P. We deduce the constants a2 and b2 from it from the right-hand side representing the normal with the segment
1
P 2
P passing by m2:

a2 = (VM 2 - VM 2) (UM 2 - U m2)

éq
5.2.1-6
2
B = (U m2 V
M2 - U M2 VM 2) (U m2 - U m2)

In the particular case *where (U m2 - U M) = 0*
2
, we force a2 and b2 with zero and we obtain them
coordinated center O of the circle circumscribed as in points 0
P, 1
P and 2
P by a specific method
described in the paragraph [§5.2.2].
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Stage 3
In the case general, we deduce from the constants a0, 0
B, 1
has, 1
B, a2 and b2 co-ordinates of
center O of the circle circumscribed as in points 0
P, 1
P and 2
P of three manner different. Let us note 0
O, 1
O
and O2 the same center O, obtained in three different ways, and the U.K. and Vk, where K = 0
O, 1
O, O2,
the components U and v of items 0 represent
O, 1
O and O2:

UO = (1
B - 0
b) (a0 - 1
has)
0

éq
5.2.1-7
VO = (a0 1
B - 1
0 have
b) (a0 - 1
has)
0

UO = (2
B - 0
b) (a0 - a2)
1

éq
5.2.1-8
VO = (a0 2
B - a2 0

b) (a0 - a2)
1

UO = (2
B - 1
b) (1
- a2 has)
2

éq
5.2.1-9
VO = (1
has 2
B - a2 1
b) (1
- a2 has)
2

After having checked that equalities: U
U
and V
V
are satisfied us
0
O
U
1
O
2
O
0
O
V
1
O
2
O
let us determine the ray of the circle circumscribed by calculating the distance between O and one of three items 0
P,
1
P or 2
P.

5.2.2 Case
private individuals

In this paragraph we treat the three particular cases of stage 2 of the paragraph [§5.2.1].
Particular case where *(U M1 - U M) = 0*
1

In this case we immediately obtain the components U and v of the center O by:

U = U
O
M1

éq
5.2.2-1
VO = (1
has 2
B - a2 1
b) (1
- a2 has)

Particular case where *(U M 0 - U M) = 0*
0

Here the components U and v of the center O are given by:

U 0 = U M 0

éq
5.2.2-2
VO = (a0 2
B - a2 0
b) (a0 - a2)

Particular case where *(U m2 - U M) = 0*
2

In this last case, U and v of the center O are given by:

U0 = U m2
éq
5.2.2-3
VO = (a0 1
B - 1
0 have

b) (a0 - 1
has)

The value of the ray of the circumscribed circle is obtained same manner as in the case general; i.e., by calculating the distance between O and one of three items 0
P, 1
P or 2
P.
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5.3
Criteria with critical plans

In this paragraph we give the list of the criteria with critical plans, cf [bib3], which are programmed as well as a brief description.
Notation:

N ***
: normal in the plan in which the amplitude of shearing is maximum;

*(****N****)*
: amplitude of shearing in a plan of normal NR;
NR
( )
max ***N***
: maximum normal constraint within normal NR during the cycle;
0

: limit of endurance in alternate pure shearing;
d0
: limit of endurance in alternate pure traction and compression;
*NR (**N**)*
m

: average normal constraint within normal NR during the cycle;

( )
*max **N***
: maximum normal deformation within normal NR during the cycle;
*(**N**)*
m

: average normal deformation within normal NR during the cycle;
P
: hydrostatic pressure;
C p
: harmful effect of pre-work hardening in controlled deformation, C p 1.

Criterion of MATAKE
*(*)*
***N** + has NR (*)*
*max **N***
B

éq 5.3-1
2

where has and B are two constant data by the user, they depend on the characteristics materials and are worth:

d0 d0
has = -

B =
.
0

2
2
0

Moreover, we define an equivalent constraint within the meaning of MATAKE, noted (*)
N
eq
:

*(*)*
N
F
*(*)*
***N** = C*
+ NR has
*(*)*
N
,
eq
p

2
max
T

where F T represents the report/ratio of the limits of endurance in alternate inflection and torsion.

Criterion of DANG VAN
*(*)*
N** + has P B **éq 5.3-2
2

where has and B are two constant data by the user, they depend on the characteristics materials and are worth:

3
(
2 -
1)
m

1
= × has

B =
×

2 (
1 -
2) - 2 m
(
2 -
1)
.
+ 2
2
m

Moreover, we define an equivalent constraint within the meaning of DANG VAN, noted (*)
N
eq
:

*(*)*
N
C
*(*)*
***N** = C*
+ P has
eq
p
,

2
T
where C T represents the report/ratio of the limits of endurance in alternate shearing and traction.
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5.4
A number of cycles to the rupture and damage

From (*)
N
eq
and from a curve of Wöhler we deduct the number of cycles to the rupture:
NR (*)
***N**, then the damage corresponding to a cycle: D (*)*
***N** = 1 NR (*)*
***N**.*

5.5
Size and components introduced into Code_Aster

The computed values are stored at the points of Gauss or the nodes according to the option selected. One
sizes and of the components were introduced into the catalogue of the sizes (file: grandeur_simple__.cata). The components of size FACY_R (Cyclic Tiredness) are described in [Table 5.5-1].

DTAUM1
***first** value of the half amplitude max of shearing in the critical plan*
VNM1X
component X of the normal vector in the plan criticizes dependent A DTAUM1
VNM1Y
component of the normal vector in the plan criticizes dependent A DTAUM1 there
VNM1Z
component Z of the normal vector in the plan criticizes dependent A DTAUM1
SINMAX1
normal maximum constraint in the plan criticizes correspondent with DTAUM1
SINMOY1
normal average constraint in the plan criticizes correspondent with DTAUM1
EPNMAX1

normal maximum deformation in the plan criticizes correspondent with DTAUM1
EPNMOY1
average maximum deformation in the plan criticizes correspondent with DTAUM1
SIGEQ1
Constraint equivalent within the meaning of the criterion selected correspondent with DTAUM1
NBRUP1
a number of cycles before rupture (function of SIGEQ1 and a curve of Wöhler)
ENDO1
damage associated with NBRUP1 (ENDO1=1/NBRUP1)
DTAUM2
second *value of the half amplitude max of shearing in the critical plan*
VNM2X
component X of the normal vector in the plan criticizes dependent A DTAUM2
VNM2Y
component of the normal vector in the plan criticizes dependent A DTAUM2 there
VNM2Z
component Z of the normal vector in the plan criticizes dependent A DTAUM2
SINMAX2
normal maximum constraint in the plan criticizes correspondent with DTAUM2
SINMOY2
normal average constraint in the plan criticizes correspondent with DTAUM2
EPNMAX2
normal maximum deformation in the plan criticizes correspondent with DTAUM2
EPNMOY2
average maximum deformation in the plan criticizes correspondent with DTAUM2
SIGEQ2
Constraint equivalent within the meaning of the criterion selected correspondent with DTAUM2
NBRUP2
a number of cycles before rupture (function of SIGEQ2 and a curve of Wöhler)
ENDO2
damage associates NBRUP2 (ENDO2=1/NBRUP2)
Table 5.5-1: Components specific to multiaxial cyclic tiredness

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6
Criteria with variable amplitude

The criteria with variable amplitude are implemented when the loading is not periodic. When the loading is not periodic it is necessary to break up the way of loading undergone by structure in elementary under-cycles using a method of counting of cycles. If it loading is nonradial it does not have there tested multiaxial method of counting. Consequently we choose, as in the literature, to use the method of counting RAINFLOW [bib7] which has requirement in entry for a scalar. This is why we reduce to a dimension the cission, which is orthogonal projection of the vector forced on a plan, by projecting the point of the vector cission on one or two axes. Another important difference with the criteria in critical plan is that it is not the amplitude of shearing which makes it possible to select the critical plan but the office plurality of damage which result from the elementary under-cycles.
The method of projection that we use is clarified in chapters 7 and 8. In the continuation us let us describe the way in which we made evolve/move the criteria of MATAKE and DANG VAN for to adapt to the cases where the loading is not periodical.

6.1
Criterion of modified MATAKE

In the context of the office plurality of damage and a periodic loading, the criterion of MATAKE [bib6], is written in the following way:
*(r**
N)
=

C
+ NR has
R
eq
p
*(**

max N)
éq
6.1-1
2

where eq represents the constraint equivalent within the meaning of the criterion of MATAKE and with:

*

NR
normal in the plan for which the amplitude of shearing is maximum;
*(*
NR

) 2 maximum half-amplitude of shearing;
has
constant which perhaps defined by a pure shear test alternated and in
alternate traction and compression or by a test in alternate traction and compression and in
nonalternate traction and compression;
NR
*(r**
*max N) maximum normal constraint within normal **
NR during the cycle;
C p
harmful effect of pre-work hardening in controlled deformation C p 1.

To calculate the cumulated damage if the loading is not periodical the first stage
consist in determining the cission (vector shearing) in a plan of normal NR at every moment
loading. The technique which is used with this intention is described in the reference [bib6]. In
second stage we start by reducing the history of the cission to a function
unidimensional of time by projecting the point of the vector cission on one or two axes defined in
the plan of normal NR considered, cf chapter 7 and 8. Thus the evolution of the projected cission is
summarized with
the relation:
F (T)
p =
what makes it possible to use the method of counting RAINFLOW. On the figure
[Figure 6.1-a] we show the values reached by the end of the vector shearing in a plan
of normal NR before projection on an axis or two axes and the figure [Figure 6.1-b] these same
values after projection on an axis. This stage it is necessary for us to introduce the concept of constraint
equivalent elementary I
eq. Practically this concept with the same significance as the concept of
equivalent constraint defined by the relation [éq 6.1-1], but it applies to the under-cycles

elementary resulting from the method of counting RAINFLOW. Thus starting from the projected cission
p
we calculate elementary equivalent constraints I
eq.
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Cission in a plan of normal N (MPa)
v
U

Appear 6.1-a: Points of the vector cission before projection

Cission projected on axis 1 (MPa)
p
Sequence number

Appear 6.1-b: Points of the vector cission after projection on an axis

Method RAINFLOW breaks up
F (T)
p =
in periodic elementary under-cycles and breaks
the history of the loading, as we show it on the figure [Figure 6.1-c]. Thus, for one
normal NR given method RAINFLOW provides for each elementary under-cycle two values,
R
R

points high and low, of the point of the vector cission I
(N) and I
(N) associated two values of
p1
p2
R
R
maximum normal constraint NR I
(N) and Ni (N).
1
max
max2
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Under elementary cycles (MPa)
38
Sequence numbers or
Numbers of under cycles
35
numbers of the moments
1
32
14
13
14
11

22
33
7
16
37
45
26
12
8

8
p
41 43
47
11
12
1
3
15
9
15
6
2
44
46
9
10
4
4
5
42
29
48
3
5
40
20
25
Numbers of the points

Appear 6.1-c: Fifteen elementary under-cycles after treatment by method RAINFLOW

For the criterion of MATAKE we define the elementary equivalent constraint in the manner following:

Max
R
R
R
R

-

I
R
(I (N,) I (N) Min I N I N
1
p
p2
) (( ), ( )
1
p
p2
)
(N) = C
+ max has
R
R
eq
p
(Neither (N,) Nor (N), 0 ***éq 6.1-2***
1
max
max2
)
2

For the office plurality of damage, this elementary equivalent constraint is to be used with a curve of tire in shearing. If one uses a curve of tiredness in traction compression it is necessary to multiply [éq 6.1-2] by a corrective coefficient which corresponds to the report/ratio of the limits of endurance in inflection and in
alternate torsion and which we note:

I
R I R

I
R I R

I
R

Max ((N,) (N) Min
N
N
1
p
p2
)- (( ), ( )
1
p
2
p
)
(N)
R
R
eq
= CP
+ max (Neither (N has), Nor (N), 0
1
max
max2
)

2

éq 6.1-3
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R
From I
(N)
eq
and from a curve of Wöhler we deduct the number of cycles to the rupture
NR I (NR)
R
R
and the elementary damage Di (N) = 1 NR I (N) correspondent with an elementary under-cycle.
We use a linear office plurality of damage. That is to say K the number of elementary under-cycles, for a normal NR fixed, the cumulated damage is equal to:

K
D (NR) = I
D (NR) ***éq***
6.1-4
i=1
*To determine the normal vector **
NR corresponding to the maximum cumulated damage it is enough to make
*to vary NR and to calculate [éq 6.1-4]. The normal vector **
NR corresponding to the maximum cumulated damage
is then given by:
*D (r**
N) = max (D (NR))
NR

6.2
Criterion of modified DANG VAN

Within the framework of the damage and a periodic loading, the criterion of DANG VAN is written:

R
*(nr**
**
)
(N) = C
+ P has
eq
p

2

where eq represents the constraint equivalent within the meaning of the criterion of DANG VAN and with:

**

NR
normal in the plan for which the amplitude of shearing is maximum;
*(**
NR

) 2 maximum half-amplitude of shearing;
has
constant which perhaps defined by a pure shear test alternated and in
alternate traction and compression or by a test in alternate traction and compression and in nonalternate traction and compression;
P
maximum hydrostatic pressure during the cycle;
C p
harmful effect of pre-work hardening in controlled deformation C p 1.

When the loading is not periodical, we calculate the damage by the same process that that used for the criterion of MATAKE. The only difference lies in the definition of elementary equivalent constraint:

Max
R
R
R
R

-

I
R
(I (N,) I (N) Min I N I N
1
p
p2
) (( ), ( )
1
p
p2
)
(N) = C
+ max has
R
R
eq
p
(Pi (N,) pi (N), 0
1
2
) ***éq***
6.2-1
2
where
I
P1 and I
P2 represent the two values of the hydrostatic pressure attached to each elementary under-cycle. This elementary equivalent constraint is to be used with a curve of tire in shearing. If one must employ a curve of tiredness in traction compression it is necessary to multiply [éq 6.2-1] by the corrective coefficient:

I
R I R
I
R I R

I

R

Max ((N,) (N) Min
N
N
1
p
2
p
)- (( ), ( )
1
p
2
p
)
(N)
R
R
eq
= CP
+ max (pi (N) has, pi (N), 0
1
2
)

2

After having defined the criteria of MATAKE and DANG VAN within the framework of the office plurality of damage and
of a nonperiodic loading, it remains us to specify the technique of projection that us let us propose.
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7
Choice of the axes of projection

With regard to the projection of the end of the vector cission we propose two options:

1) a projection on an axis,
2) a projection on two axes.

The axis of option 1 is in the same way given that the first axis of option 2. The second axis of option 2 is orthogonal with the first axis of this option.

7.1
Projection on an axis

We place ourselves in a plan of normal NR given where each point represents the position of point vector shearing at one moment, for more details to see the reference [bib6]. In this plan we build the smallest framework which contains all the points representing the end of the vector cission at every moment. The two diagonals of the framework enable us to define two axes: axis 1 corresponds to the segment AC, and centers it 2 corresponds to the segment dB, cf [Figure 7.1].

Sector Sector
v
With
B
U
axe2
Sector Sector
WITH B
axe2
P4
O
P1
O
P5

axe1
P3
axe1
P2
D C
D
C
Sector
Sector
Sector Sector
Appear 7.1-a
Appear 7.1-b

Figure 7.1: Definition of the axes of projection

We choose a priori the axis of projection among axes 1 and 2 because the diagonal of the framework is larger than the large side of the framework what has as a virtue to dilate a little the projected points. In addition the loadings which interest us are of thermal origin with the result that the points representing the evolution of the point of the vector cission, in the plans of normal NR, are more often aligned on an axis, as we show it on the figure [Figure 6.1-a].

Sectors 1, 2, 3 and 4 are built same manner as in the reference [bib6]. Only them points which are in these sectors are projected orthogonally on axes 1 and 2.

We define the axis of projection as being the axis on which the distance between two points projected is largest.

For example, on [Figure 7.1-a] the axis of projection is axis 1 since the length of segment 3
P 4
P
is greater than the length of segment 1
P 2
P. This definition of the axis of projection allows
to make sure that the axis of projection retained will make it possible to account for the amplitude of shearing
projected largest.

According to the presence or absence of points in sectors 1, 2, 3 and 4 determination of the axis of projection can be immediate, it is then not necessary to implement the procedure of selection described above. For more details the reader will be able to refer to appendix 1.

A second axis is necessary to distinguish the case where points representing the point of the vector cission are aligned on an axis of the case where these points describe a circle.
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7.2
Construction of the second axis

The second axis of projection is orthogonal with the initial axis of projection and it passes by the point O.

Since we know the co-ordinates of the points A, B, C and D, to characterize it completely second axis it is enough to determine the punctual coordinates M such as:

DB. OM = 0 if the initial axis is axis 1,
AC. OM = 0 if the initial axis is axis 2.

8
Projection of shearing

In this chapter we describe the process of projection on the initial axis, or first axis, and the second center. We point out that projection on these two axes is orthogonal.

8.1
Case where axis 1 is the initial axis

R R R
This case is represented on [Figure 8.1-a]. We in reference mark place (O, U, v, N). Definitions
R R
of ur, vr and NR are given in the reference [bib6]. In the plan (U, v) of normal NR points A,

B, C, D and O have respectively, for co-ordinates (U min, Vmax), (U max, Vmax), (U max, Vmin), (Umin, Vmin) and (UO, O
V).

Initial axis
With
B
Pi
v
Second axis
O
P
U
PS
M
D
C

Appear 8.1-a: Projection if axis 1 is the initial axis

8.1.1 Determination of the second axis

Here to determine the second axis we solve the equation:

DB. OM = 0
éq
8.1.1-1
where the co-ordinates U M, VM of the point M are the unknown factors.
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The equation [éq 8.1.1-1] is also written in the following form:

(Umax - Umin) (UM - UO) + (Vmax - V) (VM - O
V) = 0
min

what leads to:

(Umax - Umin)
VM = O
V - (
-

Vmax - Vmin) (UM UO)

By giving each other a value of U M different of UO we obtain VM immediately.

8.1.2 Projection of an unspecified point on the initial axis

Starting from an unspecified point P known, the first stage consists in calculating the co-ordinates of one
not P 'such as:
DB. PP = 0

While proceeding like previously, we obtain the relation:

(Umax - Umin)
V = V -
U - U
P
P
(Vmax - Vmin) (P
P)

where V
U
U.
P results from a value from
P different from
P

In the plan (U, v) the initial axis and the PP segment are lines closely connected respectively described by
v = U + B has and v = U + B has, therefore to know the co-ordinates of the point projected on the initial axis P
I
I
P
P
p
we solve the equation:
U + B has = has U + B
I
I
P
P
where
(V - V
(U V - U V
max
min
min
max)
max
min)
has =
,
B =
,
I
(U - U
I
(U - U
max
min)
max
min)

(V
(U V

- U V
P
P

P
P)
- V
P
P)
has =
,
B =
.
P
(U
P
(U - U
P
P)
- U
P
P)

One obtains:
B - B
p
I
U =

Pi
has - has
I
p
B has - has B
I
P
P
I
V =

Pi
has - has
I
p

The projection of an unspecified point on the second axis is described in appendix 2.
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8.2
Case where axis 2 is the initial axis

R R
This case is represented on [Figure 8.2-a]. As previously, in the plan (U, v) points A, B, C, D and O have respectively, for co-ordinates (U
, V
, (U
, V
, (U
, V
,
max
min)
max
max)
min
max)
(U, V and (U, V.
O
O)
min
min)
Initial axis
With
B

M
PS
v
Second axis
O
P
U
Pi
D
C

Appear 8.2-a: Projection if axis 2 is the initial axis

8.2.1 Determination of the second axis

Here to determine the second axis we solve the equation:

AC. OM = 0
éq
8.2.1-1
where the co-ordinates (U, V) of the point M are the unknown factors.
M
M

The equation [éq 8.2.1-1] is also written in the following form:

(U - U
U
U
V
V
V
V

M -
O +
-
M -
O
=
max
min) (
) (min
)(

) 0
max

what leads to:
(Umax - Umin)
V = V +
U - U
M
O
(Vmax - Vmin) (M
O)

By giving each other a value of U different of U we obtain V immediately.
M
O
M

8.2.2 Projection of an unspecified point on the initial axis

Starting from an unspecified point P known, the first stage consists in calculating the co-ordinates of one
not P such as:
AC. PP = 0

While proceeding like previously, we obtain the relation:

(Umax - Umin)
V = V +
U - U
P
P
(Vmax - Vmin) (P
P)

where for a value of U
U we calculate V
P different from
P
P.
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In the plan (U, v) the initial axis and the PP segment are lines closely connected respectively described by
v = U + B has and v = U + B has, therefore to know the co-ordinates of the point projected on the initial axis P
I
I
P
P
p
we solve the equation:
U + B has = has U + B
I
I
P
P
where
(V - V
(U V - U V
max
max
min
min)
max
min)
has = -
,
B =
,
I

(U - U
I
(U - U
max
min)
max
min)
(V
(U V

- U V
P
P
P
P)
- V
P
P)
has =
,
B =
.
p
(U
P
(U - U
P
P)
- U
P
P)

One obtains:
B - B
B has - has B

P
I
U =
,
I
P
P
I

V =
.
Pi
has - has
Pi
has - has
I
P
I
P

The projection of an unspecified point on the second axis is described in appendix 2.

8.3
Definition of the module and orientation of the axis of projection

We propose to define the sign of the module of the point projected compared to the initial axis. That is to say the reference mark
(O, ur, vr, NR) in which the cission evolves/moves. In this reference mark if the component U of the projected point is
I
P
higher or equal to zero the sign of the module is positive, if not it is negative. In short the module and the sign of the module of the projected point are in the following way defined:

2
2
P
= COp
COp
if U
I
+
S
P 0,
MOD
I

2
2
P
= - COp
COp
if U

I
+
S
P < 0.
MOD
I

The definition of the module differentiates the loadings closely connected from the circular loadings. Conformément
with the experiment a circular loading will be regarded as being more damaging that one loading refines [bib1].

8.4 Components of Code_Aster used

VNM1X
component X of the normal vector in the plan of greater damage
VNM1Y
component there of the normal vector in the plan of greater damage
VNM1Z
component Z of the normal vector in the plan of greater damage
ENDO1
The most important damage

9 Conclusion

In this document we presented the criteria of MATAKE and DANG VAN adapted to the office plurality
of damage under periodic and nonperiodic loading.

When the loading is periodic the criteria of MATAKE and DANG VAN are tested by case tests SSLV135a and SSLV135b. The cases tests SSLV135c and SSLV135d test these two criteria if the loading is not periodical.

The key words which make it possible to use these two criteria are described in the document [U4.83.02]
devoted to order CALC_FATIGUE. One will be able to also consult the key word factor CISA_PLAN_CRIT of order DEFI_MATERIAU [U4.43.01].
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Appendix 1

The various situations are summarized in [A1-1 Table]. In [A1-1 Table], "0" and "1" mean respectively that there are no points and that there is at least a point in the indicated sectors.

Sector 1 Sector 3 Sector 2 Sector 4
Center projection
0 0 0 0
Case impossible.
0 0 0 1
Case impossible.
0 0 1 0
Case impossible.
0 0 1 1
Center
1.
0 1 0 0
Case impossible.
0 1 0 1
Use of

procedure of selection.
0 1 1 0
Use
of

procedure of selection.
0 1 1 1
Center
1.
1 0 0 0
Case
impossible.
1 0 0 1
Use
of

procedure of selection.
1 0 1 0
Use
of

procedure of selection.
1 0 1 1
Center
1.
1 1 0 0
Center
2.
1 1 0 1
Center
2.
1 1 1 0
Center
2.
1 1 1 1
Use
of

procedure of selection.
A1-1 table: Summary of the situations

The impossible cases result from the way in which the framework and the sectors are built. This construction

makes impossible the presence of points in no or only one sector.
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Appendix 2

The projection of an unspecified point on the second axis is quickly described in this appendix. From one
not P unspecified known, we calculate the punctual coordinates P such as:

$$\overrightarrow{OM}.\overrightarrow{PP} = 0$$

After simplification it comes the relation:

$$(U - U_M)O)V = V - U - U_P P(V - V_M)O)(P P)$$

where a value of U

U gives us V
P different from
P
P.

In the plan (U, v) the second axis and the PP segment are lines closely connected respectively described by
v = U + B has and v = U + B has, therefore to know the co-ordinates of the point projected on the second axis P
S
S
P
P
p
we solve the equation:
U + B has = has U + B
S
S
P
P
where
(V - V
(U V - U V
M
O
O
M)
M
O)
has =
,
B =
,
S
(U - U
S
(U - U
M
O)
M
O)
(V
(U V

- U V
P
P
P
P)
- V
P
P)
has =
,
B =
.
P
(U
P
(U - U
P
P)
- U
P
P)

One obtains:
B - B
P
S
U =
,
PS
has - has
S
P
B has - has B
S
P
P
S
V =
.
PS
has - has
S

p

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Document: R7.04.10
Operator of calculation of wear

Summary:
This note presents three laws of wear which make it possible to evaluate the volume used starting from the quantities resulting from one
dynamic calculation carried out with operator DYNA_TRAN_MODAL [U4.54.03] and the key word SHOCK.
· The law of Archard,
· Law KWU_EPRI,
· Law EDF_MZ.
The coefficients of wear necessary for these calculations are provided by the user or specified in one base data.
From worn volume and geometry of the contact, it is possible to calculate the depth of wear for mobile or its obstacle.
An angular figure division of play authorizes the operator to calculate the sizes relating to wear by sectors.

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1 Introduction

The evaluation of the damage by wear requires a thorough knowledge of the bodies in presence at the time
contact, loadings and kinematics. Investigations led to the Mechanical Department
and Technology of the Components make it possible to provide coefficients for laws of wear relative to configurations of wear affecting the components of the nuclear thermal power stations. A transitory calculation by
modal recombination, using operator DYNA_TRAN_MODAL [U4.54.03] makes it possible to know kinematics and the dynamics of the contact for telegraphic structures such as the control rods
and the tubes of steam generator which impact and slip against their guidance.
To calculate the power of wear, the module of postprocessing of the wear of Code_Aster®, (POST_USURE [U4.67.03]), uses, in a node of shock, the result in generalized co-ordinates (tran_gene) resulting from DYNA_TRAN_MODAL. It combines the normal forces and speeds of slip according to the definite method in the following paragraph. From the knowledge of the power of wear, it is
possible to go up with the volumes used by using one of the laws of wear suggested in POST_USURE. The coefficients to be used are to be defined by the user or to seek in an integrated data base
with the operator.
In the second time, the knowledge of the geometry of the structures intern nuclear thermal power stations allows to calculate the depths of wear starting from worn volumes.
Operator POST_USURE allows to cut out the figure of play in sectors in order to affect several coefficients of wear at the same zone of shocks to take account of complex geometries. For example, the contact on edge leads to matter losses more important than the contact conformel in the case control rods.
The table generated by POST_USURE gives the value of the volumes used for several values of time. It can be used in entry of operator MODI_OBSTACLE to know the evolution of the figure of play due to the wear of the mobile and the obstacle. That gives the possibility of carrying out iterative calculations which
couple the evolution of dynamics with the wear of the mechanisms.

2 Laws of wear

In its initial form, the law of Archard [bib1] expresses, for a configuration of adhesive wear, in slip, a relation between worn volume and of the quantities characteristic of the contact:

$$\frac{V}{N} = \frac{K.F.L}{H}$$

where
V : used volume,
K

:
coefficient of wear without dimension,
Fn
:
modulate normal force of contact, presumedly constant,
L
:
slipped length,
H
:
hardness.
The coefficient *K* is different for each involved body. It depends on the conditions
geometrical and thermodynamic at the time of the contact.
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It was shown that the law of Archard can be extended to other mechanisms, in slip dominating.
With the help of a redefinition of certain parameters, the preceding equation can be written:
V = K.W
K
where
K
:
is equal to
,
H
W
:
is equal to *F*

L
N.
.
W with the dimension of a work. By convention, it is called “work of wear”.
If the normal force of contact varies in the course of time (for example, in a situation impact-slips, *Fn* presents very strong variations of short duration at the time of the shocks), definition of *W* becomes:

$$W = \int_{t0}^{T1} F\,V\,dt$$

where *W*
:
work of wear,
Fn
:
modulate normal force during the contact,
Vt
:
modulate speed of slip during the contact,
t0
:
moment of beginning of calculation,
T1
:
moment of end of calculation.
Consequently, by analogy with the usual laws of mechanics, it is possible to define a “power of wear " while posing:

$$P = F\,V$$

N
T
where *P*: power of wear.
If a stationary regime is reached, the power of wear is supposed to be constant with run from time. In order to be ensured of this stationnarity, interval [*t0*, *T1*] can be cut out in several blocks in operator POST_USURE [U4.67.03]. For each one of these blocks, it is appropriate of to check that the power of wear evolves/moves little (in any rigour, the use of the laws of wear below suppose that the power of wear is constant).

2.1 Law of wear “ARCHARD”

Law is of the linear type [bib1]: $V = K\,P\,T$
where

V
: volume of wear,
K
: coefficient of wear,
P
: power of wear,
T
: interval of time.
The coefficient *K* is provided by the user or is taken in a data base (see [§3]). It is different for the two involved bodies and depends on the geometrical conditions and thermodynamic in the contact. The interval of time *T* used for the calculation of wear does not correspond to the effective time of simulation but to the interval of time over which the user wishes
to evaluate wear.
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2.2
Law of wear "KWU_EPRI"

The step of the model consists in determining a coefficient of wear *K*, within the meaning of the law of Archard, in
taking into account the particular conditions of the studied contact [bib2].
The normal forces *Fi* (NR) are divided into 5 classes, as well as speeds of slip
V J (m/)
S.
One obtains 25 classes whose location is indicated as follows:
F I
F max
5

1.
1.4
2.5
3
4
5
1.
2.
3.
2
5
1.
2.3
3.4
4.
1
2
3
4
1.
2.
3.
4.
5.5
0
1
2
4
2.
3.
4.3
5.
1
2
3.
4.
5.3
1
2
4.
5.
1
5.

V max
V J
For a given calculation, one determines the percentages obtained for each of the 25 classes.
The treatment is done by applying suitable factor loadings for each class, which
give an account of its particular contribution in the total process of wear.
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In the case of the pure impacts (classes 1.1 to 1.5), the contribution of these classes is modelled in calling upon a factor loading *mh* defined by:
ij
F 3
m = k1 K
I
H

ij

C
where
mh: adimensional factor of intensity of impact-work hardening
ij
K
:
dimensional coefficient of correction
1
K
:
experimental adimensional constant

C
:
experimental adimensional constant
F
:
average value of the normal force for the class ij
I
In the case of the slip (class 1.1 and classes 2.1 to 5.5), the contribution of these classes is modelled by calling upon a factor loading *MW* defined by:
ij
2
m
= k2 F (*V*
W
I
J
ij
)
where
MW
: adimensional factor of intensity of wear by slip
ij
K
: dimensional coefficient of correction
2
F
: average value of the normal force for the class ij
I
V
: average value the speed of slip for the class ij
J
It is then necessary to calculate the percentages balanced for each class of the two categories impact-work hardening and wear by slip:
P = m p
H
H
ij
ij
ij
P = m p
W
W
ij

ij
ij
where *pij* is the percentage of elements of the class ij.
What leads to a total factor of intensity of wear
(2
Pwij)
W = P + P
H
W
ij
ij
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The total factor of intensity *W* is used as factor of correction of the coefficient of wear within the meaning of
the law of ARCHARD according to the expression:
K
= K W/W
KWU
R
R
V = K
Pt
KWU
where *Kr*: is the coefficient of wear of reference obtained in experiments for conditions of test conventional in oscillating slip,
and *W*
is the total factor of intensity evaluated for this same test.

R
2.3
Law of wear “EDF_MZ”
It is currently developed for the only case of the control rods.
The experience feedback shows that the kinetics of wear slows down with time *T*; a manner of holding count observations is to express the volume used in the form:
S0 - S
V
(1 E-*NT*
=
) *S T*

N -
+
where *S0* is initial speed and *S the* speed of wear asymptotic (see
below),
N is a parameter of the model.
The values of *N* and *S* are deduced from the experience feedback.
Tests on simulators, of short duration compared to that of a cycle of operation of one
engine, show that the speed of initial wear *S0* follows a law of the type:
B
S = *A*
0
(*P0*)
where *P0* is the power of initial wear
With
and *B* are coefficients determined by tests on simulators
[bib4]
The experience feedback shows that the speed of wear reaches an asymptotic value *S. the* relation
the preceding one, observed on simulator is supposed to be valid for every moment of the phenomenon of wear.
B
That supposes a power of wear *P* which makes it possible to reach *S* = *A* (*P*), for the values
raised time *T* (typically, one or more cycles of operation).
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The corresponding evolution of the volume used according to time is form:
V
S0 slope
Slope S
T
Worn volume *V* calculated with the assistance operator POST_USURE is written:
B
With

(*P0*) - *S*
V
(1 E-*NT*
=
) *S T*

N

-
+
where *V*
: volume of wear,
P0
: power of wear calculated by *Code_Aster*®,
With, B, S, N
: coefficients of the model defined above.
This model is described in detail by the reference [bib4].
3
Base data
The materials are located by a letter followed by alphanumerics. The codes are indicated below with a usual name and between brackets, standard AFNOR.
A304L
:
Steel 304L (Z2 CN 18-9),

A304LNI
:
Steel 304L nitrided,
A304LCR
:
Chrome steel 304L,
A304LLC1C
:
Steel 304L covered with chromium carbide,
A316L
:
Steel 316L (Z2 NDT 17-12),
A347
:
Steel 347 (Z6 CNNb 18-11),
A405
:
Steel 405 (Z6 CA 13),
A42
:
Steel A42 (A 42),
Z10C13
:
Z10C13 (Z10 C13),
Z6C13
:
Z6C13 (Z6 C13),
I600
:
Inconel 600 (NC 15 Fe),
I600CR
:
Inconel 600 chrome,
I600TT
:
Inconel 600 treaty thermically,
I690
:
Inconel 690 (NC 30 Fe),
I690TT
:
Inconel 690 treaty thermically,
I800

:
INCOLOY 800 (Z5 NC 35-20),
I800CR
:
INCOLOY 800 chrome,
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The tables below give the coefficients of wear for the mobiles and the obstacles for
several material couples (mat1 is the material of the variable component and mat2 that of the obstacle).
The empty boxes correspond to null coefficients. A certain number of situations is
currently envisaged without all the coefficients being available because this data base will be able
to be supplemented with the results of the tests carried out at Department MTC.
Tables of the coefficients for the control rods for the model of ARCHARD:
CONTACT
:
“GRAPPE_ALESAGE” (cf [§4.1])

mat1	**mat2**	**Coef_mobile**	**Coef_obst**	**References**
A304L	A304L	2.6E15	3.7E15	[bib5]
A316L	A304L			

4.2E15
4.1E15
[bib5]
A304LNI
A304L
0.1E15
4.1E15
[bib5]
A304LCR
A304L
0.1E15
5.5E15
[bib5]
A304LLC1C
A304L
0.1E15
5.5E15
[bib5]
CONTACT
:
“GRAPPE_1_ENCO” and “GRAPPE_2_ENCO” (cf [§4.2] and [§4.3])
mat1
mat2
Coef_mobile
Coef_obst
References
A304L
A304L
30.E15
17.E15
[bib5]
A316L
A304L
40.E15
29.E15
[bib5]
A304LNI
A304L
1.E15
124.E15
[bib5]
A304LCR
A304L

1.E15
43.E15
[bib5]
A304LLC1C
A304L
1.E15
34.E15
[bib5]
Tables of the coefficients for the control rods for model EDF-MZ:
CONTACT
:
“GRAPPE_ALESAGE” (cf [§4.1])
mat1
mat2
Coef_mobile
Coef_obst
References
A304L
A304L
With = 2.6E15
With = 3.7E15
B = 1.
B = 1.
[bib5] [bib6]
NR = 2.44E8
NR = 2.44E8
S = 1.14E16
S = 1.14E16
A316L
A304L
With = 11.E15
With = 4.1E15
B = 1.61
B = 1.
[bib5] [bib6]
NR = 2.44E8
NR = 2.44E8
S = 1.14E16
S = 1.14E16
CONTACT
:
“GRAPPE_1_ENCO” and “GRAPPE_2_ENCO” (cf [§4.2] and [§4.3])
mat1

mat2
Coef_mobile
Coef_obst
References
A304L
A304L
With = 20.E15
With = 23.E15
B = 1.05
B = 1.19
[bib5] [bib6]
NR = 2.44E8
NR = 2.44E8
S = 1.14E16
S = 1.14E16
A316L
A304L
With = 500.E15
With = 490.E15
B = 1.78
B = 1.91
[bib5] [bib6]
NR = 2.44E8
NR = 2.44E8
S = 1.14E16
S = 1.14E16
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Tables of the coefficients for the steam generators for the model of ARCHARD:
CONTACT
:
“TUBE_BAV” (cf [§4.4])
mat1
mat2
Coef_mobile
Coef_obst
References
I600
I600
1.2E13
[bib6]
I600TT
I600
4.5E14
[bib6]
I600TT
I600TT
1.4E15
[bib6]
I600
I600CR
7.2E14
[bib6]
I600TT
I600CR
9.1E16
[bib6]
I690TT
I600CR
1.2E15
[bib6]
I600
Z10C13
9.9E14
[bib6]
I600
A405
6.2E14
[bib6]
I690
A405

4.1E16
[bib6]
I600TT
Z6C13
9.2E15
[bib6]
I600
Z6C13
7.1E15
[bib6]
I690TT
Z6C13
7.7E15
[bib6]
I600
A347
1.0E13
[bib6]
CONTACT
:
“TUBE_ALESAGE” (cf [§4.5])
mat1
mat2
Coef_mobile
Coef_obst
References
I690
Z10C13
6.0E17
[bib6]
I600
I600
1.6E13
[bib6]
I690
I600
5.2E14
[bib6]
I600
I600CR
2.2E15
[bib6]
I690

I600CR
4.4E15
[bib6]
I600
A42
2.2E15
[bib6]
CONTACT
:
“TUBE_3_ENCO” (cf [§4.6])
mat1
mat2
Coef_mobile
Coef_obst
References
I600
Z10C13
2.5E16
[bib6]
I690
Z10C13
2.4E16
[bib6]
CONTACT
:
“TUBE_4_ENCO” (cf [§4.7])
mat1
mat2
Coef_mobile
Coef_obst
References
I600
Z10C13
2.4E16
[bib6]
I690
Z10C13
8.2E17
[bib6]
I600
A405
6.5E14
[bib6]

I600TT
A405
1.4E15
[bib6]
I690
A405
7.8E15
[bib6]
I600
I800
1.3E15
[bib6]
I600TT
I800
3.6E16
[bib6]
I690TT
Z10C13
1.2E15
[bib6]
I600
I800CR
2.2E15
[bib6]
I600
A347
2.6E16
[bib6]
CONTACT
:
“TUBE_TUBE” (cf [§4.8])
mat1
mat2
Coef_mobile
Coef_obst
References
I600
I600
1.8E13
[bib6]
I690
I690
1.0E12

[bib6]
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The values indicated above correspond to averages of the values recorded in
references for temperatures as close as possible to the conditions REFERENCE MARK. It should be noted that
reference [bib6] does not give a value of coefficient of wear for the antagonists.

4

Relation between worn volume and the depth of wear

From the power of wear, operator POST_USURE calculates worn volumes then them
depths of wear. Geometrical relations between worn volumes and the worn depths
depend on the type of contact.

Are: D_m : worn depth of the mobile tube,

D_O : worn depth of the obstacle,

R_O : ray external of the mobile tube,

R_m : interior ray of the obstacle,

L_O : width of the obstacle,

: mobile angle/obstacle,

V_m : worn volume of the mobile tube,

V_O : worn volume of the obstacle.

O

4.1

Situation “BUNCH - BORING”

The key word used is “GRAPPE_ALESAGE”. The bunch is centered in a boring. The trace of wear has a section in the shape of lunule [bib6]. Worn volume is brought back to a surface used in a section, multiplied by the worn height *L*

Worn volumes are written [bib3]:

VM = r2 (- (

sin 2) - *R2* - sin 2

m (

( )

L

Vo = R2 - sin 2 - 2 - sin 2

O (

() *R* (

( )

L

R sin = sin

m

() *R* ()

R

(

sin) = *R* sin

O

( )

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The variables *R*, and are intermediate variables of calculation defined on the figure

below:
Nonworn obstacle
Nonworn mobile
Ro
R
Rm

Zone of contact
after wear
A numerical solvor integrated into *Code_Aster*® allows to pass to solve this system of equations coupled to 4 unknown factors, *R*. The depths of wear are then given by the relations following:
D = *R* - *R*
O
T - (*R*
(
cos) - *Rt* C (
bone))

D = *R* - *R*
m
O
- (*Ro* (
cos) - *R* C (
bone)

4.2

Situation “BUNCH - NOTCH SIMPLE”

The key word used is “GRAPPE_1_ENCO”.
The chart of guidance comprises only one notch. Worn volume is brought back to a surface used in a section, multiplied by the worn height *L*.
V
m

= *A D* 3 + *B D* 2 + *C D* + *D*
Worn volumes are written [bib7]: *L*
m m
m m
m m
m
V = 0 4
, 7 *H R D*

O
O
O
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With = -2,76
m

B = 10,30
m
with [bib7]: *C = 0,83*
m

D = 0
m
These coefficients are founded the experience feedback. They apply only to the bunches of order whose characteristics are:
· diameter external of the pencil of bunch: 9,7 mm
· internal diameter of the chart of guidance: 10,5 mm
A solvor integrated into POST_USURE allows to determine *D* according to *V*
m
m
4.3
Situation “BUNCH - NOTCH DOUBLES”
The key word used is “GRAPPE_2_ENCO”.
The chart of guidance is made of 2 notches diametrically opposite. Worn volume is brought back to a surface used in a section, multiplied by a worn height *L*.
V
m

$= A\ D\ 3 + B\ D\ 2 + C\ D + D$
Worn volumes are written [bib7]: L
$m\ m$
$m\ m$
$m\ m$
m
$V = 0\ 9$
$, 4\ H\ R\ D$

O
O
O
$To = -5\ 5$
$, 2$
m

$B = 20,60$
m
with [bib7]: $C = 1\ 66$
,
m

$D = 0$
m
These coefficients are founded the experience feedback. They apply only to the bunches of order whose characteristics are:
· diameter of the pencil: 9,7 mm
· diameter of the chart: 10,5 mm
A solvor integrated into POST_USURE allows to determine D according to V
m
m

4.4

Situation “Tubes of steam generator - Bar antivibratory”

The key word used is “TUBE_BAV”.

Case 1:

The tube is presented vertically, the bar impacts perpendicular to the tube, one supposes that bar does not wear.

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1

2
1 3 *V*
3
3
D

m
=

The depths of wear are written [bib3]:
m

2R

4l
m
D

O = 0
Case 2:
The bar is presented tilted (angle) compared to the tube, the bar impacts perpendicular to tube, one supposes that the bar does not wear.
· if
D < *L*
m

1

2
1 5 15 *V* 5

D
m

=

The depths of wear are written [bib3]:
m

2R

8

m
D

O = 0
· if
D L
m
The relations between worn volume and depths of wear are written [bib3]
:

8 2R
5
52
V
m

=
D
m
m2 - (*D*
- *L*
m
)

15

D

$O = 0$

A solvor integrated into POST_USURE allows to determine *DM* according to *VM*

Case 3:

The tube is presented vertically, the bar impacts perpendicular to the tube, one takes into account the wear of the bar. is an unknown factor to be determined.

The relations between worn volume and depths of wear are written [bib3]:

$$\frac{1}{2}\,V\,\frac{1}{3}\,\frac{3}{3}+3\,m\,(V\,V_m\,O)\,D$$

$$m = \frac{V + V\,2R}{4\,L}\,\frac{m}{O}\,\frac{m}{V}\,+V\,\frac{m}{O} = R2 - R2\,\frac{m}{m}$$

(
sin) C (
bone)
L
D

= R
O
T (1 -
(
cos)) - DM

A solvor integrated into POST_USURE allows to determine
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Case 4:

The bar is presented tilted (angle) compared to the tube, the bar impacts perpendicular to tube, one takes into account the wear of the bar. is an unknown factor to be determined.

· if
(D + D) < L
m
O

The relations between worn volume and depths of wear are written [bib3]:

1
2

V
1
5 15
+
5
m

(V V
m
O)
D

m =

V +V 2R

m
O
m

8

V

+V
m
O

= R2 - R2
m
m
(
sin) C (
bone)
L

L
D = R
O
m (1 -

(
cos)) - DM + if (
N)

2

A solvor integrated into POST_USURE allows to determine
· if
(D + D) L
m
O
The relations between worn volume and depths of wear are written [bib3]:

8 2 R

52
5 2
V
m
m =

+
1+
-
+
1+ -
15 (
D
D
K
D
D
K
L
1 + K)
(m O) ())
(m O) ()
)

V

+V
m
O

=. R2 - R2 sin
() cos
( )
L
m
m

L
D = R
O
m (1 -
(
cos)) - DM + if (
N)
2

V
where K is the relationship between worn volumes of the bar and the tube (K
O
=
)
VM
A solvor integrated into POST_USURE allows to determine D according to V. In the same way, a solvor
m
m
allows to determine.
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4.5
Situation "Tubes of steam generator - Boring"
The key word used is "TUBE_ALESAGE".
Case 1:
The tube is centered perfectly in an animated boring of a pure orbital movement which wears of uniform manner on all the periphery in contact with the obstacle.

V
D
m
m =

2. .l. Rm
The worn depths are written [bib3]:
V
D
O
O =

2. .l. R

O
Case 2:
The tube is centered in an animated boring of a movement of impact-slips of the elliptic type which conduit with the formation of traces of wear of the cylindrical type diametrically opposite on the tube and
having a section in the shape of lunule.
Worn volumes are written [bib3]:
VM = r2 (- (
sin 2) - R2 - sin 2
m (
( )
L
Vo = R2 - sin 2 - 2 - sin 2
O (
() R (
( )

L
R sin = sin
m
() R ()
R
(
sin) = R sin
O
( )
system of equations coupled to four unknown factors to determine: R,
D = R - R
O
m - (R
(
cos) - Rm C (
bone))
D = R - R
m
O
- (Ro (
cos) - R C (
bone)
These formulas have the same origin as those of the paragraph [§4.1].
Case 3:
The tube, animated of a movement of impact-slips, presents this time a slope by report/ratio with the support. One obtains two symmetrical traces of wear on the tube.
VM = r2 (- (
sin 2) - R2m (- S (
in 2))
L
Vo = R2
2
O (-
(
sin 2) - R (- S (
in 2)
L
Rm
(
sin) = R S (
in)
R
(

sin) = Ro S (
in)
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system of equations coupled to four unknown factors to determine: R,
L
D = R - R
O
m - (R
(
cos) - Rm C (
bone)) + sin ()
2
L
D = R - R
m
O
- (Ro (
cos) - R C (
bone) + sin ()
2

These formulas have the same origin as those of the paragraph [§4.1].

4.6

Situation "Tubes of steam generator - Trifoliate"

The key word used is "TUBE_3_ENCO".
That is to say an angle characteristic of the isthmus of the trifoliate boring, defined by the figure below:

Case 1:

The initial contact is carried out against an edge of one of the isthmuses of trifoliate boring. The tube is supposed
centered perfectly compared to its obstacle. The trace of wear does not extend to the entire isthmus.
One does not take into account the wear of the obstacle.
The relations between worn volume and the depth of wear are written [bib3]:

L
X
X

V

= R2
-1
sin
- R2
-1
sin
+ X
2
m
m
O
(R - R + D
O
m
m) + D tg

2

R
R
m

m
O

D
O = 0
2
2
2 2
2

(R - R - - +
O
m
(R R D
O
m
m))
with X = R -
m
(R - R + D 2
4 O
m
m)
A solvor integrated into POST_USURE allows to determine D according to V
m
m
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Case 2:
Same assumptions as for case 1 except the position of the tube compared to the obstacle. One supposes this time that the tube presents an angle of inclination.
· if
D < L
m
The relations between worn volume and the depth of wear are written [bib3]:

D
X

X

V
m

=
R2
-1
sin
- R2
-1
sin
+ X
2
m
m
O
(R - R + D
O
m
m) + D tg

6

R
R
m

m
O

D
O = 0
2
2
2 2
2
(R - R
O
m - (R - R + D
O
m
m))

with X = Rm -
(
2
4 R - R + D
O
m
m)
A solvor integrated into POST_USURE allows to determine D according to V
m
m
· if
D L
m
The relations between worn volumes and depths of wear are written [bib3]:

L
V
m = (V1+ V1V 2 +V 2)

6
D

O = 0
2
2
2 2
-
-
-
+
2
(R R
O
m
(R R D
O
m
m))
with x1 = Rm -
(
2
4 R - R + D
O
m

m)
x1
x1
V1 R2
1
-
- R2
1
-
sin
sin
+ X
2
=
O
(1R - R +d
O
m
m) + D
m
tg
R
R
m
m
O
2
2
2 2
-
-
-
+
-
2
(R R
O
m
(R R D L
O
m
m
))

x2 = Rm -
(
2
4 R - R + D - L
O
m
m
)
x2
x2
V 2
R2
1
-
- R2
1
-
sin
sin
+ X
2
=
O
(2R - R + D - L
O
m
m
) + (D - L
m
m
) tg
R
R
m
O
A solvor integrated into POST_USURE allows to determine D according to V.
m
m
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Case 3:

The contact is carried out against an edge of one of the isthmuses of trifoliate boring. The tube is supposed
centered perfectly compared to its obstacle. One takes into account the wear of the obstacle. is an angle characteristic of the isthmus of trifoliate boring.
Worn volumes are written [bib3]:

1
X
X
2

V

+V = R2
-1
- R2
-1
sin
sin
+ X
m
O
m
O
(R - R +d + D
O
m
m
O) + (D +d
m
O) tg

2

R
R

m
O

V
= 14
. 1 R D
O
O
O
2
2
2 2
2
(R - R
O
m - (R - R + D
+ D
O
m
m
O))
with X = Rm -
(
2
4 R - R + D + D
O
m
m
O)

Case 4:

The contact is carried out against an edge of one of the isthmuses of trifoliate boring. One supposes this time that
the tube presents an angle of inclination compared to its obstacle. One takes into account wear obstacle. is an angle characteristic of the isthmus of trifoliate boring.

· if
(D + D) < L
m

O
Worn volumes are written [bib3]:

D + D
X
X
2

V

+V
m
O
=
R2
-1
sin
- R2
-1
sin
+ X
m
O
m
O
(R - R +d + D
O
m
m
O) + (D
+ D
m
O) tg

6

R
R

m
O

V

= 14
. 1 R D
O
O
O
2
2
2 2
2
(R - R
O
m - (R - R + D
+ D
O
m
m
O))
with X = Rm -
(
2
4 R - R + D + D
O
m
m
O)
A solvor integrated into POST_USURE allows to determine DM according to VM.
· if
(D + D) L
m
O
L
VM = (V1+ V1V 2 +V 2)
Worn volume is written [bib3]:
6
V = 14
. 1 R D
O
O
O
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2
2
2 2
-
-
-
+
+
2
(R R
O
m
(R R D D
O
m
m
O))
with x1 = Rm -
(
2
4 R - R + D + D
O
m
m
O)
x1
x1
V1 R2
1
-

- R2
1
=
-
sin
sin
+ X
2
O
(1R - R + D + D
O
m
m
O) + (D + D
m
m
O) tg
R
R
m
O
2
2
2 2
-
-
-
+
+
-
2
(R R
O
m
(R R D D L
O
m
m
O
))
x2 = Rm -
(4R - R + D + D - L 2
O

m
m
O
)
x2
x2
V 2
R2
1
-
- R2
1
2
=
-
sin
sin
+ X
O
2 (R - R + D + D - L
O
m
m
O
) + (D + D - L
m
m
O
) tg
R
R
m
O
A solvor integrated into POST_USURE allows to determine D according to V
m
m
4.7
Situation "Tubes of steam generator - Quadrifoliate"
The key word used is "TUBE_4_ENCO".
That is to say an angle characteristic of the isthmus of quadrifoliate, definite boring in the same manner as with
paragraph [§4.6]:
Case 1:

The initial contact is carried out against an edge of one of the isthmuses of quadrifoliate boring. It is supposed
tube perfectly centered compared to its obstacle. One does not take into account the wear of the obstacle. Worn volume is written [bib3]:
L
X
X

V = R2
-1
sin
- R2
-1
sin
+ X
2
m
m
O
(R - R + D
O
m
m) + D tg
2

R
R
m

m
O

C = 0
2
2
2 2
2
(R - R - - +
O
m
(R R D
O
m

m))
with X = R -
m
(R - R + D 2
4 O
m
m)
A solvor integrated into POST_USURE allows to determine DM according to VM
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Case 2:

Same assumptions as for case 1 except the position of the tube compared to the obstacle. One supposes this time that the tube presents an angle of inclination.

· if
$D < L$
m

The relations between worn volumes and depths of wear are written [bib3]:

$$D X X V = R2 \text{-}1 \sin - R2 \text{-}1 \sin + X 2 m m O (R - R + D O m m) + D \text{ tg}$$

6

R
R
m

m
O

D
O = 0
2
2
2 2
2
(*R* - *R* - - +
O
m
(*R R D*
O
m
m))
with *X* = *R* -
m
(*R* - *R* + *D* 2
4 *O*
m
m)
A solvor integrated into POST_USURE allows to determine *DM* according to *VM*
· if
D L
m
The relations between worn volumes and depths of wear are written [bib3]:

L
V

=
1 +
1 2 + 2
m
(*V V V V*)

6
D

= 0
O
2
2
2 2
-
-
-
+
2
(*R R*
O
m
(*R R D*
O
m
m))
with *x1* = *Rm* -
(
2
4 *R* - *R* + *D*
O
m
m)
x1
x1
V1 R2
1
-
- *R2*
1
-
sin
sin
+ *X*
2
=
O
(1R - *R* +d
O

m
m) + *D*
m
tg
R
R
m
m
O
2
2
2 2
-
-
-
+
-
2
(*R R*
O
m
(*R R D L*
O
m
m
))
x2 = *Rm* -
2
4 (*R* - *R* + *D* - *L*
O
T
m
)
x2
x2
V 2
R2
1
-
- *R2*
1
2
=

-
sin
sin
+ *X*
O
(2R - *R* +d - *L*
O
m
m
) + (*D* - *L*
m
m
) tg
R
R
m
O
A solvor integrated into POST_USURE allows to determine *D* according to *V*
m
m
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Case 3:

The contact is carried out against an edge of one of the isthmuses of quadifolié boring. The tube is supposed
centered perfectly compared to its obstacle. One takes into account the wear of the obstacle.
Worn volumes are written [bib3]:

1
X
X

V

+V = *R2*
-1
sin
- *R2*
-1
sin
+ *X*
2
m
O
m
O
(*R* - *R* +D
O
m
m) + *D* tg

2

R
R
m

m
O

V
= 18
. 8 *R D*
O
O
O
2
2
2 2
2
(*R* - *R* - - + +

O
m
$(R\ R\ D\ D$
O
m
m
$O))$
with $X = R -$
m
$($
2
$4\ R - R + D + D$
O
m
m
$O)$

Case 4:

The contact is carried out against an edge of one of the isthmuses of quadrifoliate boring. One supposes this time
that the tube presents an angle of inclination compared to its obstacle. One takes into account
the wear of the obstacle.

· if
$(D + D) < L$
m
O
Worn volumes are written [bib3]:

$D + D$
X
X
2

V

+V
m
O
=
$R2$
-1
sin
$- R2$
-1
sin

+ *X*
m
O
m
O
(*R* - *R* + *D* +d
O
m
m
O) + (*DM* + C). tg

6

R
R

m
O

V
= 18
. 8 *R D*
O
O
O
2
2
2 2
2
(*R* - *R*
O
m - (*R* - *R* + *D*
+ *D*
O
m
m
O))
with *X* = *Rm* -
(
2
4 *R* - *R* + *D* + *D*
O
m

m
O)
· if
(*D* + *D*) *L*
m
O

L
V
m = (*V1* + *V1V* 2 +V 2)
Worn volumes are written [bib3]:
6
V

= 18.8 *R D*
O
O
O
2
2
2 2
-
-
-
+
+
2
(*R R*
O
m
(*R R D D*
O
m
m
O))
with *x1* = *Rm* -
(
2
4 *R* - *R* + *D* + *D*
O
m
m
O)

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x1
x1
2
V1 R2
1
-
- *R2*
1
=
-
sin
sin
+ *X*
O
(1R - *R* + *D*
O
m
m) + (*D* + *D*
m
m
O). tg
R
R
m
O
2

2
2 2
-
-
-
+
+
-
2
(*R R*
O
m
(*R R D D L*
O
m
m
O
))
x2 = Rm -
(4R - *R* + *D* + *D* - *L* 2
O
m
m
m
)
x2
x2
V 2
R2
1
-
- *R2*
1
2
=
-
sin
sin
+ *X*
O
2 (*R* - *R* + *D* + *D* - *L*
O
m

m
O
) + (*D* + *D* - *L*
m
m
O
) tg
R
R
m
O

4.8 Situation `Tube of steam generator - Tube of generator of vapeur'

The key word used is “TUBE_TUBE”. Following the rupture of a stopped tube, there can be contact between it
tube and one of its neighbors. The wear of the two tubes by accommodation of surfaces leads to the contact
with the creation of two plane surfaces. This assertion is confirmed by tests carried out on
machine of wear.

1

2
1 5 15 *V* 5
D

m
T =

2 *R*

8

m
The worn depths are written [bib3]:
1

2
1 5 15 *V* 5
D
O

$O =$

$2 R$

8

O
5

Division the figure of play in sectors

The user with the possibility of defining a figure division of play in angular sectors for which it gives a type of contact (GRAPPE_1_ENCO…), a coefficient of wear and angles of beginning and of end of cutting (these angles must be increasing between -180° and +180°). Power of wear for each sector is then calculated like the arithmetic mean over the moments, beforehand cut out in blocks, of the product of the standards of the normal force of shock and the speed of slip by holding account only contacts which take place in the angular sector concerned. From this power, it is possible to define a volume used by multiplying the power of wear of the sector by coefficient of wear of the sector and by an operating time given by the user. He is also possible to calculate the depth of wear for this sector, by supposing that extension angular of the defect that of the sector does not exceed where it is detected.
In fact the key word SECTOR makes it possible to define the whole of these modifications.
It is not envisaged to check the total coherence of calculations carried out. In particular, a wear can to be distributed on several sectors and in this case, the calculation depth of wear does not have any more a smell.
It is up to the operator to be ensured a posteriori of the validity of its results. A new calculation with another cutting must be possibly carried out to obtain the value depth of wear. This choice is not constraining because of the speed of postprocessing considered. Interest to carry out these calculations in CONTINUATION is obvious, taking into account what precedes.
A particular case deserves a development. It is the case of the control rods for which results of POST_USURE are used in entry of order MODI_OBSTACLE. In this case, it a number of sectors is fixed at 10, as it is explained in the reference [bib8]. The operator MODI_OBSTACLE uses data resulting from the experience feedback to calculate wears which can extend on several sectors while setting out again from the worn volumes obtained using POST_USURE. In this case, the worn depths of POST_USURE necessarily do not have physical significance.

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Actualization of the table

Operator POST_USURE extrapolates the worn volume obtained in a few seconds of simulation with durations defined by the user (typically a few months, even a few years).
He restores a table which contains worn volumes and the depths of wear for all the sectors and every moment defined by the user by cumulating them since the initial moment of simulation.
It is possible to give a table to be reactualized by using key word ETAT_INIT. That allows to hold of the evolution of the geometries related to wear:

· Starting from a figure of play, the user carries out a dynamic calculation.
· It obtains volumes and depths of wear at exit of POST_USURE.
· Using geometrical considerations, it evaluates the evolution of the figure of play connected to this wear thanks to operator MODI_OBSTACLE.
· It carries out a new dynamic calculation with the figure of play modified.
· It deduces some from new sizes related to wear and cumulates them in the table result of POST_USURE.

By reiterating the process a certain number of times [bib9], it is possible to take into account the evolution
geometries according to wear and to deduce the impact from it from this phenomenon on dynamics from studied system.

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Criteria of structural stability

Summary:

This document presents the various criteria of stability, with the directions buckling of structure, available in
Code_Aster. One can classify them according to two categories:

· criterion of Euler on linearized problem,
· nonlinear criteria.

These criteria make it possible to detect the loss of unicity in solution of the quasistatic problem.
They are directly transposable with the framework of dynamics, but as they do not take account nor of stamp of mass nor of that of damping, one cannot speak about dynamic criterion of stability to the direction
traditional (for example, of negative or null damping becoming).

The nonlinear choice of criteria fulfills the requirements of:

· versatility (general method for any relation of behavior and being able to accept all
tensor of deformation available in the code),
· minimization of cost CPU and the additional obstruction memory.

The criterion presented is a generalization of the criterion of Euler, based on the analysis of the matrix of total stiffness
reactualized. It is called within operator STAT_NON_LINE, to be able to be evaluated with each step of nonlinear quasi-static incremental resolution.
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Count

matters

---

1 Introduction

Code_Aster allows the research of the modes of linear buckling, qualified method of Euler.
It is enough to solve a problem generalized with the eigenvalues (thanks to operator MODE_ITER_INV
or MODE_ITER_SIMULT and key word TYPE_RESU=' MODE_FLAMB'). The two matrices arguments of
generalized problem are the matrix of rigidity and the matrix of geometrical rigidity, resulting from a calculation
linear elastic precondition (operator MECA_STATIQUE).

In all the cases where one cannot neglect nonthe linearities, which they are geometrical or behavioral, the Euler approach is not valid any more.
We thus propose an ad hoc criterion, that one can regard as a generalization of the criterion of Euler on reactualized configuration. This criterion is built on the matrix of tangent stiffness assembly, which is calculated in the algorithm of the Newton type to solve the problems quasi nonlinear statics (operator STAT_NON_LINE).
This criterion, into nonlinear, makes it possible to treat the elastic relations of behavior rigorously nonlinear. On the other hand, the laws which present a dissipative aspect are treated rigorously that if the loading, in any point of the structure, follows a monotonous evolution (that corresponds to the assumption of Hill [bib4]).

2
Study of the stability of a structure

2.1
General concept of buckling

Buckling is a phenomenon of instability [bib6]. Its appearance can be observed in particular on slim elements of low stiffness of inflection. Beyond of a certain level of loading,
structure undergoes an important change of configuration (which can appear by the appearance sudden of undulations, for example).
One distinguishes two types of buckling: buckling by junction and buckling by limiting point ([bib1], [bib7], [bib8]). To describe the behavior of these two types of buckling, one is considered structure of which the parameter μ is characteristic of the loading and whose parameter is characteristic of displacement.

A'

stable
μ
unstable
B
μcr
With

B'
unstable
stable
O

Appear 2.1-a: Buckling by junction
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Between the point O and point A, the structure admits only one family of curve (μ,), it can, for example to act of traditional linear elasticity or elastoplasticity, where if the problem is well posed (cf [§ 2.2]), there is the traditional result of existence and unicity of the solution.
On the other hand, beyond point A, several families of curves are solution of the problem of balance. This loss of unicity is accompanied by an instability of the initial branch (known as fundamental). connect secondary can be stable (curved AB) or unstable (curve AB'). The load beyond which there is junction is called the critical load μcr.
Buckling by junction is characterized by the fact that the mode (or direction of buckling), which initiate the secondary branch, does not generate additional work in the loading applied: mode of buckling being orthogonal to him.
An example of buckling per junction with instability of the secondary branch is in case of a circular cylindrical hull under axial compression [bib10]. Examples of buckling by junction with stability of the secondary branch are in elastic beams in axial compression, of the circular rings in radial compression and the rectangular plates in longitudinal compression.

µ
With
stable
unstable
O

Appear 2.1-b: Buckling by limiting point

µ
With
µcr
A'
stable
unstable
stable
O

Appear 2.1-c: Buckling by point limits with breakdown
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On figures [Figure 2.1-b] and [Figure 2.1-c], which illustrates buckling by limiting point, the structure does not admit that only one family (µ,) of solution of the equilibrium equations. At point A, there is loss of
stability of the solution with total loss of rigidity in the case of the figure [Figure 2.1-b] and with one

phenomenon of breakdown in the case of the figure [Figure 2.1-c] (the solution becomes again stable after one
discontinuity of displacement; case of a segment of a sphere under external pressure). Point A is then called not limits.

The problem thus amounts in all the cases seeking the load from which the branch
fundamental of balance becomes unstable or of dubious stability. That generally mobilizes them great displacements.
One can finally have the case of the ruin by plastic flow which is connected at the limiting point [Figure 2.1-b].

2.2
Writing of the mechanical problem

This chapter aims to introduce the formalism general of structural analysis adapted to
nonlinear mechanical problem which we wish to tackle.
To start, we thus briefly will point out the setting in equation of a standard problem of
structural analysis. To simplify, we place ourselves, all at least at the beginning, within the framework of
small disturbances.

fd
S1
Fd
Ud
S
S2
Appear 2.2-a: Representation of a problem of structural analysis

*The structure S is subjected to imposed voluminal efforts **fd**, surface efforts D*
***F** on*
edge
*S2 and of the displacements imposed **Ud** on the remainder of the edge of S, noted*
S1.
*The unknown factors of the problem of reference on the solid are the field of displacement **U** and the field*
*of constraint of Cauchy. The solution (**U**,) of the problem structure where the heating effects are neglected is defined as:*

*To find (**U**) 1*
,
***H** (*

2
*S) × **L** (S) which checks:*

· Equations of connections:
U
*= **Ud***

éq 2.2-1
S1
· Relation of behavior:
= F () with which is the tensor of deformation

éq 2.2-2
1
= (
T
U** + **U**) in assumption of small disturbances **éq

2.2-3
2
If a linear elastic behavior is supposed:
=
C

éq 2.2-4
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· Equilibrium equations:

2
D ***U***
= + ***fd*** *with =*

D T 2

éq 2.2-5

N
= ***F***

D
S2

2.3
Study of stability of the system

The object of this chapter is to present the methods making it possible to determine the stability of balance
quasi-static nonlinear of a structure. To start, we are interested only in
detection of instability, or more exactly with the loss of unicity of the solution [bib6]. Among work of

synthesis recent, one can quote [bib9] or [bib7] and [bib8] which presents very complete papers on nonlinear analysis of stability of the structures.
The calculation of the post-critical solution will not be approached.

To make the analysis of stability, we introduce an initial configuration of reference S0, one current configuration S and a disturbed configuration S1:

S1
u1

S
S0
u0

U
Appear 2.3-a: Definition of the various configurations
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*That is to say **U the** field of displacement of the points of the structure. The behavior is supposed, for moment, linear elastic isotropic. The structure subjected to imposed displacements and efforts goes to deform and become the structure located by the current configuration S. We seek with to determine a state of balance characterized by the field of displacement between the initial configuration*
S0 and current configuration S, as well as a stress field of Cauchy, noted, or of Piola-Kirchhoff II, noted:
***F** = **U** + **I**:*
tensor gradient

transformed

of

tion

-1

*= det **F** **F***
0
with
*det **F** =*

éq

2.3-1
:
tensor
Piola

of

- Kirchhoff I

0 -1
T
-
=
F F

In this expression, one sees appearing the relationship between the initial density 0 and masses it voluminal current.
The following stage is the prediction of the stability of this balance.

To this end, we will seek a criterion allowing to determine if there is only one field of displacement balancing the efforts applied. We will suppose that the efforts increase gradually and we will seek to find as from which moment there are two configurations S and S1 which respect the equations of the problem: we seek a point of junction, i.e. a loss of unicity of the solution. This moment will be described as moment of buckling.

2.3.1 Writing of the elastic geometrical nonlinear problem

The solution U, of the problem structure without heating effects checks ([bib1], [bib7], [bib2]):

· Equations of connections:
U
= U

éq 2.3.1-1
D
S10

· Elastic Relation of behavior:

=, () with
tensor

is

who
die

of

formation éq

2.3.1-2
comporteme

one

suppose

one

If
liné

rubber band

NT
surface:
=
C

éq 2.3.1-3
· Equilibrium equations:

2
D U
= + F

with

D
=

dt2

éq 2.3.1-4
F n0
=

Fd
S 20
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The associated tensor of deformation is that of Green-Lagrange (referred with the initial configuration):

(
1
U) = (FT F - I) with

F = U + I
2
(
1
U)
L
= (U) +
Q
(U, U)

éq 2.3.1-5
2

1
L
(U) = (U + You)
linear

part
:
With
:
2

Q
(U, U) = T U U
quadratiqu

part
:
E

We can now write the Principle of the Virtual Powers in nonlinear elasticity geometrical and into quasi-static:

Pint - Pext =,
0 *
U CA

0
int
P

= Tr (
)
L
1 Q

D =
L
****
Q
****
Tr U
U, u.a. U
U, U
D

( )+
( ) (( )+ (
)

S 0
S0
:
With

2

ext.
****
****
P
=
D
F U dS +
fd U

D

S 0
S0
éq 2.3.1-6

In order to obtain a discretized formulation, one can rewrite the tensor of deformation:

L 1 NL

(U) = B + B (U) U

2

éq

2.3.1-7

=
C (U) with

tensor

is

who
Piola

of

- Kirchhoff II

The power of the internal efforts becomes:

int
P
= Tr
T
L

NL
****
B
B
U U D

éq 2.3.1-8

[[+ ( )]]
S 0

By taking account of the relation of behavior [éq 2.3.1-3]:

int
L 1 NL
T
P
= Tr
L
NL
****
B
B
U
C B
B
U U U D

éq 2.3.1-9

+
( ) [+

( )]

S 0

2

After discretization by the finite elements, one can put this equation in matric form:

****
U [
L
K

éq 2.3.1-10
0 + K (U)
Q
+ K (U)]
ext.
U = P
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The matrix
L
K is symmetrical and there are the following expressions:

T
K
B CB
0 =
L
L

D

S0

1
L

NL
T
T
L
L
NL

K (U) =
B
U CB
B CB
U D

éq 2.3.1-11

( )
+
( )

S 0 2

1
Q
K (U) =
NL
B (U) T

NL
CB (U)
D

2 S0

One obtains directly what precedes the writing in matric form by balance:

[
L
K

éq 2.3.1-12
0 + K (U)
Q
+ K (U)]
ext.
U = F

That is to say still, in an equivalent way:

Fint =
T
ext.
F with Fint = [L
B + NL
B
(U)]
D

éq 2.3.1-13
S0

We can just as easily formulate the Principle of the Virtual Powers starting from the state of constraint of Cauchy and the tensor of deformation of Almansi (thus on the current configuration). One
obtains then:

((*

Tr
U) D =
****
****
D
F U dS +
fd U
D

éq 2.3.1-14

S

S
S

That one can also put in the form, after discretization:

T
int
ext.
B D F
=
= F

éq 2.3.1-15
S

That is to say still, by supposing the elastic relation of behavior:

Ku = ext.
F with K
= T
B CB
D

éq 2.3.1-16
S

The integrals of these equations are calculated on volume running S which depends, of course, field of displacement solution U. In the same way, the operator B must be calculated on the configuration
current S and not on the initial configuration S0, as it was the case previously.
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2.3.2 Study of stability into nonlinear geometrical

One will seek if there is a second field of displacement kinematically acceptable which checks equilibrium equations: one thus seeks to know if there will be junction.
This second field will be written as the sum of a disturbance added to the first solution,
that is to say: U + u1, with which is a very small reality and that one will make tend towards 0. The field u1 is
chosen kinematically acceptable to 0.

The Principle of the Virtual Powers will be then written for this new field.
The field of deformation is put in the form:
(

L
1
U + U = U + U
Q
+ U, U
Q
+ U, U
Q
+
U, U éq

2.3.2-1
1)
( ) (1) ((1) ( )
2
1

(1 1)

2

2
The virtual deformations are given by:

L
= (*
U
Q
+ U, U
Q
+
U, U = U
Q
+
U, U

éq 2.3.2-2
1
) (*)
(*
1
) (*)
(*

1
)

In the same way, if we choose S0 like configuration of reference, the constraints become:

L
1
= + C U
Q
+ U, U
Q
+ U, U +
C Q
U, U éq 2.3.2-3
1
(1)
((1) ( )
2
1

(1 1)

2

2

We can now express the Principle of the Virtual Powers for the field of disturbed displacement. Let us take as assumptions that the imposed forces do not depend on displacement and that the initial configuration is selected like reference.

int
int
P
P
1
=

Q
****

****

L
1 Q
Q

+ Tr ((U, U D
Tr u.a. U
U, U
U, U
D
O
1
) +
1

( ) (1)+ ((
1) +
(
)

+ ()

S0
S0

2

ext.
ext.
P
P
1
=

int
ext.
P
P
1
- 1 = 0
éq 2.3.2-4

For sufficiently small, it will be enough that the term proportional to in the expression [éq 2.3.2-4] is no one so that the Principle of the Virtual Powers is checked for the field U + u1. In this case, it more unicity of the solution will not thus have there, which will translate the loss of stability of the system.

When the imposed efforts do not depend on the geometrical configuration, the study of stability thus state yourself like:

Knowing the current state, i.e the field of displacement U kinematically acceptable and the field of constraint, if there is a field of u1 displacement kinematically acceptable to 0 and such as, for any displacement *
U kinematically acceptable with 0, one has:

Tr

(Q
(U, *
1 U)
D
S0
+
Tr

*[L (*U) L*
C
(U) Q
*+ (U, **
L
L
Q
Q
Q
1
U)
C
*(U) + (**
1
U)
C
*(U, U) + (U, **
1
U)
C
(U, u1)]
D
S0
= 0
éq 2.3.2-5
Then the problem considered is unstable.

One can express this condition of junction in matric form while introducing, moreover, stamp geometrical stiffness K () which discretizes the first term of it:

T
U
** CA 0, u* K U*
T1 = 0

éq 2.3.2-6
With K
= K0 + K L
T
(U) + KQ (U) + K ()
stiffness

is

who
tangent

If one writes the condition of junction on the current configuration S, then one a:

****
U

CA0,

****
U T [K + K ()]U = 0

éq 2.3.2-7
1

The constraint to be considered is then the constraint of Cauchy and all the integrals are evaluated on the field running S.

2.3.2.1 Stability condition of a nonlinear elastic balance

It comes immediately, that if there is a state such that the tangent matrix K T defined above is singular, we will have exhibé a field of u1 displacement well not no one which shows the loss of unicity of the solution of the mechanical problem. This field of displacement is the mode of buckling.
One can notice that the condition of junction is well checked, whatever the standard and it sign u1: in this direction, one thus speaks about mode of buckling, like direction, because one is limited in [éq 2.3.2-4] to the first order in.

2.3.2.2 Case of small displacements: charge of Euler

When displacements can be qualified the small ones before buckling, one can confuse initial configuration with the current geometry. Matrices
L
K and Q
K can then be
neglected. Moreover, the constraint can be confused with the usual constraint; equations of buckling are written then:
[K + K U =

éq
2.3.2.2-1
0
( )]
0
1
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It is advisable to notice that the matrix K () is proportional to and thus to the loading applied to the structure. If one multiplies the constraint by, one obtains:
[K + K
U =
éq
2.3.2.2-2
0
( )]
0
1
This equation immediately makes think of a problem generalized with the eigenvalues, the same one type that in the case of the research of the modes of vibration, which is written:
[
2
K - M v =
éq

2.3.2.2-3
0
] 0
1
The matrix K () is replaced by the matrix of mass M, and one sees appearing the own pulsation , whereas v1 is the associated mode of vibration.

If one wishes to study buckling under loading of which only a part is controlled (part variable of the loading), by a principle of superposition, the contribution, constant, loading not controlled must be added at the end K 0 and only the constraint generated by the loading controlled will be in the term in. Formally, the following problem is thus posed:

[K0 + K (cte) + K (VAr)]u1 = 0

géné

constraint
:
rée by

loading

controlled

not

éq
2.3.2.2-4
cte
:
With
gén

constraint
:
érée by

loading

controlled

VAr

The two stress fields are obtained by resolution of two linear problems, one for loading not controlled, the other for the controlled part of the total loading. Documentation [U2.08.04] [bib17].

2.3.2.3 Case of the imposed forces depend on the geometry

Example of the following pressures:
When the external forces depend on the configuration, that involves that the work of the forces external intervenes under the stability condition. Let us take the example of a pressure applied to structure. This pressure will be supposed to be constant during buckling: in other words, the value of pressure does not change during displacement.
This assumption corresponds to two types of real problems. The first type is that where volume fluid imposing the pressure on the structure is very large in front of the variations of volume generated by the displacement of the solid. The problems of pressure tanks intern where them displacements of walls are considerable compared to dimensions of the structure itself thus do not return within this framework.
The second case corresponds to the existence of a source of fluid which makes it possible to maintain the pressure with a constant value. It is not then necessary any more to worry about the amplitude of the displacement of solid.
The value of pressure being taken fixes, the variation of the normal in the course of time are to be taken in count. This variation is due to the field of displacement which modifies the surface of the structure. Of even, if one reasons in terms of resultant and thus of integral, the element of surface can too to change surface. Consequently, the resultant of the compressive forces will vary and it is advisable to hold some count.
We see quickly that the power of the efforts, expressed on the current configuration, associated to a pressure is given by the following equation (see for example [bib11]):
ext.
P
N
1
***
N
U
éq
2.3.2.3-1
pressure =

dS

p
+
1

dS
SP

dS

In this equation, we notice that the power of the external efforts is modified in
u1 displacement. We will have then:
ext.
P
P
N U
éq
2.3.2.3-2
1

*= ext. + p 1 *dS*

1
SP
Finally, the matrix K T is enriched by an additional term, function of the pressure:
L
K = K + K U + K U + K + K

éq
2.3.2.3-3
T
0
( )
Q ()
() (p)
If one writes the operators on the current geometry, one leads to:
K = K + K () + K (p
éq
2.3.2.3-4
T
)
When we are in the presence of following compressive forces, same methods as those presented previously will be able to apply to calculate the buckling loads: it will be enough to to supplement the matrix K T with the new term K (p). One can show that the matrix K (p) is symmetrical if the compressive forces do not work on the "edge" of the model.

2.3.2.4 Vibrations under prestressing

Same methodology can also apply to the study of the vibrations of the structure in configurationcourante S. This structure is prestressed and deformed. It is enough to write the Principle
virtual Powers nonlinear geometrical [éq 2.3.1-6] by taking account of the effects of inertia and by injecting the assumption there that displacements are of the periodic functions of the type:
U
= v

éq
2.3.2.4-1
1 (T)
sin
1
(T)
It results from this:
[K +KL (U) +KQ (U) +K () +K (p) 2

- M v = éq 2.3.2.4 - 2
0
] 0
1

Let us interest in this equation.
First of all, we notice that when we have a state the Eigen frequency criticizes then of vibration of the structure corresponding to the mode of buckling is null.
Moreover, we observe that the Eigen frequencies of the structure charged are different from those initial structure for two reasons:
The own pulsation is modified by prestressing p: it is the principal effect which is used, by example, to grant a violin. The tension of the cord exploits the height of the note corresponding, therefore on its Eigen frequency.
A second effect is the variation of the frequency by modification of the geometry: the matrix of geometrical starting stiffness K 0 is replaced by the matrix of stiffness on the current geometry:
L
Q
K 0 + K + K. What causes to modify the vibratory equations.

2.3.2.5 Treatment of the elastoplastic behavior (plastic buckling)

Far from any exhaustiveness, we will present only the simplest approaches here, for their easy establishment in the code.
When the structure functions in an elastoplastic mode, buckling is affected by the loss of resistance due to plasticity [bib2]. The modification comes from the relation of behavior during additional displacement u1.
The constraint becomes, in incremental form:
=

+ C [L

+
+

2.3.2.5-1
T
(U)
Q (U, U)]
2
C
Q U, U
1

T
(1 1)
2
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In this expression, the matrix of behavior is tangent matrix CT. The choice of this matrix is not immediate: indeed, the matrix depends on u1 and is thus not known as long as it mode is unknown. One can, for example, to discharge during buckling if the mode develops in a direction and to charge if it develops in the opposite direction. It is thus necessary to do one assumption for the behavior during plastic buckling. , We will start to apply the assumption of Hill [bib4] who leaves the principle that the structure continues to charge plastically during buckling.
Let us consider an elastoplastic law of Von Mises type. We define the three modules: E which is the Young modulus, AND the tangent module, and ES the secant module. These modules are recalled on following figure:

AND
ES
E

Appear 2.3.2.5 - has: Representation of the various modules on a traction diagram 1D

Then we propose three possible methodologies.
The assumption of the tangent module simply consists in replacing the Young modulus by the module tangent in the relation of behavior. One obtains then:
E
C =
C
2.3.2.5-2
T
AND
This method is very rudimentary, but it is always pessimistic, which can constitute one favour, if one places oneself from the point of view of dimensioning.

The method used usually consists in using the tangent matrix of incremental calculation (operator STAT_NON_LINE [bib14]). We thus have the following equation in the case of plasticity of Von Mises [bib15]:

T
D
D
With
AC

CT = CI -

T
D
D

AA

H +

D

VM

2.3.2.5-3
D
vector
:
diverter

constraint

S

T

With
D
D
D

With
intervenan

stamp
:
Settings

Von
of

normalizes

in
T

=

With

VM

E E

T
H
by

defined

plastic

slope
:
: h=

E AND
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This method is perfectly rigorous only in nonlinear elasticity or if one respects
the assumption of Hill: it does not make it possible to predict the junctions in the ways of loading. As of
that the relation of behavior is dissipative, the critical loads calculated will not be exact that
if one can check that the loading is monotonous, in any point of the structure (Hill [bib4]).

The most realistic method consists in using the finished theory of the deformation only for
to calculate the plastic load of buckling. The tangent matrix of behavior is given by
the equation below:

1
-
1
1
T
D
D

C
2.3.2.5-4
T =
With [
]

With

1
-
1
1
-
+ C +
-
With
E
E
D

E
E
T
S
S

VM

Compared with the method based on the matrix of tangent stiffness [éq 2.3.2.5 - 3], this criterion requires
construction and assembly of a specific total matrix. This expensive operation comes to weigh down the incremental resolution.
For considerations of general information and minimization of the development cost and cost of calculation (CPU and memory), we thus choose the criterion based on the tangent module [éq 2.3.2.5 - 3].

3
Establishment in the code

In any rigour, in order to secure the analysis of stability of a nonlinear quasi-static calculation, it is necessary
to use the criterion of ad hoc stability to each step of incremental calculation. Any criterion of stability not
linear must thus be intrinsically the least expensive possible in time CPU and place memory.
Speaking Algorithmiquement, it appears judicious to establish the call to the criterion inside even of routine corresponding to operator STAT_NON_LINE [bib14]. Indeed, the principle of call to each step
put up badly with a completely externalized operation of the incremental method of resolution of the nonlinear mechanical problem.

3.1 Criterion
of Euler

This criterion (cf [§ 2.3.2.2]) requires only the resolution of a linear static problem, then construction and assembly of the geometrical matrix of stiffness. This one and stamps it stiffness assembly are then to pass like argument of a solvor [bib12] for the problem to the values clean [éq 2.3.2.2 - 2].

At exit one thus recovers the modes of buckling and the critical loads corresponding. For more details, the user will be able usefully to consult the document [U2.08.04] [bib17].
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3.2
Nonlinear criterion

3.2.1 Impact on operator STAT_NON_LINE

Let us start by briefly pointing out the operation of the incremental method of resolution nonlinear problems of structure [bib14].

3.2.1.1 Algorithm
of
STAT_NON_LINE

One will use index I (like "moment") to note the number of an increment of load and exhibitor N (like "Newton") to note the number of the iteration of Newton in progress. The algorithm used in operator STAT_NON_LINE can then be written schematically in the following way:

(U, and known
0
0)
0

Buckle over moments T (or increments of load): loading L = L T
I
(I)
I

.
(U,
known
I

1
-
I 1
-)
· Prédiction: calculation of
0
U and
0

I
I
· Boucle on iterations of Newton: calculation of a continuation (N
N
U,
I
I)
.
(N N
U, and (N
N
U, known
I
I)
I
I)
· Calcul of the matrices and vectors associated with the following loads
· Expression of the relation of behavior
.
calculation of constraints N
and of internal variables N
starting from the values
I
I
1

-
and with preceding balance (T) and of the increment of displacement
1
-
I 1
-
N
U
U
U since this balance
I =
N
I -
I 1
-
·
calculation of the “nodal forces”:

N
T
N
Q + B
I
I
·
possible calculation of the matrix of tangent stiffness:
N
K = K (N
U)
I
I
· Calcul of the direction of research
N 1
+
(U,
N 1
+
) by resolution of a system
I
I
linear
· Itérations of linear research:
· Actualisation of the variables and their increments:

n+1
N
n+1
U
=u + U
U =u + U
I
I
I

n+1
N
n+1

and
I
I
I

n+1
N
n+1
= +

= +
I
I
I

n+1
N
n+1
I
I
I
· Test of convergence
· Archivage of the results at the moment T
I

U = U
U
1 +

I
I
I
= 1 +
I
I
I
I

I

It is noticed that there are three overlapping levels of loops: a loop external on the steps of time, a loop of iterations (qualified the total ones) of Newton and subloops possible for linear research (if it is asked by the user) and certain relations of behavior requiring iterations (known as interns), for example for elastoplasticity in plane constraints.

If one chooses the criterion based on the assembled tangent matrix [éq 2.3.2.5 - 3], it is necessary to have this stamp reactualized with each step where one wants to make the analysis of stability.
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It is the case when one uses a method of the Newton type, and not a method of the Newton type modified.

One leads then to the following algorithm:

*(**U**, and known*
0
0)
0

*Buckle over moments T (or increments of load): loading **L** = **L** T*
I
(I)
I

.
*(**U**,*
known
I

1
-
I 1
-)
· Prédiction: calculation of
0
***U** and*

0

I
I
· Boucle on iterations of Newton: calculation of a continuation (N
N
***U**,*
I
I)
·
(N N
***U**, and (N*
N
***U**, known*
I
I)
I
I)
· Calcul of the matrices and vectors associated with the following loads
· Expression of the relation of behavior
·
calculation of constraints N
and of internal variables N
starting from the values
I
I
1
-
and with preceding balance (T) and of the increment of displacement
1
-
I 1
-
N
U
U
***U** since this balance*
I =
N
I -
I 1
-
·

calculation of the “nodal forces”:

N
T
N
Q *+* ***B***
I
I
·
possible calculation of the matrix of tangent stiffness:
N
K *=* ***K*** *(N*
U*)*
I
I
· Calcul of the direction of research
N 1
+
*(****U****,*
N 1
+
) by resolution of a system
I
I
linear
· Itérations of linear research:
· Actualisation of the variables and their increments:

n+1
N
n+1
U
=u + ***U***
U *=u +* ***U***
I
I
I

n+1
N
n+1

and

I
I
I

n+1
N
n+1
= +

= +
I
I
I

n+1
N
n+1
I
I
I
· Test of convergence
· Archivage of the results at the moment T
I

U** = **U
U
1 +
I
I
I
= 1 +
I
I
I
I

I

.
Criterion of stability, function of the reactualized tangent stiffness:
N
***K** = **K** (N*
***U**)*

I
I

The criterion is calculated at the end of the step, just after filing. It thus has like arguments the quantities
converged with the current step. Moreover, this choice of position of call makes it possible to hold account correctly
following loadings, since their calculation is done at the time of the iterations of Newton. The criterion could thus be called before the end of these iterations.

3.2.1.2 Impact on the structure of data result of STAT_NON_LINE

The call of the nonlinear criterion of stability will induce the resolution of a problem to the eigenvalues.
The result of this calculation will be thus a whole of couples critical load/mode of buckling.
The critical loads are scalars and the associated modes are fields of displacement, which will come to enrich the structure of data result by STAT_NON_LINE.
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3.2.2 Characteristics related to the tensor of deformation

In the code, it is advisable to distinguish two great families from description of the deformations.

On the one hand the linearized tensor corresponds to the case of the small disturbances (SMALL argument of the word

key DEFORMATION), but also with the case of the small disturbances reactualized (Lagrangian reactualized with
each step of incremental calculation: argument PETIT_REAC of the key word DEFORMATION).

The tensor of deformation is written then (like [éq 2.2-3]):
1
= (U
T
+ U) éq
3.2.2-1
2
The use of PETIT_REAC implies a resolution of the balance of the structure on its geometry current with a tensor of deformations linearized. One thus calculates the increment of deformation by report/ratio with position X, displacement U and the increment of displacement U in the following way:

1 U
U

=
I

éq
3.2.2-2
ij
+
J

2 (X + U)
X U
J
(+)
I

In addition, the code offers tensors of deformation of the type Green-Lagrange (GREEN or GREEN_GR) for the treatment of great displacements (and the rotations finished for certain elements of structure) but under assumption of small deformations. The tensor used is the traditional tensor according to [éq 2.3.1-5]:
1
(U) =
U
+ U + U U
ij

(I, J J, I K, I K, J) éq
3.2.2-3
2
Key word GREEN corresponds to modelings in 3D whereas key word GREEN_GR applies to modelings beam or hull.

Lastly, the framework of modeling in great transformations most complete accessible in Code_Aster results from the theory of Simo and corresponds to key word SIMO_MIEHE. It takes into account
great rotations and great deformations since the law of behavior is written in
great deformations. For more precise details on the fundamental differences between the different ones
types of deformations, the documentation [bib16] of Code_Aster presents in detail modeling SIMO_MIEHE.

Code_Aster does not allow calculations in configuration eulerienne: as with the tensor of Almansi, for example. All the tensors of deformation available are of Lagrangian type.

The fundamental difference, as for the writing of the criterion, is between the linearized deformations (SMALL and PETIT_REAC) and deformations GREEN, GREEN_GR and SIMO_MIEHE. Indeed, Code_Aster has need to make its search for balance of the tangent matrix. This one is written according to the equation ([§ 2.2.2.1] of documentation on STAT_NON_LINE [bib14]):

T

Q
T
K
éq
3.2.2-4
T = Q
:
+
:
U

U

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T
T

Q
However, Q:
corresponds at the end traditional of material rigidity and
: corresponds at the end
U

U

of geometrical rigidity which is present only in great displacements. Thus when one wants to apply one
criterion of buckling of the type (formally assimilable to [éq 2.3.2.2 - 2]): (K + K ()) = 0.
This criterion is valid only in small deformations since the geometrical term of rigidity is regarded as negligible in the tangent matrix.
One thus can, with reason, to make a traditional research of the eigenvalues and clean vectors of buckling type of Euler.

On the other hand in great transformations, the evaluation of this criterion consequently method is problems for two reasons. On the one hand, in the tangent matrix, the term of rigidity geometrical is already calculated and, in addition, the matrix K () which would possibly have to be added is
obtained under Code_Aster in small deformations. For these reasons, it is necessary to evaluate manner different the criterion according to the type of tensor of deformation asked by the user.

If one made the choice of a description eulerienne, the development of a criterion of the Euler type reactualized would be facilitated on the level of the calculation of the term K (), whatever the tensor of deformation.

3.2.2.1 In linearized deformations: SMALL and PETIT_REAC

As we said previously, this case of figure does not pose a major problem. It is enough to to calculate the geometrical matrix of rigidity and to make a traditional search for modes and values clean, of type Euler [éq 2.3.2.2 - 2]:

(K + K ()) = 0
éq
3.2.2.1-1

K is the tangent matrix reactualized at the end of the step of time.
In this case, one can thus speak indeed about criterion of reactualized the Euler type.
As one is in small deformations, the matrix of geometrical rigidities is proportional to loading. Therefore, when the critical coefficient is obtained, it is enough to multiply it by the load real with the step of current time to obtain the critical load buckling. The case =1 corresponds thus with the loss of stability.

3.2.2.2 In great displacements: GREEN, GREEN_GR and SIMO_MIEHE

The traditional method does not apply any more in this case. Indeed, Code_Aster calculates like stamp tangent the matrix of material rigidity plus the geometrical matrix of rigidity (and possibly, the contribution due to the following pressures).
One in the manners of checking buckling then is to make a research of the eigenvalues of only stamp tangent. If one of the eigenvalues is negative, it is that the matrix became singular and that an instability occurred between the moment when all its eigenvalues were positive and moment when one of it became negative.
The problem to be treated is thus slightly different since in the case of the small deformations (SMALL and PETIT_REAC), there is the following system to solve [éq 3.2.2.1 - 1]: , (K + K ()) = 0 then
that in case GREEN, GREEN_GR and SIMO_MIEHE it is necessary to solve:

(K + I) = 0 éq
3.2.2.2-1

With I which is the matrix identity and is, this time, of physical size equivalent to K, then that in the case of the small deformations, the eigenvalue is adimensional (from where sound direct interpretation as a multiplying coefficient of the loading).
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One of the defects inherent in this method compared to more explained more traditional research high [§ 2.3.2] is that one can have forecasts of buckling only when one approaches “close” critical load, even when it is exceeded. Far from this load, the first eigenvalue found does not have really physical significance since nonlinearities can appear between the step running and the calculated critical load. The coefficient report/ratio criticizes on load at moment I is thus different from that at the moment i+1 whereas in small deformations this report/ratio remains constant.
Moreover, for all the steps of time, all the eigenvalues and clean vectors except more low do not have any physical significance since, for a clean couple vector eigenvalue (V, one a:
I
I)
(K U
() + K ())V = V éq
3.2.2.2-2
I
I
I
This has clear direction only as from the moment when , in which case one finds the critical load and
I 0
the clean vector criticizes associated.
Always compared with the criterion of Euler (reactualized [éq 3.2.2.1 - 1] or not [éq 2.3.2.2 - 2]), it is noticed that the eigenvalue of the problem [éq 3.2.2.2 - 1]: (K + I) = 0 are not adimensionnée. It results from this a greater difficulty of interpretation as for knowing if the value is “small” or not. Otherwise known as, when can one say that one is close to a junction?

To define a relevant interval and of general use, in order to limit the vicinity of an instability, it would be interesting of adimensionner the eigenvalues.

3.2.2.3 Case of mixed modelings

As Code_Aster makes it possible to assign several types of deformations to the same structure, it is necessary
to consider the case where one uses several types of tensors of deformation in same calculation. The differentiation of the various elementary matrices being of no utility, it is appropriate of to solve to slice at the total level enters a method or the other. One chose to extract the values and clean vectors of the tangent matrix without adding geometrical matrices of stiffnesses. All pass as if the structure were in deformation of the Green-Lagrange type from the point of view of the criterion.
Indeed, let us consider an unspecified solid made up of two parts I and II. On part I, the tensor of deformation which was adopted is the SMALL linearized tensor and on part II that of Green-Lagrange. The tangent matrix resulting from the assembly of the two submatrices becomes:

K
*
0
I

*
*
*
éq
3.2.2.3-1
0
**** K***
K
II +
(
II)

The spangled terms represent the nodes common to both parts and are thus a combination linear of the values of the two matrices. In this configuration, it appears that none the solutions is not satisfactory but that less penalizing is to make a search for "
type
Green-Lagrange" [§ 3.2.2.2] i.e. to use (K + I) = 0 [éq 3.2.2.2 - 1].

This solution not being exact but nevertheless the only able one to be carried out simply, it is

envisaged to add a message of alarm informing the user who the results obtained are not guaranteed due to the juxtaposition of several types of tensors of deformations.

3.2.3 Improvement of the performances of the criterion

During the incremental resolution of a nonlinear quasi-static problem, in the ideal and if one admits that the discretization in time is sufficiently fine, it would be necessary to make an analysis of stability with
each step of calculation. With each step, that induces the resolution of a problem to the eigenvalues, admittedly limited in the search of some modes. The analysis of stability thus brings a overcost CPU important, with a nonlinear calculation already being able to be long.

The idea is to call upon the resolution of a problem to the eigenvalues only when it is really necessary, therefore when the current configuration is “close” to an instability. If one can define this vicinity by a preset interval, then one can call upon a test of Sturm [bib12].
This test makes it possible to know if there is at least an eigenvalue on the interval of research. In the affirmative, one will be able to then carry out modal research. In the contrary case, one continues quasi-static incremental resolution, without solving problem with the eigenvalues.
The cost of a test of Sturm is notably lower than the cost of research of the critical loads.
The interval of research for the test of Sturm can, either to be given by the user, or to have one default value in the code.
In the case of a criterion of reactualized Euler (case of the small deformations [§ 3.2.2.1]), where the problem with

to solve is written: (K + K ()) = 0 [éq 3.2.2.1 - 1], the interval of research must be centered on eigenvalue = 1 (which corresponds to value -1 for the algorithm of MODE_ITER_SIMULT, because it
solves in fact a problem of the type: K = µK ())).
The terminals of the interval are the terminals of the multiplying coefficient of the loading, therefore adimensional quantities, which are a function of the safety coefficients and the evaluation of uncertainties for the problem given. The test of Sturm is implemented within this framework.
In the specific case adapted to the tensor of Green-Lagrange [§ 3.2.2.2], where one solves:
(K + I) = 0 [éq 3.2.2.2 - 1], the interval are centered on 0. Moreover, terminals of the interval of test , contrary to the preceding case, are not adimensionnées [§ 3.2.2.2]. It is thus more difficult to identify relevant and general values (for the case of the default values). The test of Sturm is not currently established for this case.

3.3 Generalization with dynamics

We will not approach here the framework of the criteria of dynamic instability (negative damping…). It
just acts to announce that the nonlinear criterion presented here can completely apply directly in nonlinear dynamics. It will then detect any potential buckling of the structure, within the meaning of singularity of the total matrix of reactualized tangent stiffness.
In order to be exhaustive in terms of analysis of stability on a nonlinear dynamic study, the user should use two criteria:

· a criterion of buckling (criterion on the stiffness),
· a dynamic criterion (criterion on damping or the quadratic linearized problem total [bib13], for example).

For the moment, the criterion is not available in DYNA_NON_LINE.

3.4 Syntax of call of the criterion

In operator STAT_NON_LINE, the call to the criterion is done in the following way:

```
CRIT_FLAMB = _F (
CHAR_CRIT = (-1.1, 0. ),
NB_FREQ
=
3
.
)
,
```

Key word CHAR_CRIT defines the field on which one will make the test of Sturm, into small linearized deformations. If one finds at least an eigenvalue on the interval, then, one carries out resolution of the problem to the eigenvalues corresponding, if not, one does not do anything and calculation
incremental can continue.
If one uses modelings GREEN, GREEN_GR or SIMO_MIEHE, the modal resolution has inevitably place, and one seeks the eigenvalues smallest.
Key word NB_FREQ makes it possible to specify the number of modes which one seeks (default value:
3). It can be useful to seek more than one mode, mainly to be able to detect the cases "pathological" such as multiple modes or very close relations.

The mode of buckling corresponding to the smallest eigenvalue (in absolute value) is stored in the structure of data RESULT (eigenvalues named CHAR_CRIT, fields of displacement named MODE_FLAMB, which one can visualize via IMPR_RESU).
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3.5
Validation of the developments

The cases tests of validation are: SSNL126 and SSLL105D.
More precisely, the cases tests SSNL126 treat the case of a beam fixed at an end and subjected to a compression at the other end. Modeling is three-dimensional, with relation of elastoplastic behaviour with linear isotropic work hardening. Two representations kinematics are

presented:

· modeling a: linearized deformations,
· modeling b: deformations of Green-lagrange.

The case test SSLL105D is based on a problem of beam in L, which one studies buckling rubber band. The finite elements are of beam type.

4 Conclusions

Code_Aster offers two criteria of stability, within the meaning of buckling, for the structural analyses.

On the one hand, whenever a linearized approach is enough, one can apply a criterion of the Euler type
([bib12] and [bib17]), by call to an operator of resolution of the problem to the eigenvalues generalized
(for example MODE_ITER_SIMULT with key word TYPE_RESU=' MODE_FLAMB').

In addition, for all the cases where it is essential to take account of nonthe linearities, which they are had with the relation of behavior or the great transformations, the user can employ one adapted criterion, of generalized Euler type. The call of this criterion is done during the resolution incremental of the quasi-static problem (operator STAT_NON_LINE [bib14]).
With each step of time, the criterion is based on the resolution of a problem to the eigenvalues [bib12] on the matrices of brought up to date total stiffnesses.
This criterion, which is declined in two different forms, according to the tensor of deformation chosen,
base on a linearization around the step of current calculation. It accepts any type of tensor of deformation, as any type of relation of behavior for which one is able of
to build the matrix of total stiffness, at every moment. Moreover, the selected criterion is perfectly rigorous in the case of relations of nonlinear elastic behavior, and in the case of elastoplasticity associated with the assumption with Hill [§ 2.3.2.5].

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Calculation of the cyclic limiting states with Method ZAC

Summary

This method is based on linear kinematic work hardening. It gives an approximation for the adapted state and two approximations for the adapted state of the characteristics in constraint and deformation of the cycles limits, for a structure under thermomechanical periodic loading in the field of plasticity. That is made using three thermoelastic calculations at weak costs calculation.

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1 Introduction

Under a periodic loading, evolution of the deformation, or displacement, according to a many cycles of the various points of a structure are concretized in three forms: the adaptation, where all the points of the structure reach an elastic stable absolute limit; accommodation, where at least a point of the structure reaches an elastoplastic stable absolute limit; and the ratchet where for at least one not there is a constant increment of the deformation to each cycle. For a material describes by a law from linear kinematic work hardening one can obtain only accommodation or adaptation.

To study the problems of tiredness or progressive deformation, for a structure, one has generally need to know the stress and strain state to the absolute limit, i.e. them values of the amplitudes of strain and stress as well as the average constraint for tiredness, and the value of the maximum deformation for the progressive deformation. These values can be obtained using a cyclic law of behavior. Nevertheless if the absolute limit is obtained after one a significant number of cycles calculations can be very long.

Post-processor POST_ZAC provides an approximation of the characteristics in constraint and in deformation quoted above. More precisely it proposes two estimates for each characteristic in constraint or deformation. The duration of calculations corresponds to 3 or 4

calculations
rubber bands. It should be noted that in the model suggested the constants are independent of temperature. If the temperature varies it is necessary to choose the constants of material to one optimal temperature for calculations.

2 Materials with linear kinematic work hardening under periodic loading

One in the case of places an elastoplastic structure at linear kinematic work hardening strictly positive and the following assumptions are made:
· Quasi-static Evolution;
· Infinitesimal Déformations;
· The material is with linear elasticity independent of the temperature;
· Periodic Chargements (thermics T, forces intern F, forces external F on,
F
displacements imposed U on) on the structure and of border;
U
· In addition, one supposes that the field of elasticity is defined by the criterion of Von Mises:
3 (~ -)(~ -)
~
y -
2
y,
with the diverter of the constraints ~
= ij -1 3tr () ij, elastic limit, the internal variable
y
= C p where C = 2 3 H (H = E E/(E - E) where E is the slope of the traction diagram
T
T
T
during the work hardening and where p is the plastic deformation.
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F
F

U

Appear 2-a: Types of loadings of the structure

Within this framework and under the assumptions above there is the following theorem [bib1]:

Theorem: (Generalized Melan). Any solution of the problem of evolution tends towards a solution periodical in constraint and deformation. If the local amplitude of plastic deformation is null one known as that there is local adaptation, if not it is said that there is local accommodation. So at least a point of
structure is adapted one says that the structure is adapted.

3

Presentation of the simplified method, ZAC

Method ZAC [bib2], [bib3] is a method which is based on the kinematic model of work hardening linear and which gives in a simplified and inexpensive way of the approximations of the characteristics of
cycle limit in constraint and plastic deformation.

For the adapted case, one has for each component the values of: the amplitude of constraint, of average constraint and of the limiting plastic deformation.

For the case adapted for each component, one has two values for the amplitude of the constraint, two values for the amplitude of the plastic deformation, a value for the average constraint and a value for the average plastic deformation.

One can summarize the method in the following way:

One breaks up the constraint as the sum of a first term representing the calculated constraint with an elastic behavior *el* and of a second called residual stress. In term of
diverters, that is written:

~
~el
~

= + .
The use of the modified internal parameter will make it possible to build in each point the absolute limit of
structure, **independently** of what occurs for the other points.
The criterion of plasticity being:
~
- *y*
the idea-key of the method is to make the calculations uncoupled in each point from the structure. For it to make, one introduces the modified parameter:
!
~
= - .
is written then:
~
el -! *y.*
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The simplification introduced by method ZAC lies in the way of calculating!
with the absolute limit, in
each point independently from/to each other and to bring back calculation to 3 or 4 elastic designs.
The approximation of!
with the absolute limit! *lim*, is differently given according to whether there is adaptation or accommodation. This approximation is obtained starting from the initial value of the tensor!
, which is
noted!
.
0
Thus, to summarize, if the thermoelastoplastic problem of origin is written:

div = *F*,
N = *F*, on *F*

(*P*)
1

= 1 (
2 *U*
T
+ *U*), *U* = 0 on *U*

= *K* (
p
- -),
HT
+ standard law of elastoplastic behavior in linear kinematics.
The associated thermoelastic problem is written:

div el
= *F*,
el
N = *F*, on *F*

(*P2*)
el

= 1 (
2 *U el*
T
+ *U el*), *U el* = 0 on *U*

el

= *K* (*el*
-),
HT
And by making the difference between these two sets of equations by using the relation!
p
~
= *C* -,
as well as the following definitions:

= - *el*, *ine el*
= -
and *U ine*
U U el
= -
the following problem is found:

div = 0,
N = 0, on *F*

(*P*)
ine

= 1 (
2 *U ine*
T

+ *U ine*), *U ine* = 0 on *U*
ine
-1

= (
~
!
K +/*C)*
el
+
= - = *K* (
el
p
- -)

C
!
It is an elastic problem with an initial deformation equalizes with
where, *ine* and *U ine* are them
C
unknown factors.
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4 Passage in the space of the modified parameters and local construction of the solution

In the continuation one will distinguish the cases where there is an **initial state** no one or not within the meaning of method ZAC.

In the case of an initial state no one the tensor!
is taken equal to zero. One will distinguish in the same way the cases where
0
the way loading in each point of the solid **closely connected** (is often named radial in the case of this method) or not **closely connected**. Consequently, the definition of these two concepts is given.

Definition 1: Initial State not no one within the meaning of method ZAC.

It is said that one has an initial state not no one when one makes an elastoplastic incremental calculation until one
level of loading given and which one calculates!
p
~
0 = *C* -. The level of the loading considered
is generally the level of the maximum loading (It is desirable to take into account a state initial not no one when, for example, the first cycle of loading involves a deformation important of the structure.)

Definition 2: Way of loading refines within the meaning of method ZAC.

It is said that a way loading is closely connected if in each point of the structure, for a behavior rubber band, the way of the constraints is closely connected.

4.1
Adaptation and accommodation in the case of a loading closely connected

In an item *X* of the structure, one defines *F el* (*X*), such as:
F el X
el
=

X T

el
( )
max ~ (,)
~
el
el
0 -
(*X*, *T*)
~
1

(*X*)
~
max
-
(*X*)
min
,
T, *T*
0 1
where ~
el (*X*, *T*) is the tensor deviatoric of the elastic constraints as in point *X* and and where them moments *t0* and *T1* correspond to the **extrema** of the cycle of loading. The moment *t0* can be equal to zero. One defines *F el* such as:
F el
max *F el* (*X*
el
)
max ~

(*X*
~el
=
=
)
max
-
(*X*)
min
.
X
X
The comparison between *F el* and the value of the elastic limit of material makes it possible to know if

the state
limit of the structure is of **adapted** or **adapted** type:
F el 2 *y*
adaptation
F el > 2 *y*
accommodation.

4.1.1 Case of the adaptation

If the structure is adapted, there is a limiting, fixed field in time, modified parameters!
, noted
!

el
el
lim, such as!
lim belongs to the intersection *CL* of the two convex ones of center ~
min and ~
max and of
ray *y*, to see the figure [Figure 4.1-a], [bib3], [bib4].

---

y
y
~
el
~
el
min
CL

max
Appear 4.1-a: The intersection of two convex *C*, in the case of the adaptation.
L
In the case of the **adaptation the** value limits!
, noted! *lim* is determined by projection
orthogonal of initial, noted!
on convex *C* according to rules' presented on the figures
0
L
[Figure 4.1-b], [Figure 4.1-c], [Figure 4.1-d].
CL
~ *el*
el

~

min
max
! =
lim
! 0
Appear 4.1.b: Cas where!
0 **are strictly included in convex** *CL*
!
el
lim
~
el

~

min
max
! 0
CL
Appear 4.1-c: Case where!
0 **do not belong to convex** *CL*,
but belongs to the cone of top ~
el
el
max, **(or** ~
*min***)**! *lim* **is on the edge of convex** *CL*
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~ el
~ el

min
max
CL
! *lim*
! 0
Appear 4.1-d: Case where!
0 **do not belong nor to convex** *CL***,**
nor with the cone of top ~
el
max!
lim **is on the edge of convex** *CL*
In the case of the adaptation, one can take an **initial state no one**!
0 = 0, or one make a calculation
elastoplastic incremental which makes it possible to have an **initial state not no one**!
p
~
0 = *C* - to find
often a better result.

4.1.2 Case of accommodation

When the structure is adapted, i.e. in at least one of the points of the structure them
two convex of center ~
el
el
min and ~

max have an empty intersection, one is led to define the three
sizes which follow, at the points where there is accommodation:
1) the parameter of average modified internal work hardening, noted!
moy;
2) the amplitude of the parameter of modified internal work hardening lower, noted!
inf;
3) the amplitude of the parameter of modified internal work hardening higher, noted!
sup.
These three sizes are represented on the figure [Figure 4.1-e].
! *max*
! *sup*
el
~
el

~
min
max
! *moy*
! *min*
! *inf*
Appear 4.1-e: Representation of!
moy! *inf* **and**! *sup*
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The first two sizes are calculated!
moy and! *inf* as follows:
~el

~

2
min + *el*

!

max
el
el
y

moy =
and
! inf = (~
~
max - min) 1
2

~el
~

el

max - min
Calculation of!
sup
The calculation of!
sup represented on the figure [Figure 4.1-e] is given below:
In each point of the structure, if there is plasticization, one a:
~
p
-
" = ~
- with > 0
One can then write, in term of modified parameter, that:
el
p
! ~
-
" = -

, with > 0.
! ~
- *el*
Like ~ -
! ~

- *el* = *y*, in each point where there is plasticization! is on the edge of one
swell of center ~
el and of ray *y* [Figure 4.1-e]. The size “*p* is the interior normal with this
swell at the point!
.
It is supposed that if one goes very far in work hardening, one a:! “
~
= "
el, [bib3].
For a loading refines, in other words, when one passes from *el*
el
min with *max* one has
! ~el
~
=
el
el
max - *min* = ~

, i.e. direction of displacement of! is equal to ~
el.
Resolution of the homogeneous elastic problem (*P*) with the modified constants and the deformation
initial ~
el C, cf [§3], one deduces *p* and thus a direction for “*p*. By holding the same one
reasoning when one passes from ~
el
el
~el
max with ~
min, one obtains! = -, which gives to “*p*
direction opposed to the preceding case, (cf [Figure 4.1-e]). One takes finally!
sup =! *min* -! *max*.
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4.2 Adaptation and accommodation in the case of a loading not closely connected

When the loading is **nonclosely connected** method ZAC is operational only in the case of **the adaptation**.

Case of the adaptation

As in point *X* one considers the intersection of the balls of center ~ *el* (*X*, *T*) over one period. In the case of the adaptation this intersection is nonempty and convex. One thus defines, as in the case refines! *lim* by the projection of! 0 on this convex, [bib3], [bib4]. Method of projection used in *Code_Aster* is successive projection on the intersection two to two of the spheres.

Case of accommodation

In this case the rules suggested are not rigorous [bib5] and there is no option associated in *Code_Aster*. One will use approximations closely connected of the loading in this case.

5

Return in the space of the parameters of origin

Once one calculated! *lim*! *moy*! *inf* and! *sup* one has an initial deformation for problem (*P*) of [§3] which will be able to take the following values in turn:

!

!

!

!

lim
moy
inf
sup
,

,
and
.
C
C
C
C
One then solves the problem (*P*) with the initial deformations to obtain above:
p
1) in the case of the adaptation: *U lim*, *lim*, *lim* and *lim*;
2) in the case of accommodation: *U*
p
moy, *moy*, *moy*, *moy*, *U inf*, *inf*, *inf*,
p
p
inf, *U sup*, *sup*, *sup* and *sup*.
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6 Operator
POST_ZAC in *Code_Aster*

The operator post-processor POST_ZAC needs the following data: the model, the material, and them two moments of the loading. If the initial state is null (!
= 0) one use the concept
0
p
EVOL_ELAS. If the initial state is not null!
~
0 = *C* - being obtained starting from a calculation

elastoplastic, one uses in more concept EVOL_NOLI.
p
To exit POST_ZAC gives: *U lim*, *lim*, *lim* and *lim* if it there has adaptation and gives:
U
p
p
p
moy, *moy*, *moy*, *moy*, *U inf*, *inf*, *inf*, *inf*, *U sup*, *sup*, *sup* and *sup* in
case where there is accommodation. The document U associated is [U4.74.05].
The case test associated with the method is: SSNA100 [V6.01.100].

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R7.07.01 document
Calculation of load limits by the method
of Norton-Hoff-Friaâ
Summary:
The limiting analysis makes it possible to determine the acceptable loadings D `a structure, of geometry fixes given,
composed of a material having a criterion of resistance. One considers the case of loadings made up of summon of a continued load and of another parameterized by the load factor.
After a recall of the theoretical formulation, one presents the regularized kinematic approach applied to the criterion
of resistance of Von Mises (method of Norton-Hoff-Friaâ) and implemented in *Code_Aster*. One will be able
to refer to [bib4] for the various possible methods of regularization suggested in the literature. One expose then the calculation of the solutions of this nonlinear problem and postprocessing providing one estimate by higher value of the limiting load.
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Contents

1
Theoretical formulation of the limiting analysis
1.1
Definition of the limiting load

One considers a solid occupying a limited field subjected to surface loadings
F + **F** on the edge and of the loadings of volume **F** + **F** on. The loading is distinguished
0
F
0
(**F**, **F**), parameterized by positive reality, and the permanent loading (**F**, **F**). Conditions of
0
0
Dirichlet homogeneous are applied to the complementary edge *U* of (an imposed displacement or an initial anelastic deformation - thermics, plastic… - do not have an effect on the field of working loads). One can find in [bib5] several other useful properties.

The material constitutive of the solid has a criterion of resistance expressed by a scalar function of constraints, negative for working stresses. The criterion used for a material of the type elastoplastic perfect with threshold of von Mises and selected here is:

(
2
2
2
2
G) = J () -
3
D
D
y =
. .
-

2
$y =$
. (-) + (2 - 3) + (1 - 3) - y
2
1
2
D
is the diverter of the tensor of the constraints,

is the threshold of resistance in simple traction (like an elastic limit), possibly
y
variable according to zones' of the solid considered.

being principal constraints of.
I
Being given this criterion of resistance one seeks to calculate the value limits, called limiting load
, for which the structure can support the loadings **F** + **F** and **F** + **F**.
lim
lim
0
lim
0
Strictly speaking, the value indicates the limit of the bearable loadings, but for
lim
materials obeying the Principle of Maximum Plastic Work, this value is the limit of
supported loadings.

1.2

Calculation of the load limits by a kinematic approach

In design the collapse two approaches are possible: static approach (in variables of constraints) and kinematic approach (in variables speeds). These approaches provide terminals of charge limit: undervaluing for the approach static and raising for the kinematic approach. When them two provide the same result, the limiting load obtained is exact.

The kinematic approach is that used in *Code_Aster* using finite elements in displacements.

For the loading given (**F**, **F**), one defines the space speeds kinematically acceptable and standardized by:

V 1 = **v** acceptable, **v** = **0** on, L
has
U
(**v**) =
F. v D + **F. v** ds =
1

F

The power of the permanent loading (**F**, **F**) is noted: L (**v**).
0
0
0
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From the criterion of resistance in constraints G
(), one defines:
· the whole of working stresses by: *G*
= **X**, G **X** 0
X
{ ( ) (( )) }
( )

((
G is convex for the criterion *G*)
X)
0, *if* (**X**) (
G **X**)
· the indicating function: **X** =

G (
()) +, *if* (**X**)

(
G **X**)
· the function of support: () = Sup [**.** - *G*
( ])
R6
Sup in () can be reached only if is selected in
D
(
G, such as: = + µId
X)
/
1 2
(what ensures//*D*). The optimum corresponds to (
G) =
=
-
0

2. *D D*
y
3 (.)
D2
()
G
· *D*

· *D*
0
D

1
0
Appear 1.2-Error! Argument of unknown switch. **: Optimum and graph of the function** () **in 1D**
2
From where the function of support: ((**v**)) =
.
y
(**v**)**.** (**v**) +

Sup (µ.div **v**). It is observed that
3
µR
function () is not differentiable into **0**.

One to date does not treat in *Code_Aster* possible internal surfaces of discontinuity with the centre solid [bib 4].
The kinematic approach is defined using the convex functional calculus S **v**, positively homogeneous
E ()
of degree one, for **v**
1
V
defined on the whole field:
has
S (**v**) = ((**v**))*D* - L (**v**)
E

0
This functional calculus is the integral on the field of the function of support of convex (
G, calculated
X)
in (**v**) and is interpreted like the maximum resistant power in the field speeds **v** (
contribution of resistance of interface on surfaces of discontinuity is supposed to be null). The function support is positively homogeneous degree 1, and thus the functional calculus (**v**) also by consequence.
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With the criterion of Von Mises the functional calculus of power S **v** is:
E ()

2

S (**v**) =.
(**v.**)(**v**) +Sup (*q.div* **v**) *D* - L

E
y
(**v**)
éq 1.2-1

3
0
Q

R

where it is noted that only fields **v** pertaining to *C* = {**v** V 1a, div **v** = 0 in}
provide finished values. Fields **v** must thus check the condition known as of incompressibility
div **v** = tr (**v**) = 0. This is why it is necessary to use the incompressible elements for a calculation of charge limit with the criterion of Von Mises. The treatment of the incompressibility is detailed in [R3.06.05].
The load limits given by the kinematic approach is:
lim
S **v**

E
lim Inf
=
(**V**)
( )

=
Inf
S **v**
L **v**
E
1
v 1
v
V
V
L (**v**)
=
Sup Inf (() - (() -)
1
has
>0 **vV**

has
has
L (**v**) >0
With the optimum one obtains a solution **U** and the limiting load (not unicity of **U** but unicity of).
lim
lim
Thus, any loading L (**v**) + L
(**v**) with 0 is bearable. Beyond, it
0
lim
lim
problem of balance does not have a solution.
Note:
There are situations where, even if
L (**v**) *is not bearable alone,*
0
combination L (**v**) + L
(**v**), *for, becomes it on a certain interval, and not*
1

0
2
only for two parallel loadings.
Note:
The limiting load calculated for a two-dimensional problem, in plane deformations, is necessarily higher than that obtained for this problem modelled in plane constraints. It result thus provides one raising. If one wishes to deal with the problem in plane constraints, it is necessary then to make the kinematic approach on a three-dimensional modeling.

1.3 Regularization of the kinematic approach by the method of Norton-Hoff-Friaâ

Implementation the numerical of the kinematic approach requires the minimization of the functional calculus
not-differentiable S **v**. Many techniques of regularization exist [bib4]. Method of
E ()
Norton-Hoff-Friâa is used here [bib2], [bib7]. It rests on precursory work of Casciaro in 1971. It consists in replacing the function of support
() by the function of support regularized and
differentiable *NH* (). It is adjustable by a parameter of regularization *m* (1 *m2*), of which value limits *m* +
1 conduit with convergence towards the function of support
( ) :
-
1 *m*

NH ()
K
=
(())*m*
m
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2
1

In *Code_Aster* one uses the parameter of regularization *N* with *N* =
and *K* is chosen
y
=
(*M-1*)
µ
3
to find the incompressible elastic problem when *m* = 2 is *N* =1 (2µ being the coefficient of rigidity of shearing).
One notes the space acceptable speeds adapted to the problem of viscous flow for the law of Norton-Hoff of order *m*:
V *m* 1 = {
m
v *L* (), *and* (**v**)
m
L
(), **v** = 0 *on*, L
has

U
(**v**) = 1}
One defines on this space the regularized functional calculus Sm
E (**v**):
-
1 *m*
K
m
Sm (**v**) =
((**v**)) *D* - L
E
(**v**)
m
0
The problem of Inf minimization
m
is well posed thanks to the properties of
m 1 [(**v**)]
vV *A*
Lm spaces () and has a solution **um**, for which the value of Inf reached is: *Mr*. One shows
that this problem can be also written in the form of the research of the point-saddle (*m*, **um**, *pm*)
Lagrangian following:

(
With)
m
m
Max Inf
Sup
. ((**v**)**.** (**v**)) *D*
.
Q div **v** *D* L (**v**) (L (**v**))
1

0
V
2
m
Q has

+
-

-
-

R **v**
L ()

éq 1.3-1
m/2
/2
1
2
-
with: (
With)
m
K - *m* (
2
1 2

3µ
. In practice the continuation is taken:
3)
m
- *m*
m
m
=
y
= *y*
( ) (3)
m
I
= + -
1 10
2
1,1
1,01
1,001
...
1
1
1
10

100
1000
...

N = M-1
(
With)
m
2µ
...

2
y
3
It is noticed that (
With m) is increasing with *m* (if *E y*) and homogeneous with a constraint, and remains
m/2
limited when *N* +. If one chooses *E = y* then (
With)
m = 2
y (
. This Lagrangian allows
3)
to impose directly in the operator the condition of incompressibility and standardization on 1 of power of the loadings. One then builds a decreasing succession of *m* and the load limits lim is the limit of this continuation when *m* +
1 (either *N* +):

m

m
lim Li
= m
Inf
S
v
S
U
éq 1.3-2

E

E
m
m 1
m1 [
()] lim
=
(( )

m 1
v Goes

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For the demonstration one will refer to [bib4] and [bib7]. One shows also the following property of solutions of [éq 1.3-1]. Thus if one ampifie the loading L L
, when L0 = 0, the solutions have
following dependence according to:
U
-1
-
1 *m*
m () =
um ()
1
;
pm () =
pm ()
1

D
(**U**
-
1 *m D*
m
- *m m*
m () =
(**um** () 1);
E
S (**um** () =
E
S (**um** ()
1)
One of the essential assets of this method of regularization resides in the properties of fitment
Lm spaces (), which makes it possible to have properties interesting for the continuation of the *m*, to see it
paragraph 2.3. Thus one shows the following properties [bib7], for a limited field, in
calling =

D and = (A)
m D

:

m

For all 1 *m* and 1 *R S* and any function **U** of V *m*
has, one a:
1
1
1
1
-
1
-
1
(
With)
m
(**U**)**.** (**U**) *D*
R

((*A*) *m* ((**U**)**.** (**U**) *R*) /2d
2

) *R*
S
S
S

(**U**).

m (
(
With)
m (
(**U**)
)
D
m

)
éq 1.3-3
1

1
m

m

2
1-

m
2

y.
(**U**). (**U**) *D m*
y
.
(**U**). (**U**)
D
U

éq 1.3-4
3

3

These properties are interesting because applicable if the material is heterogeneous, and one can to consider the limit of resistance either as measures (density) or like belonging to the deformation energy.

2
Numerical aspects of the calculation of the limiting load
2.1
Establishment in *Code_Aster*

To carry out a calculation in *Code_Aster* in analysis limits with the method of regularization of Norton-Hoff-Friaâ with the criterion of resistance of Von Mises, it is necessary:
· to define the model 2D (plane or axis) or 3D with the incompressible finite elements,
· to define the characteristics of the materials (Young modulus *E y*, Poisson's ratio near to 0 5
, to ensure the operation of the incompressible finite elements [R3.06.03], limit of resistance *y* and coefficient of Norton-Hoff *N*), the coefficient of shearing is deduced: 2µ = 2nd/3. It should be noted that the limiting load is independent of *E* and,
· to define the permanent loading and that which is parameterized by,
· to define the standardization of the power of the loading parameterized (key word LIAISON_CHAMNO order AFFE_CHAR_MECA),
· to carry out a non-linear calculation with the relation of behavior of Norton-Hoff with order STAT_NON_LINE [U4.32.01],
· post-to treat calculation to obtain the load limits with order POST_ELEM [U4.61.04].

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2.2

Relation of behavior of Norton-Hoff in STAT_NON_LINE

The problem is written in variational form in the following way:
For $N = 1/(m -)$
1 given, to find (m, **um**, m
p) R ×V
2
× L *has* () such as:

(
m2
With m).

((**um**). (**um**)) (**um**). **v** +
v
( )

D
p .div D
m
- ml (**v**) = L (**v**) **vVa**
0

$q.divu\ D$
= 0 Q

2
m
L ()

L (**U**

m) = 1

éq 2.2-1
This problem admits a single solution for any *N* 1 (see [bib4]). For *m* = 2 or *N* =1 the problem is of incompressible linear elasticity type.
One thus obtains an estimate of the limiting load by higher value, the field **um** giving one idea of a mode of ruin.
For the treatment of the incompressibility, one will refer to the document [R3.06.05]. The treated equation is
in fact:
Q. *p*
Q. *p*
q.div **U** *D* +
D =, *Q*
L ()

D

allowing to avoid putting

0
2
, the term

at fault the solvor employed and correspondent with a choice of the Poisson's ratio = 0 4999
.
…
solutions are thus only quasi-incompressible.
The principle of operation of the algorithm general of STAT_NON_LINE is described precisely in [R5.03.01]. One leads to the following incremental problem:
To find (, **U**, *p*) R × V × Q
0
such as:
(**U** + **U**
)**.** (**v**) +

(
B,
v
+) -

D

p
p
(+) L1 (**v**) = (L + L
0
0) (**v**)
v V

0

(
B U +,
U *Q*) = **D** + **D**

Q Q

L1 (**v**) = 1

.
B is a linear operator who contains the boundary conditions homogeneous of Dirichlet, incompressibility,
.
D describes the data imposed on the solution (boundary conditions of Dirichlet, incompressibility),
.
L is the permanent loading and L the loading controlled by the parameter,
0
1
.
V and Q are spaces of functions discretized on the basis of finite element, and are thus
0
defined by a vector (**U**, **P**) of degrees of freedom.
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The tensor of the constraints (**U**) checks the relation of behavior of Norton-Hoff. The diverter of constraints associated at the speed of deformation is:
m2
-
1 *N*
D (**U**) = (). (*D* (**U.**)*D* (**U**)). *D* (**U**) *D* (**U**) = () *N*. (*D.D*). *D*
With m
With m
(**U**)
éq 2.2-2
The problem is solved by the method of Newton, after direct implicit discretization of the relations of behavior [R5.03.02].
The phase of prediction consists in solving the following system, starting from the current state (**U**, *p*) for to obtain the reiterated first: (**U**, *p*.
0
)
0

D
. (**U**
)**.**

0 (**v**) *D*

(
B,
v *p*
)
0

1
L (**v**)
L (**v**)
v V

+
-

=

D
O
0

(**U**)

(
B U
, *Q*) = **D**
0

Q
Q

1

L (**U** + **U**
0)
=
1

D
Option RIGI_MECA_TANG provides the tangent operator
.
D (**U**)
One uses the tangent rigidity, applied to a tensor deviatoric **E**:
D D

-2
(**U**)
(**U**)
.e = (
R
D
D

With R)
D
D
. (**U**)**.** (**U**)

. **E** + *R*
- 2
.e
D D

(
)
(
) *D*
D

(**U**)**.** (**U**)
Then one treats the phase of correction, for iteration *I*:

D

.

(**U**

- U
+1)**.** (**v**) *D*
I
I
+ **B** (**v**, *p*
i+1) **-** (
+) 1
L (**v**) = L
+ L (**v**)
D
(0 0)

(**u+ui**)

-

[(**u+ U**
)**.** (**v**) *D*

I
+ **B** (**v**, *p*)] **v**
Vo

B

(**U**

, *Q*
i+1
) = **D**+ **D**
- **B** (,
U *Q*)

Q Q

1
L (**u**+ **U**
i+1) = 1
Option RAPH_MECA provides the second member
D
(**U**
U
+)**.**
. This one is built by call
I
(**v**)
with the subroutine of *Code_Aster* of the type NICOMP, which calculates the stress field suitable for
D
law of behavior used. Option FULL_MECA provides in more the tangent operator
. One
D (**u**+**u**) *I*
can only decide actualization of this operator with certain iterations *I*, to avoid one
too frequent expensive assembly.
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In our case, the resolution can be made in a nonincremental direct way, but there is interest with to reactualize the tangent rigidity of time to other to accelerate convergence. The solvor to be used is: “LDLT”, while having taken care to have specified that standardization is placed at the end of the system
(key word: NUME_LAGR: “AFTER” in LIAISON_CHAMNO).
One can also decide to make recoveries, starting from a solution (**U**, *p*) obtained previously, even coming from a resolution for another parameter *N* (calculations less expensive and better convergence).
In all the cases, it is advised to start to make a calculation on a coarse grid for to evaluate the effect of parameter *N* on *Mr*.

2.3
Postprocessing of the calculation of the limiting load

Having obtained the solution (*m*, **um**, *m*
p), for a *N* given, it remains to use the continuation of the *m* for
to build the approximation of the limiting load. For that one exploits the properties [éq 1.3-2], [éq 1.3-3], the fact that (
With)
m is increasing and the property resulting from minimization [éq 1.3-1] (see [bib7]).
From these two last, with 1 *R S*, one deduces that for and **ur** and **custom** respective solutions (checking also the condition of incompressibility and standardization) of [éq 1.3-1] for *m* = *R* and *m* = *S*:
((*Ar*)

(
/2
/2
(**U**)**.** (**U**) *R*
)
D) ((
With S)

(
S

(**custom**)**.** (**custom**)) *D*

R
R

)
Associated the property [éq 1.3-2], one draws for 1 *R S*:
1
1
1
1
-
1
-
1
(
With R)
/2
/2
(**U**)**.** (**U**) *D*
R

((*Ar*)

(
R
R
S
S
S

(**ur**)**.** (**ur**)) *D*

)
(**U**)**.**

S
((*Ace*)

(*S* (**custom**)) *D*
R
R
R

)
éq 2.3-1
~
One notes the terms *m* of the continuation, which one calculates in practice by postprocessing using **um** (external power being unit:
1
1
~
-
1 *m*
m2
m
m =
U. U

-
m
((A)
m (
/

(
)
m (*m*)
)
D
L **U**
éq 2.3-2

) () *m*
0
~
This continuation *m* is thus decreasing for *N* + and it is shown [bib7] that it converges towards lim, what allows a good control. As one can undervalue (knowing that (
With)
1
2
= *y*
) the first term
3
of [éq 2.3-1]:

2

~
lim *y*
(**U**)
Mr. (**U**)
m D -

L (**U**)
m
m
3
0

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the continuation is thus calculated!
m decreasing for *N* + (parameter “CHAR_LIMI_SUP” of the table
exit of POST_ELEM [U4.61.04]) and also converging towards lim,

!
2
lim
m = y (**U**) *Mr.* (**U**) *Mandelevium* -

L (**U**)
m
éq 2.3-3
3
0

One judges quality of the approximation of the limiting load by comparison of different
lim
values of!
m which converge towards by excess (out of *N* +). These terms are calculated by
lim
numerical integration at the points of Gauss of the finite elements.
Another interpretation of the interest which this continuation brings lies in the fact that it exploits directly the expression of the function of support of convex of resistance, i.e. power dissipable in the modes of potential ruin, applied to the incompressible and standardized solutions calculated **um**.
If the permanent loading (parameter “PUIS_PERMANENTE”) is null: L = 0, one can
0
easily to exploit the stress field (almost statically acceptable) calculated with
solution **um** and to obtain a value by estimate of the limiting load, which would be him would necessarily be
lower if balance were checked exactly (see [bib4]). One thus calculates the continuation *m* (parameter “CHAR_LIMI_ESTIMEE” of the table resulting from POST_ELEM [U4.61.04]), which does not have on the other hand
properties of monotony:

1
3
-
(

With)
m

U.
U
2

=
.
U. U
D. Sup
!

éq 2.3-4
m (

D
D
() ())*m*
(*m*) (*m*)

m
m
m

X

m
y

This maximization (of the function called gauge of convex of resistance) is calculated only with points of Gauss of the finite elements. Also the value obtained, for each *m*, lower than!
m [bib4],
can be regarded only as one indication.
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3
An example of validation

3.1
Problem of reference
A rectangular plate or a hexahedron or an axisymmetric cylinder are considered. The criterion of resistance of homogeneous constitutive material checks the criterion of von Mises (with for threshold *y*). structure is subjected to pressures on the horizontal edge - *F* and the vertical edge
- (1 -) *F* with 0.5. On this very simple problem, an analytical calculation makes it possible to obtain charge exact limit in the direction of the loading, as well as the estimates produced by
method of regularization. For more details one will refer to [bib4] and [bib5]. This example of validation corresponds to test SSNV124 [V6.04.124].
The geometry is defined by:
· Interior Rayon: = *m has*
1 *m*,
· External Rayon: *B* = 2mm, thickness *B* - *has* = *m*
1 *m*,
· Hauteur
:
H = 4mm.
Z or y
D
C
H
B
has
R or X
0
With
B
3.2 Case
plan
The structure is subjected to pressures on the edges horizontal: - *F* and vertical: - 1
(-) *F*, with:
1/2, and a blocking in *Z* is exerted. One considers two ways of controlling the loading:
· cas1: the two pressures horizontal and vertical are parameterized by,
· cas2: the horizontal pressure is parameterized by, while the vertical pressure is
constant - (1 -) *f0*, with *F* = *F*
0
0
.
3.2.1 Solution analyzes limit of it
The solution is homogeneous (biaxées constraints:
(
)
xx = *F*, *yy* = 1 - *F*, *xy* = 0,

plane deformations). One obtains [bib4] the limiting load in these directions of loading, for criterion of von Mises, in plane deformations, with the threshold *y*:

2

3 *y*

cas1: lim. *F* =

éq 3.2.1-1

.

3

2 - 1

2

3 *y* 1 -

cas2: lim. *F* =

+

. *F*

.

3

0

éq 3.2.1-2

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It is checked that if one takes 0 = lim in the cas2, one finds the cas1 then.

3.2.2 Solution analyzes regularized limit of it

The solution is homogeneous. The plane deformations are necessarily of the form:

1 0
0
(**U**) =

0 -1 0
;
(**U**). (**U**) 2
éq 3.2.2-1

0 0
=
0

By the law of Norton-Hoff, one obtains the deviatoric constraints:

1 0
0
D = () *m2 m2*

-2
-
2
1
. 0 -1 0 ;
D
() *m*
2
m
With m
With m

=
3
éq 3.2.2-2
VM
0 0
0
The standardization of the loading leads to:
1
cas1: *F* =
éq 3.2.2-3
H (*B* - *has*) (
2 -)
1
1
cas2: *F* =
éq 3.2.2-4
H (*B* - *has*)
Terms of the continuation!
m of limiting load in these two parameter settings of the loading is then:
2
3 *y*
cas1: !
.
m F =
m

éq 3.2.2-5
.
3
2 -1
2
3 *y* 1 -
cas2: !
.
m F =
+
. *F*
m
0

éq 3.2.2-6
.

3

Invariance according to *m* observed here (what is a particular case) results owing to the fact that one is in an isostatic situation. In the cas1, one can also exploit the continuation of the *m*:

2

3 *y*

cas1: . *F* =

m

éq 3.2.2-7

m

3 .

2 -1

One thus obtains the load limits lim exact when *m* +

1 .

3.3 Case

axisymmetric

In axisymmetric 2D one considers the same geometry, but the solid, on which one imposes a bloquage axial complete, is only subjected to a pressure on the internal wall: *F* parameterized par.

3.3.1 Solution analyzes limit of it

One obtains [bib5] the limiting load in this direction of loading, for the criterion of von Mises, in axisymmetric and null axial deformations, with the threshold *y*:

2 3

B

lim

. *F* =

ln

y
éq 3.3.1-1
3
has
3.3.2 Solution analyzes regularized limit of it
The solution is homogeneous. Displacement being only radial, the isochoric deformations are necessarily of the form:
-1 0

0

U

R (*R*) =
;
(**U**) =
0
0 0
;
(**U**). (**U**)
2
éq 3.3.2-1
R
R 2

= *R*

2
0
0

1
By the law of Norton-Hoff, one obtains the deviatoric constraints:
-1 0
0
D = () *m2 m2* -2m+2

2
1
2
2
.
. 0 0 0

; *D*
() *m*
2
m - *m*
With m
R
With m
.r
+
2

=
. 3 **éq 3.3.2-2**
VM
0 0
1
The equilibrium equations axial and radial result in determining the average constraint:
m2
m2 -2 + 2
2
- *m*
tr (*R*) 3 (
With)
m2
. .
.r *m*
=
.
+
3
éq 3.3.2-3
1 - *m*
where is a constant, which is calculated starting from the boundary condition of null pressure in wall external. The components of the constraints then are obtained:

-2m+2
-2m+2
rr (*R*) = (*B*
- *R*
)
m

-2
2

-2m+2
-2m
With m
+2
zz (*R*) = (*B*
- (2 -)
m R
)
( )
with: =
éq 3.3.2-4
m

-1
-2m+2
-2m
m
2
-1 *fH*

+
(*R*) =

(*B*
- (3- 2)
m R
)
(
)( )
1
The standardization of the loading leads to:
F =
.
H
Terms of the continuation!
m of limiting load for this loading is then:
!
2 3
B

2 3
B
.
m F =
y H
rdr =
ln
m

éq 3.3.2-5
3
has 2
y
R
3
has
The terms of the continuation *m* of limiting load for this loading are:
-1
-2 +2
-2 +
2 3
B
2
2

2
2

m
m
3 *B*
-
-
+
- *has*

. *F =*

m
m
y
R
rdr.

Max R
m
y
=
éq 3.3.2-6
m
3
has
(
)
] *has,* [*B*

3 (
m 1 -)
-2m
m
has
+2
2 3
B
In *m* +
1, one finds: +
. *F* =
ln
1
3
y
, i.e. the same value as!

has
m and lim.
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3.4 Case
three-dimensional

In 3D one considers the same geometry, but the solid, unit thickness, is free in the direction antiplane *Z*. The solid is subjected to pressures on the walls horizontal: - *F* and vertical:
- 1
(-) *F*, with: 1/2. The two pressures horizontal and vertical are parameterized par.

3.4.1 Solution analyzes limit of it

The solution is homogeneous (biaxées constraints:
(
)
xx = *F*, *yy* = 1 - *F*, *xy* = 0, *zz* = 0,
deformations). One obtains the limiting load in this direction of loading [bib5], for the criterion of von Mises, with the threshold *y*:

y
lim. *F* =
éq 3.4.1-1

3 2 -
3 + 1

3.4.2 Solution analyzes regularized limit of it

The solution is homogeneous. The isochoric deformations are necessarily of the form:
1 0
0
(**U**) =
2
0

0
;
(**U**). (**U**)

=
(21+ +)
éq 3.4.2-1
0 0 - -
1
By the law of Norton-Hoff, one obtains the deviatoric constraints:

1 0
0
m2
D =
m

2
2
-
.
2
0

0
; *D*
3 1
with:
With m

=
+ +
=
2 1+ +
VM
( )

( ) ( )

0 0 -1
-
éq 3.4.2-2
2
+
0

0
One deduces from

zz = 0: tr = -
3 (1+). From where constraints: =. 0
+
1
2

0 .

0 0 0

3 - 2
The balance of the solid imposes that *xx*. (1 -) = *yy*
. One deduces the parameter from it =
.
1-
3
The standardization of the loading leads to:

1
F =
éq 3.4.2-3
H (*B* - *has*) (+ (1 -))
Terms of the continuation!
m of limiting load in this case of loading is thus identical to:
2
3
2 1+ + 2
y
( )

!
. *F* =
.
y
m
=
éq 3.4.2-4
3
+ (1-)

3 2 -
3 +1
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R7.10 booklet: Statistical processing
Document: R7.10.01
Examination of the random answers
Summary:
The introduction of a “stochastic approach of seismic calculation” to solve a problem of mechanics vibratory under random excitation requires a particular postprocessing.
Order POST_DYNA_ALEA [U4.76.02] allows, starting from the spectral concentration of power of one interspectre-answer, to evaluate its standard deviation, its apparent frequency, the distribution of its peaks. It allows
also, in a first approach, to calculate the useful function of Vanmarcke in the case of an analysis seismic.
NB:
This order also makes it possible to carry out the statistical estimates for any type of interspectre of response to a random excitation not necessarily seismic (for example: effect of the swell or of a turbulent flow).
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A. DUMOND

1 Introduction

For a structure subjected to a random excitation of type swells, turbulent flow, or seism… the loading is not known in a deterministic way, but is generally described by probabilistic or spectral information like the spectral concentration of power. For linear structures it is possible to use a method of calculation stochastic which allows to determine the spectral concentrations of power of response to these random excitations.

Operator POST_DYNA_ALEA has as a function to carry out the statistical analyses of the density spectral of power of answer. He thus provides probabilistic information of the answer of structure. Statistical calculations of parameters are carried out on the basis of calculation of the moments spectral of the spectral concentration of power considered.

These statistical parameters are: the standard deviation, the apparent frequency, distribution of the peaks. It is also possible in the operator to calculate, a first approach, the function of Useful VANMARCKE in the case of a seismic analysis.

Note:

Operator POST_DYNA_ALEA, designed initially for the seismic approach, after a calculation with operator DYNA_ALEA_MODAL [U4.56.06] ([bib1], [bib2]), can also carry out postprocessings of operator DYNA_SPEC_MODAL developed by department TTA within the framework of resorption of FLUSTRU. This operator carries out the calculation of the response of a structure of the type uniformly tube Steam Generator excited by a transverse flow.

2 Spectrum - Interspectre - Interspectrale Matrix

2.1 Treatment of the signal - Conventions selected

2.1.1 Introduction

A signal can have two representations: a temporal representation of the form X = F (T) or one frequential representation of the form X = (F F). These two representations are connected between them by the **Transformation of Fourier.**

There exists in the numerical field and the experimental field various manners of calculating spectral sizes relating to a temporal signal X (T) (dimensional representation or not, factor 1/2 or not for the Transformation of FOURIER).

However, if various definitions of the DSP (cf [§2.2.2] and [Annexe1]) starting from the Transformation of Fourier of the signal do not change anything with the calculation carried out by CALC_INTE_SPEC [U4.56.03], it is important in revenge, in the calculations carried out by the operator of postprocessing POST_DYNA_ALEA, that

them
data are coherent so that the results produced by this operator are with dimension
physique of the starting signal.
It is also necessary to know, for a quantitative comparison between calculation and experiment,
which are the conventions adopted for the calculation of the spectral quantities. The whole of these
conventions is recalled in [Annexe1] for each type of signals. We give again only them here
general formulas.
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2.1.2 Transformation of Fourier

For the **Transformation of FOURIER into frequency** (*F*) **of a signal** (of unit U), expressed in u/Hz
+
we adopt the following definition: (
X *F*)
(
X *T*) E *I ft*
=
-
dt

2
-
+
The reverse transformation is expressed then by: (
X *T*)
(
X *F*) E 2i *ft*
=

+
df
-
One can also express the **Transformation of Fourier into pulsation** (= 2 F), by the definition following:
+
1
X *p* ()
(
X) E *I T*
T

=
-
dt

2
-
+
The reverse transformation is expressed by: (
X *T*)
X *p* () E *it*
=
+
D

-
1
What leads to equivalence: X *p* () = X *p* (2 *F*) =
(
X *F*)

2
2.2
Concept of Power - Spectral concentration of Power
2.2.1 Power of a signal - Spectrum of Power of a signal
Just like the signal itself, the power of the signal can be expressed according to the time or of the frequency:
· the instantaneous temporal power is simply called power:
(
p *T*) = (
X *T*). x* (*T*)
where x* (*T*) is the complex quantity combined of (

X *T*).
· the frequential power is commonly called spectral concentration of power or spectrum:
2
S () = (
X). X* () = (
X
xx F
F
F
F)
This definition is not possible that when the transform of Fourier of the signal exists.
+
+
2
One can then express the total energy of the signal by *E* = S (*F*) *df* = (
X *F*) *df*
xx
-
-
The expression of this DSP for the various types of signals is given in [Annexe1]. One will see later on [§3.3] another definition - equivalent according to the theorem of Wiener-Kinchine - but more general, of the spectral concentration of power based on the statistical approach.
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2.2.2 Power of interaction - Spectral concentration of interaction of two signals - Interspectre

· One defines also the **instantaneous power of interaction of two signals** *X* (*T*) and *y* (*T*):

pxy (*T*) = (
X *T*) .y* (*T*) and pyx (*T*) = x* (*T*) (
.y *T*)
connected by pxy (*T*) = p *
yx (*T*)
· If the two signals admit a transform of Fourier (
X *F*) and (
Y *F*), one can express
frequential power of interaction or **interspectre** by S
( ) = (
X). Y*
XY F
F
(*F*)
· If

two signals are real then the power of interaction p (*T*) = p (*T*) = (
X *T*).
xy
yx
y (*T*)
is real. But there is no reason so that S *XY* (*F*) is also real; on the other hand
S *XY* (*F*) is complex with square symmetry, namely:
even real part and odd imaginary part or even module and odd phase
· If (
X *F*) = (
Y *F*), one speaks then about **autospectre.**

2.2.3 Stamp
interspectrale

A matrix interspectrale of order NR is a matrix NR, NR complex, whose each term depends on the frequency in the form of a function of *F*. The diagonal terms are the autospectres, them extra-diagonal terms are the interspectres between the points considered (each line or column representing a point in physical grid or a mode in modal calculation). Handled interspectres
NR (N+)
1
in practice being square, only them
terms of triangular higher (or lower)
2
are sufficient to define the matrix interspectrale completely.

2.3 Establishment
in
Aster

The matrices interspectrales handled by operator POST_DYNA_ALEA consist of

complex functions of the frequency: **S XY** (*F*).
These matrices are stored in tables of concept tabl_intsp.
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3
Recalls on the statistical laws [bib4]
3.1 Definitions

T discrete parameter (tn)
or continuous (time or a variable of space).
N 1
=, NR
X (*T*) random process.
At every moment *tn* is associated a random variable *Xn*, random variable of *xn* realization.
Then (
X *T*) = (*X* = *X* (*T*
N
N)
is a realization of the process (
X *T*), process made up of *NR*
N 1
=, *NR*
random variables a priori independent.
Each *Xn* variable is characterized by its **function of distribution** F (*X*, *T*) = Prob
N
N
(*Xn X*) or
F
by its **density of probability** (

p *X*, *T*)
N
=
(*X*, *T*
N
N)

.
X
The random process is also characterized by its **functions moments**, the first two moments have a particular importance. It is about the **expectation** or **average** μ (*T*) noted also *E* [(
X *T*)] and for any couple (*T*, *T*
1 2) of the **function of autocorrelation** R (*T*, *T*
1 2) or
R
(*T*, *T*
E [X *T* X *T*
1
2]
XX 1 2) noted too
( ) ( ) .
μ (*T*) = [
E (
X *T*)] = *X*
(px, *T*) *dx*
R
(*T*, *T*
1 2) = [
E (
X *T1*) (
X *t2*)] =
X X

p
1 2
(*X*, *T*; *X*, *T*
1
1
2
2) *dx dx*
XX
1

2
One defines also a **function of intercorrelation** for two processes *X* (*T*) and *Y* (*T*).
R *XY* (*T*, *T*) = [
E (
X *T*) (
Y *T*)] = *X y*

(px, *T*; *y*,
1
2
T) *dx Dy*
1 2
1 2
1
1
2
2
1
2
The “spreading out” of the process is characterized by the **variance**:
2 () = E ((
X) - µ ()
[
) 2
T
T
T
]
For a process with null average (µ = 0.), the **variance** which characterizes the “intensity then of phenomenon " (square of the standard deviation or average quadratic value) is equal to the function of autocorrelation at time *T = T1 = t2***:**
2 () = E [(
X) (
X)] = R
( )
2
T
T
T
T, T = X
p (*X*, *T*) *dx*
XX
3.2

Assumptions in random dynamics

Very classically several assumptions are posed within the framework of random dynamics. One admits as well as the studied processes are **stationary, with average null and ergodic**.

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3.2.1 Stationary processes with null average - variance

A process is known as **stationary** if the whole of its “probabilistic characteristics” is invariant during a translation t0 of the parameter T. What implies:

μ (*T*) = *Cte*

R

(*T*, *T*) = R (*T* - *T*

XX

XX

) = RXX () = R

1 2

2

1

XX (-)

For a process with null average 2 = R *XX* ()

0 .

3.2.2 Ergodicity

This concept comes from a reasoning of Gibbs (1839-1903) for whom time from observation from one physical phenomenon can be regarded as infinite in front of the scale of time on the level molecular. The system passes then by all the possible states while remaining longest possible, or while generally passing, in the states which are most probable, so that the **average temporal** becomes **equal to the statistical average on the states**, i.e. the hope mathematics. This is prolonged for the functions of correlation and intercorrelation.

+T/

1

2

μ = lim

(

X *T*) *dt*

T

T+

- *T/2*
+T/
1
2
R XX () = lim
(
X *T* -) (
X *T*) *dt*
T
T+
- *T/2*

Note:

For the continuation of the document one will suppose that the random process is stationary with average
null and ergodic. The whole of the developments carried out in Code_Aster checks these assumptions.

3.3

Spectral concentration of power

Within the framework of this statistical approach, one can give a very general definition of the **density spectral of power** or **DSP.** One will retain for *Code_Aster the* expressed following definitions
in frequency or pulsation:

+
+
S
-2i *F*

-2if
XX (*F*) = R *XX* () E
D
; **GXX** (*F*) = R *XX* () E
D
-
.
0
+
+
1
1
Sp
- *I*
p
- *I*
XX () =

;

2
RXX () E *D*
G XX () = 2 R *XX* () E
D

-
.
0
1
GpXX () =
GXX (*F*)
who lead to the following relations:
2
S
F = 2
p
p
XX ()
GXX (*F*) **SXX** () = 2 **G XX** ()
One can show that **GXX** (*F*), which is equal to the Transformation of Fourier of R *XX* (*T*), is real, positive. One will refer to [Annexe1] who contains all conventions adopted to ensure coherence of the results.
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3.4 Moments
spectral

One calls spectral moments the following quantities (which one defined in pulsation):
+
+

I
p
I
I =

S *XX* ()
D =

SXX (*F*) *df*
-
-
One has in particular:
2
2
2
0 =
2 =

! !
XX
4 =! *X*! *X* which are the standard deviations of *X* and of its
first derived.
These moments are systematically calculated until order 4; using the key word MOMENT it is possible to ask the calculation of the higher modes. In *Code_Aster*, calculation is carried out for a DSP expressed according to the frequency *F*.
Code_Aster calculates the spectral moments on the basis of field of definition of the functions such as they are provided to him.

4

Measurements of going beyond of threshold and reliability

The traditional methods give access only the maximum of displacement (or of acceleration) by summation “adapted” of the maxima on each mode. Essential interest of the stochastic approach of random vibratory calculation lies in the statistical knowledge of response of the structure which can thus be converted into statistical data of reliability. It titrate, two modes of ruin can be taken into account:

· ruin by going beyond of threshold: this type of ruin occurs when the response of the system exceed a limiting value. That amounts seeking the probability that the values of the process remain in lower part of an extreme value (**peak Factor** or **factor of peaks**) during the duration of observation T.

· ruin by tiredness or accumulation of damage.

This second approach could also be treated starting from the first calculated statistical elements

in POST_DYNA_ALEA. It is carried out in order POST_FATI_ALEA [U4.67.05] [R7.04.02].
Within the framework of the studies under seismic excitations, we are interested primarily in problem of going beyond of threshold. From where initially the calculation of a certain number of statistical parameters which make it possible to characterize the signal to study (spectral moments and formulas of Rice [§4.1]), provided with these characteristics we will be able to then estimate the probabilities of
going beyond of threshold using traditional models of probability [§4.2], as well as a criterion of reliability (law of Vanmarcke [§4.3]).

4.1
Spectral moments and characteristic parameters

+
The spectral moments are defined by:
I
I =

SXX (*F*) *df*
-
The infinite whole of these spectral moments characterize the interspectre perfectly and allow thus to determine a certain number of numerical results. In the particular case of an oscillator with 1 ddl or of a signal with only one peak, the first three spectral moments are enough to find the autospectre *S XX*. It is the case which we retain in *Code_Aster* since it is supposed that them values are distributed according to a law of GAUSS.

4.1.1 Formulas of Rice

For a random signal such as definite previously: stationary with **null average (centered)** and ergodic, one supposes moreover than the measured values are distributed according to a **normal law**

profile
of type Gauss (cf [§4.2.1]).
The analysis of a stationary Gaussian random loading has the advantage of leading to simple analytical expressions [bib8] - known under the name of formulas of Rice - and to represent many real phenomena.
The following statistical parameters are obtained as from the various spectral moments connected to different derivative from *X* (cf [§3.4]):
· Écart-type
:
X = 0
Note:
If only the positive part of the spectrum is provided, Code_Aster multiplies by 2 the 1st moment spectral X = 20.
A extremum (maximum or minimum) of amplitude *X* is defined by the probability of having a derivative null!
X = 0 associated a derivative second! *X* unspecified.
1
1 4
·
!

Average extrema numbers a second: *NR*
X
E =
=

X!
2
The going beyond of a *X0* level is defined by the probability of having *X* = *X0* with a slope! *X* unspecified: one thus counts the passages of this level with the positive **and** negative slopes.
Taking into account the assumptions of Gaussian laws, the number of passage by *X0* and a second
2
X0
1
-

2
!
express yourself by: *NR*
X
=
E 2X
X0

X
What leads to the following expressions:
1
· Nombre of goings beyond of level **with positive slope** a second: *NR* + = *NR*
X
X
0
0
2
1
1 2
·
!

A number of passages **by zero** (*X* 0 = 0) a second: *NR*
X
0 =
=
X 0
· Nombre of passages **by zero with positive slope** a second:
1
1
2
NR + = *NR*
0
0 =
2
2 0
NR +
0 represent an average statistical frequency of passage by zero with positive slope.
In the case of a “simple” signal, i.e. with **only one peak**, *NR* +
0, a number of passages by zero
can be compared to a **frequency connect** also noted *Fe*. In the case much more general
of an unspecified signal, the physical interpretation of the value *NR* +
0 are more of doubtful validity!
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The factor of irregularity translates the frequential pace of the signal. Ranging between 0 and 1, it tends towards 1
when the process is with narrow band, on the other hand it tightens towards 0 for a broad band process. Its expression is:
NR
2
2
!

I
0
X
=
=
=
2
X

X!
0
4
Three parameters - *NR*
NR, I
0 ,
E
- characterize the signal entirely. One can, in particular, estimate
the average number of positive peaks a second: *NR*
+ = 1/ (
4 1+ *I*) *NR*
peak
E.
The whole of these parameters is calculated and stored in a “printable” table on the file RESULT using order IMPR_TABLE.
4.2

Distributions of the positive peaks

One of principal knowledge interesting the originators of structures starting from his answer estimated at a random excitation is the determination of the goings beyond of threshold and in particular **them**

probabilities of goings beyond of certain critical points.

The formulas of Rice (preceding paragraph) make it possible to know the average rate of crossings of certain levels. The following approach makes it possible to give a law of probability of presence of such or such peak. One is thus interested in maximum positive of the answer.

A maximum occurs when! X (T) = 0 with! X (! T) < 0. One is thus interested in the density of probability joint (

$p\ X, X! =,$

0 $X!$, T) of X (T), $X!$ (T), $X!$ (T). (It is necessary thus that the process is twice derivable, what is acquired when one admits a Gaussian distribution of the signal.)

This density of probability of the **positive peaks** makes it possible for example to calculate the proportion of peaks

understood between A and B (or probability that the next peak lies between A and b) which is worth:

B

(px, 0x! , T) dx

has

The stationary Gaussian signal, being centered compared to its average value (null in analysis seismic), the distribution of the peaks is symmetrical compared to this average. One is thus interested in **distribution of the positive peaks**. In the case general, the distribution of the peaks of amplitude X positive

is written in the form [bib5]:

X2

X2

2

-

T

2

2

2

2

2

2

2

X -

-

1 I

IX

-

$p+$
2

X
2
peak (**X**)
( )
=
1
2

X (
$- I E$
+
E
E
dt

1 + *I*)
-

X

2

I
X
=
!

X
If
< 0 then $p+$
X
X!
peak (*X*) = 0
with
X
I
=

X

1 - *I* 2
It is about the so known formula under the name of LONGUET-HIGGINS [bib7], whose step is also clarified in [bib11]. We present, Ci after, the chart of this formula
for 4 values of *I*.
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Probability
0,80
I = 0 law of Gauss
0,70
I = 0.4
0,60
I = 0.7
I = 1 law of Rayleigh
0,50
0,40
0,30
0,20
0,10
0,00
Amplitude/Standard deviation
0,00
0,40
0,80
1,20
1,60

2,00
2,40
2,80
3,20
3,60
4,00

Appear 4.2-a: Distribution of peaks of standardized positive amplitude compared to the standard deviation of the signal

This distribution of the positive peaks is in the case of simplified the signals for which the factor of irregularity $I = 0$ or I are worth = 1.

4.2.1 *I = 0* Signal with broad band: law of Gauss or normal law

In the case of a broad band signal, the positive peaks are distributed according to a law of GAUSS:

$$p+(X) = \frac{E}{peak} \frac{2X}{2} \sqrt{2} e^{-\frac{X^2}{2}}$$

4.2.2 *I = 1* Signal with narrow band: law of Rayleigh

In the case of a signal with narrow band, the positive peaks are distributed according to a law of RAYLEIGH:

$$p+(X) = \frac{E}{peak} \frac{2X}{2} X e^{-\frac{X^2}{2}}$$

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4.2.3 Calculation of the values in *Code_Aster*

The values of these two laws are calculated in *Code_Aster* under the key words RAYLEIGH factor or GAUSS.

From the standard deviation $X = 0$ calculated précédement one calculates the values of probability of the peaks

6

$p+ (X)$ for X

X

peak

[0 6, X] with a step by defect of

.

200

If the user wishes to refine his analysis, it can provide values VALMIN and VALMAX of the field of variation of X. It can also provide the step value of calculation, if not this one will be taken with the 200ème

bandage reserve.

The figure [Figure 4.2-a] shows that the field chosen by defect, to 6 X, covers totality well values of X with nonnull probabilities.

4.3

Seismic answer: law of Vanmarcke [bib8]

In the case of the response to a seism of a primary structure (i.e. excited at its base by ground) having a **dominating mode**, IE which answers (taking into account the exiting frequencies) on one

only mode, one uses the law of reliability of VANMARCKE [bib8] which makes it possible to estimate, over one **duration of**

operation T probability that the process exceeds the threshold of ruin.

The concept of dominating mode is very important here, if the structure answers on several modes formulate in its current expression is not appropriate more.

That is to say $X (T)$ the response to a Gaussian white vibration, of a slightly deadened linear oscillator. One defines

probability $W (T)$ that the process remains in the field of safety. $W (T)$ represents the fraction of sample which did not cross the threshold of ruin after one duration T; it is a measurement of reliability.

dW T

It can be written in the form

$(T) =$

{*X* () < *X0* 0 *T* < *T*} *p1* (*T*)
( )
W
Prob
T
;
:
= -
is
D *T*
density of probability of crossing of the threshold.
For the high values of *T* one will take: *p*
-
1 (*T*) = *A*
T

E
where *A* depends on the initial conditions
and is the limiting rate of decrease.

4.3.1 Assumption of independent crossings

With the assumption that the goings beyond of threshold with a positive slope are **events independent**, the number of crossings on [0, [
T constitutes a process of Poisson of rate
of arrival *NR*
= 2 *NR* +
X
X (a number of going beyond of *X*
0
0
0 defined in [§4.1.1]). Probability that *N*
passages occur over the duration *T* is written by application of the law of Poisson (see [§4] [bib8]):
(
N
NR
T
X0)
{
- *NR T*
X
P
0
passages on [,
0 [

T}
N!
N
= E
The structure is “reliable” if the threshold is not exceeded during the duration *T*. Reliability *W* (*T*) corresponds
thus with *N* = 0 passage from where *W* (*T*) *E NR T*
X
= - 0 .
The limiting rate of decrease is thus worth here =
=
+
NR
2 *NR*
X
X.
0
0
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4.3.2 Law of Vanmarcke

For a Gaussian stationary process, the probability of exceeding value *X* is worth [§4.1.1]:
X 2
-
2
NR
= NR E 2X
X

0
; one deduces from it that the probability that the initial value of the envelope is lower
2
X
-
NR
2
with threshold *X* is: 1 -
X = 1
2
- *E*.
N0
One then combines this expression with the limiting law of decrease obtained with the assumption of independent crossings, which leads to the **expression of reliability**:
(-1e-hs)
- *NR T*
0
s2
2
W (*T*) = *Ae- T* = (- *S*
1-
2) *E*
E 2 -1
E
/
1
with *NR*
2
0 = rate of passage by 0 and *T* lasted of observation
0
X
2
1 2
.

1
where *S* =
H =
= 1-

2
0
0

2
is an estimator of bandwidth of the DSP of X.
This relation has the immense advantage of providing an explicit estimator of reliability according to reduced value of the threshold *S*, the number of equivalent semi-cycles *N0*, and of the parameter of width of
bandage.
NB:
"The agreement between the estimator and simulations can be improved if one replaces by 1.2*"*
[bib8] "correction" introduced into the formula written above compared to the expression of the rate of decrease limits given in the preceding paragraph.
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The following chart is carried out in the case of a process of frequency
connect 15 Hz, that is to say *N0* = 30 Hz, for one duration of observation of *T* = 1s. The estimator of width
of band is taken equal to 0.30. As in the preceding illustrations, the amplitude is standardized compared to the standard deviation.
Reliability
1
0,9
0,8
0,7
Vanmarcke
0,6
0,5
0,4
0,3
0,2

0,1
0
Amplitude/Standard deviation
Appear 4.3.2-a: Evolution of reliability according to the law of Vanmarcke according to the amplitude of the standardized process compared to the standard deviation of the signal
Recall:
This statistical analysis is carried out starting from relatively restrictive assumptions, namely that the process must be with "narrow band"; it will thus have to be checked that the factor of irregularity
I is not too different from 1 and that the signal comprises only one principal peak.
4.3.3 Establishment in *Code_Aster*
It is exactly the expression of $W(T)$ which is established in operator POST_DYNA_ALEA, under key word factor VANMARCKE. The field of definition of the calculation of the function of reliability is here too
6
by defect [0 6
,
X
X] with a step of
. As for the laws of GAUSS and RAYLEIGH
200
[§ 4.2.3], it can be restricted by the user.
The calculation of reliability is made for one duration T (in S) of operation: it is taken by defect with T = 10 S what is appropriate well for a seismic calculation.
The figure [Figure 4.3.2-a] shows well that for 6 X, $W(T)$ tends towards asymptote 1, the process has surely exceeded the threshold of ruin.
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5 Remarks

This postprocessing is carried out on interspectres stored in tabl_intsp. It provides statistical elements of the response of the structure which can thus be converted into information statistics of reliability, or to be useful then for calculations of damage by tiredness (POST_FATI_ALEA).

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Appendix
1
Conventions for the Spectral concentrations of
Power

A1.1 Introduction

In order to preserve coherence necessary for the whole of calculations and the comparisons with the experiment (cf [§2.3] and [§3.3]), we develop hereafter the two sets of definitions coherent with calculations of random answer and postprocessing such as they were retained for *Aster*:

· the first starting from spectral data expressed according to the frequency. It is this together which is coherent with the calculation carried out in operator CALC_INTE_SPEC [U4.56.03].

· the second starting from spectral data expressed according to the pulsation.

These two sets return validates postprocessing such as it is expressed in POST_DYNA_ALEA. We will each time specify the unit in which the various quantities are expressed handled according to the unit U of the signal of reference. The explanations given are brief. One will be able for more details to refer to the reference [bib10].

A1.2 Types of signals and definition of the power

We consider four types of signals:

· signals of finished energy,

· signals
periodicals,

· signals of finished power and signals deterministic,

· random satisfying the assumption of ergodicity and stationary signals.

In random dynamic calculation the signals are random. For the interpretation of results experimental, the signals are either periodic, or of finished power (deterministic).

We define for each type of signal an energy quantity which is either an energy, or a power and which we will indicate in the following paragraphs under the single term of power:

· Them

signals of finished energy are defined by their energy *E* expressed in u2 S:

+

E =

X (*T*) *dt* < +

2

éq An1.2-1

-

· Them

periodic signals are defined by the power *P* of the signal expressed in u2:

1

P =

X (*T*) 2dt

éq An1.2-2

T [*T*]

T indicates the period of the signal. [*T*] is an interval length *T*.

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· Them
signals of finished power are defined by the average power *P* of the signal

expressed in u2:
+T

/
1
2

P = lim
X (*T*) 2 *dt*

< +
éq An1.2-3
T+ *T*
- *T/2*

· Them
random signals are defined by the average power *P* of the signal expressed in u2:
+T

/2
1

P = *E* [(
X *T*) 2] = lim
X (*T*) 2dt

< +
éq An1.2-4
T+ *T*
- *T/2*

One is useful oneself here of the assumption of ergodicity which underlies that the average statistics and temporal
carried out on a realization of a process are identical.

A1.3 Autocorrelations

Taking into account the statistical recalls carried out in the body text one has for each type of
signals previously definite:
· Autocorrélation

signals of finished energy, expressed in u2/Hz:
R
() = *X* (*T*) *X* (*T* +) *dt*
XX

éq An1.3-1
· Autocorrélation

periodic signals, expressed in u2:
1
R
() =
X (*T*) *X* (*T* +) *dt*
XX

éq An1.3-2
T [*T*]
· Autocorrélation

signals of finished power, expressed in u2:
+T/
1
2
R
() = lim
X (*T*) *X* (*T* +) *dt*
XX

éq An1.3-3

T+ T - T/2
· Autocorrélation

random signals, expressed in u2:
+T/
1
2
R
() = *E* [(
X *T*) (
X *T* +)] = lim
X (*T*) *X* (*T* +) *dt*
XX

éq An1.3-4
T+ T - T/2
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A1.4 Definition of the spectral concentration of power.

A1.4.1 Expression in frequency

One defines the spectral concentration of power by:
+
+
S
(*F*)
R
() -2i *F D* or *G*
-2i *F*
=

E
XX (*F*) =
R
XX () E
D
XX
XX
éq An1.4.1-1
-
0
The mechanic being interested only in the positive values of the frequency and time, function *GXX* is more often used.
One can show, if the Transformations of Fourier of the signals exist, that this definition is equivalent (theorem of Wiener-Kinchine) to the definitions of the spectral concentration of power following.
· For the signals of finished energy:
2
G
(*F*) *X* (*F*
2
2
XX
=
) expressed out of U/Hz
éq An1.4.1-2
· For the periodic signals:
n=+
n=+
If *X* (*F*) = *C* (*F* - *N F*
2
0) then *G XX* (*F*) = *Cn* (*F* - *N F*
N
0)
éq An1.4.1-3
n=-
n=-
G
(*F*
XX
) expresses itself in u2/Hz.
f0 is the reverse of the period of the signal.
Cn coefficient of the Dirac functions.

· For the signals of finished power:

$$G_{XX}(F) = \lim_{T} \frac{1}{T} [X_T](F)^2 \text{ out of U/Hz}$$

éq An1.4.1-4

$$+$$

where X_T indicates the restriction of (X T) with [- T 2; T 2].

· For the random signals:

$$G_{XX}(F) = \lim_{T} \frac{1}{T} E[X_T](F)^2 \text{ out of U/Hz}$$

éq An1.4.1-5

$$+$$

where X_T indicates the restriction of $X(T)$ with $[-T/2; T/2]$.

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Bond between the DSP and the power.

With the definitions given above for the spectral concentrations of power, one has for all signals, the relation:

+
P = G

(*F*) *df*
XX

éq An1.4.1-6

-

This relation is established by using the theorem of PARSEVAL.

A1.4.2 Expression in pulsation

In pulsation, one defines the spectral concentration of power by:

+
1
G'
- I
XX () =
R

éq An1.4.2-1

2
XX () E *D*
-

Just as for the expression in frequency, one can show, if the Transformations of Fourier of the signals exist, that this definition is equivalent (theorem of Wiener-Kinchine) with the definitions of the spectral concentration of power following

· For the signals of finished energy:

2
G'
'
2
2
XX () =
2 *X* () expressed out of U/Hz
éq An1.4.2-2
· For the periodic signals:
n=+
n=+
If *X* '() = *C* (- *N*
2
0) then *G* '

XX () = *Cn* (- *N*
N
0) **éq An1.4.2-3**
n=-
n=-

2
G' XX () is expressed in u2/Hz, and 0 =
where *T* is the period of the signal.
T
Cn coefficient of the Dirac functions.
· For the signals of finished power:

2
2
G'
'
2
XX () =

lim
X [*T*] () out of U/Hz
éq An1.4.2-4
T

T

+
X ['*T*] indicates the restriction of (
X T) with [- T 2; T 2].
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· For the random signals:
2
2
G'
'
2
XX () =

lim *E*
X [*T*] () out of U/Hz
éq An1.4.2-5
T

T

+
X ['*T*] indicates the restriction of *X* (*T*) with [- T 2; T 2].
Bond between the DSP and the power.
In the same way, there is for all the signals the relation - which rises from the theorem of PARSEVAL -:
+
P = G

'*XX* () *D*
éq An1.4.2-6
-
A1.4.1 Relation between DSP in frequency and DSP in pulsation
For the four types of signals:
1
G' XX () =
GXX (*F*)

éq An1.4.3-1
2
Appendix 2 Transformation of Hilbert
That is to say *X* (*T*)
a real signal of transform of Fourier (
X).
That is to say *H* ()
J
> 0

the Transfer transfer function: H () = *J* sign () = - *J* < 0
0

= 0
H () transforms *X* (*T*) into its transform of Hilbert noted "*X* (*T*). The system of transfer transfer function *H* () produced a dephasing of +90° for the positive frequencies and 90° for the frequencies negative. It follows theorem of convolution that "
X (*T*) can also be defined like the convolution of
X (*T*) by the corresponding impulse response, is (
H T) = 1 *T*.
"*X* (*T*) is also real, one second application of the transform of Hilbert restores the initial signal, changed of sign and cut down by its possible continuous component.
Example: *X* (*T*) = *A* cos *T X* " (*T*) = - *A* sin *T*
This property is at the base of the use of the transform of Hilbert to define the envelope of one process in narrow band.
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Postprocessing of modal calculations with shock

Summary:

This document presents the principle of postprocessing of transitory calculations by modal recombination with
non-linearities of shock available in operator POST_DYNA_MODAL.

Two options of postprocessing can be employed, the first usable one for problems of

vibration-wear determines average values and RMS of displacements, forces of shock and power of wear dissipated on the level of the supports with plays, second is applicable for the fine analysis of the impacts
occurring at the time of transitory requests, the instantaneous maximum force, duration of the time of shock,
the impulse exchanged, speed before impact are given for each shock.

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Count

matters

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1 Introduction

Numerical developments were carried out in Code_Aster to allow calculation transient of structures presenting of the vibrations with shock in certain points. In certain cases, forces of friction can also appear and lead to a phenomenon of localised wear.

That it is about damage by pure impact or impact-friction, the engineer wishes to reach with the sizes associated with this damage, which requires a specific postprocessing behind non-linear transitory calculation.

This information of postprocessing is also invaluable when one wishes to validate the module of non-linear calculation by comparing its results with what can be measured on a test bench specific. Test routines (SOLID MASS and MULTICHOC) were implemented to this end and were the first users of these functionalities of postprocessing.

The objective of this note consists in specifying the sizes to be analyzed in the vibrations with shock and
their specificity. It is then a question of determining the suitable statistical treatments to apply to these signals to release from the instantaneous sizes or the most characteristic averages.

Initially one will in the case of see the treatment applied a problem with shock and friction (option “WEAR” of order POST_DYNA_MODAL).

The following chapter will be devoted to the treatments applied in the case of a phenomenon of vibration
pure impact, where the sizes of each impact are more finely analyzed (option “IMPACT” of order POST_DYNA_MODAL).

2
Sizes considered in the vibrations with shocks

The primary sizes considered in the vibrations with shock are identical that it is about experimental measurements or of numerical calculation, they relate to the forces of shock and them displacements on the level of the points of shock. The experimental results present one however additional difficulty of analysis due to the errors or skews introduced by the systems of measurement. We will examine the two sizes quoted successively previously.

2.1
Forces of shock

The first concern concerning the structures vibrating with shocks is better to know them efforts received by the structure at the time of the shocks on its supports with plays or between the structures. These
data are calculated in a temporal way by the algorithm of DYNA_TRAN_MODAL, they are then filed with a step defined in this same operator. Data of shock having contents
frequential very important one will take care to have a sufficient filing (not to exceed PAS_ARCH: 10).
These forces expressed in a local reference mark with the obstacle (Yloc, Zloc) are traditionally broken up into a normal part with the obstacle (Fn on the figure below) and a tangential part (Ft) if friction is taken into account between the structures. The conditions of shock make that the force
normal of shock has a constant sign taken conventionally positive in Code_Aster.

F
Fzloc
Fn
Ft
P
Fyloc
O

2.2
Displacements of shock

Displacements of the structure on the level of its supports with play are another information important calculated. Its analysis poses however less problems because the spectral contents are less rich. In the case of circular or described obstacles in a polar way, a polar description of displacement can be interesting.
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2.3 Sizes
secondaries

2.3.1 Time of shock

The time of contact between the structure and the supports with plays is a characteristic indirect size movement of vibration with shock. It can be deduced from various ways, on a criterion of displacement, of force of positive reaction. A concept of time of total shock, broken up into shock elementary (or rebound) will be introduced in [§3.4].

2.3.2 Sizes
calculated

Other secondary sizes can be important in the analysis of the conditions of shock, it
acts of the impulse at the time of the impact (integral of the exchanged force), the power of wear, the force
maximum at the time of an impact,… These sizes are specific to each postprocessing and they will be specified in the two chapters which follow for postprocessing option "WEAR" and "IMPACT".

3
Modal transitory postprocessing: option "WEAR"

The characterization of transitional measures is the goal of the treatment of the signal. It teaches us that one signal is entirely determined by the data of all its statistical moments. In practice it is out of question of calculating every statistical moment, one limits itself in postprocessing to sizes calculated classically in treatment of the signal (average simple, standard deviation and value RMS). They are characteristic of the signals which one wishes to analyze and compare. Signals similar must necessarily have these first close statistical moments (the reciprocal one being false). The statistical sizes selected here are well appropriate to the analysis, comparison or classification of signals of vibrations under random excitation with not linearities of shock.

We thus will examine the realised sizes and their calculation, by distinguishing the different ones sizes quoted in the preceding chapter:

· displacements,
· forces of shock,
· determination of the contact and the time of contact.

Other made up information could be calculated starting from the preceding ones in particular power of wear.

3.1
Statistical processing per blocks

In order to analyze the stationnarity of the signals and the statistical treatments carried out on signals, one carries out a cutting per blocks of the temporal signals. Thus duration of postprocessing defined between the initial moment (INST_INIT) and the final moment (INST_FIN) is cut out in a number of temporal blocks (NB_BLOC) of identical duration. The calculation of the statistics: average, standard deviation… is carried out for each block, a general value for the signal for the unit blocks is also calculated.

In the case of a calculation of response of a structure to a random loading, this technique of calculation per blocks makes it possible to make sure that the transitional stage of calculation is finished and that the value announced is quite stationary over a time of observation associated with the duration with calculation.
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3.2
Statistical treatments applied to displacements of shock

Let us consider the temporal signal Depl_ X (T), which one carries out a filing at a certain frequency Facquis on NR points. The starting data is thus a vector Depl_ X (I) with NR components.

Average displacement is defined in this case by:

NR
Depl_ X (I)
Depl_ X = 1

NR

This average value characterizes the central value around which the signal of displacement evolve/move. For displacements, it will thus make it possible to determine if a configuration is observed
centered (displacements with null average), or offset (average nonnull).

The variance of displacement is by definition:

NR
(
2
Depl_ X (I) - Depl_ X)
VAr (Depl_ X) = 1

NR

The standard deviation of displacement is worth then:

(Depl_ X) = VAr (Depl_ X)

The standard deviation of a signal characterizes its dispersion around its average value. A weak standard deviation
will concern rather a signal with weak variations of amplitude, a strong standard deviation of the variations more
strong.

For a centered variable i.e. with null average, the standard deviation is equal to average RMS of signal (Root Mean Public garden).

For an unspecified variable one defines average RMS of the signal by:

$$RMS(Depl\_X) = \sqrt{\frac{1}{NR}\sum_{NR} Depl\_X(I)^2}$$

The minimum and maximum absolutes of the signal are also interesting information and very simple to obtain, who determines the extent of the signal.

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Depl_x
max
Depl_x +
Depl_x
Depl_x -
time
min

Appear 3.2-a: Example of signal of displacement and visualization
statistical sizes

A polar representation *of the whole of the signals Depl_ X and Depl_ is there also*
interesting to analyze an obstacle of circular or close geometry in the case of. Let us be appropriate
to call R radial displacement and angular displacement, equivalents of Depl_ X and
Polar Depl_ there of.

By definition one a:

$$R(I) = Depl\_X(I)^2 + Depl\_y(I)^2$$

(
Depl_ there I
I)
( )
= Arctg

Depl_ X (I)

This representation makes it possible inter alia things to distinguish:

· of the orbital movements with permanent contact (average radial displacement about play and standard deviation of weak radial displacement),
· of the movements of pure impact (standard deviation of important radial displacement, variation of weak angular displacement),
· of other configurations: orbital movement with impacts…

Note:

In the selected local reference mark for the obstacles of shock, the sizes called here Depl_ X and Depl_ are there in fact DYloc and DZloc, the axis Xloc having been chosen by convention perpendicular in the plan of the obstacle.
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In short*, the option of postprocessing* ***"WEAR"*** *of operator* ***POST_DYNA_MODAL*** *will determine for local displacements DXloc, DYloc, DZloc like for their polar decomposition R and them statistical sizes per blocks with the principle stated above:*

· average value,
· value RMS,
· standard deviation,
· minimal value,
· maximum value.

3.3

Statistics for the forces of shock

*One supposes to lay out as for displacements of a discrete signal on NR points: Fx_*
(
shock I).
The signal obtained should be composed of temporal beaches where the force of shock is null (not contact) and others where the force of shock is significant (effective contact), which is the case at the time of
*numerical calculations. In fact, for experimental signals, because of the dynamics of the system of measure, a level of noise can be observed except period of shock (cf [Figure 3.3-a]). It is thus necessary only to carry out the statistical processing when the signal leaves the sound level. That requires the introduction of a threshold of detection (**SEUIL_FORCE**) which, although superfluous in the field numerical, was reproduced in the postprocessing of Code_Aster.*

Fchoc
Smax
time
Appear 3.3-a: Example of signal of force of experimental shock

That is to say the value S max, determining the level max of the noise considered, one then will calculate:

*· The **number of moments in shock**:*

*Nchoc = card {I/Fx_*
(
shock I) > S my}
X

*· The **average of force of shock** over **total time**:*

NR

1
Fx_ shock =
.

*Fx_*
(
shock I)
NR

I Fx_choc (I) >S max

· *The* ***average of force of shock*** *brought back to the* ***time of shock*** *is worth:*

NR
Fx_ shock = Fx_ shock.

Nchoc

· ***Average RMS of force of shock*** *over* ***total time*** *is calculated in the following way:*

/

1 2
NR

1
RMS (Fx_ shock) =

*Fx_*
(
shock I) 2

NR

I Fx_choc (I) >S max

· ***Average RMS*** *brought back to the* ***time of shock*** *is worth:*

NR
RMS (Fx_ shock) = RMS (Fx_ shock).

Nchoc

As for the signals of displacements, one can also be interested to the ***maximum or absolute minimum*** *of the signal of force, thus determining its extent. For the normal force, it minimum is always equal to zero, whereas the tangential force is alternate.*

In short*, the option of postprocessing* ***"WEAR"*** *of operator* ***POST_DYNA_MODAL*** *will determine for normal and tangential forces of shock statistical sizes per blocks with the principle statement above:*

· *average value calculated over the time of shock or total time,*
· *value RMS calculated over the time of shock or total time,*
· *maximum value of the signal.*

3.4
Statistics for times of shock

The percentage of time of shock *is defined by:*

%Tchoc = Nchoc/NR

If one looks at information of which one lays out on an experimental system, the signal of force of shock is adapted the most to determine in a precise way the occurrence of a contact. Like one has it evoked with the top one tests the need to introduce a maximum level of noise, and to count them phases of shock when the signal exceeds this threshold (SEUIL_FORCE).
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*On the figure below, one can distinguish a concept of given **elementary shock** like one successive passage to the top then with the lower part of the threshold, and a more general concept of **total shock**,*
gathering several elementary shocks separated by short moments from return under the threshold.

Fn (T)
DUREE_REPOS
THRESHOLD FORCES
A total shock = 2 shocks
A total shock = 3 shocks

Appear 3.4-a

*One thus introduces a time characteristic of rest Tr (**DUREE_REPOS**); end of a time of shock total occurring if the signal remains during a time at least higher than Tr at rest. This concept of time characteristic of Tr rest is well heard rather fuzzy and will have to be given by the user within sight of the transitory results. It is nevertheless essential because it only allows to gather a train of constituting very brought closer impact makes only one phase of contact of it.*

Concept of elementary time of shock being defined, statistical processing over the time of shock will consist in determining following information:

*· a **number of elementary shocks**: Nb_ choc_ elem*
*· a **number of total shocks**: Nb_ choc_ glob*
Nb_ choc_ elem
*· a **number of elementary shocks per total shock**:*

Nb_ choc_ glob
*· **time of average elementary shock**:*

Nchoc. T

T choc_elem =

Nb_ choc_ elem

· ***time of average total shock***

Nchoc. T

T choc_glob =

Nb_ choc_ glob

· ***time of maximum total shock the*** *greatest time of total shock noted on the block analyzed.*

In short*, the option of postprocessing* ***"WEAR"*** *of operator* ***POST_DYNA_MODAL*** *will determine for times of shock statistical sizes per blocks with the principle stated above:*

· average value of the time of total shock, `
· maximum value of the time of total shock,
· average value of the elementary time of shock,
· the number of total shocks a second,
· the average number of shocks elementary per total shock.

3.5 Power of wear

The size generally calculated in the vibrations with shock and friction is the power of wear defined by ARCHARD [bib1], which translates the average power developed by the forces of

friction at the time of the movement. These forces are the engine of wear by friction. Power of wear in the case of discrete signals is calculated as follows:

NR

(
Fn I) V
. T (I)
P
I Fn S max
wear =
>

NR

*This power can for example be correlated with a wear or removal of matter by the intermediary of a coefficient of wear KT by a relation of the type: V (T) = K * P * T*
T
wear
where
V (T) is the volume removed for the length of time T.

Other more sophisticated laws of wear can be used in another operator of postprocessing: ***POST_USURE*** *describes in [R7.01.10].*

3.6 Structure of data counts POST_DYNA associated with postprocessing option "WEAR"

3.6.1 Count POST_DYNA_MODAL

A structure of the type counts for the option ***WEAR*** *of operator* ***POST_DYNA_MODAL*** *gathers them results previously described.*

This table contains the names of the statistical under-tables of results associated different analyzed sizes: displacements, forces of shock, counting of the shocks and power of wear.

The variables of access of this table are 10:

· ***for the variables displacement****:* ***DEPL_X****,* ***DEPL_Y****,* ***DEPL_Z****,* ***DEPL_RADIAL****,* ***DEPL_ANGULAIRE****, which correspond respectively to displacements in X, Y and Z local and their cylindrical decomposition in the plan of the obstacle.*
· ***for the variables forces of shock****:* ***FORCE_NORMALE****,* ***FORCE_TANG_1****,* ***FORCE_TANG_2****, who correspond respectively to the normal, tangential forces with the obstacle the first*

being in the plan of the obstacle, the second orthogonal one in the plan of the obstacle.
*· **for the variables counting of shock**: **STAT_CHOC**.*
*· **for the variables power of wear**: **PUIS_USURE**.*
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Under tables associated with the 10 sizes above, contain a certain number of variables access for each connection of shock:

*· **for the variables displacement**: **MEANS**, **ECART_TYPE**, **RMS**, **MAXIMUM**, **MINI**, which correspond respectively to the values average, standard deviations, value RMS or effective, maximum and minimal value of the corresponding variable displacement.*
*· **for the variables forces of shock**: **MOYEN_T_TOTAL**, **MOYEN_T_CHOC**, **RMS_T_TOTAL**, **RMS_T_CHOC**, **MAXIMUM**, which correspond respectively to the values average over time total, average over the time of shock, value RMS or effective average over total time, value RMS or effective over the time of shock, maximum value of the variable forces corresponding.*
*· **for the variables of counting of the shocks**: **NB_CHOC_S**, **NB_REBON_CHOC**, **T_CHOC_MOYEN**, **T_CHOC_MAXI**, **T_CHOC_MINI**, **T_REBON_MOYEN**, **%_T_CHOC**, which correspond respectively to the values of the number of shocks a second, of the number of rebounds by shock, of the time of average shock, time of maximum shock, time of minimal shock, time of average rebound and percentage of time of shock.*

· ***for the variable power of wear**: **PUIS_USURE** which corresponds to the power of wear calculated according to ARCHARD.*

4
Modal transitory postprocessing option "IMPACT"

4.1
Practical current of postprocessing of calculations of heart

The SEPTEN made use, before with the development of postprocessing in Code_Aster, for its needs for checking of dimensioning, the code CLASH [bib2] developed by BELGONUCLEAIRE. This software calculates the seismic response of a file of assemblies. This code provides a whole of information detailed for each point of shock and each impact.

Each result consists of a table by point of shock whose example is in appendix 1. It table comprises following information:

· the moment of the peak of impact,
· the maximum force of impact reached,
· the exchanged impulse, defined as the integral of the force of shock over time,
· total duration of the shock,
· relative speed before impact.

These elements are particularly interesting for the SEPTEN bus in addition to contractual information very limited, they make it possible to know the number and the composition of the impacts, as well as essential physical sizes which theirs are associated. Relative speed before impact, the impulse are for example very invaluable information in the specification of experimental tests of dynamic buckling of the grids of assemblies.

4.2
Calculations for the postprocessing of the impacts

One regards as for preceding postprocessing that the conditions of impact are given as previously by going beyond of a force Smax threshold and one distinguishes in the same way total shock and elementary shock by the concept of rest period.

Calculation carries out a loop on all non-linearities of shock and an identical treatment for each one.

Then for each identified total shock, one will determine the following sizes:

· Temps of beginning of shock: T
such as F
(T) > S
beginning
shock
beginning
max

· Temps of end of total shock:

T
such as F
end
shock (Tfin) S
, F
max
shock (T
- T
end
) Smax
and
T
[T, T + T
end
end
rest] Fchoc (T) Smax

where T
is the step of time of integration

· Total Durée of the shock: T
= T - T
shock
end
beginning

· Maximum of force at the time of the shock: Fmax = max
( )
T [
T
, T
beginning
end] (F
T
shock
)

· The moment of maximum of force of shock,

T
·
end
The impulse exchanged at the time of the shock: I =
F
(T) .dt

shock

t=Tdébut

· Relative normal speed before impact: V
= V (T
- T
shock
beginning
)

· The number of elementary impacts cumulated in the total shock:

NR
= card {T [T, T]/F (T) > S and F

max
(T + T
) S
elementary impacts
beginning
end
shock
shock
max}

In order to synthesize information, one will moreover determine:

· the absolute maximum of force of shock, on a connection of shock given, for the duration of analysis,

The maximum of force of shock to be more precisely given will not be obtained like the max in time on the whole of the shocks for each node of shock (to avoid it skew of the precision of filing) but given in transitory calculation on all the steps of calculation and filed in the concept ***tran_gene*** *result. It is this information which will be used.*
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· the average value of the extréma of force of shock like their standard deviation.

· a histogram of the density of probability of the maximum forces of impacts.

This histogram will be relatively summary and will give for NC classes the density of probability of the maximum force of shock.

The classes will be in the following way defined:

I - 1
I

classify 1.
= F
/
F absolute
max
max
Fmax
F absolute
i= NR
max

C
NR
NR

C
C

4.3 Structure of data counts POST_IMPACT associated with postprocessing option "IMPACT"

4.3.1 Count POST_IMPACT

*A structure of data of the type counts for the option **IMPACT** of operator **POST_DYNA_MODA_T** of Code_Aster is prduite.*

*The structure of result will be a subscripted table by the names of connections of shock, of type **POST_IMPACT**, container of the names of tables which it contains.*

*The contents of each cell of this table are a name of table stored in CHARACTER*24. Three type tables are contained a table known as IMPACT, a table known as TOTAL and a table known as PROBA.*

*It thus has 3 parameters: **IMPACT**, **TOTAL** and **PROBA**. The variable of access corresponds to the name*

connection of shock considered.

4.3.2 Count
IMPACT

*The table "**IMPACT**" is of type **TABL_IMPACT** and has 6 parameters of access: **INST**, **F_MAX**, **T_CHOC**, **IMPULS**, **V_IMPACT**, **NB_IMPACT**.*
*The contents of each cell of this table are a **REAL*8**.*

4.3.3 Count
TOTAL

*The table `TOTAL `is of type **TABL_FMAX** and has 3 parameters of access:*

*· **F_MAX_ABS**, which gives access the absolute maximum of force of shock on all the shocks noted,*
*· **F_MAX_MOY**, which gives access the average value of maximum of force of shock noted,*
*· **F_MAX_ETYP**, which gives access the standard deviation extrema of forces of shock.*

*The contents of each cell of this table are a **REAL*8**.*
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4.3.4 Count
PROBA

*The table `PROBA `is of type **TABL_HISTO** and has 3 parameters of access:*

· ***BEGINNING**, which gives access the value of minimal force of class I,*
· ***FINE**, which gives access the value of maximum force of class I,*
· ***PROBA**, which gives access the density of probability of the variable forces maximum for classify I.*

*The contents of each cell of this table are a REAL*8.*

5 Conclusion

One presented in this document the methods of postprocessing applicable to the transients with shock calculated by modal synthesis on structures with play. According to concerns', one can to carry out a postprocessing directed towards a diagnosis of the wear undergone by the components at the time of shocks, a whole of important statistical sizes are then given. If the concern rather relate to the impacts and their level, another option allows a detailed analysis of each impact.

These two functionalities make it possible to synthesize the transitory results obtained by integration temporal, to classify by level of severity of the digital simulations different or from to compare ends of validation of the calculated and measured sizes.

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Appendix 1

ASTER 3.05.29 CONCEPT TT CALCULATES THE 20/12/95 A 17:32: 16 OF TYPE
TABL_POST_DYNA
TOTAL IMPACT
NO1 TT _I_NO1 TT _G_NO1
PROBA
NO1 TT _P_NO1

----->
IMPRESSION OF THE TABLE: TT _I_NO1
MOMENT F_MAX IMPULSE T_CHOC V_IMPACT
SHOCK 1 1.55000E-02 9.95269E+03 1.98051E+02 3.10000E-02 -1.00000E+00
SHOCK 2 3.61000E-01 9.95478E+03 1.98093E+02 3.15000E-02 -1.00031E+00
NB_IMPACT
SHOCK 1 1.00000E+00
SHOCK 2 1.00000E+00

----->
IMPRESSION OF THE TABLE: TT _G_NO1
F_MAX_ABS F_MAX_MOY F_MAX_ETYPE
NO1 9.95478E+03 9.95373E+03 1.04759E+00

----->
IMPRESSION OF THE TABLE: TT _P_NO1
BEGINNING FINE PROBA
CLAS 1 9.95269E+03 9.95295E+03 5.00000E-01
CLAS 2 9.95295E+03 9.95321E+03 0.00000E+00
CLAS 3 9.95321E+03 9.95347E+03 0.00000E+00
CLAS 4 9.95347E+03 9.95373E+03 0.00000E+00
CLAS 5 9.95373E+03 9.95400E+03 0.00000E+00
CLAS 6 9.95400E+03 9.95426E+03 0.00000E+00
CLAS 7 9.95426E+03 9.95452E+03 0.00000E+00

CLAS 8 9.95452E+03 9.95478E+03 5.00000E-01

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Handbook of Reference
R7.20 booklet: Projection of results or measurement
Document: R7.20.01
Projection of a field on a grid
Summary:
Order PROJ_CHAMP makes it possible “to project” fields known on the nodes of a grid (ma1) on nodes of another grid (ma2).
In this document, one describes the 3 accessible methods of projection in this order.
The paragraph [§5] gives some elements of validation of these methods.
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1
General information on order PROJ_CHAMP

Order PROJ_CHAMP makes it possible to project a field known with nodes (CHAM_NO) of a grid

(ma1) on the nodes of another grid (ma2) [U4.72.05]. In general, the 2 grids (ma1 and ma2) are “incompatible”; i.e. the nodes of ma2 are not confused geometrically with the nodes of ma1. One does not treat the fields “by element” (CHAM_ELEM).
3 methods of projection are currently available:
METHOD: “ELEM”
METHOD: “NUAGE_DEG_0”
METHOD: “NUAGE_DEG_1”
Method “ELEM” uses the functions of form of the elements of the grid ma1. It is detailed with paragraph [§3].
The 2 other methods use a smoothing of the values of the field in the vicinity of the point where one wants to project the field. These 2 methods are detailed in the paragraph [§4].

2

Operation general of the order

For 2 methods “NUAGE_DEG_0/1” the order makes it possible to project one field. In revenge, with method “ELEM” one projects the whole of the fields of a structure of data result (evol_ther, evol_noli,…).
Whatever the method, the user with the possibility of projecting only one “piece” of field on one “piece” of the grid ma2. This possibility is offered by the key word factor VIS_A_VIS. A piece of field is the restriction of the field on a whole of nodes (or meshs) of the grid ma1. One piece of the grid ma2 is a subset of the nodes of ma2.
The basic problem to solve is thus the following:
That is to say a field ch1 known, on the nodes of a subset of a grid ma1, how to calculate it field ch2 on the nodes of a subset of another grid ma2?
In the continuation to simplify the talk one will not speak any more a subset of a grid, one will make as if one projected all the grid ma1 on all the grid ma2.

Notice on the vocabulary:

The word “to project” is sometimes ambiguous in this document.
When one says “to project” the field of ma1 towards ma2, one seeks the knowing field on ma2 that on ma1: projection goes from 1 towards 2.
For method “ELEM”, it is necessary to find for each node N2 of ma2 which is the point of ma1 which occupies the same position that N2, for that one projects the node N2 on the grid ma1: projection goes from 2 towards 1.

3 Method "ELEM"

3.1 algorithm implemented

One buckles on all the nodes of the grid ma2:
for each node (N2), one proceeds in 3 stages:
1) One seeks which is the mesh m1 of ma1 which "contains" the node N2 geometrically,
2) one determines the N2 position in m1 (i.e his co-ordinates in the element of reference associated the mesh m1),
3) one uses the functions of form of the mesh m1 to determine the value of the field on N2 knowing the value of the field on the nodes of m1.

3.1.1 Notice

The third stage shows that this method supposes that all the nodes of the mesh m1 know the field to be projected. For example, one could not project a field which would carry degrees of freedom different on its nodes "tops" and its nodes "mediums from edge". The projection of such field would be possible on the other hand with the 2 other methods of projection [§4].
To use method "ELEM", it is thus important that all the nodes of the same mesh carry same components in the field to be projected. Practically (because of continuity of the meshs connected between them), that wants to say that the field must be "homogeneous" on the piece of field intended to be projected.

3.2 Encountered difficulties and their treatment

For each one of these 3 stages, one will see that difficulties obliged us with simplified the problem and thus to solve it only imperfectly.

3.2.1 Stage 1

· one does not treat the possible curve of the edges of the elements. For example, in the figure below (plane problem), the node N2 will be stated to belong to the mesh m1a whereas it belongs to the mesh m1b,

m1b
N2
m1a

· if the node N2 is actually "external" with the grid ma1, one will affect the mesh m1 more to him near to him. This behavior makes it possible to project, without stopping in fatal error, a field on a grid whose border differs slightly from that of the initial grid (what is always the case in practice).

3.2.2 Stage
2

To find the point of the element of reference which would give by the geometrical transformation the node
N2, it is in general necessary to solve a non-linear problem, because there is only for the triangle with 3 nodes (in
2D) and for the tetrahedron with 4 nodes (in 3D) that the geometrical transformation is linear.
Not to solve this non-linear problem. One “linearizes it” by cutting out the meshs of ma1 in linear triangles or tetrahedrons. The resolution of the problem arising is thus approximate.

Note:
To solve the non-linear problem exactly, one could think of using a method iterative of Newton, but it would be necessary to call into question stage 1 above because the mesh m1 selected east cannot be not the maid.

3.2.3 Stage
3

One always does not use the true functions of form of the elements of the initial grid. Indeed, in *Code_Aster*, in fact the finite elements choose their functions of form: a triangle of thermics is not obliged to choose the same functions of form as a triangle of mechanics. One element can also not need functions of form, or it can choose functions different according to the variables to be interpolated. The formulation of the element either always does not make
to appear of element of reference and associated geometrical transformation.
For all these reasons and so that the programming of PROJ_CHAMP is independent of finite elements present in the model, one assigns to all the meshs ma1, the functions of form isoparametric elements 2D or 3D [R3.01.01].

Note:

The linear meshs are not treated today. One cannot thus project a field known on 1 linear model (beam or linear hull).

3.3

Details concerning stages 1 and 2

By means of computer, stages 1 and 2 are carried out simultaneously. We will discuss successively in the way of dealing with the 3 following problems:

· treatment of a node N2 finding inside the border of the grid ma1 (case more frequent),

· treatment of a node N2 outside the border of ma1,

· treatment of the case of the grids of the type “hull” (surfaces plunged in R3).

3.3.1 Node

“interior” N2

To include/understand the treatment of an interior node, let us take the case of a mesh 2D QUAD8 (abcd). One start by forgetting its nodes mediums (and thus their possible curve) then one cuts out it into 2 triangles (ABC and acd). This cutting is arbitrary (it depends on the local classification of the nodes on QUAD8). Let us note that other possible cutting (another diagonal) would give another point in general in the element of reference.

C
D
C
D
B
N2
N2

With
B

has
Element of reference
Real element
N2 belongs to the triangle ABC. One seeks his barycentric co-ordinates in this triangle. It is
the 3 teststemxà numbers, xb, xc such as one can write: n2= teststemxà * has + xb * B + xc * C
The point N2 of the element of reference retained by the algorithm will be:
N2 = teststemxà * A + xb * B + xc * C
For the voluminal meshs (Hexahedrons, Pentahedrons, Pyramids and Tetrahedrons), one proceeds of even way: one forgets the nodes mediums and one cuts out the nontetraedric meshs in tetrahedrons.

3.3.2 Node

“external” N2

A node N2 is declared external with the grid ma1 if one found no mesh for which it is
interior. When this is noted, one seeks the mesh m1 nearest to N2. The distance
calculated is that which separates the node N2 and the border from the mesh m1. Let us call p2 the point of m1 it
nearer to N2. This point can be on a face of a voluminal element or on an edge or one
top.
Example (in 2D):
With
Pa
T4
T2
B
T1
Pb
T3

The node B, one associates the point Pb obtained here by orthogonal projection of B on an edge of T1. Node A, one associates point Pa of triangle T1. One could just as easily have associated point Pa to him T2 triangle, but that would not have changed anything since the field to be projected is known with the nodes of
grid. It is thus continuous between the adjacent elements.
Once found the point p2 of the mesh m1 which carries out the minimum of the distance with N2, one brings back oneself
with the preceding problem: the point N2 of the element of reference which will be associated N2 will be it
correspondent of p2 by the procedure of the paragraph [§3.3.1].

Note:

For a triangle (or a tetrahedron) given T, there is only one point p2 carrying out the distance minimal between N2 and T bus T is convex. This property disappears if one took account of
curve of the edges of the meshs. It is seen there that two simplifications of the implementation (lapse of memory of the nodes mediums and cutting of the meshs in linear triangles) are dependent enters they.

An external point will always have a value interpolated between the values of the nodes of the grid ma1 and ever extrapolated; what is not the case of 2 methods NUAGE_DEG_0/1.

3.3.3 Case of the grids “hull”

When one seeks to project the nodes of a surface grid on another surface grid,
one falls in general systematically on the case from the “external” points above. Indeed,
the inaccuracy on the co-ordinates of the nodes makes that a node N2 is never rigorously in plan of the triangles of the meshs of ma1.
It is thus the procedure of [§3.3.2] which applies:
For each node N2:
· research of the triangle (or of the tetrahedron) which carries out the minimum of distance with N2. Identification of the point p2 which carries out this distance,
· calculation of the point N2 of the element of reference which corresponds to p2 by the procedure of [§3.3.1].

4 Method “NUAGE_DEG_0” or “NUAGE_DEG_1”

4.1 Principle of the method

These 2 methods are based on the same principle: one chooses a priori basic functions Fi (X, y, Z) (here of the polynomials of degree 0 or 1). One seeks in the vector space generated by these functions of base, the function F= (I Fi) which carries out the distance minimum (within the meaning of least squares) with
“cloud” of the known points. Once this found function, one evaluates it at the sought point.
To reduce the notations, one places oneself in 2D (but calculations can be made in the same way in dimension 3 or more…). The field to be projected is a whole of couples (Xj, Vj) where Xj= (xj, yj) is one

node of the grid ma1 and Vj is a reality (value of the field on this node). This field constitutes the cloud known points.
That is to say a node N2 (X, y) of the grid ma2 for which one wants to calculate the value of the field (V).
Choice of the basic functions:
NUAGE_DEG_0:
only one function: F1= 1
NUAGE_DEG_1:
3 functions: F1= 1; F2=x; F3=y
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In the continuation of this paragraph, one will choose NUAGE_DEG_1 so that the formulas do not degenerate
too much.
The sought function is F = 1 F1 + 2 F2 + 3 F3. One defines a kind of “distance” between F and them couples (Xj, Vj):
D = wj (F (Xj) - Vj) 2
where the wj are the weights assigned to each couple (Xj, Vj).
One wants to minimize D compared to 3 variables 1, 2, 3. D is a quadratic function of 1, 2,
3. To minimize D amounts cancelling its derivative and thus solving a linear system with 3 unknown factors:
1, 2, 3.
When this problem is solved, one calculates:
V= 1 + 2 X + 3 y

4.2

Choice of weights of the points of the cloud

All the “easy way” of these methods is in the choice (difficult) weights wj assigned to the points of cloud.

· one decides a priori that the weight of a point (Xj) depends only on the distance (D) separating it not node N2 (isotropic weighting),
· if wj is a constant, the problem does not depend any more of the unknown node N2. The function F is single (for all the nodes of ma2): it is “the linear straight regression line” of the cloud,
· if wj (D) is a function too not very decreasing, the function F “is smoothed too much”: “accidents” buildings “are gummed” by the great number of remote points taken into account,
· if wj (D) is a too decreasing function, one takes the risk “to catch” any point of cloud. The numerical consequence is that the linear system to solve becomes singular (and thus insoluble).

We chose to write W (D) like exponential decreasing parameterized by 2 parameters: *dref* and:

W D
E D dref
( )
(/
)
= -
2

dref is a distance from reference (depend on the node N2). We will see below how it is calculated. is a constant chosen to cancel more or less quickly the weight of the distant N2 points. In the code, was selected to 0.75.
dref is the distance from which one wishes to see the weight of the points decreasing quickly. In the programming, *dref* is calculated like the product of a distance d1 by a C1 coefficient. Today, we chose C1 = 0.45.
d1 is defined as follows:
· in 3D, d1 is the ray of the smallest ball of center N2 which contains 4 points of cloud noncoplanar,
· in 2D, d1 is the ray of the smallest ball of center N2 which contains 3 points of cloud not aligned.

The problem is known as “2D” if all the nodes of the grid ma1 have same co-ordinate Z, it is known as “3D” if not.

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4.3

Current restriction of the programming

The 2 methods of projection NUAGE_DEG_0/1 are programmed only for the real fields (and not for the complex fields)

5

elements of validation

To validate the 3 methods suggested, we will treat the example of the projection of a field of temperature "unidimensional" $T = T(X)$.

The field to be projected is worth:

$T(X) = \sin(3x) + Heavysi($
X -)
1

This field is affected on the nodes of a grid (ma1) linear rather coarse (14 meshs) segment [0,2]. This field has the property to be discontinuous (in theory) at the point x=1. Because of discretization on the grid ma1, the field only seems to vary very brutally between the 2 points x=0.99 and x=1.01

One projects this field on a grid (ma2) very fine of the same segment (300 elements length 2/300).

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5.1 Method
:
"ELEM"

EDF
Electricity
from France
Method: "ELEM"
1.0
0.8
0.6
before projection
Temperature
after projection
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
co-ordinate X
agraf 13/10/2000 (c) EDF/DER 1992-1999
It is noted that with this method, the projected field is without surprised: the value obtained by projection the is always interpolated linear one between the 2 nodes of ma1 where the field is known.

5.1.1 Quadratic initial grid

If one remakes same calculation by replacing the linear grid ma1 by a quadratic grid (containing approximately 2 times less meshs), one finds:

EDF
Electricity
from France
Method: "ELEM"
1.2
1.0
0.8
é
erase
0.6
before projection
Temp

after projection
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
co-ordinate X
agraf 13/10/2000 (c) EDF/DER 1992-1999
It is noted that the interpolation of the field is now parabolic between the nodes of ma1. One thus approach better the form of the initial function.
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5.2 Method
:
“NUAGE_DEG_0”
EDF
Electricity
from France
Method: “NUAGE_DEG_0”

1.0
0.8
0.6
é
erase
before projection
Temp
after projection
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
co-ordinate X
agraf 13/10/2000 (c) EDF/DER 1992-1999

It is noted that with this method, the projected field is presented in the form of a succession of small horizontal “stages” connected to each other. This aspect in staircase is related to the parameters (and C1) selected in
“hard” in the code for the form of the exponential decay of the weight of the points. One sees also that the discontinuity of the initial field is very strongly gummed.

5.2.1 Influence form of exponential decreasing

On the following figure, we modified a parameter of exponential decreasing of the method “NUAGE_DEG_0”: the parameter dref (or C1 what returns to same) was multiplied by 0.5 or 2. by report/ratio with the value retained by the code.

EDF
Electricity
from France
Method: NUAGE_DEG_0
1.4
1.2
1.0
0.8
R
E

0.6
é
R
has
you
before projection
mp
you
0.4
dref=0.5 dref
dref= 1. dref
0.2
dref= 2. dref
0.0
-0.2
-0.4
0.0
0.2 0.4
0.6 0.8
1.0 1.2
1.4 1.6
1.8 2.0
agraf 13/10/2000 (c) EDF/DER 1992-1999
co-ordinate X

It is noted that the choice of *dref* influences the result much. If it is too large, one does not see any more discontinuity. If it is too small, the form in stair is accentuated.

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5.3 Method
:
“NUAGE_DEG_1”
EDF
Electricity
from France
Method: “NUAGE_DEG_1”
1.0
0.5
é
erase
0.0
before projection
Temp
after projection
-0.5
-1.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
co-ordinate X
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It is noted that with this method, the projected field is correct in the 2 regular zones (x<0.99 and x>1.01).
On the other hand the discontinuity of the field between these 2 values is strongly exaggerated. This famous example
well faculty that with this method to extrapolate the values of the initial points (because of estimate of gradient of the field).
In the 2 regular parts, the projected field is rather close to that obtained with the method "ELEM". It is noticed simply that method "NUAGE_DEG_1" rounds a little the angles

---

5.3.1 Influence form of exponential decreasing

On the following figure, we modified a parameter of exponential decreasing of the method "NUAGE_DEG_1": the parameter *dref* (or C1 what returns to same) was multiplied by 0.5 or 2. by report/ratio with the value retained by the code.

before projection
mp
you
0.4
dref=0.5 dref
dref= 1. dref
0.2
dref= 2. dref
0.0
-0.2
-0.4
0.0.0.2.0.4 0.6.0.8.1.0 1.2.1.4.1.6 1.8 2.0
agraf 13/10/2000 (c) EDF/DER 1992-1999
co-ordinate X
There still, it is noted that the choice of *dref* is crucial for the result: too much large, discontinuity is gummed, too small, discontinuity is exaggerated.

5.4

Choice of the best method of projection

Within sight of the some preceding curves, it appears clear that method “ELEM” is in general preferable with methods NUAGE_DEG_0/1. This method is “natural” within the framework of the elements
stop and it does not depend on any numerical coefficient of adjustment.
Methods NUAGE_DEG_0/1 must be reserved, in our opinion, for uses a little
special:
· case
of one
grid
ma1 “non-existent”: one has only of the nodes but not the meshs (by
example, the “nodes” of ma1 are in fact of the transmitters),
· case of a field (result of a calculation or obtained by measurements) that one wants to smooth voluntarily. But in this case, it would be necessary that the 2 numerical parameters (C1 and) are accessible to the user what is not the case today.

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Extrapolation of measurements on a digital model in dynamics

Summary:

A step of extrapolation of experimental results of measurement in dynamics (displacements, speeds, accelerations, strains, stresses) on a digital model is presented. Based on one representation of the structure on a basis of projection chosen beforehand, it consists of the resolution

opposite problem defined by the identification of the generalized co-ordinates relating to the base of projection.
The resolution suggested uses a minimization, within the meaning of least squares, by using the decomposition LU
or decomposition in singular values (SVD), of a functional calculus possibly regularized via addition of a criterion of proximity of a solution known a priori. In the case of a temporal identification,
an explicit formulation of information a priori is proposed.
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1 Problems

One wishes to estimate numerically, the behavior in any point of a structure from measurements taken in some points of the structure. Taking into account the costs and constraints of accessibility, experimental measurements in a limited number and are generally located in places which inevitably are not requested. Thus, in dynamics, the knowledge of the zones of stress and local value concentration of constraints is crucial to check the behaviour mechanics of the material. One is then brought to extrapolate results of measurement located, on the whole of the numerical grid of the structure.
The step of extrapolation suggested is based on a representation of the structure on a basis of judiciously selected projection (clean modes, static answer,…). It consists of determination of the generalized co-ordinates relating to this base of projection. The resolution proposed uses a minimization, within the meaning of least squares, by LU decomposition or in singular values (SVD), of a functional calculus possibly regularized via the addition of a criterion of proximity of a solution known a priori. In the case of a temporal identification, a formulation explicit of information a priori is proposed.

2 Notations

Q, q&, q&: vectors of displacements, speeds and accelerations in the physical reference mark
, &, &: vectors of displacements, speeds and accelerations generalized

: stamp formed of the basic vectors of projection (displacements)
: stamp formed of the basic vectors of projection (deformations)
: stamp formed of the basic vectors of projection (forced)
: stamp basic vectors (displacements), restricted with the measured degrees of freedom
: stamp basic vectors (deformations), restricted with the measured degrees of freedom
: stamp basic vectors (forced), restricted with the measured degrees of freedom
I: stamp identity
Nnum: a number of basic vectors of projection, Nexp: a number of degrees of freedom measured
T: variable time: variable pulsation
TF: transform of Fourier, TF -1: opposite transform of Fourier
A+: pseudo-opposite of matrix A
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3 Step of extrapolation

The step which one wishes to set up in order to extrapolate of the vibratory results of measurement on a digital model breaks up into 4 stages [bib1]:

Experimental measurement of a physical size

Numerical calculation of (displacement, speed, acceleration, Nnum first

strain or stress) in NR clean modes of the structure

exp points

Calculation of the relative generalized co-ordinates

at the base of projection (according to time

or of the frequency) by minimization

Restitution of the results in physical base

This step is based on the concept of projection of a field in a space of dimension lower than the dimension of the space of the digital model and then extrapolation on the space of digital model. The fact of projecting the field in a space of reduced size generates necessarily a loss of information during extrapolation. One thus sees here the importance of the choice of the base of projection. This base can made of modal answer and/or static answer. One suppose here that the digital model is linear.

4
Relations between the physical sizes and the sizes generalized

It is supposed that the intrinsic behavior of the structure is represented in a generated space by the NR num basic vectors of projection. The transformation of Rayleigh-Ritz establishes the relation between the degrees of freedom of the structure in the physical reference mark and its generalized co-ordinates:

Q =

In this formulation, the matrix of the basic vectors contains all space information; generalized co-ordinates, as for them, depend:

· of time, in the case of a calculation of temporal answer: Q (M, T) = (M) (T)
· of the pulsation, in the case of a harmonic calculation of answer: Q (M,) = (M) ()

One can thus deduce from them very simply the following relations:

Harmonic answer
Temporal answer
Displacement
Q (M,) = (M) ()
Q (M, T) = (M) (T)
Speed
& (
Q M,) = J (M) ()
& (
Q M, T) = (M) & (T)
Acceleration
& (
Q M,) = - 2 (M) ()
& (
Q M, T) = (M) & (T)
Deformations
(M,) = (M)

()
(M, T) = (M)

(T)

Constraints
(M,) = (M)

()
(M, T) = (M)

(T)

All these formulations thus present an equivalent form: in the continuation of the document, us will treat primarily the case of temporal displacement, but the results obtained are transposable with all the other sizes: speed, acceleration, strain and stress.
In the same way, the relations established according to time are applicable in the spectral field:

TF (Q (M, T)) = (M) TF ((T)) = (M) ()

TF -1 (Q (M,)) = (M) TF -1 (()) = (M) (T)

5
Calculation of the generalized co-ordinates

5.1
Formulation of the problem

The calculation of the generalized co-ordinates is carried out on the matrix of displacements (respectively speeds, accelerations, strains, stresses) restricted with the degrees of freedom measured, by resolution of the matric system:
qexp =

Dimensions of the matrix "to be reversed" are (NR
, NR
exp
num).
It is seen here that the calculation of the generalized co-ordinates is carried out in a restricted space: dimension of the space generated by the basic vectors is lower than the dimension of the model numerical, one exploits only information with the measured degrees of freedom.

5.2
Determination of a quasi-solution

For the resolution of the opposite problem, 3 cases can arise:

.
NR
= NR

exp
num: the number of measured degrees of freedom is equal to the number of vectors of base projection which one wishes to identify the generalized co-ordinates.
In this case, there is a single solution with the problem of inversion: = -1qexp
.
NR
> NR
exp
num: the number of measured degrees of freedom is higher than the number of basic vectors of projection of the digital model which one wishes to identify them co-ordinates generalize.
In this case, there is not exact solution with the problem of inversion. A quasi-solution can however be defined, which minimizes the distance
: qexp -. The formula
+
= [T] Tq then provides the solution (single) within the meaning of least squares. In
exp
+
this expression, the matrix [T] T indicates the opposite matrix generalized of.
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calculation of pseudo-opposite can be carried out by using the decomposition LU or decomposition in singular values (SVD).
.
NR
< NR
exp
num: the number of measured degrees of freedom is lower than the number of vectors

basic of projection which one wishes to identify the generalized co-ordinates (what corresponds to the case more running).
In this case, there is an infinity of solutions to the problem of inversion and the objective is of to determine an acceptable solution by introducing an additional condition (standard minimal of the solution or application of methods known as "of regularization").

5.3
Determination of a regularized opposite solution

5.3.1 Principles of the methods of regularization

The goal of the methods of regularization [bib4], [bib5] is to propose an approximate and stable solution
with respect to the variations of the data input. One does not seek any more to solve the equation of minimization resulting from the formulation: qexp =, but to determine an approximate solution (or regularized) answering two requirements:
· it satisfies a condition of proximity: one seeks such as qexp - num <,
· it answers additional a condition a priori known as "information".

The methods of regularization thus consist in supplementing the statement of the problem by introducing one
information to extract a priori, in the family of the solutions which are compatible with the data experimental, that which corresponds best to the problem. This is done while amalgamating in a criterion
single a measurement of the fidelity of the solution compared to the experimental data and a measurement
of its fidelity to information a priori [bib2].
An approach which can be easily implemented in finished dimension is the regularization by optimization. To bring closer the method of regularization of Tikhonov [bib3], it consists with to consider a solution a priori priori problem of minimization and to seek the solution of approximate system nearest to this solution. One then seeks to minimize the functional calculus following:

2
2
qexp - + has - priori

Parameter A determines the affected weight with information a priori.
The solution of the equation of minimization is given by:

-1
= [T + have] (Tqexp +
a priori)

or, while revealing explicitly the variation compared to the solution a priori:

-1
=
T
T
priori + [+ have] (qexp - priori)

If priori = 0 is posed, this formulation consists in seeking the solution known as of "standard minimal" (or Tikhonov of order 0).
The regularizing addition of the term related to the matrix have has as a role to shift the spectrum of T so
to ensure the stage of matric inversion. This step of calculation thus makes it possible to implement a procedure of calculation conditioned better, which softens the effects of the noise and which provides a solution
physically acceptable.
In addition, the choice of the values of the matrix have results from a compromise between the stability of
required solution and the confidence which one can grant to the solution a priori.
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5.3.2 Choice of information a priori

In the case of the methods of regularization, the choice of information a priori constitutes a stage-key who determines the representativeness of the final results. This choice can be based on a knowledge physics of the solution or on a knowledge of its evolution according to the parameter selected.

We provide, in the continuation, an example applied to the determination by minimization of one temporal variable [bib1].
The minimization of a variable according to time can be carried out with each step of time independently of the step of previous time. The introduction of information a priori allows however to enrich the functional calculus by supposing a slow evolution by the given variables:

priori (T) = (T - dt)

This assumption is acceptable only when the step of sampling is sufficiently weak. In effect, the solution at a given moment is approached by (development of Taylor):

(T) = (T - dt) +
dt & (T - dt) + O (dt)

The maximum frequency of response of the structure is determined by the pulsation of the mode of order it
higher max taken in modeling. One thus has:

(T) - (T - dt)

<
dt
max
+ O (dt)
(T - dt)

So that the corrective term is weak (and thus that information a priori constitutes an approximation with
first order of the required solution), the step of sampling must check:

1
dt <<
max

At the initial moment (t=0), since one does not have any information a priori on the solution, it calculation is carried out by seeking the solution of minimal standard. In order to avoid propagating the error which
in results, it can be necessary to assign a weak confidence to information a priori on first steps of time (via the parameter) and to exploit the results only from the moment when one can consider that the errors sufficiently attenuated. If necessary, of complementary studies will be undertaken in order to determine the optimal parameters of use of functionality developed in Code_Aster.

In the frequential field, many possibilities are offered to determine information has priori. They rest is on a physical knowledge of the solution (highlighted

experimental of resonances or forced answers), that is to say on a formulation of displacements generalized according to the frequency (standard: functions profit), in which case minimization led finally to characterize the dynamic stresses.

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6 Implementation in Code_Aster

The base of projection is made up is dynamic modes calculated by the order MODE_ITER_SIMULT [U4.52.03] stored in a concept of the mode_meca type, are modes dynamic and of the static modes calculated by order DEFI_BASE_MODALE [U4.64.02] stored in a concept of the base_modale type.
The phase of calculation of the generalized co-ordinates is treated by order PROJ_MESU_MODAL [U4.73.01]. The data are gathered there under 4 key words factors.
The data relating to the digital model (projection bases) are gathered under the key word factor MODELE_CALCUL. The digital model there is specified and projection bases.
The data relating to measurements are gathered under the key word factor MODELE_MESURE. One y specify in particular the model associated with the structure and the measurement read by order LIRE_RESU.
The possible manual space association of the nodes is given under the key word factor CORR_MANU.
The data concerning the resolution of the problem reverses are gathered under the key word factor RESOLUTION. One specifies there the method of decomposition employed (LU, SVD) and the taking into account of term of regularization.
The restitution of the results in physical base can then be carried out by the order

REST_BASE_PHYS [U4.63.21].

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